Design for Conversion of UCF's Supersonic Wind Tunnel to a Wind Tunnel with Hypersonic Capability

1987

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DESIGN FOR CONVERSION OF UCF'S SUPersonic WIND TUNNEL 
TO A WIND TUNNEL WITH HYPERSONIC CAPABILITY

BY
HUSEYIN S. SIVASLIGIL
B.S.M.E., University of Uludag, 1983

RESEARCH REPORT
Submitted in partial fulfillment of the requirements
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ABSTRACT

A supersonic wind tunnel, currently in existence at the University of Central Florida in Orlando, Florida, is examined to determine feasibility for conversion to a wind tunnel with hypersonic capability.

Computational data are presented, and the determination is made that the existing wind tunnel can be converted if alterations and additions are made. These changes include replacement of the existing two dimensional nozzle with a two dimensional nozzle specifically designed for operation at Mach number 7, and the addition of a vacuum tank and heater.

The use of an axisymmetric nozzle design is recommended for conversion to Mach numbers higher than 7. It alleviates the throat design problems caused by the combination of high stagnation enthalpies and large area ratios.
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NOMENCLATURE

A
Area, ft$^2$

A*$
Area at Mach 1

a
Speed of sound, ft/s

C$p$
Specific heat at constant pressure, ft$^2$/s$^2$-$R$

d
Pebble diameter, ft

e
Porosity of pebble bed

g
Acceleration of gravity, 4.165 x 10$^8$ft/hr$^2$

H
Enthalpy, Btu/lb

k
Thermal conductivity of air, Btu/ft-hr-$R$

M
Mach number

$m$
Flow rate, lb/ft-hr

P
Pressure, lb/ft$^2$

Q
Heat transfer rate, Btu

R
Gas constant, ft-lb/s-l-$R$

S
Area of pebble surface per unit volume of pebble bed, ft$^{-1}$

T
Temperature, $R$

t$p$
Pump time, s

t
Wind tunnel run time, s

U
Flow velocity, ft/s

V
Tank volume, ft$^3$
<table>
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<tr>
<td>( \rho )</td>
<td>Density, sl/ft(^3)</td>
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<tr>
<td>( \gamma )</td>
<td>Specific heat ratio</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Deflection angle, deg.</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Shock angle, deg.</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Diffuser half turning angle, deg.</td>
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INTRODUCTION

The University of Central Florida campus, located at Orlando, Florida, is equipped with a four inch by four inch supersonic blowdown wind tunnel. The information in this research report has been gathered for the purpose of determining if the existing wind tunnel can be converted to a hypersonic wind tunnel, and if so, to provide the data necessary to execute the conversion.

The basic theoretical concepts for the compressible flow are derived in the first section of the report. In the second part, individual components that comprise the tunnel are described. Finally, the design of the hypersonic nozzle, diffuser, vacuum tank, and storage heater are presented in the Appendix.
Development of Equations Defining Compressible Flow

It has been shown that air flow is basically governed by the five following laws:

a) Equation of state

The equation of state gives the relationship between pressure, density, and temperature at any point in a flow field.

\[ P = \rho RT \quad (1) \]  

where

\( P \) = pressure, pound/square feet  
\( \rho \) = density, slug/cubic feet  
\( T \) = temperature, \( ^\circ R \)  
\( R \) = gas constant, feet-pound/slug-\( ^\circ R \)

b) Continuity equation

For continuous flow in a circuit, mass rate of flow is constant.

\[ \rho_1 A_1 U_1 = \rho_2 A_2 U_2 \quad (2) \]  

where

\( A \) = the cross sectional area of the duct at any given station, square feet  
\( U \) = flow velocity, feet/second  
subscripts 1 and 2 describe two stations in the system
c) Energy equation

Under the assumption that no energy is added or lost between two stations, the energy equation can be written as:

\[ C_p T_1 + \frac{U_1^2}{2} = C_p T_2 + \frac{U_2^2}{2} = C_p T_0 \]  

where

\( C_p \) = the specific heat at constant pressure, square feet/square second \(^\circ\)R

Subscript 0 denotes stagnation conditions

d) Equation for isentropic flow

For a perfect gas, with constant specific heat, undergoing an isentropic process

\[ \frac{T_1}{(P_1)^{\gamma-1}/\gamma} = \frac{T_2}{(P_2)^{\gamma-1}/\gamma} \]  

\( \gamma \) = the ratio of specific heat at constant pressure \( C_p \) to specific heat at constant volume \( C_v \)

e) Momentum equation

Assuming a constant duct area with no friction, the momentum equation is expressed:

\[ P_1 + \rho_1 U_1^2 = P_2 + \rho_2 U_2^2 \]  

The following definitions are also needed for the development of the desired relations for compressible flow:
\[ U = aM \quad \text{(6)} \]
\[ a = \sqrt{\gamma RT} \quad \text{(7)} \]
\[ C_p = \frac{\gamma R}{\gamma - 1} \quad \text{(8)} \]

where

\[ a = \text{the speed of sound, feet/second} \]
\[ M = \text{the Mach number} \]
\[ R = \text{the gas constant} \]

From the energy equation (3) and the definitions (6) and (8) we obtain

\[ \frac{T_1}{T_2} = \frac{1 + \left[ \frac{(Y-1)}{2} \right] M_2^2}{1 + \left[ \frac{(Y-1)}{2} \right] M_1^2} \quad \text{(9)} \]

Combining equation (9) with the equation for isentropic flow (4) yields:

\[ \frac{P_1}{P_2} = \left( \frac{1 + \left[ \frac{(Y-1)}{2} \right] M_2^2}{1 + \left[ \frac{(Y-1)}{2} \right] M_1^2} \right)^{\frac{\gamma}{\gamma - 1}} \quad \text{(10)} \]

Combining equations (9) and (10) with the equation of state (1) yields:

\[ \frac{\rho_1}{\rho_2} = \left( \frac{1 + \left[ \frac{(Y-1)}{2} \right] M_2^2}{1 + \left[ \frac{(Y-1)}{2} \right] M_1^2} \right)^{\frac{1}{(Y-1)}} \quad \text{(11)} \]

Adding in the continuity equation (2),
From the preceding equations and the knowledge that stagnation conditions will exist at \( M = 0 \), the following isentropic flow relations are obtained:

\[
\frac{A_1}{A_2} = \frac{M_2}{M_1} \left( \frac{1+[(\gamma-1)/2]M_2^2}{1+[(\gamma-1)/2]M_1^2} \right)^{3/2} \quad [Y/(Y-1)]
\]

Using an area at \( M = 1 \) as a reference, we obtain the following from equation (12)

\[
\frac{P}{P_0} = \left( 1 + \frac{\gamma-1}{2} \frac{M_2^2}{M_1^2} \right)^{1/2} \quad [-\gamma/(\gamma-1)]
\]

\[
\frac{\rho}{\rho_0} = \left( 1 + \frac{\gamma-1}{2} \frac{M_2^2}{M_1^2} \right)^{1/2} \quad [-1/(\gamma-1)]
\]

\[
\frac{T}{T_0} = \left( 1 + \frac{\gamma-1}{2} \frac{M_2^2}{M_1^2} \right)^{-1} \quad [1/(-1)]
\]

These equations are tabulated in many gas dynamics textbooks such as those of John and Zucrow. If the flow contains a normal shock, an entropy change occurs across the shock. Consequently, the preceding isentropic flow equations are not valid. The equation of state, the energy equation, and the momentum equation are used in the derivation of normal shock equations.
From the energy equation, we obtain the same result which we obtained for isentropic relations between temperature and Mach number (9).

The combination of equations (1) and (5) gives

\[
\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{17}
\]

The combination of the continuity equation with (9) and (17) gives

\[
M_2^2 = \frac{[2/(\gamma - 1)] + M_1^2}{[2 M_1^2/(\gamma - 1)] - 1} \tag{18}
\]

The stagnation pressure downstream of a normal shock is less than that upstream of the shock. A relation for the total pressure downstream of the normal shock is obtained as follows:

\[
\frac{P_{02}}{P_{01}} = \frac{(P_1/P_{01})(P_2/P_1)}{(P_2/P_{02})} = \left(\frac{\gamma + 1}{2\gamma M_1^2 - (\gamma - 1)}\right)^{1/\gamma - 1} \left(\frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}\right)^{\gamma/\gamma - 1} \tag{19}
\]

Real Gas Effects

The equations and relationships derived in the preceding
section are based on the assumption of an invariable specific heat ratio $\gamma$. At lower temperatures this holds true. At higher temperatures, a vibrational degree of freedom is added to the translational and rotational degrees of freedom. A significant portion of the heat goes into excitation of the vibrational degree of freedom. This results in values of $\gamma$ that vary with temperature.

