Injection-Locked Vertical Cavity Surface Emitting Lasers (VCSELs) for Optical Arbitrary Waveform Generation

Sharad Bhooplapur
University of Central Florida

Part of the Electromagnetics and Photonics Commons, and the Optics Commons

Find similar works at: https://stars.library.ucf.edu/etd

This Doctoral Dissertation (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Electronic Theses and Dissertations, 2004-2019 by an authorized administrator of STARS. For more information, please contact STARS@ucf.edu.

STARS Citation
INJECTION-LOCKED VERTICAL CAVITY SURFACE EMITTING LASERS (VCSELS) FOR OPTICAL ARBITRARY WAVEFORM GENERATION

by

SHARAD BHOOPALAPUR
B.S. Washington University in St. Louis, 2005
M.S. University of Central Florida, 2008

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Optics and Photonics at the University of Central Florida Orlando, Florida

Fall Term
2014

Major Professor: Peter J. Delfyett
ABSTRACT

Complex optical pulse shapes are typically generated from ultrashort laser pulses by manipulating the optical spectrum of the input pulses. This generates complex but periodic time-domain waveforms. Optical Arbitrary Waveform Generation (OAWG) builds on the techniques of ultrashort pulse-shaping, with the goal of making non-periodic, truly arbitrary optical waveforms. Some applications of OAWG are coherently controlling chemical reactions on a femtosecond time scale, improving the performance of LADAR systems, high-capacity optical telecommunications and ultra wideband signals processing.

In this work, an array of Vertical Cavity Surface Emitting Lasers (VCSELs) are used as modulators, by injection-locking each VCSEL to an individual combline from an optical frequency comb source. Injection-locking ensures that the VCSELs’ emission is phase coherent with the input combline, and modulating its current modulates mainly the output optical phase. The multi-GHz modulation bandwidth of VCSELs updates the output optical pulse shape on a pulse-to-pulse time scale, which is an important step towards true OAWG. In comparison, it is about a million times faster than the liquid-crystal modulator arrays typically used for pulse shaping!

Novel components and subsystems of Optical Arbitrary Waveform Generation (OAWG) are developed and demonstrated in this work. They include:

1. Modulators

An array of VCSELs is packaged and characterized for use as a modulator for rapid-update pulse-shaping at GHz rates. The amplitude and phase modulation characteristics of an injection-locked VCSEL are simultaneously measured at GHz modulation rates.
2. Optical Frequency Comb Sources

An actively mode-locked semiconductor laser was assembled, with a 12.5 GHz repetition rate, \( \sim 200 \) individually resolvable comblines directly out of the laser, and high frequency stability. In addition, optical frequency comb sources are generated by modulation of a single frequency laser.

3. High-resolution optical spectral demultiplexers

The demultiplexers are implemented using bulk optics, and are used to spatially resolve individual optical comblines onto the modulator array.

4. Optical waveform measurement techniques

Several techniques are used to measure generated waveforms, especially for spectral phase measurements, including multi-heterodyne phase retrieval. In addition, an architecture for discriminating between ultrashort encoded optical pulses with record high sensitivity is demonstrated.
I dedicate this work to my family, parents, sister and grandparents, for their support, encouragement, and understanding during my PhD.
I am grateful for all the people who have helped make this work possible:

Dr. Peter Delfyett for his patience, and guidance and belief in me during my thesis work;

The members of my dissertation committee, Dr. Demetrios Christodoulides, Dr. Guifang Li and Dr. Donald Malocha, for their advice and help in making the thesis possible.

The professors at CREOL, from whom I have learned a great deal over the years. In particular, I thank Dr. Winston Schoenfeld, Dr. Patrick LiKamWa and Dr. Xun Gong (in the Electrical Engineering Dept. at UCF) for their advice and help in packaging the VCSEL array. I thank Dr. Mercedeh Khajavikhan for her advice and help on the oral presentation of the thesis defense.

CREOL staff, in particular Rachel Agerton-Franzetta, Gail Drabczuk and Amy Perry, for their great help in navigating the administrative requirements of the University.

My parents, family and many friends, for encouraging and sustaining me during the entire process of completing my PhD, and for being so understanding.
TABLE OF CONTENTS

LIST OF FIGURES............................................................................................................................... xi

LIST OF TABLES......................................................................................................................................... xxi

LIST OF ACRONYMS (or) ABBREVIATIONS .......................................................................................... xxii

CHAPTER 1: INTRODUCTION.................................................................................................................. 1

1.1 Shaping Ultrashort Laser Pulses and Generating Arbitrary Optical Waveforms ...................... 1

1.1.1 Spectral Resolution of the Pulse Shaper ....................................................................................... 3

1.1.2 Performance of the Modulator Array ........................................................................................... 4

1.2 Applications of Pulse Shaping and OAWG ..................................................................................... 6

1.3 Design Considerations for an OAWG System ................................................................................ 6

1.3.1 Requirements for Scaling the Optical Bandwidth for OAWG .................................................... 7

1.4 The Subsystems of an OAWG System ............................................................................................. 8

1.4.1 Optical Sources ........................................................................................................................... 9

1.4.2 Wavelength Demux/Mux Technologies ...................................................................................... 10

1.4.3 Modulator Array Technologies .................................................................................................. 10

1.5 Optical Arbitrary Waveform Measurement ..................................................................................... 11

1.5.1 Spectrally-Sliced, Four Quadrature Coherent Detection ........................................................... 12

1.5.2 Spectral Interferometry ............................................................................................................. 13

1.5.3 Self-Referenced Temporal Interferometry .................................................................................. 15

CHAPTER 2: RAPID-UPDATE PULSE SHAPING USING FIBER-PIGTAILED VCSELS ............. 17
CHAPTER 1: EXPERIMENTAL SETUP

2.1 Experimental Setup................................................................................................................18
    2.1.1 The Optical Frequency Comb Source ..................................................................................19
    2.1.2 Wavelength Demux And Mux Modules ..............................................................................20
    2.1.3 Array of Injection-Locked VCSEL Modulators .................................................................21
    2.1.4 Diagnostics and Waveform Measurement ...........................................................................22

2.2 Experimental Results ................................................................................................................24
    2.2.1 Pulse Shaping with Microsecond Update Rates ...............................................................24
    2.2.2 Pulse Shaping with Nanosecond Update Rates ...............................................................26

2.3 Conclusions and Discussion .....................................................................................................28

CHAPTER 3: OPTICAL MODULATION CHARACTERISTICS OF INJECTION-LOCKED VCSELS

3.1 Fundamentals of Optical Injection-Locking .............................................................................33
3.2 Fundamentals of Coherent Optical Demodulation .................................................................37
3.3 Experimental Setup ..................................................................................................................39
3.4 Experimental Results ...............................................................................................................41
    3.4.1 AM and PM vs. Modulation Frequency – Thermal Regime ...............................................42
    3.4.2 AM and PM vs. Modulation Frequency – Electronic Regime .............................................44
    3.4.3 Depth of AM and PM vs. RF power .......................................................................................46
    3.4.4 Depth of AM and PM vs. Bias Current ..................................................................................49
3.5 Conclusions .............................................................................................................................52
CHAPTER 4: RAPID-UPDATE PULSE SHAPING USING A FREE-SPACE VCSEL ARRAY

4.1 Optical Waveform Generation System Design Considerations ........................................... 54

4.2 Experimental Setup ............................................................................................................. 56

4.2.1 The Modulator – VCSEL Array .................................................................................... 57

4.2.2 Setup for Wavelength Multiplexing .............................................................................. 71

4.2.3 Measurement Techniques for Characterizing Rapidly Changing Pulse Shapes ............ 81

4.3 Optical Frequency Comb Source ....................................................................................... 92

4.4 Experimental Results - Waveforms Generated ................................................................. 95

4.4.1 Demonstration of an Injection-Locked VCSEL Array .................................................. 95

4.4.2 Static Pulse-Shaping Using the Injection-Locked VCSEL Array .................................. 97

4.4.3 Pulse Shaping at MHz Update Rates ........................................................................... 104

4.4.4 Pulse Shaping at GHz Update Rates ........................................................................... 106

4.5 Discussion And Conclusions ............................................................................................. 109

4.5.1 Merits of Using an Injection-Locked VCSEL Array for Pulse Shaping ...................... 109

4.5.2 Drawbacks of Using an Injection-Locked VCSEL Array for Pulse Shaping ............... 110

4.5.3 Future work .................................................................................................................. 113

CHAPTER 5: A LINEAR TECHNIQUE FOR DISCRIMINATION OF OPTICALLY CODED WAVEFORMS ............................................................................................................ 114

5.1 Coherent Detection Scheme for Code Discrimination ....................................................... 117
5.2   Experimental Setup and Data ................................................................. 120

5.3   Discussion and Conclusions........................................................................ 124

APPENDIX: PERMISSIONS FOR USE OF COPYRIGHTED MATERIAL................ 126

REFERENCES....................................................................................................... 132
LIST OF FIGURES

Figure 1-1: The time and frequency domain representations of a periodic pulse train. Reprinted by permission from Macmillan Publishers Ltd: Nature Photonics [3], © 2010.................................................................2

Figure 1-2: Two ultrashort pulse shaping regimes, their corresponding frequency and time domain pictures (a) Low resolution pulse shaping where groups of comblines are independently modulated. M is the number of comblines in each group (b) High resolution pulse shaping where individual comblines are independently modulated. Figure reproduced with permission from ref. [2], (c) 2011 Elsevier .................................................................3

Figure 1-3: The difference between static OAWG and dynamic OAWG. (a) In Static OAWG, the individual comblines are independently modulated, but their update rate is negligible. The waveforms repeat every period. (b) In dynamic OAWG, the individual comblines are modulated at update rates comparable to the frequency spacing of the comblines, resulting in a continuous optical spectrum and non-periodic waveforms. Figure reproduced with permission from ref. [9], © 2010 OSA .................................................................................................................................5

Figure 1-4: Schematic of the spectrally-sliced, four quadrature coherent detection technique. Reprinted by permission from Macmillan Publishers Ltd: Nature Photonics[39], © 2010.................12

Figure 2-1: Experimental setup for proof-of-concept demonstration of rapid-update pulse shaping using an array of injection-locked VCSELs as modulators. Mod – Modulator, OC – Optical Circulator, PC – polarization controller .........................................................................................................................18

Figure 2-2: Architecture of optical comb generator, which produces 5 comblines with 6.25 GHz spacing. α – Attenuator, φ – Phase shifter, IM – intensity modulator .................................................................19
Figure 2-3: Spectral and temporal characteristics of the optical frequency comb source. (a) The optical spectrum of the comb source, showing the five comblines with equal power. (b) Photodetected signal on a sampling oscilloscope .................................................................20

Figure 2-4: Photograph of the Essex Hyperfine filters, which are a matched pair of wavelength demultiplexers and multiplexers ..............................................................................................................21

Figure 2-5: Sample photodetected signal when the four VCSELs are independently modulated as shown in Table 2-1, resulting in an envelope with 1 μs periodicity (b) A close-up of the waveform from (a) showing two periods, with a time span of 2 μs (c)–(e) Close-ups corresponding to the colored regions in (b), which show the changing pulse shape within the 1 μs period of the envelope. Note that the time span is now 2 ns. Note that the pulse train shown in (c) is very similar to the input pulse train to the demux shown in Figure 2-3(b). All graphs have the same vertical scale........25

Figure 2-6: Optical spectrum of output waveform, which shows modulation sidebands on each individual channels. Each channel corresponds to an injection-locked VCSEL modulated at the frequency shown in Table 2-2.........................................................................................................................27

Figure 2-7: (a) Sample photodetected signal when the four VCSELs are independently modulated, as shown in Table 2-2 and Figure 2-6, resulting in an envelope with 1.28 ns periodicity. (b) Close up of the waveform from (a) showing the changing pulse shape within a single period of the envelope (c) Input pulse train from the comb source, prior to modulation from the VCSELs. We can observe the shape of the pulse changing on a pulse-to-pulse basis in (b), compared to the input pulses in (c)......27

Figure 3-1: The arcsine shape of the optical phase response, for α =1..................................................35

Figure 3-2: (a) Sinusoidal modulation of Δω_inj (b) The resultant phase modulation: notice the sharp peak at the top of the excursion. A pure sine wave is also plotted to highlight the difference between the shape of the driving signal (not to scale) and the phase modulation produced.........................36
Figure 3-3: A schematic of a setup for coherent optical .................................................................37
Figure 3-4: Experimental setup for measurement of the optical phase and amplitude modulation of an injection-locked VCSEL subject to current modulation. The modulated light from the VCSEL is interfered with the unmodulated CW laser using a 90° optical hybrid. The diagnostics for the injection-locked VCSEL consists of a polarization analyzer and a hi-resolution optical spectrum analyzer (Agilent 83453B), used to ensure co-polarized injection into the VCSEL, and to monitor the stability of injection-locking, respectively. EDFA – Erbium Doped Fiber Amplifier, OBPF – Optical Band Pass Filter, PC – Polarization Controller, OC – Optical Circulator, VOA – Variable Optical Attenuator ................................................................................................................40
Figure 3-5: Experimentally measured stable injection-locking range (shaded area) as a function of injection ratio. The inset shows a close up of the locking range for small injection ratios, where the stable locking range is almost symmetric. \(I_{\text{DC}} = 5.4\) mA, with co-polarized optical injection.........41
Figure 3-6: (a),(b) - The variation of AM and PM with RF frequency, between 0.98 MHz and 20 MHz, for RF power -29 dBm. .................................................................................................................................42
Figure 3-7: (a),(b) AM and PM as function of time with sinusoidal modulation of the VCSEL’s current at 0.98 MHz, -29 dBm (c) A close-up of the PM is shown, along with a sine wave, which is plotted to highlight the difference................................................................................................................43
Figure 3-8. (a),(b) - The variation of the depth of AM and PM with RF frequency, for frequencies between 14 MHz and 4 GHz, RF power -13 dBm. (c) The phase difference between the PM and AM time domain waveforms ................................................................................................................44
Figure 3-9: The variation of the depth of AM and PM with RF power for various modulation frequencies ................................................................................................................46
Figure 3-10: AM and PM as function of time with sinusoidal modulation of the VCSEL’s current at 1 GHz, $P_{RF} = -5$ dBm.

Figure 3-11: (a) Maximum depth of PM, $\beta_{\text{max}}$, experimentally achieved at various frequencies. (b) RF power required to achieve the depths of modulation in (a).

Figure 3-12: Variation in optical power of the free-running VCSEL with current. Its modulation characteristics are measured at $I = 5.4$ mA and at 10 mA.

Figure 3-13: (a),(b) The depth of AM and PM as a function of modulation frequency (c) The phase difference between the PM and AM. Data is shown for $I = 5.4$ mA (linear part of the P-I curve), and $I = 10$ mA (rollover point).

Figure 3-14: Variation of the depth of AM and PM as a function of RF power for two different modulation frequencies, for two different bias currents.

Figure 4-1: Schematic of the overall experimental setup.

Figure 4-2: Structure of the VCSEL, from [82]. © 2006, IEEE.

Figure 4-3: Schematic of a 12 channel VCSEL array on a submount.

Figure 4-4: Cross-section of a single (a) coplanar waveguide (b) microstrip waveguide.

Figure 4-5: Schematics of the microstrip waveguide array PCB used for connecting to the VCSEL array. (a) and (b) show the overall layout of the board. (c) Shows a close-up of the region where the waveguides converge, to align with the VCSEL array wirebond pads. The bends in some the traces are deliberately included to ensure path length matching between channels.

Figure 4-6: (a), (b) Photographs of the VCSEL array wirebonded to the PCB.

Figure 4-7: Picture of the high-precision DACs used as current sources for the VCSELs. (a) Three DAC evaluation boards, with four channels per board, driving a test array of LEDs. (b) Packaged DAC array, connected to battery power source and computer via an USB cable (not shown), and to
12 SMA cables, which connect to a PCB board with the VCSEL array. (c) The software interface created that permits simultaneous control of 12 channels of current with 0.3 μA precision..............64

Figure 4-8: Sample optical spectrum showing the two modes, corresponding to the dominant and suppressed polarization modes of the VCSEL.........................................................................................65

Figure 4-9: Orthogonal polarization mode suppression and frequency offset across VCSEL array M10. It is not suitable for use in the experiment because Ch 10 and 11 are orthogonally polarized .66

Figure 4-10: (a) Variation of the optical frequency with current, across the VCSEL array M4. (b) The variation of the optical frequency with channel number, and two different linear fit to it, for 6 mA per channel (c) The residuals of the linear fit ........................................................................................................67

Figure 4-11: Difference in between experimental frequency variation across VCSEL array and a target variation of 12.5GHz/ channel and offset frequency at channel 6 =194.481 THz ...............68

Figure 4-12: The variation of optical power output with current, of each VCSEL in array M4. The variation of power vs. channel number, for the currents chosen to produce 12.5 GHz/channel optical frequency variation across the array..............................................................................................68

Figure 4-13: Variation of VCSEL beam waist diameter and NA across the array..................................69

Figure 4-14: (a) Experimental setup to measure the frequency response of free-running VCSELs subject to current modulation. (b) Normalized frequency response of each VCSEL in the array......70

Figure 4-15: Wavelength demux using diffraction gratings and lenses. The light is coupled in and out of the VCSEL array using a microlens array........................................................................................................71

Figure 4-16: Photograph of the microlens array used in the experiment, mounted on an aluminum holder........................................................................................................................................72

Figure 4-17: Geometry of the setup used in the simulation to locate the focal spots for the comblines..........................................................................................................................................................74
Figure 4-18: Simulation showing the sensitivity of the focal spot spacing to incidence angle on the diffraction grating. For (a),(c) and (e), the slope of the linear fit is a fit parameter. For (b),(d) and (f), the slope of the linear fit is fixed at 250 μm/channel. ..........................................................75

Figure 4-19: Modified setup used for aligning the focal spot spacing to be 250μm per 12.5 GHz combline spacing..........................................................76

Figure 4-20: (a) Peaks in the reflected optical spectrum using an ASE input source to the demux. (b) The locations of the peaks are fit to straight line with slope of 12.5 GHz/channel. (c) The residuals to the linear fit. (d) The passband width of each channel. Two plots are shown in each of (b), (c) & (d), before and after tuning the grating angle to minimize the residuals. .........................77

Figure 4-21: (a) Optical power per channel coupled into fiber after the mux from the VCSEL array, in comparison to the power in free space. (b) Coupling efficiency vs. channel number across the VCSEL array..................................................................................................................78

Figure 4-22: (a) Photograph of the wavelength demux and VCSEL array setup. The VCSEL and microlens arrays are each mounted on an assembly that provides all 6 degrees of freedom for alignment. (b) A close-up of the VCSEL and microlens arrays as seen through the microscope. The gap between them is about 0.75 mm. ...........................................................................................................79

Figure 4-23: A commercially available optical mux/demux module from Kylia, with fiber input and outputs, along with a schematic of its operation. Source: http://www.kylia.com..........................80

Figure 4-24: A schematic of the experimental setup for multi-heterodyne measurement of optical phase ..........................................................................................................................85

Figure 4-25: Optical and RF spectra of the multi-heterodyne measurement setup. The first set of RF frequencies, shown in the close-up, is filtered from the complete RF signal produced by
photodetection. The optical beat tones that contribute to RF beat tone $h_0$ are shown, as an example.

Figure 4-26: Schematic of the experimental setup for measuring spectral phase using the two combline filtering method.

Figure 4-27: Optical spectrum showing the MLL laser output, directly out of the laser and after non-linear broadening. The VCSEL array’s wavelength range is indicated by the shaded region.

Figure 4-28: Optical Frequency comb source generated by modulation of a CW laser, using cascaded intensity and phase modulators. The programmable optical filter allows independent control of the phase and attenuation of each combline. This enables us to flatten the spectral phase, and compress the pulse to its transform limit.

Figure 4-29: (a) Optical spectra before and after the programmable optical filter. (b) Autocorrelations, before and after phase correction to compress the pulse to its transform-limit. (c) Photodetected traces, before and after phase correction.

Figure 4-30: Spectral and temporal data before and after injection-locking, for all eleven channels of the VCSEL array. Column 1, free-running VCSEL array. Column 2, injection-locked VCSEL array.

Figure 4-31: Spectral and temporal characteristics for flat spectral phase. (a) Spectral power, measured on OSA. (b) Spectral phase, measured. (c),(d) standard deviation of retrieved spectral magnitude and phase, for 50 samples. (e) Retrieved and measured spectral power. (f) Measured time-domain multi-heterodyne RF signal, and the calculated optical pulse electric field envelope, from the measured spectral power and phase. (g) Intensity autocorrelation, measured and calculated traces.

Figure 4-32: Spectral and temporal characteristics for cubic spectral phase. (a) Measured spectral power (b) Measured spectral phase, as well as a cubic curve fit to the data. (c)RF waveform, showing
the classic pulse shape for a cubic spectral phase. (d) Retrieved and measured spectral power. (e) Intensity autocorrelation, measured and calculated traces. (f) Deviation of the currents profile from the flat phase case discussed in section 4.4.2.1.

Figure 4-33: Spectral and temporal characteristics for quadratic spectral phase. (a) Measured spectral power (b) Measured spectral phase. (c) RF waveform, showing a slightly broadened pulse shape for quadratic spectral phase. (d) Retrieved and measured spectral power. (e) Intensity autocorrelation, measured and calculated traces. (f) Deviation of the currents profile from the flat phase case discussed in section 4.4.2.1.

Figure 4-34: Spectral and temporal characteristics for alternating spectral phase. (a) Measured spectral power (b) Measured spectral phase. (c) RF waveform, showing a two pulse sequence within one pulse period of the transform limited case. (d) Retrieved and measured spectral power. (e) Intensity autocorrelation, measured and calculated traces. (f) Deviation of the currents profile from the flat phase case discussed in section 4.4.2.1.

Figure 4-35: A comparison of pulse shapes created by shaping one set of channels in a alternating channel phase pattern. (a) shows the computed optical field pulse envelopes for the two different patterns resulting from the two patterns shown in (b). (c) The autocorrelation trace of the phase pattern with the quadratic shape.

Figure 4-36: (a) Optical spectrum of 4 injection locked VCSEL modulated at MHz frequencies. (b) Photodetected RF signal showing 1 μs periodicity of the intensity profile.

Figure 4-37: Multi-heterodyne signal (a), the spectral amplitude (b) and phase (c) of the signal.

Figure 4-38: Optical spectrum showing modulation sidebands.

Figure 4-39: Unmodulated pulse shapes, for modulation power to each channel turned off. (a) Span = 5*1.28 ns. (b) Span = 1.6 ns, which shows the individual pulses on a 80 ps pulse period.
Figure 4-40: Three sample waveforms showing a periodic envelope of the pulses (a1,a2, a3), each with 1.28 ns periodicity. The corresponding close-ups (b1, b2, b3) show the pulse shape changing on ns timescale.

Figure 4-41: A 2D VCSEL array at 980 nm, on a 250 micron pitch. Source: http://www.princetonoptronics.com/products/pdfs/AA64-BC-SM-W0975.pdf

Figure 5-1: (a) Typical system architecture for matched filtering using ultrashort optical pulses, where the encoder and decoder are matched filters. (b) System architecture for demonstrating code matching using a coherent detection approach. (c) Experimental setup for proof-of-concept demonstration of coherent detection architecture. SOA: semiconductor optical amplifier. OFCS: optical frequency comb source. VOA: variable optical attenuator. VOD: variable optical delay.

Figure 5-2: Using coherent detection and differential balanced photodetectors to distinguish between orthogonal codes encoded on optical spectra. Each ‘1’ bit in a WH code is encoded as phase value 0, ‘-1’ bit as phase value π. In (a), the WH codes encoded on the spectra at both inputs to the interferometer are the same, code (1,1,-1,-1). In (b), the codes are orthogonal, (1,1,-1,-1) and (1,-1,1,-1).


Figure 5-4: Spectral and temporal characteristics of the OFCS. (a) Optical Spectrum. (b) Close-up of 1 nm of the optical spectrum, which shows the individual comblines. Note that the observed
lineshape is due to the optical spectrum analyzer. The real linewidth of each combline is about 10 kHz. (c) Autocorrelation pulse widths, directly out of the laser, and after compression.................. 121

Figure 5-5: Experimental setup used for spectral phase encoding of the OFCS. PBS – Polarization Beam Splitter, PC – Polarization Controller.......................................................... 122

Figure 5-6: (a) Input spectrum to the phase encoders, showing the shape of the optical spectrum from OFCS in linear scale, and subset of 126 comblines encoded. WH codes are 16 bits long, with 6 comblines encoded per bit, and 2 comblines per bit in the crosstalk region (b) Power at each output of the interferometer, Port A and B (c) Normalized differential power, whose value is large when the codes on the SLMs match (d) Histogram of the normalized differential signal shown in (c).......... 123
LIST OF TABLES

Table 1-1: Comparison of the various technologies for wavelength demultiplexers.................................................10
Table 2-1: Modulation of VCSELs at low frequencies, along with the applied RF power.................................25
Table 2-2: Modulation of VCSELs at high frequencies, along with the applied RF powers...............................26
Table 2-3: Limitations of the proof-of-concept experiment for rapid-update pulse shaping using injection-locked VCSELs.................................................................................................................28
Table 4-1: Characteristics of the Raycan 1x12 VCSEL array.................................................................................58
Table 4-2: Variation of optical frequency and power across each VCSEL array sample........................................59
Table 4-3: Theoretical characteristics of designs for PCB.........................................................................................61
Table 4-4: Design parameters of the pulse-shaping setup.........................................................................................72
Table 4-5: Description of the electric field of the optical comblines and RF..........................................................87
Table 4-6: The phase and amplitude of the comblines, when optical comb 2 has flat spectral amplitude and phase........................................................................................................................................87
Table 4-7: Description of the four comblines ........................................................................................................88
Table 4-8: Description of the photodetected signal for each pair of the filtered comblines...............................89
Table 4-9: Photodetected, and the retrieved spectral phase...................................................................................89
Table 4-10: Spectral phase after subtracting the retrieved phase from the original.............................................89
Table 4-11: Modulation of VCSEL at MHz frequencies......................................................................................104
Table 4-12: Modulation of VCSEL at GHz frequencies.......................................................................................106
Table 4-13: Merits and drawbacks of the proposed architecture for rapid-update pulse shaping using injection-locked VCSELs.................................................................................................................112
LIST OF ACRONYMS (or) ABBREVIATIONS

AM – Amplitude Modulator or Modulation,

AR – Anti Reflection

CW – Continuous Wavelength

EDFA – Erbium Doped Fiber Amplifier

IM – Intensity Modulator or Modulation

OC – Optical Circulator

OAWG – Optical Arbitrary Waveform Generation

OAWM – Optical Arbitrary Waveform Measurement

PC – Polarization Controller

PD – Photodiode

PM – Phase Modulator or Modulation

SLM – Spatial Light Modulator

TEC – Thermo Electric Cooler (Peltier element)

VCSEL – Vertical Cavity Surface Emitting Laser

VIPA – Virtually Imaged Phased Array
CHAPTER 1: INTRODUCTION

In this chapter, we will consider:

1. The fundamentals of ultrashort laser-pulse shaping and Optical Arbitrary Waveform Generation (OAWG)
2. The applications of pulse shaping and OAWG
3. The key subsystems of an OAWG system. The requirements for each subsystem and the various implementations demonstrated in the literature are considered. The subsystems are:
   3.1. The wavelength demux/mux
   3.2. Modulator array
   3.3. Optical sources
4. Optical Arbitrary Waveform Measurement (OAWM) techniques

The discussion serves two purposes, to introduce the basic concepts and technologies of OAWG, and to motivate the future work by providing benchmarks for comparison.

