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ROLE OF FORCE RESULTANT INTERACTION ON FIBER REINFORCED CONCRETE

by

TITCHENDA CHAN
B.S. Institute of Technology of Cambodia, 2008

A thesis submitted in partial fulfilment of the requirements
for the degree of Master of Science
in the Department of Civil, Environmental and Construction Engineering
in the College of Engineering and Computer Science
at the University of Central Florida
Orlando, Florida

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2014

Major Professor: Kevin Mackie
ABSTRACT

Ultra-high performance concrete (UHPC) is a recently developed concrete gaining a lot of interest worldwide, and a lot research has been conducted to determine its material properties. UHPC is known for its very high strength and high durability. Association Francaise de Genie Civil (AFGC) has defined UHPC as a concrete exhibiting compressive strength greater than 150 MPa (22 ksi). To utilize the full compressive strength of UHPC, complementary tension reinforcement is required. A recent research study to find light weight yet high strength alternative deck systems for Florida movable bridges demonstrated that a composite UHPC and high strength steel (HSS) reinforcement deck system is a viable alternative. However, failure modes of the deck system observed during experimental testing were shear failures rather than flexural failures. Interestingly, the shear failures were ductile involving large deformations and large sectional rotations.

The purpose of this research is to quantify the sensitivity of UHPC structural member mechanical response to different shear and normal stress demands, and investigate the underlying failure modes. An experimental investigation on small-scale prisms without reinforcement, prisms reinforced with ASTM Grade 60 steel, and prisms reinforced with high strength steel was carried out to capture load-deflection behavior as well as modes of failure of the UHPC specimens. Numerical analysis based on modified compression field theory (MCFT) was developed to verify experimental results at the section level, and further verification using continuum methods was performed using MCFT/DSFM (disturbed stress field method) based finite element analysis software (VecTor2).

Results from the numerical analysis could reasonably predict the load-displacement as well as the failure modes of the experimental specimens. Obvious flexural failure was observed on unreinforced UHPC specimens where wide crack opening gradually widened at the bottom fiber of the concrete to the loading position. Whereas UHPC-Grade 60 steel specimens experienced ductile flexural failure with similar wide crack opening after the rebar yielded. On the other hand, UHPC-
MMFX specimens largely failed in shear from a diagonal tension crack and crush of concrete top fiber.
To my great parents,

who have always provided me the best of their love, encouragement and supported me financially.
ACKNOWLEDGMENTS

Back to one year ago, while I was doing well with my coursework classes, I thought that doing research would not be a different case. However, after this whole one year of research in which I started from preparing test specimens, conducting experimental investigation, doing analysis, and at the same time experiencing unexpected problems and trying to find solutions, I have realized that research is way more valuable and difficult, and I have learned a lot from it.

First, I would like to express my sincere gratitude to my advisor, Dr. Kevin Mackie, who has given me an opportunity to work on this research project, and provided me with support, inspiration, and valuable guidance. This thesis would not be possible without him.

Second, I would like to thank my committee members, Dr. Necati Catbas and Dr. Nicos Makris for their continuous help and essential suggestions.

Finally, my thank goes to research mates Munaf Al-Ramahee and Haider Al-Jelawy for their assistance as well as Juan Cruz, Structures Lab coordinator, for his help and explanation on using the test equipment.
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CHAPTER 1: INTRODUCTION

1.1 Introduction

In the early 1960s, the introduction of steel fibers resulted in development of fiber reinforced cement or concrete composites (FRC). FRC can be defined as a composite material incorporating two main components such as matrix and steel fiber. The matrix, a composite material itself, including either cement paste, mortar or concrete plus aggregates and additives, represents the first main component of FRC composites. The discontinuous steel fiber represents the second main component. Naaman [1] proposed two simple classifications of FRC based on their stress-strain response in tension, either strain-softening or strain-hardening. Strain-softening FRC are characterized by their immediate localization after first cracking along with increasing strain and stress reduction. As for stain-hardening FRC, after first cracking, the stress increases with strain until the peak, post-cracking stress and multiple cracks are visible. After that localization, stress reduction with increasing strain occurs, reflecting strain-softening FRC cases.

Ultra-high performance concrete (UHPC), a recently developed type of concrete having very high strength and high durability, falls into the strain-hardening concrete category. Defined by the Association Francaise de Genie Civil (AFGC)[2], UHPC is a type of concrete exhibiting at least 21.7 ksi compressive strength and 1 ksi of tensile strength with steel fiber to ensure ductile behavior of the concrete. In addition, UHPC has very low water content, but appropriate granular packing plus addition of high-range water reducing admixtures ensure its good rheological properties. Despite its relatively recent introduction, UHPC has been applied to various construction projects including public highway bridges, pedestrian bridges, and many other projects. Although there are many types of UHPC available around the globe, Ductal- a product developed by Lafarge is one of the only few commercially available UHPC in the United States, and it is therefore the UHPC to be discussed.
Due to several downsides of existing open grid steel deck system of movable bridges in Florida, such as poor skid resistant surface, noise disturbance, high corrosion rate, and high maintenance cost, research was funded by FDOT to find alternative deck systems [3]. Based on thorough experimental and analytical results, the Ductal - MMFX waffle deck system even without shear reinforcement provided desired load capacity and light-weight requirement, and it is; therefore, one the the potential alternatives to replace the existing deck system. However, shear failure was usually the main modes of failure of the deck system, yet acceptable ductility was observed due to the ultra strength of the concrete as well as the bridging effect of steel fibers.

1.2 Research Objectives and Plans

The overall objective of this research is to quantify the sensitivity of UHPC flexural members under different shear and normal stress demands and investigate their failure mechanisms. To accomplish the research objective, the following research plans are required.

1. Perform experimental tests of UHPC prisms under different condition of reinforcement: without transverse reinforcement, reinforced with ASTM Grade 60 rebar, and reinforced with high strength steel (MMFX Grade 100).

2. Apply Modified Compression Field Theory (MCFT) [4] which is capable of analyze reinforced concrete elements under bi-axial stresses, shear and normal stresses, to predict UHPC flexural members responses.

3. Use Finite Element Analysis software VecTor2 [5] based on continuum methods to simulate the UHPC structural members and make comparisons with experimental and MCFT analysis results.
1.3 Thesis Outline

In total, this thesis is organized into 6 different chapters with content summarized as follows:

Chapter 1, this Introduction chapter, briefly describes about history and definition of Fiber Reinforced Composites (FRC) as well as Ultra-high performance concrete (UHPC) specifically Ductal which is commercially available in the United States. Furthermore, specific objectives of the thesis and the thesis outlines are listed.

Chapter 2 presents a literature review of materials used in this research such as UHPC, MMFX steel, and ASTM steel. In addition, the chapter provides background information regarding the response and failure mechanisms of reinforced concrete (RC), steel fiber reinforced concrete (SFRC), and UHPC flexural members. The chapter ends by presenting analysis methods in the sectional level, global, and FEM software.

Chapter 3 discusses in details the experimental programs investigated starting from specimens dimension, properties of UHPC in compression and tension, and those of MMFX and ASTM Grade 60, as well as instrumentation and testing setup; while Chapter 4 summarizes experimental results of the test specimens.

Chapter 5 describes the proposed analytical method from material model to structural level along with its results. Description and results of finite element analysis done in VecTor2 software is also presented herein.

Finally, several conclusions are drawn and presented in Chapter 6 together with the discussion of possible future researches.
 CHAPTER 2: LITERATURE REVIEW

To predict the responses of RC beams, a lot of different methods can be used depending on the analysis purposes. Linear analysis; for example, is a very simple and robust solution using widely available codes, yet the solution is limited to simple-shape homogeneous cross section, small deflection, and most importantly, the RC beams are assumed to be uncracked. However, if the analysis purpose is to identify the RC beams by including the contribution of concrete in tension, concrete in compression as well as reinforcement effect or some other complex phenomena of RC beams shear deformation effects, bond slip, crack shear slip...etc, nonlinear analysis has to be used. In particular, figuring out the response of UHPC beam which is known to be so critical in shear, a simple linear or even the 1D moment-curvature nonlinear analysis is incapable of providing realistic solution because the analysis model is only for unixial stress/strain problem. As a consequence, this research adopts sectional analysis MCFT which is a biaxial stress approach as the main analysis tool.

Nevertheless, with the development of powerful computers, numerical finite element method is also the option to investigate the nonlinear behavior of RC beams and RC structures as a whole. Within finite element method, nonlinear analysis of RC beams can be overcome by using one dimensional (1D) sectional analysis, two dimensional (2D) membrane analysis using finite element continuum method or even three dimensional models. However, the continuum methods do not guarantee good result for all cases and for structural problems, stress lock or concentration is a big concern when using FEA tools.

This chapter; therefore, presents the basis of the chosen analytical methods to investigate the force resultant interaction of Ultra-High Performance Concrete (UHPC) which is normally identified through its load-displacement behavior and failure mechanisms. The chapter starts by firstly introducing the material properties used in this research including UHPC, high strength steel (HSS), and ASTM grade 60 steel. Then, failure mechanisms of normal RC and UHPC beams are
briefly presented. Sectional MCFT and global Nonlinear analysis program (NAP) developed by [6] are next to be presented followed by a suitable FEA plane stress software-VecTor2.

2.1 Ultra-High Performance Concrete

Ultra-high performance concrete comes with several different brands worldwide with relatively different compositions and mechanical properties. Furthermore, even Ductal UHPC jointly developed by Lafarge, Rhodia, and Bouygues is mixed differently depending on types of construction applications. Below is constituent materials, production and typical mechanical properties of UHPC used in this research, which has been thoroughly tested by Graybeal [7].

2.1.1 Constituent Materials and Mixing Method of UHPC

The proportion of constituent materials of this UHPC is presented in the following Table 2.1 in forms of percentage by weight and weight relative to cement obtained from optimization of granular mixture.

<table>
<thead>
<tr>
<th>Constituent Material</th>
<th>Percentage by Weight(%)</th>
<th>Weight Relative to Cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland Cement</td>
<td>28.6</td>
<td>1.00</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>41.1</td>
<td>1.44</td>
</tr>
<tr>
<td>Ground Quartz</td>
<td>8.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Silica Fume</td>
<td>9.3</td>
<td>0.33</td>
</tr>
<tr>
<td>Super-plasticizer</td>
<td>0.5</td>
<td>0.02</td>
</tr>
<tr>
<td>Steel Fibers</td>
<td>6.4</td>
<td>0.22</td>
</tr>
<tr>
<td>Water</td>
<td>5.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The UHPC does not include any coarse aggregate; only finely graded granular materials are used to obtain highly homogeneous concrete matrix. The biggest grain is fine sand with the dimension between 150 and 600 µm, followed by cement with average dimension of 15µm, average-diameter crushed quartz of 10µm, and so tiny silica fume that helps increase compressive
strength by increasing cementitious material content as well as fill in the void spaces created by the larger aggregates. By adding extraordinarily small particles, like those which make up silica fume, the ability for water to infiltrate, or even escape the material is reduced. A low water to cement ratio (w/c) only 0.2 is used in UHPC to improve strong and high quality concrete, while w/c for normal concrete is usually between 0.4 to 0.5; therefore, super-plasticizer plays very essential roles to ensure good flow, consolidation and workability of UHPC. Moreover, accelerator may be employed to help reduce the set up time due to the use of large dosages of super-plasticizer.

Steel fiber used in UHPC has extremely high yielding strength up to 3150 MPa, ultimate strength of 3250 MPa and 210 GPa of modulus of elasticity. All fibers have the same dimension with a length of 12.7mm (0.5 inch), a diameter of 0.2mm (0.008 inch). 2% volume fraction of steel fiber is used in this UHPC which is a suitable amount to achieve a homogeneous distribution. Vodicka [8] mentioned that to achieve a homogeneous distribution, critical fiber volume fraction is between 0.4% to 0.5%; while homogeneous distribution was also obtained by Taerwe et al. [9] with 8% of steel fiber volume fraction.

Although more materials are needed to produce UHPC than normal concrete, mixing of UHPC is not time consuming and even more convenient because UHPC comes with premix either 50 lb or 80 lb per bag and one only needs to weight and add water (or ice), super-plasticizer, and steel fibers. Ice is preferably used instead of water to control the mixing temperature and working properties of UHPC and it is obligatory if the surrounding temperature is higher than 25°C. Besides, a high shear mixer is recommended when mixing UHPC to ensure homogeneity. UHPC is a self-consolidating concrete where mild or no vibration is needed; casting method commonly used is flow cast. The full detail procedures for mixing UHPC is given by Graybeal [7].

