Two Point Resolution of a Defocused Multi-Aperture System Eyelet

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TWO POINT RESOLUTION OF A DEFOCUSED
MULTI-APERTURE SYSTEM EYELET

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ABSTRACT

Multi-aperture optical systems based on the insect eye offer an alternative to the common optical system based on the human eye. Some of the advantages of a multi-aperture system include the ability to perform parallel processing, have super resolution and have available large amounts of system redundancy.

An individual eyelet of a multi-aperture system consists of a gradient index lens coupled to optical fibers which transfer the incident light on the lens to individual detectors.

A mathematical model of an individual eyelet was developed. It is a flexible model allowing various system parameters to vary. Computer based algorithms were developed to locate and resolve two points in space. The model was exercised with experimental data and found to have a resolution of 3.1°. The algorithm was also exercised with the computer model and the results compared favorably.
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CHAPTER ONE

INTRODUCTION

Insect Eye

Almost all conventional imaging systems are patterned after a human eye; a single large aperture optical system coupled to a large number of detectors to obtain an image. Since each detector is addressed individually, a large amount of time and processing is required for image formation or analysis. In contrast, consider an optical system modeled after the insect eye, a multi-aperture optical system. Insect eyes have an extremely wide field-of-view, as much as 270 degrees. It has been shown that most insect eyes cannot image an object. However, the insect eye is optimized for performing certain tasks such as locating and tracking a target (searching for food, mating, normal flight and defense). The insect eye also processes in parallel; therefore, a short amount of time is required to perform certain tasks.

Most entomologists subscribe to the theory that there are two types of insect eyes, the apposition eye and the superposition eye. In the apposition eye, each eyelet has a small field of view that overlaps with the neighboring eyelets. The sensed information is reconstructed in the brain from tiny segments of information obtained by each eyelet. The superposition eye is based on the superposition principle; the actual image is formed from a layer of images that are superimposed.

The single eyelet or omatidia is shown in Figure 1. The omatidia
consists of 3 distinct structures: (1) lens system (corneal lens and the crystalline cone); (2) a rhabdom which acts like a detector and converts the light to an electric signal; and (3) a transparent, hose-like medium to transfer the light. The omatidia is surrounded by pigment cells which serve as optical insulation between neighboring eyelets. Some insects have up to 20,000 of such eyelets. The most important items learned from insect physiology are: (1) an insect does not need to form an image to perform complex tasks; and (2) multi-aperture systems are perfect platforms for application of parallel processing.
Figure 1. Single Eyelet (Basic Omatidia)
Multi-Aperture Optical System

To develop a multi-aperture optical system it is not necessary to emulate the insect eye, but use it as a guide to assemble a system to perform certain tasks. It is important to remember that an image is not required to perform many tasks.

Some typical characteristics of a multi-aperture optical system are:

1) super resolution capability
2) parallel processing
3) built-in redundancy

There are three general configurations for multi-aperture optical systems. (4)

1) Those that superimpose the image from each lens; i.e., one generates an image from overlapping images from each lens.
2) Those that place each lens in apposition; i.e., each lens has a unique field of view, where their fields of view overlap by a fixed prescription.
3) Those that discard the unified optical image and spatially sample the field of view. Reconstruction of the image information is done by calculational techniques and no alignment is necessary.

The third configuration is the only system considered, since it requires no alignment and is extremely simple to construct.

A random apposition multi-aperture optical system consists of a set of lenses focused onto pixel dividers. This system was first described by
Walters.(4) The lenses are gradient index lenses and the pixel dividers are step index optical fibers. A characteristic of this system is that any point of a concentric circle in space, viewed by a pixel, has a unique detector response.(3)

Kellog(5) compared the resolution and detection characteristics of multi-aperture vs. single aperture systems, and concluded that a multi-aperture system resolution improves by the square root of the number of overlapping pixels. Mathews(6) has shown that overlap can be controlled with excellent statistical results and need not be carefully aligned into place. Walters(4) has described the data path in mathematical nomenclature as a set of array operators.

The conventional optical system utilized for point source detection and location usually employs a quadrant detector placed at the focal plane of the system. The quadrant detector divides the focal plane into four quadrants. The object point location is determined by differencing the output signals from the detectors on each side of the axis of interest.

Consider utilizing a single eyelet of a multi-aperture optical system defocused to provide the field of view redundancy. A single Gradient Index (GRIN) lens is used with 16 optical fibers arranged in a hexagonal array and placed far enough out of the focal plane to allow at least three fibers to be illuminated by a single object point. The output from the three fibers can be utilized to determine the point source location in azimuth and elevation with respect to the GRIN lens optical axis. The single eyelet is shown in Figure 2.
The purpose of this study was to mathematically model a GRIN eyelet similar to the experimental hardware developed at the University of Central Florida by Dr. R. Walters. The configuration shown in Figure 2 was utilized to characterize the eyelet. A source model, GRIN model and fiber model were developed to simulate the eyelet.
CHAPTER TWO

THE EYELET MODEL

A mathematical eyelet model allows the system designer to change certain aspects of his system to determine their effects in a fast and efficient manner without changing actual hardware.

The eyelet model is composed of three submodels, a point source, a gradient index lens, and optical fibers.

Ray matrix optics is used to propagate the point source through the system to the fiber plane, locate the centroid and define the spot size. Separate intensity models have been developed to determine the intensity distribution due to the point source. The ray matrix models and intensity models are discussed in detail in the following sections.

The Source

The point source model is allowed to vary in divergence, wavelength, position (azimuth, elevation and distance from the GRIN lens).

Consider a point source located in the field of view of the GRIN lens. To determine the photon flux collected by the GRIN lens, the geometry of Figure 3 is used.
The total power received by the entrance aperture of the GRIN lens is given by:

\[ P = E A \cos \theta_{AZ} \cos \theta_{EL} \]  

Figure 3. Geometry of Source and GRIN Lens

The derivation of equation 1 is given in Appendix A.

The irradiance of the source can be determined by the intensity of the source and the range from source to GRIN lens.
where

\[ I = \text{intensity of source (watts/sr)} \]
\[ R = \text{range from source to GRIN lens (cm}^2\text{)} \]

Consider the geometry of Figure 3 to calculate the intensity of the source. (8) Intensity of a source is given by

\[ I = \frac{P_L}{\Omega} \]  \hspace{1cm} (3)

where

\[ P_L = \text{power of laser in watts} \]
\[ \Omega = \text{solid angle of laser beam with diverging lens in steradians} \]

The assumptions made in the model are that the laser is placed far enough from the GRIN lens that a diverging lens placed in front of the laser fills the GRIN lens with a uniform plane wave.
The total power received by the GRIN lens aperture is found by substituting equations 2 and 3 into equation 1.

\[ P = \frac{P_L A \cos \theta \cos \phi}{\Omega R^2} \]  

where

\[ P = \text{watts collected by the GRIN lens,} \]

Substituting in the equations for the lens area and solid angle of the laser, the power, \( P \), becomes

\[ P = \frac{P_L \pi r^2 \cos \theta \cos \phi}{4\pi \sin^2(\theta_D/2) R^2} \]  

where

\[ A = \text{has been replaced by } \pi r^2 \text{ and } r \text{ is the lens radius} \]

\[ \Omega = 4\pi \sin^2(\theta_D/2) \text{ where } \theta_D \text{ is the laser beam and diverging lens divergence angle} \]

and

\[ \tau = \text{transmission of diverging lens} \]

Coupled into \( \tau \), are the estimates of Fresnel losses in the optical train.
In review, $P$ is the power transferred from a point laser and diverging lens source to a lens with collecting area $A$ with $\theta_{AZ}, \theta_{EL}$ being the position of the point source with respect to the optical axis of the GRIN lens.

The ray matrix model to propagate the rays from the point source to the front of the GRIN lens is a simple transfer matrix. (9)

\[
\begin{bmatrix}
    r_1 \\
    \theta_1
\end{bmatrix} =
\begin{bmatrix}
    1 & d \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    r_s \\
    \theta_s
\end{bmatrix}
\]  \hspace{1cm} (6)

where

- $r_1 = \text{ray height at lens}$
- $\theta_1 = \text{ray angle at lens}$
- $d = \text{distance from source to lens}$
- $r_s = \text{ray height at source}$
- $\theta_s = \text{ray angle at source}$

Rays are traced from the point source to the optical fibers to locate and define the center of the spot image on the front face of the fibers.

**The GRIN Lens**

The gradient index lens is a glass rod whose refractive index decreases quasi-quadratically from the axis to the periphery along the
radius. (10)

In the model the lens is allowed to vary in index of refraction, length, diameter, pitch, and numerical aperture (NA) and also have a flat or spherical radius on one end.

