Marketing Partnerships: Impact of Monitoring Schemes and Cooperative Advertising Agreements

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by

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ABSTRACT

Marketing partnerships may involve either horizontal relationships (e.g., a co-marketing alliance between firms selling different products) or vertical relationships (e.g., between an upstream manufacturer and its downstream retailers). Either type of partnership often includes multiple members and the marketing efforts (e.g., level of advertising) of any member typically affect the profitability of the other members. When selecting their effort levels, however, the individual members of the partnership do not account for such externalities. Consequently, the overall effort on behalf of the partnership is not optimal. This dissertation investigates the value of contractual mechanisms such as monitoring schemes (for horizontal partnerships) and cooperative advertising programs (for vertical partnerships) that may provide better incentives to the partners to invest into the relationship.

The first part of this dissertation focuses on horizontal marketing partnerships and examines the relative effectiveness of outcome- and action-based contracts in providing the alliance partners with the incentives to invest appropriately. A mathematical model is developed in which a focal firm (e.g., Sony) contracts with two partners (e.g., McDonald's and Old Navy), when each of these partners is privately informed about the impact of the alliance on its demand. The analysis evaluates the strengths and weaknesses of outcome- (or output-) and action- (or input-) based contracts in several settings including those with no demand externality, a positive externality and a negative externality. The analysis shows that when there is (a) no externality, (b) negative externality, or (c) a relatively weak positive externality, there is a strict preference for output-based contracts; that preference, however, is reversed with a sufficiently strong positive externality. The rationale for these findings, along with the implications and directions for further work are discussed.

The second part of this dissertation focuses on a vertical marketing relationship where multiple retailers sell the products from a common manufacturer. Here, each retailer's level of advertising affects the demand for the other retailers. This positive externality, however,

allows any retailer to free-ride on the other retailers' efforts and leads to an overall reduction in the level of advertising by all the retailers. In this context, a manufacturer can use a cooperative advertising contract to reimburse part of the advertising expenses of its retailers in order to induce them to raise their levels of advertising. Observed terms in a cooperative advertising contract include either a participation rate, a participation rate and a variable accrual rate, or a participation rate and a fixed accrual rate. This dissertation analyzes the relative effectiveness of the above three types of cooperative advertising contracts in minimizing or eliminating the free-riding problem.

More specifically, a mathematical model is developed to analyze the relative impact of these contractual terms when the downstream retailers face either symmetric or asymmetric demand and cost structures. The analysis shows that with symmetric retailers, the three types of contracts are equally effective. With asymmetric retailers, though, including some form of accrual stipulations typically adds value to a contract that specifies only a participation rate. Further, using a variable accrual stipulation may be preferred to the fixed accrual stipulation under certain conditions and vice versa. The two types of accrual stipulations affect retail prices and efforts in distinct ways and these differences may tip the scale in favor of one contract versus the other under the appropriate circumstances. These conditions and the intuition behind the results are discussed. Overall, this dissertation contributes to the literature on horizontal and vertical marketing relationships and enhances our understanding of distinct contractual mechanisms that can help align the actions of various members involved in such partnerships.

I dedicate this dissertation to my mother Ch. Usha Rani, my father Ch. Ravinder Rao, my grandfather K. Achuta Rao and my sister Ch. Meghana Rao. Without the support and love of my family, I would not have been able to accomplish this task.

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CHAPTER 1: INTRODUCTION

Marketing partnerships may involve either horizontal relationships (e.g., a co-marketing alliance between firms selling different products) or vertical relationships (e.g., between an upstream manufacturer and its downstream retailers). Either type of partnership often includes multiple members and the marketing efforts (e.g., level of advertising) of any member typically affect the profitability of the other members. When selecting their effort levels, however, the individual members of the partnership do not account for such externalities. Consequently, the overall effort on behalf of the partnership is not optimal. This dissertation investigates the value of contractual mechanisms such as monitoring schemes (for horizontal partnerships) and cooperative advertising programs (for vertical partnerships) that may provide better incentives to the partners to invest into the relationship.

The first part of this dissertation analyzes contractual issues in horizontal marketing partnerships and the second part analyzes vertical partnerships. In the next few paragraphs, the relevant issues pertaining to horizontal partnerships, the contribution to the extant literature, the results of this work and the corresponding intuition are introduced. Subsequently, analogous issues dealing with vertical marketing partnerships are introduced.

Co-marketing alliances, such as those between Sony Pictures and Old Navy, or between Disney and McDonald's, aim to enhance the value of partner firms' offerings to consumers. In the Disney-McDonald's alliance, for instance, McDonald's was contractually required to spend around \$20 million a year on promoting Disney's movies along with the McDonald's Happy Meal[1]; and Disney received licensing fees to the tune of \$100 million per year (for ten years) for granting the right to use its characters as part of the promotion.

[1]As a part of this promotion, Disney-movie-character toys are given to consumers who purchase a McDonald's-Happy-Meal, and the popularity of the movie raises the effectiveness of McDonald's promotion (Howard 1996).

Co-marketing alliances typically involve multiple partners —for example, Sony formed alliances with McDonald's, Old Navy, ConAgra Foods and many other partners to promote the movie Surf's Up. In such alliances, each partner's marketing actions are likely to affect the outcomes generated by the other partners. For instance, Old Navy's efforts to promote Surf's Up can raise the awareness/popularity of the movie; since McDonald's sales are also linked to the movie's success, Old Navy's promotional investments can indirectly benefit the sales of McDonald's products. The individual partners of an alliance typically do not consider the impact of such indirect linkages while deciding on their levels of investment; this can lead to an overall under-investment in marketing efforts[2] (for a discussion of this under-investment problem in other contexts see, e.g., Lal 1990, Amaldoss et al 2000) and can lower the value of the alliance. Therefore, it is useful to understand the relative strengths of different types of contractual agreements in providing the appropriate incentives to the partners to increase their marketing efforts. Hence, the first part of this dissertation examines the relative effectiveness of outcome- and action-based contracts in providing the alliance partners with the incentives to invest appropriately in marketing efforts.

Extant research on co-marketing alliances has focused primarily on issues such as the firms' motivation for forming these partnerships (e.g., Rao et al 1999, Rao and Ruekert 1994), selecting suitable alliance partners (Venkatesh and Mahajan 1997) and identifying factors that lead to a successful alliance (Venkatesh et al 2000, Bucklin and Sengupta 1993). While this stream of work has generated valuable insights, little research attention has been devoted to understanding the relative effectiveness of alternative contractual agreements on the performance of co-marketing alliances. This dissertation contributes to the literature by focusing on co-marketing alliances involving multiple partners and compares alternative contractual mechanisms in their effectiveness to induce appropriate marketing investments.

[2]Hereafter, we use the term "effort" to refer to include marketing actions such as promoting the alliance or investing in advertising.

Many factors affect the terms of the contracts employed in a co-marketing alliance and the optimal contract will likely require a careful resolution of several important tradeoffs. Here, the focus is on the impact of information asymmetry. In the Sony-McDonald's alliance, for example, compared to Sony, McDonald's is likely to have better information on how customers in the fast food business react to promotions and therefore can more accurately estimate the benefit (e.g., the boost in demand) it would receive from the alliance. Such information-asymmetry issues will likely arise between Sony and the other alliance partners as well, and can affect how the partners share any surplus arising out of the alliance.

In practice, contracts are often made contingent on one or more performance criteria, and when information asymmetry is a principal feature of the market setting, extant literature (e.g., Basu et al 1985, Maskin and Riley 1985, Desai and Srinivasan 1995, Khalil and Lawarree 1995, Raju and Srinivasan 1996, Desiraju and Moorthy 1997, Mishra and Prasad 2004) suggests that varying the performance-criterion can alter the partners' incentives to exert effort. Accordingly, chapter 2's goal is to compare the relative merits of two types of agreements: those contingent on (1) the partner's marketing actions (e.g., level of Happy Meal promotions by McDonald's) and (2) the partner's outcomes (e.g., sales levels of Happy Meals). In this comparison, the interest is in the role played by any demand linkage or externality that exists among the partners.

In the absence of any demand externality, when contracting with an asymmetrically informed partner (or agent), Khalil and Lawarree (1995) (KL) demonstrate that contracts based on outputs generate superior returns to the principal. The logic underlying KL's result is as follows: Suppose the agent were a residual claimant—as is typically the case in any co-marketing alliance—and the contract were contingent on monitored outputs (KL refer to this setup as output monitoring). In such settings, the agent has discretion over the level of effort expended on behalf of the partnership (this feature is noted in the many analyses of agency relationships studied in marketing). Now, by contrast, suppose that inputs are contracted upon. In such a setting (aka input monitoring), if an agent in a more productive state of nature were to claim to be in a less productive state, then by virtue of being the residual claimant, that agent can appropriate some of the output generated from the alliance without sharing it with the principal (since the output is not monitored). This additional output that the agent can pocket under input monitoring serves as a deciding factor in arriving at KL's main finding that the principal prefers monitoring the agent's output rather than the level of effort. Interestingly, though, anecdotal evidence suggests that the contractual terms between partners in a co-marketing alliance are typically based on marketing actions rather than on marketing outcomes: As noted earlier, in its alliance with Disney, McDonald's was contractually required to invest in promoting the partnership while paying Disney a fee for the right to use its characters. Since such terms seem counter to the findings in KL, the goal here is to explore when such contracts may be preferred.

Accordingly, chapter 2 builds on KL's adverse selection setting and develop a simple multiple agent model—in which a focal firm contracts with two privately informed partners—to identify when a contract contingent on inputs may be preferred over an output-based contract. The analysis reveals that the nature of the demand externality among the alliance partners plays a critical role in the choice between these two contract forms. Intuitively, as in the single agent setting, when only the outputs are monitored, agents have an opportunity to limit the efforts put forth on behalf of the alliance. Further, the presence of a positive demand externality in the multi-agent setting tends to result in even lower effort levels than in the single agent setting. As the strength of the positive demand externality goes up, the multiple agents' effort levels shrink to correspondingly lower levels, and the principal's losses (along with those of the alliance) to correspondingly higher levels.

In contrast, by monitoring inputs, the principal effectively raises the effort (or marketing investment) levels that the multiple agents put forth. The positive demand externality, in turn, generates more output; since the partners are residual claimants, and the principal only monitors inputs, there is more output that can be appropriated by the partners without sharing it with the principal. At lower strengths of the positive externality, the principal continues to prefer output monitoring as noted in KL. When the positive demand externality is sufficiently strong, however, the lower effort levels induced under output monitoring can prove to be too costly for the principal; and input monitoring becomes the preferred contract form. Chapter 2 develops and analyzes a mathematical model and present more precise conditions (involving either a positive or a negative externality) where one type of contract may be preferred over the other. Chapter 2 also highlights the conditions under which the effort levels under one type of contract dominate those in the other type of contract. By evaluating the relative effectiveness of alternative contractual terms, the analysis adds to both the co-marketing alliance literature as well as to the agency literature dealing with monitoring issues in adverse selection settings.

The second part of this dissertation focuses on a vertical marketing partnership where multiple retailers sell the products from a common manufacturer. Here, each retailer's level of advertising affects the demand for the other retailers. This positive externality, however, allows any retailer to free-ride on the other retailers' efforts and leads to an overall reduction in the levels of advertising by all the retailers. In this context, a manufacturer can use a cooperative advertising contract to reimburse part of the advertising expenses of its retailers in order to induce them to raise their levels of advertising. Observed terms in a cooperative advertising contract include either a participation rate, a participation rate and a variable accrual rate, or a participation rate and a fixed accrual rate. This dissertation analyzes the relative effectiveness of the above three types of cooperative advertising contracts in minimizing or eliminating the free-riding problem.

While all cooperative advertising contracts offer to reimburse a portion (called the participation rate) of the retailer's advertising expenses, many of these contracts also specify limits (called the accrual rate) on the total reimbursement offered. These accrual rates are either set as a fraction of wholesale receipts from the retailer or are simply set as a fixed dollar amount. While extant research (Bergen and John 1997, Huang et al 2002, He et al 2007) in this area has focused only on the participation rate, there is no research that investigates the impact of the two accrual rates on retail behavior. Chapters 3 and 4 of this dissertation fill this gap in the literature and investigate the use of the cooperative advertising participation rate and the two types of accrual rates in their effectiveness to provide the retailers with appropriate incentives to increase their levels of advertising.

Chapter 3 considers a manufacturer selling its products through two symmetric independent retailers. The retailers face positive advertising externalities and in the absence of any cooperative advertising contract, under-invest in advertising. The analysis shows that the manufacturer can use a cooperative advertising contract that includes only a single participation rate offered to both the retailers to increase the overall levels of advertising. When the manufacturer offers the retailers a cooperative advertising contract that reimburses part of their advertising expenses (participation rate contract), the retailer's marginal cost of advertising is lowered and they find it attractive to increase their investments in advertising. The manufacturer can choose the participation rate such that the retailers increase their advertising levels to that preferred by the manufacturer. Hence, the manufacturer uses the participation rate to align the interests of the retailers with those of the channel. The optimal participation rate required to align the retailers' interest with that of the channel are derived for a general demand structure, without making specific assumptions regarding the functional form of the demand faced by the retailers. Hence, this provides a useful analytical tool to determine the participation rates in a variety of settings.

Next, the analysis shows that the manufacturer can also achieve coordination in the channel by using a combination of the participation rate and either of the two types of accrual rates. When the manufacturer uses an accrual rate in addition to the participation rate, the manufacturer can set the participation rates at higher levels than that required to coordinate the channel under the participation rate contract. This provides the retailers with an incentive to increase their advertising levels by a greater amount than under a single participation rate contract. The manufacturer then limits the total reimbursement to the retailers by using an accrual rate that is either a fixed amount or a fraction of the wholesale receipts from the retailer. While the retailers have an incentive to increase advertising to levels beyond what is sufficient to coordinate the channel, the manufacturer uses the cap on reimbursement to limit the retailer's advertising to the channel coordinating levels. Hence, the manufacturer can use both the participation rate and the accrual rate to achieve the same result achieved through the use of a participation rate alone.

While the two types of accrual rates can help coordinate the advertising levels in the distribution channel, each type of accrual rate has a unique impact on retail prices and advertising levels. The accrual rate that is set as a fixed amount has a positive impact on retail prices. As the manufacturer increases the fixed cap, retailers tend to increase prices. Since the increases in the fixed cap induce the retailers to increase their advertising levels, the increase in the advertising levels in turn leads the retailers to increase retail prices. While this indirect impact of the increase in advertising levels on retail prices also exits when the manufacturer uses an accrual rate linked to wholesale receipts, the increase in the accrual rate also has a negative direct impact on retail prices.

When the manufacturer uses an accrual rate linked to wholesale receipts, unlike the fixed accrual rate scenario, the retailers can influence the cap on reimbursements through their choices of price and effort levels. As retailers decrease prices (or increase effort), retail demand goes up and in turn the wholesale receipts go up. This leads to an increase in the cap on cooperative advertising reimbursements. Hence, as the manufacturer increases the accrual rate linked to wholesale receipts, the retailers may find it attractive to decrease retail prices and thereby further increase the cap on reimbursements. As the accrual rate increases, the increase in the cap for a unit decrease in retail prices is higher (compared to when the accrual rate is lower). Hence, the net effect of an increase in accrual rate linked to wholesale receipts may lead to a reduction in retail prices as opposed to an increase in retail prices (that occurs under the fixed accrual rate contract).

While chapter 3 investigates the use of various cooperative advertising contracts by a manufacturer selling through two symmetric retailers, downstream retailer asymmetry is likely to arise when selling to multiple retailers who differentiate themselves from each other. Retailer asymmetry can also arise due to differences in cost structures. In addition, retailers may also differ in their target customer segments' size and/or valuations of the products sold and these segment differences may result in the retailers facing asymmetric demand. As Iyer (1998) notes, retail differentiation has important implications for upstream manufacturers and the extant literature seldom accounts for such asymmetry in investigating channel issues. The use of a 'one size fits all' strategy that works well under symmetry may fail under asymmetry. Hence, chapter 4 extends the analysis in chapter 3 to investigate the effectiveness of cooperative advertising contracts in coordinating the channel in the presence of asymmetric downstream retailers.

With asymmetric retailers, while all three types of contracts can coordinate the channel under very stringent conditions, contracts that include some form of accrual stipulation can coordinate the channel under less stringent conditions compared to the contract that includes only a participation rate. When the cooperative advertising contract includes an accrual stipulation, the manufacturer can use the accrual amount to coordinate the efforts of one retailer while using the participation rate to coordinate the other retailer's efforts. Further, using a variable accrual stipulation may be preferred to the fixed accrual stipulation under certain conditions and vice versa. The two types of accrual stipulations affect retail prices and efforts in distinct ways and these differences may tip the scale in favor of one contract versus the other contract under the appropriate circumstances. These conditions and the intuition behind the results are discussed.

Overall, this dissertation contributes to the literature on horizontal and vertical marketing relationships and enhances our understanding of distinct contractual mechanisms that can help align the actions of various members involved in such partnerships.

CHAPTER 2: CO-MARKETING ALLIANCES

2.1 Introduction

Co-marketing alliances, such as those between Sony Pictures and Old Navy, or between Disney and McDonald's, are growing in popularity and involve considerable sums of money (Ebenkamp 2007). These alliances aim to enhance the value of partner firms' offerings to consumers, and a notable feature is that these partnerships are governed mainly by contractual agreements (see e.g., Bucklin and Sengupta 1993, Venkatesh et al 2000, Simonin and Ruth 1998). In the Disney-McDonald's alliance, for instance, McDonald's was contractually required to spend around $20 million a year on promoting Disney's movies along with the McDonald's Happy Meal[1]; and Disney received licensing fees to the tune of $100 million per year (for ten years) for granting the right to use its characters as part of the promotion.

Extant research on co-marketing alliances has focused primarily on issues such as the firms' motivation for forming these partnerships (e.g., Rao et al 1999, Rao and Ruekert 1994), selecting suitable alliance partners (Venkatesh and Mahajan 1997) and identifying factors that lead to a successful alliance (Venkatesh et al 2000, Bucklin and Sengupta 1993). While this stream of work has generated valuable insights, little research attention has been devoted to understanding the relative effectiveness of alternative contractual agreements on the performance of co-marketing alliances. An exception is the work by Amaldoss et al (2000) which examines, in the context of R&D alliances, how a partner's investments in new product development depend on the terms of the contract—they consider equal versus proportional profit sharing agreements.

[1]As a part of this promotion, Disney-movie-character toys are given to consumers who purchase a McDonald's-Happy-Meal, and the popularity of the movie raises the effectiveness of McDonald's promotion (Howard 1996).

We add to this literature by focusing on co-marketing alliances in which there are multiple partners—e.g., Sony formed alliances with McDonald's, Old Navy, ConAgra Foods and many other partners to promote the movie Surf's Up. In such alliances, each partner's marketing actions are likely to affect the outcomes generated by the other partners. For instance, Old Navy's efforts to promote Surf's Up can raise the awareness/popularity of the movie; since McDonald's sales are also linked to the movie's success, Old Navy's promotional investments can indirectly benefit the sales of McDonald's products. The individual partners of an alliance typically do not consider the impact of such indirect linkages while deciding on their levels of investment; this can lead to an overall under-investment in marketing efforts[2] (for a discussion of this under-investment problem in other contexts see, e.g., Lal 1990, Amaldoss et al 2000) and can lower the value of the alliance. Therefore, it is useful to understand the relative strengths of different types of contractual agreements in providing the appropriate incentives to the partners.

Many factors affect the terms of the contracts employed in a co-marketing alliance and the optimal contract will likely require a careful resolution of several important tradeoffs. Here, we focus on the impact of information asymmetry. In the Sony-McDonald's alliance, for example, compared to Sony, McDonald's is likely to have better information on how customers in the fast food business react to promotions and therefore can more accurately estimate the benefit (e.g., the boost in demand) it would receive from the alliance. Such information-asymmetry issues will likely arise between Sony and the other alliance partners as well, and can affect how the partners share any surplus arising out of the alliance.

In practice, contracts are often made contingent on one or more performance criteria, and when information asymmetry is a principal feature of the market setting, extant literature (e.g., Basu et al 1985, Maskin and Riley 1985, Desai and Srinivasan 1995, Khalil and Lawarree 1995, Raju and Srinivasan 1996, Mishra and Prasad 2004) suggests that varying

[2]Hereafter, we use the term "effort" to refer to include marketing actions such as promoting the alliance or investing in advertising.

the performance-criterion can alter the partners' incentives to exert effort. Accordingly, our goal here is to compare the relative merits of two types of agreements: those contingent on (1) the partner's marketing actions (e.g., level of Happy Meal promotions by McDonald's) and (2) the partner's outcomes (e.g., sales levels of Happy Meals). In this comparison, we are particularly interested in the role played by any demand linkage or externality that exists among the partners.

In the absence of any demand externality, when contracting with an asymmetrically informed partner (or agent), Khalil and Lawarree (1995) (KL) demonstrate that contracts based on outputs generate superior returns to the principal. The logic underlying KL's result is as follows: Suppose the agent were a residual claimant—as is typically the case in any co-marketing alliance—and the contract were contingent on monitored outputs (KL refer to this setup as output monitoring). In such settings, the agent has discretion over the level of effort expended on behalf of the partnership (this feature is noted in the many analyses of agency relationships studied in marketing). Now, by contrast, suppose that inputs are contracted upon. In such a setting (aka input monitoring), if an agent in a more productive state of nature were to claim to be in a less productive state, then by virtue of being the residual claimant, that agent can appropriate some of the output generated from the alliance without sharing it with the principal (since the output is not monitored). This additional output that the agent can pocket under input monitoring serves as a deciding factor in arriving at KL's main finding that the principal prefers monitoring the agent's output rather than the level of effort. Interestingly, though, anecdotal evidence suggests that the contractual terms between partners in a co-marketing alliance are typically based on marketing actions rather than on marketing outcomes: As noted earlier, in its alliance with Disney, McDonald's was contractually required to invest in promoting the partnership while paying Disney a fee for the right to use its characters. Since such terms seem counter to the findings in KL, our goal here is to explore when such contracts may be preferred.

Accordingly, we build on KL's adverse selection setting and develop a simple multiple agent model—in which a focal firm contracts with two privately informed partners—to identify when a contract contingent on inputs may be preferred over an output-based contract. Our analysis reveals that the nature of the demand externality among the alliance partners plays a critical role in the choice between these two contract forms. Intuitively, as in the single agent setting, when only the outputs are monitored, agents have an opportunity to limit the efforts put forth on behalf of the alliance. Further, the presence of a positive demand externality in the multi-agent setting tends to result in even lower effort levels than in the single agent setting. As the strength of the positive demand externality goes up, the multiple agents' effort levels shrink to correspondingly lower levels, and the principal's losses (along with those of the alliance) to correspondingly higher levels.

In contrast, by monitoring inputs, the principal effectively raises the effort (or marketing investment) levels that the multiple agents put forth. The positive demand externality, in turn, generates more output; since the partners are residual claimants, and the principal only monitors inputs, there is more output that can be appropriated by the partners without sharing it with the principal. At lower strengths of the positive externality, the principal continues to prefer output monitoring as noted in KL. When the positive demand externality is sufficiently strong, however, the lower effort levels induced under output monitoring can prove to be too costly for the principal; and input monitoring becomes the preferred contract form.

In what follows, we develop and analyze a mathematical model and present more precise conditions (involving either a positive or a negative externality) where one type of contract may be preferred over the other. We also highlight the conditions under which the effort levels under one type of contract dominate those in the other type of contract. By evaluating the relative effectiveness of alternative contractual terms, our analysis adds to both the co-marketing alliance literature as well as to the agency literature dealing with monitoring issues in adverse selection settings.

The rest of the paper is organized as follows: the next section reviews the relevant literature and section 2.3 develops the model. Section 2.4 presents our analysis and results while the final section concludes the paper. All proofs are confined to an appendix.

2.2 Literature Review

Co-marketing alliances are contractual relationships that are undertaken by firms that intend to enhance the value of their offerings to consumers (Bucklin and Sengupta 1993). Disney-McDonald's, Coke-NutraSweet, Coke-Bacardi Rum, Intel-Compaq, etc., are examples of such alliances in the marketplace. Researchers have noted that these alliances can help signal product quality to consumers, reach new segments, or even enhance the appeal to current market segments (Rao and Ruekert 1994, Rao et al 1999, Venkatesh et al 2000 and Bucklin and Sengupta 1993). Since our focus here is on the relative effectiveness of alternative contractual forms in a co-marketing alliance, we now discuss the literature related to the success of these alliances.

Many factors affect the success or failure of co-marketing alliances. Bucklin and Sengupta (1993), for example, find that higher payoffs (strategic value of the partnership net of development cost) from an alliance and greater organizational compatibility between the partnering firms impact the effectiveness of the co-marketing alliance positively. Venkatesh et al (2000) investigate co-marketing alliances that produce a series of co-branded products (e.g., Compaq PCs with Intel Inside), and find that such dynamic partnerships are attractive only if the alliance can help expand the market size significantly. Next, partner selection can clearly affect success, and in the context of a co-marketing alliance that involves a product with branded components, Venkatesh and Mahajan (1997) discuss an analytical approach for optimal pricing and partner selection—their focus is on whether to align with another branded component manufacturer or with an unbranded component manufacturer.

It is worth noting that the impact of contractual agreements on the partners' investment (along with partner selection and other issues) has been investigated in the context of R&D alliances (see Veugelers 1998 for a review of this literature, and Amaldoss et al 2000 and Jap 2001 for more recent work). Amaldoss et al (2000) find that the partner's investments in a new product development alliance depend on the type of profit sharing (equal or proportional) agreements between the partners. Similarly, in the context of complex collaborations (such as R&D alliances), Jap (2001) finds that the profit sharing principle (equity or equality) impacts the outcomes of the alliance. Such analyses, however, are absent in the context of co-marketing alliances. As noted in Simonin and Ruth (1998), co-marketing alliances can arise when the partners' brands or products are combined either physically (e.g. Compaq computers with Intel microprocessors) or symbolically (e.g., McDonald's promotions involving Disney movie characters[3]).

When forming co-marketing alliances, firms typically contract with multiple partners. When multiple partners are involved in such an alliance, each partner's marketing actions are likely to affect the outcomes generated by the other partners. Past research has discussed the impact of such an externality in different contexts. Amaldoss et al (2000), for instance, show that in a joint product development alliance, each partner can free ride on the investments made by the other partners, thereby leading to an under-investment problem in the alliance. Other research on R&D alliances (Veugelers 1998), too, shows that the presence externalities among the partners lead to lower R&D investments. Similarly, in the context of franchising, Lal (1990) finds an analogous under-investment problem when the actions of one franchisee benefits other franchisees. In light of such analyses, we expect that when there is a positive demand externality, the co-marketing alliance will also experience an under-investment problem. Our interest, therefore, is to understand which of two types of contractual agreements can provide the appropriate incentives to the alliance partners and limit the under-investment problem.

As noted in the introduction, contracts are typically structured to be contingent on various aspects of performance—this can be on marketing actions (e.g., certain amount of funds should be devoted to advertising) and/or on marketing outcomes (e.g., on achieved sales). Under information asymmetry though, varying the performance-criterion is likely to

[3]The hamburgers sold by McDonald's do not involve any ingredient from Disney.

alter the partners' incentives (e.g., see Basu et al 1985, Raju and Srinivasan 1996, Holmstrom and Milgrom 1991, Maskin and Riley 1985, Khalil and Lawarree 1995, Desai and Srinivasan 1995, Anderson and Oliver 1987 and Mishra and Prasad 2004). This stream of work employs the agency theory framework to examine incentive problems in a variety of organizational settings, including salesforces, distribution channels and regulatory contexts.

Notice that when proposing the alliance to potential partners, Disney, for example, offers the right to use its movie in return for a fee and a performance requirement that partners invest in promoting the alliance. In this setting, Disney can be viewed as a principal and the other partners as agents who work on behalf of the principal. From that perspective, our work is closely related to that of Khalil and Lawarree (1995)(KL). KL compare the relative attractiveness of using a contract contingent on marketing actions to a contract contingent on marketing outcomes in the context of a single principal-single agent setting. When the agents are residual claimants—i.e., the output generated by an agent accrues to that agent minus a lump sum transfer payment to the principal, as, for example, in a co-marketing alliance—KL find that a contract contingent on marketing outcomes is always preferred to a contract contingent on marketing actions. Our goal here is to explore the impact of a demand externality on the optimality of such contracts.

2.3 The Model

We consider an adverse selection model in which a risk neutral principal, P, enters into an alliance with two risk neutral partners A and B who exert efforts e^A and e^B respectively[4]. The output from partner k, depends on the efforts exerted by both partners and a random state of nature[5], θ_m^k, $m \in \{L, H\}$, with $0 < \theta_L^k < \theta_H^k$, $k \in \{A, B\}$. Given an effort choice e^l by partner l, we denote the output from partner k by $Q(e^k, e^l, \theta^k)$ where $k, l \in \{A, B\}, k \neq l$, and this output function is assumed to satisfy the conditions[6] $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_{11} \leq 0$ and $Q_{13} \geq 0$. The cost of exerting effort e^k under state θ^k is given by $C(e^k, \theta^k)$ and this cost function is assumed to satisfy the conditions $C_1 > 0$, $C_2 < 0$, $C_{11} > 0$, $C_{12} < 0$ and $C(0, \theta^k) = 0\ \forall k \in \{A, B\}$.

The sequence of events in the model are as follows:

(i) Nature moves first and chooses the productivity parameter θ^k and partner k privately observes the realization of θ^k; the probability that partner k is in state θ_m^k, denoted ϕ_m^k, however, is common knowledge, $\forall\, k \in \{A, B\}$ and $m \in \{L, H\}$.

(ii) The principal chooses the monitoring instrument, Z, where $Z \in \{I, O\}$ with I and O denoting input and output respectively, and offers each partner a contract that specifies either the pair ($e^{k,I}$,$T^{k,I}$)—i.e., effort $e^{k,I}$ and transfer payment $T^{k,I}$ under input monitoring—or the pair ($Q^{k,O}$,$T^{k,O}$)—i.e., output $Q^{k,O}$ and transfer payment $T^{k,O}$ under output monitoring—contingent on the monitored instrument[7].

[4] As noted earlier, we use the term "effort" to refer to include marketing actions such as promoting the alliance or investing in advertising.

[5] We assume that the price of the product sold by the partners is constant and is unaffected by the alliance. This feature is observed in several co-marketing alliances. For example, the price set by McDonald's for its product (the 'Happy Meal') that is part of the joint promotion remains constant and does not depend on the specific Disney movie that is promoted.

[6] Here and throughout the paper, we use numerical subscripts to denote partial derivatives: e.g., Q_1 denotes the partial derivative with respect to partner k's own effort e^k, while Q_{11} denotes its second partial derivative and Q_{13} denotes the cross partial derivative with θ^k.

[7] Here, if both input and output could be observed costlessly and contracted upon, then information asymmetry will not affect the principal's profitability. Therefore, to highlight the relative strengths of the two instruments, we assume that the principal can monitor only one instrument without cost. While

(iii) The partners decide simultaneously and noncollusively whether or not to sign the contract; if the contract is not signed, each partner obtains a reservation profit normalized at zero. If the contract is signed, then each partner chooses its effort to perform as stipulated in the contract; subsequently, monitoring and transfers occur as promised.

Under the above structure, each partner is a residual claimant and enjoys any output that remains after making the transfer payment to the principal. In this setting, the principal's profits Π^P, can be expressed as:

$$\Pi^P = \phi_H^A E[T_H^A] + (1-\phi_H^A)E[T_L^A] + \phi_H^B E[T_H^B] + (1-\phi_H^B)E[T_L^B] \qquad \text{(Eq. 2.1)}$$

where $E[T_m^k] = \phi_H^l T_{mH}^k + (1-\phi_H^l)T_{mL}^k$, with the subscripts mL and mH on T^k accommodating the possibility of varying the transfer payments based on the states of both partners, $k, l \in \{A, B\}, k \neq l$ and $m \in \{L, H\}$.

Given a monitoring instrument Z, $Z \in \{I, O\}$, we use $\pi_{n|m}^{k,Z}(.)$ to denote k's profit when that partner is in state m but reports to be in state n, $\forall\, m, n \in \{H, L\}$. As in standard adverse selection settings, for a given Z, the principal's problem here is to design a contract that maximizes Π^P subject to the conditions that each partner receives at least its reservation profit (equivalently, $\pi_{m|m}^{k,Z} \geq 0$ $k \in \{A, B\}$) while revealing the private information truthfully as a best response to the other partner doing the same (equivalently, $\pi_{m|m}^{k,Z} \geq \pi_{n|m}^{k,Z}$ $\forall\, k \in \{A, B\}$). Ultimately, the principal will select that monitoring instrument which will generate the highest expected profit.

For ease of exposition, we now focus on the following simple demand and cost structures (in the Appendix, we show how our analysis holds for the more general demand and cost

monitoring may be costly even with one instrument, normalizing those costs to zero allows us to simplify the exposition.

structures outlined above):

$$Q(e^k_m, e^l, \theta^k_m) = e^k_m \theta^k_m + \delta e^l \qquad \text{(Eq. 2.2)}$$

and

$$C(e^k_m, \theta^k_m) = \frac{(e^k_m)^2}{2\theta^k_m} \qquad \text{(Eq. 2.3)}$$

where $k, l \in \{A, B\}, k \neq l, m \in \{L, H\}$ and $\delta > 0$ is the extent to which partner l's effort impacts partner k's output.

With the above setup in mind, before deriving the optimal contracts under input and output monitoring, we first discuss the principal's optimization problem under full information (where both the partners' efforts and realized states are costlessly observed by the principal). This 'first-best' solution serves as a benchmark and here, the principal only needs to ensure that each partner receives its reservation utility; the corresponding individual rationality conditions are as follows:

$$\pi^k_{L|L} = e^k_L \theta^k_L + \delta E[e^l] - \frac{(e^k_L)^2}{2\theta^L_n} - E[T^k_L] \geq 0 \qquad (IR^k_L)$$

$$\pi^k_{H|H} = e^k_H \theta^k_H + \delta E[e^l] - \frac{(e^k_H)^2}{2\theta^k_H} - E[T^k_H] \geq 0 \qquad (IR^k_H)$$

where we use $E[e^l]$ to denote $\phi^l_H e^l_H + (1 - \phi^l_H) e^l_L$.

The properties of the first-best solution are summarized in the following lemma (all proofs are in the Appendix):

Lemma 2.1 *The optimal effort levels induced under full information are such that* $\forall\, k, l \in \{A, B\}, k \neq l$ *and* $m \in \{L, H\}$*:*

$$\frac{e^k_m}{\theta^k_m} - \delta = \theta^k_m \qquad \text{(Eq. 2.4)}$$

or equivalently:

$$e^k_m = \theta^k_m(\theta^k_m + \delta), \tag{Eq. 2.5}$$

and the optimal transfer payments are given by

$$E[T^k_m] = e^k_m\theta^k_m + \delta E[e^l] - \frac{(e^k_m)^2}{2\theta^k_m}. \tag{Eq. 2.6}$$

At the above solution, notice that $C_1 = \frac{\partial C}{\partial e^k_m} = \frac{e^k_m}{\theta^k_m}$ is the marginal cost of effort and $Q_1 = \frac{\partial Q}{\partial e^k_m} = \theta^k_m$ is its marginal revenue. Since the externality is positive ($\delta > 0$), we can see from equation (Eq. 2.4) that the principal optimally induces each partner to exert an effort level such that its marginal revenue is less than the marginal cost. Without the principal's intervention, however, the partners would prefer to exert an effort level such that its marginal cost equals the marginal revenue. Hence, under a positive (negative) externality, the principal induces the partners to exert more (less) effort than the partners would prefer to exert when left to themselves. Finally, notice that each partner is restricted to exactly its reservation profit.

2.4 Analysis and Results

In this section, we first develop the principal's optimal contract under input monitoring and output monitoring in that order. Subsequently, we compare the two monitoring regimes and characterize when the principal will prefer input monitoring over output monitoring. This comparison is first conducted when the partners' demand functions exhibit a positive externality. Later, we show how the optimal monitoring regime changes when the demand functions exhibit either no externality or a negative externality.

2.4.1 Input Monitoring

When the principal uses input monitoring, each partner can commit to a certain effort level while lying about the state. For example, a partner in the higher state could report being in the lower state, and thereby benefit from being required to commit to a lower effort level (corresponding to the lower state). In the Disney-McDonald's alliance, for instance, it is in McDonald's interest to lower the payments made to Disney. One way to effect this is to claim that the promotion may not be too effective—thereby reducing Disney's estimate of the boost in demand and subsequently on how much of the combined profit is transferred to Disney. The partner's profits under the various state-disclosure conditions are given below:

$$\pi^{k,I}_{n|m} = e^k_n \theta^k_m + \delta E[e^l] - \frac{(e^k_n)^2}{2\theta^k_m} - E[T^{k,I}_n] \qquad \text{(Eq. 2.7)}$$

where $k, l \in \{A, B\}, k \neq l$ and $\pi^{k,I}_{n|m}$ is partner k's profits when the partner is in state m and reports state n $(m, n \in \{H, L\})$. The principal's problem under input monitoring, [P-IM], is:

$$\begin{array}{lll} \underset{\{e,T\}}{\text{Max}} & \Pi^P & \text{(Eq. 2.8)} \\ \text{subject to} & \pi^{k,I}_{m|m} \geq 0 & (IR^{k,I}_m) \\ \text{and} & \pi^{k,I}_{m|m} \geq \pi^{k,I}_{n|m} \quad \forall \quad k \in \{A, B\}, m, n \in \{L, H\} & (IC^{k,I}_m) \end{array}$$

The properties of a solution to the above problem are summarized in the following Lemma.

Lemma 2.2 *At the solution to [P-IM], $\forall\, k, l \in \{A, B\}, k \neq l$ and $m \in \{L, H\}$, a partner in the lower state is restricted to its reservation utility, while a partner in the higher state accrues the following rents, denoted $R_H^{k,I}$:*

$$R_H^{k,I} = [e_L^{k,I}\theta_H^k - e_L^{k,I}\theta_L^k] + \left[\frac{(e_L^{k,I})^2}{2\theta_L^k} - \frac{(e_L^{k,I})^2}{2\theta_H^k}\right]$$

Further, the optimal effort levels satisfy these conditions:

$$\frac{e_L^{k,I}}{\theta_L^k} - \phi_L^k\delta + \phi_H^k\left(\theta_H^k - \frac{e_L^{k,I}}{\theta_H^k}\right) = \theta_L^k, \tag{Eq. 2.9}$$

$$\frac{e_H^{k,I}}{\theta_H^k} - \delta = \theta_H^k, \tag{Eq. 2.10}$$

or equivalently:

$$e_L^{k,I} = \frac{\theta_H^k\theta_L^k(\delta\phi_L^k + \theta_L^k - \phi_H^k\theta_H^k)}{\theta_H^k - \phi_H^k\theta_L^k} < \theta_L^k(\theta_L^k + \delta), \tag{Eq. 2.11}$$

$$e_H^{k,I} = \theta_H^k(\theta_H^k + \delta), \tag{Eq. 2.12}$$

and the optimal transfer payments are given by

$$E[T_L^{k,I}] = e_L^{k,I}\theta_L^k + \delta E[e^{l,I}] - \frac{(e_L^{k,I})^2}{2\theta_L^k}, \tag{Eq. 2.13}$$

$$E[T_H^{k,I}] = e_H^{k,I}\theta_H^k + \delta E[e^{l,I}] - \frac{(e_H^{k,I})^2}{2\theta_H^k} - [e_L^{k,I}\theta_H^k - e_L^{k,I}\theta_L^k] - \left[\frac{(e_L^{k,I})^2}{2\theta_L^k} - \frac{(e_L^{k,I})^2}{2\theta_H^k}\right]. \tag{Eq. 2.14}$$

As in the solution to the standard adverse selection problem, notice from the above lemma too that the principal induces the higher type partner to exert the first-best level of effort while inducing a partner in the lower state to exert lower than the first-best level. This result arises because, first the rents accruing to the partners reduce the principal's profit; further, these rents are positively affected by the effort exerted by the partner in the lower state (since $\frac{\partial R_H^k}{\partial e_L^{k,I}} > 0$). Consequently, the principal optimally lowers the effort exerted by the partner in the lower state. It is also worth pointing out (for later comparison) that the rents can be sorted into two components:

(i) rent due to extra output $[e_L^{k,I}\theta_H^k - e_L^{k,I}\theta_L^k]$ that is not shared with the principal when misrepresenting the state and

(ii) rent due to lower cost of effort $\left[\frac{(e_L^{k,I})^2}{2\theta_L^k} - \frac{(e_L^{k,I})^2}{2\theta_H^k}\right]$ when misrepresenting the state.

Finally, from the effort levels listed in Lemma (4.7) we can see that when the externality δ becomes larger, the principal would require the partners to exert higher levels of effort (i.e., $\frac{\partial e_L^{k,I}}{\partial \delta} > 0$ and $\frac{\partial e_H^{k,I}}{\partial \delta} > 0$). Further, when the effort levels under the low state increase, each of the rent components listed above also increase; hence, an increase in δ would lead to an increase in rents.

2.4.2 Output Monitoring

Here, the principal specifies and monitors the output produced by each partner. Recall that the output $Q(.)$ produced by partner k depends on the effort exerted by both partners as well as the state of θ^k. Consequently, given a level of effort exerted by the other partner, and truthful revelation of private information, we can compute partner k's required effort to produce any specified output level. In other words, the pair of contracts $(Q^{A,O}, T^{A,O})$ and $(Q^{B,O}, T^{B,O})$ can be expressed equivalently in terms of efforts as $(e^{A,O}, T^{A,O})$ and $(e^{B,O}, T^{B,O})$, where the $e^{k,O}$s are obtained by simultaneously inverting the demand functions of the two partners[8]. This equivalence facilitates comparing the two monitoring regimes (KL follow a similar approach) and in what follows, we will assume that the principal monitors the output and specifies the required (but unmonitored) input for the two partners.

Now, since partner k observes the realization of θ^k privately, there is an opportunity for that partner to misrepresent its private information. For example, when a partner is in the higher state but reports being in the lower state, it benefits from being required to produce a

[8]For more details on this equivalence, please see the Appendix.

lower output; the required output will be $Q(e^k_L, e^l, \theta^A_L) = e^k_L\theta^k_L + \delta e^l$ instead of $Q(e^k_H, e^l, \theta^A_H) = e^k_H\theta^k_H + \delta e^l$. Since $Q_3 > 0$, k selects an effort level, $\tilde{e}^k_H$, such that $Q(\tilde{e}^k_H, e^l, \theta^k_H) = Q(e^k_L, e^l, \theta^k_L)$, or equivalently, $\tilde{e}^k_H\theta^k_H + \delta e^l = e^k_L\theta^k_L + \delta e^l$, with $\tilde{e}^k_H = \frac{e^k_L\theta^k_L}{\theta^k_H} < e^k_L$. Hence, the partner's profits under the various state-disclosure conditions are:

$$\pi^{k,O}_{m|m} = Q(e^k_m, e^l, \theta^k_m) - C(e^k_m, \theta^k_m) - E[T^k_m] = e^k_m\theta^k_m + \delta E[e^l] - \frac{(e^k_m)^2}{2\theta^k_m} - E[T^k_m], \qquad \text{(Eq. 2.15)}$$

$$\pi^{k,O}_{n|m} = Q(\tilde{e}^k_m, e^l, \theta^k_m) - C(\tilde{e}^k_m, \theta^k_m) - E[T^k_n] = e^k_n\theta^k_n + \delta E[e^l] - \frac{(e^k_n\theta^k_n)^2}{2(\theta^k_m)^3} - E[T^k_n]. \qquad \text{(Eq. 2.16)}$$

The principal's problem under output monitoring, denoted [P-OM], can be expressed as follows:

$$\underset{\{Q,T\}}{\text{Max}} \quad \Pi^P \qquad \text{(Eq. 2.17)}$$

$$\text{subject to} \quad \pi^{k,O}_{m|m} \geq 0 \qquad (IR^{k,O}_m)$$

$$\text{and} \quad \pi^{k,O}_{m|m} \geq \pi^{k,O}_{n|m} \quad \forall \quad k \in \{A, B\}, m, n \in \{L, H\} \qquad (IC^{k,O}_m)$$

The properties of a solution to the above problem are summarized in the following Lemma.

