Polyurethane Fiber Reinforced Polymer Strengthening of Shear Deficient Reinforced Concrete Beams

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POLYURETHANE FIBER REINFORCED POLYMER STRENGTHENING OF SHEAR DEFICIENT REINFORCED CONCRETE BEAMS

by

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A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the Department of Civil, Environmental, and Construction Engineering in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

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ABSTRACT

The use of externally-bonded fiber-reinforced polymer (FRP) composites has been established as an effective means for the strengthening of shear-deficient reinforced concrete (RC) flexural members. Epoxy-based wet layup systems were predominantly employed in previous studies. In this study, carbon FRP pre-impregnated with polyurethane resin is utilized in strengthening shear-deficient RC beams and compared to an epoxy resin. Fourteen small-scale (96 in span, 6 in width, and 12 in height) and five large-scale (132 in span, 12 in width, and 17 in height) flexural specimens were tested, considering FRP system type (polyurethane versus epoxy), size effect, shear span-to-depth ratio, FRP configuration (U-wraps versus side bonding), and FRP scheme (sheets versus strips with 45° or 90°). Experimental strength testing under four-point loading demonstrated similar or enhanced shear capacity when strengthening by the polyurethane compared to the epoxy composite systems.

The shear behavior of polyurethane-based FRP composite system is investigated in this research using analytical and numerical approaches. A closed-form mechanics-based analytical model, utilizing the principle of effective FRP stress and upper-bound theorem, illustrated that the shear behavior and debonding mechanism were dependent on both FRP composite and bond characteristics. The analytical model is expressed in terms of shear crack opening crossed by the FRP laminate and gives good agreement with experimental results. The finite element analysis (FEA) model shows that the stresses in the FRP are not in single direction as in the coupon tests, and the biaxial stress states should be taken into consideration.

The structural behavior of RC members strengthened with externally-bonded FRP composites is mobilized through the composite action technique. Bond stress can be defined as the shear stress acting in the interface between FRP and concrete. It is of crucial importance
to evaluate the failure mode behavior. Debonding (loss of adhesion) failure is one of the
most common modes of failure encountered in shear strengthening RC members in practice.
Numerous constitutive bond-slip models have been proposed and derived numerically and
mathematically based on experimental data with an assumption that the FRP width $b_p$ is
taken as a variable and all stresses or strains at the same longitudinal coordinate ($L$ direc-
tion) are uniform. No attention has been given to study the bond states of stress which are
mainly altered by the inclined shear cracks in concrete. A new bond-slip law was proposed
to address the biaxial two-dimensional (2D) states of stress problem. Numerical solution by
finite difference (FD) was conducted to solve four partial differential equations per node (2
for FRP and 2 for concrete in each direction) with appropriate boundary conditions to obtain
the stresses, slips, and strains based on the proposed bond-slip model. A new experimental
setup was proposed to represent the 2D bond-slip model by lap shear tests in both directions
by laminating two perpendicular strips on concrete blocks with the proposed strain profile.
Experimental calibration has been carried out by using nonlinear least-squares regression
(fitting) of the experimental strain data with the numerical FD equations to obtain the
bond-slip parameters for the 2D FRP-to-concrete polyurethane interface system.
This dissertation is dedicated to the memory of my mother. I miss her everyday, but I am glad to know she saw this process through to its completion.

To my father Farouk with eternal love. My words cannot express my appreciation. To my amazing wife, Maram, and my daughters Mariam and Fatima for their love, support, and encouragement to pursue this degree. Without you, I would not be who I am today.

I also dedicate my five years of hard work on this research to my siblings Noora and Maher.
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# TABLE OF CONTENTS

LIST OF FIGURES ................................................................. x

LIST OF TABLES ................................................................. xv

CHAPTER 1: INTRODUCTION .................................................. 1

1.1 Motivation and Background ........................................... 1

1.2 Objective ........................................................................... 3

1.3 Plan ..................................................................................... 4

1.4 FRP Composites ............................................................... 6

1.4.1 Reinforcing Fibers ......................................................... 6

1.4.2 Matrix ............................................................................. 9

1.5 FRP Systems ................................................................. 10

1.6 FRP Bond-Slip Studies ..................................................... 10

1.7 Behavior of Beams Failing in Shear ................................. 12

1.7.1 B- and D-Regions ......................................................... 16

1.7.2 Inclined Cracking .......................................................... 17

1.7.3 Internal Forces in Beams with Shear Reinforcement .......... 18
1.7.4 Truss Model ................................................. 19
1.7.5 Beams without Stirrups .................................. 21

CHAPTER 2: EXPERIMENTAL METHODS ......................... 23
  2.1 Background .................................................. 23
  2.2 Beam Design .............................................. 24
  2.3 Materials .................................................. 28
  2.4 Testing Matrix ............................................. 30
  2.5 Test Setup and Instrumentation Plans ....................... 32
  2.6 Experimental Results ................................... 33
    2.6.1 Failure mode and cracking pattern .................. 34
    2.6.2 Parameter effects ................................... 38
    2.6.3 Load-deflection behavior ............................ 39
    2.6.4 Load-strain behavior ................................ 41
  2.7 Discussion ................................................ 48

CHAPTER 3: ANALYTICAL METHODS .............................. 54
  3.1 Background ................................................ 54
  3.2 Analytical Model ........................................ 63
LIST OF FIGURES

1.1 Material properties of FRP and steel reinforcement [1] .................. 8
1.2 Free body diagram of 1D bonded joint [2] ............................... 11
1.3 Effect of a/d ratio on shear strength of beams without stirrups [3] ... 13
1.4 Modes of failure in deep beams (a/d ≤ 1) [3] ............................ 14
1.5 Modes of failure in short beams (1 < a/d ≤ 2.5) [3] .................... 15
1.6 Diagonal tension failure (tooth cracking failure) in intermediate beams
   (2.5 < a/d ≤ about 6) [4] .................................................. 16
1.7 Types of inclined cracks [3] .................................................. 18
1.8 Internal forces in cracked beams with shear reinforcement [3] ........ 19
1.9 Shear transfer mechanism of slender beams [5] .......................... 20
1.10 Shear transfer mechanism and crack pattern of deep beams [5] ...... 20
1.11 Truss model [6] ............................................................... 21
1.12 Reinforcement ratio effect on shear capacity of beams without web
     reinforcement [3] .......................................................... 22
1.13 Beam size effect on shear capacity of beams without web reinforcement
     [3] ................................................................. 22
2.1 Details of small-scale specimens ........................................... 26
2.2 Details of large-scale specimens ........................................... 27
2.3 Specimens concreting ....................................................... 30
2.4 Configurations and schemes of small-scale specimens with $a/d = 3$ . 32
2.5 Configurations and schemes of large-scale specimens ................. 33
2.6 Failure mode and cracking pattern of small-scale specimens with $a/d = 3$ 36
2.7 Failure mode and cracking pattern of small-scale specimens with $a/d = 4$ 37
2.8 Failure mode and cracking pattern of large-scale specimens ............ 38
2.9 Midspan load-deflection behavior ....................................... 40
2.10 Support deflection behavior ............................................. 40
2.11 Load strain behavior of midspan longitudinal tension steel reinforcement 42
2.12 Load strain behavior of SG-B1 ........................................... 43
2.13 Load strain behavior of stirrups of small-scale specimens with $a/d = 3$ 44
2.14 Load strain behavior of FRP and stirrups of small-scale specimens with $a/d = 3$ 46
2.15 Load strain behavior of FRP and stirrups of small-scale specimens with $a/d = 4$ 47
2.16 Load strain behavior of FRP and stirrups of large-scale specimens .... 48
4.5 Principal cracking strain contour of S-Ch4 .......................... 81

4.6 Principal cracking strain contour of S-Cf4 .......................... 82

4.7 Experimental, analytical and numerical load-displacement results of S-Uh4-PU and L-Sh4-PU .......................... 84

4.8 Experimental and numerical midspan tension rebar strain results of S-Uh4-PU and L-Sh4-PU .......................... 84

4.9 Interface damage profile of S-Uh4-PU at 44.3 kips .................. 85

4.10 Numerical FRP load-strain relations of S-Uh4-PU .................. 86

4.11 Principal cracking strain contours of S-Uh4-PU .................. 87

4.12 FRP strains at different load levels of S-Uh4-PU .................. 88

4.13 Curvature and deflection analyses of specimen S-Uh4-PU ........ 89

5.1 Biaxial state of stress for different load levels of S-Ch4 ........... 96

5.2 Free body diagram of 2D bonded joint .................. 98

5.3 Biaxial constitutive model .................................. 102

5.4 Yield surface of the biaxial constitutive model .................. 102

5.5 Strain profile of biaxial bond model .......................... 104

5.6 Experimental setup of biaxial bond model .................. 105

5.7 Different response behaviors of 2D-SP1 .................. 106
5.8 Comparison of experimental and calibrated specimen average results of biaxial and uniaxial bond models . . . . . . . . . . . . . . . . 107

A.1 Cracking patterns of small- and large-scale shear deficient control specimens . . . . . . . . . . . . . . . . . . . . . . 113

A.2 Cracking patterns of S-Uh3-PU and S-Uh3-EP . . . . . . . . . . 114

A.3 Cracking patterns of S-Sh3-PU and S-Ih3-PU . . . . . . . . . 115

A.4 Cracking patterns of S-Uh4-PU and S-Uh4-EP . . . . . . . . 116

A.5 Cracking patterns of S-Th4-PU and S-Th4-EP . . . . . . . . 117

A.6 Cracking patterns of S-Sh4-PU and S-Ih4-PU . . . . . . . . 118

A.7 Cracking patterns of large-scale FRP-strengthened specimens . . . . . 119
LIST OF TABLES

1.1 Properties of common carbon fibers [10] ..................................... 7

2.1 Designed FRP-PU shear contribution ........................................ 28

2.2 Testing matrix ........................................................................... 31

2.3 Experimental results ............................................................... 34

5.1 Calibrated parameter results ..................................................... 105

5.2 Comparisons between specimen average results of biaxial and uniaxial bond models .................................................. 107
CHAPTER 1: INTRODUCTION

1.1 Motivation and Background

Many older reinforced concrete (RC) structures have deteriorated under environmental and mechanical loads and are considered under-designed with respect to current standards. Degradation of RC structures is caused by several factors, such as design and construction faults, increased service loads, environmental exposure and deterioration, aging, and rudimentary maintenance methods [11]. Limiting or decreasing the service loads, which is often an impractical treatment, is one of the available solutions for the deteriorated structures. Another option is to deconstruct the existing structures and build new ones. This solution is usually not considered feasible due to extremely high cost, stability of neighbor structures, rubble works, and the creation of traffic flow or access problems. In the last few decades, various strengthening and retrofitting solutions have been proposed for rehabilitating existing RC elements, for example introducing external post-tensioning, adding or replacing structural members, and adding new materials by composite action techniques. Historically, steel, which is considered the oldest strengthening composite action technique, was utilized as sheets or strips to strengthen concrete surfaces by bolting or by a proper adhesive. The shortcomings of using steel as strengthening material are the added dead weight, the required corrosion protection, and construction difficulties. As a result, an innovative system which eliminates the aforementioned disadvantages is required.

During the last few decades, the use of fiber-reinforced polymer (FRP) composites in RC members has arisen as one of the most promising innovations for utilizing in rehabilitation of civil infrastructure. FRP composites have a wide range of applications in both new construction and existing structures. FRP composites have emerged in the concrete rehabilitation
market due to ease of use, light weight, and superior physical and mechanical properties. Typically FRP is utilized in RC repair through externally-bonded laminates (wet layup or pre-cured sheets) or near-surface mounted reinforcement. Wet layup systems, comprised of carbon fibers and epoxy resin (denoted FRP-EP in this study), are the most common externally-bonded FRP composite system used for repairing and rehabilitating RC structural elements, such as beams, columns, slabs, and walls. The composite repairs are used to increase the flexural and shear strengths and to enhance the ductility and confinement of compression members [12]. The FRP-EP composite system exhibits desirable properties including high strength, low shrinkage during cure, high resistance to chemicals and solvents, low viscosity, and excellent adhesion to many types of fibers and substrates. However, it has some disadvantages such as high cost, quality control dependence on the installer, and release of potentially harmful volatiles during curing [10].

Despite shear behavior of RC beams being studied for more than 100 years, determination of shear strength is still an open problem for discussion. The shear capacities predicted by different design codes for various sections may differ from each other by factors of more than two, while in flexure, same design codes predict flexural strengths with no more than 10% [13]. In flexure, plane sections assumption construct the rational basis and globally accepted theory for predicting the flexural strengths. Also, simple experimental tests can be performed on RC beams under pure flexure and the results from these tests can be utilized to improve and modify the theory. In shear, no such agreed basis is found because it is difficult to use the test results to generate a general theory for shear behavior. Unlike flexure, it is not easy to perform experimental tests on beams subjected to pure shear, and although the shear is constant along the shear span in the traditional shear tests on beams, the behavior is varying from section to section for such a member. Current RC design codes require that the limiting failure mode should be controlled by flexure (yielding of reinforcement
prior to crushing of concrete), which is a ductile mechanism. Therefore, providing safety
margins against other types of more brittle failure modes is important. In shear-deficient
RC members, FRP shear strengthening is an efficient method for providing additional shear
strength in flexural members and avoiding unpredictable failures due to shear deficiencies.
The study of RC beams strengthened in shear by externally-bonded FRP composites is
complicated by the brittle nature of shear failure [14].

An alternative composite system recently utilized for flexural strengthening of RC elements
was a carbon FRP pre-impregnated (prepreg) with polyurethane resin [15] (denoted FRP-PU
in this study). The impregnation process by the manufacturer ensures high saturation level
of the fibers, and produces higher quality and controlled laminates. The FRP-PU composite
system is delivered in hermetically sealed foil pouches since the curing is dependent on
moisture. It has other desirable properties including low cost and fast curing. Depending
on the polyurethane chemistry employed, drawbacks may include low hydraulic stability
resistance and the formation of voids due to the release of CO$_2$ during cure. Experimental
studies showed the polyurethane adhesive interface had lower bond strength and stiffness
than the epoxy adhesive interface. However, similar flexural strengths were achieved in
FRP-strengthened RC members due to stress redistribution enabled by the flexible interface
[16, 17, 15, 18]. No studies exist on FRP-PU for shear strengthening of RC beams. In fact,
all the design guidelines for external shear strengthening with FRP are limited to FRP-EP
systems.

1.2 Objective

The objective of this research is to evaluate the mechanical behavior of shear-deficient RC
beams strengthened by an externally-bonded FRP-PU composite system via experimental,
analytical, and numerical techniques. The biaxial polyurethane interface investigations will be utilized to characterize the significance of considering the state of stress and stress interaction of the FRP and the interface. The objective will be accomplished by:

- Conducting an experimental program with different parameters to evaluate the shear behavior of shear strengthening RC beams by FRP-PU system.

- A closed-form mechanics-based analytical model was derived to identify the effects of various properties on shear strength capacity considering a linear brittle bond-slip constitutive law, the effective FRP stress principle, and the theorem of upper-bound limit analysis of plasticity. These equations gave scientific reasons behind different shear strength responses.

- Developing an FEA numerical model by which more details about shear transfer mechanism that could not be experimentally captured are studied. Also, it can explain the FRP and concrete states of stress corresponding to the element location related to the beam geometry and the location of diagonal shear crack.

- Evaluating the interfacial biaxial states of stress on the polyurethane shear behavior. The nonlinear bond characteristics are studied utilizing both numerical and experimental techniques.

1.3 Plan

This dissertation consists of six chapters and two appendices. Chapter 1 covers a brief motivation and introduction to the research area and expresses the need of this study. The objective, plan, FRP composites, and the behavior of beams failing in shear are also presented in this chapter. Chapter 2 presents the experimental small- and large-scale specimens with a
brief background on the literature experimental work. Different parameters were taken into consideration: adhesive type (polyurethane and epoxy), size effect (small-scale and large-scale), shear span-to-depth ratio (3 and 4), FRP configuration (two sides and U-wraps), and FRP scheme (sheets and strips with $45^\circ$ or $90^\circ$ inclination angles. Six small-scale beams with $a/d = 3$, eight small-scale beams with $a/d = 4$, and five large-scale beams with $a/d = 4$ beams were experimentally tested to failure under four-point loading.

Chapter 3 presents a closed-form mechanics-based analytical model derived considering a linear brittle bond-slip constitutive law, the effective FRP stress principle, and the theorem of upper-bound limit analysis of plasticity. The analytical equations are able to predict the shear strengths and obtain the load-displacement responses of externally strengthened shear deficient RC beams by FRP composites. Different parameters, which govern the shear strengths based on the specimen size and the FRP configuration, are also concluded. Chapter 4 deals with numerical finite element analysis (FEA) created of the experimental beam specimens using MSC.Marc. The numerical load-deflection and load-strain behaviors provide additional details about the shear transfer mechanism and the FRP stress analyses, which are significant in explaining the interfacial biaxial state of stress. The interfacial bond-slip under biaxial stress is presented in chapter 5 by proposing a new constitutive model that takes into consideration the biaxial state of stress within the interface. A new experimental setup was developed for the purpose of performing lap shear tests in two dimensions by applying two perpendicular forces using hydraulic jacks controlled by an electric pump. Experimental calibration using nonlinear least-squares fitting of the experimental strain data with the numerical finite difference (FD) equations was carried out to obtain the bond-slip parameters for the two-dimensional (2D) polyurethane interface system.
1.4 FRP Composites

FRP composites that used in civil infrastructure applications are comprised in a controlled way of high tensile strength and modulus fibers and thermosetting matrices. The achieved mixture, which cannot be reformed to its original components, has more beneficial properties than its own constituent properties such as high stiffness with low weight, and better resistance to corrosion than its raw materials. The composite laminate is one of the most popular techniques used in structural applications as retrofitting in order to increase the load carrying capacity of the deficient structural elements instead of replacing them. The responsibility of the major portion of carrying loads is for reinforcing fibers whereas the matrix keeps the fibers in the right place and direction, transfers loads and stresses between them, and protects them from mechanical degradation and adverse environment like temperature and humidity.