2.1.2 Mechanical Properties of UHPC

To fully characterize concrete behaviors, mechanical properties of the concrete have to be tested and UHPC is not a different case. For the purpose of using UHPC in bridge application,
GrayBeal tested UHPC specimens and grouped them into three categories including strength test (compressive and tensile strength), durability test (UHPC under standardized aggressors), and stability test (long-term dimensional stability). In this research, only mechanical properties of UHPC strength test is of interest. Compressive strength, modulus of elasticity, strain at peak stress and tensile strength (both first-crack and post-crack) are the necessary mechanical properties to be reviewed.

2.1.2.1 Compressive Strength

Based on 44 batches of approximately 300 UHPC cylinders compression test by Graybeal, the average 28-day compressive strength of UHPC was 193 MPa (28 ksi) for steam-treated specimens, and 126 MPa (18.3 ksi) for untreated specimens both with very close 95 percent confidence intervals. However, the compressive strength of steam treated UHPC reported by the manufacturer can reach up to 225 MPa (32.6 ksi). Two factors influenced the UHPC compressive strength tested by Graybeal. Firstly, the tested UHPC was demolded so early that it might be permeable and susceptible to moisture loss. Secondly, the studied UHPC received only 44 hours steam treatment level, while the recommended period was 48 hours. Association Francaise de Genie Civil also specifies in its "Ultra-High Performance Fibre-Reinforced Concretes Interim Recommendations report" [2] that compressive strength of UHPC is greater than 150 MPa (21.75 ksi) and possibly reaches 250 MPa (36.25 ksi).

2.1.2.2 Modulus of Elasticity and Strain at Peak Stress

The test to find out the modulus of elasticity of UHPC was also conducted from between 20 and 30 cylinders by Graybeal. The following values were found: 52.8 GPa (7650 ksi) for steam treated UHPC, and 42.8 GPa (6,200 ksi) for untreated UHPC. The cylinder test also provided another information on average strain at peak stress where the steam-treated and untreated cylinders gave overall strain at peak stress of 0.0041 and 0.0035, respectively.
Additional tests on UHPC strength accumulating up to 1000 cylinders in total led to three primary findings: UHPC is stabilized after steam or delayed steam treatment; untreated UHPC continues to gain strength for at least 8 weeks after casting whereas its increase in stiffness and decrease in strain at peak stress leveled off at 1 month; and lastly there was a dramatic increase in strength and stiffness of UHPC at early ages, but it became less ductile with a large decrease in strain at peak load.

2.1.2.3 Tensile Strength of UHPC

To study the characteristics of tensile properties of Ductal UHPC which are the first-crack strength and the post-crack strength, Chanvillard et Riguad [10] performed two different tests, four-point flexural test and direct tensile test, on both unnotched and notched specimens. Results showed that UHPC first-crack strength from direct tension test was 11.5 MPa (1.67 Ksi), compared to 18.8 MPa (2.73 Ksi) from flexural test. By considering the scale effect of flexural testing prisms with the coefficient $\alpha = 2$, Chanvillard recommended the design first-crack tensile strength of 11.5 MPa (1.67 Ksi). As for post-crack strength of UHPC, since the specimens were notched and multi-cracks were seen, not exact strength (15.5 MPa was found) was determined, but strain-hardening behavior of UHPC was observed.

However, Graybeal determined first cracking tensile strength of UHPC from four different tests including Split Cylinder, Prism Flexure, Mortar Briquette, and Direct Tension tests, where he recommended the average values of 9.0 MPa (1.3 Ks) and 6.2 MPa (0.9 Ks) for steam-treated and untreated UHPC, respectively; whereas, post-cracking strength of UHPC was taken from average strength of Mortar Briquette test to be 9.0 MPa for steam-treated and 6.2 MPa for untreated specimens.

AFGC; on the other hand, recommends the design value of 8.0 MPa (1.16 Ks) of tensile strength at first crack determined from direct tensile tests of 20 7x7x28 cm (2.75x2.75x1.1”) prisms, and flexural tests of 196 4x4x16 cm (1.6x1.6x6.4”) specimens of Chinon and Cattenom.
nuclear power plants project. AFGC also proposed one stress-strain relationship model as shown in 2.1 where for compressive side $\epsilon_{bc}$ is the elastic compressive strain, $\epsilon_u$ and $\sigma_{bcu}$ are the ultimate compressive strain and stress, respectively. As for tension side, $\epsilon_e$ is the elastic tensile strain, $\epsilon_{e0.3}$ is the tensile strain at crack width of 0.3mm, $\epsilon_{u1\%}$ is the tensile strain at crack width of 1% of specimen height, $\epsilon_{lim}$ is the tensile strain limit, $\sigma_{u1\%}$ is the tensile stress corresponding to $\epsilon_{u1\%}$, $f_{tj}$ is the first-crack tensile strength of UHPC, $\gamma_{bf}$ is a safety factor, and $\sigma_{bu}$ is the ultimate tensile strength.

![Figure 2.1: UHPC Stress-Strain Diagram Recommended by AFGC [2]](image)

Using stress-strain diagram recommended by AFGC can ensure more realistic behavior of UHPC; however, determining correct values of post-cracking tensile stress/strain is troublesome. Therefore, Eric [11] simplified AFGC stress-strain diagram of UHPC as shown in Figure 2.2.

Due to the lack of experimental tests of UHPC post-cracking tensile properties in his previous report and meanwhile to facilitate the development of a standardized quality assurance test program on UHPFRC structural elements, Graybeal [12] developed simple direct tension test (DTT) method based on 7 different groups of UHPC prisms.
Results show that the tensile behavior of UHPC experiences four different phases. Phase 1: Elastic behavior until first cracking is first observed. Phase 2: UHPC cementitious matrix repeatedly cracks, and this multi-cracking is bridged by steel fiber; nearly constant stress level is seen within this region. Phase 3- crack straining: In this phase steel fiber undergoes a combination of elastic straining and interface debonding; resulting in larger crack opening and the end of strain-hardening behavior of UHPC. Phase 4: crack localization occurs due to fiber debonding and pulling out of the matrix. The prism in this phase unloads elastically. The results of 2 set of specimens F2A-Long 5x5x42.5 cm (2x2x17”) and F2A-Short 5x5x30 cm(2x2x12”) having exactly the same composition as UHPC Ductal investigated in this research are shown as in Figure 2.3. Long specimens are recommended because the increased of prism length reduces the magnitude of bending stresses imparted during the initial gripping of the specimen.
2.2 High Strength Steel MMFX Compared with ASTM Steel

Defined by MMFX Steel corporation of America in [13], Micro-composite Multi-structural Formable Steel (MMFX) are uncoated, corrosion-resistant, high-strength steel-reinforcing products that meet or exceed the mechanical properties of ASTM A615 Grades 75 and 80, exceeding the requirement of ASTM A1035 and AASHTO MP 18. MMFX is a stronger but maintain the same ductility as Grade 60, 75, and 80 by almost totally eliminating carbide, which is the main corrosion initiator. In the United States, MMFX #3 through #11, #14 and #18 are standard bar sizes with standard length of either 18 m (60 ft) or 12 m (40 ft). MMFX has been widely used in various applications such as buildings, bridges, retaining walls, marine facilities, pavement, and other related cast-in-place and precast reinforced concrete members.

2.2.1 Corrosion Resistance

MMFX steel corrosion resistance, in terms of its critical chloride threshold level (CCTL), has been found to be more than 3 times compared to ASTM A615 conventional carbon steel bar. Studied by Texas A&M University, 2000, MMFX rebar in highly corrosive saline solution has very
little tendency of corrosion compared to A615 steel; therefore, thinner concrete cover is required resulting in higher flexural capacity of structural elements. Berke [14] described MMFX rebar had service life greater than 100 years; while Williamson and Weyers [15] mentioned by using MMFX in conjunction with low-permeability concrete, its service life is at least 200 years.

2.2.2 Strength, Bond with UHPC, and Advantages

MMFX steel bars exhibit very high strength because of two primary factors: chemical composition and manufacturing process. MMFX Gr. 100 has a yield strength of 690 Mpa (100 ksi) but its ultimate strength climbs up between 1138 Mpa (165 ksi) and 1205 Mpa (175 ksi) which is much higher than conventional steel Gr. 60 with approximately 690 Mpa (100 ksi) of ultimate strength (Figure 2.4). Wiss, Janney, Elstner Associates, Inc [16] performed tension test on MMFX Gr. 100 and Gr. 120 and results showed that MMFX Gr. 100 and Gr. 120 had average yield strength of 870 Mpa (126.2 ksi) and 945 Mpa (137 ksi) along with tensile strength of 1090 Mpa (158.1 ksi) and 1192 Mpa (172.9 ksi), respectively. Due to the high strength of MMFX, actual stress-strain curve in tension of MMFX rather than the assumed elastic-perfectly plastic stress-strain, is recommended for flexural design of concrete members.

Furthermore, MMFX has been investigated to have a very good bonding with UHPC. During flexural tests of reinforced UHPC with hooked MMFX, perfect bond was observed by Xia, J. [17]. Due to the high strength of MMFX, a few obvious advantages are observed. First, amount of steel within structural elements can be reduced, resulting in low labor cost. Second, rebar congestion can be avoided.
2.3 Failure Mechanisms of RC and UHPC Flexural Members

2.3.1 Failure Modes of RC Flexural Members

Reinforced concrete beam is the simplest structural element commonly used to carry both flexural bending and shear forces. Flexural-dominant RC beams are mostly encountered, and both the simplified and nonlinear analysis of flexural beams were well documented ensuring high accuracy of predicted responses. Flexural failure of RC beams can be identified through either yielding of longitudinal reinforcement or crush of concrete in compression. However, regarding shear critical RC beams, there have been a great number of uncertainty in predicting shear capacity as well as shear failure modes. Therefore, this section focuses on shear failure mechanism for RC beam.

Commonly there are four types of shear failure encountered in RC beam including flexural
shear failure, shear diagonal tension failure, web crushing due to compression strut, and yielding of transverse reinforcement as shown in Figure 2.5.

![Flexural shear failure](image)

![Shear tension failure](image)

![Web crushing](image)

![Yielding of the shear reinforcement](image)

Figure 2.5: Shear failure modes of RC beam (adopted from CLAUS [18])

Flexural shear failure usually happens as a result of a higher bending stresses comparing to shear stresses. This failure mechanism induces a slight crack rotation from the vertical plane. Four-point bending test is a good example of this shear failure mode where the middle portion of the RC beam is only under constant bending moment; while the beam portions between support and loading point are under combined $M$ and $V$.

Shear diagonal tension failure occurs for beam structures having high level of shear stress but a relatively low level of flexural stress. With this failure mode, cracking angle usually varies between $30^\circ$ and $45^\circ$ to the longitudinal reinforcement. The failure also happens between the loading position and the support; however, brittle failure is expected.
Web crushing happens when the RC beam has low amount of web surface but high ratio of unyielding shear reinforcement. This failure mode is equivalent to compression failure in the heavily reinforced structures loaded in flexure. Reinforced concrete I-beam with large flanges and small web dimensions tends to experience this failure mode, but there are no typical loading situation inducing this failure mode.

Yielding of shear reinforcement; on the other hand, is a failure mechanisms to be expected when the beam design is correct. This means that stirrup starts to yield before failure occurs involving a considerable deformation and provide warning before reaching the moment of failure. The failure type has a smeared crack pattern with a small number of dominating cracks. There are no typical load situation where only this type of failure occurs.

2.3.2 Failure Modes of UHPC Flexural Members

Failure modes of UHPC or SFRC have not yet been well distinguished and documented based on the state of loading configuration or flexural members sectional shapes. Therefore, this section is based mainly on experimental test data from three different researchers on SFRC beams and UHPC deck systems including Imam, M. et al. [19], Minelli, F. [20], and Xia, J. et al. [17].