Lemma 2.3 *At the solution to [P-OM], $\forall\, k, l \in \{A, B\}, k \neq l$ and $m \in \{L, H\}$, a partner in the lower state is restricted to its reservation utility, while a partner in the higher state accrues the following rents, denoted $R^{k,O}_H$:*

$$R^{k,O}_H = \left[\frac{(e^{k,O}_L)^2}{2\theta^k_H} - \frac{(e^{k,O}_L\theta^k_L)^2}{2(\theta^k_H)^3}\right] + \left[\frac{(e^{k,O}_L)^2}{2\theta^k_L} - \frac{(e^{k,O}_L)^2}{2\theta^k_H}\right]$$

Further, the optimal effort levels satisfy these conditions:

$$\frac{e_L^{k,O}}{\theta_L^k} - \phi_L^k \delta + \phi_H^k \left(\theta_L^k - \frac{e_L^{k,O} (\theta_L^k)^2}{(\theta_H^k)^3} \right) = \theta_L^k, \quad \text{(Eq. 2.18)}$$

$$\frac{e_H^{k,O}}{\theta_H^k} - \delta = \theta_H^k, \quad \text{(Eq. 2.19)}$$

or equivalently:

$$e_L^{k,O} = \frac{\phi_L^k (\theta_H^k)^3 \theta_L^k (\delta + \theta_L^k)}{(\theta_H^k)^3 - \phi_H^k (\theta_L^k)^3} < \theta_L^k (\theta_L^k + \delta), \quad \text{(Eq. 2.20)}$$

$$e_H^{k,O} = \theta_H^k (\theta_H^k + \delta), \quad \text{(Eq. 2.21)}$$

and the optimal transfer payments are given by

$$E[T_L^{k,O}] = e_L^{k,O} \theta_L^k + \delta E[e^{l,O}] - \frac{(e_L^{k,O})^2}{2\theta_L^k}, \quad \text{(Eq. 2.22)}$$

$$E[T_H^{k,O}] = e_H^{k,O} \theta_H^k + \delta E[e^{l,O}] - \frac{(e_H^{k,O})^2}{2\theta_H^k} - \left[\frac{(e_L^{k,O})^2}{2\theta_L^k} - \frac{(e_L^{k,O} \theta_L^k)^2}{2(\theta_H^k)^3} \right]. \quad \text{(Eq. 2.23)}$$

Analogous to the input monitoring case, here too the principal induces a partner in the higher state to exert the first-best effort level while inducing a partner in the lower state to exert lower than the first best effort level. Further, the rents accruing to the higher state partner can be sorted as follows:

(i) rent due to lower effort $\left[\frac{(e_L^{k,O})^2}{2\theta_H^k} - \frac{(e_L^{k,O} \theta_L^k)^2}{2(\theta_H^k)^3} \right]$ needed to generate the output when misrepresenting the state and

(ii) rent due to lower cost of effort $\left[\frac{(e_H^{k,O})^2}{2\theta_L^k} - \frac{(e_L^{k,O})^2}{2\theta_H^k} \right]$ when misrepresenting the state.

Again, we can see that as the externality parameter becomes larger, the principal would require the partners to exert higher levels of effort (i.e., $\frac{\partial e_L^{k,O}}{\partial \delta} > 0$ and $\frac{\partial e_H^{k,O}}{\partial \delta} > 0$). Finally, when the effort levels under the low state go up, each of the rent components listed above also go up; hence, an increase in δ would lead to an increase in rents.

2.4.3 Comparing Input and Output Monitoring

Begin by noting that the principal can induce the optimal effort levels of output monitoring (i.e., $e_m^{k,O}$ reported in Lemma 3) under the input monitoring regime with appropriate transfers. More specifically, under input monitoring, $\forall\, k, l \in \{A, B\}, k \neq l$ and $m \in \{L, H\}$, the following transfers will induce partner k to exert $e_m^{k,O}$:

$$E[T_L^{k,I}] = e_L^{k,O}\theta_L^k + \delta E[e^{l,O}] - \frac{(e_L^{k,O})^2}{2\theta_L^k}, \tag{Eq. 2.24}$$

$$E[T_H^{k,I}] = e_H^{k,O}\theta_H^k + \delta E[e^{l,O}] - \frac{(e_H^{k,O})^2}{2\theta_H^k} - [e_L^{k,O}\theta_H^k - e_L^{k,O}\theta_L^k] - \left[\frac{(e_L^{k,O})^2}{2\theta_L^k} - \frac{(e_L^{k,O})^2}{2\theta_H^k}\right]. \tag{Eq. 2.25}$$

Next recall the transfer payments for the output monitoring regime from Lemma 3:

$$E[T_L^{k,O}] = e_L^{k,O}\theta_L^k + \delta E[e^{l,O}] - \frac{(e_L^{k,O})^2}{2\theta_L^k}, \tag{Eq. 2.26}$$

$$E[T_H^{k,O}] = e_H^{k,O}\theta_H^k + \delta E[e^{l,O}] - \frac{(e_H^{k,O})^2}{2\theta_H^k} - \left[\frac{(e_L^{k,O})^2}{2\theta_L^k} - \frac{(e_L^{k,O}\theta_L^k)^2}{2(\theta_H^k)^3}\right]. \tag{Eq. 2.27}$$

With the transfers specified in equations (Eq. 2.24)-(Eq. 2.25) and in equations (Eq. 2.26)-(Eq. 2.27), identical effort levels are exerted by the partners under the two regimes. If input monitoring is more profitable than output monitoring when $e_m^{k,O}$ is induced, then input monitoring will be at least weakly better than output monitoring at the optimal effort levels under input monitoring (i.e., at $e_m^{k,I}$ from Lemma 2). Note that this approach generates a sufficient condition for input monitoring to be preferred. Comparing the principals' profits (using the transfer payments in (Eq. 2.24)-(Eq. 2.25) and (Eq. 2.26)-(Eq. 2.27)), we see that the transfer payments are identical under the low state condition and the difference in profits comes from the transfers under the high state.

$$\Pi^{P,I} - \Pi^{P,O} = \phi_H^A \left(\left[\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O})^2(\theta_L^A)^2}{2(\theta_H^A)^3} \right] - e_L^{A,O}(\theta_H^A - \theta_L^A) \right) + \\ \phi_H^B \left(\left[\frac{(e_L^{B,O})^2}{2\theta_H^B} - \frac{(e_L^{B,O})^2(\theta_L^B)^2}{2(\theta_H^B)^3} \right] - e_L^{B,O}(\theta_H^B - \theta_L^B) \right). \qquad \text{(Eq. 2.28)}$$

From equation (Eq. 2.28) we can see that input monitoring is more profitable than output monitoring if the following conditions hold simultaneously.

$$\left[\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O})^2(\theta_L^A)^2}{2(\theta_H^A)^3} \right] - e_L^{A,O}(\theta_H^A - \theta_L^A) > 0 \quad \Rightarrow \quad \delta > \frac{2\left[(\theta_H^A)^3 - (\theta_L^A)^3\right] - \phi_L^A(\theta_L^A)^2(\theta_H^A - \theta_L^A)}{\phi_L^A\theta_L^A(\theta_H^A + \theta_L^A)}, \qquad \text{(Eq. 2.29)}$$

$$\left[\frac{(e_L^{B,O})^2}{2\theta_H^B} - \frac{(e_L^{B,O})^2(\theta_L^B)^2}{2(\theta_H^B)^3} \right] - e_L^{B,O}(\theta_H^B - \theta_L^B) > 0 \quad \Rightarrow \quad \delta > \frac{2\left[(\theta_H^B)^3 - (\theta_L^B)^3\right] - \phi_L^B(\theta_L^B)^2(\theta_H^B - \theta_L^B)}{\phi_L^B\theta_L^B(\theta_H^B + \theta_L^B)}. \qquad \text{(Eq. 2.30)}$$

Define:

$$\delta^* = \max\left\{ \frac{2\left[(\theta_H^A)^3 - (\theta_L^A)^3\right] - \phi_L^A(\theta_L^A)^2(\theta_H^A - \theta_L^A)}{\phi_L^A\theta_L^A(\theta_H^A + \theta_L^A)}, \frac{2\left[(\theta_H^B)^3 - (\theta_L^B)^3\right] - \phi_L^B(\theta_L^B)^2(\theta_H^B - \theta_L^B)}{\phi_L^B\theta_L^B(\theta_H^B + \theta_L^B)} \right\}. \qquad \text{(Eq. 2.31)}$$

Proposition 2.1 *When the positive externality parameter is sufficiently large (formally, such that $\delta > \delta^*$), input monitoring is more profitable than output monitoring.*

From Lemma 2 we know that the rents accrued to the partner under input monitoring have two components : rents due to extra output, and rents due to lower cost of effort. Similarly, from Lemma 3, we know that the rents under output monitoring have two components: rents due to lower effort, and rents due to lower cost of effort. Now the rents from the lower cost of effort are identical (for a given level of effort across regimes). Consequently, we can see from (Eq. 2.29) and (Eq. 2.30) that the difference in the principal's profit arises from the difference in rents due to lower effort (which is the source of additional rents under output monitoring) and extra output (which is the source of additional rents under input monitoring). We also know that the principal induces the partners to exert higher effort as the magnitude of the demand externality parameter becomes larger. At higher levels of effort, the rents under output monitoring (arising out of cost savings due to lower effort) become larger at a faster rate than the rents under input monitoring. Therefore, when the demand externality is sufficiently strong (i.e., $\delta > \delta^*$), the rents under output monitoring dominate those under input monitoring, and the principal prefers the latter regime.

It is worth recording how the parameters, θ^k_m (denoting the productivity state of partner k) and ϕ^k_m (the probability of being in the lower state), $k = A, B,$ and $m \in \{L, H\}$, affect the critical externality parameter δ^*. It is easy to show that δ^* becomes smaller as (a) θ^k_L becomes larger; (b) θ^k_H becomes smaller; and (c) ϕ^k_L becomes larger. The intuition underlying these comparative statics results can be explained as follows.

Under output monitoring, when the productivity level in the lower state of nature, θ^k_L, becomes larger, the rents arising due to lower effort (denoted here by $\bar{R}^{kO} = \left[\frac{(e^{k,O}_L)^2}{2\theta^k_H} - \frac{(e^{k,O}_L)^2(\theta^k_L)^2}{2(\theta^k_H)^3}\right]$) are affected in two ways:

(i) First, an increase in θ^k_L would raise the partner's effort when lying about its state (recall: $\tilde{e}^k_H = \frac{e^k_L\theta^k_L}{\theta^k_H}$ and $\frac{\partial\tilde{e}^k_H}{\partial\theta^k_L} > 0$). Hence, the direct impact of an increase in θ^k_L is to lower $\bar{R}^{kO}$ (in other words, $\frac{\partial\bar{R}^{kO}}{\partial\theta^k_L} < 0$).

(ii) Further, at a higher θ^k_L, the partner will be induced to exert a higher level of effort in the lower state as well (since $\frac{\partial e_L}{\partial\theta^k_L} > 0$), and we know from Lemma (2.3) that $\frac{\partial\bar{R}^{kO}}{\partial e_L} > 0$.

Consequently, a raise in θ_L^k has the effect of indirectly raising $\bar{R}^{kO}$ (because, $\frac{\partial \bar{R}^{kO}}{\partial e_L}\frac{\partial e_L}{\partial \theta_L^k} > 0$).

Analogously, under input monitoring, the rents arising due to extra output (evaluated at the optimal output-monitoring effort levels and given by $\bar{R}^{kI} = e_L^{k,O}(\theta_H^k - \theta_L^k))$ are affected in two ways:

(i) First, an increase in θ_L^k would directly lower $\bar{R}^{kI}$ since $(\theta_H^k - \theta_L^k)$ goes down.

(ii) At the same time, however, the principal would require the partners to exert higher effort levels (since $\frac{\partial e_L}{\partial \theta_L^k} > 0$); and from Lemma (4.7) we know that $\frac{\partial \bar{R}^{kI}}{\partial e_L} > 0$. Consequently, raising θ_L^k indirectly raises $\bar{R}^{kI}$ (because, $\frac{\partial \bar{R}^{kI}}{\partial e_L}\frac{\partial e_L}{\partial \theta_L^k} > 0$).

Interestingly, the rates at which $\bar{R}^{kO}$ and $\bar{R}^{kI}$ change with e_L and θ_L^k follow this pattern: $\frac{\partial \bar{R}^{kO}}{\partial e_L} > \frac{\partial \bar{R}^{kI}}{\partial e_L}$ and $\left|\frac{\partial \bar{R}^{kO}}{\partial \theta_L^k}\right| < \left|\frac{\partial \bar{R}^{kI}}{\partial \theta_L^k}\right|$. The implication of these patterns is that the difference in rents, $\bar{R}^{kO} - \bar{R}^{kI}$, is increasing in θ_L^k. Since $\delta > \delta^*$ ensures that $\bar{R}^{kO} - \bar{R}^{kI} > 0$, it follows that a smaller δ^* is sufficient to achieve the same result when θ_L^k is larger. Hence, δ^* is decreasing in θ_L^k. Using an analogous set of arguments and comparisons, the impact of θ_H^k on δ^* can be explained (it is in a direction opposite to the effect of θ_L^k).

Consider what happens when the probability ϕ_L^k, that the partner k is in the lower state θ_L^k, becomes larger: first, the principal would induce the partners in the lower state to exert greater effort (since $\frac{\partial e_L}{\partial \phi_L^k} > 0$); this in turn would raise the rents under both input and output monitoring (because $\frac{\partial \bar{R}^{kO}}{\partial e_L}\frac{\partial e_L}{\partial \phi_L^k} > 0$ and $\frac{\partial \bar{R}^{kI}}{\partial e_L}\frac{\partial e_L}{\partial \phi_L^k} > 0$). Next, recall that a given increment in effort would lead to a greater increment in the rents under output monitoring than under input monitoring (since $\frac{\partial \bar{R}^{kO}}{\partial e_L} > \frac{\partial \bar{R}^{kI}}{\partial e_L} > 0$). Hence, the difference in rents, $\bar{R}^{kO} - \bar{R}^{kI}$, is increasing in ϕ_L^k. Consequently, a smaller δ^* is sufficient to ensure $\bar{R}^{kO} - \bar{R}^{kI} > 0$. Finally, in our setting, an increase in ϕ_L^k is identical to a decrease in ϕ_H^k (since $\phi_H^k = 1 - \phi_L^k$); therefore, an increase in ϕ_H^k would result in raising δ^*.

Translating the above to the multi partner co-marketing alliance (such as the alliance between the focal firm Disney and the partnering firms, McDonald's, Old Navy, Kellogg's, and others), we note the following: when $\delta > \delta^*$, marketing efforts (e.g., promotion and advertising investments) by one partner accrue significant benefits to the other partners as well. In such a setting, if Disney were to employ contracts contingent on the partner's sales, it will be detrimental to the health of the alliance because the partners will limit their efforts. Further, either if the states of productivity are not too different from one another, or if the chance of the lower productivity state is higher, then Disney should be that much more inclined to adopting contracts contingent on marketing actions.

2.4.4 Either Negative Externality or No Externality

So far, we considered the impact of a positive externality between the partners in a co-marketing alliance; here, we discuss the impact of either a negative externality or no externality(i.e., $\delta \leq 0$). Such settings can arise, for example, when the partner firms are in closely related product categories[9].

Proposition 2.2 *When there is either a negative externality or no externality among the partners, a contract that is contingent on marketing outcomes is more profitable than a contract that is contingent on marketing actions.*

[9] In the co-marketing alliance between Lucasfilm, Burger King, Kraft Foods, Dr Pepper, Expedia and others (that involved Lucasfim's movie "Indiana Jones and the Kingdom of the Crystal Skull"), some of the alliance partners are in closely related product categories—Burger King and Kraft Foods (Kraft Lunchables) cater to consumers (more specifically children) in the fast food category. Consumers often operate on limited budgets and limit the number of purchases in any given category—anecdotal evidence from the fast food industry suggests that families limit Happy Meal purchases to about 3 times in a month. Therefore, when two firms operating in related product categories invest in promoting the alliance, each firm's promotions can take the sales away from the other firm, and can counter the benefit arising from promoting the movie. If the negative impact of the partners' marketing efforts is sufficiently strong, it can result in a net (zero or) negative externality.

This result stems from the fact that when the effort exerted by the partners exhibits a (weakly) negative externality, the principal finds it attractive to induce the partners in the lower state to exert (weakly) lower marketing effort, thereby reducing the negative impact of each other's effort on their respective outputs. At these lower levels of marketing efforts, the marginal benefit from output dominates the marginal cost of effort irrespective of the magnitude of the externality (as long as the externality parameter, δ is weakly negative). Consequently, the rents to the partners resulting from extra output (under input monitoring) are greater than the rents to the partners resulting from lower effort (under output monitoring). Hence, from the principal's perspective, a contract contingent on marketing outcomes is preferred to a contract contingent on marketing actions. This result is consistent with the findings of Khalil and Lawarree (1995) who compare input and output monitoring in the context of a single principal contracting with a single agent (and therefore no externality).

2.4.5 Comparing the Effort Levels Under Input and Output Monitoring

In the previous sub-sections, we have shown that when the externality amongst the partners is positive and sufficiently strong, the principal in a co-marketing alliance would prefer a contract contingent on marketing actions over a contract contingent on marketing outcomes. Here, we focus on the conditions when a monitoring regime will result in higher levels of effort on behalf of the alliance. From the discussion in the previous sections, we know that the principal, under both input and output monitoring regimes, induces the partner in the lower state to exert lower effort compared to first best levels (in contrast to the partners in the higher state who always exert the first-best levels).

Defining

$$\tilde{\delta} = \frac{(\theta_H^k)^3 - (\theta_L^k)^3 - \phi_L^k \theta_H^k (\theta_L^k)^2}{\phi_L^k \theta_L^k (\theta_H^k - \theta_L^k)}, \tag{Eq. 2.32}$$

we show in the appendix that when $\delta > \tilde{\delta}$, the optimal effort levels under input monitoring dominate those under input monitoring. It is worth recalling that, as shown in equation (Eq. 2.28), the difference in the principal's profits under input and output monitoring depends on the difference in the rents due to lower effort and the rents due to extra output. In contrast, it is sufficient[10] for the marginal-rents-due-to-lower-effort to dominate the marginal-rents-due-to-extra-output for input monitoring to induce greater effort levels. Since the rents due to either the lower effort or the extra output are monotonic, a smaller positive externality, $\tilde{\delta}$ ($< \delta^*$), is sufficient for $e_L^{k,I} > e_L^{k,O}$ (than for $\Pi^{P,I} > \Pi^{P,O}$). In terms of the co-marketing alliance setting (such as the alliance between Sony and say the partners McDonald's and Old Navy), this result indicates that there are two levels of thresholds for the strength of the positive demand externality. When the lower of the two thresholds is exceeded, the partners can be expected to invest more heavily into the alliance under input monitoring; this however is not sufficient for Sony to prefer input monitoring; instead, other conditions (e.g., breaching the higher threshold) also need to be satisfied.

2.5 Conclusion

In forming co-marketing alliances, companies such as Sony often contract with multiple partners (e.g., McDonald's, Old Navy). In such alliances, there are often demand externalities, in the sense that the marketing investments of one partner (e.g., Old Navy) on behalf of the alliance (e.g., involving Sony's Surf's Up movie) are likely to impact the sales of the other partner (e.g., McDonald's who is selling its Happy Meals with Surf's Up toys). When these partners benefit from each other's investments, they all tend to under invest in promoting the alliance and thereby reduce the payoffs from the alliance. Clearly, the alliance's

[10]See the appendix for all proofs.

promoter (e.g., Sony or Disney) has an incentive to ensure that the alliance partners put forth appropriate levels of marketing effort. Accordingly, the central question we addressed in this research is: "What is the relative effectiveness of two types of contracts in ensuring that the partners invest appropriately in promoting the alliance?"

We developed an analytical model to compare two types of contracts—one that hinges on observing the partner's marketing actions (input monitoring) versus another that hinges on observing the marketing outcomes generated by the partners (output monitoring)—between a single principal (e.g., Disney) and multiple partners (e.g., McDonald's and Kellogg's). We find that the presence of a substantial positive externality among the partners in a co-marketing alliance is likely to tip the balance in favor of input monitoring. When the magnitude of the externality is not too strong (or for that matter negative), then output monitoring is likely to be preferred. This research suggests that firms such as Disney and Sony must be mindful of the magnitude of any externality among the (potential) partners in determining the terms of the contract offered to the partners.

Directions for further work include examining settings when the externality among the agents is asymmetric, i.e., when the actions of some partners present a positive externality while the actions of other agents do not; in such contexts, we anticipate that the required externality among the partners has to be even larger (than what we reported) to make a contract contingent on marketing actions to be more profitable. It will also be worthwhile to investigate the impact of correlated private information (e.g., see Demski and Sappington 1984) among the partners.

CHAPTER 3: COOPERATIVE ADVERTISING WITH SYMMETRIC RETAILERS

3.1 Introduction

Cooperative advertising is an common arrangement between members of a distribution channel whereby upstream channel members (e.g., manufacturers) reimburse a portion of the advertising costs of downstream members (e.g., retailers). Manufacturers often offer such cooperative advertising contracts as an incentive for the downstream retailers to increase their local advertising levels. Cooperative advertising is widely used by manufacturers and by one estimate, about $30 billion was used in 1998 for cooperative advertising (Davis 1999).

Cooperative advertising contracts specify the percentage (called the participation rate) of advertising costs that are reimbursable and also specify limits (called the accrual rate) on the total reimbursement offered. The accrual rate specified by the manufacturers can either be a fraction of the purchases made by the retailer (wholesale receipts) or a fixed dollar amount. Dutta et al (1995) report that participation rates vary between 25 and 100 percent, with 50 and 100 percent being commonly used. With regards to the accrual rate linked to wholesale receipts, Dutta et al find that participation rates vary between 0.003 to 33 percent of wholesale receipts. While Dutta et al are silent on the fixed accrual rates, there are examples[1] of manufacturers using a fixed dollar amount as an accrual rate as well.

When multiple retailers sell products from a common manufacturer, each retailer's advertising levels affect the demand faced by the other retailers. When each retailer selling the same end product advertises the product, other retailers can also benefit from this effort.

[1]For example, Aquarium Pharmaceuticals Inc. offers a cooperative advertising program to pet stores selling its products. As part of this program, Aquarium Pharmaceuticals reimburses 50% of the cost of advertisements that include its brands and this reimbursement limited to a maximum of $75 per retailer.

This positive externality causes each retailer to free-ride on other retailer's effort and leads to an overall reduction in the levels of advertising. The reduction in the overall levels of advertising decreases the total demand for the manufacturer's product and in turn hurts the manufacturer's profits. In order to combat the free-riding of the retailers, manufacturers can offer cooperative advertising contracts that offset some of the retailer's advertising expenses and thereby provide an incentive to increase their overall levels of advertising. In this paper, we investigate the use of the cooperative advertising participation rate and the two types of accrual rates in their effectiveness to provide the retailers with appropriate incentives to increase their levels of advertising.

While previous research on cooperative advertising focused on the use of a single participation rate (Bergen and John 1997, Huang et al 2002, He et al 2007), to our knowledge, our work is the first attempt to understand the impact of the two types of accrual rates on retailer's advertising levels. As Dutta et al (1995) report, many firms use a combination of participation and accrual rates in their cooperative advertising contracts. Given the prevalence of the use of these two types of contracts, it is important to investigate the impact of these contractual variables. Hence this work contributes to the literature by providing an understanding of the various types of contractual mechanisms available to manufacturers and provides guidelines for the use of these contracts.

In this research, we consider a manufacturer selling its products through two symmetric independent retailers. The retailers face positive advertising externalities and in the absence of any cooperative advertising contract, under invest in advertising. We then show that the manufacturer can use a cooperative advertising contract that only includes a single participation rate offered to both the retailers to increase the overall levels of advertising. The retailers, left to themselves, only consider their own benefit from the advertising investments they make (while ignoring the overall benefits to the channel). This leads to the retailers investing in lower levels of advertising than what is preferred by the manufacturer. When the manufacturer offers the retailers a cooperative advertising contract that reimburses part of their advertising expenses (participation rate contract), the retailer's marginal cost of advertising is lowered and they find it attractive to increase their investments in advertising. The manufacturer can choose the participation rate such that the retailers increase their advertisements levels to that preferred by the manufacturer. Hence, the manufacturer uses the participation rate to align the interests of the retailer with the interests of the channel. We derive the optimal participation rate required to align the retailers' interest with that of the channel for a very general demand structure, without making specific assumptions regarding the functional form of the demand faced by the retailers. Hence, this provides a useful analytical tool to determine the participation rates in a variety of settings.

Next, we show that the manufacturer can also achieve coordination in the channel by using a combination of the participation rate and either of the two types of accrual rates. When the manufacturer uses an accrual rate in addition to the participation rate, the manufacturer can set the participation rates at higher levels than that required to coordinate the channel under the participation rate contract. This provides the retailers with an incentive to increase their advertising levels by a greater amount than under a single participation rate contract. The manufacturer then limits the total reimbursement to the retailers by using an accrual rate that is either a fixed amount or a fraction of the wholesale receipts from the retailer. While the retailers have an incentive to increase advertising to levels beyond what is sufficient to coordinate the channel, the manufacturer uses the cap on reimbursement to limit the retailer's advertising to the channel coordinating levels. Hence, the manufacturer can use both the participation rate and the accrual rate to achieve the same result achieved through the use of a participation rate alone.

While the two types of accrual rates can help coordinate the advertising levels in the distribution channel, each type of accrual rate has a unique impact on retail prices and advertising levels. The accrual rate that is set as a fixed amount has a positive impact on retail prices. As the manufacturer increases the fixed cap, retailers tend to increase prices. Since the increases in the fixed cap induces the retailers to increase their advertising levels, the increase in the advertising levels in turn leads the retailers to increase retail prices. While this indirect impact of the increase in advertising levels on retail prices also exits when the manufacturer uses an accrual rate linked to wholesale receipts, the increase in the accrual rate also has a negative direct impact on retail prices.

When the manufacturer uses an accrual rate linked to wholesale receipts, unlike the fixed accrual rate scenario, the retailers can influence the cap on reimbursements through their choices of price and effort levels. As retailers decrease prices (or increase effort), retail demand goes up and in turn the wholesale receipts go up. This leads to an increase in the cap on cooperative advertising reimbursements. Hence, as the manufacturer increases the accrual rate linked to wholesale receipts, the retailers may find it attractive to decrease retail prices and thereby further increase the cap on reimbursements. As the accrual rate increases, the increase in the cap for a unit decrease in retail prices is higher (compared to when the accrual rate is lower). Hence, the net effect of an increase in accrual rate linked to wholesale receipts may lead to a reduction in retail prices as opposed to an increase in retail prices (that occurs under the fixed accrual rate contract). In what follows, these and other such unique effects of each type of accrual rate are discussed in detail.

The rest of the chapter is organized as follows: the next section reviews the relevant literature and section 3 develops the model. Section 4 presents our analysis and results while the final section concludes the paper. All proofs are confined to an appendix.

3.2 Literature Review

The literature on cooperative advertising is part of the broader stream of literature on vertical control in distribution channels. In a distribution channel, the downstream firms (e.g., retailers) do not always make decisions (prices, quantities and efforts) that are favorable to the upstream players (e.g., manufacturers). Hence, upstream channel members tend to use various contractual provisions like nonlinear wholesale prices, territorial restrictions, quantity limits and retail price restrictions to induce the downstream members to make decisions favorable to the upstream members. The various vertical controls used by upstream channel members can be broadly classified (Ray and Tirole 1986, Ray and Verge 2005) into two categories : 1) payment schemes and 2) provisions limiting the parties' rights. Contractual provisions in the payment schemes category include non-linear tariffs, royalty payments, cooperative advertising, among others. Contractual provisions that limit some of the downstream members rights include Resale price maintenance (RPM), quantity fixing and exclusive territories.

The need to exert control on downstream channel members may arise due to presence of three sources[2] of externalities in the distribution channel. The first externality arises due to standard double marginalization problem. In a typical distribution channel, each channel member's selling price includes a mark up over their costs. The mark up added by downstream channel members makes final prices to consumers to be higher than what upstream channel members prefer. This coordination problem is termed as the "double marginalization" problem and was formally analyzed by Spengler (1950). Since each channel member does not take into account the impact of their mark up on the other channel member's profits, the mark up by downstream channel members results in an externality to the upstream

[2]see Mathewson and Winter (1984) for a detailed discussion of these externalities.

channel members. The second externality can arise due to intrabrand competition. When multiple downstream members (retailers) sell the same product to end consumers, the price charged by each retailer presents a positive externality to the other retailers. Hence, an increase in price by one retailer increases the demand for the other retailers selling the same product. This horizontal price externality tends to push retail prices to levels that are lower than what upstream channel members (manufacturers) prefer. The third source of externality can arise due to advertising/effort spillovers amongst downstream channel members. When each retailer selling the same end produce exerts sale effort or advertise the product, the other retailers can benefit from this effort. This positive effort/advertising externality causes each retailer to free-ride on the other retailers effort and leads to an overall reduction in advertising/sales effort.

The use of two-part tariffs — that involve charging a fixed fee and a constant per unit wholesale price — and quantity discounts that provide increasing rebates on the quantity purchased have been shown to reduce or eliminate the double marginalization problem (Jeuland and Shugan 1983, Zusman and Etgar 1981, Ingene and Parry 1995 and others). The use of resale price maintenance provisions (RPM) that limit the maximum retail price or the minimum retail price charged by downstream retailers is also shown (Ray and Tirole 1986) to be useful in combating the double marginalization problem when the downstream demand is uncertain. Limiting retail prices restricts downstream members from changing the retail price due to a demand shock (change in demand). Ray and Tirole (1986) show that the use of two-part tariff is preferred to RPM under cost uncertainty while RPM is preferred to two-part tariff under demand uncertainty.

When manufacturers face horizontal price externalities amongst retailers as well as the externality due to double marginalization, Mathewson and Winter (1984) suggest that manufacturers can use either a two-part tariff with exclusive territory clause or a two-part tariff with a price floor restriction (a form of RPM) to mitigate the effects of the two externalities. Under exclusive territory clauses, each retailer is assigned the exclusive rights to all consumers within a territory. The use of exclusive territories removes the retailer's incentive to cut prices and the two-part tariff can resolve the double marginalization issue. On the other hand, the manufacturer can use a price floor contract that restricts the retailers from lowering prices below a level set by the manufacturer to combat the horizontal price externality and then use a two-part tariff to eliminate the double marginalization problem.

Ray and Stiglitz (1995) show that exclusive territory clauses also affect interbrand competition amongst competing producers (or manufacturers). The authors show that the use of exclusive territories raises equilibrium prices and profits for each producer. While the use of a two-part tariff combined with exclusive territory clause can coordinate the channel, the extent to which the two-part tariff, specifically the fixed fee, can extract the surplus from downstream retailers can sometimes be legally limited due to limited liability clauses. In this scenario, Desiraju (2004) shows that upstream channel members can sometimes prefer to induce intrabrand competition by the use of nonexclusive territories rather than try to coordinate the channel by reducing intrabrand competition by imposing exclusive territories. The author shows that when the downstream member's surplus cannot be fully extracted, the use of nonexclusive territories may be preferred over exclusive territories.

In scenarios where manufacturers face effort free riding by downstream retailers (along with double marginalization and horizontal price externalities), Mathewson and Winter (1984) show that the manufacturer can coordinate the channel by using either a two-part tariff and a retail price floor (RPM) or by using a retail price floor along with a restriction on the quantity purchased by the retailers (quantity fixing). While the use of a RPM and exclusive territory contracts seems to be useful in achieving coordination, several researchers (Mathewson and Winter 1983,Dutta et al 1999) have pointed out conditions under which these restrictions have a positive impact on social welfare and others (Ray and Tirole 1986,Mathewson and Winter 1983,Ray and Stiglitz 1988) have pointed out conditions under which the restrictions have a negative impact on social welfare. In terms of the U.S. competition policy towards vertical restraints, resale price restraints are illegal per se and territorial restrictions are evaluated on a case by case basis[3]. Two-part tariff and quantity discounts are deemed legal as long as all downstream channel members are treated equally.

When upstream manufacturers cannot include territorial restricts due to legal restrictions (or due to prohibitive monitoring costs), manufacturer's can use cooperative advertising (coop) contracts that offset downstream retailer's advertising expenses to combat the free-riding externality. Under the cooperative advertising contract, manufacturers offer to reimburse a portion of the retailers advertising expenses. The fraction of advertising cost being reimbursed is referred to as the participation rate. Berger (1972) was the first to analyze the benefits of cooperative advertising and provided a quantitative methodology to determine the parameters of a cooperative advertising contract. In his model, Berger models the cooperative advertising contract as a wholesale price discount. Dutta et al (1995) conduct an empirical investigation into the variations in the contractual terms of cooperative advertising contracts. They find that manufacturers use a participation rates for cooperative advertising contracts vary between 25 and 100 percent of the retailer's costs, with 50 and 100 percent being most commonly used. The authors note that in addition to the participation

[3]See Ray and Verge (2005) and Dutta et al (1999) for a discussion of the legal issues relating to vertical restraints.

rate, manufacturers often specify an accrual rate — a limit on the total reimbursement that the manufacturer is willing to offer to the retailers. The authors note that these accrual rates are part of almost all cooperative advertising plans and that the accrual rate is set as a percentage of purchases made by the retailers (wholesale receipts). The authors find that both the accrual and participation rates are higher for consumer products versus industrial products and within consumer products, the rates are higher for convenience products compared to non convenience products.

Bergen and John (1997) study the impact of advertising spillover, retailer differentiation and manufacturer differentiation on the participation rates in cooperative advertising contracts. The authors show that manufacturers can use a two-part tariff and a coop participation rate to achieve vertical integration profits. The authors consider symmetric retailers that are vertically differentiated and do not consider the impact of the accrual rate on retail behavior. Huang et al (2002) investigate the use of cooperative advertising in a manufacturer-retailer supply chain by considering both national brand name investments by the manufacturer and local advertising by the retailer. In this context, the authors analyze both manufacturer-as-leader and partnerships advertising structures. Huang and Li (2001) investigate a cooperative advertising model in the context of a non cooperative simultaneous move game between the manufacturer and the retailer (in addition to the game structures considered in Huang et al 2002). Karray and Zaccour (2007) investigate the role of cooperative advertising contracts when both upstream and downstream competition exists. In this context, the authors find that cooperative advertising contracts increase retail advertissment levels. In addition, the authors show that cooperative support rates offered by manufacturers to retailers increase as brand competition intensifies and store competition decreases.

In a dynamic setting, Jorgensen et al (2001) show that the use of a cooperative advertising contract can coordinate the channel. The authors assume that advertising has both short and long terms effects on retail sales and the manufacturer in addition to subsidizing the retailers local advertising, also invests in national advertising. He et al (2007) investigate cooperative advertising strategies in a stochastic Stackelberg differential game setting. The authors provide in feedback form, the optimal advertising and pricing policies for the manufacturer and the retailer.

While the two streams of literature (static and dynamic) on cooperative advertising investigate several important issues, none of the articles studies the impact of the two types of accrual rates. In this sense, our work contributes to the literature on cooperative advertising and in turn to the literature on vertical control by systematically investigating, in a general setting, the role of accrual rates in cooperative advertising contracts.

3.3 The Model

We first describe the model that incorporates no cooperative advertising strategies and later discuss how the basic model changes with the use of cooperative advertising contracts. We consider a single manufacturer M that sells its products through two retailers R^1 and R^2. The demand faced by retailers R^1 and R^2 is denoted by $Q^1 = Q^1(p_1, p_2, e_1, e_2)$ and $Q^2 = Q^2(p_2, p_1, e_2, e_1)$ respectively. The demand functions have the following properties

$$\begin{gathered} \tfrac{\partial Q^i}{\partial p_i} < 0, \tfrac{\partial Q^i}{\partial p_j} > 0, \left|\tfrac{\partial Q^i}{\partial p_i}\right| > \left|\tfrac{\partial Q^i}{\partial p_j}\right|, \\ \tfrac{\partial Q^i}{\partial e_i} > 0, \tfrac{\partial Q^i}{\partial e_j} > 0 \text{ and } \tfrac{\partial Q^i}{\partial e_i} > \tfrac{\partial Q^i}{\partial e_j} \ \forall i,j \in \{1,2\}, i \neq j \end{gathered} \qquad \text{(Eq. 3.1)}$$

Where p_i and e_i are the price charged and effort (advertising) expended by retailer R^i. The cost of effort to retailer R^i is given by G^i and is assumed to be increasing and convex in the effort ($\frac{\partial G^i}{\partial e_i} > 0$ and $\frac{\partial^2 G^i}{\partial e_i^2} > 0$). Note that here we assume that the manufacturer sells to two identical(symmetric) retailers. Hence, we must have

$$\begin{gathered} \frac{\partial Q^i}{\partial p_i} = \frac{\partial Q^j}{\partial p_j}, \frac{\partial Q^i}{\partial p_j} = \frac{\partial Q^j}{\partial p_i}, \frac{\partial Q^i}{\partial e_i} = \frac{\partial Q^j}{\partial e_j}, \frac{\partial Q^i}{\partial e_j} = \frac{\partial Q^j}{\partial e_i}, \\ \frac{\partial G^i}{\partial e_i} = \frac{\partial G^j}{\partial e_j} \text{ and } \frac{\partial^2 G^i}{\partial e_i^2} = \frac{\partial^2 G^j}{\partial e_j^2} \ \forall i,j \in \{1,2\}, i \neq j \end{gathered} \qquad \text{(Eq. 3.2)}$$

The manufacturer sells the products to retailer R^i at a wholesale price w and also charges a fixed fee F. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - G^1 - F, \qquad \text{(Eq. 3.3)}$$

$$\pi^2 = Q^2(p_2 - wt) - G^2 - F. \qquad \text{(Eq. 3.4)}$$

The manufacturer's profits can be expressed as

$$\Pi = Q^1(w-c) + Q^2(w-c) + 2F. \qquad \text{(Eq. 3.5)}$$

where c is the per unit cost incurred by the manufacturer. The manufacturer acts as a Stackelberg leader and sets the wholesale price and fixed fees for the product before the retailers set retail prices and effort levels. Given a wholesale price set by the manufacturer, the retailers simultaneously and independently choose retail price and effort levels in order to maximize their profits. The manufacturer anticipates the retailers' actions and choose the wholesale prices and fixed fees in order to maximize his profits, taking into consideration the retailers actions.

3.4 Analysis and Results

We begin by understand the manufacturer's choices under a vertically integrated channel (where the manufacturer makes the pricing and effort decisions). We denote this setting as the first best setting. The retail profits under vertical integration are given by

$$\pi^1 = Q^1(p_1 - c) - G^1, \qquad \text{(Eq. 3.6)}$$

$$\pi^2 = Q^2(p_2 - c) - G^2. \qquad \text{(Eq. 3.7)}$$

Since the manufacturer owns both the retail channels, the manufacturer's profits can be expressed as

$$\Pi = \pi^1 + \pi^2 = Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2. \qquad \text{(Eq. 3.8)}$$

The manufacturer's optimization problem (denoted by [M-FB]) can be expressed as

$$\underset{\{p_1, p_2, e_1, e_2\}}{\text{Max}} \quad \Pi = Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2 \qquad \text{(Eq. 3.9)}$$

The properties of the solution[4] to [M-FB] are summarized in the following Lemma.

Lemma 3.4 *The retail price ($p_1 = p_2 = p^*$) and the effort ($e_1 = e_2 = e^*$) that solve [M-FB] simultaneously satisfy the following equations*

$$\frac{\partial \Pi}{\partial p} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = 0, \qquad \text{(Eq. 3.10)}$$

$$\frac{\partial \Pi}{\partial e} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = 0. \qquad \text{(Eq. 3.11)}$$

[4] All proofs are relegated to the appendix

From equations (Eq. 3.10) and (Eq. 3.11), we can see that the manufacturer chooses price and effort such that the difference between the total marginal benefit(to the entire channel) from price/effort and the total marginal cost(to the entire channel) of price/effort, denoted by $DMRMC_i^{fb} \quad \forall i \in \{p, e\}$ is equal to zero ($DMRMC_p^{fb} = 0$ and $DMRMC_e^{fb} = 0$). Next, we consider the independent retailer case where the retailers make the price and effort choices while the manufacturer decides the wholesale price and fixed fees.

3.4.1 Independent Retailers with No Cooperative Advertising

In this case, while the manufacturer charges a wholesale price and a fixed fee, there is no cooperative advertising contract offered to the retailers. We denote this case as the 'second best' case. The retail profits under this setting are given by equations (Eq. 3.3)-(Eq. 3.4) and the manufacturer's profits are given by (Eq. 3.5). The manufacturer chooses the wholesale prices and fixed fees while anticipating the retailer's responses to these choices. Also, the manufacturer ensures that the retailers at least make their reservation profits (assumed to be zero). Hence, the manufacturer's optimization problem (denoted by [M-SB]) can be stated as

$$\underset{\{w, F, p_1, p_2, e_1, e_2\}}{\text{Max}} \quad \Pi \tag{Eq. 3.12}$$

$$\text{subject to} \quad \pi^1 \geq 0, \pi^2 \geq 0, \frac{\partial \pi^1}{\partial p_1} = 0, \frac{\partial \pi^1}{\partial e_1} = 0, \frac{\partial \pi^2}{\partial p_2} = 0 \text{ and } \frac{\partial \pi^2}{\partial e_2} = 0$$

The properties of the solution to [M-SB] are summarized in the following Lemma.

Lemma 3.5 *The retail price and the effort that solve [M-SB] simultaneously satisfy the following equations*

$$\frac{\partial Q^1}{\partial p_1}(p_1-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c)+Q^1=\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c), \quad \text{(Eq. 3.13)}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c)-\frac{\partial G^1}{\partial e_1}=\frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c), \quad \text{(Eq. 3.14)}$$

where $p_1 = p_2 = \hat{p}$ is the price charged and $e_1 = e_2 = \hat{e}$ is the effort expended by the two symmetric and independent retail outlets. Also, the price and effort levels under the second best solution are lower than the price and effort levels under the first best setting ($\hat{p} < p^*$ and $\hat{e} < e^*$) and hence the profits under the second best setting are lower than the first best profits ($\hat{\Pi} < \Pi^*$).

From lemma 2, we can see that the retailer's choice of effort is such that the difference between the marginal benefit to the channel and the marginal cost to the channel is positive ($DMRMC_e^{sb} = \frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_1-c) > 0 \quad \forall w > c$). Also, the retailer's choice of price is such that the difference between the marginal benefit and marginal cost is given by $DMRMC_p^{sb} = \frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_1-c)$. Note that when $w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p_1-c)}{\frac{\partial Q^1}{\partial p_1}} > c$, $DMRMC_p^{sb} = 0$ but since $w > c$, we have $DMRMC_e^{sb} > 0$. Hence, $DMRMC_e^{sb}$ and $DMRMC_p^{sb}$ cannot be zero at the same time. Hence, the first best cannot be achieved. Also, as the manufacturer increases the wholesale price, the price charged by the retailer increases[5] ($\frac{\partial p_1}{\partial w} > 0$) and the effort expended by the retailer decreases ($\frac{\partial e_1}{\partial w} < 0$). Since the manufacturer has only one instrument (the wholesale price) to control both the price and effort charged by the retailers, and since the wholesale price has an opposite effect on price and effort, first best profits cannot be achieved. Comparing with the first best price and effort levels, we can show that in equilibrium, the retail prices and effort are such that

[5] See appendix for the proof.

$\hat{p} < p^*$ and $\hat{e} < e^*$. Since the wholesale price has an opposite effect on the price and effort, inducing $\hat{p} < p^*$ and $\hat{e} > e^*$ or $\hat{p} > p^*$ and $\hat{e} < e^*$ is not a potential equilibrium — increasing or decreasing the wholesale price can move the second best solution closer to the first best solution. Also, inducing $\hat{p} > p^*$ and $\hat{e} > e^*$ would require a very low wholesale price ($w < c$) that would also induce $p_1 < c$ and thereby leading to negative profits.

3.4.2 Cooperative Advertising Contract That Specifies a Participation Rate With No Accrual Rate Specified

When the manufacturer uses a cooperative advertising contract that only specifies a participation rate, both retailers are reimbursed a fraction α of their advertising expenses. Hence, the retail profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - G^1 - F + \alpha G^1, \tag{Eq. 3.15}$$

$$\pi^2 = Q^2(p_2 - w) - G^2 - F + \alpha G^2. \tag{Eq. 3.16}$$

Correspondingly, the manufacturer's profits can be expressed as

$$\Pi = Q^1(w - c) + Q^2(w - c) + 2F - \alpha G^1 - \alpha G^2. \tag{Eq. 3.17}$$

In this setting, the manufacturer declares the participation rate α along with the wholesale price and fixed fees. Given this contract, the retailers simultaneously and independently choose price and effort levels. Anticipating the retailer's behavior, the manufacturer chooses α, w and F to maximize his profits. Also, the manufacturer ensures that the retailers at least make their reservation profits (assumed to be zero). Hence, the manufacturer's optimization problem (denoted by [M-α]) can be stated as

$$\max_{\{w, F, p_1, p_2, e_1, e_2, \alpha\}} \quad \Pi \tag{Eq. 3.18}$$

subject to $\quad \pi^1 \geq 0, \pi^2 \geq 0, \dfrac{\partial \pi^1}{\partial p_1} = 0, \dfrac{\partial \pi^1}{\partial e_1} = 0, \dfrac{\partial \pi^2}{\partial p_2} = 0$ and $\dfrac{\partial \pi^2}{\partial e_2} = 0$

The properties of the solution to [M-α] are summarized in the following Proposition.

