Versatile Gasdynamics Computer Program for One-Dimensional Isentropic Rayleigh and Fanno Flows

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A VERSATILE GASDYNAMICS COMPUTER PROGRAM FOR ONE-DIMENSIONAL ISENTROPIC, RAYLEIGH AND FANNO FLOWS

BY

ROBERT KENNETH SMITH
B.S., University of Tennessee, 1977

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the Master of Science in Engineering in the Graduate Studies Program of the College of Engineering University of Central Florida Orlando, Florida

Fall Term
1987
ABSTRACT

The purpose of this research report is to discuss the development and results of a computer program (GASDYNAMICS) that solves a variety of compressible one-dimensional (1-D) steady gas dynamics problems utilizing the algebraic equations that result from the governing differential equations assuming perfect gas conditions. This report shows how the governing differential equations that are derived from the ideal gas law, the definition of Mach number, and the fundamental conservation laws of mass, energy, and momentum can be developed into their algebraic forms when additional simplifying assumptions are made.

GASDYNAMICS solves problems for either a converging or a converging-diverging nozzle followed by a constant diameter duct where either friction and/or heating occur. Any ideal gas may be assumed. The program performs analysis necessary to find the location of shocks in either the nozzle or the duct. This analysis requires an evaluation of the backpressure to inlet stagnation pressure relationship in addition to friction and/or heating terms to determine which, if either, factors will drive the shock location.
GASDYNAMICS requires that the operator respond to a series of questions concerning identification of the gas, initial stagnation properties, and duct to nozzle throat geometry. GASDYNAMICS outputs a tabulation of static pressure, stagnation pressure, static temperature, Mach number, mass flowrate per unit area, velocity and density at discrete points along the flowpath. These values are expressed in metric units.

GASDYNAMICS was written using Applesoft Basic language on an Apple IIE Computer. The program has been modified by changing the program syntax to Microsoft Basic language. This modification, along with proper disk format, allows the program to run on either the Apple MacIntosh or IBM PC computers.
ACKNOWLEDGEMENT

I would like to thank all those who have contributed to this project. I am grateful to Dr. Eno for his leadership in making this project successful and also to Dr. Bishop and Professor Beck for their suggestions, comments, and encouragement. Special thanks are extended to Dr. Wilkerson of the University of Tennessee, who suggested the Zucrow book as a good reference and provided a large portion of my fundamental understanding of the subject.

I am grateful to Mr. Joe Letosky, who was extremely helpful in providing access to the GALE editing program as well as other vital reference material in Applesoft and Microsoft Basic programming. More importantly Joe was the person that would listen to my programming problems and very often suggest solutions that eventually worked.

Special thanks are extended to Ms. Pat Green and Ms. Susan Brinkley, who endured several revisions and spent many hours on the MacIntosh. They both did a great job typing this report. I am also grateful to Ms. Shirley Pencka for her expertise in producing the figures.
Most importantly I would like to thank my wife, Suzanne, and my children, Jason and Rachel. They were patient, understanding, and encouraging as I spent many hours working on this project and the courses that preceded it.
PRESSURE

P  STATIC PRESSURE
PB BACK PRESSURE
PC CRITICAL PRESSURE RATIO
PE CALCULATED EXIT PLANE PRESSURE
PF STATIC PRESSURE RATIO FOR FANNO FLOW
PG STAGNATION PRESSURE RATIO FOR FANNO FLOW
PJ STAGNATION PRESSURE RATIO FOR RAYLEIGH FLOW
PK STATIC PRESSURE RATIO FOR RAYLEIGH FLOW
PL EXIT PLANE STATIC PRESSURE FOR A NORMAL SHOCK AT THE EXIT PLANE
EP EXIT PLANE STAGNATION PRESSURE RATIO FOR EXIT PLANE FANNO FLOW
KF EXIT PLANE STATIC PRESSURE RATIO FOR EXIT PLANE FANNO FLOW
PO INLET STAGNATION PRESSURE (GIVEN INPUT)
PR RATIO OF BACKPRESSURE TO STAGNATION PRESSURE
PSTA STATIC PRESSURE @ WHICH M =1 FOR RAYLEIGH FLOW
PSTARF STATIC PRESSURE @ WHICH M =1 FOR FANNO FLOW
PT STAGNATION PRESSURE RATIO ACROSS A NORMAL SHOCK
PW STAGNATION PRESSURE RATIO ON SUBSONIC SIDE OF SHOCK FOR FANNO FLOW
PY STATIC PRESSURE RATIO FOR FANNO FLOW @ SHOCK LOCATION
PZ STATIC PRESSURE RATIO ACROSS A NORMAL SHOCK

TEMPERATURE

T  STATIC TEMPERATURE
TE EXIT PLANE STATIC TEMPERATURE.
TF STATIC TEMP. RATIO FOR FANNO FLOW
TK STATIC TEMP. RATIO FOR RAYLEIGH FLOW
TO STAGNATION TEMPERATURE @ INLET (GIVEN INPUT)
TQ STAGNATION TEMP. RATIO FOR RAYLEIGH FLOW @ INLET
TR STAGNATION TEMP. RATIO FOR RAYLEIGH FLOW @ EXIT
TSTAR STAGNATION TEMPERATURE RATIO ACROSS A RAYLEIGH DUCT
TT CALCULATED STAGNATION TEMP. @ DUCT EXIT
TZ STATIC TEMP. RATIO ACROSS A NORMAL SHOCK

THERMODYNAMIC PROPERTIES

CP SPECIFIC HEAT
GA SPECIFIC HEAT RATIO
GM SPECIFIC HEAT RATIO - 1
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NOMENCLATURE

**Computer Program**

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THERMODYNAMIC PROPERTIES (CONTINUED)

GP  SPECIFIC HEAT RATIO + 1
R   GAS CONSTANT
DE  DENSITY

MISCELLANEOUS

BB  PSEUDO FOR AREA RATIO
MD  MASS FLOW RATE/AREA
QQ  HEAT TRANSFER (GIVEN INPUT)
QH  UPPER LIMIT FOR HEAT TRANSFER RATE THAT WILL ALLOW SONIC FLOW AT THE NOZZLE THROAT
OZ$ PREDETERMINED INPUT CONDITIONS OPTION VARIABLE
F$  GASEOUS MEDIUM INPUT VARIABLE
CS  CASE NUMBER (GIVEN INPUT)
IN  PSEUDO FOR TYPE OF NOZZLE

Governing Equation

GEOMETRY

Lmax  MAXIMUM DUCT LENGTH FOR FANNO FLOW
dx   DIFFERENTIAL LENGTH
D    DIAMETER
A    AREA

FRICITION

f    DARCY-WEISBACH FRICTION FACTOR
f    MEAN VALUE OF FRICTION FACTOR

MACH NUMBER

M    MACH NUMBER

PRESSURE

p    STATIC PRESSURE
Pb   STATIC BACK PRESSURE
Po   STAGNATION PRESSURE
Pc   CRITICAL PRESSURE
Pr   RATIO OF BACKPRESSURE TO INLET STAGNATION PRESSURE

TEMPERATURE

T    STATIC TEMPERATURE
To   STAGNATION TEMPERATURE
INTRODUCTION

In the study of gas dynamics many problems are encountered which require iterative solutions. The student’s overall understanding of the subject increases as the number and variety of these problems are solved. Although large computers have been used to solve these problems in the past, the present-day use of personal computers as a learning tool has provided the potential for a quantum step in the learning experience.

The objective of this research has been to develop a personal computer model that will solve a variety of problems that a college senior or graduate student in gas dynamics would encounter. The design goals for this project have been to make the program versatile, fast, accurate, and user-friendly. Thus the program provides appropriate feedback during the calculation process to allow the student to understand each series of steps.

As a part of the effort to define the scope of this project, a literature search was performed to determine the extent of work that has been done in this area. Most of the work surveyed involved numerical analysis to solve compressible Navier-Stokes equations in
the field of computational fluid dynamics. The growth in this field has been primarily due to the utilization of faster and larger computers as opposed to the smaller personal computers.

As examples, the relative merits of explicit finite difference techniques were evaluated for normal shock waves in converging-diverging nozzles by Holst [6]. Also, an evaluation of cell type finite difference methods for solving viscous flow problems was performed by Taylor [7]. Other aspects of these computational problems were addressed by authors of several other papers. All of this work was done on the larger computers.

At the conclusion of the literature search, it was decided to limit the scope of this project to the solution of the algebraic equations of a wide variety of problems involving converging and converging-diverging nozzles with attaching ducts. Flow parameters were calculated at discrete points rather than continuously as in time-marching finite differencing techniques. Furthermore, it was decided that the program would assume ideal gas law behavior. This assumption as well as the assumptions for steady one-dimensional flow allow the design goals of versatility and minimization of CPU time to be realized. The discussion by Holst [6] on maximum error
versus CPU processing time was helpful in evaluating the magnitude of the incremental steps and convergence criteria used in this program.

The resulting program, GASDYNAMICS, could be used in several different teaching applications. Many universities have small scale subsonic or supersonic wind tunnels. GASDYNAMICS would provide a standard for all the important flow parameters at any point in the nozzle or test section when tare values are obtained. Thus measured parameters such as pressure or velocity could be checked against the values predicted by the GASDYNAMICS model. This application would also be useful for engineers working at commercial wind tunnel facilities such as those located at the Arnold Engineering Development Center (AEDC). A significant cost savings could be realized by using GASDYNAMICS to perform tare calculations on the wind tunnel prior to installation of the test model. Even after model installation GASDYNAMICS would provide theoretical values for pressures, velocities, and densities throughout the nozzle and test section if oblique shocks are not present.

From an academic viewpoint, GASDYNAMICS provides an opportunity for students to run several cases where all given inputs
are held constant except for one parameter which is allowed to vary. Thus, as an example, backpressure may be allowed to vary so that a student can gain a better appreciation for the effects on other parameters for these varying backpressures. This is because trends are generally easier to identify when more data is available.

The primary references for the development of the governing equations shown by the next section were John [1], Emanual [2] and Zucrow [3]. GASDYNAMICS was written using these resulting equations. Both John [1] and Emanual [2] contain clear examples of each type of problem solved. Emanual [2] was particularly useful for defining the limits associated with shock waves and Rayleigh ducts.


GASDYNAMICS is written in the Applesoft Basic language using the text by Einstein [5] as the primary reference for developing the subroutines. GASDYNAMICS was converted to Microsoft Basic language to be compatible with IBM PCs also.
The subject of gas dynamics is the study of the motion of a compressible gas. Compressible flow requires an understanding of the fundamental principles of thermodynamics and fluid mechanics. The two primary contributions of mechanics are the principles of conservation of momentum and mass. The first and second law of thermodynamics and the equations of state complete the foundation for the development of the governing equations of compressible gas flow. The following differential equations applicable to the control volume of Figure 1 summarize the laws of thermodynamics and mechanics discussed above.

The one-dimensional mass rate of flow can be expressed as

\[ m = \rho AV \]  

For steady flow the mass rate is conserved and the mass conservation principle (continuity) can be written in the differential form of Figure 1 as
Figure 1: Control Volume for Varying Area Flow with Heat Transfer and Friction.
\[
\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \tag{2}
\]

In steady state Newton's law establishes a balance between pressure and shear forces and the momentum change of the fluid. In terms of the differential control volume of Figure 1 this is

\[-\tau_w \pi D dx = Adp + \rho AVdV \tag{3}\]

Defining a friction factor \( f \) from the Darcy-Wiesbach equation

\[\tau_w = f \rho V^2 \tag{4}\]

the momentum equation may be written

\[dp + f \rho V^2 \frac{dx}{D} + \rho VdV = 0 \tag{5}\]

The equation of state for an ideal gas is

\[p = \rho RT \tag{6}\]

The Mach number is defined as

\[M = \frac{V}{a} \tag{7}\]

where \( a \) = speed of sound. For an ideal gas \( a = \sqrt{kRT} \) so that

\[M = \frac{V}{\sqrt{kRT}} \tag{8}\]

Utilizing equations (6) and (8), equation (5) can be rewritten as

\[\frac{dp}{P} + f \frac{M^2}{2} + f dx \sqrt{kM^2 \frac{dV}{V}} = 0 \tag{9}\]
Also equation (1) may be rewritten utilizing equations (6) and (8) to obtain the following expression for mass flow rate:

\[ m = \frac{PM_kRT}{A} \cdot \frac{RT}{RT} \tag{10} \]

Equation (6) can be rewritten in differential form as

\[ \frac{dp}{p} = d\frac{\rho}{\rho} + \frac{dT}{T} \tag{11} \]

From the definition of Mach number

\[ \frac{dM}{M} = \frac{dV}{V} - \frac{dT}{2T} \tag{12} \]

Neglecting mechanical work the energy conservation principle establishes a balance between heat input and enthalpy and kinetic energy changes. For steady flow

\[ dq = dh + VdV \tag{13} \]

or, for an ideal gas with constant specific heats

\[ dq = C_p dT_o \tag{14} \]

where stagnation temperature is defined as

\[ T_o = T + \frac{V^2}{2C_p} \tag{15} \]
By substitution from equation (8), equation (15) can be rewritten as

\[ T_0 = T \left( 1 + \frac{(k-1)M^2}{2} \right) \]  

(16)

or in the differential form of Figure 1

\[ \frac{dT_0}{T_0} = \frac{dT}{T} + \frac{(k-1)MdM}{1 + \frac{(k-1)M^2}{2}} \]  

(17)

In view of the relationship in equation (14), equation (17) may be seen as the energy equation. Equations (2), (9), (11), (12), and (17) constitute the differential forms of the continuity, momentum, state, Mach number, and energy equations consistent with Figure 1.

Substituting equation (11) into equation (9)

\[ \frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} = -kM^2 \frac{dV}{V} - \frac{kM^2}{2} \frac{f}{dx/D} \]  

(18)

Substituting equation (2)

\[ \frac{dT}{T} = \frac{dA}{A} + (1-kM^2)\frac{dV}{V} - \frac{kM^2}{2} \frac{f}{dx/D} \]  

(19)

Substituting equation (12)

\[ \frac{dT}{T} = \frac{dA}{A} + (1-kM^2)(\frac{dM}{M} + \frac{1}{2} \frac{dT}{T}) - \frac{kM^2}{2} \frac{f}{dx/D} \]  

(20)

or rearranging

\[ \frac{dT}{T} = \frac{2dA}{(1+kM^2)A} + \frac{2(1-kM^2)}{(1+kM^2)} \frac{dM}{M} - \frac{kM^2}{1+kM^2} \frac{f}{dx/D} \]  

(21)
Substituting from the energy equation (17)

\[
\frac{dT}{T_0} + \frac{kM^2}{1+kM^2} \frac{dx}{D} \cdot \frac{2}{1+kM^2} \frac{dA}{A} = \frac{2(1-M^2)dM}{(1+kM^2)(1+k-1M^2)M} \tag{22}
\]

Equation (22) displays how the Mach number can be affected by heating, friction, and area change in the compressible flow of an ideal gas. Since the variables cannot be separated, equation (22) cannot be solved explicitly but would have to be numerically integrated. The equation can be solved, however, when the effects are separated. These cases are considered: (1) isentropic flow with area change, (2) friction (without heating) in a constant area duct, and (3) simple heating (without friction) in a constant area duct.

**Isentropic Flow**

In the absence of friction and heating equation (22) becomes

\[
\frac{dA}{A} = - \frac{1-M^2}{1+(k-1)M^2} \frac{dM}{M}\tag{23}
\]

or upon integrating

\[
A = A^* \left[ \frac{2}{k+1} \frac{(1+k-1M^2)}{(k+1)/(2(k-1))} \right]^{(k+1)/(2(k-1))}\tag{24}
\]

where \(A^*\) refers to a throat condition of \(M=1\). The combined first and second laws of thermodynamics yield
\[ T_{ds} = dh - \frac{dP}{\rho} \]  

(25)

Assuming an ideal gas with constant specific heats

\[ ds = C_p \frac{dT}{T} - R \frac{dp}{p} \]  

(26)

For isentropic flow

\[ \frac{dT}{T} = \frac{k-1}{k} \frac{dp}{p} \]  

(27)

Integrating between static and stagnation conditions

\[ T_o = \frac{(P_o)^{(k-1)/k}}{T_o} \]  

(28)

By substituting equation (15) into equation (28) the following equation can be written

\[ P_o = P(1 + \frac{[k-1]}{2} M^2) \frac{k}{k-1} \]  

(29)

Equations (16) and (29) are used to calculate static temperature and pressure from Mach number and stagnation conditions. Knowing static temperature and pressure the gas density may be found using equation (6) and the mass flow rate per unit area can be found using equation (10). The gas velocity is found using equation (8). Equation (24) is used to calculate \( \frac{A}{A^*} \) at the nozzle exit plane when Mach number is known.
**Fanno Flow**

In the absence of heating and area change equation (22) becomes

\[
\frac{f dx}{D} = \frac{2}{kM^2(1+\frac{k-1}{2}M^2)} \frac{(1-M^2)}{dM} \quad (30)
\]

Assuming \( f \) is a constant mean value equation (30) may be integrated along the Fanno duct to a reference length \( L_{\text{max}} \) where \( M = 1 \) to yield

\[
f_{L_{\text{max}}} = \frac{1-M^2}{D} + \frac{k+1}{kM^2} \ln \left( \frac{(k+1)M^2}{2(k+1)M^2} \right) \quad (31)
\]

Since no heating is present equation (17) can be written as

\[
\frac{dT}{T} = \frac{(k-1)MdM}{1+\frac{(k-1)M^2}{2}} \quad (32)
\]

Substitution of equations (30) and (32) into equation (12) yields

\[
\frac{dV}{V} = \frac{1}{1+\frac{k-1}{2}M^2} \frac{dM}{M} \quad (33)
\]

Substituting equations (30) and (32) into equation (18) yields

\[
\frac{dp}{P} = -\frac{1+(k-1)M^2}{1+\frac{(k-1)M^2}{2}} \frac{dM}{M} \quad (34)
\]

Integrating yields
where $P^*$ is the static pressure at $M=1$ on the Fanno line as shown by Figure 2. If we retain the definition of stagnation pressure as given by equation (29), the stagnation pressure ratio for Fanno flow may be written as

$$\frac{p_o}{P^*} = \frac{1}{M^2} \left( \frac{2}{k+1} \right)^{1/2} \left( 1 + \frac{k-1}{2} M^2 \right)^{1/2}$$

Equations (31), (35), and (36) constitute a set of algebraic equations by which the individual properties can be found at positions within a constant area duct with Fanno friction. GASDYNAMICS uses the FL/D expression to find the Mach number in an iterative technique when FL/D is known. Since stagnation temperature does not change as a result of Fanno flow, the static temperature at any point in the Fanno duct is found by using equation (16). Static and stagnation pressure ratios for Fanno flow as given by equations (35) and (36) are calculated and are used to determine downstream pressure. Velocity in the Fanno duct is calculated from the Mach number definition and density is calculated from the ideal gas law expression.
Figure 2: Fanno Line on Temperature – Entropy Diagram.
\section*{Rayleigh Flow}

In the absence of area change and friction equation (22) becomes

\[ \frac{dT_0}{T_0} = \frac{2(1-M^2)}{(1+kM^2)(1+k-1M^2)^2} \frac{dM}{M} \]  

This equation can be integrated along the Rayleigh duct between an arbitrary point and the point where \( M = 1 \) to obtain

\[ \frac{T_0}{T_0^*} = \frac{2(k+1)M^2}{2(1+kM^2)^2} \]  

where \( T_0^* \) is stagnation temperature at \( M=1 \). Also equation (14) can be integrated to obtain

\[ q = C_p(T_{o2} - T_{o1}) \]  

Since no area change or friction occurs equation (20) can be rewritten as follows

\[ \frac{dT}{T} = \frac{2(1-kM^2)}{1+kM^2} \frac{dM}{M} \]  

This can be integrated using \( T^* \) as the static temperature where \( M=1 \) on the Rayleigh line as shown by Figure 3 to obtain

\[ \frac{T}{T^*} = \frac{(1+k)^2M^2}{(1+kM^2)^2} \]
Figure 3: Rayleigh Line On Temperature - Entropy Diagram.
Substitution of equation (40) into equation (12) yields

\[ \frac{dV}{V} = \frac{1+(k-1)M^2}{1-M^2} \frac{dT}{T} \]  \hspace{1cm} (42)

From equation (18) since \( f = 0 \)

\[ \frac{dP}{P} = -\frac{kM^2}{V} \frac{dV}{V} \]  \hspace{1cm} (43)

or

\[ \frac{dP}{P} = -\frac{2kM^2}{1+kM^2} \frac{dM}{M} \]  \hspace{1cm} (44)

This can be integrated to obtain

\[ \frac{P}{P^*} = \frac{1+k}{1+kM^2} \]  \hspace{1cm} (45)

where \( P^* \) is static pressure at \( M=1 \). By utilizing the definition of stagnation pressure as given by equation (29), the following expression can be written

\[ P_o = \frac{(k+1)[(2)(1+kM^2)]^{k/(k-1)}}{1+kM^2 k+1 2} \]  \hspace{1cm} (46)

As in the Fanno flow development, a series of algebraic expressions has been developed by which the individual properties can be found at positions within a constant area duct with Rayleigh heating. GASDYNAMICS utilizes the \( T_o/T_{o^*} \) expression to find
the Mach number in an iterative technique when $T_o/T_o^*$ is known. Stagnation temperature is found by utilizing equation (39) where $C_p$ is assumed constant. Static temperature is found by using equation (16) again. Static and stagnation pressure ratios for Rayleigh flow as given by equations (45) and (46) are calculated and are used to determine downstream pressure. As in Fanno flow, velocity and density are calculated from Mach number definition and ideal gas law, respectively.