<table>
<thead>
<tr>
<th>Group</th>
<th>Beam</th>
<th>a/d</th>
<th>Failure type*</th>
<th>I</th>
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*SC = shear-compression failure; DT = diagonal tension; FL = flexural failure; SF = combination of shear and flexural failure

$f_c = 110$ MPa; $f_y = 550$ MPa, $d = 300$ mm, $d_f = 14$ mm

Figure 2.6: Test data of high strength concrete beams without stirrups [19]

Imam has done extensive research on high-strength concrete beams without transverse reinforcement and with or without hooked steel fiber. The research aimed to investigate the significant
role of steel fibers in increasing the beam strength up to its full flexural capacity. In addition, he also developed an analytical approach to determine the domain of shear effect by which the beam failure modes can be predicted. Figure 2.6 shows his experimental tests on four different groups of specimens with different reinforcement ratio of 1.87% and 3.00%, without steel fiber and with steel fiber of 0.5625% where beam failure modes such as shear-compression failure, diagonal tension, and flexural failure were captured based on $a/d$ ratio.

Based on the research, Imam drew several conclusions which are worth to be mentioned here. Inclusion of steel fibers in HSC beam without stirrups helps improve shear resistance significantly and increase ultimate strength of the beam to reach nominal flexural capacity. Failure modes of HSC can be determined analytically by means of fiber ratio or longitudinal ratio versus $a/d$ ratio. Last but not least, inclusion of optimal percentage of steel fiber for a singly RC beam without stirrup, shear failure can be avoided regardless of $a/d$ ratio.

Minelli, on the other hand, did experimental test on two FRC shear critical large-scale beams (Figure 2.7) with the same dimension, fiber content of 50 kg/m$^3$ ($V_f = 0.64\%$) but different aspect ratio $\frac{l_f}{d_f} = 45/30$ and 80/30, respectively. The HSC-FRC1 beam has the compressive strength of 61.1 Mpa and tensile strength of 3.48 Mpa; while those of FRC2 are 58.3 Mpa and 3.20 Mpa, respectively.

![Figure 2.7: Comparison of final crack of different HSC](20)
The test showed that at the beginning when loading was half the failure load, both specimens experience micro flexural crack; however, as the ultimate loading reached, specimen HSC-FRC1 developed diagonal tension crack from the loading point to the supports, while HSC-FRC2 developed flexural crack propagated from the bottom fiber of the beam vertically to the loading points. The purpose of mentioning this test is only to show another effect of fiber aspect ratio resulting in two different failure modes.

Whereas Xia [17] did the experimental tests on large-scale UHPC waffle deck system whose material constituents were the same to the ones used in this research. The deck system also used MMFX steel rebars. The purpose of the project was to develop light-weight deck system as the alternative to the existing troubling open grid steel deck. Many full-scale decks were tested and shear failure modes were observed for most of the deck as shown in 2.8.

![Figure 2.8: Failure mode of four 1 span UHPC deck with: a)180° hook MMFX rebar, b) unhook MMFX, c) 180° hook Grade 60, d) 180° hook MMFX with stirrup (Adopted from Xia [17])](image-url)
Based on the above experiment, diagonal shear failure was all seen as the failure modes of the deck system with different anchorage conditions and reinforcing bar types. However, the failures were not abrupt involving large deformation and sectional rotation. For UHPC deck without stirrup, dowel action was seen to provide some contribution to shear resistance.

2.3.3 Shear Transfer mechanisms in cracked RC and UHPC beams

Shear transfer mechanisms in RC and SFRC beam highly depends on the states of stress, the opening of crack and the restraint conditions. The mechanisms include several number of complex phenomena as distinguished by [21] such as shear in the uncracked zone, aggregate interlock, dowel action, residual tensile stresses across the crack, and axial steel stress (Only for normal RC beam) as illustrated in Figure 2.9 which are to be briefly described as follows:

(a) Aggregate interlock  (b) Dowel action  (c) Axial steel stress

Figure 2.9: Shear transfer mechanisms in cracked concrete (adopted from CLAUS [18])

Shear stresses that are present in the compression zone of the concrete contribute to the shear resistance in a concrete member. The magnitude of that shear resistance is limited by the depth of the compression zone. Hence, in a relative slender beams without axial compression, the shear contribution becomes relatively small, due to the minimal depth of the compress zone.

Shear transfer through aggregate interlock was extensively researched by Walraven [22].
For normal concrete, cracking happens between cement matrix and aggregate due to the difference of the materials’ strength. As shown in Figure 2.9a, the interface between the matrix and aggregate is considered as rough; therefore, as long as the aggregate size is bigger than the crack width, shear slip can be prevent and shear stress can be transferred. This local shear crack transfer mechanism as defined by [22] is in functions of concrete compressive strength, crack width, and aggregate size.

Dowel action refers to the shear resisting force provided by longitudinal reinforcement bars. In the case of normal RC beam with shear reinforcement, the resulting vertical displacements of the bottom reinforcing bars near the diagonal crack surface are restrained and dowel action contributes considerably to the shear resistance. However, when in absence of shear reinforcement as in the case of UHPC, dowel action is less significant because the maximum shear in the reinforcement is instead limited by the tensile strength of concrete cover supporting the bars.

Tensile strength of concrete does not vanish immediately after the first crack. As described in [23], after cracking is formed, tensile stresses gradually drops by the effect of tension stiffness on reinforcement. On top of this, because generally concrete does not crack by a clean break, concrete itself can still transfer tensile stress across the crack face until macro cracks happen as results of increasing strain.

Shear transfer mechanism through axial steel stress mainly refers to RC members with transverse reinforcement. The component of the steel stress normal to the crack plane provides a contribution to the transfer of stresses across a crack. The magnitude of this force is highly dependent on amount of reinforcement as well as the bond properties. Shear reinforcement provides not only a large transfer of shear forces but also a great level of restraint against the growth of inclined cracks and thus helps to ensure a more ductile behavior.
2.4 Modified Compression Field Theory Sectional Analysis

Modified Compression Field Theory (MCFT) is a smeared rotating approach treating cracked concrete as a new orthotropic material with its own average stress-strain [23]. Different from the fixed crack approach where crack orientation is fixed after first cracking happens, crack orientation in MCFT changes based on the loading intensity. The method is capable for predicting the behavior of two dimensional RC structure with complex geometry under combined shear and normal stresses which is a big advantage over standard sectional analysis where normal and shear forces are uncoupled and there is no effect of shear deformation on normal deformation is considered. The method is based on three principles of solid mechanics of the concrete including compatibility, equilibrium, and constitutive relations as well as additional local crack check.

2.4.1 Compatibility

The compatibility requires that any deformation experienced by the concrete must be identical to the deformation of reinforcement. This means that concrete and reinforcing steel are assumed to have perfect bond. Also, the angles of inclination of principal strain and stress are assumed to coincide. With the known global strains \([\epsilon_x \epsilon_y \gamma_{xy}]^T\) shown in Figure 2.10, principal tensile strain \(\epsilon_1\), principal compressive strain \(\epsilon_2\) and cracking direction can be determined from Mohr’s circle.

\[
\epsilon_x = \epsilon_{cx} = \epsilon_{sx} \tag{2.1}
\]

\[
\epsilon_y = \epsilon_{cy} = \epsilon_{sy} \tag{2.2}
\]

\[
\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\sqrt{(\epsilon_y - \epsilon_x)^2 + \gamma_{xy}^2}}{2} \tag{2.3}
\]

\[
\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\sqrt{(\epsilon_y - \epsilon_x)^2 + \gamma_{xy}^2}}{2} \tag{2.4}
\]

\[
\theta_\sigma = \theta_\epsilon = \frac{1}{2} \arctan \left( \frac{\gamma_{xy}}{\epsilon_y - \epsilon_x} \right) \tag{2.5}
\]
2.4.1.1 **Equilibrium Conditions**

Forces applied to the reinforced concrete element are resisted by both concrete stresses $f_{Cx}$ and $f_{Cy}$ and reinforcement stresses $f_{sx}$ and $f_{sy}$ (Figure 2.11); while the applied shear stress $\tau_{cxy}$ has to be balanced by average concrete shear stress $v_{xy}$. The equilibrium conditions in both the x- and y-directions are:

\[
\begin{align*}
    f_x &= f_{Cx} + \rho_{sx} f_{sx} \\
    f_y &= f_{Cy} + \rho_{sy} f_{sy} \\
    v_{xy} &= \tau_{cxy}
\end{align*}
\]
2.4.2 Constitutive Relations of Concrete

Generally, different stress-strain curves can be chosen for different types of concrete and reinforcement. Originally, Hognestad model with compression softening was suggested for normal reinforced concrete in compression proposed by Vecchio and Collins 1986 [23]. While in tension, concrete deformed elastically up to the first cracking strength, after that stress of concrete dropped following concrete tension stiffness behavior also proposed by [23], whereas elastic-perfect plastic response was assumed for reinforcing bars. After that many other stress-strain curves for both concrete in compression and in tension have been proposed by different researchers for different types of concrete used, and correct selection of the model (especially for concrete in tension) is necessary to obtain reliable RC response. Currently, available concrete stress-strain behaviors to be used with MCFT can be found in [5].
2.4.3 Local Crack Conditions

A complete MCFT, as mentioned early, considers average tensile stresses of reinforced concrete elements even after cracking through tension stiffening. However, at the crack opening location, concrete does not carry any tensile stresses. To satisfy average stresses equilibrium, reinforcement additionally carries the average tensile stresses, and stresses of reinforcement at the cracking location is; therefore, to be maintained under the yielding strength. Meanwhile, interface shear stress is also developed on the crack surface and carried by aggregate interlock mechanism, and it is limited by the allowable interface shear stress given by Walraven [22]. However, for Steel Fiber Reinforced Concrete (SFRC) or UHPC, stress at cracking location is also carried by tensile strength of the concrete due to fiber pull-out strength. A realistic computation of fiber contribution to local crack condition has not yet been proved in MCFT for SFRC. Therefore, for UHPC analysis in this research, no local crack is checked. Figure 2.13 provides a summary of complete MCFT equations for normal RC elements.

![Figure 2.12: a) Average stress at uncrack, b) Local stresses at crack, and c) Aggregate interlock](image)

Figure 2.24 – Crack check in MCFT: a) average and b) local stresses at a crack (Wong and Vecchio)
2.4.4 Secant Stiffness Formulation

With MCFT sectional analysis, a beam section is discretized into $m$ fibers and $n$ longitudinal reinforcing bars. Each concrete fiber is treated separately as a new orthotropic material having its own stress-strain characteristics, and satisfaction of three conditions are required including equilibrium, compatibility, and constitutive relations. In the originally developed method implemented in program SMALL by Vecchio, finding stress/strain values of each fiber involved two-loop iteration process of square root and trigonometric equations occasionally causing instability and is time-consuming. Therefore, a very robust secant stiffness method proposed by Krpan [25] is adopted in this research.

In the consideration of shear effect in the section, Vecchio (1988) proposed three different
approaches: rigorous dual section, approximate constant shear flow distribution, and approximate parabolic shear strain distribution. Dual section provides exact solution with varying shear strain and shear flow distribution across the section; however, the approach involves the analysis of second section at a small distance \( \frac{h}{3} \) from the first section to find shear flow equilibrium. Assumed constant shear flow and parabolic shear strain distribution; on the other hand, were reported as shown in Fig. 5.1 to provide very close results and much quicker computation time. As the consequence, parabolic shear strain distribution is assumed herein.

With the assumption of parabolic shear strain \( \gamma_{xy} \) and the known longitudinal strain \( \epsilon_x \) determined from global analysis to be discussed in the next section, all strains and stresses of

![Figure 2.14: Shear flow and shear strain based on Vecchio rigorous and approximate approaches](image)

<table>
<thead>
<tr>
<th></th>
<th>Dual Section Analysis</th>
<th>Constant Shear Flow Analysis</th>
<th>Parabolic Shear Strain Analysis</th>
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</thead>
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<tr>
<td>Shear Flow Distribution (N/mm)</td>
<td>1.20</td>
<td>1.18</td>
<td>1.77</td>
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</table>
concrete can be determined from the following procedures as described by Guner [26]:

First of all, assume that only concrete net strain due to mechanical load exists; while concrete elastic offset strains (due to lateral expansion, thermal, shrinkage and prestrain effects), concrete plastic offset strains (due to cyclic loading and damage), and concrete crack slip offset strains (due to shear slip) are zero.

\[
[\epsilon] = [\epsilon_c] = \begin{bmatrix} \epsilon_x & \epsilon_y & \gamma_{xy} \end{bmatrix}^T
\]

(2.9)

Secondly, transverse strain \(\epsilon_y\) is assumed as zero for the first iteration and updated to the latest value of each iteration. With the known values of \(\epsilon_x\), \(\epsilon_y\), and \(\gamma_{xy}\), strain states can be obtained based on Mohr’s circle:

\[
\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\sqrt{(\epsilon_y - \epsilon_x)^2 + \gamma_{xy}^2}}{2}
\]

(2.10)

\[
\epsilon_2 = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\sqrt{(\epsilon_y - \epsilon_x)^2 + \gamma_{xy}^2}}{2}
\]

(2.11)

\[
\theta = \frac{1}{2} \arctan\left[ \frac{\gamma_{xy}}{\epsilon_y - \epsilon_x} \right]
\]

(2.12)

Based on the adopted concrete constitutive models, the corresponding principal tensile and compressive stresses \(f_{c1}\) and \(f_{c2}\) respectively can be calculated.