Proposition 3.3 *The manufacturer can achieve the first best solution by choosing the participation rate (α) and the wholesale price w such that*

$$\alpha = \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}} \quad \textit{and} \tag{Eq. 3.19}$$

$$w = c + \frac{\alpha\frac{\partial G^1}{\partial e_1} - \frac{\partial Q^2}{\partial e_1}(p^* - c)}{\frac{\partial Q^1}{\partial e_1}}. \tag{Eq. 3.20}$$

The retail price ($p_1 = p_2 = p^\alpha = p^*$) and the effort ($e_1 = e_2 = e^\alpha = e^*$) that solve [M-$\alpha$] simultaneously satisfy the following equations

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) = 0, \tag{Eq. 3.21}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} = 0. \tag{Eq. 3.22}$$

From Proposition 1, we can see that the retailer's choice of effort is such that the difference between the marginal benefit to the channel and the marginal cost to the channel is given by

$$DMRMC_e^\alpha = \left(\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1}. \qquad \text{(Eq. 3.23)}$$

Also, the retailer's choice of price is such that the difference between the marginal benefit and marginal cost is given by

$$DMRMC_p^\alpha = \frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c). \qquad \text{(Eq. 3.24)}$$

The manufacturer can therefore choose a wholesale price $w^\alpha = c + \frac{\alpha\frac{\partial G^1}{\partial e_1} - \frac{\partial Q^2}{\partial e_1}(p_1 - c)}{\frac{\partial Q^1}{\partial e_1}}$ that would result in $DMRMC_p^\alpha = DMRMC_p^{fb} = 0$. Similarly, the manufacturer can choose a participation rate $\alpha^* = \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}$ that will result in $DMRMC_e^\alpha = DMRMC_e^{fb} = 0$. We can show that as the manufacturer increases the wholesale price, the retailers increase the retail price ($\frac{\partial p_1}{\partial w} > 0$) and decrease the retail effort ($\frac{\partial e_1}{\partial w} < 0$). On the other hand, as the manufacturer increases the participation rate α, the retailers increase both the retail price and effort ($\frac{\partial p_1}{\partial \alpha} > 0$ and $\frac{\partial e_1}{\partial \alpha} > 0$). Hence, the manufacturer can choose an appropriate wholesale price w^α and participation rate α^* that leads to the retailers choosing retail prices and efforts that induce $DMRMC_p^\alpha = DMRMC_p^{fb} = 0$ and $DMRMC_e^\alpha = DMRMC_e^{fb} = 0$. Therefore, the use of a cooperative advertising contract that specifies a participation rate α can induce the first best solution if the retailers are symmetric.

Recall that in the second best solution [M-SB], the manufacturer was unable to induce the retailers to expend first best effort and price levels. This resulted from the fact that the retailer's marginal benefit from effort was less than the marginal benefit of effort to the total channel. Hence, the retailer preferred to exert lower effort compared to the first best. In contrast, when the manufacturer offers a cooperative advertising contract that only specifies a participation rate, the manufacturer reduces the retailers marginal cost of advertising (compared to second best) and thereby induces the retailers to exert greater effort. The participation rate is set such that the costs of the retailers are reimbursed sufficiently enough to increase their effort to first best levels. Since the manufacturer can coordinate the retail efforts using the participation rate, the retail prices are coordinated by choosing the appropriate wholesale price.

3.4.3 Cooperative Advertising Contract That Includes a Participation Rate as well as Variable Accruals

Under this contract, both retailers are reimbursed a fraction α of their advertising expenses and this reimbursement is capped at a fraction (δ) of the wholesale receipts from the retailer ($\delta w Q^i$). Given that the cost of advertising for the retailers is G^i, depending on the effort levels exerted by the retailers, we can have $\alpha G^i \leq \delta w Q^i$ or $\alpha G^i > \delta w Q^i$. If retailers expend effort such that $\alpha G^i \leq \delta w Q^i$, then the retailers are reimbursed αG^i by the manufacturer. On the other hand, if retailers expend effort such that $\alpha G^i > \delta w Q^i$, then the total reimbursement is only $\delta w Q^i$.

Therefore, the retail profits can be expressed as

$$\pi^1 = \left\{ \begin{array}{ccc} Q^1(p_1 - w) - G^1 + \alpha G^1 - F & \text{if} & \alpha G^1 \leq \delta w Q^1 \\ Q^1(p_1 - w) - G^1 + \delta w Q^1 - F & \text{if} & \alpha G^1 > \delta w Q^1 \end{array} \right\} \qquad \text{(Eq. 3.25)}$$

$$\pi^2 = \left\{ \begin{array}{ccc} Q^2(p_2 - w) - G^2 + \alpha G^2 - F & \text{if} & \alpha G^2 \leq \delta w Q^2 \\ Q^2(p_2 - w) - G^2 + \delta w Q^2 - F & \text{if} & \alpha G^2 > \delta w Q^2 \end{array} \right\} \qquad \text{(Eq. 3.26)}$$

Correspondingly, the manufacturer's profits can be expressed as

$$\Pi = \left\{ \begin{array}{cc} Q^1(w-c) + Q^2(w-c) + 2F - \alpha G^1 - \alpha G^2 \quad \text{if} \quad \alpha G^i \leq \delta w Q^i \\ Q^1(w-c) + Q^2(w-c) + 2F - \delta w Q^1 - \delta w Q^2 \quad \text{if} \quad \alpha G^i > \delta w Q^i \end{array} \right\} \qquad \text{(Eq. 3.27)}$$

In this setting, the manufacturer declares the participation rate α, the accrual rate δ, the wholesale price w and the fixed fees F. Given this contract, the retailers simultaneously and independently choose price and effort levels. Anticipating the retailer's behavior, the manufacturer chooses α, δ, w and F to maximize his profits. Hence, the manufacturer's optimization problem $[PM - \delta]$ can be expressed as

$$\underset{\{w, F, p_1, p_2, e_1, e_2, \alpha, \delta\}}{\text{Max}} \quad \Pi \qquad \text{(Eq. 3.28)}$$

subject to $\pi^1 \geq 0$, $\pi^2 \geq 0$, $\dfrac{\partial \pi^1}{\partial p_1} = 0$, $\dfrac{\partial \pi^1}{\partial e_1} = 0$, $\dfrac{\partial \pi^2}{\partial p_2} = 0$ and $\dfrac{\partial \pi^2}{\partial e_2} = 0$

The properties of the solution to $[PM - \delta]$ are summarized in the following proposition

Proposition 3.4 *When the manufacturer offers a cooperative advertising contract that includes an accrual rate δ and a participation rate α, the manufacturer can achieve the first best solution by choosing w, α and δ such that either of the following sets of conditions are satisfied*

$$\alpha = \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}, \tag{Eq. 3.29}$$

$$w = c + \frac{\alpha\frac{\partial G^1}{\partial e_1} - \frac{\partial Q^2}{\partial e_1}(p^* - c)}{\frac{\partial Q^1}{\partial e_1}} \quad \textit{and} \tag{Eq. 3.30}$$

$$\delta \geq \frac{\alpha G^{1^*}}{w Q^{1^*}}, \tag{Eq. 3.31}$$

or

$$w = \frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p^* - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1 - \theta_1\delta)}, \tag{Eq. 3.32}$$

$$\delta = \frac{\alpha G^{1^*}}{w Q^{1^*}} \quad \textit{and} \tag{Eq. 3.33}$$

$$\alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1^*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} > \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}, \tag{Eq. 3.34}$$

where $\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}$, G^{1^*} is the cost of effort and Q^{1^*} is the retail demand at first best price and effort levles (p^* and e^*).

When the retailers expend marketing effort such that the reimbursement through participation rate is greater than the cooperative dollars accrued, the retailers are only reimbursed $\delta w Q^i$. In this scenario, the retailer's choice of price and effort are governed by the following first order conditions

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \delta w \frac{\partial Q^1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - (1-\delta)w) + Q^1 = 0, \qquad \text{(Eq. 3.35)}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1} + \delta w \frac{\partial Q^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - (1-\delta)w) - \frac{\partial G^1}{\partial e_1} = 0. \qquad \text{(Eq. 3.36)}$$

As we can see from the above equations, the cooperative dollars amount to a discount in wholesale price. Since the cooperative dollars do not affect the retailer's advertising costs, the retailers continue to under invest in advertising. Hence, the manufacturer cannot achieve the first best setting. Substituting $\tilde{w} = w(1-\delta)$, we can see that the solution to the manufacturer's problem in this case is identical to the second best setting. Next, when the retailer choose to expend effort such that the reimbursement through participation rate is less than the cooperative dollars accrued, the retailers are reimbursed αG^i. In this scenario, the retailer's choice of price and effort are governed by the following first order conditions

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0, \qquad \text{(Eq. 3.37)}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0. \qquad \text{(Eq. 3.38)}$$

We can see from the above equations that this solution is identical to the case where the manufacturer offers a cooperative advertising contract that only includes a participation rate. Hence, the results in proposition 1 apply here. In order for the manufacturer to induce this solution, the manufacturer needs to choose an accrual rate δ such that the total cooperative dollars accrued are greater than the reimbursement to the retailers ($\delta w Q^1 > \alpha G^1$). Hence, the manufacturer can choose an appropriate participation rate α and a sufficiently high accrual rate δ (as detailed in the first part of proposition 2) to achieve the first best solution.

In addition, the retailers also have the option to expend advertising effort such that the total reimbursement through the participation rate is equal to the total accrued advertising dollars. In this scenario, the retailer's choice of price and effort are governed by the following first order conditions

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0, \tag{Eq. 3.39}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0, \tag{Eq. 3.40}$$

$$\delta w Q^1 - \alpha G^1 = 0. \tag{Eq. 3.41}$$

Equation (Eq. 3.39) governs the retailers choice of price and equation (Eq. 3.41) governs the retailers choice to effort. Since the retailer's advertisement cost reimbursement is equal to the total accruals, θ_1 denotes the incremental value to the retailer from increasing the total accrued cooperative dollars and is obtained by solving equation (Eq. 3.40). Hence, the manufacturer can induce the retailer to charge first best prices (p^*) and expend first best advertising levels (e^*) by choosing a wholesale price w and accrual rate δ such that equations (Eq. 3.39) and (Eq. 3.41) are satisfied for $p_1 = p^*$ and $e_1 = e^*$.

Since the retailers would choose prices and efforts using equations (Eq. 3.39) and (Eq. 3.41) only when the incremental value from increasing the total accrued dollars is positive ($\theta_1 > 0$), the manufacturer must choose the appropriate levels of participation rate that would ensure $\theta_1 > 0$. Since a low participation rate would require the retailers to expend large amounts of advertising effort in order to reach the cap set by the accrual limit, the retailers would choose to increase their advertising to levels that would meet the cap only for sufficiently high levels of the participation rate. Hence, the manufacturer must ensure that the participation rate is high enough to ensure $\theta_1 > 0$. Hence, the appropriate choice of δ, w and α (as noted in the second part of proposition 2) can achieve the first best solution. Note that since the retailers are charged a fixed fee in addition to the wholesale price, the retail profits excluding the fixed fee are higher when the retailers choose the solution leading to the first best prices and advertising levels. Since the manufacturer extracts the retailer's surplus using the fixed fee, the retailer's profits are identical under the three scenarios discussed above. Hence, the retailers are likely to choose the scenario that leads to the first best solution.

3.4.4 Cooperative Advertising Contract That Includes a Participation Rate as well as Fixed Accruals

Under this contract, both retailers are reimbursed a fraction α of their advertising expenses and this reimbursement is capped at a fixed amount (A). Given that the cost of advertising for the retailers is G^i, depending on the effort levels exerted by the retailers, we can have $\alpha G^i \leq A$ or $\alpha G^i > A$. If retailers expend effort such that $\alpha G^i \leq A$, then the retailers are reimbursed αG^i by the manufacturer. On the other hand, if retailers expend effort such that $\alpha G^i > A$, then the total reimbursement is only A.

Therefore, the retail profits can be expressed as

$$\pi^1 = \left\{ \begin{array}{ccc} Q^1(p_1 - w) - G^1 + \alpha G^1 - F & \text{if} & \alpha G^1 \leq A \\ Q^1(p_1 - w) - G^1 - F + A & \text{if} & \alpha G^1 > A \end{array} \right\} \quad \text{(Eq. 3.42)}$$

$$\pi^2 = \left\{ \begin{array}{ccc} Q^2(p_2 - w) - G^2 + \alpha G^2 - F & \text{if} & \alpha G^2 \leq A \\ Q^2(p_2 - w) - G^2 - F + A & \text{if} & \alpha G^2 > A \end{array} \right\} \quad \text{(Eq. 3.43)}$$

Correspondingly, the manufacturer's profits can be expressed as

$$\Pi = \left\{ \begin{array}{ccc} Q^1(w - c) + Q^2(w - c) + 2F - \alpha G^1 - \alpha G^2 & \text{if} & \alpha G^i \leq A \\ Q^1(w - c) + Q^2(w - c) + 2F - 2A & \text{if} & \alpha G^i > A \end{array} \right\} \quad \text{(Eq. 3.44)}$$

In this setting, the manufacturer declares the participation rate α, the accrual amount A along with the wholesale price w and fixed fees F. Given this contract, the retailers simultaneously and independently choose price and effort levels. Anticipating the retailer's behavior, the manufacturer chooses α, A, w and F to maximize his profits. Hence, the manufacturer's optimization problem $[PM - A]$ can be expressed as

$$\underset{\{w, F, p_1, p_2, e_1, e_2, \alpha, A\}}{\text{Max}} \quad \Pi \tag{Eq. 3.45}$$

subject to $\quad \pi^1 \geq 0, \pi^2 \geq 0, \dfrac{\partial \pi^1}{\partial p_1} = 0, \dfrac{\partial \pi^1}{\partial e_1} = 0, \dfrac{\partial \pi^2}{\partial p_2} = 0$ and $\dfrac{\partial \pi^2}{\partial e_2} = 0$

The properties of the solution to $[PM - A]$ are summarized in the following proposition

Proposition 3.5 *When the manufacturer offers a cooperative advertising contract that includes a fixed accrual rate A and a participation rate α, the manufacturer can achieve the first best solution by choosing w, α and A such that either of the following sets of conditions are satisfied*

$$\alpha = \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}, \tag{Eq. 3.46}$$

$$w = c + \frac{\alpha\frac{\partial G^1}{\partial e_1} - \frac{\partial Q^2}{\partial e_1}(p^* - c)}{\frac{\partial Q^1}{\partial e_1}} \quad \textit{and} \tag{Eq. 3.47}$$

$$A \geq \alpha G^{1^*}, \tag{Eq. 3.48}$$

or

$$w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p^* - c)}{\frac{\partial Q^1}{\partial p_1}}, \tag{Eq. 3.49}$$

$$A = \alpha G^{1^*} \quad and \tag{Eq. 3.50}$$

$$\alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^{1^*}}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}} > \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}, \tag{Eq. 3.51}$$

where G^{1^*} is the cost of effort and Q^{1^*} is the retail demand at first best price and effort levles (p^* and e^*).

When the retailers expend marketing effort such that the reimbursement through participation rate is greater than the cooperative dollars accrued, the retailers are only reimbursed A. In this scenario, the retailer's choice of price and effort are governed by the following first order conditions

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0, \tag{Eq. 3.52}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1} = 0. \tag{Eq. 3.53}$$

As we can see from the above equations, the cooperative dollars do not impact the retailer's price and effort choices. Since the retailers are reimbursed a fixed amount A, the amount of reimbursement does not affect the retailer's marginal costs. Hence, the solution is identical to the second best solution.

Next, when the retailer choose to expend effort such that the reimbursement through participation rate is less than the cooperative dollars accrued, the retailers are reimbursed

αG^i. In this scenario, the retailer's choice of price and effort are governed by the following first order conditions

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0, \tag{Eq. 3.54}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0. \tag{Eq. 3.55}$$

We can see from the above equations that this solution is identical to the case where the manufacturer offers a cooperative advertising contract that only includes a participation rate. Hence, the results in proposition 1 apply here. In order for the manufacturer to induce this solution, the manufacturer needs to choose the fixed accrual rate A such that the total cooperative dollars accrued are greater than the reimbursement to the retailers ($A > \alpha G^1$). Hence, the manufacturer can choose an appropriate participation rate α and a sufficiently high fixed accrual rate A (as detailed in the first part of proposition 2) to achieve the first best solution.

In addition, the retailers also have the option to expend advertising effort such that the total reimbursement through the participation rate is equal to the total accrued advertising dollars. In this scenario, the retailer's choice of price and effort are governed by the following first order conditions

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0, \tag{Eq. 3.56}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\left[\alpha\frac{\partial G^1}{\partial e_1}\right] = 0, \tag{Eq. 3.57}$$

$$A - \alpha G^1 = 0. \tag{Eq. 3.58}$$

Equation (Eq. 3.56) governs the retailers choice of price and equation (Eq. 3.58) governs the retailers choice to effort. Since the retailer's advertisement cost reimbursement is equal to the total accruals, θ_1 denotes the incremental value to the retailer from increasing the total accrued cooperative dollars and is obtained by solving equation (Eq. 3.57). Hence, the manufacturer can induce the retailer to charge first best prices (p^*) and expend first best advertising levels (e^*) by choosing a wholesale price w and accrual rate A such that equations (Eq. 3.56) and (Eq. 3.58) are satisfied for $p_1 = p^*$ and $e_1 = e^*$.

Since the retailers would choose prices and efforts using equations (Eq. 3.56) and (Eq. 3.58) only when the incremental value from increasing the total accrued dollars is positive ($\theta_1 > 0$), the manufacturer must choose the appropriate levels of participation rate that would ensure $\theta_1 > 0$. Since a low participation rate would require the retailers to expend large amounts of advertising effort in order to reach the cap set by the accrual limit, the retailers would choose to increase their advertising to levels that would meet the cap on cooperative advertising dollars only for sufficiently high levels of the participation rate. Hence, the manufacturer must ensure that the participation rate is high enough to ensure $\theta_1 > 0$. Therefore, the appropriate choice of A, w and α (as noted in the second part of proposition 2) can achieve the first best solution. Note that since the retailers are charged a fixed fee in addition to the wholesale price, the retail profits excluding the fixed fee are higher when the retailers choose the solution leading to the first best prices and advertising levels. Since the manufacturer extracts the retailer's surplus using the fixed fee, the retailer's profits are identical under the three scenarios discussed above. Hence, the retailers are likely to choose the scenario that leads to the first best solution.

3.4.5 Comparing the Fixed and Variable Accruals

While the previous sections described how the two types of accrual rates, in conjunction with the participation rate can induce the first best solution, each type of accrual rate has a very different impact on the retailer's price and effort choices. To understand these differences, let us first investigate the fixed amount accrual rate. When the manufacturer uses the fixed amount accrual rate to coordinate the channel, the first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0, \tag{Eq. 3.59}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\left[\alpha\frac{\partial G^1}{\partial e_1}\right] = 0, \tag{Eq. 3.60}$$

$$A - \alpha G^1 = 0. \tag{Eq. 3.61}$$

Since the choice of advertising levels is governed by equation (Eq. 3.61), we can see that as the fixed accrual amount is increased, the retailers increase their advertising levels. Also, since the accrual amount is fixed and is not impacted by the choice of wholesale price, advertising levels are not impacted by the choice of wholesale price. Next, the retailer's price levels are governed by equation (Eq. 3.59). As we can see from (Eq. 3.59), an increase in the wholesale price leads to an increase in the price charged by the retailer. Also, since an increase in advertising levels increases the marginal benefit of price, an increase in the fixed accrual amount indirectly increases the retail prices through its effect on advertising levels. With regards to the participation rate, we can see from equation (Eq. 3.61) that an increase in the participation rate would reduce advertising levels. A higher participation rate requires lower levels of advertising to satisfy equation (Eq. 3.61). Since prices are indirectly affected by the effort levels, an increase in the participation rate also lowers prices. Hence, the comparative statics[6] with respect to the wholesale price, participation rate and fixed accrual rate are given by

- $\frac{\partial p_1}{\partial w} > 0$,
- $\frac{\partial e_1}{\partial w} = 0$,
- $\frac{\partial p_1}{\partial \alpha} < 0$,
- $\frac{\partial e_1}{\partial \alpha} < 0$,
- $\frac{\partial p_1}{\partial A} > 0$,
- $\frac{\partial e_1}{\partial A} > 0$.

When the manufacturer uses the accrual rate linked to wholesale receipts to coordinate the channel, the first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0, \tag{Eq. 3.62}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0, \tag{Eq. 3.63}$$

$$\delta w Q^1 - \alpha G^1 = 0. \tag{Eq. 3.64}$$

Since the choice of advertising levels is governed by equation (Eq. 3.64), we can see that an increase in the accrual rate has a direct positive impact on the retailers advertising levels. In addition to this direct impact of the accrual rate, the increase in accrual rate also impacts the advertising levels indirectly through its impact on price and θ_1. When an increase in

[6] Detailed proofs available in the appendix

effort increases the reimbursement through the participation rate more than the increases in the total accruals, the net impact of an increase in the accrual rate δ is an increase in the retailer's advertising levels.

In contrast to the impact of the accrual rate on the advertising levels, an increase in the accrual rate (δ) has a direct negative impact on the retail prices. As the accrual rate increases, the total cap on reimbursement increases. The retailers can also increase the cap by decreasing the price. The reduction in price leads to greater demand and therefore higher wholesale receipts and in turn higher cap on reimbursement. When the accrual rate is higher, the marginal increase in the cap on reimbursements due to a reduction in price is greater than when the accrual rate is lower. Hence, the retailers tend to decrease the retail prices. The increase in the accrual rate also has an indirect positive impact on retail prices through its positive direct impact on advertising levels. As advertisement levels go up, retailers can increase prices to benefit from the increased demand from higher advertising. The increase in the accrual rate also has an indirect impact on retail prices through its impact on θ_1. When the following condition holds,

$$\begin{aligned}\Delta^1 = &-\frac{\partial Q^1}{\partial e_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})wQ^1 + \theta_1 w\frac{\partial Q^1}{\partial p_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})^2 + \\ &\delta w\frac{\partial Q^1}{\partial p_1}\left[wQ^1(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2}) - \theta_1 w\frac{\partial Q^1}{\partial e_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})\right] < 0\end{aligned} \tag{Eq. 3.65}$$

the accrual rate has a net negative impact on retail prices. Similar arguments hold for the comparative statics with respect to the participation rate and the wholesale price and the various comparative statics results[7] are listed below

- $\frac{\partial p_1}{\partial w} > 0$ when $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$,
- $\frac{\partial e_1}{\partial w} > 0$ when $\Delta^6 > 0$,

[7] Detailed proofs are available in the appendix.

- $\frac{\partial p_1}{\partial \alpha} < 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$,

- $\frac{\partial e_1}{\partial \alpha} < 0$ when $\Delta^5 < 0$,

- $\frac{\partial p_1}{\partial \delta} < 0$ when $\Delta^1 < 0$,

- $\frac{\partial e_1}{\partial \delta} > 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$.

where Δ^1 is defined in (C259) and

$$\Delta^5 = -\left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right] G^1(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}) + \delta w \frac{\partial Q^1}{\partial p_1}\left[\delta w \frac{\partial Q^1}{\partial p_1}(1-\theta_1)\frac{\partial G^1}{\partial e_1} + G^1 \frac{\partial Q^1}{\partial e_1}\right], \tag{Eq. 3.66}$$

$$\Delta^6 = +(1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1}\delta w \frac{\partial Q^1}{\partial p_1}(\delta w \frac{\partial Q^1}{\partial p_1} - \alpha \frac{\partial G^1}{\partial e_1}) + \left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right]\delta Q^1(\delta q \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1})$$
$$\delta w \frac{\partial Q^1}{\partial p_1}\left[-\delta w \frac{\partial Q^1}{\partial p_1}(1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1} - \delta Q^1 \frac{\partial Q^1}{\partial e_1}\right]. \tag{Eq. 3.67}$$

Comparing the impact of the fixed and variable accrual rates on retail prices, we can see that while an increase in the fixed accrual rate also increases the retail prices, an increase in the accrual rate linked to wholesale receipts can lead to a lowering of retail prices. Additionally, while an increase in the wholesale price has no impact on advertising levels under the fixed accrual rate contract, an increase in the wholesale price can lead to an increase in the advertising levels under the accrual rate contract linked to wholesale receipts. These differences stem from the fact that the retailers can alter the total cooperative dollars accrued under the variable accrual rate contract by changing retail prices and advertising levels. Also, a change in the wholesale price directly impacts the accrual dollars under this contract (in contrast to having no impact on the accrual dollars under the fixed accrual rate contract).

3.5 Conclusion

Cooperative advertising contracts are frequently used by manufacturers to induce retailers to increase their advertising intensity. In this essay, we provide guidelines for the use of various cooperative advertising strategies and detailed the conditions under which each type of cooperative advertising contract can lead to greater profits to the manufacturer. While previous research only considered a single type of cooperative advertising contract — one that only includes a participation rate — in practice, we observe cooperative advertising contracts that also include accruals that limit retailer reimbursement. While the various cooperative advertising contracts can be used to achieve coordination, we have shown that each contract has a unique effect of retail prices and efforts. We provide guidelines to manufacturers for using each type of cooperative advertising contract to achieve channel coordination.

While this essay investigates a channel setting involving a manufacturer selling through two symmetric retailers, downstream retailers may be asymmetric. This asymmetry may arise due to differences in cost and demand parameters. Hence, in the next essay, we extend our analysis to include asymmetric retailers.

CHAPTER 4: COOPERATIVE ADVERTISING WITH ASYMMETRIC RETAILERS

4.1 Introduction

In the previous chapter, we investigated the use of various cooperative advertising contracts by a manufacturer selling through two symmetric retailers. Downstream retailers may be symmetric when there is little or no differentiation between the retailers and/or when the customer segments that these retailers serve are similar. While symmetric retailers are plausible, downstream retailer asymmetry is more likely to arise when selling to multiple retailers who differentiate themselves from the competition. Retailer asymmetry can arise due to several sources. Retailers may have different cost structures that may result in asymmetric pricing and advertising behavior. Also, retailers may differ in their target customer segments' size and/or valuations of the products sold and these segment differences will result in the retailers facing asymmetric demand. As Iyer (1998) notes, retail differentiation has important implications for upstream manufacturers and extant literature seldom accounts for such asymmetry in investigating channel issues. The use of a 'one size fits all' strategy that works well under symmetry may fail under asymmetry.

This chapter investigates the effectiveness of cooperative advertising contracts in coordinating the channel in the presence of asymmetric downstream retailers. While these contracts allow the manufacturer to coordinate the channel when retailers are symmetric, we find that coordination can be achieved in the presence of asymmetry only when stringent conditions are met. As in the previous chapter, we compare three types of cooperative advertising contracts: contracts that only include a participation rate, contracts that include a participation rate as well as variable accruals and contracts that include a participation rate as well as a fixed accrual. While all three types of contracts can coordinate the channel under very stringent conditions, contracts that include some form of accrual can coordinate the channel under less stringent conditions compared to the contract that only includes a participation rate. When the cooperative advertising contract includes accruals, the manufacturer can use the accrual amount to coordinate the efforts of one retailer while using the participation rate to coordinate the other retailer's efforts.

Since the conditions for achieving coordination using a cooperative advertising are stringent, they are less likely to be satisfied. When coordination cannot be achieved, we compare the relative attractiveness of the three types of contracts. Since the manufacturer can control retail efforts with two instruments (participation rate and the accrual), contracts that include some form of accruals are superior to contracts that only include a participation rate. Amongst the contracts that include accruals, using a variable accrual contract may be preferred to the fixed accrual contract under certain conditions and vice versa. The two type of accruals impact retail prices and advertising levels in distinct ways and these differences may tip the scale in favor of one contract versus the other contract under the appropriate circumstances. In what follows, these conditions and the intuition behind the results are discussed in detail.

The rest of the chapter is organized as follows: the next section develops the model, section 3 presents our analysis and results while the final section concludes the paper. All proofs are confined to an appendix.

4.2 The Model

We first describe the model that incorporates no cooperative advertising strategies and later discuss how the basic model changes with the use of cooperative advertising contracts. We consider a single manufacturer M that sells its products through two retailers R^1 and R^2. The demand faced by retailers R^1 and R^2 is denoted by $Q^1 = Q^1(p_1, p_2, e_1, e_2)$ and $Q^2 = Q^2(p_2, p_1, e_2, e_1)$ respectively. The demand functions have the following properties

$$\frac{\partial Q^i}{\partial p_i} < 0, \frac{\partial Q^i}{\partial p_j} > 0, \left|\frac{\partial Q^i}{\partial p_i}\right| > \left|\frac{\partial Q^i}{\partial p_j}\right|, \quad \frac{\partial Q^i}{\partial e_i} > 0, \frac{\partial Q^i}{\partial e_j} > 0 \text{ and } \frac{\partial Q^i}{\partial e_i} > \frac{\partial Q^i}{\partial e_j} \ \forall i, j \in \{1, 2\}, i \neq j \qquad \text{(Eq. 4.1)}$$

Where p_i and e_i are the price charged and effort (advertising) expended by retailer R^i. The cost of effort to retailer R^i is given by G^i and is assumed to be increasing and convex in the effort ($\frac{\partial G^i}{\partial e_i} > 0$ and $\frac{\partial^2 G^i}{\partial e_i^2} > 0$). Note that here we assume that the manufacturer sells to two asymmetric retailers. The manufacturer sells the products to retailer R^i at a wholesale price w and also charges a fixed fee[1] F^i. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - G^1 - F^1, \qquad \text{(Eq. 4.2)}$$

$$\pi^2 = Q^2(p_2 - wt) - G^2 - F^2. \qquad \text{(Eq. 4.3)}$$

The manufacturer's profits can be expressed as

$$\Pi = Q^1(w - c) + Q^2(w - c) + F^1 + F^2. \qquad \text{(Eq. 4.4)}$$

[1]Here, we assume that the manufacturer is able to charge two separate fixed fees. The assumption of two separate fixed fees allows us to investigate the incentives created by the use of a cooperative advertising contract in a simpler setting, without having to consider rent extraction issues. In order to resolve both the rent extraction incentive and the incentive to improve retail efforts requires the use of a specific demand formulation. Also, O'Brien and Shaffer (1994) note that manufacturers may use a two-part tariff that includes a single wholesale price and separate fixed fees for each retailer.

where c is the per unit cost incurred by the manufacturer. The manufacturer acts as a Stackelberg leader and sets the wholesale price and fixed fees for the product before the retailers set retail prices and effort levels. Given a wholesale price set by the manufacturer, the retailers simultaneously and independently choose retail price and effort levels in order to maximize their profits. The manufacturer anticipates the retailers' actions and choose the wholesale prices and fixed fees in order to maximize his profits, taking into consideration the retailers actions.

We begin by understand the manufacturer's choices under a vertically integrated channel (where the manufacturer makes the pricing and effort decisions). We denote this setting as the first best setting. The retail profits under vertical integration are given by

$$\pi^1 = Q^1(p_1 - c) - G^1, \qquad \text{(Eq. 4.5)}$$

$$\pi^2 = Q^2(p_2 - c) - G^2. \qquad \text{(Eq. 4.6)}$$

Since the manufacturer owns both the retail channels, the manufacturer's profits can be expressed as

$$\Pi = \pi^1 + \pi^2 = Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2. \qquad \text{(Eq. 4.7)}$$

The manufacturer's optimization problem (denoted by [M-FB]) can be expressed as

$$\underset{\{p_1, p_2, e_1, e_2\}}{\text{Max}} \quad \Pi = Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2 \qquad \text{(Eq. 4.8)}$$

The properties of the solution[2] to [M-FB] are summarized in the following Lemma.

[2]All proofs are relegated to the appendix

Lemma 4.6 *The retail price ($p_1 = p_1^*$, $p_2 = p_2^*$) and the effort ($e_1 = e_1^*$, $e_2 = e_1^*$) that solve [M-FB] simultaneously satisfy the following equations*

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = 0 \tag{Eq. 4.9}$$

$$\frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 = 0 \tag{Eq. 4.10}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{Eq. 4.11}$$

$$\frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} = 0 \tag{Eq. 4.12}$$

From equations (Eq. 4.9)-(Eq. 4.12), we can see that the manufacturer chooses price and effort such that the difference between the total marginal benefit(to the entire channel) from price/effort and the total marginal cost(to the entire channel) of price/effort, denoted by $DMRMC_i^{fb} \quad \forall i \in \{p, e\}$ is equal to zero ($DMRMC_p^{fb} = 0$ and $DMRMC_e^{fb} = 0$). In the next section, we consider the independent retailer case where the retailers make the price and effort choices while the manufacturer decides the wholesale price and fixed fees, in addition to any cooperative advertising that the manufacturer may offer.

4.3 Analysis and Results

In this section, we investigate channel settings where the manufacturer sells through two independent retailers who make the retail price and effort decisions. In this context, we investigate scenarios where 1) there is no cooperative advertising contract offered to the retailers 2) the retailers are offered a cooperative advertising contract that only includes a participation rate 3) the retailers are offered a cooperative advertising contract that includes a participation rate as well as an accrual rate that is set as a fraction of the wholesale receipts and 4) the retailers are offered a cooperative advertising contract that includes a participation rate as well as an accrual rate that is set as a fixed amount. We compare these channel settings with that of the vertical integration (first best) case to understand the effectiveness of the cooperative advertising contract in achieving the first best outcomes.

4.3.1 Independent Retailers with No Cooperative Advertising

In this case, while the manufacturer charges a wholesale price and a fixed fee, there is no cooperative advertising contract offered to the retailers. We denote this case as the 'second best' case. The retail profits under this setting are given by equations (Eq. 4.2)-(Eq. 4.3) and the manufacturer's profits are given by (Eq. 4.4). The manufacturer chooses the wholesale prices and fixed fees while anticipating the retailer's responses to these choices. Also, the manufacturer ensures that the retailers at least make their reservation profits (assumed to be zero). Hence, the manufacturer's optimization problem (denoted by [M-SB]) can be stated as

$$\underset{\{w,F^1,F^2,p_1,p_2,e_1,e_2\}}{\text{Max}} \quad \Pi \tag{Eq. 4.13}$$
$$\text{subject to} \quad \pi^1 \geq 0,\, \pi^2 \geq 0,\, \frac{\partial \pi^1}{\partial p_1} = 0,\, \frac{\partial \pi^1}{\partial e_1} = 0,\, \frac{\partial \pi^2}{\partial p_2} = 0 \text{ and } \frac{\partial \pi^2}{\partial e_2} = 0$$

The properties of the solution to [M-SB] are summarized in the following Lemma.

Lemma 4.7 *The retail price and the effort that solve [M-SB] simultaneously satisfy the following equations*

$$\frac{\partial Q^1}{\partial p_1}(p_1-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c)+Q^1 = \frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c) \tag{Eq. 4.14}$$
$$\frac{\partial Q^2}{\partial p_2}(p_2-c)+\frac{\partial Q^1}{\partial p_2}(p_1-c)+Q^2 = \frac{\partial Q^2}{\partial p_2}(w-c)+\frac{\partial Q^1}{\partial p_2}(p_1-c) \tag{Eq. 4.15}$$
$$\frac{\partial Q^1}{\partial e_1}(p_1-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c)-\frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c) \tag{Eq. 4.16}$$
$$\frac{\partial Q^2}{\partial e_2}(p_2-c)+\frac{\partial Q^1}{\partial e_2}(p_1-c)-\frac{\partial G^2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(w-c)+\frac{\partial Q^1}{\partial e_2}(p_1-c) \tag{Eq. 4.17}$$

where $p_1 = \hat{p_1}$ *and* $p_2 = \hat{p_2}$ *are the prices charged,* $e_1 = \hat{e_1}$ *and* $e_2 = \hat{e_2}$ *are the efforts expended by the two asymmetric independent retail outlets. The manufacturer's profits under the second best setting are lower than the first best profits (*$\hat{\Pi} < \Pi^*$*).*

From lemma 2, we can see that the retailer's choice of effort is such that the difference between the marginal benefit to the channel and the marginal cost to the channel is positive ($DMRMC_e^{sb} = \frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1-c) > 0 \quad \forall w > c$). Also, the retailer's choice of price is such that the difference between the marginal benefit and marginal cost is given by $DMRMC_p^{sb} = \frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1-c)$. Note that when $w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p_1-c)}{\frac{\partial Q^1}{\partial p_1}} > c$, $DMRMC_p^{sb} = 0$ but since $w > c$, we have $DMRMC_e^{sb} > 0$. Hence, $DMRMC_e^{sb}$ and

$DMRMC_p^{sb}$ cannot be zero at the same time. Hence, the first best cannot be achieved. Also, as the manufacturer increases the wholesale price, the price charged by the retailer increases[3] ($\frac{\partial p_1}{\partial w} > 0$) and the effort expended by the retailer decreases ($\frac{\partial e_1}{\partial w} < 0$). Since the manufacturer has only one instrument (the wholesale price) to control both the price and effort charged by the retailers, and since the wholesale price has an opposite effect on price and effort, first best profits cannot be achieved.

In the next section, we investigate the impact of a cooperative advertising contract that only specifies a participation rate.

4.3.2 Cooperative Advertising Contract That Specifies a Participation Rate With No Accrual Rate Specified

When the manufacturer uses a cooperative advertising contract that only specifies a participation rate, both retailers are reimbursed a fraction α of their advertising expenses. Hence, the retail profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - G^1 - F^1 + \alpha G^1, \qquad \text{(Eq. 4.18)}$$

$$\pi^2 = Q^2(p_2 - w) - G^2 - F^2 + \alpha G^2. \qquad \text{(Eq. 4.19)}$$

Correspondingly, the manufacturer's profits can be expressed as

$$\Pi = Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - \alpha G^1 - \alpha G^2. \qquad \text{(Eq. 4.20)}$$

In this setting, the manufacturer declares the participation rate α along with the wholesale price and fixed fees. Given this contract, the retailers simultaneously and independently choose price and effort levels. Anticipating the retailer's behavior, the manufacturer chooses

[3]See appendix for the proof.

α, w and F to maximize his profits. Also, the manufacturer ensures that the retailers at least make their reservation profits (assumed to be zero). Hence, the manufacturer's optimization problem (denoted by [M-α]) can be stated as

$$\begin{array}{ll} & \underset{\{w, F^1, F^2, p_1, p_2, e_1, e_2, \alpha\}}{\text{Max}} \quad \Pi \\ \text{subject to} & \pi^1 \geq 0, \pi^2 \geq 0, \dfrac{\partial \pi^1}{\partial p_1} = 0, \dfrac{\partial \pi^1}{\partial e_1} = 0, \dfrac{\partial \pi^2}{\partial p_2} = 0 \text{ and } \dfrac{\partial \pi^2}{\partial e_2} = 0 \end{array} \tag{Eq. 4.21}$$

The properties of the solution to [M-α] are summarized in the following Proposition.

Proposition 4.6 *The manufacturer can simultaneously induce both the retailers ($R1$ and $R2$) to exert first best efforts (e_1^* and e_2^*) and charge first best retail prices (p_1^* and p_2^*) if the following conditions are simultaneously satisfied*

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \quad \textit{and} \tag{Eq. 4.22}$$

$$\frac{(p_2^* - c)(\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1})}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} = \frac{(p_1^* - c)(\frac{\partial Q^2}{\partial p_2}\frac{\partial Q^1}{\partial e_2} - \frac{\partial Q^2}{\partial e_2}\frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{Eq. 4.23}$$

When the above conditions are not satisfied, the manufacturer cannot use the wholesale price (w) and participation rate (α) induce the retailers to charge first best prices and exert first best efforts.

When the manufacturer offers a cooperative advertising contract that includes a participation rate, the first order conditions that determine the retailer's price and effort choices are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c)+Q^1=\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c) \quad \text{(Eq. 4.24)}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2-c)+\frac{\partial Q^1}{\partial p_2}(p_1-c)+Q^2=\frac{\partial Q^2}{\partial p_2}(w-c)+\frac{\partial Q^1}{\partial p_2}(p_1-c) \quad \text{(Eq. 4.25)}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c)-\frac{\partial G^1}{\partial e_1}=\left(\frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c)\right)-\alpha\frac{\partial G^1}{\partial e_1} \quad \text{(Eq. 4.26)}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2-c)+\frac{\partial Q^1}{\partial e_2}(p_1-c)-\frac{\partial G^2}{\partial e_2}=\left(\frac{\partial Q^2}{\partial e_2}(w-c)+\frac{\partial Q^1}{\partial e_2}(p_1-c)\right)-\alpha\frac{\partial G^2}{\partial e_2} \quad \text{(Eq. 4.27)}$$

As we can see from equation (Eq. 4.24), the manufacturer can induce $R1$ to exert first best price by choosing the wholesale price such that

$$\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_2^*-c)=0 \quad \Rightarrow w=c-\frac{\frac{\partial Q^2}{\partial p_1}(p_2^*-c)}{\frac{\partial Q^1}{\partial p_1}} \quad \text{(Eq. 4.28)}$$

Similarly, we can see from equation (Eq. 4.25) that the manufacturer can induce $R2$ to exert first best price by choosing the wholesale price such that

$$\frac{\partial Q^2}{\partial p_2}(w-c)+\frac{\partial Q^1}{\partial p_2}(p_1^*-c)=0 \quad \Rightarrow w=c-\frac{\frac{\partial Q^1}{\partial p_2}(p_1^*-c)}{\frac{\partial Q^2}{\partial p_2}} \quad \text{(Eq. 4.29)}$$

Since the manufacturer can only charge a single wholesale price, in order for the manufacturer to induce both retailers to charge first best prices, we must have

$$c-\frac{\frac{\partial Q^2}{\partial p_1}(p_2^*-c)}{\frac{\partial Q^1}{\partial p_1}}=c-\frac{\frac{\partial Q^1}{\partial p_2}(p_1^*-c)}{\frac{\partial Q^2}{\partial p_2}} \quad \Rightarrow \quad \frac{\frac{\partial Q^2}{\partial p_1}(p_2^*-c)}{\frac{\partial Q^1}{\partial p_1}}=\frac{\frac{\partial Q^1}{\partial p_2}(p_1^*-c)}{\frac{\partial Q^2}{\partial p_2}} \quad \text{(Eq. 4.30)}$$

Hence, the manufacturer can induce the retailers to charge first best prices only when equation (Eq. 4.30) is satisfied. Next, we can see from equation (Eq. 4.26) that the manufacturer can induce $R1$ to exert first best effort by choosing the participation rate such that

$$\left(\frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_2^*-c)\right)-\alpha\frac{\partial G^1}{\partial e_1}=0 \quad \Rightarrow \quad \alpha=\frac{(p_2^*-c)(\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}-\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1})}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{Eq. 4.31}$$

Similarly, we can see from equation (Eq. 4.27) that the manufacturer can induce $R2$ to exert first best effort by choosing the participation rate such that

$$\left(\frac{\partial Q^2}{\partial e_2}(w-c)+\frac{\partial Q^1}{\partial e_2}(p_1-c)\right)-\alpha\frac{\partial G^2}{\partial e_2}=0 \quad \Rightarrow \quad \alpha=\frac{(p_1^*-c)(\frac{\partial Q^2}{\partial p_2}\frac{\partial Q^1}{\partial e_2}-\frac{\partial Q^2}{\partial e_2}\frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{Eq. 4.32}$$

Since the manufacturer must offer the same participation rate to both the retailers, in order ofr the manufacturer to induce both retailers to exert first best efforts, we must have

$$\frac{(p_2^*-c)(\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}-\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1})}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}=\frac{(p_1^*-c)(\frac{\partial Q^2}{\partial p_2}\frac{\partial Q^1}{\partial e_2}-\frac{\partial Q^2}{\partial e_2}\frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{Eq. 4.33}$$

Hence, only when equations (Eq. 4.30) and (Eq. 4.33) are simultaneously satisfied, the manufacturer can induce the first best prices and efforts. When the retailers are symmetric, equations (Eq. 4.30) and (Eq. 4.33) are always satisfied, and the manufacturer achieves the first best profits. When the retailers are asymmetric, the marginal benefits of price and effort and the corresponding marginal costs are different for each retailer. Hence, the incentives (wholesale price and participation rate) that are required to induce first best price and effort levels differ for each retailer. Only when equations (Eq. 4.30) and (Eq. 4.33) are satisfied, the manufacturer can simultaneously induce both retailers to charge first best prices and exert first best effort levels.

In the next section, we investigate a cooperative advertising contract that includes an accrual rate in addition to a participation rate. In this setting, the total reimbursements that the retailers can get from the manufacturer are limited to a fraction (δ) of the wholesale receipts from the retailer. Since the limit on reimbursement is affected by both the manufacturer and retailer choices, the total accruals to the retailers is variable.

