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A THEORETICAL EVALUATION OF WATER
SURFACE CHANGES IN A CIRCULAR
RESERVOIR

BY

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RESEARCH REPORT

Submitted in partial fulfillment of the requirements
for the degree of Master Of Science
in the Graduate Studies Program Of the College of Engineering
University Of Central Florida
Orlando, Florida

Spring Term
1986

ABSTRACT

The purpose of this project is primarily to develop an equation to estimate the time that it will take for the water surface to decrease to another depth in an inverted conical-shaped reservoir behind an earth dam by means of the falling head permeability concept. This equation can be applied to the earth embankment either with or without underdrain. Since the derived equation for time span is a complicated and tedious matter, a computer program has been developed for solving this equation.

The computer program is written in BASIC language and executed on a Tektronix 4051 microcomputer. Primary effort has been used to set up the program in a universal BASIC so that it can be easily executed on any microcomputer with the BASIC language. This program also has been tested on an IBM P.C.

ACKNOWLEDGEMENT

The author would like to express his appreciation for the contributions made to the successful completion of this program provided by Dr. Shiou-San Kuo, Committee Chairman, and the members of the committee, Dr. David R. Jenkins and Dr. Martin P. Wanielista for their guidance. Also thanks to my parents and my best friend, Hossein Dayi, for their moral support and encouragement.

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CHAPTER I

INTRODUCTION

The early dams built by man were low earth or rock structures designed to impound and divert water for agricultural purposes. Today the earth surface is dotted with small and large dams and reservoirs contributing in many ways to the complex requirements of an expanding, technologically advancing civilization. Among them, the earth dams are built for the purpose of controlling water flow. This water flow can be from natural water sources, ocean water, or from artificial water reservoirs.

With rapidly increasing demands for irrigation water and flood control, controlling water flow has become much more important. To control water flow, earth or rock structures have been placed to impound and divert water, but the amount and distribution of moisture in soil have a sufficient effect on the physical properties of the soil. The ability of a soil to support a load may vary, depending on the percent of moisture and the type of soil. Seepage flow through an earth dam must be considered in evaluating the dam's ability to impound water. The stability of artificial and natural slopes is greatly affected by the presence of water pressure.

The objective of this research study is to develop an equation from the theoretical seepage equation, so one can estimate the time required for a given drop of the water surface in the earth embankment reservoir. Since derivation of such equation is a tedious and

complicated process of integration, a computer program is developed so one can calculate the time required for the drop of water surface in the reservoir in a matter of minutes.

There are some assumptions applied for the following computer program such as the reservoir has an inverted conical shape and the phreatic surface has been considered for case of $\beta \geq 30$ where β is the slope of the discharge face. This is the case for the two-dimensional problem of ground water flow over a horizontal impervious surface which continues at a given point into a horizontal discharge face.

CHAPTER II

LITERATURE REVIEW

The classic problem of estimating the free water surface in a homogeneous isotropic earth dam has been documented by many papers and reports in literature. In all cases, either a domain method (such as finite differences or finite element) (Desia, 1972) or a boundary element technique (such as a boundary integration equation formulation or complex variable approach) (Finnemore, 1968) is used to develop the approximate seepage face and corresponding approximate water surface elevations.

One of these techniques for estimating the free water surface is the Complex Variable Boundary Element Method (CVBEM) (Hromadka, 1984). By expanding the CVBEM approximation geometric functions into a first order Taylor series, the geometry of the unknown phreatic surface can be approximated without iteration by solving a single matrix system. The developed technique provides for the numerical solution of the inverse problem of determining the phreatic surface coordinates. The basis of the method is the Cauchy integral theorem which determines the values of an interior point from a line integral of the values along the problem boundary. Because the approximator is analytic, the numerical model solves the Laplacian exactly for various boundary condition configurations. Because the seepage problem can be mathematically modeled by the Laplace equation, it is possible to apply the CVBEM directly to the subject problem.

Another approach for determining the free water surface is Dupuit's theory (Harr, 1967). In 1863 Dupuit based his theory on the following two assumptions:

1. For small inclinations of the line of seepage the streamlines can be determined as horizontal. Therefore, the equipotential lines approach the vertical.
2. The hydraulic gradient was equal to the slope of the free surface and was invariant with depth.

With Dupuit's assumptions, the constant seepage rate q (per unit width) through any vertical section of the earth dam can be expressed as:

$$q = KY \, dy/dx \quad (1)$$

where: K = coefficient of permeability

dy/dx = slope of phreatic surface at any point with
vertical coordinates equal to y .

By integrating and substituting the boundary condition, one can obtain Dupuit's formula as follows:

$$q = K (H_0^2 - H_1^2)/2L \quad (2)$$

where: H_0 = the depth of upstream reservoir

H_1 = the depth of downstream reservoir

L = distance of soil body between upstream and downstream
reservoir (Figure 1).

Equation (2) specifies a parabolic free surface, which is usually referred to as Dupuit's parabola.

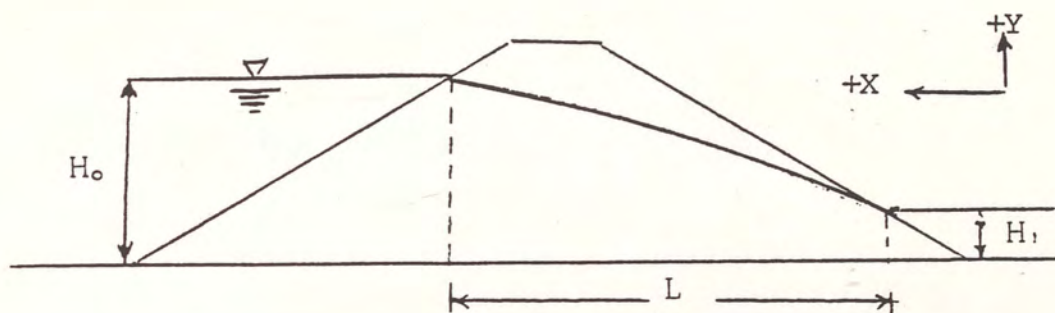


Figure 1. Cross Section of an Earth Embankment.

There have been other approaches to this type of problem. Cassa-grande developed an approximation to the position of the phreatic surface in seepage through an earth dam (1937). Polubarinova-Kochina (1962) also presented a solution to a similar problem by using the methods of velocity hodograph and conformal mapping.

Jeppson (1966) used an inverse formulation in conjunction with the finite differences method to solve this problem in the complex potential plane. Shang and Bruch (1976) used a similar approach but used the finite element method as the basis for solving the problem. Their formula proved to be as accurate as Jeppson's, while it was more flexible and easier to apply.

All of the above approaches can be categorized as follows:

1. Approximate
2. Analytic
3. Numerical - iterating along the free surface
4. Numerical - using the inverse formulation method

An approximation method is derived to estimate the phreatic line through the earth dam. In order to determine the phreatic line position, one assumes that the flow through earth dam has a parabolic shape. The determination of its focus is one of the major tasks for the investigation of flow through earth dams. To locate phreatic focus, one has to decide whether or not to place an underdrain. The placement of an underdrain helps the discharging of the flow and controlling seepage through the dam. The discharge from the drain is generally collected by pipes (Figure 2) and led into the spillway basin or into the river channel below the dam. Soils used between the dam proper and the pipe (or pipes) are designed as a graded filter.

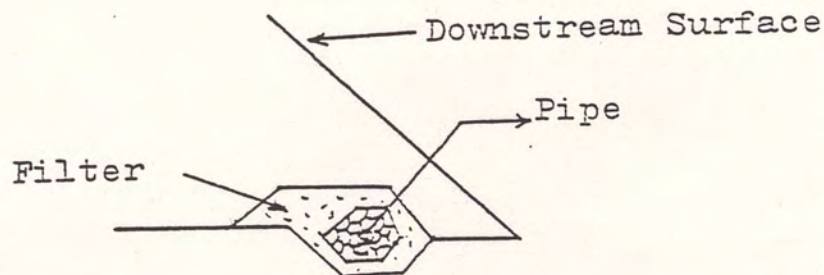


Figure 2. Cross Section of the Underdrainage.

The other important feature of the design of the earth embankment is the side-slope angle (or ratio). This angle depends on the height of the embankment, the shearing resistance of the soil, and whether or not the embankment will be flooded by impounded water, as in the case of earth dams. Embankment and earth dams of considerable height, in excess of 50 feet, should be individually analyzed to determine the slope stability and the factor of safety against shear slides. The side-slope ratio may also depend on the shearing strength of the soil foundation. The side-slope ratio given in Table 1 may be used as a guide for the design of embankments less than 50 feet. If the foundation is weak, the side-slopes should be made flatter in order to spread the weight of the embankment over a wide area. This would reduce the level of shearing stresses in the foundation material.