**Shock Wave Analysis**

A normal shock in a duct is a discontinuity across which there are abrupt changes in mechanical and thermodynamic properties. Actually, the normal shock is an approximation for a more complex system of oblique shocks which may oscillate back and forth in the axial direction. The abrupt normal shock, however, is a good approximation for modeling the property changes resulting from the actual shock pattern.

The strength of the normal shock and the magnitude of property changes depends on the Mach number ahead of the
may stand in the divergent portion of a nozzle or in a duct subject to friction. Its location, and thus its strength, are a function of the resistance to flow.

Figure 4 shows a shock wave surrounded by a control volume. Since $A_x = A_y$ and the shock has infinitesimal thickness the continuity equation can be simplified to

$$\rho_x V_x = \rho_y V_y$$  \hspace{1cm} (47)

Then neglecting surface friction for the thin shock the momentum equation becomes

$$p_x A_x - p_y A_y = \rho_y A_y V_y^2 - \rho_x A_x V_x^2$$  \hspace{1cm} (48)

or since $A_x = A_y$

$$p_x + \rho_x V_x^2 = p_y + \rho_y V_y^2$$  \hspace{1cm} (49)

There is no external heat transfer across the control volume boundaries. Therefore, the process is adiabatic. This allows simplification of the energy equation to

$$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2}$$  \hspace{1cm} (50)

By utilizing the ideal gas law equation (50) can be expressed as the following
Figure 4: Control Volume for Normal Shock Wave.
This expression is used to calculate the static pressure ratio across a normal shock. Substitution of equations (49) and (51) into the continuity equation (47) provides the following equation for $M_x$ as a function of $M_y$

$$M_y^2 = \frac{[M_x^2 + \frac{2}{k-1} \left( \frac{2k}{k-1} M_x^2 \right) - 1]}{k-1}$$

This equation is solved for $M_y$ explicitly after $M_x$ has been found using an iterative technique.

As stated earlier, the shock process is adiabatic; thus the stagnation temperature does not change across the shock. Equation (16) can be used to state that

$$T_x \left( 1 + \frac{(k-1)M_x^2}{2} \right) = T_y \left( 1 + \frac{(k-1)M_y^2}{2} \right)$$

or rearranging

$$\frac{T_x}{T_y} = \frac{\left[ 1 + \frac{(k-1)M_x^2}{2} \right]}{\left[ 1 + \frac{(k-1)M_y^2}{2} \right]}$$
Stagnation pressure variations across a normal shock can be related directly to the entropy rise across the shock as stated by the following equation

\[ \frac{s_y-s_x = C_p \frac{R}{R} \ln \frac{T_y}{T_x} - \ln \frac{P_y}{P_x} }{ } \] (55)

By substitution, then, the following expression for the ratio of stagnation pressure across the shock can be written

\[ \frac{P_{oy}}{P_{ox}} \left( \frac{k+1}{2} \right) \left[ \frac{1}{2k} \frac{M_x^{2-k-1}}{k+1} \frac{1}{k+1} \right] \frac{1}{k-1} \] (56)

For converging/diverging and converging nozzles, the throat section of these nozzles is choked (i.e., flowrate has reached a maximum) when the Mach number at the throat is sonic. The ratio of static pressure at the throat to inlet stagnation pressure is termed the critical pressure ratio \( (P_c) \). Thus for \( M=1 \) equation (29) simplifies to

\[ P_c = \left( 1 + \frac{(k-1)}{2} \right)^{\frac{k}{(k-1)}} \] (57)

For converging nozzles only the given backpressure to inlet stagnation pressure ratio \( (P_c) \) is compared with the calculated \( P_c \) to
determine if the throat flow conditions are sonic.

The following equation shows the relationship between backpressure and area ratios that is true when the converging/diverging nozzle throat section transitions to subsonic flow

\[
\frac{P_b}{P_o}^2 - \frac{P_b}{P_o}^{(k+1)/k} = k-1 \left(\frac{2}{k+1}\right) \frac{A_t^2}{A_e^2} \tag{58}
\]

Then if \( P_b/P_o = P_r \) and air is assumed as the medium this equation reduces to

\[
P_r^{1.43} - P_r^{1.71} = 0.0668 \left(\frac{A_t}{A_e}\right)^2 \tag{59}
\]

GASDYNAMICS utilizes equation (52) to calculate the Mach number change across the shock. Also, since the static and stagnation pressures are known upstream of the shock, the static and stagnation pressure ratio given by equations (51) and (56) can be used to calculate the static or stagnation pressure downstream of the shock. The static temperature ratio across the shock is also calculated using equation (54). Equations (58) and (59) are utilized to determine if the throat of the nozzle is sonic when backpressures exceed the lower limit for pressure that will cause a normal
shock to stand in the nozzle.

The relationships shown for the shock wave analysis are utilized in both the nozzle and duct sections of the GASDYNAMICS model. GASDYNAMICS assumes that flow is isentropic up to the differential length of the nozzle where the shock wave stands and utilizes the relationships shown by equations (23 through 29) for this section. Shock wave analysis is utilized to cross the differential section that contains the shock wave. Then isentropic flow analysis is utilized from that point to the nozzle exit. GASDYNAMICS does not utilize shock wave analysis relationships in conjunction with Rayleigh flow sections since in most practical circumstances a shock will not stand in the Rayleigh duct. If a duct with friction is attached to an isentropic nozzle, the Fanno flow relationships as shown by equations (30 through 36) are utilized up to the differential length of the duct where the shock wave stands. Shock wave analysis is utilized again for the section where the shock wave stands. Then Fanno flow relationships are utilized from that point to the duct exit.
COMPUTER MODEL DEVELOPMENT

The GASDYNAMICS program was written in Applesoft Basic language with the Global Apple Line Editor (GALE) program loaded into memory. An Apple IIE computer with EPSON printer and a Grappler Interface Card was utilized to perform this task. GASDYNAMICS is also compatible with Imagewriter printers with minor software modifications. The Apple IIE has a memory capacity of 128 kilobytes. The final version of the program is contained in Appendix 2. The flowchart describing the logic that was used to design this program is shown in Appendix 1.

In addition to its use on the Apple IIE computer the program was adapted to both IBM PC and Apple MacIntosh. This required a change of program syntax to Microsoft Basic Language and proper disk format. GASDYNAMICS is available in this form also.

GASDYNAMICS begins with a series of inputs that the operator must make concerning inlet stagnation pressure and temperature and static backpressure as shown by Figure 5. In addition, the operator must choose between a converging-diverging
$P_B = 70 \text{ kpa for Case 1 (Choked Throat)}$

$P_B = 690 \text{ kpa for Case 5 (Unchoked Throat)}$

**Figure 5.** Converging/Diverging Nozzle. Reference Cases 1 and 5.
nozzle or simply a converging nozzle. For the converging-diverging nozzle, both a throat diameter and an exit plane to throat diameter ratio are required inputs whereas the converging nozzle does not require a diameter input since mass flowrate is given per unit throat area.

GASDYNAMICS provides the option to include a constant diameter duct attached to either of these nozzle types. If the operator chooses this option, inputs for friction and heat transfer are required. The dimensionless term FL/D where F is the Darcy-Weisbach friction factor, L is the duct length and D is the duct diameter is the required Fanno flow input whereas the program requires a heat transfer input, QQ, in kilojoules per kilogram to satisfy Rayleigh flow. Finally, the operator must input the gaseous medium. If air, helium, or argon is chosen as the gaseous medium the ratio of specific heats (k) and the gas constant (R) are automatically assigned. Otherwise values of the gas constant and ratio of specific heats must be provided for the gaseous medium chosen. The gas constant inputs must be expressed in metric units (kj/kg °K).
**Converging Nozzle Flow**

For a converging nozzle without a duct, GASP DYNAMICS performs a comparison of static backpressure to inlet stagnation pressure ratio (PR) to the critical pressure ratio (PC) in a supporting subroutine called (PCRIT). PR and the specific heat ratio are the only input for this subroutine. The critical pressure is calculated using equation (57). If PR is less than PC the throat is choked and GASP DYNAMICS sets M=1 at the throat. Since Mach number is known static pressure may be calculated using equation (29) and static temperature may be calculated using equation (16).

If PR is greater than PC, the throat is subsonic and the throat Mach number is determined by calling subroutine MCALC. The input for this subroutine is PR and the output is throat Mach number. Mach number is determined by incrementally increasing Mach number in .01 steps using equation (29) to calculate P/PO. Then the calculated value of P/PO is compared with the actual value of PR. When the absolute difference between these two parameters is less than an allowable convergence the correct value of Mach number has been found. The flowchart in Appendix 1 details this process.
Converging Nozzle with Rayleigh Duct

For a converging nozzle followed by a heated duct, GASDYNAMICS calls a subroutine (CONVERGE) to calculate Mach number at the duct exit and at the throat. This subroutine begins by assuming the duct is thermally choked and compares the corresponding P* value with the actual backpressure to determine if backpressure affects this assumption. This is done by using equation (39) to find the stagnation temperature at the duct exit, TT*. Since the stagnation temperature at the nozzle throat is known, the corresponding nozzle throat Mach number for the ratio of these temperatures may be found by decreasing M in 0.01 increments from an initial value of 1.0 to search for the correct value that will satisfy equation (38). Next, P/P* is calculated at the duct entrance using equation (45). Then, since the actual static pressure is also known from equation (29), the P* value can be found. CONVERGE now compares P* with PB. If PB is less than P*, the system is thermally choked as previously assumed and the duct exit plane Mach number =1.

If PB is greater than P*, the operator is given a message that the backpressure is too high to allow choked flow in the nozzle or
duct. At this point a new subroutine (PSTAR) is called to calculate the Mach number for a specific P/P* at the duct exit plane. P/P* is known since P=PB and P* was previously calculated. PSTAR decreases Mach number in increments of 0.01 from an initial value of 1.0 to search for the correct Mach number that will satisfy equation (45). Thus the duct exit plane Mach number is found using subroutine PSTAR and the program returns to CONVERGE.

TT/TT* is calculated using equation (38) at the duct exit plane. Then this ratio is multiplied by the stagnation temperature ratio across the Rayleigh duct to obtain TT/TT* at the converging nozzle throat. The subroutine returns to incrementally search out the correct Mach number at the nozzle throat as previously described. Since the Mach number is known at both ends of the duct, static and stagnation pressure ratios may be calculated using equations (45) and (46).

**Converging Nozzle with Fanno Duct**

For a converging nozzle followed by an adiabatic duct with friction, GASDYNAMICS calls a subroutine (FANCONVERG) to calculate Mach number at the duct exit and at the throat. This subroutine begins by assuming the duct is choked by friction and compares the
corresponding $P^*$ value with the actual backpressure to determine if backpressure affects this assumption. This is done by using equation (31) to find the throat Mach number. Mach number is decreased in 0.01 increments from an initial value of 1.0 to search for the correct value that will satisfy equation (31). Since the throat Mach number is known, the corresponding static pressure can be calculated using equation (29). Next, $P/P^*$ is calculated at the throat using equation (35). Then, since the actual static pressure is also known from equation (29), the $P^*$ value can be found. Then FANCONVERG compares $P^*$ with PB. If PB is less than $P^*$, the system is choked by friction as previously assumed.

If PB is greater than $P^*$, the operator is given a message that the backpressure is too high to allow choked flow in the nozzle or duct. At this point a new subroutine (PSTARFAN) is called to calculate the Mach number for a specific $P/P^*$ at the duct exit plane. $P/P^*$ is known since $P=PB$ and $P^*$ was previously calculated. PSTARFAN decreases Mach number in increments of 0.01 from an initial value of 1.0 to search for the correct Mach number that will satisfy equation (35). Thus the duct exit plane Mach number is
found using subroutine PSTARFAN and the program returns to FANCONVERG.

FL/D at the duct exit plane is calculated using equation (31). Since the total FL/D for the duct is known, the sum of these values is the FL/D at the nozzle throat. Now the subroutine returns to incrementally search out the correct Mach number at the nozzle throat for a given FL/D as previously described. Since the Mach number is known at both ends of the duct static and stagnation pressure ratios may be calculated using equations (35) and (36).

**Converging/Diverging Nozzle Flow**

If the converging/diverging nozzle is chosen, GASDYNAMICS initially assumes that any following duct has no influence on the nozzle flow variables. For a converging-diverging nozzle without a Rayleigh or Fanno duct GASDYNAMICS determines the range of backpressures over which a normal shock will occur. This is done by a supporting subroutine (BPRANG). BPRANG finds the lower value of backpressure that will allow a shock to occur in the nozzle by placing a shock at the nozzle exit plane and calculating the resulting downstream pressure. This is accomplished by utilizing subroutine ASTAR to find the supersonic upstream Mach number.
Then the downstream Mach number is calculated directly using equation (52). The pressure ratio across the shock is found using equation (51). This pressure ratio multiplied by the upstream static pressure yields the downstream static pressure. BPRANG compares this calculated pressure with the given backpressure. If the given backpressure is less than the calculated static pressure, the operator receives a message that the backpressure is not high enough to force a shock to occur and calculations for the exit plane Mach number proceed. If higher, GASDYNAMICS proceeds to determine the highest backpressure that will allow a shock to occur utilizing equation (58).

If the backpressure is high enough to cause subsonic conditions to exist at the nozzle throat, Mach number is first calculated at the nozzle exit plane using subroutine MCALC as previously described for the converging nozzle. Since the exit plane Mach number is known, the exit plane area to $A^*$ ratio ($AA$) can be found using equation (24). Then the nozzle throat area to $A^*$ ratio can be calculated by multiplying $AA$ by the throat to exit plane area ratio. Once this area ratio is known subroutine ASTAR is called to
find the Mach number at the nozzle throat. An explanation of the ASTAR subroutine follows.

Consider now the conditions where a Rayleigh or Fanno duct is attached to the converging/diverging nozzle. Although previous calculations may have shown that backpressure alone is not high enough to force a shock to occur in the nozzle, the addition of heating and/or friction may provide enough additional resistance to flow to cause the shock to occur. Thus the operator must decide whether a shock exists based on backpressure and the relative magnitude of duct heating and/or frictional terms.

If a shock is assumed or dictated by the backpressure check, an initial normal shock location must be provided by indicating a value of shock diameter ratio (DY). This assumption is checked by first comparing the shock diameter ratio (DY) input with the throat diameter and exit plane diameter ratio. When this check has been satisfied, the supersonic upstream Mach number for this location is found by utilizing a subroutine (ASTAR). ASTAR uses an iterative technique to incrementally change M in 0.01 increments calculating each A/A* corresponding to these Mach numbers using equation (24). This calculated A/A* is compared with the actual
$A/A^*$ where the actual $A/A_1^*$ is equal to the ratio of the area where the shock is assumed ($A_x$) to $A_1^*$. The absolute value of this difference is compared to an allowable error. When the convergence criterion has been satisfied, the subroutine sets $M=M_x$ as shown by Figure 4 and returns to the mainline program. The corresponding subsonic $M_y$ is calculated using equation (52). Static and stagnation pressure ratios as given by equations (51) and (56) are also calculated.

At this point GASDYNAMICS finds the nozzle exit plane Mach number as follows. Since $M_y$ is known $A_x/A_y^*$ may be calculated directly from equation (24). Now $A_e/A_y^*$ can be found by the multiplication of the nozzle exit to throat diameter, $A_e/A_t$, the inverse of $DY$, and $A_x/A_y^*$. With $A_e/A_y^*$ known ASTAR is utilized again as previously described to find the exit plane Mach number.

The exit plane static pressure is found using equation (29) and another subroutine (PCHEK) is called to compare this calculated pressure with the actual backpressure. If the calculated pressure is not within allowable convergence the operator is notified. A new
value of DY is automatically assigned by GASDYNAMICS and the calculations for the shock location begin again. DY is larger if the calculated backpressure is greater than the actual pressure and smaller if the calculated backpressure is less than the actual pressure. This iterative process continues until the shock is found. In the event that the actual backpressure and the calculated exit plane pressure become increasingly divergent, GASDYNAMICS would require the operator to reevaluate the decision concerning a shock in the nozzle.