4.3.3 Cooperative Advertising Contract That Includes a Participation Rate as well as Variable Accruals

Under this contract, both retailers are reimbursed a fraction α of their advertising expenses and this reimbursement is capped at a fraction (δ) of the wholesale receipts from the retailer ($\delta w Q^i$). Since the wholesale receipts are affected by the wholesale price, retail price as well as effort, the total accrual is dependent on the choices of both the manufacturer and the retailer. Given that the cost of advertising for the retailers is G^i, depending on the effort levels exerted by the retailers, we can have $\alpha G^i \leq \delta w Q^i$ or $\alpha G^i > \delta w Q^i$. If retailers expend effort such that $\alpha G^i \leq \delta w Q^i$, then the retailers are reimbursed αG^i by the manufacturer. On the other hand, if retailers expend effort such that $\alpha G^i > \delta w Q^i$, then the total reimbursement is only $\delta w Q^i$.

Therefore, the retail profits can be expressed as

$$\pi^1 = \left\{ \begin{array}{ll} Q^1(p_1 - w) - G^1 + \alpha G^1 - F^1 & \text{if} \quad \alpha G^1 \leq \delta w Q^1 \\ Q^1(p_1 - w) - G^1 + \delta w Q^1 - F^1 & \text{if} \quad \alpha G^1 > \delta w Q^1 \end{array} \right\} \qquad \text{(Eq. 4.34)}$$

$$\pi^2 = \left\{ \begin{array}{ll} Q^2(p_2 - w) - G^2 + \alpha G^2 - F^2 & \text{if} \quad \alpha G^2 \leq \delta w Q^2 \\ Q^2(p_2 - w) - G^2 + \delta w Q^2 - F^2 & \text{if} \quad \alpha G^2 > \delta w Q^2 \end{array} \right\} \qquad \text{(Eq. 4.35)}$$

Correspondingly, the manufacturer's profits can be expressed as

$$\Pi = \left\{ \begin{array}{ll} Q^1(w-c) + Q^2(w-c) + F^1 + F^2 - \alpha G^1 - \alpha G^2 & \text{if} \quad \alpha G^i \leq \delta w Q^i \\ Q^1(w-c) + Q^2(w-c) + F^1 + F^2 - \delta w Q^1 - \delta w Q^2 & \text{if} \quad \alpha G^i > \delta w Q^i \end{array} \right\} \tag{Eq. 4.36}$$

In this setting, the manufacturer declares the participation rate α, the accrual rate δ, the wholesale price w and the fixed fees F^1 and F^2. Given this contract, the retailers simultaneously and independently choose price and effort levels. Anticipating the retailer's behavior, the manufacturer chooses α, δ, w, F^1 and F^2 to maximize his profits. Hence, the manufacturer's optimization problem $[PM-\delta]$ can be expressed as

$$\underset{\{w, F^1, F^2, p_1, p_2, e_1, e_2, \alpha, \delta\}}{\text{Max}} \quad \Pi \tag{Eq. 4.37}$$

subject to $\pi^1 \geq 0$, $\pi^2 \geq 0$, $\dfrac{\partial \pi^1}{\partial p_1} = 0$, $\dfrac{\partial \pi^1}{\partial e_1} = 0$, $\dfrac{\partial \pi^2}{\partial p_2} = 0$ and $\dfrac{\partial \pi^2}{\partial e_2} = 0$

The properties of the solution to $[PM - \delta]$ are summarized in the following proposition

Proposition 4.7 *The manufacturer can induce both the asymmetric retailers to charge first best prices and exert first best effort levels under the following three scenarios*

(i) *The demand parameters for the asymmetric retailers are such that the conditions listed in proposition 1 are satisfied and the manufacturer chooses a very high accrual rate (δ) such that the total reimbursement to either of the retailers is less than each retailer's accrual ($\delta w Q^{i^*} > \alpha G^{1^*}$). In this case, the manufacturer only uses the participation rate and the wholesale price to coordinate the channel.*

(ii) *The demand parameters of the asymmetric retailers are such that the following conditions are simultaneously satisfied*

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = \frac{c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}}}{(1-\theta_2\delta)} \tag{Eq. 4.38}$$

$$\frac{G^{1^*}}{Q^{1^*}} = \frac{G^{2^*}}{Q^{1^*}} \tag{Eq. 4.39}$$

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1^*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2}Q^{1^*}}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}}\} \tag{Eq. 4.40}$$

where $\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1^*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}$, $\theta_2 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^2}{\partial e_2}Q^{1^*}}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}$, Q^{1^*} *and* Q^{1^*} *denote the demand faced by the retailers when exerting first best efforts* (e_1^*, e_2^*) *and charging first best prices* (p_1^*, p_2^*), G^{1^*} *and* G^{2^*} *denote the cost of effort incurred by the retailers when exerting first best effort levles.*

(iii) *The demand parameters of the asymmetric retailers are such that the following conditions are simultaneously satisfied*

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \qquad \text{(Eq. 4.41)}$$

$$\frac{(p_1^* - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \qquad \text{(Eq. 4.42)}$$

$$G^{1^*}Q^{1^*} - G^{2^*}Q^{1^*} \geq 0 \qquad \text{(Eq. 4.43)}$$

where $\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1^*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}$.

The manufacturer can set a very high accrual rate δ (and a low participation rate α)such that both the retailers expend marketing effort such that their reimbursement through the participation rate is less than the total accruals. In this scenario, the retailers are reimbursed αG^i. Here, the participation rate is the only parameter that influences retail behavior. Hence, when conditions[4] listed in proposition 1 are satisfied, the manufacturer can induce the retailers to exert first best efforts and charge first best prices.

Alternatively, the manufacturer can[5] set a lower accrual rate δ (and a higher participation rate α) such that both the retailers may expend marketing effort such that their reimbursement through the participation rate is equal to the cooperative dollars accrued. In this case, the manufacturer uses the accrual rate to influence retail effort and the wholesale price to influence retail prices. Since the manufacturer charges a single wholesale price, the following condition must be satisfied in order to induce both retailers to charge first best

[4]See appendix for proof.
[5]See appendix for proof.

prices simultaneously

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = \frac{c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}}}{(1-\theta_2\delta)} \quad \text{(Eq. 4.44)}$$

Also, in order for the manufacturer to induce both retailers to exert effort such that their reimbursement through the participation rate is equal to their accruals, the following conditions must be simultaneously satisfied

$$\frac{G^{1^*}}{Q^{1^*}} = \frac{G^{2^*}}{Q^{1^*}} \quad \text{(Eq. 4.45)}$$

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1^*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2}Q^{1^*}}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}}\} \quad \text{(Eq. 4.46)}$$

Hence, when the conditions in (Eq. 4.44)-(Eq. 4.46) are simultaneously satisfied, the manufacturer can induce both the retailers to charge first best prices and exert first best efforts.

Alternatively, the manufacturer can choose the participation rate and the accrual rate such that one retailer (say, R1) expends effort such that his reimbursement through the participation rate is lower than the total accrual and the other retailer (R2) expends effort such that his reimbursement through the participation rate is equal to the total accruals. In this setting, while R1 is reimbursed $\alpha G^{1^*} < \delta w Q^{1^*}$, R2 is reimbursed $\alpha G^{2^*} = \delta w Q^{1^*}$. Here, the manufacturer can use the participation rate to influence R1's effort and can use the accrual rate to influence R2's effort. The manufacturer then uses the single wholesale price to influence retail prices of both R1 and R2. The single wholesale price can induce both the asymmetric retailers to exert first best prices only when the following condition is satisfied

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{Eq. 4.47}$$

In order for the manufacturer to be able to induce R1 to exert effort such that R1's reimbursement less than the accruals while R2's reimbursement is equal to R2's accrual, the following conditions must be simultaneously satisfied

$$\frac{(p_1^* - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{Eq. 4.48}$$

$$G^{1^*}Q^{1^*} - G^{2^*}Q^{1^*} \geq 0 \tag{Eq. 4.49}$$

Hence, when conditions (Eq. 4.47)-(Eq. 4.49) are simultaneously satisfied, the manufacturer can induce first best efforts and prices.

In addition to the above described strategies, the manufacturer can offer a very low accrual rate δ that can induce one or both retailers to exert effort such that their reimbursement through the participation rate is greater than the accruals. In this scenario, the retailers are reimbursed $\delta w Q^i$. Since this reimbursement amounts to a reduction in wholesale price, the manufacturer cannot induce the retailer to exert first best efforts. Hence, the manufacturer would prefer not to offer a very low accrual rate.

When the asymmetry between the retailers is such that the conditions listed in proposition 2 are not satisfied, the manufacturer cannot simultaneously induce both the retailers to exert first efforts and prices. In the next section, we investigate a cooperative advertising contract that includes a participation rate and a fixed accrual amount.

4.3.4 Cooperative Advertising Contract That Includes a Participation Rate as well as a Fixed Accrual

Under this contract, both retailers are reimbursed a fraction α of their advertising expenses and this reimbursement is capped at a fixed amount (A). Given that the cost of advertising for the retailers is G^i, depending on the effort levels exerted by the retailers, we can have $\alpha G^i \leq A$ or $\alpha G^i > A$. If retailers expend effort such that $\alpha G^i \leq A$, then the retailers are reimbursed αG^i by the manufacturer. On the other hand, if retailers expend effort such that $\alpha G^i > A$, then the total reimbursement is only A.

Therefore, the retail profits can be expressed as

$$\pi^1 = \left\{ \begin{array}{lll} Q^1(p_1 - w) - G^1 + \alpha G^1 - F^1 & \text{if} & \alpha G^1 \leq A \\ Q^1(p_1 - w) - G^1 - F^1 + A & \text{if} & \alpha G^1 > A \end{array} \right\} \qquad \text{(Eq. 4.50)}$$

$$\pi^2 = \left\{ \begin{array}{lll} Q^2(p_2 - w) - G^2 + \alpha G^2 - F^2 & \text{if} & \alpha G^2 \leq A \\ Q^2(p_2 - w) - G^2 - F^2 + A & \text{if} & \alpha G^2 > A \end{array} \right\} \qquad \text{(Eq. 4.51)}$$

Correspondingly, the manufacturer's profits can be expressed as

$$\Pi = \left\{ \begin{array}{lll} Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - \alpha G^1 - \alpha G^2 & \text{if} & \alpha G^i \leq A \\ Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - 2A & \text{if} & \alpha G^i > A \end{array} \right\} \qquad \text{(Eq. 4.52)}$$

In this setting, the manufacturer declares the participation rate α, the accrual amount A along with the wholesale price w and the fixed fees F^1 and F^2. Given this contract, the retailers simultaneously and independently choose price and effort levels. Anticipating the retailer's behavior, the manufacturer chooses α, A, w, F^1 and F^2 to maximize his profits.

Hence, the manufacturer's optimization problem $[PM-A]$ can be expressed as

$$\underset{\{w, F^1, F^2, p_1, p_2, e_1, e_2, \alpha, A\}}{\text{Max}} \quad \Pi \tag{Eq. 4.53}$$

subject to $\pi^1 \geq 0$, $\pi^2 \geq 0$, $\dfrac{\partial \pi^1}{\partial p_1} = 0$, $\dfrac{\partial \pi^1}{\partial e_1} = 0$, $\dfrac{\partial \pi^2}{\partial p_2} = 0$ and $\dfrac{\partial \pi^2}{\partial e_2} = 0$

The properties of the solution to $[PM-A]$ are summarized in the following proposition

Proposition 4.8 *The manufacturer can induce both the asymmetric retailers to charge first best prices and exert first best effort levels under the following three scenarios*

(i) *The demand parameters for the asymmetric retailers are such that the conditions listed in proposition 1 are satisfied and the manufacturer chooses a very high fixed accrual (A) such that the total reimbursement to either of the retailers is less than each retailer's accrual ($A > \alpha G^{i^*}$). In this case, the manufacturer only uses the participation rate and the wholesale price to coordinate the channel.*

(ii) *The demand parameters of the asymmetric retailers are such that the following conditions are simultaneously satisfied*

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{Eq. 4.54}$$

$$G^{1^*} = G^{2^*} \tag{Eq. 4.55}$$

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^{1^*}}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2} Q^{1^*}}{\frac{\partial Q^2}{\partial p_2} \frac{\partial G^2}{\partial e_2}}}\} \tag{Eq. 4.56}$$

where Q^{1^*} and Q^{1^*} denote the demand faced by the retailers when exerting first best efforts (e_1^*, e_2^*) and charging first best prices (p_1^*, p_2^*), G^{1^*} and G^{2^*} denote the cost of effort incurred by the retailers when exerting first best effort levles.

(iii) *The demand parameters of the asymmetric retailers are such that the following conditions are simultaneously satisfied*

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{Eq. 4.57}$$

$$\frac{(p_1^* - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{Eq. 4.58}$$

$$G^{1^*} \geq G^{2^*} \tag{Eq. 4.59}$$

The manufacturer can use the wholesale price, the participation rate α and the fixed accrual amount A to influence retail price and effort levels. The manufacturer can set a very high accrual A (and a low participation rate α) such that both the retailers expend marketing effort such that their reimbursement through the participation rate is less than the total accruals. In this scenario, the retailers are reimbursed αG^i. Here, the participation rate is the only parameter that influences retail efforts while the wholesale price is used to influence retail price. Hence, when conditions listed in proposition 1 are satisfied, the manufacturer can induce the retailers to exert first best efforts and charge first best prices.

Alternatively, the manufacturer can set a lower accrual amount A (and a higher participation rate α) such that both the retailers may expend marketing effort such that their reimbursement through the participation rate is equal to the cooperative dollars accrued. In this case, the manufacturer uses the accrual rate to influence retail effort and the wholesale price to influence retail prices. Since the manufacturer charges a single wholesale price, the following condition must be satisfied in order to induce both retailers to charge first best prices simultaneously

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \qquad \text{(Eq. 4.60)}$$

Also, in order for the manufacturer to induce both retailers to exert effort such that their reimbursement through the participation rate is equal to their accruals, the following conditions must be simultaneously satisfied

$$G^{1^*} = G^{2^*} \qquad \text{(Eq. 4.61)}$$

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1^*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2}Q^{1^*}}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}}\} \qquad \text{(Eq. 4.62)}$$

Hence, when the conditions in (Eq. 4.60)-(Eq. 4.62) are simultaneously satisfied, the manufacturer can induce both the retailers to charge first best prices and exert first best efforts.

Alternatively, the manufacturer can choose the participation rate and the accrual amount such that one retailer (say, R1) expends effort such that his reimbursement through the participation rate is lower than the total accrual and the other retailer (R2) expends effort such that his reimbursement through the participation rate is equal to the total accruals. In this setting, while R1 is reimbursed $\alpha G^1 < A$, R2 is reimbursed $\alpha G^2 = A$. Here, the manufacturer can use the participation rate to influence R1's effort and can use the accrual amount to influence R2's effort. The manufacturer then uses the single wholesale price to influence retail prices of both R1 and R2. The single wholesale price can induce both the asymmetric retailers to exert first best prices only when the following condition is satisfied

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2^* - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1^* - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{Eq. 4.63}$$

In order for the manufacturer to be able to induce R1 to exert effort such that R1's reimbursement less than the accruals while R2's reimbursement is equal to R2's accrual, the following conditions must be simultaneously satisfied

$$\frac{(p_1^* - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^{1*}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{Eq. 4.64}$$

$$G^{1^*} \geq G^{2^*} \tag{Eq. 4.65}$$

Hence, when conditions (Eq. 4.63)-(Eq. 4.65) are simultaneously satisfied, the manufacturer can induce first best efforts and prices.

In addition to the above described strategies, the manufacturer can offer a very low accrual amount A that can induce one or both retailers to exert effort such that their reimbursement through the participation rate is greater than the accruals. In this scenario, the retailers are reimbursed A. Since this reimbursement amounts is fixed, the retailers marginal costs are not altered by the reimbursement and hence will not influence retail behavior. Therefore, the manufacturer would not prefer to offer a very low accrual amount A.

When the asymmetry between the retailers is such that the conditions listed in proposition 3 are not satisfied, the manufacturer cannot simultaneously induce both the retailers to exert first efforts and prices.

In the next section, we investigate the conditions under which the use of variable accruals may be preferred to the use of fixed accruals when either of the contracts cannot achieve the first best solution (i.e., when the conditions in propositions 2 and 3 are not satisfied).

4.3.5 Comparing the Fixed and Variable Accrual Contracts

While the previous sections described how the two types of accrual rates, in conjunction with the participation rate can induce the first best solution, the conditions required for each contract to achieve the first best solution are less likely to be satisfied when the retailers are asymmetric. In the previous chapter, we noted that each type of accrual has a significantly different impact on retail prices and efforts than the other contract. When the manufacturer offers a cooperative advertising contract that includes a participation rate and variable accruals, the manufacturer can use the accrual rate δ to coordinate the efforts of one retailer (say R1) and use the participation rate α to coordinate the efforts of the other retailer (R2). In this context, the comparative statics of retail price and effort are given by

- $\frac{\partial p_1}{\partial w} > 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$,
- $\frac{\partial e_1}{\partial w} > 0$ when $\Delta^6 > 0$,
- $\frac{\partial p_1}{\partial \alpha} < 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$,
- $\frac{\partial e_1}{\partial \alpha} < 0$ when $\Delta^5 < 0$,
- $\frac{\partial p_1}{\partial \delta} < 0$ when $\Delta^1 < 0$,
- $\frac{\partial e_1}{\partial \delta} > 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$,
- $\frac{\partial p_2}{\partial w} > 0$, $\frac{\partial e_2}{\partial w} < 0$, $\frac{\partial p_2}{\partial \alpha} > 0$, $\frac{\partial e_2}{\partial \alpha} > 0$, $\frac{\partial p_2}{\partial \delta} = 0$ and $\frac{\partial e_2}{\partial \delta} = 0$.

where

$$\Delta^1 = -\frac{\partial Q^1}{\partial e_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})wQ^1 + \theta_1 w\frac{\partial Q^1}{\partial p_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})^2 +$$
$$\delta w\frac{\partial Q^1}{\partial p_1}\left[wQ^1(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2}) - \theta_1 w\frac{\partial Q^1}{\partial e_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})\right] < 0 \tag{Eq. 4.66}$$

$$\Delta^5 = -\left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right]G^1(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \delta w\frac{\partial Q^1}{\partial p_1}\left[\delta w\frac{\partial Q^1}{\partial p_1}(1-\theta_1)\frac{\partial G^1}{\partial e_1} + G^1\frac{\partial Q^1}{\partial e_1}\right], \tag{Eq. 4.67}$$

$$\Delta^6 = +(1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1}\delta w\frac{\partial Q^1}{\partial p_1}(\delta w\frac{\partial Q^1}{\partial p_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right]\delta Q^1(\delta q\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})$$
$$\delta w\frac{\partial Q^1}{\partial p_1}\left[-\delta w\frac{\partial Q^1}{\partial p_1}(1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1} - \delta Q^1\frac{\partial Q^1}{\partial e_1}\right]. \tag{Eq. 4.68}$$

When the manufacturer uses a cooperative advertising contract that includes a fixed accrual, the manufacturer can use the fixed accrual A to coordinate the efforts of one retailer (say R1) and use the participation rate α to coordinate the efforts of the other retailer (R2). In this context, the comparative statics of retail price and effort are given by

- $\frac{\partial p_1}{\partial w} > 0$, $\frac{\partial e_1}{\partial w} = 0$, $\frac{\partial p_1}{\partial \alpha} < 0$, $\frac{\partial e_1}{\partial \alpha} < 0$, $\frac{\partial p_1}{\partial A} > 0$, $\frac{\partial e_1}{\partial A} > 0$,
- $\frac{\partial p_2}{\partial w} > 0$, $\frac{\partial e_2}{\partial w} < 0$, $\frac{\partial p_2}{\partial \alpha} > 0$, $\frac{\partial e_2}{\partial \alpha} > 0$, $\frac{\partial p_2}{\partial A} = 0$, $\frac{\partial e_2}{\partial A} = 0$.

As we can see from the above comparative statics, the fixed and variable accrual contracts can have a significantly different impact on retail prices and efforts. Hence, when the manufacturer cannot achieve the first best solution using either of these contract, the following proposition notes the conditions under which one contract may be preferred to the other.

Proposition 4.9 *The manufacturer would prefer the variable accrual contract over the fixed accrual contract when the following conditions are satisfied*

$$p_1^\alpha > p_1^*, \qquad \text{(Eq. 4.69)}$$

$$\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0, \qquad \text{(Eq. 4.70)}$$

$$\Delta^1 < 0, \Delta^5 < 0 \; and \; \Delta^6 > 0 \qquad \text{(Eq. 4.71)}$$

where p_1^α is the retail price charged by $R1$ under a cooperative advertising contract that only includes a participation rate.

When the own effort and cross effort effects on the retailer's demand are high and when the cost of effort is low, the manufacturer using a cooperative advertising contract that only includes a participation rate may induce the retailers to charge prices that are greater than the first best prices. Since the manufacturer uses the participation rate to increase retail efforts, and since the participation rate also increases retail prices, the manufacturer may find it profitable to induce retail prices that are greater than first best levels. In this context, since the retailers are asymmetric, the manufacturer can use an additional instrument (accruals) to better coordinate retail efforts. The question then arises as to which accrual contract would result in greater profits.

As we can see from the comparative statics results, while both types of accruals have a similar effect on $R2$'s prices and efforts, an increase in the accrual rate δ induces $R1$'s to lower his price (when $\Delta^1 < 0$) while an increase in the fixed accrual A induces $R1$ to increase his price. Hence, when the manufacturer, using a participation rate only contract, prefers to induces $R1$ to exert prices greater than first best levels, the addition of the variable accrual rate to the contract induces the retailers to charge prices closer to first best levels. In addition, under the variable accrual contract, an increase in the wholesale price may induce (when $\Delta^6 > 0$) $R1$ to increase his effort levels. A similar increase in wholesale price under the fixed accrual contract would induce $R1$ to lower his effort levels.

Hence, the manufacturer, by using the variable accrual contract, can simultaneously induce $R1$'s price and effort levels to be more closer to the corresponding first best levels compared to the case where only a participation rate contract is used. This cannot be achieved using the fixed accrual contract. In fact, the retail prices and efforts of $R1$ are further way from the first best levels under the fixed accrual contract compared to the participation rate only contract. Since $R2$'s efforts are coordinated using the participation rate, and since the impact of the participation rate and wholesale price on $R2$'s choices are identical under the two types of accruals, using either of the contracts will not change $R2$'s effort and price choices. Hence, when the conditions listed in proposition 3 are satisfied, the manufacturer prefers the variable accrual contract over the fixed accrual contract.

4.4 Conclusion

Cooperative advertising contracts are frequently used by manufacturers to induce retailers to increase their advertising intensity. In this essay, we provided guidelines for the use of various cooperative advertising strategies and detailed the conditions under which each type of cooperative advertising contract can lead to greater profits to the manufacturer. When dealing with asymmetric retailers, the manufacturer can achieve coordination only when stringent conditions are met. When such conditions are not met, the manufacturer may prefer one type of cooperative advertising to the other. The preference of the type of contract depends to the nature of asymmetry and the extent of free-riding amongst the retailers. Hence, manufacturers designing cooperative advertising contracts must take these considerations into account.

APPENDIX A: PROOFS FOR RESULTS IN CHAPTER 2

Proof of Lemma 1 (First Best Setting)

Recalling IR_L^k, IR_H^k and equation (Eq. 2.1) from the main text, the principal's problem in this first best setting, denoted by [P-FB] is:

$$\max_{\{e,T\}} \quad \Pi^P \tag{A1}$$

$$\text{subject to} \quad \pi^k_{m|m} \geq 0 \quad \forall \quad k \in \{A, B\}, m \in \{L, H\} \tag{IR^k_m}$$

The Lagrangian for [P-FB] is given by:

$$\begin{aligned} L = {} & \phi_H^A E[T_H^A] + (1-\phi_H^A)E[T_L^A] + \phi_H^B E[T_H^B] + (1-\phi_H^B)E[T_L^B] + \\ & \lambda_1 \left[e_L^A \theta_L^A + \delta E[e^B] - \frac{(e_L^A)^2}{2\theta_L^A} - E[T_L^A] \right] + \lambda_2 \left[e_H^A \theta_H^A + \delta E[e^B] - \frac{(e_H^A)^2}{2\theta_H^A} - E[T_H^A] \right] + \\ & \lambda_3 \left[e_L^B \theta_L^B + \delta E[e^A] - \frac{(e_L^B)^2}{2\theta_L^B} - E[T_L^B] \right] + \lambda_4 \left[e_L^B \theta_L^B + \delta E[e^A] - \frac{(e_H^B)^2}{2\theta_H^B} - E[T_H^B] \right]. \end{aligned} \tag{A2}$$

Assuming an interior solution, the first order conditions for the principal's optimization problem are:

$$\frac{\partial L}{\partial e_L^A} = \lambda_1 \left[\theta_L^A - \frac{e_L^A}{\theta_L^A} \right] + \delta \phi_L^A \left[\lambda_3 + \lambda_4 \right] = 0, \tag{A3}$$

$$\frac{\partial L}{\partial e_H^A} = \lambda_2 \left[\theta_H^A - \frac{e_H^A}{\theta_H^A} \right] + \delta \phi_H^A \left[\lambda_3 + \lambda_4 \right] = 0, \tag{A4}$$

$$\frac{\partial L}{\partial e_L^B} = \lambda_3 \left[\theta_L^B - \frac{e_L^B}{\theta_L^B} \right] + \delta \phi_L^B \left[\lambda_1 + \lambda_2 \right] = 0, \tag{A5}$$

$$\frac{\partial L}{\partial e_H^B} = \lambda_4 \left[\theta_H^B - \frac{e_H^B}{\theta_H^B} \right] + \delta \phi_H^B \left[\lambda_1 + \lambda_2 \right] = 0, \tag{A6}$$

$$\frac{\partial L}{\partial T^A_{LL}} = \frac{\partial L}{\partial E[T^A_L]}\frac{\partial E[T^A_L]}{\partial T^A_{LL}} = (\phi^A_L - \lambda_1)\phi^B_L = 0, \tag{A7}$$
$$\frac{\partial L}{\partial T^A_{LH}} = \frac{\partial L}{\partial E[T^A_L]}\frac{\partial E[T^A_L]}{\partial T^A_{LH}} = (\phi^A_L - \lambda_1)\phi^B_H = 0, \tag{A8}$$
$$\frac{\partial L}{\partial T^A_{HL}} = \frac{\partial L}{\partial E[T^A_H]}\frac{\partial E[T^A_H]}{\partial T^A_{HL}} = (\phi^A_H - \lambda_2)\phi^B_L = 0, \tag{A9}$$
$$\frac{\partial L}{\partial T^A_{HH}} = \frac{\partial L}{\partial E[T^A_H]}\frac{\partial E[T^A_H]}{\partial T^A_{HH}} = (\phi^A_H - \lambda_2)\phi^B_H = 0, \tag{A10}$$
$$\frac{\partial L}{\partial T^B_{LL}} = \frac{\partial L}{\partial E[T^B_L]}\frac{\partial E[T^B_L]}{\partial T^B_{LL}} = (\phi^B_L - \lambda_3)\phi^A_L = 0, \tag{A11}$$
$$\frac{\partial L}{\partial T^B_{LH}} = \frac{\partial L}{\partial E[T^B_L]}\frac{\partial E[T^B_L]}{\partial T^B_{LH}} = (\phi^B_L - \lambda_3)\phi^A_H = 0, \tag{A12}$$
$$\frac{\partial L}{\partial T^B_{HL}} = \frac{\partial L}{\partial E[T^B_H]}\frac{\partial E[T^B_H]}{\partial T^B_{HL}} = (\phi^B_H - \lambda_4)\phi^A_L = 0, \tag{A13}$$
$$\frac{\partial L}{\partial T^B_{HH}} = \frac{\partial L}{\partial E[T^B_H]}\frac{\partial E[T^B_H]}{\partial T^B_{HH}} = (\phi^B_H - \lambda_4)\phi^A_H = 0, \tag{A14}$$
$$\frac{\partial L}{\partial \lambda_1} = e^A_L\theta^A_L + \delta E[e^B] - \frac{(e^A_L)^2}{2\theta^A_L} - E[T^A_L] \geq 0 \quad \text{and} \qquad \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \tag{A15}$$
$$\frac{\partial L}{\partial \lambda_2} = e^A_H\theta^A_H + \delta E[e^B] - \frac{(e^A_H)^2}{2\theta^A_H} - E[T^A_H] \geq 0 \quad \text{and} \qquad \lambda_2\frac{\partial L}{\partial \lambda_2} = 0, \tag{A16}$$
$$\frac{\partial L}{\partial \lambda_3} = e^B_L\theta^B_L + \delta E[e^A] - \frac{(e^B_L)^2}{2\theta^B_L} - E[T^B_L] \geq 0 \quad \text{and} \qquad \lambda_3\frac{\partial L}{\partial \lambda_3} = 0, \tag{A17}$$
$$\frac{\partial L}{\partial \lambda_4} = e^B_H\theta^B_H + \delta E[e^A] - \frac{(e^B_H)^2}{2\theta^B_H} - E[T^B_H] \geq 0 \quad \text{and} \qquad \lambda_4\frac{\partial L}{\partial \lambda_4} = 0. \tag{A18}$$

From equations (A7)-(A14), we can see that $\lambda_1 = \phi^A_L$, $\lambda_2 = \phi^A_H$, $\lambda_3 = \phi^B_L$ and $\lambda_4 = \phi^B_H$. Substituting the values of the Lagrange multipliers into equations (A3)-(A6), we get

$$\theta^k_L = \frac{e^k_L}{\theta^k_L} - \delta \quad \Rightarrow \quad e^k_L = \theta^k_L(\theta^k_L + \delta), \tag{A19}$$
$$\theta^k_H = \frac{e^k_H}{\theta^k_H} - \delta \quad \Rightarrow \quad e^k_H = \theta^k_H(\theta^k_H + \delta). \tag{A20}$$

Subsequently, equations (A15)-(A18) reduce to

$$e^k_L\theta^k_L + \delta E[e^l] - \frac{(e^k_L)^2}{2\theta^k_L} - E[T^k_L] = 0 \quad \Rightarrow \quad E[T^k_L] = e^k_L\theta^k_L + \delta E[e^l] - \frac{(e^k_L)^2}{2\theta^k_L}, \tag{A21}$$

$$e^k_H\theta^k_H + \delta E[e^l] - \frac{(e^k_H)^2}{2\theta^k_H} - E[T^k_H] = 0 \quad \Rightarrow \quad E[T^k_H] = e^k_H\theta^k_H + \delta E[e^l] - \frac{(e^k_H)^2}{2\theta^k_H}, \tag{A22}$$

and the statement of the Lemma 1 follows. ■

Proof of Lemma 2 (Input Monitoring Setting)

Here, the partner can commit to a certain effort level while misrepresenting its private information. For example, a partner in the high (or more favorable) state could lie and report a low state and thereby be required to exert a lower effort level (corresponding to the lower state). Hence, the partner's profits under the various state-disclosure[6] conditions are given by

$$\pi^{k,I}_{L|L} = e^k_L\theta^k_L + \delta E[e^l] - \frac{(e^k_L)^2}{2\theta^k_L} - E[T^k_L], \tag{A23}$$

$$\pi^{k,I}_{H|L} = e^k_H\theta^k_L + \delta E[e^l] - \frac{(e^k_H)^2}{2\theta^k_L} - E[T^k_H], \tag{A24}$$

$$\pi^{k,I}_{H|H} = e^k_H\theta^k_H + \delta E[e^l] - \frac{(e^k_H)^2}{2\theta^k_H} - E[T^k_H], \tag{A25}$$

$$\pi^{k,I}_{L|H} = e^k_L\theta^k_H + \delta E[e^l] - \frac{(e^k_L)^2}{2\theta^k_H} - E[T^k_L]. \tag{A26}$$

Recalling [P-IM] from the main text, we anticipate that the conditions $IR^{A,I}_H$, $IR^{B,I}_H$, $IC^{A,I}_L$ and $IC^{B,I}_L$ are not likely to bind at the solution. Accordingly, we focus on the setting where the principal considers only $IR^{A,I}_L$, $IR^{B,I}_L$, $IC^{A,I}_H$ and $IC^{B,I}_H$ and refer to that 'reduced' problem

[6] We assume that each partner reports its private information as a best response to the other partner reporting its private information truthfully.

as [P-IM-R]. Later, we verify that the remaining constraints, $IR_H^{A,I}$, $IR_H^{B,I}$, $IC_L^{A,I}$ and $IC_L^{B,I}$ are indeed satisfied at the solution to [P-IM-R]. The Lagrangian for [P-IM-R] is:

$$
\begin{aligned}
L = {} & \phi_H^A E[T_H^A] + (1-\phi_H^A)E[T_L^A] + \phi_H^B E[T_H^B] + (1-\phi_H^B)E[T_L^B] + \\
& \lambda_1 \left[e_L^A \theta_L^A + \delta E[e^B] - \frac{(e_L^A)^2}{2\theta_L^A} - E[T_L^A] \right] + \lambda_2 \left[e_L^B \theta_L^B + \delta E[e^A] - \frac{(e_L^B)^2}{2\theta_L^B} - E[T_L^B] \right] + \\
& \mu_1 \left[e_H^A \theta_H^A + \delta E[e^B] - \frac{(e_H^A)^2}{2\theta_H^A} - E[T_H^A] - \left(e_L^A \theta_H^A + \delta E[e^B] - \frac{(e_L^A)^2}{2\theta_H^A} - E[T_L^A] \right) \right] + \\
& \mu_2 \left[e_H^B \theta_H^B + \delta E[e^A] - \frac{(e_H^B)^2}{2\theta_H^B} - E[T_H^B] - \left(e_L^B \theta_H^B + \delta E[e^A] - \frac{(e_L^B)^2}{2\theta_H^B} - E[T_L^B] \right) \right]. \qquad \text{(A27)}
\end{aligned}
$$

Assuming an interior solution, the first order conditions are given by

$$
\begin{aligned}
\frac{\partial L}{\partial e_L^A} &= \lambda_1 \left[\theta_L^A - \frac{e_L^A}{\theta_L^A} \right] + \lambda_2 \delta \phi_L^A - \mu_1 \left(\theta_H^A - \frac{e_L^A}{\theta_H^A} \right) = 0, && \text{(A28)} \\
\frac{\partial L}{\partial e_H^A} &= \mu_1 \left[\theta_H^A - \frac{e_H^A}{\theta_H^A} \right] + \lambda_2 \delta \phi_H^A = 0, && \text{(A29)} \\
\frac{\partial L}{\partial e_L^B} &= \lambda_2 \left[\theta_L^B - \frac{e_L^B}{\theta_L^B} \right] + \lambda_1 \delta \phi_L^B - \mu_2 \left(\theta_H^B - \frac{e_L^B}{\theta_H^B} \right) = 0, && \text{(A30)} \\
\frac{\partial L}{\partial e_H^B} &= \mu_2 \left[\theta_H^B - \frac{e_H^B}{\theta_H^B} \right] + \lambda_1 \delta \phi_H^B = 0, && \text{(A31)} \\
\frac{\partial L}{\partial T_{LL}^A} &= \frac{\partial L}{\partial E[T_L^A]} \frac{\partial E[T_L^A]}{\partial T_{LL}^A} = (\phi_L^A - \lambda_1 + \mu_1)\phi_L^B = 0, && \text{(A32)} \\
\frac{\partial L}{\partial T_{LH}^A} &= \frac{\partial L}{\partial E[T_L^A]} \frac{\partial E[T_L^A]}{\partial T_{LH}^A} = (\phi_L^A - \lambda_1 + \mu_1)\phi_H^B = 0, && \text{(A33)} \\
\frac{\partial L}{\partial T_{HL}^A} &= \frac{\partial L}{\partial E[T_H^A]} \frac{\partial E[T_H^A]}{\partial T_{HL}^A} = (\phi_H^A - \mu_1)\phi_L^B = 0, && \text{(A34)} \\
\frac{\partial L}{\partial T_{HH}^A} &= \frac{\partial L}{\partial E[T_H^A]} \frac{\partial E[T_H^A]}{\partial T_{HH}^A} = (\phi_H^A - \mu_1)\phi_H^B = 0, && \text{(A35)} \\
\frac{\partial L}{\partial T_{LL}^B} &= \frac{\partial L}{\partial E[T_L^B]} \frac{\partial E[T_L^B]}{\partial T_{LL}^B} = (\phi_L^B - \lambda_2 + \mu_2)\phi_L^A = 0, && \text{(A36)}
\end{aligned}
$$

$$\frac{\partial L}{\partial T^B_{LH}} = \frac{\partial L}{\partial E[T^B_L]}\frac{\partial E[T^B_L]}{\partial T^B_{LH}} = (\phi^B_L - \lambda_2 + \mu_2)\phi^A_H = 0, \tag{A37}$$

$$\frac{\partial L}{\partial T^B_{HL}} = \frac{\partial L}{\partial E[T^B_H]}\frac{\partial E[T^B_H]}{\partial T^B_{HL}} = (\phi^B_H - \mu_2)\phi^A_L = 0, \tag{A38}$$

$$\frac{\partial L}{\partial T^B_{HH}} = \frac{\partial L}{\partial E[T^B_H]}\frac{\partial E[T^B_H]}{\partial T^B_{HH}} = (\phi^B_H - \mu_2)\phi^A_H = 0, \tag{A39}$$

$$\frac{\partial L}{\partial \lambda_1} = e^A_L\theta^A_L + \delta E[e^B] - \frac{(e^A_L)^2}{2\theta^A_L} - E[T^A_L] \geq 0 \quad \text{and} \qquad \lambda_1\frac{\partial L}{\partial \lambda_1} = 0, \tag{A40}$$

$$\frac{\partial L}{\partial \lambda_2} = e^B_L\theta^B_L + \delta E[e^A] - \frac{(e^B_L)^2}{2\theta^B_L} - E[T^B_L] \geq 0 \quad \text{and} \qquad \lambda_2\frac{\partial L}{\partial \lambda_2} = 0, \tag{A41}$$

$$\frac{\partial L}{\partial \mu_1} = e^A_H\theta^A_H + \delta E[e^B] - \frac{(e^A_H)^2}{2\theta^A_H} - E[T^A_H] - \left(e^A_L\theta^A_H + \delta E[e^B] - \frac{(e^A_L)^2}{2\theta^A_H} - E[T^A_L]\right) \geq 0$$
$$\text{and} \qquad \mu_1\frac{\partial L}{\partial \mu_1} = 0, \tag{A42}$$

$$\frac{\partial L}{\partial \mu_2} = e^B_H\theta^B_H + \delta E[e^A] - \frac{(e^B_H)^2}{2\theta^B_H} - E[T^B_H] - \left(e^B_L\theta^B_H + \delta E[e^A] - \frac{(e^B_L)^2}{2\theta^B_H} - E[T^B_L]\right) \geq 0$$
$$\text{and} \qquad \mu_2\frac{\partial L}{\partial \mu_2} = 0. \tag{A43}$$

Solving for the Lagrange multipliers from equations (A32)-(A39), gives:

$$\lambda_1 = \phi^A_H + \phi^A_L = 1, \tag{A44}$$

$$\lambda_2 = \phi^B_H + \phi^B_L = 1, \tag{A45}$$

$$\mu_1 = \phi^A_H, \tag{A46}$$

$$\mu_2 = \phi^B_H. \tag{A47}$$

Substituting the values of the Lagrangian multipliers in equations (A28)-(A31), gives:

$$\frac{\partial L}{\partial e^A_L} = \left[\theta^A_L - \frac{e^A_L}{\theta^A_L}\right] + \delta\phi^A_L - \phi^A_H\left(\theta^A_H - \frac{e^A_L}{\theta^A_H}\right) = 0, \tag{A48}$$

$$\frac{\partial L}{\partial e^A_H} = \phi^A_H\left[\theta^A_H - \frac{e^A_H}{\theta^A_H}\right] + \delta\phi^A_H = 0, \tag{A49}$$

$$\frac{\partial L}{\partial e_L^B} = \left[\theta_L^B - \frac{e_L^B}{\theta_L^B}\right] + \delta\phi_L^B - \phi_H^B\left(\theta_H^B - \frac{e_L^B}{\theta_H^B}\right) = 0, \tag{A50}$$

$$\frac{\partial L}{\partial e_H^B} = \phi_H^B\left[\theta_H^B - \frac{e_H^B}{\theta_H^B}\right] + \delta\phi_H^B = 0. \tag{A51}$$

Simplifying equations (A48)-(A51), we can see that the optimal effort levels satisfy the following conditions:

$$\theta_L^k = \frac{e_L^{k,I}}{\theta_L^k} - \phi_L^k\delta + \phi_H^k\left(\theta_H^k - \frac{e_L^{k,I}}{\theta_H^k}\right), \tag{A52}$$

$$\theta_H^k = \frac{e_H^{k,I}}{\theta_H^k} - \delta. \tag{A53}$$

Solving equations (A52)-(A53) for the optimal effort levels, gives:

$$e_L^{k,I} = \frac{\theta_H^k\theta_L^k(\delta\phi_L^k + \theta_L^k - \phi_H^k\theta_H^k)}{\theta_H^k - \phi_H^k\theta_L^k}, \tag{A54}$$

$$e_H^{k,I} = \theta_H^k(\theta_H^k + \delta). \tag{A55}$$

Using equations (A44)-(A47), we note that the inequalities in equations (A40)-(A43) will hold as equalities; that is:

$$e_L^{k,I}\theta_L^k + \delta E[e^{l,I}] - \frac{(e_L^{k,I})^2}{2\theta_L^k} - E[T_L^{k,I}] = 0, \tag{A56}$$

$$e_H^{k,I}\theta_H^k + \delta E[e^{l,I}] - \frac{(e_H^{k,I})^2}{2\theta_H^k} - E[T_H^{k,I}] - \left(e_L^{k,I}\theta_H^k + \delta E[e^{l,I}] - \frac{(e_L^{k,I})^2}{2\theta_H^k} - E[T_L^{k,I}]\right) = 0. \tag{A57}$$

Solving equations (A56)-(A57) for the optimal transfers, gives:

$$E[T_L^{k,I}] = e_L^{k,I}\theta_L^k + \delta E[e^{l,I}] - \frac{(e_L^{k,I})^2}{2\theta_L^k}, \tag{A58}$$

$$E[T_H^{k,I}] = e_H^{k,I}\theta_H^k + \delta E[e^{l,I}] - \frac{(e_H^{k,I})^2}{2\theta_H^k} - [e_L^{k,I}\theta_H^k - e_L^{k,I}\theta_L^k] - \left[\frac{(e_L^{k,I})^2}{2\theta_L^k} - \frac{(e_L^{k,I})^2}{2\theta_H^k}\right]. \tag{A59}$$

From equations (A58)-(A59) we can see that the principal extracts all rents from the partners in the lower state and either partner in the higher state accrues rents, $R_H^{k,I}$ as follows:

$$R_H^{A,I} = [e_L^{A,I}\theta_H^A - e_L^{A,I}\theta_L^A] + \left[\frac{(e_L^{A,I})^2}{2\theta_L^A} - \frac{(e_L^{A,I})^2}{2\theta_H^A}\right], \quad \text{and} \tag{A60}$$

$$R_H^{B,I} = [e_L^{B,I}\theta_H^B - e_L^{B,I}\theta_L^B] + \left[\frac{(e_L^{B,I})^2}{2\theta_L^B} - \frac{(e_L^{B,I})^2}{2\theta_H^B}\right]. \tag{A61}$$

Notice that the rents have two components : 1) rent due to extra output $[e_L^k\theta_H^k - e_L^k\theta_L^k]$ and 2) rent due to lower cost of effort $\left[\frac{(e_L^{A,I})^2}{2\theta_H^A} - \frac{(e_L^{A,I})^2}{2\theta_L^A}\right]$. Next, inspecting equations (A54)-(A55) and (A19)-(A20) we can see that under input monitoring, the principal induces the partners in the lower state to exert lower effort than in the first best scenario, while the partners in the higher state exert the first best effort levels. We now verify that the $IR_H^{A,I}$, $IR_H^{B,I}$, $IC_L^{A,I}$ and $IC_L^{B,I}$ conditions indeed hold at the above solution. Using the transfer payments in equations (A58)-(A59) and the expressions for the partner's profits in equations (A23)-(A26), and noting that $e_H^k > e_L^k$, we can see that

$$\pi_{H|H}^{A,I} = [e_L^{A,I}\theta_H^A - e_L^{A,I}\theta_L^A] + \left[\frac{(e_L^{A,I})^2}{2\theta_L^A} - \frac{(e_L^{A,I})^2}{2\theta_H^A}\right] > 0, \tag{A62}$$

$$\pi_{H|H}^{B,I} = [e_L^{B,I}\theta_H^B - e_L^{B,I}\theta_L^B] + \left[\frac{(e_L^{B,I})^2}{2\theta_L^B} - \frac{(e_L^{B,I})^2}{2\theta_H^B}\right] > 0, \tag{A63}$$

$$\pi_{L|L}^{A,I} - \pi_{H|L}^{A,I} = (e_H^{A,I} - e_L^{A,I})(\theta_H^A - \theta_L^A) + \frac{(\theta_H^A - \theta_L^A)\left[(e_H^{A,I})^2 - (e_L^{A,I})^2\right]}{2\theta_H^A\theta_L^A} > 0$$

$$\Rightarrow \quad \pi_{L|L}^{A,I} > \pi_{H|L}^{A,I}, \quad \text{and} \tag{A64}$$

$$\pi_{L|L}^{B,I} - \pi_{H|L}^{B,I} = (e_H^{B,I} - e_L^{B,I})(\theta_H^B - \theta_L^B) + \frac{(\theta_H^B - \theta_L^B)\left[(e_H^{B,I})^2 - (e_L^{B,I})^2\right]}{2\theta_H^B\theta_L^B} > 0$$

$$\Rightarrow \quad \pi_{L|L}^{B,I} > \pi_{H|L}^{B,I}. \tag{A65}$$

Hence, the solution to [P-IM-R] is also a solution to [P-IM] and the statement of Lemma 2 follows. ■

Proof of Lemma 3 (Output Monitoring Setting)

Here, the principal specifies and monitors the output produced by the partner. Given the output requirements (Q_H^A, Q_L^A, Q_H^B and Q_L^B) in the contract menu, we can see that

$$Q_H^A = Q(e_H^A, e^B, \theta_H^A) = e_H^A\theta_H^A + \delta(\phi_H^B e_H^B + \phi_L^B e_L^B), \tag{A66}$$

$$Q_L^A = Q(e_L^A, e^B, \theta_L^A) = e_L^A\theta_L^A + \delta(\phi_H^B e_H^B + \phi_L^B e_L^B), \tag{A67}$$

$$Q_H^B = Q(e_H^B, e^A, \theta_H^B) = e_H^B\theta_H^B + \delta(\phi_H^A e_H^A + \phi_L^A e_L^A), \tag{A68}$$

$$Q_L^B = Q(e_L^B, e^A, \theta_L^B) = e_L^B\theta_L^B + \delta(\phi_H^A e_H^A + \phi_L^A e_L^A). \tag{A69}$$

Solving for e_H^A, e_L^A, e_H^B and e_L^B from the equations above, we have

$$e_H^A = \frac{\delta\theta_L^A(\phi_L^B Q_L^B\theta_H^B + \phi_H^B Q_H^B\theta_L^B) + \delta^2\phi_L^A(Q_H^A - Q_L^A)(\phi_L^B\theta_H^B + \phi_H^B\theta_L^B) - Q_H^A\theta_L^A\theta_H^B\theta_L^B}{\delta^2(\phi_L^A\theta_H^A + \phi_H^A\theta_L^A)(\phi_L^B\theta_H^B + \phi_H^B\theta_L^B) - \theta_H^A\theta_L^A\theta_H^B\theta_L^B}, \tag{A70}$$

$$e_L^A = \frac{\delta\theta_H^A(\phi_L^B Q_L^B\theta_H^B + \phi_H^B Q_H^B\theta_L^B) - \delta^2\phi_H^A(Q_H^A - Q_L^A)(\phi_L^B\theta_H^B + \phi_H^B\theta_L^B) - Q_L^A\theta_H^A\theta_H^B\theta_L^B}{\delta^2(\phi_L^A\theta_H^A + \phi_H^A\theta_L^A)(\phi_L^B\theta_H^B + \phi_H^B\theta_L^B) - \theta_H^A\theta_L^A\theta_H^B\theta_L^B}, \tag{A71}$$

$$e_H^B = \frac{\delta\theta_L^B(\phi_L^A Q_L^A\theta_H^A + \phi_H^A Q_H^A\theta_L^A) + \delta^2\phi_L^B(Q_H^B - Q_L^B)(\phi_L^A\theta_H^A + \phi_H^A\theta_L^A) - Q_H^B\theta_L^B\theta_H^A\theta_L^A}{\delta^2(\phi_L^A\theta_H^A + \phi_H^A\theta_L^A)(\phi_L^B\theta_H^B + \phi_H^B\theta_L^B) - \theta_H^A\theta_L^A\theta_H^B\theta_L^B}, \tag{A72}$$

$$e_L^B = \frac{\delta\theta_H^B(\phi_L^A Q_L^A\theta_H^A + \phi_H^A Q_H^A\theta_L^A) - \delta^2\phi_H^B(Q_H^B - Q_L^B)(\phi_L^A\theta_H^A + \phi_H^A\theta_L^A) - Q_L^B\theta_H^B\theta_H^A\theta_L^A}{\delta^2(\phi_L^A\theta_H^A + \phi_H^A\theta_L^A)(\phi_L^B\theta_H^B + \phi_H^B\theta_L^B) - \theta_H^A\theta_L^A\theta_H^B\theta_L^B}. \tag{A73}$$

Hence, given a set of outputs, the principal can choose a set of effort levels that will result in the desired output levels. In what follows, we will assume that the principal monitors the output and specifies the required input for the partners. This allows for easy comparison of input and output monitoring.

