Automatic Q-Factor Measurement Via a Network Analyzer

1986

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AUTOMATED Q-FACTOR MEASUREMENT VIA A NETWORK ANALYZER

BY

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B.S.E., University of Central Florida, 1984

THESIS

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ABSTRACT

An algorithm is developed for the HP automated network analyzer that can determine the three basic parameters of microwave cavity resonators: the unloaded resonant frequency $f_0$, the unloaded $Q$-factor $Q_0$, and the coupling coefficient $\beta$. The input reflection coefficient of the resonator is measured via the network analyzer, which is driven by an HP desk computer. The graphical techniques (Smith Chart) are replaced by the much faster calculations on the computer using numerical techniques. Stirling and Bessel's formulas, in conjunction with the central difference interpolation method, are used to interpolate between sampled points to enhance the accuracy of the algorithm.

Experimental results are compared against theoretical calculations for different cavities to support the validity of the algorithm.
I would like first to acknowledge the help of my advisor, Dr. Christos Christodoulou, especially to thank him for his encouragement, advice, and time spent assisting with this research. I also thank the Electromagnetic Group for reviewing my progress as I completed the project and the members of my committee for their advice in completing this document. Most of all, I thank my family, especially my father, for their complete support throughout my entire education.
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Resonant circuits are used in the design of oscillators, tuned amplifiers, filter networks, wavemeters for measuring frequency, etc. The parameter of most importance in designing resonators is the quality factor, Q. This is a parameter that specifies the frequency selectivity and the performance of the resonator in general.

For frequencies above 1GHz cavity resonators are used rather than transmission-line resonators. A cavity can be described as a volume enclosed by conducting surfaces within which electric and magnetic energies can be stored. This kind of cavity can be excited, or coupled to an external circuit by a variety of methods, such as a small coaxial line probe or a loop.

There are a number of experimental techniques that can be utilized to find the quality factor and coupling coefficient of microwave resonators [1]. Today, with the advent of modern network analyzers, the experimental process has been simplified to a great extent [2]. The purpose of this thesis is to develop a computer algorithm which can be used along with a network analyzer so that any Q-
measurements can be done in a completely automatic and easy fashion. An HP-85 computer is used to drive a HP-8084A vector network analyzer. This particular software program can be used for any computer and HP network analyzer. Experimental results are compared with theoretical results to support the validity of the algorithm.
A microwave cavity can be coupled to one, two, or more transmission lines. The complete analysis of the cavity characteristics and the effect of the coupling with transmission lines can be evaluated by performing as many separate experiments as there are coupled transmission lines. The behavior of the total system can be studied in various ways. The simplest way is to observe the changes in the power delivered to the load as the frequency of the signal source is varied or the cavity is tuned. In general, the known resonance phenomenon occurs, causing the power output at resonance to be different from the detuned condition in a substantial way. The exact behavior of the system depends upon the characteristics of the cavity and degree of coupling between the cavity and the transmission lines.

Sometimes it is desired to measure the cavity parameters and the coefficients of coupling between the cavity and the transmission lines. Usually, it is simpler to determine the characteristics of the cavity by assuming that the cavity is not disturbed by the presence of the coupled transmission lines. Thus, it is easier to determine
experimentally the degree of coupling between the cavity and the transmission lines by specifying $Q_0$, $Q_L$, and $Q_{ext}$; the unloaded, the loaded, and the external quality factors, respectively.

Consider a cavity coupled to a source whose internal impedance is equal to the characteristic impedance of the transmission line as shown in Figure 2-la. The equivalent circuit of the cavity and the transmission line is depicted in Figure 2-lb, where the terminals of the coupling system are assumed to be at some arbitrary location 1-1 close to the cavity. $L_1$ is the self inductance of the coupling mechanism and $M$ is the mutual inductance between it and the cavity inductance $L$. Any resistive losses in the coupling network are neglected. A more simplified version is shown in Figure 2-1c and d.

(a) The circuit of the cavity-coupling system
(b) The equivalent circuit of the cavity in (a).

(c) Equivalent circuit of the cavity impedance with referred to the primary

\[ R_s + j \left( \frac{(2 \pi f M)^2}{2 \pi f L - \frac{1}{2 \pi f c}} \right) \]

(d) Equivalent circuit of the cavity with the impedance referred to the secondary

Figure 2-1: Cavity coupled to a voltage source through a standing-wave detector.

The impedance coupled in series with cavity parameters due to a matched generator is given by \([1]\).
The loaded Q-factor of the system is defined as the ratio of total reactance to total series loss. That is:

\[ Q_L = \frac{2 \pi f L - \beta R_s X_1/Z_o}{R_s (1 + \beta)} \]  

(2-3)

\[ = \frac{2 \pi f L}{R_s} \left( 1 - \frac{\beta R_s / Z_o}{1 + \beta} \right) \frac{X_1 / 2 \pi f L}{1 + \beta} \]  

(2-4)

where

\[ X_1 = 2 \pi f L_1 \]  

(2-5a)

\[ = \frac{(2 \pi f M)^2}{Z_o R_s} \frac{1}{1 + (X_1 / Z_o)^2} \]  

(2-5b)

The second term in the numerator of Equation (2-4), representing the ratio of coupled reactance to the cavity, is usually less than one and can be neglected. Equation (2-4) then becomes:

\[ Q_L = \frac{Q_o}{1 + \beta} \]  

(2-6)

where

\[ Q_o = \frac{2 \pi f L}{R_s} \]  

(2-7)
Equation (2-5b) can be written as

$$\beta = \beta_1 \frac{1}{1 + \left(\frac{fM}{Z_0}\right)^2}$$  \hspace{1cm} (2-8)

where

$$\beta_1 = \frac{(2 \pi fM)^2}{Z_0 R_s}$$  \hspace{1cm} (2-9)

is the ratio of the coupled resistance to the cavity resistance $R_s$. If $\beta_1 = 1$, then the coupled resistance and cavity losses are equal, and the cavity is critically coupled. When $\beta_1 < 1$, the cavity is undercoupled, and finally, when $\beta_1 > 1$, the cavity is called overcoupled. Usually, the second term in Equation (2-8) is close to unity and $\beta \approx \beta_1$. Hence, at critical coupling, $Q_L \approx Q_o/2$.

Equation (2-6) can be written as

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{\beta}{Q_o}$$  \hspace{1cm} (2-10)

or

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$  \hspace{1cm} (2-11)

where $Q_{ext} = Q_o/\beta$  \hspace{1cm} (2-12)

**Types of Q Measurements**

The various experimental techniques that can be used to determine experimentally the $Q$-factor of a resonant cavity can be divided into four categories:

1. Transmission method
2. Impedance measurement
3. Transient decay or the decrement method
4. Dynamic method.

The transmission method [1] shown in Figure 2-2a is the simplest measurement of Q. In Figure 2-2b, the transmission resonance curve is depicted. The output cavity signal is measured as a function of frequency, resulting in the known resonance curve which can be used to determine the bandwidth and Q factor. Although it is a seemingly simple approach, there are practical difficulties in its application. That means that it is necessary to pay serious attention to tuning and calibrating the devices under test, and the coupling between the cavity and the generator to assure accurate results.