The profile of the refractive index can be expressed by:

\[ n(r) = n_0(1 - A r^2) \]  \hspace{1cm} (7)

where

- \( A \) = quadratic constant
- \( r \) = radial variable
- \( n_0 \) = on axis index of refraction

A typical refractive profile is shown in Figure 4.
The ray matrix equations that characterize a spherical GRIN lens are:

\[
\begin{bmatrix}
    r_2 \\
    \theta_2
\end{bmatrix}
= 
\begin{bmatrix}
    \cos(\sqrt{A}Z) - \frac{Q_1}{N_0/A} & \sin(\sqrt{A}Z) & \frac{1}{N_0/A} & \sin(\sqrt{A}Z) \\
    -\left(\frac{Q_1}{N_0/A}\cos(\sqrt{A}Z) + N_0\sqrt{A}\sin(\sqrt{A}Z)\right) & \cos(\sqrt{A}Z)
\end{bmatrix}
\begin{bmatrix}
    r_1 \\
    \theta_1
\end{bmatrix}
\]

where:
- \( r_1 \) = distance between incident ray and optical axis
- \( \theta_1 \) = incident angle in radians
- \( A \) = quadratic constant
- \( Z \) = lens length
- \( N_0 \) = on-axis index of refraction
- \( r_2 \) = distance between exiting ray and optical axis
- \( \theta_2 \) = exiting ray angle in radians
- \( R \) = radius of curvature of GRIN lens
- \( Q_1 \) = \((N_0-1)/R\)

Figure 5 shows this relationship graphically.
If a flat lens is used, $Q_1 = 0$, the equations reduce to the more familiar

$$
\begin{bmatrix}
  r_2 \\
  \theta_2
\end{bmatrix} =
\begin{bmatrix}
  \cos(\sqrt{\lambda}Z) & \frac{1}{N_0\sqrt{\lambda}} \sin(\sqrt{\lambda}Z) \\
  -N_0\sqrt{\lambda} \sin(\sqrt{\lambda}Z) & \cos(\sqrt{\lambda}Z)
\end{bmatrix}
\begin{bmatrix}
  r_1 \\
  \theta_1
\end{bmatrix}
$$

In the experimental device a 3mm diameter, quarter-pitch flat GRIN lens was used. For a quarter-pitch GRIN lens, $\sqrt{\lambda}Z$ is equal to $\pi/2$ and the ray matrix equations reduce to equations 10 and 11.

$$
r_2 = \frac{\theta_1}{N_0\sqrt{\lambda}}
$$
From equation 10 it is evident that \( r_2 \) is not dependent on \( r_1 \), and all parallel rays entering the quarter pitch GRIN lens focus to one point on the rear surface of the lens. Therefore, the focal plane is located at the rear surface of the lens, with a focal length defined by

\[
f = \frac{1}{n_0 \sqrt{\sin(\frac{\theta}{2})}}
\]  

(12)

To determine the intensity distribution of the monochromatic light beam either at the focal plane of the GRIN lens, or out of the focal plane, the following diffraction integral for intensity and phase is utilized. (11)

\[
U(x_0, y_0) = \frac{\exp(jkz)}{jz} \exp[jk/2z(x_0^2 + y_0^2)] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x_1, y_1) \exp[-j\frac{\pi}{\lambda z}(x_0 x_1 + y_0 y_1)] \, dx_1 \, dy_1
\]

(13)

where

\[
k = \frac{2\pi}{\lambda}
\]

\[
z = \text{distance from aperture to observation region}
\]

\[
\lambda = \text{wavelength}
\]
The coordinates for this equation are defined in Figure 6. (11)

![Diffraction Geometry Diagram](image)

**Figure 6. Diffraction Geometry**

Since we are only concerned with intensity, which is given by \(UU^*\), we can ignore the phase components outside the integrand and re-write equation 13.

\[
U(x_0, y_0) = A \int \frac{t(x_1, y_1) \exp[-jk/2f(x_1^2+y_1^2)] \exp[jk/2z(x_1^2+y_1^2)]}{\lambda z} \exp[-j2\pi/\lambda z (x_0 x_1 + y_0 y_1)] \, dx_1 \, dy_1
\]

or rearranging terms

\[
U(x_0, y_1) = A \int \frac{t(x_1, y_1) \exp[jk/2(x_1^2+y_1^2)(\frac{1}{f} - \frac{1}{f})]}{\lambda z} \exp[-j2\pi/\lambda z (x_0 x_1 + y_0 y_1)] \, dx_1 \, dy_1
\]
let $e = \frac{1}{Z} - \frac{1}{f}$

where $f =$ focal length of the lens

$A =$ electric field amplitude

(and equation 14 becomes)

$$U(x_0, y_0) = \frac{A}{j\lambda z} \int_{-\infty}^{\infty} \int [t(x_1, y_1) \exp\left(\frac{jk}{2} \epsilon (x_1^2 + y_1^2)\right)]$$

$$\exp\left[-j\frac{2\pi}{\lambda z} (x_0 x_1 + y_0 y_1)\right] dx_1 dy_1$$

From equation 16, $U(x_0, y_0)$ is found as a Fourier transform of $t(x_1, y_1) \exp[k/2] \epsilon (x_1^2 + y_1^2)]$ where the transform must be evaluated at frequencies $(f_x = \frac{x_0}{\lambda z}, f_y = \frac{y_0}{\lambda z})$ to assure correct space scaling in the observation plane.

When $\epsilon$ goes to zero, i.e. the observation plane is the focal plane, equation 16 gives the Fraunhofer diffraction integral and when $\epsilon$ is finite, equation 16 gives the Fresnel integral.

Now letting

$$\exp[(jk/2)\epsilon (x_1^2 + y_1^2)] = g(x, y)$$

$$f_x = \frac{x_0}{\lambda z} \quad & \quad f_y = \frac{y_0}{\lambda z} \quad \quad x_1 = x \quad & \quad y_1 = y$$
Equation 16 becomes

\[ U(x_0, y_0) = A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \exp[-j2\pi(xf_x + yf_y)] dxdy \]  (17)

To exploit the circular symmetry of the GRIN lens, the transform to polar coordinates is accomplished by

\[
\begin{align*}
    r &= \sqrt{x^2 + y^2} \quad x = r \cos \theta \\
    \theta &= \tan^{-1}(y/x) \quad y = r \sin \theta \\
    \rho &= \sqrt{fx^2 + fy^2} \quad fx = \rho \cos \theta \\
    \phi &= \tan^{-1}(fy/fx) \quad fy = \rho \sin \phi
\end{align*}
\]

Applying the coordinate transformations to Equation 17;

\[
U(r) = A \int_{0}^{2\pi} d\phi \int_{0}^{\infty} dr \cdot r \cdot g(r) \exp[-j2\pi \rho \cos \phi \cos \theta + \sin \theta \sin \phi] \]  (18)

or

\[
A \int_{0}^{2\pi} d\phi \int_{0}^{\rho} dr \cdot g(r) \int_{0}^{2\pi} d\theta \exp[-j2\pi \rho \cos(\theta - \phi)]
\]

where

\[ g(r) = \exp[jkr^2/2] \]  and

\[ J_0(a) = \frac{1}{2\pi} \int_{0}^{2\pi} \exp[-ja \cos(\theta)] d\theta \]
So rewriting equation 18

\[ U(r) = A2\pi \int_{0}^{\infty} r g(r) J_0(2\pi r) dr \]

For a circular aperture \( t(r) = \text{circ}(r) \)

where

\[ \text{circ}(r) = \begin{cases} 1 & r < 1 \\ 0 & \text{Otherwise} \end{cases} \]

Equation 18 becomes

\[ U(r) = A2\pi \int_{0}^{R} r \exp(jk\epsilon r^2/2) J_0(2\pi r) dr \]

To simplify the integral let

\[ \alpha = \frac{k\epsilon r^2}{2} \& \beta = 2\pi r \]

Substituting into Equation 19: \( A = P^{1/2} \)

where \( P \) is the power injected into the lens (equation 5)

\[ \text{and } \exp[jk\epsilon r^2/2] = \cos(k\epsilon r^2/2) + j\sin(k\epsilon r^2/2) \]

(20)
\[ U(r) = p^{1/2} \frac{2\pi}{j\lambda z} \int_0^R r \cos(\alpha + j\sin(\alpha)) J_0(\beta r) dr \]

using a change of variables

\[ r_1 = \beta r \text{ and } dr_1 = \beta dr \] (21)

\[ U(r_1/\beta) = p^{1/2} \frac{2\pi}{j\lambda z \beta} \int_0^R r_1 \cos(\alpha + j\sin(\alpha)) J_0(r_1) dr_1 \]

so

\[ U(r_1/\beta) = p^{1/2} \frac{2\pi}{j\lambda z \beta^2} \left[ \int_0^R r_1 \cos(\alpha) J_0(r_1) dr_1 + j\int_0^R r_1 \sin(\alpha) J_0(r_1) dr_1 \right] \] (22)

In the simplifying case where the observation plane is the focal plane, \( \epsilon = 0 \), now contained in \( \alpha \) and equation 22 reduces to equation 23

\[ U(r_1/\beta) = p^{1/2} \frac{2\pi}{j\lambda z \beta^2} \int_0^R r_1 \cos(\alpha) J_0(r_1) dr_1 \] (23)

which reduces upon integration to the familiar Fraunhofer intensity distribution referred to as the Airy Pattern

\[ I = \frac{P(KR)^2}{Z} \left( \frac{J_1(2\pi \beta R)^2}{2\pi \beta} \right) \] (24)
When the observation plane is not the focal plane numerical integration is performed on equation 22 using the trapezoid method to determine the intensity at the input plane of the fibers.

The Optical Fiber

The optical fiber unit that was modeled is multimode. The fibers can be placed either at the focal plane of the GRIN lens or out of the focal plane at another observation plane.

The assumptions made in the fiber model are:

1) The fibers are parallel to optical axis

2) The illuminated single fiber is uniform in core index

The model does not constrain the numbers of fibers that can be used. The fibers are stored in an \((x, y)\) grid in the image space with \((0, 0)\) being the optical axis. Two different alignments containing 16 fibers placed in square and hexagonal array patterns are shown in figures 7 and 8 respectfully. It should be noted that a symmetrical array is not required, random placement is acceptable.

The ray matrix equation used to transfer the rays from the rear of the lens to the fiber plane is

\[
\begin{bmatrix}
    r_f \\
    \theta_f
\end{bmatrix} = \begin{bmatrix}
    1 & L \\
    0 & 1
\end{bmatrix} \begin{bmatrix}
    r_2 \\
    \theta_2
\end{bmatrix}
\] (25)
where

\[ r_f = \text{ray height at fiber plane} \]
\[ \theta_f = \text{ray angle at fiber plane} \]
\[ L = \text{distance from rear of lens to fiber plane} \]
\[ r_2 = \text{ray height at rear surface of GRIN lens} \]
\[ \theta_2 = \text{exit ray angle at rear of GRIN lens} \]

Rays are traced to define the centroid of the spot in the observation plane (front of fibers).

A simple search algorithm is utilized in the fiber model to determine which fibers are illuminated from the point source. It determines the separation of each fiber centroid from the spot centroid and compares that value with the radius of the spot plus fiber radius to ascertain whether or not that fiber is illuminated.
Figure 8. Hexagonal Fiber Array Arrangement

Separation of centers = \sqrt{(x_f-x_s)^2+(y_f-y_s)^2} \quad (26)

- $x_f$ = azimuth coordinate of fiber center
- $x_s$ = azimuth coordinate of spot center
- $y_f$ = elevation coordinate of fiber center
- $y_s$ = elevation coordinate of spot center

If the separation of centers is less than $RS+Rf$, where $RS =$ radius of spot and $Rf =$ fiber radius, then that fiber is illuminated. This is shown in Figure 9.
An area calculation is then performed to determine the proportion of the spot that falls on each illuminated fiber.

