An Analysis of Undergraduate Elementary School Pre-Service Teachers' Ability to Contextualize Fraction Expressions and Decontextualize Fraction Word Problems

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AN ANALYSIS OF UNDERGRADUATE ELEMENTARY SCHOOL PRE-SERVICE TEACHERS' ABILITY TO CONTEXTUALIZE FRACTION EXPRESSIONS AND DECONTEXTUALIZE FRACTION WORD PROBLEMS

by

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ABSTRACT

The focus of this research was to expand on existing literature by providing information on elementary school pre-service teachers' ability to contextualize fraction expressions and decontextualize fraction word problems. The elementary school pre-service teachers who participated in this study were enrolled in a mathematics for elementary school teachers content course in a college of education at a large university during the Spring 2016 semester. In this mixed-method study, the participants were given the Contextualization and Decontextualization of Fractions Instrument (CDFI) which assessed elementary school pre-service teachers' ability to solve fraction word problems and identify decontextualized fraction word problems into expressions and contextualized fraction expressions into word problems. The elementary school pre-service teachers were given the CDFI before and after they completed a unit on fractions. Of the 52 participants who completed both the pre- and post- CDFI, 11 were selected to participant in think aloud interviews in which they decontextualized fraction expressions from word problems and solved and contextualized fraction word problems from expressions.

Quantitative results showed an overall statistically significant difference in the elementary school pre-service teachers' pre- and post- test scores. With the exception of two questions, all questions on the CDFI showed a statistically significant difference between the pre- and the post- test scores. No statistical significance was found in the responses to the question that required the elementary school pre-service teachers to identify the expression that matched the given fraction subtraction word problem. A large number of participants correctly identified the correct subtraction expression on the pre-test, and only slightly more of them were able to identify the correct subtraction expression on the post-test. No statistical significance was

found in the responses to the question that required the elementary school pre-service teachers to explain their selection of a contextualized fraction multiplication expression. Though there was an increase in the elementary school pre-service teachers' ability to explain their selection of the contextualized fraction multiplication expression, it was not statistically significant. The qualitative analysis of the think aloud interview data showed that some of the elementary school pre-service teachers struggled with contextualizing fraction expressions. Most of the elementary school pre-service teachers did not struggle with solving the fraction word problems, but did struggle with decontextualizing fraction multiplication word problems.

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CHAPTER 1: INTRODUCTION

Background of the Study

Mathematics is a main part of education in the world (Barnwell, 2005). Mathematics in school covers many topics areas including fractions. Fractions are not only used by students in the classroom, but can be applied in their daily life. People use fractions for managing their money, calculating distances, and determining how much gasoline they will need. Even young children learn to apply rudimentary fraction concepts while sharing treats, spending their allowance, saving money for special purchases, telling time, and using a recipe to help cook dinner. According to the National Mathematics Advisory Panel (NMAP, 2008), the push for schools to place more emphasis on mathematics will not be effective without ensuring that pre-service teachers have received the mathematical content knowledge required to help students be successful in mathematics.

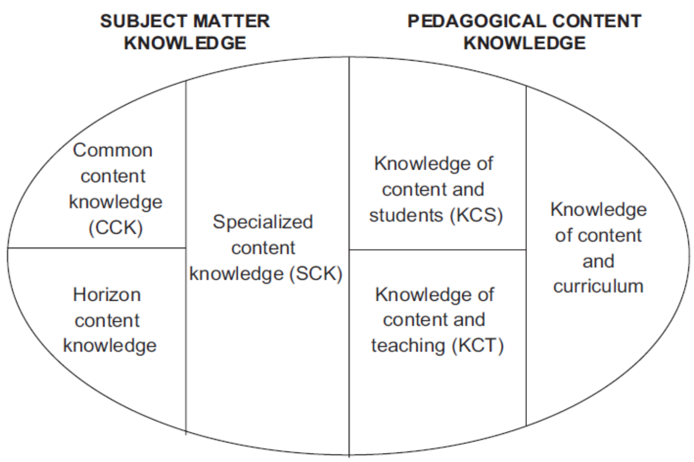
Within the field of mathematics, problem solving has become an important way of helping students to learn about different mathematical topics (National Research Council [NRC], 2001), as well as a way to expand their mathematical knowledge in those topics (National Council of Teachers of Mathematics [NCTM], 1991). Problem solving in mathematics includes solving word problems and applying mathematics to problems in the real world (Branca, 1980). In the current study, elementary school pre-service teachers engaged in problem solving as they completed the Contextualization and Decontextualization of Fractions Instrument (CDFI) which involved solving fraction word problems, and identifying the expression that matched the given fraction word problem, and identifying the word problem that matched a given fraction expression. The current study also required elementary school pre-service teachers to engage in problem solving as they participated in a think aloud interview process. In this process, they created a fraction word problem for different given expressions, solved fraction word problems, and identified an expression that matched the given word problem.

Understanding fractions is important in the development of mathematics knowledge. In a report commissioned by the U.S. Department of Education, it was noted that fractions are critical in the foundation of algebra. In the report, it was recommended that teacher education programs help teachers develop fraction understanding (NMAP, 2008). According to Siegler and Lortie-Forgues (2015), the influence of fractions is not simply identified as having an impact within a single mathematics area. Rather, it has been incorporated in many mathematics topics. The panel reported that fraction understanding was important as a foundation for future mathematics courses. "Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem solving performance" (NMAP, 2008, p. 28).

Pre-service teachers have often been found to lack competence in this area. Utley and Reeder (2012) determined, with a pre-test at the beginning of a mathematics education course, that the pre-service teachers they studied ($N = 42$) entered into elementary education programs with very little understanding of fractions and had misconceptions about fractions even though they had taken several college level courses in mathematics. Based on their findings, they determined that providing pre-service teachers with more opportunities to work with fractions could help them develop a deeper understanding of this important area of mathematics (Utley & Reeder, 2012).

In the review of the literature, the importance of fractions for future success in the area of mathematics was noted as was the need for further research into how elementary school pre-service teachers can be assisted in better understanding fractions and avoiding possible misconceptions. This provided, in part, the rationale for the present research. In the current study, the researcher investigated elementary school pre-service teachers' abilities to contextualize and decontextualize fraction word problems and fraction expressions before and after a unit on fractions presented in a mathematics for elementary school teachers content course. The pre-service teachers were provided several opportunities to contextualize and decontextualize fraction problems in a conceptually-based learning environment.

Integral to the current study was the research of Ball, Thames, and Phelps (2008) who identified the kinds of knowledge teachers require for teaching and proposed a framework for examining teachers' mathematical knowledge for teaching (MKT). Using the work of Shulman (1986) and the Mathematics Teaching and Learning to Teach Project (1993), Ball and colleagues defined domains of mathematical knowledge for teaching. The six subcategories comprising the domains are as follows: (a) common content knowledge (CCK) involves using the correct operation and being able to solve the problems that are given to students, (b) horizon content knowledge (HCK) refers to the knowledge of how different mathematics topics are related across the curriculum, (c) specialized content knowledge (SCK) refers to the mathematical knowledge and skill unique to teaching, including the ability to contextualize expressions and decontextualize word problems, understand different interpretations of operations, and understand why students make a particular error, (d) knowledge of content and students (KCS) involves understanding how students think about different mathematics topics and what they may get confused about, (e) knowledge of content and teaching (KCT) involves having an understanding of mathematics and how to teach and design instruction, and (f) knowledge of content and curriculum (KCC) refers to the combining of mathematics knowledge and the knowledge needed to present and evaluate the abilities of students and assist struggling students. It was this framework, displayed graphically in Figure 1, which served as the interpretive framework for the current study.



Note. Reproduced with permission (Appendix A) from “Content Knowledge for Teaching: What Makes it Special?” by D. L. Ball, M. H. Thames, & G. Phelps, 2008, *Journal of Teacher Education 59*(5), 403.

Figure 1. Domains of Mathematical Knowledge for Teaching

In the current study, the researcher investigated aspects of elementary school pre-service teachers’ CCK and SCK knowledge by assessing (a) 52 elementary school pre-service teachers’ pre-and post- Contextualization and Decontextualization Fractions Instrument (CDFI) results and (b) think aloud interviews with 11 of the elementary school pre-service teachers. The elementary school pre-service teachers’ SCK was evaluated when they identified the correct decontextualizations of the fraction word problems into expressions. It was also evaluated when they identified the correct contextualization of fraction expressions into word problems. Their

CCK was evaluated when they solved the given fraction word problems and explained their selection of the fraction word problem that matched the given fraction expression. During the interview process the elementary school pre-service teachers demonstrated their SCK by creating word problems from given fraction expressions and their CCK by solving the given fraction word problems.

Ball and colleagues (2008) claimed that teachers needed to have high content knowledge in order to help students in their understanding of fractions. Harrell and Eddy (2012) expressed the belief that programs designed to assist pre-service teachers need to help those pre-service teachers develop a deep understanding of mathematics content such as fractions. Shulman (1986) and Cochran (1991) had earlier observed that helping students to develop a deeper understanding of mathematics requires that teachers have both a strong content knowledge and a strong pedagogical knowledge of mathematics. In this blending of content and pedagogy known as pedagogical content knowledge (PCK), Shulman (1986), identified the skills that teachers need to know to help their students. One such area was in solving problems. Dixon et al. (2014) conducted a study involving pre-service teachers' self-authored word problems for fraction subtraction expressions and found several errors. Results of the initial survey indicated that pre-service teachers experienced difficulties with fraction subtraction (Dixon et al., 2014). Based on their results, Dixon and colleagues indicated the need for pre-service teachers to be able to decontextualize a problem situation as well as to contextualize computations and solutions. They also indicated that this level of reasoning may be difficult for teachers to facilitate without providing particular attention to their own contextualizing and decontextualizing knowledge as related to fraction subtraction.

In the current study, CCK and SCK of fractions were identified when the elementary school pre-service teachers took the CDFI and participated in the think aloud interviews. During both of the data collection procedures, the elementary school pre-service teachers were able to demonstrate CCK when they solved fraction word problems and SCK when they explained how to solve a fraction word problem as if they were helping a struggling student, contextualized fraction expressions and decontextualized fraction word problems.

In the current study, the researcher was able to identify some misconceptions that elementary school pre-service teachers had when contextualizing and decontextualizing fractions. Having CCK and SCK of fractions is important when working with students because teachers need to be able to solve the problems assigned to their students, but they also need to recognize if word problems created by students match the operation given in the expressions. If the created word problems do not match the given expression, the teacher needs to have the SCK to understand where the student made an error and how to address the error so that conceptual understanding is supported.

To be able to help students, pre-service teachers also need to strengthen their subject matter knowledge. This is evident in the study by Borko et al. (1992), who conducted observations and interviews to enhance data gathered from questionnaires and written course work from eight pre-service teachers during four different teaching placements. Borko et al. focused on one of the pre-service teachers, Ms. Daniels, and found that even though Ms. Daniels had several mathematics courses and knew how to solve the problems procedurally, she had limited knowledge of ways to represent the topic or what her students understood about a particular topic. During the interviews, Ms. Daniels explained that teachers need to make mathematics relevant to students and create problems with real world situations, but she was unable to create problems for particular examples involving fractions, and demonstrated difficulty in explaining how to use pictorial representations to explain problems. Ms. Daniels' difficulties seemed to demonstrate that she had limited conceptual understanding. Borko and colleagues (1992) determined that pre-service teachers need to have the opportunity, through course work, to increase their subject matter knowledge. It was also suggested that pre-service teachers need to be given the opportunity to discuss their solution strategies in groups in order to ensure that understanding is achieved. In the current study, elementary school pre-service teachers engaged in activities that helped them to decontextualize fraction word problems into the matching expression and participated in discussions to explain how they knew that they had solved the problem correctly, found the correct solution, and why their expression matched the given fraction word problem.

Osana and Royea (2011) conducted a study of eight pre-service teachers using a pre-test post- test design analyzing fraction knowledge. They provided evidence that the pre-service teachers they studied had a weak ability to contextualize fraction expressions and decontextualize fraction word problems. They used a type of conceptually-based environment where pre-service teachers worked in small groups to solve fraction word problems, and after solving the word problems, the groups discussed the different solution strategies that were used to solve the problems. By having the pre-service teachers work in small groups to solve problems and discuss the solutions, the instructor attempted to help the pre-service teachers make the connection between their own strategies and more common representations. Osana and Royea

(2011) found that this type of learning environment helped students to develop a deeper understanding of fractions.

Arikan and Unal (2015) conducted a study of 46 eighth-grade students to analyze their ability to contextualize problems. They determined that the learning environment was important to students' learning, that students needed to feel relaxed, be able to ask questions, and share ways of solving problems (Arikan & Unal, 2015). Similar findings were reported in from McAllister and Beaver's (2012) study of 72 pre-service teachers ($N = 72$) who were asked to solve fraction expressions and to contextualize the fraction expression. It was determined that teachers need to create activities that have students contextualizing word problems from given fraction expressions and from decontextualizing word problems into fraction expressions (McAllister & Beaver, 2012).

In the current study, the elementary school pre-service teachers were asked to share not only their solution strategies for solving the different problems and the expressions that matched a given word problem, but they were able to share and discuss their created word problems. This was similar to previous studies, in that the elementary school pre-service teachers increased their ability to identify contextualized fraction expressions and identify decontextualized fraction word problems. By providing the elementary school pre-service teachers with this experience, they might be able to help their students develop the use of symbolic notation when writing the arithmetic expression for the problem (NMAP, 2008).

Schulman (1986) observed that because pre-service teachers would not be comfortable teaching a topic that they themselves had not learned, pre-service teachers needed to develop a conceptual understanding of mathematics. This was evident in Shulman's (1986) discussion related to knowledge growth in teaching. In another study, Toluk-Ucar (2009) analyzed pre-and post- fraction tests of 50 pre-service teachers in which pre-service teachers were asked to contextualize fraction expressions. The results of the fraction pre-test demonstrated that the pre-service teachers believed that they could not create word problems from fraction multiplication expressions as "it was impossible to relate multiplication of fractions to real life" (p. 170). Toluk-Ucar (2009) proposed that pre-service teachers could develop conceptual understanding of fractions by creating word problems with real world context in their mathematics content courses. By doing so, they could improve conceptual understanding and discover possible misconceptions that might exist involving fractions. Similarly, McAllister and Beaver (2012) found that the 72 pre-service teachers in their study had difficulty writing word problems for fraction multiplication expressions, indicating that "they did not know how to write such a problem" (p. 96). These researchers posited that when pre-service teachers practice writing word problems with real world contexts they might also learn to recognize their own misconceptions about fractions so that they can be addressed.

El Sayed (2002) conducted a study to examine problem posing strategies of pre-service teachers (N = 50). He determined that pre-service teachers who spent more time exploring the process of how to solve problems were better at contextualizing problems than students who spent less time studying the problem solving process. When discussing problem posing activities, El Sayed (2002) suggested that teachers should create word problems with different types of contexts for their students. By doing this, according to El Sayed, students would find mathematics classes more interesting, as the teacher could create the word problems and relate those problems to mathematics concepts being taught (e.g., fractions).

Teachers should also require students to create word problems with real world contexts from given expressions. Having students learn how to solve real world problems is not a new idea. One of the first recommendations of "An Agenda for Action" produced by NCTM (1980) was that schools need to have students focus on problem solving. It was recommended that teachers have their students not only solve word problems, but create word problems to help them to develop conceptual understanding (NCTM, 2000). To help students become skilled at problem solving, teachers also need to be able to contextualize and decontextualize problems that contain fractions.

Arikan and Unal (2015) investigated eighth graders' (*N* = 46) ability to problem pose fraction expressions. They determined that providing activities where students create word problems from given fraction expressions helps students to think creatively and critically about how to phrase the problem so that it matches the given expression. Problem posing was also seen as a way for teachers to observe the students' thinking (Arikan & Unal, 2015). Having students contextualize problems can also help teachers to determine if students have any misconceptions about fractions. In the current study, elementary school pre-service teachers were engaged in activities that called for them to contextualize fraction expressions and share the created problems with their classmates. When the elementary school pre-service teachers shared their word problems, the class discussed whether the word problem matched the given fraction expression and, if it did not, how the word problem could be corrected.

Alibali, Brown, Stephens, Kao, & Nathan (2009) conducted a study to examine middle school students' (*N* = 257) ability to solve problems and to contextualize expressions. Through evaluation of word problems that students created from given expressions, teachers could assess students' conceptual understanding of the operations presented in the problems. It was determined that if students were able to solve the given expression, but were not able to create a word problem to match the given expression, they had a procedural knowledge but not a conceptual knowledge (Alibali et al., 2009). Capraro and Joffrion (2006) conducted a study examining middle school students' (*N* = 668) ability to decontextualize word problems. They determined that students needed to have both a conceptual and a procedural understanding to be successful at decontextualizing word problems that contained fractions into expressions and to be successful with contextualizing word problems from given expressions that contained fractions.

Ma (1999) conducted a study in which she compared the mathematical understanding of elementary school teachers in the United States and China. She found that teaching students fractions conceptually and not simply procedurally was difficult when both practicing teachers and pre-service teachers struggle to understand fractions. In a study conducted 10 years later, Huang, Liu, and Lin (2009) found that when pre-service teachers (*N* = 47) were tested on their fraction knowledge, they could solve the fraction problems procedurally, but they were not able to explain what they were doing when they solved the problems. This finding led Huang and colleagues to determine that the pre-service teachers' problem solving abilities were linked to their procedural knowledge and not their conceptual knowledge of fractions. They did, however, suggest that pre-service teachers needed to enrich their conceptual knowledge of fractions and that the problems that pre-service teachers have with fractions might carry over into their future teaching (Huang, Liu, & Lin, 2009).

The conceptually-based learning opportunities utilized in the present study emulate the types of learning opportunities that practicing teachers in general hopefully provide for their own students and that pre-service teacher will provide to their future students as they grow professionally. This approach includes the use of discourse in small groups and in whole class settings. The small group and whole class discussions are student-driven with the teacher acting as a facilitator to encourage deeper thinking. This, in turn, can help students construct meaning for themselves instead of waiting and watching teacher demonstrations of solutions (Wachira, Pourdavood, & Skitzki, 2013). Wachira et al. studied a single introductory calculus course at a public high school in which the teacher provided the students in the class with word problems to solve, and encouraged students to discuss the problems with others. Over the course of the school year, the high school students transitioned from simple discussions about the problems to using proper mathematical terms when discussing problems. The students in the current study were also given open-ended questions to solve. After solving the word problems, the students shared the different strategies that they used to solve the problems first in small groups and then in whole class discussions.

Kazemi and Stipek (2001) conducted a study involving fourth- and fifth- grade students' participation in conceptual and procedural discourse. In their study, four elementary school teachers were videotaped and observed as they interacted with their students during several fraction lessons. The teachers moved around the classroom and asked the students questions about their solutions while they were in small groups. The teachers also had the students present their solutions and strategies to the whole class. During the presentations of the solutions, the teachers queried the students as to their strategies, requesting explanations about why they performed certain steps when solving the given problem and to illustrate the students' verbal responses about their pictorial representations of the problem. Kazemi and Stipek determined that having students discuss their solutions in small groups and in whole class discussions improved students' participation in the activities and helped to influence students' mathematical understanding. Similar to the procedures followed by Kazemi and Stipek, instructors in the classrooms in the current study moved among the small groups, facilitated sharing of ideas, and discussed the different strategies that the students were using to solve the problems. The instructors also questioned the participants on their strategies to ensure that they could explain the steps in their solution strategies.

In the current study, pre-service teachers had the opportunity to show that they had developed a conceptual understanding of fraction word problems following conceptually based learning experiences. They were expected to not only solve the problems, but also to be able to explain how they solved the problem, and how they knew that they had chosen the correct contextualization for the fraction expression.

Statement of the Problem

The fact that fractions have traditionally proven to be a difficult concept for students to understand (Unlu & Ertekin, 2012) and that being able to solve problems that contain fractions has been determined to be a "major obstacle to further progress in mathematics" (NMAP, 2008, p. 28) has resulted in a dilemma for educators. Researchers have noted the struggles of both students and pre-service teachers in understanding fractions (Huang et al., 2009; Li & Kulm, 2008; Van Steenbrugge, Lesage, Valcke, & Desoete, 2014). If pre-service teachers struggle with their understanding of fractions, they are likely to struggle in teaching fractions to their future students (Van Steenbrugge et al., 2014). If teachers struggle in teaching fractions to students, their students, in turn, are likely to struggle with understanding fractions and have difficulty contextualizing and decontextualizing word problems that contain fractions.

