Saturn's Rings: Measuring Particle Size Distributions Using Cassini UVIS Occultation Data

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SATURN’S RINGS: MEASURING PARTICLE SIZE DISTRIBUTIONS USING CASSINI UVIS OCCULTATION DATA

by

TRACY M. BECKER
B. S. Lehigh University, 2010

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics in the Department of Physics, Planetary Science in the College of Sciences at the University of Central Florida Orlando, Florida

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ABSTRACT

Since its arrival to Saturn in 2004, the Cassini spacecraft has utilized its suite of sophisticated instruments to further our understanding of the Saturnian ring system. We analyze occultation data from Cassini’s Ultraviolet Imaging Spectrograph (UVIS) in order to measure the particle size distribution and place limits on the minimum particle sizes in Saturn’s rings.

Throughout the ring system, particle accretion is countered by collisional and tidal disruption and Keplerian shear. Therefore, the particle size distribution of the rings is continually evolving. The presence of sub-centimeter particles, which have short lifetimes due to these processes, is indicative of ongoing dynamics in the rings. Sub-centimeter-sized particles efficiently diffract light at ultraviolet wavelengths, and thus produce signatures of diffraction in the occultation data. The shape and intensity of the diffraction signatures are indicative of the sizes of the particles that produce them. The UVIS wavelength bandpass, 51.2 - 180 nm, contains the shortest wavelengths of the Cassini instruments, making it most sensitive to the smallest particles in the rings.

We have developed a computational model that reconstructs the geometry of a UVIS observation and produces a synthetic diffraction signal for a given truncated power-law particle size distribution, which we compare with the observed signal. We implement this model for two sets of observations: (1) diffraction spikes at sharp ring edges during stellar occultations and (2) the light curve due to attenuated and diffracted sunlight by particles in Saturn’s F ring during solar occultations.
Near sharp ring edges, ring particles can diffract light such that there is a measurable increase in the signal of an unocculted star exterior to the ring. In Saturn’s A ring, diffracted light can augment the stellar signal by up to 6% and can be detected tens of kilometers radially beyond the edge. The radial profile of the diffraction signal is dependent on the size distribution of the particle population near the ring edge. These diffraction signals are observed at sharp edges throughout Saturn’s rings, although in this work we focus on diffraction at the outer edge of Saturn’s A ring and at the edges of the Encke Gap. We find an overall steepening of the power-law size distribution and a decrease in the minimum particle size at the outer edge of the A ring when compared with the Encke Gap edges. This suggests that interparticle collisions caused by satellite perturbations in the region result in more shedding of regolith or fragmentation of particles in the outermost parts of the A ring.

We rule out any significant population of sub-millimeter-sized particles in Saturn’s A ring, placing a lower limitation of 1-mm on the minimum particle size in the ring.

We also model the light curves produced as Saturn’s F ring occults the Sun. We consider both the attenuated signal and the light diffracted by the particles in the ring during the occultation. Five of the eleven solar occultations analyzed show a clear signature of diffracted light that surpasses the unocculted solar signal. This includes a misaligned solar occultation that placed most of the solar disk outside of the instrument’s field of view, reducing the solar signal by 97.5% and resulting in the serendipitous detection of diffracted light. We measure a large variation in the the size distribution of the particles that fill the broad, \( \sim 500 \text{ km} \) region surrounding the F ring core. We find that smaller particles \((\leq 50\mu\text{m})\) are present during solar occultations for which diffraction was detected, and place a lower limit on the minimum particle size of 100\(\mu\text{m}\) for occultations during which diffraction was not detected. A comparison with images of the F ring observed by the Cassini Imaging Science Subsystem near the times of the occultations reveals that the detections of small particles in the UVIS data correspond with locations of collisional events in the F ring. This implies that collisions
within the F ring core replenish the sub-millimeter-sized dust in the 500-km region that encompasses the F ring core.
I dedicate this work to my grandmother, a truly inspirational woman.
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LIST OF SYMBOLS

UVIS  Ultraviolet Imaging Spectrograph
VIMS  Infrared Mapping Spectrometer
FOV   Field of View
ISS   Imaging Science Subsystem

\( a \)  Particle radius
\( a_{\text{min}} \)  Minimum particle size
\( a_{\text{max}} \)  Maximum particle size
\( q \)  Index for the power-law size distribution
\( \lambda \)  Wavelength of the light
\( \theta \)  Scattering angle. This is 180° minus the phase angle.
\( B \)  Ring-opening angle. This is the angle between the ring plane and the incident radiation, or the ring plane and the line-of-sight vector of the observer.
\( \mu \)  \( \sin(|B|) \)
\( \tau_n \)  Normal optical depth.
\( Q_{\text{ext}} \)  Extinction efficiency of a particle.
\( Q_{\text{occ}} \)  Effective extinction efficiency of a particle.
\( D_{\text{LOS}} \)  The line-of-sight distance from Cassini to the rings.
CHAPTER 1: INTRODUCTION

Since the advent of the telescope, curious minds have marveled at the beauty and complexity of the Saturnian ring system. Beginning in 1859, when James Clerk Maxwell first proved that the rings must be composed of numerous particles orbiting the planet (rather than solid disks as previously believed), efforts have been made to determine the distribution of the sizes of the particles that make up the rings. Ground-based observations and spacecraft fly-bys have contributed to our understanding of this fundamental property of the rings; however, the Cassini orbiter’s 11-year expedition (at the time of this writing) has enabled unprecedented, high-resolution observations by a suite of sophisticated, complimentary instruments at a variety of geometries, revolutionizing our understanding of the ring system.

Knowledge about the particle size distribution is critical for discerning current particle interactions and for modeling the origin and evolution of the ring system. The presence of small particles, for example, is indicative of on-going collisions or bombardment of larger clumps or aggregates, since small particles have short lifetimes in the rings. Additionally, the dynamics of planetary rings are analogous to those of protoplanetary disks and spiral galaxies. Therefore, the study of Saturn’s rings not only provides insight into processes within our own solar system, but also sheds light on more distant or more ancient astronomical structures.

In this dissertation we analyze Cassini data with the intent of placing strong constraints on the particle size distribution of Saturn’s rings. We investigate signatures of light diffracted by ring particles in ultraviolet (UV) data from the Cassini Ultraviolet Imaging Spectrograph (UVIS) and compare these signatures with computational models. The characteristics of the diffraction light curves are diagnostic of the sizes of the particles that produce them. The UVIS instrument observes at the shortest wavelengths of any instrument on Cassini,
which makes it the most sensitive instrument to light diffracted by sub-centimeter-sized ring particles, enabling us to constrain the minimum particle sizes in regions across Saturn’s rings. The methodology of this analysis, including a detailed description of the development of the computational models and their applications in measuring the ring particle size distribution is discussed in the following chapters. First, we begin this chapter with an introduction to the Cassini spacecraft. We then present a brief history of the study of the particle sizes in Saturn’s rings, followed by descriptions of the different ring regions and their defining characteristics and features. Finally, we present a complete outline of the dissertation.

1.1 The Cassini Mission

The Cassini mission was established in order to perform an in-depth investigation of Saturn, its magnetosphere, moons, and vast ring system. The spacecraft was launched October 15, 1997 and, after fly-bys of Venus, Earth and Jupiter, began orbiting Saturn on June 30, 2004. The successful mission is a joint effort of NASA, the European Space Agency (ESA) and the Agenzia Spaziale Italiana (ASI), with a total of 17 countries involved.

The orbiter is equipped with twelve instruments designed to address the mission’s scientific objectives focused on Saturn, its magnetosphere, the rings, the icy satellites, and Saturn’s largest moon, Titan (Matson et al., 2003). Table 1.1 lists each instrument along with a brief description of its objectives. The specific Cassini objectives for ring science include the investigations of (1) the interactions between rings and the planet’s magnetosphere, ionosphere and atmosphere, (2) the configurations of the the rings and the dynamical processes that create the ring structure, (3) dust and meteoroid distribution near the rings, (4) interrelation of rings and satellites, and (5) the composition and size distribution of ring material (Matson et al., 2003). The work presented in this dissertation directly addresses the mission objective of mapping the particle size distribution in the rings. To achieve this goal,
we analyze data from the Ultraviolet Imaging Spectrograph (UVIS). There are four additional remote sensing instruments on Cassini that we will discuss throughout this work: the Visual and Infrared Mapping Spectrometer (VIMS), the Imaging Science Subsystem (ISS), the Radio Science Subsystem (RSS) and the Composite Infrared Spectrometer (CIRS). The spacecraft also carried the ESA Huygens probe, which landed on the surface of Saturn’s largest moon, Titan, on January 14, 2005. Figure 1.1 is a diagram of the Cassini spacecraft indicating the locations of the various instruments.

Figure 1.1: Diagram of the Cassini spacecraft and its suite of instruments. Image credit NASA/Jet Propulsion Laboratory (JPL)/Space Science Institute (SSI).

The initial 4-year mission for Cassini ended in 2008 and has been followed by two successful extended missions: the Cassini Equinox mission and the Cassini Solstice mission. As the end of the Solstice mission approaches, Cassini will begin its “Grand Finale”; the spacecraft will engage in a unique set of orbits, flying through the ring plane just beyond the planet’s main rings and then through the region interior to the rings, before plummeting into the planet’s atmosphere to end its 13-year mission in September of 2017. The long extent of Cassini’s expedition will have allowed for observations spanning 45% of Saturn’s orbital period, enabling measurements of seasonal variations on the planet as well as other temporal changes in the ring system.
1.2 Particle Size Distribution of Saturn’s Rings

After Maxwell determined that Saturn’s rings must be comprised of individual particles, a natural and important question arose: what are the sizes of those particles? With the exception of the largest ring “particles” – the ring moons Pan and Daphnis – individual ring particles are too small to be distinguishable, even with the high-resolution observations of Cassini. Therefore other techniques to determine the particle sizes must be applied. Throughout this work we consider particles to be free-floating, individual entities; however they may be covered in a regolith of significantly smaller particles. An individual particle can be any size, ranging from microscopic dust to kilometer-sized bodies. Objects on the order of tens to hundreds of meters are typically referred to as embedded moonlets, while even larger objects like Pan and Daphnis are officially ring moons; however, such bodies can still be considered a part of the particle size distribution. Individual particles can clump and cluster to form larger objects such as the observed tens-of-meters-across self-gravity wakes (Colwell et al., 2006) and other ropy or straw-like structures seen in the rings (Porco et al., 2005). These structures, which extend azimuthally due to Keplerian shear (Colwell et al., 2009), would not be considered ring particles.

For over half a century, a multitude of observations have been used to characterize the sizes of the ring particles. As described in detail by Orton et al. (2009), a variety of experiments, including multiple observations with different instruments and telescopes, and ultimately the presence of spacecrafts and an orbiter, would be necessary to accurately describe the range of particle diameters found throughout the rings. In the 1960’s, ground-based microwave observations of the rings did not detect the expected 100 K thermal emission that would be emitted by particles with sizes on the order of the observational wavelength. This was interpreted to mean that the particles were sub-centimeter grains (Berge and Read, 1968; Berge and Muhleman, 1973). However, strong radar backscattering proved that the
particles had to be at least several centimeters in size (Goldstein and Morris, 1973; Goldstein et al., 1977; Ostro et al., 1980, 1982). The discrepancy was remedied by Pollack et al. (1977), who showed that pure water ice could have strong radar backscattering but weak thermal emission if the particles were comparable in size to the microwave wavelengths. The Voyager 1 and Voyager 2 flybys in 1980 and 1981, respectively, significantly contributed to measurements of the particles; diffracted light in Voyager radio occultation data was analyzed in order to determine the distribution of particle sizes, placing an upper limit of 1–5 meters in the main rings (Marouf et al., 1982; Tyler et al., 1983). The radio occultations also revealed that the particle size distribution could be modeled using a power-law. A power-law size distribution states that the number of particles $N(a)$ with a radius $a$ between $a$ and $a + da$, where $da$ is an infinitesimally small radius increase, is

$$N(a)da = Ca^{-q}da$$  \hspace{1cm} (1.1)

for a constant, $C$. The power law index, $q$, determines the steepness of the slope (large $q$ is a steep slope) of the size distribution. The size distribution measured by the radio occultations indicated a power law with an index of $\sim 3$. Such a distribution indicates equal surface area per decade of particle sizes. This value is comparable to the index of $q \sim 3.5$ observed in the asteroid belt and is also the value measured for the shattering of an object in a laboratory, suggesting that such size distributions are likely derived through collisional evolution (Dohnanyi, 1969). The Voyager 2 stellar occultation provided measurements of the surface mass density, which was used to measure a several-meter-radii upper size-cutoff throughout the rings (Cooper et al., 1985). Additionally, Showalter and Nicholson (1990) utilized the variance in the Voyager 2 stellar occultation data to measure 1-10’s of meters as the upper cut-off of the particle size distribution. In 1989, several ground-based telescopes observed the stellar occultation of 28 Sagittarius (Sgr) by Saturn’s rings. French and Nicholson (2000)
measured the signal of scattered light from the occultation and, through a comparison with the optical depth profile of the rings as determined by Voyager 2 (Lane et al., 1982; Esposito et al., 1983), determined a lower cut-off of the particle size distribution on the order of a centimeter. Additional constraints on the size distribution have been made as a result of Cassini’s extensive observational campaign and will be discussed throughout this work.

![Figure 1.2: Saturn’s main rings. This natural color mosaic from Cassini shows Saturn’s three main rings (A, B, and C), the Cassini Division that separates the A and B rings. Image Credit: NASA/JPL/SSI.](image)

The particle size distribution, however, is variable across the rings. Previous results indicate that the maximum particle size may vary from a few meters in the C ring, to 20 meters in the B ring, and a variable maximum particle size in the A ring between 10 and 20 meters (Zebker et al., 1985; French and Nicholson, 2000; Cuzzi et al., 2009). The minimum particle sizes range from 1-10 millimeters in the C ring and parts of the A ring, to 30 centimeters in the B ring and other parts of the A ring (Zebker et al., 1985; French and Nicholson, 2000; Cuzzi et al., 2009). The local distribution is indicative of active processes and possibly the origin of that ring or ring region. Additionally, observations from the era of Voyager to the era of Cassini, and even observations from within the decade of Cassini, have revealed that a given ring or region is not stagnant; the rings are changing with time and
are not azimuthally symmetric. Measuring the variations in the particle size distribution of the rings leads to a deeper understanding of the dynamics, particle interactions, and general evolution of planetary ring systems. Micron-sized dust particles seen in the F ring and other dusty rings have short lifetimes in the rings due to radiative forces (Burns et al., 1984), so their persistence in the rings is indicative of recent or on-going collisions.

As discussed above, different experiments probe various aspects of the particle size distribution; divergences in measurements made at different wavelengths can be indicative of the sizes of the particles interacting with electromagnetic radiation. In the case of occultation data, sub-centimeter-sized particles do not interact with radio waves, making such particles, which are apparent in UV occultations, invisible to the radio occultations. Additionally, varying the observational geometry can expose hidden details; for example, stellar occultation measurements made perpendicular to the ring plane can readily identify small gaps in the rings and radial variations at ring edges, while observations directed along the ring plane can reveal the physical thickness of the ring (Jerousek et al., 2011; Scharringhausen and Nicholson, 2013). Moreover, determining information about the canted gravity wakes necessitates the use of observations at a variety of azimuthal geometries (Colwell et al., 2006). Furthermore, observations of reflected sunlight by the rings provide details about the particle composition, size, and packing density, while measurements of the rings as they block sunlight can illuminate micron-sized dust due to scattered light (Chapter 2) that would otherwise be invisible. It is necessary to have a range of experiments in order to develop a complete picture of the origin and evolution of Saturn’s rings.

1.3 Saturn’s Rings

The Saturnian ring system is composed of distinct rings and ring regions. Figure 1.2 is a natural color mosaic of images from the wide-angle camera on the Cassini ISS. The ring
regions are visibly distinguishable, indicating distinct properties and possibly unique origins of each ring region. In Figure 1.2 we have labeled the A, B and C rings (main rings), as well as the Cassini Division that separates the A and B rings. The fainter, dusty rings can be seen in Figure 1.3, which is an observation of the rings at a phase angle (the observation angle between the star, target, and spacecraft) close to 180°, where the Sun is directly behind Saturn. At high phase angles, the micron- and sub-micron-sized dust particles forward-scatter a significant amount of light, illuminating faint regions and entire rings that are not otherwise easily visible. At this phase angle, the faint D ring, F ring, G ring, and E ring are prominent. Utilizing the scattering properties of particles to measure their sizes is at the crux of this dissertation and will be discussed throughout this work.

Figure 1.3: Saturn's faint rings. This composite of visible images (with exaggerated contrast for clarity) was taken near 180° phase angle, when the Sun was located behind, and was therefore blocked, by Saturn. At high phase angles, the micron- and sub-micron-sized particles that make up the faint E ring, G ring, D ring, and F ring forward-scatter the sunlight, making them visible in these images. The main rings are also labeled. Image credit NASA/JPL/SSI.

1.3.1 Main Rings

Saturn’s main rings are composed of nearly pure, crystalline water ice, with varying amounts of an unidentified UV absorber (Cuzzi et al., 2009). They consist of particles ranging in size
from a few millimeters (as shown in this work) to several meters in radius (Cuzzi et al., 2009), and are well-modeled with a power-law size distribution indicative of collisional evolution (Charnoz et al., 2009; Cuzzi et al., 2010). Larger populations of centimeter and sub-centimeter sized particles have been detected in the C ring and outer parts of the A ring than in the B ring or inner A ring (Zebker et al., 1985; Nicholson et al., 2000; Cuzzi et al., 2009).

![Figure 1.4: Optical depth profile of Saturn’s rings as it corresponds with a Cassini ISS image, similar to that found in Colwell et al. (2009). The top plot is the normal optical depth profile of Saturn’s rings as observed by a UVIS stellar occultation of the star β Centauri at 10 km radial resolution. The different ring regions are noted by the colors. We cut the optical depth at 4.5. The bottom image is a natural color mosaic taken by the Cassini ISS. The regions of high optical depth roughly correlate with some of the brighter regions of the ring. Ring gaps correspond with zero optical depth in the optical depth profile. Bottom image credit NASA/JPL/SSI.](image)

Saturn’s main rings, although vast in extent, are considerably thin; at most the main rings are a few tens of meters thick (Schmidt et al., 2009), with the exception of the vertically extended, kilometer-sized structures observed at the B ring outer edge during equinox (more information about this edge is discussed later in this section). The inelastic ring particle
collisions, although slow at 0.01-0.1 cm/second, work to dampen the energy of the particles and circularize orbits, flattening the system (Cuzzi et al., 2010).

One of the most defining characteristics of the rings is their optical depth. How optically-thick a ring region is may indicate particle packing density, size, or composition. The ring optical depth can be determined, indirectly, through stellar occultations. The occultation data is a direct measurement of the transparency of the rings: i.e., how much light is able to pass through the rings when they occult a star. We discuss the calculations for the optical depth in Chapter 3, but for this context we clarify that high optical depths indicate opaque ring regions, while low optical depths indicate more transparent ring regions. Figure 1.4 shows the optical depth profile of the rings as viewed from directly above the rings (normal optical depth) at UV wavelengths. The figure compares the optical depth profile with a natural color mosaic of Saturn’s rings. The B ring has the highest optical depth, with some regions essentially blocking all light from the background star. The A ring has a slightly more intermediate optical depth while the C ring and Cassini Division are optically-thin. Ring gaps are identifiable in the occultation data as regions with an optical thickness equal to 0. The high optical depth regions of the B ring inhibit characterizations of the particles and the mass of those regions.

The main rings are a testament to the gravitational influence of satellites on ring particles. Once thought to be the orderly display of a cloud of particles calmly orbiting Saturn, observations by the Voyager spacecrafts and now Cassini data have revealed that the rings exhibit a number of complex features, many of which result from the influence of the satellites. There are a number of resonances with satellites throughout the rings. Satellite resonances are locations in the rings where there is a periodic, gravitational influence on the ring particles by a satellite. Specifically, Lindblad resonances – where the epicyclic frequency of the ring material is in an integer-number-ratio with the forcing frequency of a satellite – can strongly influence the rings, and are even responsible for the delineation between ring regions; the
2:1 resonance with Mimas sculpts the outer edge of the B ring while the 7:6 resonance with the co-orbital moons Janus and Epimetheus maintains the sharp outer edge of the A ring. Strong satellite resonances are also responsible for the Bond Gap and the Dawes Gap in the C ring (Nicholson et al., 1990).

Figure 1.5: ISS image of the Encke Gap. The wavy edges induced by a recent encounter with the gap-moon Pan and a faint ringlet of particles are visible in the image. Density waves created by resonances with Saturn’s moons Prometheus and Pandora are also labeled. Figure from Colwell et al. (2009). Image credit NASA/JPL/SSI.

Figure 1.6: ISS image of the Keeler Gap, the ring-moon Daphnis, and the gravitationally-induced edge waves. Image credit NASA/JPL/SSI.

Satellite resonances throughout the rings also launch density waves and bending waves.
Density waves, as seen in Figure 1.5, are radial compression waves that propagate radially from the orbital resonance location where they are initiated. Bending waves are transverse waves conceived at locations where the ring particles’ vertical frequency is in resonance with that of the satellite (Colwell et al., 2009). The phase of the waves are longitude-dependent, which gives rise to the spiral structures (Colwell et al., 2009). These waves are produced by the same mechanisms that create the spiral structure in galaxies (Goldreich and Tremaine, 1978a,b, 1980; Lin and Shu, 1964).

Embedded satellites also have a significant impact on the rings. The most prominent features in the A ring are the two large gaps created and maintained by embedded moons. The Encke Gap, which is 320 km wide, is cultivated by Pan, a 28-km sized moon (Showalter, 1991), while the Keeler Gap, which is only 40 km across, is preserved by the 8-km sized moon, Daphnis (Porco et al., 2007; Colwell et al., 2009). These moons influence the regions around them, causing wavy edges in the gaps and moon-induced wakes in the ring material beyond the gaps (Figure 1.5) (Showalter et al., 1986; Colwell et al., 2009). In the case of the Keeler Gap, the influence of Daphnis causes ~ 15 km variations in the radial location of the inner edge in addition to the edge waves (Figure 1.6) (Colwell et al., 2009). Additionally, within the larger Encke Gap, faint ringlets can be detected (Showalter, 1991; Porco et al., 2005; Hedman et al., 2013).

Embedded moonlets too small to clear a circumferential gap have been discovered in the central A ring. These objects are detected due to the small, propeller-shaped openings they produce in the rings (Figure 1.7). These moonlets have diameters between 40 - 500 meters (Tiscareno et al., 2008). In 2014, Murray et al. (2014) revealed the discovery of an embedded, sub-kilometer-sized object at the outermost part of the A ring, which they named “Peggy”. Objects with a significant vertical extent have also been detected at the outer edge of Saturn’s B ring. Figure 1.8 shows the shadows cast from objects extending approximately 2.5 km above the ring plane. This image was captured just before Saturn’s equinox, enabling
the unique observation of the ring edge structure.

The rich variety of ring features and satellite-induced dynamical activity in the rings have a significant impact on the local particle size distribution. As material is stirred, particles can coalesce into larger objects, or larger objects can collide and release smaller particles.

Figure 1.7: ISS narrow angle image of the propeller object named Bleriot (found within the circle). Propeller objects are small, embedded moonlets detectable by the small, propeller-shaped features they produce in the rings. Image credit NASA/JPL/SSI.

Figure 1.8: Saturn’s B ring edge as observed near Saturn’s equinox. Structures extending up to 2.5 km above the ring plane cast long shadows in this image. Image credit NASA/JPL/SSI.
1.3.2 F Ring

Saturn’s F ring, discovered by Pioneer 11 in 1979 (Gehrels et al., 1980), continues to be a region of interest and active research. The narrow ring has a semi-major axis of $\sim 140,223$ km, approximately 3,000 km beyond the main ring system (Bosh et al., 2002). The F ring core is a low-optical depth, 10 - 50 km wide ring (Murray et al., 2008; Albers et al., 2012) consisting of mostly micron-sized dust particles (Showalter et al., 1992; Vahidinia et al., 2011; Hedman et al., 2011). A discontinuous, optically-thick, $\sim 1$ km wide “true core” exists within this region (Murray et al., 2008; Albers et al., 2012). It is this true core that was likely observed by the photopolarimeter occultations (Lane et al., 1982) and radio experiments by Voyager 1 (Tyler et al., 1983; Marouf et al., 1986), which revealed a very narrow core that must consist of centimeter-sized particles or larger (in order to be detected at radio wavelengths). In fact, it is possible that the majority of the mass of the F ring is contained in this narrow region, which could house the opaque, meter- to kilometer-sized moonlets observed in occultation data (Esposito et al., 2008; Meinke et al., 2012). Collisions of these objects may lead to the replenishment of the small particles that constitute the
rest of the ring (Barbara and Esposito, 2002). The entire F ring core lies within a very low-optical depth, extended envelope of sub-millimeter-sized dust particles (Showalter et al., 1992; Vahidinia et al., 2011; Hedman et al., 2011; French et al., 2012). In addition to the F ring core, faint strands of material, varying in shape and in number, as well as longitude and time, have been observed to straddle the core (Charnoz et al., 2005; Murray et al., 2008). They appear as concentric ringlets orbiting on either side of the F ring core, extending hundreds of kilometers radially; however, Charnoz et al. (2005) show that the strands are in fact one spiral structure that crosses the core.

In addition to its complex structure, the ring itself is extremely dynamic. Some of the structural intricacies and variability of the ring are attributed to the massive moon Prometheus, whose orbit brings the satellite between 180 km - 790 km at closest approach (e.g. Showalter and Burns (1982); Borderies and Goldreich (1983); Kolvoord et al. (1990); Colwell et al. (2009); Cuzzi et al. (2010); French et al. (2012)). The closest approach varies over a timescale of 19 years as a result of the differential precession of the orbits of Prometheus and the F ring core. There is a corresponding 19-year peak in distortions during the period when Prometheus and the F ring become anti-aligned (Cuzzi et al., 2010). As seen in the Cassini image in Figure 1.9, the Saturnian satellite regularly perturbs the core, drawing material from the ring as it orbits. Additionally, the F ring’s inclination to the main ring plane is attributed to excitation from the moon (Bosh et al., 2002). Still, the moon’s gravitational influence cannot explain all of the ring’s features. Interactions with the moon fail to explain the overall brightening of the ring observed since the Voyager flybys (French et al., 2012). Although the moon’s proximity to the ring has varied substantially throughout the extent of the Cassini mission, the relative brightness of the ring has not changed dramatically during the first several years of Cassini observations, suggesting the moon’s influence is not responsible for the overall brightening of the ring. Furthermore, Charnoz (2010) determined that scattering of the tiny particles in the core due to Prometheus’ gravity would result in only
a few kilometers of dispersion, not the observed hundreds of kilometers of dispersion.

The small dust particles scattered throughout the core and strands should have very short lifetimes due to Poynting-Robertson drag (Burns et al., 1984; Charnoz et al., 2005; French et al., 2012). Their persistence implies a replenishing source, most likely from the larger particles in the core (Showalter et al., 1992; Barbara and Esposito, 2002). Charnoz (2010) demonstrates that if loose clumps exterior to the ring collide with larger particles within the core, the amount of material ejected could form a spiral arm. With time, the arm would evolve to become parallel to the core, as observed early in the Cassini mission (Charnoz et al., 2005). Observations of large masses passing through the core (Esposito et al., 2008; Charnoz, 2010; Cuzzi et al., 2010; French et al., 2012) support this theory of spiral arm formation. Moonlets and clumps, which are large, opaque conglomerates on the order of several hundred meters in size, have been observed in Cassini images and in stellar occultations of the F ring. Analyses by Esposito et al. (2008); Meinke et al. (2012) show there may be 1500 - 140,000 moonlets approximately 600 meters in size. A second population of large (0.1 - 10 km) aggregates was predicted to exist in a 2,000 km-wide band around the F ring by Cuzzi and Burns (1988) to account for the sudden depletion of trapped magnetospheric electrons. Direct observations of moonlets and elongated clumps crossing the F ring's core have since been made (Charnoz et al., 2005; French et al., 2012). As these objects pass through, they may collide with the large aggregates within the core (Esposito et al., 2004; Charnoz, 2010; French et al., 2012). The cloud of dust that would be released due to disruptive collisions between these unconsolidated bodies may result in the observed temporary bursts of brightness seen throughout the ring (Showalter, 1998; Esposito et al., 2008; Cuzzi et al., 2010; French et al., 2012). A very large brightening event was observed by Cassini in 2006 and is believed to be associated with the transient object S/2004 S6 that was observed on a core-crossing orbit at the intersection of the spiral with the ring (Charnoz et al., 2005; Murray et al., 2008). Such collisions, with impact velocities on the order of
several tens of meters per second, would create “jets” (Figure 1.10) near the core and would scatter small particles that replenish the strands (Charnoz, 2010). Slower collisions (on the order of 1 meter per second) produce the many smaller “mini-jets” analyzed by Attree et al. (2012) throughout the ring.

Figure 1.10: ISS image of a “jet” in the F ring taken on June 20, 2013 from the unlit side of the rings. This jet is likely composed of several smaller jets, which are created by collisions between moonlets and the F ring core (Murray et al., 2008; Attree et al., 2012). Image credit NASA/JPL/SSI.

The size and distribution of the small particles throughout the core and strands could constrain theories on the formation of the F ring structures. Their properties better describe the masses of the moonlets responsible for their production, putting a constraint on the mass of the ring itself. The abundance of these small particles, which generally have short lifetimes, has implications for the rate of interactions and collisions within the ring. Just as spiral density waves and gaps in the main rings can be used as analogues to spiral galaxies and cavities in circumstellar disks, respectively, the dynamics in the F ring can be used as a laboratory to study currently unobservable phenomena like planetesimal formation (French et al., 2012). The F ring’s location at the edge of Saturn’s Roche zone highlights the everlasting battle between accretion and disruption of aggregates like that faced by planets as they form around stars.
1.3.3 Faint Rings

In addition to the main rings, Saturn has several faint and tenuous, though extensive, rings and a series of faint ringlets found within the large gaps located in the A ring, C ring, and Cassini Division. Horányi et al. (2009) provides a complete overview of these diffuse rings. We do not focus on these faint ring systems in this work; however, we do provide a brief overview of these systems here.

The diffuse rings are composed of particles smaller than 100 microns in size, making them susceptible to non-gravitational forces such as Poynting-Robertson drag (Burns et al., 1984; Horányi et al., 2009). Such forces therefore impact the evolution of these rings. The major diffuse rings include the D ring, which is located closer to Saturn than the main ring system and extends from 65,000 km - 74,500 km radially from Saturn’s center (Showalter, 1996; Hedman et al., 2007a), the G ring, which lies outside the main rings and extends from 165,000 km - 175,000 km (Hedman et al., 2007b), and the E ring which spans from 180,000 km to 700,000 km (Showalter et al., 1991). There are several tenuous rings and ring arcs coinciding with the small Saturnian satellites Pallene, Janus/Epimetheus, Anthe, and Methone in the region that extends from the F ring to the E ring (Porco et al., 2005; Porco and team, 2006; Hedman et al., 2007b; Murray et al., 2008; Hedman et al., 2009b).