1.1 Shaping Ultrashort Laser Pulses and Generating Arbitrary Optical Waveforms

Efforts to control the shapes of ultrashort laser pulses have resulted in a large body of work, which is reviewed in [1, 2]. If the laser pulses are too short (less than ~10 picoseconds) to be directly shaped by modulation in the time domain, shaping is achieved by modulation in the frequency domain. Since the ultrashort pulses are usually emitted as a periodic train of pulses, its optical spectrum consists of a set of lines spaced by the repetition frequency, $f_r$, beneath an envelope that corresponds to the spectrum of a single pulse. This is shown in Figure 1-1. Since the optical spectrum consists of set of equidistant lines like in a comb, it is called an optical frequency comb. To
know the absolute frequency location of each combline, one must also know the offset frequency, \( f_0 \). The offset frequency is related to the change in the carrier-envelope phase from pulse-to-pulse, \( \Delta \varphi_{ce} \).

Figure 1-1: The time and frequency domain representations of a periodic pulse train. Reprinted by permission from Macmillan Publishers Ltd: Nature Photonics [3], © 2010.

Pulse shaping in the frequency domain is usually achieved by spatially dispersing the optical spectrum and using a spatial light modulator. The purpose of the spatial dispersion (followed by focusing) is to convert the spectral variation of the optical electric field onto a spatial variation. The spatial pattern is incident on a modulator array with independently controlled elements/pixels. Depending on the type of modulator array used, both the magnitude and phase of the optical spectrum can be modulated. Some key characteristics of a pulse-shaping setup for ultrashort pulses are:

1. Spectral resolution of the setup, which depends on the ability to spatially resolve the different frequencies.

2. The performance of the spatial light modulator:
   2.1. The update rate of the modulator array
   2.2. The type of modulation on each channel, which can be a combination of amplitude and phase modulations.
   2.3. The number of independent channels available for modulation

Each of the characteristics is explored in the following sections.
1.1.1 Spectral Resolution of the Pulse Shaper

For a given spectrum from an optical frequency comb source, the spectral resolution of the pulse-shaper determines the regime of operation, group-of-lines or line-by-line pulse shaping. Figure 1-2 shows the difference between the two regimes, in the time and frequency domains. In line-by-line pulse shaping, the spectral resolution is high enough for each combline to be spatially separated and independently modulated. In group-of-lines pulse shaping, the spectral resolution is lower and groups of adjacent comblines are modulated together. As the number of lines in a group decreases to one, more complex waveforms may be generated in the time domain that can occupy a greater portion of the pulse period. In line-by-line pulse shaping, the generated pulse can occupy as the whole pulse period, as illustrated in Figure 1-2(b). But it still repeats every pulse period. For example, for line-by-line pulse shaping using a 10 GHz repetition rate optical source, the complex waveforms generated will still repeat every 100 ps. The spectral filters implemented are static, hence the label static Optical Arbitrary Waveform Generation (OAWG). Line-by-line control is necessary for arbitrary waveform generation.

![Diagram showing two ultrashort pulse shaping regimes](image)

**Figure 1-2:** Two ultrashort pulse shaping regimes, their corresponding frequency and time domain pictures (a) Low resolution pulse shaping where groups of comblines are independently modulated. M is the number of comblines in each group (b) High resolution pulse shaping where individual comblines are independently modulated. Figure reproduced with permission from ref. [2], (c) 2011 Elsevier.

The pulse-shaping setup must have a sufficiently high enough resolving power to spatially separate each combline. The smaller the frequency separation between comblines, $f_r$, the more technically
challenging it is to spatially resolve it. Although pulse-shapers with resolutions of down to \( \sim 360 \) MHz have been demonstrated [4] using two-dimensional spectral dispersion with bulk optics, such systems have disadvantages such as large size and high insertion loss that limit their usage. For the more widely used one-dimensional bulk-optics pulse shapers, 5 GHz resolution have been demonstrated for arbitrary waveform generation [5]. Integrated implementations of pulse-shapers such as arrayed waveguide gratings [6] and ring resonators [7], are attractive because of their small size and stability. However, the smallest resolvable frequency that has been demonstrated for arbitrary waveform generation is around \( \sim 10 \) GHz, and is limited by the fabrication errors and size of the wafer. An important consequence of the resolution of the pulse-shaper: it sets the minimum repetition rate, \( f_r \), of the optical source necessary to achieve line-by-line pulse shaping.

1.1.2 Performance of the Modulator Array

1.1.2.1 Update Rates

In the preceding discussion, we assumed that the spectral filter implemented by the pulse-shaper is static. This results in an arbitrary optical waveform that repeats every period of the optical source. In practice, this is easily satisfied because the update rates of spatial light modulator technologies are much slower than the repetition rates of the lasers. Conventional modulators (SLMs), like liquid-crystal SLMs Spatial Light Modulators [2], acousto-optic modulators and microelectromechanical systems (MEMS) [8], have slow update rates of up to several kHz. Compared to the repetition rate of typical mode-locked lasers, which are at range from tens of MHz to tens of GHz, the filter shape is updated so slowly as to be considered static.

To change the pulse shape on a pulse-to-pulse time scale, the spectral phase and amplitude must be rapidly updated. If the each combline is modulated at rates equal to the frequency spacing of the
comblines, truly arbitrary waveforms over durations much longer than the period of input is possible. This regime is labeled dynamic OAWG. The concept is illustrated in Figure 1-3, which comes from [9], where the design of a dynamic OAWG system is described. Special attention is paid to the passband shape of the wavelength multiplexer and type of modulation of each channel, which become important when combining the spectra generated from the modulation of adjacent comblines.

Figure 1-3: The difference between static OAWG and dynamic OAWG. (a) In Static OAWG, the individual comblines are independently modulated, but their update rate is negligible. The waveforms repeat every period. (b) In dynamic OAWG, the individual comblines are modulated at update rates comparable to the frequency spacing of the comblines, resulting in a continuous optical spectrum and non-periodic waveforms. Figure reproduced with permission from ref. [9], © 2010 OSA.

1.1.2.2 Modulation Type and Channel Count

To generate arbitrary waveforms, one needs to be able to modulate the phase and amplitude independently. This has been demonstrated using liquid-crystal SLMs, MEMS-based SLMs and acousto-optic modulators, which are the three widely used technologies. However for several classes of devices, independent control is not possible for intrinsic reasons. An example is optically injection-locked semiconductor lasers under current modulation, which produce a combination of amplitude and phase modulation at GHz rates. More details are provided in section 0. Other
examples where a combination of amplitude and phase modulation is produced include electro-absorption modulators [10].

Another requirement is the depth of modulation provided by the modulator. A large extinction ratio is required of the amplitude modulator (> 20dB) and at least \(2\pi\) radian phase excursion is required of the phase modulator. If the modulator falls short of these requirements, the fidelity of the generated waveforms will be compromised and only a subset of waveforms can be generated.

The larger the channel count, the more complex the waveform that may be generated, assuming that the spectrum of the optical source is wide enough to use all the pixels available. With the availability of dense, 2-dimensional liquid crystal SLMs, the channel count and the potential complexity of the waveforms of the technology has increased [2, 11].

### 1.2 Applications of Pulse Shaping and OAWG

It is important to consider the applications that will be enabled by dynamic OAWG. There are numerous applications of static OAWG, some of which are listed in [2]. These applications will benefit from the ability to produce truly arbitrary optical waveforms, as is discussed in the review paper on OAWG [3]. They include:

1. High-capacity telecommunications [12]
2. Coherent control of reactions [13, 14]
3. Generation and processing of ultra broadband RF signals [15]
4. Laser Detection and Ranging (LADAR) and metrology [16]

### 1.3 Design Considerations for an OAWG System

A bandwidth-scalable approach to dynamic OAWG is discussed in [9]. The key idea is to slice the optical spectrum of the desired waveform into smaller chunks of around 10-20 GHz each. Each
The individual slices are then coherently summed in a mux to generate the desired optical waveform. The electrical signals required (from electronic arbitrary waveform generators) to achieve the target optical waveform are determined. They depend on the filter characteristics of the mux and the modulation format used in each channel (I-Q modulation vs. amplitude-phase modulation). The system can generate continuous arbitrary optical waveforms and is scalable to THz of instantaneous optical bandwidth by increasing the number of channels. The demonstration of dynamic OAWG in [17] is impressive for the complexity and fidelity of the waveforms generated, with 30 GHz instantaneous optical bandwidth and 6ns record length. This is the highest performance demonstration of dynamic OAWG system at the time of this writing.

The ultimate system performance is limited by integration challenges related to large scale modulator arrays and demux/mux. In [17], which is an implementation of the technique in [9], it is restricted to 3 channels due to the choice of modulator. Discrete fiber-pigtailed LiNbO$_3$ modulators were chosen because they offer high performance I-Q modulation, in terms of modulation bandwidths (10 GHz) and waveform fidelity. The absence of crosstalk due to discrete packaging (vs. an integrated arrayed solution) is also important. However, the fiber pigtails for each discrete modulator introduce phase drift between the channels, which makes the output differ from the predicted waveform. The key challenges in dynamic OAWG are scalability and integration, while maintaining a sufficiently large modulation bandwidth ($\sim 10$ GHz) for each channel.

1.3.1 Requirements for Scaling the Optical Bandwidth for OAWG

The requirements are:

1. A demux/mux technology that supports a large number of channels at 10-20 GHz frequency spacing of adjacent channels
2. A modulator technology with a large channel count that can be driven with broadband electrical signals with 10-20 GHz bandwidth, with low electrical crosstalk.

3. System integration and related issues, such as

3.1. Reducing electrical cross talk in delivering GHz modulation signals to dense array of modulators.

This is a common challenge for all densely packed modulator arrays

3.2. Determining the electrical signals needed to produce a particular optical waveform.

3.3. Maintaining high stability and small size.

This may be achieved by integrating the mux/demux and modulators on a wafer. However, this conflicts with the electrical cross-talk requirement, and limits the number of channels due to finite wafer sizes. Conversely, free-space coupling schemes are more easily scaled to a large number of channels, a trade-off for a larger size and sensitivity to alignment.

1.4 The Subsystems of an OAWG System

There are three important subsystems in any OAWG system.

1. A optical frequency comb source

   One with a high enough is repetition rate desired so that individual frequencies can be resolved by the wavelength demultiplexer technology.

2. A wavelength demultiplexer (demux) and multiplexer (mux)

   The demux must spatially resolve the individual comblines of the optical source, and must have enough output channels to support the desired optical bandwidth. A mux is needed to recombine the comblines into a single spatial channel after the modulator array.

3. A modulator array to modulate the phase and amplitude of the optical spectrum.
If the update rate of the array is much slower than the repletion rate of the source, static OAWG is implemented. If the update rate is equal to the repetition rate of the source, then dynamic OAWG is possible.

Equally important is the Optical Arbitrary Waveform Measurement (OAWM) system for characterizing the generated optical waveforms. Measuring the electric field amplitude and phase of multiple comblines as they are rapidly updated is challenging. More information on measurement techniques is presented in section 1.5.

1.4.1 Optical Sources

Mode-locked lasers are the most commonly used sources for optical arbitrary waveform generation. The repetition rate of the lasers needs to be around 10 GHz for line-by-line shaping, as detailed in section 1.1.1. Commercially available fiber- and solid-state passively mode-locked lasers have repetitions below 1 GHz, although researchers have increased the repetition rates up to 10 GHz [18], [19]. Semiconductor mode-locked lasers naturally operate at GHz repetition rates, due to the intrinsic short cavity lengths of monolithic lasers. Several demonstrations of on-chip pulse-shaping systems with semiconductor mode-locked lasers have been reported [20, 21]. Typically they are electrical pumped, but optically pumped semiconductor mode-locked lasers at GHz repetition rates have also been demonstrated[22]. An actively mode-locked semiconductor laser at 12.5 GHz repetition frequency is used experimentally in the dissertation work.

By a combination of strong phase and amplitude modulation of a CW laser using electro-optic modulators and nonlinear broadening in optical fibers, an optical frequency comb may be generated, as demonstrated in [23, 24]. The optical frequency combs generated this way are attractive because of their flexibility and tunability of both center frequency and repetition rate. This technique is used experimentally in the dissertation work.
Microresonators pumped with high-power CW lasers show great potential for generating optical frequency combs, especially at very repetition rates at tens to hundreds of GHz [25]. Although pulse shaping has been demonstrated with microresonators [26], much work remains to be done to make them practical sources for OAWG.

1.4.2 Wavelength Demux/Mux Technologies

The largest number of demultiplexed channels that have been demonstrated using various technologies is shown in Table 1-1. For arrayed waveguide gratings, as the number of channels increase, or the channel spacing decreases, the number of waveguides required increases. The size of the substrate wafer ultimately restricts the maximum number of channels and minimum frequency spacing of the channels. Free-space implementations of demuxes have potentially far greater channel count than integrated solutions. The number of channels is limited currently by the spot size of the demultiplexed beams on the modulator array and by the number of pixels of the modulator array. Both of them may be optimized for much higher channel counts. Free-space demuxes are used experimentally in the dissertation work, due to ease of availability and design flexibility.

<table>
<thead>
<tr>
<th>Number of channels</th>
<th>Channel Spacing</th>
<th>Wavelength (nm)</th>
<th>Demultiplexer technology</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10 GHz</td>
<td>1550</td>
<td>Arrayed Waveguide Grating, on InP substrate</td>
<td>[27]</td>
</tr>
<tr>
<td>400, 512</td>
<td>25 GHz</td>
<td>1550</td>
<td>Arrayed Waveguide Grating, on silica on silicon substrate</td>
<td>[28, 29]</td>
</tr>
<tr>
<td>640</td>
<td>21 GHz</td>
<td>965</td>
<td>Free-space, 1-dimensional dispersion</td>
<td>[30]</td>
</tr>
<tr>
<td>~1000</td>
<td>~ 3 GHz</td>
<td>1550</td>
<td>Free-space, 2-dimensional dispersion</td>
<td>[11, 31]</td>
</tr>
</tbody>
</table>

1.4.3 Modulator Array Technologies

Various modulator technologies have also been studied for pulse shaping[1], of which the ones that have multi-GHz modulation bandwidths (necessary for dynamic OAWG) are listed below:
1. Free-space modulator arrays:
   1.1. Asymmetric Fabry-Perot reflectivity quantum well reflectivity modulators [32]

2. Waveguide devices
   2.1. GaAs waveguide devices [33]
   2.2. Ring resonators [7]
   2.3. InP waveguide devices [34], [20]

In this thesis, we propose a new modulator array towards dynamic OAWG, VCSEL arrays. The interest in using VCSEL arrays for pulse shaping lies in two aspects:

1. 2-D arrays are easily produced with hundreds of individual VCSELs. Due to the VCSELs’ surface normal operation, the larger channel capacity of free-space demux techniques may be used to advantage.

2. Individual VCSELs can be easily modulated at GHz modulation rates.

Both these characteristics are attractive for arbitrary optical waveform generation. A larger channel count of the modulator array permits the modulation of more comblines and the generation of more complex, wider bandwidth optical waveforms. Similarly, a higher modulation bandwidth of each combline generates more complex waveforms. The VCSEL array is at the heart of experiment, and its characteristics are discussed in greater detail in 1.5.2.2 and CHAPTER 4:

1.5 Optical Arbitrary Waveform Measurement

As a counterpart to OAWG, an Optical Arbitrary Waveform Measurement (OAWM) system for characterizing the generated optical waveforms is equally important. Measuring the electric field amplitude and phase of multiple comblines as they are rapidly updated is challenging. Measuring the phase is more challenging than measuring the amplitude. There are many techniques to characterize the phase (spectral or temporal) of ultrashort laser pulses, both for repetitive pulse trains and single-
shot pulses, as detailed in [35] and [36]. However, not all of them are suited for OAWG because of the complexity of the generated pulses, low peak powers and continuous recording time required. Three techniques are discussed below, of which two are implemented experimentally. All of them use only linear optical devices, which is attractive because of higher sensitivity compared to techniques which use non-linear optical devices, like FROG[37] and SPIDER[38]. This is important for low power signals from an injection-locked VCSEL array. Two of the three techniques use interferometry and require a well-characterized reference against which the signal is measured. The fourth is a self-referenced temporal interferometric technique. It is used to measure the spectral phase of the reference, again using only linear optical devices.

1.5.1 Spectrally-Sliced, Four Quadrature Coherent Detection

A technique for full-field characterization of the optical waveform with large instantaneous bandwidths (greater than 160 GHz) and long record length (up to 2 μs) has been demonstrated in [39], from which Figure 1-4 is reproduced.

Figure 1-4: Schematic of the spectrally-sliced, four quadrature coherent detection technique. Reprinted by permission from Macmillan Publishers Ltd: Nature Photonics[39], © 2010.
The total optical bandwidth to be measured was divided into four slices. For each slice, the full electric field was characterized by interfering it with a single combline from the reference, using a four quadrature coherent balanced receiver. The temporal I/Q signals are digitized and then superposed with the signals from the other slices using digital signals processing to recreate the measured optical waveform. It requires a well characterized reference optical frequency comb for measurement, which can be characterized using self-referenced methods such as FROG.

The capabilities of this technique are impressive, but it requires specialized, expensive hardware: one 90° optical hybrid, two ~ 10 GHz balanced photodetectors, and two 10 GHz bandwidth electronic digitizers per spectral slice. This method is not implemented experimentally in this thesis because of lack of access to such hardware.

The experimental measurements of the generated waveforms reported in the dissertation are limited in the ability to measure spectral phase when each channel is modulated, as constrained by the measurement techniques discussed next.

1.5.2 Spectral Interferometry

Interferometry has long been used to measure phase of optical signals. For broadband coherent signals generated by mode-locked lasers, many techniques that use spectral interferometry have been demonstrated [35]. These techniques are divided into two classes, those that measure the signal two using a well characterized reference, and self-referenced techniques which do not require a reference signal. The category of measurements which require a reference signal can be further divided into homodyne and heterodyne techniques.

1.5.2.1 Homodyne Interferometry with a Well-Characterized Reference

For homodyne techniques, the reference and signal contain the same optical frequencies. Four quadrature [40] and dual quadrature spectral interferometry [41] have been implemented to measure
both the spectral amplitude and phase of the generated waveforms (signal). In [40, 41], the reference is an unmodulated optical frequency comb. The signal and reference are interfered, both in phase and in quadrature, and the interfered signals are spectrally resolved. The spectral phase and amplitude vs. frequency of the signal are calculated from the spectra. The refresh rates are limited by the scan rate of optical spectrum analyzers to about once a second, or by the update rates of a 2D CCD array, which can be up to a MHz.

The advantage of homodyne over heterodyne spectral interferometry is that homodyne interferometry does not need a second comb source at a different repetition rate. The disadvantage is the added complexity in the interferometer and subsequent signals processing, where at least two quadratures must be measured to calculate the spectral phase.

1.5.2.2 Multi-Heterodyne Interferometry with a Well-Characterized Reference

In multi-heterodyne interference, the optical frequency comb to be measured is interfered with a reference optical frequency comb. The reference comb has a different comb spacing frequency and offset frequency compared to the signal comb. The comb spacing and offset frequencies of the signal and reference combs are deliberately controlled such that each combline in the signal comb produces a unique beat frequency in the RF domain after photodetection. The technique has been applied for characterizing waveforms generated via static, line-by-line pulse shaping in [42]. This has been extended to the case where the reference comb is also not well characterized, where the optical spectral phase and amplitude of both the signal and the reference can be deduced via post-processing of the digitized photocurrent [43, 44].
In the experimental work reported in the dissertation, we use heterodyne interferometry to measure the spectral phase and amplitude of the generated waveforms. This technique is most useful for measurement of spectral phase of static waveforms. The maximum measurable update rate depends on the comb spacing and offset frequencies of the two optical combs. For the repetition rates around 12.5 GHz proposed in the experiment, reliable data extraction is likely for update rates less than about 1 MHz. A real time scope with bandwidth of at least ~ 1 GHz will be required for this technique.

While multi-heterodyne interferometry requires a second comb source, it has several advantages over homodyne spectral interferometry: simpler interferometer setup and signal processing, and very fast acquisition times. In the experimental data shown in the dissertation, an acquisition times of 20 μs is used, which could be further reduced.

Both homodyne and heterodyne interferometry require a well-characterized reference comb. The technique used to characterize the reference comb is discussed next.

1.5.3 Self-Referenced Temporal Interferometry

For an optical frequency comb source with a large enough comb spacing (~ 10 GHz) where individual comblines can be resolved, two adjacent comblines can be filtered out, and resulting the beat signal can be measured using a fast photodiode and oscilloscope. The phase of the sinusoidal RF signal observed on the oscilloscope contains the relative phase of the two filtered optical comblines. By moving the center frequency of the filter to select other pairs of comblines, the relative phase variation across the optical frequency comb can be measured. Computing a cumulative sum using the measured relative phase values gives the optical spectral phase of the comb. This method was first reported in the literature in [45, 46], and was then used to measure the
spectral phase of complex optical pulse shapes (static OAWG) in [47]. A more detailed, mathematical description of this technique is given in chapter 0.

Since the method involves filtering two adjacent comblines to measure their relative spectral phase, it is called the ‘two combline filtering’ method hereafter. This method is suitable for the experiment undertaken in the dissertation, because the combline spacing of 12.5 GHz of the reference frequency comb is large enough for individual comblines to be filtered. The filtering is performed using the Finisar Waveshaper, a commercial product which acts as programmable optical filter, with minimum channel resolutions of about 10 GHz. A description of the Waveshaper is found in [48].
CHAPTER 2: RAPID-UPDATE PULSE SHAPING USING FIBER-PIGTAILED VCSELS

Please note that the work presented in this chapter was previously published. © 2011, OSA. Reprinted, with permission, from S. Bhooplapur, N. Hoghooghi, and P. J. Delfyett, Pulse shapes reconfigured on a pulse-to-pulse time scale by using an array of injection-locked VCSELs, Optics Letters, May 2011, [49]. Some of the material was also presented at conferences [50, 51].

Shaping of ultrashort optical pulses has numerous uses, from telecommunications to studies of ultrafast chemical reactions. Various approaches and devices have been demonstrated for pulse shaping, among which the Fourier-Transform (FT) pulse-shaping technique is the most common and versatile. Independent phase and amplitude control of each of the comb lines of a mode-locked laser was first demonstrated in a FT pulse-shaping setup in [52]. The technique has been further developed and demonstrated using Liquid Crystal Modulators (LCM) [5], thermo-optic modulators and Arrayed Waveguide Gratings (AWGs) [53] and ring-resonators [7]. The update rate of the pulse-shape is slow (less than 1 kHz) in each of these cases, limited by the response times of the liquid crystal material or the thermo-optic modulators used. Rapid update rates (greater than 1 GHz) have been demonstrated using a combination of electro-absorption and electro-optic modulators in AWGs [54, 55], and using fiber-pigtailed quadrature modulators [17]. Amplitude modulators with ~1 ns update time, based on an asymmetric Fabry–Perot quantum-well structure, have been demonstrated in direct space-to-time pulse-shaping [32]. Here we conclusively demonstrate a new approach: injection-locked VCSELs as modulators in a FT pulse-shaping experiment. In this chapter, we show both the modulation of the envelope of the pulses and a change in the pulsed-shapes within, at rates approaching half the repetition rate of the OFCG. This is a significant improvement over the results presented in [56]. Independent line-by-line modulation of four
channels at frequencies of up to 3.125 GHz is demonstrated, yielding pulse shape changes on a pulse-to-pulse timescale.

In the experiment presented in this chapter, four fiber-pigtailed VCSELs were each injection-locked to a separate comb line from an Optical Frequency Comb Generator (OFCG). Each injection-locked VCSEL functioned primarily as a phase modulator. The phase response is predicted from the theory of injection-locked lasers [57] and experimentally confirmed in [58] using a VCSEL similar to the ones used in this paper. The current to each VCSEL was modulated independently, and the modulated comb lines are then recombined into a single channel. Due to coherent addition of the different comb lines, a pulse is synthesized (Fourier synthesis) whose shape depends on the time-varying modulation of each of the comb lines.

2.1 Experimental Setup

The experimental setup for the demonstration of pulse shaping using injection-locked VCSELs is shown in Figure 2-1.
The main components of the setup are:

1. Optical frequency comb source
2. Wavelength demultiplexer (demux) and multiplexer (mux) modules
3. Array of modulators, and
4. Diagnostics and waveform measurement

Each component is explained in greater detail in the following sections.