![Diagram](a)

Figure 2-2: Measurement of Q-factor using the transmission method. (a) Experimental set-up (b) Output transmission curve.
The decrement method [1], which is more suitable for high Q cavities, utilizes transient decay of the natural oscillations in the cavity. Figure 2-3 shows the experimental set-up necessary for this measurement method. If the cavity under study is excited by a pulsed signal, during the off period, the natural fields in the cavity decay exponentially with time and the time constant of decay determines the Q.

![Diagram of transient decay method]

Figure 2-3: The transient decay method
(a) Typical experimental set-up
(b) The display on the oscilloscope of the transient decay method
Next, the dynamic method [1] makes use of the dynamic observation of the cavity characteristics. Figure 2-4 shows an experimental set-up which permits the transmission resonance curve of a high Q cavity to be compared directly against the transmission curve of a low frequency resonance circuit.

This technique is useful for two reasons: a) the frequency stability requirements of the source are decreased, and b) the Q-factor can be obtained much faster.

![Diagram](image)

**Figure 2-4:** The set-up for dynamic method

The final method, the impedance method, was used for developing the algorithm for finding the resonant Q using the automated network analyzer. This method will be presented and discussed in detail in the following chapter.
CHAPTER 3
FORMULATION OF IMPEDANCE MEASUREMENT USING AUTOMATIC NETWORK ANALYSIS

Impedance Formulation

By measuring the input impedance of a cavity as a function of frequency, the cavity characteristics can be determined.

The input impedance, as a function of frequency of a typical cavity, describes a nearly perfect circle. Ginzton [1] describes in detail the analysis of measured circles on the Smith chart in order to calculate the values of $Q_0$ and $\beta$. Ginzton's procedure was developed when impedances were measured with the use of slotted lines. Today, the impedance is measured much faster with the network analyzer. The reference plane of such a measurement coincides with the end of transmission line leading to the cavity. In the measurement method to be presented, the cavity is located at the end of the transmission line which leads to the network analyzer. The input impedance $Z_I$ of an inductance-coupled cavity is shown in the equivalent circuit Figure 3-1. The transmission line with characteristic impedance $R_0$ to the
Figure 3-1: Simplified equivalent circuit for the vicinity of frequency \( f_0 \) left of the input terminals represents the adapter leading to the network analyzer.

The impedance measurement is taken in the narrow range of frequencies around the value \( f_0 \), the resonant frequency under observation. For simplicity, \( R_{ext} + jX_{ext} \) in Figure 3-1 represents the equivalent impedance of the other parallel resonant circuits. In our case, \( R_{ext} \) is neglected [1].

The frequency detuning parameter \( \delta \) is defined as follows [1]:

\[
\delta = \frac{f - f_0}{f_0} \tag{3-1}
\]

The input impedance measured by the network analyzer is given by [2].

\[
Z_I = j X_{ext} + \frac{R}{1 + jQ_0 \left( \frac{f}{f_0} - \frac{f_0}{f} \right)} \tag{3-2}
\]
Where $Q_o$ is the unloaded $Q$ factor of the resonator, is determined by measurement

$$Q_o = \frac{R}{2 \pi f_o L}$$  \hfill (3-3)

Let $X_{ext}$ be represented by the first two terms of a Taylor series as given by [2]

$$X_{ext} = X_1 + 2R_oQ_1 \delta$$  \hfill (3-4)

and for the entire range of measured frequency, the following approximation is valid given by [2].

$$\frac{f}{f_o} - \frac{f}{f_0} = 2 \delta$$  \hfill (3-5)

Substitute equation (3-4) and equation (3-5) into equation 3-2.

$$Z_I = j(X_1 + 2R_oQ_1 \delta) + \frac{R}{1 + j2Q_o \delta}$$  \hfill (3-6)

Equation (3-6) can be divided into two terms. The first term in the equation is slowly varying with respect to $\delta$. Add $R_o$ to the slowly varying part of equation (3-6) to obtain

$$Z_s = R_o \left[ 1 + j \left( \frac{X_1}{R_o} + 2Q_1 \delta \right) \right]$$  \hfill (3-7)
The second term of equation (3-6) is a fast varying function with respect to $\delta$.

\[ Z_f = \frac{R}{1 + j2Q_0 \delta} \quad (3-8) \]

The input reflection coefficient is defined as

\[ \Gamma_i = \frac{Z_I - R_0}{Z_I + R_0} \quad (3-9) \]

Substituting equations (3-7) and (3-8) into equation (3-9) yields:

\[ \Gamma_i = \frac{Z_f - Z_s^*}{Z_f + Z_s} \quad (3-10a) \]

$Z_f$ in the above equation goes to zero at $f_o$ when the coupling coefficient goes to zero. Then the reflection coefficient for a decoupled resonator becomes:

\[ \Gamma_D = - \frac{Z_s^*}{Z_s} \quad (3-10b) \]

The difference between $\Gamma_i$ and $\Gamma_D$ can be cast in the form:

\[ \Gamma_i - \Gamma_D = \frac{Z_F - Z_s^*}{Z_F + Z_s} + \frac{Z_s^*}{Z_s} \quad (3-11) \]

The above equation can be simplified to [2]

\[ \Gamma_i - \Gamma_D = \frac{2R}{Z_s^2} \left( \frac{1}{Z_s} + \frac{1}{Z_F} \right) \quad (3-12) \]
Equation (3-12) can be written as [2]

\[
\Gamma_i - \Gamma_D = \frac{2e^{-j\tan^{-1}x_1}}{(1 + \frac{1}{\beta})(1 + j2Q_L \delta_L)}
\]  

(3-13)

In the above equation the \( \delta \) dependance in equation (3-7) was neglected and the loaded detuning parameter is defined [2]

\[
\delta_L = \frac{f - f_L}{f_o} = \delta - \frac{x_1}{2Q_o}
\]  

(3-14)

\( x_1 \) the normalized reactance given by

\[
x_1 = \frac{X_1}{R_o}
\]  

(3-15)

\( Q_L \) is defined in equation (2-6)

\[
Q_L = \frac{Q_o}{1 + \beta}
\]

and the coupling coefficient given

\[
\beta = \frac{R}{R_o (1 + x_1^2)}
\]

(3-16)

Equation (3-14) defines the detuning due to coupling.

When the input impedance from Figure 3-1 is plotted on a Smith chart, the loop still resembles a circle as given in equation (3-13) and shown in Figure 3-2. The magnitude of the reflection coefficient \( \Gamma_i \) is smallest at the frequency \( f_L \).

Equation (3-13) gives a vector that describes a circle on a Smith chart as shown in Figure 3-2. In the
neighborhood of $f_L$, the circle is a good approximation of actual measured loop, but when $Q_L \delta_L >> 1$, the approximation breaks down. For good results, it is wise to perform the measurement within the portion of the loop where $Q_L \delta_L \leq 1$. At $f = f_L$ equation (3-13) reaches its maximum.

\[
\Gamma_i - \Gamma_D = \ell = \frac{2}{1 + \frac{1}{\beta}}
\]

(3-17)

Then, the coupling coefficient can be calculated from equation (3-17) or

\[
\beta = \frac{2}{\ell} - \ell
\]

(3-18)

The value of $\ell$ can be accurately measured on the Smith chart of Figure 3-2.

![Figure 3-2: Input reflection coefficient as a function of frequency](image-url)
The value of $x_1$ can be found using the Smith chart in Figure 3-2 as the normalized reactance corresponding to the point $\Gamma_D$. The value of $x_1$ is of secondary importance in the entire measurement, since it does not appear in equation (3-18).