1.4.1 Reinforcing Fibers

Fibers are principal constituents of the FRP composites and have its vast volume fraction and responsible for carrying the tensile forces. An appropriate selection of the fiber type, volume fraction, length, and orientation has played an important role that affecting the mechanical properties of the FRP composites. Fiber composite materials have two major classes recognized as continuous and short fiber materials. The continuous fibers commonly used in the form of laminated or layered structure whereas the short fibers appear in the form of flakes or chips. There are several types of fibers used in different applications of civil infrastructure such as carbon, glass, and aramid [10]. Some properties of the common carbon fibers are listed in Table 1.1.
Table 1.1: Properties of common carbon fibers [10]

<table>
<thead>
<tr>
<th>Fiber type</th>
<th>Density (g/cm³)</th>
<th>Tensile strength (ksi)</th>
<th>Tensile modulus (Msi)</th>
<th>Failure strain (%)</th>
<th>Poisson’s ratio</th>
<th>Coefficient of thermal expansion (⁰C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-300</td>
<td>1.76</td>
<td>530</td>
<td>33.5</td>
<td>1.40</td>
<td>0.20</td>
<td>-0.60</td>
</tr>
<tr>
<td>AS-1</td>
<td>1.80</td>
<td>450</td>
<td>33.0</td>
<td>1.32</td>
<td>0.20</td>
<td>-0.75</td>
</tr>
<tr>
<td>AS-4</td>
<td>1.80</td>
<td>590</td>
<td>36.0</td>
<td>1.65</td>
<td>0.20</td>
<td>-0.75</td>
</tr>
<tr>
<td>T-40</td>
<td>1.81</td>
<td>820</td>
<td>42.0</td>
<td>1.80</td>
<td>0.20</td>
<td>-0.75</td>
</tr>
<tr>
<td>IM-7</td>
<td>1.78</td>
<td>770</td>
<td>43.6</td>
<td>1.81</td>
<td>0.20</td>
<td>-0.75</td>
</tr>
<tr>
<td>HMS-4</td>
<td>1.80</td>
<td>360</td>
<td>50.0</td>
<td>0.70</td>
<td>0.20</td>
<td>-0.75</td>
</tr>
<tr>
<td>GY-70</td>
<td>1.96</td>
<td>220</td>
<td>70.0</td>
<td>0.38</td>
<td>0.20</td>
<td>-0.75</td>
</tr>
<tr>
<td>Pitch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-55</td>
<td>2.00</td>
<td>275</td>
<td>55.0</td>
<td>0.50</td>
<td>0.20</td>
<td>-1.30</td>
</tr>
<tr>
<td>P-100</td>
<td>2.15</td>
<td>350</td>
<td>110.0</td>
<td>0.32</td>
<td>0.20</td>
<td>-1.45</td>
</tr>
</tbody>
</table>

The first successfully produced carbon fibers was in the early of 1960s which was utilized in aerospace industry. Later, carbon fibers have been employed widely in other applications of fields such as civilian aircraft and structural retrofitting. When high tensile strength, high tensile stiffness, light weight, low (negative) coefficient of thermal expansion, and superior fatigue strength are required, carbon fibers are used as a form of composites with lightweight matrix or resin. In civil infrastructural applications, carbon fibers are used to enhance the flexural and shear capacities of RC members by wrapping or attaching the carbon fibers or fabric to and around the part that wants to be strengthened. Carbon fibers unfortunately have some disadvantages including high cost, low failure strain, low impact resistance, and high electrical conductivity. They have higher confining performance compared to other types of fibers due to the higher stiffness and the improved durability characteristics [10].

Another common kind of fibers and widely utilized in different civil applications is glass fibers. Although it is not as stiff as carbon fiber, glass fiber has high tensile strength, low price, good chemical resistance, and good insulating properties. The main components of the raw material of the glass fiber are limestone, silica sand, and soda ash. Some disadvantages of
glass fiber are low tensile stiffness, high density, high sensitivity to abrasion, and low fatigue resistance. The most common types of glass fibers include E-glass and S-glass. Figure 1.1 shows several types of material properties including GFRP (Isorod) manufactured by Pultrall Inc., Canada; GFRP (C-Bar) manufactured by Marshall Industries Companies Inc., USA; CFRP (Leadline) manufactured by Mitsubishi Kasei, Japan; and conventional steel [1].

Aramid Fibers are considered the lowest density and the highest tensile strength-to-weight ratio among all other FRP composites, and also have negative coefficient of thermal expansion. The most disadvantages include low compressive strength, and machining or cutting difficulties. They are manufactured in some traditional names such as Kevlar 49, Kevlar 149, and Technora [10].

Figure 1.1: Material properties of FRP and steel reinforcement [1]
Matrix materials include polymeric, metallic, and ceramic. Polymeric matrix is divided into two major classes: thermoplastic and thermosetting. In thermoplastic matrices, the molecules do not have strong cross-links due to lack of chemically joins among them. They just have weak secondary bonds which are broken when applying temperature because the applied pressure resulting in moving the molecules relative to each other and a new configuration be formed. When decreasing temperature, the molecules can be frozen and another configuration could be formed with restoring the secondary bonds. Thermoplastics, therefore, are not used in Civil infrastructure applications.

In thermosetting matrices, on the other hand, the molecules during the polymerization reaction are joined with each other by strong cross-links forming rigid structure. Thermosetting bonds can not be broken or melted by applying heat. Therefore, thermosetting matrices are widely utilized in civil infrastructure applications. The most common types of thermosetting matrices in structural applications include epoxy and polyurethane. Epoxy Resin is the most dominant matrix utilized in structural applications. It can be utilized with a variety of curing agents and starting materials or as commonly known as hardeners to compose a wide range of properties. However, epoxy resin has some disadvantages such as relatively expensive and slow curing. Polyurethane Resin was produced first time in Leverkusen, Germany in 1937 by Otto Bayer and his coworkers by reacting polyol with polyisocyanate, and Goodyear in 1968 announced the first usage in structural application [19]. Polyurethane matrices are used in many applications such as furniture, automotive, medical devices, etc. Polyurethane-based adhesive has several advantages include fast curing, negligible shrinkage, relatively cheap compared to other adhesives, perfect at low temperature and good interface bond to most fibers and substrates. On the other hand, polyurethane has some drawbacks such as high
sensitivity to high temperature, requiring the use of a primer for some substrates, and low resistance to water immersion.

1.5 FRP Systems

FRP systems include several forms depending on how the FRP composites are delivered and installed in the site. Selection of one category of FRP systems among others depends on the simplicity of application and the potential transfer of structural loads. The most common FRP systems available in strengthening structural members include [20]: Wet layup systems: composed of dry fiber sheets (unidirectional or multidirectional) impregnated to be saturated and cured with a saturating resin on site. Prepreg systems: composed of partially cured and fully saturated fiber sheets (unidirectional or multidirectional) pre-impregnated with a saturating resin at the factory site. Depending on the system requirements, an additional resin may or may not be needed on site. Pre-cured systems: compose of lots of composite shapes fully cured at the factory site (off site) installed to the concrete surfaces through the use of adhesives. Near-surface-mounted (NSM) systems: compose of circular or rectangular bars or plates installed into grooves on the surfaces of concrete to be bonded by using an appropriate adhesive.

1.6 FRP Bond-Slip Studies

In RC members strengthened with FRP composites, the external loads are usually applied directly to the concrete and the FRP composites are engaged by load or stress transfer mechanism through the composite action between the FRP and the concrete. Bond is idealized as a shear stress (which develops between the adherent and the substrate) that
causes slips under significant loadings. Slip is defined as a relative displacement between the FRP and the concrete. The bond-slip relationship is an idealization of material-point behavior of the continuous stress field. Various numerical and analytical derived constitutive models have been proposed in the literature based on experimental data with one-dimensional (1D) assumption that the stresses and strains are uniform along the FRP width at the same coordinate of the longitudinal fiber direction.

Equations 1.1, 1.2, 1.3, and 1.4 represent the linear brittle constitutive law, the general differential equation, the analytical slip and strain expressions of one-dimensional (1D) bonded joint considering the segment $dx$ as shown in Figure 1.2 (more details are in Yuan et al. [2]).

$$\tau = \begin{cases} \frac{\tau_f \delta}{\delta_f} & \text{when } 0 \leq \delta \leq \delta_f \\ 0 & \text{when } \delta > \delta_f \end{cases}$$  \hspace{1cm} (1.1)

$$\frac{d^2 \delta}{dx^2} - \lambda_p^2 \delta = 0$$ \hspace{1cm} (1.2)
\[
\delta = \frac{\delta_f P \lambda_p \cosh (\lambda_p x)}{\tau_f b_p \sinh (\lambda_p L)}
\]  
(1.3)

\[
\epsilon = \frac{d\delta}{dx} = \frac{\delta_f P \lambda_p^2 \sinh (\lambda_p x)}{\tau_f b_p \sinh (\lambda_p L)}
\]  
(1.4)

where \( \lambda_p^2 = \frac{\tau_f}{\delta_f} \left( \frac{1}{E_p t_p} + \frac{b_p}{b_c E_c t_c} \right) \). The thickness of the adherent (FRP) and the substrate (concrete) are \( t_p \) and \( t_c \), respectively. \( b_p \) and \( b_c \) are the widths of the FRP and the concrete, respectively. \( E_p \) and \( E_c \) are the Young’s moduli of the FRP and the concrete, respectively.

1.7 Behavior of Beams Failing in Shear

In general, flexural failure and shear failure are the two major failure modes in RC beams. The shear failure though is brittle and sudden in nature. It can not be precisely predicted as in flexural failure, and it provides no advance caution prior to failure. Although it can generally be predicted by using any current design code [21, 22], but a profound understanding of the shear behavior of RC beams and the different possible failure modes should be given to be able to strengthen or design such beams for shear.

The cracking patterns, of a simply supported rectangular RC beam shown in Figure 1.3a with four-point loading configuration, are represent for different possible failures. At inclined cracking and failure of beams having rectangular shaped without web reinforcement, the moments and shears are plotted in Figures 1.3b and 1.3c as functions of shear span-to-depth ratio (see Figure 1.3a). As the span is changed, the cross section remains constant. The maximum moment which can be developed corresponding to the nominal moment capacity of the cross section is plotted as a horizontal line as shown in Figure 1.3b. The shaded area shows the strength reduction due to shear. Web shear reinforcement is provided to ensure
that the nominal flexural capacity be reached. Experimentally, the shear span-to-depth ratio has been proven to be a highly influencing factor on shear strength capacity.

From Figure 1.3b, four general failure modes have been established [4].

- **Very Short Beams (Deep) \((a/d \leq 1)\):** Inclined cracks that developed in very short shear spans join the support and the point load. Due to the inclined cracks, the horizontal shear flow is destroyed or changed from the longitudinal steel (tension tie) to the compression zone (compression strut) and the beam action behavior is changed to arch action. After inclined cracking occurs, the simply supported beam behaves as
a tied arch in which the load is carried by the compression strut spreading around the shaded area of Figure 1.4a and by the tension tie in the longitudinal steel. The tensile force in the beam is uniform from support to support and the reinforcement serves as a tension tie in the tied-arch system. Several failure modes are possible for the tied-arch system, as shown in Figure 1.4b, and the most common one is the anchorage failure.

Figure 1.4: Modes of failure in deep beams ($a/d \leq 1$) [3]

- Short Beams (Deep) ($1 < a/d \leq 2.5$): After redistribution of internal forces to carry additional loads by arch action, inclined cracks are developed when the shear exceeded the inclined cracking strength. After the flexure-shear crack develops, the crack progresses further into the compression zone as the load increases. It also extends as a secondary crack toward the tension reinforcement and then progresses horizontally along that reinforcement. Eventually, failure resulted from either a bond, splitting, or dowel failure to cause an anchorage failure at tension reinforcement, called a ‘shear-tension failure’ shown in Figure 1.5(a), or a crushing failure in the concrete near the compression face, called a ‘shear compression failure’ as in Figure 1.5(b).

- Slender Beams (Intermediate) ($2.5 < a/d \leq$ about 6): For intermediate span length beams, the first cracks to be formed are the vertical flexural cracks followed by the inclined flexure-shear cracks. At the beginning, many flexural cracks go to bend over and create beam portions between cracks called ‘teeth’ shown in Figure 1.6. After
the tooth is reduced in size so that it becomes unable to carry the moment emerging from $\triangle T$ as a result of the increasing number of the flexural cracks, it breaks to form the inclined flexural-shear crack. Suddenly at inclined flexure-shear crack, the beam is unable to redistribute the loads as in short beams (smaller $a/d$ ratio). Therefore, the beam shear strength in this mode is represented by the formation of inclined crack. This mode is considered the usual category for beam design, and the term ‘diagonal tension failure’ has arisen.

- Very Slender Beams (Long) ($a/d >$ about 6): The long span length beams start to fail with the tension reinforcement yielding and end by the concrete crushing at the maximum bending moment section. Prior to failure, and besides the vertical flexural cracks at the maximum bending moment section, slightly inclined cracks (from the vertical cracks) might be formed between the maximum bending moment section and the support.
In brief, shear tends to form inclined cracks. The effect of shear is negligible if no such inclined cracks were formed before the nominal flexural strength has been reached. After formation of an inclined crack, beams may reach failure as for the so-called ‘diagonal tension failure’ if it is not deep \((a/d > \text{about 2})\), or otherwise (for deep beams), they may reserve strength. After the formation of an inclined crack, and to maintain a state of equilibrium for beams with reserve strength capacity, the forces must be redistributed. Present knowledge of how the redistribution of the forces is limited; and therefore, for the design of all but except deep beams, the shear strength is assumed to be reached upon the inclined crack forms.

1.7.1 B- and D-Regions

To pay more attention to a physical significance of the analysis and design for shear strength, it is important to more clearly study and examine the mechanism of shear transfer in RC members. Any RC member can be divided into two regions. Longer shear spans carry load by beam action and are called B-regions, where B refers to beam or to Bernoulli who supposed linear strain distribution in the beam theory (plane sections remain plane). Shorter shear spans carry load generally by arch action including in-plane forces and are called D-regions,
where D refers to discontinuity or disturbed. A discontinuity in strain and stress distribution occurs at the point of concentrated load or reaction or at the change in the geometry of a structural element. According to St. Venant’s principle, a local disturbance like a point of reaction or concentrated load is dissipated within one height of the beam from the applied point. The shear resisting mechanisms and consequently the strength estimation and the modes of failure are distinct due to the difference in distribution of strains and stresses in the B- and D-regions [5].

1.7.2 Inclined Cracking

Before shear failure can occur, there must be inclined cracks existed, and these inclined cracks may be formed in two different ways. The first scenario is the ‘web-shear cracks’ as shown in Figure 1.7a, that happens in thin-walled I beams with small a/d ratios in which the web shear stresses are high and the flexural stresses are low. In few extreme situations, the principal shear stresses at the beam neutral axis exceed the flexural stresses at the bottom of the beam, and the relating inclined cracking shear force can be calculated as the shear necessary to make the principal tensile stress and the tensile strength be equal at the beam centroid. In most RC beams, however, the flexural cracks propagate to become flexure-shear cracks as in Figure 1.7b. The flexural cracks change the state of stress in the beam and cause stress concentration near the crack head. Empirical equations have been derived to determine the flexure-shear cracking load because that knowing the principal stresses in an uncracked beam do not predict the flexure-shear cracking.
1.7.3 Internal Forces in Beams with Shear Reinforcement

The shear transfer in RC beams occurs by a combination of the following mechanisms (shown in Figure 1.8):

- Uncracked concrete shear resistance, $V_{cy}$.
- Aggregate interlock (or interface transfer) force $V_a$, tangentially along a crack, and analogous to a frictional force because of irregular aggregate interlocking along the rough concrete surfaces on both sides of the crack.
- Dowel action, $V_d$, the longitudinal reinforcement resistance to transverse force.
- The shear reinforcement resistance, $V_s$, from stirrups (does not exist in beams without shear reinforcement).
1.7.4 Truss Model

To make use of the behavior of RC beams failing in shear in design, a mathematical-mechanical model should be derived to express that behavior. Historically, Ritter (a Swiss engineer) and Morsch (a German engineer) independently published papers in 1899 and 1902, respectively, to propose the truss model in RC beams for shear design. These proposals represented a good model to explain the forces that exist in cracked concrete beams. Since then, this model has been relied on for the development of different codes and standards for shear design of RC beams.

In slender beams, the applied shear force cannot be transferred directly through the diagonal compression struts from the applied load to the support as shown in Figure 1.9. The applied shear force is transferred first by compression fans and then the equilibrium is achieved between compression fans and tensile forces of the stirrups. If the stirrups exist, the direction of the applied shear force is reversed and transferred to the compression field located between the compression fans. This compression field can be classified as B-region because it contains parallel diagonal struts with much less complicated stress variation than in D-region.
In contrast, in deep beams as demonstrated in Figure 1.10, all compression fans act in D-region and the shear-compression failure can occur due to concentration of stress. The diagonal compression flows directly from the applied load to the support with an inclination angle. The concrete segments between the inclined cracks act as compression struts, and therefore, the inclined cracks angle and the compression struts angle are the same [5]. The truss model, which is known as the strut and tie model, assumes that after the concrete cracks, the behavior of the RC beams becomes identical to a truss with top compression and bottom tension of longitudinal cords, vertical steel rebar ties, and diagonal concrete struts as shown in Figure 1.11.