2.1.1 The Optical Frequency Comb Source

Figure 2-2 shows the experimental setup used to generate 5 comblines of equal power separated by 6.25 GHz. The comblines are achieved by intensity modulation at 6.25 GHz of a tunable CW laser. The RF power delivered to each intensity modulator and the relative RF phase shift is optimized to achieve five comb lines of equal optical power. Five comblines are adequate for the planned demonstration using 4 injection-locked VCSELs. In the time domain, pulses with 30 ps FWHM, 160 ps period are observed. The spectral and temporal characteristics of the comb source are shown in Figure 2-3.

![Figure 2-2: Architecture of optical comb generator, which produces 5 comblines with 6.25 GHz spacing. α – Attenuator, φ – Phase shifter, IM – intensity modulator](image)
Figure 2-3: Spectral and temporal characteristics of the optical frequency comb source. (a) The optical spectrum of the comb source, showing the five comblines with equal power. (b) Photodetected signal on a sampling oscilloscope.

### 2.1.2 Wavelength Demux And Mux Modules

Each combline was demultiplexed to a separate optical fiber channel using an Essex Hyperfine WDM filter [59]. The Hyperfine filter is based on a Virtually Imaged Phased Array (VIPA), which is essentially an etalon where the light is incident at an angle to the surface through an AR coated aperture. [60]. A picture of the Essex Hyperfine demux and mux filters, which are a matched pair, is shown in Figure 2-4. Some of the salient features of the filters are:

1. There are 1 input channel and 16 output channels per filter. All channels are fiberized.
2. A frequency spacing of 6.25 GHz per channel
3. 100 GHz free-spectral range (FSR) for each channel.
4. Gaussian shaped passband for each channel, with a 3dB bandwidth of about 3.5 GHz.
5. Minimum insertion loss per channel of least 6 dB at the peak of the passband.
6. The isolation from neighboring channels is about 22 dB.

In the experiment, only four of the output channels (channels 7-10) of the Hyperfine filters are used. The wavelength of the CW laser is tuned to align the comblines with the peak transmission of the demux. The modulated signals from the injection-locked VCSELS are multiplexed using the second Hyperfine filter.

![Photograph of the Essex Hyperfine filters, which are a matched pair of wavelength demultiplexers and multiplexers](image)

Figure 2-4: Photograph of the Essex Hyperfine filters, which are a matched pair of wavelength demultiplexers and multiplexers

2.1.3 Array of Injection-Locked VCSEL Modulators

Individual fiber-pigtailed VCSELS are used in the experiment, commercially available from RayCan Ltd. VCSELS can be used as modulators by injection-locking them to the light from a master laser. Upon injection-locking, the VCSELS emit at the same optical frequency as the master laser. Modulating the current to the injection-locked VCSEL modulates the light, producing both optical phase and amplitude modulation. The optical modulation characteristics are discussed in greater detail in 0.

The VCSELS are injection-locked in reflection mode, as shown in Figure 2-1. They emit at wavelengths around 1538 nm, and bias currents are fine tuned such that each VCSEL locks to its
injected tone. The injected optical power is around $2 \mu W$ to each VCSEL, while the output powers of the free-running VCSELs vary between 0.6 - 1 mW. Hence the injection ratios are small, between -25 dB and -27 dB. The power variation is due to differences in fabrication, which necessitates differences in bias currents of each VCSEL to attain the required wavelength. Additional loss is then introduced in three of the channels to ensure equal power of the four comb lines at the output of the mux, with less than 5% variation. Each VCSEL is then independently modulated at a different frequency.

The VCSELs emit linearly polarized light. The polarization controller before each VCSEL is set such that the state of polarization (SOP) of the injected comb line coincides with the preferred SOP of each VCSEL. The SOP is monitored using a polarization analyzer. The polarization controller after each VCSEL is set such that the SOP of each of the four channels is identical at the output of the mux.

2.1.4 Diagnostics and Waveform Measurement

The diagnostics equipment consists of a High Resolution Optical Spectrum Analyzer (Hi-Res OSA) and a polarization analyzer. The Hi-Res OSA (Agilent 83453B) has a minimum frequency resolution of 15 MHz and is very useful in monitoring injection locking of the VCSEL. At the low injection powers used in the experiment, the injection locking bandwidth is on the order of 2 GHz. The VCSELs can fall out of lock due to variety of reasons, due to temperature changes and the application of large RF signals for example, and the Hi-Res OSA is essential for monitoring the injection-locked status for each channel.

The polarization analyzer (Optellios PS2000B) displays the State Of Polarization (SOP) on a Poincare sphere. It is necessary to ensure that the injected light to each VCSEL matches the
polarization of the VCSEL’s emission, and to match the SOP of each of the four channels at the output of the mux. The presence of optical fiber pigtails in each channel results in the SOP of the channels varying randomly after the multiplexer. Matching the SOP in each channel is vital to observing a good depth of modulation on the generated waveform.

It is challenging to completely characterize the generated waveforms because both the amplitude and phase of the input pulses are modulated at GHz rates. An experimental technique has been demonstrated to fully characterize truly arbitrary optical waveforms [39], but it requires an extensive experimental setup. In this demonstration, we measure only the intensity profile of the generated waveforms using a fast photodiode and a high bandwidth real-time oscilloscope. Since the four channels are separated by 6.25 GHz and each channel is modulated at up to 3.125 GHz, the maximum modulation bandwidth is less than $6.25 \times 4 = 25$ GHz. Using a 33 GHz bandwidth photodiode (Discovery Semiconductor DSC 20H) and a 30 GHz real-time oscilloscope (LeCroy WaveMaster 830Zi), we can capture the complete intensity profile of the entire waveform.

The exact shape of the optical waveform depends on:

1. The amounts of AM and PM generated on each channel by the VCSEL, and
2. The relative optical phase between the four modulation channels
3. The RF power and phase of the modulation signals

The relative phase of the four channels drifts due to environmental perturbations on the fiber-pigtailed components between the demux and mux. This causes the shape of the optical waveform to change on a much longer time scale ($> 50 \mu s$) than the changes due to the modulation ($< 1 \mu s$). Such relative drift may be eliminated by using a linear array of VCSELs in a common-mode setup, i.e. a free-space grating and lens configuration similar to [5].
2.2 Experimental Results

Waveforms are generated by modulating the each VCSEL at a different frequency. Since the modulation is periodic (sinusoidal), the resultant optical waveform will also be periodic. The repetition frequency of the waveform is equal to the Highest Common Factor (HCF) of the modulation frequencies. For example, for the frequencies shown in Table 2-1, the HCF is 1 MHz, resulting in a periodicity of 1 \mu s. Two different frequency ranges are explored:

1. The VCSELs are modulated at frequencies below 10 MHz.
   In this regime, carrier density modulation causes temperature change within the VCSEL. The thermal modulation mechanism is much stronger than the electronic modulation response [61]. As a result, low RF powers are sufficient to achieve a large depth of optical phase modulation in this regime, with very low optical amplitude modulation.

2. The VCSELs are modulated at frequencies between 700 MHz and 3.2 GHz.
   In this regime, changes in the carrier density alone change the refractive index within VCSEL. A larger RF power is now needed compared to the thermal modulation regime to achieve a similar depth of modulation. Significant amounts of optical amplitude and phase modulation are produced as a result.

In the experiment, the RF powers quoted are output powers of the RF frequency synthesizers. The RF power entering the VCSEL is hard to determine precisely because of parasitic losses in coupling signals from the SMA cable to the TO-packaged VCSEL.

2.2.1 Pulse Shaping with Microsecond Update Rates

Each VCSEL is modulated at the RF frequencies and powers shown in Table 2-1. Notice that the RF power applied is very small, because of the efficiency of the thermal modulation mechanism.
Table 2-1: Modulation of VCSELs at low frequencies, along with the applied RF power.

<table>
<thead>
<tr>
<th>VCSEL</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency (MHz)</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>RF power (dBm)</td>
<td>-30</td>
<td>-30</td>
<td>-25</td>
<td>-25</td>
</tr>
</tbody>
</table>

Figure 2-5: Sample photodetected signal when the four VCSELs are independently modulated as shown in Table 2-1, resulting in an envelope with 1 μs periodicity (b) A close-up of the waveform from (a) showing two periods, with a time span of 2 μs (c)–(e) Close-ups corresponding to the colored regions in (b), which show the changing pulse shape within the 1 μs period of the envelope. Note that the time span is now 2 ns. Note that the pulse train shown in (c) is very similar to the input pulse train to the demux shown in Figure 2-3(b). All graphs have the same vertical scale.

The frequencies are too low for the sidebands to be resolved on an optical spectrum analyzer. The depth of Phase Modulation (PM) index was measured to be about 0.15π rad. The residual Amplitude Modulation (AM) was around 1%. Hence, PM is dominant. The resulting waveforms are expected to have a periodic envelope because the VCSEL currents are sinusoidally modulated. The envelope’s period corresponds to the Highest Common Factor (HCF) of the modulation frequencies.

Figure 2-5 shows a sample photodetected pulse train. Figure 2-5(a) shows a periodic envelope of the pulse train with 1 μs periodicity, corresponding to the 1 MHz HCF of the modulation frequencies shown in Table 2-1. The shape of the envelope within a period depends on the modulation by the
VCSELs, and relative optical phase of the comb lines in the four channels. The former changes quickly (< 1 μs), while the latter changes slowly (>50 μs) due to optical phase drift in each channel due to the fiber-pigtailed components. The individual pulses under the envelope cannot be resolved on this time scale. Figure 2-5(b) shows a close-up within one period of the envelope, from which three further close-ups are examined in Figure 2-5(c)-(e). The individual pulses under the envelope can now be observed, and their shape changes significantly within the 1 μs period of the envelope. This demonstrates rapid update pulse-shaping on the microsecond time scale. The pulse period is clearly seen to be 160 ps, corresponding to the 6.25 GHz repetition rate of the OFCG.

2.2.2 Pulse Shaping with Nanosecond Update Rates

To increase the update rates of the pulse shapes to nanosecond time scales, the VCSELs are modulated at frequencies up to 3.125 GHz, as shown in Table 2-2. The frequencies are all multiple of 781.25 MHz, which results in a periodicity of the envelope of 1.28 ns. The RF powers to each VCSEL are set to ensure that the sideband heights in the optical spectrum (as shown in Figure 2-6) are equal for all frequencies. In general, larger RF powers are required at higher frequencies to compensate for the higher losses, both electrical and optical. At higher RF frequencies, RF losses are higher in the cables and connectors. The Gaussian shaped passband of the wavelength mux attenuates optical sidebands at larger frequency offsets.

Table 2-2: Modulation of VCSELs at high frequencies, along with the applied RF powers.

<table>
<thead>
<tr>
<th>VCSEL</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency (MHz)</td>
<td>1562.5</td>
<td>3125</td>
<td>781.25</td>
<td>2343.75</td>
</tr>
<tr>
<td>RF power (dBm)</td>
<td>-9</td>
<td>+3</td>
<td>-16</td>
<td>-11</td>
</tr>
</tbody>
</table>
Figure 2-6: Optical spectrum of output waveform, which shows modulation sidebands on each individual channels. Each channel corresponds to an injection-locked VCSEL modulated at the frequency shown in Table 2-2.

Figure 2-7(a) shows a sample photodetected pulse train with a 1.28ns periodicity of the envelope. A close-up of two periods of the pulse train in Figure 2-7(b) shows that the pulse shape changes dramatically within 1.28 ns. Comparing Figure 2-7(b) to Figure 2-7(c) (the input pulse train), we observe that the shaped pulses change on a pulse-to-pulse time scale and fill the 160 ps pulse period. This is proof of line-by-line pulse shaping at update rates of up to half the repetition rate of the input pulse train.

Figure 2-7: (a) Sample photodetected signal when the four VCSELs are independently modulated, as shown in Table 2-2 and Figure 2-6, resulting in an envelope with 1.28 ns periodicity. (b) Close up of the waveform from (a) showing the changing pulse shape within a single period of the envelope (c) Input pulse train from the comb source, prior to modulation from the VCSELs. We can observe the shape of the pulse changing on a pulse-to-pulse basis in (b), compared to the input pulses in (c).
We thank Ms. Yamilet Ardavin of LeCroy Corporation for the loan of the LeCroy Wavemaster 830Zi real-time scope, which enabled the measurement of the waveforms in Figure 2-7.

2.3 Conclusions and Discussion

In this Chapter, we have conclusively demonstrated independent, line-by-line modulation of four comb lines using injection-locked VCSELs. The VCSELs are modulated at frequencies of up to 3.125 GHz, which generate waveforms that change their shape on a pulse-to-pulse timescale, and fill the pulse period. To produce specific waveforms with accuracy, the phase drift between the channels of demux/mux due to fiber-pigtailed components must be controlled. An alternative mux/demux technology must be used, such as a free-space grating and lens setup. By carefully characterizing the amplitude and phase response of the VCSEL, the appropriate electrical signals to produce the desired waveforms can be determined.

The main limitations in the experiment are listed in Table 2-3, along with the proposed solutions.

<table>
<thead>
<tr>
<th>Limitations &amp; Challenges</th>
<th>Proposed Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The phase between the individual channels fluctuated randomly due to optical fiber</td>
<td>Eliminate optical fiber by coupling light in free-space to a VCSEL array</td>
</tr>
<tr>
<td>The depth of amplitude and phase modulation produced by the injection-locked VCSEL in each channel is not well known</td>
<td>Measure the optical amplitude and phase modulation characteristics of an injection-locked VCSEL via coherent demodulation technique</td>
</tr>
<tr>
<td>Only the intensity profile of the generated optical waveform was measured</td>
<td>Use a multi-heterodyne system to measure the optical amplitude and phase modulation of each combline.</td>
</tr>
</tbody>
</table>

The limitations of the experiments and the solutions are discussed in detail below. Solutions to overcome all three deficiencies have been proposed and some of them have also been implemented.

1. The phase between the individual channels fluctuated randomly.

   This was due to the presence of optical fiber in each channel, which is susceptible to environmental perturbations. The phase fluctuations between each channel caused the observed
optical pulse shapes to also randomly drift with time. To address this problem, the optical fiber in each channel has to be eliminated, which impacts both the VCSEL array and the wavelength mux/demux technology used. Light can be coupled via free-space optics to the VCSEL array. Either free-space or planar integrated implementations of wavelength mux/demux technologies can be used. A free-space wavelength mux/demux is implemented in CHAPTER 4: to provide control over the generated waveform shapes.

2. The depth of modulation on each channel (amplitude and phase) was not well characterized. Current modulation of an injection-locked VCSEL produces both amplitude and phase modulation of the optical frequency. The depth of amplitude is easily measured, but phase modulation from an injection-locked VCSEL is hard to measure at GHz modulation rates, especially when accompanied by amplitude modulation. Data on the phase modulation characteristics of injection-locked VCSELs at GHz modulation frequencies had not been reported in the literature. A decision was taken to characterize the modulation characteristics of an injection-locked VCSEL. The experimental setup and results are detailed in CHAPTER 3:

3. Only the intensity profile of the optical waveform was characterized. For this demonstration, we characterized only the intensity profile of the pulses by using a photodiode and real-time oscilloscope with bandwidths greater than the modulation bandwidth of the optical signal. Characterizing both the phase and amplitude modulations at GHz modulation rates of multiple optical frequencies simultaneously is challenging. An architecture has been proposed in [39], and requires an extensive experimental setup. A simpler measurement technique involving multi-heterodyne detection is proposed in CHAPTER 0, for measurement of the amplitude and phase modulation of each combline, at modulation frequencies up to a few MHz.
CHAPTER 3: OPTICAL MODULATION CHARACTERISTICS OF INJECTION-LOCKED VCSELS

Please note that the work presented in this chapter was previously published. © 2013, IEEE. Reprinted with permission, from S. P. Bhooplapur and P. J. Delfyett, Characterization of the Phase and Amplitude Modulation of Injection-Locked VCSELs at 1550 nm Using Coherent Optical Demodulation, Selected Topics in Quantum Electronics, IEEE Journal of, Nov/Dec 2013” [62]. Some of the material was also presented at various conferences [63-66].

Optically injection-locked semiconductor lasers have been extensively studied before, and a good overview of their properties and applications is given in [67]. When the current to a semiconductor laser is modulated, both the optical power and the frequency change, due to the coupling of the real and imaginary parts of the refractive index, via the linewidth enhancement factor, $\alpha$. An injection-locked semiconductor laser emits at the same optical frequency as the master laser, but maintains a finite phase difference to the injected master light. Modulating the current of the locked laser produces optical Amplitude Modulation (AM) and Phase Modulation (PM). PM from injection-locked semiconductor lasers has been examined theoretically in [68-71], but there is very little experimental data reported in the literature about the depth of PM, especially at GHz modulation frequencies, which are of interest in telecommunications applications. Some reasons for the scarcity of the data are:

1. Many applications have been in systems with power detection, where AM is of primary interest, and PM is neglected.

2. The depth of PM from injection-locked lasers is theoretically limited to less than $\pi$ radians, which is not enough for binary $(0, \pi \text{ rad})$ optical PSK modulation in telecommunications.

3. The coupling of AM & PM in semiconductor lasers makes it difficult to independently and accurately measure each.
Nevertheless, there are several applications where the PM properties of an injection-locked semiconductor laser are useful and must be well known. They are:

1. Coherent beam combining and steering in free space, which relies on precise control of the phase of each emitter (laser) in a 2-D array. Coherent combination of the individual beams from an array of injection-locked VCSELs was shown in [72, 73].
2. The linear interferometric intensity modulator [58], which uses the arcsine phase response of an injection-locked laser in an interferometer to obtain a perfectly linear intensity transfer function.
3. Rapid-update pulse shaping of ultrashort pulses, where an array of injection-locked VCSELs modulate the optical amplitude and phase of each combline [49].

For these applications, accurate characterization of the depth of PM and the accompanying AM from the injection-locked laser is important. The limited depth of PM from injection-locking is the biggest constraint in each of the three applications. The coupling of AM and PM also affects performance, by limiting the spur-free-dynamic-range in application 2. and the types of waveforms that can be generated in application 3. listed above. Since VCSELs with large modulation bandwidths have been demonstrated (up to 55 GHz at 850 nm [74]), the bandwidth requirements of the applications can be met.

In this chapter, we utilize an interferometric technique to independently measure AM and PM from an optically injection-locked VCSEL subject to current modulation. It was first proposed in [75] for characterizing optical modulators, but there is no reported usage of the technique for characterization of optical modulators at GHz rates. The optical signal to be measured is interfered with an unmodulated optical reference, in phase & in quadrature. The photodetected signals are electronically processed to calculate the AM and PM on the signal. The advantages of this approach
over the others are that PM and AM are independently measured without any intrinsic assumptions about the shapes of the modulation, for small or large signal modulation. The availability of balanced photodetectors and oscilloscopes exceeding 30 GHz bandwidth, developed for coherent communications, enable measurement of the AM and PM of optical modulators at GHz rates in real time.

The coupled intensity and phase modulation have been independently measured for a free-running VCSEL at 1.55 μm [76], by measuring a sonogram of a repetitive pulse train. But such measurements have not been reported for injection-locked VCSELs. The advantage of the coherent demodulation technique used in this article is that the phase and amplitude of the electric field are displayed in real time without the need for extensive processing for phase retrieval. A two-part technique has been reported to measure AM and PM of an optical carrier. The depth of AM is calculated from the photodetected signal. The depth of PM is calculated by measuring the relative heights of the peaks in the optical spectrum, assuming a functional form for the PM and AM and using the measured AM. PM has been measured using this technique for an injection-locked laser in [68], and for a free-running VCSEL in [77]. The technique is limited to small-signal modulation for frequencies larger than 500 MHz.

Interferometry with homodyne detection was used in [78] to characterize the PM from an injection-locked semiconductor laser under current modulation. The optical power at the output of the interferometer changes due to the combination of 1) inherent intensity modulation, and 2) the phase-modulated light from the injection-locked laser interfering with the unmodulated reference light. Hence homodyne interference alone is unable to distinguish the relative contributions of AM and PM, and additional information is needed. Interferometry with heterodyne detection has been demonstrated for simultaneously measuring the amplitude and phase response of an optical
modulator [10, 79]. But these techniques are limited by the bandwidth or update rates of the RF demodulation techniques used, to less than 100 Hz.

In this chapter, we explore the basics of optical injection-locking in semiconductor lasers, followed by the basics of coherent optical demodulation, to show how AM and PM of an optical carrier can be independently measured. The experimental setup is then described, followed by results where we explore the variation of the depth of AM & PM of an injection-locked VCSEL, vs. modulation frequency and RF power, for two different bias currents and for small- and large-signal modulation.

3.1 Fundamentals of Optical Injection-Locking

The fundamentals of injection-locking in semiconductor lasers and their modulation properties have been well-explored, though the focus has been on the small-signal intensity modulation response. A set of three differential equations for the electric field magnitude, phase and carrier density are widely used to describe an injection-locked semiconductor laser, as clearly presented in [67]. The differential equations are non-linear and coupled, and the AM and PM are inherently non-linear as a function of carrier density. Under small-signal modulation conditions, the equations can be linearized, and amplitude and phase modulation transfer functions can be derived [68-70]. Under large signal current modulation, the differential equations must be solved numerically to obtain the phase and amplitude response, as was done in [71] to theoretically examine the PM from square wave current modulation.

Some insight into the modulation properties can be gleaned from the steady state solutions of the differential equations. The steady state phase response is given by:
\[
\phi = \sin^{-1}\left(-\frac{\Delta \omega_{inj}}{\omega_{LR}}\right) - \tan^{-1} \alpha
\]  
\tag{3-1}

\[-0.5\pi \leq \phi \leq \cot^{-1} \alpha \]  
\tag{3-2}

\[
\Delta \omega_{inj} = \omega_{ML} - \omega_{f.r.-SL}
\]  
\tag{3-3}

\[
\omega_{LR} = \kappa \sqrt{\left(1 + \alpha^2\right) \frac{A_{inj}}{A_0}}
\]  
\tag{3-4}

Equation (3-1) describes the phase of the injection-locked laser electric field, \(E_s\), with respect to the injected field, \(E_{inj}\). Equation (3-2) describes the range of phase modulation achievable. The parameters described in the equations are:

1. \(\Delta \omega_{inj}\) is labeled the detuning frequency
2. \(\omega_{ML}\) is the master laser frequency
3. \(\omega_{f.r.-SL}\) is the free-running slave laser frequency
4. \(\omega_{LR}\) is the frequency range within which the slave laser frequency locks to the master laser
5. \(\kappa\) is the coupling rate, the rate at which the injected photons enter into the cavity
6. \(\alpha\) is the linewidth enhancement factor
7. \(A_{inj}\) is the electric field magnitude of the injected field
8. \(A_0\) is the steady-state electric field magnitude within the injection-locked laser

The extrinsic injection ratio, which is experimentally controlled, is defined as the ratio of the injected optical power to the VCSEL’s free-running power, i.e. \(R_{inj} = P_{inj}/P_{fr}\). Both powers are measured at the output fiber of the VCSEL.

For injection-locking in reflection, the observed field is:

\[
E_T = E_s + E_r
\]  
\tag{3-5}

\(E_s\) is the injection-locked laser electric field.

\(E_r\) is the reflected portion of the injected field, \(E_{inj}\).
For small injection ratios explored in this article, \( E_r \) is negligible and (3-1) is valid for \( E_T \). For large injection ratios where \( E_r \) is comparable to \( E_s \), the interference of the two fields causes the phase of \( E_T \) to differ from (3-1), as discussed in [80]. The AM and PM of \( E_T \) in this regime would also differ from those explored in this paper. The modulation properties of \( |E_T|^2 \) are detailed in [80].

![Phase response diagram](image)

Figure 3-1: The arcsine shape of the optical phase response, for \( \alpha = 1 \).

Figure 3-1 shows the shape of the phase response under steady state conditions, for \( \alpha = 1 \). It shows the maximum phase modulation achieved by modulating the detuning frequency, \( \Delta \omega_{\text{inj}} \), over the full locking range. Modulating the slave laser’s current would modulate \( \omega_{\text{f.r.-SL}} \) and hence \( \Delta \omega_{\text{inj}} \). The range of \( \varphi \) under steady state conditions is given by equation (3-2). The upper limit is \( \phi_{\text{max}} = \cot^{-1}(\alpha) \). Since \( \alpha > 0 \) for semiconductor lasers, \( \cot^{-1}(\alpha) < \frac{\pi}{2} \). The lower limit is \( \phi_{\text{min}} = -0.5\pi \) rad is due to constraint on the gain change within the slave laser upon injection-locking [81]. Hence values of \( \varphi \) less than \(-0.5\pi \) rad are not physically realizable, as is indicated by the shaded region in Figure 3-1. This restricts the full range of phase excursion available due to injection locking to less \( \pi \) rad, depending on the value of \( \alpha \) for the laser, under steady state conditions.
Figure 3-2: (a) Sinusoidal modulation of $\Delta \omega_{inj}$ (b) The resultant phase modulation: notice the sharp peak at the top of the excursion. A pure sine wave is also plotted to highlight the difference between the shape of the driving signal (not to scale) and the phase modulation produced.

With sinusoidal modulation of $\Delta \omega_{inj}$ shown in Figure 3-2(a), the expected phase modulation is shown in Figure 3-2(b). A perfect sinusoid is also plotted, to highlight the shape of the phase modulation, which deviates from the sinusoidal shape more near the top of the phase excursion. There the arcsine shape of the phase response in equation (3-1) ‘cancels’ the sinusoidal shape of the $\Delta \omega_{inj}$ modulation, resulting in a linear phase variation with time. Near the bottom of the excursion, the PM resembles a sinusoid because the phase transfer function is locally almost linear. The PM is asymmetric about 0 rad because the locking range is asymmetric about $\Delta \omega_{inj} = 0$. The shape of the PM is also asymmetric about its mean value in Figure 3-2(b), because the modulation of $\Delta \omega_{inj}$ was chosen to be equal to the full locking range, which is asymmetric about zero. A smaller maximum depth of PM may be traded-off for a symmetric modulation of $\Delta \omega_{inj}$ and shape of PM.

