Design and Analysis of a 4GHz Low Noise Amplifier

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DESIGN AND ANALYSIS OF A 4 GHz LOW NOISE AMPLIFIER

BY

ABDULLA I. AL-RAWAHY
B.E.T., University of Central Florida, 1983

THESIS
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ABSTRACT

The Gallium Arsenide Field Effect Transistor (GaAs FET) is experiencing a widespread acceptance in space and terrestrial systems due to its low noise, high gain and frequency characteristics unmatched by bipolar devices. This paper describes a design of a 4 GHz low noise amplifier with GaAs FETs using the scattering parameters method. Special attention is given to overall noise/gain optimization in the band of interest. The Smith Chart is used extensively to match the two-port device with microstrip networks. Analysis and performance of the amplifier are presented.
ACKNOWLEDGEMENTS

I would first like to acknowledge the help and encouragement of my advisor, Dr. M. Belkerdid. I also wish to thank the other members of my committee for their advice and comments. A special thank you to the members of the Solid State Devices Laboratory for their cooperation. Last, but not least, to my family who sacrificed everything to make it possible.
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CHAPTER I
INTRODUCTION

The GaAs MESFET has demonstrated far superior performance than the silicon bipolar transistor and the tunnel diode in terms of noise, gain and power amplification efficiency in the higher frequencies since its appearance in the early 1970s.

Most small signal applications are in low noise amplifiers and several designs have been reported (Liechti et al. 1974). These designs were based on experimental, theoretical or both network topologies, often with computer optimization of the circuit elements to achieve maximum gain.

The objective of this thesis is to design a low noise amplifier with minimum matching networks; hence, minimizing impedance losses. A step-by-step procedure for designing an amplifier using a conditionally stable GaAs MESFET with minimum noise figure and a reasonably constant gain will be described. The Smith Chart is used to accomplish the above with the aid of an HP 41CX calculator (Appendix C). The construction of matching networks using MASCAD and etching facilities available at the University of Central Florida is outlined. Finally, amplifier tuning and experimental results are discussed.
CHAPTER II
GALLIUM ARSENIDE METAL SEMICONDUCTOR FIELD-EFFECT TRANSISTORS (GaAs MESFET)

The GaAs FET is generally referred to as a "normally on" device whose current is controlled by an electric field. The electric field is applied at the source and drain electrodes and is controlled at the gate electrode. The Metal Semiconductor FET (MESFET) uses a metallic Schottky barrier at the gate instead of an oxide layer. The width of the depletion layer under the gate controls the current flowing from drain to source.

The fabrication of most microwave FETs on gallium arsenide instead of silicon resulted in a substantial improvement in their characteristics such as lower noise figure, higher gain and higher cut-off frequencies, mainly because of the following reasons: GaAs has a higher electron bulk mobility (six times) and a larger peak drift velocity (twice). This results in smaller parasitic resistance, larger transconductance and a shorter electron transit time in the high field region.

Principles of Operation

In Figure 2, the electrons flow from the source to the drain through the active thin layer due to a $V_{DS}$ supply that accelerates them to the maximum drift velocity of about $2 \times 10^7$ cm/sec as shown
Figure 1. Equilibrium Electron Drift Velocity Versus Electric Field in GaAs and Silicon.

Figure 2. GaAs MESFET.
in Figure 1. When a negative voltage is applied between the gate and the source, the gate to channel junction is reverse biased and a depletion layer is created which varies the channel thickness. Thus, the MESFET is controlled by modulating the width of the channel (at microwave frequencies) with an input signal voltage applied across the input capacitance (or depletion layer capacitance) $C_{gs}$.

The small signal equivalent circuit for the GaAs MESFET can be represented by lumped elements up to about 12 GHz (Liechti 1976). The common-source configuration equivalent circuit is shown in Figure 3. This arrangement usually provides the design engineer with the most stable arrangement and a low drain to gate feedback capacitance.

![Figure 3. GaAs MESFET Small Signal Equivalent Circuit.](image-url)
Frequency Characteristics

The superior electron mobility of GaAs over silicon means GaAs FETs offer high frequency characteristics unmatched by bipolar devices. The limiting factor is the gate length (normally in microns) which should be as short as possible. The frequency at unity current gain is (Liechti 1976):

\[ f_T = \frac{g_m}{2\pi C_{gs}} \]  \hspace{1cm} (1)

and the maximum frequency of oscillation is:

\[ f_{\text{max}} = \frac{f_T}{2} \sqrt{\frac{R_o}{R_c}} \]  \hspace{1cm} (2)

where:

\[ R_o = \frac{C_{gs}}{g_m R_g C_{dg}} \]

\[ R_c = \frac{R_g + R_i + R_s}{R_{ds}} \]

Decreasing the gate length decreases the capacitance \( C_{gs} \) and increases the transconductance \( g_m \). Therefore, from equation (1) it is seen that the current-gain bandwidth \( f_T \) is improved. For microwave MESFETs, \( f_T \) is inversely proportional to the gate length.
Noise Characteristics

The noise behavior of the intrinsic MESFET can be described with the aid of a noiseless two-port with noise current generators connected across the input and output ports (see Figure 4) (Liechti 1976).

![Image of a simplified noise equivalent circuit of the intrinsic GaAs MESFET.](image)

Figure 4. Simplified Noise Equivalent Circuit of the Intrinsic GaAs MESFET.

The current generator $i_{nd}$ represents the channel noise and its mean square value can be expressed by (Baechtold 1972):

$$i_{nd}^2 = 4k_T \Delta f g_m P$$  \hspace{1cm} (3)

where:

- $k$ is Boltzmann's constant
- $T_0$ is the lattice temperature
- $\Delta f$ is the bandwidth
\( g_m \) is the transconductance

\( P \) is the factor depending on device geometry and the bias conditions.

For positive drain voltages, the noise generated in the channel is larger than the thermal noise generated by \( 1/R_d \). The noise voltage fluctuations along the channel cause fluctuations in the depletion layer which induce noise variations on the gate. This is represented by the noise current \( i_{ng} \) as (Baechtold 1972):

\[
\frac{i^2}{i_{ng}} = 4kT \Delta f \omega^2 \frac{C_{gs} R}{g_m}
\]  

(4)

where \( R \) is a factor depending on device geometry and the bias conditions.

The two noise currents \( i_{ng} \) and \( i_{nd} \) are caused by the same noise voltages in the channel, hence they are correlated by a factor \( C \). The minimum noise figure of the intrinsic MESFET, \( F_{\text{min}} \), can then be expressed using these terms. For a GaAs MESFET operated in the microwave region below the cutoff frequency and at room temperature, the optimal value of \( F_{\text{min}} \) is (Liechti 1976):

\[
F_0 = 1 + 2.5 \frac{f}{f_T} \sqrt{g_m (R_g + R_s)}
\]  

(5)

From equation (5), it can be seen that the minimum noise figure increases linearly with frequency in MESFETs while in bipolar transistors it increases quadratically as indicated below (Fukui 1966):
\[ F_{\text{min}} \sim 1 + b f^2 \left( 1 + \sqrt{1 + \frac{2}{b f^2}} \right) \] (6)

where:

\[ b = 40 \, I_c \, r_b / f_T^2 \]

**DC Bias**

There are several biasing methods available to the designer to bias a GaAs FET. The dual power source method, shown in Figure 5, has the source lead connected directly to ground, thereby reducing source inductance, increasing gain and lowering the noise figure at higher frequencies.