Under output monitoring, a partner in state m can lie to the principal and claim to be in state n (where $m, n \in \{L, H\}, m \neq n$). When partner k declares to be in state n and given a particular effort level by partner l (denoted by e^l), the principal would require partner k

to produce $Q(e_n^k, e^l, \theta_n^k) = e_n^k\theta_n^k + \delta e^l$. For instance, if partner k is in the higher state but reports to be in the lower state, the required output will be: $Q(e_L^k, e^l, \theta_L^k) = e_L^k\theta_L^k + \delta e^l$. However, k's effort to produce that output will be determined as follows: $Q(\tilde{e}_H^k, e^l, \theta_H^k) = \tilde{e}_H^k\theta_H^k + \delta e^l = e_L^k\theta_L^k + \delta e^l$. Since $Q_3 > 0$, it follows that $\tilde{e}_H^k = \frac{e_L^k\theta_L^k}{\theta_H^k} < e_L^k$. Similarly, we can see that $\tilde{e}_L^k = \frac{e_H^k\theta_H^k}{\theta_L^k} > e_H^k$. Hence, the partners' profits under the various state-disclosure conditions are given by:

$$\pi_{L|L}^{k,O} = e_L^k\theta_L^k + \delta E[e^l] - \frac{(e_L^k)^2}{2\theta_L^k} - E[T_L^k], \tag{A74}$$

$$\pi_{H|L}^{k,O} = Q^k(\tilde{e}_L^k, E[e^l], \theta_L^k) - \frac{(e_H^k\theta_H^k)^2}{2(\theta_L^k)^3} - E[T_H^k] = e_H^k\theta_H^k + \delta E[e^l] - \frac{(e_H^k\theta_H^k)^2}{2(\theta_L^k)^3} - E[T_H^k], \tag{A75}$$

$$\pi_{H|H}^{k,O} = e_H^k\theta_H^k + \delta E[e^l] - \frac{(e_H^k)^2}{2\theta_H^k} - E[T_H^k], \tag{A76}$$

$$\pi_{L|H}^{k,O} = Q^k(\tilde{e}_H^k, E[e^l], \theta_H^k) - \frac{(e_L^k\theta_L^k)^2}{2(\theta_H^k)^3} - E[T_L^k] = e_L^k\theta_L^k + \delta E[e^l] - \frac{(e_L^k\theta_L^k)^2}{2(\theta_H^k)^3} - E[T_L^k]. \tag{A77}$$

As in the input monitoring setting, here too, since the partners can lie, the principal needs to design the contract (effort/transfer payment pair) such that truthful revelation is induced while guaranteeing reservation utility to the partners. Recalling [P-OM] from the main text, here too, we focus on the simpler problem [P-OM-R] where we consider $IR_L^{A,O}$, $IR_L^{B,O}$, $IC_H^{A,O}$ and $IC_H^{B,O}$, and later verify that the other constraints ($IR_H^{A,O}$, $IR_H^{B,O}$, $IC_L^{A,O}$ and $IC_L^{B,O}$) are automatically satisfied at the solution to [P-OM-R]. The Lagrangian for [P-OM-R] is:

$$\begin{aligned} L = {} & \phi_H^A E[T_H^A] + (1-\phi_H^A)E[T_L^A] + \phi_H^B E[T_H^B] + (1-\phi_H^B)E[T_L^B] + \\ & \lambda_1 \left[e_L^A\theta_L^A + \delta E[e^B] - \frac{(e_L^A)^2}{2\theta_L^A} - E[T_L^A] \right] + \lambda_2 \left[e_L^B\theta_L^B + \delta E[e^A] - \frac{(e_L^B)^2}{2\theta_L^B} - E[T_L^B] \right] + \\ & \mu_1 \left[e_H^A\theta_H^A + \delta E[e^B] - \frac{(e_H^A)^2}{2\theta_H^A} - E[T_H^A] - \left(e_L^A\theta_L^A + \delta E[e^B] - \frac{(e_L^A\theta_L^A)^2}{2(\theta_H^A)^3} - E[T_L^A] \right) \right] + \\ & \mu_2 \left[e_H^B\theta_H^B + \delta E[e^A] - \frac{(e_H^B)^2}{2\theta_H^B} - E[T_H^B] - \left(e_L^B\theta_L^B + \delta E[e^A] - \frac{(e_L^B\theta_L^B)^2}{2(\theta_H^B)^3} - E[T_L^B] \right) \right]. \end{aligned} \tag{A78}$$

Assuming an interior solution, the first order conditions are given by

$$\frac{\partial L}{\partial e^A_L} = \lambda_1 \left[\theta^A_L - \frac{e^A_L}{\theta^A_L}\right] + \lambda_2 \delta \phi^A_L - \mu_1 \left(\theta^A_L - \frac{e^A_L (\theta^A_L)^2}{(\theta^A_H)^3}\right) = 0, \tag{A79}$$

$$\frac{\partial L}{\partial e^A_H} = \mu_1 \left[\theta^A_H - \frac{e^A_H}{\theta^A_H}\right] + \lambda_2 \delta \phi^A_H = 0, \tag{A80}$$

$$\frac{\partial L}{\partial e^B_L} = \lambda_2 \left[\theta^B_L - \frac{e^B_L}{\theta^B_L}\right] + \lambda_1 \delta \phi^B_L - \mu_2 \left(\theta^B_L - \frac{e^B_L (\theta^B_L)^2}{(\theta^B_H)^3}\right) = 0, \tag{A81}$$

$$\frac{\partial L}{\partial e^B_H} = \mu_2 \left[\theta^B_H - \frac{e^B_H}{\theta^B_H}\right] + \lambda_1 \delta \phi^B_H = 0, \tag{A82}$$

$$\frac{\partial L}{\partial T^A_{LL}} = \frac{\partial L}{\partial E[T^A_L]} \frac{\partial E[T^A_L]}{\partial T^A_{LL}} = (\phi^A_L - \lambda_1 + \mu_1)\phi^B_L = 0, \tag{A83}$$

$$\frac{\partial L}{\partial T^A_{LH}} = \frac{\partial L}{\partial E[T^A_L]} \frac{\partial E[T^A_L]}{\partial T^A_{LH}} = (\phi^A_L - \lambda_1 + \mu_1)\phi^B_H = 0, \tag{A84}$$

$$\frac{\partial L}{\partial T^A_{HL}} = \frac{\partial L}{\partial E[T^A_H]} \frac{\partial E[T^A_H]}{\partial T^A_{HL}} = (\phi^A_H - \mu_1)\phi^B_L = 0, \tag{A85}$$

$$\frac{\partial L}{\partial T^A_{HH}} = \frac{\partial L}{\partial E[T^A_H]} \frac{\partial E[T^A_H]}{\partial T^A_{HH}} = (\phi^A_H - \mu_1)\phi^B_H = 0, \tag{A86}$$

$$\frac{\partial L}{\partial T^B_{LL}} = \frac{\partial L}{\partial E[T^B_L]} \frac{\partial E[T^B_L]}{\partial T^B_{LL}} = (\phi^B_L - \lambda_2 + \mu_2)\phi^A_L = 0, \tag{A87}$$

$$\frac{\partial L}{\partial T^B_{LH}} = \frac{\partial L}{\partial E[T^B_L]} \frac{\partial E[T^B_L]}{\partial T^B_{LH}} = (\phi^B_L - \lambda_2 + \mu_2)\phi^A_H = 0, \tag{A88}$$

$$\frac{\partial L}{\partial T^B_{HL}} = \frac{\partial L}{\partial E[T^B_H]} \frac{\partial E[T^B_H]}{\partial T^B_{HL}} = (\phi^B_H - \mu_2)\phi^A_L = 0, \tag{A89}$$

$$\frac{\partial L}{\partial T^B_{HH}} = \frac{\partial L}{\partial E[T^B_H]} \frac{\partial E[T^B_H]}{\partial T^B_{HH}} = (\phi^B_H - \mu_2)\phi^A_H = 0, \tag{A90}$$

$$\frac{\partial L}{\partial \lambda_1} = e^A_L \theta^A_L + \delta E[e^B] - \frac{(e^A_L)^2}{2\theta^A_L} - E[T^A_L] \geq 0 \quad \text{and} \qquad \lambda_1 \frac{\partial L}{\partial \lambda_1} = 0, \tag{A91}$$

$$\frac{\partial L}{\partial \lambda_2} = e^B_L \theta^B_L + \delta E[e^A] - \frac{(e^B_L)^2}{2\theta^B_L} - E[T^B_L] \geq 0 \quad \text{and} \qquad \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0, \tag{A92}$$

$$\frac{\partial L}{\partial \mu_1} = e^A_H \theta^A_H + \delta E[e^B] - \frac{(e^A_H)^2}{2\theta^A_H} - E[T^A_H] - \left(e^A_L \theta^A_L + \delta E[e^B] - \frac{(e^A_L \theta^A_L)^2}{2(\theta^A_H)^3} - E[T^A_L]\right) \geq 0$$
$$\text{and} \qquad \mu_1 \frac{\partial L}{\partial \mu_1} = 0, \tag{A93}$$

$$\frac{\partial L}{\partial \mu_2} = e^B_H \theta^B_H + \delta E[e^A] - \frac{(e^B_H)^2}{2\theta^B_H} - E[T^B_H] - \left(e^B_L \theta^B_L + \delta E[e^A] - \frac{(e^B_L \theta^B_L)^2}{2(\theta^B_H)^3} - E[T^B_L]\right) \geq 0$$
$$\text{and} \qquad \mu_2 \frac{\partial L}{\partial \mu_2} = 0. \tag{A94}$$

Solving for the Lagrange multipliers from equations (A83)-(A90), gives:

$$\lambda_1 = \phi_H^A + \phi_L^A = 1, \tag{A95}$$

$$\lambda_2 = \phi_H^B + \phi_L^B = 1, \tag{A96}$$

$$\mu_1 = \phi_H^A, \tag{A97}$$

$$\mu_2 = \phi_H^B. \tag{A98}$$

Substituting the values of the Lagrangian multipliers in equations (A79)-(A82), gives:

$$\frac{\partial L}{\partial e_L^A} = \left[\theta_L^A - \frac{e_L^A}{\theta_L^A}\right] + \phi_L^A\delta - \phi_H^A\left(\theta_L^A - \frac{e_L^A(\theta_L^A)^2}{(\theta_H^A)^3}\right) = 0, \tag{A99}$$

$$\frac{\partial L}{\partial e_H^A} = \phi_H^A\left[\theta_H^A - \frac{e_H^A}{\theta_H^A}\right] + \phi_H^A\delta = 0, \tag{A100}$$

$$\frac{\partial L}{\partial e_L^B} = \left[\theta_L^B - \frac{e_L^B}{\theta_L^B}\right] + \phi_L^B\delta - \phi_H^B\left(\theta_L^B - \frac{e_L^B(\theta_L^B)^2}{(\theta_H^B)^3}\right) = 0, \tag{A101}$$

$$\frac{\partial L}{\partial e_H^B} = \phi_H^B\left[\theta_H^B - \frac{e_H^B}{\theta_H^B}\right] + \phi_H^B\delta = 0. \tag{A102}$$

Simplifying equations (A99)-(A102), we can see that the optimal effort levels satisfy the following conditions:

$$\theta_L^k = \frac{e_L^{k,O}}{\theta_L^k} - \phi_L^k\delta + \phi_H^k\left(\theta_L^k - \frac{e_L^{k,O}(\theta_L^k)^2}{(\theta_H^k)^3}\right), \tag{A103}$$

$$\theta_H^k = \frac{e_H^{k,O}}{\theta_H^k} - \delta. \tag{A104}$$

Solving equations (A103)-(A104) for the optimal effort levels, gives:

$$e_L^{k,O} = \frac{\phi_L^k(\theta_H^k)^3\theta_L^k(\delta + \theta_L^k)}{(\theta_H^k)^3 - \phi_H^k(\theta_L^k)^3}, \tag{A105}$$

$$e_H^{k,O} = \theta_H^k(\theta_H^k + \delta). \tag{A106}$$

Given the values of the Lagrange multipliers, equations (A91)-(A94) will hold as equalities:

$$e_L^{k,O}\theta_L^k + \delta E[e^{l,O}] - \frac{(e_L^{k,O})^2}{2\theta_L^k} - E[T_L^{k,O}] = 0, \tag{A107}$$

$$e_H^{k,O}\theta_H^k + \delta E[e^{l,O}] - \frac{(e_H^{k,O})^2}{2\theta_H^k} - E[T_H^{k,O}] - \left(e_L^{k,O}\theta_L^k + \delta E[e^{l,O}] - \frac{(e_L^{k,O}\theta_L^k)^2}{2(\theta_H^k)^3} - E[T_L^{k,O}]\right) = 0. \tag{A108}$$

Solving equations (A107)-(A108) for the transfer payments, gives:

$$E[T_L^{k,O}] = e_L^{k,O}\theta_L^k + \delta E[e^{l,O}] - \frac{(e_L^{k,O})^2}{2\theta_L^k}, \tag{A109}$$

$$E[T_H^{k,O}] = e_H^{k,O}\theta_H^k + \delta E[e^{l,O}] - \frac{(e_H^{k,O})^2}{2\theta_H^k} - \left[\frac{(e_L^{k,O})^2}{2\theta_L^k} - \frac{(e_L^{k,O}\theta_L^k)^2}{2(\theta_H^k)^3}\right]. \tag{A110}$$

From equations (A109)-(A110) notice that the principal extracts all rents from either partner in the lower state while either partner (i.e., A and/or B) in the higher state accrues rents $R_H^{A,O}$ and $R_H^{B,O}$, respectively as follows:

$$\begin{aligned} R_H^{A,O} &= \left[\frac{(e_L^{A,O})^2}{2\theta_L^A} - \frac{(e_L^{A,O}\theta_L^A)^2}{2(\theta_H^A)^3}\right] \\ &= \left[\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O}\theta_L^A)^2}{2(\theta_H^A)^3}\right] + \left[\frac{(e_L^{A,O})^2}{2\theta_L^A} - \frac{(e_L^{A,O})^2}{2\theta_H^A}\right], \end{aligned} \tag{A111}$$

$$\begin{aligned} R_H^{B,O} &= \left[\frac{(e_L^{B,O})^2}{2\theta_L^B} - \frac{(e_L^{B,O}\theta_L^B)^2}{2(\theta_H^B)^3}\right] \\ &= \left[\frac{(e_L^{B,O})^2}{2\theta_H^B} - \frac{(e_L^{B,O}\theta_L^B)^2}{2(\theta_H^B)^3}\right] + \left[\frac{(e_L^{B,O})^2}{2\theta_L^B} - \frac{(e_L^{B,O})^2}{2\theta_H^B}\right]. \end{aligned} \tag{A112}$$

$R_H^{A,O}$ and $R_H^{B,O}$ are comprised of two components: 1) rent due to lower effort $\left[\frac{(e_L^{k,O})^2}{2\theta_H^k} - \frac{(e_L^{k,O}\theta_L^k)^2}{2(\theta_H^k)^3}\right]$ and 2) rent due to lower cost of effort $\left[\frac{(e_L^{k,O})^2}{2\theta_L^k} - \frac{(e_L^{k,O})^2}{2\theta_H^k}\right]$. Next, inspecting equations (A105)-(A106) and (A19)-(A20) we can see that the partner in the higher state is induced to exert the first best effort levels while the partner in the lower state is induced to exert lower effort than in the first best setting.

We now verify whether the $IR_H^{A,O}$, $IR_H^{B,O}$, $IC_L^{A,O}$ and $IC_L^{B,O}$ conditions indeed hold at the above solution. Using the transfer payments in equations (A109)-(A110) and the expressions for the partner's profits in equations (A74)-(A77), and noting that $e_H^{k,O} > e_L^{k,O}$, we can see that

$$\pi_{H|H}^{A,O} = \left[\frac{(e_L^{A,O})^2}{2\theta_L^A} - \frac{(e_L^{A,O}\theta_L^A)^2}{2(\theta_H^A)^3}\right] > 0, \tag{A113}$$

$$\pi_{H|H}^{B,O} = \left[\frac{(e_L^{B,O})^2}{2\theta_L^B} - \frac{(e_L^{B,O}\theta_L^B)^2}{2(\theta_H^B)^3}\right] > 0, \tag{A114}$$

$$\pi_{L|L}^{A,O} - \pi_{H|L}^{A,O} = \frac{\left[(\theta_H^A)^3 - (\theta_L^A)^3\right]\left[(\theta_H^A)^2(e_H^{A,O})^2 - (\theta_L^A)^2(e_L^{A,O})^2\right]}{2(\theta_H^A)^3(\theta_L^A)^3} > 0$$

$$\Rightarrow \quad \pi_{L|L}^{A,O} > \pi_{H|L}^{A,O}, \quad \text{and} \tag{A115}$$

$$\pi_{L|L}^{B,O} - \pi_{H|L}^{B,O} = \frac{\left[(\theta_H^B)^3 - (\theta_L^B)^3\right]\left[(\theta_H^B)^2(e_H^{B,O})^2 - (\theta_L^B)^2(e_L^{B,O})^2\right]}{2(\theta_H^B)^3(\theta_L^B)^3} > 0$$

$$\Rightarrow \quad \pi_{L|L}^{B,O} > \pi_{H|L}^{B,O}. \tag{A116}$$

Hence, the solution to [P-OM-R] is also a solution to [P-OM] and the statement of Lemma 3 follows. ■

Proof of Proposition 1 (Comparing Input and Output Monitoring)

We begin by noting that the principal can induce the optimal effort levels of output monitoring under input monitoring with appropriate transfers (recall that $e_m^{k,O}$ represent the optimal effort levels under output monitoring). More specifically, the following transfers under input monitoring will induce the partner to exert effort that is optimal under output monitoring.

$$E[\tilde{T}_L^{A,I}] = e_L^{A,O}\theta_L^A + \delta E[e^{B,O}] - \frac{(e_L^{A,O})^2}{2\theta_L^A}, \tag{A117}$$

$$E[\tilde{T}_L^{B,I}] = e_L^{B,O}\theta_L^B + \delta E[e^{A,O}] - \frac{(e_L^{B,O})^2}{2\theta_L^B}, \tag{A118}$$

$$E[\tilde{T}_H^{A,I}] = e_H^{A,O}\theta_H^A + \delta E[e^{B,O}] - \frac{(e_H^{A,O})^2}{2\theta_H^A} - [e_L^{A,O}\theta_H^A - e_L^{A,O}\theta_L^A] - \left[\frac{(e_L^{A,O})^2}{2\theta_L^A} - \frac{(e_L^{A,O})^2}{2\theta_H^A}\right], \tag{A119}$$

$$E[\tilde{T}_H^{B,I}] = e_L^{B,O}\theta_L^B + \delta E[e^{A,O}] - \frac{(e_H^{B,O})^2}{2\theta_H^B} - [e_L^{B,O}\theta_H^B - e_L^{B,O}\theta_L^B] - \left[\frac{(e_L^{B,O})^2}{2\theta_L^B} - \frac{(e_L^{B,O})^2}{2\theta_H^B}\right]. \tag{A120}$$

The transfer payments for the output monitoring case are given by

$$E[T_L^{A,O}] = e_L^{A,O}\theta_L^A + \delta E[e^{B,O}] - \frac{(e_L^{A,O})^2}{2\theta_L^A}, \tag{A121}$$

$$E[T_L^{B,O}] = e_L^{B,O}\theta_L^B + \delta E[e^{A,O}] - \frac{(e_L^{B,O})^2}{2\theta_L^B}, \tag{A122}$$

$$\begin{aligned} E[T_H^{A,O}] &= e_H^{A,O}\theta_H^A + \delta E[e^{B,O}] - \frac{(e_H^{A,O})^2}{2\theta_H^A} - \left[\frac{(e_L^{A,O})^2}{2\theta_L^A} - \frac{(e_L^{A,O}\theta_L^A)^2}{2(\theta_H^A)^3}\right], \\ &= e_H^{A,O}\theta_H^A + \delta E[e^{B,O}] - \frac{(e_H^{A,O})^2}{2\theta_H^A} - \left[\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O}\theta_L^A)^2}{2(\theta_H^A)^3}\right] - \left[\frac{(e_L^{A,O})^2}{2\theta_L^A} - \frac{(e_L^{A,O})^2}{2\theta_H^A}\right], \end{aligned} \tag{A123}$$

$$\begin{aligned} E[T_H^{B,O}] &= e_L^{B,O}\theta_L^B + \delta E[e^{A,O}] - \frac{(e_H^{B,O})^2}{2\theta_H^B} - \left[\frac{(e_L^{B,O})^2}{2\theta_L^B} - \frac{(e_L^{B,O}\theta_L^B)^2}{2(\theta_H^B)^3}\right], \\ &= e_L^{B,O}\theta_L^B + \delta E[e^{A,O}] - \frac{(e_H^{B,O})^2}{2\theta_H^B} - \left[\frac{(e_L^{B,O})^2}{2\theta_H^B} - \frac{(e_L^{B,O}\theta_L^B)^2}{2(\theta_H^B)^3}\right] - \left[\frac{(e_L^{B,O})^2}{2\theta_L^B} - \frac{(e_L^{B,O})^2}{2\theta_H^B}\right]. \end{aligned} \tag{A124}$$

Since the above two sets of transfers (from equations (A117)-(A120) and (A121)-(A124)) induce the same levels of effort under the two monitoring regimes, we now compare the principal's profits under input monitoring and output monitoring (with output optimal effort levels under both cases). Let $\Pi^{P,I}(e^{A,O}, e^{B,O})$ denote the principal's profit under input monitoring when the principal induces the optimal effort levels under output monitoring. Further, let $\Pi^{P,I}(e^{A,I}, e^{B,I})$ and $\Pi^{P,O}(e^{A,O}, e^{B,O})$ denote the principal's profit under output and input monitoring respectively when the principal induces the partners to exert the corresponding optimal effort levels. Notice that $\Pi^{P,I}(e^{A,I}, e^{B,I}) \geq \Pi^{P,I}(e^{A,O}, e^{B,O})$ since $e^{A,I}$ and $e^{B,I}$ are optimal under input monitoring. Hence, if the principal's profit is higher under input monitoring when the principal induces output optimal effort levels, then the principal's profit is higher with input monitoring under input optimal effort levels (i.e., if $\Pi^{P,I}(e^{A,O}, e^{B,O}) \geq \Pi^{P,O}(e^{A,O}, e^{B,O})$, then $\Pi^{P,I}(e^{A,I}, e^{B,I}) \geq \Pi^{P,O}(e^{A,O}, e^{B,O})$).

Comparing the principals' profits (using the transfer payments in (A117)-(A120) and (A121)-(A124)), we see that the transfer payments are identical under the low state condition and the difference in profits comes from the transfers under the high state. Define

$$\Delta^A = \left[\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O})^2(\theta_L^A)^2}{2(\theta_H^A)^3}\right] - e_L^{A,O}(\theta_H^A - \theta_L^A), \tag{A125}$$

$$\Delta^B = \left[\frac{(e_L^{B,O})^2}{2\theta_H^B} - \frac{(e_L^{B,O})^2(\theta_L^B)^2}{2(\theta_H^B)^3}\right] - e_L^{B,O}(\theta_H^B - \theta_L^B). \tag{A126}$$

Then,

$$\begin{aligned}\Pi^{P,I}(e^{A,O}, e^{B,O}) - \Pi^{P,O}(e^{A,O}, e^{B,O}) &= \phi_H^A\left(\left[\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O})^2(\theta_L^A)^2}{2(\theta_H^A)^3}\right] - e_L^{A,O}(\theta_H^A - \theta_L^A)\right) + \\ &\quad \phi_H^B\left(\left[\frac{(e_L^{B,O})^2}{2\theta_H^B} - \frac{(e_L^{B,O})^2(\theta_L^B)^2}{2(\theta_H^B)^3}\right] - e_L^{B,O}(\theta_H^B - \theta_L^B)\right). \\ &= \phi_H^A\Delta^A + \phi_H^B\Delta^B \end{aligned} \tag{A127}$$

Now, $\Delta^A > 0$ if the following condition is satisfied

$$\frac{(e_L^{A,O})^2}{2\theta_H^A} - \frac{(e_L^{A,O})^2(\theta_L^A)^2}{2(\theta_H^A)^3} - e_L^{A,O}(\theta_H^A - \theta_L^A) > 0$$
$$\Rightarrow \frac{e_L^{A,O}(\theta_H^A - \theta_L^A)(e_L^{A,O}(\theta_H^A + \theta_L^A) - 2(\theta_H^A)^3)}{2(\theta_H^A)^3} > 0,$$
$$\text{or when} \quad e_L^{A,O} > \frac{2(\theta_H^A)^3}{\theta_H^A + \theta_L^A}. \tag{A128}$$

Recalling $e_L^{A,O}$ from (A105), notice that $\Delta^A > 0$ if:

$$e_L^{A,O} = \frac{\phi_L^A(\theta_H^A)^3\theta_L^A(\delta + \theta_L^A)}{(\theta_H^A)^3 - \phi_H^A(\theta_L^A)^3} > \frac{2(\theta_H^A)^3}{\theta_H^A + \theta_L^A}$$
$$\Rightarrow \delta + \theta_L^A > \frac{2\left[(\theta_H^A)^3 - \phi_H^A(\theta_L^A)^3\right]}{\phi_L^A\theta_L^A(\theta_H^A + \theta_L^A)}$$
$$\Rightarrow \delta > \frac{2\left[(\theta_H^A)^3 - (\theta_L^A)^3\right] - \phi_L^A(\theta_L^A)^2(\theta_H^A - \theta_L^A)}{\phi_L^A\theta_L^A(\theta_H^A + \theta_L^A)}. \tag{A129}$$

Hence, when the externality parameter δ satisfies inequality (A129), transfer payments from partner A under input monitoring are greater than the payments under output monitoring. A similar exercise results in the following condition for the principal's profits from partner B under input monitoring to be greater than those under output monitoring

$$\delta > \frac{2\left[(\theta_H^B)^3 - (\theta_L^B)^3\right] - \phi_L^B(\theta_L^B)^2(\theta_H^B - \theta_L^B)}{\phi_L^B\theta_L^B(\theta_H^B + \theta_L^B)}. \tag{A130}$$

Since the principal's profits are the sum of the transfer payments from both the partners, the following inequality is a sufficient condition for input monitoring to be more profitable than output monitoring.

$$\delta > \delta^* = \max\left\{\frac{2\left[(\theta_H^A)^3-(\theta_L^A)^3\right]-\phi_L^A(\theta_L^A)^2(\theta_H^A-\theta_L^A)}{\phi_L^A\theta_L^A(\theta_H^A+\theta_L^A)},\frac{2\left[(\theta_H^B)^3-(\theta_L^B)^3\right]-\phi_L^B(\theta_L^B)^2(\theta_H^B-\theta_L^B)}{\phi_L^B\theta_L^B(\theta_H^B+\theta_L^B)}\right\}. \tag{A131}$$

Hence, the statement of Proposition 1 follows. We now show how our analysis extends to more general demand and cost functions.

Extending the Analysis to a More General Setting

Here, we show that the qualitative insights arrived at so far also hold for a larger class of demand and cost functions. Recall that the general demand for partner k, $Q(e_m^k, e^l, \theta_m^k)$, satisfies the conditions $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_{11} \leq 0$ and $Q_{13} \geq 0$; and the cost function $C(e_m^k, \theta_m^k)$, satisfies $C_1 > 0$, $C_2 < 0$, $C_{11} > 0$ and $C_{12} < 0$. Using the same solution procedure employed with the specific functions, we can show[7] that the first order conditions (for agent k, $k \in \{A, B\}$) under input monitoring are given by

$$\begin{aligned} Q_1(e_L^{k,I}, E[e^{l,I}], \theta_L^k) &= C_1(e_L^{k,I}, \theta_L^k) - \phi_L^k Q_2(e_L^{l,I}, E[e^{k,I}], \theta_L^l) + \phi_H^k\left[Q_1(e_L^{k,I}, E[e^{l,I}], \theta_H^k) - C_1(e_L^{k,I}, \theta_H^k)\right] \\ &\quad - \phi_L^k\phi_H^l\left[Q_2(e_H^{l,I}, E[e^{k,I}], \theta_H^l) - Q_2(e_L^{l,I}, E[e^{k,I}], \theta_H^l)\right], \qquad \text{(A132)} \\ Q_1(e_H^{k,I}, E[e^{l,I}], \theta_H^k) &= C_1(e_H^{k,I}, \theta_H^k) - Q_2(e_L^{l,I}, E[e^{k,I}], \theta_L^l) \\ &\quad - \phi_H^l\left[Q_2(e_H^{l,I}, E[e^{k,I}], \theta_H^l) - Q_2(e_L^{l,I}, E[e^{k,I}], \theta_H^l)\right]. \qquad \text{(A133)} \end{aligned}$$

The principal extracts all rents from any agent in the low state and the rents accruing to the high type agent have two components:

[7] Complete proof available with the authors.

(i) Rent due to extra output $[Q(e_L^{k,I}, E[e^{l,I}], \theta_H^k) - Q(e_L^{k,I}, E[e^{l,I}], \theta_L^k)]$, and

(ii) Rent due to lower cost of effort $[C(e_L^{k,I}, \theta_L^k) - C(e_L^{k,I}, \theta_H^k)]$.

The total rents given by:

$$R_H^{k,I} = [Q(e_L^{k,I}, E[e^{l,I}], \theta_H^k) - Q(e_L^{k,I}, E[e^{l,I}], \theta_L^k)] + [C(e_L^{k,I}, \theta_L^k) - C(e_L^{k,I}, \theta_H^k)]\,. \tag{A134}$$

Note here that, since $Q_{13} \geq 0$ and $C_{12} \leq 0$, the rents accruing to the partners are increasing in the effort exerted by the partners. Next, the first order conditions under output monitoring are:

$$\begin{aligned} Q_1(e_L^{k,O}, E[e^{l,O}], \theta_L^k) &= C_1(e_L^{k,O}, \theta_L^k) + \phi_H^k \left(Q_1(e_L^{k,O}, E[e^{l,O}], \theta_L^k) - C_1(\tilde{e}_H^{k,O}, \theta_H^k)\right) \\ &\quad - \phi_L^k \left[\phi_H^l Q_2(e_H^{l,O}, E[e^{k,O}], \theta_H^l) + \phi_L^l Q_2(e_L^{l,O}, E[e^{k,O}], \theta_L^l)\right], \end{aligned} \tag{A135}$$

$$\begin{aligned} Q_1(e_H^{k,O}, E[e^{l,O}], \theta_H^k) &= C_1(e_H^{k,O}, \theta_H^k) - Q_2(e_L^{l,O}, E[e^{k,O}], \theta_L^l) \\ &\quad - \phi_H^l \left(Q_2(e_H^{l,O}, E[e^{k,O}], \theta_H^l) - Q_2(e_L^{l,O}, E[e^{k,O}], \theta_L^l)\right). \end{aligned} \tag{A136}$$

Again, the principal extracts all rents from the low type agent and the net rent to the high type agent is:

$$\begin{aligned} R_H^{k,O} &= [C(e_L^{k,O}, \theta_L^k) - C(\tilde{e}_H^{k,O}, \theta_H^k)] \\ &= [C(e_L^{k,O}, \theta_H^k) - C(\tilde{e}_H^{k,O}, \theta_H^k)] + [C(e_L^{k,O}, \theta_L^k) - C(e_L^{k,O}, \theta_H^k)]\,. \end{aligned} \tag{A137}$$

Analogous to the earlier settings, these rents, too, can be sorted into two components: (1) rent due to lower effort $[C(e_L^{k,O}, \theta_H^k) - C(\tilde{e}_H^{k,O}, \theta_H^k)]$, and (2) rent due to lower cost of effort $[C(e_L^{k,O}, \theta_L^k) - C(e_L^{k,O}, \theta_H^k)]$.

Since $C_1 \geq 0$, $\tilde{e}_H^{k,O} < e_L^{k,O}$ and $C_{12} \leq 0$, the rents under output monitoring are also increasing in the effort exerted by the partners. Next, from (A137) and (A134) we can see that the difference in rents under output and input monitoring (at output effort levels) is given by

$$\Delta^R = [C(e_L^{k,O}, \theta_H^k) - C(\tilde{e}_H^{k,O}, \theta_H^k)] - [Q(e_L^{k,O}, E[e^{l,O}], \theta_H^k) - Q(e_L^{k,O}, E[e^{l,O}], \theta_L^k)]\,. \tag{A138}$$

The above difference in rents arises due to the difference in rents due to lower effort (which is a source of additional rents under output monitoring) and extra output (which is a source of additional rents under input monitoring). Note that $[\phi_H^l Q_2(e_H^{l,O}, E[e^{k,O}], \theta_H^l) + \phi_L^l Q_2(e_L^{l,O}, E[e^{k,O}], \theta_L^l)]$ is the marginal impact of partner $k's$ effort on partner $l's$ output and hence denotes the strength of the externality.

Further, notice from the first order conditions in (A135) that the marginal cost of effort is greater than its marginal benefit when $\phi_H^l Q_2(e_H^{l,O}, E[e^{k,O}], \theta_H^l) + \phi_L^l Q_2(e_L^{l,O}, E[e^{k,O}], \theta_L^l) > \frac{\phi_H^k}{\phi_L^k} (Q_1(e_L^{k,O}, E[e^{l,O}], \theta_H^k) - C_1(\tilde{e}_H^{k,O}, \theta_H^k))$. In other words, as the strength of the externality is enhanced (i.e., the LHS of the above inequality is larger in magnitude), the difference between the marginal cost of effort and the marginal revenue becomes larger; this essentially induces the partners to exert higher levels of effort. Also, the rate of change of the difference in rents with respect of the effort exerted by the partners is given by[8]

$$\frac{\partial \Delta^R}{\partial e_L^{k,O}} = \left[C_1(e_L^{k,O}, \theta_H^k) - C_1(\tilde{e}_H^{k,O}, \theta_H^k)\frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}}\right] - \left[Q_1(e_L^{k,O}, E[e^{l,O}], \theta_H^k) - Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_H^k)\frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}}\right] \tag{A139}$$

$$\Rightarrow \frac{\partial \Delta^R}{\partial e_L^{k,O}} = [C_1(e_L^{k,O}, \theta_H^k) - Q_1(e_L^{k,O}, E[e^{l,O}], \theta_H^k)] - [C_1(\tilde{e}_H^{k,O}, \theta_H^k) - Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_H^k)]\,\frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}} \tag{A140}$$

[8]Recall that $Q(e_L^{k,O}, E[e^{l,O}], \theta_L^k) = Q(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_H^k)$ and $\tilde{e}_H^{k,O} < e_L^{k,O}$.

Since $Q_{11} \leq 0$ and $C_{11} \geq 0$, $[C_1(e, \theta_H^k) - Q_1(e, E[e^{l,O}], \theta_H^k)]$ is increasing[9] in e. Suppose for some $e_L^{k,O}$ we have

$$[C_1(e_L^{k,O}, \theta_H^k) - Q_1(e_L^{k,O}, E[e^{l,O}], \theta_H^k)] > 0. \tag{A141}$$

Then, since[10] $\frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}} < 1$ and $\tilde{e}_H^{k,O} < e_L^{k,O}$, we have

$$\frac{\partial \Delta^R}{\partial e_L^{k,O}} = [C_1(e_L^{k,O}, \theta_H^k) - Q_1(e_L^{k,O}, E[e^{l,O}], \theta_H^k)] - [C(\tilde{e}_H^{k,O}, \theta_H^k) - Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_H^k)] \frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}} > 0 \tag{A142}$$

It follows that when the effort exerted by the partners increases around $e_L^{k,O}$, the rents under output monitoring increase by a greater amount than the rents under input monitoring. Therefore, it is feasible that for sufficiently high levels of effort (which arise due to a stronger externality), the rents under output monitoring can dominate the rents under input monitoring (i.e., $\Delta^R > 0$). Recall from the previous section that a sufficient condition for the profits under input monitoring to dominate the profits under output monitoring is that the rents under output monitoring are greater than those under input monitoring when output monitoring optimal effort levels are induced under the two regimes. Therefore, in such a case, input monitoring will be preferred to output monitoring. The specific functions employed in the earlier section essentially help illustrate this idea. ■

[9] Also note from the first order conditions in (A136) that $Q_1(e_H^{k,O}, E[e^{l,O}], \theta_H^k) < C_1(e_H^{k,O}, \theta_H^k)$.

[10] Note that $\frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}} = \frac{Q_1(e_L^{k,O}, E[e^{l,O}], \theta_L^k)}{Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_H^k)}$ and $Q_{11} \leq 0$ implies $\frac{Q_1(e_L^{k,O}, E[e^{l,O}], \theta_L^k)}{Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_L^k)} < 1$. Since $Q_{13} \geq 0$ implies $Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_H^k) > Q_1(\tilde{e}_H^{k,O}, E[e^{l,O}], \theta_L^k)$, we have $\frac{\partial \tilde{e}_H^{k,O}}{\partial e_L^{k,O}} < 1$.

Proof of Proposition 2 (No Externality or Negative Externality)

When the partners face a negative externality from the efforts exerted by the other partners, we can see (when $\delta < 0$ in equations (A19)-(A20)) that the optimal effort levels under first best are such that the partners exert less effort than what they prefer. Under input and output monitoring, the principal continues to induce the partners in the higher state to exert first best effort levels and the partners in the lower state to exert lower effort than under the first best conditions. This can be seen by setting $\delta < 0$ in equations (A54)-(A55) and (A105)-(A106). Defining,

$$\hat{\Delta}^A = e_L^{A,I}(\theta_H^A - \theta_L^A) - \left[\frac{(e_L^{A,I})^2}{2\theta_H^A} - \frac{(e_L^{A,I})^2(\theta_L^A)^2}{2(\theta_H^A)^3}\right], \tag{A143}$$

$$\hat{\Delta}^B = e_L^{B,I}(\theta_H^B - \theta_L^B) - \left[\frac{(e_L^{B,I})^2}{2\theta_H^B} - \frac{(e_L^{B,I})^2(\theta_L^B)^2}{2(\theta_H^B)^3}\right]. \tag{A144}$$

We note that the difference in the principal's profits under output and input monitoring (at optimal effort levels under input monitoring) are given[11] by

$$\begin{aligned}
\Pi^{P,O}(e^{A,I}, e^{B,I}) - \Pi^{P,I}(e^{A,I}, e^{B,I}) &= \phi_H^A \left(e_L^{A,I}(\theta_H^A - \theta_L^A) - \left[\frac{(e_L^{A,I})^2}{2\theta_H^A} - \frac{(e_L^{A,I})^2(\theta_L^A)^2}{2(\theta_H^A)^3}\right]\right) + \\
&\quad \phi_H^B \left(e_L^{B,I}(\theta_H^B - \theta_L^B) - \left[\frac{(e_L^{B,I})^2}{2\theta_H^B} - \frac{(e_L^{B,I})^2(\theta_L^B)^2}{2(\theta_H^B)^3}\right]\right). \\
&= \phi_H^A \hat{\Delta}^A + \phi_H^B \hat{\Delta}^B
\end{aligned} \tag{A145}$$

[11] Using the transfer payments under (A58)-(A59) and (A109)-(A110)

Consequently, $\hat{\Delta}^A > 0$ if the following condition is satisfied

$$\begin{aligned} & e_L^{A,I}(\theta_H^A - \theta_L^A) - \frac{(e_L^{A,I})^2}{2\theta_H^A} - \frac{(e_L^{A,I})^2(\theta_L^A)^2}{2(\theta_H^A)^3} > 0 \\ & \Rightarrow \frac{e_L^{A,I}(\theta_H^A - \theta_L^A)(e_L^{A,I}(\theta_H^A + \theta_L^A) - 2(\theta_H^A)^3)}{2(\theta_H^A)^2} < 0, \\ \text{or when} \quad & e_L^{A,I} < \frac{2(\theta_H^A)^3}{\theta_H^A + \theta_L^A}. \end{aligned} \tag{A146}$$

Recalling, (A19), notice that the effort exerted by the low state partner under first best levels is [12] $\theta_L^A(\theta_L^A + \delta)$. Since $\frac{2(\theta_H^A)^3}{\theta_H^A + \theta_L^A} > \theta_L^A(\theta_L^A + \delta)\ \forall\ \delta \leq 0$, inequality (A146) always[13] holds. Hence, output monitoring is always more profitable than input monitoring when $\delta \leq 0$ and the statement in Proposition 2 follows. ■

Comparing Effort Levels

Here, we derive the necessary condition for optimal effort levels under input monitoring to dominate those under output monitoring (see the discussion in section (2.4.5) in the main text) . The optimal effort levels exerted by the partners in the low state under input and output monitoring are given (from equations (A54) and (A105)) by

$$e_L^{k,I} = \frac{\theta_H^k \theta_L^k (\delta \phi_L^k + \theta_L^k - \phi_H^k \theta_H^k)}{\theta_H^k - \phi_H^k \theta_L^k} \tag{A147}$$

$$e_L^{k,O} = \frac{\phi_L^k (\theta_H^k)^3 \theta_L^k (\delta + \theta_L^k)}{(\theta_H^k)^3 - \phi_H^k (\theta_L^k)^3} \tag{A148}$$

[12] Note that $\delta < 0$.