Both $A_0$ and $\Delta \omega_{inj}$ in equation (3-1) depend on the carrier density within the slave laser. For small signal current modulation, if one assumes that $A_0$ does not change, and that $\omega_{f.r.-SL}$ changes linearly with current density, then $\varphi$ is still described by equation (3-1). As shall be seen later, a small signal modulation of the current in the thermal regime (at frequencies up to a few MHz) can indeed produce large modulation of the phase, as suggested by Figure 3-2(b). For modulation in the
electronic regime (at frequencies greater than 100 MHz), large signal current modulation is required to produce similar depth of PM. In this case, $A_0$ also changes significantly and $\varphi$ will not have an arcsine dependence on the carrier density. The rate equations must be numerically solved to evaluate the shape of the phase response.

3.2 Fundamentals of Coherent Optical Demodulation

In this chapter, we explore the basics of coherent optical demodulation, which is an interferometric technique used to measure AM and PM on an optical carrier, typically used in optical communications. The optical signal to be measured is interfered with an unmodulated optical carrier (reference), in phase & in quadrature. The photodetected signals are electronically processed to calculate the AM and PM on the signal. A schematic of the demodulation setup is shown in Figure 3-3.

Figure 3-3: A schematic of a setup for coherent optical
demodulation.

Equations (3-6) to (3-9) explain how the optical phase and amplitude modulation on the signal is measured using a 90° optical hybrid and balanced differential photodetectors.

$$E_{\text{sig}}(t) = A_1(t)e^{i(\omega t + \phi_1(t))}$$
$$E_{\text{ref}}(t) = A_0e^{i(\omega t + \phi_0)} \quad (3\text{-}6)$$

Where $A_1(t)$ is the amplitude and $\phi_1(t)$ is the phase of the optical carrier frequency of the signal field. Both are vary as function of time. $A_0$ is the amplitude and $\phi_0$ is the phase of the reference
field; both are assumed to be constant. The total electric field at each of the outputs of an ideal 90° optical hybrid is expressed in equations (3-7). For \( E_3 \) and \( E_4 \), the ‘\( i \)’ indicates the 90° phase shift between the reference and signal.

\[
E_1 = \frac{1}{2} \left( E_{\text{sig}} + E_{\text{ref}} \right) \\
E_2 = \frac{1}{2} \left( E_{\text{sig}} - E_{\text{ref}} \right) \\
E_3 = \frac{1}{2} \left( E_{\text{sig}} + jE_{\text{ref}} \right) \\
E_4 = \frac{1}{2} \left( E_{\text{sig}} - jE_{\text{ref}} \right) \tag{3-7}
\]

Expressions for the differential photocurrents \( I_1(t) \) and \( I_2(t) \) are shown in equations (3-8) and (3-9), and simplified by substituting the function forms for \( E_{\text{sig}}(t) \) and \( E_{\text{ref}}(t) \) from equation (3-6). The photocurrents are in quadrature, i.e. they have a phase difference of 90°. Notice that both photocurrents depend on both \( A_1(t) \) and \( \phi_1(t) \).

\[
I_1(t) \propto \left| E_1 \right|^2 - \left| E_2 \right|^2
= \frac{1}{4} \left| E_{\text{sig}} + E_{\text{ref}} \right|^2 \left| E_{\text{sig}} - E_{\text{ref}} \right|^2
= A_1(t) A_0 \cos(\phi(t) - \phi_0) \tag{3-8}
\]

\[
I_2(t) \propto \left| E_3 \right|^2 - \left| E_4 \right|^2
= \frac{1}{4} \left| E_{\text{sig}} + jE_{\text{ref}} \right|^2 \left| E_{\text{sig}} - jE_{\text{ref}} \right|^2
= A_1(t) A_0 \sin(\phi(t) - \phi_0) \tag{3-9}
\]
\( A_1(t) \) and \( \phi(t) \) can be calculated from \( I_1(t) \) and \( I_2(t) \) as shown:

\[
\frac{I_2}{I_1} = \tan(\phi(t) - \phi_0) \tag{3.1}
\]

\[
\phi(t) = \tan^{-1} \left( \frac{I_2}{I_1} \right) + \phi_0
\]

\[
I_1^2 + I_2^2 = A_1^2(t)A_0^2 \tag{3.2}
\]

\[
A_1(t) \propto \sqrt{\frac{I_1^2 + I_2^2}{A_0^2}}
\]

The equations show that the PM and AM are independently measured without any intrinsic assumptions about the shapes of the modulation. The measurement bandwidth is limited only by the photodiodes and oscilloscope.

### 3.3 Experimental Setup

The VCSEL characterized in the experiment is a commercially available device (Raycan Ltd.) with InAlGaAs quantum wells on a InP substrate [82]. The free-running VCSEL emits 0.6 mW at 1550nm for a bias current of 5.4 mA (\( \approx 2.7 \) \( I_{\text{threshold}} \)). It is fiber-pigtailed and has a 3 dB modulation bandwidth of \( \sim 3.5 \) GHz. Figure 3-4 shows the experimental setup used to measure the modulation dynamics of an injection-locked VCSEL. The master laser is a narrow linewidth (< 1 kHz) fiber CW laser at 1550 nm. It is amplified using an EDFA and the majority of the power is diverted to the reference input of the 90° optical hybrid. This is to provide coherent gain at photodetection to compensate for the low output power of the VCSEL. An optical band pass filter with a 1nm wide 3 dB bandwidth filters out ASE. A variable optical attenuator is used to set the injected power to \( \sim 0.7 \) \( \mu \)W, resulting in \( R_{\text{inj}} \approx -30 \) dB. Sinusoidal modulation of the VCSEL’s current causes AM and PM of the emitted light, which enters the 90° optical hybrid at the signal input. The same sinusoidal input is
used as a trigger for the real-time oscilloscope (with 8 GHz analog bandwidth). Triggering permits averaging and improves the SNR of the recorded photocurrents.

Figure 3-4: Experimental setup for measurement of the optical phase and amplitude modulation of an injection-locked VCSEL subject to current modulation. The modulated light from the VCSEL is interfered with the unmodulated CW laser using a 90° optical hybrid. The diagnostics for the injection-locked VCSEL consists of a polarization analyzer and a high-resolution optical spectrum analyzer (Agilent 83453B), used to ensure co-polarized injection into the VCSEL, and to monitor the stability of injection-locking, respectively. EDFA – Erbium Doped Fiber Amplifier, OBPF – Optical Band Pass Filter, PC – Polarization Controller, OC – Optical Circulator, VOA – Variable Optical Attenuator

The 90° optical hybrid (Celight Inc.) is implemented on a LiNbO₃ substrate, with a total of six voltages to control the coupling ratios and phase shifts. Voltage control of the coupling ratios allows us to minimize variation in optical powers among the outputs of the hybrid, which in turn improves the accuracy of the measurements. A disadvantage of the hybrid is that the output powers and phase drift with time. This makes it necessary to periodically adjust the control voltages for the device, on the order of every half hour. The combination of drift and residual imperfections in the hybrid contribute to measurement errors. The smallest observable modulation is \( m = 2.5 \% \) for AM, and \( \beta = 0.015\pi \) rad for the PM, which is the noise floor.

The variation of the depth of the AM and PM are explored as a function of RF frequency, RF power and for two bias currents. This includes thermal and electronic modulation regimes, as well as small and large signal modulation levels.
3.4 Experimental Results

The linewidth enhancement factor, $\alpha$, has a strong influence on the modulation properties of a semiconductor laser. For the VCSEL used in the experiment, $\alpha$ was measured using the injection-locking technique proposed in [83]. The stable locking range is experimentally mapped as a function of the injection ratio, as shown in Figure 3-5. We observe that the stable locking range (in GHz) becomes smaller as the injection ratio decreases. For small injection ratios, $R_{inj} < -27\text{dB}$, the stable locking range is almost symmetric about zero, due to the absence of dynamic instability [57]. From this region, $\alpha = 0.7\pm0.4$.

![Figure 3-5: Experimentally measured stable injection-locking range (shaded area) as a function of injection ratio. The inset shows a close up of the locking range for small injection ratios, where the stable locking range is almost symmetric. $I_{DC} = 5.4$ mA, with co-polarized optical injection.](image)

A smaller injection ratio is desirable for operating the injection-locked VCSEL as a phase modulator. First, the stable locking range occupies a larger fraction of the full locking range at smaller injection powers. For low enough injection ratios, the full-locking range is stable. Since the maximum depth of PM is defined over the full locking range, the larger the fraction of the stable locking range, the larger the maximum depth of PM achievable in practice. Second, a smaller locking range requires less RF power to modulate $\Delta\omega_{inj}$ over the full range. Lower RF power results in smaller AM. Thus to
achieve a given to depth of PM while minimizing the AM, it is desirable to use a smaller injection ratio. In the experiment, the injection ratio is $R_{inj} \approx -30$ dB, where the full, stable locking range is 0.93 GHz.

### 3.4.1 AM and PM vs. Modulation Frequency – Thermal Regime

For frequencies below 100 MHz, there is a strong thermal contribution to the modulation response of the VCSEL. An increase in the carrier density within the laser increases heat generation, which increases temperature. This changes the refractive index and length of the laser cavity. Both effects lead to PM in the injection-locked VCSEL.

![Figure 3-6: (a),(b) - The variation of AM and PM with RF frequency, between 0.98 MHz and 20 MHz, for RF power -29 dBm.](image)

Figure 3-6 shows that the observed AM and PM vs. modulation frequency, for a fixed RF power of -29 dBm. The RF power was chosen to allow for a maximum depth of PM within the stable locking range at $\sim$1 MHz. The measured AM is the noise floor of our experimental setup, corresponding to a minimum measurable peak-to-peak modulation of 5% ($m = 2.5\%$). The true depth of AM is expected to be lower. The PM drops dramatically with increasing frequency as the efficacy of
thermal modulation drops at higher frequencies. For frequencies less than 0.98 MHz, the VCSEL falls out of injection-lock for the RF power used, due to the increasing strength of the thermal modulation mechanism at lower frequencies. The maximum peak-to-peak depth of modulation is $0.73\pi$ rad at 0.98 MHz ($\beta = 0.36\pi$ rad), which corresponds to modulation over the full, stable locking-range. Using equation (3-2), we can estimate a value of $\alpha = 1.1$, which is consistent with the value deduced from the stable locking range in Figure 3-5.

Figure 3-7: (a),(b) AM and PM as function of time with sinusoidal modulation of the VCSEL's current at 0.98 MHz, -29 dBm (c) A close-up of the PM is shown, along with a sine wave, which is plotted to highlight the difference. Figure 3-7 shows the observed AM and PM as a function of time when the VCSEL is modulated at 0.98 MHz (left most data point in Figure 3-6). Figure 3-7(a) shows that the AM is very small ($m < 2.5\%$, noise-floor limited). Figure 3-7(b) shows a large depth of PM ($\beta = 0.36\pi$ rad). A close-up of the PM in Figure 3-7(c) shows that the shape of the PM deviates from a perfect sinusoid near the top of the excursion, where it is more triangular. This matches the simulated shape shown in Figure 3-2(b) very well. We conclude that in the thermal regime of modulation, where a very small amount
of current modulation produces a significant amount of PM, the shape of the PM is arcsine as predicted by equation (3-1).

3.4.2 AM and PM vs. Modulation Frequency – Electronic Regime

Figure 3-8(a),(b) show the variation of AM and PM vs. modulation frequency for frequencies from 14 MHz to 4 GHz, for RF power of -13 dBm. The power is chosen to produce the largest possible modulation while maintaining stability of the injection locking over the whole range of frequencies.

In Figure 3-8 (a), the AM remains fairly constant at around $m = 12\%$ from 14 MHz to 1.4 GHz. For frequencies below 14 MHz, the VCSEL is not stably injection-locked at the RF power used. Beyond 1.5 GHz, the AM drops due to a combination of intrinsic VCSEL modulation bandwidth limitation and electrical parasitic losses. It has not been possible to separate their relative contributions, but based on the highly correlated variations in the AM and PM for $f \geq 1.4$ GHz, it is hypothesized that the dominant contributor is parasitic loss, which reduces the RF power reaching the VCSEL. The
bulk of the parasitic loss comes from the connection of the TO-package to the SMA cable and from the TO-56 package itself [84].

Figure 3-8 (b), the PM drops with increasing frequency until 100 MHz because of the diminishing efficacy of thermal modulation. Since the thermal response time is typically in microseconds, the temperature cannot change as rapidly as the current. For frequencies above 100 MHz, PM caused by thermal contribution becomes negligible, and modulation of the carrier density within the VCSEL is the dominant mechanism. The depth of PM rises with frequency between 100 MHz and 1.4 GHz. For frequencies beyond 1.4 GHz, the PM falls again due to parasitic electrical losses. From the depth of PM in the thermal and electronic modulation regimes, it is clear that the thermal modulation regime is stronger in producing PM than the electronic regime. Note that the RF power is constant for all the frequencies. The AM, in contrast has no appreciable difference in the two regimes, as it depends only on the carrier density.

It should be noted that change in phase with temperature, $\frac{d\varphi}{dT}$, is opposite in sign to the change in phase with carrier density, $\frac{d\varphi}{dN}$. The two mechanisms counteract each other, leading to a minimum value for $\beta$ at around 100 MHz. This is supported by Figure 3-8(c), which shows the electrical phase difference between the PM and AM signals. The phase is determined from sinusoidal curve fits to the AM and PM time domain data. The phase is negative in the thermal regime, where the PM lags behind the AM. The lag increases with frequency because the PM is caused by temperature change, which cannot keep up with the increasing frequency. The phase goes through zero between 100 and 200 MHz because the thermal and electronic modulations have opposing effects on the PM. The phase is positive in the electronic modulation regime for frequencies above 200 MHz. The mechanism which causes the PM to lead the AM in this frequency
range is currently not understood. The phase approaches zero with increasing frequency which means that the PM and AM are in phase.

### 3.4.3 Depth of AM and PM vs. RF power

Figure 3-9 shows the variation of the depth of AM and PM with RF power, for various frequencies from 20 kHz to 1 GHz. For each frequency, the upper limit of the RF power is set by stability: the injection-locking is not stable for higher powers: the injection-locking is not stable for higher powers.

![Figure 3-9: The variation of the depth of AM and PM with RF power for various modulation frequencies](image)

In Figure 3-9(a), for \( P_{RF} < -30 \) dBm, the AM measurement is noise floor limited, and the true AM is expected to drop with decreasing RF power. For \( P_{RF} > -30 \) dBm, the depth of AM rises with RF power and is independent of modulation frequency, as shown by the good overlap of the data for various frequencies. Since the depth of AM is largely determined by the change in gain caused by current modulation, it is independent of frequency in the range considered.
In contrast, the depth of PM is strongly dependent on modulation frequency, as seen in Figure 3-9(b). In general, the higher the modulation frequency, the weaker the PM and the larger the RF power required to achieve a specific depth of modulation. For \( f < 10 \text{ MHz} \), significant PM is observed for very low RF power due to thermal modulation of the refractive index. The I.L. VCSEL acts as a phase modulator in this regime with negligible AM. The RF power needed increases rapidly with increasing frequency as the efficiency of the thermal mechanism drops at higher frequencies. At 100 MHz, the \( \beta \) is a minimum, which is consistent with Figure 3-8(b). The required RF power to achieve a specified \( \beta \) is the largest because the thermal and electronic modulation mechanisms counteract each other. At \( f \geq 100 \text{ MHz} \), modulation is due to changes in the carrier density within the VCSEL, which is significantly weaker in producing PM than the thermal mechanism. Hence larger RF powers are required to produce a given depth of PM \( \beta \), which then causes a larger AM.

![Figure 3-10: AM and PM as function of time with sinusoidal modulation of the VCSEL's current at 1 GHz, \( P_{RF} = -5 \text{ dBm} \).](image)

The shapes of the AM and PM are examined for large depth of modulation in the electronic modulation regime, as shown in Figure 3-10. Clearly the shape of the PM is not arcsine for large \( \beta \) in the electronic modulation regime, in contrast to the thermal modulation regime in Figure 3-7(c). This is due to the large modulation of the current density; the arcsine shape of the PM in equation (3-1) is valid only under steady state conditions or small signal current modulation. We observe a
significant deviation from a pure sinusoidal shape in both the AM and PM on the rising part of the modulation. Further investigation is necessary to explain this feature. The shape of the PM may be examined by numerically solving the coupled, nonlinear rate equations under large signal modulation. This is, however, beyond the scope of the experiment.

Figure 3-11: (a) Maximum depth of PM, $\beta_{\text{max}}$, experimentally achieved at various frequencies. (b) RF power required to achieve the depths of modulation in (a).

Figure 3-11(a) shows the maximum observed depth of PM, $\beta_{\text{max}}$, for various frequencies and the RF power required for them. This is based on data in Figure 3-9. In the thermal regime of modulation ($f \leq 1$ MHz), $\beta_{\text{max}}$ is relatively constant around $0.35\pi$ rad, and is limited by the value of $\alpha$ as given in equation (3-2). Notice that the RF power needed decreases without saturation as frequency is reduced, indicating that thermal modulation strength increases further at lower frequencies. For $1 \text{ MHz} < f < \sim 100$ MHz, $\beta_{\text{max}}$ drops with frequency because the efficacy of thermal modulation decreases, that of electronic modulation increases, and the two counteract each other. This is accompanied by a corresponding increase in RF power. For $f = 1, 1.5$ GHz, $\beta_{\text{max}}$ is close to $0.45\pi$ rad. While $\beta_{\text{max}}$ is larger than in the thermal modulation regime, the required RF power is significantly higher. The phase range for small current modulation in equation (3-2) does not apply here as it is under large current modulation. The RF power saturates at -5 dBm for $f \geq 1$ GHz.
A note about modulation at high RF powers: the temperature of the VCSEL had to be successively reduced to maintain injection-lock at RF powers greater than -11 dBm. This is likely to be caused by parasitic device heating at high RF powers, which is compensated by temperature reduction.

3.4.4 Depth of AM and PM vs. Bias Current

The variation of the depth of AM and PM are examined for two different bias currents, both vs. frequency and RF power. For all the data shown so far, the VCSEL was operated in the middle of the linear part of the P-I curve, at \( I = 5.4 \) mA and \( T = 23 \) °C. With increasing current, the optical power begins to saturate, reaches a maximum and decreases. This is attributed to the combination of higher leakage currents and higher non-radiative recombination rates at higher currents [85]. In this section, the VCSEL is biased at the rollover point in the P-I curve where the output power of the VCSEL is a maximum. Since the optical power of the free-running VCSEL changes very little as a function of current at the rollover point, the amount of AM is expected to be lower for small signal modulation of an injection-locked VCSEL. Hence the operation at the rollover point is of interest as a means of reducing AM.

![Graph showing optical power vs. current](image)

**Figure 3-12**: Variation in optical power of the free-running VCSEL with current. Its modulation characteristics are measured at \( I = 5.4 \) mA and at 10 mA.

From Figure 3-12, rollover occurs at 10 mA, when the VCSEL is maintained at a temperature of 2° C. The temperature must be decreased as the current is increased to ensure that the VCSEL’s
emission frequency coincides with the master laser to within ±1 GHz. This is necessary because the frequency of the master laser is not tunable.

![Figure 3-13: (a),(b) The depth of AM and PM as a function of modulation frequency (c) The phase difference between the PM and AM. Data is shown for $I = 5.4$ mA (linear part of the P-I curve), and $I = 10$ mA (rollover point).](image)

The variation of AM and PM as a function of modulation frequency for two different bias currents of the VCSEL is shown in Figure 3-13. For both sets of the data, the RF power applied to the VCSEL is the same, at -13 dBm. Compared to operation in the linear regime of P-I curve, the AM is reduced by about 50-60% by biasing the VCSEL at the rollover point (Figure 3-13(a)). A smaller fraction of the injected current leads to radiative recombination, which reduces the depth of AM.

For frequencies below 200 MHz, the depth of PM is larger for $I = 10$ mA. This indicates that the thermal modulation mechanism is stronger and extends to higher frequencies. This is likely due to a higher junction temperature of the VCSEL for a larger injected current, even though the overall VCSEL temperature is lower. It explains why injection locking is not stable for frequencies below 57 MHz at $I = 10$ mA, while the lower limit is 16 MHz at $I = 5.4$ mA. For frequencies above 200 MHz, the depth of PM is smaller for $I = 10$ mA. At 1 GHz, the PM is reduced by 40% from $I = 5.4$ mA.
This is likely due to a smaller fraction of the injected current contributing to radiative recombination. The depth of PM is constant for frequencies between 200MHz and 1.4 GHz at I = 10 mA, in contrast to data at I = 5.4 mA. Parasitic losses dominate at frequencies beyond ~ 2 GHz.

Unlike the trend for I = 5.4 mA, the phase difference between the measured PM and AM is negative for the entire frequency range for I = 10 mA (Figure 3-13(c)). This is attributed to the weaker electronic modulation mechanism in producing PM at I = 10 mA, which is supported by Figure 3-13(b). PM lags behind the AM over the whole range. For both bias currents, the phase approaches zero as modulation frequency increases, albeit from opposite sides, indicating that the AM and PM are almost in phase at high modulation frequencies.

The variation of the depth of AM and PM of the VCSEL as a function of RF power for two different bias currents and frequencies is shown in Figure 3-13. For modulation at 1 MHz, the AM is noise floor limited for both bias currents. The PM is stronger for I = 10 mA at 1 MHz, since a smaller RF power is required to achieve the same depth of PM. For modulation at 1 GHz, the trend is reversed – the depth of both the AM and PM are reduced by about 50 % and 40 % respectively, compared to I = 5.4 mA. For a chosen depth of PM, biasing at I = 10 mA results in less than 5%
reduction in AM than I = 5.4 mA. Hence biasing the VCSEL at the rollover current is not an effective strategy for reducing the relative amount of AM to PM from injection-locked VCSELs.

3.5 Conclusions

The optical phase and amplitude modulation properties of an injection-locked VCSEL under current modulation have been experimentally explored. This was done using a coherent optical demodulation technique that allows us to independently measure the AM and PM of an optical carrier as a function of time. The technique is versatile enough to measure the VCSEL’s modulation response over a very wide range, from kHz to potentially 10s of GHz, and for small and large depths of modulation. The AM and PM were measured as a function of frequency. In the low frequency, thermal regime, for frequencies less than 1 MHz, the injection-locked VCSEL produces predominantly PM. A large depth of PM (β = 0.34π rad) is accompanied by minimal AM (m<2.5%) at ~ 1MHz, and corresponds to modulation over the full locking range. A good match for the arcsine shape of the PM is found, as predicted by theory. In the high frequency modulation regime, electronic modulation regime for frequencies greater than 100 MHz, the injection-locked VCSEL produces significant amounts of both AM and PM. The electronic modulation is weaker than the thermal modulation in producing PM, necessitating a much larger RF power to achieve the same depth of PM. A maximum β = 0.48π rad and m = 35% was observed at 1.5 GHz. Increasing the VCSEL’s bias current to the roll-over point resulted in only a small decrease of the AM to PM ratio at GHz modulation rates. The characterization of the AM & PM properties of injection-locked VCSELs presented in the paper will be important for its applications in telecommunications, coherent beam combining and rapid-update pulse shaping.
CHAPTER 4: RAPID-UPDATE PULSE SHAPING USING A FREE-SPACE VCSEL ARRAY

We demonstrated in 1.5.2.2 that an array of injection-locked VCSELs could update pulse-shapes at nanosecond timescales. Some of the main limitations in that demonstration are addressed in this chapter. The improvements are:

1. Generation of controlled, repeatable waveforms, by eliminating phase drift between channels. In the previous work, the random phase drift introduced by the optical fiber patch cords made it impossible to predict the waveforms generated.

2. Scaling up of the optical bandwidth of the waveforms, by increasing the number of channels from 4 to 11. This enables generation of more complex waveforms.

3. Quantitative measurements of the phase and amplitude of the generated waveforms, for modulation frequencies up to a few MHz.

As a result of these improvements, the generated waveforms conclusively demonstrate the capabilities of VCSEL arrays in generating different pulse shapes and rapidly updating them at GHz rates. The chapter is organized as follows:

1. Overall system design considerations are presented first.

2. The packing and performance characteristics of the VCSEL array at the heart of experiment is described in greater detail in chapter 4.2.

3. The design, assembly and characteristics of the free-space wavelength demux/mux are presented in chapter 4.2.2.

4. The conceptual and experimental details of the various optical waveform measurement techniques used are described in chapter 0.
5. Different optical waveforms that are experimentally generated are presented. Three different update rates of the waveforms regimes for the are explored:
   a. Static
   b. Update rates of up to 4 MHz
   c. Update rates up to 3 GHz

4.1 Optical Waveform Generation System Design Considerations

To generate arbitrary optical waveforms, one needs to completely fill the optical bandwidth between the comblines of the input optical frequency comb source, as indicated by Figure 1-3(b), which comes from [9]. This is achieved by suitable modulation of each input optical combline. This implies that the electrical modulation bandwidth of each modulator must be comparable to the frequency spacing of the optical comblines, i.e. the repetition frequency of the input optical source. Another requirement is the ability to demultiplex the individual comblines, i.e. the demux must be able to spatially resolve the individual frequency comblines to send them to their respective modulators.

There is a tradeoff between the optical frequency-spacing of the channels of the demux and the modulation bandwidth of the electrical each channel. A demux with larger frequency-spacing is easier to fabricate/assemble, but it requires each channel to be modulated with larger bandwidth electrical signals to fill the full optical bandwidth, which is more demanding on the electronics. Conversely, reducing the optical channel spacing of the demux channels increases the difficulty of meeting the target, while reducing the demands on the electronics. A suitable tradeoff, between the optical frequency spacing of the demux and the bandwidth of the electronics, occurs at around 10 GHz with the current technology.
For the experiment, a 12.5 GHz combline spacing is chosen, based on the following reasons:

1. The ease of designing & assembling a wavelength demux

   A 12.5 GHz wavelength demux is implemented using bulk optics, as detailed in section 4.2.1.1. Reducing the channel separation to ~ 6 GHz becomes challenging because one needs a lens with ~ 2m focal length, which makes the size of the setup impractical.