The value of $Q_L$ can be found using equation (3-13). $\delta_L$ is the only variable in equation (3-13), as the frequency varies. The angle $\theta$ in Figure 3-2 is given by [2].

$$\tan\theta_L = -2Q_L \delta_L \quad (3-19)$$

From equation (3-19) $\theta_L$ equal to zero when $\delta_L = 0$ or when $f = f_L$. If one selects two frequencies $f_1$ and $f_h$, then equation (3-14) can be rewritten as

$$\delta_L = \frac{f_h - f_1}{f_0} \approx \frac{f_h - f_1}{f_1} \quad (3-20)$$

Solve for $Q_L$ in equation (3-19) and substitute $\delta_L$ from equation (3-20), $Q_L$ will take the following form

$$Q_L = \frac{f_L}{f_h - f_1} \tan\theta_L \quad (3-21)$$

The above equation for arbitrary choice of $\theta_L$. If one wants to minimize possible errors to the fact that the loop locus differs from the idealized circle, select a small angle $\theta_L$ for measurement purpose. A convenient choice may be the angle $26.26^\circ$ [2] which gives $\tan\theta_L = 0.5$. But in the following measurements, one will have to take much smaller
angles to perform the measurement since the computer memory limits the choice. That fact depends on each individual measurement and has to be selected by the operator.

Knowing $\beta$ from equation (3-18) and $Q_L$ from equation (3-21), using equation (2-6) one can solve for $Q_0$

$$Q_0 = Q_L (1 + \beta) \quad (3-22)$$

One can calculate $Q_0$ using the above equation by measuring the input reflection coefficient using the automatic vector network analyzer. In the following measurement, the Hewlett-Packard (HP) 8408A network analyzer controlled by an HP86 computer.

**Impedance Measurement Using Automated Network Analysis**

In the following measurement, the 1183A Accuracy Enhancement PAC (for the 8408-Series Automatic Network Analyzer with HP-85A desk top computer) has been used. In Figure 3-3, the set-up for the measurement used is shown.

![Figure 3-3: Measurement Set-up](image)
The 1183A Accuracy Enhancement PAC program includes an eight term error model that provides directivity, source match, and frequency response vector error correction for reflections measurements. This model is appropriate for fully error corrected reflections measurements. For measurements on devices using APC-7, Type-N, APC-3.5, and SMA connectors, the standard reflections calibration sequence uses a short circuit, a shielded open circuit, and a sliding load and/or fixed load at the reference plane. The 11873A software provides broadband (500MHz - 18GHz) reflection measurements.

To illustrate the method that was used to measure the Q-factor using the Automated Network Analyzer, consider a waveguide cavity shown in Figure 3-4.

![Figure 3-4: A Waveguide Resonant Cavity](image)

After calibrating the HP8408A Network Analyzer and taking the measurements, the input reflection coefficient of the waveguide cavity is shown on Smith Chart in Figure 3-5 and in Table 3-1. The measurements were taken between 6.590GHz and 6.630 GHz with frequency increment of 0.01 GHz.
The input reflection coefficients are usually taken with respect to the center of the Smith chart, i.e., the reference point is the center of the Smith chart.

| Frequency Step | Frequency $f_i$ (GHz) | Amplitude $|\Gamma_i|$ | Angle (Deg) $\phi_i$ |
|----------------|-----------------------|------------------------|---------------------|
| 1              | 6.590                 | 0.975                  | 69.                 |
| 2              | 6.600                 | 0.932                  | 63.                 |
| 3              | 6.610                 | 0.424                  | -4.                 |
| 4              | 6.620                 | 0.960                  | -35.                |
| 5              | 6.630                 | 0.977                  | -75.                |

Table 3-1: The input reflection coefficient ($\Gamma_i$) of waveguide cavity of Figure 3-4.

![Figure 3-5: The input reflection coefficient of waveguide cavity of Table 3-1](image-url)
To calculate the unloaded Q-factor ($Q_0$) using Equation (3-22), it is required first to find the unloaded Q-factor ($Q_L$) and the coupling coefficient ($\beta$) of the waveguide cavity using Equation (3-1). To find $Q_L$ using Equation (3-21), it is required to find $f_L$ and $f_h$ and $f_1$. To find $f_L$, the nearest point on the reflection coefficient circle to the center of the Smith chart in Figure 3-5 or the minimum input reflection coefficient in Table 3-1 will correspond to $f_L$. By examining Table 3-1 and Figure 3-4, it is found that $f_L = 6.61$ GHz. To find $f_1$ and $f_h$, one should change the reference plane from the center of the Smith chart to the loaded frequency $f_L$, and represent the input reflection coefficient angle with respect to load reflection coefficient angle.

To change the reference plane and to find the value of each angle with respect to the load angle, first the input reflection coefficient should be changed from polar coordinates to cartesian coordinates. These changes are made using the following equations:

$$\vec{f}_i = |\vec{f}_i| \cos \theta_i \vec{a}_x + |\vec{f}_i| \sin \theta_i \vec{a}_y$$ (3-23)

and

$$\vec{f}_L = |\vec{f}_L| \cos \theta_L \vec{a}_x + |\vec{f}_L| \sin \theta_L \vec{a}_y$$ (3-24)
and the decoupled reflection coefficient becomes:

\[ \vec{r}_D = |\vec{r}_D| \cos \phi_D \, \vec{a}_x + |\vec{r}_D| \sin \phi_D \, \vec{a}_y \]  \hfill (3-25)

By examining Equation (3-10b), \(|\vec{r}_D| = 1\) and by examining Figure 3-2, we get \(\phi_D = \phi_L + 180^\circ\). Then Equation (3-25) becomes:

\[ \vec{r}_D = \cos (\phi_L + 180^\circ) \vec{a}_x + \sin (\phi_L + 180^\circ) \vec{a}_y \]  \hfill (3-26)

To find the reflection coefficient angle \(\phi_{\text{in}}\) with respect to the load angle \(\phi_L\), we use:

\[ \phi_{\text{in}} = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|} \right) \]  \hfill (3-27)

where

\[ \vec{A} = \vec{r}_L - \vec{r}_D \]  \hfill (3-28a)

and

\[ \vec{B} = \vec{r}_i - \vec{r}_D \]  \hfill (3-28b)

Substituting Equations (2-28a) and (3-28b) into Equation (3-27) yields

\[ \phi_{\text{in}} = \cos^{-1} \left( \frac{(\vec{r}_L - \vec{r}_D)(\vec{r}_i - \vec{r}_D)}{|\vec{r}_L - \vec{r}_D| \cdot |\vec{r}_i - \vec{r}_D|} \right) \]  \hfill (3-29)

The angle \(\phi_{\text{in}}\) is expressed with respect to the load angle. By examining Equation (3-29), it can be seen that when \(\vec{r}_i = \vec{r}_L\), then \(\phi_{\text{in}}\) equals zero.
Using the data from Table 3-1 and Equation (3-29), $\phi_i$ in Table 3-1 becomes $\phi_{in}$ in Table 3-2.

| Frequency Step | Frequency $f_i$ (GHz) | $|\Gamma_i|$ | $\phi_{in}$ (Deg) |
|---------------|-----------------------|--------------|------------------|
| 1             | 6.590                 | 0.975        | 36               |
| 2             | 6.600                 | 0.932        | 32               |
| 3             | 6.610                 | 0.424        | 0                |
| 4             | 6.620                 | 0.960        | 15               |
| 5             | 6.630                 | 0.977        | 35               |

Table 3-2: The input reflection coefficients angle $\phi_{in}$ with respect to the load angle $\phi_L$

The angle $\phi_L$ should be specified first and from that $f_1$ and $f_h$ can be found. Let's choose $\phi_L = 26^\circ$. By examining Table 3-2 there is not frequency point which corresponding to $\phi_L = 26^\circ$. Hence, an interpolation scheme should be devised to interpolate between the existing angles, to obtain the frequency that corresponds to $\phi_L = 26^\circ$.