Thirdly, the concrete material secant modulus can be determined as the followings:

\[
\bar{E}_{c1} = \frac{f_{c1}}{\epsilon_{c1}}
\]

(2.13)

\[
\bar{E}_{c2} = \frac{f_{c2}}{\epsilon_{c2}}
\]

(2.14)

\[
\bar{G}_c = \frac{\bar{E}_{c1} \cdot \bar{E}_{c2}}{\bar{E}_{c1} + \bar{E}_{c2}}
\]

(2.15)

Because reinforced concrete is considered as orthotropic material in the principal stress
directions, the concrete material stiffness can be written as:

\[
[D_c]' = \begin{bmatrix}
\bar{E}_{c1} & 0 & 0 \\
0 & \bar{E}_{c2} & 0 \\
0 & 0 & \bar{G}_c
\end{bmatrix}
\]  \hspace{2cm} (2.16)

The principal material matrix is then transformed to global axis x and y through transformation matrix \([T_c]\):

\[
[T_c] = \begin{bmatrix}
cos^2\theta & sin^2\theta & cos\theta sin\theta \\
sin^2\theta & cos^2\theta & -cos\theta sin\theta \\
-2cos\theta sin\theta & 2cos\theta sin\theta & cos^2\theta - sin^2\theta
\end{bmatrix}
\]  \hspace{2cm} (2.17)

\[
[D_c] = [T_c]^T[D_c]'[T_c]
\]  \hspace{2cm} (2.18)

Similarly, smeared reinforcement secant modulus in x and y direction as well as the reinforcement stiffness matrix can be determined as follows:

\[
\bar{E}_{sx} = \frac{f_{sx}}{\varepsilon_{sx}} \hspace{2cm} (2.19)
\]

\[
\bar{E}_{sy} = \frac{f_{sy}}{\varepsilon_{sy}} \hspace{2cm} (2.20)
\]

\[
[D_s] = \begin{bmatrix}
\rho_x \bar{E}_{sx} & 0 & 0 \\
0 & \rho_y \bar{E}_{sy} & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  \hspace{2cm} (2.21)

Each fiber stress can then be determined through the multiplication of composite material stiffness \([D] = [D_c] + [D_s]\) with the known strain matrix \([\varepsilon]\).
By assuming that there is no clamping stresses in the transverse direction $\sigma_y = 0$, calculated transverse strain $\epsilon_y$ can be found and compared with the assumed $\epsilon_y$ at the beginning. If the results converge, fiber longitudinal stress $\sigma_x$ and shear stress $\tau_{xy}$ can be calculated.

$$
\epsilon_y = -\frac{D_{21}\epsilon_x - D_{23}\gamma_{xy}}{D_{22}}
$$

(2.22)

$$
\sigma_x = D_{11}\epsilon_x + D_{12}\epsilon_y + D_{13}\gamma_{xy}
$$

(2.23)

$$
\tau_{xy} = D_{31}\epsilon_x + D_{32}\epsilon_y + D_{33}\gamma_{xy}
$$

(2.24)

In frame analysis MCFT-based program VecTor5, 100 iterations of $\epsilon_y$ is the default; however, based on the analysis in this thesis, frequently fewer than 25 iterations will result in convergence.

After longitudinal stresses $\sigma_x$ and shear stresses $\tau_{xy}$ are found, section forces $N, M, V$ can be determined from sectional equilibrium of concrete and longitudinal reinforcement as follows:

$$
N = \sum_{i=1}^{m} \sigma_x b_i h_i + \sum_{j=1}^{n} f_{sxj} A_{sxj}
$$

$$
M = \sum_{i=1}^{m} \sigma_x b_i h_i (y_{ci} - \bar{y}) + \sum_{j=1}^{n} f_{sxj} A_{sxj} (y_{sj} - \bar{y})
$$

$$
V = \sum_{i=1}^{m} v_{xyi} b_i h_i
$$
2.4.5 Global Analysis

MCFT is a good sectional analysis approach to predict responses of shear critical beams. In addition, because behavior of UHPC beams highly depends on its post tensile strength, analysis approach at the global/structural level which is capable of capturing the beam’s post-peak response is necessary. Therefore, Nonlinear Analysis Program (NAP) developed by Mackie, K. [6] which can simulate responses of various RC structural elements including beam, column, frame, etc. by employing displacement-based or force-based loading is chosen as a global analysis tool. MCFT is then integrated into NAP to predict the full response of UHPC beams. The MCFT-NAP displacement-based loading implementation is described in this section.

In NAP as other nonlinear analysis software, there are one global system and two local systems including local basic and local complete as shown in Figure 2.15.

At first the imposed displacement in the global system is transformed into local complete system and then transformed from local complete system to local basic system. In the local basic
system, with known end displacements \( v_1, v_2, \) and \( v_3, \) sectional deformation \( \epsilon \) can be determined as follows:

\[
\begin{bmatrix}
\epsilon_a \\
k \\
\gamma_a
\end{bmatrix} = \begin{bmatrix}
\frac{dN_1}{dx} v_1 \\
\frac{d^2N_2}{dx^2} v_2 + \frac{d^2N_3}{dx^2} v_3 \\
\frac{d^3N_2}{dx^3} v_2 + \frac{d^3N_3}{dx^3} v_3
\end{bmatrix}
\]

Utilizing displacement-based elements requires the displacement and curvature field along the element be determined from the same shape functions (and their derivatives). In classical Euler-Bernoulli flexural theory, there are no shear strains or rotations of the beam relative to the section. So, in this thesis, an approximate shear strain field is derived from the third derivative of the shape function. In this case \( \gamma_a \) is then constant within each element (as with the shear force), and by dividing the global beam into many numbers of elements and iterating, a more representative shear strain distribution along the span of the beam can be obtained.

Shape functions \([N]\) of each element in local basic system is given by:

\[
[N] = \begin{bmatrix}
N_1 & 0 & 0 \\
0 & N_2 & N_3
\end{bmatrix}
\]

(2.25)

\[
[N] = \begin{bmatrix}
\frac{1}{L} x & 0 & 0 \\
0 & \frac{1}{L^2} x^3 - \frac{2}{L} x^2 + x & \frac{1}{L^2} x^3 - \frac{1}{L} x^2
\end{bmatrix}
\]

(2.26)

With the known section deformations \( \epsilon_a, k, \) and \( \gamma_a, \) axial strains of each concrete fiber \( \epsilon_x \) can be determined by the basic assumption of plane section remains plane after deformation, while layer shear strain \( \gamma_{xy} \) is calculated from the parabolic function (Figure 2.16).

\[
\epsilon_x = \epsilon_a + k.y
\]

(2.27)

\[
\gamma_{xy} = \gamma_a \left(1 - \frac{4y^2}{h^2}\right)
\]

(2.28)
At this part is where Modified Compression Field theory is used to determine section forces and after that a finite difference approach was adopted to obtain the resulting section stiffness $k_s$. Because the normal force $N$ and bending moment $M$ already take into effect the stress $V$ due to strain interaction within MCFT sectional analysis, only section forces $M$ and $N$ and the first $2 \times 2$ components of section stiffness matrix are used to determine end forces $q_b$ and stiffness matrix $K_b$ in local basic system. Thereafter, the forces and stiffness matrix are transformed to local complete system $q_c$ and $K_c$, then to global system as $q_e$ and $K_e$ simply through transformation matrix from local basic to local complete $[T_{BC}]$ and from local complete to global level $[T]$. 

\[
[s] = \begin{bmatrix} N \\ M \\ V \end{bmatrix}
\]  

(2.29) 

\[
[k_S] = \begin{bmatrix} \frac{dN}{de_a} & \frac{dN}{dk} & \frac{dN}{d\gamma_a} \\ \frac{dN}{dM} & \frac{dN}{dk} & \frac{dN}{d\gamma_m} \\ \frac{dV}{de_a} & \frac{dV}{dk} & \frac{dV}{d\gamma_a} \end{bmatrix}
\]  

(2.30)
\[ q_b = \int [N]^T [s] dx \]  
(2.31)

\[ K_b = \int [B]^T [k_s] [B] dx \]  
(2.32)

\[ q_e = [T_{BC}] \cdot [q_b] \]  
(2.33)

\[ K_e = [T_{BC}]^T [K_b] [T_{BC}] \]  
(2.34)

\[ q_e = [T^T] q_e \]  
(2.35)

\[ K_e = [T] [K_e] [T] \]  
(2.36)

Finally, equilibrium of global forces are to be checked and if residual force existing, iterative process with increment of the applied global displacement through global stiffness matrix \( K_g \) will be used. \( K_g \) is simply the summation of element stiffness.

\[ P - P_r (u) = 0 \]  
(2.37)

\[ u_{i+1} = u_i + K_g^{-1} P_u \]  
(2.38)

An alternative approach to using displacement-based elements with assumed internal deformation fields is to use force-based elements that accurately capture the internal force resultants along the length of the beam. These force-based elements use force-interpolation functions instead of the shape functions \([N]\). The MCFT section lends itself naturally to the use of the force-based elements as the shear and moments equilibrium are satisfied directly, and the corresponding shear deformations are also included in the predicted displacements.

Because displacement-based elements is mainly the 1D finite element method where accurate result can be obtained through more discretization of elements. It is expected that using forced-based elements will result in slightly different prediction of loading capacity. As the force-based elements is naturally the nature of Modified Compression Field Theory, better responses
maybe captured. However, the post-peak softening responses provided by displacement-based elements would be more suitable for FRC behavior like UHPC. As a result, this thesis would base the analysis on displacement-based elements, but a comparison of load-displacement responses between force-based and displacement elements is also illustrated at the end of Chapter 5.

2.5 Finite Element Analysis using Continuum Method

In the analysis of RC structural elements under biaxial stresses, finite element using continuum method is always one the plausible option if correct meshing element size to avoid stress concentration can be made. VecTor2, a 2D finite element analysis program developed by University of Toronto based on the smeared, rotating cracks Modified Compression Field Theory (MCFT) and Disturbed Stress Field Model (DSFM) [5], is the second analysis tool in this research.

The program has the ability to simulate various types of reinforced concrete structures under monotonic, cyclic and reverse cyclic loading. Furthermore, VecTor2 is capable of modeling concrete compression softening, tension softening and stiffening, concrete expansion and confinement, hysteretic response, bond slip, crack shear slip deformations, reinforcement dowel action and crack allocation. Most importantly, the program contains user-defined tension stress-strain diagram for concrete which enables the modeling of steel fiber reinforced concrete (SFRC) which is material of interest in this research.

In VecTor2, element types (Figure 2.17) available for modeling concrete and smeared concrete are three-node constant strain triangle elements, four-node plane stress rectangular elements, and four-node quadrilateral elements; whereas two-node truss bar elements are available for modeling discrete reinforcement together with a two-node link and a four node contact element for modeling bond-slip effects. Therefore, the program is good option for this research purpose.
VecTor2 algorithm for nonlinear finite element analysis is summarized by the flow chart in Figure 2.18 where the procedure is very similar to secant stiffness modulus as described earlier; while more details of the procedure can be found in [5].
Determine average concrete and reinforcement stresses $f_{c1}$, $f_{c2}$, $f_{si}$

Determine local concrete and reinforcement stresses at cracks $v_{ci}$, $f_{scri}$

Determine crack slip strain components $[\varepsilon']$

Determine average concrete and reinforcement stresses $f_{c1}, f_{c2}, f_{si}$

Determine local concrete and reinforcement stresses at cracks $v_{ci}, f_{scri}$

Determine crack slip strain components $[\varepsilon']$

Determine secant moduli $E_{c1}, E_{c2}, G_{c}, E_{si}$

Determine material component stiffness matrices $[D_c], [D_s]$.