The amount of energy that is coupled into each fiber is also dependent on the fiber's numerical aperture (12-14).

For a step index fiber, the NA is defined as follows, refer to Figure 10. The ray shown transversing the fiber strikes the core-cladding interface at the critical angle, $\theta_c$. $\theta_c$ is the largest external angle for which a mode will propagate in this fiber. The quantity $n_0 \sin \theta_c$ is the NA of the fiber. When the medium is air, $n_0 = 1$ and $NA = \sin \theta_c$ or

$$NA = (n_1^2 - n_2^2)^{1/2}$$

(27)

Since the fiber accepts only rays contained within the cone defined by $\theta$, an input coupling loss occurs if some fraction of the incident light strikes the fiber at angles greater than $\theta$. Similarly, if the detector at the output of the fibers cannot receive all angles of light
up to θ, power is lost. In this study the detector is not modeled since relative intensity values are used in the detection algorithms, and thus the power emitted at the output of the fibers is the desired quantity to be measured.

To calculate the power transferred into a fiber from the point source (1), the intensity calculated via equation 22 is multiplied by the illuminated area of the front surface of the fiber for rays that have θ < θ_external critical. For rays outside this angle, no modal excitation is assumed. Further improvements in this model would include a scaling of modal excitation due to the diverging lens input rays from the defocused spot.

![Figure 10. Maximum Entrance Angle Definition of Numerical Aperture For An Optical Fiber](image)

This chapter discussed the power transferred from a point source through an eyelet of a multi-aperture optical system. Physical optics and geometrical optics were combined to develop a flexible model useful in predicting the performance of an eyelet. The implementation of this model in software is listed in Appendix A.
One of the purposes of this research was to develop algorithms to determine the absolute position of two point sources located in the field-of-view of the eyelet. This was to be based on input data from either a real eyelet system or the model described in Chapter 3. This permits direct comparisons between simulation and real data. This chapter discusses the development of the detection algorithms.

Traditionally, the Rayleigh criterion has been used as a measure of optical system resolution or resolving power. According to the Rayleigh criterion (15), two images are just resolved when the principal maximum of one coincides with the first minimum of the other. This is shown in Figure 11.

Using this criterion, for a single aperture optical imaging system of focal length, \( f \), and clear aperture diameter of \( D \), two point sources are resolved when separated by a distance \( r \) equal to the distance to the first zero of the Airy pattern (\( J_1 \) Bessel function), this is

\[
r = \lambda(1.22 \frac{f}{D})
\]

where

- \( r \) = radius of diffraction spot
- \( \lambda \) = wavelength
- \( f \) = focal length
- \( D \) = aperture diameter
From equation 28, one can see that as the aperture diameter increases the minimum resolvable distance between the two point sources decreases. Two objects can be brought closer together and still be resolved.

But, unless the detector size approaches the Airy disc size, detector size is the limiting factor for resolution. Two objects whose images fall on the same detector cannot be resolved.

An object's position will not be resolved to a new location until it moves from one detector to another. An object which moves within the field-of-view of a detector cannot be further resolved by that detector within its field-of-view.

Due to this fact, the fibers in the eyelet system are moved out of the focal plane to a position where at least three fibers are illuminated by a single point. This allows the equivalent pixel to be smaller than the images of the point source. Therefore, the fibers
(detector) are not the limiting factor for resolution. The resolution is now limited by the accuracy of the algorithms developed utilizing the output from the three illuminated fibers.

**Single-Point Detection Algorithm**

The single-point detection algorithm, "LASER3", modified in this research for 2 point detection, is a scaling model using the three highest intensity values from a fiber array. This algorithm and its associated hardware were developed by Dr. Roy Walters at the University of Central Florida. The software listing is given in Appendix C.

The algorithm first sorts the fibers based on intensity; the three largest intensity values are used to determine the position of the spot centroid. The algorithm determines a line both in azimuth and elevation which is based on the separations of the centroids of the three fibers. Once the line is determined, for example in elevation, the relative normalized intensities of the fibers, are used as scaling factors to determine the position of the centroid along the elevation line. This is illustrated in Figure 12.
Figure 12. Determination of Single Point Location

The equations used to determine the azimuth and elevation of the point source are:

\[ \Delta A = \Theta_{EL1} - \Theta_{EL0} \]
\[ \Delta B = \Theta_{EL0} - \Theta_{EL2} \]
\[ \Delta Z_A = \Theta_{AZ1} - \Theta_{AZ0} \]
\[ \Delta Z_B = \Theta_{AZ2} - \Theta_{AZ0} \]

where

\( \Theta_{EL0} \) = elevation position of highest intensity fiber
\( \Theta_{EL1} \) = elevation position of second highest intensity fiber
\( \Theta_{EL2} \) = elevation position of third highest intensity fiber
\( \Theta_{AZ0} \) = azimuth position of highest intensity fiber
\( \Theta_{AZ1} \) = azimuth position of second highest intensity fiber
\( \theta_{AZ2} = \) azimuth position of third highest intensity fiber

Linear mode excitation scale factors (first approximation) are given by:

\[
F_1 = \frac{I_1}{2} \\
F_2 = \frac{I_2}{2}
\]  

(30)

where

\( F_1 = \) first scale factor
\( F_2 = \) second scale factor
\( I_1 = \) normalized intensity of second highest intensity fiber
\( I_2 = \) normalized intensity of third highest intensity fiber

Equations 29 and 30 are then combined to determine the centroid of the spot.

\[
\theta_{EL} = \frac{[(\theta_{EO} + F_1 \Delta A) + (\theta_{EO} + F_2 \Delta B)]}{2} \\
\theta_{AZ} = \frac{[(\theta_{AZ0} + F_1 \Delta ZA) + (\theta_{AZ0} + F_2 \Delta ZB)]}{2}
\]  

(31)

where

\( \theta_{EL} = \) elevation position of source
\( \theta_{AZ} = \) azimuth position of source

In the case of a single point the position can be resolved better than the fiber field-of-view. A position change in the source will cause an intensity change in the three illuminated fibers resulting in a new scale factor and a new object position location prediction.
Two-Point Detection Algorithm

The two-point detection algorithm developed in this research is in three logical parts. TDA assumes that one of the following three conditions could exist.

1) a single point (or two unresolvable points) is present,
2) two well resolved points are present
3) two close but resolvable points are present (the marginal case)

If three fibers are considered as a "single detector," i.e., when a single point source is in the field-of-view of the system three fibers are illuminated, the second point must be separated a defined resolved distance from the first point in order to cause a fourth fiber to be illuminated. The intensity of the spot in the fiber plane is not the airy pattern and therefore the Rayleigh criterion cannot be invoked.

If only three fibers are illuminated, (condition #1) the single-point detection algorithm is used to determine the location of the point source.

If six or more fibers are illuminated then condition 2 exists and two well resolved points are present. The algorithm uses the six fibers with the highest intensity values. It sorts the fibers according to position in the image plane. The fiber optic object space centroid positions are known in azimuth and elevation. These values are converted to image space coordinates by the following expression derived from equation 10.

\[
X = \frac{\Theta_{AZF}}{N_{VA}}
\]  
(32)
\[ Y = \frac{\Theta_{ELF}}{N_o \sqrt{A}} \]

where

\( X, Y \) = image plane coordinates in mm

\( \Theta_{AZF} \) = azimuth angle of fiber in object space (radians)

\( \Theta_{ELF} \) = elevation angle of fiber in object space (radians)

\( N_o \) = on-axis index of refraction of GRIN lens

\( A \) = quadratic gradient constant (mm\(^{-1}\))

The separation between illuminated fibers is calculated by:

\[ S = \sqrt{\left( (X_1 - X_2)^2 + (Y_1 - Y_2)^2 \right)}^{1/2} \quad (33) \]

where

\( S \) = separation between fiber 1 and fiber 2

\( X_1, Y_1 \) = centroid of fiber 1

\( X_2, Y_2 \) = centroid of fiber 2

The separation between all six fibers is calculated and the two fibers that are farthest apart are then separated into two groups. The remaining four fibers are then compared to the two separated fibers, fiber 1 and fiber 2. The fibers closest to fiber 1 are placed in group 1 and the fibers closest to fiber 2 are placed in group 2. The single-point detection algorithm is first used on group 1, then on group 2 to determine the location of the two points.
If five fibers are illuminated, the same logic is utilized. The two fibers farthest apart are found and separated into two groups, the remaining three fibers are then placed into group 1 and 2 depending on the centroid locations. One fiber will be shared by both groups introducing a slight inaccuracy.

When four fibers are illuminated, condition (3) exists and two close but resolvable points exist. The four illuminated fibers are sorted according to intensity. The separation between the fibers is calculated using equation 33 and the two fibers with the greatest separation are placed into two separate groups. The remaining two fibers are placed in both groups. Thus, two fibers are shared by the two groups for object point location. The single-point detection algorithm is then applied to the two groups to determine the point source locations. This again introduces a small amount of error. The software listing for the two-point detection algorithms in Appendix D.

This chapter discussed the criterion for single-point and two-point detection. The algorithms developed can be utilized with either the actual hardware or the model based computer simulation.
CHAPTER 4

EXPERIMENTAL PROCEDURES AND RESULTS

This chapter explains the experimental procedures and methods used to validate the eyelet model and the two-point detection algorithms.

Experimental Setup

The eyelet system consists of a quarter pitch, SLW series, three millimeter diameter, NSG America SELFOC MICRO lens, on 16 step-index multimode optical fibers. The eyelet system is shown in Figure 13. The important parameters for those components are listed in Table 1.

Figure 13. Experimental Hardware Setup
TABLE 1.