From the estimated phreatic line and assuming homogeneous and isotropic soil in the earth dam, one can determine the seepage rate for flow through the earth dam providing that the hydraulic gradient is constant. Finally, on the basis of falling head permeability theory, one can estimate the time required if the water level in the reservoir decreases.

The coefficient of permeability may be determined in one of several ways. An approximate value may be obtained by using a fall-head permeability test. This test is more economical for tests of long duration.

TABLE 1

MINIMUM SIDE-SLOPE RATIO FOR EARTH EMBANKMENTS
ON ADEQUATE FOUNDATIONS

SIDE SLOPE-RATIO HORIZONTAL-VERTICAL	TYPE OF SOIL AND OTHER CONDITIONS
1 1/4 : 1	Unsaturated and permeable silt with some cohesion and no water table.
1 1/2 : 1	All sand fills whether inundated or not, fills of cohesive soil less than 50' height and not subjected to inundation.
2 : 1	Fills of cohesive soils more than 50' in height and not subjected to inundation.
3 : 1	All fills of cohesive soils not exceeding 50' in height and subjected to total or partial inunda- tion.

(Harr, 1962)

From Darcy's equation:

$$-a dH = dq = K A H/L dt \quad (3)$$

$$\int_{H_1}^{H_2} -dH/H = \int_0^t (K A/(L a)) dt \quad (4)$$

$$K = a L/(A t) \ln(H_1/H_2) \quad (5)$$

where: a = area of tube

A = area of soil sample

dQ = flow volume in time dt

L = length of soil sample

H₁ = water level in the tube at the beginning
of the test

H₂ = water level at the end of the test

CHAPTER III

DEVELOPMENT OF THEORETICAL EQUATION

Basic Theorem

The flow problem considered in this paper is that of seepage flow through an idealized earth dam. Figure 3 shows a typical cross section of an idealized earth dam on an impervious foundation with a downstream underdrain. The soil in earth dam is assumed to be homogeneous and isotropic permeable material, but may be anisotropic, i.e. $K_x \neq K_y$, where K_x and K_y are the coefficients of permeability in x and y directions respectively.

The top flow line (phreatic surface) is the upper stream line in the flow domain. It separates the saturated region of flow and the part of the soil body where no flow occurs. As discussed previously, the phreatic surface is a basic parabola. The determination of its focus is one of the major tasks for the investigation of flow through the earth dams. The property of a basic parabola is illustrated in Figure 4.

After the length of downstream toe drain and earth dam side-slopes are set, one can estimate the focus of the phreatic surface through the earth dam. The approximate trace of phreatic surface is also depicted in Figure 3. All the flow lines through the earth dam are parabolic curves with the common focus F . However, analysis has shown that the top flow line of Figure 3 is close to parabolic except for a short distance of reserved curvature near point B . This parabola

produced backwards reaches head water elevation at Point A. Cassa-grande (1937) suggested the use of the approximate relationship that the distance AB equaled 0.3 times the distance BE for the dams with reasonably flat upstream slopes. Therefore, the top flow line may be constructed by the parabola which has its focus at point F and passes through point A, which is called a corrected entrance point. Once the parabola is obtained, the reversed curvature starting at Point B can be sketched in. From Figure 4, by taking the origin of coordinates at the focus and letting the distance from focus to directrix be equal to S, and equating expressions for the equal distances FD and DE, it results

$$\sqrt{X^2 + Y^2} = X + S \quad (6)$$

or

$$X = (Y^2 - S^2) / (2 S) \quad (7)$$

If Equation 7 is applied to the flow through the earth dam as depicted in Figure 3, with X equals to L, when Y equals to H_0 (the initial water level in the reservoir), Equation 7 becomes:

$$S = \sqrt{L^2 + H_0^2} - L \quad (8)$$

Since L and H are known values, the value of S can be easily obtained. Once the value of S is determined, the vertex C of the parabola can be located, i.e. $FC = S/2$ and a point G on the parabola right above the focus F can be located also, i.e. $FG = S$. After both points C and G have been located, the basic parabola forming the top flow line can easily

be drawn from Equation (6). This flow line will represent the free water surface through the earth dam for the specified height of water in the reservoir. Since the position of the uppermost flow line in the cross section of an earth dam has been determined, the quantity of seepage q can be determined by:

$$q = K S \quad (9)$$

The above equation is only valid for a horizontal discharge surface. It is indeed fortunate that the problem of seepage with a horizontal discharge face has such a simple solution, not only because of the fact that in modern earth dams horizontal drainage blankets in the downstream section are assuming considerable importance, but also because this solution permits fairly reliable and simple estimates for the position of the line of seepage. For other cases that β is less than 30 degrees, the quantity of seepage q per unit width of earth dam can be calculated in the form of:

$$q = K a \sin \beta \tan \beta \quad (10)$$

where:
$$a = L/\cos \beta - \sqrt{L^2/\cos^2 \beta - H_o^2/\sin^2 \beta}$$

= wetted surface

and K is coefficient of permeability of the soil in the earth embankment. Some typical values of the coefficient of permeability are given in Table 2.

TABLE 2
SOME TYPICAL VALUES OF COEFFICIENT
OF PERMEABILITY

SOIL TYPE	COEFFICIENT OF PERMEABILITY (cm/sec)
Clean gravel	1.0 or greater
Clean sand (coarse)	1.0 - 0.01
Sand (mixture)	0.01 - 0.005
Fine sand	0.05 - 0.001
Silty sand	0.002 - 0.0001
Silt	0.0005 - 0.00001
Clay	0.000001 and smaller

(Harr, 1962)

CHAPTER IV

DERIVATION OF EQUATION FOR TIME WHEN WATER SURFACE DECREASES IN THE RESERVOIR

The equation related to the time parameter and decreasing of water level in the reservoir due to the seepage of water through the earth dam is derived in this chapter. It is assumed the upstream reservoir has an inverted conical-shape as shown in Figure 5 and the soil beneath the earth dam is impervious.

The decreasing water volume, dV , in the reservoir due to seepage flow through the earth dam can be expressed as:

$$dV = \pi X^2 dy \quad (11)$$

where: dy = decreasing water level

X = radius of the reservoir at the time when the reservoir drops to height y

From the geometry of the reservoir, the following relationships can be derived:

$$X/r_0 = (y + r_b \tan \alpha) / (H_0 + r_b \tan \alpha) \quad (12)$$

$$H_0 + r_b \tan \alpha = r_0 \tan \alpha \quad (13)$$

where: H_0 = height of water in reservoir before any drops

r_b = bottom radius of the reservoir

r_0 = top radius of the reservoir at height H_0

α = the slope of upstream surface of the earth dam

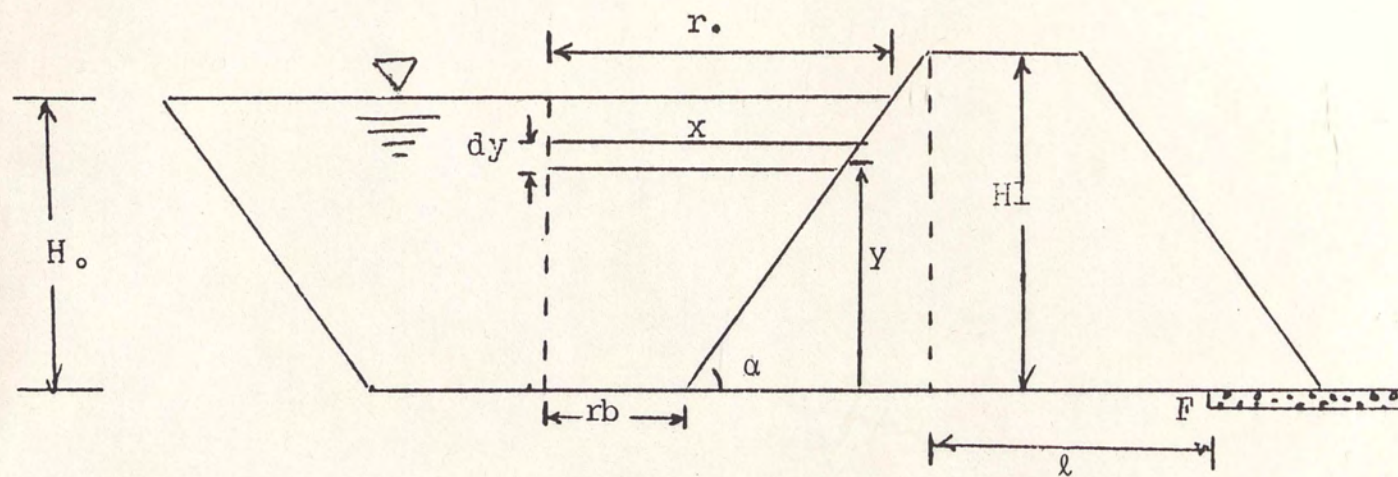


Figure 5. Cross Section of an Earth Dam and the Reservoir Behind It.