The following equation for specific heat ratio, including the effects of molecular vibration is obtained:

$$\gamma = 1 + \frac{\gamma_p - 1}{\theta^2} \frac{e^{\theta/T}}{(e^{\theta/T} - 1)^2}$$

where

$\theta = a$ constant, $5500^\circ R$ for air
$\gamma =$ specific heat ratio
$\gamma_p = $ perfect gas value of specific heat ratio, $1.4$ for air
$T =$ temperature, $^\circ R$

This equation may be used for engineering purposes up to $5000^\circ R$.

**Actual Flow in the Tunnel**

There is a friction force developed between the air and the walls as air flows through a tunnel. This causes a loss in velocity and momentum of air in a layer next to the wall called the "boundary layer." The boundary layer thickness
and the total loss of momentum increase with increasing distance from the first throat of the nozzle, and become quite important in the test section.

Viscous effects between the first throat and the test section of a nozzle are not usually of much importance during the steady-state operation of the tunnel. The growth of the boundary layer thickness with distance from the first throat is fairly predictable, and can be accounted for in nozzle design so that the desired flow outside the boundary layer can be achieved.4

During the transient process in which the tunnel is started, viscous effects are extremely important. These effects are so important that compression ratios required to start most high Mach number tunnels now in operation are usually at least one hundred percent greater than the normal shock pressure ratio $P_01/P_02$. In effect, losses due to viscous effects during the starting process are usually at least equal to losses through the normal shock.

The range of actual compression ratios for starting and running, as obtained from a number of wind tunnels, is shown in Figure 1. The starting compression ratios shown in this figure may be reduced by using an adjustable nozzle and adjusting it to a higher Mach number after the tunnel has started.5
Figure 1. Pressure Ratios for Starting and Running
[Reference 10]
Condensation

The amount of moisture that can be held by a cubic foot of air increases as the temperature rises, but is independent of pressure. Moist atmospheric air normally cools as it expands isentropically through a wind tunnel and may become supercooled (cooled to a temperature below dew point temperature). Moisture will then condense out and, if the moisture content is high enough, will appear in the form of a dense fog in the tunnel.

Changes, such as in the local Mach number, resulting from condensation may cause wind tunnel data to be erroneous. The flow changes are, naturally, a function of the amount of heat released through condensation. These are expressed as:

\[
\frac{dM^2}{M^2} = \frac{1+\gamma M^2}{(1-M^2)} \left( \frac{dQ}{H} - \frac{dA}{A} \right) \tag{21}
\]

\[
\frac{dP}{P} = -\frac{\gamma M^2}{(1-M^2)} \left( \frac{dQ}{H} - \frac{dA}{A} \right) \tag{22}
\]

where

\[ M = \text{Mach number} \]
\[ \gamma = \text{specific heat ratio} \]
\[ dQ = \text{heat added through condensation, Btu/pound} \]
\[ H = \text{enthalpy per unit mass, Btu/pound} \]
\[ A = \text{duct area, square feet} \]
\[ P = \text{static pressure, pound/square feet} \]

These equations indicate that at high speeds, the Mach
number decreases, and the pressure increases with condensation. It should be noted that the presence of water vapor without condensation is of no significance as far as temperature ratio, pressure ratio, and Mach number determined from isentropic relations are concerned.\(^7\)

There are two approaches to solving the problem of condensation in hypersonic tunnels. The first is to heat the air so that upon expansion to desired Mach number its static temperature will be above the temperature corresponding to fifty-five degrees Fahrenheit of supercooling. This approach turns out to be impractical because of high temperature requirements.

The second approach is to dry the air, and this is the common procedure. Equipment for drying the air to dewpoints around negative fifty degrees Fahrenheit is commercially available and fairly inexpensive.\(^8\)

**Liquefaction**

The components of air liquefy when proper temperature and pressure conditions are met, in a manner resembling the condensation of moisture in an airstream cooled below its saturation point. The Clasius-Clapeyron equation matches these data fairly closely:

\[
\log_{10} P = \frac{A}{T} + B \quad (23) \quad \text{where}
\]

\[
P = \text{condensation pressure in atmospheres}
\]

\[
T = \text{condensation temperature in degrees Rankine}
\]
A and B are experimentally determined constants.

Experimental data indicate that \( A = 648 \) and \( B = 445 \) over most of the normal range of hypersonic wind tunnel pressures. The stagnation pressure and temperature can easily be calculated from the isentropic flow relation and the test section conditions. For air (\( \gamma = 1.4 \)) the required stagnation temperature in the test section is given in terms of the stagnation pressure and Mach number by:

\[
\frac{A}{T_0} \left(1 + \frac{M^2}{5}\right) = B + 3.5 \log_{10} \left(1 + \frac{M^2}{5}\right) - \log_{10} P_0 \tag{24}
\]

or the curves of Figure 2.

Liquefaction problems may start around \( M = 4.0 \) for high-pressure air expanded from room temperature, although somewhat higher Mach numbers may be used without difficulty if the stagnation pressure is lowered. Figure 3 shows a comparison of experimentally determined liquefaction temperatures in wind tunnels with static saturation temperatures through a range of pressures.\(^9\)
Figure 2. Equilibrium Condensation Curves for Air  [Reference 14]
LIQUEFACTION BEGINS

STATIC SATURATION

Figure 3. Experimental Data for Beginning of Liquefaction
[Reference 6]
COMPONENTS OF THE TUNNEL

The hypersonic wind tunnel consists of several interdependent sections. The primary sections are the compressor, air storage tank and heater, settling chamber, nozzle, test section, diffuser, and vacuum tank. These are arranged and connected in a specific order to maximize efficiency and promote optimum operating conditions.

Additional elements include aftercoolers, an oil filter, an air drier, back pressure valves, a wide angle diffuser, and a silencer.

The function and a description of each of these parts are presented in the following section of this report.
Figure 4. Schematic Diagram of Hypersonic Wind Tunnel
Compressor

There are many possibilities to consider when selecting the type of compressor to use for pumping up the storage tanks. The most frequently chosen type has been the piston compressor, selected for economy and commercial availability in many sizes. Piston compressors are available with one stage of compression for providing up to about 150 psia of discharge pressure, with two stages of compression for providing up to about 500 psia of discharge pressure, and with a third stage of compression for providing still higher pressure.

Once the minimum allowable operating pressure has been calculated for the Mach number at which the tunnel is to be operated, the designer can determine if a single-stage or double-stage compressor is needed. For the pressure control system to work satisfactorily, the air storage pressure, and consequently, the discharge pressure of the compressor, must naturally be greater than the maximum tunnel operating pressure. As the Mach number increases, the required operating pressure also increases, but the nozzle throat area decreases at a more rapid rate resulting in lowest mass flow requirements for the highest Mach numbers. 10

The temperature of the air is increased by compression. Therefore, cooling water is normally required to keep the temperature of the working parts of any large compressor at
an acceptably low value. In compressors with more than one stage, cooling water is also used between stages to take away the heat added by one stage before the air enters the next stage of compression. This is known as intercooling. Special controls are also provided to turn off the compressor if the cooling water stops flowing, if the lubricating oil level gets too low, or if the discharge pressure becomes too high. Additionally, it is generally desirable to provide controls which can shut the compressor down when its storage tank reaches its design pressure, and start the compressor again when the tank pressure falls below some limit.