Determine composite material stiffness matrix $[D]= [D_c] + \Sigma [D_s]$.

Input analysis control data

Input structure data

Input external load data

Update external loads for load stage, $[F]$.

Update external loads for load stage $[F]$.

Determine element stiffness matrix, $[k]$.


Determine total load vector $[F^\prime] = [F] + \Sigma [F^\ast]$.

Determine structure stiffness matrix $[K] = \Sigma [k]$.

Determine nodal displacements $[r] = [K]^{-1} [F^\prime]$.

Determine element strains $[\varepsilon_e], [\varepsilon_s], [\varepsilon_i]$.

Determine element stresses, $[\sigma]$.

Update secant moduli $E_{c1}, E_{c2}, G_{c}, E_{si}$.

Secant moduli converged?

Steps to be performed for each element.

Figure 2.18: VecTor2 nonlinear finite element analysis algorithm

Output load stage results.

Determine pseudo nodal load vector $[F^\ast]$. Proceed to next load stage?

Analysis complete.

Update strain and stress parameters.
CHAPTER 3: EXPERIMENTAL PROGRAM

To interpret the behavior of UHPC Ductal under combined flexural and shear load, experimental test evidence on different sizes of small scale prisms as well as material compression and tension test are essential. The chapter begins from describing different dimensions of the test specimens to material characterization tests of UHPC, ASTM steel grade 60, and MMFX grade 100 steel. In addition, specimen preparation is discussed and finally followed by instrumentation and testing setup.

3.1 Test Specimens

The experiments consisted of a total number of 26 UHPC prisms where 6 prisms reinforced with MMFX steel were obtained from old experimental tests conducted by previous researcher [17], and 20 other prisms were prepared and casted in Structures Laboratory of University of Central Florida. The specimens were designed to have different cross-sectional areas and shear span ratio $a/d$ in the way to induce and capture different failure modes, shear or flexural failure. All specimens were designed without transverse shear reinforcement.

The test included three different groups of UHPC specimens. Group1 specimens were unreinforced UHPC with the dimensions as follows: three 2x2x12”, three 1.5x5.0x12”, and two 2x2x8” prisms. The specimens were tested under 3-point bending test. Group 2 specimens consists of three 1.5x4.0x26”, three 1.5x4.0x18”, and two 1.5x3.0x7.5” prisms subjected to 3-point bending test, and two 1.5x3.0x26” plus two 1.5x4.5x18” prisms under 4-point bending. The specimens were singly reinforced with Grade 60 steel. Group3 specimens were prisms reinforced with #3 high strength steel MMFX grade 100. The group contained three 1.5x3.5x26” and three 1.5x3.5x18” prisms tested under 3-point bending test. The specimen dimensions and loading configurations of each group are illustrated in Figure 3.1 to 3.2 and tabulated in table 3.1 as below:

36
Figure 3.1: Group 1 Unreinforced UHPC specimens

<table>
<thead>
<tr>
<th>No</th>
<th>Loading Type</th>
<th>a/d Ratio</th>
<th>Dimension bxdxL</th>
<th>No of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1-01</td>
<td>3-point</td>
<td>3.00</td>
<td>2x2x12</td>
<td>3</td>
</tr>
<tr>
<td>G1-02</td>
<td>3-point</td>
<td>1.20</td>
<td>1.5x5x12</td>
<td>3</td>
</tr>
<tr>
<td>G1-03</td>
<td>3-point</td>
<td>2.00</td>
<td>2x2x8</td>
<td>2</td>
</tr>
</tbody>
</table>

Specimen Details

Figure 3.2: Group 2 and Group 3 UHPC specimens reinforced with Gr. 60 and MMFX steel

<table>
<thead>
<tr>
<th>No</th>
<th>Loading Type</th>
<th>a/d Ratio</th>
<th>Dimension bxdxL</th>
<th>No of Spec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>G2-01</td>
<td>3-point</td>
<td>3.25</td>
<td>1.5x4x26</td>
<td>3</td>
</tr>
<tr>
<td>G2-02</td>
<td>3-point</td>
<td>2.25</td>
<td>1.5x4x18</td>
<td>3</td>
</tr>
<tr>
<td>G2-03</td>
<td>4-point</td>
<td>2.75</td>
<td>1.5x3x26</td>
<td>2</td>
</tr>
<tr>
<td>G2-04</td>
<td>4-point</td>
<td>1.55</td>
<td>1.5x4.5x18</td>
<td>2</td>
</tr>
<tr>
<td>G2-05</td>
<td>3-point</td>
<td>1.25</td>
<td>1.5x3x7.5</td>
<td>2</td>
</tr>
<tr>
<td>G3-01</td>
<td>3-point</td>
<td>3.25</td>
<td>1x3.5x18</td>
<td>3</td>
</tr>
<tr>
<td>G3-02</td>
<td>3-point</td>
<td>2.25</td>
<td>1x3.5x26</td>
<td>3</td>
</tr>
</tbody>
</table>

Specimen Details
Table 3.1: Summary of Test Specimens

<table>
<thead>
<tr>
<th>Group</th>
<th>Prism No</th>
<th>Dimension</th>
<th>Loading</th>
<th>a/d ratio</th>
<th>Number</th>
<th>Rebar</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01</td>
<td>2.0x2.0x12&quot;</td>
<td>3-point</td>
<td>3.00</td>
<td>3</td>
<td>Unreinforced</td>
</tr>
<tr>
<td>1</td>
<td>02</td>
<td>1.5x5.0x12&quot;</td>
<td>3-point</td>
<td>1.20</td>
<td>3</td>
<td>Unreinforced</td>
</tr>
<tr>
<td>1</td>
<td>03</td>
<td>2.0x2.0x8&quot;</td>
<td>3-point</td>
<td>2.00</td>
<td>2</td>
<td>Unreinforced</td>
</tr>
<tr>
<td>2</td>
<td>01</td>
<td>1.5x4.0x26&quot;</td>
<td>3-point</td>
<td>3.25</td>
<td>3</td>
<td>#3 ASTM Gr. 60</td>
</tr>
<tr>
<td>2</td>
<td>02</td>
<td>1.5x4.0x18&quot;</td>
<td>3-point</td>
<td>2.25</td>
<td>3</td>
<td>#3 ASTM Gr. 60</td>
</tr>
<tr>
<td>2</td>
<td>03</td>
<td>1.5x3.0x26&quot;</td>
<td>4-point</td>
<td>2.75</td>
<td>2</td>
<td>#3 ASTM Gr. 60</td>
</tr>
<tr>
<td>2</td>
<td>04</td>
<td>1.5x4.5x18&quot;</td>
<td>4-point</td>
<td>1.55</td>
<td>2</td>
<td>#3 ASTM Gr. 60</td>
</tr>
<tr>
<td>2</td>
<td>05</td>
<td>1.5x3.0x7.5&quot;</td>
<td>3-point</td>
<td>1.25</td>
<td>2</td>
<td>#3 ASTM Gr. 60</td>
</tr>
<tr>
<td>3</td>
<td>01</td>
<td>2.0x3.0x26&quot;</td>
<td>3-point</td>
<td>4.33</td>
<td>3</td>
<td>#3 MMFX Gr. 100</td>
</tr>
<tr>
<td>3</td>
<td>02</td>
<td>2.0x3.0x18&quot;</td>
<td>3-point</td>
<td>3.00</td>
<td>3</td>
<td>#3 MMFX Gr. 100</td>
</tr>
</tbody>
</table>

3.2 Material Characterization

3.2.1 Concrete

3.2.1.1 Compressive Strength

Compressive strength of UHPC specimens is determined by using Instron Universal Testing Machine with the capacity of 224.8 kips (Figure 3.3) in the University of Central Florida according to ASTM C39 for concrete cylinders and ASTM C109 for cubes. Compressive strength of Group 3 specimens was tested by previous researcher [17] and results are summarized in Table 3.2 as Batch 3 (B3), whereas that of Group 1 and 2 specimens were tested by author from four 2x2x2” cubes and three 2”x4” cylinders (Figure 3.4) from two different casting batches, B1 and B2 as shown in the same Table 3.2. The compressive strengths determined were quite low comparing to those in literature review due to imperfect surface grinding.
Figure 3.3: Compression Test Using UTM

Figure 3.4: Cubes and Cylinders for Compression Test
3.2.2 First Crack and Post-Crack Tensile Strength

Tensile strength of UHPC can be determined either according to ASTM C1018 using 4x4x14” beam with Third-Point Loading, Graybeal four-point flexural loading preferably on beam with shear span ratio \( a/d \) of 1 modified with shape factor, or based on AFGC Interim Recommendation Report. However, because of limitation of accurate testing equipment, first crack tensile strength of the three batch specimens were adopted from previous research data on UHPC having similar compressive strength and results are summarized in Table 3.2.

Table 3.2: UHPC Compressive and Tensile Strength

<table>
<thead>
<tr>
<th>Batch No</th>
<th>Specimens ID</th>
<th>Cure duration (days)</th>
<th>Compressive strength ksi(Mpa)</th>
<th>First-crack tensile strength ksi(Mpa)</th>
<th>Ultimate tensile strength ksi(Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>G2-01, G2-02, G3-01</td>
<td>127</td>
<td>21(145)</td>
<td>1.16(8.0)</td>
<td>1.16(8.0)</td>
</tr>
<tr>
<td>B2</td>
<td>G2-03, G2-04, G2-05, G3-03, G3-04</td>
<td>103</td>
<td>19.5(135)</td>
<td>1.16(8.0)</td>
<td>1.16(8.0)</td>
</tr>
<tr>
<td>B3</td>
<td>G1-01, G1-02</td>
<td>28</td>
<td>16.97(117)</td>
<td>1.16(8.0)</td>
<td>1.16(8.0)</td>
</tr>
</tbody>
</table>

3.2.3 Rebar-MMFX and Grade 60

Tensile strength of MMFX and Grade 60 were tested following ASTM E8 procedures using UTM with extensometer for measuring strain corresponding to loading of UTM. Only one sample of each steel type #3 was tested (Figure 3.5) since there is very little difference of steel properties made by steel manufacturer. Their yielding and ultimate strength are summarized in Table 3.3. While the rebars’ actual tested stress-strain diagram are illustrated in Figure 3.6 and Figure 3.7. The observed stress-strain curve results of both types of reinforcing steel are very closed to the curves described in Chapter 2. Because the diagrams are not linearly elastic-perfect plastic as usually assumed to facilitate designing purpose, curve fit capturing the actual behaviors of the
reinforcing steel will be used as described in Chapter 5.

Figure 3.5: Direct Tension Test of MMFX and Grade 60 Steel

Table 3.3: MMFX and Gr. 60 reinforcing steel strength

<table>
<thead>
<tr>
<th>Steel Type</th>
<th>Rebar No.</th>
<th>Yield Strength ksi(MPa)</th>
<th>Ultimate Strength ksi(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMFX</td>
<td>#3</td>
<td>100 (689)</td>
<td>165(1138)</td>
</tr>
<tr>
<td>Grade 60</td>
<td>#3</td>
<td>60 (414)</td>
<td>105(723)</td>
</tr>
</tbody>
</table>
Figure 3.6: Stress-Strain Diagram for #3 MMFX

Figure 3.7: Stress-Strain Diagram for #3 Gr.60
3.2.4 Specimen Fabrication

This section illustrates the fabrication process of the aforementioned small scale UHPC prisms casted in Structures Laboratory of UCF, which includes formwork preparation, reinforcement and strain gauge installation, and a brief description of UHPC mixing and casting process; while the detailed procedure was earlier described in chapter 2.

Because test specimens are small scale prisms where the largest sectional area is 1.5x5.0” (38.1x127mm) and the largest length is 32” (812.8”), accurate dimensions of the specimens are necessary to avoid high different sectional capacity and out-of-plane failure. Consequently, Polystyrene Foam Board 2” (50.8mm) thickness was used as formwork sidewall and plywood was only employed as supporting base (Figure 3.8). After formwork had been prepared, for reinforced prisms #3 steel rebars were cut and bent to 90 degree hook so as to obtain perfect bonding with UHPC. A 0.2” (5mm) long- 120Ω resistance strain gauge was then attached to the smooth ground surface of the rebars at mid-span location (Figure 3.8) in order to capture its behavior corresponding to the applied load.