2. Variation of wavelength across available VCSEL arrays

   Since each VCSEL has a narrow injection-locking range measured in a few GHz, the emission wavelength of the VCSEL has to be a very close to the input combline. The emission wavelengths of the individual VCSELs intrinsically vary due to variations in the epitaxial layer structure across the wafer during the growth process. A suitable VCSEL array has to be chosen such that the intrinsic wavelength variation across the array approximately matches the demux channel spacing. As we shall see later in Table 4-2, only a few of the available VCSEL arrays have a linear variation of optical frequency across the array of less than 20 GHz per channel. Fine tuning of the bias current to each VCSEL in the array helps us achieve the desired optical frequency variation across the array.

3. Modulation bandwidth of each VCSEL

   The modulation bandwidth of the VCSEL is determined by the VCSEL structure and the injection-locking parameters, especially the injection ratio. The currently available VCSEL arrays at 1550 nm have a free-running modulation bandwidths of around 3 GHz (single side), which results in a double-sided bandwidth of 6 GHz in the optical domain. While the modulation bandwidth can be significantly increased by strong injection [86], a small injection ratio is used in the experiment to, maximize the stable locking range and the depth of optical phase modulation achieved, and to minimize the attendant amplitude modulation. It would be desirable to have a demux with 6 GHz channel spacing so that each modulated VCSEL can fill the bandwidth
between neighboring channels. It is, however, too challenging to realize a demux with 6 GHz channel spacing. Hence with the chosen 12.5 GHz channel spacing, we compromise on the ability to use the full optical bandwidth between channels to generate optical waveforms with the current VCSEL arrays.

4. Availability of optical frequency comb sources

A 12.5 GHz comb source with sufficient bandwidth is generated by modulation of a CW laser. It is straightforward to implement this using commercially available components, as detailed in chapter 4.3.

4.2 Experimental Setup

Figure 4-1: Schematic of the overall experimental setup

Figure 4-1 shows the main components of the experiment setup. They are:

1. VCSEL waveform generator, which consists of:
   a. A VCSEL array
   b. A High-resolution free-space wavelength multiplexer

2. Two optical frequency comb sources. One is used for waveform generation, and the other is used as a reference for waveform measurement.
   a. Optical waveform measurement and diagnostics. Two methods are used, multi-heterodyne detection, and photodetection. More details are given in chapter 4.2.3.
The setup and characterization of each is described in the following sections.

### 4.2.1 The Modulator – VCSEL Array

The VCSEL array is purchased from RayCan Ltd. in South Korea, which is the only commercial supplier of VCSEL arrays at 1550 nm wavelength currently that we are aware of. A schematic of the epitaxial VCSEL structure is shown in Figure 4-2.

![Figure 4-2: Structure of the VCSEL, from [82]. © 2006, IEEE.](image)

Currently, the largest VCSEL array available at 1550nm is a 1 x 12 linear array. The VCSEL array at 1550 nm is chosen over larger VCSEL arrays available at other wavelengths (8x8 array at 810 nm by Princeton Optronics Inc). This is because of the existing lab instrumentation at 1550 nm required to generate the optical frequency combs and characterize the optical waveforms. A schematic of the VCSEL array is shown in Figure 4-3. The VCSEL array has individual anodes and a common cathode for the individual lasers. The array is bonded on a submount, which is bonded to a TEC element for actively temperature stabilization.
Figure 4-3: Schematic of a 12 channel VCSEL array on a submount

The VCSEL array is mounted with the electrodes facing the submount, for emission through the substrate. This presents a challenge for the subsequent alignment of the microlens because there are no alignment marks on the substrate.

The main characteristics of the VCSEL array are shown in Table 4-1.

Table 4-1: Characteristics of the Raycan 1x12 VCSEL array

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Channels</td>
<td>12, linear</td>
</tr>
<tr>
<td>Channel pitch</td>
<td>250 μm per channel</td>
</tr>
<tr>
<td>Center wavelength per array</td>
<td>~ 1539 nm</td>
</tr>
<tr>
<td>Optical frequency variation per channel across array, at equal current to each VCSEL</td>
<td>16 – 45 GHz (See Table 4-2)</td>
</tr>
<tr>
<td>Output power per VCSEL (at I = 6 mA)</td>
<td>~ 1 mW</td>
</tr>
<tr>
<td>1/e² full angle of far field beam per emitter</td>
<td>14°</td>
</tr>
<tr>
<td>Electrical modulation bandwidth (3 dB)</td>
<td>~2.3 GHz</td>
</tr>
</tbody>
</table>

The variation of the optical frequency and power across the array for various samples are shown in Table 4-2. All the data was taken at current I = 6 mA to each element and T = 23° C. We desire the optical frequency variation across the array to be 12.5 GHz per channel, based on the design of the pulse shaper and the repetition rate of the optical frequency comb source. We also desire the optical power variation across the array to be small. Arrays M4 and M10 come closest to meeting the targets for frequency variation per element and minimal variation in optical power across the array. Fine tuning can be achieved by controlling the bias current to each VCSEL, which will change both the optical frequency and optical power. Array M4 is finally chosen for the experiment, because all the
channels are co-polarized when biased at the currents required to ensure a 12.5 GHz/Ch frequency variation across the array, as shall be seen in chapter 4.2.1.2.1.

Table 4-2: Variation of optical frequency and power across each VCSEL array sample

<table>
<thead>
<tr>
<th>Array No.</th>
<th>Optical Frequency variation, channel 1 to 12 (GHz per channel)</th>
<th>Mean wavelength (nm)</th>
<th>Mean optical power (mW)</th>
<th>Standard deviation of optical power (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M3</td>
<td>-45</td>
<td>1538.38</td>
<td>0.90</td>
<td>0.038</td>
</tr>
<tr>
<td>M4</td>
<td>-17</td>
<td>1540.73</td>
<td>1.07</td>
<td>0.025</td>
</tr>
<tr>
<td>M8</td>
<td>-18</td>
<td>1539.95</td>
<td>0.99</td>
<td>0.051</td>
</tr>
<tr>
<td>M9</td>
<td>-41</td>
<td>1537.68</td>
<td>0.99</td>
<td>0.063</td>
</tr>
<tr>
<td>M10</td>
<td>-16</td>
<td>1540.23</td>
<td>1.01</td>
<td>0.022</td>
</tr>
</tbody>
</table>

The main steps in packaging the VCSEL array, for use in the experiment, are:

1. Electrical setup
   1.1. Design of the high-bandwidth RF printed circuit board
   1.2. Wire bonding the VCSEL array on submount & TEC to the PCB
   1.3. Design and assembly of a 12 Channel high-resolution current source array

2. Characterization of the optical performance of the VCSELs, for each channel:
   2.1. Polarization of the emission
   2.2. Optical frequency vs. current (ν-I) data
      2.2.1. Linearization of the frequency variation across the array.
   2.3. Power vs. current (P-I) data
   2.4. Measurement of the beam waist diameter

3. Measurement of the modulation response of the VCSELs

Each part is described in detail below.
### 4.2.1.1 Electrical Setup

Setting up a VCSEL array to allow independent modulation of each channel at GHz frequencies was a significant challenge, and took considerable time and effort. The main steps are listed below.

#### 4.2.1.1.1 Design of a High-Bandwidth Interconnect Printed Circuit Board

Since the VCSELs need to be driven at frequencies of about 3 GHz, a Printed Circuit Board (PCB) that delivers the RF signals to the VCSELs is needed. Requirements for the PCB are:

1. Deliver high speed signals per channel, up to 5 GHz, for 12 Channels.
2. SMA connectors on outside.
4. Trace spacing of 250 microns at the laser array.
5. Trace lengths should be equal for each channel on the PCB, to minimize phase mismatch between channels. This makes the PCB design roughly semi-circular in shape.
6. Have a controlled impedance

   Ideally, the PCB should provide some impedance matching between RF signal source (typically 50 Ω) and the VCSEL array. But the VCSEL impedance is unknown. The differential resistance at 6 mA DC is ~90 Ω. So a decision was taken to make a PCB for the standard 50 Ω impedance.
Two designs were explored: Co-Planar Waveguide (CPW) array, and microstrip waveguide array.

Figure 4-4 and Table 4-3 detail the characteristics of the PCB board designs.

Figure 4-4: Cross-section of a single (a) coplanar waveguide (b) microstrip waveguide

Table 4-3: Theoretical characteristics of designs for PCB

<table>
<thead>
<tr>
<th>Design</th>
<th>Board</th>
<th>$\varepsilon_r$</th>
<th>$h$ (μm)</th>
<th>$W$ (μm)</th>
<th>$s$ (μm)</th>
<th>$G = 250 - W - 2s$ (μm)</th>
<th>$Z_0$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPW</td>
<td>Rogers 3010</td>
<td>11.2</td>
<td>1270</td>
<td>60</td>
<td>35</td>
<td>120</td>
<td>49.55</td>
</tr>
<tr>
<td>Microstrip</td>
<td>Rogers 3010</td>
<td>11.2</td>
<td>127</td>
<td>100</td>
<td>-</td>
<td>150</td>
<td>49.23</td>
</tr>
</tbody>
</table>

Figure 4-5: Schematics of the microstrip waveguide array PCB used for connecting to the VCSEL array. (a) and (b) show the overall layout of the board. (c) Shows a close-up of the region where the waveguides converge, to align with the VCSEL array wirebond pads. The bends in some the traces are deliberately included to ensure path length matching between channels.

The microstrip waveguide design was chosen because the dimensions were large enough for the fabrication to be affordable, costing ~$350 per PCB for fabrication. The CPW design cost about $650 per board because the dimensions are smaller, especially the gap, $s$, between waveguide and ground. The layout of the PCB to generate an electronic file suitable for fabrication, and the
fabrication of the boards, were done for contract by external vendors. Figure 4-5 shows the schematics of fabricated board.

Future improvements to the design of the board should include RF testing of fabricated boards to:

1. Measure impedance of the VCSELs and PCBs, and provide impedance matching between the VCSELs and the 50Ω impedance of the RF sources.

2. Measure cross talk between channels. It is expected that the CPW design will have lower crosstalk than the microstrip waveguides.

### 4.2.1.1.2 Wirebonding the VCSEL Array to the PCB

After the VCSEL array on the TEC was attached to the PCB using epoxy resin adhesive, wirebonding was done to connect the microstrip waveguides and ground to the VCSEL array. This was done using a K&S 4500 Series Digital Series Manual Wire Bonders, and 1 mil thick gold wire. This was a very challenging operation because of the height difference between the pad sites (~ 3 mm), the long length of the wirebonds, and the large number of wirebonds per device (12 Anode, 12 cathode). Figure 4-6 shows a VCSEL array after the wirebonding has been completed.

![Wirebonding the VCSEL Array to the PCB](image)

Figure 4-6: (a), (b) Photographs of the VCSEL array wirebonded to the PCB
4.2.1.1.3 12 Channel High-Resolution Current Source Array

An array of current sources with very high precision is required, with 12 independent channels. The required precision for each channel is high because the range of optical frequencies over which each VCSEL locks to the injected combline will be small in the experiment. Typical injection-locking range will be $\sim 1 \text{ GHz}$. Given an optical frequency variation of $\sim 70 \text{ GHz/mA}$ for the VCSELs used, the locking range corresponds to about $15 \mu\text{A}$ of current. To achieve at least 10 intervals within that, one needs a current source with a maximum $1.5 \mu\text{A}$ step size. This maximum current required per VCSEL will be $\sim 10 \text{ mA}$. Finding an affordable current source array that meets the technical requirements was challenging.

Companies like Newport Corp. and Thorlabs Inc. sell modular laser current sources that come close to satisfying the technical requirements, but are priced at $1000 per channel at the time of this writing. This is clearly unaffordable, so a creative solution was required. After some searching, it was discovered that precision Digital-to-Analog Converters (DACs) could be adapted for use as current sources. With extensive modifications to the software and by using the available evaluations boards, AD5755 DACs made by Analog Devices Inc. were adapted for this experiment. Some properties of the DACs include:

1. A cost of about $50 per channel for 12 channels, which is 5% of the cost of commercially available products.
2. 16 bit resolution in the value of the output current, resulting in a minimum step size of $0.3\mu\text{A}$.
3. Maximum current per channel of 20 mA
4. Computer controlled via USB.
5. All 12 channels are driven by a single 12 V battery, for low noise operation.
Figure 4-7: Picture of the high-precision DACs used as current sources for the VCSELs. (a) Three DAC evaluation boards, with four channels per board, driving a test array of LEDs. (b) Packaged DAC array, connected to battery power source and computer via an USB cable (not shown), and to 12 SMA cables, which connect to a PCB board with the VCSEL array. (c) The software interface created that permits simultaneous control of 12 channels of current with 0.3 μA precision.
4.2.1.2 Characterization of the Optical Performance of the VCSELs

Having setup the current sources and connected the VCSEL array to the PCB by wirebonding, the optical performance of the VCSELs is characterized in this section.

4.2.1.2.1 Polarization of the VCSEL

The VCSEL has a single longitudinal mode because the cavity length is on the order of the lasing wavelength. It is laterally confined to a single transverse mode by the air gap shown in Figure 4-2 within the VCSEL structure. Although the VCSELs made by Raycan are specified to have single linear polarization, the orthogonal polarization has finite suppression, typically more than 25 dB. It was observed the polarization could vary between the channels of a VCSEL, between two orthogonal modes. In addition, polarization of some channels switched with changes in bias current. For the experiment, we need each VCSEL in the array to be co-polarized at its designated bias current. Hence the need to verify the polarization state of each of the channels is the same.

This was achieved by monitoring the optical spectrum, where the suppressed peak of the orthogonal mode is observed. The emission from each VCSEL was collected, one at a time, by using a lensed fiber. A sample spectrum is shown in Figure 4-8.

![Sample optical spectrum showing the two modes, corresponding to the dominant and suppressed polarization modes of the VCSEL.](image)

Figure 4-8: Sample optical spectrum showing the two modes, corresponding to the dominant and suppressed polarization modes of the VCSEL.
The spectrum for each channel is measured one at a time by using a lensed fiber. Of particular interest is the frequency offset of the suppressed mode. It is desirable for all the channels to have the same sign of the frequency offset, which would indicate all the channels are co-polarized. However, as shown in Figure 4-9 for VCSEL array M10, channels 10 & 11 have a different sign of the side mode offset frequency. Independent measurements using a polarization analyzer indicated that those two channels have an orthogonal polarization to the other channels. This makes array M10 unsuitable for the experiment.

![Figure 4-9: Orthogonal polarization mode suppression and frequency offset across VCSEL array M10. It is not suitable for use in the experiment because Ch 10 and 11 are orthogonally polarized](image)

It was found that the entire array could be tuned for co-polarized emission by changing the temperature of the entire array, to about 32°C for array M10. The optical power per channel dropped significantly with the increased temperature. In comparison, array M4 showed co-polarized emission across the entire array at room temperature, where output power per channel was larger. Hence array M4 was selected for the experiment.

### 4.2.1.2.2 Optical Frequency vs. Current Data

The optical frequencies of VCSELs have to vary by exactly to 12.5 GHz per channel, to match the wavelength demux channel spacing. This is achieved by tuning the bias current to each channel. The VCSELs have to be tuned to within ±1 GHz of their assigned center frequency, for efficient coupling with the demux, which is a stringent target. It is achieved by characterizing the variation of
optical frequency vs. current for each channel. The experimental data is shown in Figure 4-10(a).
The intrinsic variation of optical frequency across the array M4 vs. channel number is shown in
Figure 4-10 (b) for a fixed current of 6 mA to each channel. Linear fits to the frequency variation are
also shown for two cases: where the slope of the fit unconstrained, and when the slope is fixed at -12.5 GHz/channel. Figure 4-10(c) shows the residual from the linear fit for both cases. Note that
the residual has a maximum excursion of about ±70 GHz for a fit with slope = -12.5 GHz/channel.

![Figure 4-10: (a) Variation of the optical frequency with current, across the VCSEL array M4. (b) The variation of the optical frequency with channel number, and two different linear fit to it, for 6 mA per channel (c) The residuals of the linear fit]

4.2.1.2.3 Linearization of the Frequency Variation Across the Array

Software was developed to calculate the bias current to each channel to minimize the residuals from
a linear fit, so that the optical frequency variation is exactly 12.5 GHz/channel. An iterative
approach was used: the λ vs. I data was used on the first iteration. The resulting optical frequency
variation across the array was measured, and dλ/dI was used to calculate the correction required to
the bias currents to meet the target. About 4 iterations were required to reduce the residuals to
within ± 0.15 GHz, because thermal crosstalk causes the frequency of neighboring channels to
change as the bias current to a single channel is varied. (fixed slope = 12.5 GHz/channel and fixed
offset frequency at channel 6 =194.481 THz), the experimental measured residual to the linear fit is
within ±0.15 GHz as shown in Figure 4-11. This is a vast improvement from the initial 70 GHz
maximum residual shown in Figure 4-10(c). The high degree of precision is enabled by the Agilent 83453B High Resolution Spectrometer. Note that array M4 had only 11 functioning channels. Channel 12 at the edge of the array was non-functioning.

![Graph showing residual of linear fit (GHz) vs. channel.]

Figure 4-11: Difference in between experimental frequency variation across VCSEL array and a target variation of 12.5GHz/ channel and offset frequency at channel 6 =194.481 THz.

### 4.2.1.2.4 Power vs. Current Data

The variation of the optical power output with current for each VCSEL in array M4 is shown in Figure 4-12(a). The current for one channel was varied while the rest were kept at the bias currents necessary to achieve a 12.5 GHz/channel slope. This is a more accurate power measurement compared to the case where only one VCSEL is turned on at a time. The heat generated, from neighboring channels when all VCSELs are on, changes the output power.

![Graph showing power vs. current for each VCSEL.]

Figure 4-12: The variation of optical power output with current, of each VCSEL in array M4. The variation of power vs. channel number, for the currents chosen to produce 12.5 GHz/channel optical frequency variation across the array.
Figure 4-12(b) shows the variation of currents across the array chosen to produce 12.5 GHz/channel optical frequency variation, and the resulting optical power variation. The currents vary between 5 and 7 mA across the array. The optical powers vary between 0.75 and 1.15 mW. With customized VCSEL fabrication, it is possible to achieve the desired optical frequency variation across the array for equal bias currents to the channels. This will reduce the power variation across the array.

4.2.1.2.5 VCSEL Beam Waist Diameter Data

To ensure a good coupling efficiency between the VCSEL array and demux, it is critical to have good spatial mode matching between the VCSEL demux beams. So it is necessary to characterize the beam profile of the VCSEL. This was done experimentally using the DataRay BeamMap profiler. The results of the measurement are summarized in Figure 4-13. VCSEL beam waist (1/e2) diameter varies 9.0 ± 0.2 μm in the horizontal direction, and 8.5 ± 0.2 μm in the vertical direction. It is not certain whether the asymmetry in the beam waist diameters is real or due to measurement error in the beam profiler software. Note that the VCSEL beam N.A. ≈ 0.12 is slightly larger than that of SMF 28 at 1550 nm, N.A.\textsubscript{SMF} = 0.097 using a 1/e² beam diameter definition.

![Figure 4-13: Variation of VCSEL beam waist diameter and NA across the array](image)

This concludes the optical characterization of the VCSEL array.
4.2.1.3 **Electrical Modulation Response of the VCSELs**

The frequency response of free-running VCSELs is measured using the setup shown in Figure 4-14(a). Figure 4-14(b) shows the normalized frequency response for each VCSEL in the array, measured from 100 MHz to 5 GHz. Although all VCSELs were biased with static currents, only one VCSEL was modulated at a time.

![Experimental setup](image)

![Normalized frequency response](image)

A large dip at 1.90 GHz is observed. It is hypothesized to be caused by parasitic loss due to the PCB board and wirebonds. The 3 dB modulation bandwidth is between 2.3 & 2.45 GHz for all channels. This agrees well with manufacturer specifications, which state $\tau_{\text{rise}} = \sim 90$ ps, $\tau_{\text{fall}} = \sim 120$ ps. This results in a minimum modulation period of $2(\tau_{\text{rise}} + \tau_{\text{fall}}) = 420$ ps, which corresponds to 2.38 GHz.

This concludes the characterization of the VCSEL array as a modulator array, both electrical and optical characteristics.
4.2.2 Setup for Wavelength Multiplexing

A wavelength demux/mux using diffraction gratings and lenses is implemented in the experiment. It is based on the well-known setup used for shaping ultrashort laser pulses [1]. A modified setup is shown in Figure 4-15. The grating diffracts different wavelengths at different angles, and the converging lens focuses them to different spatial locations in its focal plane. Since the input optical signal consists of discrete regularly-spaced frequencies (an optical frequency comb), the focal plane contains a linear array of distinct spots. The microlens and VCSEL arrays are placed in the focal plane such that adjacent comblines are incident on adjacent VCSELs. Each VCSEL is injection-locked to the incident combline, which means that the VCSEL’s emission is phase-locked to the incident frequency. Modulating the current to each VCSEL modulates its optical phase and amplitude. The modulated light from each channel retraces its path through the pulse-shaper and is multiplexed on to a single beam to form a shaped pulse.

![Diagram](image)

Figure 4-15: Wavelength demux using diffraction gratings and lenses. The light is coupled in and out of the VCSEL array using a microlens array

The use of a microlens array in a pulse-shaping setup was first demonstrated in [87]. The microlens array was made by Suss Micro-Optics, part no. 18-00092, a 1x48 lens array with a 250 μm pitch and a broadband AR coating covering the telecom C-band. A 1x16 section of the microlens array is
shown in Figure 4-16. The microlens array is required to match the beam size and divergence of the beam emitted by each VCSEL to that of the beam coming from the demux, after the focusing lens. This is essential for efficient coupling of light to the VCSEL array for injection locking. Since the optical input to the demux is a frequency comb, which consists of a discrete number of frequencies, the microlens allows the light for each discrete frequency to be focused onto the active region of the corresponding VCSEL. Beam matching is also required for efficient coupling of the VCSELs’ emissions back into fiber.

![Figure 4-16: Photograph of the microlens array used in the experiment, mounted on an aluminum holder.](image)

The design parameters for the demux shown in Figure 4-15 are given in Table 4-4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Chosen</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comb line spacing</td>
<td>GHz</td>
<td>12.5</td>
</tr>
<tr>
<td>Pixel pitch on VCSEL array</td>
<td>μm</td>
<td>250</td>
</tr>
<tr>
<td>Diffraction grating groove density</td>
<td>lines/mm</td>
<td>1100</td>
</tr>
<tr>
<td>Focal length of lens</td>
<td>mm</td>
<td>1160</td>
</tr>
<tr>
<td>Center wavelength</td>
<td>nm</td>
<td>1540</td>
</tr>
<tr>
<td>Input optical bandwidth</td>
<td>nm</td>
<td>1.6</td>
</tr>
<tr>
<td>Input beam diameter on grating (approx.)</td>
<td>mm</td>
<td>15</td>
</tr>
<tr>
<td><strong>Calculated</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency resolution of setup</td>
<td>GHz</td>
<td>6.4</td>
</tr>
<tr>
<td>Theta_i – incidence angle</td>
<td>Degrees</td>
<td>56.1</td>
</tr>
<tr>
<td>Theta_1 – diffraction angle</td>
<td>Degrees</td>
<td>59.7</td>
</tr>
<tr>
<td>Littrow angle</td>
<td>Degrees</td>
<td>57.9</td>
</tr>
<tr>
<td>Ratio of beam diameters, Horizontal/Vertical (Astigmatism)</td>
<td>-</td>
<td>0.905</td>
</tr>
<tr>
<td>Beam diameter at focal plane before microlens – Horizontal</td>
<td>μm</td>
<td>162</td>
</tr>
<tr>
<td>Beam diameter at focal plane before microlens – Vertical</td>
<td>μm</td>
<td>147</td>
</tr>
</tbody>
</table>
A few comments about the design of the demux, as shown in Table 4-4.

1. To achieve the very high dispersion of optical frequencies in the focal plane (12.5 GHz per 250 μm), it is necessary to use a diffraction grating with a high groove density and a lens with a long focal length. The focal length is 1.16 m, which results in a total free-space path length from the fiber collimator to the VCSEL array of about 4 m. The total path length, from collimator to VCSEL array and back, is 8 m. This is very long, and a lot of care was taken to ensure the mechanical stability of the optical mounts to maximize beam stability. The whole setup was also enclosed to reduce disturbance due to air currents.

The spatial dispersion parameter of pulse shapers reported in literature[1], $\alpha$, is calculated.

$$\alpha = \frac{\lambda^2 f}{2\pi c d \cos(\theta_D)} = \frac{3.18 \cdot 10^{-13}}{cm \ rad/s} = \frac{250 \mu m}{12.5 \ GHz}$$

(4-1)

c is the speed of light, d is the grating period, $\lambda$ is the wavelength, f is the lens focal length, and $\theta_D$ is the diffracted angle from the grating.

2. The incident and diffraction angles are selected to be close to the Littrow angle, by judicious choice of the diffraction grating groove density and a lens focal length, while still meeting the required dispersion of frequencies. This minimizes the asymmetry of the diffracted beam, and ensures good spatial mode matching between the VCSEL and demux beams, necessary for good coupling efficiency.

4.2.2.1 Precise Alignment Of Demux To Meet Linear Dispersion Requirements

The grating angle in Figure 4-15 has to be aligned very precisely to ensure that spacing of the spots in the focal plane matches the VCSEL array channel spacing. A simulation revealed the sensitivity of the focal spot spacing as a function of the grating angle. The equations used in the simulation are detailed below. Figure 4-17 shows the geometry of the setup used in the equations.
Grating Equation: $\sin(\theta_i) + \sin(\theta_m) = mG\lambda$

$$\theta_m = \arcsin(mG\lambda - \sin(\theta_i)) \quad (4-2)$$

The location of the focal spot in the transverse plane of the focusing lens is:

$$x = f_{lens} \cdot \tan(\phi) \quad (4-3)$$

$\phi$ is the angle defined w.r.t. the optical axis of the lens, which defines the location for $x=0$. The center wavelength of the diffracted light, at $\lambda_0 = 1542.25$ nm, will have a diffraction angle $\theta_{m,0}$ w.r.t. the grating.

$$\theta_{m,0} = \arcsin(mG\lambda_0 - \sin(\theta_i)) \quad (4-4)$$

$\lambda_0$ is chosen to propagate along the optical axis after diffraction, i.e. it will have $\phi_{m,0}=0$ w.r.t. the lens.