One possible safety hazard is if small amounts of oil occur in high pressure circuits. This has resulted in the occurrence of several severe air-oil explosions in wind tunnel systems. For this reason, the wind tunnel design must be made to minimize the amount of oil which enters the high pressure system. The first step in doing this can be made at the compressor section. The use of carbon or teflon compression cylinder piston rings, which can be used instead of steel at a nominally higher cost, and which do not require oil lubrication, eliminates the possibility that some oil will get into the high pressure circuit from the piston rings. The probability of such an occurrence is high with steel rings, however, a disadvantage of this substitution is that carbon and teflon rings must be replaced more frequently.11

Compressor ratings are given by a specific number of cubic feet of sea level air per minute. The time to pump a
tank from an initial pressure $p_i$ to a final pressure $p_f$ may be found from the equation:

$$t_p = \frac{V}{14.7Q} \left( p_f - p_i \right) \quad (25)$$

where

- $t_p =$ pump time, minutes
- $Q =$ compressor rating, cfm at sea level
- $p_f =$ final pressure, psia
- $p_i =$ initial pressure, psia
- $V =$ volume, cubic feet

In practical application, $p_f$ corresponds to the run start pressure and $p_i$ corresponds to the run end pressure.

Due to the fact that the present University of Central Florida compressors are adequate for this design purpose, and in the interest of eliminating unnecessary additional cost, the existing compressors are retained for this design.

**Aftercoolers**

Removal of heat from air leaving the compression chamber is usually accomplished by an aftercooler. An aftercooler is a rather simple device whereby the hot air from the compressor is allowed to flow at a low velocity through or over water cooled tubes or pipes. A typical aftercooler unit consists of a straight section of large pipe about twenty-five to fifty diameters long. Several small pipes pass through the inside. These are manifolded together at each end and provide an air flow passage. The cooling
Figure 5. Maximum Remaining Moisture After Compression and Cooling
[Reference 10]
water for this unit generally comes from the same circuit that supplies water to the compressor.

The cooling water enters at the air-exit end of the large pipe. It then flows through the pipe, over the small pipes that carry the air, and exits at the air-inlet end.

There are several purposes achieved by cooling the air immediately after it leaves the compressors. One of these purposes is to reduce the temperature to a value where the oil filter and the air drier can be effective. It also reduces the temperature of piping, valves, and other hardware between the compressor and the air storage tank until the danger of burns to personnel is limited. In addition, it eases the requirements on valves to where regular commercial valves may be used rather than high temperature valves. It reduces the volumetric flow rate and therefore the size of oil filters and air driers between the compressor and the storage tank may be decreased. Finally, it reduces the moisture content of the air. 12

**Oil Filter**

It is clear that the purpose of keeping oil with which the air comes in contact out of the system necessitates the use of an oil filter. The most important and obvious reason for keeping oil out of the compressed air systems is the danger of air-oil explosions. Other reasons include air passing through the air drier rapidly reducing its effectiveness and oil collecting on the windows of the nozzle test.
Figure 6. Oil Filter  [Reference 10]
section and causing deterioration in the quality of photographs. Figure 6 shows a sketch of an oil filter of a frequently used type. In a filter such as this, the air is forced to pass over a bed of desiccant in granule form. The granular form of the desiccant provides a large surface area for oil vapor to condense on.

A commercial oil filter is generally chosen over a homemade one for reasons of economy when considering design time. The desiccant in the filter section must be removed periodically and cleaned with an oil solvent or replaced, otherwise it will lose effectiveness when the granular surface becomes covered with oil. The amount of oil leaving the compressor will determine the frequency for cleaning or replacing the desiccant granules.

**Air Drier**

Blowdown wind tunnel air driers are usually "high-pressure" driers. They operate at some point between the maximum air storage tank pressure and the maximum compressor discharge pressure. The drying method most frequently used is the adsorption method. Moisture is collected in condensed form on the surfaces of a desiccant in this method. The desiccant may be silica gel, activated alumina, or zirconia. They are in granular form with an extremely porous structure. Capillary action draws moisture condensing on the outer surfaces of the granules down into the pores. The granules are heated to temperatures about one hundred degrees Fahrenheit above
the boiling temperature of water which removes the trapped moisture. This cycle is known as "reactivation." 

### Back Pressure Valves

A back-pressure valve is a necessary component of the air drier. Since air driers are usually operated at high pressure and air leaving the drier is discharged into the air storage tank, which will always be below design pressure when the drier is in use, a back-pressure valve is required. This valve maintains a high pressure in the drier. It is located between the drier and the air storage tank and automatically adjusts to maintain a specified pressure in the drier regardless of the pressure in the air storage tank. The valve must be compatible in size with the compressor capacity and pressure. This component is usually obtained commercially and is not a part of the design of the tunnel.

### Air Storage Tank

The capacity of the air storage tank is primarily dependent upon mass flows during a wind tunnel run and the desired run frequency. This hypersonic design will utilize the present storage tank of the University of Central Florida's existing supersonic tunnel, with a volume of 329 cubic feet. Higher pressures require smaller but somewhat stronger tanks. High pressure offers a margin of safety on compression ratios for tunnel startup, has advantages concerning air
drying, and makes possible later tunnel changes for higher Mach numbers.

Storage tanks in smaller sizes (400 to 4000 cubic feet) are usually cylindrical, and can be mounted either vertically or horizontally depending upon available space. The storage tank for this design is currently in existence, and is a cylindrical tank mounted horizontally outside of the building.

As the air temperature in the tank drops, heat is transferred from the walls of the tank to the air. Therefore the expansion of the air in the tanks is not an adiabatic process. Rather, it is a polytropic expansion process with a value of n between one (for isothermal) and one point four (for adiabatic) in the equation:

\[
\frac{T_i}{T_f} = \left( \frac{P_i}{P_f} \right)^{(n-1)/n}
\]

(26) where

T = temperature, °R
P = pressure, pound/square feet
i = initial conditions in tank
f = final conditions in tank

**Storage Heater**

Storage heaters are generally used in pressure-vacuum type hypersonic tunnels. The most common type, and the type used in this design, is the "pebble-bed" type. The pebble-
Figure 7. Pebble-bed Heater  [Reference 10]
bed heater stores thermal energy in refractory pebbles at a relatively slow rate by electrical heating or by a combustion process. The energy is then transferred from the pebbles to the air at a much higher rate during the relatively short period of wind tunnel operation.

An electrically heated pebble-bed heater, as shown in Figure 7, consists mainly of a cylindrical pressure vessel lined with insulation, twelve "Globar" (silicon carbide) heating elements, a tubular liner, a grate at the bottom of the tubular liner, and a bed of aluminum oxide pebbles retained by the tubular liner and resting on the grate at the bottom of the liner. During operation, electric power is supplied to the heating elements. Heat is transferred from the heating elements to the surrounding refractory bricks primarily by radiation, and from this refractory to the pebble-bedliner and the pebbles in it by radiation and conduction. Heating continues until the pebbles are completely heat soaked. For running, cool air is introduced at the bottom of the heater and flows upward through the pebble bed, absorbing heat, and then goes to the wind tunnel nozzle.

Pressure drop is the most important problem in the design of pebble-bed heaters. The pebbles in existing heaters of this order are restrained only by gravity. If air flow in the heater causes a pressure drop across the bed enough to lift the pebbles, severe damage may be done to the heater and perhaps to personnel as well. Carmen has given the pressure drop per unit bed length as:
\[
\frac{dp}{dx} = 2.4 \left( \frac{\rho U}{\mu S} \right)^{0.1} \frac{1-e}{e} \frac{\dot{m}^2}{ga^2d} \tag{27}
\]

where

\( a \) = flow area at any cross section of pebble bed, \( eA \), square feet

\( e \) = porosity of pebble bed, ratio of void volume to total volume, dimensionless

\( A \) = cross-sectional area of empty container at subject station, square feet

\( g \) = acceleration of gravity, \( 4.165 \times 10^8 \) feet/hour

\( \rho \) = air density, pounds/cubed feet

\( d \) = pebble diameter, feet

\( \dot{m} \) = flow rate, pounds/hour

\( U \) = air velocity in heater bed based on empty container, feet/hour

\( \mu \) = air viscosity in bed, pound/feet-hours

\( S \) = area of pebble surface per unit volume of pebble bed, feet\(^{-1}\)

The pressure drop through a pebble bed increases very rapidly with decreasing porosity \( e \). Therefore, any heater design should include an accurate experimental determination of \( e \). Values of \( e \) are likely to be near 0.33 in pebble-bed heaters with large bed to pebble diameter ratios. This has been established through experimentation. The pressure drop is highest in the upper portion of the bed, even for cold operation, because the pressure is dropping and the
density decreasing as air progresses through the pebble bed. For this reason, pressure drop calculations are critical in the upper portion of the pebble bed.\textsuperscript{17} The expression:

\[ h_C = 0.56 \frac{k}{d} \left( \frac{\rho U'd}{\mu} \right)^{0.6} \left( \frac{\mu C_p}{k} \right)^{0.33} \text{ Btu/ft}^2\text{-hr}^{-\circ}\text{R} \] (28)

gives the convection heat transfer rate between spheres and a moving fluid, where

- \( k \) = thermal conductivity of air, Btu/ft-hr-\( ^\circ \text{R} \)
- \( U' \) = apparent velocity in bed based on the flow area, \( eA \), ft/hr
- \( C_p \) = specific heat of air, Btu/lb-\( ^\circ \text{R} \)

Pebble-bed heater design is given in the Appendix.