Cassini observations have revealed a different D ring than the Voyager images, signifying that the ring has evolved significantly in the last few decades. One of the brightest ringlets in the D ring during the Voyager era no longer exists, or has possibly evolved into a broader, more diffuse feature (Hedman et al., 2007a; Horányi et al., 2009). In addition to temporal variability, the ring demonstrates significant variation in its particle size distribution as a function of radial location from Saturn. Closer to the planet, the rings are composed of a higher percentage of the smaller, $1 - 10\mu m$ particles than the larger, $10 - 100\mu m$ particles, while in the outer parts of the D ring, the distribution becomes more even (Horányi et al.,...
The ring also displays pronounced density waves with perturbations with periods corresponding to the rotation of Saturn’s atmosphere (Hedman et al., 2009a; Horányi et al., 2009). These perturbations are not likely due to gravitational effects, and therefore studies of the particle dynamics in the D ring may provide insight into the asymmetries of the planet’s magnetosphere.

Saturn’s G ring is defined by a sharp inner edge and a much more diffuse outer edge that blends into the E ring. Within the ring is a relatively bright, 60° arc that has persisted throughout the Cassini observations (Hedman et al., 2007b; Horányi et al., 2009). The arc is constrained by the 7:6 co-rotation eccentricity resonance with Mimas (Hedman et al., 2007b), providing another strong example of the relationship between the Saturnian satellites and the rings. Prior to Cassini’s arrival, ground-based and Hubble observations had revealed a spectrally redder G ring (Nicholson et al., 1996; de Pater et al., 1996; Bauer et al., 1997; de Pater et al., 2004), indicating a collisionally-evolved particle size distribution similar to other dusty rings, like those of Jupiter and Uranus. Saturn’s E ring, however, had a strong blue slope in backscattered light, suggestive of a very different size distribution and likely a different source for the particles (Showalter et al., 1991). The enhancement in the density of E ring particles in the vicinity of Enceladus had suggested a correlation between the moon and the rings; however, it was not until the arrival of Cassini and the discovery of the water-ice plumes emanating from the south pole of Enceladus (Porco et al., 2005) that the origin of the E ring could be confirmed.

Like in the main rings, the diffuse rings of Saturn are governed by the gravitational influences of the Saturnian satellites. The material that creates the rings themselves are derived from some of these moons. Additionally, the study of the small particles that constitute these rings can provide insight and understanding of the processes that form and evolve the diffuse rings of Saturn.
1.4 Organization of Thesis

The goal of this thesis work is to utilize data from the Cassini spacecraft to better constrain the particle size distribution of Saturn’s rings. We organize this thesis as follows: Chapter 2 contains a detailed discussion of the Cassini Ultraviolet Imaging Spectrograph (UVIS) and the original work we completed to characterize the instrument’s sensitivity and to correct for a pointing issue with the instrument. We present a model by which one can calculate the relative sensitivity of the observation based on the light source’s location within the instrument field of view. Chapter 3 discusses the theoretical background concerning the diffraction of light. Here we also discuss the implementation of the theoretical calculations to computational modeling of Saturn’s rings. In Chapter 4 we analyze solar occultation data of Saturn’s F ring and discuss the particle size distribution of the ring as determined through our modeling efforts. Chapter 5 primarily explores Saturn’s outer A ring and how we implement our model to reproduce diffraction signals observed at sharp ring edges and thereby determine the particle size distribution of the ring near those edges. We devote Chapter 6 to a discussion on the implications of the results recorded in this thesis work and discuss remaining questions and future work.
Table 1.1: List of instruments on board the Cassini spacecraft and a brief description of the objectives as stated by Matson et al. (2003).

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Acronym</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imaging Science Subsystem</td>
<td>ISS</td>
<td>Multispectral imaging of Saturn, Titan, rings and the icy satellites to observe their properties</td>
</tr>
<tr>
<td>Radio Science Subsystem</td>
<td>RSS</td>
<td>Study of atmospheric and ring structure, gravity fields and gravitational waves</td>
</tr>
<tr>
<td>Composite Infrared Spectrometer</td>
<td>CIRS</td>
<td>Temperature and composition of surfaces, atmospheres, and rings within the Saturn system</td>
</tr>
<tr>
<td>Visual and Infrared Mapping Spectrometer</td>
<td>VIMS</td>
<td>Spectral mapping to study composition and structure of surfaces, atmospheres, and rings</td>
</tr>
<tr>
<td>Ultraviolet Imaging Spectrograph</td>
<td>UVIS</td>
<td>Spectra and low resolution imaging of atmospheres and rings for structure, chemistry and composition</td>
</tr>
<tr>
<td>Cassini Plasma Spectrometer</td>
<td>CAPS</td>
<td>In situ study of ice and dust grains in the Saturn system</td>
</tr>
<tr>
<td>Cosmic Dust Analyzer</td>
<td>CDA</td>
<td>In situ study of ice and dust grains in the Saturn system</td>
</tr>
<tr>
<td>Ion and Neutral Mass Spectrometer</td>
<td>INMS</td>
<td>In situ compositions of neutral and charged particles within the Saturn magnetosphere</td>
</tr>
<tr>
<td>Magnetometer</td>
<td>MAG</td>
<td>Study of Saturn’s magnetic field and interactions with the solar wind</td>
</tr>
<tr>
<td>Magnetospheric Imaging Instrument</td>
<td>MIMI</td>
<td>Global magnetospheric imaging and in situ measurements of Saturn’s magnetosphere and solar wind interactions</td>
</tr>
<tr>
<td>Radio and Plasma Wave Science</td>
<td>RPWS</td>
<td>Measure the electric and magnetic fields and electron density and temperature in the interplanetary medium and within the Saturn magnetosphere</td>
</tr>
<tr>
<td>Radar</td>
<td>RADAR</td>
<td>Radar imaging, altimetry, and passive radiometry of Titan’s surface</td>
</tr>
</tbody>
</table>
Table 1.2: Location and width of Saturn’s main ring regions. The numbers indicate the ring plane radial distance from Saturn’s center. This table is adopted from Charnoz et al. (2009), with values determined by Nicholson and Dones (1991) and Burns et al. (2001).

*Ring edge varies with azimuth by 160 km due to perturbations by the 2:1 inner Lindblad resonance with Mimas (Colwell et al., 2009).

+F ring core varies by 50 km (Murray et al., 2008; Albers et al., 2012).

<table>
<thead>
<tr>
<th>Ring Region</th>
<th>Radial Distance from Saturn (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D ring</td>
<td>66,000 - 74,000</td>
</tr>
<tr>
<td>C ring</td>
<td>74,490 - 91,983</td>
</tr>
<tr>
<td>B ring</td>
<td>91,983 - 117,516*</td>
</tr>
<tr>
<td>Cassini Division</td>
<td>117,516* - 122,053</td>
</tr>
<tr>
<td>A ring</td>
<td>122,053 - 136,744</td>
</tr>
<tr>
<td>F ring</td>
<td>140,200+</td>
</tr>
<tr>
<td>G ring</td>
<td>166,000 - 173,000</td>
</tr>
<tr>
<td>E ring</td>
<td>180,000 - 450,000</td>
</tr>
</tbody>
</table>
CHAPTER 2: OBSERVATIONS FROM THE ULTRAVIOLET IMAGING SPECTROGRAPH

The Ultraviolet Imaging Spectrograph (UVIS) is one of twelve instruments aboard the Cassini spacecraft (Figure 2.1). The instrument was built at the Laboratory for Atmospheric and Space Physics (LASP) at the University of Colorado, capitalizing on the university’s experience with building ultraviolet spectrometers for the Mariner, Pioneer, and Galileo missions (Esposito et al., 2004). UVIS consists of four separate channels that are designed to operate concurrently. These include two telescope-spectrographs: the Extreme Ultraviolet (EUV) channel, which observes at wavelengths from 56 - 118 nm and the Far Ultraviolet (FUV) channel, which covers the wavelength range from 110 - 190 nm. The EUV channel has a solar occultation mode enabled by a pick-off mirror pointed 20° off-axis to spread the signal and avoid overexposing the primary telescope (Section 2.3). A third channel is the high-resolution High Speed Photometer (HSP) and the fourth channel is the Hydrogen Deuterium Absorption Cell (HDAC). Figure 2.2 is a graphic showing the configuration of the instrument. The four channels were built to execute revolutionary observations on a variety of aspects of the Saturnian system, including studies of Saturn’s rings, its atmosphere, its magnetosphere, Titan, and the so-called icy satellites (Esposito et al., 2004). UVIS operates at shorter wavelengths than any other instrument on Cassini, enabling unique observations of these science targets.

In this investigation, we utilize high temporal resolution HSP stellar occultation data of Saturn’s rings as well as solar occultation data from the EUV channel; we therefore focus on these UVIS channels in the following sections. First, we describe the procedure for modeling and analyzing spacecraft data with the proper geometry in Section 2.1. In Section 2.2 we
describe the HSP, stellar occultations, and the necessary calibrations for the instrument’s response to starlight. Finally, we describe solar occultations and our complete calibration and pointing analysis of the EUV Solar Port in Section 2.3.

Figure 2.1: Photograph of the Ultraviolet Imaging Spectrograph. Image credit LASP.

Figure 2.2: The UVIS channel configuration. The EUV Solar Occultation Aperture is marked, indicating the location and angle of the pick-off mirror used to redirect solar signal to prevent overexposure of the telescope. Figure from Esposito et al. (2004).
2.1 Spacecraft observations

NASA’s Navigation and Ancillary Information Facility (NAIF) provides SPICE, an observation geometry information system. The SPICE toolkit supplies users with the positions, orientations, time conversions, and sizes and shapes of solar system bodies and NASA spacecrafts. These tools can therefore be used to determine the relative positions of spacecrafts and celestial bodies, such as Cassini’s location relative to Saturn, and the orientation of individual instruments, like UVIS.

SPICE operates using information stored in data files called kernels. There are several types of kernels: SP (spacecraft/planet) kernels contain the spacecraft ephemeris and the location, orientation and size of the target bodies. The I (instrument) kernel contains instrument information, including the field of view (FOV) shape, size and orientation. The C (camera) kernel provides the pointing information for the spacecraft using a transformation matrix, or C-matrix, of the spacecraft orientation angles as a function of time. It is generally used to determine the orientation of the spacecraft or an object for a specified reference frame. For example, we typically need to account for the position and orientation of the spacecraft in an inertial frame. Throughout this work we use the conventional Celestial Reference Frame, J2000, to define the inertial frame. The J2000 frame is geocentric, with the x-axis aligned with the Earth’s mean equinox on January 1, 2000 at 12:00 Terrestrial Time. The z-axis is aligned with the Earth’s spin axis on that date, and the y-axis is orthogonal to the x- and z-axes. The C-kernel can therefore be used to determine the pointing of the spacecraft within the inertial frame at specified times. The E (events) kernels summarize mission events, including the science plans, sequences, and notes. Additional important kernels include the F (frames) kernel, which is used to specify and define the relationships between reference frames. Specifically, the frames kernel contains details like the orientation of each instrument onboard Cassini. The SPICE toolkit, also provided by NAIF, consists
of application program interfaces (APIs) that can be used to read the kernel files and then calculate observation geometry parameters (NAIF, 2015).

Throughout the analyses described in this work, SPICE kernels and the SPICE toolkit are used to determine the locations of solar system objects in a given reference frame, converting between coordinate systems, and to find number of other parameters including the direction of the boresight (center of field of view) look vectors for the individual channels of UVIS.

### 2.2 High Speed Photometer

The HSP is designed for high-temporal-resolution observations, and is often used to observe stellar occultations of stars by Saturn’s rings. The instrument has a short integration time of 1-8 milliseconds, providing radial spatial resolutions of $\sim 10$ meters for typical observation geometries (Esposito et al., 2004; Colwell et al., 2007, 2010). The HSP is therefore particularly useful for analyzing fine structure in the rings.

The HSP consists of a $4.4 \times 6.0$ cm telescope mirror, an MgF$_2$ lens, a field aperture, photomultiplier tube, a high voltage power supply and a pulse amplifier/discriminator (McClintock et al., 1993; Esposito et al., 2004). Its configuration is shown in Figure 2.3. The field of view size is limited to 6 mrad $\times$ 6 mrad by the field aperture. The photomultiplier tube has a CsI cathode, selected for its low sensitivity to the solar signal at visible wavelengths. This is particularly useful for stellar occultations (Section 2.2.1) that occur when the spacecraft is on the lit side of the rings. The ring-reflected solar signal would otherwise cause a background signal larger than that of the star being observed. This material, however, limits the observation wavelength to 190 nm due to the work function of CsI. The wavelengths are also limited to 115 nm by the MgF$_2$ detector window (Esposito et al., 2004). These wavelength limitations require that the stellar occultations be of bright stars with spectral class O and B.
2.2.1 Stellar Occultations

During a stellar occultation, the boresight of the HSP field of view is pointed toward a chosen star. The HSP records the number of photons from the star, observing the variations in that count as the rings pass in front of (occult) the star, as seen by the spacecraft. Stellar occultations provide at least two useful mechanisms for studying ring structure and particle sizes: first, the extinction of direct starlight provides a measurement of the optical depth of the rings, which is dependent on the sizes and number density of particles; second, in addition to the direct stellar signal, observations of stellar occultations capture diffracted starlight that has a characteristic light curve dependent on the wavelength of light and the particle size (van de Hulst, 1957). The use of diffracted light to probe the particle size distribution of Saturn’s rings will be explored throughout this dissertation.

Figure 2.4 compares a Cassini Imaging Science Subsystem (ISS) image of Saturn’s A Ring and F Ring (top of inset) with a UVIS stellar occultation dataset (bottom of inset).
Figure 2.4: Comparison of an ISS image of Saturn’s A Ring (top of inset) and F ring with a UVIS occultation of the same region (bottom of inset). The stellar occultation shown, binned to 1-km resolution, is a typical signal observed during a stellar occultation. The photon counts are high when the star is in regions devoid of ring particles, like beyond the A ring edge ($\sim 136,700$ km) and in the gaps such as the Encke Gap ($\sim 133,000$ km) and the Keeler Gap ($\sim 136,500$ km), and low when the star passes behind ring material. Image credit NASA/JPL/SSI.

In the region between $137,000$ km – $140,000$ km, we observe the direct, unocculted stellar signal. As the star passes behind the rings, the particles that constitute the rings block the incident light, attenuating the signal and causing the decrease in photon counts. The signal notably returns to the unocculted value, $I_0$, in the nearly-particle-free gaps known as the Encke Gap and Keeler Gap. The gaps, which are clearly defined in the images, are seen as regions of maximum stellar signal in the UVIS stellar occultation data. The feature at the radial location of $\sim 134,300$ km, where the stellar signal is nearly completely lost, is the 6:5 resonance with Saturn’s moon, Janus. The variations in the stellar signal as the star passes
behind the rings is indicative of the variety of features and ongoing dynamical activity in the rings.

![Figure 2.5: (a) Ingress vs. Egress](image1)

![Figure 2.5: (b) Chord Occultation](image2)

Figure 2.5: (a) Here we show the difference between an ingress and an egress observation. Ingress observations are observations during which the spacecraft moves such that the radial distance from Saturn along the ring plane is decreasing with time (the projected line of sight moves toward Saturn). Egress observations are observations such that the observed ring plane radius increases with time, so the projected observation path is moving farther from Saturn.

(b) Chord occultations are occultations that do not transverse the entire ring system. A chord occultation will view the same region of the rings twice, separated less than 180° apart in longitude. Chord occultations will also have an ingress and an egress component, as shown here. The egress observation begins when the ring plane radius of the observation begins increasing rather than decreasing.

Image credit NASA/JPL/SSI.

Throughout this dissertation, we will often refer to stellar occultations by their abbreviated name and the revolution of Cassini about Saturn during the observation (i.e. BetCen077 for β Centauri during Cassini’s 77th revolution). Some occultations traverse the entire ring system, so each region of the ring is seen twice. Additionally, chord occultations, which cut across the rings (Figure 2.5) will produce data from the same region or regions of the rings during one occultation. We distinguish the inbound and outbound parts of the occultation
as ingress and egress. Ingress observations are defined as observations when the spacecraft is moving such that the radial distance from Saturn in the ring plane is decreasing with time. During egress occultations, the radial distance from Saturn increases with time (see Figure 2.5). We denote an ingress or egress occultations with an ‘i’ or an ‘e’, respectively (i.e. BetCen077E for the egress observation).

2.2.2 HSP Calibration

2.2.2.1 Background

The HSP measures a signal \( I \) of photon counts per iteration, which includes the (possibly attenuated) starlight and a background signal, \( b \). When the occultation occurs on the sunlit side of the rings, the dominant source for the background is the sunlight reflected by the rings. The primary source for the background signal during occultations by the unlit side of the sun (or by rings in Saturn’s shadow) is interplanetary Lyman-\( \alpha \) emission (Colwell et al., 2007, 2010). The intensity of the background signal is also dependent on the filling-factor of the rings in the instrument FOV. For very bright stars, occultations by the unlit rings result in a background contribution of \(< 0.05\%\) and up to 1\% of the overall signal for occultations of the lit side. We measure the background signal directly by observing regions in the rings that are completely opaque to the UVIS instrument. The starlight is completely blocked in these regions, so any signal observed must be the background signal. We can then subtract this signal from the rest of the data.

For the stellar occultations we analyze, we remove the background signal when we determine the optical depth of the rings, as described in Chapter 3 and implemented in Chapters 4 and 5, even though the background signal contributes very little to the signal in these observations. The contribution of background noise is even less pronounced when observing the high photon rate of the star outside of the ring edges. As a result, we do not remove
the background when measuring the ratio of the diffraction signal to the direct (unocculted) signal.

2.2.2.2 Signal to Noise

The signal to noise ratio of the stellar occultations varies with the stars being observed. Very bright stars have a high signal to noise ratio, as expected. The signal in the stellar occultations is Poissonian, since the HSP counts photons from the star. The variance in the data is therefore equal to the square root of the mean of the unocculted signal. However, the data is recorded with 9-bit lossy compression to reduce data volume. This introduces additional error, and so we must calculate the true standard deviation of the counts in order to determine the noise of each occultation. The data is compressed using the following algorithm (Esposito, 1999) for the signal, $I$: IF $I > 128$

$$I_{\text{comp}} = \text{FLOOR}(\sqrt{2I} + .5) + 128 \quad (2.1)$$

where FLOOR rounds the signal down to the integer. The data is uncompressed by calculating:

$$I_{\text{uncomp}} = \text{LONG}(0.5 \times (I_{\text{comp}} - 128)^2). \quad (2.2)$$

Including the ‘LONG’ command returns a longword integer value. The rounding of the original signal causes the lossy-signal when uncompressed. This produces observations with discrete levels of photon counts, as seen in Figure 2.6. The quantization error introduced by the square-root compression is $\sim 30\%$ lower than the random error (personal comm. Greg Hosclaw).
2.2.2.3 HSP Ramp-Up

The signal response of the HSP is dependent on its exposure time to a light source (Colwell et al., 2007, 2010). This means that the instrument must be exposed to a star for an extended period of time before it will reach its maximum sensitivity to the signal. The ramp-up is variable and its shape differs for each observation, rendering the effect difficult to remove. Previous attempts to model the response were unsuccessful (personal comm. Joshua Colwell). When the instrument is first exposed to a star, the signal strength as observed by the HSP increases quickly over the first 10 - 15 seconds, then linearly increases until it plateaus to a steady, maximum signal. Figure 2.7 shows the ramp-up of the instrument during the ingress occultation of BetCen105I. The observation begins far from Saturn, and as time increases, the spacecraft moves such that the star approaches the rings (the angular distance between the rings and star shrinks), and the signal of the unocculted, direct stellar light grows stronger, ramping-up quickly in the beginning, followed by a linear increase in
signal until it nearly plateaus at the maximum sensitivity to the star.

When the star is first revealed in a gap, the photon count is again lower than the maximum unocculted signal, and restarts the ramp-up, until the star reaches the other end of the gap. In Figure 2.8 the HSP ramp-up is observed in the Encke Gap for the ingress and egress occultation of the α Virginis Rev 43. This figure clearly demonstrates that for the ingress observation, the signal is weaker at the outer edge of the Encke Gap and grows stronger towards the inner edge of the Encke Gap. The egress observation has the opposite slope. For an ingress occultation, the star is first exposed at the outer edge of the gap, and as the instrument is subjected to the starlight throughout the gap, it grows more sensitive to the signal with time, as seen at the inner edge. The opposite occurs for the egress occultation.

![Figure 2.7: HSP observation of β Centauri Rev 105I before being occulted by the outer A ring. This observation is ingress, so time moves from right to left on the plot. The signal begins (~159,000 km) with fewer photon counts, then increases quickly as the instrument is exposed to the star.](image)

In order to analyze the HSP data, we remove the ramp-up by modeling the unocculted light curve with a 4th-order polynomial fit. We divide the data by the model and apply a secondary, linear correction to remove any effects introduced by the polynomial fit within 60 km of the ring edges. The corrections enable us to find the baseline of the direct stellar signal after the instrument has been exposed to the star for several seconds.
Figure 2.8: HSP stellar occultation of $\alpha$ Virginis Rev 34, both ingress and egress, binned to 1-km radial resolution. The slope of the signal in the Encke Gap ($\sim 133,500$ km) is due to the ramp-up. The ramp-up causes the signal to increase with time, so the signal increases with radial distance from Saturn during an egress occultation and decreases with radial distance for an ingress occultation. The pattern of the ramp-up is not the same for the ingress and egress observation of the same star, which is why this systematic effect is difficult to account for.

Figure 2.9: HSP ramp-up as a function of time within the Encke Gap during the occultation of $\beta$ Centauri Rev 105 (ingress). The non-linear shape of the signal sensitivity over time can be modeled with a fourth-order polynomial, as shown by the blue fit. The data is binned to 0.5-km resolution. The dip in the signal at $\sim 50$ seconds is due to one of the faint ringlets that lie within the Encke Gap.

Figure 2.9 shows the ramp-up as a function of time in the Encke Gap during the occultation of $\beta$ Centauri Rev 105I. The signal response is typical for an occultation by the
Encke Gap. We fit a 4th-order polynomial to the data, shown in Figure 2.9 as the blue solid line. We divide the data by this fit and normalize the signal to the middle of the gap. In some cases, the removal of the polynomial fit introduces additional complicated features at the very edges of the data. For our study of stellar diffraction at ring edges (Chapter 5), it is necessary to account for this possible artifact. We therefore fit a line to the unoccluded stellar signal between 15 km and 60 km from the ring edge into the gap and divide the gap signal by the linear model. We do not include the signal closer than 15 km from the ring edge because diffracted light enhances the signal in this region - the basis for the analysis in Chapter 5. The application of the secondary correction is shown in Figure 2.10.

![Figure 2.10: Application of linear fit (red) to the processed signal (orange, dashed) after the removing the first-order effects of the HSP ramp-up during the occultation of LamSco044I by the inner edge of the Encke Gap. The blue, solid-line is the final signal used for the ring edges after both the polynomial and linear corrections have been made.](image)

We compare the signal before the ramp-up removal and after the corrections have been applied in Figure 2.11. Removing the ramp-up is particularly important for the analysis of the signal observed within the gaps due to diffracted starlight, as is discussed in Chapter 5, and for the study of the strands of dust sometimes observed in stellar occultations within the Encke Gap.
2.3 Extreme Ultraviolet Channel Solar Port

The EUV channel uses a grating spectrometer and a multi-element detector. The detector is an imaging, pulse-counting micro-channel plate with a Coded Anode Array Converter (CODACON) readout anode. The detectors have a $10^5 \text{s}^{-1}$ count rate limit (Esposito et al., 2004). The CODACON detectors enable simultaneous spectral and spatial observations, with a detector format of $1024 \times 64$ (spectral) $\times$ (spatial). For this study, we utilize the EUV solar port to observe solar occultations of the rings. To obtain the highest signal, we typically sum over all wavelengths and spatial elements.

In Section 2.3.1 we explain the utility of solar occultations. We discuss our investigation of an offset in the pointing of the EUV solar port and the full characterization of the sensitivity of the detector in Sections 2.3.2 and 2.3.3.
2.3.1 Solar Occultations

Solar occultation experiments are conducted by pointing the solar port of the UVIS instrument toward the Sun as the rings pass between the Sun and the spacecraft, as seen by Cassini. The Sun is a useful source of signal in the EUV; starlight emitted at wavelengths shorter than 91.2 nm is typically absorbed by interstellar atomic hydrogen (Esposito et al., 2004). The Sun is also an extremely bright source - in fact, it is so bright that direct observations of the Sun by the UVIS instrument would destroy the telescope detector sensitivity. In order to attenuate the signal, UVIS is fitted with a pick-off mirror that disperses the signal. During solar observations, the spacecraft points the solar port towards the Sun. The solar port is situated $\sim 20^\circ$ away from the UVIS boresight. The sunlight enters through a small aperture, which is then dispersed by the grazing incidence mirror and redirected toward the detector (Esposito et al., 2004).

At Saturn, the Sun subtends an angular size of $\sim 1$ mrad. This large size, especially compared with the distant stars observed during stellar occultations, results in the smearing of many of the features in Saturn’s rings. Solar occultations are not extremely useful for studying the fine structure in the rings; however, as discussed in detail in Chapter 4, we can use the solar occultations to learn a great deal about the ring system.

2.3.2 Instrument Misalignment

In 2005, the first solar occultation of the rings revealed that the boresight of the EUV solar port was not aligned with the nominal pointing direction. The observation placed the Sun at the very edge of the instrument’s FOV, placing $> 90\%$ of the solar disk outside of the FOV and reducing the observed solar counts per second to $1.3\%$ of the optimal solar signal. This issue was quickly identified and remedied by creating a new, adjusted rotation matrix in the Frames kernel that transforms the pointing vector from the Cassini spacecraft
coordinate frame into the solar port frame with an additional rotation about the y-axis of the spacecraft. This correction largely amended the misalignment and was thought to have aligned the nominal pointing direction with the boresight of the solar port. Our analysis of the instrument’s pointing found that the boresight remains slightly offset. Here we describe that analysis of the solar scans (listed in Table 2.1) and our changes to the Frames kernel to improve the accuracy of the instrument pointing.

Figure 2.12: Four time steps showing the model of the 2003 solar scan as the instrument slews such that the Sun travels across the slit in the x-direction. The UVIS solar port field of view is represented by the rectangular box and its boresight is indicated by the + symbol. The Sun is denoted with an X symbol when the model predicts the Sun is not within the FOV and as a diamond when the Sun is within the bounds of the FOV.

The misaligned solar occultation in 2005 resulted in the serendipitous detection of diffracted light in the vicinity of Saturn’s F ring. The discussion of the science extracted from this unique observation can be found in Chapter 4. In order to analyze the observation, a detailed understanding of the Sun’s placement in the solar port FOV is required, since the strength of an observed signal is not constant across the FOV (Section 2.3.3). To understand the strength of the signal at different locations in the FOV, we analyzed a series of solar scans.
Solar scans are observations of the Sun only, during which the solar port FOV moves such that the Sun drifts along and across the detector. Utilizing the SPICE toolkit, we developed an IDL program that would determine the position of the Sun in the FOV as a function of time. The program used the adjusted pointing of the solar port and the nominal FOV size of 4 milliradians by 4 milliradians. It was quickly noted that off-axis light could be detected along the direction of the dispersion mirror beyond the nominal FOV size, and thus the model initially incorporated a 4 milliradian by 10 milliradian FOV. Here we define a coordinate system for the solar port FOV: the long axis (the direction of the dispersion mirror) is the y-axis and the short axis is the x-axis. The goal of this analysis was to observe the total signal detected as a function of the Sun’s X-Y position in the FOV.

Figure 2.12 shows the general concept of the model. The figure consists of four snapshots of progressing time steps in the model. The FOV begins far from the Sun, then moves such that the Sun crosses, in the x-direction, completely through the FOV and then is again outside of the FOV, maintaining a constant y-position. The plots indicate when the model recognizes the Sun to be within the bounds of the FOV by marking the Sun with a diamond rather than an X. At this stage in the model we assume uniform sensitivity across the FOV and plot the resulting signal in Figure 2.13 (a). This figure clearly portrays the offset between the times when the model places the Sun in the FOV and the times when there was, in fact, a solar signal detected. This offset could have been due to an error in the C-kernel, providing an inaccurate orientation of the spacecraft; however, further analysis proved the offset to be due to a difference in the nominal pointing of the instrument and its true pointing.

As described in Section 2.1, the Frames kernel contains a rotation matrix specific to each instrument, defining the orientation of that instrument with respect to the spacecraft. Within the Frames kernel, there are two frames for the UVIS solar port FOV. The Ultraviolet Imaging Spectrograph Solar Occultation Port, CASSINI_UVIS_SOLAR (ID -82843), and the Solar Occultation Port with Offset, CASSINI_UVIS_SOL_OFF (ID -82849). The first
frame is the frame indicating the nominal pointing of the solar port before Cassini’s arrival at Saturn. In response to the detection of the pointing error in 2005 (described above), the second frame was generated. Rather than update the existing CASSINI_UVIS_SOLAR frame, for the convenience of the ground system tools, the new frame was created to account for the offset in the nominal viewing direction.

Figure 2.13: Model of a uniform solar port FOV for the 2003a solar scan before applying the modified Frames kernel (a) and after applying the modified Frames kernel (b). In (a) the model is offset from the data, predicting a signal from the sun at an earlier time. This is due to an error in the nominal pointing of the instrument recorded in the UVIS Frames kernel. After we modify the Frames kernel solar port orientation on Cassini, we are able to accurately model the timing of the solar scan (b).

At the time of this work, the most current Frames kernel is labeled cas_v40.tf. Through some trial and error, we adjusted the rotation matrix that takes the vectors represented in the CASSINI_UVIS_SOL_OFFSET frame into the spacecraft frame, CASSINI_SC_COORD frame. The original rotation matrix consisted of a rotation of -110.0 degrees about the spacecraft x-axis, 0.0 degrees about the spacecraft’s z-axis, and -0.11459 degrees about the spacecraft’s y-axis. We found that by rotating about the spacecraft’s y-axis by an additional 0.27959 degrees, the timing of the model could match the data. We updated the Frames
kernel for the CASSINI_UVIS_SOL_OFF frame from

\[ TKFRAME_{-} - 82849 \_ANGLES = (-110.0, 0.0, -0.11459) \]  \hspace{1cm} (2.3)

to

\[ TKFRAME_{-} - 82849 \_ANGLES = (-110.0, 0.0, -0.165). \]  \hspace{1cm} (2.4)

Using this rotation matrix, the instrument is pointing such that the model nicely reflects the time during the solar scan when the Sun was visible in the solar port FOV, as seen in Figure 2.13 (b). The modification to the Frames kernel was tested for two other solar scans that made repeated x-direction scans (discussed in the next section). The adjusted kernel is always required to accurately model the solar signal.