EXPERIMENTAL HARDWARE PARAMETERS

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
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</tr>
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<td></td>
<td>0.206</td>
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<tr>
<td></td>
<td>0.46</td>
</tr>
<tr>
<td>Fiber</td>
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</tr>
<tr>
<td></td>
<td>0.37</td>
</tr>
<tr>
<td>Point Source</td>
<td>0.6 µm</td>
</tr>
</tbody>
</table>

An LED was used as a point source in the experimental system.

The experimental configuration was assembled to allow variation of azimuth and elevation of the point source and location of the fibers with respect to the focal plane of the GRIN lens. To achieve angular offset of the point source, the GRIN lens and fiber assembly were mounted on a two-axis rotational stage. The point source remains stationary while the rotational stages provide angular point source offset variation in both azimuth and elevation.

Experimental Results

Utilizing the experimental configuration described above, various measurements were made to verify the detection algorithms. The detection algorithms were also exercised with eyelet model simulation data.

Various azimuth and elevation angles were used to verify the single-point detection algorithm. The measured data and the single-point location prediction are listed in Table 2. The mean error is 1.2 degrees. The eyelet simulation was then exercised using the component characteristic values listed in Table 1. The azimuth and elevation
### TABLE 2

**SINGLE POINT LOCATION DATA**

<table>
<thead>
<tr>
<th>AZ = 10 DEGREES</th>
<th>LASER3</th>
<th>ERROR (ABSOLUTE)</th>
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</tr>
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<td>0.5</td>
</tr>
<tr>
<td>11</td>
<td>10.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**EL = 5 DEGREES**

<table>
<thead>
<tr>
<th>AZIMUTH</th>
<th>MEASUREMENT (DEGREES)</th>
<th>ERROR (ABSOLUTE)</th>
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</thead>
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<tr>
<td>3</td>
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<td>17</td>
<td>15.8</td>
<td>2.1</td>
</tr>
<tr>
<td>18</td>
<td>17.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**MEAN ERROR** 1.2
angles of Table 2 were used for input with detection algorithm results listed in Table 3. (Using the two-point detection algorithm, TDA, with a single point target i.e., case #1.)

The mean error of the eyelet model input was 0.71 degrees. The mean error between eyelet model inputs and measured data inputs into TDA was 0.88 degrees. It should be noted that the two-point detection algorithm contains LASER3 as the method to locate the point source. For single points, TDA and LASER3 predict the same object coordinates.

A second point source was placed in the field-of-view of the eyelet and the coordinates were determined utilizing the single-point detection algorithm contained in TDA. With both diodes at various azimuth and elevation positions, the two-point detection algorithm was exercised. The results are listed in Table 4. The TDA had the capability to resolve two-points located within 3.1 degrees. Table 5 lists the eyelet model predictions for the same coordinates. Table 6 provides an eyelet model to measured comparison.

The model predicted that two-point sources located at (20.0, 5.0) and (22.2, 3.4) respectively could be resolved, a 2.7 degree resolution capability. The actual hardware could not resolve the two sources located at (20.0, 5.0) and (22.2, 3.4). The model also predicted that two points located at (20.0, 5.0) and (22.2, 3.5) could not be resolved. The eyelet model and actual measured 2 point resolution differed by 0.4 degrees.
<table>
<thead>
<tr>
<th>AZ = 10 DEGREES</th>
<th>LASER3 ACTUAL DATA</th>
<th>TOA MODEL DATA</th>
<th>ERROR (ABSOLUTE) MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELEVATION =</td>
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<td>0.5</td>
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<td>1.5</td>
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<td>3.9</td>
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<td>0.9</td>
</tr>
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<td>4.3</td>
<td>3.7</td>
<td>0.3</td>
</tr>
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<td>5.0</td>
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<td>5.8</td>
<td>1.9</td>
</tr>
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<td>8.7</td>
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<td>1.7</td>
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<td>9.0</td>
<td>8.6</td>
<td>1.0</td>
</tr>
<tr>
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<td>9.3</td>
<td>10.0</td>
<td>0.3</td>
</tr>
<tr>
<td>10</td>
<td>9.5</td>
<td>10.1</td>
<td>0.5</td>
</tr>
<tr>
<td>11</td>
<td>10.0</td>
<td>10.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

| EL = 5 DEGREES |                        |                |                        |
| AZIMUTH =      |                        |                |                        |
| 3               | 1.6                   | 2.4            | 1.4                    | 0.6                     |
| 4               | 2.1                   | 3.9            | 1.9                    | 0.1                     |
| 5               | 4.1                   | 4.2            | 0.9                    | 0.8                     |
| 6               | 4.6                   | 4.5            | 1.4                    | 1.5                     |
| 7               | 5.2                   | 6.2            | 1.8                    | 0.8                     |
| 8               | 6.3                   | 6.9            | 1.7                    | 1.1                     |
| 9               | 10.4                  | 10.1           | 1.4                    | 1.1                     |
| 10              | 11.0                  | 10.9           | 1.0                    | 0.9                     |
| 11              | 11.7                  | 11.5           | 0.7                    | 0.5                     |
| 12              | 12.0                  | 11.5           | 0.0                    | 0.5                     |
| 13              | 12.0                  | 11.8           | 1.0                    | 1.2                     |
| 14              | 12.6                  | 12.7           | 1.4                    | 1.3                     |
| 15              | 13.0                  | 16.2           | 2.0                    | 1.2                     |
| 16              | 12.9                  | 16.9           | 3.1                    | 0.9                     |
| 17              | 15.8                  | 17.4           | 1.2                    | 0.4                     |
| 18              | 17.0                  | 17.6           | 1.0                    | 0.4                     |

**MEAN ERROR**

|                     | 1.2 | 0.71 |

**TABLE 3**

SINGLE POINT LOCATION EYELET MODEL TO MEASURED COMPARISON MEASUREMENTS AND ERROR (DEGREES)
<table>
<thead>
<tr>
<th>PRIMAR Y DI ODE AZ EL</th>
<th>SECONDAR Y DI ODE AZ EL</th>
<th>TDA PRIMARY AZ EL</th>
<th>TDA SECONDARY AZ EL</th>
<th>ERROR PRIMARY AZ EL</th>
<th>ERROR SECONDARY AZ EL</th>
<th>SEPARATION ANGLE</th>
</tr>
</thead>
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<tr>
<td>10.0 5.0</td>
<td>12.4 3.0</td>
<td>14.8 3.1</td>
<td>18.7 1.3</td>
<td>-4.8 2.0</td>
<td>-6.3 1.7</td>
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</tr>
<tr>
<td>10.0 5.0</td>
<td>15.7 -3.9</td>
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<td>-1.7 -0.8</td>
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</tr>
<tr>
<td>10.0 5.0</td>
<td>20.5 5.5</td>
<td>15.0 3.0</td>
<td>18.4 1.7</td>
<td>-5.0 -0.5</td>
<td>2.1 3.8</td>
<td>10.5</td>
</tr>
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<td>18.8 -6.6</td>
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<tr>
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<td>15.2 -5.1</td>
<td>0.6 -0.3</td>
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<td>11.1</td>
</tr>
<tr>
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<td>PRIMARY DIODE AZ</td>
<td>SECONDARY DIODE AZ</td>
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<td>21.9 -3.5</td>
<td>0.2 2.7</td>
<td>0.9 0.1</td>
<td>8.7</td>
</tr>
</tbody>
</table>

**MINIMUM RESOLUTION ANGLE**

| MEAN ERROR       | 1.0 1.5 1.3 1.2 2.7 |
## TABLE 6

### TWO-POINT EYELET MODEL AND MEASURED COMPARISON

<table>
<thead>
<tr>
<th>PRIMARY DIODE AZ</th>
<th>SECONDARY DIODE EL</th>
<th>TDA PRIMARY AZ</th>
<th>TDA PRIMARY EL</th>
<th>TDA SECONDARY AZ</th>
<th>TDA SECONDARY EL</th>
<th>ERROR PRIMARY AZ</th>
<th>ERROR PRIMARY EL</th>
<th>ERROR SECONDARY AZ</th>
<th>ERROR SECONDARY EL</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>5.0</td>
<td>12.4</td>
<td>3.0</td>
<td>14.8</td>
<td>3.1</td>
<td>18.7</td>
<td>1.3</td>
<td>10.9</td>
<td>4.5</td>
</tr>
<tr>
<td>10.0</td>
<td>5.0</td>
<td>15.7</td>
<td>-3.9</td>
<td>11.7</td>
<td>5.8</td>
<td>11.4</td>
<td>-4.4</td>
<td>11.7</td>
<td>2.4</td>
</tr>
<tr>
<td>10.0</td>
<td>5.0</td>
<td>20.5</td>
<td>5.5</td>
<td>15.0</td>
<td>3.0</td>
<td>18.4</td>
<td>3.0</td>
<td>12.8</td>
<td>4.4</td>
</tr>
<tr>
<td>20.0</td>
<td>5.0</td>
<td>18.8</td>
<td>-6.6</td>
<td>20.6</td>
<td>5.3</td>
<td>19.5</td>
<td>-6.8</td>
<td>20.2</td>
<td>4.9</td>
</tr>
<tr>
<td>20.0</td>
<td>5.0</td>
<td>15.9</td>
<td>-5.3</td>
<td>20.6</td>
<td>5.3</td>
<td>15.2</td>
<td>-5.1</td>
<td>19.3</td>
<td>4.5</td>
</tr>
<tr>
<td>20.0</td>
<td>5.0</td>
<td>22.2</td>
<td>2.4</td>
<td>--COULD NOT RESOLVE--</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>5.0</td>
<td>16.0</td>
<td>-5.4</td>
<td>20.6</td>
<td>5.3</td>
<td>15.2</td>
<td>-5.1</td>
<td>19.0</td>
<td>2.6</td>
</tr>
<tr>
<td>20.0</td>
<td>5.0</td>
<td>14.2</td>
<td>-2.9</td>
<td>20.5</td>
<td>5.2</td>
<td>15.0</td>
<td>-2.2</td>
<td>17.8</td>
<td>1.6</td>
</tr>
<tr>
<td>20.0</td>
<td>5.0</td>
<td>21.0</td>
<td>-3.6</td>
<td>20.5</td>
<td>5.3</td>
<td>21.3</td>
<td>1.7</td>
<td>19.8</td>
<td>2.3</td>
</tr>
</tbody>
</table>

**MEAN**

<table>
<thead>
<tr>
<th>PRIMARY ERROR</th>
<th>SECONDARY ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.9</td>
</tr>
<tr>
<td>1.4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

*Model values resolved these two points
Model predicted two points located at (20.0, 5.0) and (22.2, 3.5)
Could not be resolved
The major causes of error in the experimental results can be attributed to:

1. Inaccurate centroid measurement.
2. Nonsymmetrical modal excitation in fibers.
3. Use of a linear excitation form factor.