**Wide Angle Diffuser**

When a large-area, low velocity section is provided immediately upstream of the nozzle, so that a large contraction of the flow is provided as it enters the nozzle, the uniformity of flow in a wind tunnel is greatly improved. This large-area section is called the "settling chamber." It is economically desirable to use the smallest practical pipe size to deliver air from the storage tank to the tunnel itself, and this small pipe size corresponds to high flow velocities. The device in which high-velocity flow, such as that in the small piping, is decelerated to a low-velocity flow, such as that is the settling chamber, is the diffuser.
Wide angle diffusers are typically used in blowdown tunnel designs for the transition from the pipe to the settling chamber. Angles between opposite walls range from forty-five degrees to ninety degrees. Due to the highly turbulent and non-uniform flow which usually exists at the diffuser inlet, many different devices are used to spread the flow from the inlet pipe to the settling chamber. There are many possible spreader designs.\(^{18}\)

A recommended design contains a perforated cone facing upstream from the settling chamber to allow ample perforations for a low-pressure-drop design. The perforations should be designed with a flow area large enough to keep the average velocity through the perforations below Mach number 0.5 in the most severe operating conditions. This design has been found to spread the flow satisfactorily.

**Settling Chamber**

Usually a cylindrical shell, one diameter or more in length, the settling chamber accepts air from the wide-angle diffuser, provides space for settling to obtain uniform flow and screens for promoting uniformity of flow and for reduction of turbulence in the air stream, and then exhausts into the subsonic portion (inlet) of the nozzle.

The static pressure in the settling chamber is normally much lower than that in the storage tanks and in the piping to the pressure regulator. The static pressure in the settling chamber is, however, higher than any downstream
point in the tunnel. For reasons of economy, the settling chamber and downstream sections of the tunnel are usually designed for their normal operating pressures instead of the tank pressure. Due to the possibility of malfunction of a tunnel component resulting in excessive pressures in the settling chamber, it normally contains a blowoff stack extending through the roof of the building. This blowoff stack is equipped with a commercially obtained "safety disc" or blowout diaphragm. A blowoff stack and safety disc with a flow area comparable to that of the pipe entering the wide-angle diffuser will usually be adequate to accommodate the most severe conditions expected during normal operation.

If the flow spreader in the wide-angle diffuser is properly designed, air will enter the settling chamber with a fairly uniform distribution. Wind tunnel engineers usually agree that settling chambers should be designed for flow velocities not in excess of eighty to one hundred feet per second. The lowest velocity should be no less than ten feet per second. 19

Nozzle

A two dimensional, flexible plate nozzle was used in the original design of the supersonic wind tunnel located in the Engineering Building of the University of Central Florida. Our design replaces the existing nozzle with a similar two dimensional flexible plate nozzle capable of operating at low hypersonic speeds. At high hypersonic speeds, area ratios
become extremely large necessitating the use of axially symmetric nozzles. This report outlines the design of a two dimensional nozzle specifically engineered for operation at Mach number seven.

This is a contoured nozzle, carefully designed to have a short length and yet produce uniform parallel flow at the exit. Stability of the boundary layer in the contoured nozzle was maximized by designing the nozzle to have as short a length as practical, so that large favorable pressure gradients will exist throughout its length.

The region to region method of characteristics was used to calculate flow inside the nozzle. The method of characteristics is a method of defining the properties of supersonic and hypersonic flows in the presence of varying boundaries, such as in a wind tunnel nozzle. The method, as it is normally used, requires constant entropy flow. Therefore, it cannot be used in a flow field having shock waves. In general, this restricts the method of characteristics to the case of a continually expanding flow because weak compression waves have to be widely separated in order to avoid the formation of a shock and subsequent entropy changes. Here the method of characteristics is used for defining internal contours of the hypersonic nozzle in the region between the first throat and the test section. Calculation of the inviscid flow began with an assumed centerline Mach number distribution. The flow field was calculated outward from the centerline using the characteristics method with region to region extrapolation.
These calculations are described in the Appendix, section A, and yield a nozzle length of 16.16 inches and throat area of 0.15364 square inches. The side walls are parallel, and top and bottom walls are contoured based on our calculations. In the throat area, the Mach number is equal to one and gradually increases until we attain Mach number seven at the entrance to the test section. The design also attempts to minimize incoming flow disturbances by providing smooth nozzle wall surfaces. The entrance of the nozzle requires a porous stainless steel filter designed to effectively erase the previous history of the flow. 20

It should be noted that although the design chosen for conversion of the supersonic tunnel to a hypersonic one uses a two dimensional nozzle, there is a valid argument for the design of axisymmetric nozzles when evaluating hypersonic wind tunnel conditions. A useful explanation of one such approach was shown by James Sivells. At higher Mach numbers, the use of axisymmetric nozzles alleviates the throat design problems caused by the combination of high stagnation enthalpies and large area ratios. Axial velocity distribution is divided into three parts as illustrated in Figure 7. From sonic point I to E, where radial flow is assumed to begin, velocity distribution is described by a forth degree polynomial. From point E to B radial flow is assumed. From point B to C, where uniform flow begins, the Mach number is described by a fifth degree polynomial. The coefficients of the polynomials are chosen such that the second derivatives of the
Figure 8. Inviscid Contour [Reference 12]
of the axial velocity is continuous throughout and zero at point C.

The contour from point G to A is a straight line at an angle $\theta$ relative to the axis. Two characteristics solutions are therefore needed, one to determine the throat contour from point H on the branch line to point G, and the other to determine downstream contour from point A to the theoretical end of the nozzle, point D. The characteristic CD is a straight line since H defines the beginning of the uniform flow region. Calculations were performed by Sivells and a detailed analysis is presented in his paper "Aerodynamic Design and Calibration of the VKF 50-inch Hypersonic Wind Tunnels."\textsuperscript{21}

**Test Section**

In designing the test section of the wind tunnel, the main consideration is to make sure that the model length will not be limited by the length of the uniform flow portion of the model. Another consideration is access to the test section, including doors with a rapid lock system and viewing windows.\textsuperscript{22}

This design incorporates the existing test section of the University of Central Florida supersonic tunnel into this hypersonic conversion. It has an area four inches by four inches with a glass viewing window.

**Diffuser**

The object of the diffuser section is to both improve the operation and reduce the power requirements of the wind
tunnel. To provide a longer running time at the design Mach number, a diffuser can be placed at the test section exit. The convergent angle of the diffuser must be between five and seven degrees for efficient operation. The design of diffusers for hypersonic use is quite similar to supersonic design with the addition of thermal protection on the walls to accommodate the high supply temperatures required for hypersonic operation. Longer blowdown times may result from more efficient diffuser function meaning lower storage tank pressure decreases before breakdown of the flow begins. The wind tunnel air flow must enter and pass through a shock system when it decelerates from hypersonic velocities to a state of rest. This normal shock should be produced at the lowest attainable Mach number in the diffuser since entropy gain through a normal shock is greater with greater Mach numbers. Therefore, oblique shocks will be reflected and finally terminate in a normal shock at a lower Mach number. The proper combination of oblique shocks and normal shock must be produced.