To choose which interpolation method to use, one should consider the behavior of the input reflection coefficient as a function of frequency. From Equation (3-13) and Figure 3-1, the reflection coefficient of the cavity resembles a circle. Let $x$ represent the frequency step and $f(x)$ the angle of reflection coefficient. From Table 3-2, the $x$-increments are evenly spaced whereas the $f(x)$ points
are unevenly spaced along x. For this reason, the central difference interpolation method is employed. For more details about this method, refer to Appendix A. To interpolate between frequency steps and angles, a central difference table is first generated as discussed in Appendix A. Table 3-3 depicts the results in the central difference mode.

<table>
<thead>
<tr>
<th>X</th>
<th>f(x)</th>
<th>8f</th>
<th>8^2f</th>
<th>8^3f</th>
<th>8^4f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>34</td>
<td>-4</td>
<td></td>
<td>-28</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>-18</td>
<td>9.5</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>16</td>
<td>1-32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-8.5</td>
<td>47</td>
<td>16.5</td>
<td>117</td>
</tr>
<tr>
<td>3.5</td>
<td>7.5</td>
<td>15</td>
<td>26</td>
<td>-42</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>17.5</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>25</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-3: A central difference table for data in Table 3-2, where x is the frequency step and f(x) is the angle θ in

According to Appendix A, using full lines as a base, Stirling's formula is used:

\[ f(x) = f(0) + x(8f) + \frac{x^2}{2!} (8^2f) + \frac{x(x^2-1)}{3!} (8^3f) \]

\[ + \frac{x^2(x^2-1)}{4!} (8^4f) + \frac{x(x^2-1)(x^2-4)}{5!} (8^5f) + \ldots \]  

(3-30)

If half lines are used as a base, Bessel's formula is employed:
\[ f(x) = f(0) + x(8f) + \frac{(x^2 - \frac{3}{2})}{2!} (8^2f) + \frac{x(x^2 - \frac{3}{2})}{3!} (8^3f) \]

\[ + \frac{(x^2 - \frac{3}{2})(x^2 - 9/4)}{4!} (8^4f) + \frac{x(x^2 - \frac{3}{2})(x^7 - 9/4)}{5!} (8^5f) + \]

(3-31)

Using the data in Table 3-3 and Equations (3-30) and (3-31), one can find the frequencies corresponding to \( \phi_L = 26^\circ \). The known quantity here is \( f(x) \) which is equal to \( 26^\circ \). It is desired to find \( x \). By looking at Equations (3-30) and (3-31), these equations are of the fifth order. One way to solve for \( x \) is to use a trial and error approach.

To solve for \( f_1 \) and \( f_h \), looking back at \( f(x) \) in Table 3-3, the angle \( 26^\circ \) is between \( x = 2 \) and \( x = 2.5 \) which corresponds to \( f_1 \) and between \( x = 4.5 \) and \( x = 5 \) which corresponds to \( f_h \).

To solve for \( f_1 \), the interpolation should be done on the line \( x = 2, f(x) = 32 \), this is a full line base. Since we are using a full line as a base, Stirling's formula (3-30) is used:

\[ 26^\circ = 32 + x (-18) + x^2 (-28) + \frac{x(x^2-4)}{6} (0) \]

\[ + \frac{x^2(x^2-1)}{24} (0) + \frac{x(x^2-1)(x^2-4)}{120} (0) \]

By trying a number of values for \( x \) between 0 and 0.5, it was found that \( x = 0.274 \). Hence, the frequency step \( x \) that corresponds to \( 26^\circ \) equals \( 2 + 0.274 \). The relation between
the frequency step \((x)\) and the frequency \(f\) is

\[
f = f_1 + (x-1) \Delta f
\]

(3-32)

Where \(f_1\) is the first frequency and \(\Delta f\) is the frequency increment. Using \(x = 2.274\), \(f_1\) is found to be 6.60274GHz.

To find \(f_h\), the same method of interpolation is used. From Table 3-3, we see that the angle 26° falls between \(x = 4.5\) and \(x = 5\). The interpolation should be done at the line \(x = 4.5\) to find \(f_h\) that corresponds to \(\theta_L = 26°\).

This line happens to coincide with a half line. In that case, Bessel's formula (3-31) is used.

\[
26° = 26° + x (20) + \left(\frac{x^2 - \frac{1}{2}}{2}\right)(0) + x \left(\frac{x^2 - \frac{1}{4}}{6}\right)(0)
\]

\[
= \left(\frac{x^2 - \frac{1}{2}}{24}\right)(0) + x \left(\frac{x^2 - \frac{1}{4}}{120}\right)(0)
\]

From this, \(x\) is found to be equal to 0.05. The frequency step \(x\) that corresponds to 26° equal to 4.5 + 0.05. Using Equation (3-31), \(f_h\) is found to be 6.6255GHz.

Once \(f_1\) and \(f_h\) have been found, \(Q_L\) can be evaluated next by using Equation (3-21)

\[
Q_L = \frac{6.61 \tan(26°)}{6.6255 - 6.60274}
\]

\[
Q_L = 141.64
\]

To find the unloaded Q-factor \(Q_0\), using Equation (3-22), \(Q_L\) and \(\beta\) should be found first. At this point \(Q_L\)
is found. To find the coupling factor $\beta$ using Equation (3-18) is used, the distance ($l$) between the load and the decoupled resonant $\Gamma_D$ can be found using Equation (3-17).

$$l = |\Gamma_L - \Gamma_D|$$

In this example, $l = 1.43$ the coupling factor

$$\beta = \frac{1.43}{2 - 1.43} = 2.51$$

and $Q_0$ is

$$Q_0 = 141.64(1+2.51) = 496.98$$

Appendix B includes the computer program of this method.
CHAPTER 4

RESULTS

In this chapter, Q-factor measurements for a rectangular and microstrip resonant cavity are presented. The results are compared with the theoretical values.

Rectangular Resonant Cavity

Two cavities were tested. The two rectangular cavities are made of rectangular waveguide shorted at one end. On the other end a rectangular to N-type adapter is used as shown in Figure 4-1.