Figure 5. Dual Source Bias Method.

The gate bias must range from \( I_{\text{DSS}} \) (\( V_G = 0 \)) to \( I_{\text{DS}} = 0 \) (\( V_G = V_p \)). The drain to source voltage \( V_{\text{DS}} \) has little effect on the current \( I_{\text{DS}} \).
flowing in the channel at the above gate bias range. A change in the gate-to-source voltage $V_{GS}$ produces a change in $I_{DS}$ and their ratio is the mutual conductance $g_m$.

$$I_D = I_{DSS} (1 - \frac{V_{GS}}{V_P})^2$$  \hspace{1cm} (7)$$

$$g_m = \frac{dI_D}{dV_{GS}} = -2 \frac{I_{DSS}}{V_P} (1 - \frac{V_{GS}}{V_P})$$  \hspace{1cm} (8)$$

Proper bias conditions are normally given by the manufacturer. For optimum noise figure, the bias current is varied around the figure quoted on the data sheets. Low noise amplifiers operate at a low $V_{DS}$ and $I_{DS}$, usually $I_{DS} = 0.15 I_{DSS}$ as shown by point A in Figure 6.

Figure 6. Typical GaAs FET DC Characteristics.
Other bias methods insert a bypass capacitor into the source; this may cause a problem at high frequencies since any dielectric losses would degrade the noise figure and any excess inductance could cause oscillations.
In space or satellite communications where the received signal is below -130 dBm, the major noise source is the receiver since atmospheric noise falls to a negligible value above about 30 MHz. The amplifier (receiver) considered here operates in its small signal mode, where linearity exists; the effect of signal and noise is additive. The role of noise in the amplifier can be understood by considering noise in linear two-ports.

The noise performance of an amplifier or chain of amplifiers is usually rated by its noise figure \( F \) defined as:

\[
F = \frac{Si}{So/No} = \frac{No}{Ni G} = \frac{No}{k To B G}
\]

where:

- \( Si \) = signal input power
- \( So \) = signal output power
- \( No \) = noise output power
- \( Ni \) = available noise input power = \( k To B \)
- \( G \) = the two-port power gain
- \( k \) = Boltzmann's constant = \( 1.374 \times 10^{-23} \) joule/°K
- \( To \) = 290°K
- \( B \) = noise bandwidth
From equation (9), it can be seen that the total noise output power $N_0$ is $kT_0\text{BGF}$, hence the noise added by the two-port is $(F-1)kT_0\text{BG}$.

![Figure 7. Simple Model Showing Internal Noise Source and Equivalent Resistance.](image)

A low noise amplifier in a receiving system can consist of many stages in cascade. The overall noise figure can be found by knowing the noise contribution of individual stages in the system as shown below.

![Diagram of amplifier stages](image)

The source supplies a noise input power $kT_0B$ which is amplified by $n$ stages while the noise power added by stage one is amplified likewise
and so on. The overall noise figure of the chain is then:

\[ F = \frac{\sum_{i=1}^{n} G_i + (F_1 - 1) \sum_{i=1}^{n} G_i + (F_2 - 1) \sum_{i=2}^{n} G_i + \ldots + (F_n - 1) G_n}{kT0B \sum_{i=1}^{n} G_i} \]  

(10)

\[ = F_1 + \frac{F_2 - 1}{G_1} + \ldots + \frac{F_n - 1}{\sum_{i=1}^{n-1} G_i} \]  

(11)

It is clear from the above relation that the noise figure of the first stage has the dominant effect on the overall noise figure provided \( G_1 \) is much greater than one. This means that the designer should minimize the first stage noise while maintaining a good gain. Some optimization is needed here, and the following sections will demonstrate this.

**Device Selection and Characteristics**

The NE 71083 GaAs MESFET (package) was chosen due to its low noise figure, high associated gain and reasonable cost. It employs a recessed 0.3 micron gate allowing \( f_{\text{max}} \) to be very high. It is conditionally stable up to about 8 GHz.

**Scattering Parameters**

The parameters listed in Table 1 are from the manufacturer's data sheets and are guaranteed to be close but not necessarily representing the actual device.
TABLE 1
S-PARAMETERS FOR NE 71083 (COMMON SOURCE)

<table>
<thead>
<tr>
<th>FREQ (MHz)</th>
<th>MAGNITUDE AND PHASE</th>
<th>G_{ma} (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>3500</td>
<td>S_{11}</td>
<td>0.924</td>
</tr>
<tr>
<td>4000</td>
<td>S_{21}</td>
<td>0.887</td>
</tr>
<tr>
<td>4500</td>
<td>S_{12}</td>
<td>0.847</td>
</tr>
</tbody>
</table>

G_{ma} = maximum available gain

V_D = 3 V, IDS = 10 mA

The scattering parameters of the two port can be defined with the aid of Figure 8.

![Figure 8. Two Port Network.](image-url)
\[ b_1 = S_{11} a_1 + S_{12} a_2 \] (12)

\[ b_2 = S_{21} a_1 + S_{22} a_2 \] (13)

and the reflection coefficients of the two ports are:

\[ \Gamma_{\text{in}} = \frac{b_1}{a_1} = S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22} \Gamma_L} \] (14)

\[ \Gamma_{\text{out}} = \frac{b_2}{a_2} = S_{22} + \frac{S_{21} S_{12} \Gamma_S}{1 - S_{11} \Gamma_S} \] (15)

Stability

Stability is an important parameter in the design of microwave amplifiers. To achieve maximum transducer gain, the network has to be conjugately matched to the load and source, but this is only possible in case of stable conditions.

A network is said to be unconditionally stable if the real part of its input and output impedances remain positive for all passive load and source impedances at a specific frequency. This implies that:

\[ |\Gamma_S| \text{ and } |\Gamma_L| < 1 \] (16)

where:

\[ \Gamma = \text{reflection coefficient} = \frac{Z - Z_0}{Z + Z_0} \]

\[ \Gamma_S = \text{source reflection coefficient} \]
\[ \Gamma_L = \text{load reflection coefficient} \]
\[ Z_0 = 50 \ \Omega \] unless otherwise stated

For a conditionally stable two port, the real part of its input and output impedances can be negative for some source and load impedances at a specific frequency. Hence, both \( \Gamma_s \) and \( \Gamma_L \) will be greater than one.

A stability factor, \( K \) (see Appendix A), is defined as:

\[
K = \frac{1 + |S_{11} S_{22} - S_{12} S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2 |S_{12}||S_{21}|} \quad (17)
\]

A stability factor greater than one is required for unconditional stability.

The NE 71083 is unstable at the frequencies of interest and the \( K \) values were calculated from the S-parameters given in Table 1.

<table>
<thead>
<tr>
<th>FREQ (GHz)</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>0.256</td>
</tr>
<tr>
<td>4.0</td>
<td>0.360</td>
</tr>
<tr>
<td>4.5</td>
<td>0.454</td>
</tr>
</tbody>
</table>

Before any further design calculations, it is necessary to know which load and source impedances provide stable operation.