Researchers have shown that students who are successful at creating word problems that contain fractions are also successful at decontextualizing word problems into expressions. To help students be successful with decontextualizing word problems that contain fractions into numerical expressions and contextualize expressions that contain fractions into word problems, pre-service teachers need to be able to identify when their students have correctly decontextualized fraction word problems and contextualized fraction expressions. For example, if students are given a fraction subtraction expression and are asked to create a word problem, the teacher needs to be able to recognize if the students have correctly created a subtraction word problem or if they created an incorrect multiplication word problem.

The current study sought to identify differences in the pre- and post-scores of undergraduate elementary school pre-service teachers when assessed on their ability to identify the decontextualization of fraction word problems and to identify the contextualization of fraction expressions into word problems using the ideas from the MKT framework. Also of interest was the extent to which undergraduate elementary school pre-service teachers' understanding of decontextualizing fraction word problems and contextualizing fraction expressions improved as a result of having studied fractions using a conceptually-based teaching methodology. The pre-test results provided information about the pre-service teacher's fraction knowledge as it related to contextualizing, decontextualizing, and solving fraction problems. The post-test results provided information about the change in pre-service teachers' knowledge of fractions as it related to contextualizing, decontextualizing, and solving fraction problems.

Van Steenbrugge et al. (2014) found that the pre-service teachers that they studied had limited content knowledge and limited specialized content knowledge (SCK), as defined by Ball et al. (2008). The present study was conducted to evaluate whether a conceptually-based learning experience helped increase pre-service teachers' fraction content knowledge and also increased their SCK. The Contextualization and Decontextualization of Fractions (CDFI) test was designed to assess pre-service teachers' thinking about fractions. Think aloud interviews of selected pre-service teachers were conducted to complement the findings of the CDFI and contribute to the in-depth understanding of how pre-service teachers mentally processed fraction word problems as they decontextualized and solved them. The CDFI, along with the interview portion of the study, provided insight into pre-service teachers' thinking processes related to decontextualizing fraction word problems. This type of analysis provided needed information regarding this area of research.

Research Questions

The purpose of this research study was to answer the following research questions:

1. Did undergraduate elementary pre-service teachers' ability to decontextualize and explain fraction word problems improve following a unit on fractions in a mathematics for elementary school teachers' content course?
2. Did undergraduate elementary pre-service teachers' ability to contextualize and explain fraction expressions improve following a unit on fractions in a mathematics for elementary school teachers' content course?
3. In what ways do pre-service teachers make sense of decontextualizing fraction expressions?

4. In what ways do pre-service teachers make sense of contextualizing fraction word problems?

Hypotheses

1. Undergraduate elementary pre-service teachers' will improve their ability to decontextualize and explain fraction word problems following a conceptually-based unit on fractions in a mathematics for elementary school teachers' content course
2. Undergraduate elementary pre-service teachers' will improve their ability to contextualize and explain fraction expressions following a conceptually-based unit on fractions in a mathematics for elementary school teachers' content course.

Operational Definitions

For the purposes of this study, the following terms and definitions will be used.

Arithmetic expression: A series of numbers and mathematical symbols assembled to represent the situational representation of a word problem. For example, [$\frac{1}{2} + \frac{1}{3}$], Sarah ate $\frac{1}{2}$ of a pizza then she ate another $\frac{1}{3}$ of a pizza. How much of a whole pizza did she eat?

Conceptual learning: Learning that occurs when a student understands how to solve a problem and can explain why that method works.

Contextualization: The process of creating a word problem from a given expression, also called problem posing.

Decontextualization: The process of creating an expression that matches the given word problem.

Elementary school pre-service teachers: Undergraduate college students seeking a degree in either early childhood education or elementary education.

Fluency: The ability of students to solve problems that contain fractions accurately and without difficulty.

Problem posing: The process of constructing a word problem in context from a given mathematics expression.

Procedural learning: Understanding that occurs when a student learns the equations and steps necessary to solve a problem.

Undergraduate education majors: Undergraduate college students enrolled in an institution of higher education for the purpose of earning a bachelor's degree in education.

Summary

In this chapter, the importance of pre-service teachers' conceptual understanding of fractions was presented. This chapter also addressed how pre-service teachers' need for conceptual understanding of fractions could be improved by having pre-service teachers participate in a conceptually-based learning environment using the creation of word problems in a context that contains fractions. This methodology provided an environment where pre-service teachers were able to discover possible misconceptions related to fraction concepts. Chapter 2 contains a review of the literature on the topics of mathematical knowledge for teaching, problem posing, decontextualization of word problems, problem posing, and conceptually-based learning.

CHAPTER 2: LITERATURE REVIEW

Introduction

This chapter includes a review of literature for mathematical knowledge for teaching, problem posing, decontextualizing fraction word problems, and conceptually-based instruction. The mathematical knowledge for teaching section summarizes the domains of mathematical knowledge for teaching that were designed by Ball and colleagues (2008). The problem posing section presents the benefits of giving students activities where they create problems from given expressions, along with the difficulties that pre-service teachers have with contextualizing word problems from given expressions. The section concerning decontextualizing fraction word problems highlights the difficulties that students typically have with decontextualizing word problems. These difficulties include selecting the correct operation, choosing the correct order for the numbers in the expression, and issues with the vocabulary found in the word problems. Finally, the conceptually-based instruction section contains a discussion of this approach, its connection to constructivism, and the benefits of teaching mathematics conceptually in general.

Developing Mathematical Knowledge for Teaching

Ball and colleagues (2008) defined mathematical knowledge for teaching as the knowledge a teacher needs in order to be able to teach students mathematics. Mathematical knowledge for teaching is comprised of subject matter knowledge and pedagogical content knowledge (Ball, et al., 2008). These ideas provide a guide for the researcher to determine if the participants in the study have gained the knowledge that is important for teaching students about fractions. Following is a description of each of the components of mathematical knowledge that were relevant in the present study.

Ball and colleagues (2008) divided subject matter knowledge into the following three domains: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). Common content knowledge was defined by Ball et al. as "the mathematical knowledge and skill used in settings other than teaching" (p. 399). For example, Ball et al. indicated that teachers need to "know the material they teach" (p. 399), "recognize when their students give wrong answers" (p. 399), and be able to use the correct notation when contextualizing and decontextualizing problems. Specialized content knowledge was defined by Ball et al. as "the mathematical knowledge and skill unique to teaching" (p. 400) that occurs when the teacher needs to recognize "patterns in student errors" (p. 400) and have an "understanding of different interpretations of operations" (p. 400), as well to determine if a story problem matches a given expression or if the expression matches the given story problem. Horizon content knowledge is defined as an "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p. 403). For example, teachers in lower grades need to understand how the mathematics that they are teaching to their students relates to the mathematics that the students will learn in higher grades.

Pedagogical content knowledge (PCK) was first identified by Shulman (1986, 1987) as a combination of pedagogical knowledge, "the knowledge of generic principles of classroom organization and management" (Shulman, 1986, p. 14), and content knowledge, the "amount of and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p. 9). Operationally, content knowledge means that teachers should be able to solve a given problem, explain why they solved the problem in the way they did, and why this type of problem is important for students to learn.

The pedagogical content knowledge that was identified by Shulman (1986, 1987) was enhanced by Ball et al. (2008) and further divided into three domains consisting of knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). Knowledge of content and students is "knowledge that combines knowing about students and knowing about mathematics" (Ball et al. p. 401). KCS is the information that teachers need to understand to know what students will find difficult, and be "familiar with common errors and deciding which of several errors students are most likely to make" (Ball et al., p. 401). According to these researchers, knowledge of content and teaching "combined knowing about teaching and knowing about mathematics" (Ball et al., p. 401). When teachers apply their KCT, they understand which examples to use to introduce a topic and which examples to use to deepen students' understanding. Knowledge of content and curriculum is knowledge that combines the tools that teachers use to present content and evaluate the abilities of students and assist struggling students.

This study focused on common content knowledge (CCK) and specialized content knowledge (SCK), which represented two parts of the subject matter knowledge identified by Ball et al. (2008). CCK was selected for the current study because the participants' abilities to solve the given fraction word problems were analyzed along with their ability to decontextualize fraction word problems. During the think aloud interview portion of the current study, the participants' CCK was analyzed when they decontextualized the given fraction word problems and then proceeded to solve the fraction word problems. SCK was selected for the current study because of the researcher's interest in the participants' abilities to identify the correct contextualization of fraction expressions into word problems. The participants also demonstrated their SCK when they explained their selection of the correct contextualized word problems on the CDFI. During the interview portion of the current study the participants' SCK was analyzed when they contextualized a fraction expression and explained how they knew they created a word problem that matched the given fraction expression.

Decontextualization of Word Problems to Expressions

Decontextualizing a word problem requires a student to write an expression that represents the situation presented in the word problem (Carey, 1991). The expressions that are created by students can contain numbers, variables, and symbols. Researchers have shown that school-age children have difficulty when asked to decontextualize word problems into expressions (Capraro & Joffrion, 2006; Carey, 1991; Carpenter, Hiebert, & Moser, 1983; Carpenter, Moser, & Bebout, 1988; Zweng, 1979). When looking at research involving decontextuliazxation of word problems, the researcher noticed that most of the research dealing with decontextualization revolved around elementary students. It was also noted that when students were decontextualizing word problems the problems did not contain fractions.

Carpenter and colleagues (1983) reported that when first-grade students (N = 43) were asked to decontextualize a word problem, but were unsure of the operation that occurred in the word problem, they tended to write an expression with an addition operation. Some of the first graders in the study were found to struggle more with identifying the correct operation of subtraction at the end of the study than at the beginning. The researchers believed that the first graders in the study began to think that the way to solve a word problem was to choose the correct operation with the given numbers in the word problem without looking at what the word problem was asking (Carpenter et al., 1983). They suggested that teaching children how to write expressions too early may encourage children to focus on an expression rather than what the word problem is asking. It was their belief that this would lead children to choose the incorrect operation of addition or subtraction when decontextualizing a word problem to an expression (Carpenter et al., 1983).

First-grade students also reversed the order of the numbers when decontextualizing subtraction word problems into expressions (Carpenter et al., 1983). One explanation for this was that they were modeling the actions presented in the word problem (Carey, 1991; Carpenter et al., 1983). An alternate explanation offered for middle school students' reversing the order was that students were writing the numbers in the order that they appeared in the word problem rather than interpreting the processes to be performed (Capraro & Joffrion, 2006; Carey, 1991). In a study by Tobias (2009), 33 preservice teachers were given fraction word problems and were asked to solve them, writing the expression that matched the word problem. As with elementary and the middle school students, the pre-service teachers also demonstrated difficulty with determining the correct order when working with multiplication problems.

A second reason that students have difficulty decontextualizing word problems into situational number sentences might be related to vocabulary (Capraro & Joffrion, 2006; Carpenter et al., 1983; Dunlap & McKnight, 1978). For students to be able to decontextualize a word problem into an expression, they need to understand what the problem is asking and the vocabulary that is presented in the problem (Dunlap & McKnight, 1978). Capraro and Joffrion (2006) reported that students (N = 668) in their study needed to be able to read the word problems and understand the vocabulary in order to be able to decontextualize it into an expression. Dunlap and McKnight (1978) observed that by having students decontextualize word problems, the teacher was able to (a) determine if students were having difficulty with a particular concept, (b) provide scaffolding for the students to understand what was being asked by the question, and (c) appropriately develop the solution.

When writing word problems for expressions that contain a fraction with the operation of multiplication, middle school students tended to write addition word problems (Alibali et al., 2009). Dunlap and McKnight (1978) wrote that when students are introduced to the idea of decontextualizing word problems into expressions, they may need the teacher's help to understand precisely what the problem is asking before they are able to decontextualize it into a correct expression.

Some teachers think that teaching their students about "key words" could help students to decontextualize word problems into correct expressions. According to Capraro and Joffrion (2006), when helping students who are initially learning to decontextualize word problems into expressions, care needs to be taken to accurately teach students about key words. Capraro and Joffrion did acknowledge that teaching students to use key words can cause students to be confused, particularly when one of the key words is found in a word problem, but the operation in the word problem does not match the key words. Consider for example the following word problem.

> "Skyler has 4 times as many books as Karen. If Skylar has 36 books, how many books does Karen have?" (Englard, 2010, p. 157)

Note that the key word "times" would signify a multiplication problem. However, if the students multiplied the two numbers, the correct solution would not be achieved. To prevent students from confusion with key words, teachers can help students acquire the skill to decontextualize word problems into expressions by having students practice decontextualizing word problems (Dunlap & McKnight, 1978).

Even in-service teachers have been shown to have difficulty solving problems when they contain fractions (Osana & Royea, 2011). Osana and Royea found fractions were a difficult topic to master for a group of eight pre-service teachers. Toluk-Ucar (2009) reported that pre-service teachers (*N* = 95, 50 experiment, 45 control) had difficulty connecting fractions to any concept except procedures. Similar to Osana and Royea (2011), McAllister and Beaver (2012) found that pre-service teachers were better at solving the fraction problems than they were at creating word problems to match a given fraction expression.

In the current study, the elementary school pre-service teachers were required to identify the correct decontextualization of fraction word problems in both the pre- and post CDFI. During the interview process, the participants in the study were given fraction word problems and asked to decontextualize and solve them.

Problem Posing

In traditional mathematics instruction, students are rarely asked to contextualize expressions into word problems. Rather, in the opinion of Silver (1994), the responsibility of contextualizing or posing problems has been placed on the teacher who needs to create problems or use problems supplied by the textbook. Students should be given the opportunity to create problems from given mathematical expressions (NCTM, 1989; Silver, 1994). Problem posing, as presented by Toluk-Ucar (2009), not only considers the creation of word problems from a given mathematical expression, but also sees it as a way to help students make connections between their interests and mathematics. In the current study, pre-service teachers' ability to identify the correct contextualization of a given fraction expression was assessed along with their ability to identify the correct decontextualization of a fraction word problem. This assessment provided insight into the abilities of pre-service teachers before they started a unit on fractions and after they had completed the unit on fractions.

In a study conducted by Ellerton (2013), pre-service teachers (*N* = 154) in an education mathematics content class participated in problem posing activities. During the semester, at week 12 and week 16, the pre-service teachers in the class completed a short questionnaire about problem posing. The pre-service teachers in the study stated in the questionnaires that they did not have enough experience with creating word problems from a given mathematical expression. These pre-service teachers also believed that the ability to create word problems was a skill that they would need as a classroom teacher. Ellerton (2013) determined that by having pre-service teachers create word problems from given mathematical expressions, pre-service teachers could gain confidence in themselves as problem posers and might be more willing to encourage their own students to create word problems from given mathematical expressions.

In terms of problem posing and fractions, the process of problem posing can be used to help all students make connections between fractions and fraction situations found in the real world (Isik & Kar, 2012). In their study, Isik and Kar asked 64 pre-service teachers to complete a problem posing test. After completing the problem posing test, 16 of the pre-service teachers participated in semi-structured interviews. When analyzing the results of the problem posing test, the problem posing skills of the pre-service teachers was found to be low. It was also determined that to help correct errors with fractions when problem posing, an environment that was based on problem posing was required (Isik & Kar, 2012).

Typically, problem posing in the learning environment occurs when the instructor asks questions of the students while they are in the process of solving word problems (Toluk-Ucar, 2009). By posing questions, the teacher is able to help students decontextualize their own word problems to reflect the given mathematical expression (Dunlap & McKnight, 1978). Through formulation and reformulation, students are able to intensify their understanding of the mathematical processes which they are studying (Silver, 1994). The question that directs this process is, "How can I formulate this problem so that it can be solved?" (Silver, 1994, p. 20).

When constructing word problems, teachers need to utilize real life situations that their students might encounter in the world (McAllister & Beaver, 2012). Using word problems with real world contexts helps students make connections between mathematics and the real world and will help make mathematics relevant and meaningful to students by tapping into the everyday lives of students (Capraro & Joffrion, 2006; Sharp, & Adams, 2002). The use of real word situations by teachers can also help students make connections from the word problem to a symbolic representation of the problem (Cramer, Post, & del Mas, 2002).

One way to give students problems with real world context is to have teachers create the word problems that can be used in the classroom. However, in a study conducted by Toluk-Ucar (2009), pre-service teachers (N = 95) were given a fraction test at the beginning and at the end of a semester-long mathematics teaching methods course. Toluk-Ucar found that pre-service teachers had difficulty with writing word problems for expressions with the operations of multiplication and division that contained fractions. Additionally, some of the pre-service teachers also reported that they could only create multiplication and division word problems using whole numbers (Toluk-Ucar, 2009).

Osana and Royea (2011) studied the procedural and conceptual understanding of fractions of eight pre-service teachers along with problem posing before and after five individual one-on-one sessions on fractions. The pre-service teachers were found to lack a strong conceptual understanding of fractions no matter what operation was being used in the problem both before and after the intervention. Osana and Royea suggested that, because of this lack of conceptual understanding, pre-service teachers had difficulty creating word problems that matched the expressions they were given.

In a later study, McAllister and Beaver (2012) asked pre-service teachers (N = 72) to create word problems from given fraction expressions. These researchers found that the pre-service teachers were able to create word problems in context for addition and subtraction fraction expressions, but had difficulty creating word problems in context for multiplication and division. When McAllister and Beaver analyzed the story problems, they found that the pre-service teachers had "a weak conceptual understanding of operations" (p. 95). Because they had such difficulty writing word problems for multiplication expressions that contained fractions, they concluded that the pre-service teachers did not have a deep understanding of the operations of multiplication, at least when fractions were contained in the expressions (McAllister & Beaver, 2012). Similar to the findings of Toluk-Ucar (2009) and Osana and Royea (2011), McAllister and Beaver found that the pre-service teachers in their study had difficulty creating fraction word problems that represented the different operations of addition, subtraction, multiplication, and division.

Furthermore, when posing problems for a given expression that contains fraction division, it was noted that pre-service teachers made several errors. McAllister and Beaver

(2012) reported that when writing word problems, the pre-service teachers did not struggle with writing word problems for addition expressions that contained fractions, but they did struggle with writing word problems for subtraction, multiplication and division expressions that contained fractions. When given a fraction multiplication expression, the pre-service teachers would create a word problem that reflected an addition problem (McAllister & Beaver, 2012). The pre-service teachers in the study also had problems when writing a word problem for fraction subtraction expressions. For example, if the pre-service teachers were given a fraction subtraction problem in the form “a – b”, they would create a word problem that matched the expression “a - (b · a)” (McAllister & Beaver, 2012).

Luo, Lo, and Leu (2011), conducted a study comparing pre-service teachers (N = 174) fraction knowledge. The study compared pre-service teachers in the United States (N = 89) with pre-service teachers in Taiwan (N = 85). In the study the pre-service teachers took a 15 question multiple-choice tests that covered different areas of fraction knowledge. This study found that the pre-service teachers in both the United States and Taiwan did well on problems that contained area models. The study results also showed that both groups of pre-service teachers scored low on problems involving understanding the meaning of operations. While both groups of pre-service teachers easily selected the fraction subtraction word problem that matched the given expression, they had difficulty when choosing the word problem that matched the given fraction multiplication expression. Luo and colleagues (2011) determined that the pre-service teachers in the United States had a lower level of fraction understanding than the pre-service teachers in Taiwan. They also indicated that when teaching fractions, time needs to be spent on both contextualizing fraction expressions and decontextualizing fraction word problems.

Dixon and colleagues (2014) studied graduate level pre-service teachers' (N = 19) understanding of fraction subtraction and their ability to contextualize fraction subtraction expressions. The pre-service teachers (N = 19) completed a fraction survey prior to starting a unit on fractions. Of the pre-service teachers (N = 19) who completed the fraction survey, nine of them were selected to participate in an interview prior to the unit on fractions. At the end of the course, the pre-service teachers completed a final examination. Dixon and colleagues (2014) determined from the initial fraction survey that the pre-service teachers struggled with fraction subtraction. The pre-service teachers also demonstrated confusion with subtraction and multiplication during the interview. Similar to the findings of McAllister & Beaver (2012), Dixon et al. (2014) found that when pre-service teachers were asked to contextualize a problem for the expression "a – b" the pre-service teachers created a word problem for the expression "a - (b · a)". One reason that the pre-service teachers struggled with writing subtraction word problems when given an expression that contained fractions was attributed to their "struggle with redefining the whole" and not "keeping the whole consistent" (Dixon et al., 2014, p. 13). For example, when students are asked to decontextualize a one-step fraction word problem that creates the expression $\frac{2}{3}-\frac{1}{2}$, they might struggle with determining whether they should subtract the $\frac{1}{2}$ from the $\frac{2}{3}$, or whether to they should subtract the $\frac{1}{2}$ from the whole.