Substituting Equation (13) in Equation (12), one obtains:

$$X = (y + rb \tan \alpha) / \tan \alpha \quad (14)$$

By substituting the X value from Equation (14) into Equation (11), it gives:

$$dV = \pi [(y + rb \tan \alpha) / \tan \alpha]^2 dy \quad (15)$$

Meantime, the decrease of water volume is also equal to:

$$dV = q dt \quad (16)$$

From Equation (9), one can obtain:

$$dV = K S W dt \quad (17)$$

where K and W are soil permeability and width of the underdrain respectively. By substituting Equation (8) into Equation (17), one obtains

$$dV = K (\sqrt{L^2 + y^2} - L) W dt \quad (18)$$

From geometry of the earth dam (Figure 3), the distance L can be expressed as:

$$L = \ell + (H_1 - y) \cot \alpha + 0.3M \quad (19)$$

M is the horizontal distance from the upstream toe of the earth dam to the point where the water touches the earth dam, (see Figure 3). Thus,

$$M = y \cot \alpha \quad (20)$$

Equation (19) becomes:

$$\begin{aligned} L &= l + H_1 \cot \alpha - y \cot \alpha + 0.3y \cot \alpha \\ &= l + H_1 \cot \alpha - 0.7y \cot \alpha \end{aligned} \quad (21)$$

Since l , H_1 , and α are known values for a given section of earth dam, let

$$c = l + H_1 \cot \alpha \quad (22)$$

Equation (21) is then reduced to:

$$L = c - 0.7y \cot \alpha \quad (23)$$

By substituting the value of L from Equation (23) into Equation (18), it becomes:

$$dV = K (\sqrt{y^2 + (c - 0.7y \cot \alpha)^2} - (c - 0.7y \cot \alpha)) W dt \quad (24)$$

Equation (24) states that the decrease of water volume in the reservoir is equal to the volume of water collected at the underdrain through seepage in the earth dam. Using the fact that Equation (15) is equal to Equation (24), the result is:

$$\begin{aligned} \pi ((y + r_b \tan \alpha) / \tan \alpha)^2 dy &= K (\sqrt{y^2 + (c - 0.7y \cot \alpha)^2} \\ &\quad - (c - 0.7y \cot \alpha)) W dt \end{aligned} \quad (25)$$

Equation (25) can be reformed as:

$$\begin{aligned} (y + r_b \tan \alpha)^2 / ((y^2 + (c - 0.7y \cot \alpha)^2)^{1/2} - (c - 0.7y \cot \alpha)) dy \\ = K \tan^2 \alpha W / \pi dt \end{aligned} \quad (26)$$

It has been established that the reservoir water table is initially at a height of H_0 from the bottom of the reservoir and decreases to a new height, H_2 after a period of time, t . The required time for the water surface to drop from H_0 to H_2 can be calculated by integrating Equation (26).

$$\int_{H_0}^{H_2} (y+rb \tan \alpha)^2 dy / ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} - (c-0.7y \cot \alpha)) = \int_0^t K \tan^2 \alpha W / \pi dt \quad (27)$$

The integration of the left side of Equation (27) is very tedious and lengthy. The step by step integration is taken as described in the following.

Since rb and α are known values, we now let $d = rb \tan \alpha$, and the left side of Equation (27) becomes:

$$I = \int (y+d)^2 / ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} - (c-0.7y \cot \alpha)) dy \quad (28)$$

By multiplying the numerator and denominator by a common value of

$$((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} + (c-0.7y \cot \alpha))$$

one obtains:

$$I = \int (y+d)^2 ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} + (c-0.7y \cot \alpha)) / ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} - (c-0.7y \cot \alpha)) ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} + (c-0.7y \cot \alpha)) dy \quad (29)$$

Simplification of the above equation would give us:

$$I = \int (y+d)^2 ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} + (c-0.7y \cot \alpha)) / y^2 dy \quad (30)$$

The above equation is separated into two terms:

$$I = \int (y+d)^2 (y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} / y^2 dy + \int (y+d)^2 (c-0.7y \cot \alpha) / y^2 dy \quad (31)$$

where A term is

$$A = \int (y+d)^2 (y^2 + c^2 - 1.4c \cot \alpha y + 0.49 y^2 \cot^2 \alpha)^{\frac{1}{2}} / y^2 dy \quad (32)$$

and B term is

$$B = \int (y+d)^2 (c-0.7y \cot \alpha) / y^2 dy \quad (33)$$

Equation (33) is rearranged to be:

$$A = \int (y+d)^2 ((1+0.49 \cot^2 \alpha) y^2 - (1.4c \cot \alpha) y + c^2)^{\frac{1}{2}} / y^2 dy \quad (34)$$

Now, let a, b c' respectively represent:

$$a = 1+0.49 \cot^2 \alpha \quad (35)$$

$$b = -1.4c \cot \alpha \quad (36)$$

$$c' = c \quad (37)$$

Equation (34) can be then rewritten as:

$$A = \int (y+d)^2 (ay^2 + by + c')^{\frac{1}{2}} / y^2 dy \quad (38)$$

Multiplying the numerator and denominator by $(ay^2 + by + c')^{\frac{1}{2}}$, one obtains:

$$A = \int (y+d)^2 (ay^2 + by + c') / (y^2 (ay^2 + by + c')^{\frac{1}{2}}) dy \quad (39)$$

and then removing the parentheses which results in:

$$A = \int (ay^4 + (b+2ad)y^3 + (c'+ad^2+2ad)y^2 + (2dc'+2bd^2)y + d^2c') / (y^2(ay^2+by+c')^{\frac{1}{2}}) dy \quad (40)$$

If we let $Y = ay^2+by+c'$, Equation (40) becomes:

$$A = a \int y^2 / \sqrt{Y} dy + (b+2ad) \int y / \sqrt{Y} dy + (c'+ad^2+2ad) \int dy / \sqrt{Y} + (2dc'+bd^2) \int dy / (y \sqrt{Y}) + d^2c' \int dy / (y^2 \sqrt{Y}) \quad (41)$$

By applying the integration formulas, Equation (41) is integrated respectively as follows:

$$a \int y^2 / \sqrt{Y} dy = a((y/2a - 3b/4a^2)\sqrt{Y} + ((3b^2 - 4ac')/8a^2)(1/\sqrt{a} \ln(\sqrt{Y} + y\sqrt{a+b/2}\sqrt{a}))) \quad (42)$$

$$(b+2ad) \int y dy / \sqrt{Y} = (b+2ad)((\sqrt{Y}/a - b/2a)(1/\sqrt{a} \ln(\sqrt{Y} + y\sqrt{a+b/2}\sqrt{a}))) + b/2 \sqrt{a}))) \quad (43)$$

$$(c'+ad^2+2bd) \int dy / \sqrt{Y} = (c'+ad^2+2bd)(1/\sqrt{a} \ln(\sqrt{Y} + y\sqrt{a+b/2}\sqrt{a})) + b/2 \sqrt{a})) \quad (44)$$

$$(2dc'+bd^2) \int dy / (y \sqrt{Y}) = (2dc'+bd^2)(-1/\sqrt{c'} \ln((\sqrt{Y} + \sqrt{c'})/(y+b/2\sqrt{c'}))) \quad (45)$$

$$d^2c' \int dy / (y^2 \sqrt{Y}) = d^2c'(-\sqrt{Y}/c'y - b/2c'(-1/\sqrt{c'} \ln((\sqrt{Y} + \sqrt{c'})/(y+b/2\sqrt{c'})))) \quad (46)$$

Combining all the above five terms, one obtains a solution of integration for the first term A. Now, integration of the second term B from Equation (33) becomes:

$$B = \int (y+d)^2 (c-0.7y \cot \alpha) / y^2 dy \quad (33)$$

or

$$= \int (y^2 c - 0.7y^3 \cot \alpha + 2ydc - 1.4y^2 d \cot \alpha + d^2 c - 0.7yd^2 \cot \alpha) / y^2 dy \quad (47)$$