Looking back to equation (14), a boundary could be established by setting \( \Gamma_{in} = 1 \) and solving the equation:
The right side of this equation can be changed in form to give the solution of a circle as a function of S-parameters:

\[
|\Gamma_{in}| = \left| S_{11} + \frac{S_{21} S_{12}}{1 - S_{22} \Gamma_L} \right| = 1 \tag{18}
\]

\[
\text{radius} \quad r_{out} = \left| \frac{S_{21} S_{12}}{|S_{22}|^2 - |\Delta|^2} \right| \tag{19}
\]

\[
\text{center} \quad c_{out} = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \tag{20}
\]

where:
* = conjugate

\[
\Delta = S_{11} S_{22} - S_{12} S_{21}
\]

\(r_{out}\) and \(c_{out}\) are the radius and center of the circle, respectively, which define the load impedance (or \(\Gamma_{out}\)) which make \(|\Gamma_{in}|\) stable or unstable.

For the input stability circles, equation (15) is solved and:

\[
r_{in} = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \tag{21}
\]

\[
c_{in} = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \tag{22}
\]
For this design, $Z_S$ and $Z_L$ are 50 ohms, $\Gamma_L$ and $\Gamma_S$ ideally should be zero, hence $|S_{11}| = |\Gamma_{in}|$ and $|S_{22}| = |\Gamma_{out}|$ (see Figure 8). Referring to the Smith Chart (Figure 9), the center is 50 $\Omega$ or $\Gamma_L = \Gamma_S = 0$ and as $|S_{11}|$ and $|S_{22}|$ from Table 2 are less than 1 (or $|\Gamma_{in}|$ and $|\Gamma_{out}| < 1$), then the insides of circles described by equations (19) through (22) represent the area of unstable operating conditions for the networks. As only passive loads ($\Gamma < 1$) are used to ensure stable operation of the amplifier, impedances inside the shaded region of Figure 9 as well as those close to the border line are avoided.

**TABLE 2**

**STABILITY CIRCLES**

<table>
<thead>
<tr>
<th>FREQ (GHz)</th>
<th>2</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{out}$</td>
<td>3.6</td>
<td>6.5</td>
<td>3.1</td>
<td>2.06</td>
<td>1.18</td>
</tr>
<tr>
<td>$\phi_{out}$</td>
<td>4.0</td>
<td>6.8</td>
<td>3.6</td>
<td>2.7</td>
<td>2.13</td>
</tr>
<tr>
<td>$\phi_{out}$</td>
<td>$93^0$</td>
<td>$118^0$</td>
<td>$114.4^0$</td>
<td>$110.76^0$</td>
<td>$134^0$</td>
</tr>
<tr>
<td>$r_{in}$</td>
<td>0.25</td>
<td>0.434</td>
<td>0.42</td>
<td>0.43</td>
<td>0.492</td>
</tr>
<tr>
<td>$\phi_{in}$</td>
<td>1.1</td>
<td>1.19</td>
<td>1.21</td>
<td>1.26</td>
<td>1.46</td>
</tr>
<tr>
<td>$\phi_{in}$</td>
<td>$530^0$</td>
<td>$88^0$</td>
<td>$97.85^0$</td>
<td>$105.4^0$</td>
<td>$156.17^0$</td>
</tr>
</tbody>
</table>

**Constant Gain Circles**

In the design of low noise amplifiers, optimization between gain and noise is of upmost importance. This could be done by drawing gain
(a) Shaded area indicates $|\Gamma_{\text{out}}| > 1$; therefore unshaded area indicates allowed input impedances region.

(b) The unshaded area is allowed for output impedances.

Figure 9. Input and Output (Multi-Frequency) Stability Circles.
and noise circles on the Smith Chart and choosing the best suitable stable impedance to match the input with.

The expression for the unilateral transducer power gain \( S_{12} = 0 \) as defined in the Hewlett-Packard Application Note 154 (April 1972) is:

\[
G_{TU} = \frac{(1 - |\tau_s|^2)}{|1 - S_{11} \Gamma_s|^2} \cdot |S_{21}|^2 \cdot \frac{(1 - |\tau_L|^2)}{|1 - S_{22} \Gamma_L|^2}
\]

\[
= G_s \cdot G_o \cdot G_L
\]

Figure 10. Amplifier Gain Blocks.

The input and output matching networks are passive but they contribute some gain \( G_s \) and \( G_L \) as they minimize the mismatch loss between \( Z_0 \) and \( S_{11} \), and \( Z_0 \) and \( S_{22} \), respectively. Maximum unilateral
transducer gain is achieved when $\Gamma_s = S_{11}^*$ and $\Gamma_l = S_{22}^*$ and equation (23) becomes:

$$G_{u_{\text{max}}} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2}$$ (24)

and when $\Gamma_s$ (or $\Gamma_l$) = 1, then $G_s$ ($G_l$) = 0. This means that for any arbitrary value of $G_s$, there is a value of $\Gamma_s$ which lies on a circle whose center is located on the vector going from the Smith Chart center to $S_{11}^*$. Such constant gain circles can be generated from the MESFET S-parameters by using the following formulas for both input and output, as $G_s$ has the same form as $G_l$.

$$r_i = \frac{\sqrt{1 - g_i} \cdot (1 - |S_{i1}|^2)}{1 - |S_{i1}|^2 (1 - g_i)}$$ (25)

$$d_i = \frac{G_i \cdot |S_{i1}|}{1 - |S_{i1}|^2 (1 - g_i)}$$ (26)

where:

- $g_i = G_i \cdot (1 - |S_{i1}|^2) = G_i / G_{i_{\text{max}}}$
- $i = 1$ or $2$
- $r_i = \text{the radius of the circle}$
- $d_i = \text{distance from center of Smith Chart to center of gain circle along the vector } S_{i1}^*$
Using the S-parameters in Table 1, the input and output constant gain circles at 4 GHz are listed in Table 3.

### TABLE 3
INPUT AND OUTPUT GAIN CIRCLES

<table>
<thead>
<tr>
<th>GAIN CIRCLE (dB)</th>
<th>CENTER</th>
<th>RADIUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.561</td>
<td>0.43</td>
</tr>
<tr>
<td>2</td>
<td>0.626</td>
<td>0.362</td>
</tr>
<tr>
<td>3</td>
<td>0.690</td>
<td>0.295</td>
</tr>
<tr>
<td>4</td>
<td>0.749</td>
<td>0.230</td>
</tr>
<tr>
<td>6.7115 (max)</td>
<td>0.887</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GAIN CIRCLE (dB)</th>
<th>CENTER</th>
<th>RADIUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.521</td>
<td>0.306</td>
</tr>
<tr>
<td>1.5</td>
<td>0.563</td>
<td>0.210</td>
</tr>
<tr>
<td>1.9628 (max)</td>
<td>0.603</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Constant Noise Circles

The noise figure as a function of source reflection coefficient $\Gamma_s$ can be expressed as (Hewlett Packard 1972):

$$F = F_{\text{min}} + 4 \, r_n \frac{|\Gamma_s - \Gamma_0|^2}{(1 - |\Gamma_s|^2) \, |1 + \Gamma_0|^2}$$

(27)

where:
Figure 11. Input and Output Gain Circles.
Figure 12. Noise Figure Circles.
\( F_{\text{min}} \) = minimum noise figure
\( r_n \) = normalized equivalent noise resistance = \( R_n/50 \)
\( \Gamma_0 \) = optimum source reflection coefficient for \( F_{\text{min}} \)

To determine a family of noise figure circles, a noise figure parameter, \( N_i \), is defined as:

\[
N_i = \frac{F_i - F_{\text{min}}}{4 \frac{1}{r_n}} (1 + |\Gamma_0|^2)
\]

where \( F_i \) is the desired noise figure circle. The center and radius of the circle is then given as (Hewlett-Packard 1972):

\[
C_{F_i} = \frac{\Gamma_0}{1 + N_i}
\]

\[
R_{F_i} = \frac{1}{1 + N_i} \sqrt{N_i^2 + N_i (1 - |\Gamma_0|^2)}
\]

The centers of the circles lie along the \( \Gamma_0 \) vector.