Diffusers may be either fixed or adjustable. A possible advantage of a fixed diffuser is economy, however since the Mach number range varies and the test objects are considerably variable in size or shape, a fixed diffuser may be undesirable. For optimum recovery, it is always preferable to utilize a variable diffuser. Obviously, the cost of adjustable diffusers greatly exceeds the cost of fixed ones.

A contraction angle of seven degrees was selected for
this design. Contraction of the diffuser is possible at Mach number seven to the minimum theoretical second throat height using the seven degree contraction angle. The overall length requirement of the diffuser was also taken into consideration.

As tunnel operation commences, the nozzle velocity will increase until sonic velocity is reached at the narrowest point of the tunnel circuit or nozzle throat. At this point, a normal shock wave will form and will travel from the nozzle throat into the divergent portion as the tunnel pressure ratio rises. The supersonic portion behind the nozzle throat will terminate in a normal shock wave which reduces the total air pressure. At any point in the circuit where the Mach number is sonic, the mass flow must be an equal value of the nozzle throat because the mass flow and the total temperature are unchanged by passing through the shock. For this reason, the diffuser throat area must be greater than the nozzle throat area since the total pressure was reduced by the shock. The greatest total pressure loss or worst condition occurs when the normal shock is standing at the largest section where the flow is still hypersonic, which is the test section. The minimum diffuser throat area \( (A_2) \) is expressed by the relationship:

\[
A_2 = A_1 \frac{P_{01}}{P_{02}} \]  

(29) where

\( (P_{01}/P_{02}) = \) the total pressure ratio across the normal shock at the test section Mach number
A normal shock in a convergent passage will not remain stable in one position during starting because of certain factors. A changing pressure gradient exists and as soon as the pressure ratios exceed the value required for starting by a small amount, the shock travels from the test section, passes through the second throat, and finally assumes a position in the diffuser where the area is basically equal to the test section area. For the best operation once a tunnel has been started, the shock should be at the diffuser throat, as this is the point of minimum supersonic Mach number downstream of the test section. But, in practice, the shock is maintained slightly downstream of the throat for stability.

The second throat is made just slightly larger than the design calculations suggest to allow for uncertainty in boundary layer calculation and to accommodate effects of model wakes and other disturbances to the system.

The purpose of the design is to allow flow exiting the test section of the wind tunnel to be compressed and slowed in the converging portion of the diffuser. It will then pass through the second throat at a speed considerably less than that of the test section. It will begin to increase speed back into the diverging section. A normal shock will then be established in the diverging portion of the diffuser at a Mach number considerably lower than the test section Mach number. It will also possess a correspondingly smaller loss. Design of the diffuser is presented in the Appendix. Calculations for design Mach number seven yield a diffuser length
of 16.00 inches and a diffuser throat width of 0.0696 inches. Figure 10 illustrates the regions and proposed angles and length.

Silencers

The noise levels of high-speed wind tunnels vary according to one's distance from the tunnel, but are high enough to warrant the use of a silencer. It is not necessary for the wind tunnel designer to create his own design since silencers are commercially available and generally represent only two percent or less of the tunnel cost.

The simplest silencers are made up of two concentric cylinders with roughly a six-inch gap between them. The inner one through which the air is discharged is perforated and the space between the two cylinders is filled with a sound absorbent substance such as glass wool. Typical silencers are from two and one-half to four diameters long. The silencer diameter should be large enough to permit the air to be below seventy miles per hour at the discharge end.24

Vacuum Tank

The vacuum tank is usually a sphere, and its size is a function of the size of the tunnel and the desired run time. The tank must have a space to enter for inspection and painting, and a low point drain.

If the piping from the tunnel diffuser goes directly into
the vacuum tank, it is desirable to have a "model catcher" installed in such a way as to prevent any drop in pressure and therefore any decrease in run time that may occur if a heavy model should come loose during a run. 25

Design calculations in the Appendix yield a volume of 2541.26 cubic feet, which is a sphere with a diameter of 16.93 feet.
From the research contained in this report, the author believes the existing supersonic wind tunnel at the University of Central Florida can be modified to obtain a hypersonic capability. The storage tank currently in use can be utilized in this conversion and will be sufficient for operation at higher Mach numbers with the addition of a vacuum tank and a heater. Although the vacuum tank and heater will bring additional costs, they will eliminate the need to alter or replace the existing storage tank.

It is recommended that, due to large area ratios resulting at higher Mach numbers, an axially symmetric nozzle design be utilized. This will alleviate the throat design problems encountered at high Mach numbers. The throat height of the axially symmetric nozzle is larger than that of the two dimensional nozzle with the same test section and Mach number. Structurally, fabrication is easier. With an axially symmetric nozzle, higher temperatures may be permitted at the throat since thermal expansion is small and uniform due to the axial symmetry.

It is also suggested that a computer program be developed for the nozzle design. This will simplify the calculation procedure, and enable the designer to use more
regions in the characteristics method and obtain more precise results. This will yield smoother and more accurate contours.

In addition, it is suggested that a cost analysis be done as the next step in the development of a hypersonic wind tunnel for the University of Central Florida. This analysis should include a survey of hardware sources to determine the components that are available as off-the-shelf items and those components that need to be developed as one-of-a-kind items. A cost versus capability trade-off is recommended as part of the cost analysis.

Finally, it is suggested that the possibility of using the existing compression pump to evacuate all or part of the vacuum tank be explored. If this dual use is feasible it will make possible considerable cost saving for the modification.
APPENDIX
As previously mentioned, the two dimensional nozzle presented in this report was designed using the region to region method of characteristics. The method of characteristics may be reviewed by referring to gas dynamics textbooks. The texts of John26 and Zucrow27 are recommended references on the subject. This appendix presents the equations that result from lengthy derivation and utilizes them in calculating the nozzle design.

The expansion is considered as eight waves of equal strength, and the nozzle has been designed from these waves. The waves separating regions one and two, two and three, three and four, four and five, five and six, six and seven, seven and eight, and eight and nine, are of type one, so across these waves $\Delta v = \Delta \theta$. The waves dividing regions eight and seventeen, seventeen and twenty-four, twenty-four and thirty, thirty and thirty-five, thirty-five and thirty-nine, thirty-nine and forty-two, forty-two and forty-four, and forty-four and forty-five, are type two, so $\Delta v = -\Delta \theta$. The overall $\Delta v = v_{45} - v_1 = 90.97^\circ$ for $M = 7$.

The waves are assumed to be of equal strength ($\Delta \theta = \Delta v$). Therefore, for each of these waves $\Delta v = 90.97/16 = 5.68^\circ$, since the flow crosses a total of sixteen waves between one and forty-five. The flow is horizontal at one and forty-five, therefore $\theta_1 = \theta_{45} = 0^\circ$. The flow angle $\theta$ increases across the fan and decreases to the exit. At each point
where the reflected waves impinge on the wall, the wall must be turned 5.68°. The computational method is presented below.

Region 1 to 2
Type I wave; \( \Delta \nu = \Delta \theta \)
\[
\nu_2 - \nu_1 = \theta_2 - \theta_1 \\
\nu_2 - 0 = 5.68 - 0 \\
\nu_2 = 5.68°
\]

Region 2 to 3
Type I wave; \( \Delta \nu = \Delta \theta \)
\[
\nu_3 - \nu_2 = \theta_3 - \theta_2 \\
\nu_3 - 5.68 = 11.37 - 5.68 \\
\nu_3 = 11.37°
\]
For regions 3 to 8, the procedure is repeated for each wave, noting that these waves are of type I.

Region 2 to 10
Type II wave; \( \Delta \nu = -\Delta \theta \)
\[
\nu_{10} - \nu_2 = \theta_{10} - \theta_2 \\
\nu_{10} - 5.68 = -(0 - 5.68) \\
\nu_{10} = 11.37°
\]
\( \nu \)'s in regions 18, 25, 31, 36, 40, 43, and 45 can be calculated in the same manner.