Decomposition of the above equation will result in six terms. Each term is integrated separately as follows:

$$B = \int c dy - \int 0.7y \cot \alpha dy + \int 2dc/y dy - \int 1.4d \cot \alpha dy + \int d^2 c / y^2 dy - \int 0.7yd^2 \cot \alpha / y^2 dy \quad (48)$$

The summation of the above integration yields:

$$B = ((c - 1.4d \cot \alpha)y - 0.7 \cot \alpha / 2 + 2dc \ln(y) + d^2 c / y - 0.7d^2 \cot \alpha \ln(y)) \Big|_{H_0}^{H_1} \quad (49)$$

Referring to Equation (27), which is:

$$\int (y+rb \tan \alpha)^2 / ((y^2 + (c-0.7y \cot \alpha)^2)^{\frac{1}{2}} - (c-0.7y \cot \alpha)) dy \\ = \int K \tan \alpha W / \pi dt$$

Integration of the left side of Equation (27) is the summation of terms A and B. Thus,

$$A+B = K \tan^2 \alpha W t / \pi \quad (50)$$

where the expression of A and B terms are given in Equation (41) and (49) respectively. The time t in Equation (50), therefore, can be rearranged as:

$$t = (A+B) \pi / K \tan^2 \alpha W \quad (51)$$

The procedure leading to the derivation of Equation (51) is quite complicated and tedious. In a case of numerical integration carried out through all the equations, chances of making errors are definite. A computer program in BASIC language has been developed to reduce the chance of any numerical errors and provide the answers in a short period of time.

CHAPTER V

COMPUTER PROGRAM AND EXAMPLE

In order to assist the calculation of the derived equations, a computer program had been developed in BASIC language. It requires only a small computer memory of about 30K bytes. Most of the small microcomputers should be able to run the program in a short period of time. In developing this program, a Tektronix 4051 microcomputer in the computer aided design laboratory at the University of Central Florida was used. An example problem output and a listing of the computer program are presented in this chapter.

The computer program consists of four parts. Each part performs a separate function. In the first part, the geometry of the reservoir is computed based on input data from the cross section and soil properties of the earth dam and the top radius of the reservoir. The second part is to calculate the location of the phreatic line and the focus of the parabola. The third part of the program includes all equations derived in Chapter IV. In the final part, the program will plot separate graphs for seepage quantity versus reservoir height and the time versus the change of water level in the reservoir.

Sample of Computer Program

In Figure 6 a cross section of a trapezoidal homogeneous soil dam is represented. It is assumed that the dam rests on an impervious foundation and it follows that all the seepage water flows through

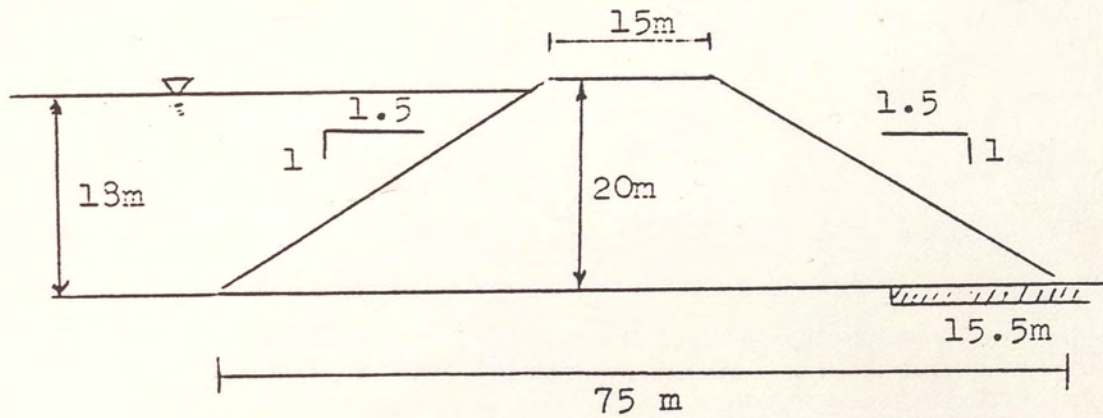


Figure 6. Cross Section of Sample Earth Dam.

the dam. At the downstream toe of the dam there is provided a horizontal drainage layer which extends some distance back under the structure and acts as a collecting gallery for the seepage water.

The dam has a width of 75.0 meters and a coefficient of permeability of 2.7×10^{-3} m/sec. Some other known properties of the earth dam are as follows:

Length of the underdrain = 15.50 m

Embankment height = 20.0 m

Starting reservoir height = 18.0 m

Right side slope ratio = 1.5

Left side slope ratio = 1.5

Top width of the earth dam = 15.0 m

Top radius of the reservoir = 100 m

Pervious width = 1.0 m

The above sample problem has been used to run the computer program. The program has option of using English or S.I. unit system. After inputting inquired data, the program will estimate the radius of the reservoir at different depths. In the next step, the program will calculate the phreatic line positions through the earth dam for different depths of water in the reservoir. The water level in the reservoir will drop each time as an increment of $1/20$ th of the original depth of water in the reservoir. The estimated time for each drop will be kept in the memory and latter graphs of discharge rate vs. reservoir drop and time vs. reservoir drop will be plotted. The following is the output of the program as it would be seen on the screen.

SELECT UNIT SYSTEM:

1 = ENGLISH SYSTEM

2 = S.I. SYSTEM

SELECT 1 OR 2 ----->? 2

INPUT THE LENGH OF UNDERDRAIN

IF NO UNDERDRAIN INPUT 0 (METER)-----> ? 15.5

INPUT EMBANKMENT HEIGHT (METER)-----> ? 20

INPUT THE WATER DEPTH (METER) -----> ? 18

INPUT RIGH SIDE SLOPE RATIO (RUN/RISE)? 1.5

INPUT LEFT SIDE SLOPE RATIO ? 1.5

INPUT THE TOP WIDTH OF THE DAM (METER)? 15

INPUT RESERVOIR TOP RADIUS (METER)? 100

INPUT COEFFICIENT OF PERMEABILITY (M/SEC)----->? 2.7E-3

INPUT EARTH DAM WIDTH (METER)----->? 1

GEOMETRY OF THE RESERVOIR

RADIUS
OF RESERVOIR

HEIGHT
FROM THE BOTTOM

-----	-----
73.00	0.00
74.50	1.00
76.00	2.00
77.50	3.00
79.00	4.00
80.50	5.00
82.00	6.00
83.50	7.00
85.00	8.00
86.50	9.00
88.00	10.00
89.50	11.00
91.00	12.00
92.50	13.00
94.00	14.00
95.50	15.00
97.00	16.00
98.50	17.00
100.00	18.00

PRESS RETURN TO CONTINUE.....