Figure 2.14 demonstrates the changes in the assumed pointing of the solar port FOV. The green FOV labeled ‘UVIS_SOLAR’ represents the nominal orientation of the FOV before the 2005 solar occultation occurred. The orange FOV labeled ‘UVIS_SOL_OFF’ indicates the nominal pointing of the solar port after the 2005 solar occultation when the first misalignment was identified. The labels reflect the naming scheme of the UVIS Frames kernel. Finally, the purple FOV, ‘UVIS_SOL_OFF (proposed)’ is the true orientation of the FOV as determined by the work described here. It is a small but noticeable shift in the pointing of the instrument. This shift is important for the analysis of the 2005 misaligned solar occultation discussed in Chapter 4 and for any analysis that requires accurate knowledge of the location of the Sun within the FOV.

The required modification to the UVIS Frames kernel was presented and discussed with the UVIS team and the appropriate rotation matrix will be included in an updated Frames kernel.
Figure 2.14: The shift in the instrument’s pointing due to the modifications to the Frames kernel. The green box represents the solar port FOV using the nominal pointing of the instrument before the 2005 solar occultation. This orientation of the instrument is referred to as ‘UVIS_SOLAR’ in the Frames kernel. ‘UVIS_SOL_OFF’ is the orange FOV and represents the orientation of the solar port after the 2005 solar occultation. The purple FOV represents the true orientation of the solar port as determined by this analysis.

### 2.3.3 Solar Port Sensitivity

As previously described, the aperture for the EUV solar port is aligned 20 degrees off-axis from the primary telescope and reflects sunlight to the telescope using a parabolic grazing incidence mirror. The mirror disperses the solar signal across the detector to protect the telescope from overexposure. As a result, the signal along the dispersion direction extends beyond the nominal bounds of the instrument FOV, increasing its effective size. Additionally, the sensitivity of the instrument is not constant across the FOV. Here we describe the effective size and the sensitivity of the solar port field of view, determined in-flight through the analysis of multiple solar scans.
Table 2.1: Solar scans used for calibrating the solar port FOV sensitivity. The first column indicates how the observation is referenced throughout this work. The second and third columns indicate the start and stop times of the observation, respectively. The fourth column is the Cassini Information Management System (CIMS) name of the observation. The data is broken into ‘a’ and ‘b’ if the scan changes between a $\Delta x$ (a) and $\Delta y$ (b) scan.

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<th>Observation Stop</th>
<th>Observation File</th>
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<td>2007 Day 108 01:02:49</td>
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<td>2013 Day 174 02:19:59</td>
<td>EUV2013_175_00_07_49_UVIS_193SU_SOLARPORT001_VIMS</td>
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2.3.3.1 Initial Calibration

Figure 2.15: (a) Normalized Solar signal for the 2003b, 2006 and 2007 y-axis solar scans. The color and shape indicate from which scan each data set was collected. (b) Piece-wise fit to the data for modeling purposes. (c) Linear extension of the fit in order to model the signal sensitivity of the FOV farther from the boresight than these scans account for.

We calibrate the sensitivity of the solar port FOV by combining multiple solar scans taken over several years during which the instrument scans in such a way that the Sun drifts along the slit (y-direction) only and across the slit (x-direction) only. Here we note that we
describe the FOV with a 2-dimensional x-y coordinate system, but this coordinate system differs from the spacecraft’s 3-dimensional coordinate system. The x-axis is equivalent in the two coordinate systems, however what we refer to as the y-axis is the spacecraft’s z-axis. Figure 2.15 (a) shows the normalized solar signal from three solar scans (2003b, 2006, and 2007) along the y-axis as a function of angular separation between the Sun and boresight ($\Delta y$) and Figure 2.15 (b) is the piece-wise fit to the data. Note that the signal never drops below 70% of the maximum solar signal, indicating that the FOV is never placed far enough from the Sun to prevent off-axis light from being detected.

These scans place the Sun more than 10 milliradians from the instrument’s boresight, meaning the effective size of the field of view along the y-axis is much larger than its nominal 4 milliradians. Figure 2.15 (c) shows the linear extrapolation we use to model the signal of the Sun when placed more than 10 milliradians from the solar port boresight. Figure 2.16 shows the data and fit for the one solar scan along the x-axis that was available during this analysis. The slow, mostly linear slope of the decreasing solar signal near the edges of the FOV.
field of view along the x-axis is consistent with the time required for the 1-mrad-sized Sun to completely exit the FOV.

![Solar Scan 2003](image)

(a) Solar Scan 2003 a & b

![Solar Scan 2006](image)

(b) Solar Scan 2006

![Solar Scan 2007](image)

(c) Solar Scan 2007

Figure 2.17: Solar scans 2003 a & b, 2006, and 2007 and models. The 2003 solar scan begins with a scan along the x-axis (2003a) and then scans along the y-axis (2003b). The 2006 data is a scan along the y-axis in the +y-direction and then reverses and scans in the -y-direction. The 2007 data is one scan along the y-axis. Here some models are scaled to match the data because the reduction in signal due to $\Delta x$ is being accounted for in the model which lowers the overall signal sensitivity from the ideal at $\Delta x = 0$. 
2.3.3.2 Solar Port Sensitivity Model

Figure 2.18: Solar scan 2013 and model. This is a portion of the data, which scanned back and forth along the x-axis of the solar port. The model was developed before this observation, so this scan confirms the model’s ability to accurately predict the signal.

Using our knowledge of the angular separation between the Sun and boresight along (Δy) and across (Δx) the FOV, we built a model that would determine the solar signal of the Sun as a percentage of its maximum signal due to its x-y position in the FOV. This model addresses any asymmetries in the scattered light within the instrument because both the Δx and Δy position must be input to determine the total decrease in optimum solar signal. Figure 2.17 shows how the fits can be applied to model the data. The model signal is produced by the knowledge of the C-kernel pointing of the instrument and the location of the Sun, as well as the equations developed to indicate the decrease in FOV sensitivity as a function of Δx and Δy. However, the model would be expected to match these data since the model was developed using fits to the 2003b, 2006, and 2007 solar scans. The model was better tested in 2013 when a new solar scan was completed. A portion of the model and data from the 2013 solar scan are shown in Figure 2.18. Again, only the information recorded in
the C-kernel for instrument pointing is used in conjunction with the sensitivity equations to
determine the model signal, and as Figure 2.18 indicates, these match well. It is important
to note that these models only match the data when the adjusted frames kernel described
in Section 2.3.2 is used.

2.3.3.3 Designed Solar Scan

Figure 2.19: Diagram used for indicating priority scans for the 2014 solar calibration. Here
the spacecraft’s z-direction is corresponds to the y-direction in our two-dimensional FOV
analysis (see Section 2.3.3.1). Because this solar observations were led by the VIMS team,
we use the VIMS boresight to describe the requested movements of the spacecraft. The
circles represent starting points for the VIMS boresight and the arrows indicate direction of
each scan.

Although we were capable of properly modeling the sensitivity as a function of solar
position within the FOV, no solar scan, including the 2013 observation, scanned far enough
in the y-direction to characterize the sensitivity > 10 mrad from the solar port boresight. For
the analysis of the 2005 misaligned solar occultation (Chapter 4), it is critical to understand
the sensitivity of the entire solar port. In 2013 we designed a series of solar scans that would
place the Sun 30 milliradians from the solar port FOV boresight. We requested that these additional, specialized UVIS scans be added to the already-planned solar scan designed for the VIMS instrument. The VIMS team agreed to incorporate our requested solar scans into their solar calibration observation. The diagram for the UVIS-requested solar scans is shown in Figure 2.19.

Figure 2.20: The 2014 a & b solar port calibration scans. (a) is a series of x-axis scans that occur at different $\Delta y$ locations in the FOV from $\Delta y = -30$ mrad - +30 mrad. The model (purple line) matches the data well when the Sun is near the center of the FOV, but at high values of $\Delta y$, the model predicts a higher solar signal. (b) shows one of the two y-axis solar scan data and model. This figure shows the clear deviation of the model and data at the beginning and end of the y-axis scan, when $\Delta y$ is large.

The solar port calibration was executed in 2014. All of the scans requested were granted, in addition to the many scans planned by the VIMS team. Figure 2.20 (a) shows the first half of the observation (2014a). This part of the observation involved a series of x-direction scans at different, but constant $\Delta y$ positions, beginning with $\Delta y = -30$ mrads and ending at $\Delta y = 30$ mrads. The scans with the highest signal are seen half-way through the observation, when $\Delta x$ and $\Delta y$ are minimized (the Sun is nearest to the center of the FOV). The model fits the data well in this region, but when the Sun was placed at large $\Delta y$ values, the model predicts
a much higher signal than observed. Figure 2.20 (b) is the data from the two y-direction scans during the 2014b observation. Here we can see the discrepancy between the model and the data along the entirety of the y-axis of the solar port. We fit the solar signal with 4th-order polynomials in order to improve our models to characterize the FOV, as seen in Figure 2.21. Note that this scan witnesses a complete drop in solar signal, indicating that the solar observation successfully scanned the entire FOV along the y-direction. This is the first solar port calibration scan to do so.

We also utilize the new data to improve upon the model along the x-axis. We implement 4th-order polynomial fits rather than the simple linear models for the x-direction edges of the FOV (Figure 2.22). The 2014a observation performed a series of scans across the x-direction along the entire field of view, measuring the signal at nearly every location. Our implementation of the modeled sensitivity captures the observed signal (see the next section), further verifying our model and removing uncertainties regarding an asymmetric off-axis signal.

![Piecewise Fit for Solar Port Sensitivity](image)

Figure 2.21: Piecewise fit of 4th-order polynomials to the two y-axis scans during the 2014b solar port calibration observation.
2.3.3.4 Signal Sensitivity

The solar calibrations scans enabled a complete characterization of the UVIS solar port FOV. Figure 2.23 (a) shows the final solar port sensitivity model compared with the 2014a solar calibration scan. Figure 2.23 (b) reflects the goodness-of-fit of the sensitivity model with the data. The percent error of the model is constrained to 5% and increases most dramatically when the Sun is at the very edge of the FOV in the dispersion direction. The coefficients for the sensitivity model are displayed in Table 2.2. We provide a contour plot displaying the characterization of the sensitivity of the entire FOV in Figure 2.24. Table 2.2 and Figure 2.24, as well a brief description of the model have been submitted to the Cassini UVIS User’s Guide, which can be found on the Planetary Data System (PDS), for other scientists to use in order to better understand the UVIS solar port.
Figure 2.23: (a) The 2014 solar port calibration scan with updated sensitivity model. (b) Percent error of updated solar port sensitivity model with the data from 2014 solar port calibration scan. The percent error is confined to be within 5%. The largest discrepancies are still found near the edges of the FOV that define the caps of the y-axis.

Figure 2.24: The complete characterization of the solar port field of view sensitivity. Dark blue to light blue indicates a 10% decrease in sensitivity relative to the maximum signal. Here the x-axis of the plot corresponds to the x-axis of the solar port FOV and the y-axis on the plot corresponds to the y-axis of the solar port FOV.
Table 2.2: Bounds indicate for which angular distances in the x- and y-directions the associated fit should be used. Fit type is the type of fit used for that region in the FOV, and the coefficients are listed in order for that fit type. The coefficients are applied as follows:

\[ y = \text{coefficient}[1] + \text{coefficient}[2] \times x + \text{coefficient}[3] \times x^2 + \text{coefficient}[4] \times x^3 + \text{coefficient}[5] \times x^4 \]

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<th>Fit</th>
<th>Coefficients</th>
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<td>(+8.186 &lt; \Delta y &lt; +23)</td>
<td>4\text{th}-order polynomial</td>
<td>([0.99457854, 1.5741268, -88.499673, -46525.346, -11337395.0])</td>
</tr>
</tbody>
</table>
The intent of this work is to garner knowledge regarding the Saturnian ring system through the analysis of occultation data. We determine the size distribution of the particles based on how particles of different sizes interact with light. Specifically, we measure the strength of the light diffracted by ring particles during stellar and solar occultations. At the UV wavelengths of the EUV channel (∼100 nm) and the HSP (∼150 nm), micron-sized dust and larger can be considered much larger than the wavelength of incident light, and are therefore efficient at diffracting the incident light. In this work, we follow van de Hulst (1957) in defining diffraction as Fraunhofer diffraction only (Section 3.2). We exclude other forms of scattering by the rings, such as reflection and refraction, from contributing to the diffraction signal. Ignoring other forms of scattering is justified; the rings are highly-absorbing at UV wavelengths, so scattering through reflections during occultations contributes very little to the observed signals. As a result, the intensity distribution of the diffracted light is guided by the ratio of the wavelength of the incident light to the sizes of the ring particles and is independent of their composition (Section 3.2). Therefore, we can utilize clear detections of diffracted light in occultation data to ascertain information regarding the particle size distribution in Saturn’s rings.

We have developed computational models using a forward-modeling technique to simulate the geometry of a given observation and determine the intensity of the diffracted signal for various particle size distributions in a given region of the rings. We describe the assumptions, applications, and results of these models in Chapter 4 and Chapter 5. In this chapter, we describe the theoretical background of diffraction in Sections 3.1 and 3.2. In Section 3.3, we
describe how we apply the theoretical description of diffraction in an analysis of planetary ring systems. In Section 3.5, we explain how we implement the diffraction theory in our models. Some of the description of the models is left to Chapters 4 and 5 where their application to different observations is discussed in more detail. This chapter provides the background information needed to understand how we measure and model diffracted light from a planetary ring system.

3.1 Fraunhofer Diffraction

3.1.1 Assumptions

Here we present the requirements and limitations of the application of Fraunhofer diffraction to the particle size distribution in Saturn’s rings.

First, the theoretical equations presented in this chapter are for independent scatterers, meaning the particles are well-defined, separate entities (van de Hulst, 1957). This is a good description of the smallest particles in Saturn’s rings. An agglomerate of small particles can be treated as one, larger particle. Second, we assume the particles are spheres. This assumption is somewhat simplistic for the particles in Saturn’s rings; however, unless the particles are significantly oblong and also have a preferential orientation (unlikely for the small particles producing the observed diffraction signal), this assumption does not affect our conclusions about the particle sizes (van de Hulst, 1957).

We begin by presenting the theory for single-diffraction only and then address multiple-diffraction processes in the rings in Section 3.4. The effects of multiple diffraction make a significant contribution to the signal at higher slant optical depths of the ring.

Fraunhofer diffraction is produced by a particle within the geometric optics regime. That is, the radius of the particle, \( a \), is much larger than the wavelength, \( \lambda \), of the incident light:
\[
\frac{2\pi a}{\lambda} \gg 1. \quad (3.1)
\]

Particles with radii that do not fall in the geometric optics regime will still scatter light, but the intensity of the scattered signal will be described by Mie theory rather than Fraunhofer diffraction. In the case of particles much smaller than the wavelength of light, we would use Rayleigh approximations to describe the intensity of scattered light. We find that Fraunhofer diffraction theory is suitable for measuring the observed scattering signal and is justified by the particle-size regime studied in this work (\(a \geq 1\mu m\) for \(\lambda \leq 150\) nm).

### 3.2 Babinet’s Principle

In the 17\(^{th}\) century, Christiaan Huygens described wave propagation with a geometric model in which each point on a wave front bore a spherical wavelet that would spread out at the wave speed. The line tangent to all of the wavelets would then form the shape of the wave front at some later time (Knight, 2004). When a plane wave front encounters a particle, that particle blocks an amount of light equivalent to its geometric area, \(G = \pi a^2\). Here, the geometric area is the 2-dimensional area of the particle (radius \(a\)) that a plane wave would encounter. Huygens’ principle causes the incomplete wave front to produce, at large distances, the Fraunhofer diffraction pattern (van de Hulst, 1957).

Fraunhofer diffraction is a phenomenon more commonly associated with the result of light passing through an aperture. Figure 3.1 shows the basic physical phenomenon of producing Fraunhofer diffraction as a plane wave passes through an aperture. When that aperture is circular, the diffraction pattern is referred to as Airy diffraction (Figure 3.2). In astronomy, an Airy disk is often seen as a result of light from a celestial object passing through the telescope’s circular aperture.

According to Babinet’s principle, if the area of the aperture is the same as the geometric
area of the particle, the resulting diffraction patterns are identical (van de Hulst, 1957). Therefore, spherical particles (like those assumed here for Saturn’s rings) will produce this characteristic diffraction pattern.

Figure 3.1: Explanation for the production of Fraunhofer diffraction. As a plane wave of light passes through the aperture, the emerging light interferes with itself, producing the Fraunhofer diffraction at a distance far from the aperture.

To first order, the angular size $\theta$ of the Fraunhofer diffraction pattern is

$$\theta = \frac{1.22\lambda}{2a}. \quad (3.2)$$

For very large particles, the diffraction cone subtends a very small angle. Small particles produce large angular diffraction patterns. When the particles are of a size comparable with the wavelength, then the assumptions described in this section have been violated and Equation 3.2 does not describe the diffraction pattern. More rigorous forms of Mie theory would be needed. It should be noted that, since the pattern is dependent exclusively on the particle size and the incident wavelength, that the composition of the particle is irrelevant to the diffracted signal. The independence from the particle composition means that the...
diffracted light will have the same polarization as the incident light (van de Hulst, 1957). It also removes the need to understand the albedo and scattering properties of Saturn’s rings in the modeling efforts presented in this work.

![Airy diffraction pattern](image)

**Figure 3.2**: Airy diffraction pattern produced by light as it passes through a telescope aperture. We mark the angle of the first minimum in the pattern that corresponds with the $\theta$ in Figure 3.1.

### 3.2.1 Extinction Paradox

Babinet’s principle gives rise to the Extinction Paradox, sometimes known as Babinet’s Paradox. The Extinction Paradox states that the total energy removed by a particle in the geometric optics regime will have an effective cross section equal to twice the geometric area, $2G = 2\pi a^2$, of the particle intercepting the light (van de Hulst, 1957). As the incident light encounters a particle, the particle will absorb and/or scatter (through reflection and refraction) an amount of energy with an effective cross section equal to $G$. Additionally, the particle will also diffract light in a way that mimics diffraction through an aperture with area $G$. If we assume *all* of the diffracted light is considered ‘removed’ from the incident signal, then diffraction removes an amount of energy with an effective cross section equal to

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G as well. Therefore, the total energy removed from the incident plane wave corresponds to a cross section equal to twice the geometric area of the particle (van de Hulst, 1957). The assumption that all of the diffracted light is removed is valid in the far-field approximation, and is therefore valid for Cassini observations of the rings.

This paradox is justly named, as the phenomenon is difficult to comprehend in everyday life. van de Hulst (1957) gives the example that a flower pot in a window blocks only the sunlight that directly hits the pot (not twice the amount of light that hits the pot). However, at astronomical distances, the same pot would remove twice that amount of light.

This has important implications for occultation data. A stellar occultation of a single ring particle will remove energy equal to twice its geometric area. If that light is not replaced (see Section 3.3.1), then in a stellar occultation, the observed optical depth will be twice as high when compared with the otherwise expected value due to blocked (absorbed/scattered) light only. For radio occultation experiments, the phase of the transmitted signal is known. As that signal passes through the rings, diffracted light will be received out of phase and is therefore distinguishable from the attenuated signal. The optical depths reported by the Radio Science Subsystem (RSS) on Cassini intentionally remove the diffracted component of the signal, and therefore also measure an optical depth that is twice what would be measured in the near-field. As will be discussed in Section 3.3.1, however, the light that is diffracted out of the UVIS FOV during a stellar occultation can also be replaced by neighboring particles, making the observed decrease in signal equal to only one geometric area of the particles. As a result, the RSS often reports an optical depth of the rings that is twice that determined by the stellar occultation experiments.
3.3 Application to Saturn’s Rings

In Chapter 1 we introduced the relevance of diffracted light in Saturn’s rings. Because small particles are efficient forward-scatterers, observations made at high phase angles observe a significant amount of diffracted (and forward-scattered) light if such particles are present.

By definition, occultations require high phase angles so that the rings block the light source. As a result, the population of small particles throughout the rings diffract light in every observation. However, as is discussed extensively in Chapters 4 and 5, it is not possible to distinguish between the direct, attenuated light and the light that has been diffracted into the field of view by particles except in a few, special cases. In Chapter 5 we discuss unambiguous detections of diffraction and the model we developed to replicate those signals. In Chapter 4 we model both the blocked (through absorption) and diffracted signal.

Throughout this work, we model Saturn’s rings according to the “classical” ring model, which assumes the particles that make up the ring are uniformly distributed in a loosely-packed, extended layer that is many-particles-thick (Cuzzi et al., 2009). In this model, the interaction between light and the particles is treated as a radiative-transfer problem. Work by Zebker et al. (1985), using the Voyager RSS, demonstrate that the rings may more aptly be described as three thin layers with multiple-scattering processes. Additionally, structure within the rings, especially the self-gravity wakes in the A ring, indicate that the ring does not conform to the classical ring model. However, the ultraviolet wavelengths observed by UVIS make the instrument most sensitive to diffraction by a population of sub-centimeter particles between the large, flattened self-gravity wakes. This population of small particles is well-described by the classical ring model.

We begin our modeling approach with the assumption of the classical ring model with a ring of low optical depth, such that only single-diffraction is considered. We model the particle size distribution of the ring by a power-law defined as (Cuzzi et al., 2009)
\[ n(a)da = Ca^{-q}da, \, a_{\text{min}} \leq a \leq a_{\text{max}} \] (3.3)

where \( a \) is the radius of the particle, \( n(a) \) is the number of particles with radius in the range \([a, a+da]\), \( da \) is an infinitesimal increment in \( a \), \( C \) is a constant, \( q \) is the power-law index, and \( a_{\text{min}} \) and \( a_{\text{max}} \) are the minimum and maximum particle sizes in the distribution, respectively.

Power-law size distributions have successfully described the ring material in other analyses (i.e. Zebker et al. (1985); Showalter et al. (1992); French and Nicholson (2000); Cuzzi et al. (2009); Charnoz et al. (2009)). Such a distribution of particles is generally expected; a power-law size distribution with an index of \( q \sim 3.5 \) is a typical distribution for the remnants of an object that is shattered in the laboratory (Dohnanyi, 1969). Saturn’s rings have likely evolved through fragmentation processes, so it is unsurprising to find that a power-law size distribution with \( q \sim 3 \) (French and Nicholson, 2000; Sremčević et al., 2007; Becker et al., 2015) describes most of the rings for the population of particles \( a < 10 \text{ m} \) (Charnoz et al., 2009). Such a distribution means that there is equal surface area per decade of particle size.

For such a particle size distribution, the intensity \( I_1(\theta, \lambda) \) of the forward-scattered light compared with the “free-space” incident power per unit area \( I_i \) is given by (Cuzzi et al., 2009)

\[
\frac{I_1(\theta, \lambda)}{I_i} = e^{-\tau_n/\mu_0} \frac{4\pi\mu_0}{\pi a^2} \int_{a_{\text{min}}}^{a_{\text{max}}} P(\theta, \phi, a)n(a)da \tag{3.4}
\]

and

\[
P(\theta, \phi, a) = \left[ \frac{2J_1(ka \sin \theta)}{\sin \theta} \right]^2 \tag{3.5}
\]

where \( P(\theta, \phi, a) \) is the phase function for Fraunhofer diffraction, \( k = \frac{2\pi}{\lambda} \), \( \lambda \) is the wavelength of the incident light, \( J_1 \) is the Bessel function of the first kind and order 1, \( a_{\text{min}} \) is the lower
bound on the radius $a$ of particles in the rings, $a_2$ is the maximum particle size contributing to
the diffracted signal, $\theta$ is the scattering angle, and $\mu_0 = \sin(|B|)$, where $B$ is the angle between
the ring plane and incident radiation (ring opening angle). $\tau_n$ is the normal optical depth of
the rings. The phase function is normalized such that $\int P(\theta)d\Omega = 2\pi \int_0^\pi P(\theta) \sin \theta d\theta = 4\pi$
(French and Nicholson, 2000).

We cannot measure $\tau_n$ directly unless the spacecraft’s line of sight is normal (perpendi-
cular) to the ring plane ($B = 90^\circ$). We can determine the normal optical depth from the
observed optical depth, $\tau$, by accounting for the slant angle ($\mu_0$) of the observation. For a
classical multi-layer ring, $\tau_n = \mu_0 \tau$.

Before we can convert $\tau$ to $\tau_n$, we first must determine $\tau$ from the data. The HSP
is a photometer; it measures the number of photons per integration period. In order to
determine the optical thickness of the ring, we must compare the unocculted stellar signal,
$I_0$, with the measured signal when blocked by the ring, $I$. It is also necessary to account
for any background signal, $b$. We follow Colwell et al. (2009) to calculate the measured
transparency ($T$) of the ring

$$T = \frac{I - b}{I_0}. \tag{3.6}$$

We measure $b$ directly in regions where $T = 0$, such as in the very opaque regions of the
B ring. In these regions, we expect no light from the star, so any detected signal is due
to contributions from other sources (such as Saturn-shine, instrumental effects, etc.). We
measure $I_0$ in regions where the star is completely unobstructed ($T = 1$). $I_0$ is easily
calculated in the large ring gaps or in the region outside of the main ring system when the
unocculted star is being observed. For bright stars, the contribution of the background signal
is very small.

The optical depth is the natural log of the inverse of the transparency (Colwell et al.,
\[
\tau = \ln(T^{-1}) = \ln\left(\frac{I_0}{I - b}\right).
\] (3.7)

We solve Equation 3.7 across the entire ring system to find the observed (slant path) optical depth. Finally we determine the normal optical depth \( \tau_n = \mu_0 \tau \), throughout Saturn’s rings. It is important to note that structures in the ring can cause variations in optical depth even at the same radial location. The observed optical depth in regions with self-gravity wakes, which are structures that are canted to the orbital direction (Colwell et al., 2007), varies depending on the geometry of the observation. Such effects must be considered when determining the true value for the optical depth of the rings.

Ultimately, we want to utilize \( \tau_n \) and the observed diffraction signal to constrain the particle size distribution. \( \tau_n \) is related to the particle size distribution by (Cuzzi, 1985; Cuzzi et al., 2009)

\[
\tau_n(\lambda) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi a^2 Q_{\text{occ}}(a, \lambda, f)n(a)da
\] (3.8)

where \( Q_{\text{occ}} \) is the effective extinction efficiency (Cuzzi and Pollack, 1978; Cuzzi, 1985; French and Nicholson, 2000) and depends on the fraction \( f \) of the scattering cone filled by ring particles, described in Section 3.3.1.

### 3.3.1 Extinction Efficiency \( Q_{\text{occ}} \)

According to the Extinction Paradox described in Section 3.2.1, the total energy removed from the incident light by a particle of radius \( a \) has an effective cross section equal to twice its geometric area \( G \). The extinction efficiency, \( Q_{\text{ext}}(a, \lambda) \), of an individual particle in the geometric optics regime is therefore equal to 2. However, when we observe Saturn’s rings, the light that is diffracted out of the instrument’s FOV by one particle can be replaced by near-by particles that are diffracting light into the FOV. If all of the light that has been
removed by each particle in Saturn’s rings is replaced by diffracted light from neighboring particles, then the overall effective $Q_{ext}$ is reduced to 1; the extinction of light is due only to the absorption/scattering properties of the ring particles. If none of the signal diffracted out of the FOV is replaced by diffraction from neighboring particles, then the effective $Q_{ext} = 2$. To refer to this phenomenon of the effective extinction efficiency of the rings, we use the term $Q_{occ}(a_c, \lambda, f)$, where $a_c$ is the critical particle size described below.

![Figure 3.3](image.jpg)

**Figure 3.3:** Schematic illustrating the dependence of $Q_{occ}$ on the particle size and the angular size of the FOV. Here the star represents the star being occulted in the observation. A particle directly between the spacecraft and the star will diffract light out of the detector. Because of the small size of the detector and the distance of Cassini from the rings during the observations, we assume *all* diffraction light is lost, even light diffracted at very small angles. The yellow circle labeled the ‘region of diffraction’ is the region for which particles of the same size can diffract light into the detector that would otherwise not be seen, replacing the light diffracted by the particle along the line-of-sight (see text). The size of this circle corresponds to the first ring of the Airy disk shown in Figure 3.2. The teal box represents the projected FOV of a detector with a large acceptance angle while the green box represents the projected FOV of a detector which a small acceptance angle. The large acceptance angle captures all of the light replaced by neighboring particles through diffraction, while the green box does not, so only some of the diffracted light is replaced. Therefore, the extinction efficiency $Q_{occ}$ of the teal FOV is 2 and that of the green FOV is between 1 and 2.
The amount of diffracted light captured by the instrument FOV is determined by the angular size of the FOV, the wavelength of the incident light, and the size of the particles responsible for the diffraction. Cuzzi et al. (2009) show that the critical particle size $a_c$ for which all light (both direct flux and the equivalent amount of light diffracted) will be captured by a detector with an angular FOV size of $\theta_{FOV}$ is given by

$$a_c = \frac{\lambda}{2\theta_{FOV}}. \quad (3.9)$$

Figure 3.3 is a schematic that shows how $Q_{occ}$ depends on the particle size and the angular size of the FOV. In this figure, the orange star represents the star being occulted by a particle. That particle will diffract light at angles and intensities as determined by its size. One of the assumptions stated at the beginning of this chapter is that all light diffracted by a particle along the line of sight is removed (making $Q_{ext} = 2$ for individual particles). The yellow circle could be interpreted as the limit of diffraction for the particle directly between the star and the spacecraft boresight; however, it is more appropriately thought of as the region for which other particles of the same size can replace the light diffracted out of the FOV through their own diffraction of starlight into the FOV. Particles beyond this region cannot diffract light at a sufficient angle for the instrument to detect that light. If the angular extent of the region of diffraction is less than the angular size of the FOV ($\theta_a < \theta_{FOV}$, where $\theta_a \approx \frac{\lambda}{2a}$), or their corresponding footprints in the 2-dimensional schematic $\theta_a D_{LOS} < \theta_{apD_{LOS}}$ (where $D_{LOS}$ is the line-of-sight distance from Cassini to the rings), then all of the light diffracted out is replaced and $Q_{ext} = 1$. This is the case for the “large” FOV represented by the teal square in Figure 3.3. The teal square is the projection of the FOV onto the rings and its size is determined by the acceptance angle of the detector and the line of sight distance $D_{LOS}$ from the spacecraft to the rings. In the case of an acceptance angle that is smaller than the diffraction cone (small, green square in the same figure), then more light has been
diffracted out of the instrument’s detector than can be replaced by the surrounding particles, and $Q_{occ}$ falls somewhere between 1 and 2. For a constant wavelength, the angular extent of the diffraction region will grow with a decreasing particle size following Equation 3.2. If all particles remain larger than the critical particle size, $a_c$, for which the entire diffraction region remains within the projected FOV, then $Q_{occ} = 1$.