The other wavelengths in the input beam will propagate at various angles, $\phi$, to the optical axis after diffraction, given by:

$$\phi = \theta_m - \theta_{m,0} \quad (4-5)$$

Combining (4-2),(4-3)and (4-5):

$$x = f_{lens} \cdot \tan(\theta_m - \theta_{m,0})$$

$$= f_{lens} \cdot \tan(\arcsin(mG\lambda - \sin(\theta_i)) - \theta_{m,0})$$

$$= f_{lens} \cdot \tan\left(\arcsin\left(\frac{mGc}{f} - \sin(\theta_i)\right) - \theta_{m,0}\right) \quad (4-6)$$

Equation (4-6) shows the location of the focal spot as a function of the input frequency, $f$, which is clearly nonlinear. Over a small range of frequencies, the deviation from linearity will be small. In the
simulation, 12 comblines from an optical frequency comb source with 12.5 GHz spacing are used. The incidence angle of the beam onto the grating, $\theta_i$, is varied around 56.5°. The locations of the focal spots of the comblines are tracked. The center wavelength is chosen to be $\lambda_0 = 1542.25$ nm. The results of the simulation are shown in Figure 4-18.

![Simulation showing the sensitivity of the focal spot spacing to incidence angle on the diffraction grating. For (a),(c) and (e), the slope of the linear fit is a fit parameter. For (b),(d) and (f), the slope of the linear fit is fixed at 250 $\mu$m/channel.](image)

In column 1, a linear fit of the spot locations vs. channel number is performed, where the slope of linear fit is unconstrained. We observe that the offset of the linear fit remains largely unchanged (
Figure 4-18(a), while the slope decreases with $\theta_i$ (Figure 4-18(c)). The residual of the linear fit vs. channel number is shown in Figure 4-18(e). We observe that the residuals from the linear fits are almost identical for all $\theta_i$. It represents the minimum attainable error in the location of the focal spots. The maximum value of the residuals is about 1 $\mu$m which occurs at the edge of the array. In column 2, the slope of the linear fit is fixed at 250 $\mu$m/channel, which is the VCSEL channel spacing. We observe that the offset of the linear fit decreases with $\theta_i$ (Figure 4-18(b)), while the slope is fixed (Figure 4-18(d)). The residual of the linear fit vs. channel number is shown in Figure 4-18(f). The green zone shows a region of acceptable error, within $\pm \omega_0$, the VCSEL beam waist radius. For incidence angles between $56.405^\circ < \theta_i < 56.550^\circ$, the error for all 12 channels is constrained within the green zone. So the grating angle has a maximum range $\Delta \theta_i = 0.145^\circ$, to meet the focal spot spacing of 250 $\mu$m/channel and the error in the focal spot location of any one channel to be less than the VCSEL beam waist radius, for all 12 channels.

The focal spot spacing was monitored experimentally by using an ASE optical source as the input to the demux shown in Figure 4-19, and placing a mirror in the focal plane after the microlens array.

![Figure 4-19: Modified setup used for aligning the focal spot spacing to be 250$\mu$m per 12.5 GHz combline spacing.](image)

The reflected spectrum shows peaks corresponding to each channel of the microlens array, as seen in Figure 4-20 (a). The center optical frequency of the peaks were determined by Gaussian curve fit to each peak, and the residuals from a linear fit (with fixed slope of 12.5 GHz/channel) are found.
The curve fitting also provides the bandwidth of each channel. Software was developed to automate
the process of Gaussian curve fitting using the measured optical spectrum, and the extraction of
residuals from the linear fit. The software makes it possible to iteratively minimize the residuals to
the linear fit, by tuning the grating angle.

The comparison of the residuals before and after the grating angle tuning is shown in Figure 4-20(c),
and shows the improvement. The maximum residual is about 0.07 GHz, which corresponds to an
error in the location of the beam center of = 0.07 GHz/12.5 (GHz per 250 μm) = 1.4 μm. Again,
the high spatial precision is enabled by the high precision frequency measurement using the Agilent
83453B High Resolution Spectrometer. The alignment of the demux is now complete.

The mirror is removed and the VCSEL array M4 aligned within the pulse shaper, first using ASE,
and then the emission from the VCSEL array itself. The optical power in free space and the coupled
optical power in fiber are seen in Figure 4-21(a). The fiber coupled power and the coupling
efficiency are shown in Figure 4-21(b). The fiber-coupled power is measured at port 3 of the circulator. The minimum coupling efficiency is about -1.75 dB, which is excellent, considering the large size of the demux. It includes 0.7 dB of loss due to the circulator itself, so the free-space coupling loss from the VCSEL array to the fiber is about 1 dB. The excellent coupling efficiency is due to the careful alignment of the demux and VCSEL array, which resulted in good match between the demux and VCSEL beams. The roll angle between the VCSEL and microlens array is deliberately misaligned to flatten the power coupled into fiber, to within 1 dB for the different channels. Hence the coupling efficiency is lowered for smaller channel numbers.

Figure 4-21: (a) Optical power per channel coupled into fiber after the mux from the VCSEL array, in comparison to the power in free space. (b) Coupling efficiency vs. channel number across the VCSEL array.
Figure 4-22: (a) Photograph of the wavelength demux and VCSEL array setup. The VCSEL and microlens arrays are each mounted on an assembly that provides all 6 degrees of freedom for alignment. (b) A close-up of the VCSEL and microlens arrays as seen through the microscope. The gap between them is about 0.75 mm.

Figure 4-22 shows the complete assembled setup in the lab. The enclosure is necessary to block air currents given the long beam path. It covers the entire setup after the microscope is removed, after completion of alignment. The VCSEL array and demux are now aligned.
4.2.2.2 Alternative Demux Technologies

Although the bulk optics demux shown in Figure 4-15 and Figure 4-22 has the disadvantages of large size and high sensitivity to optical alignment, it is chosen because of ready availability of the components and the flexibility of design.

Alternatively, Arrayed Waveguide Gratings (AWGs) can be used to demultiplex the optical frequency comblines. Although AWGs with 10 GHz channel separation have been demonstrated [27], they are not commercially available. The VCSEL array would then be butt-coupled to the waveguides of the AWG, greatly reducing the size and complexity of the experiment.

A demux using bulk optics can be packaged into a much smaller unit (150 x 80 x 20 mm), as sold by the company Kylia. The typical product has optical fiber on the input and an array of optical fibers on the output, as shown in Figure 4-23, but removing the fiber pigtailed on the output would allow demultiplexed beams to be coupled to the VCSEL array in free-space. This product too would greatly reduce the size and complexity of the experiment. Due to the high cost of the product, it is not used in the experiment.

![Figure 4-23: A commercially available optical mux/demux module from Kylia, with fiber input and outputs, along with a schematic of its operation. Source: http://www.kylia.com](http://www.kylia.com)
4.2.3 Measurement Techniques for Characterizing Rapidly Changing Pulse Shapes

This section discusses the measurement techniques used to characterize the optical waveforms generated experimentally. The complexity of the optical waveforms generated in the experiment depends on two factors:

1. The depth of PM and AM achievable by the injection-locked VCSEL modulators
   
   As detailed in CHAPTER 3, the physics of optical injection locking restricts the depth of achievable PM to less than π rad. For arbitrary waveform generation, one needs a phase range of 0 to 2π rad. With the limited phase range available from injection-locking, one can generate a limited subset of waveforms. Furthermore, there is significant AM, especially when a large depth of PM is required at frequencies greater than 100 MHz.

2. The total optical bandwidth of the signal. This is determined by many parameters:
   
   2.1. The number of channels available for modulation
   
   The larger the number of channels, the larger the potential optical bandwidth of the waveform. However, the experimental complexity increases with the channel count. In the experiment, the number of channels available is limited by:
   
   2.1.1. The size of the VCSEL array, to 12 channels
   
   2.1.2. The number of independent RF sources available
   
   2.1.3. The bandwidth of the waveform measurement technique, especially for high modulation frequencies.

   2.2. The passband width of each channel of the demux.

   Since the VCSEL acts as a reflective modulator, the demux also performs as a mux. For the high-resolution pulse-shaping setup outlined in Figure 4-15 and Table 4-4, the passband for
each channel is measured to be around 3 GHz (3 dB width). It is the strictest restriction on the optical bandwidth of the modulated combines.

2.3. Modulation bandwidth of the modulators, in this case, VCSELs.

The currently available VCSEL arrays at 1550 nm have a free-running modulation bandwidths of around 3 GHz, which results in a double-sided bandwidth of 6 GHz in the optical domain.

2.4. The type of RF signal applied to each channel.

By using RF arbitrary waveform generators for each channel with output bandwidths equal to the channel spacing, the entire modulation bandwidth between the optical combines can be used. With the channel spacing chosen to be 12.5 GHz, it is challenging to generate multiple independent channels of arbitrary electronic waveforms each with 12.5 GHz bandwidth. In the experiment, the VCSELs will be driven with single frequency sinusoidal signals rather than arbitrary waveform generators due to the availability of RF sources.

Full field characterization of the generated optical waveform, where each channel is modulated at GHz rates, is challenging.

4.2.3.1 Modulation Regimes in the Experiment

In our experiment, the bandwidth of the optical waveforms generated is limited primarily by the narrow bandwidth demux/mux used to couple light to the VCSEL array. Several different regimes of update rates for pulse-shaping will be explored, and the measurement options for each regime are discussed. The details of each measurement technique are discussed in section 1.5. The details of the modulation characteristics of injection-locked VCSELs are presented in detail in CHAPTER 3:

1. Static pulse shaping
The phases of the different comblines are independently controlled (line-by-line control) and any changes will be manually done. It is achieved by controlling the bias currents (DC) to the VCSELs. The purpose is to generate different user specified pulse shapes using the setup. The update rate of the waveforms is not the focus of this part. Waveforms generated in this case may be measured using multi-heterodyne measurement, using a second well-characterized optical frequency comb source as a reference. More details are given in section 3.2.

2. Pulse shaping at ~ 1 MHz update rates

The currents to the VCSELs are modulated at rates of up to about 1 MHz. In this frequency regime, the VCSEL acts predominantly as a phase modulator, producing a peak-to peak depth of modulation of up to ~0.7π rad. The VCSELs can be driven by arbitrary RF waveforms to generate complex pulse shapes that are updated on a microsecond time scale.

The experimental constraint of available RF sources limits modulation to 4 channels, with each channel modulated at a single frequency. Since the RF modulation is periodic, the optical waveform will also be periodic, with a period equal to the highest common factor of the modulation frequencies.

Multi-heterodyne measurement technique could be used to measure the modulation of each comblines, provided the modulation is sufficiently slow.

The intensity profile of the output waveform can be captured by photodetection using a fast photodiode and a sampling scope with a bandwidth of 50 GHz. More details on this are provided in section 0.

3. Pulse shaping at > 100 MHz update rates

The currents to the VCSELs are modulated at frequencies above 100 MHz. In this frequency regime, the VCSEL generates significant optical AM and PM, producing a maximum peak-to peak depth of modulation of up to π rad (PM) and 30% (AM). The maximum update rate of
waveforms is constrained chiefly by the channel passband width of the wavelength mux of about 3.5 GHz (3 dB width) (see Figure 4-20).

Spectrally sliced, four quadrature, coherent detection is the only technique reported in the literature[39] for complete characterization of these waveforms, but without access to the required hardware, we will be unable to utilize it in our experiment.

A high-bandwidth photodiode and sampling oscilloscope are used to measure the intensity of the rapidly changing pulse shapes. This was possible under the following conditions:

1. The total bandwidth of the optical waveform is less than that of the photodiode and scope.
2. The modulation on each channel is periodic, so that the generated waveform is periodic. A real-time scope with 50 GHz bandwidth is required for non-periodic modulation of four channels, which we do not have access to.
4.2.3.2 **Multi-Heterodyne Measurement**

The multi-heterodyne technique measures the spectral phase of one optical frequency comb by interferometry, with reference to a second, well-characterized comb. The technique was previously used for optical arbitrary waveform measurement in [42]. In the experiment, the reference optical frequency comb has flat spectral phase and intensity, which greatly simplifies the processing of the RF waveform, because then the RF waveform is a scaled version of the optical signal being measured. This is shown in Figure 4-24, a schematic of the experimental setup. In general, however, the reference comb need not have flat intensity and phase, as discussed in chapter 1.5.2.2.

![Figure 4-24: A schematic of the experimental setup for multi-heterodyne measurement of optical phase](image)

Comb sources 1 and 2 are both generated by modulating a CW laser, and are discussed in greater detail in chapter 4.3. The CW frequency is shifted using an Acousto-Optic Modulator (AOM) by 200 MHz, which is the offset frequency between the two combs. By generating both combs from the same CW laser, the combs are phase coherent with each other. This is important for the spectral
phase retrieval. Independent RF sine wave generators (synthesizers) are used to set the repetition frequencies of the comb sources. All three RF sine waves sources shown in Figure 4-24 are time-base synchronized using a 10 MHz signal.

The retrieval of the optical spectral information from the RF signal is explained next, using an example optical spectra consisting of 4 comblines.

\[
\Delta = g_r - f_r
\]

\[
f_{\text{offset}} = g_0 - f_0 \text{ (due to AOM)}
\]

Table 4-5 shows the electric field description of the comblines for both optical combs and the resulting RF comb after photodetection, for the example shown in Figure 4-25.
Software to automate the entire process of data acquisition experimentally attained by using the... to ensure that there is no mixing between the different sets of RF tones generated. Under these conditions there is no mixing between the different sets of RF tones generated.

Significant effort was spend to develop software to automate the entire process of data acquisition and compute the optical spectral phase and the resultant optical pulse shape.

Table 4-5: Description of the field of the optical comblines and RF

<table>
<thead>
<tr>
<th>Line</th>
<th>Optical comb 1</th>
<th>Optical comb 2</th>
<th>Photodetected comb</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>$A_{-2}e^{i((f_{0}+2f_{r})t+\phi_{-2})}$</td>
<td>$B_{-2}e^{i((g_{0}+2g_{r})t+\theta_{-2})}$</td>
<td>$\propto A_{-2}B_{-2}e^{i((f_{\text{offset}}-2\Delta)t+(\theta_{-2}-\phi_{-2}))}$</td>
</tr>
<tr>
<td>-1</td>
<td>$A_{-1}e^{i((f_{0}-f_{r})t+\phi_{-1})}$</td>
<td>$B_{-1}e^{i((g_{0}-g_{r})t+\theta_{-1})}$</td>
<td>$\propto A_{-1}B_{-1}e^{i((f_{\text{offset}}-\Delta)t+(\theta_{-1}-\phi_{-1}))}$</td>
</tr>
<tr>
<td>0</td>
<td>$A_{0}e^{i(f_{0}t+\phi_{0})}$</td>
<td>$B_{0}e^{i(g_{0}t+\theta_{0})}$</td>
<td>$\propto A_{0}B_{0}e^{i((f_{\text{offset}})t+(\theta_{0}-\phi_{0}))}$</td>
</tr>
<tr>
<td>1</td>
<td>$A_{1}e^{i((f_{0}+f_{r})t+\phi_{1})}$</td>
<td>$B_{1}e^{i((g_{0}+g_{r})t+\theta_{1})}$</td>
<td>$\propto A_{1}B_{1}e^{i((f_{\text{offset}}+\Delta)t+(\theta_{1}-\phi_{1}))}$</td>
</tr>
</tbody>
</table>

If the reference comb has flat spectral phase and amplitude, i.e. $B_{n} = B_{0}$ and $\theta_{n} = \theta_{0}$ for all $n$, then the spectral amplitude of the photodetected comblines is equal to that of the signal optical comb, except for constant amplitude and phase factors. The spectral phase of the photodetected comblines is the negative of the signal optical comb phase. This is shown in Table 4-6. The flat spectral phase condition is experimentally attained by using the 2 combline filtering method shown in the next section. The flat spectral amplitude is attained by suitably attenuating the comblines.

Table 4-6: The phase and amplitude of the comblines, when optical comb 2 has flat spectral amplitude and phase

<table>
<thead>
<tr>
<th>Optical Comb 1</th>
<th>Optical Comb 2</th>
<th>RF comb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line</td>
<td>Amplitude</td>
<td>Phase</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>-2</td>
<td>$A_{-2}$</td>
<td>$\phi_{-2}$</td>
</tr>
<tr>
<td>-1</td>
<td>$A_{-1}$</td>
<td>$\phi_{-1}$</td>
</tr>
<tr>
<td>0</td>
<td>$A_{0}$</td>
<td>$\phi_{0}$</td>
</tr>
<tr>
<td>1</td>
<td>$A_{1}$</td>
<td>$\phi_{1}$</td>
</tr>
</tbody>
</table>

There are constraints on the difference in the repetition frequencies, $\Delta$, the offset frequency and the number of optical comblines (optical bandwidth), to ensure that there is no mixing between the different sets of RF tones generated. Avoiding mixing is important, to be able to easily match each RF tone to the pair of optical comblines that generated it. In the experiment, the AOM frequency shift is $f_{\text{offset}} = 200$ MHz, $\Delta = 1$ MHz, and number of comblines is 12 (limited by VCSEL array size). Significant effort was spent to develop software to automate the entire process of data acquisition and compute the optical spectral phase and the resultant optical pulse shape.
4.2.3.3 Two Combline Filtering Measurement

To measure the spectral phase of the reference optical frequency comb, the two combline filtering method is used to measure the relative spectral phase between neighboring comblines. The method was published in [45, 47], and is described in greater detail in this section. The conjugate of the measured spectral phase is then imposed on the spectrum, to flatten the spectral phase. Both the phase measurement and compensation use the Finisar Waveshaper, a commercial product which acts as programmable optical filter, with minimum channel resolutions of around 10 GHz. A description of the Waveshaper is found in [48].

![Figure 4-26: Schematic of the experimental setup for measuring spectral phase using the two combline filtering method.](image)

Figure 4-26 shows the experimental setup used to measure the spectral phase of the reference comb, with four comblines shown as an example. The calculation of the spectral phase for the four channels is worked out next.

Table 4-7: Description of the four comblines

<table>
<thead>
<tr>
<th>Combine</th>
<th>Optical Freq</th>
<th>Amplitude</th>
<th>Phase</th>
<th>Electric Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f_1 )</td>
<td>( A_1 )</td>
<td>( \phi_1 )</td>
<td>( E_1 = A_1 \ast \exp(i(2\pi f_1 t + \phi_1)) )</td>
</tr>
<tr>
<td>2</td>
<td>( f_2 = f_1 + f_r )</td>
<td>( A_2 )</td>
<td>( \phi_2 )</td>
<td>( E_2 = A_2 \ast \exp(i(2\pi (f_1 + f_r) t + \phi_2)) )</td>
</tr>
<tr>
<td>3</td>
<td>( f_3 = f_1 + 2f_r )</td>
<td>( A_3 )</td>
<td>( \phi_3 )</td>
<td>( E_3 = A_3 \ast \exp(i(2\pi (f_1 + 2f_r) t + \phi_3)) )</td>
</tr>
<tr>
<td>4</td>
<td>( f_4 = f_1 + 3f_r )</td>
<td>( A_4 )</td>
<td>( \phi_4 )</td>
<td>( E_4 = A_4 \ast \exp(i(2\pi (f_1 + 3f_r) t + \phi_4)) )</td>
</tr>
</tbody>
</table>

Each pair of comblines is filtered and photodetected. The photodetector and oscilloscope bandwidth exceeds the combline spacing. The photodetected signal is a sinusoid, and its phase is measured. Since the time origin is of the observed sine wave is arbitrary on the scope, there is an additional phase term introduced, \( \Delta \), after photodetection. It is constant for each pair of filtered channels, because the setup is physically unchanged, and the RF frequency is also the same.
Table 4-8: Description of the photodetected signal for each pair of the filtered comblines

<table>
<thead>
<tr>
<th>Filtered lines</th>
<th>Total Electric field</th>
<th>Photodetected signal on scope</th>
<th>Phase of photodetected signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>( E_t = E_1 + E_2 = e^{i2\pi f_r t} (A_1 e^{i\phi_1} + A_2 e^{i(2\pi f_r t + \phi_2)}) )</td>
<td>( V_1(t) \propto</td>
<td>E_1 + E_2</td>
</tr>
<tr>
<td>2, 3</td>
<td>( E_t = E_2 + E_3 = e^{i(2\pi (f_{3}+f_r) t)} (A_2 e^{i\phi_2} + A_3 e^{i(2\pi f_r t + \phi_3)}) )</td>
<td>( V_2(t) \propto</td>
<td>E_2 + E_3</td>
</tr>
<tr>
<td>3, 4</td>
<td>( E_t = E_3 + E_4 = e^{i(2\pi (f_{4}+f_r) t)} (A_3 e^{i\phi_3} + A_4 e^{i(2\pi f_r t + \phi_4)}) )</td>
<td>( V_3(t) \propto</td>
<td>E_3 + E_4</td>
</tr>
</tbody>
</table>

The process is repeated for each adjacent pair of comblines, by changing the center frequency of the optical filter while keeping its bandwidth constant. A cumulative sum is calculated using the measured photodetected phase values of the different combline pairs, which yields a phase that can be assigned to a combline, as shown in Table 4-9. Note that the cumulative sum includes two terms: the desired phase difference between the selected channel and Channel 1, and an unknown phase term of \( \Delta \).

Table 4-9: Photodetected, and the retrieved spectral phase.

<table>
<thead>
<tr>
<th>Filtered lines</th>
<th>Phase of photodetected signal</th>
<th>Cumulative phase</th>
<th>Value</th>
<th>Retrieved phase assigned to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>( \theta_{1,2} = -(\phi_1 - \phi_2 + \Delta) )</td>
<td>( \theta_{1,2} )</td>
<td>( -(\phi_1 - \phi_2 + \Delta) )</td>
<td>( \phi_{2,R} = -(\phi_1 - \phi_2 + \Delta) )</td>
</tr>
<tr>
<td>2, 3</td>
<td>( \theta_{2,3} = -(\phi_2 - \phi_3 + \Delta) )</td>
<td>( \theta_{1,2} + \theta_{2,3} )</td>
<td>( -(\phi_1 - \phi_3 + 2\Delta) )</td>
<td>( \phi_{3,R} = -(\phi_1 - \phi_3 + 2\Delta) )</td>
</tr>
<tr>
<td>3, 4</td>
<td>( \theta_{3,4} = -(\phi_3 - \phi_4 + \Delta) )</td>
<td>( \theta_{1,2} + \theta_{2,3} + \theta_{3,4} )</td>
<td>( -(\phi_1 - \phi_4 + 3\Delta) )</td>
<td>( \phi_{4,R} = -(\phi_1 - \phi_4 + 3\Delta) )</td>
</tr>
</tbody>
</table>

The spectral phase of the comb source may be flattened by applying the conjugate of retrieved spectral phase to the comb. The resulting spectral phase may be observed in Table 4-10.

Table 4-10: Spectral phase after subtracting the retrieved phase from the original

<table>
<thead>
<tr>
<th>Combineline</th>
<th>Original Phase ( \phi_n )</th>
<th>Retrieved phase 1 ( \phi_{n,R1} )</th>
<th>After subtracting the retrieved phase 1 from original ( \phi_{n,1} = \phi_n - \phi_{n,R1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \phi_1 )</td>
<td>( \phi_{1,R1} = 0 ), (by definition)</td>
<td>( \phi_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( \phi_2 )</td>
<td>( \phi_{2,R1} = -(\phi_1 - \phi_2 + \Delta) )</td>
<td>( \phi_1 + \Delta )</td>
</tr>
<tr>
<td>3</td>
<td>( \phi_3 )</td>
<td>( \phi_{3,R1} = -(\phi_1 - \phi_3 + 2\Delta) )</td>
<td>( \phi_1 + 2\Delta )</td>
</tr>
<tr>
<td>4</td>
<td>( \phi_4 )</td>
<td>( \phi_{4,R1} = -(\phi_1 - \phi_4 + 3\Delta) )</td>
<td>( \phi_1 + 3\Delta )</td>
</tr>
</tbody>
</table>
The optical waveform produced after phase compensation is given by

$$E_t = E_1 + E_2 + E_3 + E_4$$
$$= A_1 * \exp \left( i \left( 2\pi f_1 t + \phi_1 + 0 \right) \right) + A_2 * \exp \left( i \left( 2\pi f_1 t + f_1 t + \phi_2 + \phi_{2R} \right) \right) + A_3 * \exp \left( i \left( 2\pi f_1 + 2f_1 t + \phi_3 + \phi_{3R} \right) \right) + A_4 * \exp \left( i \left( 2\pi f_1 + 3f_1 t + \phi_4 + \phi_{4R} \right) \right)$$

$$= e^{i(2\pi f_1 t + \phi_1)} \left( A_1 + A_2 * e^{i\Delta e^{i(2\pi f_1 t)}} + A_3 * e^{i2\Delta e^{i(2\pi f_1 t)}} + A_4 * e^{i3\Delta e^{i(2\pi f_1 t)}} \right)$$

$$= e^{i(2\pi f_1 t + \phi_1)} \left( A_1 + A_2 * e^{i2\pi f_1 \left( t + \frac{\Delta}{2\pi f_r} \right)} + A_3 * e^{i2\pi f_2 \left( t + \frac{\Delta}{2\pi f_r} \right)} + A_4 * e^{i2\pi f_3 \left( t + \frac{\Delta}{2\pi f_r} \right)} \right)$$

$$= e^{i(2\pi f_1 t + \phi_1)} \left( A_1 + A_2 * e^{i2\pi f_1 (t + t_0)} + A_3 * e^{i2\pi f_2 (t + t_0)} + A_4 * e^{i2\pi f_3 (t + t_0)} \right)$$

where $$t_0 = \frac{\Delta}{2\pi f_r}$$

(4-8)

The carrier wave can be rewritten:

$$e^{i(2\pi f_1 t + \phi_1)} = e^{i(2\pi f_1 (t + t_0) - 2\pi f_1 t_0 + \phi_1)} = e^{-\frac{f_1 \Delta}{f_r}} e^{i(2\pi f_1 (t + t_0) + \phi_1)}$$

(4-9)

Note that the transform-limited pulse is:

$$E_{T,L}(t) = e^{i(2\pi f_1 t + \phi_1)} \left( A_1 + A_2 * e^{i2\pi f_1 t} + A_3 * e^{i2\pi f_2 t} + A_4 * e^{i2\pi f_3 t} \right)$$

(4-10)

Using equations (4-9) and (4-10), the optical waveform in equation (4-8) is:

$$E_t = e^{-\frac{f_1 \Delta}{f_r}} \cdot E_{T,L}(t + t_0)$$

(4-11)

Conclusion: After adding the retrieved phase to the original, the spectral phase varies linearly with frequency, with a slope of $$\Delta$$ per combline. A linearly varying spectral phase results in a time delay of the pulse, by an amount $$t_0 = \frac{\Delta}{2\pi f_r}$$, but does not change the shape of the pulse. Even without knowledge of $$\Delta$$, a transform-limited pulse is created. From a phase measurement perspective, the retrieved phase is accurate, excluding ambiguities of a constant phase and a linear variation of phase with frequency. The technique is self-referenced, i.e. it does not need a second optical source for the phase measurement.