[13] Note that the partner in the lower state is always induced to exert lower effort compared to the first best level.

Comparing the effort levels, we see that the effort under input monitoring is greater when

$$
\begin{aligned}
e_L^{k,I} - e_L^{k,O} &= \frac{\theta_H^k \theta_L^k (\delta \phi_L^k + \theta_L^k - \phi_H^k \theta_H^k)}{\theta_H^k - \phi_H^k \theta_L^k} - \frac{\phi_L^k (\theta_H^k)^3 \theta_L^k (\delta + \theta_L^k)}{(\theta_H^k)^3 - \phi_H^k (\theta_L^k)^3} > 0 \\
&\Rightarrow \delta > \tilde{\delta} = \frac{(\theta_H^k)^3 - (\theta_L^k)^3 - \phi_L^k \theta_H^k (\theta_L^k)^2}{\phi_L^k \theta_L^k (\theta_H^k - \theta_L^k)}
\end{aligned}
\tag{A149}
$$

Hence, when $\delta > \tilde{\delta}$, the effort exerted by the agents in the low state is higher under input monitoring.

Alternatively, recall that the rents under input monitoring that arise due to extra output are given by $\bar{R}_H^{k,I} = [e_L^k \theta_H^k - e_L^{k,I} \theta_L^k]$ and the rents under output monitoring that arise due to lower effort are given by $\bar{R}_H^{k,O} = \left[\frac{(e_L^k)^2}{2\theta_H^k} - \frac{(e_L^k \theta_L^k)^2}{2(\theta_H^k)^3}\right]$. Note that $\frac{\partial \bar{R}_H^{k,I}}{\partial e_L^k} = (\theta_H^k - \theta_L^k) > 0$ and $\frac{\partial \bar{R}_H^{k,O}}{\partial e_L^k} = \left[\frac{(e_L^k)}{\theta_H^k} - \frac{e_L^k (\theta_L^k)^2}{(\theta_H^k)^3}\right] > 0$—i.e., the rents are increasing in the effort exerted by the partners. Define η as follows:

$$
\eta = \left[\frac{(e_L^{k,I})}{\theta_H^k} - \frac{e_L^{k,O} (\theta_L^k)^2}{(\theta_H^k)^3}\right] - (\theta_H^k - \theta_L^k). \tag{A150}
$$

Now, from the first order conditions (Eq. 2.9) and (Eq. 2.18), it is easy to show that $e_L^{k,I} > e_L^{k,O}$ if $\eta > 0$. Further, notice that whenever $e_L^{k,I} > e_L^{k,O}$, this condition holds:

$$
\eta > \left[\frac{\partial \bar{R}_H^{k,O}}{\partial e_L^k} - \frac{\partial \bar{R}_H^{k,I}}{\partial e_L^k}\right]_{e_L^{k,I}} \tag{A151}
$$

It follows that $\left[\frac{\partial \bar{R}_H^{k,O}}{\partial e_L^k} - \frac{\partial \bar{R}_H^{k,I}}{\partial e_L^k}\right]_{e_L^{k,I}} > 0 \Rightarrow \delta > \tilde{\delta}$ is a sufficient condition for $e_L^{k,I} > e_L^{k,O}$. Hence, it is sufficient for the marginal-rents- due-to-lower-effort to dominate the marginal-rents-due-to-extra-output for input monitoring to induce greater effort levels.

Comparing $\tilde{\delta}$ and δ^*, we have

$$\delta^* - \tilde{\delta} = \frac{2\left[(\theta_H^k)^3 - (\theta_L^k)^3\right] - \phi_L^k(\theta_L^k)^2(\theta_H^k - \theta_L^k)}{\phi_L^k\theta_L^k(\theta_H^k + \theta_L^k)} - \frac{(\theta_H^k)^3 - (\theta_L^k)^3 - \phi_L^k\theta_H^k(\theta_L^k)^2}{\phi_L^k\theta_L^k(\theta_H^k - \theta_L^k)}$$
$$= \frac{(\theta_H^k)^3 - \phi_L^k(\theta_L^k)^3}{\phi_L^k\theta_L^k(\theta_H^k - \theta_L^k)} > 0. \tag{A152}$$

APPENDIX B: PROOFS FOR RESULTS IN CHAPTER 3

Proof of Lemma 1 (First Best Setting)

The manufacturer's profits are given by

$$\Pi = Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2. \tag{B1}$$

The manufacturer's optimization problem (denoted by [M-FB]) can be expressed as

$$\underset{\{p_1, p_2, e_1, e_2\}}{\text{Max}} \quad \Pi = Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2 \tag{B2}$$

The first order conditions for [M-FB] are given by

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}p_1 + \frac{\partial Q^2}{\partial p_1}p_2 + Q^1 - \left[\frac{\partial Q^1}{\partial p_1} + \frac{\partial Q^2}{\partial p_1}\right] c = 0, \tag{B3}$$

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial Q^1}{\partial p_2}p_1 + \frac{\partial Q^2}{\partial p_2}p_2 + Q^2 - \left[\frac{\partial Q^1}{\partial p_2} + \frac{\partial Q^2}{\partial p_2}\right] c = 0, \tag{B4}$$

$$\frac{\partial \Pi}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}p_1 + \frac{\partial Q^2}{\partial e_1}p_2 - \left[\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^2}{\partial e_1}\right] c - \frac{\partial G^1}{\partial e_1} = 0, \tag{B5}$$

$$\frac{\partial \Pi}{\partial e_2} = \frac{\partial Q^1}{\partial e_2}p_1 + \frac{\partial Q^2}{\partial e_2}p_2 - \left[\frac{\partial Q^1}{\partial e_2} + \frac{\partial Q^2}{\partial e_2}\right] c - \frac{\partial G^2}{\partial e_2} = 0. \tag{B6}$$

When the retailers are symmetric, we must have

$$\frac{\partial Q^1}{\partial p_1} = \frac{\partial Q^2}{\partial p_2}, \tag{B7}$$
$$\frac{\partial Q^1}{\partial p_2} = \frac{\partial Q^2}{\partial p_1}, \tag{B8}$$
$$\frac{\partial Q^1}{\partial e_1} = \frac{\partial Q^2}{\partial e_2}, \tag{B9}$$
$$\frac{\partial Q^1}{\partial e_2} = \frac{\partial Q^2}{\partial e_1}, \tag{B10}$$
$$\frac{\partial G^1}{\partial e_1} = \frac{\partial G^2}{\partial e_2}. \tag{B11}$$

Hence, we can see from (B3)-(B6) that for symmetric retailers, $p_1 = p_2$ and $e_1 = e_2$ and the the first order conditions simplify to

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) + Q^1 = 0, \tag{B12}$$
$$\frac{\partial \Pi}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(e_1 - c) + \frac{\partial Q^2}{\partial e_1}(e_1 - c) - \frac{\partial G^1}{\partial e_1} = 0. \tag{B13}$$
$$\tag{B14}$$

Hence, under vertical integration, the manufacturer's choice of price ($p_1^* = p_2^* = p^*$) and effort ($e_1^* = e_2^* = e^*$) satisfy equations (B12)-(B13) and the statement of the Lemma 1 follows. ■

Proof of Lemma 2(Second Best Setting)

Here, the retailers decide price and advertising. The retailer's profit functions are given by

$$\pi^1 = Q^1(p_1 - w) - G^1 - F, \tag{B15}$$

$$\pi^2 = Q^2(p_2 - w) - G^2 - F. \tag{B16}$$

The retailers choose p_i and e_i ($i \in \{1, 2\}$) to maximize profits. The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F + F. \tag{B17}$$

The manufacturer chooses the wholesale prices such that $\pi^1 \geq 0$, $\pi^2 \geq 0$, $\frac{\partial \pi^1}{\partial p_1} = 0$, $\frac{\partial \pi^1}{\partial e_1} = 0$,$\frac{\partial \pi^2}{\partial p_2} = 0$ and $\frac{\partial \pi^2}{\partial e_2} = 0$ (the retailer's surplus is extracted and the first order condition of the retailer are satisfied) The manufacturer's optimization problem can be expressed as

$$\begin{gathered}\underset{\{w, w, F, F, p_1, p_2, e_1, e_2\}}{\text{Max}} \quad \Pi \\ \text{subject to} \quad \pi^1 \geq 0, \pi^2 \geq 0, \frac{\partial \pi^1}{\partial p_1} = 0, \frac{\partial \pi^1}{\partial e_1} = 0, \frac{\partial \pi^2}{\partial p_2} = 0 \text{ and } \frac{\partial \pi^2}{\partial e_2} = 0\end{gathered} \tag{B18}$$

The Lagrangian for the manufacturer's optimization problem can be expressed as

$$\begin{aligned} L =& Q^1(w - c) + Q^2(w - c) + F + F + \\ & \lambda_1 \left[Q^1(p_1 - w) - G^1 - F\right] + \lambda_2 \left[Q^2(p_2 - w) - G^2 - F\right] + \\ & \mu_1 \left[\frac{\partial Q^1}{\partial p_1} p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1} w\right] + \mu_2 \left[\frac{\partial Q^2}{\partial p_2} p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2} w\right] + \\ & \eta_1 \left[\frac{\partial Q^1}{\partial e_1} p_1 - \frac{\partial Q^1}{\partial e_1} w - \frac{\partial G^1}{\partial e_1}\right] + \eta_2 \left[\frac{\partial Q^2}{\partial e_2} p_2 - \frac{\partial Q^2}{\partial e_2} w - \frac{\partial G^2}{\partial e_2}\right]. \end{aligned} \tag{B19}$$

The first order conditions are given by

$$\frac{\partial L}{\partial w} = Q^1 - \lambda_1 Q^1 - \mu_1 \frac{\partial Q^1}{\partial p_1} - \eta_1 \frac{\partial Q^1}{\partial e_1} = 0, \tag{B20}$$

$$\frac{\partial L}{\partial w} = Q^2 - \lambda_2 Q^2 - \mu_2 \frac{\partial Q^2}{\partial p_2} - \eta_2 \frac{\partial Q^2}{\partial e_2} = 0, \tag{B21}$$

$$\frac{\partial L}{\partial F} = 1 - \lambda_1 = 0, \tag{B22}$$

$$\frac{\partial L}{\partial F} = 1 - \lambda_2 = 0, \tag{B23}$$

$$\begin{aligned}\frac{\partial L}{\partial p_1} =& \frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(w-c) + \lambda_1 \left[\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial p_1}(p_2 - w)\right] + \\ & 2\mu_1 \left[\frac{\partial Q^1}{\partial p_1}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial p_1}\right] + \eta_1 \left[\frac{\partial Q^1}{\partial e_1}\right] = 0,\end{aligned} \tag{B24}$$

$$\begin{aligned}\frac{\partial L}{\partial p_2} =& \frac{\partial Q^1}{\partial p_2}(w-c) + \frac{\partial Q^2}{\partial p_2}(w-c) + \lambda_1 \left[\frac{\partial Q^1}{\partial p_2}(p_1 - w)\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2\right] + \\ & \mu_1 \left[\frac{\partial Q^1}{\partial p_2}\right] + 2\mu_2 \left[\frac{\partial Q^2}{\partial p_2}\right] + \eta_2 \left[\frac{\partial Q^2}{\partial e_2}\right] = 0,\end{aligned} \tag{B25}$$

$$\begin{aligned}\frac{\partial L}{\partial e_1} =& \frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(w-c) + \lambda_1 \left[\frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1}\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial e_1}(p_2 - w)\right] + \\ & \mu_1 \left[\frac{\partial Q^1}{\partial e_1}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial e_1}\right] - \eta_1 \left[\frac{\partial^2 G^1}{\partial e_1^2}\right] = 0,\end{aligned} \tag{B26}$$

$$\begin{aligned}\frac{\partial L}{\partial e_2} =& \frac{\partial Q^1}{\partial e_2}(w-c) + \frac{\partial Q^2}{\partial e_2}(w-c) + \lambda_1 \left[\frac{\partial Q^1}{\partial e_2}(p_1 - w)\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial e_2}(p_2 - w) - \frac{\partial G^2}{\partial e_2}\right] + \\ & \mu_1 \left[\frac{\partial Q^1}{\partial e_2}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial e_2}\right] - \eta_2 \left[\frac{\partial^2 G^2}{\partial e_2^2}\right] = 0,\end{aligned} \tag{B27}$$

$$\frac{\partial L}{\partial \lambda_1} = Q^1(p_1 - w) - G^1 - F = 0, \tag{B28}$$

$$\frac{\partial L}{\partial \lambda_2} = Q^2(p_2 - w) - G^2 - F = 0, \tag{B29}$$

$$\frac{\partial L}{\partial \mu_1} = \frac{\partial Q^1}{\partial p_1}p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1}w = 0, \tag{B30}$$

$$\frac{\partial L}{\partial \mu_2} = \frac{\partial Q^2}{\partial p_2}p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2}w = 0, \tag{B31}$$

$$\frac{\partial L}{\partial \eta_1} = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - \frac{\partial G^1}{\partial e_1} = 0, \tag{B32}$$

$$\frac{\partial L}{\partial \eta_2} = \frac{\partial Q^2}{\partial e_2}p_2 - \frac{\partial Q^2}{\partial e_2}w - \frac{\partial G^2}{\partial e_2} = 0. \tag{B33}$$

From (B22) and (B23) we can see that $\lambda_1 = 1$ and $\lambda_2 = 1$. Substituting in equations B20,B21, B24,B25,B26, B27 and simplifying, we have

$$\frac{\partial L}{\partial w} = \mu_1 \frac{\partial Q^1}{\partial p_1} + \eta_1 \frac{\partial Q^1}{\partial e_1} = 0, \tag{B34}$$

$$\frac{\partial L}{\partial w} = \mu_2 \frac{\partial Q^2}{\partial p_2} + \eta_2 \frac{\partial Q^2}{\partial e_2} = 0, \tag{B35}$$

$$\frac{\partial L}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 + 2\mu_1 \left[\frac{\partial Q^1}{\partial p_1}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial p_1}\right] + \eta_1 \left[\frac{\partial Q^1}{\partial e_1}\right] = 0, \tag{B36}$$

$$\frac{\partial L}{\partial p_2} = \frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 + \mu_1 \left[\frac{\partial Q^1}{\partial p_2}\right] + 2\mu_2 \left[\frac{\partial Q^2}{\partial p_2}\right] + \eta_2 \left[\frac{\partial Q^2}{\partial e_2}\right] = 0, \tag{B37}$$

$$\frac{\partial L}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} + \mu_1 \left[\frac{\partial Q^1}{\partial e_1}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial e_1}\right] - \eta_1 \left[\frac{\partial^2 G^1}{\partial e_1^2}\right] = 0, \tag{B38}$$

$$\frac{\partial L}{\partial e_2} = \frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} + \mu_1 \left[\frac{\partial Q^1}{\partial e_2}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial e_2}\right] - \eta_2 \left[\frac{\partial^2 G^2}{\partial e_2^2}\right] = 0. \tag{B39}$$

From (B34) , we can see that μ_1 and η_1 must either both be zero, both positive or both negative (since $\frac{\partial Q^1}{\partial p_1} < 0$ and $\frac{\partial Q^1}{\partial e_1} > 0$). Similarly, from (B35) , we can see that μ_2 and η_2 must either both be zero, both positive or both negative (since $\frac{\partial Q^2}{\partial p_2} < 0$ and $\frac{\partial Q^2}{\partial e_2} > 0$).

The equations that govern the retailer's price and effort choices are given by

$$\frac{\partial Q^1}{\partial p_1} p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1} w = 0, \tag{B40}$$
$$\frac{\partial Q^2}{\partial p_2} p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2} w = 0, \tag{B41}$$
$$\frac{\partial Q^1}{\partial e_1} p_1 - \frac{\partial Q^1}{\partial e_1} w - \frac{\partial G^1}{\partial e_1} = 0, \tag{B42}$$
$$\frac{\partial Q^2}{\partial e_2} p_2 - \frac{\partial Q^2}{\partial e_2} w - \frac{\partial G^2}{\partial e_2} = 0. \tag{B43}$$

The above equations can be expressed as

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c), \tag{B44}$$
$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c), \tag{B45}$$
$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c), \tag{B46}$$
$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c). \tag{B47}$$

When the two retailers are symmetric, (B44)-(B47) are reduced to

$$\frac{\partial Q^1}{\partial p_1}(p_1-c)+\frac{\partial Q^2}{\partial p_1}(p_1-c)+Q^1=\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_1-c), \tag{B48}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1-c)+\frac{\partial Q^2}{\partial e_1}(p_1-c)-\frac{\partial G^1}{\partial e_1}=\frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_1-c). \tag{B49}$$

Recall that the price and effort under first best (with symmetric retailers) are governed by the following expressions

$$\frac{\partial \Pi}{\partial p_1}=\frac{\partial Q^1}{\partial p_1}(p_1-c)+\frac{\partial Q^2}{\partial p_1}(p_1-c)+Q^1=0, \tag{B50}$$

$$\frac{\partial \Pi}{\partial e_1}=\frac{\partial Q^1}{\partial e_1}(e_1-c)+\frac{\partial Q^2}{\partial e_1}(e_1-c)-\frac{\partial G^1}{\partial e_1}=0. \tag{B51}$$

Notice from (B48) and (B49) that the manufacturer controls the price charged and the effort expended by the retailer by choosing the wholesale price w. Note that $\frac{\partial Q^1}{\partial p_1}<0$, $\frac{\partial Q^1}{\partial e_1}>0$ and $\frac{\partial Q^2}{\partial e_1}>0$. Comparing (B48)-(B49) and (B50)-(B51) that the first best solution can be achieved if the manufacturer sets w such that $\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_1-c)=0$ and $\frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_1-c)$. Hence, we must have

$$w=c-\frac{\frac{\partial Q^2}{\partial p_1}(p_1-c)}{\frac{\partial Q^1}{\partial p_1}} \quad \text{and} \tag{B52}$$

$$w=c-\frac{\frac{\partial Q^2}{\partial e_1}(p_1-c)}{\frac{\partial Q^1}{\partial e_1}}. \tag{B53}$$

Hence, we must have

$$c - \frac{\frac{\partial Q^2}{\partial p_1}(p_1 - c)}{\frac{\partial Q^1}{\partial p_1}} = c - \frac{\frac{\partial Q^2}{\partial e_1}(p_1 - c)}{\frac{\partial Q^1}{\partial e_1}}, \tag{B54}$$

$$\Rightarrow \quad \frac{\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^2}{\partial e_1}}{\frac{\partial Q^1}{\partial e_1}}, \tag{B55}$$

$$\Rightarrow \quad \frac{\partial Q^2}{\partial p_1}\frac{\partial Q^1}{\partial e_1} - \frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} = 0. \tag{B56}$$

Since $\frac{\partial Q^1}{\partial p_1} < 0$, the above equation can never be satisfied[14]. Hence, the first best cannot be achieved.

In order to understand the impact of w on the price and effort choice under second best, we can express (B48) - (B49) as

$$\frac{\partial \pi^1}{\partial p_1} = f(p_1, e_1, w) = \frac{\partial Q^1}{\partial p_1}p_1 - \frac{\partial Q^1}{\partial p_1}w + Q^1 = 0, \tag{B57}$$

$$\frac{\partial \pi^1}{\partial e_1} = h(p_1, e_1, w) = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - \frac{\partial G^1}{\partial e_1} = 0. \tag{B58}$$

Note that the second order conditions for the retailer's optimization problem are given by

[14]Unless $\frac{\partial Q^2}{\partial p_1} = 0$ and $\frac{\partial Q^2}{\partial e_1} = 0$ or $\frac{\partial Q^1}{\partial e_1} = 0$ and $\frac{\partial Q^2}{\partial e_1} = 0$ — i.e., both the cross price and cross effort (free riding) effects are nonexistent or the effort does not affect demand. One other scenario where the equation can be satisfied is when the retailers exhibit a negative externality in effort, i.e., $\frac{\partial Q^2}{\partial e_1} < 0$.

$$\frac{\partial^2 \pi^1}{\partial p_1^2} = \frac{\partial f}{\partial p_1} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0, \tag{B59}$$

$$\frac{\partial^2 \pi^1}{\partial e_1^2} = \frac{\partial h}{\partial e_1} = \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - \frac{\partial^2 G^1}{\partial e_1^2} < 0, \tag{B60}$$

$$\frac{\partial^2 \pi^1}{\partial p_1^2}\frac{\partial^2 \pi^1}{\partial e_1^2} - (\frac{\partial^2 \pi^1}{\partial e_1 \partial p_1})^2 = \frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0. \tag{B61}$$

Using the implicit function theorem, we have

$$\frac{\partial p_1}{\partial w} = -\frac{det\left(\frac{\partial(f,h)}{\partial(e_1,w)}\right)}{det\left(\frac{\partial(f,h)}{\partial(e_1,p_1)}\right)} \tag{B62}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial w} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial w} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial p_1} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial p_1} \end{pmatrix}} \tag{B63}$$

$$= \frac{\frac{\partial f}{\partial e_1}\frac{\partial h}{\partial w} - \frac{\partial h}{\partial e_1}\frac{\partial f}{\partial w}}{\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - \frac{\partial f}{\partial e_1}\frac{\partial h}{\partial p_1}} \tag{B64}$$

$$\Rightarrow \quad \frac{\partial p_1}{\partial w} = \frac{-\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial h}{\partial e_1}\frac{\partial Q^1}{\partial p_1}}{\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2} \tag{B65}$$

Since we know from second order conditions that $\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$,

$$\frac{\partial p_1}{\partial w} > 0 \quad \text{if} \quad \frac{\partial h}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_1} = \left(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - \frac{\partial^2 G^1}{\partial e_1^2}\right)\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_1} > 0 \tag{B66}$$

$$\Rightarrow \quad \frac{\partial p_1}{\partial w} > 0 \quad \text{if} \quad \frac{\partial h}{\partial e_1} > \frac{(\frac{\partial Q^1}{\partial e_1})^2}{\frac{\partial Q^1}{\partial p_1}} \tag{B67}$$

Note that $\frac{\partial Q^1}{\partial p_1} < 0$ and we know from the second order conditions that $\frac{\partial h}{\partial e_1} = \left(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - \frac{\partial^2 G^1}{\partial e_1^2}\right) < 0$. Also, we know from the second order conditions that $\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$. Hence, we must have $\frac{\partial h}{\partial e_1} > \frac{(\frac{\partial Q^1}{\partial e_1})^2}{\frac{\partial f}{\partial p_1}}$. We can see that if $\frac{\partial Q^1}{\partial p_1} > \frac{\partial f}{\partial p_1}$, $\frac{\partial p_1}{\partial w} > 0$ is always satisfied. Hence, we need

$$\frac{\partial Q^1}{\partial p_1} > \frac{\partial f}{\partial p_1} \tag{B68}$$

$$\Rightarrow \quad \frac{\partial Q^1}{\partial p_1} > 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) \tag{B69}$$

$$\Rightarrow \quad \frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0 \tag{B70}$$

Since $\frac{\partial Q^1}{\partial p_1} < 0$, inequality (B70) is always satisfied when $\frac{\partial^2 Q^1}{\partial p_1^2} \leq 0$. The corresponding comparative static of effort is given by

$$\frac{\partial e_1}{\partial w} = -\frac{det\left(\frac{\partial(f,h)}{\partial(p_1,w)}\right)}{det\left(\frac{\partial(f,h)}{\partial(p_1,e_1)}\right)} \tag{B71}$$

$$= -\frac{det \begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial w} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial w} \end{pmatrix}}{det \begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial e_1} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial e_1} \end{pmatrix}} \tag{B72}$$

$$= \frac{\frac{\partial h}{\partial p_1}\frac{\partial f}{\partial w} - \frac{\partial f}{\partial p_1}\frac{\partial h}{\partial w}}{\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - \frac{\partial h}{\partial p_1}\frac{\partial f}{\partial e_1}} \tag{B73}$$

$$\Rightarrow \quad \frac{\partial e_1}{\partial w} = \frac{-\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial p_1} + \frac{\partial f}{\partial p_1}\frac{\partial Q^1}{\partial e_1}}{\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2} \tag{B74}$$

Since we know from the second order conditions that $\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$,

$$\frac{\partial e_1}{\partial w} < 0 \quad \text{if} \quad -\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial p_1} + \frac{\partial f}{\partial p_1}\frac{\partial Q^1}{\partial e_1} < 0 \tag{B75}$$

Simplifying the above condition (and substituting for $\frac{\partial f}{\partial p_1}$), we have

$$\frac{\partial Q^1}{\partial e_1}(\frac{\partial f}{\partial p_1} - \frac{\partial Q^1}{\partial p_1}) < 0 \tag{B76}$$

$$\Rightarrow \quad \frac{\partial f}{\partial p_1} < \frac{\partial Q^1}{\partial p_1} \tag{B77}$$

$$\Rightarrow \quad 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < \frac{\partial Q^1}{\partial p_1} \tag{B78}$$

$$\Rightarrow \quad \frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0 \tag{B79}$$

Hence, when $\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0$, $\frac{\partial e_1}{\partial w} < 0$. Note that if $\frac{\partial^2 Q^1}{\partial p_1^2} < 0$, $\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0$ is always satisfied and hence $\frac{\partial e_1}{\partial w} < 0$. Assuming $\frac{\partial e_1}{\partial w} < 0$ and $\frac{\partial p_1}{\partial w} > 0$, let us compare the levels of price and effort under second best with those under first best. First denote $\hat{p}$ and

$\hat{e}$ as the price and effort under the second best solution. Next, denote p^* and e^* as the price and effort under the first best solution. The manufacturer sets retail prices and efforts under first best such that the difference between marginal revenue and marginal cost is zero. Hence, denoting the difference between marginal revenue and marginal cost as $DMRMC_i^{fb}$ $\forall i \in \{p, e\}$, for symmetric retailers we have

$$DMRMC_p^{fb} = \frac{\partial Q^1}{\partial p_1}(p_1^* - c) + \frac{\partial Q^2}{\partial p_1}(p_1^* - c) + Q^1 = 0 \tag{B80}$$

$$DMRMC_e^{fb} = \frac{\partial Q^1}{\partial e_1}(e_1^{fb} - c) + \frac{\partial Q^2}{\partial e_1}(e_1^{fb} - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{B81}$$

Under second best, the above expressions can be expressed as

$$DMRMC_p^{sb} = \frac{\partial Q^1}{\partial p_1}(\hat{p}_1 - c) + \frac{\partial Q^2}{\partial p_1}(\hat{p}_1 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(\hat{p}_1 - c) \tag{B82}$$

$$DMRMC_e^{sb} = \frac{\partial Q^1}{\partial e_1}(\hat{p}_1 - c) + \frac{\partial Q^2}{\partial e_1}(\hat{p}_1 - c) - \frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(\hat{p}_1 - c) \tag{B83}$$

Comparing the price and effort levels under first best with those under second best, we can have the following potential cases

(i) $\hat{p} > p^*$ and $\hat{e} > e^*$

(ii) $\hat{p} > p^*$ and $\hat{e} < e^*$

(iii) $\hat{p} < p^*$ and $\hat{e} > e^*$

(iv) $\hat{p} < p^*$ and $\hat{e} < e^*$

We now investigate the viability of each of the cases listed above. When $\hat{p} > p^*$ and $\hat{e} > e^*$, we must have

$$DMRMC_p^{sb} < DMRMC_p^{fb} = 0 \quad \text{and} \tag{B84}$$

$$DMRMC_e^{sb} < DMRMC_e^{fb} = 0 \tag{B85}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(\hat{p}_1 - c) < 0 \quad \text{and} \tag{B86}$$

$$\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(\hat{p}_1 - c) < 0 \tag{B87}$$

Since $\frac{\partial Q^1}{\partial e_1} > 0$ and $\frac{\partial Q^2}{\partial e_1} > 0$, for equation (B87) to be satisfied, we must have $w < c$. Since, $\frac{\partial Q^1}{\partial p_1} < 0$ and $\frac{\partial Q^2}{\partial p_1} > 0$, for equation (B86) to be satisfied when $w < c$, we must also have $\hat{p}_1 < c$. Since $p^* > c$, $\hat{p}_1 > p_1^*$ will be violated when $\hat{p}_1 < c$. Hence, $\hat{p} > p^*$ and $\hat{e} > e^*$ is not possible (also note that when $w < c$ and $\hat{p}_1 < c$, we will have $\Pi < 0$).

Next, when $\hat{p} > p^*$ and $\hat{e} < e^*$, we must have

$$DMRMC_p^{sb} < DMRMC_p^{fb} = 0 \quad \text{and} \tag{B88}$$

$$DMRMC_e^{sb} > DMRMC_e^{fb} = 0 \tag{B89}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(\hat{p}_1 - c) < 0 \quad \text{and} \tag{B90}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(\hat{p}_1 - c) > 0 \tag{B91}$$

While we can see that the above equations can be simultaneously satisfied, we can show that the manufacturer has an incentive to decrease the price and increase the effort exerted by the retailer by lowering the wholesale price. Since $\frac{\partial e_1}{\partial w} < 0$ and $\frac{\partial p_1}{\partial w} > 0$, a reduction in wholesale price will reduce the price and at the same time increase the effort exerted by the retailers and thereby move the second best solution closer to the first best solution. Hence, $\hat{p} > p^*$ and $\hat{e} < e^*$ cannot be an equilibrium outcome.

Next, when $\hat{p} < p^*$ and $\hat{e} > e^*$, we must have

$$DMRMC_p^{sb} > DMRMC_p^{fb} = 0 \quad \text{and} \tag{B92}$$

$$DMRMC_e^{sb} < DMRMC_e^{fb} = 0 \tag{B93}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(\hat{p}_1 - c) > 0 \quad \text{and} \tag{B94}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(\hat{p}_1 - c) < 0 \tag{B95}$$

While we can see that the above equations can be simultaneously satisfied (by charging $w < c$), we can show that the manufacturer has an incentive to increase the price and decrease the effort exerted by the retailer by increasing the wholesale price. Since $\frac{\partial e_1}{\partial w} < 0$ and $\frac{\partial p_1}{\partial w} > 0$, an increase in wholesale price will increase the price and at the same time lower the effort exerted by the retailers and thereby move the second best solution closer to the first best solution. Hence, $\hat{p} < p^*$ and $\hat{e} > e^*$ cannot be an equilibrium outcome.

Finally, when $\hat{p} < p^*$ and $\hat{e} < e^*$, we must have

$$DMRMC_p^{sb} > DMRMC_p^{fb} = 0 \quad \text{and} \tag{B96}$$

$$DMRMC_e^{sb} > DMRMC_e^{fb} = 0 \tag{B97}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(\hat{p}_1 - c) > 0 \quad \text{and} \tag{B98}$$

$$\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(\hat{p}_1 - c) > 0 \tag{B99}$$

We can see that the above equations can be simultaneously satisfied. Also, since $\frac{\partial e_1}{\partial w} < 0$ and $\frac{\partial p_1}{\partial w} > 0$, an increase in wholesale price will increase the price and at the same time lower the effort exerted by the retailers. Hence, an increase in the wholesale price moves the second best price closer to the first best price and the second best effort farther from the first best effort. Therefore, increasing the wholesale price will not necessarily move the second best solution any closer to the first best solution. Similarly, a decrease in wholesale price will decrease the price and at the same time increase the effort exerted by the retailers. The lowering of wholesale price moves the second best price farther from the first best price and the second best effort closer to the first best effort. Hence, decreasing the wholesale price will not necessarily move the second best solution any closer to the first best solution. Therefore, $\hat{p} < p^*$ and $\hat{e} < e^*$ is the only viable equilibrium outcome and the statement of the Lemma 2 follows. ■

Proof of Proposition 1

Here, the retailers decide price and advertising and the manufacturer reimburses part (fraction α) of the retailer's advertising costs (the manufacturer does not set any upper bound on the reimbursement). The retailer's profit functions are given by

$$\pi^1 = Q^1(p_1 - w) - (1-\alpha)G^1 - F \tag{B100}$$

$$\pi^2 = Q^2(p_2 - w) - (1-\alpha)G^2 - F \tag{B101}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w-c) + Q^2(w-c) + F + F - \alpha G^1 - \alpha G^2 \tag{B102}$$

The manufacturer chooses the wholesale prices such that $\pi^1 = 0$, $\pi^2 = 0$, $\frac{\partial \pi^1}{\partial p_1} = 0$, $\frac{\partial \pi^1}{\partial e_1} = 0$, $\frac{\partial \pi^2}{\partial p_2} = 0$ and $\frac{\partial \pi^2}{\partial e_2} = 0$ (the retailer's surplus is extracted and the first order condition of the retailer are satisfied) The manufacturer's optimization problem can be expressed as

$$\begin{aligned}
&\underset{\{w, w, F, F, p_1, p_2, e_1, e_2, \alpha\}}{\text{Max}} \quad \Pi \\
&\qquad\qquad\text{subject to} \quad \pi^1 \geq 0,\, \pi^2 \geq 0,\, \frac{\partial \pi^1}{\partial p_1} = 0,\, \frac{\partial \pi^1}{\partial e_1} = 0,\, \frac{\partial \pi^2}{\partial p_2} = 0 \text{ and } \frac{\partial \pi^2}{\partial e_2} = 0
\end{aligned} \tag{B103}$$

The Lagrangian for the manufacturer's optimization problem can be expressed as

$$\begin{aligned}
L =& Q^1(w-c) + Q^2(w-c) + F + F - \alpha G^1 - \alpha G^2 + \\
& \lambda_1 \left[Q^1(p_1 - w) - (1-\alpha)G^1 - F\right] + \lambda_2 \left[Q^2(p_2 - w) - (1-\alpha)G^2 - F\right] + \\
& \mu_1 \left[\frac{\partial Q^1}{\partial p_1} p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1} w\right] + \mu_2 \left[\frac{\partial Q^2}{\partial p_2} p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2} w\right] + \\
& \eta_1 \left[\frac{\partial Q^1}{\partial e_1} p_1 - \frac{\partial Q^1}{\partial e_1} w - (1-\alpha)\frac{\partial G^1}{\partial e_1}\right] + \eta_2 \left[\frac{\partial Q^2}{\partial e_2} p_2 - \frac{\partial Q^2}{\partial e_2} w - (1-\alpha)\frac{\partial G^2}{\partial e_2}\right]
\end{aligned} \tag{B104}$$

The first order conditions are given by

$$\frac{\partial L}{\partial w} =Q^1 - \lambda_1 Q^1 - \mu_1 \frac{\partial Q^1}{\partial p_1} - \eta_1 \frac{\partial Q^1}{\partial e_1} = 0 \tag{B105}$$

$$\frac{\partial L}{\partial w} =Q^2 - \lambda_2 Q^2 - \mu_2 \frac{\partial Q^2}{\partial p_2} - \eta_2 \frac{\partial Q^2}{\partial e_2} = 0 \tag{B106}$$

$$\frac{\partial L}{\partial F} =1 - \lambda_1 = 0 \tag{B107}$$

$$\frac{\partial L}{\partial F} =1 - \lambda_2 = 0 \tag{B108}$$

$$\begin{aligned}\frac{\partial L}{\partial p_1} =&\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(w-c) + \lambda_1 \left[\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial p_1}(p_2 - w)\right] + \\ &2\mu_1 \left[\frac{\partial Q^1}{\partial p_1}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial p_1}\right] + \eta_1 \left[\frac{\partial Q^1}{\partial e_1}\right] = 0\end{aligned} \tag{B109}$$

$$\begin{aligned}\frac{\partial L}{\partial p_2} =&\frac{\partial Q^1}{\partial p_2}(w-c) + \frac{\partial Q^2}{\partial p_2}(w-c) + \lambda_1 \left[\frac{\partial Q^1}{\partial p_2}(p_1 - w)\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2\right] + \\ &\mu_1 \left[\frac{\partial Q^1}{\partial p_2}\right] + 2\mu_2 \left[\frac{\partial Q^2}{\partial p_2}\right] + \eta_2 \left[\frac{\partial Q^2}{\partial e_2}\right] = 0\end{aligned} \tag{B110}$$

$$\begin{aligned}\frac{\partial L}{\partial e_1} =&\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(w-c) - \alpha \frac{\partial G^1}{\partial e_1} + \lambda_1 \left[\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1}\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial e_1}(p_2 - w)\right] + \\ &\mu_1 \left[\frac{\partial Q^1}{\partial e_1}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial e_1}\right] - \eta_1 \left[(1-\alpha)\frac{\partial^2 G^1}{\partial e_1^2}\right] = 0\end{aligned} \tag{B111}$$

$$\begin{aligned}\frac{\partial L}{\partial e_2} =&\frac{\partial Q^1}{\partial e_2}(w-c) + \frac{\partial Q^2}{\partial e_2}(w-c) - \alpha \frac{\partial G^2}{\partial e_2} + \lambda_1 \left[\frac{\partial Q^1}{\partial e_2}(p_1 - w)\right] + \lambda_2 \left[\frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2}\right] + \\ &\mu_1 \left[\frac{\partial Q^1}{\partial e_2}\right] + \mu_2 \left[\frac{\partial Q^2}{\partial e_2}\right] - \eta_2 \left[(1-\alpha)\frac{\partial^2 G^2}{\partial e_2^2}\right] = 0\end{aligned} \tag{B112}$$

$$\frac{\partial L}{\partial \alpha} = - G^1 - G^2 + \lambda_1 G^1 + \lambda_2 G^2 + \eta_1 \frac{\partial G^1}{\partial e_1} + \eta_2 \frac{\partial G^2}{\partial e_2} = 0 \tag{B113}$$

$$\frac{\partial L}{\partial \lambda_1} = Q^1(p_1 - w) - (1-\alpha)G^1 - F = 0 \tag{B114}$$

$$\frac{\partial L}{\partial \lambda_2} = Q^2(p_2 - w) - (1-\alpha)G^2 - F = 0 \tag{B115}$$

$$\frac{\partial L}{\partial \mu_1} = \frac{\partial Q^1}{\partial p_1}p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1}w = 0 \tag{B116}$$

$$\frac{\partial L}{\partial \mu_2} = \frac{\partial Q^2}{\partial p_2}p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2}w = 0 \tag{B117}$$

$$\frac{\partial L}{\partial \eta_1} = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{B118}$$

$$\frac{\partial L}{\partial \eta_2} = \frac{\partial Q^2}{\partial e_2}p_2 - \frac{\partial Q^2}{\partial e_2}w - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{B119}$$

From (B107) and (B108) we can see that $\lambda_1 = 1$ and $\lambda_2 = 1$. Substituting in equations B105,B106, B109,B110,B111 and B112 and simplifying, we have

$$\frac{\partial L}{\partial w} = \mu_1\frac{\partial Q^1}{\partial p_1} + \eta_1\frac{\partial Q^1}{\partial e_1} = 0 \tag{B120}$$

$$\frac{\partial L}{\partial w} = \mu_2\frac{\partial Q^2}{\partial p_2} + \eta_2\frac{\partial Q^2}{\partial e_2} = 0 \tag{B121}$$

$$\frac{\partial L}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 + 2\mu_1\left[\frac{\partial Q^1}{\partial p_1}\right] + \mu_2\left[\frac{\partial Q^2}{\partial p_1}\right] + \eta_1\left[\frac{\partial Q^1}{\partial e_1}\right] = 0 \tag{B122}$$

$$\frac{\partial L}{\partial p_2} = \frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 + \mu_1\left[\frac{\partial Q^1}{\partial p_2}\right] + 2\mu_2\left[\frac{\partial Q^2}{\partial p_2}\right] + \eta_2\left[\frac{\partial Q^2}{\partial e_2}\right] = 0 \tag{B123}$$

$$\frac{\partial L}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} + \mu_1\left[\frac{\partial Q^1}{\partial e_1}\right] + \mu_2\left[\frac{\partial Q^2}{\partial e_1}\right] - \eta_1\left[(1-\alpha)\frac{\partial^2 G^1}{\partial e_1^2}\right] = 0 \tag{B124}$$

$$\frac{\partial L}{\partial e_2} = \frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} + \mu_1\left[\frac{\partial Q^1}{\partial e_2}\right] + \mu_2\left[\frac{\partial Q^2}{\partial e_2}\right] - \eta_2\left[(1-\alpha)\frac{\partial^2 G^2}{\partial e_2^2}\right] = 0 \tag{B125}$$

$$\frac{\partial L}{\partial \alpha} = \eta_1\frac{\partial G^1}{\partial e_1} + \eta_2\frac{\partial G^2}{\partial e_2} = 0 \tag{B126}$$

From (B120) , we can see that μ_1 and η_1 must either both be zero, both positive or both negative (since $\frac{\partial Q^1}{\partial p_1} < 0$). Similarly, from (B121) , we can see that μ_2 and η_2 must either both be zero, both positive or both negative (since $\frac{\partial Q^2}{\partial p_2} < 0$) But, from (B126) we can see that η_1 and η_2 cannot have the same sign. Using (B120)-(B121), the first order conditions can be expressed as

$$\frac{\partial L}{\partial p_1} = 0 \text{ implies}$$

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \eta_1 \left[\frac{\partial Q^1}{\partial e_1} - \frac{\frac{\partial Q^2}{\partial e_2}\frac{\partial Q^2}{\partial p_1}\frac{\partial G^1}{\partial e_1}}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \right] \tag{B127}$$

$$\frac{\partial L}{\partial p_2} = 0 \text{ implies}$$

$$= \frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 = \eta_2 \left[\frac{\partial Q^2}{\partial e_2} - \frac{\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial p_2}\frac{\partial G^2}{\partial e_2}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \right] \tag{B128}$$

$$\frac{\partial L}{\partial e_1} = 0 \text{ implies}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \eta_1 \left[\frac{(\frac{\partial Q^1}{\partial e_1})^2}{\frac{\partial Q^1}{\partial p_1}} + (1 - \alpha)\frac{\partial^2 G^1}{\partial e_1^2} - \frac{\frac{\partial Q^2}{\partial e_2}\frac{\partial Q^2}{\partial e_1}\frac{\partial G^1}{\partial e_1}}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \right] \tag{B129}$$

$$\frac{\partial L}{\partial e_2} = 0 \text{ implies}$$

$$\frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} = \eta_2 \left[\frac{(\frac{\partial Q^2}{\partial e_2})^2}{\frac{\partial Q^2}{\partial p_2}} + (1 - \alpha)\frac{\partial^2 G^2}{\partial e_2^2} - \frac{\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_2}\frac{\partial G^2}{\partial e_2}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \right] \tag{B130}$$

$$\frac{\partial L}{\partial \lambda_1} = Q^1(p_1 - w) - (1-\alpha)G^1 - F = 0 \tag{B131}$$

$$\frac{\partial L}{\partial \lambda_2} = Q^2(p_2 - w) - (1-\alpha)G^2 - F = 0 \tag{B132}$$

$$\frac{\partial L}{\partial \mu_1} = \frac{\partial Q^1}{\partial p_1}p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1}w = 0 \tag{B133}$$

$$\frac{\partial L}{\partial \mu_2} = \frac{\partial Q^2}{\partial p_2}p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2}w = 0 \tag{B134}$$

$$\frac{\partial L}{\partial \eta_1} = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{B135}$$

$$\frac{\partial L}{\partial \eta_2} = \frac{\partial Q^2}{\partial e_2}p_2 - \frac{\partial Q^2}{\partial e_2}w - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{B136}$$

$$\frac{\partial L}{\partial w} = \mu_1\frac{\partial Q^1}{\partial p_1} + \eta_1\frac{\partial Q^1}{\partial e_1} = 0 \tag{B137}$$

$$\frac{\partial L}{\partial w} = \mu_2\frac{\partial Q^2}{\partial p_2} + \eta_2\frac{\partial Q^2}{\partial e_2} = 0 \tag{B138}$$

$$\frac{\partial L}{\partial \alpha} = \eta_1\frac{\partial G^1}{\partial e_1} + \eta_2\frac{\partial G^2}{\partial e_2} = 0 \tag{B139}$$

The first order conditions that determine the retailer's price and effort choices are given by

$$\frac{\partial L}{\partial \mu_1} = f(p_1, e_1, p_2, e_2) = \frac{\partial Q^1}{\partial p_1}p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1}w = 0 \tag{B140}$$

$$\frac{\partial L}{\partial \mu_2} = f(p_2, e_2, p_1, e_1) = \frac{\partial Q^2}{\partial p_2}p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2}w = 0 \tag{B141}$$

$$\frac{\partial L}{\partial \eta_1} = h(p_1, e_1, p_2, e_2) = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{B142}$$

$$\frac{\partial L}{\partial \eta_2} = h(p_2, e_2, p_1, e_1) = \frac{\partial Q^2}{\partial e_2}p_2 - \frac{\partial Q^2}{\partial e_2}w - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{B143}$$

Re-arranging the above equations, we have

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{B144}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{B145}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} \tag{B146}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \left(\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c)\right) - \alpha\frac{\partial G^2}{\partial e_2} \tag{B147}$$

When the retailers are symmetric, we can see from the above equations that $p_1 = p_2$ and $e_1 = e_2$ must be the solution and hence the above equations reduce to the following

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) \tag{B148}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} \tag{B149}$$

The above equations can be expressed as

$$f(p_1, e_1, p_2, e_2) = \frac{\partial Q^1}{\partial p_1}p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1}w = 0 \tag{B150}$$

$$h(p_1, e_1, p_2, e_2) = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - (1 - \alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{B151}$$

Note that the second order conditions for the retailer's optimization problem (for symmetric retailers) are given by

$$\frac{\partial^2 \pi^1}{\partial p_1^2} = \frac{\partial f}{\partial p_1} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0 \tag{B152}$$

$$\frac{\partial^2 \pi^1}{\partial e_1^2} = \frac{\partial h}{\partial e_1} = \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - (1-\alpha)\frac{\partial^2 G^1}{\partial e_1^2} < 0 \tag{B153}$$

$$\frac{\partial^2 \pi^1}{\partial p_1^2}\frac{\partial^2 \pi^1}{\partial e_1^2} - (\frac{\partial^2 \pi^1}{\partial e_1 \partial p_1})^2 = \frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0 \tag{B154}$$

Using the implicit function theorem, we have

$$\frac{\partial p_1}{\partial w} = -\frac{det\left(\frac{\partial(f,h)}{\partial(e_1,w)}\right)}{det\left(\frac{\partial(f,h)}{\partial(e_1,p_1)}\right)} \tag{B155}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial w} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial w} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial p_1} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial p_1} \end{pmatrix}} \tag{B156}$$

$$= \frac{\frac{\partial f}{\partial e_1}\frac{\partial h}{\partial w} - \frac{\partial h}{\partial e_1}\frac{\partial f}{\partial w}}{\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - \frac{\partial f}{\partial e_1}\frac{\partial h}{\partial p_1}} \tag{B157}$$

$$\Rightarrow \quad \frac{\partial p_1}{\partial w} = \frac{-\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial h}{\partial e_1}\frac{\partial Q^1}{\partial p_1}}{\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2} \tag{B158}$$

$$\tag{B159}$$

Since we know from second order conditions that $\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$,

$$\frac{\partial p_1}{\partial w} > 0 \quad \text{if} \quad \frac{\partial h}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_1} = \left(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - (1-\alpha)\frac{\partial^2 G^1}{\partial e_1^2}\right)\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial e_1} > 0 \tag{B160}$$

$$\Rightarrow \quad \frac{\partial p_1}{\partial w} > 0 \quad \text{if} \quad \frac{\partial h}{\partial e_1} > \frac{(\frac{\partial Q^1}{\partial e_1})^2}{\frac{\partial Q^1}{\partial p_1}} \tag{B161}$$

Note that $\frac{\partial Q^1}{\partial p_1} < 0$ and we know from the second order conditions that

$$\frac{\partial h}{\partial e_1} = \left(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - (1-\alpha)\frac{\partial^2 G^1}{\partial e_1^2} \right) < 0. \tag{B162}$$

Also, we know from the second order conditions that $\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$. Hence, we must have $\frac{\partial h}{\partial e_1} > \frac{(\frac{\partial Q^1}{\partial e_1})^2}{\frac{\partial f}{\partial p_1}}$. Therefore, if $\frac{\partial Q^1}{\partial p_1} > \frac{\partial f}{\partial p_1}$, $\frac{\partial p_1}{\partial w} > 0$ is always satisfied. Hence, we need

$$\frac{\partial Q^1}{\partial p_1} > \frac{\partial f}{\partial p_1} \tag{B163}$$

$$\Rightarrow \quad \frac{\partial Q^1}{\partial p_1} > 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) \tag{B164}$$

$$\Rightarrow \quad \frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0 \tag{B165}$$

Since $\frac{\partial Q^1}{\partial p_1} < 0$, inequality (B165) is always satisfied when $\frac{\partial^2 Q^1}{\partial p_1^2} \leq 0$.