The instruments on board Cassini have detectors with different FOV angular sizes, which results in the collection of different amounts of light from the rings for particles of certain sizes. The UVIS HSP has an aperture of $6 \text{ mrad} \times 6 \text{ mrad}$. If we were to model the aperture as a circle with the same area $A$ and FOV angular size $\theta_{\text{aperture}} = \sqrt{\frac{A}{\pi}}$, we find an aperture with an effective FOV angular size of $\theta_{\text{FOV}} = 3.39$ milliradians. For an aperture this size, we find the critical particle size $a_c \sim 22 \mu m$, indicating that for particles as small as a few tens of microns, the HSP will capture enough diffracted light from nearby particles to entirely replace the light diffracted out of the FOV by the particles directly in the line of sight to the star, effectively capturing the diffraction lobe and therefore both the direct and diffracted components of the light (assuming the star is centered in the instrument FOV). Thus, if the HSP field of view is filled by particles larger than 22 microns, $Q_{occ} = 1$. If there are particles smaller than 22 $\mu m$ or if the FOV is not filled with ring particles, then $Q_{occ} > 1$, with the exact value depending on the abundance of small particles and the fraction of the FOV that is filled.

The occultation mode of the Cassini VIMS instrument has a significantly smaller FOV than UVIS and measures starlight at $\lambda \sim 2.9 \mu m$ (Brown et al., 2004). The larger wavelength increases the value of $a_c$, and for particles smaller than $a_c$, the light that is diffracted out of the field of view cannot be entirely replaced by near-by particles. The resulting $Q_{occ}$ lies somewhere between 1 and 2, depending on the sizes of the those particles. This causes a difference in the observed optical depths between VIMS and UVIS in regions with very small particles; UVIS collects more light from the ring due to the diffraction component.
and therefore measures a smaller optical depth than VIMS does. This difference in signal must be accounted for when comparing optical depth measurements. The differences in the optical depths between the instruments can also be used to study the population of small particles throughout the rings, as demonstrated by Jerousek et al. (in prep).

The Cassini RSS transmits a coherent signal through the rings to receivers on Earth. The diffracted component of the radio signal is phase-shifted and therefore distinguishable from the direct signal (unlike in the stellar occultations). The RSS data is analyzed by completely removing the scattered component so $Q_{occ} = 2$. Therefore the RSS publishes optical depths that are twice that of the UVIS optical depth measurements, since they consider both the extinguished and diffracted light to be removed.

The values of $Q_{occ}$ discussed so far have been determined assuming that the entire field of view of the instrument is filled by ring particles; however, this is not always the case. We must also account for the filling factor of the FOV to measure the total extinction of light. The value of $Q_{occ}(a, \lambda, f)$ is determined by subtracting the fractional area $f$ of the region of diffraction covered by the ring material from the maximum $Q_{occ}(a, \lambda, f)$ value of 2, assuming that all particles are larger than $a_c$ (Cuzzi, 1985; French and Nicholson, 2000):

$$Q_{occ} \approx 2 - f. \quad (3.10)$$

Here $f$ is determined by

$$f = \frac{2w}{\pi \theta_a D_{LOS}}. \quad (3.11)$$

where $w$ is the width of the ring. This equation is derived from Equation 2 by Cuzzi (1985), who calculates $f$ for multiple, thin rings in the Uranian system.
Figure 3.4: Schematic illustrating the dependence of $Q_{occ}$ on the particle size, angular size of the FOV, and the fraction of the region of diffraction containing ring material. In this schematic we show the thin F ring. The material extends $\sim 500$ km. Light diffracted out of the instrument’s FOV is only partially replaced by other ring particles, because there is simply not enough material to replace all of the light. We account for this by calculating $Q_{occ}(a, \lambda, f)$. The yellow region and teal square represent the region of diffraction and projected FOV, respectively, as described in Figure 3.3. Here the purple hashed section represents the fractional region ‘$f$’ of material capable of replacing the light diffracted out of the FOV. Image without annotations credit to NASA/JPL/SSI.

Figure 3.4 is a schematic that shows how $Q_{occ}$ depends on the fractional area of diffraction in which ring particles exist. In this scenario, the particles blocking the starlight also diffract an additional amount of light equal to their geometric area $G$. As described above, neighboring particles can replace that diffracted light, reducing $Q_{occ}$ to unity. However, for
a thin ring or rings, if there are not enough neighboring particles distributed around the occulting region, then not all of the light diffracted out can be replaced, and so $Q_{occ}$ falls between 1 and 2 according to Equation 3.10. This can be the case for Saturn’s F ring in stellar occultations, as well as occultations at ring edges where only half of the instrument FOV is filled with ring particles.

The angular size of the light source is also important in determining the effective extinction efficiency of the ring particles. For extended sources, such as the Sun during solar occultations, a particle that diffracts light out of the detector can replace its own light by diffracting light from the rest of the extended source into the FOV (Cuzzi and Pollack, 1978). If the diffraction cone of the particle is smaller than the angular size of the extended source, then $Q_{occ} = 1$.

In this thesis we explore several of these scenarios. In Chapter 4 we discuss the thin F ring as it occults the Sun and in Chapter 5 we have a scenario in which half of the instrument FOV is filled by ring particles as a “point source” star approaches ring edges. We address how we implement $Q_{occ}$ in each of these scenarios in those respective chapters.

### 3.4 Multiple Diffraction

In the case of a ring with very low optical depth ($\tau << 1$), we expect light to be diffracted one time, without additional interference from other particles. However, for higher optical depths, additional interactions can make a significant contribution to the observed signal. We implement the methods of multiple scattering described by Marouf et al. (1982); Tyler et al. (1983); Cuzzi et al. (2009). However, we introduce the term “multiple diffraction”, which we use in place of “multiple scattering” to avoid confusion with the more complex, indirect paths that multiple scattering typically refers to. In the case of multiple diffraction, the path of the diffracted light will continue in the near-forward direction (very small scattering
angles) and can be diffracted again in the same manner. Multiple diffraction in the rings broadens the angular distribution of diffracted light (Zebker et al., 1985), so a larger particle could produce a diffraction signal similar to that of a smaller particle if only single-diffraction is accounted for. Therefore, throughout this work where only single-diffraction is assumed, our results may be an underestimate of $a_{min}$ but provide a strong lower limit to the size distribution.

Marouf et al. (1982) derive a model for near-forward multiple scattering (multiple diffraction) in Saturn’s rings for the classical ring case. The derivation expands upon the solutions for the forward-scattered (single scattering) approximation expressed in Wang and Guth (1951). Here we use $\alpha$ to represent the scattering angle, as is used by Wang and Guth (1951) and Zebker et al. (1985). The parameter is a function of $\theta$ and $\phi$, the polar and azimuthal angles in a coordinate system whose $z$-axis is perpendicular to the ring plane (Zebker et al., 1985). We calculate the intensity of the diffraction as a function of $\alpha$ because we must take into account the two-dimensionality of the diffraction signature in order to properly describe multiple diffraction.

For consistency with the derivation presented by Marouf et al. (1982), we introduce the terms $\hat{u}_0$ and $\hat{u}$ to represent the direction of the incident plane wave and the direction of the scattered signal, respectively. In spherical coordinates, the direction of the incident velocity and scattered velocity are described by the angles $\theta, \phi$ and $\theta', \phi'$, respectively, and $\hat{u} = \cos \theta$, $\hat{u}' = \cos \theta'$ (Wang and Guth, 1951). Their relationship to $\alpha$ is given by

$$\cos \alpha = \hat{u} \hat{u}' + \left[(1 - \hat{u}^2)(1 - \hat{u}'^2)\right]^{1/2} \cos(\phi - \phi'). \quad (3.13)$$

In order to determine the effects of multiple scattering, Marouf et al. (1982) step through the optical depth of the ring, solving the small-angle approximation of the Boltzmann equation as a function of optical depth (Equation 9 in Wang and Guth (1951)), written by Marouf
et al. (1982) as

$$u_0 \frac{\delta I(\tau, \hat{u})}{\delta \tau} = -I(\tau, \hat{u}) + J(\tau, \hat{u}). \quad (3.14)$$

In the generalized formula (without the small-angle approximation), the term $u_0$ is written as $u \equiv \cos \theta$ (Wang and Guth, 1951; Marouf et al., 1982). $J(\tau, \hat{u})$ is the source function and is defined as

$$J(\tau, \hat{u}) = \frac{\omega}{4\pi} \int d\hat{u}' \Phi(\hat{u}, \hat{u}') I(\tau, \hat{u}') \quad (3.15)$$

where $\Phi(\hat{u}, \hat{u}')$ is the phase function, which is equal to that defined in Equation 3.5, and $\omega$ is the single scattering albedo, which is the ratio of the particle’s scattering and extinction cross-sections ($\frac{Q_{\text{scatt}}}{Q_{\text{occ}}}$). For values of $Q_{\text{occ}} = 2$, $\omega = 0.5$, because $Q_{\text{scatt}} = 1$.

Marouf et al. (1982) determine the expression for multiple diffraction by solving Equation 3.14 for the boundary condition $I(0, \hat{u}) = 0$ for $-1 \leq \hat{u} \leq 1$, starting from the single-scattering approximation

$$4\pi I_1(\tau, \hat{u}) \simeq (\tau \exp^{-\tau}) \omega \Phi(\hat{u}, \hat{u}_0). \quad (3.16)$$

Equation 3.16 is used to approximate the source function, which can then be used to determine the intensity of the diffraction signature in Equation 3.14. That result can then be used to determine the source function for the next iteration, which gives the second approximation of the intensity, and so on. The dependence on $\tau$ and $\hat{u}$ are decoupled in these iterations, such that the resulting expression for the intensity of the multiply-diffracted signal is given by amplitude functions $X_k(\tau)$ and angular functions $F_k(\hat{u}, \hat{u}_0)$ (Marouf et al., 1982):

$$4\pi I(\tau, \hat{u}) \simeq \sum_{k=1}^{\infty} X_k(\tau)F_k(\hat{u}, \hat{u}_0) \quad (3.17)$$
where the amplitude functions are defined as

\[ X_k(\tau) = \frac{\tau^k}{k!} e^{-\tau} \]  

(3.18)

and the angular functions are defined as

\[ F_k(\hat{u}, \hat{u}_0) = \frac{1}{4\pi} \int_{4\pi} d\hat{u}' F_{k-1}(\hat{u}, \hat{u}') F_1(\hat{u}', \hat{u}_0) \]  

(3.19)

starting from the initial condition

\[ F_1(\hat{u}, \hat{u}_0) = \bar{\omega}\Phi(\hat{u}, \hat{u}_0). \]  

(3.20)

This resulting expression for multiple diffraction is written explicitly (for constant wavelength and optical depth) as Equation 15.4 by (Cuzzi et al., 2009)

\[ \frac{I_s(\alpha)}{I_i} = \sum_{k=1}^{\infty} \frac{I_k(\alpha)}{I_i} = \sum_{k=1}^{\infty} \left[ \frac{1}{k!} \left( \frac{\tau_n}{\mu_0} \right)^k e^{-\tau_n/\mu_0} \right] \left[ \frac{1}{4\pi} \bar{\omega}_0 P(\alpha) \right]^{(k-1)} \]  

(3.21)

The term \([x]^{*k-1}\) indicates a convolution of \(x\) with itself \(k-1\) times. Note that in Equation 15.4 from Cuzzi et al. (2009), the convolution is listed as \(*k\), which we found to be an error and should be written as presented here. The correction is derived from the simplification of this expression to single scattering, which is explicitly written in Cuzzi et al. (2009) as Equation 15.5. To obtain the single scattering approximation from the multiple scattering equation (Equation 3.21), no convolution should occur. For the second-order diffraction, only one convolution should occur, and so on. Therefore, the expression should state that the convolution occurs with itself \(k-1\) times rather than \(k\) times. This error also appears in Zebker et al. (1985). We also note that in Cuzzi et al. (2009) this equation is expressed as \(P(\theta)\) however the convolution is 2-dimensional and should be written as \(P(\alpha) \equiv P(\theta, \phi)\).
We solve this Equation 3.21 computationally. Details of the implementation of multiple diffraction into our computer model are included in Section 3.5. Figure 3.5 shows how the strength of the multiply-diffracted signal is dependent on the optical depth of the ring as a function of scattering angle. For larger ring optical depths, light can be diffracted multiple times, broadening the overall signal by increasing the signal intensity at larger scattering angles.

### 3.5 Implementation

Here we describe the basic implementation of the diffraction theory into our models of Saturn’s particle size distribution. Details of the models, such as the majority of the assumptions, are provided in Chapters 4 and 5.

Generally, the objective of the models is to solve Equation 3.21 and therefore determine the intensity of the light diffracted by the various particles in the ring, given their angular separation from the light source. The model result is then compared with the observation to determine if our guess for the particle size distribution parameters produces a similar diffraction signature and is therefore a valid description of the distribution. We input different particle size distributions to find the best-fit model.

We numerically solve Equation 3.21 to find the total signal. In the case of an optically-thin ring, where single-diffraction is assumed, the equation reduces to Equation 3.4. We substitute Equation 3.3 into the equation 3.4 and convert the integral to a summation

\[
\frac{I_1(\theta, \lambda)}{I_i} = \frac{e^{-\tau_n / \mu_0}}{4\pi \mu_0 \int_{a_{\min}}^{a_{\max}}} \pi a^2 \left[ \frac{2J_1(ka \sin \theta)}{\sin \theta} \right]^2 Ca^{-q} da
\]

(3.22)

\[
\frac{I_1(\theta, \lambda)}{I_i} = \frac{e^{-\tau_n / \mu_0}}{\mu_0} \sum_{j=0}^{N_{\text{max}}^a} \frac{J_1(ka_j \sin \theta)}{\sin \theta} \left[ J_1(ka_j \sin \theta) \right]^2 C\Delta a_j \quad (3.23)
\]
where \( j \) is the \( j^{th} \) element in a range of particle sizes from \( a_{\text{min}} \) to \( a_2 \) and \( a_{N_{\text{max}}} = a_2 \) and \( \Delta a_j \) is the incremental increase in the range \([a_j, a_{j+1}]\). In this equation, \( \tau_n \) is known, either by using the observed optical depth from the occultation of interest or is a free parameter. The parameters of interest, \( a_{\text{min}}, a_{\text{max}}, \) and \( q \), are inputs into the models. We can solve for \( C \) by combining Equation 3.8 and Equation 3.3 to find \( \tau_n \):

\[
\tau_n(\lambda) = \int_{a_{\text{min}}}^{a_{\text{max}}} \pi Q_{\text{occ}}(a, \lambda, f) C a^{2-q}.
\] (3.24)

We solve for the constant

\[
C = \frac{\int_{a_{\text{min}}}^{a_{\text{max}}} \tau_n(\lambda)}{\int_{a_{\text{min}}}^{a_{\text{max}}} \pi Q_{\text{occ}}(a, \lambda, f) a^{2-q}}.
\] (3.25)

Finally we can solve Equation 3.4, given \( q, a_{\text{min}}, a_{\text{max}}, \) and \( \tau_n \). Figure 3.6 shows the results of solving for \( \frac{l}{l_h} \) and the effect of varying the four parameters described here.

In the case of an optically-thick ring where the contributions from multiple diffraction are non-negligible, we follow the steps described in this section as well. The term \( \tilde{\omega}_0 P(\alpha) \) in Equation 3.21 is equal to the summation term in Equation 3.23 divided by \( \tau_n \). We substitute these values into the term \( \left[ \frac{1}{4\pi} \tilde{\omega}_0 P(\alpha) \right] \) and convolve the term with itself using the IDL function ‘convol’ to implement the convolution calculation numerically over the entire range of scattering angles. For higher-order diffraction, we convolve the initial term with the convolved result, and continue to do so \( k-1 \) times. The remaining terms in Equation 3.21 are constants. Again, Figure 3.5 shows the computational models that result from including multiple diffraction.
3.6 Summary of Chapter

In this chapter we outlined the theoretical description of diffracted light by particles in the geometric optics regime \( \frac{2\pi a}{\lambda} >> 1 \), specifically explaining the distribution of the intensity of the light and the extinction of the light due to Babinet’s principle and paradox. We explored the relevance of light diffraction by particles in Saturn’s rings and described how to characterize the ring’s optical depth (normal and line-of-sight). We included the equations that characterize the intensity of the diffracted light and how they are calculated in the context of our ring model, including for multiple diffraction. We also discussed the effective extinction efficiency \( Q_{occ} \) of rings in a variety of observational geometries.

This chapter lays the foundations for the physical principles implemented in our two ring models: the F ring solar occultation model (Chapter 4) and the A ring stellar occultation model (Chapter 5). These equations are common to both the computational models, however the models themselves differ in their implementation, so we leave the remaining details of the ring models to the respective chapters.
Figure 3.5: Multiple diffraction as a function of scattering angle for the Rev 9 F ring solar occultation using different of normal optical depth for the lowest B angle observation, $B = 7.2^\circ$. Here we have solved Equation 3.21 using a single particle size of 100$\mu$m and 1 cm. The intensity of the total diffracted signal depends on the optical depth and the particle size. For low optical depths, the single-scattering component is dominant for any particle size, whereas for higher slant optical depths, the contributions from second and third-order diffraction become significant.
Figure 3.6: The intensity of the diffraction signal (Eq. 3.4) as a function of scattering angle due to variations in parameters. In each of these plots, the initial parameters are as follows: $a_{\text{min}} = 0.01$ mm, $a_{\text{max}} = 10$ mm, $q = 3.5$, and $\tau = 0.1$. The results are determined for $\mu_0 = 0.35$. In a) we vary the optical depth $\tau$ between 0.01 - 0.2. In b) we change the power-law index $q$. In c) we vary the maximum particle size $a_{\text{max}}$. Note that this parameter does not affect the overall shape of the curve except for at very small scattering angles. In d) we vary the minimum particle size $a_{\text{min}}$. 
CHAPTER 4: SATURN’S F RING

Saturn’s F ring is a unique and dynamic ring, displaying variability with longitude and time. Our understanding of the ring has developed through spacecraft and ground-based observations of the F ring that cover a range of observational wavelengths, geometries, longitudes, and times. The F ring was first detected by Pioneer 11 in 1979 (Gehrels et al., 1980). In 1980, higher-resolution Voyager 1 images revealed clumps and kinks in the ring, uncovering some of the longitudinal variations of the F ring (Smith et al., 1981, 1982). Radio occultation experiments conducted by the radio instrument on Voyager 1 observed the F ring as a single ringlet with a width of 2 km in the 3.6 cm data but did not detect the ring in the 13.6 cm data (Tyler et al., 1983; Marouf et al., 1986). The stellar occultation of δ Sco conducted at ultraviolet wavelengths from the Voyager 2 Photopolarimeter (PPS) exposed an F ring ~ 40 km across with strand-like features and an optically thicker, 3-km embedded component. An analysis of the Voyager photometric data revealed a much broader envelope consisting of micron-sized (or smaller) dust with an embedded, 1-km core containing a higher population of centimeter-sized particles (Showalter et al., 1992). In addition to the direct observations of the F ring, Pioneer 11 indirectly detected a belt of objects 0.1 – 1 km in size spanning a 2000-km-wide region surrounding the ring. These objects were proposed to explain the observed depletion in flux of trapped magnetospheric electrons in the region (Cuzzi and Burns, 1988).

Additional observations made from ground-based telescopes and the Hubble Space Telescope (HST) during Saturn’s ring plane crossing enabled measurements of the F ring thickness of 1 - 3 km (Nicholson et al., 1996). Poulet et al. (2000) found that the dust particles of the ring dominate the signal in backscattered light as well, and estimated the fraction of dust
in the F ring to be $> 0.80$. Bosh et al. (2002) measured the inclination of the F ring to be $0.0065 \pm 0.0014$ degrees through an analysis of the ring occultation of the star GSC5249-01240 observed by the Faint Object Spectrometer on the HST and other ground-based instruments.

Cassini observations detect an F ring core that is 10 - 50 km wide (Murray et al., 2008; Albers et al., 2012) and is comprised of a significant population of sub-millimeter-sized dust (Showalter et al., 1992; Hedman et al., 2011; Vahidinia et al., 2011) (Section 4.1.2). Murray et al. (2008) identified a narrow, non-continuous, ring with a radius of $\sim 1$ km located 15 – 50 km inward from the known F ring core in imaging data from Cassini. The orbital parameters of this narrow component match the predicted orbital solution for the F ring better than the known ring material, suggesting that this narrow component may be the “true” core containing most of the F ring’s mass (Murray et al., 2008; Colwell et al., 2009). This argument is strengthened by the existence of 1-km-diameter objects observed within the narrow ring component (Murray et al., 2008). Ultraviolet Imaging Spectrograph (UVIS) stellar occultation data confirm the transient presence of a 1 to few-km-wide narrow ring with an optical depth greater than 0.5 (Albers et al., 2012) embedded in the larger F ring core. This “true” core is likely what the Voyager 1 RSS and Voyager 2 Photopolarimeter detected during their observations of the ring.

In addition to the F ring core, strands of material varying in shape, optical depth and number are seen in imaging and UV data (Charnoz et al., 2005; Murray et al., 2005; Charnoz, 2010; Albers et al., 2012). A more detailed discussion of the strands and their formation is discussed in Section 4.1.1. In the UVIS occultation data, the strands are measured to be approximately 10 times as wide as the F ring core (which has a mean FWHM of 10 km) and with optical depths 10 times smaller (on the order of $\tau \sim .03$ (Albers et al., 2012). These strands are in fact a spiral that emanate from the core (Charnoz et al., 2005; Charnoz, 2010).
4.1 F Ring Particle Size Distribution and Evolution

4.1.1 Embedded Objects and Collisions

The micron-sized ice particles observed in the ring (Showalter et al., 1992; Charnoz et al., 2009; Hedman et al., 2011; Vahidinia et al., 2011) should have a very short lifetime in the ring; they would be expected to sputter away in $10^4$ years or be eroded by micrometeoroid bombardment in $10^6$ years (Grün et al., 1984; Burns et al., 1984; Showalter et al., 1992). This led to the natural assumption that these particles are replenished through collisions in the ring, resulting in a release of small grains (Showalter et al., 1992; Barbara and Esposito, 2002). Evidence for such large, progenitor objects has been mounting. As described in the beginning of this chapter, objects on the order of a kilometer in size have been directly imaged in the narrow component of the F ring core (Murray et al., 2008). Additionally, large moonlets or ephemeral clumps were observed during the HST observations during the ring plane crossing (Nicholson et al., 1996; Poulet et al., 2000; McGhee et al., 2001). Furthermore, an analysis of UVIS occultation data by Esposito et al. (2008) and Meinke et al. (2012) revealed the existence of up to 140,000 ephemeral clumps or moonlets on the order of 600 meters in size. “Fans” detected in the Imaging Science Subsystem (ISS) images are believed to be created by a mechanism similar to that of propellers, implying objects up to 5 kilometers in size are also embedded within the core (Murray et al., 2008).

The presence of these large objects distributed throughout the F ring core could provide the source material for large impact events and the replenishing of small ring particles. As described above, Cuzzi and Burns (1988) predicted a belt of 0.1 – 1 km-sized moonlets spanning a 2000-km wide region surrounding the F ring. Cassini images have provided direct evidence of some large moonlets in the vicinity of the F ring (Porco et al., 2005). One such object, called S/2004 S6, is on an eccentric, core-crossing orbit (Porco et al., 2005;
Spitale et al., 2006). Large objects colliding with moonlets embedded within the core could be responsible for producing a variety of dynamical features observed in Saturn’s F ring, including the F ring strands. Charnoz (2010) finds that a particularly energetic collision ($\sim 30$ km/s) of a km-sized object with clumps in the core can release and scatter material over hundreds of kilometers. With time, this material will evolve to form a spiral arm parallel to the F ring core. Murray et al. (2008) suggests that the spiral structure observed early in the Cassini mission may have resulted from the impact of S/2004 S6 into the F ring core. At the locations where the spiral intersects the F ring and at locations where the jets are observed, the structures brighten with phase angle, which is indicative of the presence of small particles (Showalter et al., 1992; Charnoz et al., 2005), further supporting the notion that the structures are the products of small-particle-releasing collisions. Additionally, the long-lasting bright feature observed in 2006 was likely due to a very large collision, again possibly with S/2004 S6. French et al. (2012) note that this feature was three times brighter than the rest of the ring combined, and slowly stretched longitudinally and dimmed over the following 2 years.

Figure 4.1: Image of a mini-jet in Saturn’s F ring observed June 20, 2013 by the ISS narrow-angle camera. The mini-jets are due to slow (centimeter per second) collisions in the F ring core. Image credit: NASA/JPL/SSI.
The spiral-inducing collisions are likely the most energetic of the continual collisions that produce “jets” throughout the F ring. These dynamic features are seen as extensions of material inward and outward of the ring and are consistent with the impacts between 10 kilometer-sized objects and the core with collisional velocities on the order of tens of meters per second (Murray et al., 2008; Attree et al., 2012). Hundreds of smaller features, called “mini-jets” (Figure 4.1), thought to be the product of 1 meter/second collisions in the core, have been catalogued by Attree et al. (2012). The range of energetic collisions likely explains the short-lived “burst” events observed in the Voyager data by Showalter (1998, 2004).

4.1.2 Observed Particle Size and Distribution

The particle size distribution of Saturn’s F ring is a reflection of the on-going, accretion processes countered by disruptive, collisional processes. As previously noted, the short lifetime of small particles in the rings indicates that these particles are replenished. The frequency and velocity of impacts that replenish the particles can be extrapolated from the particle size distribution.

Showalter et al. (1992) analyzed photometric data from the Voyager flybys and found that the ring’s radial integrated brightness increased with increasing phase angle, indicative of scattering from small dust particles. They modeled the particles as randomly-oriented, non-spherical particles and found that the contribution to the optical depth of the ring due to dust was over 98%. They found a steep power law of $q = 4.6 \pm 0.5$. Additionally, they conclude through the analysis of the optical depth profile at $\lambda = 0.264\mu m$, $3.6\mu m$ and 13 cm, that the optical depth of the core is dominated by centimeter-sized particles and that micron-sized dust particles dominate the surrounding envelope. French et al. (2012) reanalyzed the Voyager data studied by Showalter et al. (1992) and expanded upon it by comparing the data with Cassini photometry of the ring. They found that F ring is brighter, more optically thick, and wider during the Cassini era than it was during the Voyager flybys by factors of 2-
3, but find that these properties of the ring have been stable throughout Cassini’s lifetime at the Saturn system. They suggest that the F ring objects stirred up by Prometheus’ previous encounter with the ring have been brightening and widening the ring, and a only handful of the large objects that have been impacting the ring still remain to provide occasional outburst events. French et al. (2012) also note the increase in brightness and width of the F ring with increasing phase angle, suggestive of a higher percentage of small, forward-scattering particles that have persisted in the region for over 30 years. Assuming the ring width consists of 90% of the intensity of the total equivalent width, French et al. (2012) find the F ring to have a photometric width of 580 ± 70 km.

The particle size distribution of the F ring was also derived by Bosh et al. (2002). In addition to describing the orbital parameters used HST and ground-based occultation data, they analyze the spectra obtained by the Faint Object Spectrometer during the stellar occultation by the rings of the star GSC5249-01240. They find that the equivalent depth that describes the F ring does not vary across the instrument’s bandpass of 0.27 – 0.74μm. Since the transparency of the ring particles should vary when the observed wavelength is on the order of the particle sizes (Hedman et al., 2011), Bosh et al. (2002) conclude that the particles are at least as large as 10μm.

Spectral analysis by Hedman et al. (2011) and Vahidinia et al. (2011) do show variations in the transparency of the F ring in the VIMS wavelengths of 1 – 5μm. Vahidinia et al. (2011) report a broad spectral trend with a narrow dip at 2.87 μm. The spectral signal peaks between 2 - 3 μm. Their modeling efforts of the spectra reveal a narrow size distribution of particles with radii between 10 – 30μm. The narrow dip is due to the Christiansen frequency of water ice, enabling Vahidinia et al. (2011) to conclude the particles’ composition of crystalline water ice. Hedman et al. (2011) find that a complex, broken power law is required to reproduce the VIMS spectral data from stellar occultations. They model the majority of the ring as having two size distributions: a steep distribution ($q = 3.5$) for
particles between 10 microns and 1 millimeter, and a shallow distribution \((q = 2)\) for particles up to 10 microns in size.

### 4.1.3 Intent of this work

In this work, we explore the particle size distribution of the F ring through the analysis of 13 solar occultations observed by the Extreme Ultraviolet (EUV) solar port channel on UVIS and compare the results with data from the Imaging Science Subsystem (ISS) taken at similar ring longitudes and times. In 5 of the solar occultations, clear signatures of diffraction were observed, indicating a substantial population of sub-millimeter-sized particles. We explore the potential connection of these diffraction signatures with collisional events in the rings as observed in the ISS data that could produce the elevated number of small particles in ring.

We investigate a particularly interesting solar occultation that occurred during Rev 9. During this occultation, the boresight of the instrument was not properly aligned with the Sun and resulted in an occultation during which over 95% of the solar disk was placed outside of the instrument’s field of view. During this observation, the solar signal was decreased to \(\sim 2\%\) its nominal value, enabling the detection of strong diffraction signals in the data. The ISS images reveal that the misaligned solar occultation serendipitously occurred near a collisional event in Saturn’s F ring. We discuss two additional solar occultations, Rev 172 and Rev 181, during which the Sun is purposefully placed near the edge of the instrument’s FOV in an attempt to repeat the observation of diffracted light observed during the Rev 9 occultation.

We develop a computational model to replicate the UVIS observations in order to calculate the amount of diffracted light the instrument would detect given a particle size distribution and optical depth of the ring. We run this model for all solar occultations to explore the variations in the required size distribution of the ring to reproduce the observations. This includes solar occultations for which no diffracted light was detected. We determine the
best-fit model of the particle size distribution for each model and then compare these results with the ISS images to search for correlations in the size distribution and recent collisional events in Saturn’s F ring.

4.2 Observations

4.2.1 Cassini UVIS Solar Occultations

We analyze solar occultation data observed by the UVIS Extreme Ultraviolet (EUV) channel. As described in detail in Chapter 2, this channel simultaneously records spectral and spatial information. The solar occultations measure the variation in intensity of the signal from the Sun at wavelengths between 512 – 1180 Å as the rings occult the Sun. As of 2015, the instrument has observed 13 solar occultations by the F ring, which we refer to by the Cassini revolution during which the occultation occurred (Table 4.1). ‘I’ and ‘E’ are used to denote if the observation occurred during ingress or egress, respectively. The integration periods vary for each solar occultation and are included in Table 4.1. These observations occur at varying ring longitudes and at different ring opening angles - the angles at which the line of sight from the spacecraft intersects the ring plane ($B$). For observations of a star during which the spacecraft’s line of sight is perpendicular to the ring plane, $B = 90^\circ$. In an edge-on observation, $B = 0^\circ$. The geometry of the solar occultations result in very low $B$ angles.

For each occultation model, we calculate a percent increase in unocculted solar signal due to diffraction and a percent decrease due to the F ring particles occulting the Sun. Because the analysis is comparative, it is unnecessary to further process the data (remove background, etc.). As we describe in later sections, we simply find the relative increase and decrease in signal due to the particles present in the ring.
4.2.1.1 Rev 9 Misaligned Solar Occultation

Figure 4.2: Schematic of the UVIS solar port FOV during the Rev 9 occultation. The black region represents the UVIS FOV. The white ‘X’ is the boresight of the instrument. The orange circle is the angular size and location of the Sun during the Rev 9 observation. Over 95% of the solar disk is outside of the FOV, reducing the observed solar signal during this observation. The teal line is the location of the F ring at one snapshot in time. At subsequent time steps, the ring will move up along the FOV, crossing in front of the Sun.