UHPC mixing and casting process started after the preparation of formwork and rebars. Two batches were casted separately, in which batch 1 specimens were casted on 10/25/2013 and batch 2 specimens on 11/26/2013. Five 50-lb (22.68 kg) bags of UHPC premix were used. All other materials including steel fiber, HRWA, and ice were weighted according to the proportion given in Chapter 2, while mixing procedures were adopted from [7]. A high shear core capacity mixer was needed to ensure uniform distribution of steel fiber as well as the premix; therefore, an MQ Whiteman Plaster/Mortar Mixer as shown in Figure 3.9 was employed.

Approximately 20 minutes of mixing process, the concrete was casted (Figure 3.10). The newly casted specimens were left 3 days for hardening before curing by submerging into water for 7 days. No heat treatment was used, and testing process can begin after UHPC is 28 days old.
Figure 3.8: Strain Gauge Installation on Rebar at Mid-Span

Figure 3.9: High Shear Core Capacity Whiteman Mixer
3.3 Instrumentation and Test Setup

The test setup for this research was 3-point and 4-point flexural loading on small scale UHPC prisms with varying shear span ratio $a/d$. To test varying $a/d$ prisms, adjustable support base from 6” (152.4 mm) to 32” (812.8 mm) as well as replaceable one-point or two-point loading cell were chosen.

The main purposes of the experimental test were to observe load-deflection behavior of the prisms at mid span as well as its modes for future comparison with MCFT analytical model and finite element analysis software VecTor2. However, twisting or displacing of the supports is frequently observed resulting in a large increase of beam deflection and reduction of beam initial stiffness.

To reduce the effects, two $\frac{1}{2}”$ LVDTs in addition to the $1\frac{1}{2}”$ mid-span LVDT were mounted on top of the UHPC specimens at the support locations; therefore, the difference of mid-span and
average support deflection monitored is the net mid-span deflection. Again the controlled loading was done by using available UTM machine; while the accurate real-time load-deflection along with reinforcement strain were captured by using National Instrument (NI) data acquisition (DAQ) system. The details of the test setup are shown in Figure 3.11 and 3.12.

Figure 3.11: Schematic of the Bending Test Setup

Figure 3.12: Bending Test Setup
CHAPTER 4: EXPERIMENTAL RESULTS

This chapter describes the results of the experiment from the three different groups of UHPC specimens as mentioned in Chapter 3 where Group 1 UHPC specimens without reinforcement, Group 2 reinforced with Gr. 60 steel, and Group 3 reinforced with MMFX. The results included observed failure patterns, load-deflection curves plus reinforcement load-strain curves.

4.1 Unreinforced UHPC Specimens

The failure patterns of the tested unreinforced UHPC specimens were predominantly in flexure even though shear span ratio $a/d$ was as low as 1.20. For all the specimens of group 1 as illustrated in Figure 4.1, 4.2, and 4.3, no single shear failure pattern was witnessed. All specimens failed with a single large crack opening propagating vertically from the bottom fiber to the top fiber under the loading position at mid span. The failures were fairly fast because of the absence of longitudinal reinforcement to help transfer loading, but they were less brittle than normal concrete due to the interface shear strength between steel fiber and UHPC.

Figure 4.4, 4.5, and 4.6; on the other hand, showed the load-deflection curves of the tested specimens. After the peak loads were reached, the specimens unloaded gradually. G1-01 specimens provided the most consistent results among the group; while G1-02 and G1-03 specimens experienced approximately 20% difference of peak load capacity. Different fiber distribution, slight difference of section dimension, and rough loading surface were the contribution to the peak load variation. As a consequence, the load-deflection diagram of G1-03 specimens where only two specimens were tested, will not be used for comparisons with the analytical findings. No strain was recorded for this unreinforced UHPC specimens.
Figure 4.1: Flexural Failure of specimen G1-01

Figure 4.2: Flexural Failure of specimen G1-02
Figure 4.3: Flexural Failure of specimen G1-03

Figure 4.4: Load-Deflection Diagram for G1-01: 2x2x12 (a/d=3.00)
Figure 4.5: Load-Deflection Diagram for G1-02: 1.5x5x12 (a/d=1.20)

Figure 4.6: Load-Deflection Diagram for G1-03: 2x2x8 (a/d=2.00)
4.2 UHPC Specimens reinforced with Grade 60 steel

Similar to unreinforced UHPC specimens whose failure modes were mostly encountered in flexure, UHPC specimens reinforced with Gr. 60 steel failed generally in flexure. As illustrated in Figures 4.7, 4.8, and 4.9, specimens G2-01, G2-02, and G2-03 with shear span ratio $a/d$ ranging between 2.25 and 3.25, the cracks started to open at the bottom fiber close to loading location and propagated to the top fiber, and the specimens completely failed with very large crack patterns due to steel yielding or fracture. As for specimens G2-04 with shorter $a/d = 1.55$ ratio shown in Figure 4.10, despite failing in flexure due to reinforcement yielding, micro diagonal shear cracks were also observed. The flexural failure modes of the four specimens can also be explained from their load-deflection diagrams in Figure 4.12 to 4.15 and rebars load-strain curves in Figure 4.17 to 4.18 where in the nonlinear region, the specimens largely deformed with constant or gradual increasing load. On the other hand, shear failure pattern was captured on specimens G2-05 with shear span ratio $a/d$ as low as 1.25 as shown in Figure 4.11. Large diagonal tension crack between the support and the applied load location due to compression strut effect was seen. Its load-deflection in Figure 4.16 indicated the specimen was less ductile with a sudden drop of loading after reaching its peak.

Figure 4.7: Flexural Failure of specimen G2-01
Figure 4.8: Flexural Failure of specimen G2-02

Figure 4.9: Flexural Failure of specimen G2-03
Figure 4.10: Flexural Failure of specimen G2-04

Figure 4.11: Shear Failure of specimen G2-05
Figure 4.12: Load-Deflection Diagram for G2-01: 1.5x4x26 (a/d=3.25)

Figure 4.13: Load-Deflection Diagram for G2-02: 1.5x4x18 (a/d=2.25)
Figure 4.14: Load-Deflection Diagram for G2-03: 1.5x3x26 (a/d=2.75)

Figure 4.15: Load-Deflection Diagram for G2-04: 1.5x4.5x18 (a/d=1.55)
Figure 4.16: Load-Deflection Diagram for G2-05: 1.5x3x7.5 (a/d=1.25)

Figure 4.17: Reinforcement Load-Strain Diagram for all specimens of G2-01 and G2-02
4.3 UHPC Specimens reinforced with MMFX steel

For UHPC with MMFX specimens as shown in Figure 4.19 and 4.20 with big shear span ratio of 3.00 to 4.33, most of the specimens still failed in shear from a wide diagonal tension crack and crushing of concrete top. Micro diagonal cracks with the cracking angle of approximately $45^\circ$ were first observed, and as the load reached the peak, brittle shear failure occurred with a sudden drop of loading as shown in the load-deflection diagram of Figure 4.21 to 4.22. However, prism 2 of G3-01 was exceptional because the specimen instead experienced ductile flexural failure with yielding of reinforcement as shown in Figure 4.22 where after the peak load the prism kept deforming with approximately the same loading intensity. For each prism G3-01 and G3-02 specimens, 3 strain gauges were attached to its reinforcement at 3 different locations— at mid span, at 1/4 of prism length, and at the support. Because the reinforcement load-strain behaviors were similar, only load-strain curves of one specimen from each group were shown. Load-strain in Figure 4.23
and 4.24 indicated that while rebars at mid span and at 1/4 of span highly engaged from the begin-
ning of loading, rebar at the support was not activated until the bond between UHPC and MMFX
failed at mid span location.

Figure 4.19: Shear Failure of specimen G3-01

Figure 4.20: Shear Failure of specimen G3-02
Figure 4.21: Load-Deflection Diagram for G3-01: 1x3.5x26 (a/d=3.70)

Figure 4.22: Load-Deflection Diagram for G3-02: 1x3.5x18 (a/d=2.57)
Figure 4.23: Reinforcement Load-Strain Diagram for Prism 3 of G3-01

Figure 4.24: Reinforcement Load-Strain Diagram for Prism 1 of G3-02
CHAPTER 5: NUMERICAL ANALYSIS

5.1 MCFT Analysis

This so-called MCFT Analysis is implemented in MATLAB program where the sectional level analysis is based on modified compression field theory; while structure level analysis is based on displacement-controlled Nonlinear Analysis Program (NAP) as described in Chapter 2.

Adopted material models are next to be illustrated, and finally MCFT-analysis results compared with experimental results are presented. In this MCFT analysis, the load-displacement diagram can be directly generated for comparison, while the failure modes are analyzed through the sectional principal tensile strains and principal compressive strains along the beam length.

5.1.1 Material Constitutive Models

**Concrete:** Since overall performance of SFRC is known to highly depend on its tensile behavior and meanwhile there are only a slight deviation of the compressive strength of group 1 and group 2 specimens, the lower bound value of $f'_c = 135$ MPa is taken as compressive strength for all specimens from group 1 and 2 in the modeling; while $f'_c = 117$ MPa is used for group 3 specimens. For concrete in compression both pre- and post-peak, Hogbenstad model as described in chapter 2 is adopted; while Vecchio-Collins 1982 is selected for compression softening behavior. A summary UHPC compression characteristics is tabulated in Table 5.1.

<table>
<thead>
<tr>
<th>$f'_c$ for G1 and G2</th>
<th>$f'_c$ for G3</th>
<th>Young’s modulus $E_c$</th>
<th>Strain at Peak stress $\varepsilon'_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>135 MPa (19.5 ksi)</td>
<td>117 MPa (17 ksi)</td>
<td>42000 MPa (6090 ksi)</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

For concrete in tension, direct tension test data from Graybeal on UHPC specimen F2A-
Long is chosen by assuming trilinear response. After cracking localization the specimens unloaded elastically. The tensile stress/strain values are tabulated in Table 5.2 and shown in Figure 5.1.

### Table 5.2: UHPC Tensile stress-strain response

<table>
<thead>
<tr>
<th>Stress Mpa (ksi)</th>
<th>First Cracking</th>
<th>Localization</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0 (1.16)</td>
<td>8.0 (1.16)</td>
<td>0 (0)</td>
<td></td>
</tr>
<tr>
<td>Strain (mm/mm)</td>
<td>$f_{tcr}/E_c$</td>
<td>0.003</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Figure 5.1: Assumed UHPC stress-strain diagram

**Reinforcement:** As shown in Chapter 3, both ASTM Gr. 60 and MMFX steel deformed elastically up to the yield strength; thereafter, nonlinear behavior were observed. For ASTM Gr. 60, trilinear elastic-hardening curve has been known to be a close assumption and therefore was chosen as the reinforcement model where its yield strength $f_y = 414$ MPa (60 ksi), ultimate strength $f_u = 723$ MPa (105 ksi), offset strain $\epsilon_{sh} = 0.006$ and strain at failure $\epsilon_u = 0.06$. MMFX stress/strain...
response; on the other hand, is represented by curve-fitting equation \( f_s = 1138(1 - e^{-236.27e_s}) \); the value is in MPa. The adopted reinforcement responses are shown in Figure 5.2 and 5.3.