Toluk-Ucar (2012) reported that on the pre-test completed by 95 pre-service teachers, the subjects had difficulty writing word problems for division expressions and demonstrated difficulty on the post-test with writing mathematically appropriate word problems. Unlu and Ertekin (2012) had similar results when most of the 82 pre-service teachers in their study either created inappropriate fraction division word problems or could not create any fraction division word problems. These pre-service teachers were given four fraction expressions and asked to create word problems. The pre-service teachers who created inappropriate word problems also participated in an interview and were asked to explain and justify their answers. Though the pre-service teachers were able to solve the fraction division problems using the method of invert and multiply, their fraction knowledge did not seem to go beyond a procedural understanding (Unlu & Ertekin, 2012). In an earlier study conducted by Forrester and Chinnappan (2010), pre-service teachers (N = 186) were given a fraction test at the beginning of the semester. The results of the fraction test determined that the pre-service teachers' content knowledge of fractions was mostly procedural when they solved subtraction and multiplication problems. Forrester and Chinnappan observed that pre-service teachers who have procedural knowledge of fractions and not a great amount of conceptual understanding might make mistakes when solving word problems that contain fractions.

In the pre- and post-test on fractions that Osana and Royea, (2011) administered in their study, it was found that the eight pre-service teachers had misconceptions when dealing with division of fractions. One of the misconceptions identified by Osana and Royea was that pre-service teachers, when asked to construct word problems with a real world context for a fraction division expression, would switch the order of the dividend and the divisor in the expression when they wrote a word problem. For example, given the expression $6 \div \frac{3}{4}$, pre-service teachers might construct the following incorrect word problem.

> "There are six children who want to share $\frac{3}{4}$ of a giant cookie. How much of the cookie will each child get?" (Osana & Royea, 2011, p. 340).

Furthermore, Unlu and Ertekin's (2012) found that some pre-service teachers, when asked to write a word problem from a given a fraction division expression, wrote a multiplication word problem. For example, given the expression $\frac{5}{3} \div \frac{1}{2}$, pre-service teachers might construct the following incorrect word problem.

> "If someone has $\frac{5}{3}$ of a loaf of bread and he wants to give half of this to his friend. How much will the friend get?" (Unlu & Ertekin, 2012, p. 493)

McAllister and Beaver (2012) reported that pre-service teachers could not create word problems when presented with a fraction division expression. Pre-service teachers who were able to create a word problem often made "division-specific errors related to the partitive model for division" (McAllister & Beaver, 2012, p. 94). This error caused the pre-service teachers to create a word problem in which the number of groups was represented by a fraction (McAllister & Beaver, 2012).

In a study conducted by Isik and Kar (2012), 64 pre-service teachers completed a problem posing test and 16 also participated in an interview. Isik and Kar noted that when writing division word problems, the pre-service teachers would use units that would not work with fractions. For example, if given a number sentence such as $? \div \frac{1}{4} = \frac{16}{5}$, the pre-service teacher might create a word problem as follows:

> "If we have $\frac{16}{5}$ units of trees, when we cut each tree into $\frac{1}{4}$ parts, how many tree parts will we have?" (Isik & Kar, 2012, p. 2305).

Through their use of inappropriate units, pre-service teachers demonstrated confusion when trying to solve the problem. They also constructed word problems that reflected the operation of multiplication when they intended to write a division word problem (Isik & Kar, 2012). Researchers such as Osana and Royea (2011) demonstrated that pre-service teachers have difficulty with creating word problems from given fraction expressions, as well as understanding fractions expressions with different operations. Newton studied 85 pre-service teachers who took pre- and post-tests examining their knowledge of fractions at the beginning and end of a semester. The fraction test consisted of both fraction expressions and fraction word problems for the pre-service teachers to solve. The pre-service teachers in the study demonstrated misconceptions at the beginning of the course. The pre-service teachers demonstrated better understanding at the end of the course. Newton suggested that the pre-service teachers should be taught multiple strategies to solve problems so that they are able to choose an appropriate model or strategy to solve problems.

Toluk-Ucar (2012) also reported that when pre-service teachers (*N* = 95) were given multiplication expressions with fractions, they had difficulty writing the corresponding word problems. The pre-service teachers in Osana and Royea's (2011) study, when given a fraction subtraction problem, created a word problem that reflected the operation of multiplication. For example, given the expression of $\frac{4}{5} - \frac{1}{2}$, they constructed the following word problem:

> "Sandra has $\frac{4}{5}$ of a watermelon in her fridge. Her mom comes to visit and eats half of the watermelon in the fridge. How much of a watermelon did she eat?" (Osana & Royea, 2011, p. 340).

According to Dixon et al. (2014), when pre-service teachers read and then decontextualize word problems that contain fractions or contextualize fraction expressions, misconceptions with fractions can be identified. If the misconceptions with fractions are not corrected, "it is unlikely that the pre-service teachers with these sorts of misconceptions will become teachers who will adequately support student engagement . . . regarding fractions" (Dixon et al., 2014, p. 20). In the current study, the elementary school pre-service teachers were required to identify the correct contextualization of fraction expressions when they took both the pre- and post CDFI. During the interview process, participants were given a fraction expression and asked to contextualize it. Through analysis of the CDFI and contextualization of fraction expressions during the interview process, the elementary school pre-service teachers' misconceptions were exhibited.

Conceptually-Based Learning

Conceptually-based learning is modeled after the constructivist theory of learning (Windschitl, 2002). Constructivist teaching has students construct knowledge within small groups or as a whole class while being guided by the teacher to elicit understanding (Noddings, 1990). Cobb (1988) determined that constructivism views both the teacher and the student as giving each other "contextually based meaning" (p. 88) as they interact in the classroom.

In a conceptually-based learning environment, students are engaged in problem-based activities and work together to solve problems (Windschitl, 2002). While solving problems, students are provided with different tools to help with the problem solving task. In this learning environment, students are asked to explain their solution strategies to their small groups and the whole class instead of focusing on the correct answer (Windschitl, 2002). Through small group and whole class discussions, the instructors are able to understand some of the thinking of their students. Instructors can then guide both small group and whole class discussions to facilitate different strategies in solving problems and support students as they develop a deeper understanding of fractions (Windschitl, 2002).

Rittle-Johnson and Alibali (1999), who studied fourth- (N = 60) and fifth-grade (N = 29) students found that students who were taught conceptually had a greater increase in knowledge when compared to students who were taught procedurally. In the study, students were divided into three groups. Group 1 received addition instruction that was taught conceptually. Group 2 received addition instruction that was taught procedurally. The researchers found that students who were part of the conceptual group improved their understanding of mathematics when compared to the procedural group. The students in the conceptual group, also were able to develop more flexible problem solving procedures. Rittle-Johnson and Alibali (1999) concluded that learning conceptually may help students to recognize when problem solving procedures are incorrect.

In a study conducted by Sharp and Adams (2002), 23 fifth-grade students were taught fractions through solving word problems. The students were asked to solve fraction word problems, write their thinking using words or pictures, and engage in a whole class discussion about the problems. Sharp and Adams found that, by giving the students fraction word problems with real world context and having discussions about the different problems, students were better able to solve the problems and to begin to create procedures that mirrored what they saw with whole numbers.

Hecht (1998) studied seventh and eighth graders' (N = 83) fraction skills by administering a test to examine their procedural knowledge of fractions, conceptual knowledge of fractions, and general mathematics knowledge. As part of the testing, students were asked to create word problems to match given expressions. Hecht determined that students needed a conceptual understanding of fractions in order to create a fraction word problem that matched the given expression.

Cramer et al. (2002) studied fourth- (N = 953) and fifth-grade (N = 713) students' understanding of fractions after studying a unit on fractions. During the unit, the students were placed into one of two groups. Group 1 focused more on learning procedures, and developing computational skills. Group 2 spent more time developing an understanding of fractions and understanding what the fraction numbers meant. The teaching styles of the two groups also differed. Students who were in Group 1 classrooms learned fractions in a more traditional way: they solved problems from a book and had few manipulatives available for their use. The goal of the students in Group 1 was to develop competency with fraction operations. Students who were in Group 2 classrooms participated in small group and whole class discussions. During the small group discussions, students used manipulatives, pictures, and written symbols to solve word problems with a real world context. As the students worked on solving fraction word problems, they made connections to different representations of the problems and learned that symbols could be used to represent a given word problem. Cramer and colleagues (2002) found that the students who were in Group 2 classrooms scored better than Group 1 students on the test following the fraction unit. The Students in Group 2 also showed stronger conceptual understanding of fractions than the students in Group 1. There was, however, no difference found between the two groups when it came to solving fraction problems without context.

As many students find fractions difficult, educational institutions should use the development of conceptual knowledge of fractions to help pre-service teachers when decontextualizing word problems (NMAP, 2008). However, in the study conducted by Osana and Royea (2011), pre-service teachers had difficulty in explaining their solutions to problems. It was determined that the pre-service teachers could use previous remembered procedures to solve the fraction problems, but could not conceptually explain the procedures that they used. Based on the results of their study, Osana and Royea determined that the use of procedures by pre-service teachers was so ingrained that it prevented them from making sense of fraction word problems. They also found that pre-service teachers would revert to using procedures to solve word problems if they struggled with conceptually explaining the problems.

Rayner, Pitsolantis, and Osana (2009) conducted a study to understand the connection between pre-service teachers ($N = 32$) mathematics anxiety and their knowledge of fractions. The pre-service teacher's mathematics anxiety levels were measured using the Revised Mathematics Anxiety Rating Scales (RMARS). Their fraction knowledge was assessed using a paper and pencil test. The two tests were administered after the pre-service teachers had completed two class periods of fractions. They determined that conceptually-based learning experiences help pre-service teachers to develop a deeper understanding of mathematics.

In a study conducted by Tchoshanov (2011) on in-service middle school teachers ($N = 102$), it was determined that teachers who have a conceptual content knowledge of mathematics tend to teach their students more conceptually, but that teachers with a more procedural understanding teach their students procedures. Tchoshanov (2011) indicated that students who had teachers with conceptual content knowledge of mathematics were higher achieving students than those who had teachers with procedural knowledge. Having teachers with conceptual content knowledge can help to determine if a teacher will be successful at helping student learn mathematics (Tchoshanov, 2011).

In the current study, the elementary school mathematics content course established a conceptually-based environment. The elementary school pre-service teachers in the study solved problems in their small groups and discussed those problems during whole class discussions. During whole class discussions the elementary school pre-service teachers would present and explain their solutions to the different problems. During the CDFI the think aloud interview participants were given an opportunity to share their solution strategies to the different problems presented.

Problem

Both students and pre-service teachers struggle with understanding fractions. When students struggle with understanding fractions, they can also experience difficulty contextualizing and decontextualizing word problems that contain fractions. When pre-service teachers are unable to accurately address student misconceptions about fractions, they may have difficulty in assisting their future students when they are contextualizing fraction expressions and decontextualizing fraction word problems. Researchers have shown that when students are more successful at creating word problems that contain fractions, they are also more successful at contextualizing and decontextualizing word problems and expressions, and that this is a valuable skill to have in the real world (Alibali et al., 2009).

In this chapter, a review of the literature was presented. The review included the areas of problem posing, decontextualizations of word problems, and mathematical knowledge for teaching, and conceptually-based learning. Overall, the research indicated that pre-service teachers have misconceptions regarding the areas of fractions, (Dixon et al., 2014; McAllister & Beaver, 2012; Osana & Royea, 2011). Pre-service teachers can improve competence when provided an environment where they have the opportunity to practice contextualizing fraction expressions into word problems and decontextualizing fraction word problems into mathematical expressions. Such practices have been shown to improve competence in fraction and problem posing which was also identified as a valuable tool to assist pre-service teachers in mastering fractions (Dunlap & McKnight, 1978; Silver, 1994; Toluk-Ucar, 2009). Finally, it was inferred that if pre-service teachers do not master the concepts of contextualizing and decontextualizing fractions, overall weakness in the area of fractions would continue (Dixon et al., 2014). Chapter 3 presents the methods and procedures used to conduct the study. These include the methodology, questions, hypothesis, design, population, sample, data collection, and instrumentation/data gathering procedures.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

Introduction

This chapter includes a discussion of the methodology, research questions, hypothesis, design, population, sample, data collection, and instrumentation/data analysis used in this research study. The chapter also includes a discussion of (a) items for both the pre- and post-tests, correct answers, and research connections for the Contextualization and Decontextualization of Fractions Instrument (CDFI) developed for the current study; (b) the procedures used to improve the instrument's reliability and validity; and (c) items that were used during the interviews along with correct answers and research connections.

Methodology

The current study involved mixed methods. The quantitative part of the study involved the administration of pre- and post-tests using the Contextualization and Decontextualization of Fractions Instrument (CDFI). The instrument evaluated the ability of participants to identify contextualizations of fraction expressions and decontextualization of fraction word problems involving the operations of subtraction and multiplication. The qualitative part of the study involved think aloud interviews with selected participants. The participants in the current study consisted of undergraduate elementary education majors enrolled in a course focused on mathematics for elementary school teachers.

Research Questions

The purpose of this research was to determine possible answers to four questions.

1. Did undergraduate elementary school pre-service teachers' ability to decontextualize and explain fraction word problems improve following a unit on fractions in a mathematics for elementary school teachers' content course?
2. Did undergraduate elementary school pre-service teachers' ability to contextualize and explain fraction expressions improve following a unit on fractions in a mathematics for elementary school teachers' content course?
3. In what ways do pre-service teachers make sense of decontextualizing fraction expressions?
4. In what ways do pre-service teachers make sense of contextualizing fraction word problems?

Hypotheses

1. Undergraduate elementary pre-service teachers will improve their ability to decontextualize and explain fraction word problems following a conceptually-based unit on fractions in a mathematics for elementary school teachers content course.
2. Undergraduate elementary pre-service teachers will improve their ability to contextualize and explain fraction expressions following a conceptually-based unit on fractions in a mathematics for elementary school teachers content course.

Design

This study used a mixed-methods design involving quantitative and qualitative approaches. The researcher used a convenience sampling to obtain the participants for the study for the following reasons:

1. The elementary school pre-service teachers in a mathematics for elementary school teachers content course fit the criteria of the study.
2. The mathematics for elementary school teachers content course was offered at the university where the researcher studied and was also a course that the researcher had taught.
3. The researcher was familiar with the faculty members who taught the mathematics for elementary school teachers content course and was able to obtain access to the elementary school pre-service teachers in the class (Gall, Gall, & Borg, 2007; Johnson & Christensen, 2004).

For the quantitative part of the study, the researcher administered the CDFI pre-test version to the participants prior to the unit on fractions. The researcher also administered the CDFI as a post-test version after the completion of the fraction unit. The post-test contained the same questions as the pre-test, but the fractions used on the post-test were different. The pre-test data were collected during the sixth week of the semester, and the post-test data were collected during the 12th week of the semester. The data collection for this study took place in the spring 2016 semester during the participants' scheduled class time of their course section.

For the qualitative part of the study, using the CDFI pre-test scores the researcher identified participants who demonstrated low and medium ability levels when identifying and explaining fraction decontextualization and contextualization involving expression and word problems. When analyzing the CDFI post-test scores, the researcher identified participants who demonstrated low, medium, or high ability levels of identifying and explaining fraction decontextualization and contextualization involving expression and word problems. Based on the participants' understanding from the CDFI pre- to post-tests, six groups were identified: (a) started low and stayed low; (b) improved from low to medium; (c) improved from low to high; (d) decreased from medium to low; (e) stayed the same from medium to medium; (f) and improved from medium to high. The low group consisted of participants who were able to answer zero to two questions correctly. The medium group was composed of participants who were able to answer three to five questions correctly. The high group contained participants who were able to answer six to eight questions correctly. During the pre-test none of the participants were able to answer six to eight questions correctly, so a group that started high was not created. The researcher selected participants from each of the six groups and interviewed them using a think aloud process to obtain more information about their thoughts while contextualizing and decontextualizing fraction problems. The qualitative data were collected in the two weeks following the completion of the fraction unit during the spring 2016 semester.

Population

The population discussed in this research was comprised of undergraduate students who were enrolled as education majors in a college located in the southeast United States. All participants in this study were elementary school pre-service teachers enrolled in the mathematics for elementary school teachers content course, during the spring 2016 semester.

Learning Environment

Two sections of the mathematics for elementary school teachers content course were selected, and each of the sections was taught using a conceptually-based learning approach. The mathematics for elementary school teachers content course was designed and modified using the research of several studies. Only the fraction portion of the learning activities developed based on the research of Wheeldon (2008) and Tobias (2009) were used in this study. This provided previously examined learning activities developed to help pre-service teachers learn about fractions. The sequence of the activities was as follows: (a) defining fractions based on the whole, (b) composing and decomposing fractions, (c) unitizing, (d) equivalent fractions, (e) ordering fractions, (f) adding and subtracting fractions, (g) multiplying fractions, and (h) dividing fractions. Both instructors used the same learning activities.

In her research, Wheeldon (2008) video recorded all the class sessions during the fraction unit and took notes of her classroom observations. She also had the pre-service teachers in the course complete questionnaires at the beginning and at the end of the fraction unit. Her goal was to analyze the classroom experiences in order to develop a method of teaching fraction concepts to pre-service teachers.

Tobias (2009) built on the dissertation of Wheeldon (2008) by documenting ways that fraction knowledge was developed by pre-service teachers. Tobias (2009) video recorded all of the fraction unit class sessions during her research. She also conducted interviews with five of the pre-service teachers in the course before and after the fraction unit. She determined that there was not enough time spent on fraction operations and suggested that more time be spent on fraction operations in future classes.

The classroom (in the present research) where both mathematics for elementary school teachers content course sections met, was organized by using eight tables where groups of three to six elementary school pre-service teachers collaborated. The researcher attended and observed three class lessons for each of the two sections of the course. While observing each of the class sessions, the researcher took field notes, documenting what occurred during each of the attended lessons. The researcher observed the class sessions for addition and subtraction of fractions, and multiplication of fractions. The researcher sat at one of the tables and observed the conversations that occurred during the lesson both in the small group and during the whole class discussions. The researcher did not participate in the discussions that occurred in the small group or during the whole class discussions.

The current study focused on the elementary school pre-service teachers' ability to contextualize fraction expressions and decontextualize fraction word problems. The conceptually-based learning experiences involved the activities relating to subtraction and multiplication of fractions which were developed by Wheeldon (2008) and Tobias (2009). These activities included the following: (a) Pizza Parlor Situation 1, (b) Subtraction, (c) Pizza Parlor Situation 2, and (d) Multiplication.

The Pizza Parlor Situation 1 page included in the course pack contained four problems. The elementary school pre-service teachers were to read the given fraction word problems, write the expression that matched the given word problems, and solve the given word problem while justifying their solution. The Pizza Parlor Situation 1 word problems contained fraction addition and fraction subtraction problems.

The Subtraction page in the course pack contained five fraction subtraction expressions. The elementary school pre-service teachers were asked to create a word problem in context for the given expressions and to solve the problem while justifying their solution.

The Pizza Parlor Situation 2 page in the course pack contained four fraction multiplication word problems. The elementary school pre-service teachers were to read the given fraction multiplication word problems, write the expression that matched the given word problems, and solve the given word problem while justifying their solution.

The last page in the course pack was the Multiplication page. This page contained five fraction multiplication expressions. The elementary school pre-service teachers were asked to create a word problem in context for the given expressions and to solve the problem while justifying their solution. For each of these pages, the elementary school pre-service teachers had discussions in their small groups and participated in whole class discussions about the problems.

For all of the activities the elementary school pre-service teachers were encouraged to use pictorial representations as they solved the problems, and to collaborate within their groups. Collaboration was established at the beginning of the semester when the elementary school pre-service teachers started the unit on whole numbers. Continued collaboration was encouraged by the course instructor who circulated around the classroom, talking with the different groups. When the instructor stopped at a group, she would question the group members on the solutions for one of the problems. If all of the participants had the same solution, the instructor would query them as to how they arrived at the solution, requesting that the group members compare and contrast the possible different solution strategies. If different solutions were obtained, the instructor would question the group members, asking them to explain how they arrived at their various solutions. If some of the group members were unsure of how to obtain the solution, the instructor would encourage one of the group members who had a solution to explain the strategy that was used to obtain their answer.

It was established in the mathematics for elementary school teachers content course that the elementary school pre-service teachers were not to use algorithms to solve the problems given to them in class. Rather, they were instructed to use other problem-solving methods such as pictorial representations. Students were also expected to explain and justify their solutions to problems. Andreasen (2006) conducted a study with pre-service teacher (*N* = 16) in a mathematics education class. This study examined the social aspects found in the classroom that facilitated learning. One social norm that emerged in this study was the idea that students needed to explain and justify their solutions. Another social norm established in the classroom was the idea of making sense of others reasoning. Both of these norms were integrated into the mathematics for elementary school teachers content course used in the current study.