The noise parameters for the NE 71083 from data sheets at 4 GHz (\( V_{DS} = 3 \) V, \( I_{DS} = 10 \) mA):

\( F_{\text{min}} = 0.6 \) dB

\( \Gamma_0 = 0.64|61^\circ | \)

\( r_n = R_n/50 = 0.69 \) ohms
From equations (28) through (30), the following noise circles are calculated and are shown in Figure 12 (see Table 4 below):

**TABLE 4**

**NOISE CIRCLES**

<table>
<thead>
<tr>
<th>$F_i$ (dB)</th>
<th>$CF_i$</th>
<th>$RF_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.59</td>
<td>0.216</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>0.54</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>0.63</td>
</tr>
</tbody>
</table>

**Amplifier Specifications**

From Figure 9(a), it can be seen that $\Gamma_0$ is in the stable region and when Figures 11(a) and 12 are overlayed, $\Gamma_0$ falls on the 3 dB gain circle. This means that if the input matching network is such that the MESFET "sees" $\Gamma_0$, then the input will be matched for $F_{min}$ and the matching will reduce the mismatch losses by 3 dB (i.e., a gain of 3 dB). Figure 9(b) indicates that designing the output matching network may cause oscillations. $S_{22}^*$, which gives maximum power transfer, will make the amplifier oscillate; hence, the amplifier has to be detuned a little, as will be shown in the next chapter.
The design goals could now be set after the above analysis.

**Initial Design Goals:**

1. Amplifier noise figure < 2 dB
2. Associated gain: Device $(S_{21})^2 \approx 8$ dB
   
   \[ \text{Input Matching} = 3.0 \text{ dB} \]
   \[ \text{Output Matching} = 1.0 \text{ dB} \]

3. Frequency Range: \(4 \pm 0.25\) GHz

**Matching Networks**

**Input Matching**

The input circuit matching is accomplished by matching to optimum source impedance \(Z_{NF}\) for minimum noise figure. There are numerous matching networks, but the most used is the Tchebycheff filter type multi-stage impedance matching circuit (Liechti and Tillman 1974). This network has a large number of components and is more suited to wide band amplifiers. The trend recently is to go to minimum component networks for low noise amplifiers requiring a band ratio of 10 to 15 percent (this design goal is a $\text{BW}/f_0 = 12.5\%$).

The impedance \(Z'_{F_{opt}}\) corresponding to $\Gamma_0 = 0.64 \ \text{[610]}$ is given by:

\[
Z_{NF} = \frac{(1 - |\Gamma_0|^2) 50}{1 + |\Gamma_0|^2 - 2|\Gamma_0| \cos |\Gamma_0|} + j \frac{(2|\Gamma_0| \sin |\Gamma_0|) 50}{1 + |\Gamma_0|^2 - 2|\Gamma_0| \cos |\Gamma_0|} \tag{31}
\]

\[
= 37.412 + j 70.94
\]
Figure 13. Input Matching.

By matching the input to $\Gamma_0$ instead of $S_{11}^*$, the amplifier is stabilized, noise is optimized and gain is degraded by: $10 \log \left[ \frac{1}{1 - |S_{11}|^2} \right] - G_{\text{ref}} \, \text{dB} = 3.71 \, \text{dB}$ due to mismatch.
Figure 13 shows the matching process. \( Z_{NF} \) is converted to its equivalent admittance:

\[
Y_{NF} = \frac{1}{Z_{NF}} = 0.00582 - j0.011
\]

A short-circuited stub an eighth-wave length long is added to remove the imaginary part of the admittance. This stub looks like a shunt inductor of impedance:

\[
Z = jZ_0 \tan \beta l
\]

\[
= jZ_0 \tan \left( \frac{2}{g} \cdot \frac{g}{8} \right)
\]

\[
Z = jZ_0
\]

Therefore, the characteristic impedance \( jZ_0 \) of the stub is:

\[
jZ_0 = \frac{j}{0.011} = 91 \text{ ohms}
\]

The real part of the admittance is transformed to 50 ohms by a quarter wave transducer of characteristic impedance:

\[
Z = \sqrt{50 \times \frac{1}{0.00582}} = 92.7 \text{ ohms}
\]

Figure 14. Input Matching Sections.
Output Matching

The amplifier output is matched to the load to provide a low VSWR and a good power gain over the bandwidth. The output reflection coefficient \( S'_{22} \) is dependent on the input as was seen previously:

\[
S'_{22} = S_{22} + \frac{S_{21} S_{12} \Gamma_0}{1 - S_{11} \Gamma_0}
\]

at 4 GHz,

\[
S'_{22} = 0.603 \angle -65^\circ + \frac{2.8 \angle 99^\circ 0.073 \angle 28^\circ 0.64 \angle 61^\circ}{1 - 0.887 \angle -86^\circ 0.64 \angle 61^\circ}
\]

\[
= 0.603 \angle -65^\circ + \frac{0.1308 \angle 188^\circ}{0.5415 \angle -26.3^\circ}
\]

\[
= 0.685 \angle -85.37^\circ
\]

For a conjugate match, the load reflection coefficient should be:

\[
\Gamma_L = (S'_{22})^* = 0.685 \angle 85.37^\circ
\]

and its corresponding impedance from equation (31) is:

\[
Z_L = 19.533 + j50.254 \text{ ohms}
\]

This impedance will cause \( \Gamma_{in} \) to be greater than one (Figure 9); hence, it causes oscillations. A \( \Gamma'_L = 0.52 \angle 86^\circ \) was selected. This corresponds to \( Z_L = 30.454 + j43.305 \) ohms which falls on the 1.5 dB constant gain circle.
Figure 15. Output Matching.

By matching the output to $\Gamma'_L$ instead of $S_{22}^*$, the amplifier is stabilized, but the mismatch causes a gain degradation of at least

$$10 \log \left[ \frac{1}{1 - |S_{22}|^2} \right] - G_{T_L} \text{ dB} = 0.6 \text{ dB}.$$
From equation (31), a $\Gamma_L'$ of $0.52\,^\circ$ corresponds to:

$$Z_L = 30.454 + j 43.305$$

which equals to an admittance of:

$$Y_L = \frac{1}{Z_L} = 0.0109 - j 0.0154$$

A shunt inductive stub an eighth-wave length long of impedance $jZ_0$ is added to tune out the imaginary part.

$$jZ_0 = \frac{1}{0.0154}$$

$$Z_0 = 64.9 \text{ ohms}$$

A quarter-wave transformer of characteristic impedance:

$$Z = \sqrt{50 \times \frac{1}{0.0109}} = 67.73 \text{ ohms}$$

transforms the 92 ohms real part to the load impedance of 50 ohms.