The corresponding comparative static of effort is given by

$$\frac{\partial e_1}{\partial w} = -\frac{det\left(\frac{\partial(f,h)}{\partial(p_1,w)}\right)}{det\left(\frac{\partial(f,h)}{\partial(p_1,e_1)}\right)} \tag{B166}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial w} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial w} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial e_1} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial e_1} \end{pmatrix}} \tag{B167}$$

$$= \frac{\frac{\partial h}{\partial p_1}\frac{\partial f}{\partial w} - \frac{\partial f}{\partial p_1}\frac{\partial h}{\partial w}}{\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - \frac{\partial h}{\partial p_1}\frac{\partial f}{\partial e_1}} \tag{B168}$$

$$\Rightarrow \quad \frac{\partial e_1}{\partial w} = \frac{-\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial p_1} + \frac{\partial f}{\partial p_1}\frac{\partial Q^1}{\partial e_1}}{\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2} \tag{B169}$$

Since we know from the second order conditions that $\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$,

$$\frac{\partial e_1}{\partial w} < 0 \quad \text{if} \quad -\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^1}{\partial p_1} + \frac{\partial f}{\partial p_1}\frac{\partial Q^1}{\partial e_1} < 0 \tag{B170}$$

Simplifying the above condition (and substituting for $\frac{\partial f}{\partial p_1}$), we have

$$\frac{\partial Q^1}{\partial e_1}(\frac{\partial f}{\partial p_1} - \frac{\partial Q^1}{\partial p_1}) < 0 \tag{B171}$$

$$\Rightarrow \quad \frac{\partial f}{\partial p_1} < \frac{\partial Q^1}{\partial p_1} \tag{B172}$$

$$\Rightarrow \quad 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < \frac{\partial Q^1}{\partial p_1} \tag{B173}$$

$$\Rightarrow \quad \frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0 \tag{B174}$$

Hence, when $\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0$, $\frac{\partial e_1}{\partial w} < 0$. Note that if $\frac{\partial^2 Q^1}{\partial p_1^2} \leq 0$, $\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0$ is always satisfied and hence $\frac{\partial e_1}{\partial w} < 0$.

The impact of the coop rate on the retail price is given by

$$\frac{\partial p_1}{\partial \alpha} = -\frac{det\left(\frac{\partial(f,h)}{\partial(e_1,\alpha)}\right)}{det\left(\frac{\partial(f,h)}{\partial(e_1,p_1)}\right)} \tag{B175}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial \alpha} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial \alpha} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial p_1} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial p_1} \end{pmatrix}} \tag{B176}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & 0 \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial \alpha} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial e_1} & \frac{\partial f}{\partial p_1} \\ \frac{\partial h}{\partial e_1} & \frac{\partial h}{\partial p_1} \end{pmatrix}} \tag{B177}$$

$$= \frac{\frac{\partial f}{\partial e_1}\frac{\partial h}{\partial \alpha}}{\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - \frac{\partial f}{\partial e_1}\frac{\partial h}{\partial p_1}} \tag{B178}$$

$$\Rightarrow \quad \frac{\partial p_1}{\partial \alpha} = \frac{\frac{\partial Q^1}{\partial e_1}\frac{\partial G^1}{\partial e_1}}{\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2} \tag{B179}$$

$$\tag{B180}$$

Since we know from second order conditions that $\frac{\partial h}{\partial e_1}\frac{\partial f}{\partial p_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$, $\frac{\partial Q^1}{\partial e_1} > 0$ and $\frac{\partial G^1}{\partial e_1} > 0$, we must have $\frac{\partial p_1}{\partial \alpha} > 0$.

The corresponding comparative static of effort is given by

$$\frac{\partial e_1}{\partial \alpha} = -\frac{det\left(\frac{\partial(f,h)}{\partial(p_1,\alpha)}\right)}{det\left(\frac{\partial(f,h)}{\partial(p_1,e_1)}\right)} \tag{B181}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial \alpha} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial \alpha} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial e_1} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial e_1} \end{pmatrix}} \tag{B182}$$

$$= -\frac{det\begin{pmatrix} \frac{\partial f}{\partial p_1} & 0 \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial \alpha} \end{pmatrix}}{det\begin{pmatrix} \frac{\partial f}{\partial p_1} & \frac{\partial f}{\partial e_1} \\ \frac{\partial h}{\partial p_1} & \frac{\partial h}{\partial e_1} \end{pmatrix}} \tag{B183}$$

$$= \frac{-\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial \alpha}}{\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - \frac{\partial h}{\partial p_1}\frac{\partial f}{\partial e_1}} \tag{B184}$$

$$\Rightarrow \quad \frac{\partial e_1}{\partial \alpha} = \frac{-\frac{\partial f}{\partial p_1}\frac{\partial G^1}{\partial e_1}}{\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2} \tag{B185}$$

Since we know from the second order conditions that $\frac{\partial f}{\partial p_1}\frac{\partial h}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0$, $\frac{\partial f}{\partial p_1} < 0$ and since $\frac{\partial G^1}{\partial e_1} > 0$, we must have $\frac{\partial e_1}{\partial \alpha} > 0$.

Assuming $\frac{\partial e_1}{\partial w} < 0$, $\frac{\partial p_1}{\partial w} > 0$, $\frac{\partial p_1}{\partial \alpha} > 0$ and $\frac{\partial e_1}{\partial \alpha} > 0$, let us compare the levels of price and effort under second best with those under first best. First denote p^α and e^α as the price and effort under the coop rate solution. Next, denote p^* and e^* as the price and effort under the first best solution. The manufacturer sets retail prices and efforts under first best such that the difference between marginal revenue and marginal cost is zero. Hence, denoting the difference between marginal revenue and marginal cost as $DMRMC_i^{fb}$ $\forall i \in \{p, e\}$, for symmetric retailers we have

$$DMRMC_p^{fb} = \frac{\partial Q^1}{\partial p_1}(p_1^* - c) + \frac{\partial Q^2}{\partial p_1}(p_1^* - c) + Q^1 = 0 \tag{B186}$$

$$DMRMC_e^{fb} = \frac{\partial Q^1}{\partial e_1}(e_1^{fb} - c) + \frac{\partial Q^2}{\partial e_1}(e_1^{fb} - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{B187}$$

Under the coop rate plan, the above equations can be expressed as

$$DMRMC_p^{\alpha} = \frac{\partial Q^1}{\partial p_1}(p_1^{\alpha} - c) + \frac{\partial Q^2}{\partial p_1}(p_1^{\alpha} - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_1^{\alpha} - c) \tag{B188}$$

$$DMRMC_e^{\alpha} = \frac{\partial Q^1}{\partial e_1}(p_1^{\alpha} - c) + \frac{\partial Q^2}{\partial e_1}(p_1^{\alpha} - c) - \frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_1^{\alpha} - c) - \alpha\frac{\partial G^1}{\partial e_1} \tag{B189}$$

Comparing the price and effort levels under first best with those under the coop rate plan, we can have the following potential cases

(i) $p^{\alpha} > p^*$ and $e^{\alpha} > e^*$

(ii) $p^{\alpha} > p^*$ and $e^{\alpha} < e^*$

(iii) $p^{\alpha} < p^*$ and $e^{\alpha} > e^*$

(iv) $p^{\alpha} < p^*$ and $e^{\alpha} < e^*$

(v) $p^{\alpha} = p^*$ and $e^{\alpha} = e^*$

We now investigate the viability of each of the cases listed above. When $p^{\alpha} > p^*$ and $e^{\alpha} > e^*$, we must have

$$DMRMC_p^{\alpha} < DMRMC_p^{fb} = 0 \quad \text{and} \tag{B190}$$

$$DMRMC_e^{\alpha} < DMRMC_e^{fb} = 0 \tag{B191}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1^{\alpha}-c) < 0 \quad \text{and} \tag{B192}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1^{\alpha}-c) - \alpha\frac{\partial G^1}{\partial e_1} < 0 \tag{B193}$$

The manufacturer can choose w and α to satisfy the above equations. But, since $\frac{\partial e_1}{\partial w} < 0$, $\frac{\partial p_1}{\partial w} > 0$, $\frac{\partial p_1}{\partial \alpha} > 0$ and $\frac{\partial e_1}{\partial \alpha} > 0$, the manufacturer can lower the wholesale price and the coop rate α to move p^{α} and e^{α} closer to the first best levels. Hence, $p^{\alpha} > p^*$ and $e^{\alpha} > e^*$ cannot be an equilibrium outcome.

Next, when $p^{\alpha} > p^*$ and $e^{\alpha} < e^*$, we must have

$$DMRMC_p^{\alpha} < DMRMC_p^{fb} = 0 \quad \text{and} \tag{B194}$$

$$DMRMC_e^{\alpha} > DMRMC_e^{fb} = 0 \tag{B195}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1^{\alpha}-c) < 0 \quad \text{and} \tag{B196}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1^{\alpha}-c) - \alpha\frac{\partial G^1}{\partial e_1} > 0 \tag{B197}$$

While we can see that the above equations can be simultaneously satisfied, we can show that the manufacturer has an incentive to decrease the price and increase the effort exerted by the retailer by lowering the wholesale price. Since $\frac{\partial e_1}{\partial w} < 0$ and $\frac{\partial p_1}{\partial w} > 0$, a reduction in wholesale price will reduce the price and at the same time increase the effort exerted by the retailers and thereby move the solution closer to the first best solution. Hence, $p^\alpha > p^*$ and $e^\alpha < e^*$ cannot be an equilibrium outcome.

Next, when $p^\alpha < p^*$ and $e^\alpha > e^*$, we must have

$$DMRMC_p^\alpha > DMRMC_p^{fb} = 0 \quad \text{and} \tag{B198}$$

$$DMRMC_e^\alpha < DMRMC_e^{fb} = 0 \tag{B199}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1^\alpha - c) > 0 \quad \text{and} \tag{B200}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1^\alpha - c) - \alpha\frac{\partial G^1}{\partial e_1} < 0 \tag{B201}$$

While we can see that the above equations can be simultaneously satisfied, we can show that the manufacturer has an incentive to increase the price and decrease the effort exerted by the retailer by increasing the wholesale price. Since $\frac{\partial e_1}{\partial w} < 0$ and $\frac{\partial p_1}{\partial w} > 0$, an increase in wholesale price will increase the price and at the same time lower the effort exerted by the retailers and thereby move the solution closer to the first best solution. Hence, $p^\alpha < p^*$ and $e^\alpha > e^*$ cannot be an equilibrium outcome.

Finally, when $p^\alpha < p^*$ and $e^\alpha < e^*$, we must have

$$DMRMC_p^{\alpha} > DMRMC_p^{fb} = 0 \quad \text{and} \tag{B202}$$

$$DMRMC_e^{\alpha} > DMRMC_e^{fb} = 0 \tag{B203}$$

Hence, we must have the following conditions being satisfied simultaneously

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1^{\alpha} - c) > 0 \quad \text{and} \tag{B204}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1^{\alpha} - c) - \alpha\frac{\partial G^1}{\partial e_1} > 0 \tag{B205}$$

We can see that the above equations can be simultaneously satisfied. But, since $\frac{\partial e_1}{\partial w} < 0$, $\frac{\partial p_1}{\partial w} > 0$, $\frac{\partial p_1}{\partial \alpha} > 0$ and $\frac{\partial e_1}{\partial \alpha} > 0$, the manufacturer can increase the coop rate α to increase both the price and effort levels. Hence, $p^{\alpha} < p^*$ and $e^{\alpha} < e^*$ cannot be an equilibrium outcome.

Hence, $p^{\alpha} = p^*$ and $e^{\alpha} = e^*$ is the equilibrium outcome. We compute below the wholesale price (w) and the coop rate α required to achieve the first best.

When the retailers are perfectly symmetric, the retailer's price and effort choices are determined by

$$\frac{\partial Q^1}{\partial p_1}(p_1-c) + \frac{\partial Q^2}{\partial p_1}(p_2-c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_2-c) \tag{B206}$$

$$\frac{\partial Q^1}{\partial p_1}(p_2-c) + \frac{\partial Q^2}{\partial p_1}(p_1-c) + Q^2 = \frac{\partial Q^1}{\partial p_2}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_1-c) \tag{B207}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1-c) + \frac{\partial Q^2}{\partial e_1}(p_2-c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_2-c)\right) - \alpha\frac{\partial G^1}{\partial e_1} \tag{B208}$$

$$\frac{\partial Q^1}{\partial e_1}(p_2-c) + \frac{\partial Q^2}{\partial e_1}(p_1-c) - \frac{\partial G^2}{\partial e_2} = \left(\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1-c)\right) - \alpha\frac{\partial G^2}{\partial e_2} \tag{B209}$$

substituting $e_1 = e_2$, $p_1 = p_2$ and $w = w$, we have[15]

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) \tag{B210}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} \tag{B211}$$

If the manufacturer sets w and α such that

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) = 0, \tag{B212}$$

$$\left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} = 0, \tag{B213}$$

equations (B210) and (B211) simplify to

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_1 - c) + Q^1 = 0 \tag{B214}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{B215}$$

This is identical to the first order conditions for the first best setting (with symmetrical retailers). The manufacturer's profits are given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F + F - \alpha G^1 - \alpha G^2 \tag{B216}$$

[15]note that when $e_1 = e_2$, $p_1 = p_2$ and $w = w$, we must have $Q^1 = Q^2$ and $\frac{\partial G^1}{\partial e_1} = \frac{\partial G^2}{\partial e_2}$

The fixed fees are obtained by solving equations (B114) and (B115) and are given by

$$F = Q^1(p_1 - w) - (1-\alpha)G^1 \tag{B217}$$

$$F = Q^2(p_2 - w) - (1-\alpha)G^2 \tag{B218}$$

Substituting in the manufacturer's profit function, we have

$$\Pi = Q^1(w-c) + Q^2(w-c) + F + F - \alpha G^1 - \alpha G^2 \tag{B219}$$

$$= Q^1(w-c) + Q^2(w-c) + Q^1(p_1 - w) - (1-\alpha)G^1 + Q^2(p_2 - w) - (1-\alpha)G^2 - \alpha G^1 - \alpha G^2 \tag{B220}$$

$$= Q^1(p_1 - c) + Q^2(p_2 - c) - G^1 - G^2 \tag{B221}$$

This is identical to the first best. We now show that (B212) and (B213) can be solved. Solving for $(w-c)$ from (B212), we have

$$w - c = -\frac{\frac{\partial Q^2}{\partial p_1}(p_1 - c)}{\frac{\partial Q^1}{\partial p_1}} \tag{B222}$$

Solving for α from (B213),

$$\alpha = \frac{1}{\frac{\partial G^1}{\partial e_1}}\left[\frac{\partial Q^1}{\partial e_1}(w-c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c)\right] \tag{B223}$$

Substituting for $w - c$ from (B222), we have

$$\alpha = \frac{p_1 - c}{\frac{\partial G^1}{\partial e_1}} \left[\frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}} \right] \tag{B224}$$

Note that when (B213) is satisfied, we can see from (B211) that $\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_1 - c) - \frac{\partial G^1}{\partial e_1} = 0$. Hence, we must have

$$\frac{p_1 - c}{\frac{\partial G^1}{\partial e_1}} = \frac{1}{\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^2}{\partial e_1}} \tag{B225}$$

Substituting the above in (B224), we have

$$\alpha = \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}} \tag{B226}$$

Since $|\frac{\partial Q^1}{\partial p_1}| > |\frac{\partial Q^2}{\partial p_1}|$, $\alpha < 1$ is always satisfied. Now, we ensure that $w - c \geq 0$. From (B213), we have

$$w = c + \frac{\alpha\frac{\partial G^1}{\partial e_1} - \frac{\partial Q^2}{\partial e_1}(p_1 - c)}{\frac{\partial Q^1}{\partial e_1}} \tag{B227}$$

Hence, we must have

$$\alpha > \frac{\frac{\partial Q^2}{\partial e_1}(p_1 - c)}{\frac{\partial G^1}{\partial e_1}} \tag{B228}$$

Substituting for α from (B226) and for $\frac{(p_1-c)}{\frac{\partial G^1}{\partial e_1}}$ from (B225), we must have

$$\left[\frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}\right] > \frac{\frac{\partial Q^2}{\partial e_1}}{\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^2}{\partial e_1}} \tag{B229}$$

$$\Rightarrow \quad \left[\frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}(\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^2}{\partial e_1})}\right] > \frac{\frac{\partial Q^2}{\partial e_1}}{\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^2}{\partial e_1}} \tag{B230}$$

$$\Rightarrow \quad \left[\frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}}\right] > \frac{\partial Q^2}{\partial e_1} \quad (\text{since} \quad \frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^2}{\partial e_1} > 0) \tag{B231}$$

$$\Rightarrow \quad \frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1} < \frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} \quad (\text{since} \quad \frac{\partial Q^1}{\partial p_1} < 0) \tag{B232}$$

$$\Rightarrow \quad -\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1} < 0 \quad \text{which is always true} \tag{B233}$$

Hence, the first best can be achieved by setting $\alpha = \frac{\frac{\partial Q^2}{\partial e_1}\frac{\partial Q^1}{\partial p_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1}}{\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^1}{\partial e_1} + \frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1}}$ and the statement of proposition 1 follows. ■

Proof of Proposition 2

When the manufacturer offers the retailers a cooperative advertising contract that includes a participation rate α and an accrual rate δ, a fraction of the retailer's advertising costs(αG^i) are reimbursed and the total cost reimbursement is caped at a fraction of the wholesale receipts(δ). Hence, the advertising cost reimbursement is given by

(i) αG^i if $\alpha G^i \leq \delta w Q^i$

(ii) $\delta w Q^i$ if $\alpha G^i > \delta w Q^i$

Given this cooperative advertising contract, the retailers have two options:

(i) expend effort such that the total cost reimbursement is less than or equal to the total accrual($\alpha G^i \leq \delta w Q^i$)

(ii) expend effort such that the total cost reimbursement is greater than the total accrual($\alpha G^i > \delta w Q^i$)

We consider each scenario separately

Effort Levels are Chosen Such That the Total Cost Reimbursement is Less than or Equal to the Total Accrual

When the retailers choose effort such that $\alpha G^i \leq \delta w Q^i$, the retailers are reimbursed αG^i. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F - G^1 + \alpha G^1 \tag{B234}$$

$$\pi^2 = Q^2(p_2 - w) - F - G^2 + \alpha G^2 \tag{B235}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F + F - \alpha G^1 - \alpha G^2 \tag{B236}$$

We first consider[16] the retailer 1's optimization problem given by

$$\begin{aligned} &\underset{\{p_1, e_1\}}{\text{Max}} \quad \pi^1 \\ &\text{subject to} \quad \delta w Q^1 - \alpha G^1 \geq 0 \end{aligned} \tag{B237}$$

The Lagrangian for the retailer's optimization problem is given by

$$LR1 = Q^1(p_1 - w) - F - G^1 + \alpha G^1 + \theta_1 \left[\delta w Q^1 - \alpha G^1\right] \tag{B238}$$

[16]Note that since the retailers are symmetric, the solution to retailer 2's optimization problem is identical to retailer 1's problem.

The first order conditions for the retailer's optimization problem are given by

$$\frac{\partial LR1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{B239}$$

$$\frac{\partial LR1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{B240}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 \geq 0 \tag{B241}$$

$$\theta_1(\delta w Q^1 - \alpha G^1) = 0 \tag{B242}$$

$$\theta_1 \geq 0 \tag{B243}$$

With respect to the Lagrange multiplier, we have two possibilities

- $\theta_1 = 0$
- $\theta_1 > 0$

In the next few section, we consider each of the above cases separately.

<u>$\theta_1 = 0$</u>

If the Lagrange multiplier $\theta_1 = 0$, then we must have $\delta w Q^1 - \alpha G^1 \geq 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B244}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{B245}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 \geq 0 \tag{B246}$$

Notice that these first order conditions are identical to the case where the manufacturer offers a cooperative advertising contract that only comprises of a single participation rate α. As we have shown earlier, the first best solution cannot be achieved in this case. The accrual rate δ can be set high enough to satisfy (B246).

<u>$\theta_1 > 0$</u>

If $\theta_1 > 0$, then we must have $\delta w Q^1 - \alpha G^1 = 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{B247}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1\left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{B248}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 = 0 \tag{B249}$$

From (B247), we can see that

$$(p_1 - (1 - \theta_1\delta)w) = -\frac{Q^1}{\frac{\partial Q^1}{\partial p_1}} \tag{B250}$$

Substituting the above in (B248), and solving for θ_1, we have

$$\frac{\partial Q^1}{\partial e_1}(p_1 - (1-\theta_1\delta)w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\alpha\frac{\partial G^1}{\partial e_1} = 0 \tag{B251}$$

$$\Rightarrow \quad \theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{B252}$$

The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{B253}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 = 0 \tag{B254}$$

Re-arranging the retailers' first order conditions, we have

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) \tag{B255}$$

$$\delta w Q^1 - \alpha G^1 = 0 \tag{B256}$$

In order for the manufacturer to induce the first best solution, the manufacturer must choose w, α and δ such that

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{B257}$$

$$\delta w Q^1 - \alpha G^1 = 0 \tag{B258}$$

We can see that (B257) can be satisfied if the wholesale price is chosen such that

$$w = \frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1 - \theta_1 \delta)} \tag{B259}$$

With regards to the retail effort, we can see from equations (B258) that the manufacturer can induce the retailer to expend first best effort by choosing the participation rate α and the accrual rate δ such that

$$\delta = \frac{\alpha G^{1^*}}{wQ^{1^*}} \tag{B260}$$

where G^{1^*} is the cost of effort incurred when exerting first best effort levels and Q^{1^*} is the demand when first best price and effort levels are exerted by both retailers. Also, we can see from equation (B252) that inorder for $\theta_1 > 0$, we must have

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}} > 0 \tag{B261}$$

$$\Rightarrow \quad \alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}} \tag{B262}$$

Hence, the first best can be achieved if the manufacturer chooses the wholesale prices, accrual rate and the participation rate such that the following equations are satisfied

$$w = \frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1 - \theta_1 \delta)} \tag{B263}$$

$$\delta = \frac{\alpha G^{1^*}}{wQ^{1^*}} \tag{B264}$$

$$\alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}} \tag{B265}$$

Hence, the first best solution can be achieved only when the above conditions are satisfied.

Effort Levels are Chosen Such That the Total Cost Reimbursement is Greater than the Total Accrual

When the retailers choose effort such that $\alpha G^i > \delta w Q^i$, the retailers are reimbursed $\delta w Q^i$. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F - G^1 + \delta w Q^1 \tag{B266}$$

$$\pi^2 = Q^2(p_2 - w) - F - G^2 + \delta w Q^2 \tag{B267}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F + F - \delta w Q^1 - \delta w Q^2 \tag{B268}$$

The first order conditions for $R1$'S optimization problem are given by

$$\frac{\partial \pi^1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \delta w \frac{\partial Q^1}{\partial p_1} = 0 \tag{B269}$$

$$\frac{\partial \pi^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1} + \delta w \frac{\partial Q^1}{\partial e_1} = 0 \tag{B270}$$

Re-arranging the above equations in terms of the first best first order conditions, we have

$$\frac{\partial \pi^1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \delta w \frac{\partial Q^1}{\partial p_1} \tag{B271}$$

$$\frac{\partial \pi^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \delta w \frac{\partial Q^1}{\partial e_1} \tag{B272}$$

As we can see from the above equations, in order for the first best to be achieved, the following equations must be satisfied simultaneously.

$$\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c)-\delta w\frac{\partial Q^1}{\partial p_1}=\frac{\partial Q^1}{\partial p_1}(w(1-\delta)-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c)=0 \quad \text{(B273)}$$

$$\frac{\partial Q^1}{\partial e_1}(w-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c)-\delta w\frac{\partial Q^1}{\partial e_1}=\frac{\partial Q^1}{\partial e_1}(w(1-\delta)-c)+\frac{\partial Q^2}{\partial e_1}(p_2-c)=0 \quad \text{(B274)}$$

Since $\frac{\partial Q^1}{\partial e_1} > 0$ and $\frac{\partial Q^2}{\partial e_1} > 0$, we can see that equations (B274) cannot be satisfied for any choice of w and δ. Hence, the first best cannot be achieved when the retailers choose to expend effort such that the total cost reimbursement through the participation rate is greater than the total accrual. Hence, the first best can be achieved only when the retailers expend effort such that the total cost reimbursement through the participation rate is less than or equal to the total accrual. In this scenario, the manufacturer can achieve the first best in two way: 1) choosing w, α and δ such that equations ()-() are satisfied or 2) choosing w, α and A such that equations ()-() are satisfied. In case 1, the accrual rate does not impact retail behavior and in case 2, the participation rate and the accrual rate both impact the retailer's behavior. Also, since the manufacturer uses a two-part tariff, the retail profits under all scenarios are equal to their reservation profit levels and hence the retailers would prefer the equilibrium that results in first best profits to the manufacturer. We now investigate the impact of the variable accrual rate on the retailer's price and effort choices by computing the comparative statics of price and effort with respect to the accrual rate.

Understanding The Impact of The Accrual Rate Linked to Wholesale Receipts

The first order conditions for the retailer $R1$ under the fixed accrual rate contract (derived in section(4.4)) are given by

$$\frac{\partial LR1}{\partial p1} = F1 = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{B275}$$

$$\frac{\partial LR1}{\partial e1} = F2 = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{B276}$$

$$\frac{\partial LR1}{\partial \theta_1} = F3 = \delta w Q^1 - \alpha G^1 = 0 \tag{B277}$$

The second order conditions for R1's optimization problem are given by

$$\frac{\partial^2 LR1}{\partial p_1^2} < 0, \quad \begin{bmatrix} \frac{\partial^2 LR1}{\partial p_1^2} & \frac{\partial^2 LR1}{\partial e_1 \partial p_1} \\ \frac{\partial^2 LR1}{\partial p_1 \partial e_1} & \frac{\partial^2 LR1}{\partial e_1^2} \end{bmatrix} > 0, \quad \begin{bmatrix} \frac{\partial^2 LR1}{\partial p_1^2} & \frac{\partial^2 LR1}{\partial e_1 \partial p_1} & \frac{\partial^2 LR1}{\partial \theta_1 \partial p_1} \\ \frac{\partial^2 LR1}{\partial p_1 \partial e_1} & \frac{\partial^2 LR1}{\partial e_1^2} & \frac{\partial^2 LR1}{\partial \theta_1 \partial e_1} \\ \frac{\partial^2 LR1}{\partial p_1 \partial \theta_1} & \frac{\partial^2 LR1}{\partial e_1 \partial \theta_1} & \frac{\partial^2 LR1}{\partial \theta_1^2} \end{bmatrix} < 0 \tag{B278}$$

where

$$\frac{\partial^2 LR1}{\partial p_1^2} = \frac{\partial F1}{\partial p_1} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) \tag{B279}$$

$$\frac{\partial^2 LR1}{\partial e_1^2} = \frac{\partial F2}{\partial e_1} = \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} \tag{B280}$$

$$\frac{\partial^2 LR1}{\partial \theta_1^2} = \frac{\partial F3}{\partial \theta_1} = 0 \tag{B281}$$

$$\frac{\partial^2 LR1}{\partial p_1 \partial e_1} = \frac{\partial F1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1} \tag{B282}$$

$$\frac{\partial^2 LR1}{\partial e_1 \partial p_1} = \frac{\partial F2}{\partial p_1} = \frac{\partial Q^1}{\partial e_1} \tag{B283}$$

$$\frac{\partial^2 LR1}{\partial p_1 \partial \theta_1} = \frac{\partial F1}{\partial \theta_1} = \delta w\frac{\partial Q^1}{\partial p_1} \tag{B284}$$

$$\frac{\partial^2 LR1}{\partial \theta_1 \partial p_1} = \frac{\partial F3}{\partial p_1} = \delta w\frac{\partial Q^1}{\partial p_1} \tag{B285}$$

$$\frac{\partial^2 LR1}{\partial e_1 \partial \theta_1} = \frac{\partial F2}{\partial \theta_1} = \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \tag{B286}$$

$$\frac{\partial^2 LR1}{\partial \theta_1 \partial e_1} = \frac{\partial F3}{\partial e_1} = \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \tag{B287}$$

Hence, the second order conditions are given by

$$\frac{\partial^2 LR1}{\partial p_1^2} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) < 0 \tag{B288}$$

$$\begin{bmatrix} 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) & \frac{\partial Q^1}{\partial e_1} \\ \frac{\partial Q^1}{\partial e_1} & \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} \end{bmatrix} > 0 \tag{B289}$$

$$\begin{bmatrix} 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial Q^1}{\partial e_1} & \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial p_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & 0 \end{bmatrix} < 0 \tag{B290}$$

Using the implicit function theorem, we have

$$\frac{\partial p_1}{\partial \delta} = -\frac{det\frac{\partial(F1,F2,F3)}{\partial(e_1,\delta,\theta_1)}}{det\frac{\partial(F1,F2,F3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial \delta} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial \delta} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial \delta} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}} \tag{B291}$$

$$= -\frac{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & \theta_1 w\frac{\partial Q^1}{\partial p_1} & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \theta_1 w\frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & wQ^1 & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}}$$

$$
= - \frac{\Delta^1}{det \begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) & \delta w \frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} \\ \delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} & \delta w \frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}} \tag{B292}
$$

where

$$
\begin{aligned}
\Delta^1 = & - \frac{\partial Q^1}{\partial e_1}(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}) w Q^1 + \theta_1 w \frac{\partial Q^1}{\partial p_1}(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1})^2 + \\
& \delta w \frac{\partial Q^1}{\partial p_1}\left[w Q^1 (\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2}) - \theta_1 w \frac{\partial Q^1}{\partial e_1}(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}) \right]
\end{aligned} \tag{B293}
$$

Hence, when $\Delta^1 < 0$, we must have $\frac{\partial p_1}{\partial \delta} < 0$. We now consider the comparative static with respect to the participation rate α. Using the implicit function theorem, we have

$$
\frac{\partial p_1}{\partial \alpha} = - \frac{det \frac{\partial(F1,F2,F3)}{\partial(e_1,\alpha,\theta_1)}}{det \frac{\partial(F1,F2,F3)}{\partial(e_1,p_1,\theta_1)}} = - \frac{det \begin{bmatrix} \frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial \alpha} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial \alpha} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial \alpha} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}}{det \begin{bmatrix} \frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}} \tag{B294}
$$

$$= -\frac{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 0 & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & (1-\theta_1)\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & -G^1 & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}}$$

$$= -\frac{\Delta^2}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}} \tag{B295}$$

where

$$\begin{aligned}\Delta^2 =& \frac{\partial Q^1}{\partial e_1}G^1(\delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1})+ \\ & \delta w\frac{\partial Q^1}{\partial p_1}\left[-G^1(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2})-(1-\theta_1)\frac{\partial G^1}{\partial e_1}(\delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1})\right]\end{aligned} \tag{B296}$$

Hence, when $\Delta^2 < 0$, we must have $\frac{\partial p_1}{\partial \alpha} < 0$. Notice from (B296) that when $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$, $\Delta^2 < 0$ is always satisfied. We now consider the comparative static with respect to the wholesale price w. Using the implicit function theorem, we have

$$\frac{\partial p_1}{\partial w} = -\frac{det\frac{\partial(F1,F2,F3)}{\partial(e_1,w,\theta_1)}}{det\frac{\partial(F1,F2,F3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix}\frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial w} & \frac{\partial F1}{\partial \theta_1}\\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial w} & \frac{\partial F2}{\partial \theta_1}\\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial w} & \frac{\partial F3}{\partial \theta_1}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1}\\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1}\\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1}\end{bmatrix}} \tag{B297}$$

$$= -\frac{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & -(1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1} & \delta w\frac{\partial Q^1}{\partial p_1}\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & -(1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1}\\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & \delta Q^1 & 0\end{bmatrix}}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1}\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1}\\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0\end{bmatrix}}$$

$$= -\frac{\Delta^3}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1}\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1}\\ \delta w\frac{\partial Q^1}{\partial e_1}-\alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0\end{bmatrix}} \tag{B298}$$

where

$$\Delta^3 = -\frac{\partial Q^1}{\partial e_1}\delta Q^1(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}) - (1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})^2 +$$
$$\delta w\frac{\partial Q^1}{\partial p_1}\left[\delta Q^1(\frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2}) + (1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1}(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})\right] \tag{B299}$$

Hence, when $\Delta^3 > 0$, we must have $\frac{\partial p_1}{\partial w} > 0$. Notice from (B299) that when $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$, $\Delta^3 > 0$ is always satisfied. We now consider the comparative statics for the effort. Using the implicit function theorem, we have

$$\frac{\partial e_1}{\partial \delta} = -\frac{det\frac{\partial(F1,F2,F3)}{\partial(\delta,p_1,\theta_1)}}{det\frac{\partial(F1,F2,F3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix}\frac{\partial F1}{\partial \delta} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1}\\ \frac{\partial F2}{\partial \delta} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1}\\ \frac{\partial F3}{\partial \delta} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1}\\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1}\\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1}\end{bmatrix}} \tag{B300}$$

$$= -\frac{det\begin{bmatrix}\theta_1 w\frac{\partial Q^1}{\partial p_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1}\\ \theta_1 w\frac{\partial Q^1}{\partial e_1} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}\\ wQ^1 & \delta w\frac{\partial Q^1}{\partial p_1} & 0\end{bmatrix}}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1}\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}\\ \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0\end{bmatrix}}$$

$$= -\frac{\Delta^4}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}} \tag{B301}$$

where

$$\Delta^4 = -\theta_1 w^2(\frac{\partial Q^1}{\partial p_1})^2\delta(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right] wQ^1(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}) +$$
$$\delta w\frac{\partial Q^1}{\partial p_1}\left[\theta_1 w^2\frac{\partial Q^1}{\partial e_1}\delta\frac{\partial Q^1}{\partial p_1} - w\frac{\partial Q^1}{\partial e_1}Q^1\right] \tag{B302}$$

Hence, when $\Delta^4 < 0$, we must have $\frac{\partial e_1}{\partial \delta} < 0$. Note that when $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$, we must have $\Delta^4 > 0$ and hence $\frac{\partial e_1}{\partial \delta} > 0$. We now consider the comparative static with respect to the participation rate α. Using the implicit function theorem, we have

$$\frac{\partial e_1}{\partial \alpha} = -\frac{det\frac{\partial(F1,F2,F3)}{\partial(\alpha,p_1,\theta_1)}}{det\frac{\partial(F1,F2,F3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial F1}{\partial \alpha} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial \alpha} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial \alpha} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}} \tag{B303}$$

$$= -\frac{det\begin{bmatrix} 0 & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ (1-\theta_1)\frac{\partial G^1}{\partial e_1} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ -G^1 & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}}$$

$$= -\frac{\Delta^5}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}} \tag{B304}$$

where

$$\Delta^5 = -\left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right]G^1(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \delta w\frac{\partial Q^1}{\partial p_1}\left[\delta w\frac{\partial Q^1}{\partial p_1}(1-\theta_1)\frac{\partial G^1}{\partial e_1} + G^1\frac{\partial Q^1}{\partial e_1}\right] \tag{B305}$$

Hence, when $\Delta^5 < 0$, we must have $\frac{\partial e_1}{\partial \alpha} < 0$. We now consider the comparative static with respect to the wholesale price. Using the implicit function theorem, we have

$$\frac{\partial e_1}{\partial w} = -\frac{det\frac{\partial(F1,F2,F3)}{\partial(w,p_1,\theta_1)}}{det\frac{\partial(F1,F2,F3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial F1}{\partial w} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial w} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial w} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial F1}{\partial e_1} & \frac{\partial F1}{\partial p_1} & \frac{\partial F1}{\partial \theta_1} \\ \frac{\partial F2}{\partial e_1} & \frac{\partial F2}{\partial p_1} & \frac{\partial F2}{\partial \theta_1} \\ \frac{\partial F3}{\partial e_1} & \frac{\partial F3}{\partial p_1} & \frac{\partial F3}{\partial \theta_1} \end{bmatrix}} \tag{B306}$$

$$= -\frac{det\begin{bmatrix} -(1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ -(1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ \delta Q^1 & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}}$$

$$= -\frac{\Delta^6}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-(1-\theta_1\delta)w) & \delta w\frac{\partial Q^1}{\partial p_1} \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-(1-\theta_1\delta)w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} \\ \delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} & \delta w\frac{\partial Q^1}{\partial p_1} & 0 \end{bmatrix}} \tag{B307}$$

where

$$\Delta^6 = +(1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1}\delta w\frac{\partial Q^1}{\partial p_1}(\delta w\frac{\partial Q^1}{\partial p_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right]\delta Q^1(\delta q\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})$$
$$\delta w\frac{\partial Q^1}{\partial p_1}\left[-\delta w\frac{\partial Q^1}{\partial p_1}(1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1} - \delta Q^1\frac{\partial Q^1}{\partial e_1}\right] \tag{B308}$$

Hence, when $\Delta^6 > 0$, we must have $\frac{\partial e_1}{\partial w} > 0$. Hence, summarizing the comparative statics, we have

- $\frac{\partial p_1}{\partial w} > 0$ when $\Delta^3 > 0$ (Note that $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$ ensures $\Delta^3 > 0$),
- $\frac{\partial e_1}{\partial w} > 0$ when $\Delta^6 > 0$,
- $\frac{\partial p_1}{\partial \alpha} < 0$ when $\Delta^2 < 0$ (Note that $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$ ensures $\Delta^2 < 0$),
- $\frac{\partial e_1}{\partial \alpha} < 0$ when $\Delta^5 < 0$,
- $\frac{\partial p_1}{\partial \delta} < 0$ when $\Delta^1 < 0$,
- $\frac{\partial e_1}{\partial \delta} > 0$ when $\Delta^4 > 0$ (Note that $\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1} < 0$ ensures $\Delta^4 > 0$).

Proof of Proposition 3

When the manufacturer offers the retailers a cooperative advertising contract that includes a participation rate α and an fixed accrual rate A, a fraction of the retailer's advertising costs(αG^i) are reimbursed and the total cost reimbursement is capped at A. Hence, the advertising cost reimbursement is given by

(i) αG^i if $\alpha G^i \leq A$

(ii) A if $\alpha G^i > A$

Given this cooperative advertising contract, the retailers have two options:

(i) expend effort such that the total cost reimbursement is less than or equal to the total accrual($\alpha G^i \leq A$)

(ii) expend effort such that the total cost reimbursement is greater than the total accrual($\alpha G^i > A$)

We consider each scenario separately

Effort Levels are Chosen Such That the Total Cost Reimbursement is Less than or Equal to the Total Accrual

When the retailers choose effort such that $\alpha G^i \leq A$, the retailers are reimbursed αG^i. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F - G^1 + \alpha G^1 \tag{B309}$$

$$\pi^2 = Q^2(p_2 - w) - F - G^2 + \alpha G^2 \tag{B310}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F + F - \alpha G^1 - \alpha G^2 \tag{B311}$$

Retailer 1's optimization problem given by

$$\begin{aligned} &\underset{\{p_1, e_1\}}{\text{Max}} \quad \pi^1 \\ &\text{subject to} \quad A - \alpha G^1 \geq 0 \end{aligned} \tag{B312}$$

The Lagrangian for the retailer's optimization problem is given by

$$LR1 = Q^1(p_1 - w) - F - G^1 + \alpha G^1 + \theta_1 \left[A - \alpha G^1\right] \tag{B313}$$

The first order conditions for the retailer's optimization problem are given by

$$\frac{\partial LR1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B314}$$

$$\frac{\partial LR1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1 - \alpha)\frac{\partial G^1}{\partial e_1} - \theta_1 \left[\alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{B315}$$

$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 \geq 0 \tag{B316}$$

$$\theta_1(A - \alpha G^1) = 0 \tag{B317}$$

$$\theta_1 \geq 0 \tag{B318}$$

With respect to the Lagrange multiplier, we have two possibilities

- $\theta_1 = 0$
- $\theta_1 > 0$

In the next few sections, we consider each of the above cases separately.