The technique is used in the experiment to flatten the spectral phase of two comb sources, using two independent Finisar Waveshaper products. One of the comb sources serves as the reference for
the multi-heterodyne measurement technique, while the other comb source is used for injection-locking the VCSEL array.

Software was developed to automate the entire process:

1. Select a pair of optical comblines using the Waveshaper.
2. Measure the photodetected sine wave for each combline pair using the sampling scope.
3. Extract the RF phase of the sine wave, and calculate the optical spectral phase.
4. Write the conjugate of the measured optical phase back onto the optical spectrum, to flatten the phase and compress the pulses to their transform-limited pulse widths.

4.2.3.4 Photodetection

In the proof-of-concept experiment shown in 1.5.2.2, a high-bandwidth photodiode and real-time scope are used to measure the intensity of the rapidly changing pulse shapes. This was possible because the total bandwidth of the optical waveform was smaller than that of the photodiode and scope. Only four comblines spaced at 6.25 GHz were used. In the proposed experiment the combline spacing is at 12.5 GHz, and the number channels is 12, so the total optical bandwidth = 150 GHz. Hence simple photodetection and sampling will not be able to capture the generated pulse shapes accurately. However, by reducing the number of channels to four, such that the total bandwidth of the optical signal lies within sampling scope bandwidth of 50 GHz, we will still be able to capture the complete intensity profile.
4.3 Optical Frequency Comb Source

There are three criteria an optical frequency comb source must satisfy for it to be used in the experiment:

1. The repetition rate must be 12.5 GHz.
   This is largely determined by the wavelength demux specifications
2. There must be at least 12 channels with equal power per combline.
   This is determined by the size of the VCSEL array
3. The optical frequencies of the comblines must overlap with the emission of the VCSEL array.

There are two available optical frequency comb sources at 12.5 GHz repetition rate:

1. Using a mode-locked laser at 12.5 GHz repetition rate
   An actively mode-locked, external cavity, semiconductor laser at 12.5 GHz repetition rate can be used. It has the same architecture as the laser described in [88, Section 5]. The output of the laser has a spectral width of about 17 nm (10 dB width) centered at 1555nm, shown in Figure 4-27.

Figure 4-27: Optical spectrum showing the MLL laser output, directly out of the laser and after non-linear broadening. The VCSEL array’s wavelength range is indicated by the shaded region.
While the laser provides more than 160 comblines, they do not overlap with the VCSEL array’s emission wavelength of around 1540 nm. Despite extensive efforts, the center wavelength of the laser could not be tuned to provide spectral overlap. To overcome this, the spectrum of the laser may be non-linearly broadened using highly non-linear fiber, as shown in Figure 4-27. However, this approach was not used because of the increased complexity compared to the alternative discussed next.

2. By modulation of a CW laser

By modulating a CW laser with a combination of phase and amplitude modulators, a flat optical frequency comb may be generated at 12.5 GHz spacing [24, 89], with more than 12 comblines. The advantage of a frequency comb generation via modulation of a CW laser is flexibility, in terms of repetition rate and wavelength tunability. By tuning the wavelength of the CW laser, the comblines can coincide with the VCSELs wavelengths. This technique will be used in the experiment.

For the multi-heterodyne measurement technique, discussed in chapter 0, two comb sources are needed. A second identical comb source is setup. The repetition frequency of the second comb is different from the first.
4.3.1.1 Experimental Data for CW Comb Source

A 12 channel optical frequency comb source is assembled, as shown in Figure 4-28.

![Figure 4-28: Optical Frequency comb source generated by modulation of a CW laser, using cascaded intensity and phase modulators. The programmable optical filter allows independent control of the phase and attenuation of each combline. This enables us to flatten the spectral phase, and compress the pulse to its transform limit.](image)

Sample optical spectra, autocorrelations and photodetected waveforms are shown in Figure 4-29, before and after the programmable optical filter. The filter imparts the conjugate of the phase measured using the 2 combline filtering method discussed in 0, resulting in a compressed pulse at its output. The measured autocorrelation shows excellent agreement with the calculated autocorrelation using the measured optical spectrum and flat spectral phase.

![Figure 4-29: (a) Optical spectra before and after the programmable optical filter. (b) Autocorrelations, before and after phase correction to compress the pulse to its transform-limit. (c) Photodetected traces, before and after phase correction.](image)
4.4 Experimental Results - Waveforms Generated

Experimental waveforms for three different regimes of modulation are presented, along with the different measurement techniques are used in each regime:

1. Static, where the bias currents to the VCSEL are set, but no modulation is present. Multiheterodyne detection is used to measure the spectral phase of the comblines.

2. Modulation of VCSEL currents at MHz frequencies. Photodetection is used to measure the intensity envelope of the waveform. Multi heterodyne detection is also useful for phase retrieval, but under limited configurations.

3. Modulation of VCSEL currents at GHz frequencies. Photodetection is used to measure the intensity envelope of the waveform.

4.4.1 Demonstration of an Injection-Locked VCSEL Array

The graphs in Figure 4-30 demonstrate conclusively that the entire VCSEL array is injection-locked to the input optical frequency comb source. Column 1 shows the data for the free-running VCSEL array, without any external injection. Column 2 shows the same data upon optical injection, and each VCSEL is injection locked. The optical spectrum in Figure 4-30(a) and (b) show the narrowing of the free-running VCSEL linewidth upon optical injection. In the free-running case, the VCSELs are not phase coherent, hence no pulse is formed in the time domain, shown in Figure 4-30(c). Upon injection-locking with an optical frequency comb, the VCSEL array emission is phase coherent, resulting in short pulse formation, shown in Figure 4-30(d). The RF spectrum of the photodetected signals are shown in Figure 4-30(e)-(f). For the free-running case, weak spurs are seen at harmonics of 12.5 GHz, but after the injection-locking the tones at the harmonics of 12.5 GHz, become sharp and powerful, because of the optical spectrum is now a phase coherent comb at 12.5 GHz spacing.
The extrinsic injection ratio, defined as the ratio \( \frac{P_{\text{injected laser}}}{P_{\text{free-running VCSEL}}} \approx -30 \text{ dB} \). The injection ratio is valid for all the data presented in this chapter.
4.4.2 Static Pulse-Shaping Using the Injection-Locked VCSEL Array

Different waveforms generated by the injection-locked VCSEL array are now presented.

4.4.2.1 Transform-Limited Pulses

Transform-limited pulses are produced by adjusting the VCSELs’ bias currents to flatten the spectral phase (measured using the multi-heterodyne method), as shown in Figure 4-31(a)-(g).

![Graphs and images](image)

Figure 4-31: Spectral and temporal characteristics for flat spectral phase. (a) Spectral power, measured on OSA. (b) Spectral phase, measured. (c),(d) standard deviation of retrieved spectral magnitude and phase, for 50 samples. (e) Retrieved and measured spectral power. (f) Measured time-domain multi-heterodyne RF signal, and the calculated optical pulse electric field envelope, from the measured spectral power and phase. (g) Intensity autocorrelation, measured and calculated traces.
Figure 4-31(b) shows that spectral phase is flat, with deviations from flatness less than 0.005π rad, which is extremely flat. Figure 4-31(c),(d) show the standard deviations of retrieved spectral magnitude and phase, for 50 samples. Both are on the order of 10^{-3} or smaller, which shows that both the retrieved values are very precise. Figure 4-31(e) compares the experimental measures spectral power per channel with the retrieved values; again we find excellent agreement. Figure 4-31(f) shows the measured time-domain RF waveform. Superimposed on it is the calculated optical pulse electric field envelope, from the measured spectral power and phase. Since the spectrum has a rectangle-like power spectrum and flat phase, the pulse has a sinc(t)-like pulse profile, as expected from basic Fourier transforms. Figure 4-31(g) shows the experimentally measured intensity autocorrelation, and the theoretically calculated trace using the measured spectral power and phase. There is excellent agreement between the autocorrelation and the measured data from the multi-heterodyne retrieval. The autocorrelation shows a transform-limited pulse, with FWHM =8.8 ps.

4.4.2.2 Pulses With Cubic Spectral Phase

A cubic spectral phase is created by tuning the currents to the VCSELs appropriately. The spectral and temporal data are presented in Figure 4-32. The measured time domain RF waveform in Figure 4-32(c) shows the classic pulse shape for a cubic spectral profile. Again, there is excellent agreement between the retrieved and measured spectral power in Figure 4-32(d), and between the measured and calculated autocorrelation traces in Figure 4-32(e). Figure 4-32(f) shows the deviation of the currents for each channel, from the currents for producing flat spectral phase in section 4.4.2.1. This shows a cubic variation as expected. The maximum deviation is about ±12 μA. The range of phase values is restricted to about ±0.3π rad, for a full range of 0.6 π rad. This is consistent with the measured depth of PM achieved for slow modulation within the stable locking range, as explained in Chapter 3.4.1.
Figure 4.32: Spectral and temporal characteristics for cubic spectral phase. (a) Measured spectral power (b) Measured spectral phase, as well as a cubic curve fit to the data. (c) RF waveform, showing the classic pulse shape for a cubic spectral phase. (d) Retrieved and measured spectral power. (e) Intensity autocorrelation, measured and calculated traces. (f) Deviation of the currents profile from the flat phase case discussed in section 4.4.2.1.
4.4.2.3  Pulses With Quadratic Spectral Phase

A quadratic spectral phase is created by tuning the currents to the VCSELs, and the resulting spectral phase is shown in Figure 4-33(b).

![Figure 4-33](image-url)

Figure 4-33: Spectral and temporal characteristics for quadratic spectral phase. (a) Measured spectral power (b) Measured spectral phase. (c) RF waveform, showing a slightly broadened pulse shape for quadratic spectral phase. (d) Retrieved and measured spectral power. (e) Intensity autocorrelation, measured and calculated traces. (f) Deviation of the currents profile from the flat phase case discussed in section 4.4.2.1.
The measured time domain RF waveform in Figure 4-33(e) shows a slight broadening of the pulse shape compared to the transform limited case. The change is small because the total quadratic phase excursion is $\sim 0.4\pi$ rad, limited by the range of phase change under stable injection-locking conditions. The autocorrelation FWHM is about 1.8 ps wider than the transform limited pulse due to the quadratic phase. Again, there is excellent agreement between the retrieved and measured spectral power in Figure 4-33 (d), and between the measured and calculated autocorrelation traces in Figure 4-33(e). Figure 4-33 (f) shows the deviation of the currents for each channel, from the currents for producing flat spectral phase in section 4.4.2.1.

### 4.4.2.4 Alternating Spectral Phase for a Two-Pulse Sequence

Another common pulse pattern for pulse-shaping demonstrations is a two pulse sequence, created by alternating the phase periodically, as shown in Figure 4-34. Alternating the phase for every channel in Figure 4-34(b), within the limits allowed by stable injection-locking, creates a two pulse sequence, as seen in both Figure 4-34(c), the RF time domain signal and Figure 4-34(e), the autocorrelation. The second pulse is observed at exactly halfway between the original 40 ps pulses period. Note that the measured autocorrelation deviates from the theoretical trace for times less than about 30 ps, because of experimental error in the calibration of the time axis for large offsets from zero. Apart from this error, there is excellent agreement between the traces.
Figure 4-34: Spectral and temporal characteristics for alternating spectral phase. (a) Measured spectral power (b) Measured spectral phase. (c) RF waveform, showing a two pulse sequence within one pulse period of the transform limited case. (d) Retrieved and measured spectral power. (e) Intensity autocorrelation, measured and calculated traces. (f) Deviation of the currents profile from the flat phase case discussed in section 4.4.2.1.
4.4.2.5 Two-Pulse Sequence where One Pulse has Quadratic Spectral Phase

As an extension of the pattern shown in section 4.4.2.4, a quadratic shape is imposed on one of the double pulses while the other is unmodified. Thus we expect one of the two pulses to be broadened. This is indeed observed, as shown in Figure 4-35 (a), where the secondary pulse is now broadened due to the quadratic-like shape imposed on one set of the spectral phase. The autocorrelations also show excellent agreement, apart from the time-base calibration error discussed previously.

Figure 4-35: A comparison of pulse shapes created by shaping one set of channels in an alternating channel phase pattern. (a) shows the computed optical field pulse envelopes for the two different patterns resulting from the two patterns shown in (b). (c) The autocorrelation trace of the phase pattern with the quadratic shape.

4.4.2.6 Summary of Pulse-Shaping Results with Static Currents

Some of the canonical pulse shapes were demonstrated using the injection-locked VCSEL array as a modulator array. This demonstrates the stability and pulse shaping capability of both the VCSEL array and the measurement system. The range of waveforms that can be generated are limited by:

1. The phase-only change for each channel

2. The range of phase change. It is experimentally limited to about $0.6\pi$ rad peak-to-peak across the array, limited by the phase change available by injection-locking.
4.4.3 Pulse Shaping at MHz Update Rates

Similar to the work in Chapter 2.2.1, the VCSELs are modulated at MHz frequencies. At these frequencies, it will be challenging to obtain reliable spectral phase and amplitude data using the multi-heterodyne measurement technique. Hence only the intensity profile of a subset of the channels is measured. Please refer to section 0 for the complete rationale. A subset of four channels is chosen, because the total optical bandwidth of four channels (4*12.5 GHz = 50 GHz) matches the bandwidth of the available sampling scope.

Table 4-11 shows the RF frequencies and powers applied to each of the four VCSELs. For larger RF powers, the injection locking was not stable. Hence the VCSELs are being modulated over the full stable locking range, corresponding to the maximum achievable depth of phase modulation in the thermal modulation regime.

<table>
<thead>
<tr>
<th>VCSEL</th>
<th>RF frequency (MHz)</th>
<th>RF power (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>-23</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>-20</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>-22</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>-22</td>
</tr>
</tbody>
</table>

The Highest Common Factor (HCF) of the frequencies is 1 MHz, so we expect the photodetected waveform to have a periodicity of 1 μs. This is indeed seen in Figure 4-36(b).

![Figure 4-36: (a) Optical spectrum of 4 injection locked VCSEL modulated at MHz frequencies. (b) Photodetected RF signal showing 1 μs periodicity of the intensity profile.](image)
When the span on the scope is set to a few microseconds to see the periodicity in the envelope, it is not possible to display the individual pulses (80 ps period) within that envelope. This is due to the limited memory of the sampling scope; it does not record sufficient number of points to display both the individual pulses and the envelope, unlike the real-time scope used to record Figure 2-5.

The optical spectrum displayed in Figure 4-36(a) has a span of 50 GHz, and does not display the MHz modulation sidebands, because that is below the resolution bandwidth of the spectrum analyzer.

Figure 4-37 shows the multi-heterodyne signal captured in this case. It too has a 1 μs periodicity. We are able to resolve the modulation sidebands corresponding to each VCSEL in the spectrum of the signal. However, retrieval of the optical spectral phase from this data is much more complex, and a modified algorithm is required.

![Figure 4-37: Multi-heterodyne signal(a), the spectral amplitude(b) and phase(c) of the signal.](image-url)
4.4.4 Pulse Shaping at GHz Update Rates

Similar to the work in Chapter 2.2.2, the VCSELs are modulated at GHz frequencies. At these frequencies, the multi-heterodyne measurement technique will not work. Hence only the intensity profile of a subset of the channels is measured. Please refer to section 0 for the complete rationale.

Table 4-12 shows the RF frequencies and powers applied to each of the four VCSELs. For larger RF powers, the injection locking was not stable. Hence the VCSELs are being modulated over the full stable locking range, corresponding to the maximum achievable depth of phase modulation in the electronic modulation regime. There is significant accompanying amplitude modulation per channel at these RF powers, as shown in Chapter 3.4.3

Table 4-12: Modulation of VCSEL at GHz frequencies

<table>
<thead>
<tr>
<th>VCSEL</th>
<th>RF frequency (MHz)</th>
<th>RF power (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1562.5</td>
<td>-5</td>
</tr>
<tr>
<td>6</td>
<td>3125</td>
<td>-6</td>
</tr>
<tr>
<td>7</td>
<td>781.25</td>
<td>-5</td>
</tr>
<tr>
<td>8</td>
<td>2343.75</td>
<td>-5</td>
</tr>
</tbody>
</table>

Figure 4-38: Optical spectrum showing modulation sidebands

Figure 4-38 shows the optical spectrum corresponding to Table 4-12. There are several interesting features in the spectrum:

1. We can observe the sidebands on each channel corresponding to its modulation frequency.
2. We can observe considerable electrical crosstalk. For example, we observe sidebands at 781.25 MHz on channel 5, which is modulated at 1562.5 MHz. There is crosstalk between all channels, however, it is typically at least 10 dB lower than the main modulation frequency. The origin of the crosstalk is most likely due to the long wirebonds and the close-traces on the PCB near the wirebond pads.

3. The filtering effect due to the limited passband width of the demux is obvious, because each channel seems to have a similar envelope. From Chapter 0, we expect the demux passband FWHM to be ~ 3.5 GHz. The finite passband width limits the update rate of the pulse shapes.

Figure 4-39 shows the unmodulated pulse train recorded on the sampling scope, showing both the flat envelope and the individual pulses within it, with a pulse period of 80 ps.

![Unmodulated pulse shapes](image)

**Figure 4-39**: Unmodulated pulse shapes, for modulation power to each channel turned off. (a) Span = 5*1.28 ns. (b) Span = 1.6 ns, which shows the individual pulses on a 80 ps pulse period.

The Highest Common Factor (HCF) of the frequencies is 781.25 MHz, which corresponds to a period of 1.28 ns. So we expect the envelope of the photodetected waveform to have a periodicity of 1.28 ns. This is indeed observed in all the sample waveforms shown in Figure 4-40. Each waveform was generated by changing the RF phase of the modulation, by using a manual phase shifter. Each has a different shape of the envelope. The close-up of each waveform shows
conclusively that the pulse shape changes dramatically within 1.28 ns. For example, the pulse shape changes from a pulse doublet type to a transform-limited type within 0.32 ns in Figure 4-40(b1).

Note that improvement in clarity of these results compared to the data shown in Figure 2-7. This is due to the highly stable waveforms due to the free-space coupling, which minimizes the relative phase drift that the data in Figure 2-7 suffered from. Furthermore, 50 GHz sampling oscilloscope has greater sensitivity, lower noise floor and finer temporal resolution compared to the 30 GHz real-time oscilloscope used to acquire Figure 2-7.

![Graphs showing pulse shape changes](image)

Figure 4-40: Three sample waveforms showing a periodic envelope of the pulses (a1, a2, a3), each with 1.28 ns periodicity. The corresponding close-ups (b1, b2, b3) show the pulse shape changing on ns timescale.
4.5 Discussion And Conclusions

Pulse shaping using an array of injection-locked VCSELs have been demonstrated in three different regimes. In the static current regime, the use of the multi-heterodyne measurement technique allows us to fully characterize the various pulse shapes generated. In this regime, the VCSEL array acts as a phase-only modulator.

When currents to the VCSELs are modulated at MHz and GHz frequencies, we were limited by the available characterization techniques in measuring only the intensity profile of waveforms for four modulated channels. At GHz modulation frequencies, we showed conclusively that the pulse shapes are updated on a nanosecond time scale.

The merits and drawbacks of using an array of injection-locked VCSELs for pulse shaping are discussed next.

4.5.1 Merits of Using an Injection-Locked VCSEL Array for Pulse Shaping

1. VCSEL arrays with large channel counts can be easily made, especially in 2-D arrays.

   Larger number of channels permits generation of more complex waveforms.

2. The electrical modulation bandwidth of VCSELs is large, at tens of GHz.

   This allows filling of bandwidth between the input comblines, again allowing more complex waveforms to be generated. This permits the use of multi-GHz repetition rate optical sources, which reduces the demands on the design of the wavelength demux.

3. Injection-locking can provide gain to the injected signal

   In this work, the injected power was -30 dB lower than VCSEL’s emitted power. This makes it suitable for waveform generation using low power optical sources.
4.5.2 Drawbacks of Using an Injection-Locked VCSEL Array for Pulse Shaping

The limitations of using injection-locked lasers (and not just VCSELs) for dynamic OAWG, are the limited phase range and coupled amplitude modulation. They limit the type of waveforms that may be generated. True arbitrary waveform generation requires a range of phase modulation of $2\pi$ rad of the modulator and the ability to independently control the amplitude and phase modulation imparted. Thus injection-locked VCSEL arrays are not adequate for full dynamic OAWG. An OAWG system with the VCSEL’s limited modulation capabilities may still have other applications.

The limitations are

1. Range of phase modulation limited to less than $\pi$ rad, peak-to-peak.
   This is a fundamental constraint due to the physics of injection-locking.

2. Phase and amplitude modulation cannot be controlled independently for a single modulator at GHz rates.
   The coupling of phase and amplitude modulation is significant at GHz modulation rates because of the coupling of gain and refractive index with carrier density in a semiconductor laser. At modulation rates less than $\sim 1$ MHz, an injection-locked VCSEL produces largely phase modulation. Both 1. and 2. limit the complexity of waveforms that can be generated.

3. Optical bandwidth of each modulator is narrow, within a few nm.
   For injection-locking to occur, the optical frequencies of the injected tone and the free-running VCSEL must lie within a few GHz of each other. The optical frequency of the VCSEL may be tuned with bias current for $\sim 350$ GHz/3 nm for the RayCan VCSELs, which limits range of optical frequencies that can be modulated. It is very limited compared to modulator technologies like electro-optic waveguide modulators on InP or LiNbO$_3$, which have an optical range of many tens or hundreds of nm.
4. Low power handling capacity of the modulators.

Typically, the desired optical injection power to the VCSELs is in the few microwatts (small injection ratio regime), to maximize the PM to AM ratio, and to obtain a large depth of PM. If the input power is large, it must be attenuated. Modulation characteristics of injection-locked VCSEL change with injected power. At large injection ratios, larger current modulation is required to generate a specified depth of PM, which also generates large AM.

5. The output optical power is low, a few mW, due to the low power output of VCSELs.

The VCSELs used in the experiment emit a maximum of around ~ 1 mW per emitter. This may be alleviated by external optical amplification.

6. The frequency response of the injection-locked VCSEL depends on many parameters.

The depth of PM and AM as function of modulation frequency depend on injection-ratio, the frequency detuning between master and slave laser, and bias current to the VCSEL. The three parameters determine the resonance frequency, damping and shape of the frequency response.

Each of them has to be controlled experimentally, which makes this modulator more complicated to use than electro-optic modulators, especially for broad-band (many GHz) applications.

7. Electrical crosstalk in delivering high bandwidth RF signals.

This is a challenge common to all modulator array technologies operating at GHz frequencies. Crosstalk was observed in the experimental setup, as seen in Figure 4-38. The crosstalk can be reduced significantly by improved design of the PCB and wirebonding. For a 1D VCSEL array, the electrical interconnect is easier to design because the traces can come from the direction orthogonal to the array. The channel counts of 1D arrays are limited, and one has to go to 2D arrays to increase the channel counts beyond ~ 30. For 2D VCSEL arrays, the electrical interconnect becomes more difficult given the density of channels. Figure 4-41 shows a
schematic of a 2D VCSEL array to highlight the density of the metallic traces on the array. The VCSEL shown is intended for DC operation. For high-speed operation, GSG-type electrical waveguide traces are required.

Figure 4-41: A 2D VCSEL array at 980 nm, on a 250 micron pitch. Source: http://www.princetonoptronics.com/products/pdfs/AA64-BC-SM-W0975.pdf

8. Free-space coupling of light to VCSELs makes setup bulky and sensitive to alignment.

VCSELs emit perpendicular to the wafer surface, and coupling light in and out of the array must be done in free-space. This makes it bulky compared to planar integrated implementations like Arrayed Waveguide Gratings, and becomes challenging for larger channel counts in 2D arrays.

A summary of the merits and drawbacks is provided in Table 4-13.

Table 4-13: Merits and drawbacks of the proposed architecture for rapid-update pulse shaping using injection-locked VCSELs.