Figure 16. Output Matching Sections.
Before explaining the fabrication process of the amplifier design with microstrips, a brief discussion of microstrip transmission lines is given.

**Microstrip Transmission Lines**

Truly lumped elements such as inductors and capacitors with low loss are difficult to find at high frequencies \( f \geq 4 \text{ GHz} \). Hence, transmission lines that can be fabricated on microstrip substrates with low loss are used to approximate the lumped elements.

By definition, a microstrip transmission line consists of a strip conductor and a ground plane separated by a dielectric medium. The field lines between the strip and the ground lines are not entirely contained in the substrate, but the bulk of the energy is transmitted with a field distribution closely resembling transverse electromagnetic (TEM). This is referred to as a quasi-TEM mode of propagation.

![Figure 17. Microstrip Transmission Lines: (a) General Geometry and (b) Showing a Quasi-TEM Wave.](image-url)
The important electrical parameters for designing a microstrip line are its characteristic impedance \( (Z_o) \) and guide wavelength \( (\lambda_g) \). The effective dielectric constant \( (E_{\text{eff}}) \) of the substrate material affects the physical length of the line as the wavelength is a function of \( E_{\text{eff}} \).

To implement the designed matching networks using microstrip lines, the physical widths and lengths of the lines have to be evaluated using formulas based on a quasi-TEM mode of propagation which ignores dispersion and radiation effects. At 4 GHz, this is a good static approximation of a dynamic structure. \( E_{\text{eff}} \) and the shape ratio, \( w/h \), can be expressed as (Edwards 1984):

For narrow lines \( (Z_o > \{44 - 2 E_r\}) \):

\[
\frac{w}{h} = \left( \frac{\exp H}{8} - \frac{1}{4 \exp H} \right)^{-1}
\]

\[
E_{\text{eff}} = \frac{E_r + 1}{2} \left(1 - \frac{1}{2 H} \left(\frac{E_r - 1}{E_r + 1}\right)\left(\ln \frac{\pi}{2} + \frac{1}{E_r} \ln \frac{4}{\pi}\right)\right)^{-2}
\]

where:

\[
H = \frac{Z_o \sqrt{2(E_r + 1)}}{119.9} + \frac{1}{2} \left(\frac{E_r - 1}{E_r + 1}\right)\left(\ln \frac{\pi}{2} + \frac{1}{E_r} \ln \frac{4}{\pi}\right)
\]

\( E_r \) = relative permittivity

\( \frac{w}{h} \) = ratio of strip width \( (w) \) to substrate height \( (h) \)

\( \exp \) = exponential base \( (e) \)
The wavelength, \( \lambda_g \), is then:

\[
\lambda_g = \frac{C}{F \sqrt{\varepsilon_{\text{eff}}}}
\]  

(35)

where:

\[
C = 3 \times 10^8 \text{ m/s}
\]

\[ F = \text{frequency (Hz)} \]

Finally, it is important to look at the dominant microstrip line losses, conductor loss and substrate dielectric loss. The dielectric losses are normally very low compared to conductor losses. The conductor losses are a function of surface resistivity, characteristic impedance, the width of the strip and frequency (Edwards 1984):

\[
\alpha_c = 0.072 \frac{\sqrt{f}}{w Z_0} \lambda_g \text{ dB/}\lambda_g
\]  

(36)

where \( f \) is in GHz.

The copper clad used has a polytetrafluoroethylene (PTFE) woven glass laminate. Its thickness \( h \) is 0.031 inches and it has a relative dielectric constant \( (\varepsilon_r) \) of 2.33. With this information, the strip width \( w \) and \( E_{\text{eff}} \) can be calculated from equations (32) through (34).

**Bias Lines**

To apply the gate and drain voltages, proper bias lines have to be designed so as not to effect the signal flow in the band of interest.
TABLE 5
MICROSTRIPS DIMENSIONS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>LENGTH (λg)</th>
<th>Z₀</th>
<th>E_{eff}</th>
<th>LENGTH (cm)</th>
<th>W/h</th>
<th>W (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shunt stub</td>
<td>λg/8</td>
<td>91</td>
<td>1.86</td>
<td>0.69</td>
<td>1.043</td>
<td>0.821</td>
</tr>
<tr>
<td>Transformer</td>
<td>λg/4</td>
<td>93</td>
<td>1.855</td>
<td>1.38</td>
<td>0.996</td>
<td>0.784</td>
</tr>
<tr>
<td>Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shunt stub</td>
<td>λg/8</td>
<td>65</td>
<td>1.94</td>
<td>0.673</td>
<td>1.96</td>
<td>1.54</td>
</tr>
<tr>
<td>Transformer</td>
<td>λg/4</td>
<td>68</td>
<td>1.91</td>
<td>1.35</td>
<td>1.83</td>
<td>1.44</td>
</tr>
<tr>
<td>50 Ohm lines</td>
<td>-</td>
<td>50</td>
<td>2.026</td>
<td>-</td>
<td>3</td>
<td>2.36</td>
</tr>
<tr>
<td>Bias lines</td>
<td>λg/4</td>
<td>114</td>
<td>1.8175</td>
<td>13.9</td>
<td>0.61</td>
<td>0.48</td>
</tr>
</tbody>
</table>

The above data was calculated from the program, "W/H" (see Appendix C).

For this design, very high impedance quarter wave lines were used to bias the MESFET. The following bias line was designed to see its insertion loss prior to its application to the amplifier.

Board Thickness, h = 0.7874 mm

E_r = 2.55

From equation (33):

E_{eff} = 1.944

W/h = 0.557

Therefore: W = 0.557 x 0.7874 = 0.44 mm
37

and:

\[
\frac{\lambda g/4}{4 \times 10^9 \sqrt{1.944}} = 3 \times 10^8
\]

\[
= 1.345 \text{ cm}
\]

The designed line is as shown in Figure 18. The insertion loss due to the bias line as measured at 4 GHz was 1.1 dB.

![Diagram of Bias Line](attachment:figure18.png)

**Figure 18. Bias Line.**

**Fabrication**

The amplifier layout with the bias, coupling capacitors and extra 50 ohm lines for connector connections is shown in Figure 19.

To fabricate the amplifier design on the PC board, a mask was generated with the aid of MASKCAD (a menu driven program developed at the University of Central Florida by the Solid State Devices Lab). The structure data file for the amplifier was developed on the IBM PC. The PC is connected to the HP 7580B plotter and uses India ink to draw the design layout on mylar paper.
MESFET: NE71083
RF by-pass capacitors: 1000 pF
DC blocking capacitors: 10 pF
L₁ L₂: RF chokes

Figure 19. Amplifier Design Layout.
The PC board undergoes a photoresist, exposure and etching process to transfer the designed pattern from the mask to the board, as described in Appendix B. Next, the capacitors, connectors and bias wires are soldered using very little solder. A hole just big enough to allow the bottom of the FET to slide through is cut and, using grounded tweezers (not to damage the MESFET by static discharge), the FET is placed in position, as shown in Figure 19. Using a medium hot soldering iron, the source leads are connected to the ground, and the gate and drain leads are soldered next.
CHAPTER IV
AMPLIFIER PERFORMANCE

Gain Measurements

The Wiltron Scalar Network Analyzer Model 560A was used to test the amplifier. The frequency response was poor, especially above 4 GHz; hence, two open circuit stubs were added (see Figure 20) to dampen the high frequencies as well as to facilitate for any necessary tuning. An open stub acts like a capacitor if it is less than a quarter-wave length long, thus the high frequencies are shunted.