Figure 4-1: A rectangular waveguide cavity
The two cavities have the same width \( (a) \) and the same height \( (b) \), 2.28 cm and 1.016 cm, respectively. The first cavity has a length of \( d = 18 \) cm and the second cavity a length of \( d = 28 \) cm. The input reflection coefficients of both cavities, at the resonant frequency, is given in Tables 4-1 and 4-2, and on the Smith chart in Figure 4-2 and Figure 4-3.

| Frequency (GHz) | Input Reflection Coefficient \( |\Gamma_1| \) | \( \phi_1 \) | Frequency (GHz) | Input Reflection Coefficient \( |\Gamma_1| \) | \( \phi_1 \) |
|----------------|---------------------------------|------------|----------------|---------------------------------|------------|
| 6.560          | 0.982                           | 125.       | 6.575          | 0.768                           | 84.        |
| 6.561          | 0.977                           | 125.       | 6.576          | 0.441                           | 82.        |
| 6.562          | 0.979                           | 125.       | 6.577          | 0.441                           | 81.        |
| 6.562          | 0.976                           | 124.       | 6.577          | 0.330                           | 162.       |
| 6.563          | 0.978                           | 124.       | 6.578          | 0.688                           | 158.       |
| 6.564          | 0.974                           | 123.       | 6.579          | 0.690                           | 158.       |
| 6.565          | 0.980                           | 122.       | 6.580          | 0.850                           | 147.       |
| 6.565          | 0.977                           | 122.       | 6.580          | 0.915                           | 140.       |
| 6.566          | 0.968                           | 121.       | 6.581          | 0.914                           | 141.       |
| 6.567          | 0.974                           | 120.       | 6.582          | 0.941                           | 136.       |
| 6.568          | 0.972                           | 120.       | 6.583          | 0.950                           | 133.       |
| 6.568          | 0.972                           | 118.       | 6.583          | 0.952                           | 133.       |
| 6.569          | 0.962                           | 117.       | 6.584          | 0.962                           | 131.       |
| 6.570          | 0.968                           | 117.       | 6.585          | 0.971                           | 129.       |
| 6.571          | 0.957                           | 115.       | 6.586          | 0.967                           | 129.       |
| 6.571          | 0.947                           | 112.       | 6.586          | 0.970                           | 128.       |
| 6.572          | 0.948                           | 112.       | 6.587          | 0.977                           | 127.       |
| 6.573          | 0.923                           | 108.       | 6.588          | 0.977                           | 127.       |
| 6.574          | 0.884                           | 103.       | 6.589          | 0.976                           | 125.       |
| 6.574          | 0.878                           | 103.       | 6.589          | 0.981                           | 124.       |

Table 4-1: The measured input reflection coefficient \( (|\Gamma_1|) \) for a rectangular cavity with \( d = 28 \) cm
Figure 4-2: The data from Table 4-1 on the Smith chart

Q-Factor of Waveguide Cavity

Figure 4-3: The data from Table 4-2 on the Smith chart

Q-Factor of Waveguide Cavity
| Frequency $f_i$ GHz | Input Reflection Coefficient $|\Gamma_i|$ | Angle $\phi_i$ (Deg) | Frequency $f_i$ GHz | Input Reflection Coefficient $|\Gamma_i|$ | Angle $\phi_i$ (Deg) |
|---------------------|-----------------------------|-----------------|---------------------|-----------------------------|-----------------|
| 6.602               | 0.776                       | 55.             | 6.609               | 0.796                       | 177.            |
| 6.602               | 0.781                       | 53.             | 6.609               | 0.796                       | 176.            |
| 6.603               | 0.774                       | 53.             | 6.610               | 0.800                       | 176.            |
| 6.603               | 0.698                       | 36.             | 6.610               | 0.847                       | 172.            |
| 6.603               | 0.694                       | 36.             | 6.610               | 0.851                       | 172.            |
| 6.604               | 0.695                       | 36.             | 6.611               | 0.852                       | 172.            |
| 6.604               | 0.575                       | 6.              | 6.611               | 0.880                       | 164.            |
| 6.604               | 0.572                       | 6.              | 6.611               | 0.882                       | 164.            |
| 6.605               | 0.573                       | 6.              | 6.612               | 0.883                       | 164.            |
| 6.605               | 0.465                       | -41.            | 6.612               | 0.905                       | 158.            |
| 6.606               | 0.467                       | -41.            | 6.613               | 0.905                       | 158.            |
| 6.606               | 0.468                       | -41.            | 6.613               | 0.910                       | 157.            |
| 6.606               | 0.490                       | -97.            | 6.613               | 0.927                       | 152.            |
| 6.607               | 0.597                       | -97.            | 6.614               | 0.923                       | 152.            |
| 6.607               | 0.957                       | -97.            | 6.614               | 0.925                       | 152.            |
| 6.607               | 0.597                       | -134.           | 6.614               | 0.938                       | 148.            |
| 6.608               | 0.948                       | -134.           | 6.615               | 0.938                       | 148.            |
| 6.608               | 0.712                       | -160.           | 6.615               | 0.934                       | 148.            |
| 6.608               | 0.712                       | -161.           | 6.615               | 0.949                       | 145.            |
| 6.609               | 0.712                       | -161.           | 6.616               | 0.949                       | 145.            |

Table 4-2: The measured input reflection coefficient ($\Gamma_i$) for a rectangular cavity with d = 18 cm.

The angle $\phi_L$, the load frequency $f_L$, the measured coupling coefficient, and the loaded and unloaded measured Q-factor for the rectangular cavities are tabulated in Table 4-3.

The theoretical unloaded Q-factor ($Q_o$) for a rectangular cavity and its resonant frequency $f_o$ can be found from Appendix C. These values are also given in Table 4-3.
Rec tan-gular Cavity

<table>
<thead>
<tr>
<th></th>
<th>EXPERIMENTAL</th>
<th>THEORETICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_L$ (GHz)</td>
<td>$\theta_L$ (Deg)</td>
</tr>
<tr>
<td>18 cm</td>
<td>6.60515</td>
<td>16</td>
</tr>
<tr>
<td>28 cm</td>
<td>6.57725</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4-3: The experimental and theoretical results of the rectangular cavities

From Table 4-3, we see that the experimental and theoretical values of the unloaded Q-factor for the 18 cm rectangular cavity are in very good agreement. The 2.5% difference is attributed to the surface irregularities, dielectric losses, radiation losses, and reflection from the connection between the adapter and waveguide. For the 28 cm rectangular cavity, the difference is 22.4%. This is due to the fact that more reflection and vacillation losses exist in the 28 cm cavity, since this cavity is made of an adapter and two waveguide sections.

Microstrip Cavity

As a third example, a microstrip antenna with length $L = 2.408$ cm, width $W = 1.875$ cm was chosen. The feed is located at the edge of the length of the antenna, to obtain a matched impedance between the feedline and the antenna. The microstrip antenna designed to resonate at 4GHz is shown in Figure 4-4.
The microstrip antenna on PC board with $\varepsilon_r = 2.33$, $t = 35.56$ μm, $h = 0.7874$ mm

The measured input reflection coefficient is given in Table 4-4 and its Smith chart is depicted in Figure 4-5.