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                                 **
** DEPTH OF WATER IN RESERVOIR-----> 18 METER                **
** EMBANKMENT HEIGHT -----> 20 METER                         **
** DAM'S TOP WIDTH -----> 15 METER                           **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 99.99999 METER                **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                           **
** UNDERDRAIN LENGH -----> 15.5 METER                       **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-1.905632        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE   **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      18.00                40.60                **
**      16.20                32.52                **
**      14.40                25.30                **
**      12.60                18.92                **
**      10.80                13.40                **
**      9.00                 8.72                 **
**      7.20                 4.90                 **
**      5.40                 1.92                 **
**      3.60                -0.21                **
**      1.80                -1.48                **
**      0.00                -1.91                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 17.1 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                          **
** DAM'S TOP WIDTH -----> 15 METER                            **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                                **
** RESERVOIR TOP RADIUS -----> 98.64999 METER                 **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC            **
** EARTH DAM WIDTH -----> 1 METER                             **
** UNDERDRAIN LENGH -----> 15.5 METER                        **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5           **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-1.690788         **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE   **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
** PHREATIC HEAD                DISTANCE FROM FOCUS           **
** -----                    -----                          **
    17.10                      41.54
    15.39                      33.33
    13.68                      25.98
    11.97                      19.49
    10.26                      13.87
     8.55                      9.12
     6.84                      5.23
     5.13                      2.20
     3.42                      0.04
     1.71                     -1.26
     0.00                     -1.69
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 16.2 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                         **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                             **
** LEFT SIDE SLOPE -----> 1.5                             **
** RESERVOIR TOP RADIUS -----> 97.3 METER                   **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC          **
** EARTH DAM WIDTH -----> 1 METER                          **
** UNDERDRAIN LENGH -----> 15.5 METER                      **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5         **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-1.491753       **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF          **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE  **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      16.20                42.49                **
**      14.58                34.13                **
**      12.96                26.66                **
**      11.34                20.06                **
**      9.72                 14.34                **
**      8.10                 9.50                 **
**      6.48                 5.55                 **
**      4.86                 2.47                 **
**      3.24                 0.27                 **
**      1.62                -1.05                 **
**      0.00                -1.49                 **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 15.3 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                        **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 95.95 METER                  **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                           **
** UNDERDRAIN LENGH -----> 15.5 METER                      **
** FOCOUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5         **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-1.307974        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE  **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
**      PHREATIC HEAD              DISTANCE FROM FOCUS        **
**      -----              -----              **
**      15.30              43.43              **
**      13.77              34.93              **
**      12.24              27.33              **
**      10.71              20.62              **
**      9.18               14.80              **
**      7.65               9.88              **
**      6.12               5.85              **
**      4.59               2.72              **
**      3.06               0.48              **
**      1.53              -0.86              **
**      0.00              -1.31              **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                                **
** DEPTH OF WATER IN RESERVOIR-----> 14.4 METER             **
** EMBANKMENT HEIGHT -----> 20 METER                        **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 94.6 METER                   **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC          **
** EARTH DAM WIDTH -----> 1 METER                           **
** UNDERDRAIN LENGH -----> 15.5 METER                       **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5         **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-1.13887        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF          **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE  **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      14.40                44.38                **
**      12.96                35.73                **
**      11.52                27.99                **
**      10.08                21.17                **
**      8.64                 15.25                **
**      7.20                 10.24                **
**      5.76                 6.14                 **
**      4.32                 2.96                 **
**      2.88                 0.68                 **
**      1.44                 -0.68                **
**      -0.00                -1.14                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 13.5 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                          **
** DAM'S TOP WIDTH -----> 15 METER                            **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 93.24999 METER                 **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC            **
** EARTH DAM WIDTH -----> 1 METER                            **
** UNDERDRAIN LENGH -----> 15.5 METER                        **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5           **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.9838848         **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF            **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE    **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE     **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      13.50                45.32                **
**      12.15                36.53                **
**      10.80                28.65                **
**      9.45                 21.71                **
**      8.10                 15.69                **
**      6.75                 10.59                **
**      5.40                  6.43                **
**      4.05                  3.18                **
**      2.70                  0.87                **
**      1.35                 -0.52                **
**      -0.00                 -0.98                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                                 **
** DEPTH OF WATER IN RESERVOIR-----> 12.6 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                         **
** DAM'S TOP WIDTH -----> 15 METER                           **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 91.89999 METER               **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                            **
** UNDERDRAIN LENGH -----> 15.5 METER                       **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.8424492        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE  **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE   **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      12.60                46.27                **
**      11.34                37.32                **
**      10.08                29.31                **
**      8.82                 22.24                **
**      7.56                 16.12                **
**      6.30                 10.94                **
**      5.04                  6.70                **
**      3.78                  3.40                **
**      2.52                  1.04                **
**      1.26                 -0.37                **
**      0.00                 -0.84                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 11.7 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                          **
** DAM'S TOP WIDTH -----> 15 METER                            **
** RIGHT SIDE SLOPE -----> 1.5                                **
** LEFT SIDE SLOPE -----> 1.5                                **
** RESERVOIR TOP RADIUS -----> 90.55 METER                    **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC            **
** EARTH DAM WIDTH -----> 1 METER                             **
** UNDERDRAIN LENGH -----> 15.5 METER                        **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.7140274        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF            **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE   **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      11.70                47.21                **
**      10.53                38.11                **
**      9.36                 29.96                **
**      8.19                 22.77                **
**      7.02                 16.54                **
**      5.85                 11.27                **
**      4.68                 6.95                 **
**      3.51                 3.60                 **
**      2.34                 1.20                 **
**      1.17                 -0.23                **
**      0.00                 -0.71                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                                 **
** DEPTH OF WATER IN RESERVOIR-----> 10.8 METER              **
** EMBANKMENT HEIGHT -----> 20 METER                         **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 89.2 METER                   **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                           **
** UNDERDRAIN LENGH -----> 15.5 METER                      **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.5980587        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE   **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
**      PHREATIC HEAD                DISTANCE FROM FOCUS      **
**      -----                -----                **
**      10.80                48.16                **
**      9.72                 38.90                **
**      8.64                 30.61                **
**      7.56                 23.29                **
**      6.48                 16.95                **
**      5.40                 11.59                **
**      4.32                 7.20                 **
**      3.24                 3.79                 **
**      2.16                 1.35                 **
**      1.08                 -0.11                **
**      0.00                 -0.60                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 9.900003 METER          **
** EMBANKMENT HEIGHT -----> 20 METER                        **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                             **
** LEFT SIDE SLOPE -----> 1.5                             **
** RESERVOIR TOP RADIUS -----> 87.85 METER                  **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                           **
** UNDERDRAIN LENGH -----> 15.5 METER                      **
** FOCOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5        **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.4940148        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE  **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
** PHREATIC HEAD                DISTANCE FROM FOCUS          **
** -----                -----                **
** 9.90                49.10                **
** 8.91                39.68                **
** 7.92                31.25                **
** 6.93                23.81                **
** 5.94                17.36                **
** 4.95                11.91                **
** 3.96                7.44                **
** 2.97                3.97                **
** 1.98                1.49                **
** 0.99                0.00                **
** 0.00                -0.49                **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 9.000004 METER          **
** EMBANKMENT HEIGHT -----> 20 METER                        **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                             **
** LEFT SIDE SLOPE -----> 1.5                             **
** RESERVOIR TOP RADIUS -----> 86.5 METER                   **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                           **
** UNDERDRAIN LENGH -----> 15.5 METER                      **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.4013767        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF          **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE  **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
** PHREATIC HEAD                DISTANCE FROM FOCUS          **
** -----                -----                **
**      9.00                50.05                **
**      8.10                40.46                **
**      7.20                31.89                **
**      6.30                24.32                **
**      5.40                17.76                **
**      4.50                12.21                **
**      3.60                7.67                 **
**      2.70                4.14                 **
**      1.80                1.62                 **
**      0.90                0.10                 **
**      -0.00              -0.40                 **
** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 8.100005 METER          **
** EMBANKMENT HEIGHT -----> 20 METER                        **
** DAM'S TOP WIDTH -----> 15 METER                          **
** RIGHT SIDE SLOPE -----> 1.5                             **
** LEFT SIDE SLOPE -----> 1.5                             **
** RESERVOIR TOP RADIUS -----> 85.15 METER                  **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                          **
** UNDERDRAIN LENGH -----> 15.5 METER                      **
** FOCOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5        **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.3196468        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF          **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE  **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
**      PHREATIC HEAD      DISTANCE FROM FOCUS      **
**      -----      -----      **
**      8.10      50.99      **
**      7.29      41.25      **
**      6.48      32.52      **
**      5.67      24.82      **
**      4.86      18.15      **
**      4.05      12.51      **
**      3.24      7.89       **
**      2.43      4.30       **
**      1.62      1.73       **
**      0.81      0.19       **
**      0.00      -0.32      **
**
** PRESS RETURN TO CONTINUE.....

```

** CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE **

** **

** DEPTH OF WATER IN RESERVOIR-----> 7.200004 METER

** EMBANKMENT HEIGHT -----> 20 METER

** DAM'S TOP WIDTH -----> 15 METER

** RIGHT SIDE SLOPE -----> 1.5

** LEFT SIDE SLOPE -----> 1.5

** RESERVOIR TOP RADIUS -----> 83.8 METER

** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC

** EARTH DAM WIDTH -----> 1 METER

** UNDERDRAIN LENGH -----> 15.5 METER

** FOCOUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5 **

** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.2483254 **

** **

** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF **

** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE. **

POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE

** **

** **

** PHREATIC HEAD DISTANCE FROM FOCUS **

** ----- **

7.20 51.94

6.48 42.03

5.76 33.15

5.04 25.32

4.32 18.54

3.60 12.80

2.88 8.10

2.16 4.45

1.44 1.84

0.72 0.27

0.00 -0.25

** PRESS RETURN TO CONTINUE.....

```

** PRESS RETURN TO CONTINUE.....

```

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 5.400004 METER          **
** EMBANKMENT HEIGHT -----> 20 METER                          **
** DAM'S TOP WIDTH -----> 15 METER                            **
** RIGHT SIDE SLOPE -----> 1.5                                **
** LEFT SIDE SLOPE -----> 1.5                                **
** RESERVOIR TOP RADIUS -----> 81.1 METER                     **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC            **
** EARTH DAM WIDTH -----> 1 METER                             **
** UNDERDRAIN LENGH -----> 15.5 METER                        **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-.1350861        **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF           **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE   **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
** PHREATIC HEAD                DISTANCE FROM FOCUS          **
** -----                -----                **
** 5.40                53.83                **
** 4.86                43.58                **
** 4.32                34.40                **
** 3.78                26.31                **
** 3.24                19.29                **
** 2.70                13.36                **
** 2.16                8.50                **
** 1.62                4.72                **
** 1.08                2.02                **
** 0.54                0.40                **
** 0.00                -0.14                **
** PRESS RETURN TO CONTINUE.....