![Graph showing frequency response](image)

Frequency = 10 MHz to 18.6 GHz
Markers = 4 + 0.25 GHz
Offset = 21 dB

Vertical = 5 dB/division
Horizontal Center line = + 18 dBm

Figure 21. Broad Band Response.
Figure 21 shows the wide band (10 MHz to 18.6 GHz) gain performance. The out-of-band response at about 1.5 GHz (due to the fact that the NE71083 has $S_{21} > 3$ at low frequencies) and 8 GHz is only 10 dB down.

Frequency = 4 + 0.25 GHz
Horizontal Centerline = +18 dBm
Vertical = 5 dB/division

Figure 22. Pass Band Frequency Response.

Figure 22 shows the pass band with a 3 dB bandwidth just over 500 MHz. The roll-off is not quick enough at 4.2 GHz.
Tuning the open stubs improved the response above the passband as can be seen in Figure 23. Below the passband, the shorted shunt inductor and the D.C. blocking capacitor form a high-pass filter at the amplifier input, by tuning either of these two the low frequency roll-off could have been improved but instead the MESFET was substituted with another one which had a longer gate lead. This added some series inductance which helped to resonate the MESFET at 4 GHz and stop all the low frequencies.
Frequency = 4 ± 0.25 GHz  \hspace{1cm} \text{Vertical} = 5 \text{ dB/division} \\
Horizontal Centerline = +18 \text{ dBm}

Figure 24. Pass Band Response.

Figure 24 shows that the 3 dB bandwidth has been reduced to about 380 MHz and the gain slightly higher due to the higher Q caused by the series inductance.

The Smith charts indicate that the input is better matched to 50 ohms than the output. The measured impedances around 4 GHz are given in Table 6.
**TABLE 6**

**INPUT AND OUTPUT IMPEDANCES**

<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>INPUT (ohms)</th>
<th>OUTPUT (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.98</td>
<td>39.7 + j 0.12</td>
<td>68.8 - j 7.0</td>
</tr>
<tr>
<td>4.00</td>
<td>41.3 + j 0.22</td>
<td>92.4 - j 8.9</td>
</tr>
<tr>
<td>4.02</td>
<td>53.2 - j 0.3</td>
<td>65.0 - j 0.5</td>
</tr>
<tr>
<td>4.04</td>
<td>61.2 - j 1.5</td>
<td>57.0 - j 0.52</td>
</tr>
</tbody>
</table>

**Figure 25. Isolation (S_{12}).**
Figure 26. Input Reflection.
Noise Figure Measurement

The noise factor of a device is the change in signal-to-noise ratio which occurs as a signal passes through that device. Hence, it is a figure of merit ideally equal to one which can be used to compare different amplifiers.

Noise figure measurements are normally made on a noise figure meter. In the absence of such equipment, the spectrum analyzer could be used with a good accuracy. The noise figure equation can be arranged to read as:

\[ F = 10 \log \frac{N_0}{\text{Output Noise Power}} - 10 \log G_d - 10 \log kTB \]  

where:

\[ G_d = \text{device gain} \]

The term for the noise input can be reduced to \(-[10 \log KT + 10 \log B]\) which equals \(-[-174 \text{ dBm} + 10 \log B]\). Thus, the noise figure can now be determined by knowing the device's noise power output, gain and bandwidth. The noise figure measurement procedure with a spectrum analyzer, as given by the Hewlett Packard Spectrum Analyzer Series, is:

Step One: Determine System Gain. The total gain is determined by the change in signal power displayed with switches on position one and then two as shown in Figure 28.
Step Two: Noise Power Measurement. The signal generator is disconnected and the device input terminated in its characteristic impedance. The analyzer input attenuator is set to 0 dB. The average noise power is read on the display by using sufficient video smoothing. The analyzer's IF bandwidth setting, B, is recorded. The noise figure is then calculated as:

\[
F = 10 \log N_0 - (G_d + G_p) - 10 \log B + 174 \text{ dB} + CF \text{ dB} \quad (39)
\]

The correction factor, CF, is the sum of two things, depending on the type of analyzer being used. For an HP Spectrum Analyzer, this is made up of:

1. Bandwidth: The noise power bandwidth, B, is set by the resolution bandwidth of the spectrum analyzer (set sufficiently narrow). But, the noise power bandwidth of an HP spectrum analyzer is about 1.2 times the resolution bandwidth as shown in Figure 28.

2. The (HP) Spectrum Analyzer's log amplifier and detector perform non-linear processes that cause the noise level to be degraded by 2.5 dB.

Therefore, \( CF = 2.5 - 10 \log 1.2 = 1.7 \text{ dB} \).
The equipment was set up as shown in Figure 28, but a mixer was inserted after the device (low noise amplifier) to down convert the frequency. The noise figure with the amplifier inserted was measured using the above procedures and then the noise figure without the amplifier was measured.

At 4 GHz:

Total Gain = 42.5 dB
Output Noise Power = -86 dBm

If bandwidth setting = 30 KHz = 44.7 dB, then:

\[ F = -87 - 42.5 - 44.77 + 174 = 1.43 \text{ dB} \]

Gain without Amplifier = 33.5 dB
Output Noise without Amplifier = -95 dBm

\[ F = -95 - 3.5 - 44.7 + 175.7 = 2.43 \text{ dB} \]

Therefore, low noise amplifier noise figure is:

\[ F_{\text{LNA}} = 2.43 - 1.43 = 1 \text{ dB} \]
CHAPTER V
CONCLUSION

A comparison of the design specifications with measured amplifier performance is shown in Table 7.

TABLE 7
EXPERIMENTAL AND THEORETICAL DATA

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>MEASURED PERFORMANCE</th>
<th>DESIGN GOAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>14 dB</td>
<td>12 dB</td>
</tr>
<tr>
<td>Noise Figure</td>
<td>1 dB</td>
<td>&lt;2 dB</td>
</tr>
<tr>
<td>Input VSWR</td>
<td>1.21</td>
<td>-</td>
</tr>
<tr>
<td>Output VSWR</td>
<td>1.87</td>
<td>-</td>
</tr>
<tr>
<td>Isolation</td>
<td>-21 dB</td>
<td>-</td>
</tr>
<tr>
<td>Bandwidth (3 dB)</td>
<td>380 MHz</td>
<td>500 MHz</td>
</tr>
</tbody>
</table>

A 4 GHz low noise amplifier was designed and fabricated. The input circuit consists of a quarter wave transformer and a shunt inductive stub. A shunt inductive stub and a quarter wave transformer match the MESFET's output to the load. The MESFET was biased for minimum noise and the input network was designed to synthesize the optimum source impedance for a low noise figure.
The stability regions for the conditionally stable MESFET were established and with constant gain and noise circles the best compromise between amplifier gain and noise figure was achieved.