Figure 1.9: Shear transfer mechanism of slender beams [5]

Figure 1.10: Shear transfer mechanism and crack pattern of deep beams [5]
1.7.5 Beams without Stirrups

Beams without stirrups fail when inclined cracking happens or shortly later. Therefore, the inclined cracking shear must be equated to the shear capacity of such members. Five principal variables affect the inclined cracking load, and some are included in the shear design equations and the others are not [3]. The variables include: tensile strength of concrete which is directly related to the shear strength of concrete member, longitudinal reinforcement ratio $\rho_{w}$ as shown in Figure 1.12, $a/d$ ratio as shown in Figure 1.3c, lightweight aggregate concrete which has lower tensile strength than the normal weight concrete and is handled in the ACI Code (through introducing the factor $\lambda$), and beam size as shown in Figure 1.13.
Figure 1.12: Reinforcement ratio effect on shear capacity of beams without web reinforcement [3]

Figure 1.13: Beam size effect on shear capacity of beams without web reinforcement [3]
2.1 Background

Numerous experimental studies demonstrated the efficacy of carbon FRP-EP shear strengthening of rectangular RC flexural specimens. Uji [23], Chaallal et al. [24], Adhikary et al. [25], Carolin and Täljsten [26] studied the effect of strengthening scheme on shear strength with fixed shear span-to-depth ratio \(a/d\). Test results showed that the FRP debonding was the predominant failure mode, and the increase in shear strength by using U-wraps was greater than using side bonding of the FRP. Chaallal et al. [24] studied the performance of eight RC beams strengthened in shear with FRP strips bonded on the beam sides. The parameters considered were the orientation angle of the reinforcing FRP strips (90° versus 135°) with respect to the specimen longitudinal axis. The failure mode was FRP strip debonding, and the laminate did not have any effect on the rigidity of the beams in the first phase of loading. The effect of FRP reinforcement, however, was apparent at the initial of cracks formation and was more significant as increasing the applied load. The authors asserted that shear strengthening both increased the shear capacity and also the overall rigidity of the specimens through the prevention of cracks propagation.

Khalifa [27] performed experimental tests on twelve RC beams with dimensions of 150×305×3050 mm strengthened with FRP for shear. The objective of the study was to understand the following parameters: the existence of transverse steel stirrups, \(a/d\) ratio, and FRP configuration. The specimens failed in shear by FRP debonding. The results showed that the FRP contributed to shear strength to a specific level of the FRP axial stiffness after which no FRP contribution to shear strength. The results confirmed that the transverse steel strain would be less in the presence of the FRP reinforcement. Conclusions included the following:
The FRP contribution to the shear capacity is affected by the $a/d$ ratio, the FRP contribution to shear strength remains constant above a certain amount of FRP ratio, and the FRP contribution to shear capacity is more effective in beams without stirrups.

Leung et al. [28] studied the size effect on shear capacity on geometrically similar small and large specimens strengthened with FRP. Three depths (180, 360, and 720 mm) and constant span-to-depth and span-to-width ratios were considered. Specimens were strengthened with U-wraps with variable thickness, width, and spacing and compared to fully-wrapped and control cases. Results showed that the shear capacity of specimens strengthened with U-wraps exhibited a size effect (the smaller the beam the higher the size effect); however, fully wrapped specimens were not dependent on size. Grande et al. [29] investigated the effect of $a/d$ ratios (2.5, 3, and 4). Results indicated that the FRP contributions to shear capacity were maximum with $a/d = 3$, intermediate with $a/d = 4$, and minimum with $a/d = 2.5$.

2.2 Beam Design

A series of specimen cross sections and steel reinforcement ratios were proposed for designing small-scale beams (span length 96 in, width 6 in) and large-scale beams (span length 132 in, width 12 in). The design criteria were to obtain minimum dimensions and minimum steel reinforcement ratios that ensure the ratio between the nominal design flexural load and the nominal design shear load ($P_m/P_v$) was greater than 2.0 for shear-deficient specimens. The compression zone of both small- and large-scale specimens was reinforced with two 1/2 in diameter (US # 4). Transverse steel rebars, 3/8 in diameter (US # 3) with spacing of 4 in on center, were used in half the span to ensure shear failure on one side. Control specimens contained stirrups along the full span.
Fourteen small-scale beams were designed with height of 12 in and two span-to-depth ratios (3 and 4). Four 3/4 in diameter (US # 6) steel rebars were utilized as longitudinal reinforcement in the tension zone with two layers of two rebars in each layer with clear cover of 1.5 in by steel chairs, and clear distance of 1 in between the two layers, with a reinforcement ratio $\rho_w$ of 3.3%. The nominal design bending moment of the small-scale specimens was 749.7 kips-in, with nominal design flexural load $P_m$ of 56.0 kips for $a/d = 3$ and 42.0 kips for $a/d = 4$. The nominal concrete shear strength was calculated based on the equation for ultimate shear force without transverse reinforcement derived by Reineck [30]. For small-scale specimens, $P_v$ was 20.9 kips for $a/d = 3$ and 16.1 kips for $a/d = 4$.

Five large-scale beams were designed with height of 17 in and $a/d = 4$. Five 9/8 in diameter (US # 9) steel rebars were utilized as longitudinal reinforcement in the tension zone with two layers of three rebars in the bottom and two rebars in the second layer with clear cover of 1.5 in by steel chairs, and clear distance of 1 in between the two layers, with a reinforcement ratio $\rho_w$ of 3.0%. The nominal design bending moment of the large-scale specimens was 3316.4 kips-in, with $P_m$ of 121.0 kips and $P_v$ of 59.0 kips. Figures 2.1 and 2.2 show the detailed information regarding the small-scale and large-scale specimens, respectively.
Figure 2.1: Details of small-scale specimens
Figure 2.2: Details of large-scale specimens

Regarding the FRP composite design, the FRP-PU specimens were designed using the model
of Triantafillou and Antonopoulos [31]. The equations for effective FRP strain $\epsilon_{pe}$ were based on the available experimental test data of FRP-EP system only, but were used here since no literature about FRP-PU system shear strengthening RC beams are available yet. Table 2.1 shows the design entries for the FRP-PU shear contribution $P_p$ of both small- and large-scale composite systems. For the specimen configurations tested with both the FRP-PU and FRP-EP composite systems, the strengthening details were taken to be the same as that designed for the FRP-PU specimen (no independent experimental design for FRP-EP).

Table 2.1: Designed FRP-PU shear contribution

<table>
<thead>
<tr>
<th>Specimen size</th>
<th>Scheme</th>
<th>$\epsilon_{pe}$ debonding</th>
<th>$\epsilon_{pe}$ rupture</th>
<th>$P_p$ (kips)</th>
</tr>
</thead>
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<td>0.0060</td>
<td>25.4</td>
<td></td>
</tr>
<tr>
<td>strip</td>
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<td>0.0083</td>
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<td></td>
</tr>
<tr>
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<td>0.0074</td>
<td>57.9</td>
<td></td>
</tr>
<tr>
<td>strip</td>
<td>0.0087</td>
<td>0.0102</td>
<td>35.9</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ S = 96×6×12 in and L = 132×12×17 in

2.3 Materials

Concrete was obtained from a local supplier with target mix strength of 5.0 ksi, water-cement ratio of 0.46, maximum course aggregate of 3/8 in, and a slump of 5.5 in. All the reinforced concrete specimens were poured in October 27$^{th}$, 2014, and Figure 2.3 shows the forms preparations and concrete pouring. Standard cylinder compression tests were carried out based on ASTM C39 [32] utilizing an Instron SATEC universal testing machine (UTM). The 28-day average compressive strength was 5.8 ksi. The average day of test compressive strengths were 8.75 ksi and 9.51 ksi for small-scale and large-scale specimens, respectively. All the steel reinforcing bars were ASTM A615 Gr. 60 steel. Three steel reinforcing bars were tested in uniaxial tension for each steel bar size following ASTM E8 [33] and the average yield stress, ultimate stress, yield strain were 63.82 ksi, 97.18 ksi, and 0.00253 respectively.
The FRP-PU composite was comprised of a moisture-cured polyurethane matrix and unidirectional carbon fibers pre-impregnated at the factory. The carbon fiber used was 17.4 oz/yd² unidirectional fabric with dry thickness of 0.026 in. The prepreg system uses the same polyurethane as resin and primer. The one-part primer was applied separately to the surface of the beams prior to installing the laminates. Curing was catalyzed by misting the laminates with water, applying to the primed surface, and rolling to prevent accumulation of bubbles during cure. The FRP-EP composite was a wet lay-up system composed of a two-part epoxy saturating resin and dry unidirectional carbon fibers (same as the carbon fibers used in the FRP-PU system). The same epoxy was used as a primer, and was applied to the beam surface prior to impregnating the dry fiber with the saturating resin and rolling. The FRP-PU and FRP-EP systems are the same as that employed in the previous PU study [15].

The bond behavior of the composite materials was previously performed by El Zghayar et al. [15]. A linear brittle bond-slip model was assumed in the analytical portion of this study (Chapter 3), with parameters maximum interfacial stress $\tau_f$ and the maximum interfacial slip $\delta_f$. The bond parameters were obtained from experimental calibration by using nonlinear least-squares fitting of the experimental strain data [15] with the analytical equations derived by Wu et al. [34], Yuan et al. [2]. The parameters for FRP-PU system are $\tau_f$ of 0.87 ksi and $\delta_f$ of 0.0078 in, and for FRP-EP system are $\tau_f$ of 1.35 ksi and $\delta_f$ of 0.0056 in.

Tensile properties of the cured laminates were obtained according to ASTM D7565 [35] for each FRP composite system. Natural G10 FR4 fiberglass epoxy sheets were utilized as end tabs with beveled edges to avoid gripping failure. Six tensile coupons from a 12 × 12 in panel were prepared and tested. Longitudinal strain was obtained from one 120 Ω strain gauge at the center of the gauge length. For FRP-PU system, the maximum tensile force per unit width was 3876 lbf/in and the tensile stiffness per unit width was 325470 lbf/in. For
FRP-EP system, the maximum tensile force per unit width was 8809 lbf/in and the tensile stiffness per unit width was 814314 lbf/in.

![Specimens concreting](image)

Figure 2.3: Specimens concreting

### 2.4 Testing Matrix

The small-scale and large-scale flexural specimens are identified by the convention S-###-# and L-###-#, respectively. FRP-strengthened specimens are indicated by first placeholder ‘U’ for U-wraps, ‘T’ for sheets, ‘S’ for side strips, and ‘I’ for strips inclined 45° from the longitudinal direction. Control specimens not strengthened by FRP are indicated by ‘C’. The second placeholder designates internal transverse steel reinforcement over one half ‘h’ or the full span length ‘f’. The third placeholder designates the $a/d$ ratio. The final place-
holder indicates what composite system was used (PU or EP), and is omitted for the control specimens.

The FRP reinforcement ratio $\rho_p$ is presented as a percentage of the beam cross section: $2t_p/b_w$ for FRP sheets and $2t_pb_w/s_p$ for FRP strips [31], where $t_p$, $b_p$, and $s_p$ are the FRP thickness, width, and spacing respectively. The $\rho_p$ is twice in small-scale specimens as in large-scale specimens, because the specimen width $b_w$ is one-half of the large-scale $b_w$. Table 2.2 and Figures 2.4 and 2.5 show the testing matrix and explain the strengthening configurations and schemes with detailed information about the FRP dimensions, orientations, and positions for both small- and large-scale specimens.

### Table 2.2: Testing matrix

<table>
<thead>
<tr>
<th>ID</th>
<th>Size</th>
<th>FRP configuration</th>
<th>FRP orientation</th>
<th>Stirrups</th>
<th>$a/d$ ratio</th>
<th># of strips or sheets / side</th>
<th>$\rho_p$ (%)</th>
<th>Composite system</th>
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</thead>
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<td>S</td>
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<td>...</td>
<td>half</td>
<td>3</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<tr>
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<td>...</td>
<td>full</td>
<td>3</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>S-Uh3-PU</td>
<td>S</td>
<td>U-wraps</td>
<td>90</td>
<td>half</td>
<td>3</td>
<td>3</td>
<td>0.68</td>
<td>PU</td>
</tr>
<tr>
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<td>U-wraps</td>
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<td>half</td>
<td>3</td>
<td>3</td>
<td>0.95</td>
<td>EP</td>
</tr>
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<td>half</td>
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<td>3</td>
<td>0.68</td>
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<td>3</td>
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<tr>
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<td>4</td>
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</table>

$^a$ $S = 96 \times 6 \times 12$ in and $L = 132 \times 12 \times 17$ in
2.5 Test Setup and Instrumentation Plans

All specimens were tested using a 110 kips capacity servo-controlled MTS hydraulic actuator with a four-point loading configuration. An adjustable steel spreader beam was used to obtain the required $a/d$ ratio based on the variable load points. The specimens were tested monotonically under displacement control with loading rate of 0.05 in/min. The vertical displacements were monitored and recorded at the midspan of each specimen from lower points on both sides using two string potentiometers. A linear variable differential transformer (LVDT) was installed on the support to record the support displacement and calculate the relative displacement. Several strain gauges were installed on the stirrups, along the longitudinal tension and compression reinforcement, on the concrete surfaces, and on the FRP composites as shown in Figures 2.1, 2.2, 2.4 and 2.5. The wrap/strip number and the FRP strain gauge number are the same as shown in Figures 2.4a, 2.4c, 2.5a and 2.5c.

Figure 2.4: Configurations and schemes of small-scale specimens with $a/d = 3$
2.6 Experimental Results

Table 2.3 summarizes the experimental results. The load values are based on the superposition method, with the limitation that the shear resistance due to concrete $P_c$ is constant with or without strengthening with FRP/transverse steel. The percentage of shear force gained by FRP along with the maximum recorded experimental strains attained by the FRP composites at debonding are also presented. The FRP contribution to shear strength $P_p^*$ was calculated from the model of Triantafillou and Antonopoulos [31] based on the maximum recorded experimental FRP strains at debonding. The strain gauges were limited to specific locations and do not reflect the actual maximum strain in the laminate at debonding.
Table 2.3: Experimental results

<table>
<thead>
<tr>
<th>ID</th>
<th>( P ) (kips)</th>
<th>( P_c ) (kips)</th>
<th>( P_p ) (kips)</th>
<th>Shear gained due to FRP (%)</th>
<th>Maximum ( \epsilon_p )</th>
<th>( P_p^* ) (kips)</th>
<th>Failure mode&lt;sup&gt;a&lt;/sup&gt;</th>
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<sup>a</sup> S = shear failure, F = flexure failure, D = shear with debonding failure, N = shear without debonding failure, C = shear with debonding and fracture failure, and P = test stopped

2.6.1 Failure mode and cracking pattern

All controls with stirrups over half the span and FRP-strengthened specimens failed in shear, as expected given the design philosophy. The control failure modes are shown in Figures 2.6a, 2.7a, and 2.8a. Diagonal shear cracks formed and continued to widen in the portion of the specimens with no stirrups. The control specimens with stirrups across the full span failed in flexure as shown in Figures 2.6b and 2.7b, except L-Cf4 where the test was stopped when the actuator capacity was reached.
In the small-scale specimens, the first flexural cracks were observed at a total load of approximately 12.4 kips. Diagonal shear cracks started to form at the middle of the shear span at mid depth at a load of approximately 18.9 kips. As the load increased in the FRP-PU specimens, multiple shear cracks formed and the original shear cracks in the shear deficient side widened progressively until shear failure occurred. In the large-scale specimens, the flexural cracks were finer than small-scale specimens and not obvious in the early loading stages, whereas single diagonal shear crack appeared at approximately 42.7 kips.

The FRP-strengthened beams exhibited concrete shear failure with FRP debonding in all specimens except S-Uh3-PU. FRP-EP specimens debonded within a thin layer of concrete substrate, whereas FRP-PU specimens consistently debonded in the adhesive layer. In S-Uh3-PU, the failure mode was concrete crushing in the compression zone and a diagonal crack forming between the top midspan and the first U-wrap (Figure 2.6c). The U-wraps did not prevent the diagonal shear cracks from forming, but prevented them from widening before debonding occurred, as seen in S-Uh3-EP (Figure 2.6d) and S-Uh4-EP (Figure 2.7d). Specimen S-Uh4-PU failed by FRP debonding of the third wrap followed by transverse fracture from one side of the bottom corner of the second wrap as shown in Figures 2.7c and 2.18. Figures 2.6, 2.7, and 2.8 show the failure mode and the cracking of the small- and large-scale specimens (refer to Table 2.3). The cracking patterns of the small- and large-scale shear deficient specimens are explained in Appendix A.
Figure 2.6: Failure mode and cracking pattern of small-scale specimens with $a/d = 3$
Figure 2.7: Failure mode and cracking pattern of small-scale specimens with $a/d = 4$
2.6.2 Parameter effects

Table 2.3 shows the total resistance of all specimens and the FRP contribution to the shear strength. When compared to control specimens, the strengthening effect of all strengthened specimens had a crucial importance with different degree depending on the different parameters used in this program. Regarding the adhesive types, it was found that the FRP-PU system always provided higher strength than the FRP-EP system for U-wrap strengthening. The shear force increased due to FRP composites for specimens S-Uh4-PU and S-Uh4-EP by 53.1 % and 17.3 %, respectively, whereas a small difference in shear force gained has been found for specimens S-Uh3-PU and S-Uh3-EP with 40.7 % and 38.6 %, respectively.
Strengthening with sheets for both adhesive systems had very close and comparable results. Specimens S-Th4-PU and S-Th4-EP had shear force due to FRP sheets with 31.0 % and 32.0 %, respectively.