During the classroom visits in the current study, the following general learning processes were observed. The researcher was able to observe conceptually-based learning experiences, recommended by Windschitl (2002), when the participants were given problem-based activities to work on in their small groups. While the participants worked on the problems in their small groups, the instructor for the course moved around to the different groups and questioned the participants about their solutions and strategies. As advocated by Windschitl, the instructor provided time for the small groups to work collaboratively in solving the word problems and discussing their solutions. Sharp and Adams (2002) and Cramer and colleagues (2002) established their conceptually-based classroom by having small group and whole class discussions about fraction problems. Similarly, in the current study, the participants participated in whole-class discussions about the problems after their small group problem-solving activities. During the whole class discussion, the instructor for each class guided the class discussion focusing on different solutions and strategies. The students were familiar with this teaching/learning style because it had been used throughout the semester for other topic areas.

Sample

A convenience sample was used for the current research. A minimum sample size of 34 students was identified using the G* Power software (Faul, Erdfelder, Lang, & Buchner, 2007). The significance level for the current study was established at .05 with an effect size of .5 and a power of .8 (Faul et al., 2007). For the purposes of the current study, the researcher administered the CDFI pre-test and post-test versions to two sections of the mathematics for elementary school teachers content course. Each class contained 35 students. During administration of the CDFI pre-test version to both of the identified course sections, the researcher was able to test 60 of the elementary school pre-service teachers during the pre-test. When the CDFI post–test version was administered to the participants, 54 of the participants completed both the pre- and post-test. Two of the 54 participants who completed both the CDFI pre-and post-test were eliminated from the sample because they were not education majors. Therefore, the sample for the current study was 52 elementary school pre-service teachers. The two selected sections of the mathematics for elementary school teachers content course involved instructors who used the same course pack which consisted of a workbook developed specifically for this course. The researcher was able to interview 11 of the 52 elementary school pre-service teachers who completed both the pre- and post-tests for further participation in the think aloud interview portion of the study. The 11 participants represented each of the six groups.

Think Aloud Interviews

Think aloud interviews were used for the qualitative part of the study. As described by Young (2005), the think aloud process was used to evaluate and observe each of the 11 randomly selected participants while they were solving problems. Wade (1990) determined that the use of a think aloud process can help teachers assess a student's comprehension and give the teacher information about their strengths and weakness in a particular area, such as the ones included in the current study involving contextualizing fraction expressions and decontextualizing fraction word problems. Also, Wade (1990) indicated that the information gained from a think aloud process can help the teacher to adjust instruction so that students' weaknesses can be addressed and become strengths. This information was beyond the scope of the current study because the researcher was not concerned with adapting instruction, but it could be useful in understanding the impact of the conceptually-based approach on participants' learning.

In a study conducted by Secolsky et al. (2016) involving 30 high school students, it was determined that the process of using think alouds has the potential to assist teachers to address misconceptions that their students have in mathematics. Secolsky et al. (2016) also determined that the think aloud process helps students who struggle in mathematics. Teachers who use the think aloud process with their students need to have the ability to determine if a student has given a correct or incorrect solution to a problem (Secolsky et al., 2016). By determining what the students know, teachers are able to develop the lessons that are taught in mathematics classes to address any misconceptions that students might have (Secolsky et al., 2016). Rosenzweig, Krawec, and Montague (2011) conducted a similar study with eighth-grade students (N = 73) and found that using think alouds helped the students to find solutions to problems they were working on.

According to Kuusela and Paul (2000), the think aloud process, when used with students, needs to be recorded and transcribed. There are two types of think aloud processes that Kuusela and Paul discussed in their study: concurrent and retrospective think alouds. A concurrent think aloud occurs when the data are collected with the participants talking aloud as they solve the problem (Kuusela & Paul, 2000). A retrospective think aloud occurs after participants have arrived at a solution and are asked what they were thinking as they solved the problem (Kuusela & Paul, 2000). According to the study conducted by these researchers, the concurrent think aloud process was more effective in determining students' thought processes and comprehension. In the present study, the researcher used the concurrent type of think aloud process, a seemingly more appropriate fit, for the present study. Each of the think aloud interviews were recorded and transcribed. The data analysis procedures used for both quantitative and qualitative data gathered in the current study are discussed in detail in the following section.

Data Collection

Before beginning the current study, the researcher secured Institutional Review Board (IRB) approval for research with human subjects (Appendix B). The researcher visited both sections of the mathematics for elementary school teachers content course at the start of the fraction unit to inform the students about the study and to ensure them that all participant information would remain confidential. After obtaining informed consent (Appendix C), the researcher administered the CDFI pre-test to the participants. After the completion of the fraction unit the researcher administered the CDFI post-test version. The post-test retained the same question structure but substituted alternative fractions within the questions. Completion time of the CDFI for participants was 15-20 minutes. After the pre- and post-tests, the researcher analyzed the results and placed the participants into groups of Low-Low (LL), Low-Medium (LM), Low-High (LH), Medium-Low (ML), Medium-Medium (MM), and Medium-High (MH) designations based on their demonstrated ability to decontextualize fraction word problems and contextualize fraction expressions on the pre- and post-tests. All the groups except for the Medium-High (MH) group contained multiple students. The Medium-High (MH) group only contained one student.

The researcher used a stratified random sampling to obtain participants from each group to participate in the interview phase of the research. The Medium-High (MH) group only contained one member, and that member was included in the selection of participants for the think aloud interview portion of the study. If the participant initially identified did not wish to be interviewed, an alternate group member was selected. The interview consisted of two parts. In the first part, interviewees were asked to create two word problems about pizza, one for subtraction and one for multiplication. In the second part of the interview, interviewees were asked to solve two problems, one multiplication and one subtraction and to write the expression for the given word problem.

Prior to initiating the interviews, the 11 participants were instructed in how to do a think aloud interview: they were instructed to talk aloud as they solved the problems. During the think aloud interview, the researcher asked the interview participants to describe their thinking as they (a) created the word problems, (b) solved the fraction word problems, and (c) wrote the fraction expressions. For the word problem creation activity, the interview participants were asked to explain how they knew they created a word problem that matched the given fraction expression. For the problem-solving activity, interview participants were asked to explain any strategies that they used while they solved the two fraction word problems and to explain how they knew that the fraction expression they created matched the word problem they were given.

Instrumentation/ Data Gathering

The following section will include information about the CDFI and the think aloud interviews. The CDFI consisted of a pre-test given prior to the unit on fractions and a post-test given after the unit on fractions. The think aloud interviews were obtained from participants after the unit on fractions.

CDFI Instrument

The instrument that was used in the current study was the Contextualization and Decontextualization of Fractions Instrument (CDFI). The CDFI was designed to assess participants' abilities to correctly identify and explain (a) decontextualization of subtraction and multiplication one-step word problems into arithmetic expressions that matched the situation presented in the word problems, and (b) contextualization of subtraction and multiplication fraction arithmetic expressions into one-step word problems that matched the given arithmetic expression.

The CDFI consisted of two parts. The first part contained two questions, the pizza and the brownie problems. The brownie problem addressed the operation of fraction subtraction, and the pizza problem addressed the operation of fraction multiplication. Each question was designed to

(a) evaluate the participants' abilities to correctly solve the problem, and (b) provide an explanation of the solution process as they would to a student in their classroom using words and pictures. Participants matched the correct decontextualization of each fraction word problem with one of eight mathematical expressions provided.

The second part of the CDFI also contained two questions. The first question, the pie problem, contained the operation of fraction subtraction. The second question, the candy problems, included the operation of fraction multiplication. From the five provided word problems, each question was designed to evaluate participants' abilities to identify the word problem that correctly matched the given mathematical expression. Participants also were asked to provide a written explanation of why the word problem they selected was correct.

The CDFI pre-test version was composed of four questions. Two of the questions were presented as word problems for which the participants need to explain how to solve the problem and then identify the expression that matches the given word problem. The subtraction word problem given on the CDFI pre-test version was:

> Tami brought $\frac{1}{2}$ a pan of brownies to school to share with her classmates. If Tami's classmates ate $\frac{3}{7}$ of a pan of brownies, how much of the pan of brownies does Tami have left?

For this problem, the correct decontextualization would be "Student D translated the problem into $\frac{1}{2} - \frac{3}{7}$". The expression that was used in this problem originated from a study conducted by McAllister and Beaver (2012). The multiplication word problem that was given on the CDFI pre-test version was:

There was $\frac{3}{4}$ of a pizza left over from dinner last night. For lunch Jim ate $\frac{1}{3}$ of the leftover pizza. How much of the whole pizza did Jim eat for lunch?

For this problem, the correct decontextualization would be "Student F translated the problem into $\frac{1}{3} \times \frac{3}{4}$". The expression that was used in this problem originated from a study conducted by Toluk-Ucar (2009).

The other two questions on the CDFI pre-test version required the participants to select a word problem that matched the given expression. The subtraction expression that was given was "$\frac{5}{6} - \frac{1}{3}$". The expression that was used in this problem originated from a study conducted by Saxe and Gearhart (2001). For this problem the correct selection of the word problem that matches the expression was:

> Student A wrote: Sally had $\frac{5}{6}$ of an apple pie leftover in her refrigerator. For dessert Sally and her husband David ate $\frac{1}{3}$ of a whole apple pie. How much of the apple pie is left?

The multiplication expression that was given in the CDFI pre-test version was "$\frac{1}{2} \times \frac{3}{4}$". The expression for this problem originated from a study conducted by Osana and Royea (2011). The correct selection of the word problem that matches the expression was:

> Students B wrote: Juan has $\frac{3}{4}$ of a bag of candy. If Juan eats $\frac{1}{2}$ of the candy, how much of a bag of candy did Juan eat?

The four problems that comprised the CDFI pre-test version can be found in Appendix D.

The CDFI post-test version also consisted of four questions. As with the pre-test version, two of the questions were presented in word problem format, and participants needed to explain how to solve the problem. The final two questions required participants to identify the expression that matched the given word problem. The subtraction word problem given in the CDFI post-test version was:

> Tami brought $\frac{3}{4}$ a pan of brownies to school to share with her classmates. If Tami's classmates ate $\frac{1}{2}$ of a pan of brownies, how much of the pan of brownies does Tami have left?

For this problem, the correct decontextualization selection was "Student C translated the problem into $\frac{3}{4} - \frac{1}{2}$". The expression that was used in this problem originated from a study conducted by Toluk-Ucar (2009). The multiplication word problem that was given on the CDFI post-test version was:

> There was $\frac{3}{4}$ of a pizza left over from dinner last night. For lunch Jim ate $\frac{2}{3}$ of the leftover pizza. How much of the whole pizza did Jim eat for lunch?

For this problem, the correct decontextualization selection was "Student F translated the problem into $\frac{2}{3} \times \frac{3}{4}$". The expression that was used in this problem originated from a study conducted by Lin (2010).

The other two questions on the CDFI post-test version required participants to select a word problem that matched the given expression. The subtraction expression was "$\frac{2}{3} - \frac{1}{2}$". This expression originated from a study conducted by Saxe and Gearhart (2001). For this problem, the correct selection of the word problem that matched the expression was:

Student A wrote: Sally had $\frac{2}{3}$ of an apple pie leftover in her refrigerator. For dessert Sally and her husband David ate $\frac{1}{2}$ of a whole apple pie. How much of the whole apple pie is left?

The multiplication expression that was given on the CDFI post-test version was "$\frac{1}{2} \times \frac{3}{4}$". The expression used in this problem originated from a study conducted by Newton (2008). The correct selection of the word problem that matches the expression was:

Students B wrote: Juan has $\frac{3}{4}$ of a bag of candy. If Juan eats $\frac{1}{2}$ of the candy, how much of a bag of candy did Juan eat?

The four problems that comprised the CDFI post-test version may be found in Appendix E. For the post-test version, only the fractions in the questions were changed; the context of the questions remained the same as the pre-test.

Think Aloud Interviews

The think aloud interview portion of the data collection consisted of a dialog between the researcher and each interviewed participant. The interview was focused on the processes of contextualizing fraction expressions and decontextualizing fraction word problems along with solving fraction word problems, and each interview consisted of two parts. During the first part of the interview, the researcher provided the participants with two expressions, one subtraction and one multiplication. Participants were asked to create a word problem with the context of pizza for each of the two given expressions. In the second part of the interview, the researcher posed two additional problems: (a) the cake problem, addressing fraction subtraction; and (b) the brownie problem, addressing fraction multiplication. For the brownie and cake problems, the participants were given a word problem for which they provided a solution, the mathematical expression for the given word problem, and then discussed the rationale for each step of their analysis and solution process with the researcher.

The think aloud interview portion of the current study consisted of two parts. The first part required the interviewer to give participants two fraction expressions and ask them to create a word problem about pizza. The first expression was "$\frac{3}{5} \times \frac{5}{6}$"; the second expression was "$\frac{3}{4} - \frac{1}{2}$". These expressions were retrieved from a study conducted by Toluk-Ucar (2009). The second part of the interview portion of the current study contained two fraction word problems, one subtraction and one multiplication. The participants were asked to solve a given fraction word problem and explain how to solve it with pictures and words. The participants were then asked to write the fraction expression that matched the given word problem. The fraction subtraction word problem was:

> John left $\frac{5}{6}$ of a cake on the kitchen counter. His dog came in and ate an amount equal to $\frac{1}{3}$ of the whole cake before John chased him from the room. How much of the cake is left after the dog left the room?

The expression represented in this word problem was obtained from a study conducted by Saxe and Gearhart (2001). For this problem, the correct decontextualization selection was "Student D: $\frac{5}{6} - \frac{1}{3}$". The fraction multiplication word problem was:

David brings $\frac{3}{4}$ of a pan of brownies to school to share with his classmates. If David's classmates eat $\frac{1}{3}$ of the brownies, how much of a whole pan of brownies did David's class eat?

The expression represented in this word problem was obtained from a study conducted by Toluk-Ucar (2009). For this problem the correct decontextualization selection was "Student F: $\frac{1}{3} \times \frac{3}{4}$". The four participant interview questions and the researcher's questions posed to the interviewees may be found in Appendix F.

Three rubrics were used to assess participants' responses to the various questions presented to them. Table 1 contains the rubric used to assess participants' responses to the decontextualization and contextualization of the Pizza, Brownie, Pie, and Candy CDFI items. The researcher assigned a zero for participants who, on the Pizza and Brownie problems, chose an incorrect representation of the fraction expression that matched the given fraction word problem or if they left the portion blank. On the Brownie and Pie problem, if the participants chose an incorrect word problem to match the given fraction expression or left the selection blank they would be assigned a zero for that problem. The researcher assigned a one for the participants, on the Pizza and Brownie problems, who chose the correct representation of the fraction expression that matched the given fraction word problem, and on the Brownie and Pie problem, if the participants chose the correct word problem that matched the given fraction expression. Table 2 contains the rubric used to assess participants' answers to the Pizza and Brownie CDIF items. Table 3 contains the rubric used to assess participants' explanations provided for the Pie and Candy items.

Table 1

Rubric: Responses to the Decontextualization and Contextualization of Pizza, Brownie, Pie, and Candy CDFI items.

Score	Meaning
0	Blank or incorrect answer
1	Correct Answer

Note. CDFI = Contextualization and Decontextualization of Fractions Instrument

Table 2

Rubric: Participants' Responses to the Pizza and Brownie CDFI Instrument Items

Score	Meaning
0	No answer, blank, or answered none
1	Used words or diagrams/pictures with no clear understanding either conceptually or procedurally
2	Procedural with incorrect answer or unclear answer/understanding
3	Conceptual but incorrect answer or unclear answer/understanding
4	Procedural with correct answer without conceptual connection
5	Conceptual with correct answer without procedural connection

Note. CDFI = Contextualization and Decontextualization of Fractions Instrument

Table 3

Rubric: Explanation of the Pie and Candy Problems

Score	Meaning
0	No answer, blank, or answered none
1	Incorrectly explained word problem selection
2	Correctly explain word problem selection

Reliability and Validity of CDFI

The researcher created the CDFI for the purpose of analyzing a student's ability to identify and explain contextualization of fraction expressions and decontextualization of word problems. To improve the face, content, and construct validity of the instrument, the researcher conducted cognitive interviews with doctoral students and doctoral candidates in the mathematics education track of the school of education. During the cognitive interviews, the researcher had the doctoral students and candidates share aloud their thinking processes as they answered all the questions on the CDFI. During the cognitive interviews, the doctoral students and candidates also informed the researcher of any problems with the questions, such as confusion regarding what the question was asking them to do. The cognitive interviews helped the researcher further refine the questions to make the instrument less confusing for participants. The researcher also had members of the university's mathematics education faculty review the instrument to check for validity for the purposes of this study. The mathematics education faculty helped the researcher refine the set-up for each of the questions on the CDFI.

After the cognitive interviews, a pilot study was conducted on November 4, 2015 to ensure the reliability and the validity of the instrument. After running a factor analysis involving the data collected from the pilot study, the researcher found that the items on the CDFI had two factors where absolute values less than .22 were suppressed. The first factor was composed of the items found in Part A of the CDFI, and the second factor was composed of the items found in Part B of the CDFI. The researcher conducted a factor analysis on part A of the test and found that the four items had two factors. The first factors consisted of the two items on the CDFI that related to decontextualizing fraction multiplication word problems and the second factor consisted of the two items on the CDFI that related to decontextualizing fraction subtraction word problems. Because there were two factors for Part A of the test, the researcher randomly selected one of the fraction multiplication word problems and one of the fraction subtraction word problems to create a new Part A. The researcher then completed a factor analysis of the items found in Part B of the CDFI and found that all four questions combined into one factor. The researcher randomly selected two items, one with a multiplication expression and one with a subtraction expression from the original part B of the CDFI to create the new Part B.

Data Analysis

Quantitative Data

This study used a quasi-experimental design as the two sections of the mathematics for elementary school teachers content course were not randomly selected. For the quantitative part of the study, a t-test with paired means was used to analyze the data. A t-test helped (a) to determine if there was a statistically significant difference between the ability of the participants to identify decontextualized word problems that contained fractions into expressions and (b) to identify contextualized fraction expressions in word problems before and after the participants studied the fraction unit in the mathematics for elementary school teachers content course.

Participants' ratings were judged to be reliable for all items of the CDFI given to the undergraduate education students based on the pilot study, with a reliability of .55.

For the qualitative part of the study the researcher conducted think aloud interviews with 11 participants selected from the 52 elementary school pre-service teachers who completed both the pre- and post-tests. Prior to selection of the participants to be interviewed, the researcher analyzed the CDFI pre-and post-test scores. Ability levels were determined for all participants as to (a) the identification of the contextualization of fraction expressions, (b) the decontextualization of fraction word problems, and (c) solving the fraction word problems. When possible, the researcher randomly selected participants from each level of fraction decontextualization and contextualization ability to participate in the interviews. Each of the interviews was video recorded, and the researcher took notes during the interview process. All interviews were transcribed at the conclusion of the interviews.

Qualitative Data

The interview process consisted of having the participants use a think aloud process. The think aloud process required that the participants share aloud the thoughts that were in their heads as they solved the problems (Young, 2005). During this process, the researcher generally did not speak unless the participant was quiet for an extended period of time, as advocated by Young. The use of the think aloud process helped to reduce the problem of trying to remember "why" or "how" the participant solved the fraction word problems (Ericsson & Simon, 1980). The think aloud process assisted in assessing participants' comprehension (Wade, 1990) in regard to solving fraction word problems and the process of decontextualizing the fraction word problems into expressions. During the think aloud interviews, the researcher took notes on what participants said as well as how they solved the problems. The notes taken during the think aloud process aided the researcher as the interviews were transcribed.

After all interviews were conducted and transcribed, the researcher searched through the transcripts and identified common themes within the participants' responses. Using the problem posing and mathematical knowledge for teaching (MKT) domains of common content knowledge (CCK) and specialized content knowledge (SCK) as frameworks, a protocol analysis was used to analyze possible themes. The use of protocol analysis allowed the researcher to evaluate participants' thought processes and search for patterns (Kuusela & Paul, 2000). First, the problem posing framework allowed the researcher to examine the responses as they related to common themes. Second, the domains of the MKT allowed the researcher to determine the participants demonstrated the knowledge necessary to be an effective teacher when teaching fractions.

Protocol analysis, as defined by Austin and Delaney (1998), is a method that can be used to obtain information about what people are thinking while they are performing a specific task. The role of the researcher during the interview process is only to prompt the participant if they become silent. The analysis of the interviews occurs by having the researcher group each of the interviews into sections (Austin & Delaney, 1998). In the current study, the researcher grouped all of the transcribed interviews by the eight main questions asked during the interview. For each individual question during the think aloud interview, the researcher reviewed the responses of the participants and counted the number of times a similar response was given by the participants.