Figure 5.2: ASTM Gr.60 stress-strain diagram

Figure 5.3: MMFX stress-strain diagram
5.1.2 Analysis Results

5.1.2.1 Group 1 Specimens (Unreinforced UHPC)

Figure 5.4: MCFT Load-Displacement Curve for Specimen G1-01 (2x2x12)

Figure 5.5: MCFT Load-Displacement Curve for Specimen G1-02 (1.5x5x12)
Figure 5.6: Principal tensile strain along G1-02 at final load stage

Figure 5.7: Principal compressive strain along G1-02 at final load stage
For group1 only specimens G1-01 and G1-02 are analyzed because there is a big discrepancy of experimental load-displacement curves for specimens G1-03. As illustrated in the load-displacement diagram in Figure 5.4 and 5.5, MCFT predicts quite well the nonlinear behavior of the unreinforced UHPC. In average, only 10% of peak load underestimation is made. Although the proposed method is not able to capture the whole post-peak load-displacement response, similar softening slopes to the experimental data are observed. In addition, the flexural failure modes of unreinforced UHPC can also be seen through the principal tensile strains and principal compressive strains as shown in Figure 5.6 and 5.7, respectively. Because the principal strains of all specimens within group 1 has the same trend, only those of specimen G1-02 are brought for discussion. From the figures, it is obvious that the specimen fails in flexure at mid-span because the principal tensile strain of the bottom fiber at mid-span (e1@L/2 = 0.0075) is much higher than those at one-third (e1@L/3) and one-sixth (e1@L/6) of beam length. Based on UHPC stress-strain diagram, e1 = 0.0075 represents that the tensile strength of the bottom fiber at mid-span drops from 8 MPa to approximately 6 MPa; while those at L/3 and L/6 are still able to increase. On the other hand, the compressive strain of the top fiber at mid-span ε2 = -0.001 is still far below the strain at peak stress ε′c = -0.0032, meaning the compressive stress on the specimen can still go further.

5.1.2.2 Group2 Specimens (UHPC-Gr.60)

For specimens G2-01, G2-02, G2-03, and G2-04, in regards to peak load capacity as indicated from Figure 5.8 to Figure 5.11, the analytical method estimated fairly well for UHPC-Gr.60 specimens except for specimen G2-04 where the the peak load were 20% underestimated; which can possibly be improved through more elements division; while the stiffer initial response of the analytical load-displacement curve is due to the assumption of Hognestad parabolic compression stress-strain curve. From the analysis, the failure modes of all UHPC-Gr.60 specimens are predominantly flexural failure which can be identified from the load-displacement diagram through the plateau post-peak behavior of the specimens during which reinforcement is yielding. On top
of this, the flexural failure can also be identified through the principal strains along the specimen length. Figure 5.12 and 5.13 only show the principal strains of specimen G2-02 having similar trend to all other specimens except specimen G2-05. The figures indicate that at the final load stage UHPC principal tensile strain of the bottom fiber at mid span $\epsilon_1$ is 0.0087; while strain of rebar locating slightly on top of the bottom fiber is $\epsilon_s$ is 0.0075; meaning reinforcement has already yielded. Meanwhile, the principal compressive strain of the top fiber at mid span $\epsilon_2$ = -0.002 is still far below the compressive strain at peak stress. A slightly different case happened to specimen G2-05 with very small $a/d$ ratio of 1.25, where its peak load capacity is fairly well predicted; however, a big difference of the test stiffness compared to analytical stiffness (Figure 5.14) was due to support settlement during the experiment. In addition, for this specimen flexural failure due to yielding of rebar with $\epsilon_s = 0.0107$ (Figure 5.15), $f_s = 441$ MPa is predicted to happen first, and as the rebar keeps yielding, crush of concrete top may be the last failure mode because the concrete principal compressive strain at the top fiber is as high as $\epsilon_2 = -0.003206$ (Figure 5.16).

![Figure 5.8: MCFT Load-Displacement Curve for Specimen G2-01 (1.5x4x26)](image)
Figure 5.9: MCFT Load-Displacement Curve for Specimen G2-02 (1.5x4x18)

Figure 5.10: MCFT Load-Displacement Curve for Specimen G2-03 (1.5x3x26)
Figure 5.11: MCFT Load-Displacement Curve for Specimen G2-04 (1.5x4.5x18)

Figure 5.12: Principal tensile strain along G2-02 at final load stage
Figure 5.13: Principal compressive strain along G2-02 at final load stage

Figure 5.14: MCFT Load-Displacement Curve for Specimen G2-05 (1.5x3x7.5)
Figure 5.15: Principal tensile strain along G2-05 at final load stage

Figure 5.16: Principal compressive strain along G2-05 at final load stage
5.1.2.3 Group3 Specimens (UHPC-MMFX)

Figure 5.17: MCFT Load-Displacement Curve for Specimen G3-01 (1.5x3x26)

Figure 5.18: MCFT Load-Displacement Curve for Specimen G3-02 (1.5x3x18)
Figure 5.19: Principal tensile strain along G3-02 at Peak

Figure 5.20: Principal compressive strain along G3-02 at Peak
Again for UHPC-MMFX specimens, load-displacement diagrams shown in Figure 5.17 and 5.18 are well estimated. Different from unreinforced and Gr.60 steel reinforced UHPC, UHPC-MMFX load capacity suddenly drops after reaching the peak. This results in first-hand conclusion that the specimens have failed in shear. Further investigation is made based on the fiber level of the specimens through their principal tensile and compressive strains within a distance of L/4 from mid-span at the peak load stage. Shown in Figure 5.19 and 5.20, the very high strains at +0.5” depth are resulted from no convergence of one fiber leading to zero stress on that fiber; however, sectional equilibrium can still reasonably be achieved from the remaining fibers. Neglecting no convergence effect, both the midspan maximum principal tensile strain at bottom fiber \( \varepsilon_1 = 0.0091 \) and compressive strains at the top fiber \( \varepsilon_2 = -0.0052 \). Both strains are far bigger than their starting softening strains of \( \varepsilon_1 = 0.0035 \) and \( \varepsilon_2 = -0.0032 \), respectively. From these strain states, UHPC should be going to fail in flexure and crush of concrete top at midspan with further increase of loading, instead of diagonal shear failure. Nevertheless, interestingly the corresponding stress of MMFX is only 909.4 MPa which is below the real plateau yield strength \( F_{su} = 1138 \) MPa. Most reasonable explanation leading to shear failure in the experiment is that crack widening at the bottom fiber of UHPC was restricted by MMFX. Therefore, the existing diagonal micro crack of concrete suddenly widen due to compression strut accompanied by crush of concrete top. Further investigation on this shear crack is made using FEA VecTor2.

5.2 Finite Element Analysis

5.2.1 FEM Modeling

5.2.1.1 Elements, Boundary Condition, and Loading

In modeling the experimental prisms, UHPC is modeled with four-noded plane stress rectangle elements (Figure 5.21) having uniform thickness. The element has 8 degrees of freedom (dofs) in which each node is free to translate in x and y direction. To ensure accuracy of the
results, square shape elements 0.25” by 0.25” are chosen.

Figure 5.21: Concrete Plane stress rectangular element

Because only two nodes exist along each corner of the element, the element displacement functions \( u \) and \( v \) are linear and have the following formula:

\[
\begin{align*}
  r_x(x, y) &= a_1 + a_2 x + a_3 y + a_4 xy \\
  r_y(x, y) &= a_5 + a_6 x + a_7 y + a_8 xy
\end{align*}
\] (5.1) (5.2)

By eliminating \( a_i \) and taking derivative of shape function, the element strains \([\varepsilon]\) can be expressed in relation to nodal displacement \([r]\) and coordinates \(x, y\) as follows:

\[
[\varepsilon] = [B][r] \quad (5.3)
\]

\[
[\varepsilon] = \begin{bmatrix}
  \varepsilon_x & \varepsilon_y & \gamma_{xy}
\end{bmatrix}^T \quad (5.4)
\]
\[
[r] = \begin{bmatrix}
    r_{ix} & r_{iy} & r_{jx} & r_{jy} & r_{mx} & r_{my} & r_{nx} & r_{ny}
\end{bmatrix}^T
\tag{5.5}
\]

\[
[B] = \frac{1}{4ab} \begin{bmatrix}
    -(b-y) & 0 & (b-y) & 0 & (b+y) & 0 & -(b+y) & 0 \\
    0 & -(a-x) & 0 & -(a+x) & 0 & (a+x) & 0 & 0 \\
    -(a-x) & -(b-y) & -(a+x) & (b-y) & (a+x) & (b+y) & (a-x) & -(b+y)
\end{bmatrix}
\tag{5.6}
\]

The stiffness matrix of the element is given by:

\[
[k] = \int_{vol} [B]^T[D][B]dV 
\tag{5.7}
\]

As for longitudinal reinforcement, two-noded truss elements with uniform cross-sectional area are used. Based on the experiment, no clear slippage between rebar and UHPC was observed; therefore, perfect reinforcement-to-concrete bond is assumed. With perfect bond assumption, the rebar truss elements are connected to the existing nodes of concrete.

All the tested specimens are simply supported beams with a pinned support assigned to the left side of the structure and a roller to the right side. Displacement-controlled loading is chosen and applied in this FEM modeling in order to find out the post cracking behavior of the structure which is one of most important criteria in identifying its failure modes. For specimen G2-05, under point load and above pinned and roller supports, high-stiffness steel plate elements with the width twice as big as concrete element and thickness of 0.125” are used to avoid concrete punching failure due to stress concentration. The FEM modeling of the tested specimens is shown in Figure 5.22.
5.2.1.2 Material Modeling

Concrete: The same as previous analysis assumption, compressive strength is chosen differently for group 1, 2 specimens, and group 3 specimens; Hognestad Parabola is still assumed to be the pre- and post-peak concrete compression response with Vecchio-Collins 1982 as the compression softening model. No tension stiffening is considered due to the high UHPC post-cracking tensile strength. Accurate modeling of tensile behavior of UHPC in VecTor2 can be obtained through curve-fitting of the unreinforced UHPC with the available Exponential tension softening in combination with the Simplified Diverse Embedment Model (SDEM) for monotonic loading of FRC as shown in the load-displacement curves of group 1 specimens. Below is the summary of both the Exponential tension softening and SDEM-Monotonic FRC tension model. Detailed description of the models can be found in [5].

The exponential tension softening highly depends on first cracking tensile strength $f_{cr}$, average crack width $w_{cr}$, and concrete fracture energy $G_f$ and is given by the formula as the following:

$$f_{c,t} = f_{cr} \exp\left(-\frac{f_{cr}w_{cr}}{G_f}\right)$$ (5.8)
In the FRC tension SDEM-Monotonic for straight fiber, tensile stress is obtained through frictional bond, and it is in functions of $\alpha_f$ fiber orientation factor, $V_f$ fiber volumetric ratio, $K_{st}$ bond modulus, $\tau_{f,max}$ concrete bond stress, $\frac{l_f}{d_f}$ fiber length to diameter ratio or aspect ratio, and crack width $w_{cr}$. Table 5.3 shows the adopted first cracking stress, fracture energy, and crack width of UHPC based on curve fitting with load-deflection response of unreinforced specimens.

$$f_{st} = \alpha_f V_f K_{st} \tau_{f,max} \frac{l_f}{d_f} (1 - \frac{2w_{cr}}{l_f})^2$$  \hfill (5.9)

Table 5.3: Adopted tensile stress-strain parameters based on curve-fitting

<table>
<thead>
<tr>
<th>$f_{cr}$ (MPa)</th>
<th>$G_f$ (kN/m)</th>
<th>$w_{cr}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>70</td>
<td>$\frac{80}{\cos \theta}$</td>
</tr>
</tbody>
</table>

To ensure correct output of UHPC tensile stress-strain response, a 50mm by 250mm UHPC coupon is modeled in VecTor2 where high stiffness steel plate with the thickness of 25mm are placed at both ends of the coupon to avoid high stress concentration. Element size of 10mm by 10mm are meshed. Pinned supports are assigned to all nodes of the left end of the coupon, and displacement-controlled loadings are equally applied to each node of the other end as shown in (Figure 5.23). Expected result is achieved (Figure 5.24).

Figure 5.23: Coupon modeling in VecTor2
Reinforcement: Although there are various constitutive models for many types of steel reinforcement in VecTor2 program, the available models for ductile steel reinforcement are not the perfect representation of the exact stress-strain behavior of MMFX and ASTM Gr.60 which have nonlinear behavior. Nevertheless, Based on curve fitting of the stress-strain diagrams from the direct tension test as mentioned in Chapter 3, Trilinear elastic-hardening (Figure 5.25), and Curvilinear elastic-hardening with reduced ultimate strain (Figure 5.26) are chosen for ASTM Gr.60 and MMFX, respectively. Table 5.4 shows the values of the assumed reinforcement parameters.