When the experimental hardware is used, the object space coordinates of each fiber must be determined. To determine the object space centroid in azimuth and elevation the position of the source is changed with respect to the GRIN lens. The location that produces the highest intensity output for a fiber is used as the object space centroid location for that fiber. Inaccuracies in this measurement account for the majority of errors in the experimental results.

The intensity scale factors that are used in LASER3 assume uniform modal excitation in the fibers and does not account for the variation with angle that exists.

A cause of error in the eyelet model is the arbitrary angular extinction of propagation in the fiber. The fiber model assumes all rays with an angle less than the external critical angle propagate thru the fiber and does not take into account the losses due to the incidence angle.

This chapter discussed the experimental setup that was used to validate the computer simulation model and verify the detection algorithms. The detection algorithms were exercised with experimental data and simulation data, the results were compared to validate the
computer eyelet model. The mean error of detection was 1.4 degrees overall for the eyelet model. The detection algorithm locations were compared to actual measured locations and the mean error was 1.2 degrees. The resolution of the hardware system was 3.1 degrees and that of the eyelet model 2.7 degrees.
The purpose of this research was to develop a two-point detection algorithm for an eyelet of a multi-aperture optical system. The algorithm was based on allowing three fibers to be illuminated by a single-point source. A computer simulation model of a multi-aperture eyelet was developed and validated with experimental data producing an overall mean error of 1.4 degrees.

The eyelet model was used to develop a two-point detection algorithm. The two-point detection algorithm provided a 3.1 degree resolution capability. If the system had been focused the resolution would have been limited by the size of the fiber and the system resolution point detection algorithm gives a resolution improvement of 40 percent.

The computer eyelet model is composed of three sub-models, a source model, GRIN lens model, and a fiber model. The model combines geometrical and physical optics to predict the output of the eyelet under various conditions. Ray matrices are used to propagate the rays from the point source to the fiber plane and determine the size and position of the spot at the fiber plane. Radiometric principles are applied to the point source to determine the power collected by the GRIN lens. A diffraction integral is utilized to calculate the intensity of the spot at the fiber plane. The fiber plane may be located either at the focal plane of the GRIN lens or removed from the focal plane. The fiber
Energy transfer model is a simple area calculation to determine the amount of power coupled into the fiber. The eyelet model is a flexible model giving multi-aperture researches a valuable tool in developing various algorithms. The results between model and measured values compared favorably.

There are many areas of research in multi-aperture optical systems that could utilize the results of this research. The eyelet model developed could be modified to predict the performance of many eyelets instead of a single eyelet. The detection algorithms could be exercised with 32 inputs instead of 16 to provide a larger field-of-view system. This research demonstrated the ability of an inexpensive, easily assembled system to resolve two-points within 3.1 degrees. Further research can provide a simple, low cost system that could resolve and track targets.
APPENDIX A

POWER CALCULATION
Figure 14 shows the geometry of a collection aperture and emitting source. The power collected by an optical system with a circular aperture of radius, $r$, is given by (7):

$$ P = EA_p $$

where

- $P$ = Power in watts
- $E$ = Irradiance of the source (w/cm$^2$)
- $A_p$ = Projected area (cm$^2$)
The projected area is given by the area of the aperture projected through the angle between the normal to the aperture and the line of sight.

\[ A_p = ACOS\theta \]  \hspace{1cm} \text{(35)}

\[ A = \pi r^2 \]

\[ \theta = \text{Angle between normal and line of sight} \]

This projected area can also be expressed in terms of the elevation and azimuth angles. Referring to Figure 14 for nomenclature, \( COSE \) is given by

\[ COSE = \frac{\text{R}}{\text{B}} \]  \hspace{1cm} \text{(36)}

\[ COSE_{AZ} = \frac{\text{A}}{\text{B}} \]  \hspace{1cm} \text{(37)}

\[ COSE_{EL} = \frac{\text{R}}{\text{A}} \]  \hspace{1cm} \text{(38)}

rearranging equations 37 & 38:

\[ \text{B} = \frac{\text{A}}{COSE_{AZ}} \]  \hspace{1cm} \text{(39)}

\[ \text{R} = ACOSO_{EL} \]

substituting 39 into 36

\[ COSE = \frac{ACOSO_{EL}}{A/COSE_{AZ}} \]

or

\[ COSE = COSE_{EL}COSE_{AZ} \]  \hspace{1cm} \text{(40)}

So equation 34 becomes

\[ P = \text{EACOSO}_{AZ}COSE_{EL} \]
APPENDIX B

SOFTWARE LISTING FOR THE EYELET MODEL
*PROGRAM HEX*

**MAOSIM** (MULTIAPERATURE OPTICAL SYSTEM INTENSITY MODEL)

**INPUTS**

**PROGRAM**

**CALCULATES**

**FOR A SINGLE APERTURE**

**INPUT**

**SOURCE**

**POWER**

**PROPAGATED**

**SOURCE**

**DIVERGING LENS**

**SOURCE**

**WAVELENGTH**

**SOURCE**

**POWER**

**PROPAGATED**

**OUTPUT PLANE**

**FIBERS**

**COLLECTS**

**BASIC PARAMETERS**

**NEEDED**

**PERFORM**

**CALCULATIONS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

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**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**

**COLLECTS**

**BASIC PARAMETERS**
890 INPUT "WHAT IS SOURCE 10 LENS DISTANCE (mm)? ", SD
900 INPUT "WHAT IS DIAMETER OF LENS (mm)? ", PHI
910 IF ARFL=1 THEN GOTO 1290
920 REM ************************************************************************************
930 REM END OF USER INPUTS*******************************************************************
940 REM ************************************************************************************
950 REM DEFINE FIBER POSITIONS****************************************************************
960 FC(1,1)=0
970 FC(1,2)=0
980 FC(2,1)=.707*PHI
990 FC(2,2)=-(1/2)*PHI
1000 FC(3,1)=.707*PHI
1010 FC(3,2)=(1/2)*PHI
1020 FC(4,1)=0
1030 FC(4,2)=-PHI
1040 FC(5,1)=.707*PHI
1050 FC(5,2)=-(1/2)*PHI
1060 FC(6,1)=-.707*PHI
1070 FC(6,2)=(1/2)*PHI
1080 FC(7,1)=0
1090 FC(7,2)=PHI
1100 FC(8,1)=.707*PHI
1110 FC(8,2)=(3/2)*PHI
1120 FC(9,1)=2*PHI
1130 FC(9,2)=0
1140 FC(10,1)=2*PHI
1150 FC(10,2)=-PHI
1160 FC(11,1)=.707*PHI
1170 FC(11,2)=-(3/2)*PHI
1180 FC(12,1)=O
1190 FC(12,2)=-(3/2)*PHI
1200 FC(13,1)=O
1210 FC(13,2)=-(1/2)*PHI
1220 FC(14,1)=2*PHI
1230 FC(14,2)=0
1240 FC(15,1)=.707*PHI
1250 FC(15,2)=(1/2)*PHI
1260 FC(16,1)=O
1270 FC(16,2)=2*PHI
1280 GOTO 1120
1290 FOR I=1 TO 16
1300 PRINT "WHAT IS AZIMUTH FOR FIBER #1" ", I
1310 INPUT "ENTER VALUE IN DEGREES" , AZ(I)
1320 PRINT "WHAT IS ELEVATION FOR FIBER #1" ", I
1330 INPUT "ENTER VALUE IN DEGREES" , EL(I)
1340 NEXT I
1350 FOR I=1 TO 16
1360 AZR(I)=AZ(I)*PI/180
1370 ELR(I)=EL(I)*PI/180
1380 NEXT I
1390 PI=3.141593
1400 B=PI*f/P
1410 X=COS B
1420 Y=SIN B
1430 IF RD=O THEN O=O ELSE Q=(N - 1)/RD
1440 TSRAD=TS*PI/180
1450 AZR=AZ*PI/180
1460 ELR=EL*PI/180
1470 REM GEOMETRIC CALCULATIONS (EFL,S,WL CONVERSION TO mm)
1480 EFL=1/(N*A*Y)
1490 Z=EFL*L
1500 W=WL*M*001
1510 S=(X-Y)*Y/(N*A)/(Q*X)+(N*A*Y)
1520 NALR=ATN(NAL/(1-NAL(2)), 5)
1530 IF AZR=NAE THEN PRINT "AZ IS TO LARGE" :STOP
1540 IF ELR=NAE THEN PRINT "EL IS TO LARGE" :STOP
1550 REM RAY MATRIX CALCULATIONS****************************************************************
1560 FOR I=0 TO 19
1570 HIX(I)=-PHI/2+(1-PHI/10)
1580 DIX(I)=HIX(I)*X-(HIX(I)*O)/(N*A)*Y+(AZR*Y)/(N*A)
1590 HYI(I)=-HIX(I)*X-(HIX(I)*O)/(N*A)*Y+(ELR*X)/(N*A)
1600 EIE(I)=-HIX(I)*O*X-(HIX(I)*N*A*Y)+ELR*X
1610 EIA(I)=-HIX(I)*O*X-(HIX(I)*N*A*Y)+AR*X
1620 HFX(I)=OYT+HFX(I)
1630 HFX(I)=0YT+HFX(I)
1640 NEXT I
1650 REM THIS PART OF THE CODE DETERMINES WHICH PIXELS ARE ILLUMINATED
1660 IF RS=PI=EFL+NAL3/2+NT2 THEN RS=ABS(HFX(S)-HFX(O))
1670 RF=PHI/2
PRINT "HFY (5) IS ",HFY(5)
PRINT "HFX (5) IS ",HFX(5)
FOR I=1 TO 16
  SEP(I)=FCI(I)+RE(1)-HF(I)
END FOR
FOR I=1 TO 16
  IF SEP(I)=1 THEN PIX(I)=1 ELSE PIX(I)=0
END FOR
IF AREA(I)>1 THEN PRINT "ERROR AREA IS > 1"
NEXT I
REM 10 INTENSITY CALCULATIONS
REM 10 L=ABS(TD) COS(AR) COS(ELR)+(PHI/2)*2)/((SD)12*(SIN(TSRAD/2))12+.1)
REM 10 FOR L=0 THEN EPS=0 ELSE EPS=(1/2-L/1-EFL)
REM 10 FOR JL=1 TO 16
  IF PIX(JL)=0 THEN : GOTO 2170
  RH0(JL)=SEP(JL)
END FOR
REM 10 PRINT "PIXV IS ",PIXV(I)
NEXT I
INPUT "RUN AGAIN YES=1 NO=0 ":,RFL
IF RFL=0 THEN : GOTO 3020
INPUT "ENTER AZ IN DEGREES ":AZ
INPUT "ENTER EL IN DEGREES ":EL
GOTO 1~20
REM SUBROUTINE BESSEL FUNCTION CALCULATION
REM BESSEL FUNCTION ROUTINE
REM THIS ROUTINE USES THE RECURRANCE RELATION TECHNIQUE TO
REM COMPUTE THE BESSEL FUNCTION OF THE FIRST KIND FOR
REM W AND ORDER O.
REM INPUT:
REM W=THE ARGUMENT OF THE BESSEL FUNCTION
REM O=THE ORDER OF THE BESSEL FUNCTION
REM OUTPUT:
REM BJ=THE RESULTANT BESSEL FUNCTION IF
REM W>14 THEN : GOTO 2460
REM KF(0)= 1
FOR I=3 TO 11 STEP 2
  M=I-2
  L(I)=L(M)/(8*W)12*(2*I+3)12*(2*I+5)12
  NEXT I
NEXT K
GOTO 2730
FOR I=1 TO 6
  K=2*I-1
  BJ=BJ*K*KF(I)
NEXT I
NEXT I
NEXT I
NEXT I
NEXT I
NEXT I
NEXT I
NEXT I
2650 PO(I)=37.5/((8*W))**3
2660 QU=PO(I)/((8*W))**3
2670 FOR K=3 TO 11 STEP 2
2680 PO(K)=(2*K+1)/((8*W)**2*K+3)*PO(K-2) /((8*W)**2*(K+2)*(K+1))
2690 QU=QU+PO(K)**3
2700 NEXT K
2710 REM ** END OF BESSEL SUBROUTINE *
2720 REM ** AREA SUBROUTINE **
2730 REM SUBROUTINE TO CALCULATE THE AREA OF THE SPOT ON THE FIBER
2740 REM END OF AREA SUBROUTINE **
2750 IF HFX(S)=FC(I) AND HFY(S)=FC(I,2) THEN FLAG=1 ELSE FLAG=0
2760 IF RS<RF AND FLAR=1 THEN AREA(I)=1
2770 IF RS<RF AND FLAR=1 THEN AREA(I)=1
2780 REM ************ END OF BESSEL SUBROUTINE ************
2790 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2800 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2810 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2820 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2830 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2840 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2850 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2860 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2870 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2880 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2890 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2900 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2910 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2920 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2930 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2940 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2950 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2960 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2970 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2980 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
2990 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
3000 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
3010 IF RS>RF AND HFX(I)>RF THEN AREA(I)=PI*RS**2/2
3020 END
APPENDIX C