During the Rev 9 occultation, UVIS observed its first solar occultation by Saturn’s rings. The observation resulted in a particularly low signal; the unocculted solar photon count per second is $\sim 2.5\%$ of that observed during later solar observations. The pointing of UVIS during Rev 9 placed the Sun at the very edge of the instrument’s field of view (FOV) rather than at its center (boresight) during this occultation, an error that was corrected in later solar observations. The misalignment caused over 95% of the solar disk to be located outside of the instrument FOV, reducing the signal to the observed level. Figure 4.2 is a schematic of the UVIS EUV FOV and the location of the Sun. Serendipitously, there was a strong detection of diffracted light near Saturn’s F ring during this misaligned observation. While only a small percentage of the sunlight entered the instrument’s aperture, the ring particles
within the FOV continued to diffract light from the bright source, enabling a stronger relative diffraction signal to be observed. In Figure 4.3 we show the Rev 9 occultation data. The line of sight distance from the spacecraft to the ring was approximately 200,000 km. At Saturn, the Sun’s diameter subtends an angle of $\sim 1$ mrad, so the projected diameter of the Sun during this occultation is 200 km. Due to the large projected size of the Sun, the Keeler Gap (42 km) is too narrow for the Sun to be completely unocculted by ring particles, preventing the signal from returning to the unocculted signal $I_0$ as typically observed during a stellar occultation.

Figure 4.3: Rev 9 solar occultation data. During this observation, the UVIS instrument was pointed such that the majority of the solar disk was placed outside of the instrument’s FOV, reducing the unocculted solar signal to 2.5% its nominal value. The Encke Gap is visible in the observation but the Keeler Gap is hardly detected due to the Sun’s large projected size during the occultation. The inset shows a close up of the signal near the F ring. The increase in photon counts per second is due to particles diffracting sunlight into the instrument’s detector. The dip in the signal is caused by the occultation of the Sun by the F ring.

Nearly centered on the F ring’s semi-major axis of 140,221.3 km (Bosh et al., 2002; Albers et al., 2012), there is a sudden increase in photon counts per second above the unocculted
solar signal. The explanation for this increase in light is that, in addition to the observed (although low) solar signal, small particles in the F ring diffract additional light into the detector. In the inset of Figure 4.3, we see that the diffraction signal takes on a particular ‘M’ shape. We show in this work that the central dip in the signal aligns with the occultation of the Sun by the center of the F ring. The peak in the diffraction signal represents a 10% increase from the unocculted solar signal.

4.2.1.2 Designed Misaligned Solar Occultations

Figure 4.4: Two solar occultations intentionally designed to point the instrument away from the Sun in attempt to recreate the geometry of the Rev 9 occultation. (a) Slight shifts in the pointing of the instrument during the Rev 172 solar occultation may have led to structure in the unocculted signal of the Sun. No clear signal from the occultation of the F ring is discernible. (b) The Rev 181 solar occultation placed the Sun very near on the FOV as the Rev 9 occultation, reducing the solar signal to $\sim 1800$ photon counts per second, close to the signal observed during the Rev 9 occultation. However, no obvious diffraction signal or attenuation of the solar signal is visible.

In 2012 and in 2013 we designed solar occultations intended to replicate the geometry of the Rev 9 solar occultation. Figure 4.4 shows the data from the Rev 172 and Rev 181 solar occultations. The solar signal outside of the main rings is not constant during the Rev
occultation, indicating that slight variations in the pointing of the instrument occurred
during this observation. As a result, the occultation of the F ring is not discernible through
diffracted light or the attenuation of the solar signal. The variation in the signal is on the
order of the strength of the expected diffraction signal, so we do not use this observation
for further analysis at this time. The pointing of the Rev 181 solar occultation replicates
the pointing of the Rev 9 occultation, placing the Sun in a similar location in the FOV and
therefore reduces the solar signal to a level consistent with that of the Rev 9 occultation.
However, we do not detect a clear diffraction signal or the attenuation of light at the F ring
in this occultation either. We continue to analyze this occultation, however, because the
solar signal is stable and the lack of a clear F ring detection may be a useful indication of
the particle sizes as well.

4.2.2 Cassini ISS Mosaics

We compare the results of our analysis of the UVIS data with ISS mosaics of the F ring.
The mosaics are constructed using individual images re-projected onto the equatorial plane
(Murray et al., 2008). For all mosaics presented in this work, the horizontal axis is the
longitude and the vertical axis is the orbital radius. We look at mosaics created from images
of the rings that were taken near in time to the solar occultation data. Dr. Nick Attree and
Dr. Carl Murray provided these mosaics and indicated the longitude of the observed solar
occultation in each mosaic (red vertical line). We compare the occultation data with these
mosaics, and as is discussed throughout this work, we find a correlation with detections of
diffracted light in solar occultations and ring features in the ISS images that are likely due
to collisional events.

In Figure 4.5a we show the mosaic nearest in time to the Rev 43 solar occultation, with
the longitude of the occultation marked by the red line. There is a prominent collision
feature in this mosaic, and serendipitously, where the solar occultation occurred. Murray
et al. (2008) shows a series of images of this location in the ring spanning from December 2006 through May 2007, displaying the evolution of this particularly large collision event. They assert that this feature was produced by a collision into the core by S/2004 S 6 or some fragment of it. This collision event is also the one described as drastically increasing the brightness of the ring by French et al. (2012).

The Rev 43 occultation occurred in April of 2007. As shown in images taken 6 days prior (Figure 4.5a), the disturbed region of the ring is what occults the Sun during the UVIS observation. Of all the aligned solar occultations, the Rev 43 occultation displays the most prominent diffraction signature. We show in this work that the collision visible in the ISS images likely released a significant population of sub-millimeter particles capable of producing the observed diffraction signature.

Figure 4.5: (a) ISS mosaic of the F ring 6.37 days before the Rev 43 solar occultation was observed. The red line indicates the longitude of the solar occultation. The large extent of material emanating from the F ring core is the mark of a large collisional event, possibly with the ephemeral ring-crossing object S/2004 S 6 (Murray et al., 2008). (b) UVIS solar occultation data from Rev 43. The attenuated signal due to the occultation of the Sun by the F ring is labeled, as well as the diffraction signature that extends above the unocculted solar signal.
Table 4.1: Table of Solar Occultations. The inertial longitude is the longitude of the F ring where the line-of-sight vector of the UVIS boresight intersects the ring.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Occultation Data File</th>
<th>Integration (s)</th>
<th>B angle (Degrees)</th>
<th>Inertial Longitude (Degrees)</th>
<th>Line-of-Sight Distance km</th>
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<tbody>
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<td>Rev 9</td>
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<td>20.96</td>
<td>251.3</td>
<td>207,996</td>
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<td>Rev 43</td>
<td>EUV2007_114_09_38_12_UVIS_043RI_SOLAROCC001_VIMS</td>
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<td>12.66</td>
<td>75.1</td>
<td>347,275</td>
</tr>
<tr>
<td>Rev 55I</td>
<td>EUV2008_003_20_15_55_UVIS_055RI_SOLAROCC001_VIMS</td>
<td>4.0</td>
<td>8.97</td>
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<td>353.4</td>
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<td>7.20</td>
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<td>Rev 172</td>
<td>EUV2012_207_23_31_52_UVIS_172RI_SOLAROCC001_VIMS</td>
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<td>15.91</td>
<td>299.5</td>
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<td>Rev 181</td>
<td>EUV2013_044_22_52_13_UVIS_181RI_SOLAROCC002_VIMS</td>
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<td>110.7</td>
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4.3 Data Analysis

4.3.1 Spectral Analysis

Figure 4.6: The first column is the solar occultation data. We subtract the spectra in the core, indicated by the purple star, from the mean spectra of the unocculted solar signal, marked by red triangles. The second column is the differences in the intensity at each wavelength normalized to the unocculted solar signal.

One technique to discern information regarding the particle size distribution is to analyze the ring’s spectrum. When particles are in the geometric optics regime \((a \gg \lambda)\), the transmission of light through the rings is constant with wavelength. However, when the radii of the particles are comparable to the wavelength of the incident light, the observed
transmission of that light can vary with wavelength (Hedman et al., 2011). These variations
can be used to constrain the sizes of the particles interacting with the incident light.

Vahidinia et al. (2011) used VIMS data to study the F ring and found a spectral trend
with a broad peak at $2 - 3 \mu m$, which they determine indicates a peak in the particle size
distribution of $\sim 10 \mu m$. Similarly, Hedman et al. (2011) utilized variations in a dip in the
optical depth profile of the VIMS spectra due to the Christiansen Effect to map the particle
size distribution across the F ring. However, an investigation of the ring by Bosh et al.
(2002) did not find any variations in equivalent depth of the F ring over the $0.27 - 0.74 \mu m$
bandpass of the Faint Object Spectrograph on the Hubble Space Telescope. As a result,
they conclude that the ring particles are larger than $10 \mu m$ in size.

Here we analyze the F ring spectra observed by the UVIS EUV instrument. We compare
an averaged spectrum from the unocculted Sun with the spectrum observed at the minimum
signal beyond the main rings; this is where the F ring core is attenuating the most signal. In
occultations where diffraction is evident, we also compare the mean spectrum of the diffrac-
tion signal with the unocculted solar signal. Figures 4.6 and 4.7 show the signals used for
the spectral comparison. We subtract the spectral signal in the core from the spectral signal
of the unocculted Sun and normalize by the unocculted solar signal. We do the same for
the diffracted signal if available. We complete this spectral comparison for all of the observ-
ations with the exception of Rev 181 and Rev 172 because these observations have no clear
attenuated signal due to the F ring core. We show the results from several occultations in
Figures 4.6 and 4.7. The resulting spectral comparisons for all of the occultations look very
similar; they are flat with no obvious indication of variation in transmission as a function of
wavelength. This suggests that the particles in Saturn’s rings are larger than the observa-
tional wavelength, placing a lower limit on the particle size of $0.1 \mu m$. These results are in
agreement with (Bosh et al., 2002), who also did not detect changes in the transmission at
ultraviolet wavelengths.
4.4 Computational Modeling of Solar Occultations

We have developed a computational model of solar occultations by Saturn’s F ring. In this section, we explain each component of the model. In summary, we model the UVIS FOV as a collection of discretized “pixels”. We use SPICE kernels to determine the pointing of the FOV and location of the F ring and Sun at the time of a given solar occultation. We
calculate the scattering angle between each pixel containing ring material and each pixel containing direct sunlight. We determine the diffraction signal within each of these pixels by solving Equation 3.21 and sum over all of the pixels to model the total diffraction signal observed at each time step. We also determine the decrease in signal due to the attenuation of sunlight as the ring particles occult the Sun. We combine these two signals to model the observed solar occultation data and then compare the model directly with the occultation data.

4.4.1 Occultation Geometry

In order to make comparisons between the observations and the model, we must accurately reproduce the geometry of the spacecraft, Sun, and the ring in each model. We reconstruct the trajectory of the spacecraft and the pointing of UVIS using the SPICE toolkit (Chapter 2). Many of the following steps were adapted from Geometer software, written by Dr. Joshua E. Colwell for determining the geometry of the Cassini observations in the Saturn system.

4.4.1.1 Instrument Frame

We begin the model by determining the rotation matrix from the instrument frame, which is a two-dimensional representation of the angular size of the instrument FOV, to the inertial reference frame (J2000, see Chapter 2) using the SPICE `pxform` routine for the time vector of the observations. This spice routine calls to the frames kernel, which contains the rotation matrix for the orientation of the instrument relative to the spacecraft. We use the adjusted frames kernel indicating the correct pointing of the UVIS solar port as described in Chapter 2. The resulting rotation matrix can be used to determine the pointing of the boresight or any other position in the instrument’s reference frame in J2000 coordinates, as well as take the J2000 coordinates of any celestial object and determine its location in the instrument’s reference frame.
We use the SPICE routine ‘spkpos’ to determine the location of the Sun with respect to Cassini in the inertial frame. We then apply the rotation matrix as explained above to determine the Sun’s location in the instrument’s reference frame. For the properly aligned solar occultations, this places the center of the Sun near the center of the instrument field of view. It is not exactly centered due to the slight error in the pointing of the boresight that has been addressed throughout this work. In the case of the Rev 9 occultation, the instrument pointing was significantly offset from the boresight: the $\Delta y$ position of the Sun of $\sim -11.00$ mrad. The C-kernel indicates that the $\Delta x$ position of the Sun is -4.298 mrad, however we apply an additional adjustment to this value based on the observed signal, as is discussed in Section 4.5.4.

4.4.1.2 Saturn Frame

The next step is to determine the orientation of the Saturn system in the instrument’s reference frame. We again use SPICE ‘pxform’ to determine the rotation matrix between Saturn’s reference frame and the inertial frame. We multiply the rotation matrix to the Saturn frame, centering it on the planet’s pole. For the main rings, we define the ring plane as perpendicular to the pole vector using the SPICE routine ‘NVC2PL’. This creates a plane extending radially outward from Saturn’s equator.

4.4.1.3 F Ring Frame

Saturn’s F ring is eccentric and inclined to the main rings (Bosh et al., 2002; Albers et al., 2012). The parameters that define the ring are listed in Table 4.2. In order to determine the geometry of the ring, it is necessary to define a separate plane that describes the F ring. To do this, we must create a rotation matrix that converts between the Saturn frame and
an F ring reference frame. We rotate Saturn’s frame such that its $\hat{X}$ axis is aligned with the ascending node of Saturn (where Saturn’s equatorial plane and the inertial plane intersect). We create a rotation matrix to transform between this Saturn frame and the F ring frame, with the $\hat{x}$-axis of the F ring along the F ring ascending node (where the F ring plane and Saturn’s equatorial plane intersect). We construct the matrix using Euler angles and the respective $\hat{X}$-, $\hat{Y}$-, and $\hat{Z}$-axes we need to rotate about using the ‘EUL2M’ (Euler to Matrix) SPICE routine. We rotate about Saturn’s $\hat{Z}$-axis (its pole) by the argument of pericenter $\tilde{\omega}$ (Equation 4.1). We then rotate about the newly-formed F ring $\hat{x}$-axis by the inclination angle $i$ (Equation 4.2) (Murray and Dermott, 1999). This provides the orientation of a vector normal to the F ring (F ring pole). We can then build the ring by again creating a plane normal to the F ring pole using ‘NVC2PL’.

Figure 4.8: Schematic illustrating the geometry of Saturn and the F ring. We utilize the orbital elements labeled in this figure to create rotation matrices that enable the determination of the position of the F ring in the inertial and instrument reference frames (see text). This figure is based on Figure 2.13 from Murray and Dermott (1999).
With the rotation matrix in Equation 4.3, we can then easily rotate from the F ring frame to the Saturn frame (with the rotated ascending node) using Equation 4.3. Then we can utilize the previous rotation matrices to finish converting from the F ring frame to the inertial J2000 or UVIS instrument reference frames. These calculations are executed as a function of time, since the F ring is precessing. Figure 4.8 is a schematic showing the F ring in the Saturn-centered coordinate system with labels indicating the parameters listed in Table 4.2.

\[
P_1 = \begin{bmatrix}
\cos \tilde{\omega} & -\sin \tilde{\omega} & 0 \\
\sin \tilde{\omega} & \cos \tilde{\omega} & 0 \\
0 & 0 & 1
\end{bmatrix} \quad (4.1)
\]

\[
P_2 = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos i & -\sin i \\
0 & \sin i & \cos i
\end{bmatrix} \quad (4.2)
\]

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = P_2 P_1 \begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} \quad (4.3)
\]

We account for the eccentricity of the F ring by creating an ellipse in the F ring frame using Equation 4.4:

\[
r = \frac{a_{sm}(1 - e^2)}{1 + e \cos \theta_{ta}} \quad (4.4)
\]

where \(\theta_{ta}\) is the true anomaly, defined as the angle between the longitude and the pericenter of the F ring, \(a_{sm}\) is the semi-major axis, \(e\) is eccentricity, and \(r\) is the radial distance of the ring at that true anomaly value. We use Equation 4.4 to determine where the ring material
is located for a given occultation.

### 4.4.2 Model Field of View

To determine where the instrument is pointing, we apply the rotation matrix that rotates from the instrument frame into the J2000 frame to the instrument’s boresight. We then use the SPICE routine ‘INRYPL’ to determine the intersection of the boresight vector and the ring plane of interest (main rings or F ring). This provides a vector that, when normalized, indicates the radial distance from Saturn at which the boresight ‘look vector’ intersects the ring plane. We can also determine the ring longitude at the point of intersection. The longitudes applied in our model are measured prograde from the ascending node of Saturn’s equatorial plane with the J2000 inertial frame. We then calculate the true anomaly and use Equation 4.4 to determine the radial location of the center of the F ring core.

We construct our model FOV by creating a number of “pixels” in the instrument FOV. We discuss the number and resolution of the pixels in Section 4.4.2.2. We repeat the calculation described above for the boresight for each of the FOV pixels, creating an array of vectors intersecting the ring plane. We then determine which of these look vectors intersects the ring plane at a radial distance from Saturn where ring material exists. If so, then we include this pixel in the calculation of diffracted and attenuated signal.

#### 4.4.2.1 Verification of Model Geometry

Determining the geometry of each observation is difficult, so it is important that we verify that the model accurately reproduces the geometry of observations of the F ring. We test our model by reproducing the geometry of UVIS HSP stellar occultations, assuming the star is in the center of the FOV. Figure 4.9 shows the residuals of our determination of the F ring core location based on the model parameters and the observed location in the data. The residuals are all within 50 km. More importantly, our results map very well with Figure 7
of Albers et al. (2012), who performed the same analysis. This provides verification that the basic geometric assumptions and calculations of the model are correct. Because the F ring is variable, it is not surprising that there are offsets from the predicted core location. Additionally, Albers et al. (2012) shows that the solution derived by (Bosh et al., 2002) can be improved upon. For the purposes of our study, a slight radial offset does not affect the observed or modeled diffraction signal from the ring.

Figure 4.9: The residuals of our model’s determination of the F ring core location compared with their observed radial locations in the UVIS stellar occultation data. The residuals show that our model, which uses the orbital parameters for the F ring as described by Bosh et al. (2002), determines the location of the core to within 50 km of its observed location. There is one clear outlier in the data at a true anomaly of ~140°, which is also noted in the model by Albers et al. (2012), who suggest this is an observation of a strand rather than the core.

4.4.2.2 Modeling the Sun and the F ring

For each solar occultation we create a low-resolution (large pixel) model FOV that replicates the pointing and angular extent of the UVIS FOV. We then determine, through the steps described in Section 4.4.2, the intersection of the look vector for each pixel and the F ring
plane. We subtract the radial location of the intersection for each pixel from the radial location of the F ring core. If the difference is less than the radial extent of the ring (an input into the model), then we mark those pixels as containing F ring material.

We determine the position of the Sun in the FOV as described in Section 4.4.1.1. We calculate the angular size of the Sun for each observation

\[ \theta_{\odot} = 2 \sin^{-1} \left( \frac{D_{\odot}}{2 \text{LOS}_{\odot}} \right) \] (4.5)

where \( \theta_{\odot} \) is the angle subtended by the Sun’s diameter, \( D_{\odot} \), and \( \text{LOS}_{\odot} \) is the distance between Cassini and the Sun. The Sun has an angular diameter of approximately 1 mrad as viewed by Cassini at Saturn.

We calculate the 2-dimensional scattering angle, \( \theta \), between the center of the Sun and each pixel in the model FOV. If the scattering angle is less than the angular extent of the Sun (\( \theta < \theta_{\odot} \)), then we mark those pixels as directly observing part of the Sun.

With the locations of the ring material and the Sun in the low-resolution model now known, we can create two, smaller and higher-resolution regions of the FOV that encompass the ring and the Sun. The resolution of the model is somewhat dependent on the particle sizes in the model. Figure 4.10 shows the results of the \( \theta \)-dependent part of the integral in Equation 3.4 for a single particle size. This figure indicates that for small particles, the signal is constant at small scattering angles, but for large particles, the variation in the signal is significant at very small scattering angles. Therefore, if we design a model ring that contains a maximum particle size of 1 mm, the intensity of the scattered light will be constant for scattering angles less than \( \sim 5 \times 10^{-6} \) radians. As a result, we do not need a model with a greater resolution than that to accurately determine the intensity of the diffracted light. Additionally, with the exception of a few time-steps in each model when the ring is occulting the Sun, the angular separation between the ring particles and the Sun are much greater.
than this critical scattering angle and so even lower resolutions can be applied.

However, during those time steps when the ring does occult the Sun, it is important to have the appropriate, high resolution necessary to model the diffraction signal accurately. We therefore create an additional, even higher-resolution grid that surrounds the region of the Sun and ring-filled FOV pixels where the ring is occulting the Sun.

We do not simply create an extremely high-resolution model of the entire FOV from the beginning due to computational limitations. The required resolutions have been tested extensively to assure that the choice in pixel size does not affect the resulting model.

Figure 4.10: The phase function $P(\theta)$ as a function of scattering angle, $\theta$. Small particles diffract more signal at larger scattering angles. Large particles diffract the majority of the signal at small scattering angles. For each particle size, there is a minimum scattering angle for which the intensity of the diffracted light is constant. This angle is much smaller for large particles than for smaller particles.
4.5 Computational Models of the F ring

The observed signal (photon count rate) for an occultation of the rings has two components: the direct (though possibly attenuated due to ring particles) signal from the light source and the light diffracted into instrument’s FOV by ring particles. We model these two components separately and combine their effects to directly model the observed signal during solar occultations by Saturn’s F ring.

We begin by describing the models of the diffracted signal in Section 4.5.1 and then explain our method for modeling the attenuated signal in Section 4.5.2.

4.5.1 Model of the Diffracted Signal

We model the F ring assuming an identical particle size distribution across the entire ring. We follow Showalter et al. (1992); Albers et al. (2012) in defining an equivalent depth of the ring, $W_d$, by Equation 4.6

$$W_d = \int_{r_{\text{min}}}^{r_{\text{max}}} \tau(r)dr \approx \Delta r \sum_{i=r_{\text{min}}}^{r_{\text{max}}} \tau_i.$$  

where $r_{\text{max}}$ and $r_{\text{min}}$ are the maximum and minimum radial distances of the F ring from Saturn, respectively.

While there are variations in the optical depth of the F ring, the ring itself is not resolved in the solar occultations due to the large projected size of the Sun. We therefore assume a constant $\tau$ across the ring, simplifying Equation 4.6 to $W_d = \tau(r_{\text{max}} - r_{\text{min}}) = \tau W$. This can be somewhat of a limitation of the model; however, we do also explore the more realistic scenario of an F ring core embedded in a broader envelope, each of which with different particle size distributions and optical depths.

In our models, we are supplying a value of optical depth, $\tau$, for the ring. Here, we
assume that this optical depth is the total extinction of light by the particles, such that $Q_{\text{ext}} = Q_{\text{occ}} = 2$. Our model determines how much of that light is replaced by nearby particles by using Equation 3.21, which is then added to the direct, attenuated signal (Section 4.5.2). As a result, the values of $\tau$ discussed here are not necessarily equal to the optical depths that would be reported by UVIS. Deconstructing the true optical depth of the ring from UVIS solar occultations is complicated by the large angular size of the Sun, which helps to replace light and lower the value of $Q_{\text{occ}}$ to 1, but the small angular extent of the F ring means that the FOV is not filled by light-replacing ring particles. For a more detailed description, refer to Chapter 3 and Figure 3.4.

For each model, we provide the following inputs: the radial width of the ring, $W$, the mean normal optical depth across the ring, $\tau_n$, the minimum and maximum particle sizes of the size distribution, $a_{\text{min}}$ and $a_{\text{max}}$, respectively, and the index of the power-law size distribution $q$. These parameters are first discussed in Chapter 3.

### 4.5.1.1 Diffraction Calculations

To determine the amount of light diffracted by the ring particles at each time step, we sum the diffracted signal observed in each model FOV pixel.

In the Section 4.4.2.2 we described how we determine the angular separation between the pixel and the center of the Sun. The Sun, however, is not a point source. We must account for the angular separation of all parts of the Sun and the ring particles. Since the Sun is already discretized, as explained in 4.4.2.2, we must determine the scattering angle between each pixel with ring material and every pixel observing the Sun. We do this calculation for all the pixels containing ring-particles.

Because each pixel contains the same distribution of particle sizes and the same optical depth, the only variation in signal from pixel to pixel and within a single pixel is due to the scattering angle of that pixel with the Sun pixels. Therefore, we employ Equation 3.21 as a
function of $\theta$ for each ring pixel to each Sun pixel, and sum over all the pixels. We normalize both by the solid angle of the ring pixel and the percent area of the Sun represented by each Sun pixel.

### 4.5.1.2 FOV Sensitivity

As discussed in Chapter 2, the solar port FOV is not uniformly sensitive. As discussed throughout this chapter, the Sun is significantly offset from the boresight of the FOV on the Rev 9 occultation, meaning the intensity of the diffraction signal from ring particles near the Sun will also be affected. For consistency, we must also account for the sensitivity of the FOV in the other solar occultations as well. To do this we employ the FOV sensitivity model described in Table 2.2 in Chapter 2. This method provides a value indicating the percent of the nominal signal that is observed due to each pixel’s location in the FOV. We apply this percentage to the diffracted signal, which then reduces the intensity of the signal in pixels far from the center of the FOV. We apply this reduction in intensity due to FOV location before summing the pixels for the total diffracted signal.

We also apply this to the unocculted solar signal. As described in Chapter 2, the corrected pointing of the EUV Solar Port did not entirely align the pointing of the instrument, and so the center of the FOV is still slightly offset. This causes all of the solar occultations performed after the Rev 9 occultation to be slightly offset. We find the Sun’s location in the FOV and determine the percent decrease in intensity due to its position. We then assume that the true, unocculted solar signal from the Sun, $I_0$, is the observed unocculted signal divided by the percent decrease due to the Sun’s position in the FOV.

### 4.5.2 Model of the Direct Signal

The total signal observed during the solar occultations includes the diffracted light as well as the direct solar signal. When the rings are not occulting the Sun, the direct solar signal is $I_0$,
the unoccluded signal observed outside of the ring system. For a point source, like a distant star, the attenuated signal, $I$, observed as the ring occults the star would be calculated directly from Equation 3.7. However, for solar occultations the entire signal is not occulted at once. Therefore we must account for the direct, unoccluded signal of the Sun and the direct, attenuated signal from the Sun as it passes through the rings.

![Figure 4.11: Schematic of the modeled direct solar signal. The orange circle is the Sun, with an area $A_\odot$. The F ring is represented by the blue rectangle. The cross-section of the F ring and the Sun has an area $A_{\text{occ}}$. The total, direct model signal is obtained by combining the attenuated signal and the unoccluded signal of the Sun.](image)

We determine the area of the Sun covered by ring material based on the discretized Sun in the model; we divide the total area of the pixels containing ring material that overlap with the Sun, $A_{\text{occ}}$, by the area of the Sun, $A_\odot$ (Figure 4.11). We ignore limb darkening and assume the entire Sun emits the same signal at all parts of the Sun. Therefore, we can determine that the signal we would observe from the unoccluded part of the Sun is

$$I_{\text{unoccluded}} = I_0(1 - \frac{A_{\text{occ}}}{A_\odot}).$$  \hspace{1cm} (4.7)

In the case of an extremely optically thick ring that covers 10% of the Sun, this means that the observed signal is 90% the unoccluded solar signal. The F ring is not extremely optically
thick, so we must determine the attenuated sunlight that comes through the ring. Since we assume a single optical depth for this model, we solve Equation 3.7 for the area of the ring covered in ring material:

\[ I_{occluded} = I_0 e^{-\tau_n/\mu} \frac{A_{occ}}{A_\odot}. \]  

(4.8)

The total, direct model signal is

\[ I_{direct} = I_{occluded} + I_{unoccluded}. \]  

(4.9)

Figure 4.12: Determination of the attenuated signal solar signal. Because of the large angular size of the Sun in solar occultations, the F ring does not necessarily cover the entire Sun. We must calculate the area of the Sun blocked by particles and the attenuation of light in that area (blue curve) and combine that signal with the solar signal from the area not occulted by the ring (red curve). The combination of these signals makes the orange curve, which represents the modeled direct signal. In this example of the Rev 43 occultation, we model a 200-km ring with an optical depth \( \tau = .05 \) (\( W_d = 10 \) km).
4.5.3 Final Model Occultation Signal

Figure 4.13 shows the combination of the model attenuated signal and the model diffraction signal, for two different particle sizes, combine to create the model signal, which we then compare directly with the data.

![Image of Figure 4.13 showing the solar occultation model with two components: the modeled attenuated signal due to the F ring occulting the Sun (red ‘X’) and the modeled diffraction signal (green squares and blue stars). The model signal is the combination of these two components (solid green and solid blue lines). Here we show how the diffraction signal due to two different particle size regimes affect the resulting model of the solar occultation. Small particles diffract at larger angles, so when $a_{min} = 10\,\mu m$, the total signal is above the unocculted signal at larger radial distances (blue), but the central dip due to the attenuation of sunlight is not significantly reduced. For a population devoid of smaller particles, as is the case for $a_{min} = 500\,\mu m$, the observation of diffracted light is restricted to when the particles are very near to or directly in front of the Sun, significantly reducing the decrease in light due to the attenuation of sunlight by the ring particles. This example is of Rev 43 for a 500-km ring with an optical depth $\tau = 0.05$ ($W_d = 25$ km).

The total observed signal is the addition of the diffracted light and the direct (though possibly attenuated light) at each time step

$$I_{obs} = I_1 + I_{direct}.$$  \hspace{1cm} (4.10)
4.5.3.1 Combine Rings

In the beginning of this chapter, we describe the F ring with a dominant, \( \sim 50 \) km core, embedded in a larger envelope of dust particle. We model this scenario by creating independent models of the larger envelope and the core, then combine their signals. The amount of light observed due to diffraction is simply the summation of the diffracted signal from each of the rings. To model the combined optical depth, we multiply the \( I_{\text{direct}} \) for each ring. Then we combine the new direct signal and the new diffraction signal as is done in Equation 4.10 to obtain the total modeled signal for a model of the F ring core embedded in an envelope of particles with a different set of parameters. Figure 4.14 shows the results for such a model.