```

** CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE **

** **

** DEPTH OF WATER IN RESERVOIR-----> 4.500004 METER

** EMBANKMENT HEIGHT -----> 20 METER

** DAM'S TOP WIDTH -----> 15 METER

** RIGHT SIDE SLOPE -----> 1.5

** LEFT SIDE SLOPE -----> 1.5

** RESERVOIR TOP RADIUS -----> 79.75 METER

** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC

** EARTH DAM WIDTH -----> 1 METER

** UNDERDRAIN LENGH -----> 15.5 METER

** FOCOUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5 **

** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-9.226799E-02 **

** ***** **

** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF **

** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE **

** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE **

** **

** **

** **

** PHREATIC HEAD DISTANCE FROM FOCUS **

** ----- **

4.50 54.78

4.05 44.35

3.60 35.02

3.15 26.79

2.70 19.66

2.25 13.62

1.80 8.69

1.35 4.85

0.90 2.10

0.45 0.46

0.00 -0.09

** PRESS RETURN TO CONTINUE.....

** CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE **

** **

** DEPTH OF WATER IN RESERVOIR-----> 3.600004 METER

** EMBANKMENT HEIGHT -----> 20 METER

** DAM'S TOP WIDTH -----> 15 METER

** RIGHT SIDE SLOPE -----> 1.5

** LEFT SIDE SLOPE -----> 1.5

** RESERVOIR TOP RADIUS -----> 78.4 METER

** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC

** EARTH DAM WIDTH -----> 1 METER

** UNDERDRAIN LENGH -----> 15.5 METER

** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5 **

** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-5.808068E-02 **

** **

** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF **

** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE **

POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE

** **

** **

** PHREATIC HEAD DISTANCE FROM FOCUS **

** ----- **

3.60 55.73

3.24 45.13

2.88 35.64

2.52 27.28

2.16 20.02

1.80 13.89

1.44 8.87

1.08 4.96

0.72 2.17

0.36 0.50

0.00 -0.06

** PRESS RETURN TO CONTINUE.....

```

*****
**      CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE      **
**      .....                                              **
** DEPTH OF WATER IN RESERVOIR-----> 2.700004 METER          **
** EMBANKMENT HEIGHT -----> 20 METER                          **
** DAM'S TOP WIDTH -----> 15 METER                            **
** RIGHT SIDE SLOPE -----> 1.5                               **
** LEFT SIDE SLOPE -----> 1.5                               **
** RESERVOIR TOP RADIUS -----> 77.05 METER                   **
** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC           **
** EARTH DAM WIDTH -----> 1 METER                            **
** UNDERDRAIN LENGH -----> 15.5 METER                       **
** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5          **
** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-3.214264E-02    **
**                                                              **
*****
** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF          **
** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE  **
** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE    **
**                                                              **
**                                                              **
** PHREATIC HEAD                DISTANCE FROM FOCUS          **
** -----                -----                **
** 2.70                56.67                **
** 2.43                45.90                **
** 2.16                36.26                **
** 1.89                27.75                **
** 1.62                20.38                **
** 1.35                14.14                **
** 1.08                9.04                 **
** 0.81                5.07                 **
** 0.54                2.24                 **
** 0.27                0.53                 **
** -0.00               -0.03                 **
** PRESS RETURN TO CONTINUE.....

```

** CASSAGRANDE METHOD USED TO ESTIMATE PHREATIC LINE **
**

** DEPTH OF WATER IN RESERVOIR-----> 1.800004 METER

** EMBANKMENT HEIGHT -----> 20 METER

** DAM'S TOP WIDTH -----> 15 METER

** RIGHT SIDE SLOPE -----> 1.5

** LEFT SIDE SLOPE -----> 1.5

** RESERVOIR TOP RADIUS -----> 75.7 METER

** COEFFICIENT OF PERMEABILITY -----> .0027 M/SEC

** EARTH DAM WIDTH -----> 1 METER

** UNDERDRAIN LENGH -----> 15.5 METER

** FOCUS OF PARABOLA FROM THE TOE OF EMBANKMENT 15.5 **

** DISTANCE FROM FOCUS TO O HYDROSTATIC HEAD=-1.405144E-02 **

**

** O,O COORDINATE WOULD BE AT THE DOWNSTREAM,END OF **

** UNDERDRAIN. POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE **

** POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE **

**

**

**

** PHREATIC HEAD DISTANCE FROM FOCUS **

** -----

1.80 57.63

1.62 46.68

1.44 36.88

1.26 28.23

1.08 20.74

0.90 14.40

0.72 9.21

0.54 5.17

0.36 2.29

0.18 0.56

0.00 -0.01

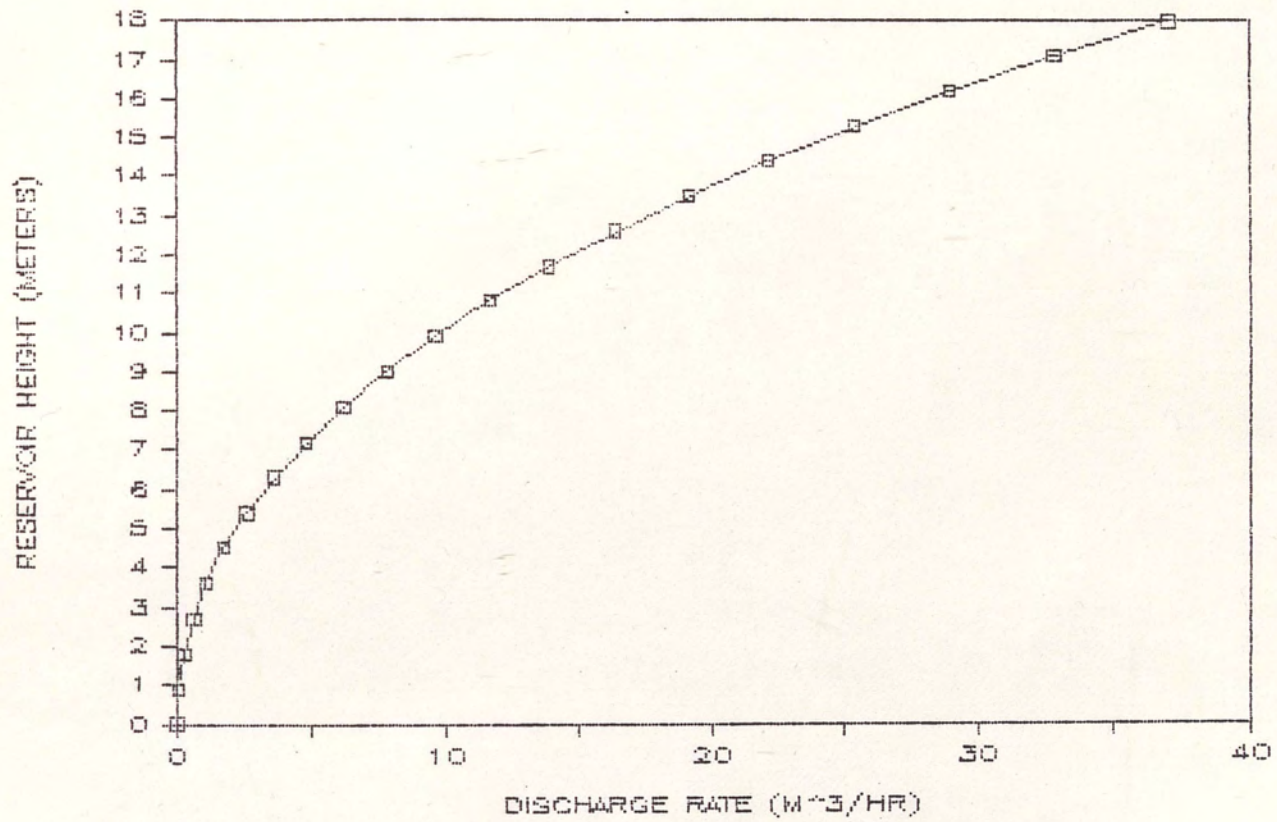
** PRESS RETURN TO CONTINUE.....