SOFTWARE LISTING FOR SINGLE-POINT DETECTION
55

60 CLS
70 CLEAR, 491521
80 SCREEN 0,0
90 KEY OFF
100 DEF SEG = 0
110 DIM TR(15)
120 SG = 256 * PEEK(&H511) + PEEK(&H510)
130 DIM DIO%(8)
140 FOR N = 0 TO 4: DIO%(N) = 0: NEXT N
150 FOR L = 1 TO AV
160 FOR S = 0 TO 15
170 CALL DASH16(MD%, DIO%(O), FLAG%)
180 NEXT S
190 MD% = 3
200 FOR S = 0 TO 15
210 CALL DASH16(MD%, DIO%(O), FLAG%)
220 NEXT S
230 FOR L = 1 TO AV
240 IF P(I) < 50 THEN PRINT "FIBER # " ; I ; " REMOVED FROM LIST"
250 IF P(I) < 50 THEN GOTO 820
260 P(I) = PEAK/P(I)
270 NEXT S
280 NEXT L
290 FOR I = 0 TO 15
300 IF P(I) > PEAK THEN PEAK = P(I)
310 NEXT I
320 FOR I = 0 TO 15
330 IF P(I) < 50 THEN PRINT "FIBER # " ; I ; " REMOVED FROM LIST"
340 IF P(I) < 50 THEN GOTO 820
350 'GENERATING THE NORMALIZATION MULTIPLIER
360 IF P(I) > PEAK/P(I)
370 PRINT 1, P(I)
380 NEXT I
390 PEAK = 0
400 NEXT L
410 PRINT: INPUT "NUMBER OF SAMPLES TO AVERAGE? ", NS
420 OPEN 1, "NORM"
430 CLOSE
440 PRINT: INPUT "NUMBER OF SAMPLES TO AVERAGE? ", NS
450 PRINT 1, "NORM", 2048
460 FOR I = 0 TO 15
470 IF P(I) > PEAK THEN PEAK = P(I)
480 NEXT I
490 OPEN 0, 1, "NORM", 2048
500 FOR I = 0 TO 15
510 IF P(I) < 50 THEN PRINT "FIBER # " ; I ; " REMOVED FROM LIST"
520 IF P(I) < 50 THEN GOTO 820
530 'GENERATING THE NORMALIZATION MULTIPLIER
540 IF P(I) > PEAK/P(I)
550 PRINT 1, P(I)
560 NEXT I
570 CLOSE
580 PRINT: INPUT "NUMBER OF SAMPLES TO AVERAGE? ", NS
590 PRINT 1, "NORM"
890 Dim AZ(16), DIM ND(16)
900 Dim EL(16)
910 For K = 0 To 15
920 Input #1, ND(K)
930 Input #2, AZ(K): Input #2, EL(K)
940 REM PRINT "normalization operator": ND(K)
950 Next K
960 Close
970 Input "DEFINE SATURATION LEVEL (~4000) = " SAT
980 GoSub 1960
990 Goto 1000
1000 "Taking data RETURN TO THIS POINT
1001 For L = 0 To 15
1002 AR(L) = 0
1003 Next L
1010 MD% = 3
1020 Dim DIO%() = 0
1030 For R = 0 To NS
1040 For Z = 0 To 15
1050 Dim DIO%(0). FLAG%
1060 AR(Z) = AR(Z) + DIO%(0)
1070 Next Z
1080 Next R
1090 For I = 0 To 15
1100 AR(I) = AR(I)/NS
1110 Next I
1111 Print AR(15)
1120 T = 0
1130 For I = 0 To 15
1140 If AR(I) > SAT Then AR(I) = 0
1150 'ZEROING OUT DEFECTIVE FIBERS
1160 If ND(I) = 0 Then AR(I) = 0
1170 If AR(I) > 1 Then T = T + 1
1180 REM PRINT "valid input data" : AR(I)
1190 Next I
1200 CLS
1210 Print " scan CHANNELS = ": T
1220 If T = 2 Then Goto 1270
1230 Print "NOT ENOUGH POINTS. YOU ARE EITHER TOO WEAK OR TOTALY SATURATED."
1240 Z$ = INKEY$ 
1250 If Z$ = "$ Then End
1260 If Z$ = "s" Then End
1270 Normalizing THE AR(I) SENSOR DATA
1280 For K = 0 To 15
1290 AR(K) = AR(K)*ND(K)
1300 REM PRINT "normalized data": AR(K)
1310 Next K
1320 'find the largest three locator in AR(K) array TOP3(0) is largest
1330 TOP3(0) = 0: TOP3(1) = 0: TOP3(2) = 0
1340 MAX = 0
1350 For J = 0 To 15
1360 TR(J) = AR(J)
1370 REM PRINT TR(J)
1380 Next J
1390 For V = 0 To 15
1400 If TR(V) > MAX Then TOP3(0) = V
1410 MAX = MAX + TR(TOP3(0))
1420 Next V
1430 TR(TOP3(0)) = 0
1440 MAX = 0
1450 For V = 0 To 15
1460 If TR(V) > MAX Then TOP3(1) = V
1470 MAX = MAX + TR(TOP3(1))
1480 Next V
1490 TR(TOP3(1)) = 0
1500 MAX = 0
1510 For V = 0 To 15
1520 If TR(V) > MAX Then TOP3(2) = V
1530 MAX = MAX + TR(TOP3(2))
1540 REM PRINT TR(V)
1550 Next V
1560 PRINT AR(TOP3(0)), AR(TOP3(1)), AR(TOP3(2))
1570 'normalize these three
1580 AR(TOP3(1)) = AR(TOP3(1))/AR(TOP3(0))
1590 AR(TOP3(2)) = AR(TOP3(2))/AR(TOP3(0))
1600 AR(TOP3(0)) = 1
1610 PRINT AR(TOP3(0)), AR(TOP3(1)), AR(TOP3(2))
1620 'algorithm for az/el of point source
1630 PO = TOP3(0): P1 = TOP3(1): P2 = TOP3(2)
1640 'differentials
1650 DELA = EL(P1) - EL(PO)
1660 DELB = EL(P2) - EL(PO)
1670 'azimuths = A2/el of point source
1680 DAZA = AZ(P1) - AZ(PO)
1690 DAZB = AZ(P2) - AZ(PO)
1700 Scaling OF POSITION ON POINT VECTORS (VERY SIMPLE SCALING)
1710 SCAL = AR(P2)/2
1720 Scaling = AR(P2)/2
1750 'FIND COORDINATE VALUES FOR EACH LEG
1760 AZA = AZ(P0) + SCAL(A)·DAZA
1770 AZB = AZ(P0) + SCAL(B)·DAZB
1780 ELA = EL(P0) + SCAL(A)·DELA
1790 ELB = EL(P0) - SCAL(B)·DEL B
1800 'FIND THE MEAN POSITION
1810 EL = (ELA + ELB) / 2
1820 AZ = (AZA + AZB) / 2
1830 'CLS
1840 PRINT "AZUMITH = " ; AZ
1850 PRINT "ELEVATION = " ; EL
1860 Z$ = INKEY$
1870 IF Z$ = "S" OR Z$ = "S" THEN GOTO 1950
1880 GOTO 1000
1890 END
1900 OPEN "I" 3, "CIR"
1910 DIM A(16), N(16)
1920 INPUT# 3, A(0), N(0)
1930 OPEN "I" 2, "CAL"
1940 FOR I = 0 TO 15
1950 INPUT# 2, AZ(I), EL(I)
1960 NEXT I
1970 CLOSE
1980 RETURN
APPENDIX D
SOFTWARE LISTING FOR TWO-POINT DETECTION
•• TWO POINT RESOLUTION ALGORITHM

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REMOV TO TAK THE INTENSITY VALUES FOR 16 FIBERS AND SORT ACCORDING TO INTENSITY FROM LOWEST TO HIGHEST.