Region 10 to 11
Type I wave; \( \Delta \nu = \Delta \theta \)
\[
\nu_{11} - \nu_{10} = \theta_{11} - \theta_{10} \\
\nu_{11} - 11.37 = \theta_{11} - 0
\]
\[ V_{11} - 11.37 = \theta_{11} \]

**Region 3 to 11**

Type II wave; \( \Delta V = -\Delta \theta \)

\[ V_{11} - V_3 = -(\theta_{11} - \theta_3) \]
\[ V_{11} - 11.37 = -(\theta_{11} - 11.37) \]
\[ V_{11} = - \theta_{11} + 22.74 \]

From these equations:

\[ V_1 - 11.37 = \theta_{11} \]
\[ V_{11} = - \theta_{11} + 22.74 \]
\[ V_{11} = 17.05 \]
\[ \theta_{11} = 5.68 \]

Calculations for the remaining regions were conducted repeating the same technique. Results are presented in Table I.
Figure 9. Regions of Two-dimensional Hypersonic Nozzle
Figure 10. Mach Number Distribution in Two-dimensional Hypersonic Nozzle
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Proposed Diffuser Design

For a diffuser-inlet Mach number $M_D = 7.0$ and a deflection angle $\delta = 7^\circ$, the calculations for determining the diffuser length are given in this appendix.

For $M_D = 7.0$ and $\delta = 7^\circ$, from NACA Report 1135, chart 2, the incident shock wave angle can be found as $\theta_I = 14^\circ$.

From chart 4 for $M_D = 7.0$ and $\delta = 7^\circ$; $M_b = 5.5$

$M_b = 5.5$ and $\delta = 7^\circ$; $M_c = 4.6$

From chart 2 for $M_b = 5.5$ and $\delta = 7^\circ$; $\theta_b = 16^\circ$

$\theta_b$ is the angle between the flow direction in region b and the reflected wave.

From geometric considerations, the reflected shock wave angle $\theta_{br}$ is given by: $\theta_{br} = \theta_b - \delta = 16^\circ - 7^\circ = 9^\circ$.

From geometric considerations, as shown in Figure 10,

$$\tan \theta_1 = \frac{H/2}{X_1} = \frac{2.0}{X_1}$$

$$X_1 = \frac{2.0}{\tan \theta_1} = \frac{2.0}{\tan 14^\circ} = \frac{2.0}{0.2488} = X_1 = 8.02156^\circ$$

Also from geometry relationships

$$\tan \delta = \frac{Y_1}{X_1+X_2} = \frac{Y_1}{8.02156+X_2} = 0.1228$$

$$\tan \theta_{br} = \frac{H/2-Y_1}{X_2} = \frac{2.0-Y_1}{X_2} = 0.19438$$
Figure 11. Diffuser Model for Oblique Shock Analysis
\[ X_2 = 3.6096 \]
\[ y_1 = 1.4283 \]

Thus, point C is located 11.631" downstream from the diffuser entrance contraction point A.

**II**

For \( M_c = 4.6 \) and \( \delta = 7^\circ \) from chart 2, \( \theta_{\text{II}} = 18^\circ \).

From chart 4 for \( M_c = 4.6 \) and \( \delta = 7^\circ \); \( M_d = 4.0 \)

For \( M_d = 4.0 \) and \( \delta = 7^\circ \); \( M_e = 3.5 \)

From chart 2 for \( M_d = 4.0 \) and \( \delta = 7^\circ \); \( \theta_d = 19.5 \)

From geometric considerations, \( \theta_{\text{dr}} = \theta_d - \delta = 19.5 - 7 = 12.5^\circ \)

\[ \tan \theta_{\text{II}} = \frac{H/2-y_1}{X_3} = \frac{2.0-1.4283}{X_3} = 0.3249 \]

\[ X_3 = 1.7596" \]

\[ \tan \delta = \frac{Y_2-y_1}{X_3+X_4} = \frac{Y_2-1.4283}{1.7596+X_4} = 0.1228 \]

\[ \tan \theta_{\text{dr}} = \frac{H/2-Y_2}{X_4} = \frac{2-Y_2}{X_4} = 0.2217 \]

\[ X_4 = 1.0323" \]
\[ Y_2 = 1.7711" \]

**III**

For \( M_e = 3.5 \) and \( \delta = 7^\circ \) from chart 2, \( \theta_{\text{III}} = 22^\circ \)

From chart 4 for \( M_e = 3.5 \) and \( \delta = 7^\circ \); \( M_f = 3.2 \)

For \( M_f = 3.2 \) and \( \delta = 7^\circ \); \( M_g = 2.8 \)

From chart 2 for \( M_f = 3.2 \) and \( \delta = 7^\circ \); \( \theta_f = 23.5 \)

From geometric considerations, \( \theta_{\text{fr}} = \theta_f - \delta = 23.5 - 7 = 16.5^\circ \)
\[
\tan \theta_{III} = \frac{H/2-y_2}{x_5} = \frac{2-1.7711}{x_5} = \tan 22
\]

\[x_5 = 0.5664\]

\[
\tan \delta = \frac{y_3-y_2}{x_5-x_6} = \frac{2-y_3}{x_6} = \tan 16.5
\]

\[x_6 = 0.3803\]

\[y_3 = 1.8873\]

IV For \(M_g = 2.8 \text{ and } \delta = 7^\circ\) from chart 2, \(\theta_{IV} = 26.2^\circ\).

From chart 4 for \(M_g = 2.8 \text{ and } \delta = 7^\circ\); \(M_h = 2.5\)

for \(M_h = 2.5 \text{ and } \delta = 7^\circ\); \(M_i = 2.2\)

From chart 2 for \(M_h = 2.5 \text{ and } \delta = 7^\circ\); \(\theta_h = 29.2^\circ\).

From geometry, \(\theta_{hr} = \theta_h - \delta = 29.2^\circ - 7^\circ = 22.2^\circ\).

\[
\tan \theta_{IV} = \frac{H/2-y_3}{x_7} = \frac{2 - 1.8873}{x_7} = \tan 26.2
\]

\[x_7 = 0.2290\]

\[
\tan \delta = \frac{y_4-y_3}{x_7+x_8} = \frac{y_4-1.8873}{0.2290+x_8} = \tan 7
\]

\[
\tan \theta_{hr} = \frac{H/2-y_4}{x_8} = \frac{2 - y_4}{x_8} = \tan 22.2
\]

\[x_8 = 0.1593\]

\[y_4 = 1.9350\]

V For \(M_i = 2.2 \text{ and } \delta = 7^\circ\) from chart 2, \(\theta_v = 33^\circ\).

From chart 4 for \(M_i = 2.2 \text{ and } \delta = 7^\circ\); \(M_j = 1.91\)
for $M_j = 1.91$ and $\delta = 7^\circ$; $M_k = 1.65$

From chart 2 for $M_j = 1.91$ and $\delta = 7^\circ$; $\Theta_j = 38$.

From geometry, $\Theta_{jr} = \Theta_j - \delta = 38 - 7 = 31^\circ$.

\[
\tan \Theta_V = \frac{H/2 - Y_4}{X_9} = \frac{2 - 1.935}{X_9} = \tan 33
\]

$X_9 = 0.1001$

\[
\tan \delta = \frac{Y_5 - Y_4}{X_9 + X_{10}} = \frac{Y_5 - 1.935}{0.1001 + X_{10}} = \tan 7
\]

\[
\tan \Theta_{jr} = \frac{H/2 - Y_5}{X_{10}} = \frac{2 - Y_5}{X_{10}} = \tan 31
\]

$X_{10} = 0.0728$

$Y_5 = 1.9562$

VI For $M_k = 1.65$ and $\delta = 7^\circ$ from chart 2, $\Theta_{VI} = 45$.