It is known that the strengthening effect is higher with $a/d = 4$ than with $a/d = 3$ [36]. In this program, the small-scale specimens with $a/d = 4$ exhibited larger increases in shear strength capacity compared with same specimens with $a/d = 3$ except for specimens S-Uh3-EP and S-Uh4-EP when the shear strength increase of specimen S-Uh3-EP was 18.3 kips with increase of 38.6 %, whereas the increase in shear strength of specimen S-Uh4-EP was 5.0 kips with increase of 17.3 %. The size parameter effect on shear strength were proportional with the FRP reinforcement ratio. Regarding configuration and scheme effects, strengthening with U-wraps was the most efficient configuration method, but on the other hand, strengthening with inclined strips at 45° with respect to the horizontal direction had the least efficiency of all schemes used in this study.

2.6.3 Load-deflection behavior

Figure 2.9 shows representative load-deflection curves for the small- and large-scale specimens. Quasi-linear behavior to shear failure is the characteristic response for all specimens except S-Uh3-PU. Small-scale specimens with $a/d = 3$ exhibited higher stiffness than the same specimens with $a/d = 4$. Figure 2.9 shows that strengthening with externally-bonded FRP composites increases the overall stiffness by a negligible amount but enhances the shear capacity (refer to Table 2.3 for percentages). The apparent ductility of S-Uh3-PU is due to the yielding of longitudinal steel (shifting of failure modes due to shear strengthening). Figure 2.10 shows the support deflection curves for representatives of small-scale specimens.
Figure 2.9: Midspan load-deflection behavior

Figure 2.10: Support deflection behavior
2.6.4 Load-strain behavior

Figure 2.11 shows the strain behavior of the longitudinal tension steel reinforcement at the midspan of small- and large-scale specimens. The longitudinal steel strains in the initial stage of loading were very small. The initial loading stage continued up to approximately (6.7-7.9) kips for small-scale specimens with $a/d = 3$, (2.9-4.0) kips for small-scale specimens with $a/d = 4$, and 7.9 kips for large-scale specimens. The second loading stage featured higher strain rates than the initial stage. The transition zone between the two loading stages indicates the beginning of the flexural cracks that appeared at midspans. The longitudinal steel appeared to have contribution in resistance at the transition zone. Flexural cracks started at the transition zones of strain data as micro cracks and experimentally appeared later at higher load levels when became wider. The strains in the second loading stage were investigated to be changed in a quasi-linear manner with applied load. Yielding of longitudinal steel is the third loading stage of all FRP strengthened small-scale specimens with $a/d = 3$ and specimen S-Sh4-PU. These specimens have a plastic response that makes them more deformable as indicated by the ductile plateau.

Figure 2.12 shows the longitudinal tension steel reinforcement response near the support (SG-B1) of the shear deficient side. In the small-scale specimens, the contribution of the longitudinal steel rebar in resistance started at approximately 16.9 kips for control specimen S-Ch3 and (18.0-20.2) kips for other retrofitted specimens with $a/d = 3$, whereas it started at approximately 11.2 kips for control specimen S-Ch4 and (15.7-18.0) kips for other strengthened specimens with $a/d = 4$. The change rate in the longitudinal steel reinforcement strains near the support in large-scale specimens was minimal, and this might be due to the crack pattern which was very horizontal and narrow from the point load to the mid shear span and steep from there to make an $s$ shape (far from SG-B1). Physically the beginning time of
Figure 2.11: Load strain behavior of midspan longitudinal tension steel reinforcement straining the longitudinal rebar near the support is the starting of the diagonal shear cracks there. Generally, the FRP composite effects seem to delay and decrease the longitudinal tension steel reinforcement strains near the support (increase the stiffnesses) and hence delay the diagonal shear cracks. The control specimens have lower load-strain stiffnesses than FRP strengthened specimens as shown in Figure 2.12.

Figure 2.13 explains the load versus strain relationships for the transverse steel reinforce-
Figure 2.12: Load strain behavior of SG-B1

ment strain gauge ST1 for small-scale specimens with $a/d = 3$. The responses showed that the behaviors featured two loading stages. In the first loading stage, no contribution of the transverse steel reinforcement to the shear resistance was observed. In the second stage of loading, the transverse steel started to strain at a load of approximately 7.9 kips for specimen S-Ch3 and 20.2 kips for specimen S-Uh3-PU, whereas it started at approximately 45.9 kips for specimens S-Sh3-PU and S-Ih3-PU. Specimens strengthened with FRP composites
behave differently from each other in the transverse steel reinforcement until failure. The control specimen S-Ch3 started to strain earlier and the transverse steel strains in the FRP strengthened specimens were substantially less than specimen S-Ch3 at any load level which means that existence of FRP composites eases the transverse steel strains. Also, yielding of transverse steel reinforcement was observed in specimen S-Ch3, but this was not the case for all strengthened specimens with FRP composites.

![Figure 2.13: Load strain behavior of stirrups of small-scale specimens with $a/d = 3$](image)

Regarding the FRP composite data, Figures 2.14, 2.15, and 2.16 present the significant maximum recorded FRP strains for all small- and large-scale specimens strengthened with FRP composites (refer to Table 2.3). The FRP strains measured for specimens strengthened with sheets were smaller than other specimens strengthened with different schemes which means that the FRP strain distribution is highly affected by the FRP width. The FRP composites did not have any contribution to the load-carrying capacity at the initial stage of loading for all specimens except for beams with inclined strips of a 45° fiber orientation.
from the longitudinal axis. This indicates that the state of FRP strain is dependent on the FRP orientation with respect to the shear crack angle. In the second loading stage, the FRP started to strain, e.g., at an applied load of approximately 27.0 kips for small-scale specimens strengthened with U-wraps of $a/d = 3$, approximately 19.1 kips for small-scale specimens strengthened with U-wraps of $a/d = 4$, and approximately 67.4 kips for large-scale specimens.

The FRP strains continue to increase when increasing the applied load until a specific threshold (different from specimen to another) at which the third stage of loading started. The FRP strain behaviors in the last stage of loading were different in FRP-PU than in FRP-EP specimens. The maximum recorded FRP strains at the last stage of loading (before failure load) of the FRP-EP specimens decreased remarkably and immediately without decreasing the applied load by reversing the curves when debonding occurred. Incidentally the applied load had no effect on the debonded FRP. The maximum recorded FRP strains at the last stage of loading of FRP-PU specimens would decrease or increase with always decreasing the applied loads.
Figure 2.14: Load strain behavior of FRP and stirrups of small-scale specimens with $a/d=3$
Figure 2.15: Load strain behavior of FRP and stirrups of small-scale specimens with $a/d = 4$
Figure 2.16: Load strain behavior of FRP and stirrups of large-scale specimens

2.7 Discussion

The FRP contribution to shear strength $P_p^*$ in FRP-PU specimens based on the maximum recorded FRP strains at debonding was significantly lower than the designed values based on the effective debonding strains. For instance, the experimental and designed the FRP contributions to shear strength for S-Uh4-PU were 8.3 kips and 15.8 kips, and S-Th4-PU
were 4.3 kips and 25.4 kips. Although not used to size the FRP-EP composite system during design, much closer agreement was observed between the hypothetical designed and those based on the maximum recorded FRP strains at debonding. For example, the experimental and designed the FRP contributions to shear strength for S-Uh3-EP were 26.0 kips and 23.7 kips, S-Uh4-EP were 19.9 kips and 23.7 kips, and S-Th4-EP were 27.9 kips and 38.2 kips. The controlling parameter that caused this lower value for FRP-PU is the low tensile stiffness of FRP-PU that resulted in high effective FRP strain design values. However, the designed $P_p$ is dependent on both tensile stiffness and effective FRP strain [31] and yields lower shear strength for FRP-PU specimens than the corresponding FRP-EP specimens. Therefore, given the experimental $P_p$ values based on superposition method are similar between FRP-PU and FRP-EP specimens, the effective FRP debonding strain equations of Triantafillou and Antonopoulos [31] based on the composite material properties only, are not appropriate for the FRP-PU system and a new model based on both bond and material properties is needed.

Specimens with side bonding configurations showed that bond is effective when sufficient available bond length is available, otherwise the bond length governs. Figures 2.8d and 2.8c show that debonding occurred on one side of the shear crack with the shorter bonded length. Figure 2.6f shows the second strip debonded from both sides of the crack since insufficient bond length occurred on either side. However, the specimens strengthened with U-wraps showed that bond is effective and the debonding occurred at the upper side of shear crack. Figure 2.7d shows the upper crack side debonding whereas Figures 2.6d and 2.7c show how the second wrap was affected by debonding (specimen S-Uh3-EP) or rupture (specimen S-Uh4-PU) failures when the shear crack passed the wrap near the bottom side.

To investigate the shear-strengthening behaviors of the FRP-PU and FRP-EP composite systems, specimens S-Uh4-PU and S-Uh4-EP are compared. Although S-Uh4-EP failed at a
lower load than expected (premature debonding of wrap three), both specimens represented the debonding progression and FRP strain behavior of other specimens strengthened with FRP-PU or FRP-EP with different schemes or a/d ratios. Figure 2.17 shows different load-strain and strain-time behaviors for three FRP strain gauge locations (the gauge closest to the support SG-F1 is omitted) and two longitudinal rebar locations (midspan and mid-shear-span).

In specimen S-Uh4-PU, the shear cracks widened until debonding initiated in SG-F3 at $A_{F3}$, as shown in Figures 2.17a and 2.17c. The strains in SG-F2 and SG-F4 at the corresponding load level are identified by $A_{F2}$ and $A_{F4}$, respectively. Figure 2.7c shows the bonded lengths of wrap three relative to the location of the two shear cracks. SG-F4 showed a decrease in strain during the slight drop in applied load at $B_{F4}$, following which the second U-wrap (SG-F2) was rapidly engaged (rapid increase in strain at $B_{F2}$). Failure occurred in SG-F2 at $C_{F2}$ when one side of the bottom corner of the second wrap ruptured and pulled out with concrete cover as shown in Figure 2.18 (although no debonding occurred on the side face of the U-wrap).

In specimen S-Uh4-EP, unexpected debonding took place in SG-F3 at $D_{F3}$, as shown in Figures 2.17b and 2.17d. Figure 2.7d shows the bonded lengths of wrap three relative to the location of the single shear crack. SG-F2 and SG-F4 showed strain increase with applied load until SG-F4 debonded at $E_{F4}$. The strain in SG-F2 at the corresponding load level is identified by $E_{F2}$. The second U-wrap (SG-F2) showed a rapid increase in strain and debonding at a slightly lower load level $F_{F2}$.

Figures 2.17e and 2.17f show the longitudinal rebar strains at midspan (SG-B4) and mid-shear-span (SG-B2) of specimens S-Uh4-PU and S-Uh4-EP. The mid-shear-span strain gauge at SG-B2 is located at the same longitudinal location as SG-F3. The longitudinal rebar
strains at SG-B4 are increasing with similar trends of both S-Uh4-PU and S-Uh4-EP specimens. However, SG-B2 strain gauge behaviors show the increasing strain rate is higher in S-Uh4-PU than S-Uh4-EP.

The FRP-PU strain results showed that when the diagonal shear crack caused a load drop, three scenarios were observed from the recorded FRP strains. The first scenario was when FRP debonding failure occurred associated with load drop causing a decrease in the maximum recorded FRP strain as shown in SG-F3 of Figure 2.14c, SG-F1 of Figure 2.14d, SG-F4 of Figure 2.16b, and SG-F3 of Figure 2.16c. The second scenario was when no FRP debonding took place and the maximum recorded FRP strain decreased when the shear crack failure occurred not in the strengthened region but in the neighbor zone as shown in SG-F2 of Figure 2.14a. The third scenario was when FRP debonding followed by FRP rupture failure mode took place and the drop of the load was not associated with a decrease in the maximum FRP strain as shown in SG-F2 of Figure 2.15a. This was due to different FRP failure mode as it was debonding in the third wrap followed by rupture at one corner of the second wrap.
(a) FRP load-strain of S-Uh4-PU

(b) FRP load-strain of S-Uh4-EP

(c) FRP strain-time of S-Uh4-PU

(d) FRP strain-time of S-Uh4-EP

(e) SG-B4 and SG-B2 of S-Uh4-PU

(f) SG-B4 and SG-B2 of S-Uh4-EP

Figure 2.17: Comparative response of specimens S-Uh4-PU and S-Uh4-EP
Figure 2.18: Failure mode of S-Uh4-PU
CHAPTER 3: ANALYTICAL METHODS

3.1 Background

Many analytical models for predicting the shear strength of RC beams strengthened with FRP have been proposed. The available models for predicting the shear capacity of RC members strengthened in shear with externally-bonded FRP composites provide largely scattered shear strength predictions. The analysis of FRP in shear strengthening of RC beams is dependent on the bond stress-slip model between FRP laminates and concrete, material properties of the FRP laminates, FRP effective bond length and width, failure mode of the FRP system, and anchorage system if available [9]. Superposition was adopted by most analytical studies on FRP shear strengthening RC beams, which divides the total strength of a member into three components: concrete shear strength, transverse steel shear reinforcement, and externally-bonded FRP shear reinforcement. The FRP contribution to shear strength is proposed to be analogous to the transverse shear steel reinforcement based on the strut-and-tie model. Different design procedures for shear strengthening RC structures by externally-bonded FRP composites are currently available in the form of guidelines, codes, or specifications including ACI 440.2R-17 [20], CNR-DT 200 R1/2013 [37], fib-TG 9.3 [38], TR55 [39], and JSCE [40]. Some of the most important models are discussed here with the same original units as in their papers:

Triantafillou [7] was one of the first researchers proposed that FRP contribution to shear strength was based on calculation of an effective FRP strain $\varepsilon_{pe}$ which was obtained from regression of 40 experimental test result data from different studies. This model is considered the first model to recognize that the effective FRP strain is based on the bond condition of FRP-concrete adhesive interface and the development length of FRP, which was assumed to
be a function of FRP axial rigidity $\rho_p E_p$. Equation 3.1 was proposed as an expression for predicting the FRP shear contribution to shear capacity:

$$P_p = \frac{0.9}{\gamma_p} \rho_p E_p \epsilon_{pe} b_w d (1 + \cot \beta) \sin \beta$$  \hspace{1cm} (3.1)

where $\gamma_p$ is partial safety factor in FRP uniaxial tension and equal to 1.15 for CFRP, $\beta$ is the angle of fiber with respect to the beam longitudinal direction, and $\epsilon_{pe}$ is the effective FRP strain. $\epsilon_{pe}$ is a function of $\rho_p E_p$ to fit both FRP debonding and FRP rupture failure modes by a single curve as in Equations 3.2 and 3.3.

$$\epsilon_{pe} = 0.0119 - 0.0205(\rho_p E_p) + 0.0104 (\rho_p E_p)^2 \text{ for } 0 \leq \rho_p E_p \leq 1\text{GPa}$$  \hspace{1cm} (3.2)

$$\epsilon_{pe} = -0.00065(\rho_p E_p) + 0.00245 \text{ for } \rho_p E_p > 1\text{GPa}$$  \hspace{1cm} (3.3)

Increasing the axial rigidity resulted in reducing the effective FRP strain as shown in Figure 3.1 (nomenclature ‘p’ is used here instead of ‘frp’ in the original paper). Therefore, the thicker and stiffer FRP laminate, the less the $\epsilon_{pe}$ and the more dominant FRP debonding over tensile fracture. This model was proposed for continuous FRP sheets, but it can be easily used in case of FRP strips by taking into account the FRP reinforcement ratio of Equation 3.4.

$$\rho_p = \frac{2t_p b_p}{b_w s_p}$$  \hspace{1cm} (3.4)

Triantafillou and Antonopoulos [31] proposed an updated model after recognizing the deficiencies of the previous model by taking into consideration the behavior differences between FRP debonding and rupture and the concrete strength effect. The effective FRP strain
expression was calibrated based on 75 experimental test data from different studies. The effective FRP strain, \( \epsilon_{pe} \) was written against \( \rho_p E_p / f_c^{2/3} \) according to their recommendations by taking \( f_c^{2/3} \) to account for the concrete strength effect.

Curve fittings to the data were performed and separate expressions for effective FRP strains were derived for FRP debonding (Equation 3.5) and FRP shear-tension failure followed by or combined with FRP rupture (Equation 3.6).

\[
\epsilon_{pe} = 0.65 \left( \frac{f_c^{2/3}}{E_p \rho_p} \right)^{0.56} \times 10^{-3} \tag{3.5}
\]

\[
\epsilon_{pe} = 0.17 \left( \frac{f_c^{2/3}}{E_p \rho_p} \right)^{0.30} \times \epsilon_{pu} \tag{3.6}
\]

where \( f_c' \) is in MPa, and \( E_p \) is in GPa, \( \epsilon_{pu} = 0.015 \) for CFRP, and maximum \( \epsilon = 0.005 \) was placed as an upper limit to account for the mechanism of aggregate interlock of concrete.
Equation 3.7 is the experimentally derived FRP contribution to shear capacity.

\[ P_p = 0.9 \epsilon_{pe} E_p \rho_p b_w d (1 + \cot \beta) \sin \beta \] \hspace{1cm} (3.7)

This model was incorporated in the European Bulletin 14 (fib-TG 9.3) [38].

Khalifa et al. [8] modified the original model of Triantafillou [7] by introducing strain limitations depending on shear crack opening and loss of aggregate interlock. A regression was made for the database that was expanded with additional data from other researchers. The modified effective strain was proposed for both FRP rupture and FRP debonding failure modes by introducing the reduction factor \( R \) for the FRP ultimate tensile stress. The reduction factor is a proportion of the FRP modified effective strain to its ultimate strain. An upper limit of 0.5 for reduction factor was proposed to limit the FRP strain up to 0.004-0.005 and prevent the loss of aggregate interlock mechanism resulted from increasing the crack widths. The contribution of FRP composites to shear strength was determined by computing the FRP tensile force resulted from the FRP tensile stress across the proposed crack. The model recognized the difference between the behaviors of the FRP debonding and FRP rupture failure modes and provided a reduction factor for each case.