Using the transcripts of the interviews, the researcher searched for themes, determined by common responses, among all participants' responses to the eight main questions asked during the interview involving the contextualization of fraction expressions and the decontextualization of fraction word problems. Each set of common responses was analyzed using the problem posing and MKT domains of CCK and SCK to determine if the 11 participants' responses aligned with the research.

Review of the common responses was influenced by previous research. Analysis of the think aloud interview portion involving problem posing was influenced by the discussion of problem posing vocabulary noted within the research of Capraro and Joffrion (2006) and of Dunlap and McKnight (1976). Assessment related to the importance of the order of fractions related to the research of Capraro and Joffrion and Tobias (2009). Recognition of participant strengths or misconceptions about fraction problems within the present study were similar to those previously described by Dixon and colleagues (2014) and McAllister and Beaver (2012).

When searching for common interview responses for the last two questions on decontextualization and solving word problems, the researcher interpreted her findings by referring to the decontextualization section of the literature review from Capraro and Joffrion (2006) and Tobias (2009) which related to the order that the participants wrote the expression for the given fraction word problems. When decontextualizing word problems in the present study, participants noted the reliance on vocabulary to identify the type of problem as previously noted in the research of Capraro and Joffrion (2006) and Dunlap and McKnight (1978). This difficulty of identifying and decontextualizing multiplication word problems also correlated with the research of Lou, Lo, and Leu (2011) and Toluk-Ucar (2009).

While analyzing all questions in the interview, the researcher was also influenced by the literature review section of the Ball et.al (2008) article on mathematical knowledge for teaching for the domains of the CCK and SCK. Examining the responses relating to the ability to solve the problems falls under the domain of CCK as does being able to decontextualize the word problems. Being able to contextualize fraction expressions falls under the domain of SCK along with the ability to recognize and explain the correct vocabulary used when contextualizing fraction expressions and decontextualizing fraction word problems.

As the researcher reviewed the responses to the main questions and found common responses, the common responses were highlighted in the transcripts. After reviewing all the responses from the participants for each question, three main themes emerged.

Summary

The design of the methodology, questions, hypothesis, population, and sample for this research study have been discussed in this chapter, and the data collection and the instrumentation used in the study have also been detailed. Chapter 4 contains a presentation of the results of the study. The final chapter consists of a discussion of the findings along with the implications, recommendations, and limitations.

CHAPTER 4: RESULTS

Introduction

This chapter contains the results of the study. The demographics of participants are presented, and the data obtained in observations from the visits to the classroom are described and discussed. This information was included to provide the general context of the study.

The remainder of the chapter has been organized to respond to the four research questions which guided this mixed methods study. Research Questions 1 and 2 were related to the quantitative data obtained from the Contextualization and Decontextualization of Fractions Instrument (CDFI). For the quantitative portion of the study, the results of the pre- and post-test obtained by using the CDFI were analyzed using a t-test. Research Questions 3 and 4 were answered using the qualitative data collected in the think aloud portion of the study. The think aloud interviews permitted the researcher to examine the selected participants' abilities to make sense of contextualizing fractions into word problems and decontextualizing fraction word problems into expressions. Twelve themes were identified based on the data from the interviews.

Demographics of Participants

The participants in the current study consisted of undergraduate elementary school pre-service teachers who were enrolled in the content course, Mathematics for Elementary School Teachers. Demographics were obtained through self-reports of the participants. Table 4 contains demographic characteristics (i.e., ethnicity, gender, and year in school) for the 52 elementary school pre-service teachers who completed both the pre- and post-tests. The percentages of Hispanic/Latino (19.2%), Black/African American (13.5%), and Asian American (1.9%) in the sample were low. However, the percentage of women represented in the sample was high

(94.2%). These percentages were comparable to 2014 statistics representative of nationwide employment trends. The U.S. Bureau of Labor Statistics (2016) provided the following percentages as evidence of these trends: Hispanic/Latino (8.5%), Black/African American (10.8%), or Asian American (2.5%). Also, the U.S. Bureau of Labor Statistics (2016) indicated that 80.7% of the elementary and middle school teachers employed in the United States in 2015 were women. Most of the elementary school pre-service teachers in the sample were in their junior year (76.9%).

Table 4

Demographics of Participant Population (N = 52)

	Frequency	Percentage
Ethnic Background		
White non-Hispanic	32	61.5
Hispanic or Latino	10	19.2
Black or African American	7	13.5
Asian or Asian American	1	1.9
Biracial or Multiracial	2	3.8
Gender		
Male	3	5.8
Female	49	94.2
Year in School		
Sophomore	12	23.1
Junior	40	76.9

Conceptually-based Classroom Observation

During the visits to the participating classrooms, the following observations were made. During each class meeting, the pre-service teachers were given a problem in context to work from the course pack. While they worked on the course pack problems, the instructor moved around the classroom and talked with different groups asking questions, helping them think about the problems in different ways, and questioning them to guide their understanding of the problems. After giving the students time to work through a few problems in their small groups, the instructor led a whole class discussion about the problems. Several students presented their strategies and the problems, including their solutions, one at a time. For each of the problems assigned, the class discussed strategies for solving the problems.

After five days of working with different fraction concepts, the students began work with fraction operations. A list of the activities and the amount of time spent on the activities is presented in Table 5. The operations of fraction addition and fraction subtraction were discussed for two days and were discussed together. While working on fraction operations, the instructors had the students work on the "Pizza Parlor Addition" and "Pizza Parlor Subtraction" pages from the course pack. These pages asked the students to write word problems for the given fraction expressions and solve the problems. While the students worked on creating word problems, the instructor moved around the classroom and discussed the different problems that the students were creating at the different tables. The instructors then had students share and discuss some of the word problems that they had created.

Table 5:

Classroom Activities and Days of Activities

Activity	Number of days on topic
Define Fractions Based on Whole; Compose and Decompose Fractions	1
Unitizing	1
Relational Thinking:	1
Comparing Fractions	1
Ordering Fractions	1
Addition and Subtractions with Fractions	2
Multiplication with Fractions	1
Division with Fractions	2
Total	10

The class as a whole discussed whether the problem made sense, and whether the problem matched the given expression. During this discussion the students also commented that many of the problems created involved food. This discussion was followed by another discussion about using standard units to make it easier to write the word problem instead of using non-standard units. The instructor then guided the discussion toward the problems that occurred with writing fraction subtraction word problems. The discussions included the issue of identifying the whole, noting that if one refers back to what was started with, a multiplication problem will be created instead of a subtraction problem.

After discussing the issue with writing fraction subtraction word problems, the instructor then had students share their created word problems with the class. As each student shared a created word problem, the class would discuss if the word problem matched the expression given. If the problem did not match, the class discussed how the problem could be changed so that it would match the given expression. The class then began working the "Pizza Parlor Situation 1" activities. These activities had the elementary school pre-service teachers solve word problems and write the expression that matched the given word problem. As with all the activities, the elementary school pre-service teachers worked in groups while the instructor moved around the classrooms, discussing the problems with the different groups. After the groups had an opportunity to work on the problems, the class had a discussion about each of the problems. The discussion included what operation occurred in each of the four problems, the expression for each problem, the solution for each problem, and how the solution was obtained.

After discussing addition and subtraction fraction word problems, the classes moved on to discuss fraction multiplication word problems. The students began this lesson by looking at "Pizza Parlor Situation 2" in the course pack and worked on the problems found on the page. While the students worked on the problems, the instructor walked around the classroom and queried the different groups as to their approaches to solving the problems. While working with the different small groups (and only if they were struggling with a word problem), the instructor would guide them to discover strategies that could help them solve the problem. Following the group work, the class as a whole discussed the problems and shared the different solution strategies that were used to solve the problems. The students also wrote the expression that matched the word problem. As the students shared their solution strategies and expressions with the class, they discussed the importance of the order in which the fractions were written in the expression. The class decided that although order did not matter for the solution of a multiplication problem, it did matter when writing an expression to match a given word problem or writing a word problem that matched a given expression.

After discussing the “Pizza Parlor Situation 2” problems, the students moved on to the “Pizza Parlor Multiplication” page in their course pack. On this page, they were asked to create word problems that matched the given fraction multiplication expression, and to solve the problems. After the students spent some time writing and solving the fraction multiplication problems in their small groups, the instructor had several students share their word problems with the whole class. As the students shared their fraction multiplication word problems, the whole class discussed whether their classmates had written a fraction multiplication word problem that matched the given expression. If it was decided that the shared word problem did not match the given expression, the class discussed how the problem could be changed so that it would. During this discussion, the students also talked about the differences between writing fraction subtraction word problems and fraction multiplication word problems. This was an important discussion because it exposed the students to the common misconceptions that can occur when writing fraction word problems.

T-test Results

This section includes the paired-sample t-test results used to analyze the quantitative data related to Research Questions 1 and 2. Paired-sample t-test analyses were used to determine if there were statistically significant differences between the pre-and post-test results of the participants as related to the overall scores and each of the first two research questions. The rubric displayed in Table 1 was used to evaluate the answers given for the decontextualization of fraction word problems and the contextualization of fraction expressions of the four problems on the CDFI. The rubric shown in Table 2 was used to evaluate how the participants solved the Pizza and Brownie problems on the CDFI. The rubric presented in Table 3 was used to evaluate how the participants responded to the explanation part of the pie and candy problem on the CDFI.

A paired-sample t-test was calculated to compare the pre-test overall scores to the post-test overall scores of the CDFI. There was a statistically significant difference (t = -5.267, df = 51, $p < .005$) in the scale between the participants' pre-test overall scores ($\bar{x}$ = 8.23, s = 3.02) and post-test overall scores ($\bar{x}$ = 11.48, s = 3.75). The 95% confidence interval suggests that the true mean difference was included in -4.36 < mu < -.1.95. Because the overall pre- and post-test scores were statistically significant, each individual test question on the CDFI was analyzed using the Bonferroni approach to correct for alpha error. A table listing the paired sample t-test can be found in Appendix G

Research Question 1

Did undergraduate elementary pre-service teachers' ability to decontextualize and explain fraction word problems improve following a unit on fractions in a mathematics for elementary school teachers' content course?

This question was addressed by two of the problems on the CDFI, the Pizza and the Brownie problems. The questions in the CDFI pre-test version (Appendix D) and the CDFI post-test version (Appendix E) were the same, but the fractions in the questions were changed. The Pizza problem on the CDFI was a fraction multiplication word problem, and the Brownie problem was a fraction subtraction word problem. For both of these problems, the participants were asked to (a) read the given word problem; (b) solve the given word problem; and (c) select the correct expression that matched the given word problem from a given list of expressions.

The researcher first evaluated the participants' abilities to identify the correct decontextualizations of the fraction word problems for both the Pizza and the Brownie problem. A paired-sample t-test was performed to compare the means of the pre- and post-test scores of the combined score for the Pizza and Brownie problem where participants identified the expression for the given fraction word problem. There was a statistically significant difference ($t = -5.267$, $df = 51$, $p < .005$) in the scale between the participants who correctly identified the expressions on the Pizza and Brownie problems on the pre-test ($\bar{x} = 6.38$, s = 2.44) and those who correctly identified the Pizza and Brownie problems on the post-test ($\bar{x} = 8.54$, s = 2.87). The 95% confidence interval suggests that the true mean difference was included in $-3.062 < mu < -1.25$.

Since the pre- and post-test scores of the combined scores for the Pizza and Brownie problem was statistically significant the researcher then evaluated each individual question on the CDFI. The first question that the researcher evaluated was the Pizza problem, which assessed the participants' abilities to identify the correct decontextualizations of the fraction multiplication word problem into the correct expression. On the pre-test version, none of the participants were able to identify the correct decontextualization of the Pizza problem; on the post-test version, however, 25% were able to do so. Frequency charts displaying this information are found in Appendix H.

A paired-sample t-test was performed to compare the means of the pre- and post-test scores of the Pizza problem where participants identified the expression for the given multiplication word problem. There was a statistically significant difference ($t = -4.123$, $df = 51$, $p < .005$) in the scale between the participants who correctly identified the expression on the Pizza problem on the pre-test ($\bar{x} = 0$, s = 0) and those who correctly identified it on the post-test ($\bar{x} = .25$, s = .437). The 95% confidence interval suggests that the true mean difference was included in $-.372 < mu < -.128$.

The researcher also evaluated participants' abilities to solve the given fraction multiplication word problem. A total of 34.6% of the participants were able to correctly solve the Pizza problem on the pre-test version. The percentage increased to 53.8% of participants who were to correctly solve the Pizza problem on the post-test. When looking at the pre-test results, 7.7% of the participants solved the problem correctly using a procedure; and 26.9% of the participants correctly solved the Pizza problem conceptually. On the post-test, 1.9% of the participants solved the problem correctly using a procedure, and 51.9% of the participants correctly solved the Pizza problem conceptually. Frequency charts of this information can be found in Appendix H.

A paired-sample t-test was calculated to compare the mean pre-test score to the mean for the post-test scores of the Pizza problem where students explained how to solve the given fraction multiplication word problem. There was a statistically significant difference ($t = -2.927$, $df = 51$, $p < .005$) in the scale between the participants who correctly explained the solution for the Pizza problem on the pre-test ($\bar{x} = 2.88$, s = 1.567) and those who correctly explained it on the post-test ($\bar{x}$ = 3.60, s = 1.648). The 95% confidence interval suggests that the true mean difference was included in -1.199 < mu < -.224.

The researcher examined the participants' abilities to identify correct decontextualization of the fraction subtraction word problem into the correct expression. On the pre-test version, 59.2% of the participants were able to identify the correct decontextualization of the Brownie problem; on the post-test version, the percentage increased to 75%. Frequency tables with this information can be found in Appendix H.

A paired-sample t-test was calculated to compare the means for the pre- and post-test scores of the Brownie problem where students identified the expression for the given fraction subtraction word problem. There was not a statistically significant difference (t = -.724, *df* = 51, *p* = .472) in the scale between the participants who correctly identified the expression on the Brownie problem on the pre-test ($\bar{x}$ = .69, s = .466) and those who correctly identified it on the post-test ($\bar{x}$ = .75, s = .437). The 95% confidence interval suggests that the true mean difference was included in -.218 < mu < .102.

After finding that the decontextualization of the brownie problem was not statistically significant, the researcher examined the participants' ability to solve the given fraction subtraction word problem. A total of 42.3% of the participants were able to correctly solve the Brownie problem on the pre-test. That percentage increased to 67.3% on the post-test version. In reviewing the pre-test results, 32.7% of the participants solved the Brownie problem correctly using a procedure, and 9.6% of the participants correctly solved the problem conceptually. On the post-test, however, only 9.6% of the participants solved the Brownie problem correctly using a procedure, and 57.7% of the participants correctly solved the problem conceptually. Frequency charts of this information can be found in Appendix H.

A paired-sample t-test was calculated to compare the means for the pre- and post-test scores of the Brownie problem where students explained how to solve the given fraction subtraction word problem. There was a statistically significant difference ($t = -4.111$, $df = 51$, $p < .005$) in the scale between the participants who correctly explained the solution for the Brownie problem on the pre-test ($\bar{x} = 2.81$, $s = 1.496$) and those who correctly explained it on the post-test ($\bar{x} = 3.94$, $s = 1.474$). The 95% confidence interval suggests that the true mean difference was included in $-1.689 < \mu < -.580$.

Research Question 2

Did undergraduate elementary pre-service teachers' ability to contextualize and explain fraction expressions improve following a unit on fractions in a mathematics for elementary school teachers' content course?

This question was addressed by two of the problems on the CDFI, the pie problem and candy problem. The questions in the CDFI pre-test version (Appendix D) and the CDFI post-test version (Appendix E) were the same, but the fractions in the questions were changed. The Candy problem on the CDFI was a fraction multiplication word problem, and the Pie problem was a fraction subtraction word problem. For both of these problems, participants were asked to (a) look at the given fraction expression, (b) select the correct word problem that matched the given word problem from a given list of word problems, and (c) explain how they knew that they had selected the correct word problem from the provided list.

When considering these two problems from the CDFI, the researcher first analyzed the participants' abilities to identify the correct contextualizations of the fraction word problems for both the Pie and the Candy problem. A paired-sample t-test was performed to compare the means of the pre- and post-test scores of the combined score for the Pie and Candy problem where participants identified the correct word problem from a list of given word problems that matched the given fraction expression. There was a statistically significant difference (t = -4.262, *df* = 51, $p < .005$) in the scale between the participants who correctly identified the word problems on the Pie and Candy problems on the pre-test ($\bar{x}$ = 1.942, s = 1.145) and those who correctly identified the word problems on the Pie and Candy problems on the post-test ($\bar{x}$ = 2.942, s = 1.552). The 95% confidence interval suggests that the true mean difference was included in -.471 < mu < -.529.

Since the pre- and post-test scores of the combined scores for the Pie and Candy problem was statistically significant the researcher then evaluated each individual question on the CDFI. The first question that the researcher evaluated was the Pie problem, which assessed the participants' ability to identify the correct fraction subtraction word problem from the provided list of word problems. A total of 23.1% of participants were able to identify the correct contextualization of the Pie problem on the pre-test, and 55.8% were able to do so on the post-test version. Frequency charts with this information can be found in Appendix H.

A paired-sample t-test was calculated to compare the means for the pre- and post-test scores of the Pie problem where students identified the fraction subtraction word problem that matched the given fraction subtraction expression. There was a statistically significant difference (t = -3.636, *df* = 51, $p < .005$) in the scale between the participants who correctly identified the word problem for the Pie problem on the pre-test ($\bar{x}$ = .23, s = .425) and those who did so on the post-test ($\bar{x}$ = .56, s = .502). The 95% confidence interval suggests that the true mean difference was included in -.507 < mu < -.146.

The researcher also analyzed participants' abilities to explain how they knew that they had selected the correct word problem that matched the provided fraction subtraction expression for the pie problem. On the pre-test version, only 7.7% of the participants were able to correctly explain their word problem selection for the Pie problem; but on the post-test version, 40.4% were able to provide a correct explanation. Frequency charts with this information are found in Appendix H.

A paired-sample t-test was calculated to compare the means for the pre- and post-test scores of the Pie problem where students explained how they knew they selected the correct subtraction word problem. There was a statistically significant difference (t = -2.911, *df* = 51, *p* < .005) in the scale between the participants who could correctly explain how they knew they selected a subtraction word problem for the Pie problem on the pre-test ($\bar{x}$ = .92, s = .479) and those who could provide a correct explanation on the post-test ($\bar{x}$ = 1.25, s = .711). The 95% confidence interval suggests that the true mean difference was included in -.552 < mu < -.101.

The researcher then analyzed the participants' answers to the Candy problem, which examined participants' ability to identify the correct fraction multiplication word problem from a list of word problems. On the pre-test version, only 1.9% of the participants were able to identify the correct contextualization of the Candy problem. This percentage increased to 23.1% on the post-test version 23.1%. Frequency charts with this information can be found in Appendix H.

A paired-sample t-test was calculated to compare the means for the pre- and post-test scores of the candy problem where students identified the fraction subtraction word problem that matched the provided fraction multiplication expression. There was a statistically significant difference (t = -3.335, *df* = 51, *p* < .005) in the scale between the participants who correctly identified the word problem for the Candy problem on the pre-test ($\bar{x}$ = .02, s = .139) and those who did so on the post-test ($\bar{x}$ = .23, s = .425). The 95% confidence interval suggests that the true mean difference is included in -.339 < mu < -.084.

The researcher further analyzed the participants' abilities to explain how they knew that they had selected the correct word problem that matched the provided fraction multiplication expression for the Candy problem. Only 5.8% of the participants were able to correctly explain their word problem selection for the Candy problem on the pre-test, but 13.5% were able to do so on the post-test. Frequency charts with this information can be found in Appendix H.

A paired-sample t-test was calculated to compare the means for the pre- and post-test scores of the Candy problem where students explained how they knew they selected the correct word problem. There was not a statistically significant difference (t = -1.413, *df* = 51, *p* = .164) in the scale between the participants who could correctly explain how they knew they selected the correct multiplication word problem for the Candy problem on the pre-test ($\bar{x}$ = .77, s = .546) and on the post-test ($\bar{x}$ = .90, s = .603). The 95% confidence interval suggests that the true mean difference was included in -.326 < mu < .057.