Converging/Diverging Nozzle with Rayleigh Duct Flow

After the nozzle flow conditions have been established, GASDYNAMICS determines if Rayleigh heating section exists. If so, calculations proceed into the Rayleigh duct. When heat transfer occurs, GASDYNAMICS utilizes equation (39) derived from the energy equation to find the stagnation temperature at the end of the Rayleigh duct. At this point GASDYNAMICS uses equation (38) to calculate the duct inlet stagnation temperature to $TT^*$ ratio since duct inlet Mach number is known. This ratio is multiplied by the ratio of the duct exit stagnation temperature to duct inlet stagnation temperature to obtain the duct exit stagnation
temperature to TT* ratio (TT/TT*). G ASDY NAMICS checks to see if this ratio is less than unity. If the ratio is greater than one, the operator receives a message that the heating rate is too high to allow the calculated flow conditions at the nozzle exit. At this point G ASDY NAMICS used subroutine (QLIM) to determine the maximum heating rate that will allow choked conditions at the nozzle throat with a thermally choked duct. This is done by using ASTAR to find the subsonic Mach number that corresponds to the A/A* at the nozzle exit plane. TT/TT* can be calculated directly at the nozzle exit using equation (38). Then the duct exit plane temperature can be obtained since the stagnation temperature at the nozzle exit plane is known. Equation (39) can be used to calculate the maximum heating rate that will allow choked flow at the nozzle throat. If the given heating rate is higher than the value calculated by QLIM, G ASDY NAMICS proceeds to calculate subsonic flow conditions at the nozzle throat and exit plane. If the given heating rate is less than the value calculated by QLIM, G ASDY NAMICS returns to request a shock location in the nozzle.

If the exit plane stagnation temperature ratio is less than one, subroutine TTSTAR is utilized for finding Mach number for a
given TT/TT* at the exit of the Rayleigh duct. TTSTAR utilizes the same iterative technique to incrementally increase Mach number as previously described for ASTAR to search for the correct Mach number at the Rayleigh duct exit. M is increased in 0.004 steps for Mach number greater than 1. Increments are 0.01 for M less than 1. TT/TT* is calculated at each M and compared with the actual value of TT/TT*. At the end of this process, TTSTAR identifies the Mach number for the exit of the Rayleigh duct and returns to the mainline program where static and stagnation pressure ratios and static temperature ratios are calculated. As in the nozzle calculations, the calculated exit plane pressure is compared with the backpressure by subroutine (PCHEK). For subsonic exit plane conditions the exit plane pressure and backpressure must be equal. If the exit plane Mach number = 1.0 GASDYNAMICS compares the calculated value of P* with PB. If PB is greater than P* the operator receives a message that the backpressure is too high to allow calculated flow conditions and upstream shock locations must be adjusted.
Converging/Diverging Nozzle with Fanno Flow

If Rayleigh and Fanno ducts appear in series following a converging/diverging nozzle, the exit plane flow conditions are the same regardless of the order in which the flow resistances occur. For simplicity, G ASDYN AM ICS assumes that the Rayleigh duct, if present, follows the nozzle and precedes the Fanno duct. Calculations of Mach number proceed as follows. The dimensionless FL/D term (CA) is calculated using equation (31) for the Mach number at the beginning of the Fanno duct. The given FL/D input (FL) for the duct is subtracted from CA. If the resulting FL/D term is negative, G ASDY N AM ICS assumes a shock in the FANNO duct and calculations proceed as described in the following section. If the resulting FL/D term is positive, a supporting subroutine (FANMACH) is called to calculate the corresponding Mach number for this FL/D term. FANMACH uses the same iterative technique to incrementally change Mach number to search for the correct Mach number at the Fanno duct exit. Mach number is changed in increments of 0.01 for supersonic and subsonic flow. FL/D is calculated at each Mach number and compared with the actual value of FL/D. At the end of this process FANMACH identifies the Mach number for the exit of
the Fanno duct and returns to the mainline program where static and stagnation pressure ratios are calculated.

**Shock in the Fanno Duct**

For cases where friction chokes the duct and a normal shock occurs, a supporting subroutine (FANSHOCK) is called to determine its location. This subroutine begins by telling the operator that if previous work by GASDYNAMICS has determined a shock location in the nozzle the location of that shock will require adjustment to meet the new frictional requirements that were previously neglected. The program then returns to a point where the operator must input a new shock location for the nozzle.

If a shock does not exist in the nozzle, the FANSHOCK subroutine begins the iterative process of finding the shock in the Fanno duct. Mach number is decreased in 0.03 steps beginning with Mach number at the entrance to the Fanno duct as an initial value in an effort to search for the supersonic Mach number at which the shock occurs. For each supersonic Mx the corresponding subsonic My is calculated from equation (52). Then FL/D terms are calculated for each of these Mach number in accordance with equation (31). After these values have been determined, the FL/D term for My is
added and the FL/D term for Mx is subtracted to calculate the FL/D term for the duct. This calculated FL/D is compared to the actual FL/D. If the absolute value difference of this comparison is less than 0.02, the correct shock location has been found.

At the conclusion of this process, the duct exit plane pressure (PE) is calculated as follows. Equation (29) is used to calculate the static pressure at the nozzle exit. Then this static pressure is multiplied by \( \frac{P}{P^*} \) pressure ratios as calculated by equation (45) for the Rayleigh duct and equation (35) for the Fanno duct and the static pressure ratio across the normal shock given by equation (51). This resultant value is compared with the given backpressure. If indeed friction controls the shock location, the exit plane pressure must be greater than the backpressure. GASDYNAMICS provides the operator feedback on this comparison and automatically calls another subroutine (BACKPRESSURE) to recalculate the shock location if PB is greater than PE.

BACKPRESSURE begins by calculating \( P^* \) for the Mach number at the beginning of the Fanno duct using equation (35). Since the backpressure must be equal to the exit pressure, \( \frac{PB}{P^*} \) for the exit plane Mach number may be calculated. Mach number is decreased
in .01 increments beginning with Mach number=1 as an initial value in an effort to search for the subsonic Mach number that will satisfy equation (35) for the known PB/P*. Once the correct exit Mach number (ME) has been found, the new shock location may be determined as previously described. If BACKPRESSURE determines a Mach number at which the shock will occur that is greater than the Mach number at the beginning of the Fanno duct, GASDYNAMICS will notify the operator that the combination of friction, heating and backpressure has forced the shock into the nozzle.

When the calculations for duct exit plane conditions have been completed and all the pressure checks satisfied, GASDYNAMICS utilizes subroutines (TAB1), (TAB2), and (TAB3) to present important results in tabular form. Static pressure, static temperature, Mach number, area, mass flowrate, stagnation pressure, velocity, and density are presented for the discrete points shown by figures 5 through 13. These results have been rounded off in the second decimal and are expressed in metric units.

TAB1 is used for calculating the isentropic flow parameters in either the converging or converging/diverging nozzle when a duct is not attached. TAB2 is used for all cases when a shock occurs in the
nozzle. TAB3 is used for cases where a shock is not present in either the duct or nozzle. Also, TAB3 is used for cases where a shock occurs in the Fanno duct.

The input for all of these subroutines is Mach number, stagnation pressure ratios, and stagnation temperature for each discrete point. The stagnation pressure is calculated at each discrete point in the flowstream by multiplication of the given inlet stagnation pressure and the stagnation pressure ratios calculated during previous subroutines. Then equation (29) is utilized to calculate the static pressure for each discrete point. Stagnation temperatures are known inputs to these subroutines. Thus equation (16) is utilized to calculate the static temperature at each discrete point. Since both pressures and temperatures have been calculated, equation (10) can be utilized to calculate mass flowrate per area. Each TAB subroutine calls subroutine VECALC to calculate velocity using equation (8) and density using equation (6). Thus the output for these subroutines are static pressure and temperature, mass flowrate per area, stagnation pressure, velocity, and density. These subroutines also contain the programming that commands the printer to tabulate the results as shown by Appendix 1.
The flow properties at other points in the ducts or nozzle may be found by reinitializing the program inputs and reperforming GASDYNAMICS with this new point of interest as one of the selected discrete planes that the program recognizes. As an example, if a flow property at a different point in the Fanno duct is required, the operator would make appropriate adjustment to the FL/D term because L has changed and input the new FL/D value for the next run. GASDYNAMICS asks the operator if another run is desired at the end of each run.
RESULTS AND DISCUSSION

As stated previously, the purpose of this work has been to provide a versatile computer program that will solve a variety of compressible flow problems that a student would encounter during the study of GASDYNAMICS. Different cases have been selected to demonstrate GASDYNAMICS' versatility in performing that task. The program is certainly not limited to the cases discussed here. These, however, have been chosen as examples of GASDYNAMICS' capabilities.

The computer model was verified by hand calculations for the cases presented here as well as others not shown. In addition, some textbook example problems were also run and used to verify the computer model.

Three different converging nozzle cases were run with air as the gaseous medium. Two of these are without a duct. The backpressures are chosen to make one throat choked whereas the other case demonstrates the results of an unchoked condition. The remaining case is an example of a choked Rayleigh duct
Ten cases are shown for converging-diverging nozzles. The first three are without ducts. Backpressures are chosen to demonstrate all possibilities for choked conditions at the exit plane and the throat. Six cases are shown for Rayleigh ducts. Three of these also have Fanno ducts attached. One additional case is shown for an adiabatic Fanno duct where backpressure controls the shock location rather than friction. Shock locations are determined for the nozzle and Fanno ducts.

Figures 5 through 13 show the configuration of the nozzles and connecting ducts analyzed by GASDYNAMICS. The results of selected cases are shown by Table I. Computer printouts for each case are shown in Appendix 3.

**Converging Nozzles**

Case 2 shows a converging nozzle with a throat that is choked due to a low backpressure to inlet stagnation pressure ratio. At the exit plane $M = 1.0$ and thus signal waves cannot propagate from the backpressure region to the inlet stagnation area. For all backpressures from 0 to $0.528P_o$ for air flow the exit plane $M = 1.0$. 
### TABLE I

**SUMMARY OF NUMERICAL RESULTS**

<table>
<thead>
<tr>
<th>CASE #</th>
<th>MEDIA</th>
<th>NOZZLE TYPE</th>
<th>SHOCK LOCAT.</th>
<th>HEATING RATE KJ/KG</th>
<th>FL/D</th>
<th>BACK PRESS KPA</th>
<th>EXIT PRESS KPA</th>
<th>EXIT TOTAL PRESS KPA</th>
<th>EXIT MACH NO.</th>
<th>EXIT TEMP. °K</th>
<th>EXIT TOTAL TEMP. °K</th>
<th>EXIT MASS FLOW RATE KG/SEC-SQRMET</th>
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<td>C/D</td>
<td>N/A</td>
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<td>CONV</td>
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<td>0</td>
<td>70</td>
<td>370</td>
<td>700</td>
<td>1.00</td>
<td>183</td>
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<td>1901</td>
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<td>569</td>
<td>581</td>
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<td>N/A</td>
<td>0</td>
<td>0</td>
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<td>570</td>
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<td>0.55</td>
<td>207</td>
<td>220</td>
<td>1515</td>
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<td>0</td>
<td>70</td>
<td>50</td>
<td>282</td>
<td>1.79</td>
<td>170</td>
<td>280</td>
<td>1900</td>
</tr>
<tr>
<td>7</td>
<td>AIR</td>
<td>CONV</td>
<td>N/A</td>
<td>60</td>
<td>0</td>
<td>70</td>
<td>340</td>
<td>644</td>
<td>1.00</td>
<td>233</td>
<td>280</td>
<td>1552</td>
</tr>
<tr>
<td>8</td>
<td>AIR</td>
<td>C/D</td>
<td>NOZ.@</td>
<td>60</td>
<td>0</td>
<td>570</td>
<td>571</td>
<td>587</td>
<td>0.20</td>
<td>278</td>
<td>280</td>
<td>1880</td>
</tr>
<tr>
<td>9</td>
<td>AIR</td>
<td>C/D</td>
<td>NOZ.@</td>
<td>60</td>
<td>0</td>
<td>128</td>
<td>128</td>
<td>217</td>
<td>0.90</td>
<td>286</td>
<td>331</td>
<td>1880</td>
</tr>
<tr>
<td>10</td>
<td>AIR</td>
<td>C/D</td>
<td>N/A</td>
<td>60</td>
<td>0.15</td>
<td>70</td>
<td>71</td>
<td>217</td>
<td>1.37</td>
<td>203</td>
<td>280</td>
<td>1880</td>
</tr>
<tr>
<td>11</td>
<td>AIR</td>
<td>C/D</td>
<td>DUCT.@</td>
<td>60</td>
<td>0.35</td>
<td>70</td>
<td>104</td>
<td>197</td>
<td>1.00</td>
<td>233</td>
<td>280</td>
<td>1880</td>
</tr>
<tr>
<td>12</td>
<td>HE</td>
<td>C/D</td>
<td>DUCT.@</td>
<td>60</td>
<td>0.35</td>
<td>70</td>
<td>87</td>
<td>179</td>
<td>1.00</td>
<td>173</td>
<td>232</td>
<td>748</td>
</tr>
<tr>
<td>13</td>
<td>AIR</td>
<td>C/D</td>
<td>DUCT.@</td>
<td>0</td>
<td>0.65</td>
<td>150</td>
<td>150</td>
<td>199</td>
<td>0.65</td>
<td>203</td>
<td>220</td>
<td>1880</td>
</tr>
</tbody>
</table>

**NOTES:**

1) **CONVERGING/DIVERGING NOZZLE EXIT PLANE AREA TO NOZZLE THROAT AREA RATIO IS 4.0.**

2) **INLET STAGNATION CONDITIONS ARE: P₀=700KPA AND T₀=220°K. THESE ARE ARBITRARY VALUES THAT WERE MAINTAINED CONSTANT FOR EACH CASE.**
Case 4, which is exhibited in Figure 6, demonstrates the effects of raising the backpressure to 570 kpa which is above the critical value of .528Po. In this instance the throat of the converging nozzle becomes subsonic and the exit plane pressure is equal to the backpressure of 570 kpa. The static temperature has also increased whereas stagnation pressure and temperature have not been affected in these isentropic flow cases. The Case 4 converging nozzle mass flowrate is 20% lower than that shown by Case 2 when the backpressure allowed sonic flow at the nozzle exit. The mass flowrate will continue to decrease as the backpressure is increased for the converging nozzle because the nozzle is no longer choked.

Case 7 shows the effects of adding a heated frictionless constant diameter duct to the nozzle exit plane as illustrated by Figure 7. As heat is added to subsonic flow the flow properties change as indicated by Table 2. Case 7 shows that the choke point changes from the throat to the duct exit plane when the heated duct is added. Thus the nozzle throat has subsonic conditions for all backpressures less than P*. The area and Mach number for cases 2 and 7 are the same but the exit plane temperature is 18% higher due to the
TABLE II

EFFECT OF HEAT ADDITION \((dT_0 > 0)\) ON THE FLOW PROPERTIES FOR AN IDEAL GAS IN A CONSTANT AREA DUCT

<table>
<thead>
<tr>
<th>Property</th>
<th>(M &lt; 1)</th>
<th>(M &gt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{dM}{M})</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(\frac{dP}{P})</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(\frac{d\rho}{\rho})</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(\frac{dT}{T})</td>
<td>+ for (M &lt; 1/\sqrt{k})</td>
<td>+</td>
</tr>
<tr>
<td>(\frac{dV}{V})</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(\frac{dP_0}{P_0})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\frac{ds}{c_p})</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
**Figure 6:** Converging Nozzle. Reference Cases 2 and 4.

- $P_B = 70$ kpa For Case 2 (Choked)
- $P_B = 570$ kpa For Case 4 (Unchoked)
Figure 7: Converging Nozzle Followed By Constant Diameter Heated Duct. Reference Case 7.
increase in exit plane stagnation temperature and the exit plane pressure is 8% lower than Case 2 conditions due to the decrease in exit plane stagnation pressure. The resulting mass flowrate for a system with a heated duct is less than that for a system without the duct as demonstrated by comparing cases 2 and 7. If the backpressure is increased to a value greater than $P^*$ the exit plane and backpressures are equal. However, the exit temperature increases while and $M$ decreases; thus the mass flowrate decreases even more.

**Converging/Diverging Nozzles**

Cases 1, 3, and 5 show a comparison of exit plane conditions for a converging-diverging nozzle as shown by figures 5 and 8. The backpressure to stagnation pressure ratio (PR) for Case 1 is less than the calculated lower limit for pressure that would cause a shock to stand in the nozzle. As a result sonic conditions exist at the throat and supersonic conditions at the exit plane.

Case 3 shows the effect of raising the backpressure to 570 kpa. The converging-diverging nozzle retains sonic flow at the throat but the increased backpressure causes a normal shock to stand in the
nozzle divergent section at a location corresponding to a diameter ratio of 1.175, as shown by Figure 8. Thus the exit plane conditions are subsonic. The actual mass flowrate is the same for cases 1 and 3 since the throat is choked for both cases. GASDYNAMICS predicts the same mass flowrate at the throat but at the duct exit plane a 1% error exists in the calculation for Case 3.

As the backpressure continues to increase the shock becomes weaker as it moves closer to the throat section. When the shock reaches the throat and backpressure continues to increase, the shock vanishes and flow becomes subsonic throughout the nozzle. The relationship between PR and area ratio shown by equation (58) provides a means of determining the backpressure that will cause subsonic conditions at the throat. Case 5 demonstrates this condition. For subsonic conditions at the throat, the mass flowrate is reduced from the choked condition. As the backpressure continues to increase to the stagnation pressure, the mass flowrate decreases to zero. As an example, the conditions shown for Case 5 cause a 5% reduction in mass flowrate compared to the choked conditions presented by Case 1.
Figure 8: Converging/Diverging Nozzle with Normal Shock. Reference Case 3.
Rayleigh Flow

Cases 6, 8, and 9 show the effect of adding a heated, frictionless, constant diameter duct to the nozzle exit plane as illustrated by figures 9 and 10. As heat is added to supersonic flow the flow properties change as indicated by Table 2 with the Mach number decreasing toward unity. This is illustrated by Case 6. As an example, if the heat transfer continued to increase to 114 kj/kg the duct exit Mach number reaches a limiting condition of 1.0. Note that although all figures show the heat transfer evenly distributed over a finite length of duct the same change in flow properties is obtained if the heat transfer occurs at one concentrated area.

Additional increases in heating will cause the flow to follow a new Rayleigh line on the temperature vs. entropy curve shown by Figure 3. This also means a decreased mass flow rate for the system. [1] The new Rayleigh line requires a larger stagnation temperature $T_T^*$. This change can only be caused by a normal shock in the nozzle if inlet stagnation conditions remain constant. Thus, increasing the heat transfer rate will not cause a shock to stand in the heated duct. For example, if the heat transfer rate is increased to 300 kj/kg, a
Figure 9: Converging/Diverging Nozzle Followed By Constant Diameter Heated Duct. Reference Case 6.
Figure 10: Converging/Diverging Nozzle Followed By Constant Diameter Heated Duct.
Reference Cases 8 And 9.
shock occurs at diameter ratio $= 1.3$ in the nozzle and the flow is subsonic in the heated duct.

Case 9 illustrated by Figure 10 represents a limiting condition where the heating rate and backpressure are sufficient to cause a normal shock to occur at the nozzle to duct interface with subsonic conditions at the duct exit plane. The governing equations can also be satisfied for the given Case 9 conditions by placing the shock in the Rayleigh duct. [2] However, this does not represent a practical solution since an infinitesimal increase in backpressure or heating would cause the shock to stand in the nozzle.