IT THEN DETERMINES IF ONE OR TWO POINTS ARE ILLUMINATING THE FIBERS. IT WILL THEN CALCULATE THE POSITION OF THE POINT(S).

INPUT VARIABLES:
- $AZ(K) =$ AZIMUTH POSITION OF FIBER $K$ IN mm.
- $EL(K) =$ ELEVATION POSITION OF FIBER $K$ IN mm.
- $AR(K) =$ IRRADIANCE VALUE FOR FIBER $K$.
- $N_0 =$ INDEX OF REFRACTION OF GRIN LENS (ON-AXIS).
- $A =$ GRADIENT LENS QUADRATIC INDEX CONSTANT.

CALCULATED VALUES:
- $S(I) =$ INTENSITY VALUES ORDERED FROM LOW TO HIGH (0-15).
- $AZR(I) =$ AZIMUTH OF FIBER $I$ IN RADIANS.

DIMENSION STATEMENTS:
- $DIMA(16)$.
- $AR(16)$.
- $AZI(16)$.
- $EL(16)$.
- $F(16)$.
- $L(16)$.
- $TR(15)$.
- $IO(16)$.
- $ND(16)$.
- $A(16)$.
- $N(16)$.

INPUT STATEMENTS:
- WHAT IS FIBER DIAMETER (mm)? $FPHI$
- DO YOU WANT MODEL (ENTER 0) OR HARDWARE CENTER 1?

CALCULATE $S(I)$=

DIM $IO$.

DASHING:
- $MD%=0$
- $DIO%(0)=0$
- $MD%=3$
- CALL DASH16 ($MD%$, $DIO%(0)$, $FLAG%$)
- FOR $S=0$ TO 15: CALL DASH16 ($MD%$, $DIO%(S)$, $FLAG%$)
- $Z=$ INKEY$

PRINT "IF YOU HAVE NOT CHANGED FIBER OPTIC CONNECTORS THIS STEP IS NOT NECESSARY."

REMOV TO NORMALIZE?

IF $NORM$ = "N" THEN GOTO 1100

PRINT "Place diffusion screen in front of lens and light it up from the front. Adjust intensity until all channels fall below saturation (about 3500), but are above about 50.

PRINT "Channels below 50 will be deleted from the file because they are defective."

PRINT "THIS PROGRAM GIVES THE AZIMUTH AND ELEVATION (DEGREES) OF A SINGLE LASER SOURCE. DATA IS BASED ON COORDINATE INPUTS TO THE CALIB PROGRAM. RUN CALIB FIRST TO ESTABLISH COORDINATE SYSTEM."

PRINT "USE ONLY STEDY STATE LIGHT SOURCES. THE PROGRAM USES SIGNAL AVERAGING FOR NOISE REDUCTION. THIS PROGRAM RUNS ON HARO DISC C.

PRINT "IF YOU HAVE NOT CHANGED FIBER OPTIC CONNECTORS THIS STEP IS NOT NECESSARY."

PRINT "DO YOU WISH TO NORMALIZE? ", $NORM$

IF $NORM$ = "N" THEN GOTO 1100

PRINT "Place diffusion screen in front of lens and light it up from the front. Adjust intensity until all channels fall below saturation (about 3500), but are above about 50.

PRINT "When all 16 channels are within bounds, enter $S$.

PRINT "Channels below 50 will be deleted from the file because they are defective."