Interview Results

This section presents the results of the interviews conducted with a subset of the population. The interviews were conducted to gather qualitative data to be used in responding to

Research Questions 3 and 4. After completing the pre- and post-testing, the tests were scored and 52 participants who completed both the pre- and post-tests were placed into six groups of fraction decontextualization and contextualization ability. As previously explained in Table 5 and an accompanying narrative, the groups consisted of Low-Low (LL), Low-Medium (LM), Low-High (LH), Medium-Low (ML), Medium-Medium (MM), and Medium-High (MH). The CDFI was composed of four questions with each question having two parts for a total of eight possible responses. The low group consisted of participants who were able to answer zero to two questions correctly. The medium group was composed of participants who were able to answer three to five questions correctly. The high group contained participants who were able to answer six to eight questions correctly. The first part of the group name was assigned based on the number of correct responses on the CDFI pre-test version, and the second part of each group name was assigned based on the number of correct responses on the CDFI post-test version.

A total of three themes, wording of the problem, order of the fractions, and misconceptions, were identified using (a) the Domains of Mathematical Knowledge for Teaching (Ball et al., 2008) and (b) problem posing as frameworks related to the qualitative data. The number of participants in each of the six established fraction decontextualization and contextualization ability groups is displayed in Table 6.

Table 6

Pre-test/Post-test Score Fraction Decontextualization and Contextualization Ability Groups by Participants

Fraction Decontextualization and Contextualization Ability Groups	Participants (N = 52)
Low-Low (LL)	15
Low-Medium (LM)	14
Low-High (LH)	10
Medium-Low (ML)	3
Medium-Medium (MM)	9
Medium-High (MH)	1

A total of 11 participants were selected for interview. All 11 of the participants were interviewed by the researcher. During the interview process each of the participants was asked a series of questions which are shown in their entirety in Appendix F.

A total of three themes emerged from the responses of the interview participants. The first theme that emerged in during the think aloud interviews was the importance of the wording in the problem. The second theme that emerged was about the order of the fractions. The third theme that emerged was about misconceptions that the elementary school pre-service teachers had with fractions. The themes were identified by the frequency of similar responses participants provided when questioned during the interview process. For example, during the interview, participants were asked to identify the operation of the Cake problem. All 11 of the participants correctly identified the Cake problem as subtraction and were able to correctly create the expression that matched the given word problem. Table 7 displays the six fraction decontextualization and contextualization ability groups, the number of participants who participated in the think aloud interviews from each of the groups, and the assigned identification numbers.

Table 7:

Pre-test/Post-test Scores Identified Participant Groups and Number of Interviews

Fraction Decontextualization and Contextualization Ability Groups	Participants Interviewed	Participant Identification Numbers
Low-Low (LL)	2	8 and 44
Low-Medium (LM)	2	11 and 40
Low-High (LH)	2	33 and 56
Medium-Low (ML)	2	3 and 16
Medium-Medium (MM)	2	5 and 7
Medium-High (MH)	1	60

Research Question 3

In what ways do pre-service teachers make sense of decontextualizing fraction word problems?

This question was addressed by the participants during the think aloud interviews when they responded to the last two problems presented to them. Participants were asked to solve a given fraction word problem and write the expression that matched the given word problem. Based on the verbal responses of the participants, three main themes were identified. Following is a presentation of three themes as they related to this research question.

Theme 1 Related to Think Aloud Interviews

The fraction word problem given to the participants for the third problem was: David brings $\frac{3}{4}$ of a pan of brownies to school to share with his classmates. If David's classmates eat $\frac{1}{3}$ of the brownies, how much of a whole pan of brownies did David's class eat? For this problem, seven of the 11 participants correctly identified the brownie problem as multiplication. The participants who correctly identified the word problem as multiplication stated that they knew that the problem was a multiplication problem because the class ate $\frac{1}{3}$ of the brownies, and the question asked how much of a pan did the class eat? The following are some examples of the participants' responses:

Participant # 44 (LL):

"They ate only $\frac{1}{3}$ of the pan that they had."

Participant # 33 (LH):

"Because the class is eating $\frac{1}{3}$ of the leftover $\frac{3}{4}$."

Participant # 60 (MH):

"I know that it is a multiplication problem because it is asking how much of a pan of brownies that the class ate?"

Of the four participants who incorrectly identified the Brownie problem as subtraction Participant # 8 (LL) and Participant # 40 (LM) solved the problem as if it was a multiplication problem. Participant #3 (ML) and Participant #16 (ML) solved the problem as if it were a subtraction problem.

The last question of the interview was a fraction subtraction word problem about cake. The fraction word problem given to the participants was: John left $\frac{5}{6}$ of a cake on the kitchen counter. His dog came in and ate an amount equal to $\frac{1}{3}$ of the whole cake before John chased him from the room. How much of the cake is left after the dog left the room? All 11 of them were able to correctly identify that the problem was a subtraction problem and were able to write an expression that matched the given word problem. When asked how they knew it was a subtraction problem, the following two responses were obtained related to this theme:

Participant # 3 (ML):

> "There was $\frac{5}{6}$ of a cake that was left, and the dog came and ate or took away $\frac{1}{3}$ of that whole cake."

Participant # 5 (MM):

> "Because the dog ate $\frac{1}{3}$of the whole cake."

Although all of the participants correctly created the expression, two of the participants were unable to correctly solve this problem. Participant # 60 (MH) took $\frac{1}{3}$ of each of the $\frac{1}{6}$ size pieces and counted the pieces that were not shaded and said that 10 of the 15 shaded pieces were left. Participant # 40 (LM) worked on the problem for several minutes, but finally stated:

> "I'm not sure how to do this one."

<u>Theme 2 Related to Think Aloud Interviews</u>

The second theme of the order of fractions can be seen by the responses of the participants who were able to identify the brownie problem as a multiplication word problem. Of the seven participants who correctly identified the word problem as multiplication, four wrote the expression for this problem as $\frac{3}{4} \times \frac{1}{3}$. When these participants were asked how they knew that they wrote an expression that matched the given fraction word problems, they shared the following four responses:

Participant # 11 (LM):

> "I kind of just did it how it was."

Participant # 7 (MM):

> "You're starting with $\frac{3}{4}$ of a pan of brownies, and the amount that David's classmates ate is $\frac{1}{3}$."

Participant # 5 (MM):

> "Because you start out with the $\frac{3}{4}$ and they eat $\frac{1}{3}$."

Participant # 60 (MH):

> "I knew that he brought ¾ of a pan of brownies and I was trying to figure out $\frac{1}{3}$ of $\frac{3}{4}$, so . . . I wrote $\frac{3}{4}$ first."

<u>Theme 3 Related to Think Aloud Interviews</u>

Four of the 11 participants incorrectly identified the brownie problem as subtraction and wrote the expression for this problem as $\frac{3}{4} - \frac{1}{3}$. The participants felt that they needed to take away or subtract what the class ate from the brownies brought to school. When these four participants were asked how they knew it was a subtraction word problem, they shared the following responses:

Participant # 8 (LL):

"Because you're given a number and told that $\frac{1}{3}$ of the original number was being taken away so you know that that is subtraction."

Participant # 40 (LM):

"I was talking about a whole pan of brownies and not leftover brownies."

Participant # 3 (ML):

"His classmates ate $\frac{1}{3}$ of the brownies so they're taking away $\frac{1}{3}$. So you're trying to figure out how much of a whole pan of brownies is left."

Participant # 16 (ML):

"They took away from what he had in, like, total already, and then it asked how much did his classmates eat?"

Research Question 4

In what ways do pre-service teachers make sense of contextualizing fraction expressions?

This research question was addressed through a think aloud interview process. During the interview process, the selected participants were asked to create a word problem about pizza for a given fraction expression. Following this word problem creation exercise, the participants were interviewed to obtain clarification of their thinking. Based on the written word problems and the verbal responses of the participants, three main themes were identified. Following is a presentation of three themes as they related to this research question.

Theme 1 Related to Think Aloud Interviews

The first theme to emerge during the think aloud interviews was the importance of the wording in the problem. This theme was first observed during the interview when the participants were given the expression $\frac{3}{5} \times \frac{5}{6}$ and were asked to create a word problem about pizza that matched the given expression. Of the 11 participants, seven were able to create a fraction multiplication problem about pizza. When reviewing the responses of the seven participants who correctly created a multiplication word problem, the importance of wording began to emerge. Two of the 11 participants responded that the question in the word problem needed to ask how much of the whole was used:

Participant # 40 (LM):

"How much of it did you use?"

Participant # 60 (MH):

"In the problem, the question's asking how much of the entire pizza did he eat".

The seven participants who created a multiplication problem, in the think aloud interviews, also explained how they knew that they had written a multiplication word problem, the following five responses were obtained from five participants:

Participant # 40 (LM):

"When it's subtraction it's how much do I have left. When it's multiplication it's like how much of it did you use?"

Participant # 56 (LH):

"They both kind of sound the same with subtractions one if you take from the whole, but I'm taking from the leftover."

Participant # 33 (LH):

"You are taking pieces out of those leftovers instead of out of a whole."

Participant # 7 (MM):

"The word problem itself is easy to tell that it is a multiplication problem because she's taking a certain amount of the pizza that was already left over."

Participant # 60 (MH):

"We want to know how much he ate. Since we're not, or I should say, how much of a pizza did Jim eat? So if I'm only referring to the leftover pizza in the problem, the question's asking how much of the entire pizza did he eat".

The second expression given to the participants in the think aloud interview was $\frac{3}{4} - \frac{1}{2}$. Of the 11 participants five were able to correctly create a problem about pizza that matched the given subtraction expression. Several of the participants expressed the idea that that one begins with $\frac{3}{4}$ and is taking $\frac{1}{2}$ of a pizza away. The following two responses were obtained:

Participant # 44 (LL):

"She has $\frac{3}{4}$ of a pizza and then I'm taking away like… half of a pizza."

Participant # 7 (LM):

". . . started with $\frac{3}{4}$ of a pizza leftover, and then he. . . eats half of a pizza."

Theme 2 Related to Think Aloud Interviews

The second theme that emerged, when analyzing the responses of the think aloud interview participants, was the order that the fractions in the problems. When the participants were asked how they knew that they had placed the fractions in the correct order in the word problem for the expression $\frac{3}{5} \times \frac{5}{6}$, they conveyed the idea that the expression given means taking $\frac{3}{5}$ of $\frac{5}{6}$. The following two responses were obtained from the participants:

Participant # 60 (MH):

"You have $\frac{5}{6}$ and Jim ate $\frac{3}{5}$ of the pizza."

Participant # 40 (LM):

"It's like $\frac{3}{5}$ of $\frac{5}{6}$, so $\frac{5}{6}$ is what you have."

When reviewing the word problems created for the expression $\frac{3}{5} \times \frac{5}{6}$ by the 11 participants in the think aloud interview, seven of the participants created a word problem that reflected the operation of multiplication. Of those seven word problems, only four of them matched the expression $\frac{3}{5} \times \frac{5}{6}$.

All the participants except for Participant 11 placed the numbers in the correct order when they wrote the word problem for subtraction expression $\frac{3}{4} - \frac{1}{2}$. When questioned about the order that the numbers were placed in the problem, Participant 11 responded that the order was correct because "$\frac{3}{4}$ is lesser than $\frac{1}{2}$". The participants who placed the fractions in the correct order in the created word problem responded that they knew the order was correct for the fractions in the word problem because they were taking $\frac{1}{2}$ from $\frac{3}{4}$. The following two responses were obtained from participants who correctly placed the fractions in the created subtraction word problem:

Participant # 33 (LH):

"We're subtracting $\frac{1}{2}$ from $\frac{3}{4}$. $\frac{3}{4}$ is my beginning quantity."

Participant # 3 (ML):

"You're subtracting $\frac{1}{2}$ from $\frac{3}{4}$, so you know that $\frac{3}{4}$ has to come first as it is the unit that you are taking away from."

Theme 3 Related to Think Aloud Interviews

The third theme that emerged, when the researcher analyzed the responses of the think aloud interview participants, was the misconceptions that the participants had about fractions. In a review of the responses of the participants who were not able to create a multiplication word problem for the expression $\frac{3}{5} \times \frac{5}{6}$ or were not able to correctly place the fractions in the correct order in the word problem they created a misconception emerged where participants placed the fractions in the word problem based on the order that they appeared in the given expression. Participants placed the first number in the expression first in the problem and the second fraction in the expression second in the word problem. When the three participants who incorrectly created a word problem to match the given expression were asked how they knew that the order they placed the numbers in the word problem were correct, they responded as follows:

Participant # 8 (LL):

> "I put the first number I was given first in the word problem multiplied by the second number that was given in the expression."

Participant # 3 (ML):

> "$\frac{3}{5}$ comes first and then $\frac{5}{6}$ is the second number."

Participant # 16 (ML):

> "Just do it in order. $\frac{3}{5}$ is the first number and it's being multiplied by $\frac{5}{6}$."

When the researcher analyzed the responses of the participants who wrote word problems for the expression $\frac{3}{4} - \frac{1}{2}$ that did not match the given expression; five of the six participants wrote a word problem that created the expression $\frac{3}{4} - \left(\frac{1}{2} \times \frac{3}{4}\right)$. The participants who created the word problem that matched the expression $\frac{3}{4} - \left(\frac{1}{2} \times \frac{3}{4}\right)$ were asked how they knew that they wrote a subtraction word problem. The participants stated that they had $\frac{3}{4}$ and they were subtracting half of it. Some examples of the participants' responses as to how they knew this was a subtraction problem follow:

Participant # 8 (LL):

> "We're taking $\frac{1}{2}$ away from $\frac{3}{4}$ of something. So, I know that the something and the taking away from is the $\frac{3}{4}$ of a pizza that Mike had, and I wrote that I'm eating half of it."

Participant # 11 (LM):

> "Because of the words 'how much pizza does he have left?'"

Participant # 5 (MM):

> "Because you are subtracting . . . the $\frac{3}{4}$ and you're subtracting half of that because you want to know how much he has left."

Participant # 60 (MH):

> "I know that Danielle had $\frac{3}{4}$ of a pizza. And Jim is eating $\frac{1}{2}$ of it."

Summary

In this chapter, to provide the general context of the study, the demographics of participants and observations from the visits to the classroom were described and discussed. Also, the analyses of the quantitative and qualitative data were included as they related to each of the research questions and the data collection methods. The results for the first two research questions were found to be of statistical significance. This indicated that the participants in this study's abilities to identify and explain (a) the decontextualization of fraction word problems and (b) the contextualization of fraction expressions improved after participation in conceptually-based learning experiences in a content course, Mathematics for Elementary School Teachers. Additionally, think aloud interviews were conducted, and three themes emerged which explained how elementary school pre-service teachers made sense of contextualizing fraction expressions and solving and decontextualizing fraction word problems presented to the participants during the interview process.

An in-depth analysis of the results of the study is provided in Chapter 5. Included is a discussion of the findings for each research question, implications, limitations, potential contributions, and recommendations for future research.

CHAPTER 5: CONCLUSION

Introduction

This chapter contains an analysis of the findings of the current research study. It presents a discussion of the findings for each of the four research questions which guided the study, implications of the current study, limitations, potential contributions, and recommendations for future research.

Discussion

The purpose of this study was to determine if undergraduate elementary school pre-service teachers' abilities to identify and explain decontextualizations of fraction word problems and contextualizations of fraction expression improved following conceptually-based learning experiences in a mathematics for elementary school teachers content course through the use of the CDFI and think aloud interviews. In this section, the researcher discusses the results and findings of this study and how they connect with the research literature related to this topic. The discussion was organized using the four research questions which guided the study.

1. Did undergraduate elementary school pre-service teachers' ability to decontextualize and explain fraction word problems improve following a unit on fractions in a mathematics for elementary school teachers' content course?
2. Did undergraduate elementary school pre-service teachers' ability to contextualize and explain fraction expressions improve following a unit on fractions in a mathematics for elementary school teachers' content course?
3. In what ways do elementary school pre-service teachers make sense of decontextualizing fraction expressions?

4. In what ways do elementary school pre-service teachers make sense of contextualizing fraction word problems?

Research Question 1

Did undergraduate elementary pre-service teachers' ability to decontextualize and explain fraction word problems improve following a unit on fractions in a mathematics for elementary school teachers' content course?

This question was addressed by both the Pizza problem and the Brownie problem on the Contextualization and Decontextualization of Fractions Instrument (CDFI). Each of these problems had two parts. The first part of the problem was designed to determine participants' abilities to identify the correct decontextualization of the presented fraction word problem into a fraction expression. The second part of each problem asked the participants to solve the given fraction word problem. The Pizza problem on the CDFI consisted of a fraction multiplication word problem, and the Brownie problem consisted of a fraction subtraction word problem. Appendices D and E contain the respective pre- and post-test version items.

In considering the data to respond to the first research question, the researcher initially analyzed the participants' responses on the CDFI pre-test version for both the Pizza and Brownie problems. The responses of the participants demonstrated a limited ability to decontextualize fraction word problems into an expression and solve the fraction word problems. These results support the findings of Utley and Reeder (2012) who found that pre-service teachers have limited understanding of fractions prior to beginning their mathematics education courses despite having taken several college level mathematics courses. While reviewing the results of the statistical analyses in the current study for the CDFI pre-test version, the researcher noted that none of the

52 participants were able to identify the correct decontextualization of the fraction multiplication word problem into a correct expression on the CDFI pre-test version, but 69% of them were able to successfully identify the decontextualization of the fraction subtraction word problem into the correct fraction expression on the CDFI pre-test version. Frequency charts can be found in Appendix H.

Utley and Reeder (2012) also stated that pre-service teachers need more opportunity to develop fraction knowledge. This led the researcher in the current study to analyze the CDFI post-test results. This showed a statistically significant difference in the elementary school pre-service teachers' abilities to identify and explain the decontextualization of fraction word problems after completing a mathematics for elementary school teachers' content course. This result seems to indicate that the conceptually-based learning experiences that occurred in the mathematics for elementary school teachers' content course provided opportunities for the participants to expand their knowledge with regard to decontextualizing fraction word problems. Similar to the findings of Sharp and Adams (2002), the conceptually-based learning experiences provided opportunities for the participants to discuss strategies for finding the correct solution to the problem and for explaining how to find the correct solution.

In the mathematics for elementary school teachers content course, participants were engaged in small group and whole class discussions while working on fraction word problems with real world contexts. By working on these problems in small groups and then discussing the problems in whole class discussions, the participants may have developed tools and different strategies as suggested by Windschitl (2002) to solve these problems. The participants in the current study, through the process of small group and whole class discussions, were able to increase their ability to decontextualize fraction word problems into fraction expressions. As shown in Table 8, participants were able to improve their ability to correctly solve the given fraction word problems as measured by the CDFI.

Table 8

CDFI Pre-/Post-test Comparison of Participants' Problem-solving Performance: Pizza and Brownie Problems

	Percentages	
Problem	Pre-Test	Post-Test
Pizza problem expression identification	0	25
Pizza problem solving	37	54
Brownie problem expression identification	69	75
Brownie problem solving	42	67

Note. CDFI = Contextualization and Decontextualization of Fractions Instrument (CDFI)

Research Question 2

Did undergraduate elementary pre-service teachers' ability to contextualize and explain fraction expressions improve following a unit on fractions in a mathematics for elementary school teachers' content course?

This question was addressed by the Pie and the Candy problems on the CDFI. Each of these problems had two parts. The first part was designed to examine participants' abilities to identify the correct contextualization of the given fraction expression into a fraction word problem. The second part queried the participants as to how they knew that they had selected the correct contextualized word problem that matched the given fraction expression. The Pie problem on the CDFI was a fraction subtraction expression, and the Candy problem was a fraction multiplication expression.

In reviewing the CDFI pre-test version results of the Pie and Candy problems, the researcher found that the scores of the participants showed limited ability to (a) identify the contextualization of a fraction expression into a word problem or to (b) explain how they knew that the word problem they selected matched the given fraction expression. The results of the pre-test matched the research findings of Luo and colleagues (2011) who found that pre-service teachers had difficulty selecting word problems that match a given fraction multiplication expression.

Upon analyzing the CDFI post-test, there was a statistically significant difference between the participants' abilities to identify the contextualization of a fraction expression into a word problem and to explain how they knew that the word problem they selected matched the given fraction expression. These results indicate that the conceptually based learning experiences in the mathematics for elementary school teachers content course helped the elementary school pre-service teachers in this study identify contextualizations of fraction expressions into word problems and explain their reasoning for their word problem selections. These findings support the findings of Luo and colleagues (2011) who found that pre-service teachers were able to identify word problems that matched a given fraction subtraction expression, and that pre-service teachers had difficulty identifying fraction multiplication word problems that matched a given expression. Although the elementary school pre-service teachers in the current study had statistically significant differences in their ability to identify the contextualization of fractions from expressions into word problems, they still struggled with identifying the multiplication word problems on the CDFI. This finding suggests that the elementary school pre-service teachers might need to spend more than one class session discussing fraction multiplication problems.