As an example, consider Case 8 where the backpressure is increased to 570 kpa and a normal shock occurs in the nozzle at diameter ratio $= 1.16$, as illustrated by Figure 10. A comparison between cases 3 and 8 shows that the Case 8 shock location is slightly closer to the nozzle throat than that of Case 3. This is because of the heated duct at the nozzle exit. The mass flowrate is unchanged as a result of adding the heated duct since the nozzle remains choked and the inlet stagnation properties remain unchanged.
Note that changes in stagnation temperature only occur where heat is added to or removed from the duct in accordance with the law of conservation of energy. Flow across a normal shock or through a duct with friction does not change the stagnation temperature of the gas. Stagnation pressure is decreased across a normal shock and through both Fanno and Rayleigh flow ducts. Stagnation temperatures and pressures at the nozzle exit plane are shown by Table 1.

Although GASDYNAMICS does not limit results based on excessively high temperatures, gases such as air may not behave as ideal gases because of molecular dissociation. Therefore, the assumptions required by the governing equations incorporated in GASDYNAMICS may be violated and the operator should exercise caution when utilizing the GASDYNAMICS results for conditions where temperature is excessive. Many sources agree that air does not behave as an ideal gas at temperatures above 1000°K. GASDYNAMICS uses a value of 1.40 for the ratio of specific heats for air. These limitations do not apply to monatomic gases such as helium.
Fanno Flow

Case 10 demonstrates the effect of adding an adiabatic constant diameter Fanno flow duct to the heated duct with supersonic flow as shown by Figure 11. A comparison of cases 6 and 10 shows that the addition of a duct with friction decreases the exit Mach number when the flow is supersonic. Table 3 shows a complete outline of how flow properties change for Fanno flow. As the FL/D term increases from zero to 0.24 in the selected example, the exit Mach number reaches a limiting condition of 1.0. Additional increases in friction will force a normal shock to occur in the Fanno duct as shown by Case 11 where a shock stands at $M_x = 1.6$. As friction continues to increase, the shock location moves closer to the exit of the heated duct. However, when the shock reaches the transition between the heated duct and the Fanno duct, additional frictional increases force the shock to occur in the nozzle.

Cases 11 and 12 demonstrate GASDYNAMICS' capability to find a shock location in the Fanno portion of a duct preceded by the converging-diverging nozzle. This is illustrated by Figure 12. The shock location in each case is determined by the friction term rather than backpressure. The duct is choked by friction and the maximum
### TABLE III

**EFFECT OF FRICTION ON THE FLOW PROPERTIES FOR AN IDEAL GAS IN A CONSTANT AREA DUCT**

<table>
<thead>
<tr>
<th>Property</th>
<th>$M &lt; 1$</th>
<th>$M &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\rho/\rho$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$dV/V$</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$ds/c_p$</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
Figure 11: Converging/Diverging Nozzle Followed By Constant Diameter Duct with Heating and Friction. Reference Case 10.
Figure 12: Converging/Diverging Nozzle Followed By Constant Diameter Heated Duct with Shockwave and Friction. Reference Cases 11 and 12.
duct length (Lmax) has been reached. This is known since the calculated exit pressure for each case is greater than the given backpressure. Note that increasing or decreasing the backpressure in the range between zero and P* does not affect the shock location. [3] The exit Mach number is 1.0 for cases 11 and 12 since the exit pressure is greater than the backpressure in each case. The exit plane Mach number must be one to satisfy the second law of thermodynamics if the duct is choked by friction. [1]

For cases where the friction is low enough to allow supersonic conditions at the duct exit GASDYNAMICS calls a subroutine (BPFAN) to determine the range of backpressures that will cause a normal shock to stand in the Fanno duct. BPFAN places a shock at the duct entrance and calculates the resulting exit plane pressure utilizing the same equations previously discussed. BPFAN also places a shock at the duct exit and calculates the resulting exit plane pressure. If the given backpressure is between these two calculated pressures, a normal shock exists in the duct and GASDYNAMICS returns to request a normal shock location. Otherwise the previously calculated exit conditions are correct.
Figure 13: Converging/Diverging Nozzle Followed By Constant Diameter Adiabatic Duct with Shockwave. Reference Case 13.
Case 13 shows the results for a condition where the backpressure rather than friction controls the location of the shock. This case represents a situation where the duct length is less than \( L_{\text{max}} \). Therefore, the supersonic flow is unable to sense changes in duct length occurring ahead of it. Thus, the flow adjusts by means of a normal shock. The given backpressure is greater than the Fanno line \( P^* \) and equal to the exit plane pressure. Note that for subsonic flow conditions the Mach number increases as friction increases as shown by Table 3. Figure 13 illustrates the location of the shock for Case 13 conditions. Further backpressure increases would force the shock back into the nozzle and the Fanno duct would be entirely subsonic.

Cases 11 and 12 were run with air and helium, respectively. Inlet stagnation conditions, heat transfer, friction nozzle geometry parameters, and backpressure are identical for these two cases. A comparison of the shock location relative to the nozzle exit shows that the shock resulting from helium (Case 12) gas flow is weaker than the shock shown by Case 11 for air. This is because the gas constant \( (R) \) and ratio of specific heats \( (k) \) for helium are lower than
those of air. The ratio of static pressure across the shock for air (Case 11) is 12.3% higher than that produced by helium (Case 12). The mass flow rate through the duct with helium as the medium is reduced by 60% when compared to air.

GASDYNAMICS does not provide solutions for situations where heating, area change, or friction occur simultaneously. The governing differential equations for ducts with area change, heating and friction are given by equations shown in the previous section. These equations have no general closed-form solution and must be numerically integrated to provide approximate solutions. However, very often one effect is much more significant than the others and an acceptable approximation can be obtained by neglecting the other two effects.
ERROR ANALYSIS

Since GASDYNAMICS utilizes a constant value of Cp for any temperature, errors for specific heat are introduced for some gases such as air. This error is magnified by calculations for mass flow rate where several parameters are affected by this error.

Since some equations must be solved by trial and error iterative techniques, GASDYNAMICS performs this task by comparing the absolute value of the difference between two calculated parameters and an arbitrary uncertainty limit. The largest allowable uncertainty limit is 0.09 when calculating Mach number less than 0.5 for a given A/A* in the ASTAR subroutine. The slope of the M vs. A/A* curve has a rapidly changing slope when Mach number approaches zero. Thus larger uncertainty allowables are necessary to avoid excessive processing time and memory utilization. Mach number is changed in increments that vary in size from 0.004 to 0.03 to achieve an accuracy of within 2%. These increments and associated accuracy apply for M less than 4. Calculations for Mach number greater than 4 require a larger increment or an expanded
dimensional matrix with associated loss of computer memory utilization. Computer processing time, memory storage capability, and computational accuracy are competing factors that were evaluated in developing GASDYNAMICS.
APPLICATION AS A TEACHING AID

GASDYNAMICS may be utilized as a teaching aid for either advanced undergraduate or graduate students. GASDYNAMICS requires that the operator provide input on thermodynamic property values, stagnation values, and assumptions about flow through the nozzle. GASDYNAMICS provides written and visual feedback on the validity of these assumptions as well as suggestions to the operator concerning changes that may be necessary to satisfy all governing equations. Because of the relative speed with which calculations are performed, inputs may be changed several times to achieve a clearer understanding of flow phenomena. Both student and professor must recognize the limitations of the program and execute different cases within the bounds of those provided by the software architecture.
GASDYNAMICS was developed as a teaching aid for college seniors or graduate students in steady compressible one-dimensional gas flow. The program can be utilized to gain a practical understanding of how backpressure, for example, may affect shock location for a variety of circumstances. It is recommended that the program be used as supplementary material in teaching the gas dynamics course. GASDYNAMICS is also useful in providing a relatively quick means of solving a wide variety of gas dynamics problems involving isentropic, Rayleigh, and Fanno flow.

The program framework can be easily modified by adding subroutines to cover additional problems such as isothermal flow. Other enhancements are also possible to expand the program's capabilities. Additional improvements could be made by incorporating imperfect gas behavior for cases where inlet stagnation temperature exceeds reasonable temperature limits depending on the accuracy needed by the operator.
APPENDICES
Appendix 1

COMPUTER PROGRAM FLOW CHART
SUMMARY OF GASDYNAMICS MESSAGES

MESSAGE ID
A  Since backpressure = stagnation pressure there is no gas flow.

B  Backpressure is low enough to allow choked flow in the nozzle. Subsonic conditions may exist at the nozzle exit plane if friction and/or heating terms in the duct are too large.

C  Backpressure is low enough to allow sonic conditions to exist at the throat.

D  Backpressure is too high to allow M=1 conditions to exist at the throat.

E  Backpressure is not high enough to force a shock to occur in the nozzle.

F  GASDYNAMICS cannot find the shock location in the nozzle: recheck assumption.

G  DY must be less than or equal to duct diameter to throat diameter ratio and greater than or equal to one; choose again.

H  Heating rate is too high to support supersonic flow @ nozzle exit. Recheck assumptions.

I  Since backpressure is less than exit pressure shock location is not affected.

J  Backpressure is too high to allow flow properties calculated assuming a duct choked by friction; the shock location is determined by the backpressure.

K  Calculated backpressure is NOT equal given backpressure; choose a different value for DY.

L  Backpressure is too high to allow choked flow in the Rayleigh duct.
M  Backpressure is too high to allow the shock to stand in the duct; assume shock is located in the nozzle.

N  Shock location as calculated to satisfy upstream conditions will not support FL/D requirements; new nozzle shock location is required.

O  Backpressure is too high to allow choked flow in the Fanno duct.

P  Backpressure is not high enough to force a shock to occur in the nozzle. However, friction and/or heating terms in the duct may be large enough to force the shock to occur.

Q  Heating rate is high enough to cause a normal shock to stand in the nozzle.

R  Heating rate is high enough to cause subsonic flow to occur throughout the nozzle.
2

IS NOZZLE CONVERGING?

Y

GOSUB B PRANG

N

GOSUB PCRIT

IS HH<GG?

Y

IS PL<PB?

N

IS DUCT ATTACHED TO NOZZLE?

N

3

3A

3B

N

Y
GOSUB MCALC
FOR M1

INPUT: DY (ASSUMED SHOCK LOC)

PRINT MESSAGE G

IS 1 < DY < DR ?

14

Y

CALCULATE AX/A1
SET F = 1

GOSUB ASTAR FOR Mx

CALCULATE MY, PT, AY
AB, PT, T2

SET L = L + 1

PRINT MESSAGE F

IS L = 6 ?

N

Y

GOSUB ASTAR FOR M1

IS Q > 0 ?

6

Y

IS PI > 0 ?

7

N

N

CALCULATE AD/A1

IS PR < PC ?

38

Y

N

IS NOZZLE CONVERGING ?

N

Y

3

3A

ASSUME A NORMAL SHOCK IN NOZZLE

CALCULATE AD/A1

38

MT

Mx

M1

My
GOSUB TAB2

GOSUB PCHECK1

18

GOSUB OLIM

GOSUB TTSTAR FOR M2

GOSUB QLIM

13

16

5

Y

IS Q = 0 AND FL/D = 0 ?

N

6

CALCULATE TT2

11

IS NOZZLE CONVERGING ?

N

CALCULATE P*

TT1

TT*

TT2

TT*

GOSUB TTSTAR FOR M2

Y

TT2 > 1

N

GOSUB QLIM

IS QQ > QH ?

Y

N
CALCULATE PF, PK, TK, PE

IS FL > 0?

IS NOZZLE CONVERGING?

CALCULATE FL

IS FL > 0?

GOSUB FANCONVERG

GOSUB FANSHOK FOR M2

GOSUB FANMACH FOR M2

GOSUB TAB2

GOSUB PCHEK1

PRINT PE, TE, ME

DO YOU WISH TO CONTINUE?

END
SUBROUTINE FANMACH FLOW CHART

BEGIN

SET M(0) = M1

I = 1 TO 300

IS M1 > 1.0 ?

Y

M(I+1) = M(I-1) + 0.01

SET ER = 0.09

CALCULATE CA(I)

SS = CA(I) - CB

IS SS < ER ?

N

Y

SET M = M(I)

NEXT I

RETURN

N

M(I) = M(I-1) - 0.01

IS M(I) < 1.0 ?

SET ER = 0.02
SUBROUTINE ASTAR FLOW CHART

BEGIN
SET M(0) = 1.01

I = 1 TO 299

IS FG = 1 ?

M(I) = M(I-1) + .01

SET EZ = .02

CALCULATE A(I)

WW = |BB - A(I)|

IS WW < EZ ?

SET M = M(I)

NEXT I

RETURN
SUBROUTINE TTSTAR FLOW CHART

BEGIN
SET M(0) = M1

I = 1 TO 199

IS M(I) > 1.0 Y

M(I+1) = M(I) + .004

IS M(I) > 1.0 Y

CALCULATE MM(I)

YY = [WK - MM(I)]

IS YY < .0003 Y

SET M = MM(I)

NEXT I

RETURN
SUBROUTINE PCRIT FLOWCHART

CALCULATE CRITICAL PRESSURE

IS PR < PC ?

Y

IS DUCT ATTACHED TO NOZZLE

PRINT MESSAGE "B"

N

PRINT MESSAGE "C"

2

PRINT MESSAGE "E"

SET MT = 1

2
SUBROUTINE PCHEK1 FLOWCHART

BEGIN

IS ME = 1 ?
  Y
  N

IS ME > 1 ?
  Y
  N

IS NOZZLE CONVERGING ?
  Y
  N

\[ LL = \frac{|P(n) - PB|}{PB} \times 100 \]

IS LL < 2% ?
  Y
  N

PRINT MESSAGE K

SET FG = 0

IS P(n) < PB ?
  Y
  N

SET DY = 0.95 * DY

GOSUB PSTAR

MZ = M

DOES A SHOCK STAND IN NOZZLE ?
  Y
  N

SET DY = 1.05 * DY

RETURN
SUBROUTINE BACKPRESSURE FLOWCHART

BEGIN

IS PB < PINI ?

PRINT MESSAGE

PRINT MESSAGE

RETURN

CALCULATE P*(PSTABIL: PB)

SET M = 1.01

K = 1 TO 99

M(K) = M(K-1) - 0.01

CALCULATE PF(K)

VV = PB - PF(K)

IS VV < 0.02 ?

SET M = M(K)

NEXT K

CALCULATE KF $\Phi$ M

17
BACKPRESSURE (Cont'd)

17

SET M(0) = 1.01

I = 1 TO 99

M(I) = M(I-1) + 0.05

CALCULATE CC, MY, FX, FY

FZ(I) = FY(I) + CA - CC - FL

RR = |FX(I) - FZ(I)|

IS RR < 0.014 ?

Y

CALCULATE PF(I), PG(1), PG(2), PW, EP, PY(I), PT, PZ, TZ

SET RG = 1

IS M(I) > MZ ?

Y

PRINT MESSAGE M

SET FG = 0

N

18
SUBROUTINE TAB1 FLOWCHART

BEGIN
  INPUT M
  @ 1 AND 2

  J = 1 TO 2

  CALCULATE
  P(J)
  T(J)
  MO(J)

  GOSUB VECALC

  PRINT
  P(J), T(J), MO(J), A(J), MO(J), PO(J), V(J), DE(J)

  NEXT J

  RETURN
BEGIN

IS Q = 0?

IS FL = 0?

N = 4

INPUT PO(J), M(J), TO(J)

GOSUB STAGP

J = 1 TO N

CALCULATE T(J), M(J), MO(J), P(J)

GOSUB VECALC

PRINT P(J), T(J), M(J), A(J), MO(J), PO(J), V(J), DE(J)

NEXT J

RETURN
SUBROUTINE TAB3 FLOWCHART

BEGIN
N = 6

IF FL = 5 THEN
N = 3

N = 0

IF PK (1) = PK (2) THEN
INPUT PU(i), TO(j, M(j))

GOSUB STAGP

J = 1 TO N

CALCULATE T(j), MU(j), PU(j)

GOSUB VECALC

PRINT PU(j), T(j), MU(j)

NEXT J

RETURN
SUBROUTINE MCALC FLOWCHART

BEGIN
SET M(0) = 1.01

i = 1 TO 100

M(i) = M(i-1) - 0.1

CALCULATE PR(i)

uu = |PR-PR(i)|

IS uu < 0.0028

Y

SET M = M(i)

NEXT i

RETURN

N
SUBROUTINE CONVERGE FLOWCHART

11
SET M2 = 1.01

J = 1 TO 99

M(j) = M(j-1) - 0.1

IS M(j) > 0 ?

Y
CALCULATE T(j)

N

00 = |T(j)| - TS

IS 00 < 0.015 ?

Y
SET M1 = M(j)

N

IS M2 < 1 ?

Y

N

SET M2 = 1

CALCULATE P1, PSTAR

15

16

IS PB < PSTAR ?

Y
PRINT MESSAGE

N
CALCULATE PB/PSTAR @ 2

GO SUB PSTAR FORM2

CALCULATE T0, TR WITH M2

16

IS NN = 1 ?

Y
CALCULATE TK, PF, T, MD
EP, PI, PK, P1, P2, TK

N
GO SUB VECALC

13

GO SUB STAGP

18
SUBROUTINE FAN CONVERG FLOW CHART

BEGIN

SET M2 = 1.01

J = 1 TO 99

M(J) = M(J-1) + .01

CALCULATE FL/D(J)

XY = FL(J) - FL/0

IS XY < .02 ?

SET M1 = M(J)

IS M2 < 1 ?

SET M2 = 1

CALCULATE P/P* @1

IS M2 < 1 ?

CALCULATE P, P*, T

GOSUB VECALC

GOSUB STAGP

15

CALCULATE P, P*, PSTARF

IS PB < PSTARF ?

PRINT MESSAGE

CALCULATE P8/P* @2

GOSUB PSTARFAN

SET M2 = M

CALCULATE FL/D @2

16
SUBROUTINE PSTARFAN FLOWCHART

BEGIN

SET
M(0) = 1.01

I = 1 TO 99

M(I) = M(I-1) - 0.01

CALCULATE
PF(I)

LB = \frac{PF(I) - P}{P^2}

IS LB < 0.01 ?

SET
M = M(I)

RETURN
SUBROUTINE VECALC FLOWCHART

BEGIN

CALCULATE VELOCITY
REQ. EQT. *8

CALCULATE DENSITY
REQ. EQT. *8

RETURN
SUBROUTINE BPRANG FLOWCHART

BEGIN

BB = AD/AI

GOSUB ASTAR FOR Mz

CALCULATE My

CALCULATE (PL)

IS PB > PL ?

IS DUCT ATTACHED TO NOZZLE ?

PRINT MESSAGE "Q"

PRINT MESSAGE "E"

PRINT MESSAGE "C"

SET MT = 1

RETURN
SUBROUTINE QLIM FLOWCHART

BEGIN

SET M (0) = 1.01

GOSUB ASTAR
FOR M < 1

CALCULATE \( \frac{T_T}{T_T^*} \)
\( T_T^* @ M1 \)

CALCULATE QH

IS QQ > QH ?