Equations 3.8 and 3.9 are the reduction factor for FRP debonding failure and FRP rupture failure, respectively.

\[ R = \frac{0.0042 f_c^{2/3} w_{pe}}{(E_p t_p)^{0.58} \epsilon_{pu} d_p} \] \hspace{1cm} (3.8)

\[ R = 0.5622 (\rho_p E_p)^2 - 1.2188 (\rho_p E_p) + 0.778 \leq 0.50 \] \hspace{1cm} (3.9)

The FRP effective strain is taken as the minimum of the two reduction factors multiplied by the ultimate strain of the FRP composite used. The equation utilized in debonding failure
mode adopted the concepts of effective bond length $L_e$ and the average of bond stress $\tau_{bu}$ which was developed by Maeda et al. [41].

Also, the model used the effective width to calculate the shear strength capacity governed by debonding shear failure and proposed only the FRP part (active zone) that extending past the shear crack by the effective bond length would be able to carry the shear. The effective width is depending on the shear crack angle which was proposed 45° in this model and the type of configuration as shown in Figure 3.2. Equations 3.10 (Figure 3.2(a)) and 3.11 (Figure 3.2(b)) are the FRP effective width (available bond length) equations for U-jacket sheet and bonded on sides sheet configurations, respectively (nomenclature ‘p’ is used here instead of ‘f’ in the original paper).

$$w_{pe} = d_p - L_e \quad (3.10)$$

$$w_{pe} = d_p - 2L_e \quad (3.11)$$

Figure 3.2: FRP effective width [8]
Khalifa and Nanni [42] proposed Equation 3.12 as a new value to the reduction factor $R$. The authors conducted six experimental tests of RC beams with four-point bending to investigate the required parameters. The parameters considered were the following: the FRP amount and distribution (strips versus continuous), wrapping scheme (U-wrap versus bonded on sides), fiber direction ($90^\circ$ versus $0^\circ - 90^\circ$), and end anchorage (U-wrap with versus without end anchorage).

$$R = \frac{f'_c^{2/3} w_{pe}}{\epsilon_{pu} d_p} [738.93 - 4.06(E_p t_p)] \times 10^{-6}$$  \hspace{1cm} (3.12)

The new model proposed the modified effective bond length developed by Miller and Nanni [43] instead of the previous model proposed by Maeda et al. [41] (Equations 3.13 and 3.14 for English and Metric units, respectively).

$$L_e = -0.00298 t_p E_p + 3.711$$  \hspace{1cm} (3.13)

$$L_e = -0.432 t_p E_p + 94.3$$  \hspace{1cm} (3.14)

Equation 3.15 is the upper limit of the reduction factor which was modified to take into consideration the differences in the ultimate strain of the CFRP sheets, that is to limit the effective strain to 0.004 for all kinds of FRP composites.

$$R = \frac{0.006}{\epsilon_{pu}}$$  \hspace{1cm} (3.15)

The drawback of this procedure according to Triantafillou and Antonopoulos [31] is that the FRP rupture equation is derived from data of experimental tests on both FRP rupture and debonding.

Chen and Teng [44, 45] proposed that the FRP stress distribution along the shear crack
is non-uniform due to the shear crack width variation along its length, the linear-elastic behavior of the FRP, and the bond characteristics between the FRP composites and the substrate concrete. The authors developed two models for each of the FRP debonding and the FRP rupture failure modes. The models were not relying on hypothetical shear crack angle of 45°, therefore, the shear crack angle was taken as one of the variables in the expression of $P_p$ in Equation 3.16. The FRP contribution to the shear capacity depends on truss analogy and varies for each model according to the effective stress of the FRP $\sigma_{pe}$. Here, a review of the FRP debonding model is considered.

$$P_p = 2\sigma_{pe} t_p b_p \frac{h_p (\cot \theta + \cot \beta) \sin \beta}{s_p}$$

(3.16)

The stress distribution factor $D_p$ was developed and used in the model to show that the effective stress at the failure is a fraction from the maximum stress achieved by the FRP debonding. The non-uniform FRP stress distribution was calculated by applying the stress distribution factor $D_p$ to the maximum FRP stress $\sigma_{p,m}$ (Equations 3.17 and 3.18).

$$\sigma_{pe} = D_p \sigma_{p,m}$$

(3.17)

$$D_p = \frac{\int_{z_b}^{z_t} \sigma_{p,z} dz}{h_p \sigma_{p,m}}$$

(3.18)

The parameters in the equations above were obtained from the bond behavior from simple shear tests.

The effective bond length is one of the most important aspects of bond behavior and is defined as the bond length after which no additional strength can be accomplished. The authors
developed models to estimate the effective bond length depending on an experimental data. Equation 3.19 is the FRP maximum stress $\sigma_{p,m}$ which was supposed to occur in the longest bond length region and was limited for debonding state with ultimate bond strength.

$$\sigma_{p,m} = \min \left\{ \sigma_p, \ 0.427\beta_w\beta_L\sqrt{\frac{E_p}{f_c}} \sqrt{t_p} \right\} \quad (3.19)$$

where the parameters in Equation 3.19 were calculated according to Equations 3.20, 3.21, 3.22, 3.23 and 3.24.

$$\beta_L = \begin{cases} 1 & \text{if } \lambda' \geq 1 \\ \sin \frac{\pi \lambda'}{2} & \text{if } \lambda' < 1 \end{cases} \quad (3.20)$$

$$\lambda' = \frac{L_{max}}{L_e} \quad (3.21)$$

$$L_{max} = \begin{cases} \frac{h_{pe}}{\sin \beta} & \text{for U-wrap} \\ \frac{h_{pe}}{2\sin \beta} & \text{for side bonded} \end{cases} \quad (3.22)$$

$$L_e = \sqrt{\frac{E_p t_p}{f_c}} \quad (3.23)$$

$$\beta_w = \sqrt{\frac{2 - b_p/(s_p \sin \beta)}{1 + b_p/(s_p \sin \beta)}} \quad (3.24)$$

The FRP maximum bond length $L_{max}$ is supposed to occur at the mid height in the case of side bonding configuration and at the lower end of the shear crack in the case of U-wrap configuration, given that the assumption of the critical shear crack being straight is verified.
Equation 3.25 is the derived stress distribution factor $D_p$ for the debonding limit state.

$$D_p = \begin{cases} 
\frac{2}{\pi \lambda'} \frac{1 - \cos \frac{\pi \lambda'}{2}}{\sin \frac{\pi \lambda'}{2}} & \text{if } \lambda' \leq 1 \\
1 - \frac{\pi - 2}{\pi \lambda'} & \text{if } \lambda' > 1 
\end{cases} \quad (3.25)$$

The available models and guidelines for predicting the shear capacity of RC members strengthened in shear with externally-bonded FRP composites provide significant variances and largely scattered shear strength predictions. Figure 3.3 explains the comparison between different available analytical models of the FRP strength contribution according to the FRP axial rigidity $\rho_p E_p$ (nomenclature ‘p’ is used here instead of ‘f’ in the original reference).

Figure 3.3: Comparison of available models by FRP axial rigidity influence on shear stress resistance [9]
3.2 Analytical Model

Different analytical models have been proposed for structural concrete members strengthened with FRP reinforcement. Unlike the mechanics-based analytical models, most of them were based on regression analysis [42, 31, 44]. Monti et al. [46, 47] derived a mechanics-based model to predict the FRP contribution to shear capacity. They proposed a sine-constant-cosine FRP stress-slip constitutive law, and included evenly-spaced shear cracks with depth equal to the internal lever arm $0.9d$. The assumed behavior was more appropriate for rebar-concrete interface assumption, and the derivation did not utilize the softening branch of the constitutive model.

Unlike the previously proposed FRP stress-slip constitutive model [46, 47], in this study an interfacial bond stress-slip constitutive model is used in the analytical derivation. A linear brittle bond-slip model was adopted, consistent with those of Neubauer and Rostasy [48], Al-Jelawy [18]. The diagonal shear crack was assumed to be inclined with an angle $\theta$ to the horizontal beam axis $x'$, and with the abscissa $x$ being along the crack beginning from the crack tip and normal to the crack width, as shown in Figure 3.4. The theoretical effective (average) FRP stress along the shear crack length $L_x = d / \sin \theta$ is given by Equation 3.26.

Figure 3.4: Proposed rigid body deformation due to diagonal shear crack in specimen with no steel stirrups in left half of span
\[
\sigma_{pe}(x, \alpha) = \frac{1}{L_x} \int \sigma_p dx
\]

(3.26)

From the first equilibrium equation of the analytical derivation of the interfacial bond mechanism conducted by Wu et al. [34], Yuan et al. [2], the FRP stress \( \sigma_p \) at the loaded end can be calculated as \( \tau(\delta) \frac{L_x}{t_p} \), where \( \tau(\delta) \) is \( \tau_f \delta(x, \alpha)/\delta_f \), and hence \( \sigma_{pe}(x, \alpha) \) is Equation 3.27:

\[
\sigma_{pe}(x, \alpha) = \int \frac{\tau_f}{t_p \delta_f} \delta(x, \alpha) dx
\]

(3.27)

The kinematic (compatibility) relation is expressed in terms of crack width \( w(x, \alpha) = \alpha x \) which is assumed to be linear with crack opening angle \( \alpha \) along the diagonal shear crack axis. Therefore, the imposed slip of FRP crossing the crack is given by Equation 3.28:

\[
\delta(x, \alpha) = \frac{w(x, \alpha)}{2} \sin(\theta + \beta) = \frac{1}{2} \alpha x \sin(\theta + \beta)
\]

(3.28)

Boundary condition constraints of the available bond length are necessary to predict the shear capacity. It was assumed only the FRP extending past the shear crack by the effective bond length resists shear. Equation 3.29 is the compatibility constraint expression which defines the slip of a point beyond which full debonding occurs.

\[
\delta_{lim}(x) = \epsilon_p \sin \beta \left( \frac{0.9 d}{\sin \beta} - x \frac{\sin \theta}{\sin \beta} \right) - \mu L_e
\]

(3.29)

where \( \epsilon_p \) is the debonding strain taken as 0.004 [49], and \( \mu \) is a multiplier on the effective bond length for different geometry and wrap configurations. For example, Khalifa et al. [8] proposed a shear crack angle of 45\(^\circ\) which passed 2\(L_e\) from top and bottom for side bonding (i.e., \( \mu = 2 \)) and \( L_e \) from the top for U-wrap (i.e., \( \mu = 1 \)).
Different models for the effective bond length were proposed in the literature, and the adopted $L_e$ was analytically given by Equation 3.30 [34, 2]:

$$L_e = 2 \left( \frac{\tau_f}{\delta_f} \left( \frac{1}{E_p t_p} + \frac{b_p}{b_c E_c t_c} \right) \right)^{-1/2}$$  \hspace{1cm} (3.30)

$L_e$ for FRP-PU and FRP-EP systems are 3.40 in and 3.63 in, respectively. The bond stiffness ($\tau_f/\delta_f$) and tensile stiffness ($E_p t_p$) ratios between the FRP-PU and FRP-EP systems are 0.47 and 0.40, respectively. That is, the higher FRP-EP bond stiffness is offset by the higher FRP-EP tensile stiffness when computing $L_e$.

Shear deflections were obtained directly from the global geometry of the beam kinematic deformed shape relations of the proposed analytical model. The midspan shear deflection can be calculated by Equation 3.31 from geometry by considering that $\alpha = \eta + \omega$ as shown in Figure 3.4.

$$\Delta_v(\alpha) = \frac{L}{2} \tan \eta = \frac{L}{2} \tan \frac{a \alpha}{L}$$  \hspace{1cm} (3.31)

Solving Equations 3.27, 3.28, and 3.29, the effective FRP stress can be found at the value of $x$ where the limiting slip occurs. Equation 3.32 shows this limiting $\sigma'_{pe}$ as a function of $\alpha$.

$$\sigma'_{pe}(\alpha) = \lambda_1 \lambda_2^2$$  \hspace{1cm} (3.32)

in which $\lambda_1$ and $\lambda_2$ are given in Equations 3.33 and 3.34, respectively.

$$\lambda_1 = \frac{\tau_f \alpha \epsilon_p^2 \sin(\theta + \beta)}{100 \left( \alpha \sin(\theta + \beta) + 2 \epsilon_p \sin \theta \right)^2 t_p \delta_f}$$  \hspace{1cm} (3.33)

$$\lambda_2 = 10 \mu L_e \sin \beta - 9 d$$  \hspace{1cm} (3.34)
The FRP-concrete interfacial bond effect appears in $\lambda_1$ whereas the FRP effective bond length or the FRP tensile stiffness effect appears in $\lambda_2$.

To investigate sensitivity of the proposed model to composite configuration and type, Figure 3.5 shows $\sigma'_{pe}(\alpha)$ assuming $\theta = 45^\circ$, $\beta = 90^\circ$, $\mu = 2$ for side bondings, and $\mu = 1$ for U-wraps [8]. The comparisons of FRP-PU versus FRP-EP systems using properties of the small-scale specimens are shown in Figures 3.5a and 3.5b for side bondings and U-wraps, respectively. The comparison of FRP-PU versus FRP-EP systems using properties of the large-scale specimens is shown in Figure 3.5c for side bonding configurations.

In small-scale specimens with side bondings, $\sigma'_{pe}$ is higher for FRP-PU because of the smaller depth $d$ and $\mu L_e$ in Equation 3.29. Limiting the available bond length governs (through $\lambda_2^2$) although the FRP-EP system has higher bond stiffness or $\lambda_1$. In small-scale specimens with U-wraps, the FRP-concrete interfacial bond is the governing parameter ($\lambda_1$ effect). The interfacial bond is the governing parameter in $\sigma'_{pe}$ for all large-scale specimens regardless the configuration due to high beam depth $d$.

Petrone and Monti [50] considered the effect of the interaction between the FRP composite and the transverse steel reinforcement. They proposed an approach to predict the effect of the transverse steel reinforcement to the shear capacity in terms of crack opening angle $\alpha$ and crack axis $x$. They assumed that the crack angle being the same as the compression strut inclination angle and the concrete contribution to shear capacity is the summation of FRP and transverse steel contributions with no mechanical meaning. Different models have been proposed for equilibrium equations to predict the shear capacity of RC beams. In this study, the model proposed was based on the upper-bound theorem of limit analysis of plasticity [51, 52, 53]. The model was successfully applied for shear capacity calculations with steel-fiber-concrete (SFC) by Lim et al. [54] and with FRP composites by Sim et al.
Figure 3.5: Effective FRP stress-crack opening angle relationships

[55]. The diagonal shear crack is mathematically idealized as a yield line with an inclination angle of $\theta$ with a relative displacement rate $u$ as shown in Figure 3.6.
The vertical equilibrium required that the rate of internal plastic work per unit area of the shear crack and the rate of external work be equal as in Equation 3.35.

\[ V_u = \gamma_p b_w d \cot \theta \cdot u + \frac{1}{2} \nu' f'_c (1 - \cos \theta) \frac{b_w d}{\sin \theta} \cdot u \quad (3.35) \]

where \( \gamma_p = 2t_p b_p \sigma_p \sin \beta / s_p b_w \); \( b_p \) is the FRP width, \( s_p \) is the FRP spacing, and \( \nu' \) is the effectiveness factor for the proposed ductility in concrete in compression. Equation 3.35 can be written as Equation 3.36:

\[ \frac{\tau_s}{\nu' f'_c} = \psi \cot \theta + \frac{1}{2} \left( \sqrt{1 + \cot^2 \theta} - \cot \theta \right) \quad (3.36) \]

where \( \tau_s = V / b_w d \) and \( \psi = \gamma_p / \nu' f'_c \). The optimum inclination of the yield line and hence the minimum upper-bound is obtained by imposing \( \frac{d\tau_s}{d\cot \theta} = 0 \) which gives Equation 3.37 of \( \cot \theta \).

\[ \cot \theta = \frac{1 - 2\psi}{2\sqrt{\psi(1 - \psi)}} \quad (3.37) \]
The beam geometry constraints require that $0 \leq \cot \theta \leq a/d$ and give Equation 3.38.

$$\frac{\tau_s}{\nu' f'_c} = \sqrt{\psi(1 - \psi)}$$  \hspace{1cm} (3.38)

The values of $\tau_s$ and $\psi$ are substituted along with Equation 3.32 for $\sigma'_{pe}$ into Equation 3.38 to obtain Equation 3.39 for the shear capacity $P = 2V$ in terms of $\alpha$.

$$P(\alpha) = \frac{\sqrt{2}}{5} b_w d \nu' f'_c \sqrt{\frac{\lambda_1 \lambda_2^2 \lambda_3}{\lambda_1 - \frac{1}{50} \lambda_1 \lambda_2^2 \lambda_3}}$$  \hspace{1cm} (3.39)

in which $\lambda_3$ is given in Equations 3.40.

$$\lambda_3 = \frac{b_p t_p \sin \beta}{s_p b_w \nu' f'_c}$$  \hspace{1cm} (3.40)

The model does not directly take the $a/d$ ratio into account, but it can be considered in terms of concrete effectiveness factor $\nu'$ through a calibration process with the experimental results. The effectiveness factor $\nu'$ is also affected by $b_w/d$ ratio [56]. $\nu'$ was obtained from the experimental control specimens by using Equation 3.35 for equilibrium by assuming that internal work is due to concrete only. The effectiveness factor values used in this study were 0.30 and 0.25 for small-scale specimens with $a/d = 3$ and $a/d = 4$, respectively. The effectiveness factor $\nu'$ used for large-scale specimens was 0.08.