PRINT "I am busy doing the normalization. DO NOT TOUCH THE LIGHT!!"
MD% = 3 : D10%(O) = 0 : AV = 100
FOR L = 1 TO AV
FOR R = 0 TO 15
CALL DASH16 (MD%, D10%(O), FLAG%)
P(I) = P(I) + D10%(O)
NEXT R
NEXT L
IF P(I) > PEAK THEN PEAK = P(I)
NEXT L
OPEN "O", 1, "NORM", 2048
IF P(I) < 50 THEN P(I) = 0
FOR L = 0 TO 15
CAL¾
OASH16 (MD%, D10%(O), FLAG%)
P(L) = P(L) + D10%(O)
NEXT L
IF P(L) > PEAK THEN PEAK = P(L)
NEXT L
OPEN "I", 1, "NORM";
OPEN "I", 2, "CAL"
FOR K = 0 TO 15
INPUT #1, ND(K)
INPUT #2, AZR(K); INPUT #2, ELR(K)
PRINT "NORMALIZATION OPERATOR ", ND(K)
NEXT K
CLOSE
INPUT "NUMBER OF SAMPLES TO AVERAGE ?", NS
PRINT
OPEN "I", 1, "NORM"
OPEN "I", 2, "CAL"
FOR L = 0 TO 15
IF P(L) < 50 THEN PRINT "FIBER #", L, " REMOVED FROM LIST"
IF P(L) < 50 THEN GOTO 1070
REM GENERATING THE NORMALIZATION MULTIPLIER
P(L) = PEAK / P(L)
PRINT "NORMALIZED DATA "; AR(I)
NEXT I
CLOSE
INPUT "DEFINE SATURATION LEVEL (4000) = ", SAT
REM GET CALIBRATION AND SENSITIVITY DATA
GOSUB 4200
REM TAKING DATA RETURN TO THIS POINT
FOR L = 0 TO 15
AR(L) = 0
NEXT L
MD% = 3
D10%(O) = 0
FOR R = 0 TO 15
CALL DASH16 (MD%, D10%(O), FLAG%)
AR(R) = AR(R) + D10%(O)
NEXT R
NEXT R
IF AR(I) < SAT THEN AR(I) = 0
REM ZEROING OUT DEFECTIVE FIBERS
IF ND(I) = 0 THEN AR(I) = 0
IF AR(I) > 1 THEN I = I + 1
REM PRINT "VALID INPUT DATA ", AR(I)
NEXT I
CLS
PRINT "SCAN CHANNELS = ": T
IF T < 2 THEN GOTO 153.
PRINT "NOT ENOUGH POINTS. YOU ARE EITHER TOO WEAK OR SATURATED."
PRINT "HIT 5 TO QUIT"
IF Z$ = "5" THEN END
IF Z$ = "S" THEN END
GOTO 1290
REM NORMALIZING THE AR(I) SENSOR DATA
FOR I = 0 TO 15
AR(I) = AR(I) / ND(I)
NEXT I
REM PRINT "NORMALIZED DATA "; AR(I)
NEXT I
GOTO 1290
REM PROCEEDING FIBER DATA
FOR K = 0 TO 15
IF K = 0 THEN GOTO 153.
PRINT "WHAT IS AZ FOR FIBER # K"; K
INPUT "ENTER VALUE IN DEGREES ", AZ(K)
PRINT "WHAT IS EL FOR FIBER # K"; K
INPUT "ENTER VALUE IN DEGREES ", EL(K)
AZR(K) = AZ(K) * 3.1415 / 180
ELR(K) = EL(K) * 3.1415 / 180
AZ(K) = AZR(K) / NG%A
EL(K) = ELR(K) / NG%A
NEXT K
GOTO 1380
FOR K = 0 TO 15
PRINT "WHAT IS PIXEL VALUE FOR FIBER # K"; K
1770 INPUT "ENTER INTENSITY VALUE", AR(k)
1780 NEXT I
1790 FOR I = 0 TO 15
1800 P(I) = AR(I)
1810 NEXT I
1820 REM
1830 REM ARRANGE S(I) INTENSITY ARRAY FROM LOWEST VALUE TO HIGHEST
1840 REM S(0)=LOWEST INTENSITY VALUE : S(15)=HIGHEST INTENSITY VALUE
1850 REM FIBER * CORRESPONDING TO THAT INTENSITY VALUE
1860 FOR I = 0 TO 14
1880 NEXT J
1890 NEXT I
1900 REM NORMALIZE THE INTENSITY VALUES
1910 FOR I = 0 TO 15
1920 S(I)=S(I)/S(15)
1930 NEXT I
1940 NEXT I
1950 REM DETERMINE SEPARATION OF HIGHEST TWO INTENSITY FIBERS
1960 REM IF SEPARATION IS GREATER THEN A FIBER DIAMETER THEN TWO POINTS
1970 REM NOT ILLUMINATING THE GRIN LENS. TWO WELL SEPARATED POINTS.
1980 J=15
1990 FOR I = 0 TO 15
2000 SEP=|((EL(J)-EL(K))|^2+(AZ(F(J))-AZ(F(K)))|^2)^(1/2)
2010 IF SEP>FPHI THEN FLAG=1 ELSE FLAG=0
2020 REM
2030 REM DETERMINE IF TWO CLOSE POINTS ARE ILLUMINATING THE FIBER
2040 REM IF IFLAG IS 1 THEN FOUR FIBERS ARE ILLUMINATED AND 2 CLOSE
2050 REM NEED TO BE DETERMINED WHICH FIBERS ARE GROUPED TOGETHER
2060 REM IF S(15)-S(14)<.27 AND S(12)>3.1 THEN IFLAG=1 ELSE IFLAG=0
2070 REM
2080 REM DETERMINE IF FIBER IS ZERO THEN WE KNOW THAT WE DO NOT HAVE TWO WELL RESOLVED
2090 REM IF FLAG=0 THEN GOTO 2980
2100 IF SEP>FPHI THEN "GOTO 2980"
2110 IF SEP>F(1) AND SEP>F(13) THEN PRINT "SEPARATED POINTS BEING CALCULATED"
2120 REM THIS DETERMINES FOR EACH OF THE 6 ILLUMINATED FIBERS HOW MANY
2130 REM FIBERS NEAR F(I)
2140 REM FIBERS NEAR F(I)
2150 REM FOR I = 10 TO 15
2160 L(I)=0
2170 FOR J = 10 TO 15
2180 SEP=|((EL(J)-EL(I))|^2+(AZ(F(J))-AZ(F(I)))|^2)^(1/2)
2190 IF SEP>FPHI THEN EN=L(I-1)+1 ELSE L(I)=L(I)
2200 NEXT J
2210 NEXT I
2220 REM NOW L(I) IS SORTED FROM LOWEST TO HIGHEST. THIS DETERMINES WHICH OF
2230 REM THE 6 FIBERS HAVE THE FEWEST ILLUMINATED FIBERS NEAR THEM
2240 FOR I = 10 TO 14
2250 IF L(J)>L(I) THEN "GOTO 2350 ELSE GOTO 2360"
2270 NEXT J
2280 NEXT I
2290 REM
2300 REM NOW L(I) IS SORTED FROM LOWEST TO HIGHEST. THIS DETERMINES WHICH OF
2310 REM THE 6 FIBERS HAVE THE FEWEST ILLUMINATED FIBERS NEAR THEM
2320 FOR I = 10 TO 14
2330 IF L(J)>L(I) THEN "GOTO 2350 ELSE GOTO 2360"
2350 NEXT J
2360 REM IF THE FIBERS ARE GROUPED TOGETHER "GOTO 1230"
2370 REM IF THE FIBERS ARE NOT GROUPED "GOTO 2630"
2380 REM IF L(I)=L(I-1) THEN LF=1 ELSE LF=0
2390 REM IF LF=1 THEN "GOTO 1230"
2400 REM IF LF=0 THEN "GOTO 2630"
2410 REM FOR TWO POINTS WELL RESOLVED AND THE TWO GROUPS OF THREE FIBERS
2420 REM SEPERATE FIND WHICH FIBERS GO TOGETHER.
2430 FOR I = 11 TO 15
2440 SEP=|((EL(F(I))|2+(AZ(F(I))|2)^(1/2)
2450 IF SEP>FPHI THEN "GOTO 2570 ELSE JL=JL+1"
2480 REM
2490 REM FOR TWO POINTS WELL RESOLVED AND THE TWO GROUPS OF THREE FIBERS
2500 REM SEPERATE FIND WHICH FIBERS GO TOGETHER.
2510 FOR I = 11 TO 15
2520 SEP=|((EL(F(I))|2+(AZ(F(I))|2)^(1/2)
2530 IF SEP>FPHI THEN "GOTO 2570 ELSE JL=JL+1"
2560 REM END OF GROUPING FIBERS
2570 GOTO 2780
2580 REM OUT OF THE SIX FIBERS THAT ARE ALL GROUPED TOGETHER FOUR ARE NOT
2590 REM OUT OF THE TWO SEPERATE GROUPS OF FIBERS IN F(12) & F(13) AND F(14) & F(15)
2600 FOR I = 13 TO 15
2610 SEP=|((EL(F(I))|2+(AZ(F(I))|2)^(1/2)
2620 IF SEP>FPHI THEN "GOTO 2570 ELSE JL=JL+1"
2650 REM END OF GROUPING FIBERS
2660 REM
2670 SEP=|((EL(F(I))|2+(AZ(F(I))|2)^(1/2)
REM ** END OF TWO POINT SIX GROUPED FIBER CALCULATION **********
REM IF FOUR FIBERS ARE ILLUMINATED GOTO 1850
IF FLAG=1 THEN GOTO 3580
REM ********
REM IF FIVE FIBERS ARE ILLUMINATED GOTO 1590
REM IF S(J)<S(I) THEN GOTO 1590
REM "ONE POINT CALCULATION"
PO=S(15) : P1=S(11) : P2=S(13)
IF SEP>FPHI THEN S(J)=S(J)+1) =S(I) =T : F=F(J) : F(J)=F(I) : F(I) =F
NEXT J
REM NOW CALCULATE THE POSITION OF EACH POINT.
REM THAT TOUCHES ALL FOUR ILLUMINATED FIBERS (L(I)=0).
FOR I=1 TO 15 STEP 3
L(I)=0
FOR J=11 TO 15
SEP=((EL(F(11))=EL(F(11))+2)+(AZ(F(J))+AZ(F(11)))+12)+1.5
IF SEP>FPHI THEN GOTO 3390 ELSE GOTO 3270
S(I)=S(J) : S(I)=S(I) =S(I)=S(I)=F(J) : F(J)=F(I) : F(I)=F
NEXT J
NEXT I
PRINT "L F 1 S = " , L(I) , F(I)
REM FIND WHICH OF THE OTHER FOUR FIBERS BELONG TOGETHER IN GROUPS OF TWO.
JL=0
FOR I=13 TO 15
SEP=((EL(F(I)))-EL(F(I))+12)+(AZ(F(12)))+AZ(F(11)))+12)+1.5
IF SEP>FPHI THEN GOTO 3390 ELSE JL+1
IF JL=2 THEN STOP
NEXT I
NEXT J
REM ARRANGE FIBERS ACCORDING TO INTENSITY
NEXT J
REM FIND THE POSITION OF THE FIRST POINT
PO=S(11) : P1=S(11) : P2=S(12)
REM SECOND POINT
NEXT J
REM END FIVE FIBER CALCULATION ************
3570 REM**************************
3580 REM TWO CLOSE POINTS BEING CALCULATED FOR FOUR ILLUMINATED FIBERS
3590 FOR I=12 TO 15
3600 L(I)=0
3610 FOR J=1 TO 15 SEP=(EL(F(J))-EL(F(I)))+1.5 IF SEP>PHI THEN L(I)=L(I)+1 ELSE L(I)=L(I)
3630 NEXT J
3640 NEXT I
3650 FOR I=12 TO 14
3680 NEXT J
3690 NEXT I
3700 FOR I=0 TO 2
3710 FOR J=I TO 3 IF AR(J)<AR(I) THEN A=AR(I):AR(I)=AR(J):AR(J)=A F=F(J):F(J)=F(I):F(I)=F
3720 NEXT J
3730 NEXT I
3740 PO=AR(3)/AR(3):P1=AR(2)/AR(3):P2=AR(1)/AR(3) EL1=ELR(3):EL2=ELR(2):EL3=ELR(1) AZO=AZR(3):AZ1=AZR(2):AZ2=AZR(1) SCALA=P1/2 SCALB=P2/2 AZA=AZO+SCALA*DAZA AZB=AZO+SCALB*DAZB ELA=EL1+SCALA*DELA ELB=EL1+SCALB*DELB EL=(ELA+ELB)/2*180/P1 A2=(AZA+AZB)/2*180/P1 PRINT "PO=",PO," P1=",P1," P2=",P2 IF P0=P1 THEN PRINT "PO=",PO else PRINT "PO=",PO, " P1=",P1, " P2=",P2 RETURN
3750 IF AR(J)<AR(I) THEN A=AR(I):AR(I)=AR(J):AR(J)=A F=F(J):F(J)=F(I):F(I)=F
3760 FOR J=1 TO 3 IF AR(J)<AR(I) THEN A=AR(I):AR(I)=AR(J):AR(J)=A F=F(J):F(J)=F(I):F(I)=F
3770 NEXT J
3780 NEXT I
3790 PO=AR(3)/AR(3):P1=AR(2)/AR(3):P2=AR(1)/AR(3) EL1=ELR(3):EL2=ELR(2):EL3=ELR(1) AZO=AZR(3):AZ1=AZR(2):AZ2=AZR(1) SCALA=P1/2 SCALB=P2/2 AZA=AZO+SCALA*DAZA AZB=AZO+SCALB*DAZB ELA=EL1+SCALA*DELA ELB=EL1+SCALB*DELB EL=(ELA+ELB)/2*180/P1 A2=(AZA+AZB)/2*180/P1 PRINT "PO=",PO," P1=",P1," P2=",P2 IF P0=P1 THEN PRINT "PO=",PO else PRINT "PO=",PO, " P1=",P1, " P2=",P2 RETURN
3800 PRINT "DO YOU WANT TO CONTINUE WITH SAME COORDINANTS" INPUT "ENTER (1=YES O=NO) ",RAG IF RAG=1 THEN :GOTO 1750 ELSE END
3810 REM************************ END OF FOUR FIBER TWO POINT CALCULATION
3820 REM************************ SUBROUTINE TO DETERMINE LOCATION OF POINT(S)
3830 REM************************ LASER3 CODE
3840 DELA=EL1-EL2 DELB=EL2-EL3 EL=(ELA+ELB)/2*180/P1 AR=AR1/AR3:AR=AR1/AR3
3860 IF EL<180 THEN PRINT "ELO:EL1:EL2" PRINT "AZO:AZ1:AZ2",
3870 PRINT "ELO:EL1:EL2" PRINT "AZO:AZ1:AZ2"
3880 IF RAG=1 THEN :GOTO 1750 ELSE
3890 REM************************ START SUBROUTINE
3900 Z$=INKEY$ IF Z$="SM THEN GOTO 4250 IF Z$="s" THEN GOTO 4250 GOTO 1230
3910 OPEN "1", 3, "CIR" INPUT#3 AF(I),NF(I) OPEN "1", 2, "CAL" FOR I=0 TO 15 PRINT AF(I),AZR(I),ELR(I)
3930 NEXT I
3940 REM************************ END OF PROGRAM
3950 REM************************
REFERENCES


REFERENCES - Continued