PRINT MESSAGE Q

RETURN

PRINT MESSAGE N
LISTS,230

5 DIM KK(100)
10 DIM PK(100)
15 DIM TQ(200)
20 DIM CA(200)
25 DIM FX(100)
30 DIM FZ(100)
35 DIM FY(100)
40 DIM MM(200)
45 DIM CB(100)
50 DIM CY(100)
55 DIM MY(100)
60 DIM A(300)
65 DIM PF(100)
70 DIM PR(100)
75 DIM M(300)
80 L = 0
85 QH = 100000
90 NN = 0
95 M1 = 0
100 MX = 0
105 M2 = 0
110 HOME : INPUT "OPTION Z?";OZ$
115 IF OZ$ = "NO" THEN 180
120 FL = 0.15
125 CS = 13
130 PO = 700
135 TO = 220
140 IN = 2
145 QQ = 60
150 DR = 2
155 DT = 2
160 DL = 1
165 F# = "AIR"
170 Z# = "YES"
175 PB = 122
180 HOME : PRINT "THE PURPOSE OF THIS PROGRAM IS TO SOLVE SUBSONIC OR SUPersonic GASDYNAMICS PROBLEMS INVOLVING FANNO AND/OR RAYLEIGH FLOW. A SERIES OF INPUTS ARE REQUIRED TO DETERMINE STAGNATION PROPERTIES AND DUCT GEOMETRY."
185 FOR I = 1 TO 2000: NEXT I
190 IF OZ$ = "YES" GOTO 290
195 HOME : INPUT "INPUT DUCT GEOMETRY TO BE ANALYZED:1)CONVERGENT NOZZLE 2)ONE CONVERGENT-DIVERGENT NOZZLE ";IN
200 IF IN = 1 GOTO 215
205 HOME : PRINT "INPUT EXIT PLANE TO THROAT DIAMETER RATIO";DR
210 INPUT DR
215 HOME : PRINT "INLET STAGNATION PRESSURE IN KPA= "
220 INPUT PO
225 HOME : PRINT "INLET STAGNATION TEMPERATURE IN DEGREES KELVIN="
230 INPUT TO
LIST 235, 490

235 IF IN = 1 GOTO 245
240 HOME : INPUT "INPUT THROAT DIAMETER IN METERS"; DT
245 HOME : PRINT "INPUT CASE#"; CS
250 INPUT CS
255 HOME : INPUT "INPUT GASEOUS MEDIA"; F$
260 HOME : INPUT "BACK PRESSURE IN KPA"; PB
265 HOME : INPUT "DOES CONSTANT AREA DUCT FOLLOW THE NOZZLE (YES OR NO)"; Z$
270 IF Z$ = "NO" GOTO 290
275 HOME : PRINT "HEATING RATE IN KILOJOULES/KILOGRAM"; QQ
280 INPUT QQ
285 HOME : INPUT "INPUT FL/D"; FL
290 PR# = 1
295 PRINT "GIVEN CONDITIONS"
300 PRINT "CASE#" ; CS
305 PRINT "******************************************************************" ;
310 PRINT "*
315 PRINT "* STAGNATION PRESSURE " ; PO ; " *
320 PRINT "* STAGNATION TEMPERATURE " ; T0 ; " *
325 PRINT "* HEATING RATE " ; QQ ; " *
330 IF IN = 1 GOTO 345
335 PRINT "* DIAMETER RATIO " ; DR ; " *
340 PRINT "* NOZZLE THROAT DIAMETER " ; DT ; " *
345 PRINT "* BACK PRESSURE " ; PB ; " *
350 PRINT "* FL/D = " ; FL ; " *
355 PRINT "* GASEOUS MEDIA " ; F$ ; " *
360 IF IN = 1 GOTO 375
365 PRINT "* NOZZLE IS CONVERGING-DIVERGING TYPE *
370 GOTO 380
375 PRINT "* NOZZLE IS CONVERGING TYPE *
380 PRINT "*
385 PRINT "******************************************************************
390 PRINT
395 PRINT
400 PR# = 0
405 PR = PB / PO
410 IF PR > .99 GOTO 420
415 GOTO 445
420 PR# = 1
425 HOME : PRINT "SINCE BACK PRESSURE =STAGNATION PRESSURE THERE IS NO GAS FLOW"
430 FOR I = 1 TO 2000: NEXT I
435 PR# = 0
440 GOTO 1885
445 IF IN = 1 GOTO 470
450 DD = DR * DT
455 AISTAR = 3.14 * (DT ^ 2) / 4
460 AD = 3.14 * (DD ^ 2) / 4
465 GOTO 480
470 AD = 1
475 AI = 1
480 REM : THIS PART OF THE PROGRAM CALCULATES OR ASSIGNS THERMODYNAMI

PROPERTY VALUES
485 IF F$ = "AIR" GOTO 515
490 IF F$ = "HELIUM" GOTO 530
495 IF F$ = "ARGON" GOTO 545
500 HOME : INPUT "INPUT R IN KJ/KG*DEG. K"; R
505 HOME : INPUT "INPUT GAMMA"; GA
510 GOTO 555
515 R = 0.287
520 GA = 1.4
525 GOTO 555
530 R = 2.077
535 GA = 1.67
540 GOTO 555
545 R = 0.208
550 GA = 1.67
555 CP = GA * R / (GA - 1)
560 GM = GA - 1
565 GP = GA + 1
570 IF IN = 2 GOTO 585
575 GOSUB 2220
580 GOTO 985
585 GOSUB 4745
590 IF HH < GG GOTO 600
595 IF PL < PB GOTO 630
600 IF Z$ = "NO" GOTO 885
605 HOME : PRINT "ASSUME A NORMAL SHOCK IN THE NOZZLE (YES OR NO)"; IS$
610 INPUT IS$
615 IF IS$ = "NO" THEN 885
620 REM : THIS IS THE ROUTINE FOR SUBSONIC FLOW AT THE DUCT
625 FG = 0
630 HOME : INPUT "INPUT NOZZLE TO THROAT DIAMETER RATIO THAT CORRESPONDS TO THE SHOCK LOCATION BY INDICATING A VALUE OF DY "; DY
635 IF DY > DR OR DY < 1 GOTO 645
640 GOTO 660
645 PRINT "DY MUST BE < OR = DUCT DIAMETER TO NOZZLE THROAT DIAMETER AND > OR = 1; CHOOSE AGAIN"
650 FOR I = 1 TO 1500: NEXT I
655 GOTO 630
660 DX = DY * DT
665 AX = 3.14 * (DX^2) / 4
670 BB = AX / AI
675 GOSUB 2015
680 MX = M
685 MY = SQRT((MX^2 + 2 / GM) / (2 * GA * (MX^2) / GM - 1))
690 PT = (((GP / 2) * MX^2) / (1 + (GM / 2) * MX^2)) / (GA / GM) * (1 / (((2 * (GA / GP) * MX^2) - GM / GP)) / (1 / GM))
695 PO(2) = PT * PO
700 AY = (1 / MY) * (((2 / GP) * (1 + (GM / 2) * MY^2)) / (GP / (2 * GM))
705 AR = AY * AI / AX
710 PZ = (1 + GA * MX^2) / (1 + GA * MY^2)
715 TZ = (1 + (GM / 2) * MX^2) / (1 + (GM / 2) * MY^2)
LIST 720, 965

720 TX = T0 / (1 + (GM / 2) * MX ^ 2)
725 TY = T0 / (1 + (GM / 2) * MY ^ 2)
730 PR# 1
735 PRINT
740 PRINT "SUBSONIC FLOW AT NOZZLE EXIT WITH A SHOCK @DIAMETER:
745 PRINT "RATIO= "; DY; ""
750 PRINT "MX= "; MX
755 PRINT "MY= "; MY
760 PRINT "AR= "; AR
765 PRINT "PZ= "; PZ
770 PRINT "TZ= "; TZ
775 PRINT "TX= "; TX
780 PRINT "TY= "; TY
785 PRINT "PT= "; PT
790 PRINT
795 PR# 0
800 IF DY = DR GOTO 860
805 L = L + 1
810 IF L = 6 GOTO 820
815 GOTO 830
820 PRINT "PROGRAM CANNOT FIND THE SHOCK LOCATION IN THE NOZZLE:
825 RECHECK THE ISENTROPIC ASSUMPTION"
830 GOTO 605
835 BB = AD * AY / AX
840 FG = 1
845 GOSUB 2025
850 M1 = M
855 GOTO 965
860 M1 = MY
865 IF QQ > 0 AND FL > 0 THEN 1075
870 IF QQ = 0 AND FL > 0 THEN 1345
875 IF QQ = 0 AND FL = 0 GOTO 1325
880 IF QQ > 0 AND FL = 0 THEN 1075
885 REM ; THIS ROUTINE CALCULATES FLOW PROPERTIES FOR 1-D ISENTROPIC
890 FLOW IN A CONSTANT AREA DUCT
895 BB = AD / AI
900 IF PR > PC GOTO 950
905 GOSUB 2015
910 M1 = M
915 IF M1 < 0.01 GOTO 930
920 IF QQ > QH GOTO 965
925 GOTO 1005
930 PR# 1
935 PRINT "M1=0 RECHECK ASSUMPTIONS"
940 PR# 0
945 GOTO 605
950 GOSUB 3405
955 M1 = M
960 IF BB = 1 GOTO 1005
965 AA = (1 / M) * ((2 / GP) * (1 + (GM / 2) * M ^ 2)) ^ (GP / (2 * GM))
LIST 970, 1200

970 BB = AI * AA / AD
975 HOME : PRINT AA, M, BB
980 IF NN = 1 GOTO 995
985 IF MT = 1 GOTO 1005
990 FG = 1
995 GOSUB 2015
1000 MT = M
1005 IF QQ > 0 THEN 1075
1010 IF FL > 0 THEN 1345
1015 IF PR > FC GOTO 1025
1020 IF IN = 1 THEN M1 = 1.0
1025 GOSUB 3260
1030 PRINT
1035 IF IN = 2 GOTO 1055
1040 ME = M(1)
1045 PE = P(1)
1050 GOTO 1065
1055 ME = M(N)
1060 PE = P(N)
1065 IF QQ > 0 GOTO 1075
1070 GOTO 1025
1075 REM : THIS PORTION OF THE PROGRAM CALCULATES FLOW PROPERTIES FOR RAYLEIGH FLOW IN A CONSTANT AREA DUCT
1080 TT = QQ / CP + T0
1085 TSTAR = T0 / TT
1090 IF IN = 1 THEN 3715
1095 PK = GP / (1 + GA * M1 ^ 2)
1100 P2 = PO(2) / (1 + (GM / 2) * (M1 ^ 2)) * (GA / GM)
1105 PSTA = P2 / PK
1110 IF ME = 1 GOTO 1250
1115 TQ = (((GP ^ 2) * (M1 ^ 2)) / ((1 + GA * M1 ^ 2) ^ 2) * (1 + (GM * M1 ^ 2) / 2) / (GM / 2 + 1))
1120 TR = (TT / T0) * TQ
1125 WK = TR / (2 * GP)
1130 GOSUB 2130
1135 PR# 1
1140 PRINT "FLOW CONDITIONS @RAYLEIGH DUCT"
1145 PRINT "TT= "; TT
1150 PRINT "TQ= "; TQ
1155 PRINT "TR= "; TR
1160 PRINT "M1= "; M1
1165 PRINT "PSTA= "; PSTA
1170 PRINT "M2= "; M2
1175 PR# 0
1180 IF TR > 1 GOTO 1190
1185 IF TR < 1 GOTO 1250
1190 PR# 1
1195 HOME : PRINT " HEATING RATE IS TOO HIGH TO SUPPORT THE FLOW CONDITIONS CALCULATED @NOZZLE EXIT. RECHECK ASSUMPTIONS"
1200 PR# 0
LIST1205,1430

1205  NN = 1 + NN
1210  IF NN > 1 GOTO 660
1215  FG = 1
1220  GOSUB 4340
1225  ME = 1
1230  M2 = ME
1235  IF DO > GQ GOTO 3725
1240  M = M1
1245  GOTO 605
1250  M(1) = M1
1255  M(2) = M2
1260  FOR J = 1 TO 2
1265  PJ(J) = (GP / (1 + (GA * M(J) ^ 2))) * ((2 / GP) * (1 + (GM / 2) * M(J) ^ 2)) ^ (GA / GM)
1270  PK(J) = GP / (1 + GA * M(J) ^ 2)
1275  TK(J) = ((GP ^ 2) * M(J) ^ 2) / (1 + GA * M(J) ^ 2) ^ 2
1280  PF(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * M(J) ^ 2)) ^ ((GM / 2) * M(J) ^ 2)) ^ (GA / (2 * GM))
1290  PRINT
1295  PRINT PJ(J),PK(J),TK(J),PF(J)
1300  PR# 0
1305  PF = PF(J)
1310  NEXT J
1315  IF FL > 0 THEN 1345
1320  IF FL = 0 THEN ME = M2
1325  GOSUB 2620
1330  GOSUB 2330
1335  PE = P(N)
1340  GOTO 1625
1345  REM :THIS PORTION OF THE PROGRAM CALCULATES FLOW PROPERTIES FOR FANNO FLOW IN A CONSTANT AREA DUCT
1350  IF QQ = 0 GOTO 1365
1355  IF IN = 1 GOTO 4470
1360  GOTO 1385
1365  IF IN = 1 GOTO 4470
1370  M2 = M1
1375  IF IN = 1 GOTO 1385
1380  PB(1) = (1 / M1) * ((2 / GP) * (1 + (GM / 2) * M1 ^ 2)) ^ (GP / (2 * GM))
1385  CA = ((1 - (M2 ^ 2)) / (GA * (M2 ^ 2))) + (GP / (2 * GA)) * LOG (GP * M2 ^ 2 / (2 + GM * M2 ^ 2))
1390  CB = CA - FL
1395  PRINT
1400  PRINT "CB= ";CB
1405  PR# 0
1410  IF CB < 0 GOTO 1560
1415  HOME : PRINT "ASSUME A NORMAL SHOCK IN THE FANNO DUCT?(YES OR NO)";OS$
1420  INPUT OS$
1425  IF OS$ = "NO" THEN 1435
1430  GOTO 1560
LIST1435,1645

1435 GOSUB 1890
1440 M(1) = M2
1445 IF QQ = 0 THEN M(1) = M1
1450 M(2) = ME
1455 FOR J = 1 TO 2
1460 PF(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * (M(J) ^ 2))) ^ (-0.5)
1465 PG(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * M(J) ^ 2)) ^ (GP / (2 * GM))
1470 TF(J) = ((2 / GP) * (1 + (GM / 2) * M(J) ^ 2)) ^ (-1)
1475 NEXT J
1480 PR# 1
1485 PRINT "CA= ";CA
1490 PRINT "CB= ";CB
1495 PRINT "M2= ";M2
1500 PRINT "TF(1)= ";TF(1)
1505 PRINT "TF(2)= ";TF(2)
1510 PRINT "PF(1)= ";PF(1)
1515 PRINT "PF(2)= ";PF(2)
1520 PRINT "PG(2)= ";PG(2)
1525 PRINT "PG(1)= ";PG(1)
1530 PRINT "ME= ";ME
1535 PR# 0
1540 GOSUB 2620
1545 GOSUB 2330
1550 PRINT "SINCE FLOW IS SUBSONIC AT THE EXIT PLANE, THE EXIT PRESSURE = THE BACK PRESSURE."
1555 GOTO 1825
1560 REM: THIS PORTION OF THE PROGRAM (FANSHOCK) FINDS THE SHOCK LOCATION IN A FANNO DUCT ASSUMING THAT THE DUCT IS CHOKED BY FRICTION
1565 IF FG = 1 GOTO 1580
1570 GOTO 1595
1575 PR# 1
1580 REM: SHOCK LOCATION AS CALCULATED TO SATISFY RAYLEIGH FLOW CONDITIONS WILL NOT SUPPORT FL/D REQUIREMENTS; NEW NOZZLE SHOCK LOCATION IS REQUIRED.
1585 PR# 0
1590 GOTO 620
1595 IF QQ = 0 GOTO 1610
1600 M(0) = M2
1605 GOTO 1620
1610 M(0) = M2
1615 PF = (1 / M(0)) * ((2 / GP) * (1 + (GM / 2) * (M(0) ^ 2))) ^ (-0.5)
1620 FOR I = 1 TO 100
1625 M(I) = M(I - 1) - 0.03
1630 IF M(I) < 0 THEN 1825
1635 MY(I) = SQR ((M(I) ^ 2 + 2 / GM) / (2 * GA * (M(I) ^ 2) / GM - 1))
1640 CA(I) = (1 - (M(I) ^ 2)) / (GA * (M(I) ^ 2)) + (GP / (2 * GA)) * LOG (GP * M(I) ^ 2 / (2 + GM * M(I) ^ 2))
1645 CY(I) = (1 - MY(I) ^ 2) / (GA * MY(I) ^ 2) + (GP / (2 * GA)) * LOG (GP * MY(I) ^ 2 / (2 + GM * MY(I) ^ 2))
1650 CB(I) = CA - CA(I) + CY(I)
1655 FF = ABS(CB(I) - FL)
1660 IF FF < 0.02 THEN 1670
1665 GOTO 1820
1670 PR# 1
1675 PF(I) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * (M(I) ^ 2)) - (0.5))
1680 PG(2) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * M(I) ^ 2)) * (GP / (2 * GM))
1685 PY(1) = (1 / MY(I)) * ((2 / GP) * (1 + (GM / 2) * (MY(I) ^ 2)) - 0.5)
1690 PW = (1 / MY(I)) * ((2 / GP) * (1 + (GM / 2) * MY(I) ^ 2)) * (GP / (2 * GM))
1695 REM :EXIT PLANE MACH# MUST BE SONIC TO SATISFY THE SECOND LAW OF THERMODYNAMICS IF DUCT IS CHOKED BY FRICTION
1700 ME = 1.0
1705 KF = (1 / ME) * ((2 / GP) * (1 + (GM / 2) * (ME ^ 2)))
1710 EP = (1 / ME) * ((2 / GP) * (1 + (GM / 2) * ME ^ 2)) * (GP / (2 * GM))
1715 EZ = (1 + GM * M(I) ^ 2) / (1 + GA * MY(I) ^ 2)
1720 PT = (((GP / 2) * M(I) ^ 2) / (1 + (GM / 2) * M(I) ^ 2)) * (GA / GM) * (1 / (GA / GP) * M(I) ^ 2) - GM / GP)
1725 TZ = (1 + (GM / 2) * MX ^ 2) / (1 + (GM / 2) * MY ^ 2)
1730 MZ = M(I)
1735 PRINT "IF DUCT CHOKED BY FRICTION ASSUMPTION IS CORRECT THEN SHOCK IS LOCATED IN THE DUCT AND THE FOLLOWING PARAMETERS ARE VALID"
1740 PRINT "CA= "; CA
1745 PRINT "MZ = "; MZ
1750 PRINT "MY(I)= "; MY(I)
1755 PRINT "CA(I)= "; CA(I)
1760 PRINT "CY(I)= "; CY(I)
1765 PRINT "PF(I)= "; PF(I)
1770 PRINT "PY(1)= "; PY(1)
1775 PRINT "PF= "; PF
1780 PRINT "PK(1)= "; PK(1)
1785 PRINT "PT= "; PT
1790 PRINT "PG(2)= "; PG(2)
1795 PRINT "PW= "; PW
1800 PR# 0
1805 GOSUB 2975
1810 GOSUB 2540
1815 GOTO 1825
1820 NEXT I
1825 PR# 1
1830 PRINT
1835 PRINT "EXIT PRESSURE= "; INT (PE * 100 + 0.5) / 100
1840 PRINT "EXIT TEMPERATURE= "; INT (T(N) * 100 + 0.5) / 100
1845 PRINT "EXIT MACH# = "; INT (ME * 100 + 0.5) / 100
1850 PR# 0
1855 TE = 0
1860 ME = 0
LIST1865,2100