From chart 4 for $M_k = 1.65$ and $\delta = 7^\circ$; $M_1 = 1.4$

for $M_1 = 1.4$ and $\delta = 7^\circ$; $M_m = 1.2$

From chart 2 for $M_1 = 1.4$ and $\delta = 7^\circ$; $\Theta_1 = 56.5$

From geometry, $\Theta_{1r} = \Theta_1 - \delta = 56.5 - 7 = 49.5^\circ$.

\[
\tan \Theta_{VI} = \frac{H/2 - Y_5}{X_{11}} = \frac{2 - 1.9562}{X_{11}} = \tan 45
\]

$X_{11} = 0.0438$

\[
\tan \delta = \frac{Y_6 - Y_5}{X_{11} + X_{12}} = \frac{Y_6 - 1.9562}{0.0438 + X_{12}} = \tan 7
\]
\[ \tan \theta_{1r} = \frac{H/2-y_6}{X_{12}} = \frac{2-y_6}{X_{12}} = \tan 49.5 \]

\[ X_{12} = 0.0297 \]
\[ y_6 = 1.9652 \]

Thus point M is located 0.0297 inches downstream from impingement point L or \( X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12} = 16.005 \) inches downstream from diffuser entrance point A. It is assumed that the reflected shock wave LM intersects with the expansion wave from the corner of the constant second throat as shown in Figure 10.

Downstream of the oblique shock wave LM, there are large variations in pressure and flow direction, and slip lines are formed. However, it is assumed that the reflected shock wave LM is exactly cancelled by the Prandtl-Meyer expansion wave and one dimensional uniform flow is obtained at a Mach number of 1.2. \(^{29}\)
Sizing of the Vacuum Tank

The chosen running time is forty-five seconds with a stagnation temperature of 1000° Rankine at Mach number seven. Before entering the vacuum tank, the air is cooled to 540° Rankine by the cooler.

At Mach number seven, the area ratio is: \( \frac{A}{A^*} = 104.14 \).

For a 4" X 4" test section, the throat area \( A^* = 4 \times 4 / 104.14 \times 12 \times 12 \), therefore \( A^* = 0.001067 \) square feet.

Since most good vacuum pumps should be capable of evacuating the vacuum tanks to 0.1 psia, a relationship can be found between the volume of the vacuum tank, exit pressure, and stagnation pressure.  

\[
\frac{V}{P_0/P_e} = \frac{28.35 A^* T_e t}{T_0^{1/2}[1-(P_i/P_e)^{1/n}]} \quad (30)
\]

\[
= \frac{(28.35)(0.001067)(540)(45)}{(1000)^{0.5}[1-(0.1/P_e)^{0.909}]} \\
= \frac{23.245}{[1-(0.1/P_e)^{0.909}]} 
\]

At higher values, it is sensible to choose \( P_e = 1.5 \) psia as a design point, as the value of \( V/(P_0/P_e) \) does not decrease much with increasing \( P_e \). At this point, the value of \( V/(P_0/P_e) = 25.4126 \). Figure 1 shows that a compression ratio of 100 is required to run the tunnel, so the resulting sphere
volume is 2541.26 cubic feet with a 16.93 feet diameter, and the operating pressure is 150 psia.

For an operating pressure of 1000 psia with a compression ratio of 100, the design value of $P_e$ is 10. At this point, the required volume is 2360.39 cubic feet. The nominally smaller volume does not justify the use of such high pressures, therefore a tank volume of 2541.26 cubic feet is selected.

Variation of sphere volume over compression ratio with sphere pressure at the end of a forty-five second run at Mach number seven is given is in Figure 12.
Figure 12. Variation of Sphere Volume Over Compression Ratio With Exit Pressure
Storage Tank Conditions

For economic reasons, this conversion design uses the 329 cubic foot storage tank already in operation as a part of the University of Central Florida's supersonic wind tunnel. A tunnel design running time of forty-five seconds has been selected. The following equations are used to derive an initial tank pressure:

Mass flow rate through the tunnel is \( \dot{m} = \rho UA \) \( \text{(2)} \)

where:  
- \( \dot{m} \) = mass flow rate of air, slug/second  
- \( \rho \) = density of air, slug/cubic feet  
- \( U \) = velocity, feet/second  
- \( A \) = duct cross sectional area, square feet

Isentropic flow relationships:

\[
\frac{P}{P_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\gamma/\gamma-1} \quad \text{(13)}
\]

\[
\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/\gamma-1} \quad \text{(14)}
\]

\[
\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \quad \text{(15)}
\]

steady state equation:

\[ P_0 = \rho_0 R T_0 \] \( \text{(1)} \)
where:

\[ R = 1716 \text{ square feet/second} \]

\[ P_0 = \text{total pressure, pounds/square feet} \]

\[ T_0 = \text{total temperature, °R} \]

Relationship between Mach number and velocity:

\[ U = aM \quad (6) \]

where:

\[ a = \sqrt{\gamma RT} \quad (7) \]

thus:

\[ U = M \sqrt{\gamma RT} \]

Substituting equation (15) into equation (6) yields:

\[ U = M \sqrt{\gamma R \frac{T_0}{(1 + (\gamma - 1/2) M^2)}} \quad (31) \]

Substituting equations (14) and (7) into equation (2) yields:

\[ \dot{m} = \frac{P_0}{RT_0} (1 + (\gamma - 1/2) M^2)^{-1/\gamma - 1} M \sqrt{\gamma R \frac{T_0}{(1 + (\gamma - 1/2) M^2)}} A \]

\[ \dot{m} = \sqrt{\frac{\gamma}{RT_0}} (MP_0A) (1 + (\gamma - 1/2) M^2)^{-\gamma - 1/2(\gamma - 1)} \quad (32) \]

At the throat; \( M = 1 \):

\[ \dot{m} = \sqrt{\frac{\gamma}{RT_0}} (P_0 A^n) (1 + \gamma - 1/2)^{-\gamma - 1/2(\gamma - 1)} \quad (33) \]

\( \dot{m}_{\text{run}} \) is the change of mass in the tank.
\[ \sqrt{\frac{\gamma}{R T_0}} (P_0 A^*) (1+ \gamma -1/2) - \gamma -1/2 (\gamma -1) t_{\text{run}} = \rho_i V - \rho_f V \]

where:

\[ V = \text{tank volume} \]
\[ f \text{ denotes final values} \]
\[ i \text{ denotes initial values} \]

\[ \sqrt{\frac{\gamma}{R T_0}} (P_0 A^*) (1+ \gamma -1/2) - \gamma -1/2 (\gamma -1) t_{\text{run}} = \rho_i V [1- \rho_f/\rho_i] \quad (34) \]

For a polytropic expansion in the tank:

\[ \frac{\rho_f}{\rho_i} = \left(\frac{P_f}{P_i}\right)^{1/n} \quad (35) \]

and steady state equation:

\[ P_i = \rho_i RT_i \quad (36) \]

By substituting equations (35) and (36) into (34):

\[ \sqrt{\frac{\gamma}{R T_0}} (P_0 A^*) (1+ \gamma -1/2) - \gamma -1/2 (\gamma -1) t_{\text{run}} = \frac{P_i}{RT_i} [1- \left(\frac{P_f}{P_i}\right)^{1/n}] V \quad (37) \]

Note: The run does not continue until the tank pressure drops to the stagnation pressure \( P_0 \). It stops when the pressure reaches some higher value \( P_f = P_0 + \Delta P \), where \( \Delta P \) denotes the losses in the duct work and in the regulator. The value of \( \Delta P \) is about 0.1 for hypersonic wind tunnels. Thus: \( P_f = P_0 - 0.1P_0 = 1.1P_0 \)
Substituting equations (38) into (37)

\[ \sqrt{\frac{\gamma}{R T_0}} \left( \frac{P_f A^*}{1.1} \right) \left( 1 + \frac{\gamma - 1}{2} \right)^{-\frac{\gamma - 1/2}{\gamma - 1}} \]

\[ t_{\text{run}} = \frac{P_i}{R T_i} \left[ 1 - \left( \frac{P_f}{P_i} \right)^{1/n} \right] V \quad (39) \]

The throat area is:

\[ A^* = \frac{4" \times (0.038410")}{(12)(12)} = 0.0011 \text{ ft}^2 \]

The volume of the storage tank is:

\[ V = 329 \text{ ft}^3 \]

Final pressure at the end of the 45 second running time, with consideration of losses in the duct and the regulator is:

\[ P_f = 165 \text{ psia} \]

The initial temperature of the storage tank is:

\[ T_i = 80^\circ \text{F} = 540^\circ \text{R} \]