Table 5.4: Adopted tensile stress-strain parameters based on curve-fitting

<table>
<thead>
<tr>
<th>Steel</th>
<th>$f_y$ MPa (ksi)</th>
<th>$f_u$ MPa (ksi)</th>
<th>Offset Strain $e_{sh}$</th>
<th>Strain at Failure $e_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gr.60</td>
<td>414 (60)</td>
<td>723 (105)</td>
<td>0.006</td>
<td>0.06</td>
</tr>
<tr>
<td>MMFX</td>
<td>689 (100)</td>
<td>1138 (165)</td>
<td>0.003445</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Figure 5.25: Assumed ASTM Gr.60 Model Trilinear Elastic-Hardening

Figure 5.26: Assumed MMFX Model Curvilinear Elastic-Hardening
5.2.2  FEM Results

5.2.2.1  Group1 Specimens (Unreinforced UHPC)

![Load-Displacement Curve](image)

Figure 5.27: Load-Displacement Curve for Specimen G1-01 (Unreinforced UHPC 2x2x12)

![Crack Pattern](image)

Figure 5.28: G1-01 Crack Pattern at Peak Load = 1.53kips

![Crack Pattern](image)

Figure 5.29: G1-01 Crack Pattern at Post Peak Load = 0.94kips
Figure 5.30: Load-Displacement Curve for Specimen G1-02 (Unreinforced UHPC 1.5x5x12)

Figure 5.31: G1-02 Crack Pattern at Peak Load = 7.10kips

Figure 5.32: G1-02 Crack Pattern at Post Peak Load = 5.17kips
As illustrated in Figure 5.27 and 5.30, the FEM predicts very close average first cracking strength of unreinforced UHPC as well as the failure modes. In average, only 7.5% underestimation is simulated. However, specimen G1-01 experiences slightly lower softening behavior in the analysis than in the experiment; while the post cracking behavior of specimen G2-02 is well predicted. As illustrated in Figure 5.29 and 5.32, the thick red lines in the middle of the specimens represent that both specimens fail in flexure from a wide crack opening propagating from the bottom to the top fiber of the prisms. The gradual decrease of loading capacity after the peak is due to the remaining post-cracking tensile strength of UHPC with the aid of fiber bridging effect. Figure 5.27 and 5.30 also show the load-displacement diagram from NAP-MCFT analysis where both the peak load capacity and softening behavior are quite close.

5.2.2.2 Group2 Specimens (UHPC-Gr.60)

With regards to UHPC specimens reinforced with ASTM Gr.60 steel, As shown in the load-displacement diagram of Figure 5.33 , 5.35, 5.38, 5.41, and 5.44, the MCFT/DSFM based FEM provides good estimation of the beams’ stiffness, load capacity and failure modes. In average, only 4.70% underestimation is predicted. Specimen G2-03 (Figure 5.38) is the least precise simulation in terms of peak load capacity with 9.70% of error. As shown with the thick red line in the mid-span, flexural failures are seen due to the yielding of reinforcement and vertical wide crack under loading position for specimens G2-01 (Figure 5.34) and G2-02 (Figure 5.37). As for specimens under 4-point bending G2-03 (Figure 5.40) and G2-04 (Figure 5.43), a few wide flexural cracks are seen to develop between the loading points; whereas in the experiment only a single wide flexural crack was noticed. The reason for this difference is because UHPC properties in VecTor2 is isotropic, but it is anisotropic by nature. On the other hand, specimen G2-05 with lowest \( a/d \) ratio of 1.25, flexural-shear failure is encountered. At first, flexural micro crack at mid-span due to the yielded rebar is observed as shown in Figure 5.45. As the displacement keeps increasing, mid-span section stiffness decreases and loading is thereafter transferred to the supports through compression.
struts resulting in shear failure (Figure 5.46). Again, in the load-displacement diagram (Figure 5.44), the large offset of initial stiffness for the specimens is due to support displacement. Again, by comparing VecTor2 with the results from NAP-MCFT analysis for all specimens in group 2, very close load-displacement behaviors are predicted, yet bigger initial stiffness are always observed in NAP-MCFT; however, for 4-point test specimens G2-03 and G2-04, NAP-MCFT generates softer peak load responses. These maybe as the results different types and numbers of elements are used. NAP-MCFT uses 1D elements with 6 members; while VecTor2 uses rectangular plane-stress with 0.25”x0.25” meshes.

![Graph showing load-displacement behavior](image.png)

**Figure 5.33: Load-Displacement Curve for Specimen G2-01(UHPC-Gr.60 1.5x4x26)**

![Image of crack pattern](image.png)

**Figure 5.34: G2-01 Crack Pattern at Post Peak Load = 5.35kips**
Figure 5.35: Load-Displacement Curve for Specimen G2-02(UHPC-Gr.60 1.5x4x18)

Figure 5.36: G2-02 Crack Pattern at Peak Load = 8.24kips

Figure 5.37: G2-02 Crack Pattern at Post Peak Load = 7.87kips
Figure 5.38: Load-Displacement Curve for Specimen G2-03(UHPC-Gr.60 1.5x3x26)

Figure 5.39: G2-03 Crack Pattern at the change of slope = 5.26kips

Figure 5.40: G2-03 Crack Pattern at Peak Load = 6.07kips
Figure 5.41: Load-Displacement Curve for Specimen G2-04(UHPC-Gr.60 1.5x4.5x18)

Figure 5.42: G2-04 Crack Pattern at the change of slope = 11.46kips

Figure 5.43: G2-04 Crack Pattern at Peak Load = 13.51kips
Figure 5.44: Load-Displacement Curve for Specimen G2-05(UHPC-Gr.60 1.5x3x7.5)

Figure 5.45: G2-05 Crack Pattern at Peak Load = 14.16kips

Figure 5.46: G2-05 Crack Pattern at Post Peak Load = 7.64kips
5.2.2.3  Group3 Specimens (UHPC-MMFX)

![Load-Displacement Curve for Specimen G3-01(UHPC-MMFX 1.5x3x26)](image)

Figure 5.47: Load-Displacement Curve for Specimen G3-01(UHPC-MMFX 1.5x3x26)

![G3-01 Crack Pattern at Peak Load = 5.93kips](image)

Figure 5.48: G3-01 Crack Pattern at Peak Load = 5.93kips

![G3-01 Crack Pattern at Post Peak Load = 1.44kips](image)

Figure 5.49: G3-01 Crack Pattern at Post Peak Load = 1.44kips
Figure 5.50: Load-Displacement Curve for Specimen G3-02(UHPC-MMFX 1.5x3x18)

Figure 5.51: G3-02 Crack Pattern at Peak Load = 8.45kips

Figure 5.52: G3-02 Crack Pattern at Post Peak Load = 6.61kips
Based on the load-displacement diagram in Figure 5.47 and 5.50, VecTor2 modeling can accurately simulate the structural behavior of UHPC prisms reinforced with MMFX where the predicted peak load for specimen G3-01 is 5.93 kips compared to the average experimental value of 5.98 kips; while that for specimen G3-02 is 8.45 kips compared to the average experimental value of 8.21 kips. Averagely, only a small estimation error of 2% is predicted because the stress-strain diagrams of UHPC and MMFX are correctly assumed.

Also, the modeling is able to accurately simulate the shear failure mode of UHPC-MMFX specimens. As shown in Figure 5.48 and 5.51 with the thick red line, formation of micro diagonal cracks are seen as the load gradually increased, and when peak load is reached, diagonal tension crack greatly widens accompanied by the crush of concrete top (Figure 5.49 and 5.52) due to the shortage of UHPC tensile strength as well as compressive strength. At the peak load, MMFX stress value are 958 MPa (138.9 Ksi) and 928MPa (134.6 Ksi) which are well below MMFX ultimate strength of 1138 MPa (165 Ksi) at which MMFX rebar will experience extensive yielding. Load-displacement results from NAP-MCFT and VecTor2 for this group of specimens are well predicted.

5.3 Comparison between Displacement-Based and Force-Based Elements

Although fairly good results for all groups of specimen, both pre-peak and post-peak responses, can be obtained using the displacement-based elements, analysis using force-based elements by satisfying sectional forces equilibrium directly, which is the nature of Modified Compression Field Theory as briefly described in Chapter 2, is worth to be simulated and compared with displacement-based element results. Specimen G2-01 (UHPC-Gr.60 group) with the dimension of 1.5x4x18” is chosen for the comparison. The material properties used are as described in 5.1.1. The result is shown as the following:
As shown in Figure 5.53, although the initial stiffness determined from both methods are very closed, peak load capacity determined by force-based element analysis is almost 14% lower than that obtained from displacement-based element analysis. The reason for this huge different capacity is mainly because in the MCFT analysis using the displacement-based elements which is the same as the approximate finite element method, only 6 elements were discretized, resulting in stiffer response. In addition, the tensile behavior of UHPC was calibrated based on Graybeal direct tension test which may not be a good representation of UHPC flexural members. However, the post-peak response from displacement-based elements provides better results. Therefore, dividing the beam element into more numbers using displacement-based elements as well as using stronger backbone stress-strain behavior of UHPC such as those provided by AFGC as described in Chapter 2 should provide close peak load prediction as the analysis using force-based elements.
CHAPTER 6: CONCLUSION AND DISCUSSION

Ultra-High Performance concrete (UHPC) is a new type of fiber reinforced concrete having very high compressive strength as well as high tensile strength up to 22 ksi and 1 ksi, respectively. Previous research had been conducted to use UHPC deck in place of the currently problematic open grid steel deck system of Florida movable bridges, and UHPC reinforced with high strength steel deck was found to be a high potential alternative. However, based on the experimental tests, the failure modes were seen to be a dominant diagonal tension shear failure even though either high strength steel or ASTM grade 60 steel were used as the complementary tension reinforcement. Quantification of the shear effect by adding shear force demanding and resisting to the uniaxial flexural analysis of the UHPC deck system had been made by previous researcher, and degradation of the flexural member capacity was seen, but a combined realistic normal-shear stresses is still absent.

Due to these limited evidences on the behavior of Ultra-High Performance Concrete (UHPC) flexural members under combined normal and shear stresses, this research performed experimental test on UHPC small scale beams with three different types of reinforcement conditions including UHPC prisms reinforced with ASTM Grade 60 steel, UHPC prisms reinforced with high-strength steel (MMFX), and UHPC prisms without reinforcement. The main objective of this research was to use available analytical methods in quantifying the sensitivity on UHPC flexural members under the condition of different shear and normal stress demands and meanwhile investigating the failure mechanisms of the structural members.

From the experimental tests, the unreinforced UHPC prsims (Group 1) failed in flexure through a widen crack propagated from the bottom fiber vertically to the loading position. Four out of the five sets of group 2 specimens also failed in flexure through yielding reinforcement, and similar crack pattern to group 1 specimens were seen but the failures were very ductile. For the 5\(^{th}\) set of group 2 specimens, flexural cracks were first seen to form, but as the load reached the
peak, the specimens failed by diagonal compression. Group 3 (UHPC-MMFX) specimens; on the other hand, failed predominantly in diagonal tension shear with wide crack propagating from the loading position. Meanwhile, crush of concrete top fiber is also seen as the combined failure.

Modified Compression Field Theory (MCFT) being capable of predicting the response of two-dimensional reinforced concrete structures subjected to in-plane shear and normal stresses was then selected as the internal nonlinear sectional analysis tool employing Secant Stiffness Formulation. While at the global level, Nonlinear Analysis Program (NAP) being able to simulate reasonably well the response of various RC structural elements beyond the peak capacity was employed interactively with MCFT using displacement-based elements and displacement-based loading. Results showed that the adopted method could closely predict the load-displacement response of the tested specimens as well as the failure modes. However, when comparing with force-based element analysis, the displacement-based element analysis provides good post-peak response but bigger peak load capacity. Discretizing the structures into more elements should result in satisfying results.

Plane stress FEM software VecTor2 is a complementary analysis tool used in this research. By choosing appropriate material models as well as element meshing size, VecTor2 could well predict both the load-displacement behavior and failure modes of all the specimens. On top of NAP-MCFT, the software could simulate the shear crack of the beam where diagonal shear failures were seen when steel reinforcement stress was well below the yielding strength. Most reasonable explanation is the rebar helped to restrain crack at mid-span of the specimens. Therefore, the already existing diagonal micro crack started to widen through compression strut effect from the loading position.
6.1 Future Work

To fully quantify the sensitivity of UHPC flexural members under different shear and normal stress demands, investigate on dowel action as well as local crack shear slip should be made.
LIST OF REFERENCES


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