1865 P(N) = 0
1870 PE = 0
1875 HOME: INPUT "DO YOU WISH TO CONTINUE WITH ANOTHER RUN (YES OR NO)?"; SY#
1880 IF SY# = "YES" GOTO 80
1885 END

1890 REM : THIS SUBROUTINE (FANMACH) CALCULATES A MACH NUMBER FOR A GIVEN FL/D
1895 M(0) = M1
1900 M(0) = M2
1905 FOR I = 1 TO 200
1910 IF M1 > 1 THEN 1935
1915 IF IN = 1 THEN 1935
1920 ER = 0.03
1925 M(I) = M(I - 1) + 0.001
1930 GOTO 1955
1935 M(0) = M1
1940 M(I) = M(I - 1) - 0.01
1945 IF M(I) < 1 THEN 2010
1950 ER = 0.002
1955 CA(I) = (1 - (M(I) ^ 2)) / (GA * (M(I) ^ 2)) + (GP / (2 * GA)) * LOG (GP * M(I) ^ 2 / (2 + GM * M(I) ^ 2))
1960 SS = ABS (CA(I) - CB)
1965 IF SS < ER GOTO 1975
1970 GOTO 2000
1975 PRINT
1980 PRINT "SS= "; SS
1985 PRINT "ME= "; M(I)
1990 PRINT "CA(I)= "; CA(I)
1995 ME = M(I)
2000 NEXT I
2005 HOME
2010 RETURN

2015 REM : THIS SUBROUTINE (ASTAR) CALCULATES MACH NUMBER FOR A SPECIFIC A/A*.
2020 M(0) = 1.01
2025 FOR I = 1 TO 299
2030 IF FG = 1 THEN 2055
2035 M(0) = 1.01
2040 M(I) = M(I - 1) + 0.01
2045 EZ = 0.02
2050 GOTO 2085
2055 M(I) = M(I - 1) - 0.01
2060 IF M(I) > 0.49 GOTO 2075
2065 EZ = 0.09
2070 GOTO 2080
2075 EZ = 0.02
2080 IF M(I) < 0 THEN 2125
2085 A(I) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * M(I) ^ 2) / (GP / (2 * GM))
2090 WW = ABS (BB - A(I))
2095 HOME: PRINT WW, A(I), M(I)
2100 PRINT
2105 IF WW < EZ THEN M = M(I): PRINT WW, A(I), M(I)
2110 PR# 0
2115 NEXT I
2120 HOME
2125 RETURN
2130 REM : THIS SUBROUTINE (TTSTAR) CALCULATES MACH NUMBER FOR
2135 A GIVEN TT/TT*
2140 M(1) = M1
2145 M(0) = M1
2150 FOR I = 1 TO 199
2155 M(I + 1) = M(I) + 0.004
2160 IF M(I) > 1.0 GOTO 2210
2165 GOTO 2180
2170 M(I) = M(I - 1) - 0.01
2175 IF M(I) < 1.0 GOTO 2210
2180 MM(I) = ((1 + (GM / 2) * M(I) ^ 2) * M(I) ^ 2) / ((1 +
2185 GA * M(I) ^ 2) ^ 2)
2190 YY = ABS (WK - MM(I))
2195 IF YY < 0.0003 THEN MZ = M(I): PRINT YY, WK, M(I), MM(I)
2200 PR# 0
2205 NEXT I
2210 HOME
2215 RETURN
2220 REM : THIS ROUTINE (PCRIT) CALCULATES CRITICAL PRESSURE
2225 RATIO AND COMPARES WITH GIVEN BACK PRESSURE TO STAGNATION PRESSURE
2230 RATIO
2235 PC = ((1 + GM / 2) ^ (GA / GM)) ^ (- 1)
2240 IF PR < = PC THEN 2245
2245 IF Z# = "YES" GOTO 2245
2250 HOME: PRINT "BACK PRESSURE IS LOW ENOUGH TO ALLOW SONIC
2255 FLOW IN THE THROAT."
2260 FOR I = 1 TO 1500: NEXT I
2265 GO TO 2270
2270 HOME: PRINT "BACK PRESSURE IS LOW ENOUGH TO ALLOW CHOKED
2275 FLOW IN THE NOZZLE HOWEVER, FRICTION AND/OR HEATING TERMS IN
2280 THE DUCT MAY BE LARGE ENOUGH TO FORCE SUBSONIC CONDITIONS TO
2285 OCCUR."
2290 FOR I = 1 TO 1500: NEXT I
2295 PRINT "PR= "; INT (PR * 100 + 0.5) / 100
2300 PRINT "CRITICAL PRESSURE RATIO= "; INT (PC * 100 + 0.5)
2305 MT = 1
2320 PR# 0
2325 RETURN
**LIST 2330, 2615**

2330 REM: THIS SUBROUTINE (PCHEK1) CHECKS BACK PRESSURE REQUIREMENTS WITH RAYLEIGH AND FANNO FLOW
2335 IF ME > 1 GOTO 2505
2340 IF IN = 1 GOTO 2530
2345 IF ME = 1 GOTO 4455
2350 IF IS# = "NO" GOTO 2360
2355 IF PB > PL GOTO 2360
2360 LL = ABS ((PE - PB) / PB) * 100
2365 PR# 1
2370 PRINT "LL = " ; LL
2375 PRINT "PB = " ; PB
2380 PRINT "PE = " ; PE
2385 IF LL < 3 GOTO 2530
2390 PR# 1
2395 PRINT "CALCULATED BACK PRESSURE IS NOT = GIVEN BACK PRESSURE; CHOOSE A DIFFERENT VALUE FOR DY"
2400 PR# 0
2430 FG = 0
2435 IF NN = 0 GOTO 2460
2440 PK(2) = PB / PSTA
2445 GOSUB 4275
2450 M2 = M
2455 GOTO 1250
2460 IF P(N) < PB THEN DY = 0.95 * DY
2465 IF P(N) > PB THEN DY = 1.05 * DY
2475 GOTO 660
2480 IF PB < PSTA GOTO 2505
2485 DY = 1.05 * DY
2490 FG = 0
2495 ME = 0
2500 GOTO 660
2505 IF FL > 0 GOTO 4900
2510 PR# 1
2515 PRINT "EXIT PLANE PRESSURE IS LOW ENOUGH TO ALLOW FLOW CONDITIONS CALCULATED"
2520 PR# 0
2530 HOME
2535 RETURN
2540 REM: THIS SUBROUTINE (BACKPRESSURE-1) CHECKS BACK PRESSURE VS. EXIT PRESSURE FOR A SHOCK IN A FANNO DUCT
2545 IF FL = 0 THEN PF(I) = 1
2550 PR# 1
2555 PE = P(2) * PK(2) * PF(I) * PZ / (PK(1) * FF * PY(1))
2560 IF PB < PE THEN 2600
2565 PRINT
2570 PRINT "BACK PRESSURE IS TOO HIGH TO ALLOW FLOW PROPERTIES CALCULATED ASSUMING A DUCT CHOKED BY FRICTION; THE SHOCK LOCATION IS DETERMINED BY THE BACKPRESSURE."
2575 PR# 0
2580 GOSUB 3455
2585 GOSUB 2975
2590 PE = PB
2595 GOTO 2610
2600 PRINT
2605 PRINT "SINCE BACK PRESSURE < EXIT PRESSURE, SHOCK LOCATION IS NOT AFFECTED"
2610 HOME
2615 RETURN
CALCULATES THE MASS FLOW RATE, PRESSURE, AND TEMP RATIO FOR VARIABLE MACH #'S.

N = 6
IF IS# = "NO" GOTO 2975
IF FL = 0 THEN N = 5
IF QQ = 0 GOTO 2650
GOTO 2690
2690 IF FL = 0 THEN N = 4
PK(1) = 1
PK(2) = 1
PO(1) = PO
PO(2) = PO
PO(3) = PT * PO
PO(4) = PO
PO(5) = INT (PO(4) * 100 + 0.5) / 100
IF QQ > 0 GOTO 2780
GOTO 2780
PO(5) = PO(4) * PT(2) / PT(1)
PO(5) = INT (PO(5) * 100 + 0.5) / 100
GOTO 2780
PO(5) = PO(4) * PT(2) / PT(1)
PO(5) = INT (PO(5) * 100 + 0.5) / 100
GOTO 2780
PO(5) = PO(4) * PT(2) / PT(1)
PO(5) = INT (PO(5) * 100 + 0.5) / 100
GOTO 2780
PO(5) = PO(4) * PT(2) / PT(1)
PO(5) = INT (PO(5) * 100 + 0.5) / 100
MT(1) = MT
MT(2) = MX
MT(3) = MY
MT(4) = M1
IF QQ = 0 GOTO 2815
M(5) = M2
M(6) = ME
IF FL > 0 THEN M(5) = ME
A(1) = AISTAR
A(2) = AX
T0(3) = T0
T0(1) = T0
T0(4) = T0
T0(2) = T0
T0(5) = TT
T0(6) = TT
FOR J = 1 TO N
A(J + 3) = AD
P(J) = PO(J) / (1 + (GM / 2) * M(J) ^ 2)) ^ (GA / GM)
T(J) = T0(J) / (1 + ((GM / 2) * M(J) ^ 2))
GOSUB 4080
MD(J) = (P(J) / (R * T(J))) * M(J) * SDR (GA * T(J) * 1000 * R)
PRINT Chr$(9)"80N"
LIST 2900, 3100

2900 IF J = 1 GOTO 2910
2905 GOTO 2925
2910 PRINT "PRESS TEMPERATURE MACH# AREA MDOT"
2915 PRINT "TOTAL PRESSURE VELOCITY DENSITY"
2920 PRINT "KPA M/SEC KG/M^2"
2925 PRINT "PRINT RIGHT$ (" + STR$ (INT (P(J) * 100 + 0.5) / 100), 7); RIGHT$
2930 PRINT "PRINT RIGHT$ (" + STR$ (INT (M(J) * 100 + 0.5) / 100), 7); RIGHT$
2935 PRINT "PRINT RIGHT$ (" + STR$ (INT (T(J) * 100 + 0.5) / 100), 6); RIGHT$
2940 NEXT J
2950 TE = T(N)
2960 PE = P(N)
2975 ME = M(N)
2980 HOME
2990 RETURN
3000 N = 6
3005 IF FL = 0 THEN N = 3
3010 GOTO 3020
3020 PK(1) = 1
3025 T0(2) = T0
3030 T0(1) = T0
3035 M(1) = MT
3040 M(2) = M1
3045 M(3) = M2
3050 IF FL = 0 GOTO 3105
3055 IF ME > 1 GOTO 3060
3060 M(4) = MZ
3065 M(5) = MY(I)
3070 M(6) = ME
3075 GOTO 3105
3080 N = 4
3085 M(4) = ME
3090 T0(1) = T0
3100 A(1) = AISTAR
3105 END

REM : THIS SUBROUTINE (TAB3) CALCULATES THE MASS FLOW RATE, PRESSURE AND TEMP RATIO S FOR VARIABLE SUPERSONIC MACH # S.
LIST3105,3300

3105 FOR J = 1 TO N
3110 T0(J + 2) = TT
3115 A(J + 1) = AD
3120 P(1) = PO / (1 + (GM / 2) * (M(1) ^ 2)) ^ (GA / GM)
3125 P(2) = PO / (1 + (GM / 2) * (M(2) ^ 2)) ^ (GA / GM)
3130 P(3) = P(2) * PK(2) / PK(1)
3135 IF FL = 0 GOTO 3170
3140 IF ME > 1 GOTO 3165
3145 P(4) = PF(I) * P(3) / PF
3150 P(5) = PZ * P(4)
3155 P(6) = P(5) * KF / PY(1)
3160 GOTO 3170
3165 P(4) = P(2) * PK(2) * PF(2) / (PK(1) * PF(1))
3170 T(J) = T0(J) / (1 + ((GM / 2) * M(J) ^ 2))
3175 GOSUB 4115
3180 MD(J) = (P(J) / (R * T(J))) * M(J) * SQRT ((GA * T(J) * 1000 * F) / (1 + ((GM / 2) * M(J) ^ 2)))
3185 GOSUB 4080
3190 PR# 1: PRINT CHR$ (9) "60N"
3195 IF J = 1 GOTO 3205
3200 GOTO 3220
3205 PRINT " PRESS  TEMP  MACH#  AREA  MDOT"
3210 PRINT " VEL  DENSITY"
3215 PRINT " KPA  DEGK  SQRMT  KG/SEC^2  KG/M^3"
3220 PRINT " KPA  M/SEC  KG/M^3"
3225 PRINT " " : PRINT FACTORS: "
3226 PRINT " " + STR$ (INT (P(J) * 100 + 0.5) / 100), 7; RIGHT$ ("
3227 PRINT " " + STR$ (INT (M(J) * 100 + 0.5) / 100), 6; RIGHT$ ("
3228 PRINT " " + STR$ (INT (A(J) * 100 + 0.5) / 100), 6; RIGHT$ ("
3229 PRINT " " + STR$ (INT (MD(J) * 100 + 0.5) / 100), 7; RIGHT$ ("
3230 PRINT " " + STR$ (P0(J)), 9; RIGHT$ ("$
3231 PRINT " " + STR$ (V(J)), 12); RIGHT$ ("
3232 PRINT " " + STR$ (D(J)), 11)
3235 IF IN = 1 GOTO 3295
3270 A(1) = AISTAR
3275 IF IN = 1 GOTO 3295
3280 M(1) = MT
3285 M(2) = M1
3295 GOTO 3305
3296 REM : THIS SUBROUTINE (TAB1) CALCULATES THE MASS FLOW RATE, PRESSURE
AND TEMP RATIOS FOR ISENTROPIC FLOW IN A NOZZLE WITH NO AND
FRICITION = 0.
3295 N = 2
3300 N = 1
LIST3305,3510

3305 FOR J = 1 TO N
3310 A(J + 1) = AD
3315 P(J) = PO / (1 + (GM / 2) * (M(J) ^ 2)) ^ (GA / GM)
3320 T(J) = TO / (1 + ((GM / 2) * (M(J) ^ 2))
3325 GOSUB 4115
3330 MD(J) = (P(J) / (R * T(J))) * M(J) * SQR (GA * T(J) * 1000 * R)
3335 GOSUB 4080
3340 PR# 1: PRINT CHR$ (9) "B0N"
3345 IF J = 1 GOTO 3355
3350 GOTO 3370
3355 PRINT " PRESS TEMP MACH# AREA MDOT"
3360 PRINT " TOTPRESS VEL DENSITY"
3365 PRINT " KPA DEGK SORMET kg/sec m^3"
3370 IF UU < 0.02 THEN M = M(J); PRINT UU,PR(I),M(I)
3375 NEXT I
3380 FR# 0
3385 NEXT J
3390 TE = T(N)
3395 HOME
3400 RETURN
3405 REM: THIS SUBROUTINE(MCALC) CALCULATES MACH NUMBER FOR
3410 A SPECIFIC P/PO.
3415 M(0) = 1.01
3420 FOR I = 1 TO 99
3425 M(I) = M(I - 1) - 0.01
3430 PR(I) = (1 + (GM / 2) * (M(I) ^ 2)) ^ (- GM / GM)
3435 UU = ABS (PR - PR(I))
3440 IF UU < 0.0028 THEN M = M(I); PRINT UU,PR(I),M(I)
3445 NEXT I
3450 HOME
3455 RETURN
3455 REM: THE PURPOSE OF THIS ROUTINE(BACKPRESSURE-2) IS TO
3460 DETERMINE THE LOCATION OF THE NORMAL SHOCK IN A DUCT THAT IS
3465 CHOKED BY BACKPRESSURE WITH FRICTION PRESENT ALSO.
3470 FSTARF = P(3) / PF
3475 BB = PB / FSTARF
3480 M(0) = 1.01
3485 FOR K = 1 TO 99
3490 M(K) = M(K - 1) - 0.01
3495 PF(K) = (1 / M(K)) * ((2 / GP) * (1 + (GM / 2) * (M(K) ^ 2)) ^ (- GM / GM))
3500 VV = ABS (BB - PF(K))
3505 PRINT
3510 IF VV < 0.02 THEN M = M(K); PRINT VV,PF(K),M(K)
LIST 3515, 3710

3515 ME = M
3520 KF = (1 / ME) * ((2 / GP) * (1 + (GM / 2) * (ME ^ 2)))
\[ ( - 0.5) \]
3525 M(0) = 1.01
3530 FOR I = 1 TO 99
3535 M(I) = M(I - 1) + 0.05
3540 CC = ((1 - (ME ^ 2)) / (GA * (ME ^ 2))) + (GP / (2 * GA))
\[ \log (GP * ME ^ 2 / (2 + GM * ME ^ 2)) \]
3545 FX(I) = (1 - (M(I) ^ 2)) / (GA * (M(I) ^ 2)) + (GP / (2 * GA)) * \log (GP * M(I) ^ 2 / (2 + GM * M(I) ^ 2))
3550 MY(I) = SQR ((M(I) ^ 2 + 2 + GM) / (2 * GA * (M(I) ^ 2) / GM - 1))
3555 FY(I) = (1 - MY(I) ^ 2) / (GM * MY(I) ^ 2) + (GP / (2 * GA)) * \log (GM * MY(I) ^ 2 / (2 + GM * MY(I) ^ 2))
3560 FZ(I) = FY(I) + CA - CC - FL
3565 RR = ABS (FX(I) - FZ(I))
3570 IF RR < 0.014 GOTO 3580
3575 GOTO 3700
3580 PF(I) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * (M(I) ^ 2)))
\[ ( - 0.5) \]
3585 PG(I) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * M2 ^ 2))
\[ (GP / (2 * GM)) \]
3590 PG(2) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * M(I) ^ 2))
\[ (GP / (2 * GM)) \]
3595 PW = (1 / MY(I)) * ((2 / GP) * (1 + (GM / 2) * MY(I) ^ 2))
\[ (GP / (2 * GM)) \]
3600 EP = (1 / ME) * ((2 / GP) * (1 + (GM / 2) * ME ^ 2))
\[ (GP / (2 * GM)) \]
3605 PY(I) = (1 / MY(I)) * ((2 / GP) * (1 + (GM/2) * MY(I) ^ 2))
\[ ( - 0.5) \]
3610 PT = (((GP / 2) * M(I) ^ 2) / (1 + (GM / 2) * M(I) ^ 2))
\[ (GA / GM) * (1 / ((2 * (GA / GP) * M(I) ^ 2) - GM / GP)) \]
\[ (1 / GM) \]
3615 FZ = (1 + GA * M(I) ^ 2) / (1 + GA * MY(I) ^ 2)
3620 TZ = (1 + (GM / 2) * MX ^ 2) / (1 + (GM / 2) * MY ^ 2)
3625 RG = 1
3630 PR# 1
3635 PRINT "MZ= "; M(I)
3640 PRINT "RR= "; RR
3645 PRINT "FX(I) = "; FX(I)
3650 PRINT "MY(I) = "; MY(I)
3655 PR# 0
3660 IF M(I) > M2 GOTO 3675
3665 MZ = M(I)
3670 GOTO 3710
3675 PR# 1
3680 PRINT "BACK PRESSURE IS TOO HIGH TO ALLOW THE SHOCK TO STAND IN THE DUCT; ASSUME SHOCK IS LOCATED IN THE NOZZLE."
3685 PR# 0
3690 FG = 0
3695 GOTO 620
3700 NEXT I
3705 HOME
3710 RETURN
LIST 3715, 3945