To investigate sensitivity of the proposed model, Figure 3.7 shows the predicted load $P$ as a function of the crack opening angle $\alpha$ using the same assumptions presented for $\sigma'_{pe}(\alpha)$ in Figure 3.5. The comparisons of FRP-PU versus FRP-EP systems of small-scale specimens are shown in Figures 3.7a and 3.7b for side bondings and U-wraps, respectively. The comparison of FRP-PU versus FRP-EP systems of large-scale specimens is shown in Figure 3.7c for
side bonding configurations. Consistent with previous observations, the interfacial bond is the governing parameter in $P$ when sufficient bond length is present, i.e., all large-scale specimens and small-scale specimens with U-wraps. The FRP effective bond length or the FRP tensile stiffness is the governing parameter in $P$ for small-scale specimens with side bonding configurations.

![Graphs showing load-crack opening angle relationships for different composite materials and bonding configurations.]

(a) Comparison of composites for small-scale with side bondings

(b) Comparison of composites for small-scale with U-wraps

(c) Comparison of composites for large-scale with side bondings

Figure 3.7: Load-crack opening angle relationships

The shear strength $P_v$ is given in Equation 3.41 by substituting the debonding strength
(maximum $\sigma'_{pe}$), which is obtained by imposing $d\sigma'_{pe}/d\alpha = 0$ in Equation 3.38. Parameter $\lambda_4$ is given in Equation 3.42.

$$P_v = \frac{1}{10} b_w d \nu' f'_c \sqrt{\lambda_2^2 \lambda_4 \left(1 - \frac{1}{400} \lambda_2^2 \lambda_4\right)}$$  \hspace{1cm} (3.41)

$$\lambda_4 = \frac{b_p \tau_f \epsilon_p \sin \beta}{s_p b_w \nu' f'_c \delta_f \sin \theta}$$  \hspace{1cm} (3.42)

### 3.3 Analytical Results

Flexural deflections were obtained based on the effective flexural stiffness $EI_e(\alpha)$ and the bending moment $M_a(\alpha)$ in the specimen at the load increment at which the deflection was calculated [21, 57]. $I_e(\alpha)$ is given by Equation 3.43.

$$I_e(\alpha) = \left(\frac{M_{cr}}{M_a(\alpha)}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a(\alpha)}\right)^3\right] I_{cr}$$  \hspace{1cm} (3.43)

in which $M_{cr}$ is the cracking moment, $I_g$ is the moment of inertia of gross concrete section neglecting reinforcement, and $I_{cr}$ is the moment of inertia of the cracked section transformed to concrete. The analytical flexural deflection is calculated from Equation 3.44 based on effective flexural stiffness $EI_e(\alpha)$ and analytical load $P(\alpha)$.

$$\Delta_m(\alpha) = \begin{cases} \frac{P(\alpha) x'}{12EI_e(\alpha)} (3La - 3a^2 - x'^2) & \text{for } x' \leq a \\ \frac{P(\alpha) a}{12EI_e(\alpha)} (3L x' - 3x'^2 - a^2) & \text{for } a < x' < (L - a) \end{cases}$$  \hspace{1cm} (3.44)

The analytical load-deflection response of the RC specimens externally-bonded with FRP composites was obtained considering both the flexural and shear deformations (Equations
For verification, the results of experimental and analytical flexural deformations were compared for specimen S-Uh4-PU. The experimental curvature profile at an applied load of 31.5 kips was obtained using the compression and tension longitudinal rebar strains, and is presented in Figure 3.8a. The analytical curvature profile was calculated based on effective flexural stiffness at the same load. Figure 3.8b shows the corresponding flexural deflection profile. The experimental profile was calculated by double integration of the Lagrange interpolation polynomial through the experimental curvature profile and applying the displacement boundary conditions. The analytical deflection profile was calculated based on Equation 3.44 for the analytical flexural deflection. The actual total experimental deflection measured at the center LVDT is shown for comparison (and includes both shear and flexural contributions). Figure 3.8c shows the shear and flexural contributions to the total analytical load-displacement relation for specimen S-Uh4-PU.

Figures 3.9a and 3.9c show the analytical and experimental results of the load-deflection responses of specimens S-Th4-PU, S-Th4-EP, and L-Sh4-PU. The FRP effective bond length or the FRP tensile stiffness governs the analytical responses of small-scale side sheet specimens shown in Figure 3.9a, whereas the FRP-concrete interfacial bond governs the analytical response of large-scale side bonding specimen shown in Figure 3.9c. Good agreement between the analytical and experimental results are obtained. In addition, the experimental data of epoxy-based U-wrap specimen SO4-2 conducted by Khalifa and Nanni [36] is compared with the analytical results in Figure 3.9b. The effectiveness factor \( \nu' \) was taken as 0.32 from the control specimen SO4-1 from the same study. The behavior is sensitive to the bond parameters, which were not reported and therefore assumed to be the same as the FRP-EP parameters in this study.
Figure 3.8: Curvature and deflection analyses of specimen S-Uh4-PU
Figure 3.9: Experimental and analytical results
CHAPTER 4: NUMERICAL METHODS

4.1 Finite Element Model

Three-dimensional (3D) finite element analysis (FEA) was performed using MSC.Marc [58] to compare with analytical and experimental measured deformations of four-point loading tests to better understand the shear behaviors and failure modes. Eight-node solid isoparametric 3D hexahedral elements were used to model the concrete material. Tension steel reinforcement was modeled using two-node solid section beam elements whereas the compression longitudinal and transverse steel reinforcement were modeled using two-node truss elements. The FRP laminates were modeled using four-node quadrilateral membrane isoparametric 3D elements.

Two line loads were used to model the two point loads at the locations similar to the experiments. The beam’s supports were modeled through fixed displacement option as pinned and roller (simply-supported beam). Figure 4.1 represents concrete finite element meshing along with load and support conditions. The finite element meshing of steel and FRP reinforcements is shown in Figure 4.2.

In order to simulate the onset and progress of debonding between the FRP composites and the substrate concrete, eight-node 3D interface hexahedral elements were used with a cohesive zone model, which associates the bond or adhesive interface properties with the cohesive model. The constitutive relation of these interface elements defines the traction (bond stress) $\tau$ in terms of the effective opening displacement (slip) $\delta$ between the top and bottom of element surfaces/edges.
A bilinear bond-slip model was used with the parameters of cohesive energy $G_f$, critical effective opening displacement associated with ultimate bond stress, and maximum effective opening displacement. The debonding started when the slip reached the maximum effective opening displacement in different locations. As the loads increased the debonding propagated to other neighbor locations of the interface until the system failed. The effective opening displacement $\delta$ is defined by Equation 4.1 through coupling the relative displacement components (one normal $V_n$, and two shears $V_s$ and $V_t$).

$$\delta = \sqrt{V_n^2 + V_s^2 + V_t^2}$$  \hspace{1cm} (4.1)

To reduce the coupling effect and in order to limit the axes of the interface relative displacement components, the FRPs were modeled as linear elastic with orthotropic material properties which have three different planes of symmetry. The cohesive zone model pa-
parameters were calibrated utilizing single-lap shear tests previously conducted on the same FRP-PU composite system used in this study [15]. To obtain the parameters of the bond-slip model, the experimental strain data were fitted with the analytical strain distribution profile (Equation 1.4) along the bonded FRP laminate derived by Wu et al. [34], Yuan et al. [2] using nonlinear least-squares regression. The bond parameters were given in Section 2.3.

The concrete tension behavior was modeled by tension damage model through the cracking criteria. Cracking is initiated when the principal tensile stress reached the critical cracking stress of concrete. After cracking of concrete, the tensile stresses decay with increase in the principal tensile strain. This phenomenon is called tension softening and was incorporated in Marc by assuming a bilinear behavior in the concrete tension region, and when it is not taken into consideration, the concrete tensile stresses drop to zero upon cracking. The tension softening modulus was calculated based on the concrete tensile fracture energy, which represents the energy required to propagate a tensile crack of unit area, according to the
CEB-FIP model code [59]. A critical stress (modulus of rupture) $f_r$ of 0.5 ksi and shear retention factor of 0.0019 were used for the concrete cracking model.

The concrete material behavior in compression was modeled through the concrete crushing strain only (ultimate compression strain of 0.003). No plasticity model was used for compression concrete due to numerical difficulties prevented obtaining convergence with the specified allowed maximum number of cut-backs in Marc. The real stress-strain behavior of concrete in compression beyond the peak ($f'_c$) is stress decreasing when strain increasing (compression softening). When not included in the model, the nonlinear concrete response is impacted (changed), especially under high confining pressure.

The steel reinforcement materials were modeled utilizing the mechanical properties of steel reinforcement by introducing the yield stress $f_y$ and the equivalent plastic strain Table built in Marc which defines the steel stress-strain relationship after yielding. Figure 4.3 shows the concrete patch tests in tension and compression, and tension steel patch test in tension. Patch test of compression concrete (Figure 4.3b) shows that the concrete crushing stress is 16 ksi (approximately twice $f'_c$). Therefore, the concrete elements in the compression zone of the numerical model might not reach the concrete crushing strain.
4.2 Finite Element Results

Specimens S-Ch4 and S-Cf4 were modeled in Marc and are presented here as control samples to check the FE modeling. Figure 4.4 shows the load versus midspan deflection curves that compare the experimental and the numerical FE results. There are three distinct stages in the numerical load-displacement behavior of specimen S-Ch4: initial stage, second stage
(after flexural cracks), and third stage (after shear cracks). Overall, the trends of the load-displacement behavior and the shear failure mode of S-Ch4 were captured by the FE model. The differences between the experimental and numerical load-displacement responses are due to material parameters (compression concrete). The tensile strain in the beam longitudinal direction of the midspan concrete element increased linearly up to 0.0001 (corresponding to 5.0 kips load). At this load the tensile strain jumped to 0.0002 announcing the beginning of flexural cracks (second loading stage).

![Figure 4.4: Load-deflection behaviors for control specimens](image)

Figure 4.4 shows the principal cracking strain contour of specimen S-Ch4 at load level of 5.0 kips where the flexural cracks started to propagate. Figure 4.5b shows the principal cracking strain contour at load level of 16.7 kips where the shear cracks began to propagate (second loading stage). The shear crack patterns are different in the beam’s left half (with stirrups) than its right half (without stirrups). The numerical stiffness was remarkably reduced (less
than experimental stiffness) when shear cracks propagated. In addition to the modeling of concrete in compression, this stiffness discrepancy between experimental and numerical responses might be due to material parameter modeling in shear (the concrete shear retention parameter). Figures 4.5c and 4.5d show the principal cracking strain contours at load levels of 22.7 kips and 27.5 kips, respectively.

![Figures 4.5c and 4.5d showing principal cracking strain contours at load levels of 22.7 and 27.5 kips, respectively.](image)

Figure 4.5: Principal cracking strain contour of S-Ch4

In specimen S-Cf4, no third load stage occurred since no shear cracks took place, and the failure mode was flexure. Figure 4.6 shows the principal cracking strain contours at similar load levels of those in S-Ch4). The shear crack patterns are similar in the beam’s left and right halves (both with stirrups). At failure load, the top midspan concrete element compressive strain and the midspan steel element tensile strain in the beam longitudinal direction were
0.00179 and 0.00303, respectively. The numerical model did not capture the experimental concrete crushing because the compression behavior in concrete was not considered in the FE model.

Figure 4.6: Principal cracking strain contour of S-Cf4

The ultimate capacity in experimental and modeling results were identical for specimen S-Ch4, and it was higher in the FEA model than the experimental for specimen S-Cf4. The numerical responses of the control specimens verified the finite element models (with some differences than experiments), and provided a reasonable basis to expand the study to specimens externally strengthened in shear with FRP-PU system. When the nonlinear behavior of concrete in compression along with its descending part (compression softening) are not considered, the numerical finite element model overestimates the flexural capacity.
Specimens S-Uh4-PU and L-Sh4-PU were analyzed by the FE model utilizing the material characterizations and interface parameters. Figure 4.7 shows the midspan load-displacement plots comparing the experimental, analytical, and the simulated FE results of S-Uh4-PU and L-Sh4-PU specimens. The experimental, analytical, and numerical peak load of specimen S-Uh4-PU are 44.5 kips, 63.0 kips, and 46.4 kips respectively. The experimental, analytical, and numerical peak load of specimen L-Sh4-PU are 85.5 kips, 93.1 kips, and 90.0 kips respectively. The FEA models captured the experimental shear strengths of FRP strengthened specimens. At failure load of specimen S-Uh4-PU, the top midspan concrete element compressive strain and the midspan steel element tensile strain in the beam longitudinal direction were 0.00137 and 0.00293, respectively. The FRP strengthened specimens have also three loading stages, however, the numerical stiffnesses in the third stage are further reduced dramatically than experimental stiffnesses. In addition to the previous reasons (in control specimens) for this stiffness variation, it might be due to the Marc modeling of the FRP-concrete interface elements. It couples the interface relative displacement components which might prevent the FRP strains from being dropped when debonding (as shown later in this chapter). Figure 4.8 shows the midspan tension rebar load-strain plots comparing the experimental and simulated FE results of S-Uh4-PU and L-Sh4-PU specimens.
Figure 4.7: Experimental, analytical and numerical load-displacement results of S-Uh4-PU and L-Sh4-PU

Figure 4.8: Experimental and numerical midspan tension rebar strain results of S-Uh4-PU and L-Sh4-PU
Figure 4.9 shows the damage profile of the concrete-FRP interface of specimen S-Uh4-PU at a load step of 44.3 kips. It shows the interface debonding at which the debonded elements were deactivated and excluded from the profile. Figure 4.10 presents the numerical FRP load-strain behavior of specimen S-Uh4-PU. The FRP strain started to increase earlier at SG-F4 (closest to point load) followed by SG-F3, SG-F2, and latest SG-F1 (farthest to point load). The progression of the FRP strains is dependent on the load step value and the locations of the strain gauges corresponding to the shear crack (at failure, SG-F2 has the highest strain reading whereas SG-F1 and SG-F4 have the lowest strain reading). The numerical model, and when the shear crack passed the FRP element under consideration (strain gauge position), does not drop the FRP strain when shear failure occurred. It appears that when an interface element is being removed (debonded), the state of stress in the FRP element is decidedly not uniaxial tensile but rather biaxial. Therefore, the FRP elements can redistribute stresses/strains to the adjacent elements.

![Figure 4.9: Interface damage profile of S-Uh4-PU at 44.3 kips](image)

Figures 4.11 and 4.12 show the principal cracking strain contours and the FRP strain profiles of specimen S-Uh4-PU with different load steps. The strains in the FRP elements transferred
from wrap to another when loads are increased as shown in Figure 4.12. Overall, the shear strengths and the deformations are captured by the FE model with some differences than the experimental responses because of several modeling issues including the concrete behavior in compression and the cohesive model that has the coupling effect.

For verification, the numerical flexural deformations (curvatures and deflections) were compared to experimental and analytical results of specimen S-Uh4-PU as shown in Figure 4.13 with same load levels utilized for the strains of the FRP presented in Figure 4.12. The experimental and analytical calculation details were presented in Section 3.3. The numerical curvature profile at different applied loads were obtained using the numerical compression and tension longitudinal rebar strains.
Figure 4.11: Principal cracking strain contours of S-Uh4-PU
Figure 4.12: FRP strains at different load levels of S-Uh4-PU
Figure 4.13: Curvature and deflection analyses of specimen S-Uh4-PU
CHAPTER 5: INTERFACIAL BOND-SLIP RELATIONSHIP UNDER BIAXIAL STRESS

5.1 Background

Bond stress (shear stress acting in the interface) between FRP adherents and concrete substrate plays a significant role in the structural behavior of RC members strengthened with externally-bonded FRP composites. One of the most common modes of failure encountered in externally-bonded FRP composites strengthening RC members in practice is debonding (loss of adhesion) failure. There are many categories for debonding failure modes, however, the most frequent failure modes are plate end debonding (Mode-I) and intermediate crack debonding (Mode-II).

Debonding starts upon reaching the bond strength according to one or more of the debonding failure modes. Debonding may take place in the adhesive interface, between the concrete and adhesive, in the concrete itself, or within the FRP laminate between the FRP plies [37]. The active zone that transfers load is shifted to the neighboring zone during the debonding process, which means that only part of the bonded zone is effective at a time. This leads to the concept of the effective bond length after which the load could not be increased, and the ultimate FRP tensile strength might never be reached. A significant step toward understanding the bond behavior is to have an assumption for the constitutive law of local bond stress versus slip relationship of the adhesive interface due to the debonding failure mechanism.

In the last few decades, many studies were performed on the bond transfer mechanism for different types of interfaces between different types of substrates and different types of
adherents. The numerical or mathematical-derived bond-slip constitutive models based on experimental data proposed the FRP width $b_p$ is a variable parameter, and the strains and stresses are uniform along the same coordinate in $L$ direction. The states of stress are altered by the location of the FRP element corresponding to the concrete inclined shear micro cracks. The in-plane stress assumption in mechanics is made when two dimensions are much higher than the third dimension (generally the thickness) whose stress can be negligible compared to the significant in-plane stresses. The shear strengthening of concrete beams by FRP composites is an application of the in-plane stress assumption. In fact, the FRP-concrete bond in the RC beams strengthened in shear is subjected to biaxial stress condition. Nonetheless, almost all studies on bond-slip models have emphasized on uniaxial state of stress, or sometimes (in flexure analysis) with taking the normal stress (constant through the thickness) into consideration.