In discussing their findings, McAllister and Beaver (2012) stated that the pre-service teachers needed more practice creating word problems with real life context. The participants in the current study were given an opportunity in the mathematics for elementary school teachers' content course to create word problems and to discuss the created word problems in their small groups and in whole class discussions. As suggested by Capraro and Joffrion (2006), students need to understand the vocabulary used to create the desired word problem in order to be successful at decontextualizing them. In the current study, when students shared their created word problems with the whole class, the students discussed each of the presented word problems and why it matched or did not match the given expression, focusing on the language contained in the question. The discussions that occurred during the mathematics for elementary school teachers content course may have increased the participants' ability to correctly identify the word problem which matched the fraction expression and may have also contributed to participants' ability to explain their word problem selection as measured by the participants' results on the post-test.

Table 9

CDFI Pre-/Post-test Comparison of Participants' Problem-solving Performance: Pie and Candy Problems

Problem	Percentages Pre-Test	Post-Test
Pie problem word problem identification	23	56
Pie problem explanation	8	41
Candy problem word problem identification	2	23
Candy problem explanation	6	14

Note. CDFI = Contextualization and Decontextualization of Fractions Instrument (CDFI)

Research Question 3

In what ways do pre-service teachers make sense of decontextualizing fraction word problems?

This question was addressed by looking at the elementary school pre-service teachers' responses when answering the two questions in the problem solving section of the think aloud interviews. In this section, participants were asked to solve the given word problem and to write the expression that matched the given word problem. Question 3 of the interview was a fraction multiplication word problem about brownies, and question 4 was a fraction subtraction word problem about cake. By looking at and analyzing the responses to the decontextualization and problem solving portion of the think aloud interviews, the researcher was able to identify the three themes as they related to the fourth research question. The following paragraphs include a discussion of these themes.

Themes 1, as it related to the fourth research question, referred to the wording in the word problems that were given to the participants. The participants in the think aloud interviews explained that they knew the Brownie problem was multiplication because of the wording of the question which stated that the class "ate $\frac{1}{3}$ of the pan of brownies" and the participants knew that the Cake problem was a subtraction problem because of the wording of the question which stated, "…$\frac{1}{3}$ of a whole cake." The understanding of correct wording by the participants demonstrated that "vocabulary plays an important role in mathematics conceptualization" (Capraro & Joffrion, 2006, p. 161). This also implied that the participants had developed both common content knowledge (CCK) and specialized content knowledge (SCK) when they discussed the questions in the given word problem and used the wording to correctly decontextualize the fraction word problem. By being able to use the correct operation when writing the fraction expression, the participants demonstrated their CCK. When they wrote the fraction expression in the correct order, they demonstrated their SCK and recognized that the expression they created matched the word problem given to them.

Theme 2, as it related to the fourth research question, referred to the order that the participants placed the fractions when writing the expression that matched the given word problem. All 11 of the participants in the think aloud interviews were able to correctly identify and create an expression that matched the given word problem. This finding matched the findings of Luo and colleagues (2011), who compared fraction knowledge in pre-service teachers from the United States (N = 89) and Taiwan (N = 85). They found that both groups of pre-service teachers (N = 174) were able to correctly identify the word problem that matched the given fraction subtraction expression. This finding implies that the elementary school pre-service teachers have developed the CCK or the SCK needed to help their future students decontextualize fraction subtraction word problems.

Theme 3, as it related to the fourth research question, referred to misconceptions that the participants had about writing an expression that matched a given word problem. Similar to the findings of Carpenter and colleagues (1983) and Carey (1991) with first graders and Capraro and Joffrion (2006) with middle-school students, participants in the current study explained that they wrote the expression for the fraction multiplication word problem in the order that the fractions were presented in the problem. Tobias (2009) also found that the pre-service teachers in her study made similar mistakes when decontextualizing fraction multiplication word problems. This misconception implies that some of the participants have not developed the CCK or the SCK needed to help their future students decontextualize fraction multiplication word problems.

Similar to the findings of Toluk-Ucar (2009) who found that some pre-service teachers had difficulty decontextualizing fraction word problems into expression, participants in the current study demonstrated that they had difficulty identifying a fraction multiplication word problem. In addition, in their research, Luo and colleagues (2011), when they compared fraction knowledge in pre-service teachers from the United States (N = 89) and Taiwan (N = 85), found that the pre-service teachers had difficulty correctly identifying the word problem that matched the given fraction multiplication expression. Some of the participants in the current study, when considering the Brownie problem in the interview, mistakenly identified the operation as subtraction instead of multiplication. This theme may show that some of the participants need more practice in decontextualizing fraction word problems into expressions. This also implies that the participants in the interview did not have a strong CCK to identify the correct operation for the problem or a strong SCK to recognize that the expression they wrote did not match the given fraction word problem.

After the conceptually-based learning activities, each of the three themes demonstrated how the participants in this study made sense of fraction word problems and their decontextualization into expressions. Most of the themes identified in the problem solving section of the interview indicate that the participants had some understanding of solving fraction word problems and decontextualizing them into expressions. Most of the participants were able to correctly identify the brownie problem as multiplication and the cake problem as subtraction. Although the expression written by several of the participants for the brownie problem did not match the given word problem, the participants were able to find the correct solution to the problem.

Research Question 4

In what ways do pre-service teachers make sense of contextualizing fraction expressions?

This question was addressed by analyzing elementary school pre-service teachers' responses in the problem posing section of the think aloud interviews. The problem posing section occurred at the beginning of the interview and consisted of two expressions: (a) a fraction multiplication expression and (b) a fraction subtraction expression. During the problem posing section, participants were asked to construct a word problem about pizza to match the given fraction expression. The use of a real world situation when creating word problems helps students and pre-service teachers to increase their understanding of fractions (McAllister & Beaver, 2012).

Problem posing has been suggested as a way to obtain more information about students thinking (Alibali et al., 2009; Arikan & Unal, 2015; Osana & Royea, 2011) and identify any misconceptions that students might have (McAllister & Beaver, 2012). The researcher in the current study was given a clearer understanding of the thinking of the participants by reviewing the problems they constructed and by asking questions about those problems. By looking at and analyzing the responses during the problem posing section of the interview, the researcher was able to identify three themes as they related to the third research question. The following paragraphs include a discussion of these themes

Themes 1, as it related to the third research question, referred to the wording in the problems that were created by the participants. The participants in the current study, when trying to formulate a question about pizza that matched the given expression, discussed the importance of the phrasing of the question along with the correct way to write the word problem. These discussions aligned with comments made by Silver (1994) who stated that when creating word problems, the creator needs to ask the question about how to create the problem so that it can be solved.

Dunlap and McKnight (1978) and Capraro and Joffrion (2006) stated that students need to develop an understanding of the vocabulary in word problems. Students need to understand the vocabulary in a word problem and be able to decontextualize it into an expression or to take an expression and contextualize a word problem (Capraro & Joffrion, 2006). Teachers therefore, also need to have an understanding of the vocabulary necessary to contextualize expressions into word problems so that they will be able to assist their students to contextualize expressions (Dunlap & McKnight, 1978; Capraro & Joffrion, 2006). The participants in the current study demonstrated that they understood the vocabulary needed to create either a multiplication or a subtraction word problem. When discussing the multiplication problem, the students identified that one takes part of “the” pizza; whereas with subtraction, one takes part of “a” pizza. By demonstrating their understanding of the correct terminology to use in the creation of the fraction word problems, the participants were demonstrating that they should be able to help their future students in creating word problems from expressions or decontextualizing word problems into expressions.

Theme 2, as it related to the third research question, referred to the order of the fractions in the created word problems. When writing the word problems for the subtraction expression ten of the 11 participants placed the fractions in the correct order, while four of the 11 participants were able to correctly place the fractions in the multiplication word problems. This finding implies that some of the participants in the interview portion of the study understood that when creating a word problem to match a given expression, the fractions in the expression influence the placement of the terms in the word problem. This also implies that the participants in the current study had developed their SCK in relation to contextualizing the fraction expressions. By creating word problems that match the given fraction expression, the participants demonstrated their ability to create story problems that fit given operations. This falls under the domain of SCK.

Theme 3, as it related to the third research question, referred to misconceptions that the participants had about writing a word problem that matched a given expression. The participants, when asked how they knew the order was correct when writing a word problem for a fraction multiplication expression, stated that they placed the numbers in the order they appeared in the expression. This misconception was noted by Capraro and Joffrion (2006) who found that students, when asked to create word problems for given fraction expression, wrote the numbers in the order they appeared in the expression. This indicates that some of the interview participants did not see that although the solution might be the same for a multiplication problem, the expression for the word problem they created did not match the given expression. This misconception in the placement of the fractions in the created multiplication word problem may also be indicative of participants' need to devote more time to learning about contextualizing fraction multiplication expressions.

Another misconception observed among some of the interviewed participants occurred when the participants were asked to create a word problem for the expression $\frac{3}{4} - \frac{1}{2}$, and instead created a problem in the form of $\frac{3}{4} - \left(\frac{1}{2} \times \frac{3}{4}\right)$. This error mirrors the errors found by McAllister and Beaver (2012) and Dixon and colleagues (2014). This suggests that elementary school pre-service teachers may need more practice with writing fraction subtraction word problems as suggested by McAllister and Beaver (2012) so that they are able to help future students contextualize fraction expressions.

Each of these themes showed how participants in this study made sense of the given fraction expressions and their contextualization into word problems. Although most of the themes demonstrate that the participants were developing an understanding of contextualizing fraction expressions into word problems, the third theme showed that some of the participants in the current study continued to have some confusion when it came to contextualizing fraction expressions into word problems. The elementary school pre-service teachers in the interview portion of the study who demonstrated that they were developing an understanding of how to contextualize fraction expressions into word problems also demonstrated that they could be successful at helping their future students (Dixon et al., 2014).

Implications

Rayner, Pitsolantis, and Osana (2009) determined that conceptually-based learning experiences help pre-service teachers to develop a deeper understanding of mathematics. In a study conducted by Tchoshanov (2011) indicated that students who had teachers with conceptual content knowledge of mathematics were higher achieving students than those who had teachers with simply procedural knowledge. The results of the current study suggest that studying fractions with conceptually-based learning experiences in a mathematics for elementary school teachers content course can help elementary school pre-service teachers to develop greater CCK and SCK. Because students need the ability to decontextualize word problems into expressions (Izsak et al., 2009), they need teachers who will be able to help them learn how to decontextualize fraction word problems (Dunlap & McKnight, 1978). The CCK and SCK that is developed by elementary school pre-service teachers will enable them to assist their future students.

A mathematics for elementary school teachers content course can also provide pre-service teachers with more opportunities to contextualize fraction expressions and decontextualize fraction word problems. By gaining practice in identifying decontextualization of word problems that contain fractions into mathematical expressions and identifying contextualizations of mathematical expressions that contain fractions into word problems, elementary school pre-service teachers could discover their own misconceptions about fractions. By discovering their own misconceptions about contextualizing fraction expressions and decontextualizing fraction word problems, elementary school pre-service teachers might be able to identify those same misconceptions in their future students and address those misconceptions (Dixon et al., 2014).

Additionally, mathematics education courses that employ opportunities for pre-service teachers to contextualize fraction expressions and decontextualize fraction word problems will also help them develop a deeper understanding of fractions (Rayner et al., 2009; Utley & Reeder, 2012). By developing a deeper understanding of fractions, elementary school pre-service teachers can gain confidence in their abilities (Ellerton, 2013) to help their future students solve fraction problems and to create word problems that would be interesting and relevant to them (Silver, 1994). In the present study, the participants used pictorial representations of the problems to assist in solving the fraction problems during the conceptually-based learning experiences. During the post-test and during the think aloud interviews the participants also used pictorial representations when they solved the problems. This implies that the participants in this study found the use of picture helpful when solving the fraction word problems given to them.

Limitations

One limitation to this study is that the participants in the study took the pre- and post-CDFI in which each question required them to select an answer from a given list and explain how they obtained their answer. Some of the participants in the study chose to not answer part or all of any question that asked them to explain how they selected their answer. The researcher was not given the opportunity to talk to all of the participants to determine what they were thinking as they completed the CDFI. If a participant chose to not answer one or all parts of a question, missing data occurred. To reduce the impact of this limitation, the researcher conducted interviews with 11 of the participants. If the participants chosen failed to answer any part of the pre- or post-test, the researcher was able to elicit the reasoning for the exclusion of a response.

Another limitation of this study is inherent within the pre-test/post-test design. With this design, some pre- and post-test score variances may be related to repetition rather than learning, as the subjects retake the same test (Gall et al., 2007; Shadish et al., 2002). To try to reduce the impact of this limitation the researcher changed the fractions in each of the problems in the post-test. Other pre-test/post-test differences in scores may be attributed to other variables that cannot be accounted for.

Student maturation can also affect the scores on the pre-tests and post-tests (Gall et al., 2007; Shadish et al., 2002). Due to the time between the administration of the pre- and post-tests, different events could have occurred for the subjects which could have affected their scores on the post-test. A final threat to the internal validity of the study was the instrument that was used (Gall et al., 2007; Shadish et al., 2002). Although the individual questions were obtained from other research studies in which reliability and validity was determined, the CDFI, as a unit, was used in a single pilot study prior to this research,

This study also had threats to its external validity. One of the threats was the type of sampling (i.e., convenience) which can cause problems in population validity (Gall et al., 2007; Shadish et al., 2002). This study had a selection bias due to the sample being a convenience sample (Gall et al., 2007; Shadish et al., 2002). Using convenience sampling severely limits the ability to generalize the results of the study (Johnson & Christensen, 2004). Another limitation due to external validity was the sample size. According to Gall et al. and Shadish et al., the sample size of 52 students may also limit the ability to generalize the results. In addition, the concurrent mathematics courses in which some participants were enrolled may have affected the achieved scores on the test.

Another limitation of external validity may be caused by information that the participants acquired during the mathematics for elementary school teachers content course, as there were two different classes (sections) involved in the study, and each class had a different instructor (Gall et al., 2007; Shadish et al., 2002). Though careful attention was given to ensuring that instruction was consistent and identical materials were used, slight differences may have inadvertently occurred in the two sections. This could have provided some students in one classroom with more or less information than students enrolled in the mathematics for elementary school teachers content course in the second classroom (Gall et al., 2007; Shadish et al., 2002).

Recommendations for Future Research

Findings in this study indicated that elementary school pre-service teachers benefitted from participation in a conceptually-based learning experiences during a mathematics for elementary school teachers' content course. This study showed an improvement in the elementary school pre-service teachers' ability to 1) decontextualize fraction word problems into fraction expressions, 2) solve fraction word problems, and 3) identify contextualizations of fraction expressions into word problems. While the participants in the current study did improve their ability to explain their reasoning for their selection of the fraction subtraction word problem matching the given fraction subtraction expression there was not a statistically significant increase in the participant's ability to explain their reasoning for the fraction multiplication word problem selection. Future research could investigate and determine if more time spent on fraction multiplication by elementary school pre-service teachers improves their ability to contextualize fraction expressions and decontextualize fraction word problems. It could also examine if more time spend on fraction multiplication improved elementary school pre-service teachers' ability to explain their reasoning as they solved fraction multiplication word problems.

On the CDFI and during the think aloud interview, the elementary school pre-service teachers had difficulty identifying the correct fraction multiplication expression that matched the given word problem. On the CDFI post-test 15 of the 39 participants who incorrectly identified the fraction multiplication expression for the given word problem chose multiplication for their operation eight of those 15 placed the fractions in an order that did not match the given word problem, while the other seven identified both multiplication expressions as matching the given word problem. While the order the fractions are placed in a multiplication problem does not change the solution to the problem, the order does matter when writing an expression that matches a given multiplication word problem. During the interview seven of the eleven participants identified the fraction multiplication word problem as multiplication, but four of those seven wrote the expression in the incorrect order. Future research could examine whether more class time spent on fraction multiplication could help elementary school pre-service teachers with writing a fraction multiplication expression to match a given word problem.

On the CDFI and during the think aloud interview, 11 participants, seven chose to solve the Brownie problem by using an area model strategy. When asked to explain their strategy, participants in the current study were able to explain their process for solving the problem and finding the correct solution. Similarly, Luo and colleagues (2011) determined that pre-service teachers found that an area model was the easiest model to use; and Toluk-Ucar (2009) had earlier determined that if pre-service teachers could pictorially represent a problem, they could explain how they solved the problem. The participants in the current study also used pictorial representations of the fraction word problems and then went on to explain how they used the pictures to help them solve the problems.

Unlike the pre-service teachers in the study conducted by Van Steenbrugge and colleagues (2014), the participants in the current study did not have difficulty solving the problems conceptually or explaining their solutions. Also, unlike Huang and colleagues (2009), who found that pre-service teachers' problem solving ability was mainly procedural, the participants in the current study solved all the problems conceptually through pictorial representations. The participants in the interview portion of the study explained how to solve the fraction word problems and used pictorial representations to assist in their explanations of the solutions. Future research could examine why an elementary pre-service teacher would choose a particular method when solving fraction word problem over another method.

Another area of research could look at just the contextualization of fraction expressions by pre-service teachers. The researcher could conduct interviews with the pre-service teachers before and after a unit on fractions in the mathematics for elementary education content course. The word problems created during the interview could be examined to determine if the participants are able to make word problems with connections to fraction situations in the real world, and whether the word problems created by the pre-service teachers could be used in a classroom. The pre- and post- interviews would also enable the researcher to determine if the pre-service teachers gained any confidence in themselves as problem posers.

Conclusion

This chapter discussed the findings of this research. This research found a statistically significant difference pre- and post-test scores of the elementary school pre-service teachers to decontextualize and explain fraction word problems and in their ability to contextualize and explain fraction expressions, following a unit on fractions in a mathematics for elementary school teachers' content course. This research also identified three main themes when analyzing the responses given by the 11 participants in the think aloud interview.

APPENDIX A: CONSENT TO REPRODUCE FIGURE 1

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Title: Content Knowledge for Teaching:What Makes It Special?

Author: Deborah Loewenberg Ball, Mark Hoover Thames, Geoffrey Phelps

Publication: Journal of Teacher Education

Publisher: SAGE Publications

Date: 11/01/2008

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APPENDIX B: INSTITUTIONAL REVIEW BOARD APPROVAL

University of Central Florida Institutional Review Board
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Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Laura E. Tapp**

Date: **February 17, 2016**

Dear Researcher:

On 02/17/2016, the IRB approved the following human participant research until 02/16/2017 inclusive:

Type of Review:	UCF Initial Review Submission Form
Project Title:	An analysis of undergraduate elementary pre-service teacher's understanding of fraction word problems and fraction expression translations as a result of conceptually based learning experiences
Investigator:	Laura E Tapp
IRB Number:	SBE-16-12016
Funding Agency:	
Grant Title:	
Research ID:	N/A

The scientific merit of the research was considered during the IRB review. The Continuing Review Application must be submitted 30days prior to the expiration date for studies that were previously expedited, and 60 days prior to the expiration date for research that was previously reviewed at a convened meeting. Do not make changes to the study (i.e., protocol, methodology, consent form, personnel, site, etc.) before obtaining IRB approval. A Modification Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at https://iris.research.ucf.edu .

If continuing review approval is not granted before the expiration date of 02/16/2017, approval of this research expires on that date. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

Use of the approved, stamped consent document(s) is required. The new form supersedes all previous versions, which are now invalid for further use. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Participants or their representatives must receive a copy of the consent form(s).

All data, including signed consent forms if applicable, must be retained and secured per protocol for a minimum of five years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained and secured per protocol. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Joanne Muratori

Signature applied by Joanne Muratori on 02/17/2016 02:48:45 PM EST

IRB Manager

APPENDIX C: INFORMED CONSENT

An Analysis of Undergraduate Elementary Pre-Service Teacher's Understanding of Fraction Word Problems and Fraction Expression Translations as a Result of Conceptually Based Learning Experiences

Informed Consent

Principal Investigator(s):	Laura Tapp, MS
Faculty Supervisor:	Enrique Ortiz, PhD
Investigational Site(s):	University of Central Florida College of Education and Human Performance

Introduction: Researchers at the University of Central Florida (UCF) study many topics. To do this we need the help of people who agree to take part in a research study. You are being invited to take part in a research study which will include about 70 student participants from UCF and about 6 people at UCF to analyze the results of the research. You have been asked to take part in this research study because you are in an undergraduate elementary pre-service teacher at UCF. You must be 18 years of age or older to be included in the research study.