ORIGINAL WATER SURFACE WAS @ 19 METER

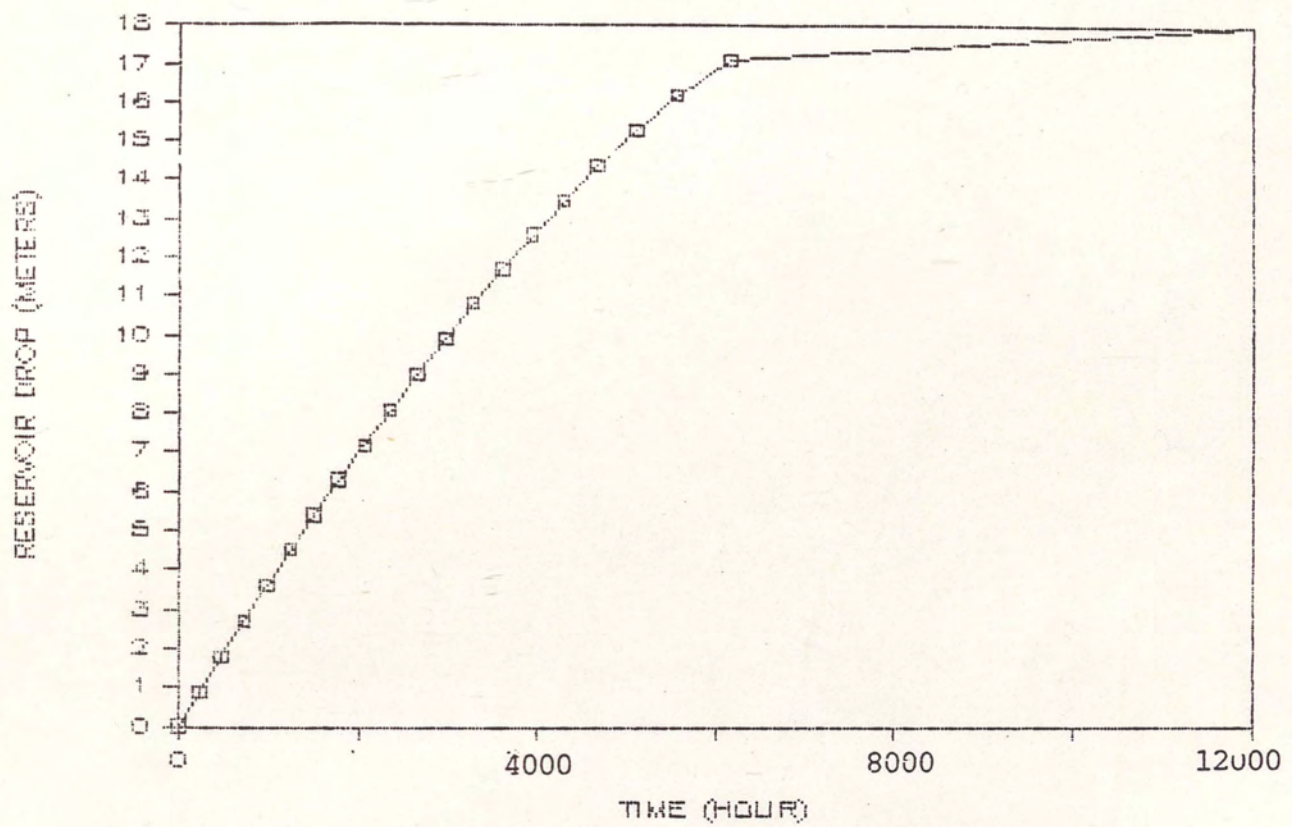
CHANGE OF DEPTH IN RESERVOIR	TIME (HOUR)	DISCHARGE RATE (q) (METER ³ /HR)
19.00- 17.10	39.48198	37.04549
17.10- 16.20	42.07146	32.86893
16.20- 15.30	44.972	28.99967
15.30- 14.40	48.23666	25.42701
14.40- 13.50	51.93682	22.13964
13.50- 12.60	56.15842	19.12672
12.60- 11.70	61.01672	16.37721
11.70- 10.80	66.66159	13.88069
10.80- 9.90	73.29525	11.62626
9.90- 9.00	81.20186	9.603648
9.00- 8.10	90.77628	7.802764
8.10- 7.20	102.6112	6.213935
7.20- 6.30	117.6087	4.827445
6.30- 5.40	137.2373	3.634433
5.40- 4.50	164.0358	2.626073
4.50- 3.60	202.8483	1.79369
3.60- 2.70	264.1697	1.129088
2.70- 1.80	376.023	.6248529
1.80- 0.90	649.0922	.2731599
0.90- 0.00	11761.72	6.714981E-02

PRESS RETURN TO CONTINUE.....

DISCHARGE RATE VS RESERVOIR HEIGHT



RESERVOIR DROP VS TIME



```

1 INIT
100 PAGE
110 PRINT "SELECT UNIT SYSTEM:"
120 PRINT ""
130 PRINT "1=ENGLISH SYSTEM"
140 PRINT "2=S.I. UNIT"
150 PRINT " SELECT 1 OR 2----->";
160 INPUT S9
161 PRINT
170 IF S9=1 THEN 210
180 S$="METER"
190 K$="m^3/sec/m"
200 GO TO 230
210 S$="FT. "
220 K$="FT^3/SEC/FT. "
230 T3=20
240 D2=10
250 DIM M(16),D9(16),H0(T3+1),A1(T3+1),B1(T3+1),T(T3+1)
260 PRINT "INPUT THE LENGH OF UNDERDRAIN "
270 PRINT " IF NO UNDERDRAIN INPUT 0 (";S$;")----->";
280 INPUT D1
281 PRINT
290 F1=D1
300 PRINT "INPUT EMBANKMENT HEIGHT (";S$;")----->";
310 INPUT H1
311 PRINT
320 PRINT "INPUT THE WATER DEPTH (";S$;")----->";
330 INPUT H0(1)
331 PRINT
340 H9=H0(1)
350 PRINT "INPUT RIGHT HAND SIDE SLOPE RATIO (RUN/RISE)";
360 INPUT S1
361 PRINT
370 PRINT "INPUT LEFT HAND SIDE SLOPE RATIO ----->";
380 INPUT S4
381 PRINT
390 PRINT "INPUT THE TOP WIDTH (";S$;")----->";
400 INPUT T1
401 PRINT
410 PRINT "INPUT RESERVOIUR TOP RADIUS (";S$;")----->";
420 INPUT R0
421 PRINT
430 IF R0<H0(1)*S4 THEN 450
440 GO TO 510
450 PRINT "#####ERROR:THIS RADIUS CAN NOT BE RIGHT!#####&&
460 PRINT "&& MIN. RADIUS IS ";H0(1)*S4;"&
470 PRINT "&& TRY IT AGAIN WITH A HIGHER VALUE&
480 PRINT "#####&&

```

```

500 GO TO 410
510 PRINT "INPUT COEFFICIENT OF PERMABILITY (";K$;")---->";
520 INPUT K
521 PRINT
530 PRINT "INPUT EARTH DAM WIDTH (";S$;")----->";
540 INPUT W
541 PRINT
550 FUZZ 4
560 FOR I=2 TO T3+1
570 H0(I)=H0(I-1)-H9/T3
580 NEXT I
590 DIM X(H0(1)+1),X4(T3+1)
600 X4=0
610 R6=(R0/S4-H0(1))/(1/S4)
620 FOR Y=1 TO H0(1)+1
630 X(Y)=(Y-1+R6/S4)/(1/S4)
640 NEXT Y
650 PAGE
700 PRINT "                                GEOMETRY OF RESERVOIR"
710 PRINT
720 PRINT "                                RADIUS                                HEIGHT"
730 PRINT "                                OF RESERVOIR                                FROM THE BOTTOM"
740 PRINT "                                -----                                -----"
750 FOR I=1 TO H0(1)+1
760 PRINT USING 770:X(I),I-1
770 IMAGE 7X,5D.2D,25X,5D.2D
780 NEXT I
790 PRINT "PRESS RETURN TO CONTINUE....."
800 INPUT A$
810 FOR J=1 TO T3+1
820 L1=H1*S1
830 L1=L1-D1
840 L1=L1+T1
850 H3=H1-H0(J)
860 L2=S4*H3
870 L3=H0(J)*S4*0.3
880 L=L1+L2+L3
890 X1=(L^2+H0(J)^2)^0.5-L
900 X4(J)=(0-X1)/2
910 D3=H0(J)/D2
920 D4=D3
930 M(1)=(H0(J)^2-X1^2)/(2*X1)
940 D9(1)=H0(J)
950 FOR I=2 TO D2+1
960 M(I)=((H0(J)-D3)^2-X1^2)/(2*X1)
970 D9(I)=H0(J)-D3
980 D3=D3+D4
990 NEXT I