The experimental results demonstrate that amplifiers with GaAs MESFETs can yield a high gain and low noise with the above matching networks over a bandwidth approximately 12 percent of the center frequency. The measured bandwidth is 120 MHz short of the design goal due to the high Q input admittance. A resistor in-series with a short-circuited shunt stub connected between gate and source (stub \(= \lambda g/4\)) lowers the Q, hence increasing the bandwidth. As this adjustment is done after tuning the amplifier, it was not possible to accomplish a bandwidth increase in the laboratory.

Finally, the output VSWR and the amplifier isolation could have been improved by computer optimization of the matching sections.
APPENDIX A

UNCONDITIONAL STABILITY

Viewing the conditions of stability graphically, it is necessary for the stability circle whose inside represents the unstable region to be completely outside the Smith Chart. This means the Smith Chart area ($|\Gamma| < 1$) is stable. Therefore:

$$|C_L| - r_L > 1 \quad (A-1)$$

or

$$|C_s| - r_s > 1 \quad (A-2)$$

Shown graphically in Figure 30.

Substituting equations (19) and (20) into equation (A-1), we have:

$$\left| \frac{|(S_{22} - S_{11}^*)| - |S_{12} S_{21}|}{|S_{22}|^2 - |\Delta|^2} \right| > 1 \quad (A-3)$$

which implies:

$$\left| |S_{22} - \Delta S_{11}^*| - |S_{12} S_{21}| \right| > \left| |S_{22}|^2 - |\Delta|^2 | \right| \quad (A-4)$$

Squaring both sides and grouping terms:
Figure 30. Unconditional Stability.
\[ 2 \left| S_{12} S_{21} \right| \left| S_{22} - \Delta S_{11} \right| < \left| S_{22} - \Delta S_{11} \right|^2 + \left| S_{12} S_{21} \right|^2 \]

\[ - \left| S_{22} \right|^2 - \left| \Delta \right|^2 \right|^2 \]

(A-5)

The term:

\[ \left| S_{22} - S_{11}^* \right|^2 = (S_{22} - \Delta S_{11}^*) (S_{22}^* - \Delta S_{11}^*) \]

\[ = S_{22} S_{22}^* - \Delta S_{11} S_{22} - \Delta S_{11}^* S_{22}^* + |\Delta|^2 \left| S_{11} \right|^2 \]

\[ = |\Delta|^2 \left| S_{11} \right|^2 + \left| S_{22} \right|^2 - \Delta S_{11} S_{22} - \Delta S_{11}^* S_{22}^* \]

But:

\[ \Delta = (S_{11} S_{22} - S_{12} S_{21}) \]

Hence:

\[ S_{11} S_{22} = \Delta + S_{12} S_{21} \]

and

\[ S_{11}^* S_{22}^* = \Delta^* + S_{12}^* S_{21}^* \]

Therefore:

\[ \left| S_{22} - \Delta S_{11} \right|^2 = |\Delta|^2 \left| S_{11} \right|^2 + \left| S_{22} \right|^2 \]

\[ - \Delta^* (\Delta + S_{12} S_{21}) - \Delta (\Delta^* + S_{12}^* S_{21}^*) \]
\[ |s_{22} - \Delta s_{11}|^2 = |\Delta|^2 |s_{11}|^2 + |s_{22}|^2 - |\Delta|^2 \]

\[- (|\Delta|^2 + \Delta^* s_{12} s_{21} + \Delta s_{12}^* s_{21} + |s_{12} s_{21}|^2) \]

\[= |\Delta|^2 |s_{11}|^2 + |s_{22}|^2 - |\Delta|^2 \]

\[- (\Delta + s_{12} s_{22})^2 + |s_{12}|^2 |s_{21}|^2 \]

Substituting for \( \Delta \) in the brackets:

\[ |s_{22} - \Delta s_{11}|^2 = |\Delta|^2 |s_{11}|^2 + |s_{22}|^2 - |\Delta|^2 \]

\[- (s_{11} s_{22} - s_{12} s_{21} + s_{21} s_{12})^2 + |s_{12} s_{21}|^2 \]

\[= |s_{12} s_{21}|^2 + (1 - |s_{11}|^2) (|s_{22}|^2 - |\Delta|^2) \quad (A-6) \]

Substituting equation (A-6) into (A-5), squaring and combining terms, we have:

\[ (|s_{22}|^2 - |\Delta|^2)^2 \{(1 - |s_{11}|^2) - (|s_{22}|^2 - |\Delta|^2)\}^2 - 4 |s_{12} s_{21}|^2 > 0 \]
\[ \begin{align*}
&= \left( |S_{22}|^4 - 2 |S_{22}|^2 |\Delta|^2 + |\Delta|^4 \right) \left( (1 - |S_{11}|^2 - 2 (1 - |S_{11}|^2) 
\right. \\
& \hspace{1cm} \left. \left( |S_{22}|^2 - |\Delta|^2 \right) + \left( |S_{22}|^2 - |\Delta|^2 \right)^2 - 4 |S_{12} S_{21}|^2 \right) > 0
\end{align*} \]

This reduces to:

\[ 2 |S_{12} S_{21}| < 1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2 \]

and the stability factor is:

\[ K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 |S_{12}| |S_{21}|} > 1 \]

for unconditional stability.
APPENDIX B

PHOTORESIST TECHNIQUE

The microstrip lines are fabricated by following the technique outlined below:

1. Preparation
   a. The mask is drawn as explained in the text.
   b. The copper clad board is cleaned using a fine emery paper to remove any oxides.

2. Set-Up Procedure
   a. The oven is switched on and set to 95°C. It takes about 30 minutes to stabilize.
   b. The vacuum pump for the spinner table and the spinner control are switched on. The control is set to approximately 4000 rpm.
   c. The ultraviolet exposure is switched on.

3. The board is placed on the spinner and photoresist (Baker PR-20, positive) is applied with a syringe. The board is spun for about 30 seconds and then soft baked in the oven for 15 minutes. The above procedure is repeated for the other side of the board.

4. The mask is placed on the microstrip plane and held firm by a piece of glass to ensure good contact. The board is set on the Kasper mask aligner and exposed for a least 90 seconds, depending on the nature of the mask.

5. The exposed photoresist is developed with Shipley Microposit Developer and deionized H₂O, 1:1 mixture. This is done at room temperature until the exposed photoresist has been removed.