The person doing this research is Laura Tapp of the Mathematics Education Track of the UCF College of Education and Human Performance. Because the researcher is a doctoral candidate, she will be guided by Dr. Enrique Ortiz a UCF faculty advisor in the UCF College of Education and Human Performance.

What you should know about a research study:

- Someone will explain this research study to you.
- A research study is something you volunteer for.
- Whether or not you take part is up to you.
- You should take part in this study only because you want to.
- You can choose not to take part in the research study.
- You can agree to take part now and later change your mind.
- Whatever you decide it will not be held against you.
- Feel free to ask all the questions you want before you decide.

Purpose of the research study: The purpose of this study is to assess pre-service teachers' ability to identify translations of fraction word problems and fraction expressions.

What you will be asked to do in the study: Participants will be asked to complete a pre-test prior to studying fraction word problems and a post-test after studying fraction word problems. After an analysis of the pre- and post-tests some of the participants will be asked to complete a brief interview.

Location: The pre-test and the post-test will occur in the usual classroom setting during class time. The interview portion of the study will be conducted in the office of the researcher before or after class.

Time required: *It is expected that the pre-test and the post-test will take 10-20 minutes to complete. The interview is expected to take 20-30 minutes to complete.*

Audio or Video Taping*: You will be videotaped during this study. If you do not want to be audio taped, you will not be able to be in the interview portion of the study. Discuss this with the researcher. If you are videotaped, the tape will be kept in a locked, safe place. The tape will be erased or destroyed after the researcher has transcribed the video.*

Risks: There are no foreseeable risks or discomforts involved in participating in this study.

Benefits: Results of this study may benefit future pre-service teachers by providing information to schools on the abilities of education majors with translating problems that contain fractions, and where pre-service teachers struggle with their fraction understanding.

Compensation or payment: There is no compensation, payment or extra credit for taking part in this study. If you choose not to participate, there will be no penalty.

Confidentiality: We will limit your personal data collected in this study to people who have a need to review this information. We cannot promise complete secrecy.

Study contact for questions about the study or to report a problem: If you have questions, concerns, or complaints, or think the research has hurt you, talk to Laura Tapp, doctoral candidate, Mathematics Education Track, College of Education and Human Performance at (832) 757-0330 or by email at laura.tapp@knights.ucf.edu or Dr. Enrique Ortiz, Faculty Supervisor, College of Education and Human Performance at by email at enrique.ortiz@ucf.edu.

IRB contact about your rights in the study or to report a complaint: Research at the University of Central Florida involving human participants is carried out under the oversight of the Institutional Review Board (UCF IRB). This research has been reviewed and approved by the IRB. For information about the rights of people who take part in research, please contact: Institutional Review Board, University of Central Florida, Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901. You may also talk to them for any of the following:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.
- You want to talk to someone besides the research team.
- You want to get information or provide input about this research.

Withdrawing from the study: If you decide to leave the study, contact the investigator so that the investigator can remove your data from the study. You can email or call using the information above, or speak to your instructor who will relay the information.

APPENDIX D: ASSESSMENT OF CONTEXTUALIZATION AND DECONTEXTUALIZATION OF FRACTIONS INSTRUMENT PRE-TEST ITEMS

Mrs. Smith asked her students to solve and translate the following problems. Several students shared their translations with the class.

a. There was $\frac{3}{4}$ of a pizza left over from dinner last night. For lunch Jim ate $\frac{1}{3}$ of the leftover pizza. How much of the whole pizza did Jim eat for lunch?

   a. Using words and pictures solve the problem as if you were helping someone in mathematics.

   b. Place an X next to the student or students who correctly represented the situation presented in the word problem. You may have no, one, or more than one X in your response

   ______ **Student A** translated the problem into $\frac{3}{4} + \frac{1}{3}$

   ______ **Student B** translated the problem into $\frac{1}{3} + \frac{3}{4}$

   ______ **Student C** translated the problem into $\frac{3}{4} - \frac{1}{3}$

   ______ **Student D** translated the problem into $\frac{1}{3} - \frac{3}{4}$

   ______ **Student E** translated the problem into $\frac{3}{4} \times \frac{1}{3}$

   ______ **Student F** translated the problem into $\frac{1}{3} \times \frac{3}{4}$

   ______ **Student G** translated the problem into $\frac{3}{4} \div \frac{1}{3}$

   ______ **Student H** translated the problem into $\frac{1}{3} \div \frac{3}{4}$

b. Tami brought $\frac{1}{2}$ of a pan of brownies to school to share with her classmates. If Tami's classmates ate $\frac{3}{7}$ of a pan of brownies, how much of the pan of brownies does Tami have left?

a. Using words and pictures solve the problem as if you were helping someone in mathematics.

b. Place an X next to the student or students who correctly represented the situation presented in the word problem. You may have no, one, or more than one X in your response

______ **Student A** translated the problem into $\frac{3}{7}+\frac{1}{2}$

______ **Student B** translated the problem into $\frac{1}{2}+\frac{3}{7}$

______ **Student C** translated the problem into $\frac{3}{7}-\frac{1}{2}$

______ **Student D** translated the problem into $\frac{1}{2}-\frac{3}{7}$

______ **Student E** translated the problem into $\frac{3}{7}\times\frac{1}{2}$

______ **Student F** translated the problem into $\frac{1}{2}\times\frac{3}{7}$

______ **Student G** translated the problem into $\frac{3}{7}\div\frac{1}{2}$

______ **Student H** translated the problem into $\frac{1}{2}\div\frac{3}{7}$

Mrs. Smith gave her students the following arithmetic expression, and asked them to create word problems that accurately reflects the given arithmetic expression. Several students shared their word problems with the class.

c. $\frac{5}{6} - \frac{1}{3}$

a. Place an **X** next to the student or students who provided a word problem that most closely reflects the given arithmetic expression

______ **Student A** wrote

Sally had $\frac{5}{6}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{1}{3}$ of a whole apple pie. How much of the apple pie is left?

______ **Student B** wrote

Sally had $\frac{1}{3}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{5}{6}$ of a whole apple pie. How much of the apple pie is left?

______ **Student C** wrote

Sally had $\frac{1}{3}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{5}{6}$ of the leftover apple pie. How much of the whole apple pie did they eat?

______ **Student D** wrote

Sally had $\frac{5}{6}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{1}{3}$ of the apple pie leftover. How much of the whole apple pie is did they eat?

______ **Student E** wrote

Sally had $\frac{5}{6}$ of an apple pie leftover in the refrigerator. Sally's husband David put another $\frac{1}{3}$ of an apple pie in the refrigerator. How much apple pie is in the refrigerator?

How did you know that the student you selected wrote a correct word problem? If you indicated no agreement in part a, please include an explanation of a correct selection and include the word problem to represent the problem.

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d. $\frac{1}{2} \times \frac{3}{4}$

a. Place an **X** next to the student or students who provided a word problem that <u>most closely</u> reflects the given arithmetic expression

_______ **Student A** wrote

Juan has $\frac{1}{2}$ of a bag of candy. If Juan eats $\frac{3}{4}$ of the candy, how much of a bag of candy did Juan eat?

_______ **Student B** wrote

Juan has $\frac{3}{4}$ of a bag of candy. If Juan eats $\frac{1}{2}$of the candy, how much of a bag of candy did Juan eat?

_______ **Student C** wrote

Juan ate $\frac{1}{2}$ of a bag of candy. Then Juan ate $\frac{3}{4}$ of a bag of candy. How much of a bag of candy did Juan eat?

_______ **Student D** wrote

Juan has $\frac{1}{2}$ of a bag of candy. If Juan eats $\frac{3}{4}$of a bag of candy, how much of a bag of candy does he have left?

_______ **Student E** wrote

Juan has $\frac{3}{4}$ of a bag of candy. If Juan eats $\frac{1}{2}$of a bag of candy, how much of a bag of candy does he have left?

How did you know that the student you selected wrote a correct word problem? If you indicated no agreement in part a, please include an explanation of a correct selection and include the word problem to represent the problem.

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Place an X on the line next to your selection.

1. What year are you in School?

   ______ Freshman
   ______ Sophomore
   ______ Junior
   ______ Senior

2. What is your gender?

   ______ Male
   ______ Female

3. What is your race?

   ______ Non-Hispanic White
   ______ Hispanic or Latino
   ______ Black or African American
   ______ Native American or American Indian
   ______ Asian or Asian American
   ______ Hawaiian or Other Pacific Islander
   ______ Other

4. What is your major

   ____

APPENDIX E: ASSESSMENT OF CONTEXTUALIZATION AND DECONTEXTUALIZATION OF FRACTIONS INSTRUMENT POST-TEST ITEMS

Mrs. Smith asked her students to solve and translate the following problems. Several students shared their translations with the class.

a. There was $\frac{3}{4}$ of a pizza left over from dinner last night. For lunch Jim ate $\frac{2}{3}$ of the leftover pizza. How much of the whole pizza did Jim eat for lunch?

  a. Explain using words and pictures how to solve the problem as if you were helping someone in mathematics.

  b. Place an X next to the student or students who correctly represented the situation presented in the word problem. You may have no, one, or more than one X in your response

  ______ **Student A** translated the problem into $\frac{3}{4} + \frac{2}{3}$

  ______ **Student B** translated the problem into $\frac{2}{3} + \frac{3}{4}$

  ______ **Student C** translated the problem into $\frac{3}{4} - \frac{2}{3}$

  ______ **Student D** translated the problem into $\frac{2}{3} - \frac{3}{4}$

  ______ **Student E** translated the problem into $\frac{3}{4} \times \frac{2}{3}$

  ______ **Student F** translated the problem into $\frac{2}{3} \times \frac{3}{4}$

  ______ **Student G** translated the problem into $\frac{3}{4} \div \frac{2}{3}$

  ______ **Student H** translated the problem into $\frac{2}{3} \div \frac{3}{4}$

b. Tami brought $\frac{3}{4}$ of a pan of brownies to school to share with her classmates. If Tami's classmates ate $\frac{1}{2}$ of a pan of brownies, how much of the pan of brownies does Tami have left?

a. Explain using words and pictures how to solve the problem as if you were helping someone in mathematics.

b. Place an X next to the student or students who correctly represented the situation presented in the word problem. You may have no, one, or more than one X in your response

_______ **Student A** translated the problem into $\frac{3}{4}+\frac{1}{2}$

_______ **Student B** translated the problem into $\frac{1}{2}+\frac{3}{4}$

_______ **Student C** translated the problem into $\frac{3}{4}-\frac{1}{2}$

_______ **Student D** translated the problem into $\frac{1}{2}-\frac{3}{4}$

_______ **Student E** translated the problem into $\frac{3}{4}\times\frac{1}{2}$

_______ **Student F** translated the problem into $\frac{1}{2}\times\frac{3}{4}$

_______ **Student G** translated the problem into $\frac{3}{4}\div\frac{1}{2}$

_______ **Student H** translated the problem into $\frac{1}{2}\div\frac{3}{4}$

Mrs. Smith gave her students the following arithmetic expression, and asked them to create word problems that accurately reflects the given arithmetic expression. Several students shared their word problems with the class.

c. $\frac{2}{3} - \frac{1}{2}$

a. Place an **X** next to the student or students who provided a word problem that most closely reflects the given arithmetic expression

______ **Student A** wrote

Sally had $\frac{2}{3}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{1}{2}$ of a whole apple pie. How much of the whole apple pie is left?

______ **Student B** wrote

Sally had $\frac{1}{2}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{2}{3}$ of a whole apple pie. How much of the whole apple pie is left?

______ **Student C** wrote

Sally had $\frac{1}{2}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{2}{3}$ of the leftover apple pie. How much of the whole apple pie did they eat?

______ **Student D** wrote

Sally had $\frac{2}{3}$ of an apple pie leftover in the refrigerator. For dessert Sally and her husband David ate $\frac{1}{2}$ of the apple pie leftover. How much of the whole apple pie is did they eat?

______ **Student E** wrote

Sally had $\frac{2}{3}$ of an apple pie leftover in the refrigerator. Sally's husband David put another $\frac{1}{2}$ of an apple pie in the refrigerator. How much apple pie is in the refrigerator?

How did you know that the student you selected wrote a correct word problem? If you indicated no agreement in part a, please include an explanation of a correct selection and include the word problem to represent the problem.

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d. $\frac{1}{5} \times \frac{2}{3}$

a. Place an **X** next to the student or students who provided a word problem that most closely reflects the given arithmetic expression

______ **Student A** wrote

Juan has $\frac{1}{5}$ of a bag of candy. If Juan eats $\frac{2}{3}$ of the candy, how much of a bag of candy did Juan eat?

______ **Student B** wrote

Juan has $\frac{2}{3}$ of a bag of candy. If Juan eats $\frac{1}{5}$ of the candy, how much of a bag of candy did Juan eat?

______ **Student C** wrote

Juan ate $\frac{1}{5}$ of a bag of candy. Then Juan ate $\frac{2}{3}$ of a bag of candy. How much of a bag of candy did Juan eat?

______ **Student D** wrote

Juan has $\frac{1}{5}$ of a bag of candy. If Juan eats $\frac{2}{3}$ of a bag of candy, how much of a bag of candy does he have left?

______ **Student E** wrote

Juan has $\frac{2}{3}$ of a bag of candy. If Juan eats $\frac{1}{5}$of a bag of candy, how much of a bag of candy does he have left?

How did you know that the student you selected wrote a correct word problem? If you indicated no agreement in part a, please include an explanation of a correct selection and include the word problem to represent the problem.

APPENDIX F: PARTICIPANT AND RESEARCHER INTERVIEW QUESTIONS

Participant Interview Questions

For the following expressions create a word problem about pizza.

1. $\frac{3}{5} \times \frac{5}{6}$

2. $\frac{3}{4} - \frac{1}{2}$

Mrs. Smith asked her students to solve and translate the following problems. Several students shared their translations with the class.

3. David brings $\frac{3}{4}$ of a pan of brownies to school to share with his classmates. If David's classmates eat $\frac{1}{3}$ of the brownies, how much of a whole pan of brownies did David's class eat?

    a. Solve the problem as if you were helping a student with mathematics.

4. John left $\frac{5}{6}$ of a cake on the kitchen counter. His dog came in and ate an amount equal to $\frac{1}{3}$ of the whole cake before John chased him from the room. How much of the cake is left after the dog left the room?

   a. Solve the problem as if you were helping a student with mathematics.

Researcher Interview Questions

Problem #1 ($\frac{3}{5}$ x $\frac{5}{6}$)

1. How do you know that you wrote a multiplication word problem?
2. How do you know that the order that you wrote the fractions in the problem matches the given expression?

Problem # 2 ($\frac{3}{4}$ - $\frac{1}{2}$)

1. How do you know that you wrote a subtraction problem?
2. How do you know that the order that you wrote the fractions in the problem matches the given expression?

Problem #3 (David brings $\frac{3}{4}$ of a pan of brownies to school to share with his classmates. If David's classmates eat $\frac{1}{3}$ of the brownies, how much of a whole pan of brownies did David's class eat?)

1. How do you know that it's a (multiplication/subtraction) word problem?
2. How do you know that the order that you wrote for the expression matches the word problem?

Problem # 4 (John left $\frac{5}{6}$ of a cake on the kitchen counter. His dog came in and ate an amount equal to $\frac{1}{3}$ of the whole cake before John chased him from the room. How much of the cake is left after the dog left the room?)

1. How do you know that it's a (multiplication/subtraction) word problem?
2. How do you know that the order that you wrote for the expression matches the word problem?

APPENDIX G: PAIRED T-TEST TABLE

Paired Samples Test with 51 Degrees of Freedom

		Mean	Std. Deviation	t	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	PT1All - PT2All	-3.15385	4.31770	-5.267	-4.35590	-1.95179
Pair 2	Q1PreTest - Q1PostTest	-2.15385	3.26229	-4.761	-3.06207	-1.24562
Pair 3	Q2PreTest - Q2PostTest	-1.00000	1.69196	-4.262	-1.47105	-.52895
Pair 4	T1 Expression Pizza 1/3 x 3/4 - T2 Expression Pizza 2/3 x 3/4	-.250	.437	-4.123	-.372	-.128
Pair 5	T1PizzaSol - T2PizzaSol	-.712	1.753	-2.927	-1.199	-.224
Pair 6	T1 Expression Brownie 1/2 - 3/7 - T2 Expression Brownie 3/4 - 1/2	-.058	.574	-.724	-.218	.102
Pair 7	T1BrownieSol - T2BrownieSol	-1.135	1.990	-4.111	-1.689	-.580
Pair 8	T1 Word Problem Pie 5/6 - 1/3 - T2 Word Problem Pie 2/3 - 1/2	-.327	.648	-3.636	-.507	-.146
Pair 9	T1PieSol - T2PieSol	-.327	.810	-2.911	-.552	-.101
Pair 10	T1 Word Problem Candy 1/2 x 3/4 - T2 Word Problem Candy 1/5 x 2/3	-.212	.457	-3.335	-.339	-.084
Pair 11	T1CandySol - T2CandySol	-.135	.687	-1.413	-.326	.057

APPENDIX H: FREQUENCY TABLES

T1 Expression Pizza 1/3 x 3/4

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	52	100.0	100.0	100.0

T2 Expression Pizza 2/3 x 3/4

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	39	75.0	75.0	75.0
	Correct	13	25.0	25.0	100.0
	Total	52	100.0	100.0	

T1PizzaSol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No answer, blank, none	2	3.8	3.8	3.8
	words/picture no clear understanding	8	15.4	15.4	19.2
	procedurally incorrect	16	30.8	30.8	50.0
	conceptually incorrect	8	15.4	15.4	65.4
	procedurally correct	4	7.7	7.7	73.1
	conceptually correct	14	26.9	26.9	100.0
	Total	52	100.0	100.0	

T2PizzaSol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No answer, blank, none	2	3.8	3.8	3.8
	words/picture no clear understanding	6	11.5	11.5	15.4
	procedurally incorrect	6	11.5	11.5	26.9
	conceptually incorrect	10	19.2	19.2	46.2
	procedurally correct	1	1.9	1.9	48.1
	conceptually correct	27	51.9	51.9	100.0
	Total	52	100.0	100.0	

T1 Expression Brownie 1/2 - 3/7

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	16	30.8	30.8	30.8
	Correct	36	69.2	69.2	100.0
	Total	52	100.0	100.0	

T2 Expression Brownie 3/4 - 1/2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	13	25.0	25.0	25.0
	Correct	39	75.0	75.0	100.0
	Total	52	100.0	100.0	

T1BrownieSol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	No answer, blank, none	3	5.8	5.8	5.8
	words/picture no clear understanding	12	23.1	23.1	28.8
	procedurally incorrect	4	7.7	7.7	36.5
	conceptually incorrect	11	21.2	21.2	57.7
	procedurally correct	17	32.7	32.7	90.4
	conceptually correct	5	9.6	9.6	100.0
	Total	52	100.0	100.0	

T2BrownieSol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	words/picture no clear understanding	8	15.4	15.4	15.4
	conceptually incorrect	9	17.3	17.3	32.7
	procedurally correct	5	9.6	9.6	42.3
	conceptually correct	30	57.7	57.7	100.0
	Total	52	100.0	100.0	

T1 Word Problem Pie 5/6 - 1/3

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	40	76.9	76.9	76.9
	Correct	12	23.1	23.1	100.0
	Total	52	100.0	100.0	

T2 Word Problem Pie 2/3 - 1/2

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	23	44.2	44.2	44.2
	Correct	29	55.8	55.8	100.0
	Total	52	100.0	100.0	

T1PieSol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	none, blank	8	15.4	15.4	15.4
	explained incorrectly	40	76.9	76.9	92.3
	explained correctly	4	7.7	7.7	100.0
	Total	52	100.0	100.0	

T2PieSol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	none, blank	8	15.4	15.4	15.4
	explained incorrectly	23	44.2	44.2	59.6
	explained correctly	21	40.4	40.4	100.0
	Total	52	100.0	100.0	

T1 Word Problem Candy 1/2 x 3/4

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	51	98.1	98.1	98.1
	Correct	1	1.9	1.9	100.0
	Total	52	100.0	100.0	

T2 Word Problem Candy 1/5 x 2/3

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Incorrect	40	76.9	76.9	76.9
	Correct	12	23.1	23.1	100.0
	Total	52	100.0	100.0	

T1CandySol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	none, blank	15	28.8	28.8	28.8
	explained incorrectly	34	65.4	65.4	94.2
	explained correctly	3	5.8	5.8	100.0
	Total	52	100.0	100.0	

T2CandySol

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	none, blank	12	23.1	23.1	23.1
	explained incorrectly	33	63.5	63.5	86.5
	explained correctly	7	13.5	13.5	100.0
	Total	52	100.0	100.0	

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