Two research paths, which were focused on numerical or mathematical models validated with experimental results of plate end debonding or intermediate debonding, can be recognized [60]. The first path is by calibration of analytical expressions (empirical, numerical, or mathematical) with experimental results by well defined mathematical procedures such as linear or nonlinear least-squares minimization. The second path is by validation of refined numerical models using a finite element procedure analysis based on limited experimental results and exploration the FEA results for different case studies.

The behavior of the FRP-to-concrete interface is often expressed in terms of fracture energy $G_f$ which is associated with the maximum force $P_{\text{max}}$ observed from lap shear tests. However, the estimation of fracture energy is considered not sufficient to produce the overall behavior of the adhesive interface. An accurate bond-slip model is considered of fundamental relevance to obtain a consistent response of RC elements strengthened with externally-bonded FRP composites. There are two methods (neither FEA nor fracture mechanics based) utilized to
produce the bond-slip curves from experimental values which are the direct identification method (DirIM) and indirect identification method (IndIM) [61].

The first method is based on experimental data to determine the shear stresses and the corresponding relative displacements evaluated at various points in the interface under different load levels until debonding failure. The experimental bond stresses and displacements are obtained from an experimental strain profile, and the bond-slip relationship is defined directly using numerical regression such as least-squares method. In fact, these local points for shear stresses and associated slips are not really available in the adhesive interface, and therefore the resulting relationship is a non-local law. This method frequently does not produce real bond-slip relations, and the fracture energy is underestimated and varies significantly from one test to another [61].

Various experimental setups have been proposed in the literature, however, a standard procedure has not been defined yet. Double shear pull or push tests, single shear pull or push tests, and beam (bending) tests are examples of the experimental setups available in the literature. Several experimental bond tests have been carried out and analyzed using different bond-slip models, and the predicted shape of the bond-slip law is determined by the experimental FRP axial strain distribution profile [60]. Specimens usually consisted of concrete blocks laminated with FRP strips placed in rigid frames to prevent the movement in the load direction. Enough bond length $L$ is usually used and the FRP laminate width is always less than the concrete width. Some strain gauges are placed onto the FRP laminate with certain distances. A tensile force is applied at one end of the bonded FRP laminate to the concrete block.

Ferracuti et al. [62] presented a calibration procedure to obtain the FRP-to-concrete interface laws from experimental results of the bond tests. At each load level, the strain distribution
of the FRP laminate is obtained. Then, between every two subsequent strain gauges, the average shear stress is obtained by Equation 5.1 as a function of differences in the measured strains.

\[ \tau_{i+1/2} = \frac{E_p A_p (\epsilon_{i+1} - \epsilon_i)}{b_p (x_{i+1} - x_i)} \]  

(5.1)

where \( E_p, A_p, \) and \( b_p \) are the elastic modulus, area, and width of FRP laminate; \( \epsilon_i \) and \( \epsilon_{i+1} \) are two subsequent strain gauge values; \( x_i \) and \( x_{i+1} \) are corresponding strain gauge positions, respectively.

The average local slip between every two subsequent gauges is obtained by Equations 5.2 and 5.3 assuming that there is no slip (perfect bonding) at the far end gauge \( s(x_0) \), no concrete strain (far from external cover) with respect to FRP, and linear strain variation between every two subsequent gauges [62].

\[ s(x_{i+1}) = s(x_i) + \frac{(\epsilon_{i+1} - \epsilon_i) (x_{i+1} - x_i)^2}{2} - \epsilon_i (x_{i+1} - x_i) \]  

(5.2)

\[ s_{i+1/2} = \frac{s(x_{i+1}) + s(x_i)}{2} \]  

(5.3)

where \( s_{i+1/2} \) is the average local slip between strain gauges \( \epsilon_i \) and \( \epsilon_{i+1} \). Several points of average bond stresses versus average slips for different load levels define the interface behavior. Experimentally, it is not possible to get a unique bond-slip curve due to existence of different trends between every two subsequent gauges. However, this aspect has been investigated by researchers and they suggested different models for bond-slip relationships that can be calibrated with experimental data. In this model, the maximum force was used to prescribe the fracture energy value which considered a constraint to obtain the interface law parameters by DirIM.
Mazzotti et al. [63] proposed a nonlinear interface law between concrete and FRP laminate which was similar to the Popovics constitutive law for concrete in compression [64]. Equation 5.4 presented the proposed model which was calibrated with the experimental results obtained from direct shear tests.

\[
\tau = \tau_{\text{max}} \frac{s}{s_{\text{max}}} (n - 1) + \left(\frac{s}{s_{\text{max}}}\right)^n
\]

(5.4)

where \(\tau_{\text{max}}\) and \(s_{\text{max}}\) are the maximum bond stress and corresponding slip, and \(n\) is a constant parameter responsible for softening branch. A least-squares minimization between experimental and theoretical data of bond stress-slip was adopted by using the fracture energy of the adhesive in the minimization procedure as a constraint to transfer a maximum experimental force \(P_{\text{max,expt}}\) by an infinite anchorage length (Equation 5.5).

\[
G_f = \frac{P_{\text{max,expt}}^2}{2E_p t_p b_p^2}
\]

(5.5)

The authors took into account the width ratio and obtained good theoretical curves from the interface law in spite of some scattered results regarding the experimental data especially for small widths.

The second method (IndIM) has measurements of shear stresses and associated relative displacements that do not come directly from the pull-out tests. In contrast to DirIM, experimental data in terms of axial strains are measured at various intervals of distance and load level indirectly. In this method, at each set of collected data (load level), corresponding theoretical axial strains can be evaluated. Closed form solutions are available and were reported for particular interface laws [65, 34, 66, 2]. A constraint of maximum force or fracture energy is imposed to identify the interface law as a constrained optimization procedure. These two methods lead to significantly diverse bond-slip relationships because the averag-
ing procedure for estimating the shear stresses from experimental strains followed in DirIM underestimates the fracture energy. Unlike the DirIM, the IndIM is less sensitive to the data size in terms of strain stations and load levels [61].

The objective of this chapter is to correctly evaluate the bond behavior between the concrete and the FRP composites by taking the biaxial states of stress into consideration. A new two-dimensional (2D) bond-slip law was proposed and derived to address the biaxial states of stress problem that takes into consideration the coupling or interaction between the in-plane stresses and the associated slips. Four partial differential equations (PDE) per node (two for FRP and two for concrete in each direction) were solved numerically (no closed-form mathematical solutions exist) by finite difference (FD) approach with appropriate boundary conditions to obtain the stresses, slips and strains based on the proposed bond-slip model. The backbone points and surfaces of a specific 2D bond-slip model were calibrated depending on the experimental strain profile data following the indirect identification method IndIM.

5.2 Biaxial Bond-Slip Model

Figure 5.1 shows the stress analysis (principal stresses) of different concrete surface elements of control specimen S-Ch4. It was analyzed for three different load steps for both left half (with stirrups) and right half (without stirrups). It can be shown the effect of stirrups and diagonal shear crack on the responses of various elements corresponding to beam geometry and loading conditions. It shows that the states of stresses of the concrete surface elements of the shear deficient side (and hence the concrete-FRP interface elements when applying FRP composites) are not unidirectional and considering the biaxial states of stresses are significance.
Figure 5.1: Biaxial state of stress for different load levels of S-Ch4
A new 2D bond-slip model was proposed and derived based on considering an infinitesimal element with dimensions of $dx$ and $dy$ shown in Figure 5.2. This model consists of an interface adhesive layer located between an FRP layer denoted by $p$ and a concrete layer denoted by $c$. The following assumptions were made to simplify of the problem:

- linear elastic and homogeneous FRP adherents.
- negligible bending effects.
- the adhesive function is to transfer shear stresses from FRP to concrete with negligible thickness.
- constant thicknesses of the adherents and the substrates along the bond area.
- The interface stresses $\tau_x$ and $\tau_y$ are principal stresses ($\tau_{xy}$ does not exist). FRP stress analysis should be performed for each load step to obtain the FRP principal stresses and hence the angle between FRP and interface principal directions.

Equations 5.6, 5.7, 5.8, and 5.9 are the equilibrium equations of forces in $x$ and $y$ directions.

\[
\frac{\partial \sigma_{px}}{\partial x} + \frac{\partial \sigma_{pxy}}{\partial y} = \frac{\tau_x}{t_p} \tag{5.6}
\]

\[
\frac{\partial \sigma_{cx}}{\partial x} + \frac{\partial \sigma_{cxy}}{\partial y} = -\frac{\tau_x}{t_c} \tag{5.7}
\]

\[
\frac{\partial \sigma_{py}}{\partial y} + \frac{\partial \sigma_{pxy}}{\partial x} = \frac{\tau_y}{t_p} \tag{5.8}
\]

\[
\frac{\partial \sigma_{cy}}{\partial y} + \frac{\partial \sigma_{cxy}}{\partial x} = -\frac{\tau_y}{t_c} \tag{5.9}
\]
Figure 5.2: Free body diagram of 2D bonded joint

where $\tau_x$ and $\tau_y$ are the principal bond stresses.

Equations 5.10 and 5.11 are the compatibility relations which are taken through the inter-
facial slips defined as the relative displacements between FRP and concrete.

\[ \delta_x(x, y) = \delta_{px} - \delta_{cx} \]  

(5.10)

\[ \delta_y(x, y) = \delta_{py} - \delta_{cy} \]  

(5.11)

Equations 5.12 through 5.23 are based on the in-plane stress state.

\[ \frac{\partial \sigma_{px}}{\partial x} = \frac{E_p}{1 - \nu_p^2} \left( \frac{\partial^2 \delta_{px}}{\partial x^2} + \nu_p \frac{\partial^2 \delta_{py}}{\partial x \partial y} \right) \]  

(5.12)

\[ \frac{\partial \sigma_{px}}{\partial y} = \frac{E_p}{1 - \nu_p^2} \left( \frac{\partial^2 \delta_{px}}{\partial y^2} + \nu_p \frac{\partial^2 \delta_{py}}{\partial x \partial y} \right) \]  

(5.13)

\[ \frac{\partial \sigma_{py}}{\partial y} = \frac{E_p}{1 - \nu_p^2} \left( \frac{\partial^2 \delta_{py}}{\partial x \partial y} + \nu_p \frac{\partial^2 \delta_{pz}}{\partial y^2} \right) \]  

(5.14)

\[ \frac{\partial \sigma_{py}}{\partial x} = \frac{E_p}{1 - \nu_p^2} \left( \frac{\partial^2 \delta_{py}}{\partial x \partial y} + \nu_p \frac{\partial^2 \delta_{pz}}{\partial x^2} \right) \]  

(5.15)

\[ \frac{\partial \sigma_{cx}}{\partial x} = \frac{E_c}{1 - \nu_c^2} \left( \frac{\partial^2 \delta_{cx}}{\partial x^2} + \nu_c \frac{\partial^2 \delta_{cy}}{\partial x \partial y} \right) \]  

(5.16)

\[ \frac{\partial \sigma_{cx}}{\partial y} = \frac{E_c}{1 - \nu_c^2} \left( \frac{\partial^2 \delta_{cx}}{\partial x \partial y} + \nu_c \frac{\partial^2 \delta_{cy}}{\partial y^2} \right) \]  

(5.17)

\[ \frac{\partial \sigma_{cy}}{\partial y} = \frac{E_c}{1 - \nu_c^2} \left( \frac{\partial^2 \delta_{cy}}{\partial y^2} + \nu_c \frac{\partial^2 \delta_{cx}}{\partial x \partial y} \right) \]  

(5.18)
\[ \frac{\partial \sigma_{cy}}{\partial x} = \frac{E_c}{1 - \nu_c^2} \left( \frac{\partial^2 \delta_{cy}}{\partial x \partial y} + \nu_c \frac{\partial^2 \delta_{cx}}{\partial x^2} \right) \] (5.19)

\[ \frac{\partial \sigma_{p_{xy}}}{\partial x} = \frac{E_p}{2 + 2\nu_p} \left( \frac{\partial^2 \delta_{px}}{\partial x \partial y} + \frac{\partial^2 \delta_{py}}{\partial x^2} \right) \] (5.20)

\[ \frac{\partial \sigma_{p_{xy}}}{\partial y} = \frac{E_p}{2 + 2\nu_p} \left( \frac{\partial^2 \delta_{px}}{\partial y^2} + \frac{\partial^2 \delta_{py}}{\partial x \partial y} \right) \] (5.21)

\[ \frac{\partial \sigma_{c_{xy}}}{\partial x} = \frac{E_c}{2 + 2\nu_c} \left( \frac{\partial^2 \delta_{cx}}{\partial x \partial y} + \frac{\partial^2 \delta_{cy}}{\partial x^2} \right) \] (5.22)

\[ \frac{\partial \sigma_{c_{xy}}}{\partial y} = \frac{E_c}{2 + 2\nu_c} \left( \frac{\partial^2 \delta_{cx}}{\partial y^2} + \frac{\partial^2 \delta_{cy}}{\partial x \partial y} \right) \] (5.23)

where \( \nu_p \) and \( \nu_c \) are the Poisson’s ratios of the FRP and the concrete, respectively.

Several constitutive relationships have been proposed for 1D interface to represent the bond stress-slip relationships [60] (refer to Section 1.6 for more information). The biaxial constitutive relations were proposed as extensions of a uniaxial linear brittle model. The 1D constitutive model was achieved in a direction when no traction in the other direction occurred. Therefore, the yield surface was assumed to degenerate into the uniaxial case in both directions. Two points were needed to obtain the yield surface: one point was related to uniaxial maximum shear stress \( (\tau_f) \) in one direction as an extreme case when no bond stress acted on the other direction, and the other point was related to biaxial maximum shear stresses \( (\tau_i \text{ in } \tau_j \text{ relation}) \). The yield surface was symmetric in all four quadrants (tension-tension, compression-compression, and tension-compression tractions).

Equations 5.24 and 5.25 are the proposed biaxial bond stresses as functions of \( \delta_x \) and \( \delta_y \).
with cutoff conditions when reaching any maximum slip in any $\delta_x$ or $\delta_y$. Three boundary conditions were needed for each $\tau_x$ and $\tau_y$ relations (Equations 5.24 and 5.25). The boundary conditions required for $\tau_x$ relation were $\delta_y = 0$ and $\delta_x = \delta_f$ at $\tau_x = \tau_f$, $\delta_x = 0$ and $\delta_y = \delta_2$ at $\tau_x = 0$, and $\delta_x = \delta_1$ and $\delta_y = \delta_2$ at $\tau_x = \tau_i$. Figures 5.3 and 5.4 show the proposed biaxial constitutive model and the yield surface, respectively.

\[
\tau_x = \begin{cases} 
\frac{(\tau_i + \tau_f) \delta_x}{\delta_f} + \frac{\tau_i \delta_x (\delta_y - \frac{\delta_2 \delta_x}{\delta_f})}{\delta_2 \delta_f} & \text{when } 0 \leq \delta_x \leq \delta_f \text{ and } 0 \leq \delta_y \leq \delta_2 \\
0 + \left(\frac{\delta_1 \tau_i - \delta_1 \delta_f \tau_f - 2\delta_1 \delta_f \tau_i + \delta_2^2 \tau_i}{\delta_1 \delta_2 \delta_f^2}\right) \delta_x \delta_y & \text{when } \delta_x > \delta_f \text{ or } \delta_y > \delta_2
\end{cases}
\]

\[
\tau_y = \begin{cases} 
\frac{(\tau_j + \tau_f) \delta_y}{\delta_f} + \frac{\tau_j \delta_y (\delta_x - \frac{\delta_3 \delta_y}{\delta_f})}{\delta_3 \delta_f} & \text{when } 0 \leq \delta_x \leq \delta_3 \text{ and } 0 \leq \delta_y \leq \delta_f \\
0 + \left(\frac{\delta_4 \tau_j - \delta_4 \delta_f \tau_f - 2\delta_4 \delta_f \tau_j + \delta_2^2 \tau_j}{\delta_3 \delta_4 \delta_f^2}\right) \delta_x \delta_y & \text{when } \delta_x > \delta_3 \text{ or } \delta_y > \delta_f
\end{cases}
\]

in which $\tau_f$ and $\delta_f$ are the 1D bond parameters; whereas $\tau_i$, $\delta_1$ and $\delta_2$ are the 2D bond parameters in $\tau_x$, and $\tau_j$, $\delta_3$ and $\delta_4$ are the 2D bond parameters in $\tau_y$. 

101
Figure 5.3: Biaxial constitutive model

Figure 5.4: Yield surface of the biaxial constitutive model
Equations 5.26, 5.27, 5.28, and 5.29 are the partial differential equations resulting from substituting the equilibrium and compatibility equations into the constitutive relations.

\[
\frac{E_p}{1 - \nu_p} \frac{\partial^2 \delta_{px}}{\partial x^2} + \frac{E_p}{2 + 2\nu_p} \frac{\partial^2 \delta_{px}}{\partial y^2} + \left( \frac{\nu_p E_p}{1 - \nu_p} + \frac{E_p}{2 + 2\nu_p} \right) \frac{\partial^2 \delta_{py}}{\partial x \partial y} - \frac{\tau_x}{t_p} = 0 \tag{5.26}
\]

\[
\frac{E_c}{1 - \nu_c} \frac{\partial^2 \delta_{cx}}{\partial x^2} + \frac{E_c}{2 + 2\nu_c} \frac{\partial^2 \delta_{cx}}{\partial y^2} + \left( \frac{\nu_c E_c}{1 - \nu_c} + \frac{E_c}{2 + 2\nu_c} \right) \frac{\partial^2 \delta_{cy}}{\partial x \partial y} + \frac{\tau_x}{t_c} = 0 \tag{5.27}
\]

\[
\frac{E_p}{2 + 2\nu_p} \frac{\partial^2 \delta_{py}}{\partial x^2} + \frac{E_p}{1 - \nu_p^2} \frac{\partial^2 \delta_{py}}{\partial y^2} + \left( \frac{\nu_p E_p}{1 - \nu_p^2} + \frac{E_p}{2 + 2\nu_p} \right) \frac{\partial^2 \delta_{px}}{\partial x \partial y} - \frac{\tau_y}{t_p} = 0 \tag{5.28}
\]

\[
\frac{E_c}{2 + 2\nu_c} \frac{\partial^2 \delta_{cy}}{\partial x^2} + \frac{E_c}{1 - \nu_c^2} \frac{\partial^2 \delta_{cy}}{\partial y^2} + \left( \frac{\nu_c E_c}{1 - \nu_c^2} + \frac{E_c}{2 + 2\nu_c} \right) \frac{\partial^2 \delta_{cx}}{\partial x \partial y} + \frac{\tau_y}{t_c} = 0 \tag{5.29}
\]

The numerical solution to the aforementioned partial differential equations is explained in Appendix B.