| Frequency (GHz) | Input Reflection Coefficient $|\Gamma_i|$ | Phase $\phi_i$ (Deg) | Frequency (GHz) | Input Reflection Coefficient $|\Gamma_i|$ | Phase $\phi_i$ (Deg) |
|----------------|-----------------|------------------|-----------------|-----------------|------------------|
| 3.850          | 0.852           | 25.              | 3.990           | 0.385           | -63.             |
| 3.857          | 0.855           | -29.             | 3.997           | 0.427           | -61.             |
| 3.864          | 0.840           | -34.             | 4.004           | 0.485           | -63.             |
| 3.871          | 0.833           | -39.             | 4.011           | 0.540           | -65.             |
| 3.878          | 0.816           | -44.             | 4.018           | 0.589           | -69.             |
| 3.885          | 0.805           | -48.             | 4.025           | 0.630           | -73.             |
| 3.892          | 0.790           | -53.             | 4.032           | 0.670           | -78.             |
| 3.899          | 0.772           | -58.             | 4.039           | 0.698           | -83.             |
| 3.906          | 0.748           | -63.             | 4.046           | 0.720           | -87.             |
| 3.913          | 0.720           | -68.             | 4.053           | 0.741           | -92.             |
| 3.920          | 0.688           | -73.             | 4.060           | 0.760           | -97.             |
| 3.927          | 0.650           | -78.             | 4.067           | 0.777           | -102.            |
| 3.934          | 0.603           | -83.             | 4.074           | 0.793           | -106.            |
| 3.941          | 0.546           | -87.             | 4.081           | 0.803           | -111.            |
| 3.948          | 0.500           | -90.             | 4.088           | 0.818           | -116.            |
| 3.955          | 0.440           | -91.             | 4.095           | 0.826           | -120.            |
| 3.962          | 0.384           | -90.             | 4.102           | 0.832           | -124.            |
| 3.969          | 0.341           | -84.             | 4.109           | 0.836           | -129.            |
| 3.976          | 0.326           | -76.             | 4.116           | 0.843           | -133.            |
| 3.987          | 0.343           | -68.             | 4.127           | 0.853           | -137.            |

Table 4-4: The measured input reflection $\Gamma_i$ of the microstrip antenna.
Figure 4-5: The data from Table 4-4 on the Smith chart

The angle $\phi_L$, the load frequency $f_L$, the coupling coefficient $\beta$, and the loaded and unloaded Q-factor of the microstrip antenna are shown in Table 4-5.

The theoretical unloaded Q-factor, $Q_o$, is given in Table 4-5. (See Appendix D for the calculation of $Q_o$ of microstrip antenna.)

<table>
<thead>
<tr>
<th>EXPERIMENTAL</th>
<th>THEORETICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_L$ (GHz)</td>
<td>$\phi_L$ (Deg)</td>
</tr>
<tr>
<td>3.976</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4-5: The experimental and theoretical results of the microstrip antenna.
We see from Table 4-5 that the experimental and theoretical value of $Q_0$ are the same. In some cases, these values may be different. The experimental values could be smaller than the theoretical values, and that could be due to the mis-match between the feedline and the input of the antenna.
CHAPTER 5
CONCLUSION

A method of evaluating the unloaded Q-factor, $Q_o$; the loaded Q-factor, $Q_L$; the resonant frequency; and the coupling coefficient of resonators from the reflection coefficient has been presented. A computer algorithm was developed to drive an HP-85 Computer to control the network analyzer. Measurements involving rectangular cavities and microstrips were measured and compared with theoretical data. The results are in excellent agreement for both types of resonators. The method is valid for both overcoupling and undercoupling cases. The only limitation to this model is the limited HP-85 memory capability. For more accurate results, the number of data points and interpolation points should be increased. The algorithm was written in Basic as it can be used with any other computer model.

It is believed that this method can easily be extended to measurements involving dielectronic resonators and high Q-factor cavities, such as cylindrical cavities.
APPENDIX A

INTERPOLATION WITH CENTRAL DIFFERENCES

Interpolation near the center of a set of evenly spaced values is best accomplished by using central differences. A central difference table is first generated as will be shown. Then, an interpolation formula must be chosen. There are many interpolation formulas using central differences, but we will use only two of the most common methods. These are; **Stirling's Formula** (full lines as base):

\[
f(x) = f(0) + x(Sy_0) + \frac{x^2}{2!} (S^2y_0) + \frac{x(x^2-1)}{3!} (S^3y_0) + \frac{x^2(x^2-1)}{4!} (S^4y_0) + \frac{x(x^2-1)(x^2-4)}{5!} (S^5y_0) + \ldots
\]  

**Bessel's Formula** (half lines as base):

\[
f(x) = f(0) + x(Sy_0) + \frac{(x^2-\frac{1}{4})}{2!} (S^2y_0) + \frac{x(x^2-\frac{1}{4})}{3!} (S^3y_0) + \frac{(x^2-\frac{9}{4})}{4!} (S^4y_0) + \frac{x(x^2-\frac{9}{4})(x^2-\frac{9}{2})}{5!} (S^5y_0) + \ldots
\]

To illustrate the central difference interpolation, we begin by considering data which are tabulated at evenly spaced intervals in \( x \). Consider, for example, Table A-1.
Let us assume that we want to find the value of \( f(2.7) \).

First, a forward difference table should be generated by taking forward differences at each point in \( x \), then taking differences of the differences, etc. A forward difference table generated from Table A-1 is shown in Table A-2.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( 8f )</th>
<th>( 8^2f )</th>
<th>( 8^3f )</th>
<th>( 8^4f )</th>
<th>( 8^5f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
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<td>2</td>
<td>38</td>
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</tr>
<tr>
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<td></td>
<td></td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-2: Forward difference table

A central difference table can be generated in a very similar fashion following a purely mechanical scheme. We leave a space between each line of data, and define the lines containing the original data as full lines and the lines between the full lines as half lines. We then take differences as in the forward difference table, but alternate the entries between half lines and full lines. The central difference operator is convenient to use in this context. The definition of the operator is
\[ f_{i+\frac{1}{2}} = f_{i+1} - f_i \]  
(A-3)

The central difference table is Table A-3.

Then, we fill in the gaps in the table A-3 by taking the arithmetic mean of the values above and below each gap. This leads us up to Table A-4.

<table>
<thead>
<tr>
<th>X</th>
<th>( f(x) )</th>
<th>( \delta f )</th>
<th>( \delta^2 f )</th>
<th>( \delta^3 f )</th>
<th>( \delta^4 f )</th>
<th>( \delta^5 f )</th>
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<td>140</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-3: The central difference table

<table>
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<tr>
<th>X</th>
<th>( f(x) )</th>
<th>( \delta f )</th>
<th>( \delta^2 f )</th>
<th>( \delta^3 f )</th>
<th>( \delta^4 f )</th>
<th>( \delta^5 f )</th>
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<tbody>
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<td>2</td>
<td>18</td>
<td>40</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>38</td>
<td>41</td>
<td>2</td>
</tr>
<tr>
<td>2.5</td>
<td>12.5</td>
<td>5</td>
<td>21</td>
<td>16.5</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>25</td>
<td>40</td>
<td>-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>37.5</td>
<td>45</td>
<td>37.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>62.5</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>100</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-4: The central difference table after taking the arithmetic mean
The coordinate \( x = 2.7 \) is nearest \( x = 2.5 \), so this half line is chosen as a base line and the coordinate shifted accordingly. The result is Table A-5.