3715 REM: THE PURPOSE OF THIS SUBROUTINE (CONVERGE) IS TO CALCULATE MACH# FOR A CONVERGING NOZZLE FOLLOWED BY A HEATED DUCT
3720 M2 = 101
3725 M(0) = 1.01
3730 BB = T0 / TT
3735 FOR J = 1 TO 99
3740 M(J) = M(J - 1) - 0.01
3745 IF M(J) < 0 THEN 3820
3750 T0(J) = ((GP * 2) * (M(J) ^ 2)) / ((1 + GA * M(J) ^ 2) ^ 2) * (1 + (GM * M(J) ^ 2) / 2) / (GM / 2 + 1)
3755 00 = ABS (T0(J) - BB)
3760 IF 00 < 0.015 GOTO 3770
3765 GOTO 3815
3770 M1 = M(J)
3775 M = M1
3780 IF M2 < 1 GOTO 3900
3785 M2 = 1
3790 PK(J) = GP / (1 + GA * M(J) ^ 2)
3795 PRINT
3800 PRINT T0(J), M(J), BB, 00
3805 PR# 0
3810 GOTO 3830
3815 NEXT J
3820 HOME
3825 IF NN = 1 GOTO 965
3830 IF M2 < 1 GOTO 3895
3835 P(1) = P0 / (1 + (GM / 2) * (M(J) ^ 2)) ^ (GA / GM)
3840 PSTA = P(1) / PK(J)
3846 IF PB < PSTA GOTO 3900
3850 PR# 1
3855 PRINT "BACK PRESSURE IS TOO HIGH TO ALLOW CHOKED FLOW IN THE RAYLEIGH DUCT"
3860 PR# 0
3865 PK(2) = PB / PSTA
3870 GOSUB 4275
3875 M2 = M
3880 T0 = ((GP * 2) * (M2 ^ 2)) / ((1 + GA * M2 ^ 2) ^ 2) * (1 + (GM * M2 ^ 2) / 2) / (GM / 2 + 1)
3885 TR = TSTAR * T0
3890 BB = TR
3895 GOTO 3735
3900 IF NN = 1 GOTO 965
3905 M(1) = M1
3910 M(2) = M2
3915 T0(1) = T0
3920 T0(2) = TT
3925 EP = (1 / M2) * ((2 / GP) * (1 + (GM / 2) * M2 ^ 2)) ^ (GM / (2 * GM))
3930 FOR J = 1 TO 2
3935 A(J) = AD
3940 PJ(J) = (GP / (1 + (GA * M(J) ^ 2))) * ((2 / GP) * (1 + (GM / 2) * M(J) ^ 2)) ^ (GA / GM)
3945 PK(J) = GP / (1 + GA * M(J) ^ 2)
LIST 3950, 4150

3950 F(1) = PK(1) * FSTA
3955 P(2) = P(1) * FK(2) / PK(1)
3960 TK(J) = (GP ^ 2) * M(J) ^ 2 / (1 + GA * M(J) ^ 2) ^ 2
3965 PF(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * (M(J) ^ 2))) ^ (1 / 0.5)
3970 T(J) = T0(J) / (1 + ((GM / 2) * M(J) ^ 2))
3975 GOSUB 4115
3980 MD(J) = (P(J) * M(J) * SQRT(GA * T(J) * 1000 * R)) / (R + T(J))
3985 GOSUB 4030
3990 PRINT PJ(J), PK(J), TK(J), PF(J), VE(J), PO(J)
3995 NEXT J
4000 PF = PF(J)
4005 PR# 1: PRINT CHR$(9): "80N"
4010 FOR J = 1 TO 2
4015 IF J = 1 GOTO 4025
4020 GOTO 4040
4025 PRINT "PRESS TEMP MACH# AREA MDOT"
4030 PRINT "TOT PRESS VELOCITY DENSITY"
4035 PRINT "KPA DEGK SQRTM KG/SEC M^-3"
4040 PRINT ""; J: PRINT ""; PRINT RIGHT$(""
" " + STR$(INT(P(J) * 100 + 0.5) / 100), 7); RIGHT$""
" " + STR$(INT(T(J) * 100 + 0.5) / 100), 8); RIGHT$""
" " + STR$(INT(M(J) * 100 + 0.5) / 100), 7); RIGHT$""
" " + STR$(INT(A(J) * 10 + 0.5) / 10), 6); RIGHT$""
4045 PRINT "": PRINT RIGHT$(""
" " + STR$""
" " + STR$(INT(MD(J) * 100 + 0.5) / 100), 8); RIGHT$""
" " + STR$(INT(P0(J)), 10); RIGHT$""
" " + STR$(INT(V(J)), 10); RIGHT$""
4050 NEXT J
4055 PR# 0
4060 ME = M(2)
4065 PE = P(2)
4070 T(N) = T(2)
4075 GOTO 1825
4080 REM: THE PURPOSE OF THIS SUBROUTINE (VECCALC) IS TO CALCULATE VELOCITY AND DENSITY
4085 VE(J) = M(J) * (SQRT(GA * R * 1000 * T(J)))
4090 DE(J) = P(J) / (R * T(J))
4095 V(J) = INT(VE(J) * 100 + 0.5) / 100
4100 D(J) = INT(DE(J) * 100 + 0.5) / 100
4105 HOME
4110 RETURN
4115 REM: THE PURPOSE OF THIS SUBROUTINE (STAGP) IS TO CALCULATE STAGNATION PRESSURE.
4120 IF IN = 1 GOTO 4130
4125 GOTO 4160
4130 P0(1) = PO
4135 P0(1) = INT(P0(1) * 100 + 0.5) / 100
4140 IF QQ = 0 GOTO 4250
4145 PO(2) = PO * FJ(2) / PJ(1)
4150 P0(2) = INT(P0(2) * 100 + 0.5) / 100
LIST4155,4395

4155 GOTO 4265
4160 PO(1) = PO
4165 PO(1) = INT (PO(1) * 100 + 0.5) / 100
4170 PO(2) = PO
4175 PO(2) = INT (PO(2) * 100 + 0.5) / 100
4180 IF QQ = 0 GOTO 4200
4185 PO(3) = PO * PJ(2) / PJ(1)
4190 PO(3) = INT (PO(3) * 100 + 0.5) / 100
4195 GOTO 4210
4200 PO(3) = PO(2)
4205 PO(3) = INT (PO(3) * 100 + 0.5) / 100
4210 IF FL = 0 GOTO 4265
4215 PO(4) = PO(3) * PG(2) / PG(1)
4220 PO(4) = INT (PO(4) * 100 + 0.5) / 100
4225 IF ME > 1 GOTO 4265
4230 PO(5) = PO(4) * PT
4235 PO(5) = INT (PO(5) * 100 + 0.5) / 100
4240 PO(6) = PO(5) * EP / PW
4245 PO(6) = INT (PO(6) * 100 + 0.5) / 100
4250 IF FL = 0 GOTO 4265
4255 PO(2) = PG(2) * PO / PG(1)
4260 PO(2) = INT (PO(2) * 100 + 0.5) / 100
4265 HOME
4270 RETURN
4275 REM :THIS SUBROUTINE(PSTAR) CALCULATES MACH NUMBER FOR A SPECIFIC P/PSTAR FOR RAYLEIGH FLOW.
4280 M(0) = 1.01
4285 FOR I = 1 TO 99
4290 M(I) = M(I - 1) - 0.01
4295 KK(I) = GP / (1 + GA * M(I) ^ 2)
4300 LA = ABS (KK(I) - PK(2))
4305 HOME : PRINT LA,KK(I),M(I)
4310 PRINT
4315 IF LA < 0.01 THEN M = M(I): PRINT LA,KK(I),M(I)
4320 PR# 0
4325 NEXT I
4330 HOME
4335 RETURN
4340 REM :THE PURPOSE OF THIS SUBROUTINE(OLIMIT) IS TO DETERMINE THE MAXIMUM HEAT TRANSFER THAT WILL ALLOW SONIC CONDITIONS AT THE CONVERGING-DIVERGING NOZZLE THROAT.
4345 BB = AD / AI
4350 GOSUB 2015
4355 M1 = M
4360 TQ = ((GP ^ 2) * (M1 ^ 2)) / ((1 + GA * M1 ^ 2) ^ 2) * (1 + (GM * M1 ^ 2) / 2) / (GM / 2 + 1)
4365 TS = TQ / TQ
4370 QH = CP * (TS - TQ)
4375 PR# 1
4380 PRINT "QH=":QH
4385 PRINT "TQ=":TQ
4390 PRINT "M=":M
4395 PR# 0
LIST 4400, 4605

4400 IF QQ = QH GOTO 4430
4405 IF QQ > QH GOTO 4430
4410 PR# 1
4415 PRINT "HEATING RATE IS HIGH ENOUGH TO CAUSE A NORMAL SHOCK TO STAND IN THE NOZZLE"
4420 PR# 0
4425 GOTO 4445
4430 PR# 1
4435 PRINT "HEATING RATE IS HIGH ENOUGH TO CAUSE SUBSONIC FLOW THROUGHOUT THE NOZZLE"
4440 PR# 0
4445 HOME
4450 RETURN
4455 REM : THIS PART OF PCHECK DETERMINES IF BACKPRESSURE IS CORRECT
4456 LQ = ABS ((PF - PSTA) / PSTA) * 100
4457 PR# 1
4458 PRINT "PSTAR= "; PSTA
4459 PR# 0
4460 IF LQ < 3 GOTO 2530
4465 GOTO 2440
4470 REM : THE PURPOSE OF THIS SUBROUTINE (FANCONVREG) IS TO CALCULATE MACH# FOR A CONVERGING NOZZLE FOLLOWED BY A FANNO DUCT
4475 M2 = 101
4480 M(J) = 1.01
4485 FOR J = 1 TO 300
4490 M(J) = M(J - 1) - 0.004
4495 CA(J) = ((1 - (M(J)^2)) / (GA * (M(J)^2))) + (GP / (2 * GA)) * LOG (GP * M(J)^2 / (2 + GM * M(J)^2))
4500 XY = ABS (CA(J) - FL)
4505 IF XY < 0.01 THEN 4515
4510 NEXT J
4515 M1 = M(J)
4520 IF M2 < 1 GOTO 4610
4525 M2 = 1
4530 PF(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * (M(J)^2))) ^ (-0.5)
4535 P(1) = PO / (1 + (GM / 2) * (M(J)^2)) ^ (GA / GM)
4540 PSTARF = P(1) / PF(J)
4545 PR# 1
4550 PRINT "PSTARF= "; PSTARF
4555 PR# 0
4560 IF PB < PSTARF GOTO 4610
4565 PR# 1
4570 PRINT "BACK PRESSURE IS TOO HIGH TO ALLOW CHOKED FLOW IN THE FANNO DUCT"
4575 PR# 0
4580 FP(2) = PB / PSTARF
4585 GOSUB 4680
4590 M2 = M
4595 CA = ((1 - (M2^2)) / (GA * (M2^2))) + (GP / (2 * GA)) * LOG (GP * M2^2 / (2 + GM * M2^2))
4600 FL = FL + CA
4605 GOTO 4480
LIST4610,4805
4610 M(1) = M1
4615 M(2) = M2
4620 FOR J = 1 TO 2
4625 T0(J) = T0
4630 T(J) = T0(J) / (1 + ((GM / 2) * (M(J) ^ 2))
4635 PF(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * (M(J) ^ 2))) ^ ( - 0.5)
4640 PG(J) = (1 / M(J)) * ((2 / GP) * (1 + (GM / 2) * (M(J) ^ 2))) ^ (GP / (2 * GM))
4645 P(2) = P(1) * PF(2) / PF(1)
4650 GOSUB 4115
4655 P(J) = PO(J) / (1 + (GM / 2) * (M(J) ^ 2)) ^ (GA / GM)
4660 MD(J) = (P(J) * M(J) * SQR (GA * T(J) * 1000 * R)) / (R * T(J))
4665 GOSUB 4080
4670 NEXT J
4675 GOTO 4005
4680 REM : THIS SUBROUTINE (PSTARFAN) CALCULATES MACH NUMBER FOR A SPECIFIC P/PSTAR FOR FANNO FLOW.
4685 M(0) = 1.01
4690 FOR I = 1 TO 99
4695 M(I) = M(I - 1) - 0.01
4700 PF(I) = (1 / M(I)) * ((2 / GP) * (1 + (GM / 2) * (M(I) ^ 2))) ^ ( - 0.5)
4705 LB = ABS (PF(I) - FP(2))
4710 HOME: PRINT LB,PF(I),M(I)
4715 PRINT
4720 IF LB < 0.01 THEN M = M(I): PRINT LB,PF(I),M(I)
4725 PRINT
4730 NEXT I
4735 HOME
4740 RETURN
4745 REM : THE PURPOSE OF THIS SUBROUTINE (BPRA0G) IS TO DETERMINE THE RANGE OF BACKPRESSURES FOR WHICH A NORMAL SHOCK WILL OCCUR IN A CONVERGING/DIVERGING NOZZLE.
4750 BB = AD / AI
4755 GOSUB 2015
4760 MX = M
4765 MY = SQR ((MX ^ 2 + 2 / GM) / (2 * GA * (MX ^ 2) / GM - 1))
4770 PZ = (1 + GA * MX ^ 2) / (1 + GA * MY ^ 2)
4775 PX = PO / (1 + (GM / 2) * (MX ^ 2)) ^ (GA / GM)
4780 PL = PX * PZ
4785 PR# 1
4790 IF PB > PL GOTO 4825
4795 IF Z# = "NO" GOTO 4815
4800 HOME: PRINT "BACK PRESSURE IS NOT HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE. HOWEVER, FRICTION AND/OR HEATING TERMS IN THE DUCT MAY BE LARGE ENOUGH TO FORCE THE SHOCK TO OCCUR."
4805 PR# 0
LIST 4810, 4970

4810 GOTO 4640
4815 HOME : PRINT "BACK PRESSURE IS NOT HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE."
4820 GOTO 4840
4825 FR# 1
4830 HOME : PRINT "BACK PRESSURE IS HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE."
4835 FR# 0
4840 GG = (GM / 2) * ((AI / AD) ^ 2) * ((2 / GP) ^ (GP / GM))
4845 HH = FR ^ (2 / GA) - FR ^ (GP / GA)
4850 IF HH < GG GOTO 4870
4855 PRINT "BACK PRESSURE IS LOW ENOUGH TO ALLOW MACH=1 CONDITIONS TO EXIST AT THE THROAT"
4860 MT = 1.0
4865 GOTO 4885
4870 FR# 1
4875 PRINT "BACK PRESSURE IS TOO HIGH TO ALLOW MACH=1 CONDITIONS TO EXIST AT THE THROAT"
4880 FR# 0
4885 HOME
4890 RETURN
4900 REM : THE PURPOSE OF THIS SUBROUTINE(BPFRANG) IS TO DETERMINE THE RANGE OF BACKPRESSURES FOR WHICH A NORMAL SHOCK WILL OCCUR IN A FANNO DUCT
4901 P3 = P0(3) / (1 + (GM / 2) * (M2 ^ 2)) ^ (GA / GM)
4902 PF = (1 / M2) * ((2 / GP) * (1 + (GM / 2) * (M2 ^ 2))) ^ (-0.5)
4903 MX = M2
4905 MY = SQRT((MX ^ 2 + 2 / GM) / (2 * GA * (MX ^ 2) / GM - 1))
4910 PZ = (1 + GA * MX ^ 2) / (1 + GA * MY ^ 2)
4912 PY = (1 / MY) * ((2 / GP) * (1 + (GM / 2) * (MY ^ 2))) ^ (-0.5)
4915 CY = (1 - MY ^ 2) / (GA * MY ^ 2) + (GP / (2 * GA)) * LOG (GP * MY ^ 2 / (2 + GM * MY ^ 2))
4920 CB = CY - FL
4922 M(0) = MY
4925 FOR I = 1 TO 100
4930 M(I) = M(I - 1) + 0.005
4932 IF M(I) > 1 THEN 5050
4935 CA(I) = (1 - (M(I) ^ 2)) / (GA * (M(I) ^ 2)) + (GP / (2 * GA)) * LOG (GP * M(I) ^ 2 / (2 + GM * M(I) ^ 2))
4945 SS = ABS(CA(I) - CB)
4950 IF SS < .05 GOTO 4965
4955 NEXT I
4960 ME = M(I)
4965 KF = (1 / ME) * ((2 / GP) * (1 + (GM / 2) * (ME ^ 2))) ^ (-0.5)
4967 FR# 1
4968 PP = KF * PZ * P3 / PY
4969 FR# 0
4970 CA = (1 - (M2 ^ 2)) / (GA * (M2 ^ 2)) + (GP / (2 * GA)) * LOG (GP * M2 ^ 2 / (2 + GM * M2 ^ 2))
LIST 4975,5100

4975 CB = CA - FL
4977 M(0) = M2
4980 FOR I = 1 TO 200
4985 M(I) = M(I - 1) - 0.01
4990 IF M(I) < 1 THEN 5050
4995 CA(I) = (1 - (M(I) ^ 2)) / (GP * (M(I) ^ 2)) + (GP / (2 * GA)) * LOG (GP * M(I) ^ 2 / (2 + GM * M(I) ^ 2))
5000 SS = ABS (CA(I) - CB)
5005 IF SS < .03 GOTO 5015
5010 NEXT I
5015 MX = M(I)
5020 MY = SQR ((MX ^ 2 + 2 / GM) / (2 * GA * (MX ^ 2) / GM - 1))
5025 PZ = (1 + GA * MX ^ 2) / (1 + GA * MY ^ 2)
5030 KF = (1 / MX) * ((2 / GP) * (1 + (GM / 2) * (MX ^ 2))) ^ (1 / 0.5)
5031 PS = (PZ * KF * P3) / PF
5047 IF PB > PS GOTO 1415
5048 IF PB > PP GOTO 605
5050 HOME
5055 GOTO 2510
Appendix 3

COMPUTER PRINTOUT FOR EACH CASE
GIVEN CONDITIONS
CASE # 1

*************
*
* STAGNATION PRESSURE 700   *
* STAGNATION TEMPERATURE 220 *
* HEATING RATE 0              *
* BACK PRESSURE 70            *
* FL/D = 0                    *
* GASEOUS MEDIA = AIR         *
* NOZZLE IS CONVERGING TYPE   *
*
*************

BACK PRESSURE IS LOW ENOUGH TO ALLOW SONIC FLOW IN THE THROAT.
PR = .1
CRITICAL PRESSURE RATIO = .53

<table>
<thead>
<tr>
<th>PRESS</th>
<th>TEMP</th>
<th>MACH#</th>
<th>AREA</th>
<th>MDOT</th>
<th>TOTPRESS</th>
<th>VEL</th>
<th>DENSITY</th>
</tr>
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<tr>
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<td>1907.51</td>
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<td>271.41</td>
<td>7.03</td>
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EXIT PRESSURE = 369.8
EXIT TEMPERATURE = 183.33
EXIT MACH# = 1
GIVEN CONDITIONS
CASE# 2
*****************************************
* STAGNATION PRESSURE 700 *
* STAGNATION TEMPERATURE 220 *
* HEATING RATE 0 *
* BACK PRESSURE 70 *
* FL/D= 0 *
* GASEOUS MEDIA AIR *
* NOZZLE IS CONVERGING TYPE *
*
*****************************************
BACK PRESSURE IS LOW ENOUGH TO ALLOW SONIC FLOW IN THE THROAT.
PR= .1
CRITICAL PRESSURE RATIO= .53

<table>
<thead>
<tr>
<th>PRESS</th>
<th>TEMP</th>
<th>MACH#</th>
<th>AREA</th>
<th>MDOT</th>
<th>TOTPRESS</th>
<th>VEL</th>
<th>DENSITY</th>
</tr>
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<tbody>
<tr>
<td>KPA</td>
<td>DEGK</td>
<td>SQRMET</td>
<td>KG/SEC</td>
<td>M/SEC</td>
<td>KG/M^3</td>
<td></td>
<td></td>
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<tr>
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<td>1</td>
<td>1</td>
<td>1907.51</td>
<td>700</td>
<td>271.41</td>
<td>7.03</td>
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EXIT PRESSURE= 369.8
EXIT TEMPERATURE= 183.33
EXIT MACH#= 1
GIVEN CONDITIONS  
CASE# 3

* STAGNATION PRESSURE 700
* STAGNATION TEMPERATURE 220
* HEATING RATE 0
* DIAMETER RATIO 2
* NOZZLE THROAT DIAMETER 2
* BACK PRESSURE 570
* FL/D= 0
* GASEOUS MEDIA AIR
* NOZZLE IS CONVERGING-DIVERGING TYPE

BACK PRESSURE IS HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE.