6. The board is then immersed in copper etchant (ferrochloride) until the exposed copper has been removed. The board is rinsed in water and blown dry with the air gun.
APPENDIX C
DESIGN PROGRAMS

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFF</td>
<td>Frequency dependent microstrip permittivity $E_{\text{eff}}(f)$</td>
</tr>
<tr>
<td>LEO</td>
<td>The equivalent extra length of a microstrip line to account for the end-effect of an open stub</td>
</tr>
<tr>
<td>$Z_{\text{NF}}$</td>
<td>The impedance corresponding to $\Gamma$ in a 50 ohms system (equation 31)</td>
</tr>
<tr>
<td>DELTA</td>
<td>Computes $S_{11}S_{22} - S_{12}S_{21}$</td>
</tr>
<tr>
<td>$K$</td>
<td>The stability factor (equation 17)</td>
</tr>
<tr>
<td>NF</td>
<td>Noise figure circles (equations 29 and 30)</td>
</tr>
<tr>
<td>GAIN</td>
<td>Constant gain circles (equations 25 and 26)</td>
</tr>
<tr>
<td>M</td>
<td>Noise measure</td>
</tr>
<tr>
<td>STAB</td>
<td>Stability circles (equations 19 through 22)</td>
</tr>
<tr>
<td>W/H</td>
<td>Finds the ratio of W/H and $E_{\text{eff}}$ (equations 32 through 34)</td>
</tr>
</tbody>
</table>
11:18  11/20
01 LBL "EFF"
02 "F GHZ=?"
03 PROMPT
04 STO 02
05 "H MM=?"
06 PROMPT
07 STO 03
08 "ZO=?"
09 PROMPT
10 STO 04
11 "ER=?"
12 PROMPT
13 STO 05
14 "E EFF=?"
15 PROMPT
16 STO 06
17 RCL 02
18 3
19 Y^X
20 .009
21 *
22 CHS
23 RCL 02
24 X^2
25 .43
26 *
27 +
28 RCL 03
29 RCL 04
30 /
31 1.33
32 Y^X
33 *
34 1
35 +
36 1/X
37 RCL 05
38 RCL 06
39 -
40 *
41 CHS
42 RCL 05
43 +
44 END
01 LBL "LED"
02 "W/H=?”
03 PROMPT
04 STO 02
05 "EFF=?”
06 PROMPT
07 STO 03
08 "H=?”
09 PROMPT
10 STO 04
11 RCL 02
12 .813
13 +
14 1/X
15 RCL 02
16 .262
17 +
18 *
19 RCL 03
20 .258
21 -
22 1/X
23 RCL 03
24 .3
25 +
26 *
27 *
28 RCL 04
29 *
30 .412
31 *
32 END
01 LBL "ZNF"
02 "REF=?"
03 PROMPT
04 STO 00
05 X<>Y
06 STO 01
07 RCL 01
08 COS
09 RCL 00
10 *
11 2
12 *
13 CHS
14 RCL 00
15 X^2
16 1
17 +
18 +
19 STO 02
20 RCL 01
21 SIN
22 RCL 00
23 *
24 100
25 *
26 RCL 02
27 /
28 STO Y
29 RCL 00
30 X^2
31 CHS
32 1
33 +
34 50
35 *
36 RCL 02
37 /
38 END
01 LBL "DELTA"
02 "S11=?"
03 PROMPT
04 P-R
05 "S22=?"
06 PROMPT
07 P-R
08 XROM "C*"
09 STO 00
10 X<>Y
11 STO 01
12 "S12=?"
13 PROMPT
14 P-R
15 "S21=?"
16 PROMPT
17 P-R
18 XROM "C*"
19 STO 02
20 X<>Y
21 STO 03
22 RCL 01
23 ENTER
24 RCL 00
25 ENTER
26 RCL 03
27 ENTER
28 RCL 02
29 XROM "C-"
30 R-P
31 STO 20
32 X<>Y
33 STO 21
34 X<>Y
35 X^2
36 BEEP
37 END
11:21  11/20
01LBL "K"
02 "MAG S22=?”
03 PROMPT
04 X^2
05 "MAG S11=?”
06 PROMPT
07 X^2
08 +
09 CHS
10 1
11 +
12 STO 05
13 XEQ "DELTA"
14 RCL 05
15 +
16 2
17 /
18 "MAG S12=?”
19 PROMPT
20 /
21 "MAG S21=?”
22 PROMPT
23 /
24 "K="
25 ARCL X
26 AVIEW
27 END
11:21  11/20
01 LBL "NF"
02 "GAMA O=?" 41 *
03 PROMPT 42 RCL 00
04 STO 01 43 X^2
05 X<>Y 44 +
06 STO 02 45 SQRT
07 X<>Y 46 RCL 00
08 P-R 47 1
09 1 48 +
10 + 49 1/X
11 R-P 50 *
12 X^2 51 "RF=
13 4 52 ARCL X
14 / 53 AVIEW
15 "RN/50=?" 54 END
16 PROMPT
17 /
18 "FI=?”
19 PROMPT
20 "F MIN=?”
21 PROMPT
22 -
23 *
24 STO 00
25 RCL 00
26 1
27 +
28 1/X
29 RCL 01
30 *
31 "CF=
32 ARCL X
33 SF 21
34 AVIEW
35 RCL 01
36 X^2
37 CHS
38 1
39 +
40 RCL 00
01 LBL "GAIN"
02 "SII=?" PROMPT
03 STO 00
05 X<>Y
06 STO 01
07 X<>Y
08 X^2
08 CHS
10 1
11 +
12 "GA REQ=?" PROMPT
13 *
15 STO 02
16 CHS
17 1
18 +
19 RCL 00
20 X^2
21 *
22 CHS
23 1
24 +
25 1/X
26 RCL 00
27 *
28 RCL 02
29 *
30 "CNT=" ARCL X
31 SF 21
33 AVIEW
34 1
35 RCL 02
36 -
37 RCL 00
38 X^2
39 *
40 CHS
41 1
42 +
43 1/X
44 1
45 ENTER^)
46 RCL 02
47 -
48 SQRT
49 1
50 ENTER^)
51 RCL 00
52 X^2
53 -
54 *
55 *
56 "RAD=" ARCL X
57 AVIEW
58 AVIEW
59 END
01 LBL "M"
02 "G=?"
03 PROMPT
04 1/X
05 CHS
06 1
07 +
08 1/X
09 "NF=?"
10 PROMPT
11 1
12 -
13 *
14 END
11:24  11/20
01 LBL "STAB"
02 "S22 MAG=?"
03 PROMPT
04 X^2
05 STO 04
06 XEQ "DELTA"
07 CHS
08 RCL 04
09 +
10 1/X
11 STO 05
12 "S12=?”
13 PROMPT
14 P-R
15 "S21=?"
16 PROMPT
17 P-R
18 XROM "C-"
19 R-P
20 RCL 05
21 *
22 "RAD="
23 ARCL X
24 SF 21
25 AVIEW
26 "S11=?”
27 PROMPT
28 P-R
29 X<>Y
30 CHS
31 X<>Y
32 RCL 21
33 RCL 20
34 P-R
35 XROM "C*"
36 "S22=?"
37 PROMPT
38 P-R
39 RDN
40 RDN
LBL "W/H"
ER=7"
PROMPT
STO 00
4
ENTER
PI
/
LN
RCL 00
1/X
PI
2
I
LN
+
RCL 00
1
-
RCL 00
1
+
2
•
SQRT
"ZO=?"
PROMPT
*
119.9
/
+
STO 01
RCL 01
E^X
8
/
RCL 01
E^X
4
*
1/X
50 -
51 1/X
52 "W/H=
53 ARCL X
54 SF 21
55 AVIEW
56 PI
57 2
58 /
59 LN
60 4
61 ENTER^  
62 PI
63 /
64 LN
65 RCL 00
66 1/X
67 *  
68 +
69 RCL 00
70 1
71 -
72 RCL 00
73 1
74 +
75 /
76 *  
77 RCL 01
78 2
79 *
80 1/X
81 *
82 CHS
83 1
84 +
85 X^2
86 1/X
87 RCL 00
88 1
89 +
90 2
91 /
92 *
93 "EeFF=
94 ARCL X
95 SF 21
96 AVIEW
97 END
REFERENCES