### 5.3 Experimental Setup and Testing Matrix

A new experimental setup was proposed to represent the biaxial bond-slip model by laminating two perpendicular strips on concrete block with the proposed strain profile. Lap shear tests in both directions were conducted by using two hydraulic jacks operated through an electric pump. Three specimens were prepared with new setup to represent the biaxial constitutive model. Each specimen consisted of a concrete block of dimensions (4×6×12) in, and prepared to conduct lap shear tests in 2D as shown in Figure 5.5.

Two hydraulic jacks operated by electric pump were used to apply tensile loads on the two strips to cause a direct shear on the interface as shown in Figure 5.6. The strip widths were
chosen to be 2.75 in to be inserted inside the opening of the jack cylinder. Therefore, the bond area is (2.75 × 2.75) in. Two grips were used in each jack to hold the FRP strips and prevent them from sliding while applying the tensile loads.

5.4 Experimental Calibration Results

Table 5.1 summarizes the experimental results of the calibrated parameters. In each specimen, the strain profile was recorded corresponding to each load step. The experimental strain results were calibrated based on the FD numerical equations using nonlinear least-squares fitting to obtain the bond-slip parameters. The system of nonlinear equations was solved by Newton’s method. All the interior nodes are governed by the partial differential equations and the proposed constitutive relations. The boundary nodes are not governed by the partial differential equations, but rather are governed by the Neumann boundary conditions which are not the dependent variables themselves (δ_x and δ_y), but they are the first order derivation of the dependent variables (σ_x and σ_y).
There are six essential polyurethane bond parameters. The experimental calibrated interface parameters are the bond stresses $\tau_i$ and $\tau_j$, and the slips in x- and y-directions: $\delta_1$ and $\delta_2$ for $\tau_i$ and $\delta_3$ and $\delta_4$ for $\tau_j$. The bond parameters corresponding to $\tau_y$ ($\tau_j$, $\delta_3$, and $\delta_4$) were assumed to be equal to the parameters corresponding to $\tau_x$. Figure 5.7 shows different response behaviors of specimen 2D-Sp1.

<table>
<thead>
<tr>
<th>ID</th>
<th>$\tau_i$ (ksi)</th>
<th>$\delta_1$ (in)</th>
<th>$\delta_2$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D-Sp1</td>
<td>0.423</td>
<td>0.00112</td>
<td>0.00318</td>
</tr>
<tr>
<td>2D-Sp2</td>
<td>0.345</td>
<td>0.00083</td>
<td>0.00273</td>
</tr>
<tr>
<td>2D-Sp3</td>
<td>0.348</td>
<td>0.00098</td>
<td>0.00248</td>
</tr>
</tbody>
</table>
5.5 Discussion

In this chapter, the biaxial bond-slip relationships between concrete and FRP-PU composites were investigated using numerical and experimental approaches. Results of bond parameters from single lap shear tests in 2D numerical procedure that derived in this chapter were obtained by experimental calibration and compared with 1D analytical procedure (Section 1.6) conducted on two test specimens (uniaxial lap shear tests) with the same bond area (length 2.75 in, width 2.75 in) to study the biaxial effect. The comparisons of average...
results of uniaxial and biaxial specimens are summarized in Table 5.2. Figure 5.8 also shows the comparisons between the experimental and calibrated results of specimens 2D-SP1 and 1D-SP1.

Table 5.2: Comparisons between specimen average results of biaxial and uniaxial bond models

<table>
<thead>
<tr>
<th>ID</th>
<th>$\tau_i$ (or $\tau_f$ 1D) (ksi)</th>
<th>$\delta_1$ (or $\delta_f$ 1D) (in)</th>
<th>$\delta_2$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D-Sp1</td>
<td>0.372</td>
<td>0.00098</td>
<td>0.00280</td>
</tr>
<tr>
<td>1D-Sp1</td>
<td>0.578</td>
<td>0.00177</td>
<td>...</td>
</tr>
</tbody>
</table>

Figure 5.8: Comparison of experimental and calibrated specimen average results of biaxial and uniaxial bond models
CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

This study presented experimental, analytical and numerical results of a pre-impregnated polyurethane FRP composite system used to externally strengthen shear-deficient RC beams. Shear responses were compared to a more commonly used epoxy-based wet layup FRP composite system. The experimental program consisted of fourteen (96 in span, 6 in width, and 12 in height) small-scale and five (132 in span, 12 in width, and 17 in height) large-scale RC beam specimens. The FRP system type, configuration and scheme, shear span-to-depth ratio, and size effect were considered in the testing matrix. The biaxial state of stress of the polyurethane interface was also studied to characterize the significance of considering the stress interaction of the interface.

The following conclusions can be drawn based on the specimens tested, the specific FRP composite systems utilized in the study, and the corresponding analytical and numerical models proposed along with the biaxial bond-slip assumption:

1. Both FRP-PU and FRP-EP systems successfully strengthened the shear deficient flexural specimens. The design strength values utilized for the FRP-PU composites, based on the effective FRP debonding strains $\epsilon_{pe}$ of Triantafillou and Antonopoulos [31], resulted in higher effective FRP debonding strains and lower strengths than those measured during the experiments.

2. Consistent with the results of Khalifa and Nanni [36], small-scale specimens with $a/d = 4$ exhibited larger shear strength increase compared to specimens with $a/d = 3$. The size effect on shear strength was proportional to the FRP reinforcement ratio. Strengthening with U-wraps was the most efficient configuration method, but unlike the previous studies, strengthening with inclined strips at 45° with respect to the hori-
zontal direction had the least efficiency of all schemes used in this study, and strength-
ening with FRP-PU side sheets and strips provided approximately similar experimental
shear strengths.

3. The experimental FRP strains measured for specimens strengthened with sheets were
smaller than other specimens. This is consistent with the findings of Mofidi and Chaal-
lal [67], and indicates the local FRP strain distribution is highly affected by the FRP
width.

4. Polyurethane debonding failure was found to occur in the interface (weaker bond),
whereas the epoxy debonding occurred in a thin layer of the concrete cover.

5. The analytical model, which is based on the effective FRP stress principle and upper-
bound theorem of limit analysis of plasticity, showed good agreement of the load-
deflection responses with the data obtained from the experiment.

6. The interfacial bond parameters govern the shear strength for all large-scale specimens
and small-scale specimens with U-wrap configurations, whereas the FRP effective bond
length or the FRP tensile stiffness governs the shear strength for small-scale specimens
with side bonding configurations.

7. The numerical FEA model provided more insightful details regarding stresses, strains,
deformations, etc. that were not captured experimentally and/or analytically.

8. The lack of modeling some material properties such as the concrete in compression
behavior affected the numerical results with accepted differences.

9. The proposed biaxial bond-slip relationships provided a better physical representation
than the available unidirectional models because the shear strengthening of concrete
beams with FRP composites is one of the in-plane stress assumption applications.
The following are recommendations to extend this study:

1. Investigations, on shear deficient small- and large-scale specimens externally strengthened with FRP-PU system, with various $a/d$ ratios are recommended to extend the range of database.

2. The effect of transverse steel reinforcement is recommended to be analytically considered by incorporating the corresponding mathematical term in the plastic work equilibrium equation.

3. Develop user subroutines of the MSC.Marc [68] regarding the implementation of interfacial bond-slip parameters under biaxial stress. This procedure is useful for implementing numerical analyses and obtaining the shear behaviors of FRP-PU strengthened specimens using the biaxial bond model parameters.

4. A new design setup for the interfacial bond-slip relationship under biaxial state of stress is recommended using UTM with sufficient bonded area since it is limited by the cylinder opening of the hydraulic jacks.
APPENDIX A: CRACK PATTERNS
Figure A.1 shows the cracking patterns of small- and large-scale shear deficient control specimens.

Figure A.2 shows the cracking patterns of S-Uh3-PU and S-Uh3-EP.

Figure A.3 shows the cracking patterns of S-Sh3-PU and S-Ih3-PU.

Figure A.4 shows the cracking patterns of S-Uh4-PU and S-Uh4-EP.

Figure A.5 shows the cracking patterns of S-Th4-PU and S-Th4-EP.

Figure A.6 shows the cracking patterns of S-Sh4-PU and S-Ih4-PU.

Figure A.7 shows the cracking patterns of large-scale FRP-strengthened specimens.
Figure A.1: Cracking patterns of small- and large-scale shear deficient control specimens
Figure A.2: Cracking patterns of S-Uh3-PU and S-Uh3-EP
Figure A.3: Cracking patterns of S-Sh3-PU and S-Ih3-PU
Figure A.4: Cracking patterns of S-Uh4-PU and S-Uh4-EP
Figure A.5: Cracking patterns of S-Th4-PU and S-Th4-EP
Figure A.6: Cracking patterns of S-Sh4-PU and S-Ih4-PU
Figure A.7: Cracking patterns of large-scale FRP-strengthened specimens
APPENDIX B: FINITE DIFFERENCE PROCEDURE
The Neumann boundary conditions (forms of derivatives of the dependent variables) are \( \sigma_{px} = 0 \) and \( \sigma_{cx} = 0 \) at \( x = 0 \), \( \sigma_{px} = \frac{P_x}{t_p} \) and \( \sigma_{cx} = \frac{P_x}{t_c} \) at \( x = L_x \), \( \sigma_{py} = 0 \) and \( \sigma_{cy} = 0 \) at \( y = 0 \), \( \sigma_{py} = \frac{P_y}{t_p} \) and \( \sigma_{cy} = \frac{P_y}{t_c} \) at \( y = L_y \). The governing partial differential equations (Equations 5.26, 5.27, 5.28, and 5.29) were solved by utilizing FD approach since no closed from solutions can be obtained in the available software programs. To solve the aforementioned partial differential equations by FD method, the continuous independent variables \( x \) and \( y \) (in which \( 0 \leq x \leq L_x \) and \( 0 \leq y \leq L_y \)) should be discretized by finite differences or step sizes \( \Delta x \) and \( \Delta y \). Equations B.1, B.2, and B.3 are the central difference symbolic forms of partial derivatives.

\[
\frac{\partial^2 \delta}{\partial x^2} = \frac{\delta_{i-1,j} - 2\delta_{i,j} + \delta_{i+1,j}}{\Delta_x^2} \tag{B.1}
\]

\[
\frac{\partial^2 \delta}{\partial y^2} = \frac{\delta_{i,j-1} - 2\delta_{i,j} + \delta_{i,j+1}}{\Delta_y^2} \tag{B.2}
\]

\[
\frac{\partial^2 \delta}{\partial x \partial y} = \frac{\delta_{i-1,j-1} - \delta_{i+1,j-1} - \delta_{i-1,j+1} + \delta_{i+1,j+1}}{4\Delta_x \Delta_y} \tag{B.3}
\]

in which the dependent variable \( \delta \) may be expressed in terms of FRP composites \( p \) or concrete \( c \) in \( x \) or \( y \) directions.

Forward and backward differences are also needed in the middle nodes at the edges (not corner nodes) with mixed partial derivatives \( \frac{\partial^2 \delta}{\partial x \partial y} \). Equations B.4, B.5, B.6, and B.7 are the difference symbolic forms for middle nodes at the left edge, middle nodes at the right edge, middle nodes at the bottom edge, and middle nodes at the top edge, respectively.

\[
\frac{\partial^2 \delta}{\partial x \partial y} = \frac{\delta_{i,j-1} - \delta_{i+1,j-1} - \delta_{i,j+1} + \delta_{i+1,j+1}}{2\Delta_x \Delta_y} \tag{B.4}
\]
\[
\frac{\partial^2 \delta}{\partial x \partial y} = \frac{\delta_{i-1,j-1} - \delta_{i,j-1} - \delta_{i-1,j+1} + \delta_{i,j+1}}{2\Delta_x \Delta_y} \quad \text{(B.5)}
\]

\[
\frac{\partial^2 \delta}{\partial x \partial y} = \frac{\delta_{i-1,j} - \delta_{i+1,j} - \delta_{i-1,j+1} + \delta_{i+1,j+1}}{2\Delta_x \Delta_y} \quad \text{(B.6)}
\]

\[
\frac{\partial^2 \delta}{\partial x \partial y} = \frac{\delta_{i-1,j-1} - \delta_{i+1,j-1} - \delta_{i-1,j+1} + \delta_{i+1,j}}{2\Delta_x \Delta_y} \quad \text{(B.7)}
\]

Equations B.8, B.9, B.10, and B.11 are the FD forms of the boundary conditions in the FRP composites or concrete for middle nodes at the left edge, middle nodes at the right edge, middle nodes at the bottom edge, and middle nodes at the top edge, respectively.

\[
\sigma_x = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_x}{\partial x} + \nu \frac{\partial \delta_y}{\partial y} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{xi+1,j} - \delta_{xi,j}}{\Delta_x} + \nu \frac{\delta_{yi,j+1} - \delta_{yi,j-1}}{2\Delta_y} \right) \quad \text{(B.8)}
\]

\[
\sigma_x = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_x}{\partial x} + \nu \frac{\partial \delta_y}{\partial y} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{xi,j} - \delta_{xi-1,j}}{\Delta_x} + \nu \frac{\delta_{yi,j+1} - \delta_{yi,j-1}}{2\Delta_y} \right) \quad \text{(B.9)}
\]

\[
\sigma_y = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_y}{\partial y} + \nu \frac{\partial \delta_x}{\partial x} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{yi,j+1} - \delta_{yi,j}}{\Delta_y} + \nu \frac{\delta_{xi,j+1} - \delta_{xi,j-1}}{2\Delta_x} \right) \quad \text{(B.10)}
\]

\[
\sigma_y = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_y}{\partial y} + \nu \frac{\partial \delta_x}{\partial x} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{yi,j} - \delta_{yi,j-1}}{\Delta_y} + \nu \frac{\delta_{xi,j+1} - \delta_{xi,j-1}}{2\Delta_x} \right) \quad \text{(B.11)}
\]

Equations B.12 and B.13 are the FD for left bottom corner node.

\[
\sigma_x = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_x}{\partial x} + \nu \frac{\partial \delta_y}{\partial y} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{xi+1,j} - \delta_{xi,j}}{\Delta_x} + \nu \frac{\delta_{yi,j+1} - \delta_{yi,j}}{\Delta_y} \right) \quad \text{(B.12)}
\]
\[ \sigma_y = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_y}{\partial y} + \nu \frac{\partial \delta_x}{\partial x} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{yi,j+1} - \delta_{yi,j}}{\Delta y} + \nu \frac{\delta_{x_{i+1,j}} - \delta_{x_{i,j}}}{\Delta x} \right) \] (B.13)

Equations B.14 and B.15 are the FD for right bottom corner node.

\[ \sigma_x = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_x}{\partial x} + \nu \frac{\partial \delta_y}{\partial y} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{xi,j} - \delta_{xi-1,j}}{\Delta x} + \nu \frac{\delta_{yi,j+1} - \delta_{yi,j}}{\Delta y} \right) \] (B.14)

\[ \sigma_y = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_y}{\partial y} + \nu \frac{\partial \delta_x}{\partial x} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{yi,j+1} - \delta_{yi,j}}{\Delta y} + \nu \frac{\delta_{xi,j} - \delta_{xi-1,j}}{\Delta x} \right) \] (B.15)

Equations B.16 and B.17 are the FD for left top corner node.

\[ \sigma_x = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_x}{\partial x} + \nu \frac{\partial \delta_y}{\partial y} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{xi+1,j} - \delta_{xi,j}}{\Delta x} + \nu \frac{\delta_{yi,j} - \delta_{yi-1,j}}{\Delta y} \right) \] (B.16)

\[ \sigma_y = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_y}{\partial y} + \nu \frac{\partial \delta_x}{\partial x} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{yi,j} - \delta_{yi,j-1}}{\Delta y} + \nu \frac{\delta_{xi+1,j} - \delta_{xi,j}}{\Delta x} \right) \] (B.17)

Equations B.18 and B.19 are the FD for right bottom corner node.

\[ \sigma_x = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_x}{\partial x} + \nu \frac{\partial \delta_y}{\partial y} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{xi,j} - \delta_{xi-1,j}}{\Delta x} + \nu \frac{\delta_{yi,j} - \delta_{yi,j-1}}{\Delta y} \right) \] (B.18)

\[ \sigma_y = \frac{E}{1 - \nu^2} \left( \frac{\partial \delta_y}{\partial y} + \nu \frac{\partial \delta_x}{\partial x} \right) = \frac{E}{1 - \nu^2} \left( \frac{\delta_{yi,j} - \delta_{yi,j-1}}{\Delta y} + \nu \frac{\delta_{xi,j} - \delta_{xi-1,j}}{\Delta x} \right) \] (B.19)
LIST OF REFERENCES