Figure 4.14: Solar occultation model of an F ring with a core embedded in a larger envelope of particles. Here the parameters of the core are: \( a_{\text{min}} = 100 \mu\text{m} \), \( \tau = 0.10 \), \( W = 50 \) km. The parameters of the envelope are: \( a_{\text{min}} = 10 \mu\text{m} \), \( \tau = 0.05 \), \( W = 500 \) km. For both rings, \( q = 4.0 \) and \( a_{\text{max}} = 1\text{mm} \). All of these parameters can be varied. As expected, embedding a narrow core decreases the model photon counts at the time that the core occults the Sun.
4.5.4 Rev 9 Solar Occultation Adjustments

The motivation behind this work is the detection of the diffraction signal observed during the misaligned Rev 9 solar occultation. Using the C-kernel with the corrected pointing (Chapter 2), we verify that the disk of the Sun is nearly entirely cut out of the FOV. However, we find that the decrease in the signal due to the Sun’s apparent position in the UVIS solar port FOV does not match the observed signal. We therefore make the reasonable assumption that the instrument pointing described in the reconstructed C-kernel is slightly offset from the true pointing. We describe how we adjust for this slight pointing offset below.

The observed solar signal near the time when the F ring occults the sun in the Rev 9 data is 2,000 photon counts per second. In all other solar occultations, the direct solar signal is measured to be between $8.0 \times 10^4 - 8.5 \times 10^4$ photon counts per second, which means that the solar signal in the Rev 9 occultation is reduced to $2.2 - 2.4\%$ its nominal signal strength. If we assume the pointing indicated by the C-kernel in conjunction with the analysis of the sensitivity of the solar port FOV described in Chapter 2, then the $[\Delta x, \Delta y]$ location of the Sun in the instrument frame of the FOV are $[-0.00429800, -0.01100224]$ and we would expect the observed Rev 9 solar signal to be $\sim 15\%$ the nominal value. Since we know the observed direct solar signal, $I_0 = 2,000$ counts/second, this would indicate a true solar signal $S_0$ of $1.3 \times 10^4$ counts/second, which is low. We assume the true solar signal during the Rev 9 occultation $S_0 = 8.5 \times 10^4$, similar to the direct signal measured for other solar occultations.

Adjustments to the $y$-direction pointing of the instrument result in a very slow decrease in signal because the fall-off of the signal itself is shallow (see Figure 2.24 in Chapter 2). In order to decrease the signal by another 1% we would have to adjust the pointing by a $\Delta y$ of 1 mrad. This is a much more significant offset in the C-kernel than expected. The more likely explanation for the decrease is due to a very small offset in the $\Delta x$, since the signal
drops off quickly with very small increases in $\Delta x$. In fact, along the $\Delta x$ direction, the fall-off in signal is accounted for by the percent of the Sun in the FOV. When 50% of the Sun is in the FOV (and $\Delta y = 0$), then the signal is reduced by 50%.

The $\Delta y$ position of the Sun in the FOV results in a signal $I_y = 0.67S_0$. In order to reduce $S_0$ to the observed $I_0 = 2000$ photon counts/second, the signal must be reduced to just $\sim 3.5\%$ of $0.67S_0$. This additional decrease in the signal is directly related to how much of the Sun falls within the bounds of the FOV. In our model, we assume that all parts of the Sun produce an equal amount of sunlight (we are ignoring any effects of limb-darkening). In order to produce the observed signal, we must place the Sun such that $\sim 3.5\%$ of the solar disk is within the FOV. We find that this requires the center of the Sun to be located at $[\Delta x, \Delta y] = [-0.004437, -0.01100424]$ (radians). This is an adjustment to the instrument pointing of $\Delta x = 0.139$ mrad, a reasonable offset for the UVIS pointing.

4.6 Results

4.6.1 Initial Model Runs

We begin by modeling each solar occultation using the parameters listed in Table 4.3. We use the results of these runs to guide the input parameters for subsequent models. Typically, we maintain the same values for $a_{min}$ and $q$, but change the values of the optical depth of the ring. We compare each model point directly the data point at the nearest radial location and use a least-squares fit to determine the goodness-of-fit for our models.

4.6.2 Ring Width

The models of the solar occultations place a strong lower bound on the radial extent of ring particles capable of attenuating the solar signal. We find that in all solar occultations, the
width of the F ring must be greater than 100 km. For nearly all of the occultations, models
with $W = 500$ provide the best fits to the data. In a few cases, a smaller ring (250 - 400
km) provided the best fit. Figure 4.15 shows examples of the attenuated signal only for a
few solar occultation models. The diffracted signal can only add more light, so these plots
show the lower limit for the radial extent of the total modeled signal. It is clear from the
figure that a smaller ring width does not attenuate the signal for enough time to replicate
the data. The majority of the best-fit models require a ring width of at least 500 km.

This finding is consistent with ISS images (Murray et al., 2008) and UVIS stellar occul-
tation data (Albers et al., 2012), that show some structure in the rings (mainly the strands)
that extend out to $\pm 250$ km radially from the core. It is important to note, however, that
here we model this large region with a constant optical depth, rather than as individual
strands with higher optical depths. This result is even more in agreement with French et al.
(2012) who find the radial extent of the ring to be $580 \pm 70$ km.

4.6.3 Optical Depth

We analyze all of the solar occultations assuming a ring width of 500 km and a normal
optical depth of 0.05. For particle size distributions for Rev 43 and Rev 9, this optical depth
is appropriate. However, for the majority of the occultations, we find it be too high of an
optical depth. Values of $\tau_n = 0.1$ are too high for any model to reproduce the observed light
curve, and therefore is an upper limit on the mean normal optical depth of this region. For
many of the other observations, an optical depth of 0.035 is suitable, however this optical
depth still appears to be too high for some of the occultations. Therefore we can conclude
that the mean optical depth in this region is indeed variable and must be considered a free
parameter in our models.
Table 4.2: Table of F Ring Orbital Parameters. Values listed are from (Albers et al., 2012) Table 3. $a_{sm}$ is the semi-major axis, $e$ is the eccentricity, $i$ is the inclination, $\Omega$ is the longitude of the ascending node, $\tilde{\omega}$ is the longitude of pericenter, $\dot{\Omega}$ is the regression rate and $\dot{\tilde{\omega}}$ is the procession rate of the F ring.

<table>
<thead>
<tr>
<th>$a_{sm}$ (km)</th>
<th>$e$</th>
<th>$i$ (deg)</th>
<th>$\Omega$ (deg)</th>
<th>$\tilde{\omega}$ (deg)</th>
<th>$\dot{\Omega}$ (deg day$^{-1}$)</th>
<th>$\dot{\tilde{\omega}}$ (deg day$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140221.3</td>
<td>2.35 $\times$ 10$^{-3}$</td>
<td>6.43 $\times$ 10$^{-3}$</td>
<td>15.0</td>
<td>24.2</td>
<td>-2.6877</td>
<td>2.7025</td>
</tr>
</tbody>
</table>

Table 4.3: Initial parameters used to model the solar occultation data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring Widths, W (km)</td>
<td>100, 250, 500</td>
</tr>
<tr>
<td>Optical Depth, $\tau$</td>
<td>0.035, 0.05</td>
</tr>
<tr>
<td>Minimum Particle Size, $a_{min}$ ($\mu$m)</td>
<td>1, 5, 10, 50, 100, 500</td>
</tr>
<tr>
<td>Power-law Index, $q$</td>
<td>2.8, 3.2, 3.5, 4.0, 5.0</td>
</tr>
</tbody>
</table>

Figure 4.15: The modeled attenuated signal only for ring widths equal to 100 km (orange circles) and 500 km (teal ‘X’s) for $\tau = 0.05$. The total observed signal will change the shape of the signal, particularly by making the dip more shallow due to diffracted light. We do not try to match the attenuation dip, but rather plot these rings to show that a ring with a width of 100 km produces a signal that is too skinny to fit the data. A larger ring, often with $W = 500$ km, creates a signal that aligns with the timing of the signal dip due to attenuated signal.
4.6.4 Variations in the Minimum Particle Size

Figure 4.16: (a) Two models for the solar occultation during Rev 43 with $\tau = 0.05, W = 500$ km, $q = 3.5$. The magenta triangles are from a model with $a_{\text{min}} = 5\mu$m and the blue circles are from a model with $a_{\text{min}} = 500\mu$m. (b) Two models of Rev 55I with $\tau = 0.035, W = 500$ km, $q = 3.5$. As in (a), magenta triangles are from a model with $a_{\text{min}} = 5\mu$m and the blue circles are from a model with $a_{\text{min}} = 500\mu$m. Small particles are needed to match the deep dip of the light curve in (a) and to match the signal above $I_0$. We do note the diffracted signal is apparent on only one side of the ring, perhaps suggesting an asymmetry in the spread of the small particles. In (b), small particles produce a dip in the light curve that is too deep to match the data.

Changes in the minimum particle size distribution of the model has the following effects: models with a large minimum particle size ($a_{\text{min}} = 500$ mm) produce a light curve with a shallower dip and little or no signal above the unocculted solar signal while models with minimum particle sizes of $a_{\text{min}} = 5 - 100\mu$m produce light curves with a deeper dip, often with signal above the unocculted solar value on either side of the dip. This difference is seen in Figure 4.13. The larger particles are diffracting light at smaller angles but with a higher intensity, essentially ‘filling in’ the light removed by the particles blocking the Sun during the occultation. The smaller particles diffract less light, but that light is spread over larger angles, so the signal can be seen before and after the ring begins to occult Sun, but does
not replace as much of the attenuated light as the larger particles do. The differences are significant, and can be used to determine if there is a significant population of small particles or not during a particular solar occultation.

Figure 4.16 shows two model light curves for Rev 43 and Rev 55I solar occultations. The figure shows that for Rev 43, a minimum particle size of 500µm does not fit the data but a minimum particle size of \( a_{min} = 5\mu m \) is a much better fit. A minimum particle size of 500µm, however, is a much better fit for the Rev 55I solar occultation. We see a range in best-fit \( a_{min} \) across all the solar occultations, indicating that this 500-km region that envelops the F ring core is also quite variable. Not only does the optical depth vary, but the size distribution and the minimum particle size change as well.

### 4.6.5 Best-Fit Models and Comparisons with ISS Images

We compare our best-fit models and our analysis of the UVIS solar occultation data with the ISS mosaics of the F ring. We look at the mosaics imaged near in time and longitude to the solar occultations. We indicate the longitude of the F ring in the ISS images with a red vertical line in the mosaics. We present the best-fit model, solar occultation data, and the ISS mosaic of the F ring. Figures 4.17 and 4.18 display all occultations during which we detected diffraction in the UVIS solar occultation data. Figures 4.19 and 4.20 display the occultation data in which no obvious signature of diffraction was detected.

The mosaics of the ISS images in these figures clearly demonstrate the complexity and variability of the F ring core. Most notably, the ISS mosaics in Figures 4.17 and 4.18 appear to display small and large features, identified as collisional features, near the longitude of the solar occultation observations. The most obvious of these can be seen in the Rev 43 and Rev 9 mosaics. In Figures 4.19 and 4.20, however, the longitude of the solar occultations appear to correspond with more quiescent regions of the F ring. These observations are consistent: collisional event in the F ring can release a population of smaller particles,
capable of diffracting a detectable signal in the solar occultation data.

In the first column of these figures, we show the solar occultation data with our best-fit $q$ for a range of minimum particle sizes. When determining the best-fit, we look both at the model’s ability to match the magnitude of the light curve dip as well as match the diffraction signature (if present). We note that many of the occultations with diffraction signatures (such as Rev 43 and Rev 55E) display diffraction signatures on only one side of the ring, suggesting an asymmetry in the spread of the small particles. Our models produce diffraction on both sides of the core since we consider an isotropic ring.

In Table 4.4 we list our best-fit parameters for each solar occultation. We find a range of best-fit minimum particle sizes throughout the occultation data. We find that occultations with obvious diffraction signatures require a smaller ($\leq 50\mu m$) $a_{min}$ than those occultations during which the there is no obvious diffraction signature. We also find that the corresponding ISS images show a collisional feature in the vicinity of the solar occultation longitude in occultations where we measure small particles. We identify which ISS mosaics display collisional features near the occultation longitude and which do not in Table 4.4. We also list which occultations contain diffraction signatures in the UVIS data.
Table 4.4: The best-fit parameters for the particle size distribution, ring width, and optical depth of the F ring for each of the modeled solar occultations. We also include whether we there is a clear diffraction signature in the UVIS data and if there is an identifiable collisional feature in the ISS mosaics. We highlight in green the observations that are consistent across the following columns: small minimum particle size, a UVIS detection of diffracted light, and a collisional feature OR larger minimum particle size, no UVIS detection of diffracted light, and no collisional feature in the ISS images. If one of these three criteria do not meet, or the detection of diffracted light is ambiguous, we highlight that column in yellow. *We find that none of our current models replicate both the light curve and diffraction signature observed in Rev 62 (E).

<table>
<thead>
<tr>
<th>Occultation</th>
<th>Width</th>
<th>$\tau_n$</th>
<th>$a_{min}$ ($\mu$m)</th>
<th>$q$</th>
<th>Diffraction</th>
<th>Collisional Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rev 9</td>
<td>250 km</td>
<td>0.05</td>
<td>5 - 10</td>
<td>3.5</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Rev 43</td>
<td>500 km</td>
<td>0.05</td>
<td>5 - 10</td>
<td>3.2</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Rev 55 (I)</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 500$</td>
<td>any</td>
<td>maybe</td>
<td>old collision</td>
</tr>
<tr>
<td>Rev 55 (E)</td>
<td>500 km</td>
<td>0.030</td>
<td>5 - 10</td>
<td>3.2</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Rev 59</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 100$</td>
<td>any</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Rev 62 (I)</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 500$</td>
<td>any</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Rev 62 (E) *</td>
<td>500 km</td>
<td>0.030</td>
<td>-</td>
<td>-</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Rev 65 (I)</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 500$</td>
<td>any</td>
<td>maybe</td>
<td>no</td>
</tr>
<tr>
<td>Rev 65 (E)</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 50$</td>
<td>any</td>
<td>maybe</td>
<td>maybe</td>
</tr>
<tr>
<td>Rev 66 (I)</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 500$</td>
<td>any</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Rev 66 (E)</td>
<td>500 km</td>
<td>0.030</td>
<td>$\geq 100$</td>
<td>any</td>
<td>maybe</td>
<td>yes</td>
</tr>
</tbody>
</table>

4.7 Conclusions and Discussion

Through our analysis and modeling of solar occultations by the F ring, we find that there is a region of small (sub-centimeter) particles that extends at least 500 km, encompassing the core, in nearly every observation, in agreement with the analysis by French et al. (2012). We use this constraint to model a ring with varying optical depths and particle size distributions. We find that we must have a population a particles $a_{min} \leq 50\mu$m in order to reproduce any solar occultation that has a diffraction signature. The values for $q$ vary from 2.8 - 3.5
to produce the best-fit light curves for these occultations. We do note that our models produce diffraction signatures before and after the occultation, whereas the data often only show diffraction on one side. This may indicate that the small particles are distributed asymmetrically about the core. For solar occultations without any diffraction, the best-fit models for the light curves have minimum particle sizes $a_{min} \geq 100 \mu m$. Since the maximum particle size modeled here is $a_{max} = 1 \text{ mm}$, variations in $q$ do not affect the models with large $a_{min}$ values.

We find that in almost all observations for which the best-fit minimum particle size is less than 100 microns, there is a collisional feature detected at that longitude in the ISS F ring mosaics. Accordingly, all observations in which we rule out a population of particles less than 100 microns correspond to a location in the F ring without any significant collisional features in the ISS mosaics.

There are several free parameters in our model that are closely related. For example, changing the optical depth will decrease the light curve dip observed in the F ring. However, decreasing the optical depth will also affect the strength of the diffraction signal above the unocculted signal. While there may be many fits to the same data, the ability to match both the light curve and create (or not create) a diffraction signal outside of the rings constrains $\tau$ and $a_{min}$. More work is needed, however, to fully characterize the combination of parameters that provide good fits to the model. We must explore a larger parameter space in $\tau_n$ and ring width, in particular.

We also discuss the possibility of adding an F ring core to the model. As shown, this will generally further deepen the dip in the light curve. As a result, slightly larger particles (and perhaps a slightly lower optical depth) in the envelope would be needed to still fit the data. Even in this scenario, we would still require small material to produce the diffraction signatures exterior to the ring, so adding a core would not qualitatively change our results. It is important to note that when modeling the core, we will need to consider the contributions...
of multiple diffraction due to the geometry of the F ring solar occultations. The low $B$ angles observed in many of these solar occultations greatly increases the observed optical depth, so for a core with even moderate optical depth ($\tau_n > 0.075$), multiple diffraction may contribute to the signal.

The consistency between the ISS images, the UVIS occultation data, and the computational models strongly suggest that the F ring envelope has a varying particle size distribution in which the smallest particles are replenished after observable collisional events occur in the F ring core. The strongest case for this scenario is the Rev 43 solar occultation, during which a prominent diffraction signal is observed in the UVIS data. In order to fit the Rev 43 light curve, we must use a ring of higher optical depth than in most other occultation models ($\tau_n = 0.05$) and a minimum particle size on the order of $a_{\text{min}} = 5\mu m$. The ISS images show that a highly disturbed region of the F ring, likely created by a large collisional event (Murray et al., 2008; French et al., 2012), occulted the Sun during Rev 43, lending further credence to this concept. The overall brightening of the ring that occurred as a result (French et al., 2012) may be related to the higher optical depth of the F ring envelope we measure during the Rev 43 solar occultation. We also note that the Rev 9 occultation requires a higher optical depth as well and point to the relatively large collisional event observed in the ISS mosaics as well. However, we find that the ring cannot be as large as 500 km in this observation. This may be a result of our adjustments of the Sun on the FOV and will require further work to understand.

The particle size distribution described in this work is consistent with those determined through analyses of Cassini VIMS observations by Vahidinia et al. (2011) and Hedman et al. (2011), though no temporal or longitudinal variation in the sizes are observed in these works. Hedman et al. (2011) report a broken power-law size distribution, centered on $a_{\text{min}} = 10\mu m$. A very small population of smaller particles than $10\mu m$ is not inconsistent with this work; $1-\mu m$ sized particles do not affect the model significantly, especially if they follow a separate,
additional power law distribution. Some more work is needed to explain why Hedman et al. (2011) always see a population of 10µm particles whereas we find that this population exists only near collisional events. We do note, however, that we do not see spectral variations in the solar spectrum at the F ring, in agreement with (Bosh et al., 2002). This rules out any significant population of sub-micron sized dust.
Figure 4.17: In the first column we present our best-fit models for all solar occultations during which we detected a diffraction signature in the UVIS data. In the second column we compare with the ISS mosaic of the F ring as near in time to the solar occultation as available. The time proximity is written at the top, where a negative time indicates the images were taken before the solar occultation. The red solid line in the images indicates the longitude of the F ring during the solar occultation. We find that at the location and time of each of the solar occultations, there appears to be a collisional feature in the occultation data.
Figure 4.18: (cont from Figure 4.17) In the first column we present our best-fit models for all solar occultations during which we detected a diffraction signature in the UVIS data. In the second column we compare with the ISS mosaic of the F ring as near in time to the solar occultation as available. The time proximity is written at the top, where a negative time indicates the images were taken before the solar occultation. The red solid line in the images indicates the longitude of the F ring during the solar occultation. We find that at the location and time of each of the solar occultations, there appears to be a collisional feature in the occultation data.
Figure 4.19: In the first column, we present our best-fit models for all solar occultations during which we did not detect a diffraction signature in the UVIS data. In the second column we compare with the ISS mosaic of the F ring as near in time to the solar occultation as available. The time proximity is written at the top, where a negative time indicates the images were taken before the solar occultation. The red solid line in the images indicates the longitude of the F ring during the solar occultation. We find that at the ring longitudes of these solar occultations, the F ring is generally more quiescent, with no obvious collision features.
Figure 4.20: (continued from Figure 4.19) In the first column, we present our best-fit models for all solar occultations during which we did not detect a diffraction signature in the UVIS data. In the second column we compare with the ISS mosaic of the F ring as near in time to the solar occultation as available. The time proximity is written at the top, where a negative time indicates the images were taken before the solar occultation. The red solid line in the images indicates the longitude of the F ring during the solar occultation. We find that at the ring longitudes of these solar occultations, the F ring is generally more quiescent, with no obvious collision features.
CHAPTER 5: SATURN’S A RING

We now focus our analysis on Saturn’s A ring. Applying a similar technique to that described in Chapter 4, we use HSP stellar occultation data to measure and model diffracted light near the sharp edges within Saturn’s A ring in order to constrain the particle size distribution in this region of Saturn’s rings. The bulk of this chapter has been published in Becker et al. (2015).

We analyze occultations in which we detect diffraction spikes at the sharp edges in Saturn’s A ring. These spikes occur during stellar occultations when the star is completely unocculted but near enough to a ring edge that the ring particles diffract starlight into the instrument field of view, augmenting the direct stellar signal. Figure 5.1 shows a clear example of a diffraction spike detected at the outer edge of the A ring during an occultation of β Centauri on Cassini’s 104th revolution around Saturn. Well beyond the ring edge, the photon count rate remains constant; we only observe direct, unocculted starlight. As the angular separation between the star and the ring edge decreases, particles in the rings diffract the light at an angle such that the detector collects the unocculted stellar signal plus the additional, diffracted light. Smaller particles diffract light at larger angles so the radial extent of the diffraction signal depends on the population of the smallest particles, while the strength and shape of the signals depend on the overall particle size distribution (see Chapter 3).

The incoherent nature of the stellar signal eliminates the ability to distinguish the direct starlight from the diffracted signal during the rest of the occultation when the star is behind the rings as seen from Cassini. In a ring gap or outside of the ring system, however, any signal that surpasses that of the unocculted stellar flux must be due to diffracted light. Thus these spikes provide a clear signal from diffracted light only, which we can use to tease out
the particle size distribution near the ring edge.

Figure 5.1: Stellar occultation of $\beta$ Centauri by Saturn’s A ring edge during Cassini Rev 104. The photon count rate within the ring (at a radial distance from Saturn of $\sim 136,770$ km) is low due to the ring particles occulting the starlight. Beyond the ring edge ($\sim 136,775$ km $+$), we measure the direct, unocculted stellar signal. The horizontal line indicates the mean stellar signal. Near the ring edge there is a clear increase in photon counts. This is the diffraction spike, which is the combination of the unocculted, direct stellar signal and the starlight diffracted into the detector by small particles in Saturn’s A ring.

We model the particle size distribution by a power-law defined as (Cuzzi et al., 2009)

$$n(a)da = Ca^{-q}da, a_{min} \leq a \leq a_{max}$$  \hspace{1cm} (5.1)

where $a$ is the radius of the particle, $n(a)$ is the number of particles with radius in the range $[a, a+da]$, $da$ is an infinitesimal increment in $a$, $C$ is a constant, $q$ is the power-law index, and $a_{min}$ and $a_{max}$ are the minimum and maximum particle sizes in the distribution, respectively.

Previous studies of the particle size distribution of Saturn’s A ring from the analysis of diffracted light were completed by Zebker et al. (1985), French and Nicholson (2000), Marouf et al. (2008), and Harbison et al. (2013). A thorough summary of studies conducted before 2010 can be found in Cuzzi et al. (2009). Zebker et al. (1985) inverted the near-forward scattered signal from the Voyager Radio Science Subsystem (RSS), comparing and modeling the X-band (3.6 cm) and S-band (13 cm) signals for particle sizes ranging from 1 millimeter
to 8.9 meters in the outermost part of the A ring and 1 millimeter to 11.2 meters in the A ring region interior to the Encke Gap. They find $q = 3.03$ in the outer A ring and $q = 2.93$ in the inner A ring. Using a similar technique, Marouf et al. (2008) analyzed the differential optical depths of the three wavelengths transmitted during the Cassini RSS ring occultations, which includes the Ka-band (0.94 cm) in addition to the X- and S-bands. As described by Cuzzi et al. (2009), the results indicate a sharp size distribution cut-off $a_{\text{min}} \sim 5$ mm for $q = 3.2$ in the outer A ring. Their findings also suggest either a depletion of particles smaller than 50 cm or $q < 2.8$ for the inner A ring. French and Nicholson (2000) used the Voyager photopolarimeter (PPS) stellar occultation data in conjunction with ground-based observations of the stellar occultation of 28 Sagittarius. They implemented the optical depth profile determined by the PPS data to separate the diffraction signal from the attenuated, direct starlight observed in the 28 Sgr occultation. Using data at the wavelengths of 0.9 $\mu$m, 2.1 $\mu$m, and 3.9 $\mu$m, they derived power-law size distributions with indices $q = 2.75$ and $q = 2.9$ for particles with radii from 30 cm-20 m and 1 cm-20 m in Saturn’s inner and outer A ring, respectively. More recently, Harbison et al. (2013) measured the diffracted signal from solar occultations observed by the Visual and Infrared Mapping Spectrometer (VIMS) on Cassini and found a population of sub-millimeter particles in the A ring interior to the Encke Gap. For the assumed value of $q = 2.75$ they find $a_{\text{min}} < 0.3$ mm and for $q = 2.9$ they determine $a_{\text{min}} = 0.56^{+0.35}_{-0.16}$ mm.

The UVIS HSP has a wavelength bandpass of 110-190 nm, a factor of $\sim 6$ times shorter than the smallest wavelength utilized in previous studies. Due to the diffraction lobe’s dependence on wavelength and particle size, these shorter wavelengths make the HSP observations more sensitive to diffraction by particles as small as a few tens of microns.

We have developed a forward modeling technique that reconstructs the geometry for each stellar occultation as the star approaches the outer edge of the A ring and the inner and outer edges of the Encke Gap. We do not include the edges of the Keeler Gap in this
analysis because of the complications at these edges due to instrumental effects that we elaborate on in Section 5.2.1. The model produces a synthetic diffraction signal consisting of the direct stellar signal and the light diffracted into the instrument’s field of view (FOV) by particles with a size distribution given by Equation 5.1. By comparing the model to the data, we measure centimeter and sub-centimeter particles in Saturn’s A ring. The extensive collection of stellar occultations spanning a decade enables us to probe azimuthal and temporal variations in the particle size distribution.

In this chapter, we describe how we apply the diffraction theory discussed in Chapter 3 in Section 5.1. In Section 5.2 we describe the observations chosen for the analysis. Section 5.3 discusses the model developed and used for this work. We present the results of our analysis in Section 5.4, followed by the preliminary results of a study of the B ring outer edge in Section 5.5. We close with a discussion of our analysis’ implications in Section 5.6.

5.1 Theory

To determine the particle size distribution of the rings near the sharp edges that produce diffraction spikes, we follow the theoretical description provided in Chapter 3. We model the ratio of the total signal near the ring edges (which contain the unocculted stellar signal \( I_0 \) and the diffracted light) to the unocculted stellar signal far from the ring edge following Equation 3.4. We measure the signal as \( I/F \), the ratio of the intensity, \( I \), and the incident solar flux density \( \pi F \) (French et al., 2012). We normalize the signal to the average, unocculted value of the stellar signal. We determine the optical depth of the A ring from the stellar occultation data being modeled and use that as the input for the slant optical depth of the ring at the time of the observation. Because we are analyzing the ring edge just as the ring begins to occult the star, approximately half of the FOV is filled by ring particles (assuming the star is centered in the FOV). As described in Chapter 3, the ring particles at the very edge fill
only half of the region of diffraction (see Figure 3.3). This reduces the value of $Q_{occ}(\lambda, a, f)$ to approximately 1.5 for all particles near the edge of the ring. We assume this constant value for $Q_{occ}$ for all ring-edge models.

5.2 Observations

The HSP’s fast sampling interval of 1-2 milliseconds gives radial spatial resolution of $\sim 10$ meters for typical observation geometries (Esposito et al., 2004; Colwell et al., 2007, 2010), making the HSP occultation data particularly useful for analyzing the fine structure of Saturn’s rings. Details of the data processing of the occultations are provided by Colwell et al. (2007) and Colwell et al. (2010). The HSP has observed more than 180 stellar occultations, though we focus only on the subset of those occultations in which the diffraction signals are detectable (Section 5.2.1). Several geometric and instrumental effects can prevent the detection of the diffraction signal at a ring edge.

5.2.1 Selection Effects

5.2.1.1 HSP Ramp-Up

As described in Chapter 2, the HSP has a ramp-up response to bright stellar signals (Colwell et al., 2007, 2010), which means the instrument requires extended exposure to the light source before the sensitivity of the instrument stabilizes. For each stellar occultation, we remove the ramp-up using the technique described in Chapter 2. We implement the ramp-up removal so that we can find the baseline for the unocculted stellar signal and can therefore measure the ratio of the intensity of the diffraction signal to the unocculted stellar signal as seen in the gap.

The shape of the ramp-up often suppressed the visibility of a diffraction signal at the beginning of the ramp-up, but does not strongly affect the detectability of that at the end.
of the ramp-up. In some occultations, the suppressed diffraction signal is still detectable, but the signal still lies well below the true unocculted stellar signal (Figure 5.2). Due to the complexity of the ramp-up when the star first becomes unocculted in conjunction with the inability to know the shape of the diffraction curve \textit{a priori}, we discard occultations at these edges where the instrument is beginning its ramp-up. This results in the use of only ingress occultations for the outer edge of the A ring and the inner edge of the Encke Gap and only egress occultations for the outer edge of the Encke Gap so that the instrument has had time to stabilize before reaching the edge being analyzed. For ingress occultations of the outer edge of the A ring, the instrument has typically been exposed to the direct stellar signal for a long period of time prior to the occultation. This enables the HSP sensitivity to stabilize, rendering the ramp-up effect negligible. We therefore do not remove the ramp-up at the outer edge of the A ring.

![Figure 5.2: Stellar occultation of BetCen105E by the outer edge of the A ring. The ramp-up effect produces the shape of the signal beginning at the edge of the ring. Despite the ramp-up, a diffraction signal is still detectable. Because we do not know the shape of the ramp-up without the diffraction signal, we cannot adequately remove the signal at an edge such as this one to measure the diffraction signal without high uncertainties. We therefore only analyze ingress occultations for the outer edge of the A ring and the inner edge of the Encke Gap and only egress occultations for the outer edge of the Encke Gap.](image)

We also find diffraction spikes near the sharp edges that define the Keeler Gap; however,
the small size of the gap (∼ 35 km) does not provide enough time for the HSP sensitivity to stabilize. The result is that both edges are affected by the ramp-up. Figure 5.3 shows the outer part of the A ring for the ingress β Centauri Rev 104 occultation. The signal in the Keeler Gap, a combination of diffracted and direct starlight, fall below the unocculted signal observed beyond the outer edge of the ring. The observations in the Keeler Gap are further complicated by the narrowness of the gap, which enables large diffraction signals from each edge to overlap. It is difficult to distinguish the diffracted light from the direct signal due to the complex ramp-up in the gap, therefore we do not attempt to model any of the diffraction signals in the Keeler Gap for this analysis. This data could be analyzed for future work, however. We could apply the model of the HSP ramp-up fit to the Encke Gap to the Keeler Gap to remove the signal, as shown in Figure 5.4. While this method can be used to remove the ramp-up, the true signal within the gap will still be unknown. Because of the narrowness of the Keeler Gap, if particles are diffracting a significant amount of light from both edges, the signal across the gap may always include diffracted light. Therefore the minimum signal in the Keeler Gap may be above the unocculted stellar signal, but this cannot be known a priori. This unknown would need to be considered in the modeling process. It would be of interest to measure the particle size distribution at the Keeler Gap edges and any variations associated with the effects of Daphnis.