SUBSONIC FLOW AT NOZZLE EXIT WITH A SHOCK @DIAMETER RATIO= 1.175

<table>
<thead>
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<th>PRESS KPA</th>
<th>TEMP DEGK</th>
<th>MACH#</th>
<th>AREA SQRMET</th>
<th>MDOT KG/SEC*M^2</th>
<th>TOTPRESS KPA</th>
<th>VEL M/SEC</th>
<th>DENSITY KG/M^3</th>
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<td>183.33</td>
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<td>1907.51</td>
<td>700</td>
<td>271.41</td>
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<td>129.49</td>
<td>135.84</td>
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<td>1365.73</td>
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<td>446.39</td>
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<td>4</td>
<td>568.78</td>
<td>218.65</td>
<td>.18</td>
<td>12.6</td>
<td>472.04</td>
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<td>52.08</td>
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EXIT PRESSURE= 568.78
EXIT TEMPERATURE= 218.65
EXIT MACH#= .18
GIVEN CONDITIONS
CASE# 4

* STAGNATION PRESSURE 700
* STAGNATION TEMPERATURE 220
* HEATING RATE 0
* BACK PRESSURE 570
* FL/D= 0
* GASEOUS MEDIA AIR
* NOZZLE IS CONVERGING TYPE

PC= .53
BACK PRESSURE ALONE IS TOO HIGH TO ALLOW SONIC CONDITIONS AT THE NOZZLE EXIT

<table>
<thead>
<tr>
<th>PRESS</th>
<th>TEMP</th>
<th>MACH#</th>
<th>AREA</th>
<th>MDOT</th>
<th>TOTPRESS</th>
<th>VEL</th>
<th>DENSITY</th>
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<td>KPA</td>
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<td>SQRMET</td>
<td>KG/SEC</td>
<td>KPA</td>
<td>M/SEC</td>
<td>KG/M^3</td>
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<td>----------</td>
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<td>---------</td>
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<td>.55</td>
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<td>1519.99</td>
<td>700</td>
<td>156.79</td>
<td>9.57</td>
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EXIT PRESSURE= 569.92
EXIT TEMPERATURE= 207.45
EXIT MACH#= .55
### GIVEN CONDITIONS
#### CASE # 5

<table>
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<td>HEATING RATE</td>
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<td>DUCT LENGTH</td>
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<td>DUCT DIAMETER</td>
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<td>NOZZLE THROAT DIAMETER</td>
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<td>BACK PRESSURE</td>
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<td>FL/D</td>
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<td>GASEOUS MEDIA</td>
<td>AIR</td>
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<td>NOZZLE IS CONVERGING-DIVERGING TYPE</td>
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---

PC = .53

BACK PRESSURE ALONE IS TOO HIGH TO ALLOW SUPERSOONIC CONDITIONS AT THE NOZZLE EXIT; HOWEVER, FRICTION AND/OR HEATING TERMS IN THE DUCT MAY BE LARGE ENOUGH TO FORCE SUPERSOONIC CONDITION TO OCCUR.

BACK PRESSURE IS TOO HIGH TO ALLOW MACH=1 CONDITIONS TO EXIST AT THE THROAT.

<table>
<thead>
<tr>
<th>PRESS</th>
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<th>MACH#</th>
<th>AREA</th>
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<th>TOTPRESS</th>
<th>VEL</th>
<th>DENSITY</th>
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<td>M/S</td>
<td>SQM/</td>
<td>KG/SEC</td>
<td>M/SEC</td>
<td>KG/M^3</td>
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EXIT PRESSURE = 691.78
EXIT TEMPERATURE = 219.26
EXIT MACH# = .13
**GIVEN CONDITIONS**

*CASE# 6*

*****************************************

* STAGNATION PRESSURE 700
* STAGNATION TEMPERATURE 220
* HEATING RATE 60
* DIAMETER RATIO 2
* NOZZLE THROAT DIAMETER 2
* BACK PRESSURE 70
* FL/D= 0
* GASEOUS MEDIA AIR
* NOZZLE IS CONVERGING-DIVERGING TYPE

*****************************************

BACK PRESSURE IS NOT HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE, HOWEVER, FRICTION AND/OR HEATING TERMS IN THE DUCT MAY BE LARGE ENOUGH TO FORCE THE SHOCK TO OCCUR.

**FLOW CONDITIONS @RAYLEIGH DUCT**

<table>
<thead>
<tr>
<th>PRESS KPA</th>
<th>TEMP DEGK</th>
<th>MACH#</th>
<th>AREA SQRTM</th>
<th>MDOT KG/SEC M²</th>
<th>TOTFRESS KPA</th>
<th>VEL M/SEC</th>
<th>DENSITY KG/M³</th>
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<tbody>
<tr>
<td>1</td>
<td>369.8</td>
<td>183.33</td>
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<td>2.94</td>
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<td>477</td>
<td>700</td>
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<tr>
<td>3</td>
<td>51.2</td>
<td>172.72</td>
<td>1.76</td>
<td>12.6</td>
<td>478.9</td>
<td>276.78</td>
<td>463.65</td>
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EXIT PRESSURE= 51.2
EXIT TEMPERATURE= 172.72
EXIT MACH#= 1.76
### Given Conditions

#### Case #7

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<th>Value</th>
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<td>Stagnation Pressure</td>
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<td>Stagnation Temperature</td>
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<tr>
<td>Heating Rate</td>
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</tr>
<tr>
<td>Back Pressure</td>
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</tr>
<tr>
<td>FL/D</td>
<td>0</td>
</tr>
<tr>
<td>Gaseous Media</td>
<td>Air</td>
</tr>
<tr>
<td>Nozzle Is Converging Type</td>
<td></td>
</tr>
</tbody>
</table>

---

Back pressure is low enough to allow choked flow in the nozzle; however, friction and/or heating terms in the duct may be large enough to force subsonic conditions to occur.

**Critical Pressure Ratio:** 0.53

<table>
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<tr>
<th>Press</th>
<th>Temp</th>
<th>Mach</th>
<th>Area</th>
<th>Mdot</th>
<th>Totpress</th>
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<td>SQRT</td>
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<td>M/SEC</td>
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<tr>
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<td>1562.63</td>
<td>646.62</td>
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</table>

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Exit Pressure: 341.6

Exit Temperature: 233.11

Exit Mach#: 1
**GIVEN CONDITIONS**

**CASE # 8**

* STAGNATION PRESSURE 700
* STAGNATION TEMPERATURE 220
* HEATING RATE 60
* DIAMETER RATIO 2
* NOZZLE THROAT DIAMETER 2
* BACK PRESSURE 570
* FL/D= 0
* GASEOUS MEDIA AIR
* NOZZLE IS CONVERGING-DIVERGING TYPE

**BACK PRESSURE IS HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE.**

**SUBSONIC FLOW AT NOZZLE EXIT WITH A SHOCK @DIAMETER RATIO= 1.1624**

<table>
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<th>TEMP</th>
<th>MACH#</th>
<th>AREA</th>
<th>MDOT</th>
<th>TOTPRESS</th>
<th>VEL</th>
<th>DENSITY</th>
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<tbody>
<tr>
<td>KPA</td>
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<td>SQRMT</td>
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<td>KPA</td>
<td>M/SEC</td>
<td>KG/M³</td>
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<tr>
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EXIT PRESSURE= 569.97
EXIT TEMPERATURE= 277.49
EXIT MACH#=.2
GIVEN CONDITIONS
CASE# 9
*******************************************************************************
* STAGNATION PRESSURE 700 *
* STAGNATION TEMPERATURE 220 *
* HEATING RATE 111 *
* DIAMETER RATIO 2 *
* NOZZLE THROAT DIAMETER 2 *
* BACK PRESSURE 128 *
* FL/D= 0 *
* GASEOUS MEDIA AIR *
* NOZZLE IS CONVERGING-DIVERGING TYPE *
*******************************************************************************
BACK PRESSURE IS NOT HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE. HOWEVER, FRICTION AND/OR HEATING TERMS IN THE DUCT MAY BE LARGE ENOUGH TO FORCE THE SHOCK TO OCCUR.

SUBSONIC FLOW AT NOZZLE EXIT WITH A SHOCK @DIAMETER RATIO= 2
MX= 2.93999996
MY= .478836262
AY= 1.38256625
AR= .345641558
PZ= 9.91753305
TZ= 2.60907599
TX= 80.623937
TY= 210.353839
PT= .34570058

FLOW CONDITIONS @RAYLEIGH DUCT
TT= 330.502738
TO= .659604075
TR= .990913421
M1= .478836262
PSTA= 113.850824
M2= .898836258

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<th>TEMP DEGK</th>
<th>MACH#</th>
<th>AREA SQRT M</th>
<th>MDOT KG/SEC M^2</th>
<th>TOTPRESS KPA</th>
<th>VEL M/SEC</th>
<th>DENSITY KG/M^3</th>
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<tbody>
<tr>
<td>1 369.8 183.33 1 3.1 1907.51 700 271.41 7.03</td>
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</tr>
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EXIT PRESSURE= 128.22
EXIT TEMPERATURE= 284.53
EXIT MACH= .9
GIVEN CONDITIONS
CASE# 1D

************************************************
STAGNATION PRESSURE 700
STAGNATION TEMPERATURE 220
HEATING RATE 60
DUCT LENGTH
DUCT DIAMETER 4
NOZZLE THROAT DIAMETER 2
BACK PRESSURE 70
FL/D= .15
GASEOUS MEDIA AIR
NOZZLE IS CONVERGING-DIVERGING TYPE

************************************************
BACK PRESSURE IS LOW ENOUGH TO ALLOW CHOKE FLOW IN THE NOZZLE
PR=.1
CRITICAL PRESSURE RATIO= .53
CA= .238548664
CB= .098548664
M2= 1.78999998
TF(1)= .731341652
TF(2)= .872406152
PF(1)= .477757075
PF(2)= .691802664
PG(2)= 1.09901548
PG(1)= 1.4281902
ME= 1.36999999

PRESS KPA TEMP DEGK MACH# AREA SQMETER MDOT KG/SEC TOTPRESS KPA VEL M/SEC DENSITY KG/M^3
--- --- ----- ----- ----- --- ----- ----- --- --- ----- --- --- --- --- --- --- --- --- ---
1 369.8 183.33 1 3.1 5989.6 700 271.41 7.03
2 20.86 80.62 2.94 12.6 5990.6 700 529.16 .9
3 49.61 170.48 1.79 12.6 5990.2 281.85 468.49 1.02
4 71.08 203.38 1.37 12.6 5990.2 216.09 391.64 1.22

EXIT PRESSURE= 71.08
EXIT TEMPERATURE= 203.38
EXIT MACH#= 1.37
GIVEN CONDITIONS
CASE 11
*****************************************
* STAGNATION PRESSURE 700 *
* STAGNATION TEMPERATURE 220 *
* HEATING RATE .60 *
* DIAMETER RATIO 2 *
* NOZZLE THROAT DIAMETER 2 *
* BACK PRESSURE 70 *
* FL/D= .35 *
* GASEOUS MEDIA AIR *
* NOZZLE IS CONVERGING-DIVERGING TYPE *
*
*****************************************
BACK PRESSURE IS LOW ENOUGH TO ALLOW CHOKED FLOW IN THE NOZZLE
PR= .1
CRITICAL PRESSURE RATIO=.53
IF DUCT CHOKE BY FRICTION ASSUMPTION IS CORRECT THEN SHOCK IS LOCATED IN THE DUCT AND THE FOLLOWING PARAMETERS ARE VALID
CA= .236548664
M2 = 1.63999998
MY(I)= .656765467
CA(I)= .186673082
CY(I)= .306294443
FF= 8.17002519E-03
PF(I)= .538616816
PY(I)= 1.60033825
PF= .477757075
PK<1>= .879920589
PG<2>= 1.28355252
PW= 1.12942428
PRESS KPA
KPA
TEMP DEG
MACH# SQM/SEC
MDOT KG/SEC
TOTPRESS KPA
VEL M/SEC
DENSITY KG/M^3
1 | 369.8 | 183.33 | 1 | 3.1 | 5990.2 | 700 | 271.41 | 7.03
2 | 20.86 | 80.62 | 2.94 | 12.6 | 5990.6 | 700 | 529.16 | .9
3 | 49.81 | 170.48 | 1.79 | 12.6 | 5990.2 | 201.85 | 468.49 | 1.02
4 | 66.15 | 101.07 | 1.64 | 12.6 | 5990.2 | 253.31 | 443.36 | 1.08
5 | 166.85 | 257.52 | .66 | 12.6 | 5990.2 | 222.89 | 211.26 | 2.26
6 | 104.26 | 233.11 | 1 | 12.6 | 5990.2 | 197.35 | 306.04 | 1.56

SINCE BACK PRESSURE < EXIT PRESSURE, SHOCK LOCATION IS NOT AFFECTED
EXIT PRESSURE= 104.26
GIVEN CONDITIONS
CASE# 12
****************************************
STAGNATION PRESSURE 700
STAGNATION TEMPERATURE 220
HEATING RATE 60
DUCT LENGTH 1
DUCT DIAMETER 4
NOZZLE THROAT DIAMETER 2
BACK PRESSURE 70
FL/D= .35
GASEOUS MEDIA HELIUM
NOZZLE IS CONVERGING-DIVERGING TYPE
****************************************

BACK PRESSURE IS LOW ENOUGH TO ALLOW CHOKED FLOW IN THE NOZZLE
PR= .1
CRITICAL PRESSURE RATIO= .49
IF DUCT CHOKED BY FRICTION ASSUMPTION IS CORRECT THEN SHOCK IS LOCATED IN THE DUCT AND THE FOLLOWING PARAMETERS ARE VALID
CA= .321761528
MZ = 1.50999996
MY(I)= .712674104
CA(I)= .100020765
CY(I)= .14986234
PF= .015703830
PF(I)= .576149128
PY(I)= 1.49875023
PF= .220750212
PK(I)= .12789087
PT= .93536765
PG(2)= 1.15364683
PW= 1.0790393

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<th>MACH#</th>
<th>AREA SQM</th>
<th>MDOT KG/SEC</th>
<th>TOTPRESS KPA</th>
<th>VEL M/SEC</th>
<th>DENSITY KG/M^3</th>
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SINCE BACK PRESSURE < EXIT PRESSURE, SHOCK LOCATION IS NOT AFFECTED
EXIT PRESSURE= 87.32
EXIT TEMPERATURE= 173.48
EXIT MACH#= 1
GIVEN CONDITIONS
CASE# 13

STAGNATION PRESSURE 700
STAGNATION TEMPERATURE 220
HEATING RATE 0
DIAMETER RATIO 2
NOZZLE THROAT DIAMETER 2
BACK PRESSURE 150
FL/D = .65
GASEOUS MEDIA AIR
NOZZLE IS CONVERGING-DIVERGING TYPE

BACK PRESSURE IS NOT HIGH ENOUGH TO FORCE A SHOCK TO OCCUR IN THE NOZZLE. HOWEVER, FRICTION AND/OR HEATING TERMS IN THE DUCT MAY BE LARGE ENOUGH TO FORCE THE SHOCK TO OCCUR.

IF DUCT CHOKED BY FRICTION ASSUMPTION IS CORRECT THEN SHOCK IS LOCATED IN THE DUCT AND THE FOLLOWING PARAMETERS ARE VALID

<table>
<thead>
<tr>
<th>PRESS</th>
<th>TEMP</th>
<th>MACH#</th>
<th>AREA</th>
<th>MDOT</th>
<th>TOTPRESS</th>
<th>VEL</th>
<th>DENSITY</th>
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BACK PRESSURE IS TOO HIGH TO ALLOW FLOW PROPERTIES CALCULATED ASSUMING A DUCT CHOKED BY FRICTION; THE SHOCK LOCATION IS DETERMINED BY THE BACK PRESSURE.

Mz = 2.36
RR = 3.69418122E-03
FX(I) = .400619291
MY(I) = .527485632
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<th>MACH#</th>
<th>AREA SQM</th>
<th>MDOT KG/SEC M^2</th>
<th>TOTPRESS KPA</th>
<th>VEL M/SEC</th>
<th>DENSITY KG/M^3</th>
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</thead>
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<td>369.8</td>
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EXIT PRESSURE= 150
EXIT TEMPERATURE= 202.86
EXIT MACH#= .65
REFERENCES