Assuming:

\[ \gamma = 1.4 \]

\[ t_{\text{run}} = 45 \text{ seconds} \]

\[ n = 1.2 \]

\[ R = 1716 \text{ ft}^2/\text{seconds}^2-^\circ \text{R} \]

\[ \sqrt{\frac{1.4}{(1716)(540)}} \left( 165 \times 0.0011 \right)^{1/2} \left( \frac{1.4 - 1}{2} \right)^{-2.4/0.8} \left( 1 + \frac{1.4 - 1}{2} \right) 45 \]

\[ P_0 = P_f/1.1 \quad (38) \]
This implicit equation must be solved using the trial and error method, and yields:

\[ P_i = 182 \text{ psia} \]

From polytropic relationships:

\[ T_i = \left( \frac{P_i}{P_f} \right)^{n-1/n} \]

\[ T_f = \frac{T_i}{(P_i/P_f)^{n-1/n}} \]

\[ T_f = \frac{540}{(182/165)^{1.2-1/1.2}} \]

\[ T_f = 531.24 \, ^\circ R \]

Mass rate of flow:

\[ \dot{m} = 0.0186 \text{ slug/second or 0.5981 pounds/second} \]
Heater Design

Since increasing the Mach number may be desired in the future, assumptions were made which allow for increases beyond Mach number seven. These higher design condition assumptions result in more costly construction, however will later enable satisfactory operation at higher Mach numbers. The author has set the following conditions, and the resulting calculations follow:

- Heat air from 470°F to 1850°F
- Mass rate of flow: 5 pounds/second or 18000 pounds/hour
- Discharge pressure: 400 psia
- Assume an initial pebble-bed temperature: 2100°F

I First, the porosity of the bed and the pebble-bed material need to be defined. For this design, alumina pebbles serve the purpose. The pebble size is arbitrarily assumed as 0.0833 feet and the bed porosity is selected to be 0.33 percent. 34

II Second, a bed diameter required to prevent lifting of the pebbles must be calculated. At the heater exit condition

\[
\rho = \frac{(400)(144)}{(53.3)(1850)} = 0.5841 \text{ pounds/cubic feet}
\]

\[
U = \frac{18000}{0.54A} = 33333.4/A \text{ feet/hour}
\]

\[
\mu = 0.104 \text{ pound/feet-hour (Figure 12)}
\]

The volume of a sphere is \((4/3)\pi (d/2)^3\) and the surface area is \(4\pi (d/2)^2\). For the sphere surface area in each cubic foot of the pebble bed:
The density of alumina is about 240 pounds/cubic feet, so the bulk density of the pebble bed is 0.67 \times 240 = 161 \text{ pounds/cubic feet}. If the pressure drop exceeds this figure, the pebbles will lift. Thus the pebble-bed area is calculated for a pressure drop of 161 pounds/cubic feet.

\[ A^{1.9} = \frac{521.63}{161} = 3.24 \]

\[ A = 1.856 \text{ square feet} \]

The bed diameter is 1.5375 feet

III Constants needed for the heat transfer calculations need to be defined. The pebble surface area in each one foot length of the bed is:

\[ S' = 1.856(48.2) = 89.45 \text{ square feet} \]

The weight of the pebbles in each one foot length of the bed is:

\[ W_p = (0.67)(1.856)(240) = 298.44 \text{ pounds} \]

The specific heat of the pebbles is:

\[ C_{pp} = 0.216 \text{ Btu/pounds}^\circ R \]

IV Heat balance equations for a one foot length of the bed are:
bed are given below. The heat added to the air in heating it from one temperature to another is:

\[ Q_a = \dot{m} t \left( \frac{h_e - h_{400}}{T_e - 400} (T_e - 400) - \frac{h_i - h_{400}}{T_i - 400} (T_i - 400) \right) \text{ Btu} \quad \text{(40)} \]

where:

\[ t = \text{run time, hours} \]
\[ h = \text{air enthalpy, Btu/pounds from Figure 15} \]
\[ \text{subscript e denotes exit conditions} \]
\[ \text{i denotes inlet conditions} \]

The heat removed from the pebbles is:

\[ Q_p = W_p C_{pp}(T_{p0} - T_{p1}) \quad \text{(41)} \]

where:

\[ 0 \text{ denotes conditions at the beginning of the run} \]
\[ 1 \text{ denotes conditions at the end of the run} \]
\[ p \text{ denotes pebbles} \]

The heat transferred from the pebbles to the air by convection is:

\[ Q = h_c S't(T_p(\text{ave}) - T_{\text{ave}}) \quad \text{(42)} \]

where:

\[ \text{subscript ave denotes average of initial and final temperatures and average of inlet and exit air temperatures.} \]

Heat transfer equations including constants are:

\[ Q_a = 225 \left( \frac{h_e - h_{400}}{T_e - 400} (T_e - 400) - \frac{h_i - h_{400}}{T_i - 400} (T_i - 400) \right) \quad \text{(43)} \]

\[ Q_p = 64.46 (T_{p0} - T_{p1}) \quad \text{(44)} \]
Q = 1.12 h_c(T_p(ave) - T_ave) \quad (45)

VI Several exit air temperatures are selected between the inlet air temperature and initial pebble temperature.

VII At each of these exit temperatures:

a) read the values of the enthalpy functions of equation (43) from Figure 15

b) solve equation (43) for Q_a

c) determine the average of inlet and exit air temperatures, T_ave

d) read a value of heat transfer corresponding to T_ave from Figure 14

e) set Q = Q_a and solve equation (45) for T_p(ave)

f) set Q_p = Q_a and solve equation (44) for T_p1

g) calculate T_p(ave) from T_p(ave) = (T_p0 + T_p1)/2

h) compare T_p(ave) from 7e and 7g. The equations are solved at the final air temperatures for which those two values are equal.

Results of the calculations are presented in Table 2.
### TABLE II
PEBBLE-BED HEATER CALCULATIONS

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<th>$T_e$</th>
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<th>$h_e/h_{400}$</th>
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<th>$T_{ave}$</th>
<th>$h$</th>
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<th>$T_{p(ave)}$</th>
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<td>$T_i = 470$</td>
<td>800 0.240</td>
<td>96 17820 635 71</td>
<td>224.1</td>
<td>859</td>
<td>276.5</td>
<td>1823</td>
<td>1962</td>
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<td>1200 0.245</td>
<td>196 40320 835 81</td>
<td>444.4</td>
<td>1279</td>
<td>625.5</td>
<td>1474</td>
<td>1787</td>
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<tr>
<td>$T_i/400$</td>
<td>1400 0.247</td>
<td>247 51684 935 83</td>
<td>555.9</td>
<td>1490</td>
<td>801.8</td>
<td>1298</td>
<td>1700</td>
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<tr>
<td>$h_i/h_{400} = 16.8$</td>
<td>1500 0.248</td>
<td>272 57600 985 84</td>
<td>612.2</td>
<td>1597</td>
<td>893.5</td>
<td>1206</td>
<td>1653</td>
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<tr>
<td>$h_i/h_{400} = 285.2$</td>
<td>1550 0.248</td>
<td>285 60390 1010 84</td>
<td>642.0</td>
<td>1651</td>
<td>936.0</td>
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<td>1631</td>
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<td>$T_i = 1550$</td>
<td>1850 0.254</td>
<td>369 15530 1700 107</td>
<td>129.6</td>
<td>1829</td>
<td>240.9</td>
<td>1859</td>
<td>1979</td>
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<tr>
<td>$h_i/h_{400} = 0.248$</td>
<td>1900 0.255</td>
<td>382 21892 1725 108</td>
<td>180.9</td>
<td>1905</td>
<td>340.2</td>
<td>1760</td>
<td>1930</td>
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"...0"
Figure 13. Viscosity Variation of Air [Reference 10]
Figure 14. Thermal Conductivity Variation of Air With Temperature [Reference 10]

Figure 15. Variation of Specific Heat at Constant Pressure With Temperature [Reference 10]
ENDNOTES


5 Ibid., p. 34.


8 Ibid., p. 57.


12 Ibid., p. 80.

13 Ibid., p. 83.

14 Ibid., p. 87.


18 Ibid., p. 97.
19 Ibid., p. 99.


25 Ibid., p. 142.
31 Ibid., p. 147.

BIBLIOGRAPHY