<table>
<thead>
<tr>
<th>No.</th>
<th>Merits</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VCSEL arrays with large channel counts are easily fabricated</td>
<td>Delivering high bandwidth signals to each emitter with low electrical crosstalk is challenging, especially for 2D VCSEL arrays</td>
</tr>
<tr>
<td>2</td>
<td>VCSELs have large electrical modulation bandwidths, at tens of GHz</td>
<td>Low optical power handling of the VCSEL as a modulator. It must be injected with very low power for favorable phase modulation characteristics.</td>
</tr>
<tr>
<td>3</td>
<td>Injection-locking can provide gain, useful for pulse shaping with weak signals</td>
<td>Range of phase modulation limited to be less than π rad, peak-to-peak</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>Both phase and amplitude modulation are significant at GHz modulation rates.</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>Optical bandwidth of each VCSEL modulator is narrow, less than 3 nm</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>Output optical power is low per VCSEL, ~ 1mW in this experiment</td>
</tr>
</tbody>
</table>
4.5.3 Future work

Several avenues can be explored to expand the capabilities of the rapid update pulse-shaping system in the future. They are:

1. Improved waveform measurement capability. The spectrally-sliced, four quadrature coherent detection system presented in [39] is best suited for complete measurement (especially the optical phase) of large instantaneous bandwidth signals generated by modulation at GHz frequencies.

2. Improved wavelength demultiplexer
   2.1. A smaller demultiplexer, like an arrayed waveguide grating, will greatly reduce the overall size of the experiment, and make coupling the light from the VCSEL easier and more robust.
   2.2. A demultiplexer with a broader passband width is required. Currently the modulation bandwidth is limited by the passband width to about 3.5 GHz FWHM. Alternative multiplexers are explored in [9], which have a broader passband.

3. Improved VCSEL array and packaging
   3.1. A VCSEL array with a larger intrinsic modulation bandwidth per channel is required. VCSELs with 26 GHz modulation bandwidth were recently reported [90], so the ~ 3 GHz modulation bandwidth of the VCSEL used in the dissertation work can easily be improved. Better packaging techniques, including the PCB interconnect board, will also increase the achievable modulation bandwidth.
CHAPTER 5: A LINEAR TECHNIQUE FOR DISCRIMINATION OF OPTICALLY CODED WAVEFORMS

Please note that the work presented in this chapter was previously published. © 2012, IEEE. Reprinted, with permission, from S. P. Bhooplapur, F. J. Quinlan, M. Akbulut, and P. J. Delfyett, Linear Technique for Discrimination of Optically Coded Waveforms Using Optical Frequency Combs, Photonics Technology Letters, IEEE, Oct 2012, [91]. Some of the material was also presented at conferences [92, 93].

Matched filters are widely used in many areas of optics, and some of the key applications include image processing [94], laser radar [95], and optical telecommunications, for example in a Optical Code Division Multiple Access (OCDMA) communications architecture [96]. Each of the applications relies on the use of matched filters to improve the detected signal-to-noise ratio, to distinguish between a jamming signal and radar return signal, and to distinguish between multiple simultaneous users, respectively. The generic system transmits a signal after encoding. In our approach, the spectral phase is encoded optically. The optical source typically used for encoding is a conventional Mode-Locked Laser (MLL). The ultrashort pulses from the MLL are transformed into noise-like bursts after encoding. At the receiver, when the decoder is a matched filter for the encoder, the output consists of short pulses similar to the input pulses. When they are not matched, the output remains a noise-like burst that is spread in time. To clearly distinguish between the properly decoded and improperly decoded pulses requires photodetectors and thresholding hardware with response times $\sim 1 \text{ ps}$ [96]. Due to the slow response time of commercially available photodetectors, photocurrents produced by the encoded and decoded optical pulses are hard to distinguish. Thus the photodetector degrades the outcome of the optical matched filtering. This is an important limitation for applications of matched filters using ultrashort optical pulses.
To overcome this limitation, a variety of high-speed nonlinear optical effects have been used to help identify the properly decoded pulses with low error, using synchronous gating and asynchronous thresholding [96]. The non-linear optical techniques rely on the difference in the peak power between the properly and improperly decoded pulses. In this work, an alternative coherent architecture is proposed that successfully decodes the data using only linear optical devices. The receiver concept was proposed earlier in [97] and preliminary results were presented in [98] as part of a range of applications of optical frequency combs. The results shown in this chapter represent a significant improvement to the earlier work.

Figure 5-1: (a) Typical system architecture for matched filtering using ultrashort optical pulses, where the encoder and decoder are matched filters. (b) System architecture for demonstrating code matching using a coherent detection approach. (c) Experimental setup for proof-of-concept demonstration of coherent detection architecture. SOA: semiconductor optical amplifier. OFCS: optical frequency comb source. VOA: variable optical attenuator. VOD: variable optical delay.

The conventional architecture using nonlinear optical gating and thresholding is shown schematically in Figure 5-1(a). The new architecture is shown in shown in Figure 5-1(b), where the transmitter is modified by replacing the MLL with an OFCS as the optical source used for encoding. The OFCS is realized from a high-repetition rate, frequency stabilized, semiconductor Mode-Locked
Laser (MLL) system [88]. The optical frequency stability of the comblines of the OFCS, provided by an intracavity etalon and Pound-Drever-Hall frequency stabilization, enable line-by-line spectral phase encoding. The ultralow noise performance of the OFCS, as characterized by the amplitude fluctuations, phase noise, and narrow optical linewidth, make the OFCS attractive as a master laser to which the receiver can be synchronized. These characteristics of the OFCS enable robust optical matched filtering that would be challenging for conventional MLLs to achieve. A drawback of the OFCS compared to conventional MLLs is the additional complexity and cost of the active optical frequency stabilization.

At the receiver, a Local Oscillator (LO) MLL synchronized to the transmitter is used to generate reference pulses encoded using the assigned code. The reference pulses are interfered with incoming pulses from the network, and the differential current of the photodetected outputs of the interferometer is observed. The differential photocurrent is high when the codes match (correct decoding), and low (ideally zero) when the codes are mismatched (incorrect decoding).

Synchronization between the transmitter and receiver can easily be achieved and maintained by using a combination of optical phase-locked loops and/or injection locking [99]. Semiconductor MLLs have been synchronized via injection locking, as demonstrated in [100], with detailed considerations of the noise introduced given in [101]. It was shown that the degradations due to the injection-locking process are small, and the synchronized optical pulses will still maintain the phase and amplitude noise of the master laser and remain locked to its phase. In particular, a relative timing jitter of ~300 fs after synchronization is achieved in [102], which is well within the temporal accuracy of ~1 ps which is required in this experiment. The ~1 ps synchronization accuracy of the pulses was determined experimentally based on the spectral width of the OFCS used in the experiment. In general, a larger optical bandwidth requires more accurate synchronization.
The most efficient passive nonlinear optical thresholding device needs a minimum average input power of around -5.5 dBm (~280 μW) for a single transmitter, from Fig. 15 in [96]. With the proposed detection architecture, ~25 μW of average optical power is used at each input to the interferometer. This number is chosen for convenience and does not represent a fundamental limit of the technique's sensitivity. The use of simpler linear optical devices and lower optical power requirements (increased receiver sensitivity) are the salient features of the proposed coherent detection system over one that uses non-linear optical gating and thresholding.

5.1 Coherent Detection Scheme for Code Discrimination

The experimental setup shown in Figure 5-1(c) is used to demonstrate the architecture shown in Figure 5-1(b) for the discrimination of optically coded waveforms. Codes from the Walsh-Hadamard (WH) set are encoded onto the spectral phase of the optical frequency comb generator, where each ‘1’ in the code is encoded as zero phase, ‘-1’ as π phase. The encoded spectra are coherently combined in a waveguide interferometer, followed by differential balanced photodetection. A variable optical delay stage is used to ensure that the two input pulses to the interferometer overlap in time, making it a synchronous detection scheme. The power at the output ports of the interferometer for each input is equalized, which is important for the rejection of mismatched codes.
Figure 5-2: Using coherent detection and differential balanced photodetectors to distinguish between orthogonal codes encoded on optical spectra. Each ‘1’ bit in a WH code is encoded as phase value 0, ‘-1’ bit as phase value π. In (a), the WH codes encoded on the spectra at both inputs to the interferometer are the same, code (1,1,-1,1). In (b), the codes are orthogonal, (1,1,-1,1) and (1,-1,1,-1).

Figure 5-2 illustrates the concept of how orthogonal codes can be distinguished by mixing the received signal with a LO and using differential balanced photodetectors. A free-space interferometer is shown with four optical frequencies (comblines), labeled 1 – 4, at both inputs. For a chosen optical frequency, when the phase difference is zero between the two inputs, all the optical power goes to one output port of the interferometer due to constructive interference. When the condition for constructive interference is met for all comblines as shown in Figure 5-2(a), the total power of the comblines is at one output of the interferometer. The large difference in photodetected currents between the two outputs of the interferometer is normalized to give a differential signal of 1. This is the output with matching codes at both inputs.

In Figure 5-2(b), orthogonal codes from the Walsh-Hadamard (WH) set are encoded on the spectral phase of the inputs. In this case, half of the optical comblines undergo constructive interference and half destructive interference. This is the salient property of orthogonal coding. The output powers at both ports of the interferometer are half the total power, resulting in equal photocurrents and a zero
differential signal. For a good null to be observed for orthogonal codes, the power of the combines at both inputs to the interferometer must be equal. When the codes match, the differential signal is high, when the codes are orthogonal, the signal is close to zero. Thus one can distinguish between orthogonal codes with high contrast.

The receiver concept can be expressed mathematically as follows. Since an ideal comb source is a set of \( N \) infinitesimally narrow, equally-spaced frequencies separated by \( f_{\text{rep}} \), it can be represented as a vector \( \vec{x} \) of length \( N \). Each element of the vector is a complex number that represents the amplitude and spectral phase of a combline. In the time-domain, an Optical Frequency Comb Source (OFCS) is a repetitive pulse-train with pulse period, \( T_0 = 1/f_r \). The time-averaged power of the comb source after photodetection is represented by the square of the norm of the vector, \( |\vec{x}|^2 \). The representation is valid for averaging durations \( T \) much longer than \( T_0 \), where beat terms at multiples of \( f_r \) average to zero. The time-averaged power is then equal to the sum of the power of each combline. If \( \vec{p} \) and \( \vec{q} \) are orthogonal vectors (codes encoded on the spectrum of a comb source), and both vectors have a dimension that is even, then the difference of the time-averaged outputs of the interferometer (shown in Figure 5-2 (b)) will always be zero.

If:

\[
\dim(\vec{p}, \vec{q}) = N, \text{ where } N \text{ is even, AND} \]
\[
\vec{p} \cdot \vec{q} = 0 \text{ (orthogonal vectors)} \] (5-1)

Then:

\[
\text{Output} = |\vec{p} + \vec{q}|^2 - |\vec{p} - \vec{q}|^2
\]
\[
= \begin{cases} 0, \text{ when } \vec{p} \neq \vec{q} \text{ (mismatched codes)} \\ 2|\vec{p}|^2, \text{ when } \vec{p} = \vec{q} \text{ (matched codes)} \end{cases} \] (5-2)
Codes from the WH code set satisfy these conditions, and are used in the experiment. Each code of length \( L \) is part of a WH set with a total of \( L \) unique codes. In applications envisioned for this architecture, such as OCDMA or laser radar, the number of unique users or coded radar signals scales linearly with the code length, limited ultimately by the number of available comblines.

5.2 Experimental Setup and Data

The OFCS used in the experiment is an external-cavity, actively mode-locked, frequency-stabilized, semiconductor laser, shown in Figure 5-3, with an almost identical architecture to that in [88]. The laser spectrum is centered about 1560nm, with a repetition rate of 12.5 GHz corresponding to a combline spacing of 0.10 nm. The Pound-Drever-Hall (PDH) frequency stabilization technique [103] is used to stabilize the optical frequencies with respect to the intra-cavity etalon.

The spectrum provides roughly 165 comblines (10 dB width) for encoding. In the time domain, the pulses directly out of the OFCS have a pulse-width of around 15 ps. The pulses have a large linear chirp, and can be externally compressed to around 0.6 ps. The results are shown in Figure 5-4.

There are three key features of the OFCS that enable the coherent detection scheme:

1. The large number of comblines with 12.5 GHz separation, which potentially enables usage of long codes.
2. The relative flatness of the power spectrum, especially in the middle.
3. The frequency stability of the frequency comb permits its use in high-resolution spectral encoding.

The WH codes are encoded using two independent spectral phase encoders. Each encoder is a fiber-coupled Fourier-transform pulse shaper [104], as shown in Figure 5-5, with a reflective, Liquid-Crystal on Silicon (LCoS), phase-only Spatial Light Modulator (SLM) used to encode the spectral
phase of the OFCS. Integrated devices for spectral phase encoding, such as Arrayed Waveguide
Gratings [54] and Ring Resonators [7], may be more attractive for commercial deployment
because they are smaller and potentially more stable than the bulk-optic encoders used in this
demonstration.

A design criterion for the phase encoders was to enable the independent encoding of each combline.
This would maximize the length of WH code that can be encoded on the spectrum of the OFCS.
Using an expanded beam size (1/e2 diameter) of about 20mm incident on the diffraction grating
(1050 lines/mm groove density) and a doublet lens with focal length of 200mm, the 1/e2 diameter
of a single combline at the focal plane is about 17 μm, measured using a beam profiler. The center-
to-center separation between adjacent spots is 30 μm. Thus the high-resolution pulse-shaper can
clearly resolve two adjacent comblines separated by 12.5 GHz.

The spot from each optical combline incident on the SLM is slightly larger than a pixel (square pixel
with a 15 μm pitch), and adjacent comblines are separated by two pixels. In an ideal SLM, each
combline could be independently modulated using two pixels. In the SLMs used in the experiment,
Boulder Nonlinear Systems HSP 512-1550 devices with dielectric mirror coatings for improved fill
factor, there is significant electrical crosstalk between neighboring pixels. Within the lateral crosstalk distance, measured to be \(\sim 8\) pixels, the encoded phase changes gradually between the target phase levels. Binary phase patterns of high spatial frequencies necessary for independent encoding of each combline are not possible on the SLM. To improve accuracy of encoded phase values, lower spatial frequency patterns are used, with 6-8 comblines encoded for each bit in the WH code, and 2-3 comblines on either side in the crosstalk region. Thus crosstalk strongly limits the maximum code length in the current setup. Limiting the encoded comblines to the middle of the spectrum, where the spectral power is flatter compared to the edges, as shown in Figure 5-6(a), improves the extinction of orthogonal codes. A maximum WH code length of 16 satisfies these constraints.

Figure 5-6: (a) Input spectrum to the phase encoders, showing the shape of the optical spectrum from OFCS in linear scale, and subset of 126 comblines encoded. WH codes are 16 bits long, with 6 comblines encoded per bit, and 2 comblines per bit in the crosstalk region (b) Power at each output of the interferometer, Port A and B (c) Normalized differential power, whose value is large when the codes on the SLMs match (d) Histogram of the normalized differential signal shown in (c).
Code 1 is loaded on spectral phase encoder 1; all codes from the WH code set are loaded sequentially on encoder 2. The SLM’s refresh rate, limited by the liquid crystal material’s response time to 47 Hz, is set to 2 Hz for convenience. The optical power at both output ports of the interferometer is shown in Figure 5-6(b), and the normalized differential signal is shown in Figure 5-6(c). The average input power at each input port is ~ 25 μW, a value chosen for convenience. It is not the sensitivity limit of the architecture. The normalized differential power is defined as the difference in optical power between the two ports divided by the total power. It is high when the codes match, and low when codes are orthogonal.

From the time-domain data in Figure 5-6(c), we can clearly distinguish between the matched and mismatched codes. There are six instances when the codes match and 90 instances where the codes do not match. A histogram of the data is plotted in Figure 5-6(d). For mismatched codes the values are close to zero, as expected. The mean value of the distribution for matching codes is ~0.6 (ideally equal to 1) because the power in the comblines at the edges of the spectrum (Fig. 4(a)), which are not encoded, is equally split between the two outputs and does not contribute to the differential signal. The clear separation between the distributions for matched and mismatched codes means that we can identify the optically coded waveform with high confidence.

5.3 Discussion and Conclusions

The separation between the two distributions is currently limited by two factors: non-uniform power of comblines across the spectrum, and phase drift between the two arms of the interferometer due to environmental perturbations on the optical fiber. Non-uniform power of comblines across the spectrum results in increased variance of the distribution for orthogonal codes. The variance can be reduced by equalizing the power of the comblines by using an SLM which can perform both amplitude and phase filtering. Phase drift increases the standard deviation of each of the two
distributions in the histogram. Drift can be greatly reduced by phase locking the transmitter and receiver, which will be achieved during the process of synchronization via injection-locking, as discussed earlier in the letter.

In conclusion, we have demonstrated a novel coherent optical detection architecture that serves as a matched filter and successfully discriminates between various encoded optical waveforms with high confidence. The waveforms are produced by encoding the spectral phase of an optical frequency comb with codes from a Walsh-Hadamard set. The encoded optical waveforms are decoded with a coherent detection technique implemented using an optical frequency comb as a set of local oscillators, an interferometer and a pair of differential balanced photodetectors. The architecture depends on the measurements of time-averaged powers of encoded ultrashort optical pulses instead of their pulse shapes. Thus we are able to use the slow response time of photodetectors and the properties of orthogonal codes to our advantage. Experimentally, we have distinguished between ultrashort optical pulses encoded using identical and orthogonal codes with high accuracy, for codes of length 16. The use of only linear optical devices in the receiver results in a higher sensitivity compared to that using nonlinear optical thresholding and gating techniques. Decoding is demonstrated at power levels of \( \sim 25 \, \mu W \), which are much lower than previously reported. Possible applications of the receiver architecture include laser radar and optical communications in an Optical Code-Division Multiple Access (OCDMA) network.
APPENDIX: PERMISSIONS FOR USE OF COPYRIGHTED MATERIAL
Copyright Permission from IEEE

The following notice was published on IEEE’s website at http://www.ieee.org/publications_standards/publications/rights/reqperm.html

“The IEEE does not require individuals working on a thesis to obtain a formal reuse license, however, you may print out this statement to be used as a permission grant:

Requirements to be followed when using any portion (e.g., figure, graph, table, or textual material) of an IEEE copyrighted paper in a thesis:
1) In the case of textual material (e.g., using short quotes or referring to the work within these papers) users must give full credit to the original source (author, paper, publication) followed by the IEEE copyright line © 2011 IEEE.
2) In the case of illustrations or tabular material, we require that the copyright line © [Year of original publication] IEEE appear prominently with each reprinted figure and/or table.
3) If a substantial portion of the original paper is to be used, and if you are not the senior author, also obtain the senior author’s approval.

Requirements to be followed when using an entire IEEE copyrighted paper in a thesis:
1) The following IEEE copyright/credit notice should be placed prominently in the references: © [year of original publication] IEEE. Reprinted, with permission, from [author names, paper title, IEEE publication title, and month/year of publication]
2) Only the accepted version of an IEEE copyrighted paper can be used when posting the paper or your thesis on-line.
3) In placing the thesis on the author's university website, please display the following message in a prominent place on the website: In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of [university/educational entity’s name goes here]'s products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to http://www.ieee.org/publications_standards/publications/rights/rights_link.html to learn how to obtain a License from RightsLink.”
Dear Sharad Bhooplapur,
Thank you for contacting The Optical Society.
OSA considers your requested use of its copyrighted material to be Fair Use under United States Copyright Law. It is requested that a complete citation of the original material be included in any publication.
Let me know if you have any questions.

Kind Regards,
Susannah Lehman

Susannah Lehman
November 14, 2014
Authorized Agent, The Optical Society

From: Sharad Bhooplapur [mailto:sharadB@creol.ucf.edu]
Sent: Sunday, November 09, 2014 11:15 PM
To: copyright@osa.org
Subject: Permission for use of copyright material

Hello,

I would like permission for use of the following material in my PhD Dissertation.

I am the primary author of this publication, and I would like to add the publication in its entirety as a chapter in my PhD dissertation thesis.
I would like permission to use Figure 1 & 2. Each figure will be used once in my PhD dissertation thesis.

Thank you,

Sharad Bhooplapur
PhD Candidate
Ultrafast Photonics Group

College of Optics and Photonics, CREOL
Building 53
University of Central Florida
4304 Scorpius Street
Orlando, FL 32816-2700
USA
Tel: 407-823-6996
Fax: 407-823-6880
Copyright Permissions from Macmillan Publishing Ltd

NATURE PUBLISHING GROUP LICENSE
TERMS AND CONDITIONS

Nov 09, 2014

This is a License Agreement between Sharad Bhooplapur ("You") and Nature Publishing Group ("Nature Publishing Group") provided by Copyright Clearance Center ("CCC"). The license consists of your order details, the terms and conditions provided by Nature Publishing Group, and the payment terms and conditions.

All payments must be made in full to CCC. For payment instructions, please see information listed at the bottom of this form.

<table>
<thead>
<tr>
<th>License Number</th>
<th>3505150573198</th>
</tr>
</thead>
<tbody>
<tr>
<td>License date</td>
<td>Nov 09, 2014</td>
</tr>
<tr>
<td>Licensed content publisher</td>
<td>Nature Publishing Group</td>
</tr>
<tr>
<td>Licensed content publication</td>
<td>Nature Photonics</td>
</tr>
<tr>
<td>Licensed content title</td>
<td>Optical arbitrary waveform generation</td>
</tr>
<tr>
<td>Licensed content author</td>
<td>Steven T. Cundiff, Andrew M. Weiner</td>
</tr>
<tr>
<td>Licensed content date</td>
<td>Oct 29, 2010</td>
</tr>
<tr>
<td>Volume number</td>
<td>4</td>
</tr>
<tr>
<td>Issue number</td>
<td>11</td>
</tr>
<tr>
<td>Type of Use</td>
<td>reuse in a dissertation / thesis</td>
</tr>
<tr>
<td>Requestor type</td>
<td>academic/educational</td>
</tr>
<tr>
<td>Format</td>
<td>electronic</td>
</tr>
<tr>
<td>Portion</td>
<td>figures/tables/illustrations</td>
</tr>
<tr>
<td>Number of figures/tables /illustrations</td>
<td>1</td>
</tr>
<tr>
<td>High-res required</td>
<td>no</td>
</tr>
<tr>
<td>Figures</td>
<td>Figure 1.a only</td>
</tr>
<tr>
<td>Author of this NPG article</td>
<td>no</td>
</tr>
<tr>
<td>Your reference number</td>
<td>None</td>
</tr>
<tr>
<td>Title of your thesis / dissertation</td>
<td>INJECTION-LOCKED VERTICAL CAVITY SURFACE EMISSING LASERS (VCSELs) FOR OPTICAL ARBITRARY WAVEFORM GENERATION</td>
</tr>
<tr>
<td>Expected completion date</td>
<td>Dec 2014</td>
</tr>
<tr>
<td>Estimated size (number of pages)</td>
<td>140</td>
</tr>
<tr>
<td>Total</td>
<td>0.00 USD</td>
</tr>
</tbody>
</table>

Terms and Conditions

Nature Publishing Group hereby grants you a non-exclusive license to reproduce this material for this purpose, and for no other use, subject to the conditions below:
1. NPG warrants that it has, to the best of its knowledge, the rights to license reuse of this material. However, you should ensure that the material you are requesting is original to Nature Publishing Group and does not carry the copyright of another entity (as credited in the published version). If the credit line on any part of the material you have requested indicates that it was reprinted or adapted by NPG with permission from another source, then you should also seek permission from that source to reuse the material.

2. Permission granted free of charge for material in print is also usually granted for any electronic version of that work, provided that the material is incidental to the work as a whole and that the electronic version is essentially equivalent to, or substitutes for, the print version. Where print permission has been granted for a fee, separate permission must be obtained for any additional, electronic re-use (unless, as in the case of a full paper, this has already been accounted for during your initial request in the calculation of a print run). NB: In all cases, web-based use of full-text articles must be authorized separately through the 'Use on a Web Site' option when requesting permission.

3. Permission granted for a first edition does not apply to second and subsequent editions and for editions in other languages (except for signatories to the STM Permissions Guidelines, or where the first edition permission was granted for free).

4. Nature Publishing Group’s permission must be acknowledged next to the figure, table or abstract in print. In electronic form, this acknowledgement must be visible at the same time as the figure/table/abstract, and must be hyperlinked to the journal’s homepage.

5. The credit line should read:
   Reprinted by permission from Macmillan Publishers Ltd: [JOURNAL NAME] (reference citation), copyright (year of publication)
   For AOP papers, the credit line should read:
   Reprinted by permission from Macmillan Publishers Ltd: [JOURNAL NAME], advance online publication, day month year (doi: 10.1038/sj.JOURNAL.ACRONYM.XXXXX)

   **Note: For republication from the British Journal of Cancer, the following credit lines apply.**
   Reprinted by permission from Macmillan Publishers Ltd on behalf of Cancer Research UK: [JOURNAL NAME] (reference citation), copyright (year of publication) For AOP papers, the credit line should read:
   Reprinted by permission from Macmillan Publishers Ltd on behalf of Cancer Research UK: [JOURNAL NAME], advance online publication, day month year (doi: 10.1038/sj.JOURNAL.ACRONYM.XXXXX)

6. Adaptations of single figures do not require NPG approval. However, the adaptation should be credited as follows:

   Adapted by permission from Macmillan Publishers Ltd: [JOURNAL NAME] (reference citation), copyright (year of publication)

   **Note: For adaptation from the British Journal of Cancer, the following credit line applies.**
   Adapted by permission from Macmillan Publishers Ltd on behalf of Cancer Research UK: [JOURNAL NAME] (reference citation), copyright (year of publication)

7. Translations of 401 words up to a whole article require NPG approval. Please visit [http://www.maccmillanmedicalcommunications.com](http://www.maccmillanmedicalcommunications.com) for more information. Translations of up to 400 words do not require NPG approval. The translation should be credited as follows:

   Translated by permission from Macmillan Publishers Ltd: [JOURNAL NAME] (reference citation), copyright (year of publication)

   **Note: For translation from the British Journal of Cancer, the following credit line applies.**
   Translated by permission from Macmillan Publishers Ltd on behalf of Cancer Research UK: [JOURNAL NAME] (reference citation), copyright (year of publication)
We are certain that all parties will benefit from this agreement and wish you the best in the use of this material. Thank you.

Special Terms:

v1.1

Questions? customercare@copyright.com or +1-855-239-3415 (toll free in the US) or +1-978-646-2777.

Gratis licenses (referencing $0 in the Total field) are free. Please retain this printable license for your reference. No payment is required.
REFERENCES