<table>
<thead>
<tr>
<th>Old X</th>
<th>New X</th>
<th>( f(x) )</th>
<th>( 8f )</th>
<th>( 8^2f )</th>
<th>( 8^3f )</th>
<th>( 8^4f )</th>
<th>( 8^5f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-2</td>
<td>5.5</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1.5</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>-1</td>
<td>8.5</td>
<td>3</td>
<td>2</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.5</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>18</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>-0</td>
<td>12.5</td>
<td>5</td>
<td>21</td>
<td>38</td>
<td>41</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>15</td>
<td>25</td>
<td>40</td>
<td>16.5</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>1</td>
<td>37.5</td>
<td>45</td>
<td>37.5</td>
<td>-5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>60</td>
<td>62.5</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>2</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A-5: The central difference table after the coordinate shifted accordingly

In the shifted coordinate, we wish to find \( f(0.2) \). Since we are using a half line as a base, Bessel's formula Equation (A-2) is used:

\[
f(0.2) = 12.5 + 0.2(5) + \frac{(0.04-0.25)}{2} \tag{21}
\]

\[
+ \frac{0.2(0.04 - 0.25)}{4} \tag{38} + \left( \frac{0.04 - 0.25}{24} \right) \left(0.04 - 2.25\right) \tag{41}
\]

\[
+ \frac{0.2(0.04 - 0.25)(0.04 - 2.25)}{120} \tag{2}
\]

\[
f(0.2) = 11.823
\]

The above answer is in terms of the new \( x \); in terms the old \( x \), the answer is \( f(2.7) = 11.823 \).
The following program is written in BASIC language. This program is a modification to the 1183A Accuracy Enhancement PAC. This modification is to calculate the unloaded Q-factor and the coupling coefficient of resonant cavity.
# Program Variables

Some variables and constants are self explanatory and are not defined.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
</table>
| F(*)     | Test Frequency range specifications in GHz  
\[ F(1) = \text{Start} \]  
\[ F(2) = \text{Stop} \]  
\[ F(3) = \text{Step} \] |
| X(1,N)   | Magnitude of the measure reflection coefficient |
| Y(1,N)   | Phase value in degrees of measured reflection coefficient with respect to the center of the Smith chart |
| N        | The measurement arrays of the present test frequency |
| M1       | Temporary variable |
| M2       | Temporary variable |
| M3       | Temporary variable |
| M4       | Temporary variable |
| M5       | Temporary variable |
| M6       | Temporary variable |
| R1       | Temporary variable |
| R2       | Temporary variable |
| R3       | Temporary variable |
| R4       | Temporary variable |
| B1       | Temporary variable |
| B(1,N)   | The phase value in degrees of the measured reflection coefficient with respect to load angle |
| N5       | The coupling effect |
| D(1,I)   | The central difference array for the interpolation |
Recalling the reflection coefficient

computing the load frequency $f_L$

computing the coupling coefficient $B$

Writing the reflection coefficient angle with respect to the load angle

Generate the central difference Table

Choose the angle bandwidth

Interpolation

If the base full line

Interpolation using Striling formula

Computing the loaded Q-factor $Q_L$

Computing the unloaded Q-factor $Q_0$

Return to the main program "MEAS"

Figure B-1: Flow chart for the program used to calculate the coupling coefficient, the loaded Q-factor, and the unloaded Q-factor.
9000 ! Q FACTOR MEASUREMENT
9010 !
9020 !
9030 IMAGE #, 5D DDD, 3x
9040 IMAGE 4X,2DZ.DDD,4X,3DZ
9050 FOR N=1 TO N2
9060 F5=F(1)+(N-1)*F(3)
9070 PRINT USING 9030 ; F5
9080 X5=X(1,N)
9090 Y5=Y(1,N)
9095 X5=10 (-(.05*X5))
9100 PRINT USING 9040 ; X5,Y5
9110 NEXT N
9111 FOR N=1 TO N2
9112 X(1,N)=10 (-(.05*X(1,N)))
9113 NEXT N
9120 U1=INF
9130 FOR N=1 TO N2
9140 IF X(1,N) U1 THEN U2=N
9150 IF X(1,N) U1 THEN U3=Y(1,N)
9160 IF X(1,N) U1 THEN U1=X(1,N)
9170 NEXT N
9171 IF Y(1,N) 0 THEN 9173
9172 IF U3 0 THEN 9177
9173 FOR N=1 TO N2
9174 IF Y(1,N) 0 THEN 9190
9175 NEXT N
9176 U4=U3 @ GOTO 9220
9177 FOR N=1 TO N2
9178 IF Y(1,N) 0 THEN 9190
9179 NEXT N
9180 U4=U3 @ GOTO 9220
9190 IF U3 0 THEN 9210
9200 U4=U3-180 @ GOTO 9220
9210 U4=U3+180
9220 R1=U1*COS(U3)-1*COS(U4)
9230 R2=U1*SIN(U3)-1*SIN(U4)
9240 FOR N=1 TO N2
9250 R3=X(1,N)*COS(Y(1,N))-1*COS(U4)
9260 R4=X(1,N)*SIN(Y(1,N))-1*SIN(U4)
9270 U6=(R3*R1+R2*R4)/(SQR(R1 2+ R2 2)*SQR(R3 2+R4 2))
9275 IF U6 1 THEN U6=1
9276 IF U6 -1 THEN U6=-1
9280 B(1,N)=ACS(U6)
9300 NEXT N
9320 R1=X(1,U2)*COS(Y(1,U2))-1*COS(U4)
9330 R2=X(1,U2)*SIN(Y(1,U2))-1*S
   IN(U4)
9340 V1=SQR(R1 2+-R2 2)
9400 U5=V1/(2-V1)
9409 PRINT ;U1,U2,U3,U4
9410 PRINT "d=";V1
9420 PRINT "k=";U5
9430 A2=10
9440 I=N
9450 M2=A2*2-1
9460 FOR I=1 TO A2
9470 FOR J=1 TO M2
9480 D(I,J)=0
9490 NEXT J
9500 NEXT I
9510 FOR I=1 TO (M2+1)/2
9520 D(1,1+(I-1)*2)=I
9530 NEXT I
9540 FOR I=1 TO (M2+1)/2
9550 D(2,1+(I-1)*2)=B(1,I+U2-6)
9560 NEXT I
9570 V1=2
9580 T=(M2-1)/2
9590 FOR I=3 TO A2
9600 FOR J=1 TO T
9610 D(I,V1+(J-1)*2)=D(I-1,V1+1+(J-1)*2)-D(I-1,V1-1+(J-1)*2)
9620 NEXT J
9630 V1=V1+1
9640 T=T-1
9650 NEXT I
9660 FOR I=1 TO (M2-1)/2
9670 D(1,I*2)=(D(1,I*2-1)+D(1,I*2+1))/2
9680 NEXT I
9690 T=(M2-1)/2
9700 V1=2
9710 FOR I=2 TO A2
9720 FOR J=1 TO T
9730 D(I,V1+(J-1)*2)=(D(I,1+V1+(J-1)*2)+D(I,-1+V1+(J-1)*2))/2
9740 NEXT J
9750 V1=V1+1
9760 T=T-1
9770 NEXT I
9780 FOR I=1 TO 2
9790 FOR J=1 TO M2
9800 PRINT ;D(I,J)
9810 NEXT J
FOR J=1 TO 11
V1=MAX(V1,D(2,J))
NEXT J
A4=0
FOR J=11 TO M2
A4=MAX(A4,D(2,J))
NEXT J
V1=MIN(A4,V1)
NEXT I
9861 \[ B2 = B2 + D(5, B1) \times N \times (N^2 - 1)/6 \]
9862 \[ B2 = B2 + D(6, B1) \times N \times 2 \times (N^2 - 1)/2 \]
9863 \[ B2 = B2 + D(7, B1) \times N \times (N^2 - 1)/120 \]
9864 IF ABS(B2-U9) .01 THEN B1=B 1+N @ GOTO 9867
9866 NEXT N
9867 RETURN
9869 FOR N=0 TO .5 STEP .001
9870 B2=0
9871 \[ B2 = B2 + D(2, B1) + D(3, B1) \times N + D(4, B1) \times (N^2 - .25)/2 \]
9872 \[ B2 = B2 + 0 + D(6, B1) \times N \times (N^2 - .25)/6 \]
9873 \[ B2 = B2 + D(7, B1) \times N \times (N^2 - 2.25) \times (N^2 - .25)/24 \]
9874 IF ABS(B2-U9) .01 THEN B1=B 1+N @ GOTO 9876
9876 NEXT N
9877 RETURN
A rectangular resonator is a hollow rectangular box of metallic walls and of dimensions $a$, $b$, and $d$ in the $x$, $y$, and $z$ directions, respectively (see Figure C-1).