5.2.1.2 Star Brightness and Geometry

In addition to the ramp-up effect, ring edge diffraction signals are typically undetectable in the occultations of dim stars. The strongest diffraction spikes create a ∼ 6% increase in the unocculted stellar signal; the noise in observations of dim stars is comparable to the increase in counts due to any diffracted light. Because of the narrow width of the diffraction spikes in the HSP data, the low amplitude of the diffracted signal makes it difficult to separate the diffraction signal from the noise in the occultations of faint stars.
Figure 5.3: Ingress stellar occultation of β Centauri Rev 104 by Saturn’s A ring. The horizontal line indicates the average, unocculted stellar signal. The diffraction signal at the A ring edge is clearly observed; however, any diffraction signals at the edges of the Keeler Gap are distorted by the instrument ramp-up effect (see text). The narrow gap does not provide enough time for the instrument to reach maximum sensitivity, so almost no signal in the Keeler Gap reaches the unocculted stellar signal.

Figure 5.4: Possible technique to remove the ramp-up effect in the Keeler Gap applied to data from the stellar occultation of BetCen104I. We apply the model of the ramp-up effect in the Encke Gap from the same occultation to the ramp-up in the Keeler Gap to remove its effects. This could be one way to improve the data for further particle-size analyses.

The geometry of the observation also plays an important role in determining if the diffraction signal is detectable. The diffraction spikes are clearly observed when the projection of the edge as seen along the line-of-sight to the star is sharp. Due to the geometry of some
occultations, the ring edge does not appear sharp, causing a slower progression from the
fully occulted to unocculted signal. The sharpness of the edge depends on two angles: the
ring opening angle $B$ and the azimuthal angle $\phi$. $B$ is the angle between the ring plane
and the line-of-sight vector from the spacecraft to the star. An observation with a $B = \pi/2$
has a line-of-sight vector orthogonal to the ring plane. The azimuthal angle, $\phi$, is the angle
measured from the radius vector to the projection of the line-of-sight onto the ring plane
(see Jerousek et al. (2011), Figure 6). An occultation for which $\phi$ is 0 or $\pi$ is an observation
where the line of sight is parallel with a radial vector in the ring plane, while $\phi = \pi/2$ is an
observation with the line of sight tangent to the ring at the occultation point. An azimuthal
angle of $\pi/2$ would make the rings appear sharp, while an angle at 0 radians would stare into
the edge and would expose a signature of the thickness of the rings. We follow Jerousek
et al. (2011) in combining the angles $B$ and $\phi$ to find the angle $\alpha$, which we use to quantify
how sharp the ring edge is for the geometry of each occultation

$$\tan \alpha = \frac{\tan |B|}{\cos |\phi|}. \tag{5.2}$$

Occultations with high values of $\alpha$ indicate observations where the ring edge will appear
sharp, increasing the chance of detecting a diffraction spike.

Figure 5.5 shows the rate of detection of a diffraction spike for occultations accounting for
the discussed biases. The occultations included are of bright stars (stars with an unocculted
photon count rate of $I_0 \geq 100$), have high $\alpha$ ($\geq 60^\circ$), and are ingress for the A ring outer edge
and Encke Gap inner edge occultations and egress for Encke Gap outer edge occultations.
When considering the selection effects, we find that nearly all the occultations of the A
ring outer edge display clear diffraction spikes. Some occultations of the Encke Gap edges,
however, meet the criteria of a bright star at a high ring-opening angle and still lack a
detectable diffraction signal. We find this particularly true of the inner edge of the Encke
Gap. Since the extent of the diffraction signal depends on the population of the smallest particles, and we find no geometric or instrumental effects that could be hiding the signal, these non-detections at the inner Encke Gap edge suggest a smaller population of sub-centimeter particles. We further discuss this conclusion in Section 5.4.

Figure 5.5: Detection of diffraction spikes for each ring edge. The gray (top bar) shows the total number of occultations of each edge that meet the criteria of 1) high star brightness \( I_0 \geq 100 \), 2) high \( \alpha \) angle \( (> 60^\circ) \) and 3) are ingress for the A ring outer edge and the inner edge of the Encke Gap and egress for the outer edge of the Encke Gap. Yellow (middle bar) represents the number of those occultations in which at least a weak or noisy diffraction signal was detected. Blue (bottom bar) represents the number of occultations of each edge that display a strong diffraction signal. A high percentage of the occultations of the A ring outer edge result in a clear diffraction signal, while the Encke Gap inner edge has a much lower rate of diffraction signal detection, even after accounting for geometric and instrumental effects.

We attempt to model nearly all of the edges that meet the star brightness and geometric requirements; however, some data in which a detection is noted are still not ideal for modeling. This is often due to poor signal to noise (lower values of \( I_0 \)) or complex edges (Section 5.2.2). We do include occultations that do not meet the diffraction requirements and still show a strong diffraction spike. For example, many occultations of dim stars result in a very
clear diffraction spike. We model those data and include them in our analysis.

5.2.1.3 Keeler Gap

As noted in the previous section, in this study we do not analyze the edges of the Keeler Gap due to the complexity of the signal from the ramp-up effect in the narrow gap. However, we do note that, like the A ring edge, there is a very high detection rate of diffraction at the Keeler Gap edges when $B > 60^\circ$ (Figure 5.6).

![Figure 5.6: (a) The number of detections and non-detections of diffraction at the inner edge of the Keeler Gap as a function of ring-opening angle $B$. (b) The number of detections and non-detections of diffraction at the outer edge of the Keeler Gap as a function of ring-opening angle $B$. For the optimal viewing geometry (high $B$ angles), nearly all occultations by these edges reveal diffraction signatures, suggestive of a significant, consistent population of small particles.](image)

5.2.2 Complex Edges

Several occultations by ring edges reveal a complicated structure within the diffracted signal rather than the characteristic diffraction spike. Figure 5.7 shows an occultation of $\delta$ Centauri Rev 66 by the outer A ring edge where the diffraction spike does not conform to the typical,
smooth signal observed at the majority of the ring edges. The strength of the high-frequency noise is comparable in size to the diffraction signal. We look for structure near the ring edge that could explain the complex signals but do not find an obvious explanation. Complex edges are most frequently seen in occultations of stars with lower signal to noise ratios. We include such edges in our list of diffraction detections, though we do not attempt to model edges with this level of complexity.

Figure 5.7: Ingress occultation by Saturn’s outer A ring of δ Centauri Rev 66. There is a consistently high signal near the ring edge, suggesting a diffraction spike; however, the signal does not match the characteristic diffraction signal. There is structure at the same level as the diffraction spike that is not likely due only to diffracted light.

5.3 Analysis

5.3.1 Occultation Model

The explicit dependence of the diffraction signal on the particle size enables us to model the observed diffracted signal using a forward-modeling technique with two parameters:
the power-law index $q$ and the minimum particle radius $a_{min}$ of the size distribution. We constrain the model wavelength to 150 nm, the center of the UVIS HSP bandpass. Our approach to modeling the diffraction signal is to calculate the intensity of the diffracted light across the instrument’s FOV at different times, and therefore at different angular separations between the star and the ring edge. We run a suite of models for each stellar occultation, varying the two parameters, and compare the model diffraction curve with the data. We vary $a_{min}$ from 0.1 mm or 0.5 mm-20 mm and $q$ from 2.6-3.6 for the A ring outer edge, $a_{min}$ from 0.5 mm-30 mm and $q$ from 2.6-3.5 for the Encke Gap outer edge, and $a_{min}$ between 0.5 mm-30 mm and $q$ from 2.4-3.5 for the Encke Gap inner edge. At the farthest observation distances of Cassini during these occultations ($\sim$ 1 million km), 1-mm particles would produce a diffraction signal detectable up to 75 km from that particle. Therefore, we are typically measuring the size distribution of a region < 100 km from each ring edge, although the majority of the signal will be diffracted by ring particles 5-10 km adjacent to the edge.

We set $a_{max} = 8.9$ m (Zebker et al., 1985) for all models. As noted by Harbison et al. (2013) and Cuzzi et al. (2009), diffraction signals at these wavelengths are not very sensitive to $a_{max}$. Because each cross-sectional area of the rings is dominated by small particles for values of $q \geq 3$, small changes to $a_{max}$ only shift the distribution of particles slightly. A large increase in $a_{max}$ would require a smaller $a_{min}$ to produce the same diffracted signal for a given value of $q$, but for the current ranges of $a_{max}$, these variations are within the errors of our results.

Our model assumes a single-scattering classical ring model of a loosely-packed, extended layer many-particles-thick (Cuzzi et al., 2009). Multiple scattering in the rings would act to broaden the angular distribution of scattered light (Zebker et al., 1985), so a larger particle could appear, through the detected signal of subsequent diffractions, to be a smaller particle. Therefore, our results may be an underestimate of $a_{min}$, but provide a strong lower limit.
5.3.1.1 Occultation Geometry

We use the Navigation and Ancillary Information Facility (NAIF) SPICE toolkit to determine the geometry, instrument pointing, and line-of-sight distance of the spacecraft for each occultation. We model all of the stars as point-sources located at the boresight of the FOV. Among the stars we observe for this analysis, $\sigma$ Sagittarius has the largest angular diameter, with $\theta \sim 0.003$ microradians. This is significantly smaller than the spatial resolution of the data (which we bin to 0.5 km, see below), as well as the resolution of the discretized field of view (Section 5.3.1.2). Some stellar occultations are designed using the boresight of the UVIS far-ultraviolet spectrograph, which is offset from the HSP boresight. We account for the occultations with off-set pointing by reducing the HSP FOV from 6 mrad $\times$ 6 mrad (Esposito et al., 2004) to 4 mrad $\times$ 4 mrad for each model.

We bin the stellar occultation data to 0.5-km resolution, which was selected because it is the optimal resolution for detecting a diffraction spike in most of the occultation data. Because the radial location of the ring edge varies, we use the data to determine where to model the ring edge for each occultation. We define the brightest data point in the diffraction peak to be the last, fully-unocculted observation of the star. In some occultations, there may be additional data points above the unocculted signal closer to the ring material but that are not as bright. We assume the full signal at these radial distances is obstructed by some particles in the ring edge, creating a non-negligible optical depth and therefore reducing the diffraction signal. We therefore define the first data point with a lower I/F than the diffraction peak to represent the ring edge. Most of the time, modeling the first point above the stellar signal as the ring edge or modeling the brightest point as the ring edge makes a small difference in the best-fit $a_{min}$ and $q$. For extreme cases, as shown in Figure 5.8, the placement of the ring edge makes a significant difference. This is typically true for occultations with a large difference in I/F from the first data point to the maximum data
point or when there are multiple data points with I/F > 1 but less than the maximum signal. In these cases, as is true for Figure 5.8, modeling the signal starting with the maximum signal results in values more consistent with the rest of our results, lower $\chi^2$ values and a visibly better fit.

Figure 5.8: Occultation of $\beta$ Centauri Rev 81 by the A ring outer edge. There is some scatter in the diffraction peak, rendering the first two points with I/F > 1 lower in signal than the maximum diffracted signal. Here we show the best-fit model assuming the edge to be near the first point with I/F > 1 (blue circles) and the best-fit model assuming the edge to be near the maximum signal (red squares). We find that for most occultations the difference in best-fit models is very small; however, in this case the best-fit $a_{\text{min}}$ and $q$ values are very different for the two models. This is because there are two points with low I/F before the maximum signal and so the First Point model must compensate for this structure, artificially lowering the $a_{\text{min}}$ and $q$ values. We find that in the context of our other results, the $\chi^2$ values for these fits, and from visual inspection, that models using the maximum signal as the edge is more appropriate. We therefore implement the Maximum Point Model for all occultations.

Jerousek et al. (2011) show that ring edges can be sharp, meaning that the distance over which the ring transitions from a high optical depth to zero optical depth is small. We therefore define the true ring edge to be within 0.1 km of the data point defining the ring edge.
We reproduce the geometry of the occultation and calculate the diffracted signal at the times corresponding to the binned data. Within 10 km of the ring, we reproduce every binned data point. Beyond 10 km, we calculate the signal for every 20 km rather than 0.5 km, as a higher time-step resolution is not required to match the model diffraction signal with the data far from the edge. The lower time resolution helps to offset the computational time required to model each ring edge.

5.3.1.2 Field of View & Particle Sizes

We use a discretized field of view in our model. For each time step, we determine the radial location of each discretized “pixel” of the FOV and correlate that radial location with the observed optical depth in the data. Each optical depth value is input into Equation 3.4 to calculate the pixel’s contribution to the total I/F signal from the entire FOV. This approach is beneficial because we can implement the observed radial optical depth profile for the determination of the diffraction signal rather than an average optical depth. This is important because particles as far as 50-100 km from the ring edge may be contributing to the diffraction signal, and the optical depth in the A ring is extremely variable over such distances.

The biggest limiting factor of the discretized field of view is the computational power required to model each FOV pixel. The FOV resolution and FOV size are dependent on the particle size, and because we calculate I/F for each particle size in the distribution, we can change the FOV size and resolution depending on the particle size regime being calculated. Figure 5.9 shows how the signal for different particle sizes varies as a function of scattering angle. From the figure, it is apparent that for a particle with a radius of 500 mm, the shape of the diffraction signal varies on the order of $< 10^{-8}$ radians, whereas for a particle radius of 1 mm, the variation is on the order of $10^{-5}$ radians. Therefore, we can model the diffraction from different particle sizes with resolutions determined by these limits. Additionally, the
diffraction lobe of larger particles is significantly smaller than the lobes of small particles. We can therefore measure larger particles with a small, higher-resolution field of view and smaller particles with a large, lower-resolution field of view. The results from each FOV add linearly to determine the total I/F over all particle sizes and FOV pixels.

We implement five fields of view, each covering a different particle size regime. The particle size range for each field of view is: \(< 1 \text{ mm}, 1 \text{ mm} \leq a < 5 \text{ mm}, 5 \text{ mm} \leq a < 10 \text{ mm}, 10 \text{ mm} \leq a < 50 \text{ mm} \text{ and } 50 \text{ mm} \leq a < 500 \text{ mm}\). We do not model the diffracted signal from particles larger than 500 mm because the diffraction lobe from particles larger than a few tenths of a meter is not large enough to be detected at our binned resolution. By implementing five fields of view, we can efficiently produce high-resolution models of the diffraction signature at ring edges.

Figure 5.9: Bessel function as a function of scattering angle for different particle sizes, \(a\). \(J_1\) is the Bessel function of the first kind and order 1, \(x = \frac{2\pi a}{\lambda}\), \(\lambda\) is the wavelength of light, and \(\theta\) is the scattering angle. For large particle sizes, a finer resolution in scattering angle is required to capture the shape of the Bessel function.
5.3.1.3 Binary Stars

Many of the stars used in this analysis are binary stars; however, the apparent spacing between the stars is typically less than the 0.5-km radial bins used for each occultation, so we are able to treat the signal as emanating from one source. The components of α Crucis and μ Scorpii are widely separated when projected onto the ring plane (> 12 km), so that the observed diffraction spikes are from only one of the stars. For these observations, we determine the approximate relative signal for each star, and calculate the strength of the diffraction spike created by only the star closest to the ring edge. We then add the other star’s contribution to the overall signal to reproduce the observations for calculating the best-fit model.

5.3.2 Best-Fit Models and Uncertainties

The uncertainties introduced by the noise in the direct stellar signal are naturally guided by Poisson statistics; however, because the stellar occultation data is recorded with lossy compression, the resulting uncertainties for each data point are slightly larger than those determined by the square root of the mean photon count. We therefore use the standard deviation of the unocculted signal beyond the diffraction spike (> 30 km) to place error bars on the data.

In order to determine the best-fit models for each stellar occultation, we compute a reduced $\chi^2$ ($\chi^2_r$) for each model light curve. The radial locations of the model points match those of the binned data so that we can directly compare the modeled and observed signal at each radial distance from Saturn. We avoid comparing model points that represent the direct stellar signal (and little or no diffracted light) with noise in the direct stellar signal by comparing each model point with the mean of 10 data points surrounding that radial location. We only implement this calculation for model points >20 km from the ring edge.
Some of the $\chi^2_\nu$ values are below 1. This suggests that our model is over-constrained. This over-constraint is a consequence of having only 2 free parameters, $a_{\text{min}}$ and $q$, but many more data points (\textasciitilde 28 per occultation), in addition to large error bars on the data with lower signal to noise ratios. For other occultations, our minimum $\chi^2_\nu$ values are quite high; this results from an underestimate of the error bars on the data points that make up the diffraction signal because we do not account for additional uncertainties introduced by the inhomogeneity of the rings from which the diffraction signal originates.

We place uncertainties on our model results using 1-$\sigma$, 2-$\sigma$ and 3-$\sigma$ confidence levels. These levels were determined using the the IDL chisqr_cvf routine, which provides a cut-off $\chi^2_\nu$ value for each confidence level based on the number of degrees of freedom. Figure 5.10 shows a contour plot of the $\chi^2_\nu$ values for the models of the diffraction spike during the A ring edge occultation of $\beta$ Centauri Rev 104. Each contour represents the three confidence levels. Figure 5.10 also shows the best-fit model and three models that lie outside of the 2-$\sigma$ confidence level. The bottom plots indicate how we determine the errors on the model fits. For each occultation, we report uncertainties determined by the range of model results with $\chi^2_\nu$ values below the 2-$\sigma$ confidence level, given the best-fit $q$ for errors on $a_{\text{min}}$ and the best-fit $a_{\text{min}}$ for errors on $q$. This is indicated in the bottom plots of Figure 5.10 by the intersection of the dashed line representing the 2-$\sigma$ cut-off value and the parabola of model $\chi^2_\nu$ values. For the occultations for which even the best-fit models have large $\chi^2_\nu$ values (discussed above), we report the uncertainties below the 3-$\sigma$ cut-off value. For noisy occultation data which results in many models with values of $\chi^2_\nu < 1$, we report the uncertainties at the 1-$\sigma$ confidence level. Essentially, we use the bounds of the contour region in which the best-fit model lies to determine the uncertainties, unless that region in $\chi^2_\nu$ space is small. We indicate the choice of confidence level for each occultation in Tables 5.1 - 5.3.
Figure 5.10: The normalized $\chi^2$ contour plot for the ingress occultation of $\beta$ Centauri Rev 104 showing the region of best-fit models for varying $a_{\text{min}}$ and $q$ (upper left). The contours represent the 1-$\sigma$, 2-$\sigma$, and 3-$\sigma$ confidence levels in the $\chi^2$ values. The upper-right plot shows the data with the best-fit model (red stars), and three model diffraction signals that lie just outside of the 1-$\sigma$ confidence range for $\chi^2$. For guidance, the symbols used for each model in the upper right plot correspond to the location on the contour plot (upper left) marked by the same symbol and color. The bottom plots show the 1-$\sigma$ (red, dotted line) and 2-$\sigma$ (blue, dashed line) and 3-$\sigma$ (magenta, dash-dot line) confidence levels compared with the lowest $\chi^2_{\nu}$ for $a_{\text{min}}$, assuming the best-fit $q$ value (bottom left) and $q$, assuming the best-fit $a_{\text{min}}$ value (bottom right). For most observations, we determine the error bars to be where 2-$\sigma$ confidence cut-off value intersects our model $\chi^2_{\nu}$ values. In some cases, we report the 1-$\sigma$ or 3-$\sigma$ error bars when more appropriate based on the $\chi^2_{\nu}$ results (see text).
5.3.3 Models

Figure 5.11: Model light curves produced by varying $a_{\text{min}}$ with a constant $q$. Sub-millimeter particles would produce detectable signal over 50 km beyond the ring edge. Large particles produce narrow, sharp diffraction peaks, while small particles produce broad diffraction signals.

Variations of the model’s $q$ and $a_{\text{min}}$ affect the shape and strength of the model diffraction curve. Figure 5.11 and Figure 5.12 show how the diffraction spike responds to variations in $a_{\text{min}}$ and $q$, respectively. Note in Figure 5.11, for $q = 3.1$, a sub-millimeter $a_{\text{min}}$ would result in the detection of diffracted light at least 50 km from the ring edge. As further discussed in Section 5.4, we do not detect diffracted light more than $\sim 25$ km from the ring edge in any occultation, so the A ring lacks a significant population of sub-millimeter-sized particles.
Figure 5.12: Model light curves produced by varying $q$ while maintaining a constant power law size distribution cutoff $a_{min}$. Variations in $q$ impact the overall scale of the diffraction signal.

5.3.3.1 Degeneracy of $a_{min}$ and $q$

As seen in Figure 5.10, there is a correlation between $a_{min}$ and $q$. The diffraction spike can be modeled well with an increase in the best-fit $q$ and an increase in the best-fit $a_{min}$ or a decrease in both of these parameters. For example, if we decrease the minimum particle size but also make the size distribution more shallow (decrease $q$), then we have fewer of those smallest particles, and the resulting model remains a good fit. In many of the strongest diffraction signals, the best-fit region is still well-defined, as is true for the Rev 104 $\beta$ Centauri occultation shown in the Figure 5.10. However, the data with lower signal-to-noise or simply a much smaller diffraction signal are more difficult to model, and the $\chi^2$ values indicate good fits for many parameters, as shown in Figure 5.13. Although we cannot place strong constraints, the region of good fit is still indicative of the particle size distribution in each ring edge region.
Figure 5.13: Contour plot of the $\chi^2_\nu$ results and the corresponding models from the occultation of $\sigma$ Sagitarius Rev 114. There is a linear dependence on the two parameters $a_{\text{min}}$ and $q$. Additionally, because of the lower signal to noise of this occultation, the models cannot place strong constraints on the parameters. However, the figure on the right shows that any model above the 2-$\sigma$ confidence level is clearly a poor fit to the data.

5.4 Results

5.4.1 A Ring Outer Edge

As discussed in Section 5.2, nearly all ingress occultations of bright stars with high $\alpha$ result in the detection of a strong diffraction spike. We plot a few of our resulting model light curves in Figure 5.14. The figure also shows the corresponding $\chi^2_\nu$ contour plots for the different stellar occultations by the A ring outer edge. These figures show that although there is a correlation between $a_{\text{min}}$ and $q$, the resulting region of best-fit models is consistent for all the observations, indicating a strong constraint on the particle size distribution parameters.

The models that produce synthetic light curves that best match the data have $a_{\text{min}}$ ranging from 1 mm - 10 mm with $q$ from 2.8 - 3.5 with a mean of $a_{\text{min}} = 4.5$ mm and $q = 3.2$. These results are shown in Table 5.1. The results with the largest $a_{\text{min}}$ also have the highest values for $q$. This could suggest some variation in particle population but is also consistent with the degeneracy of $a_{\text{min}}$ and $q$. 

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Figure 5.14: Model light curves and bounds compared with the data for the occultations by Saturn’s outer A ring edge of α Crucis Rev 92, β Centauri Rev 89 and ζ Centauri Rev 60 (first column) and their corresponding $\chi^2_\nu$ contour plots (second column). BF indicates the best-fit model. The colors and shapes of the model light curves in the first column are indicated with the same color and shape in the contour plot. Note that although some occultations have a broader range of acceptable fits, the best-fit region follows similar paths for $a_{\text{min}}$ and $q$ in the $\chi^2_\nu$ phase space.
Figure 5.15: Continued from from Figure 5.14. All parameters are the same. The occultations are: AlpAra033, AlpCru100I, BetCen096, BetCen102, and KapCen036, in order.
Figure 5.16: Continued from Figure 5.14. All parameters are the same. The occultations are: KapCen042I, LamSco044I, ZetCen112, in order.

None of the best-fit models indicate a particle size distribution with $a_{\text{min}} < 1$ mm; however, the errors do allow for some models with $a_{\text{min}} = 0.5$ mm to be considered viable fits. These models require $q < 2.8$. Such a shallow power-law further indicates that there is not a significant population of sub-millimeter particles near the outer edge of the A ring.
Table 5.1: A Ring Outer Edge Results: The first column lists the name of the star being occulted, with the revolution number in parentheses. The I signifies an ingress occultation and an E signifies an egress occultation. $I_0$ is the unocculted photon counts per millisecond for that occultation. Range is the line of sight distance from the spacecraft to the rings, given in Saturn radii. The date is the year, day and time in UTC indicating when the ring edge occults the star.

* Edges where the diffraction peak is not the last data point with I/F > $I_0$.

+ Binary stars with a wide separation such that the signal from only one star is accounted for when modeling the diffraction spike.

Occultations marked with $3\sigma$ or $1\sigma$ indicate that the errors were determined based on the 3-$\sigma$ or 1-$\sigma$ confidence levels, respectively, as described in Section 5.3.2.

$\infty$ indicates that the parameter is not constrained.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>$a_{\text{min}}$ (mm)</th>
<th>$q$</th>
<th>$B$ (deg)</th>
<th>$\phi$ (deg)</th>
<th>$I_0$ (ms$^{-1}$)</th>
<th>Range ($R_S$)</th>
<th>Date (Year-Day Time)</th>
</tr>
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<td>276.9</td>
<td>36.57</td>
<td>11.36</td>
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<td>$\alpha$ Cru (92) I$^+$</td>
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<td>-68.18</td>
<td>176.9</td>
<td>521.58</td>
<td>11.96</td>
<td>2008-312 19:40:48</td>
</tr>
<tr>
<td>$\alpha$ Cru (100) I$^+$</td>
<td>$6^{+4.73}_{-1.75}$</td>
<td>$3.1^{+10.10}_{-2.14}$</td>
<td>-68.18</td>
<td>158.1</td>
<td>439.72</td>
<td>13.72</td>
<td>2009-012 14:24:00</td>
</tr>
<tr>
<td>$\beta$ Cen (81) I* $3\sigma$</td>
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<td>$3.5^{+\infty}_{-0.08}$</td>
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<td>270.1</td>
<td>547.46</td>
<td>10.13</td>
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<td>$\beta$ Cen (89) I*</td>
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<td>10.07</td>
<td>2008-290 09:07:12</td>
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<td>267.4</td>
<td>443.06</td>
<td>11.43</td>
<td>2008-343 17:02:24</td>
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<td>$3.1^{+9.09}_{-10}$</td>
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<td>217.8</td>
<td>364.95</td>
<td>17.58</td>
<td>2009-053 16:04:48</td>
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<td>315.78</td>
<td>17.59</td>
<td>2009-065 14:52:48</td>
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<td>14.00</td>
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<tr>
<td>$\kappa$ Cen (36) I$^1\sigma$</td>
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<td>-48.54</td>
<td>238.9</td>
<td>44.89</td>
<td>18.94</td>
<td>2007-002 15:21:36</td>
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<td>$\lambda$ Sco (44) I</td>
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<td>$2.8^{+18.21}_{-12}$</td>
<td>-41.7</td>
<td>243.6</td>
<td>233.75</td>
<td>16.44</td>
<td>2007-129 11:16:48</td>
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<td>$\lambda$ Sco (114) I*</td>
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<td>$3.0^{+9.12}_{-19}$</td>
<td>-41.7</td>
<td>253.3</td>
<td>88.01</td>
<td>27.35</td>
<td>2009-195 10:04:48</td>
</tr>
<tr>
<td>$\zeta$ Cen (60) I</td>
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<td>$3.2^{+25.26}_{-28}$</td>
<td>-53.59</td>
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<td>105.28</td>
<td>15.94</td>
<td>2008-060 16:48:00</td>
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<tr>
<td>$\zeta$ Cen (112) I*</td>
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<td>-53.59</td>
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<td>20.97</td>
<td>2009-163 10:33:36</td>
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Figure 5.17: Light curve models and bounds compared with the data for the occultations of β Centauri Rev 92, β Centauri Rev 105 and ζ Centauri Rev 62 by the outer edge of the Encke Gap (first column) and their corresponding $\chi^2$ contour plots (second column). BF indicates best fit. The colors and shapes of the model light curves in the first column are indicated with the same color and shape in the contour plot. The shallow-sloped relationship between $a_{min}$ and $q$ enable models across the entire range of $a_{min}$ to fit the data well with only a slight change in $q$, making it difficult to put strong constraints on the parameters.
5.4.2 Encke Gap Outer Edge

Models of the diffraction spikes at the outer edge of the Encke Gap (ring plane radius $\sim 133,745$ km) reveal a range in $a_{\text{min}}$ from 3 mm - 30 mm with a mean of 9.3 mm. The mean $q$ is 3.1 with a range from 2.9 - 3.5. The results for each occultation are displayed in Table 5.2. These results indicate a broader range in particle sizes, often larger than the average particle size at the A ring outer edge. This is further discussed in Section 5.4.4.

Figure 5.17 displays three best-fit models and the corresponding $\chi^2_\nu$ contour plots. The diffraction spikes are typically not as strong as the spikes observed at the A ring outer edge, making them more difficult to model. This, in addition to the error involved in removing the HSP ramp-up, causes a wider range of models to be potentially good fits. The degeneracy...
between $q$ and $a_{\text{min}}$ for best-fits occurs for the Encke Gap outer edge as well, but the slope along best-fits in the contour plots is shallower than those observed for the models of the outer edge of the A ring. As a result, we often cannot put a strong constraint on both $q$ and $a_{\text{min}}$, as is seen in the plots in Figure 5.17. A comparison of the minimum $\chi^2$ regions in Figures 5.14 and 5.17 shows that the A ring edge has steeper contours in the $q - a_{\text{min}}$ space than the Encke gap outer edge. This suggests particles at the A ring edge are generally smaller even though individual occultation measurements have relatively large uncertainties. See also Section 5.4.4. Again, no models with particle sizes below a few millimeters can match the observed diffraction spike, indicating that this region in the A ring is also devoid of sub-millimeter particles.

Figure 5.19: Continuation from Figure 5.17. Parameters are the same. The occultations are: GamAra037E, GamCas100, and MuSco043, in order.
5.4.2.1 Correlation with Pan

![Figure 5.20: Minimum particle size as a function of relative longitude between the inertial longitude of the Encke Gap outer (trailing) edge where the occultation occurred and Pan’s inertial longitude. Purple asterisks indicate the results for the analyzed occultation data. The circles represent the relative longitudes of the four occultations during which diffraction signals were not originally detected (see text), and arbitrarily placed at \( a_{\text{min}} = 20 \) mm. The closed symbols indicate the two occultations for which diffraction spikes were detected at higher resolution. There appears to be a correlation in measured \( a_{\text{min}} \) and relative longitude to Pan, suggesting a recent passing of the moon has a short-lived effect on the distribution of the smallest particles.](image)

We find a possible correlation with particle size and a recent encounter with Pan. Only two models indicate \( a_{\text{min}} > 10 \) mm near the outer edge of the Encke Gap: \( \mu \) Scorpii Rev 43 and \( \zeta \) Centauri Rev 62. Both of these occultations occurred at a longitude relative to Pan \( \geq 300^\circ \), indicating the moon had recently passed by the occultation region. Sharp diffraction spikes are indicative of an absence of small particles. The minimum particle size as a function of longitude relative to Pan for all analyzed occultations is plotted in Figure 5.20.