```

```

1000 Z=S4
1010 A=1+0.49*Z^2
1020 C=L1+H1*Z
1030 C1=C^2
1040 B=-1.4*C*Z
1050 D=R6/Z
1060 I1=0
1070 Y=H0(J)
1080 I1=I1+1
1090 P=SQR(A*Y^2+B*Y+C1)
1100 P1=P+Y*SQR(A)+B/(2*SQR(A))
1110 P2=LOG(P1)
1120 F8=A*((Y/(2*A))-3*B/(4*A^2))*P+(3*B^2-4*A*C1)/(8*A^2)*1/SQR(A)*P2
1130 F2=(B+2*A*D)*(P/A-B/(2*A)*P2/SQR(A))
1140 F3=(C1+A*D^2+2*B*D)*P2/SQR(A)
1150 P3=(P+C)/Y+B/(2*C)
1160 P4=LOG(P3)
1170 F4=(2*D*C1+B*D^2)*(-P4/C)
1180 F5=D^2*C1*(-P/(C1*Y)-B/(2*C1))*(-P4/C)
1190 E1=(C-1.4*D*Z)*Y
1200 E2=0.7*Z*Y^2/2
1210 E3=2*D*C*LOG(Y)
1220 E4=D^2*C/Y
1230 E5=0.7*D^2*Z*LOG(Y)
1240 IF I1=2 THEN 1300
1250 A8=F8+F2+F3+F4+F5
1260 B8=E1-E2+E3+E4-E5
1270 Y=H0(1)-H0(2)
1280 Y=H0(J)-Y
1290 GO TO 1080
1300 A9=F8+F2+F3+F4+F5
1310 B9=E1-E2+E3+E4-E5
1320 A1(J)=A8-A9
1330 B1(J)=B8-B9
1340 T(J)=(A1(J)+B1(J))*3.14/(K*W*Z^2)
1341 T(J)=T(J)/3600
1350 IF Y<1.0E-4 THEN 1720
1360 GO TO 1390
1370 NEXT J
1380 END
1390 PAGE
1400 PRI "*****"
1410 PRINT " CASSAGRANDE METHOD USED FOR ESTIMATING PHREATIC LINE"
1420 PRINT "....."
1430 PRINT "DEPTH OF WATER----->";H0(J);S$
1440 PRINT "EMBANKMENT HEIGHT----->";H1;S$
1450 PRINT "TOP WIDTH----->";T1;S$
1460 PRINT "RIGHT SIDE SLOPE----->";S1
1470 PRINT "LEFT SIDE SLOPE----->";S4
1480 PRINT "RESERVOIR TDP RADIUS----->";(H0(J)+R6*(1/Z))/(1/Z);S$

```

```

1490 PRINT "COEFFICIENT OF PERMIBILITY->";K;K$
1500 PRINT "EARTH DAM WIDTH----->";W;S$
1510 PRINT "UNDERDRAIN LENGH----->";D1;S$
1520 PRINT "FOCUS OF PRABOLA FROM THE TOE OF EMBANKMENT ";
1530 PRINT USING 1540:F1
1540 IMAGE 5D.2D
1550 PRINT "DISTANCE FROM FOCUS TO 0 HYDROSTATIC HEAD=";
1560 PRINT USING 1570:X4(J)
1570 IMAGE 5D.2D
1580 PRINT ""
1590 PRINT "OUR 0,0 COORDINATE WOULD BE AT THE END OF UNDERDRAIN"
1600 PRINT "POSITIVE X-AXIS WILL BE TOWARD UPSTREAM FACE"
1610 PRINT "POSITIVE Y-AXIS WILL BE FROM BASIN TOWARD WATER SURFACE"
1620 PRINT ""
1630 PRINT "      PHREATIC HEAD                      DISTANCE FROM FOCUS"
1640 PRINT "      =====
1650 FOR I=1 TO D2+1
1660 PRINT USING 1670:D9(I),M(I)
1670 IMAGE 4X,6D.3D,17 X,6D.3D
1680 NEXT I
1690 PRINT "PRESS RETURN TO CONTINUE....."
1700 INPUT A$
1710 GO TO 1370
1720 PRINT "PROGRAM WILL CONTINUE IN A MOMENT!"
1730 FOR U=1 TO 999
1740 NEXT U
1750 PAGE
1760 PRINT
1770 PRINT "ORIGINAL WATER SURFACE WAS @ ";H0(1);S$
1780 PRINT
1790 PRI "-----"
1800 PRINT "|              |      TIME              | DISCHARGE RATE (q) |"
1801 PRINT "|              |      (HOUR)              |      (";S$;"^3/HR)"
1810 PRI "-----"
1820 FOR I=1 TO T3
1830 PRINT USING 1840:H0(I),H0(I+1);
1840 IMAGE "I",4D.2D,"-",4D.2D,"I",S
1850 PRINT T(I),"I ";K*2*X4(I)*-1*3600
1860 NEXT I
1870 PRINT "-----"
1880 PRINT "PRESS RETURN TO CONTINUE....."
1890 INPUT A$
1900 PAGE
1910 Q5=0
1920 FOR I=1 TO T3
1930 Q5=T(I)+Q5
1940 NEXT I
1950 VIEWPORT 30,110,15,75
1960 WINDOW 0,Q5,0,H0(1)
1970 MOVE -Q5/2.7,10

```

```

1980 PRINT "RESERVOIR"
1990 PRINT "DROP ";S$;
2000 MOVE Q5/2,-2
2010 PRINT "TIME (HOUR)"
2020 MOVE -Q5/20,-1
2030 PRINT "0"
2040 AXIS Q5/T3,H0(1)/T3
2050 Q5=0
2060 FOR I=1 TO T3
2070 Q5=T(I)+Q5
2080 DRAW Q5,H0(1)-H0(I+1)
2090 NEXT I
2100 MOVE -Q5/15,0
2110 Q6=0
2120 FOR I=1 TO T3
2130 Q6=Q6+H0(1)/T3
2140 MOVE -Q5/8.5,Q6
2150 PRINT Q6
2160 NEXT I
2170 MOVE Q5,-3
2180 PRINT USING 2190:Q5
2190 IMAGE +2E
2200 FOR I=1 TO 6000
2210 NEXT I
2220 PAGE
2230 Q7=K*-2*X4(1)*3600
2240 VIEWPORT 30,110,15,75
2250 WINDOW 0,Q7,0,H0(1)
2260 MOVE -Q7/2.7,10
2270 PRINT "RESERVOIR"
2280 PRINT "  HEIGHT"
2281 PRINT " (";S$;")"
2290 MOVE Q7/3,-2
2300 PRINT "DISCHARGE RATE (q) "
2301 PRINT " (";S$;"^3/HR. )"
2310 MOVE -Q7/20,-1
2320 PRINT "0"
2330 MOVE -Q7/20,0
2340 Q8=0
2350 FOR I=1 TO T3
2360 Q8=Q8+H0(1)/T3
2370 MOVE -Q7/8.7,Q8
2380 PRINT Q8
2390 NEXT I
2400 MOVE Q7,-3
2410 PRINT USING 2420:Q7
2420 IMAGE +2E
2430 AXIS Q7/T3,H0(1)/T3
2440 FOR I=1 TO T3+1
2450 MOVE K*-2*X4(I)*3600,H0(I)
2460 PRINT "*"
2470 NEXT I

```

CHAPTER VI

SUMMARY AND CONCLUSIONS

When water flows through the earth dam under a hydraulic gradient, the differential pressure head produces a force on the soil grains in the direction of the flow. The pressure accumulation, called seepage force, and falling head concepts have been used earlier to derive the equation of time that it would take for water surface to decrease to another depth in a reservoir and flow through the earth dam.

An approximation approach has been used to solve the two-dimensional seepage through a homogenous dam on an impervious foundation with a toe drain in this paper. The derived equation of time and the computer program developed for this matter can be an aid to an engineer because it is computationally efficient, accurate, and fast and only requires a fraction of the effort and time that the other approaches do.

The water level in the reservoir is assumed to be constant and no evaporation or rainfall effects its depth. However, in the case that evaporation and rainfall has significant effect on water level, it can be corrected by using a monthly precipitation chart available for the specific located reservoir area.

Using analytical and experimental data available in the literature, the computer method was validated with computed results being on the conservative side.

As an engineering design aid, the computer method utilizes the BASIC computer language and a small micro-computer for operation.

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