Figure C-1: A rectangular cavity and the relative coordinate system

For the rectangular cavity box of Figure C-1, if we select the $T_{E_{mnp}}$ modes only. These modes are oriented with their electric field in the $x$-$y$ plane and thus propagate along the $z$ direction. The condition that $E$ shall be zero at $Z = 0$ and $d$, as required by the perfect conductors, is satisfied if the dimension $d$ is a half-guide wavelength; i.e., or multiples of $\lambda g/2$. For the $T_{E_{101}}$ mode, we have:
\[ d = \frac{\lambda g}{2} = \frac{\lambda}{[1 - (\lambda/2a)^2]^{\frac{1}{2}}} \]

or

\[ \lambda = \frac{2ad}{(a^2 + d^2)^{\frac{1}{2}}} \] (C-1)

For the following field representation, the coordinate system of Figure (C-1) is chosen. The field patterns are obtained by superimposing incident and reflected waves for various waveguide modes propagating in the z direction. These field patterns are given by:

\[ H_x = -\frac{C}{K_c^2} \frac{P\pi}{d} \frac{m\pi}{a} \sin \left(\frac{m\pi x}{a}\right) \cos \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \] (C-2)

\[ H_y = -\frac{C}{K_c^2} \frac{P\pi}{d} \frac{n\pi}{a} \sin \left(\frac{m\pi x}{a}\right) \cos \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \] (C-3)

\[ E_x = \frac{iwmC}{K_c^2} \frac{m\pi}{a} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \] (C-4)

\[ E_y = \frac{iwmC}{K_c^2} \frac{m\pi}{a} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \] (C-5)

\[ H_z = C \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \] (C-6)

where

\[ K_c^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 \] (C-7)

\[ \beta = \left(\frac{2\pi^2}{\lambda} - K_c^2\right)^{\frac{1}{2}} = \frac{p\pi}{d} \] (C-8)

\[ K_{mnp} = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 + \left[\frac{p\pi}{d}\right]^2 \] (C-9)
The general expression for this quality factor $Q$ of any resonant system is given by:

$$Q = \frac{\omega_0}{W_L} \frac{\text{energy stored}}{\text{average power loss}} = \frac{\omega_0 U}{W_L} \quad (C-10)$$

For the rectangular cavity in Figure C-1

$$U = \frac{E}{2} \int_{0}^{d} \int_{0}^{b} \int_{0}^{a} |E_y|^2 \, dx \, dy \, dz \quad (C-11)$$

Substituting Equation (C-5) for $TE_{10p}$ mode into C-11, yield to the energy stored in the cavity equal to

$$U = \frac{Eabd}{8} E_o^2 \quad (C-12)$$

$$W_L = R_s \left( \int_{0}^{d} \int_{0}^{a} |H_x|^2 \, dx \, dy + \int_{0}^{d} \int_{0}^{b} |H_z|^2 \, dy \, dz \right) + \int_{0}^{d} \int_{0}^{a} \left[ |H_x|^2 + |H_z|^2 \right] \, dx \, dz \quad (C-13)$$

Substituting $H_x$ and $H_z$ from Equations (C-2) and (C-6), yield

$$W_L = \frac{R_s C^2}{2} \left( \frac{\pi^2 m \rho p}{K^2_c} \right)^2 \frac{b}{d^2a} + \frac{b d}{d^2a} + \frac{1}{2ad} + \frac{1}{c} \quad (C-14)$$

Substituting Equations (C-12) and (C-14) into Equation (C-10) yield to
\[ Q_o = \frac{(K_{10p} \, ad)^3 \, b \, \eta}{2\pi^2 \, R_s \, (2a^3b + 2d^3b + a^3d + d^3a)} \quad (C-15) \]

Where

- \( \eta \) = Free space impedance
- \( R_s \) = Surface resistivity
APPENDIX D
MICROSTRIP ANTENNA CAVITY

This appendix is to show how to design a microstrip antenna and calculate the theoretical Q-factor of the antenna. A microstrip antenna consists of a rectangular element that is photoetched from one side of a printed-circuit board (Figure D-1). The element is fed with a coaxial feed. The length, \( L \), is the most critical dimension and is slightly less than half wavelength in the dielectric substrate material [4].

\[
L = 0.49 \frac{\lambda}{d} = 0.49 \frac{\lambda_o}{\sqrt{\varepsilon_r}} \tag{D-1}
\]

Where

- \( L \) = length of element
- \( \varepsilon_r \) = relative dielectric constant of the printed circuit substrate
- \( \lambda_o \) = free-space wavelength

The width, \( W \), must be less than a wavelength in the dielectric substrate material.

![Figure D-1: Rectangular microstrip-antenna element](image)

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The Q-factor for the microstrip antenna element is given by [5].

\[
\frac{1}{Q_o} = \frac{\lambda_o}{1} \left( \alpha_c + \alpha_d \right) \frac{\epsilon_{re}(r)}{\epsilon_{re}(f)} \frac{\epsilon_{re} - 1}{\epsilon_{re}} \left( \tan \delta \right) \frac{\lambda_o}{\lambda_o} \]  \quad (D-2)

where

\[
\alpha_c = 6.1 \times 10^{-5} A R_s Z_{om} \epsilon_{re} \left( \frac{W_e}{h} + \frac{0.667 W_e}{h} \right) \] dB/unit length \( (W/h \geq 1) \)  \quad (D-3)

and

\[
\alpha_d = 27.3 \left( \frac{\epsilon_r}{\epsilon_r - 1} \right) \left( \frac{\epsilon_{re} - 1}{\epsilon_{re}} \right) \frac{\tan \delta}{\lambda_o} \] dB/unit length \quad (D-4)

where

\[
A = 1 + h \left( 1 + \frac{1.25 \ln 2h}{\pi} \right) \] \quad (D-3)

and

\[
\epsilon_{re} = \frac{\epsilon_r + 1}{2} + \left( \frac{\epsilon_r - 1}{2} \right) F(W/h) - C \]

where

\[
C = \frac{(\epsilon_r - 1) t}{4.6 h \left[ \frac{W_e}{h} \right]^{1/2}} \]

and

\[
F(W/h) = \left( 1 + \frac{12h}{W} \right)^{-\frac{1}{2}} \]

\[
W_e = W + \frac{1.25}{\pi} t \left( 1 + \ln 2h \right) \]

and
\[ Z_{om} = \frac{377}{\varepsilon_{re}} \left( \frac{W}{h} \right)^{0.393} + 1.393 + 0.667 \ln \left( \frac{W}{h} \right) + 1444 \]

and

\[ \varepsilon_{re}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{re}}{1 + G(f/f_p)^2} \]

where

\[ G = \frac{Z_{om} - 5}{60} + 0.004 Z_{om} \]

and

\[ f_p(\text{GHz}) = \frac{15.66 Z_{om}}{h} \]

with \( h \) in mils.
LIST OF REFERENCES