We take a second look at all of the occultations that meet the standards for detection (Section 5.2.1) at higher resolutions: \( \beta \) Centauri Rev 64, \( \beta \) Centauri Rev 77, \( \beta \) Centauri
Rev 78, and α Virginis Rev 8. These occultations are indicated in Figure 5.20 by circles. Of these four occultations we find that at higher resolutions, β Centauri Rev 77 and β Centauri Rev 78, have strong but narrow diffraction spikes. This would suggest the presence of some centimeter-sized particles during these observations, but very few smaller particles. The occultation of β Centauri Rev 78 also occurred at a large longitude relative to Pan of 306 degrees, consistent with the idea that after a recent encounter with Pan, there is an absence of particles on the millimeter-scale. The β Centauri Rev 77 occurred at a relative longitude to Pan of 34.5 degrees, where we would expect to find small particles if this correlation is true. The very sharp diffraction spike typically indicates a lack of sub-centimeter particles, however, the spacecraft was only 248,500 km from the ring edge, or ∼ 4.27 Saturn radii, during this occultation. This small line-of-sight distance between the spacecraft and the ring edge would make a diffraction signal more difficult to observe, even for millimeter-sized particles.

Of the two occultations that do not display a diffraction spike at any resolution, the β Centauri Rev 64 occultation occurs after a recent passing of Pan. The non-detection may indicate a lack of millimeter- or even centimeter-sized particles, again consistent with Pan’s possible effect on the size distribution. The α Virginis Rev 8 occultation occurs at a relative longitude of 31.4 degrees, so we would expect to observe the signature of smaller particles. Although α Virginis has a satisfactory value of α, the ring opening angle is still very low, which could suppress a strong diffraction signal and therefore explain the non-detection.

The results indicate that for every occultation during which we expect to see a diffraction spike that occurred with a longitude relative to Pan ≥ 300°, we either measure $a_{\text{min}} \geq 10$ mm or we do not detect any diffraction spike at the 0.5-km resolution, implying even larger particles. In one case we find a sharp diffraction signal indicative of a large $a_{\text{min}}$ at a longitude that does not correspond to a recent encounter with Pan, although the sharpness may be due to the spacecraft’s small line-of-sight distance to the ring edge. This result suggests
that interactions with Pan may have a short-lived effect on the cut-off of the particle size
distribution near the outer edge of the Encke Gap.

Table 5.2: Encke Gap Outer Edge Results. Parameters are the same as those in Table 5.1

<table>
<thead>
<tr>
<th>Occultation</th>
<th>(a_{\text{min}})</th>
<th>(q)</th>
<th>(B)</th>
<th>(\phi)</th>
<th>(I_0)</th>
<th>Range</th>
<th>Date</th>
</tr>
</thead>
<tbody>
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<td>(\alpha) Ara (35) E</td>
<td>(5^{+\infty}_{-3.26})</td>
<td>(2.9^{+21}_{-\infty})</td>
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<td>26.00</td>
<td>2006-352 12:11:40</td>
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<tr>
<td>(\alpha) Cru (100) E*</td>
<td>(10^{+6}_{-3.23})</td>
<td>(3.4^{+\infty}_{-23})</td>
<td>-68.18</td>
<td>92.4</td>
<td>433.83</td>
<td>13.53</td>
<td>2009-012 20:25:52</td>
</tr>
<tr>
<td>(\beta) Cen (92) E</td>
<td>(3^{+12.89}_{-1.51})</td>
<td>(3.0^{+19}_{-32})</td>
<td>-66.7</td>
<td>57.8</td>
<td>473.81</td>
<td>8.46</td>
<td>2008-313 09:36:00</td>
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<tr>
<td>(\beta) Cen (105) E</td>
<td>(5^{+6.33}_{-2.63})</td>
<td>(2.9^{+13}_{-15})</td>
<td>-66.7</td>
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<td>17.50</td>
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<tr>
<td>(\epsilon) Lup (37) E</td>
<td>(5^{+14.21}_{-2.96})</td>
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<td>33.21</td>
<td>24.44</td>
<td>2007-020 23:36:58</td>
</tr>
<tr>
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<td>2007-022 19:12:10</td>
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<td>(\gamma) Cas (100) E</td>
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<td>(3.4^{+\infty}_{-55})</td>
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<td>(\zeta) Cen (62) E</td>
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<td>67.7</td>
<td>110.05</td>
<td>12.16</td>
<td>2008-082 15:30:44</td>
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</table>

5.4.3 Encke Gap Inner Edge

Although there are many occultations of the Encke Gap inner edge (ring plane radius of 133,423 km), the diffraction spike is observed in only six occultations. Many of the occ-
cultations of bright stars that produce a strong diffraction signal at the A ring outer edge
produce little or no signal at the Encke Gap inner edge. There are two explanations for
the lack of strong diffraction signals at the inner edge: (1) the edge is not sharp or (2) the
particles interior to the Encke Gap are larger than those in the region exterior to the gap
and therefore produce a narrower, weaker diffraction signal at the HSP wavelengths. The
occultation data do not indicate that the other ring edges in this study are sharper than the
Encke Gap inner edge, thus implying that there must exist fewer millimeter-sized particles
Table 5.3: Encke Gap Inner Edge Results. Parameters are the same as those in Table 5.1.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>$a_{\text{min}}$ (mm)</th>
<th>$q$</th>
<th>$B$ (deg)</th>
<th>$\phi$ (deg)</th>
<th>$I_0$ (ms$^{-1}$)</th>
<th>Range ($R_S$)</th>
<th>Date</th>
</tr>
</thead>
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<td>$7^{+5.8}_{-5.2}$</td>
<td>$3.0^{+0.45}_{-\infty}$</td>
<td>-68.18</td>
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<td>521.58</td>
<td>11.93</td>
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<td>$\alpha$ Cru (100) I$^+$</td>
<td>$4^{+11.16}_{-2.06}$</td>
<td>$3.0^{+0.17}_{-0.27}$</td>
<td>-68.18</td>
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<td>439.72</td>
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<td>-66.7</td>
<td>217.0</td>
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<td>141.6</td>
<td>75.60</td>
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<tr>
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<tr>
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<td>233.75</td>
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<tr>
<td>$\lambda$ Sco (114) I</td>
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<td>$2.8^{+0.09}_{-0.03}$</td>
<td>-41.7</td>
<td>251.3</td>
<td>88.01</td>
<td>27.40</td>
<td>2009-195 10:42:00</td>
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<tr>
<td>$\sigma$ Sgr (114) I$^{*}$</td>
<td>$20^{+\infty}_{-16.79}$</td>
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<td>-29.1</td>
<td>329.6</td>
<td>33.10</td>
<td>37.73</td>
<td>2009-198 20:42:28</td>
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</table>

In this region. Occultations of bright stars with high $\alpha$, such as $\beta$ Centauri, result in strong diffraction spikes at the A ring outer edge but tend to result in a non-detection at the inner edge of the Encke Gap. Higher spatial resolutions still do not result in a detection for the majority of the occultations, suggesting a depletion of centimeter-sized particles as well.

We analyze the occultations during which a diffraction spike was detected and find that the mean $a_{\text{min}} = 15$ mm with a range between 5 mm - 30 mm. $q$ ranges between 2.7 - 3.2 with a mean of 2.9. These results are listed in Table 5.3. Figure 5.21 shows a sample of those results. The weak diffraction signal coupled with the model dependence between $a_{\text{min}}$ and $q$ make placing strong constraints for each occultation difficult. Still, it is clear that the pattern of best-fit models implies larger particles. In many cases, the best-fit model is a particle size of 30 mm, which is the maximum $a_{\text{min}}$ used in the modeling process. This suggests that $a_{\text{min}}$ may be even larger in the bulk of the region. We do not see a correlation between $a_{\text{min}}$ with the relative longitude of Pan as we did for the particles near the outer edge of the Encke Gap.
Figure 5.21: Model light curve models and bounds compared with the data for the occultations of \( \gamma \) Pegasus Rev 36, \( \kappa \) Centauri Rev 42 and \( \lambda \) Scorpii Rev 44 by the outer edge of the Encke Gap (first column) and their corresponding \( \chi^2 \) contour plots (second column). BF indicates best fit. The colors and shapes of the model light curves in the first column are indicated with the same color and shape in the contour plot in the second column. The shallow-sloped relationship between \( a_{\text{min}} \) and \( q \) enable models across the entire range of \( a_{\text{min}} \) to fit the data well with only a slight change in \( q \), making it difficult to put strong constraints on the parameters.
Studies conducted by French and Nicholson (2000) and Marouf et al. (2008) indicate a depletion of centimeter-sized particles and smaller in the regions interior to the Encke Gap. Our findings are largely consistent with their results; in most occultations we do not detect a diffraction signal that would be expected for a population of sub-centimeter particles. In a few occultations, however, we do measure some signal from these small particles, indicating that there is at least a small or transient population of free-floating millimeter-sized particles.

Figure 5.22: Continuation of Figure 5.22. Parameters are the same. The occultations are: AlpCru092, BetCen105I, SamSco114, SigSgr114, in order.
Our results appear to be inconsistent with the work done by Harbison et al. (2013) who found $a_{\text{min}} \sim 0.56 \text{ mm}$ for $q = 2.9$. Their model includes multiple-scattering and self-gravity wakes, which could explain some of the discrepancy; however, their models without self-gravity wakes still require a sub-millimeter $a_{\text{min}}$. As discussed in Section 5.3, single-scattering models presented here would produce an underestimate of $a_{\text{min}}$, not an over-estimate, so single-scattering vs. multi-scattering cannot explain the difference. The other significant difference between the studies is the region being explored; here we report properties of the size distribution within $< 100 \text{ km}$ of the ring edge. Harbison et al. (2013) analyze a much larger region of the ring, from $122,000 \text{ km} - 133,000 \text{ km}$. The particle population may be different across this large region of the A ring; however, if a substantial population of sub-millimeter-sized particles were to exist in the vicinity of the Encke Gap inner edge we would expect to observe a broad diffraction signal within the gap. We do not observe such a signal.

From the UVIS occultation data we place a lower bound on $a_{\text{min}}$ in this region of the rings; however, the VIMS stellar occultation data is more sensitive to the diffraction signal produced by centimeter-sized particles and will be able to set a stronger limit on $a_{\text{min}}$ near the inner edge of the Encke Gap. At the longer wavelengths observed by VIMS, the same particles will produce a broader signal that extends $\sim 20$ times farther, radially, from the edge. Therefore, a population of centimeter-sized particles will produce a signal that would be more easily detected by VIMS stellar occultation data. Combining the analysis of the diffraction signatures from both instruments will provide a more complete characterization of the particle size distribution across these locations in the A ring.

### 5.4.4 Ring Edge Comparisons

Table 5.4 summarizes the model results for each A ring edge studied in this paper. A comparison of the best-fit parameters for each occultation at each edge is shown in Figure 5.23. Although there are large errors on some of the measured $a_{\text{min}}$, particularly for the
Encke Gap edges, this figure indicates that each region studied is qualitatively different from the others. The particle size distribution of the outermost parts of the A ring has a steep \( q \) with \( a_{\text{min}} < 1 \text{ cm} \), while the \( q \) of the distribution of particles near the Encke Gap inner edge is shallow, with a large range in \( a_{\text{min}} \). Typically the smallest particles are on the order of a few centimeters. The parameters for the particles near the Encke Gap outer edge seem to lie somewhere in between the results for the inner Encke Gap and the outer edge of the A ring. These results suggest that the particle size distribution steepens and the minimum particle size decreases as a function of radial distance from Saturn, a trend that was also observed by French and Nicholson (2000) and Zebker et al. (1985).

We compare the results of this work with those of previous studies in Table 5.5 and in Figure 5.23. In the outer A ring, our findings are consistent with previous work, indicating a cut-off in particle sizes between 1 and 10 mm for \( q \sim 3.1 \). The wavelength and resolution of the HSP enables us to detect particles as small as 1 - 2 millimeters in the outermost part of the A ring, extending the mean \( a_{\text{min}} \) to 4.5 mm from the 10 mm suggested by French and Nicholson (2000) and the 5 mm reported by Marouf et al. (2008). We list the mean particle size for the region interior to the Encke Gap based on the measurements of diffracted light, but as discussed in Section 5.4.3, many occultations do not show a diffraction spike, suggesting a much larger population of particles. Therefore, the mean particle size listed for the inner edge of the Encke Gap is in fact a lower limit. It is important to note, however, that we do detect particles as small as 5 millimeters in the region in some observations. These values place a lower bound on the \( a_{\text{min}} \) cut-off than the studies of French and Nicholson (2000) and Marouf et al. (2008), but are largely consistent with the observed depletion of centimeter and sub-centimeter particles. There remains a discrepancy with the reported particle size interior to the Encke Gap described by Harbison et al. (2013) that was addressed in Section 5.4.3.
Figure 5.23: A comparison of the best-fit models for each ring edge and the results of previous studies (as described by Table 5.5). This figure indicates that the particle size distribution for each edge is different from one another, with the outer A ring edge typically consisting of the smallest $a_{min}$ values and the largest $q$ values, while the inner edge of the Encke Gap has a smaller $q$ and much larger $a_{min}$. It also shows consistency with previous studies, although it should be noted that the results for the Encke Gap Inner A ring from the Cassini RSS and the Earth-based stellar occultations do not fit in this figure because of their large $a_{min}$ values and are not shown.

Table 5.4: Results

<table>
<thead>
<tr>
<th>Ring Edge</th>
<th>mean $a_{min}$ (mm)</th>
<th>range $a_{min}$ (mm)</th>
<th>mean q</th>
<th>range q</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Ring Outer Edge</td>
<td>4.4</td>
<td>1 - 10</td>
<td>3.2</td>
<td>2.8 - 3.5</td>
</tr>
<tr>
<td>Encke Gap Outer Edge</td>
<td>9.3</td>
<td>3 - 30</td>
<td>3.1</td>
<td>2.9 - 3.5</td>
</tr>
<tr>
<td>Encke Gap Inner Edge</td>
<td>15</td>
<td>5 - 30</td>
<td>2.9</td>
<td>2.7 - 3.2</td>
</tr>
</tbody>
</table>
Table 5.5: Results Comparison: Comparison of results from various studies of the particle size distribution for the A ring region exterior and interior to the Encke Gap. The $a_{\text{min}}$ and $q$ are given for the indicated ring plane radius used in each study.

- Voyager RSS results from Zebker et al. (1985).
- Cassini RSS results from Marouf et al. (2008) and Cuzzi et al. (2009).
- VIMS solar occultation results from Harbison et al. (2013). This paper also reports $a_{\text{min}} < 0.34$ for $q = 2.75$.

<table>
<thead>
<tr>
<th>Study</th>
<th>Exterior to Encke Gap</th>
<th>Interior to Encke Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_{\text{min}}$ (mm)</td>
<td>$q$</td>
</tr>
<tr>
<td>This work</td>
<td>4.5 - 9.3</td>
<td>3.1 - 3.2</td>
</tr>
<tr>
<td>Voyager RSS$^a$</td>
<td>1</td>
<td>3.03</td>
</tr>
<tr>
<td>Cassini RSS$^b$</td>
<td>5</td>
<td>3.2</td>
</tr>
<tr>
<td>Earth-Based 28 Sgr$^c$</td>
<td>10</td>
<td>2.9</td>
</tr>
<tr>
<td>VIMS Solar Occultations$^d$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5.5 B Ring Outer Edge

Figure 5.24: BetCen104I stellar occultation by the outer edge of Saturn’s B ring binned to 500-meter resolution. The diffraction signal has a radial extent of at least 25 km, even larger than the signal observed at the A ring outer edge for the same stellar occultation, indicative of millimeter-sized particles near the ring edge.

This analysis can be easily extended to other sharp edges in Saturn’s rings. We detect diffraction spikes at the B ring outer edge (Figure 5.24) and at the edges of various gaps and
ringlets, including the outer edge of the Huygens Gap and at the Maxwell Ringlet. We have completed a preliminary analysis of the diffraction signals at the B ring outer edge. In some occultations, the diffraction signals are similar to those observed at the A ring outer edge. In order to analyze these edges, we again must remove the diffraction signal. We applied the same technique described in Section 5.2.1: we fit a polynomial to the signal as it increases in the Huygens Gap (just exterior to the B ring edge) and divide by this signal to normalize the signal in the gap (Figure 5.25).

Figure 5.25: We fit the shape of the original data (black) with a polynomial to remove the ramp-up effect. We normalize the signal (purple) and use a region far from the ring edge (orange) to determine the average unocculted stellar signal in the Huygens Gap exterior to the B ring edge, which is represented by the red line.

We guided a high school student, Nirja Shah, through the process of plotting and measuring the extent of the diffraction signals. She determined the location of the ring edge and the radial extent of the diffraction signature. To first order, she determined the particle sizes of the ring by applying the expression for the first diffraction minima for a given particle size

\[ a_{\text{min}} = \frac{\lambda}{2\theta} = \frac{\lambda D_{\text{LOS}}}{\Delta r} \]  

(5.3)
where $D_{LOS}$ is the line of sight distance to the rings from the spacecraft and $\Delta r$ is the radial extend of the diffraction signal from the outer edge of the B ring into the Huygens Gap. Her results are listed in Table 5.6. She looked through all of the occultations for which a diffraction signal was detected at the A ring outer edge. Most of the occultations that showed a strong diffraction signal at the A ring outer edge also showed one at the B ring outer edge, however there were some occultations during which only the B ring edge created a significant diffraction signal. We also compared the variation in particle size with the initial longitude of ring region occulting the star (Figure 5.26a). Because the 2:1 resonance with Mimas maintains the B ring edge, we also compare the occultation longitude relative to the longitude of Mimas (Figure 5.26b). We do not find any immediate correlation with the particle size and either of these parameters. These results must be verified through a more robust method such as the ring edge model described for the A ring.

![Graph](image)

(a) B Ring Edge Minimum Particle Size vs. Longitude
(b) B Ring Edge Minimum Particle Size vs. Longitude Relative to Mimas

Figure 5.26: (a) 1st-order minimum particle size as a function of inertial longitude of the B ring outer edge for each stellar occultation for which a diffraction signal was observed. (b) 1st-order minimum particle size as a function of the occultation longitude relative to the longitude of Mimas. Neither plot indicates an obvious correlation of particle size with these parameters, however more work must be done to constrain the minimum particle size of the B ring outer edge, including the diffraction modeling.
Table 5.6: B Ring Outer Edge: Measurements of the radial extent ($\Delta r$), the line of sight distance from the spacecraft to the ring (LOS), the inertial longitude of the location of the occultation, the longitude relative to the longitude of Mimas, and the resulting calculation of the minimum particle size for each occultation for which a diffraction signal was detected.

<table>
<thead>
<tr>
<th>Occultation</th>
<th>$\Delta r$ (km)</th>
<th>LOS (km)</th>
<th>Longitude Relative to Mimas (degrees)</th>
<th>Inertial Longitude (degrees)</th>
<th>Particle Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetCen075</td>
<td>27.52</td>
<td>478778.74</td>
<td>240.11</td>
<td>169.35</td>
<td>1.30</td>
</tr>
<tr>
<td>BetCen077I</td>
<td>38.3</td>
<td>476941.75</td>
<td>263.84</td>
<td>169.32</td>
<td>0.93</td>
</tr>
<tr>
<td>BetCen085</td>
<td>7.05</td>
<td>580990.22</td>
<td>50.88</td>
<td>163.09</td>
<td>6.18</td>
</tr>
<tr>
<td>BetCen096</td>
<td>13.5</td>
<td>660035.49</td>
<td>129.50</td>
<td>166.88</td>
<td>3.67</td>
</tr>
<tr>
<td>BetCen104I</td>
<td>24.45</td>
<td>1025766.4</td>
<td>326.80</td>
<td>225.26</td>
<td>3.15</td>
</tr>
<tr>
<td>DeCen098</td>
<td>17.45</td>
<td>871368.61</td>
<td>63.52</td>
<td>194.29</td>
<td>3.75</td>
</tr>
<tr>
<td>EpsLup037I</td>
<td>82.3</td>
<td>1421308.9</td>
<td>184.22</td>
<td>168.97</td>
<td>1.30</td>
</tr>
<tr>
<td>KapCen042I</td>
<td>82.3</td>
<td>1839851.1</td>
<td>206.28</td>
<td>300.36</td>
<td>1.68</td>
</tr>
<tr>
<td>LamSco044</td>
<td>24.4</td>
<td>935963.38</td>
<td>71.94</td>
<td>257.58</td>
<td>2.88</td>
</tr>
<tr>
<td>LamSco114</td>
<td>151.4</td>
<td>1613438.6</td>
<td>167.69</td>
<td>259.68</td>
<td>0.80</td>
</tr>
<tr>
<td>ZetCen060</td>
<td>7.38</td>
<td>917432.96</td>
<td>268.36</td>
<td>207.46</td>
<td>9.33</td>
</tr>
</tbody>
</table>

5.6 Discussion

Through the implementation of a ring-edge diffraction model, we have placed constraints on the steepness of a truncated power-law particle size distribution near the edges of the Encke Gap and the outer edge of Saturn’s A ring. We extend the lower limit of the particle size distribution determined by (French and Nicholson, 2000) to 4.4-millimeter particles in the outermost part of the A ring and to 15-millimeter-sized particles near the Encke Gap inner edge. As described by Tyler et al. (1983), Zebker et al. (1985) and French and Nicholson (2000), we also find a steepening of the power-law size distribution of the particles and a decrease in the minimum particle size as a function of radial distance from Saturn. The mean $q$ changes from 2.9 to 3.1 to 3.2 from the inner edge of the Encke Gap to the outer edge of the
Encke Gap and to the outer edge of the A ring. The mean $a_{\min}$ of the distribution for each edge is 15 mm, 9.3 mm, and 4.4 mm, respectively. The changes in the size distribution of the particles are likely a result of higher collision velocities between particles in the outermost parts of the A ring due to the high number of density waves in the region. We place strong constraints on the minimum particle size at these three locations and find no significant population of sub-millimeter particles in these regions.

Saturn’s A ring is host to a number of complex, dynamical features. The weaker tidal force in the outer A ring enables the coagulation of particles, as demonstrated by the existence of accretion features on Pan and moons just beyond the ring edge (Charnoz et al., 2007) and by the propeller objects that open small gaps in the A ring (Sremčević et al., 2007; Tiscareno et al., 2007, 2008). Lindblad resonances with the Saturnian satellites, particularly Prometheus and Pandora, excite density waves throughout the outer A ring (Esposito et al., 1983). Esposito et al. (2012) argue that in regions strongly perturbed by satellites, a predator-prey model of particle aggregation and disaggregation occurs. Large clumps have been shown to form in the rings at locations strongly perturbed by nearby satellites (Esposito et al., 2012). The gravitational influence of these larger aggregates (prey) on the surrounding material can then lead to more energetic collisions (predator), resulting in fragmentation and erosion, resupplying the region with small particles. Bodrova et al. (2012) find that the mean radius of free particles in the rings decreases with increasing relative velocity of colliding boulders, which is consistent with our results.

The observations of the outer edge of the Encke Gap and the correlation of particle size and longitude relative to Pan may be more evidence of the predator-prey influence of satellites. The region interior to the Encke Gap, however, contains fewer strong density waves, which could explain the larger $a_{\min}$, smaller $q$, and the lack of detectable diffraction signals observed at the inner edge of the Encke Gap. Our results show that Pan’s influence on the particles at the edge of the Encke Gap is less disruptive to the ring particles than we
infer for the collisions excited by density waves, though our analysis is radially limited to the \( \sim 10 \) km of ring material adjacent to each edge.

We do not find a significant population of sub-millimeter particles. However, Déau (2015) model the opposition effect as observed by the Cassini Imaging Science Subsystem (ISS) and find grain sizes below 100 \( \mu \text{m} \), and studies of the VIMS band-depths by Nicholson et al. (2008) indicate grain sizes on the order of 10 \( \mu \text{m} \). If we were to extend the size distribution to 100 \( \mu \text{m} \), we would expect to observe diffracted light 75 – 750 km from the ring edge, for spacecraft distances of 100,000 km - 1,000,000 km, respectively. For a population of 10\( \mu \text{m} \)-sized particles, the diffraction signals would extend 10 times farther. No diffracted light has been observed at these radial distances from the edges, placing a strong constraint on the lower size cutoff of the ring particle size distribution. Grain sizes deduced from VIMS band-depths may represent absorption path lengths through the ice on the surfaces of ring particles rather than the physical sizes of discrete regolith particles. If sub-millimeter particles are present in the regolith, they are not released from the regolith in sufficient numbers to extend the power-law size distribution of the ring particles to sub-millimeter sizes.

The lack of sub-millimeter particles may indicate that the collisions are not energetic enough to overcome the contact forces that more effectively hold the micron-sized dust to the larger aggregates (Albers and Spahn, 2006). Our results suggest that the smallest discrete constituents of the regolith on ring particles in the A ring are \( > 1 \text{mm} \) and any smaller grains are more permanently sintered onto larger particles until disrupted by energetic micrometeoroid impacts. Any dust particles produced by micrometeoroid impacts are too few in number to represent a continuation of the broad ring particle size distribution that extends from a few millimeters up to several meters.
CHAPTER 6: DISCUSSION

In this work, we have constrained the particle size distribution of Saturn’s rings through a comparison of computationally-modeled diffraction signatures with those observed in stellar and solar occultation data observed by Cassini.

6.1 Stellar Occultation Data

We have placed a strong lower bound of 1 mm-sized free-floating particles in Saturn’s outer A ring and found that this number increases as the radial distance from Saturn decreases. Modeling the diffraction spikes near ring edges can be a useful technique for placing lower limits on particle sizes – the radial extent of the signal is bound to the smallest particles in the ring region. This technique can only probe a region of ring particles very near to the edges (several to tens of kilometers), so it may not be useful for characterizing the general size distribution of an entire ring. However, this method can be implemented to monitor variations in a more narrowly-defined region of the rings. Ring edges can be of great interest; it is at the B ring outer edge where kilometer-sized vertically-extended structures were observed during equinox, and at the A ring outer edge where the proto-moon Peggy was imaged (Murray et al., 2014). Do large clumps preferentially form at these ring edges, perhaps due to the satellites resonances that maintain them, or do they migrate through the rings to the edges? In either case, the size distribution near the ring edges would be affected and could perhaps indicate when large clumps are in the vicinity of the ring edge. The diffraction spike measurements would be even more sensitive to a situation in which such clumps have collided or otherwise released a large population of small particles. In this work, we find a correlation between the size distribution at the Encke Gap outer edge and
a recent passing of Pan. We do not, however, explore the exact methodology by which a recent passing of Pan can cause a depletion in smaller particles. If the moon does incite clumping or a sweeping of the smaller particles, the larger material must then break apart through shear or through collisions before one complete orbit.

Although we did explore correlations of varying particle size distributions with the resonant moons, time, and longitude of the occultation, there are several other avenues worth exploring. For example, azimuthally-limited gaps extending meters to a few kilometers radially have been detected on either side the Encke Gap and Keeler Gap in UVIS occultation data and appear to be associated with the ring-moons. Perhaps the presence of these gaps is also correlated with the particle size distribution near the ring edge – the opening of the gap may incite clumping closer to the gap itself, temporarily removing some of the smaller particles. The proximity of density waves near the gaps and how that correlates with the particle size distribution may also be worth exploring.

Our computational model of the ring edge diffraction can still be improved upon. For the ring edge stellar occultations, we assume the simplistic case of spherical particles. More complex structures will affect the diffraction signature of the particles. Additionally, we assume single-diffraction at the ring edges, which may be a poor assumption. We show in this work how multiple-diffraction can be implemented for this model and thus a significant amount of work would not be required to determine the affects of including multiple diffraction.

One of the greatest advantages of the Cassini mission is the suite of instruments. The VIMS instrument can also detect and measure the diffraction signatures at ring edges. Initial comparisons with their measurements reveal general agreement for the size distribution and the lower limit of the particle sizes near these ring edges. A more thorough comparison of the data, particularly at the inner edge of the Encke Gap where VIMS is more apt at detecting the larger particles that reside there, would be useful in truly constraining the particle size distribution near these ring edges.
Additionally, there are several edges in the Saturn ring system left unexplored by this work. Future work would include a deeper analysis of the B ring outer edge and the sharp edges of gaps and ringlets observed in the B ring, C ring and Cassini Division.

6.2 Solar Occultation Data

Saturn’s F ring is a complex, dynamically-excited system. Our analysis shows that it is not only the clearly-observed F ring core that is changing; the envelope of material in that surrounds the ring is also continually evolving. UVIS solar occultations in which unambiguous diffraction signatures are observed appear to correspond with ISS images that show large collisional events in the ring, indicating the release of smaller particles during such events. Furthermore, models of the F ring require that particles on the order of 10 microns must span a 500-km region around the core near these collisional events. However, occultations of a quiescent region in the F ring appear to lack significant quantities of sub-millimeter-sized particles.

There are still several parameters to explore in this work. An even more realistic model of the F ring, that includes higher optical depth strands as well as the core may lead to a deeper understanding of the ring during these occultations. However, the continually-evolving ring makes the parameters of the ring itself difficult to constrain and complicates comparisons between occultations at different times and longitudes. It would be useful to explore a different maximum particle size in the F ring to see how this may affect the results. Comparisons with the stellar occultation data could also provide stronger constraints on the variations in the size distribution of particles in Saturn’s F ring.

The size distribution of particles and the radial extent over which they are dispersed can be used to constrain the energy in the collisions that release them. Are these collisions of moonlets formed within the F ring or is the impactor from a region exterior to the ring?
What is the source of these core-crossing exterior objects, such as S/2004 S 6? Much work is needed in order to understand the F ring, however placing constraints on the particles – particularly those released during collisional events – is an important step in constraining the velocities, sizes and frequency of collisions in the F ring.

The unique Rev 9 solar occultation during which the Sun was nearly removed from the instrument’s FOV could be used as a model for other types of observations. Very small particles, such as those that exist in the E ring, will diffract light at very large angles. If the Sun were placed entirely outside of the instrument’s FOV, it is possible that small particles within the FOV could diffract enough light to produce a detectable signal. This signal could then be used to further constrain the size distribution of the ice particles in the E ring, an essential characteristic for understanding the rate of cryvolcanic activity from Enceladus.

6.3 Concluding Remarks

The Cassini mission is set to end in 2017. Until then, more occultation data will be collected and can be used to explore the particle size distribution throughout Saturn’s rings. The methods used here can be applied to additional data sets, continuing the observation of the time-evolution of the rings.

The techniques discussed in this work – modeling diffraction in the rings – can be applied to any ring system. Diffraction signatures in the Voyager and Hubble observations of the Uranian ring system could be used to analyze the particle size distribution there. Comparisons with the Saturnian ring system can continue our understanding of the fundamental differences between these ring systems. Furthermore, the discovery of rings around the centaur 10199 Chariklo introduces a new realm for ring science. For how long are such rings sustainable? How common are rings around small bodies? More observations will be necessary to answer such questions, however determining the particle size distribution of these
rings will lead to a better understanding of their origin and evolution. Improved observations may enable the resolution needed to observe diffraction signatures in such ring systems.

The study of diffracted light can be a powerful tool for measuring the particle size distribution in a ring system – a fundamental property for understanding the origin and evolution of such a system.
REFERENCES