$\underline{\theta_1 = 0}$

If the Lagrange multiplier $\theta_1 = 0$, then we must have $A - \alpha G^1 \geq 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B319}$$
$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{B320}$$
$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 \geq 0 \tag{B321}$$

Notice that these first order conditions are identical to the case where the manufacturer offers a cooperative advertising contract that only comprises of a single participation rate α. As we have shown earlier, the first best solution can be achieved in this case. The fixed accrual rate A can be set high enough to satisfy (B321).

$\underline{\theta_1 > 0}$

If $\theta_1 > 0$, then we must have $A - \alpha G^1 = 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B322}$$
$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\left[\alpha\frac{\partial G^1}{\partial e_1}\right] = 0 \tag{B323}$$
$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 = 0 \tag{B324}$$

From (B322), we can see that

$$(p_1 - w) = -\frac{Q^1}{\frac{\partial Q^1}{\partial p_1}} \tag{B325}$$

Substituting the above in (B323), and solving for θ_1, we have

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1 \alpha \frac{\partial G^1}{\partial e_1} = 0 \tag{B326}$$

$$\Rightarrow \quad \theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{B327}$$

Hence, the first order conditions that govern the retailers price and effort choices are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B328}$$

$$A - \alpha G^1 = 0 \tag{B329}$$

Re-arranging the retailers' first order conditions, we have

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{B330}$$

$$A - \alpha G^1 = 0 \tag{B331}$$

In order for the manufacturer to induce the first best solution, the manufacturer must choose w, α and A such that

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) = 0 \tag{B332}$$

$$A - \alpha G^{1^*} = 0 \tag{B333}$$

where G^{1^*} is the cost of effort incurred by the retailer when first best effort levels are expended. We can see that (B332) can be satisfied if the wholesale price is chosen such that

$$w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} \tag{B334}$$

In terms of the accrual rate and participation rate, we can see that since A is fixed, we must have

$$A = \alpha G^{1^*} \tag{B335}$$

Notice that the Lagrange multiplier in equation (B327) depends on the participation rate α. Since the Lagrange multiplier is positive ($\theta_1 > 0$), we must have

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}} > 0 \tag{B336}$$

$$\Rightarrow \quad \alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}} \tag{B337}$$

Hence, the manufacturer can achieve the first best solution by choosing the wholesale price, the participation rate and accrual rate such that the following conditions are satisfied

$$w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} \tag{B338}$$

$$A = \alpha G^{1^*} \tag{B339}$$

$$\alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}} \tag{B340}$$

Since the retailers are symmetric, the wholesale price, accrual rate and the participation rate induce identical behavior from both the retailers.

Effort Levels are Chosen Such That the Total Cost Reimbursement is Greater than the Total Accrual

When the retailers choose effort such that $\alpha G^i > A$, the retailers are reimbursed A. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F - G^1 + A \tag{B341}$$

$$\pi^2 = Q^2(p_2 - w) - F - G^2 + A \tag{B342}$$

The first order conditions for retailer 1's optimization problem are given by

$$\frac{\partial \pi^1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B343}$$

$$\frac{\partial \pi^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1} = 0 \tag{B344}$$

Notice that the above first order conditions are identical to the second best setting. Hence, the manufacturer cannot induce the first best price and effort levels in this case. In summary, the manufacturer can induce the first best solution only when the retailers exert effort such that the total cost reimbursement is less than or equal to the total accrual. In this scenario, the manufacturer can achieve the first best in two way: 1) choosing w, α and A such that equations (B319)-(B321) are satisfied or 2) choosing w, α and A such that equations (B338)-(B340) are satisfied. In case 1, the accrual rate does not impact retail behavior and in case 2, the participation rate and the accrual rate both impact the retailer's behavior. Also, since the manufacturer uses a two-part tariff, the retail profits under all scenarios are equal to their reservation profit levels and hence the retailers would prefer the equilibrium that results in first best profits to the manufacturer. We now investigate the impact of the fixed accrual rate on the retailer's price and effort choices by computing the comparative statics of price and effort with respect to the accrual rate.

Understanding the Impact of The Fixed Accrual Rate

The first order conditions for the retailer $R1$ under the fixed accrual rate contract (derived in section(4.4)) are given by

$$\frac{\partial LR1}{\partial p_1} = f1 = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{B345}$$

$$\frac{\partial LR1}{\partial e_1} = f2 = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1 \left[\alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{B346}$$

$$\frac{\partial LR1}{\partial \theta_1} = f3 = A - \alpha G^1 = 0 \tag{B347}$$

Note that $\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}$. The second order conditions for the retailer's optimization problem are given by

$$\frac{\partial^2 LR1}{\partial p_1^2} < 0, \quad \begin{bmatrix} \frac{\partial^2 LR1}{\partial p_1^2} & \frac{\partial^2 LR1}{\partial e_1 \partial p_1} \\ \frac{\partial^2 LR1}{\partial p_1 \partial e_1} & \frac{\partial^2 LR1}{\partial e_1^2} \end{bmatrix} > 0, \quad \begin{bmatrix} \frac{\partial^2 LR1}{\partial p_1^2} & \frac{\partial^2 LR1}{\partial e_1 \partial p_1} & \frac{\partial^2 LR1}{\partial \theta_1 \partial p_1} \\ \frac{\partial^2 LR1}{\partial p_1 \partial e_1} & \frac{\partial^2 LR1}{\partial e_1^2} & \frac{\partial^2 LR1}{\partial \theta_1 \partial e_1} \\ \frac{\partial^2 LR1}{\partial p_1 \partial \theta_1} & \frac{\partial^2 LR1}{\partial e_1 \partial \theta_1} & \frac{\partial^2 LR1}{\partial \theta_1^2} \end{bmatrix} < 0 \tag{B348}$$

where

$$\frac{\partial^2 LR1}{\partial p_1^2} = \frac{\partial f1}{\partial p_1} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) \tag{B349}$$

$$\frac{\partial^2 LR1}{\partial e_1^2} = \frac{\partial f2}{\partial e_1} = \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1 - \alpha(1-\theta_1)] \frac{\partial^2 G^1}{\partial e_1^2} \tag{B350}$$

$$\frac{\partial^2 LR1}{\partial \theta_1^2} = \frac{\partial f3}{\partial \theta_1} = 0 \tag{B351}$$

$$\frac{\partial^2 LR1}{\partial p_1 \partial e_1} = \frac{\partial f1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1} \tag{B352}$$

$$\frac{\partial^2 LR1}{\partial e_1 \partial p_1} = \frac{\partial f2}{\partial p_1} = \frac{\partial Q^1}{\partial e_1} \tag{B353}$$

$$\frac{\partial^2 LR1}{\partial p_1 \partial \theta_1} = \frac{\partial f1}{\partial \theta_1} = 0 \tag{B354}$$

$$\frac{\partial^2 LR1}{\partial \theta_1 \partial p_1} = \frac{\partial f3}{\partial p_1} = 0 \tag{B355}$$

$$\frac{\partial^2 LR1}{\partial e_1 \partial \theta_1} = \frac{\partial f2}{\partial \theta_1} = -\alpha \frac{\partial G^1}{\partial e_1} \tag{B356}$$

$$\frac{\partial^2 LR1}{\partial \theta_1 \partial e_1} = \frac{\partial f3}{\partial e_1} = -\alpha \frac{\partial G^1}{\partial e_1} \tag{B357}$$

Hence, the second order conditions are given by

$$\frac{\partial^2 LR1}{\partial p_1^2} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0 \tag{B358}$$

$$\begin{bmatrix} 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & \frac{\partial Q^1}{\partial e_1} \\ \frac{\partial Q^1}{\partial e_1} & \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1 - \alpha(1-\theta_1)]\,\frac{\partial^2 G^1}{\partial e_1^2} \end{bmatrix} > 0 \tag{B359}$$

$$\begin{bmatrix} 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & \frac{\partial Q^1}{\partial e_1} & 0 \\ \frac{\partial Q^1}{\partial e_1} & \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1 - \alpha(1-\theta_1)]\,\frac{\partial^2 G^1}{\partial e_1^2} & -\alpha\frac{\partial G^1}{\partial e_1} \\ 0 & -\alpha\frac{\partial G^1}{\partial e_1} & 0 \end{bmatrix} < 0 \tag{B360}$$

Using the implicit function theorem, we have

$$\frac{\partial p_1}{\partial A}=-\frac{det\frac{\partial(f1,f2,f3)}{\partial(e_1,A,\theta_1)}}{det\frac{\partial(f1,f2,f3)}{\partial(e_1,p_1,\theta_1)}}=-\frac{det\begin{bmatrix}\frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial A} & \frac{\partial f1}{\partial \theta_1}\\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial A} & \frac{\partial f2}{\partial \theta_1}\\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial A} & \frac{\partial f3}{\partial \theta_1}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1}\\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial p_1} & -\alpha\frac{\partial G^1}{\partial e_1}\\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1}\end{bmatrix}}$$

$$=-\frac{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 0 & 0\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & 0 & -\alpha\frac{\partial G^1}{\partial e_1}\\ -\alpha\frac{\partial G^1}{\partial e_1} & 1 & 0\end{bmatrix}}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-w) & 0\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1}\\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0\end{bmatrix}}$$

$$=-\frac{\frac{\partial Q^1}{\partial e_1}\alpha\frac{\partial G^1}{\partial e_1}}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-w) & 0\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1}\\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0\end{bmatrix}} \tag{B361}$$

Notice that the denominator for (B361) is identical to the second order condition and must be negative. Hence, we must have $\frac{\partial p_1}{\partial A}>0$. We now consider the comparative static with respect to the participation rate α

$$\frac{\partial p_1}{\partial \alpha} = -\frac{det\frac{\partial(f1,f2,f3)}{\partial(e_1,\alpha,\theta_1)}}{det\frac{\partial(f1,f2,f3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial \alpha} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial \alpha} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial \alpha} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}$$

$$= -\frac{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 0 & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & (1-\theta_1)\frac{\partial G^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & -G^1 & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}}$$

$$= \frac{\frac{\partial Q^1}{\partial e_1}\alpha\frac{\partial G^1}{\partial e_1}G^1}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1-w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}} \tag{B362}$$

Notice that the denominator for (B362) is identical to the second order condition and must be negative. Hence, we must have $\frac{\partial p_1}{\partial \alpha} < 0$. We now consider the comparative static with respect to the wholesale price

$$\frac{\partial p_1}{\partial w} = -\frac{det\frac{\partial(f1,f2,f3)}{\partial(e_1,w,\theta_1)}}{det\frac{\partial(f1,f2,f3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix}\frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial w} & \frac{\partial f1}{\partial \theta_1}\\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial w} & \frac{\partial f2}{\partial \theta_1}\\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial w} & \frac{\partial f3}{\partial \theta_1}\end{bmatrix}}{det\begin{bmatrix}\frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1}\\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1}\\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1}\end{bmatrix}}$$

$$= -\frac{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & -\frac{\partial Q^1}{\partial p_1} & 0\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & -\frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1}\\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0\end{bmatrix}}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-w) & 0\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1}\\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0\end{bmatrix}}$$

$$= \frac{\frac{\partial Q^1}{\partial p_1}(\alpha\frac{\partial G^1}{\partial e_1})^2}{det\begin{bmatrix}\frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1}+\frac{\partial^2 Q^1}{\partial p_1^2}(p_1-w) & 0\\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1-w)-[1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1}\\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0\end{bmatrix}} \tag{B363}$$

Notice that the denominator for (B363) is identical to the second order condition and must be negative. Since $\frac{\partial Q^1}{\partial p_1} < 0$, we must have $\frac{\partial p_1}{\partial w} > 0$. We now consider the comparative statics for the effort.

Using the implicit function theorem, we have

$$\frac{\partial e_1}{\partial A} = -\frac{det\frac{\partial(f1,f2,f3)}{\partial(A,p_1,\theta_1)}}{det\frac{\partial(f1,f2,f3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial f1}{\partial A} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial A} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial A} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}$$

$$= -\frac{det\begin{bmatrix} 0 & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ 0 & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ 1 & 0 & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}}$$

$$= \frac{\left(2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w)\right)\alpha\frac{\partial G^1}{\partial e_1}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}} \tag{B364}$$

We know from the second order conditions that the denominator for (B364) as well as $2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w)$ are both negative. Hence, we must have $\frac{\partial e_1}{\partial A} > 0$. We now consider the comparative static with respect to the participation rate α

$$\frac{\partial e_1}{\partial \alpha} = -\frac{det\frac{\partial(f1,f2,f3)}{\partial(\alpha,p_1,\theta_1)}}{det\frac{\partial(f1,f2,f3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial f1}{\partial \alpha} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial \alpha} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial \alpha} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}$$

$$= -\frac{det\begin{bmatrix} 0 & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ (1-\theta_1)\frac{\partial G^1}{\partial e_1} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -G^1 & 0 & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}}$$

$$= \frac{-\left(2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w)\right)\alpha\frac{\partial G^1}{\partial e_1}G^1}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1-\alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}} \tag{B365}$$

We know from the second order conditions that the denominator for (B365) as well as $2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w)$ are both negative. Hence, we must have $\frac{\partial e_1}{\partial \alpha} < 0$. We now consider the comparative static with respect to the wholesale price.

$$\frac{\partial e_1}{\partial w} = -\frac{det\frac{\partial(f1,f2,f3)}{\partial(w,p_1,\theta_1)}}{det\frac{\partial(f1,f2,f3)}{\partial(e_1,p_1,\theta_1)}} = -\frac{det\begin{bmatrix} \frac{\partial f1}{\partial w} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial w} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial w} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}{det\begin{bmatrix} \frac{\partial f1}{\partial e_1} & \frac{\partial f1}{\partial p_1} & \frac{\partial f1}{\partial \theta_1} \\ \frac{\partial f2}{\partial e_1} & \frac{\partial f2}{\partial p_1} & \frac{\partial f2}{\partial \theta_1} \\ \frac{\partial f3}{\partial e_1} & \frac{\partial f3}{\partial p_1} & \frac{\partial f3}{\partial \theta_1} \end{bmatrix}}$$

$$= -\frac{det\begin{bmatrix} -\frac{\partial Q^1}{\partial p_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ -\frac{\partial Q^1}{\partial e_1} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ 0 & 0 & 0 \end{bmatrix}}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1 - \alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}}$$

$$= \frac{0}{det\begin{bmatrix} \frac{\partial Q^1}{\partial e_1} & 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) & 0 \\ \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - [1 - \alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2} & \frac{\partial Q^1}{\partial e_1} & -\alpha\frac{\partial G^1}{\partial e_1} \\ -\alpha\frac{\partial G^1}{\partial e_1} & 0 & 0 \end{bmatrix}}$$

$$= 0 \qquad \text{(B366)}$$

Hence, the effort is not affected by the wholesale price. Summarizing the comparative statics, we have

- $\frac{\partial p_1}{\partial w} > 0$,
- $\frac{\partial e_1}{\partial w} = 0$,
- $\frac{\partial p_1}{\partial \alpha} < 0$,
- $\frac{\partial e_1}{\partial \alpha} < 0$,
- $\frac{\partial p_1}{\partial A} > 0$,
- $\frac{\partial e_1}{\partial A} > 0$,

As we can see from the above comparative statics, as the manufacturer increases the accrual rate, the retailers increase both their price and effort levels. The impact of the participation rate is negative on both the price and effort. Since the retailers choose effort such that the total cost reimbursement is equal to the accrual rate, an increase in the participation rate would require a lower effort to ensure that the total reimbursement is equal to the accrual rate. Hence, the retailers reduce their effort levels. This lowering of effort causes an indirect lowering of the retail prices as well. Since the retailer's choice of effort is governed solely by the criterion to equate cost reimbursement to constant accrual rate, the wholesale price does not impact retail efforts. The retail price is affected negatively by the wholesale price due to its impact on the retailer's margin. ■

APPENDIX C: PROOFS FOR RESULTS IN CHAPTER 4

Proof of Lemma 1

From the earlier proofs in Appendix B, we know that the first order conditions under the first best setting are given by

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = 0 \tag{C1}$$
$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 = 0 \tag{C2}$$
$$\frac{\partial \Pi}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{C3}$$
$$\frac{\partial \Pi}{\partial e_2} = \frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} = 0 \tag{C4}$$

■

Proof of Lemma 2

Under the second best setting, the equations that govern the retailer's price and effort choices are given[17] by

$$\frac{\partial Q^1}{\partial p_1} p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1} w = 0 \tag{C5}$$
$$\frac{\partial Q^2}{\partial p_2} p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2} w = 0 \tag{C6}$$
$$\frac{\partial Q^1}{\partial e_1} p_1 - \frac{\partial Q^1}{\partial e_1} w - \frac{\partial G^1}{\partial e_1} = 0 \tag{C7}$$
$$\frac{\partial Q^2}{\partial e_2} p_2 - \frac{\partial Q^2}{\partial e_2} w - \frac{\partial G^2}{\partial e_2} = 0 \tag{C8}$$

The above equations can be expressed as

[17] See appendix B for the proof

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{C9}$$
$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C10}$$
$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) \tag{C11}$$
$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) \tag{C12}$$

Since $\frac{\partial Q^1}{\partial e_1} > 0$ and $\frac{\partial Q^2}{\partial e_2} > 0$, the manufacturer cannot choose the wholesale prices such that $\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) = 0$ and $\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) = 0$. Hence, the first best cannot be achieved. ■

Proof of Proposition 1

In this setting, the equations that govern the retailer's price and effort choices are given by

$$\frac{\partial L}{\partial \mu_1} = f(p_1, e_1, p_2, e_2) = f^1 = \frac{\partial Q^1}{\partial p_1}p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1}w = 0 \tag{C13}$$
$$\frac{\partial L}{\partial \mu_2} = f(p_2, e_2, p_1, e_1) = f^2 = \frac{\partial Q^2}{\partial p_2}p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2}w = 0 \tag{C14}$$
$$\frac{\partial L}{\partial \eta_1} = h(p_1, e_1, p_2, e_2) = h^1 = \frac{\partial Q^1}{\partial e_1}p_1 - \frac{\partial Q^1}{\partial e_1}w - (1 - \alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{C15}$$
$$\frac{\partial L}{\partial \eta_2} = h(p_2, e_2, p_1, e_1) = h^2 = \frac{\partial Q^2}{\partial e_2}p_2 - \frac{\partial Q^2}{\partial e_2}w - (1 - \alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C16}$$

The second order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial^2 \pi^1}{\partial p_1^2} = \frac{\partial f^1}{\partial p_1} = 2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - w) < 0, \tag{C17}$$

$$\frac{\partial^2 \pi^1}{\partial e_1^2} = \frac{\partial h^1}{\partial e_1} = \frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - w) - (1-\alpha)\frac{\partial^2 G^1}{\partial e_1^2} < 0, \tag{C18}$$

$$\frac{\partial^2 \pi^1}{\partial p_1^2}\frac{\partial^2 \pi^1}{\partial e_1^2} - (\frac{\partial^2 \pi^1}{\partial e_1 \partial p_1})^2 = \frac{\partial f^1}{\partial p_1}\frac{\partial h^1}{\partial e_1} - (\frac{\partial Q^1}{\partial e_1})^2 > 0, \tag{C19}$$

and

$$\frac{\partial^2 \pi^2}{\partial p_2^2} = \frac{\partial f^2}{\partial p_2} = 2\frac{\partial Q^2}{\partial p_2} + \frac{\partial^2 Q^2}{\partial p_2^2}(p_2 - w) < 0 \tag{C20}$$

$$\frac{\partial^2 \pi^2}{\partial e_2^2} = \frac{\partial h^2}{\partial e_2} = \frac{\partial^2 Q^2}{\partial e_2^2}(p_2 - w) - (1-\alpha)\frac{\partial^2 G^2}{\partial e_2^2} < 0 \tag{C21}$$

$$\frac{\partial^2 \pi^2}{\partial p_2^2}\frac{\partial^2 \pi^2}{\partial e_2^2} - (\frac{\partial^2 \pi^2}{\partial e_2 \partial p_2})^2 = \frac{\partial f^2}{\partial p_2}\frac{\partial h^2}{\partial e_2} - (\frac{\partial Q^2}{\partial e_2})^2 > 0 \tag{C22}$$

Using the implicit function theorem, we can show that

$$\frac{\partial p_1}{\partial w} > 0, \quad \frac{\partial p_2}{\partial w} > 0, \tag{C23}$$

$$\frac{\partial e_1}{\partial w} < 0, \quad \frac{\partial e_2}{\partial w} < 0, \tag{C24}$$

$$\frac{\partial p_1}{\partial \alpha} > 0, \quad \frac{\partial p_2}{\partial \alpha} > 0, \quad \text{and} \tag{C25}$$

$$\frac{\partial e_1}{\partial \alpha} > 0, \quad \frac{\partial e_2}{\partial \alpha} > 0. \tag{C26}$$

Re-arranging the equations (C13)-(C16), we have

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{C27}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C28}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} \tag{C29}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \left(\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c)\right) - \alpha\frac{\partial G^2}{\partial e_2} \tag{C30}$$

The manufacturer can ensure that the retailers expend first best effort levels and charge first best prices by choosing the wholesale price and accrual rate such that the following conditions hold

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) = 0 \quad \Rightarrow \quad w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}}, \tag{C31}$$

$$\frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) = 0 \quad \Rightarrow \quad w = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}}, \tag{C32}$$

$$\left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} = 0 \quad \Rightarrow \quad \alpha = \frac{\left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right)}{\frac{\partial G^1}{\partial e_1}}$$

$$\Rightarrow \quad \alpha = \frac{(p_2 - c)(\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1})}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C33}$$

$$\left(\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c)\right) - \alpha\frac{\partial G^2}{\partial e_2} = 0 \quad \Rightarrow \quad \alpha = \frac{\left(\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c)\right)}{\frac{\partial G^2}{\partial e_2}}$$

$$\Rightarrow \quad \alpha = \frac{(p_1 - c)(\frac{\partial Q^2}{\partial p_2}\frac{\partial Q^1}{\partial e_2} - \frac{\partial Q^2}{\partial e_2}\frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{C34}$$

Since the manufacturer offers a single participation rate α, in order to induce first best effort levels from both the retailers, we must have

$$\frac{(p_2 - c)(\frac{\partial Q^1}{\partial p_1}\frac{\partial Q^2}{\partial e_1} - \frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_1})}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} = \frac{(p_1 - c)(\frac{\partial Q^2}{\partial p_2}\frac{\partial Q^1}{\partial e_2} - \frac{\partial Q^2}{\partial e_2}\frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{C35}$$

Notice that the condition (C35) holds when the retailers are symmetric. When the retailers are asymmetric, and when condition (C35) does not hold, the manufacturer cannot induce the first best effort levels from both retailers simultaneously. Also, since the manufacturer charges a single wholesale price we can see from equations (C31)-(C32) that in order to induce both retailers to charge the first best prices, we must have

$$c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C36}$$

$$\Rightarrow \quad \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C37}$$

When the retailers are perfectly symmetric, the above equation is satisfied. However, when the retailers are asymmetric, and equation (C37) is not satisfied, the manufacturer cannot induce the first best price levels from both retailers simultaneously. ■

Proof of Proposition 2

When the manufacturer offers the retailers a cooperative advertising contract that includes a participation rate α and an accrual rate δ, a fraction of the retailer's advertising costs(αG^i) are reimbursed and the total cost reimbursement is caped at a fraction of the wholesale receipts(δ). Hence, the advertising cost reimbursement is given by

(i) αG^i if $\alpha G^i \leq \delta w Q^i$

(ii) $\delta w Q^i$ if $\alpha G^i > \delta w Q^i$

Given this cooperative advertising contract, the retailers have two options:

(i) expend effort such that the total cost reimbursement is less than or equal to the total accrual($\alpha G^i \leq \delta w Q^i$)

(ii) expend effort such that the total cost reimbursement is greater than the total accrual($\alpha G^i > \delta w Q^i$)

We consider each scenario separately

Effort Levels are Chosen Such That the Total Cost Reimbursement is Less than or Equal to the Total Accrual

When the retailers choose effort such that $\alpha G^i \leq \delta w Q^i$, the retailers are reimbursed αG^i. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F^1 - G^1 + \alpha G^1 \tag{C38}$$

$$\pi^2 = Q^2(p_2 - w) - F^2 - G^2 + \alpha G^2 \tag{C39}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - \alpha G^1 - \alpha G^2 \tag{C40}$$

We first consider the retailer 1's optimization problem given by

$$\begin{aligned} &\underset{\{p_1, e_1\}}{\text{Max}} \quad \pi^1 \\ &\text{subject to} \quad \delta w Q^1 - \alpha G^1 \geq 0 \end{aligned} \tag{C41}$$

The Lagrangian for the retailer's optimization problem is given by

$$LR1 = Q^1(p_1 - w) - F^1 - G^1 + \alpha G^1 + \theta_1 \left[\delta w Q^1 - \alpha G^1\right] \tag{C42}$$

The first order conditions for the retailer's optimization problem are given by

$$\frac{\partial LR1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C43}$$

$$\frac{\partial LR1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1 - \alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C44}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 \geq 0 \tag{C45}$$

$$\theta_1(\delta w Q^1 - \alpha G^1) = 0 \tag{C46}$$

$$\theta_1 \geq 0 \tag{C47}$$

The first order conditions for retailer 2's optimization problem are similar to that of retailer 1 and are given by

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 + \theta_2(\delta w \frac{\partial Q^2}{\partial p_2}) = 0 \tag{C48}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} + \theta_2\left[\delta w Q^2 - \alpha \frac{\partial G^2}{\partial e_2}\right] = 0 \tag{C49}$$

$$\frac{\partial LR2}{\partial \theta_2} = \delta w Q^2 - \alpha G^2 \geq 0 \tag{C50}$$

$$\theta_2(\delta w Q^2 - \alpha G^2) = 0 \tag{C51}$$

$$\theta_2 \geq 0 \tag{C52}$$

With respect to the Lagrange multipliers, we have four possibilities

- $\theta_1 = 0$ and $\theta_2 = 0$
- $\theta_1 > 0$ and $\theta_2 > 0$
- $\theta_1 > 0$ and $\theta_2 = 0$
- $\theta_1 = 0$ and $\theta_2 > 0$

In the next few sections, we consider each of the above cases separately.

$\theta_1 = 0$ and $\theta_2 = 0$

If the Lagrange multiplier $\theta_i = 0$, then we must have $\delta w Q^i - \alpha G^i \geq 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C53}$$
$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{C54}$$
$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 \geq 0 \tag{C55}$$

and

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C56}$$
$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C57}$$
$$\frac{\partial LR2}{\partial \theta_2} = \delta w Q^2 - \alpha G^2 \geq 0 \tag{C58}$$
$$\tag{C59}$$

Notice that these first order conditions are identical to the case where the manufacturer offers a cooperative advertising contract that only comprises of a single participation rate α. As we have shown earlier, the first best solution cannot be achieved in this case. The accrual rate δ can be set high enough to simultaneously satisfy (C50) and (C58).

$\theta_1 > 0$ **and** $\theta_2 > 0$

If $\theta_i > 0$, then we must have $\delta w Q^i - \alpha G^i = 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C60}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C61}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 = 0 \tag{C62}$$

$$\tag{C63}$$

and

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 + \theta_2(\delta w \frac{\partial Q^2}{\partial p_2}) = 0 \tag{C64}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} + \theta_2 \left[\delta w Q^2 - \alpha \frac{\partial G^2}{\partial e_2}\right] = 0 \tag{C65}$$

$$\frac{\partial LR2}{\partial \theta_2} = \delta w Q^2 - \alpha G^2 = 0 \tag{C66}$$

$$\tag{C67}$$

From (C60), we can see that

$$(p_1 - (1-\theta_1\delta)w) = -\frac{Q^1}{\frac{\partial Q^1}{\partial p_1}} \tag{C68}$$

Substituting the above in (C61), and solving for θ_1, we have

$$\frac{\partial Q^1}{\partial e_1}(p_1 - (1-\theta_1\delta)w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\alpha\frac{\partial G^1}{\partial e_1} = 0 \tag{C69}$$

$$\Rightarrow \quad \theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C70}$$

Since $\theta_1 > 0$, we must have

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}} > 0 \tag{C71}$$

$$\Rightarrow \alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}} \tag{C72}$$

Similarly, solving for retailer 2's Lagrange multiplier, we have

$$\theta_2 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2} \frac{\partial G^2}{\partial e_2}} \tag{C73}$$

and since $\theta_2 > 0$, we must have

$$\theta_2 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2} \frac{\partial G^2}{\partial e_2}} > 0 \tag{C74}$$

$$\Rightarrow \alpha > \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2} \frac{\partial G^2}{\partial e_2}}} \tag{C75}$$

From (C72) and (C75), we can see that the manufacturer's choice of participation rate must satisfy

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1} \frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2} \frac{\partial G^2}{\partial e_2}}}\} \tag{C76}$$

The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C77}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 = 0 \tag{C78}$$

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 + \theta_2(\delta w \frac{\partial Q^2}{\partial p_2}) = 0 \tag{C79}$$

$$\frac{\partial LR2}{\partial \theta_2} = \delta w Q^2 - \alpha G^2 = 0 \tag{C80}$$

$$\tag{C81}$$

Re-arranging the retailers' first order conditions, we have

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) \tag{C82}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) - \theta_2(\delta w \frac{\partial Q^2}{\partial p_2}) \tag{C83}$$

$$\delta w Q^1 - \alpha G^1 = 0 \tag{C84}$$

$$\delta w Q^2 - \alpha G^2 = 0 \tag{C85}$$

In order for the manufacturer to induce the first best solution, the manufacturer must choose w, α and δ such that

$$\frac{\partial Q^1}{\partial p_1}(w-c) + \frac{\partial Q^2}{\partial p_1}(p_2-c) - \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C86}$$

$$\frac{\partial Q^2}{\partial p_2}(w-c) + \frac{\partial Q^1}{\partial p_2}(p_1-c) - \theta_2(\delta w \frac{\partial Q^2}{\partial p_2}) = 0 \tag{C87}$$

$$\delta w Q^1 - \alpha G^1 = 0 \tag{C88}$$

$$\delta w Q^2 - \alpha G^2 = 0 \tag{C89}$$

We can see that (C86) and (C87) can be satisfied if the wholesale prices are chosen such that

$$w = \frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2-c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} \tag{C90}$$

$$w = \frac{c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1-c)}{\frac{\partial Q^2}{\partial p_2}}}{(1-\theta_2\delta)} \tag{C91}$$

Hence, in order for the manufacturer to simultaneously induce both retailers to charge first best prices, we must have

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2-c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = \frac{c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1-c)}{\frac{\partial Q^2}{\partial p_2}}}{(1-\theta_2\delta)} \tag{C92}$$

With regards to the retail effort, we can see from equations (C88) and (C89) that the manufacturer can induce the retailers to expend first best efforts by choosing the participation rate α and the accrual rate δ such that

$$\delta = \frac{\alpha G^{1^*}}{w Q^{1^*}} \tag{C93}$$

$$\delta = \frac{\alpha G^{2^*}}{w Q^{1^*}} \tag{C94}$$

$$\Rightarrow \frac{\alpha G^{1^*}}{w Q^{1^*}} = \frac{\alpha G^{2^*}}{w Q^{1^*}} \tag{C95}$$

$$\Rightarrow \frac{G^{1^*}}{Q^{1^*}} = \frac{G^{2^*}}{Q^{1^*}} \tag{C96}$$

where G^{i^*} is the cost of effort incurred when exerting first best effort levels and Q^{i^*} is the demand when first best price and effort levels are exerted by both retailers. Hence, the first best can be achieved if the manufacturer chooses the wholesale prices, accrual rate and the participation rate such that the following conditions are simultaneously satisfied

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = \frac{c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}}}{(1-\theta_2\delta)} \tag{C97}$$

$$\frac{G^{1^*}}{Q^{1^*}} = \frac{G^{2^*}}{Q^{1^*}} \tag{C98}$$

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2}Q^2}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}}\} \tag{C99}$$

$\underline{\theta_1 > 0 \textbf{ and } \theta_2 = 0}$

The first order conditions for the retailers are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C100}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1 \left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C101}$$

$$\frac{\partial LR1}{\partial \theta_1} = \delta w Q^1 - \alpha G^1 = 0 \tag{C102}$$

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C103}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C104}$$

$$\frac{\partial LR2}{\partial \theta_2} = \delta w Q^2 - \alpha G^2 \geq 0 \tag{C105}$$

Since $\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}$, retailer 1's price and effort choices are governed by (C100) and (C102) respectively. Retailer 2's price and effort are governed by (C103) and (C104) respectively. Also note that retailer 2 also ensures that (C105) is satisfied. Hence, the first order conditions can be expressed as

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C106}$$
$$\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C107}$$
$$\delta w Q^1 - \alpha G^1 = 0 \tag{C108}$$
$$\frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C109}$$
$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C110}$$
$$\delta w Q^2 - \alpha G^2 \geq 0 \tag{C111}$$

The first order conditions noted above can be expressed as

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) \tag{C112}$$
$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C113}$$
$$\delta w Q^1 - \alpha G^1 = 0 \tag{C114}$$
$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \alpha\frac{\partial G^2}{\partial e_2} \tag{C115}$$
$$\delta w Q^2 - \alpha G^2 \geq 0 \tag{C116}$$
$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C117}$$

We know that the first order conditions that induce the first best solution are given by

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = 0 \tag{C118}$$

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 = 0 \tag{C119}$$

$$\frac{\partial \Pi}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{C120}$$

$$\frac{\partial \Pi}{\partial e_2} = \frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} = 0 \tag{C121}$$

Comparing (C118) and (C112), we can see that the manufacturer can induce R1 to charge first best price by setting the wholesale price such that

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) - \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C122}$$

$$\Rightarrow \quad w = \frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1 - \theta_1 \delta)} \tag{C123}$$

Similarly, comparing (C119) and (C113), we can see that the manufacturer can induce R2 to charge first best price by setting the wholesale price such that

$$\frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) = 0 \tag{C124}$$

$$\Rightarrow \quad w = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C125}$$

Since the manufacturer can only charge a single wholesale price, we must have

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2-c)}{\frac{\partial Q^1}{\partial p_1}}}{(1-\theta_1\delta)} = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1-c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C126}$$

Comparing (C121) and (C115), we can see that the manufacturer can induce R2 to exert first best effort levels by choosing the participation rate α such that

$$\frac{\partial Q^2}{\partial e_2}(w-c) + \frac{\partial Q^1}{\partial e_2}(p_1-c) - \alpha\frac{\partial G^2}{\partial e_2} = 0 \tag{C127}$$

$$\Rightarrow \quad \alpha = \frac{(p_1-c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{C128}$$

Since $\theta_1 > 0$, we can see from (C117) that the following condition must be satisfied

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} > 0 \tag{C129}$$

$$\Rightarrow \alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{C130}$$

In order for equations (C128) and (C130) to be simultaneously satisfied, we must have

$$\frac{(p_1-c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{C131}$$

Next, the manufacturer can induce R1 to exert first best effort levels by offering an accrual rate δ such that

$$\delta w Q^{1^*} - \alpha G^{1^*} = 0 \tag{C132}$$

$$\Rightarrow \quad \delta = \frac{\alpha G^{1^*}}{w Q^{1^*}} \tag{C133}$$

where w is as given by (C123), Q^{1^*} is the demand when first best price and effort are charged by both retailers, G^{1^*} is the cost of effort incurred to expend first best levels of cost and α is the participation rate (given by (C128)) that induces R2 to exert first best effort. Also, in order for the first best solution to be achieved, the participation rate α, the wholesale price w, the accrual rate δ and the first best price and effort levels must satisfy the following inequality

$$\delta w Q^{1^*} - \alpha G^{2^*} \geq 0 \tag{C134}$$

$$\Rightarrow \quad \frac{\alpha G^{1^*}}{w Q^{1^*}} w Q^{1^*} - \alpha G^{2^*} \geq 0 \tag{C135}$$

$$\Rightarrow \quad G^{1^*} Q^{1^*} - G^{2^*} Q^{1^*} \geq 0 \tag{C136}$$

In summary, the manufacturer can induce both the retailers to charge first best prices and exert first best efforts if the following conditions are simultaneously satisfied

$$\frac{c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}}}{(1 - \theta_1 \delta)} = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C137}$$

$$\frac{(p_1 - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{C138}$$

$$G^{1^*} Q^{1^*} - G^{2^*} Q^{1^*} \geq 0 \tag{C139}$$

The solution to the case where $\theta_1 = 0$ and $\theta_2 > 0$ is symmetric to the solution described above.

Effort Levels are Chosen Such That the Total Cost Reimbursement is Greater than the Total Accrual

When the retailers choose effort such that $\alpha G^i > \delta w Q^i$, the retailers are reimbursed $\delta w Q^i$. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F^1 - G^1 + \delta w Q^1 \tag{C140}$$

$$\pi^2 = Q^2(p_2 - w) - F^2 - G^2 + \delta w Q^2 \tag{C141}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - \delta w Q^1 - \delta w Q^2 \tag{C142}$$

The first order conditions for retailer 1's optimization problem are given by

$$\frac{\partial \pi^1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \delta w \frac{\partial Q^1}{\partial p_1} = 0 \tag{C143}$$

$$\frac{\partial \pi^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1} + \delta w \frac{\partial Q^1}{\partial e_1} = 0 \tag{C144}$$

The first order conditions for retailer 2's optimization problem are similar to that of retailer 1 and are given by

$$\frac{\partial \pi^2}{\partial p_1} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 + \delta w \frac{\partial Q^2}{\partial p_2} = 0 \tag{C145}$$

$$\frac{\partial \pi^2}{\partial e_1} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - \frac{\partial G^2}{\partial e_2} + \delta w \frac{\partial Q^2}{\partial e_2} = 0 \tag{C146}$$

$$\tag{C147}$$

The above first order conditions can be expressed as

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w(1-\delta) - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{C148}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w(1-\delta) - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C149}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(w(1-\delta) - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) \tag{C150}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - w) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(w(1-\delta) - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) \tag{C151}$$

Since $\frac{\partial Q^1}{\partial e_1} > 0$, $\frac{\partial Q^1}{\partial e_2} > 0$, $\frac{\partial Q^2}{\partial e_2} > 0$ and $\frac{\partial Q^2}{\partial e_1} > 0$, from (C150) and (C151), we can see that the manufacturer cannot induce the retailers to exert first best effort levels for any choice of δ and w. ■

Proof of Proposition 3

When the manufacturer offers the retailers a cooperative advertising contract that includes a participation rate α and an fixed accrual rate A, a fraction of the retailer's advertising costs(αG^i) are reimbursed and the total cost reimbursement is capped at A. Hence, the advertising cost reimbursement is given by

(i) αG^i if $\alpha G^i \leq A$

(ii) A if $\alpha G^i > A$

Given this cooperative advertising contract, the retailers have two options:

(i) expend effort such that the total cost reimbursement is less than or equal to the total accrual($\alpha G^i \leq A$)

(ii) expend effort such that the total cost reimbursement is greater than the total accrual($\alpha G^i > A$)

We consider each scenario separately

Effort Levels are Chosen Such That the Total Cost Reimbursement is Less than or Equal to the Total Accrual

When the retailers choose effort such that $\alpha G^i \leq A$, the retailers are reimbursed αG^i. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F^1 - G^1 + \alpha G^1 \tag{C152}$$

$$\pi^2 = Q^2(p_2 - w) - F^2 - G^2 + \alpha G^2 \tag{C153}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - \alpha G^1 - \alpha G^2 \tag{C154}$$

We first consider the retailer 1's optimization problem given by

$$\begin{aligned} &\underset{\{p_1, e_1\}}{\text{Max}} \quad \pi^1 \\ &\text{subject to} \quad A - \alpha G^1 \geq 0 \end{aligned} \tag{C155}$$

The Lagrangian for the retailer's optimization problem is given by

$$LR1 = Q^1(p_1 - w) - F^1 - G^1 + \alpha G^1 + \theta_1 \left[A - \alpha G^1\right] \tag{C156}$$

The first order conditions for the retailer's optimization problem are given by

$$\frac{\partial LR1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C157}$$

$$\frac{\partial LR1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1 - \alpha)\frac{\partial G^1}{\partial e_1} - \theta_1 \left[\alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C158}$$

$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 \geq 0 \tag{C159}$$

$$\theta_1(A - \alpha G^1) = 0 \tag{C160}$$

$$\theta_1 \geq 0 \tag{C161}$$

The first order conditions for retailer 2's optimization problem are similar to that of retailer 1 and are given by

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C162}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} - \theta_2 \left[\alpha \frac{\partial G^2}{\partial e_2}\right] = 0 \tag{C163}$$

$$\frac{\partial LR2}{\partial \theta_2} = A - \alpha G^2 \geq 0 \tag{C164}$$

$$\theta_2(A - \alpha G^2) = 0 \tag{C165}$$

$$\theta_2 \geq 0 \tag{C166}$$

With respect to the Lagrange multipliers, we have four possibilities

- $\theta_1 = 0$ and $\theta_2 = 0$
- $\theta_1 > 0$ and $\theta_2 > 0$
- $\theta_1 > 0$ and $\theta_2 = 0$
- $\theta_1 = 0$ and $\theta_2 > 0$

In the next few sections, we consider each of the above cases separately.

<u>$\theta_1 = 0$ **and** $\theta_2 = 0$</u>

If the Lagrange multiplier $\theta_i = 0$, then we must have $A - \alpha G^i \geq 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C167}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} = 0 \tag{C168}$$

$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 \geq 0 \tag{C169}$$

and

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C170}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C171}$$

$$\frac{\partial LR2}{\partial \theta_2} = A - \alpha G^2 \geq 0 \tag{C172}$$

Notice that these first order conditions are identical to the case where the manufacturer offers a cooperative advertising contract that only comprises of a single participation rate α. As we have shown earlier, the first best solution cannot be achieved in this case. The fixed accrual rate A can be set high enough to simultaneously satisfy (C169) and (C172).

$\theta_1 > 0$ and $\theta_2 > 0$

If $\theta_i > 0$, then we must have $A - \alpha G^i = 0$. The first order conditions that govern the retailer's price and effort choices are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C173}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\left[\alpha\frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C174}$$

$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 = 0 \tag{C175}$$

and

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C176}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} - \theta_2\left[\alpha\frac{\partial G^2}{\partial e_2}\right] = 0 \tag{C177}$$

$$\frac{\partial LR2}{\partial \theta_2} = A - \alpha G^2 = 0 \tag{C178}$$

From (C173), we can see that

$$(p_1 - w) = -\frac{Q^1}{\frac{\partial Q^1}{\partial p_1}} \tag{C179}$$

Substituting the above in (C174), and solving for θ_1, we have

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\alpha\frac{\partial G^1}{\partial e_1} = 0 \tag{C180}$$

$$\Rightarrow \quad \theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C181}$$

Similarly, solving for retailer 2's Lagrange multiplier, we have

$$\theta_2 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{C182}$$

Hence, the first order conditions that govern the retailers price and effort choices are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C183}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C184}$$

$$A - \alpha G^1 = 0 \tag{C185}$$

$$A - \alpha G^2 = 0 \tag{C186}$$

Re-arranging the retailers' first order conditions, we have

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{C187}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C188}$$

$$A - \alpha G^1 = 0 \tag{C189}$$

$$A - \alpha G^2 = 0 \tag{C190}$$

In order for the manufacturer to induce the first best solution, the manufacturer must choose w, w, α and A such that

$$\frac{\partial Q^1}{\partial p_1}(w-c)+\frac{\partial Q^2}{\partial p_1}(p_2-c)=0 \tag{C191}$$

$$\frac{\partial Q^2}{\partial p_2}(w-c)+\frac{\partial Q^1}{\partial p_2}(p_1-c)=0 \tag{C192}$$

$$A-\alpha G^{1^*}=0 \tag{C193}$$

$$A-\alpha G^{2^*}=0 \tag{C194}$$

where G^{i^*} is the cost of effort incurred by retailer i when first best effort levels are expended. We can see that (C191) and (C192) can be satisfied if the wholesale prices are chosen such that

$$w=c-\frac{\frac{\partial Q^2}{\partial p_1}(p_2-c)}{\frac{\partial Q^1}{\partial p_1}} \tag{C195}$$

$$w=c-\frac{\frac{\partial Q^1}{\partial p_2}(p_1-c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C196}$$

Since the manufacturer charges a single wholesale price, we must have

$$c-\frac{\frac{\partial Q^2}{\partial p_1}(p_2-c)}{\frac{\partial Q^1}{\partial p_1}}=c-\frac{\frac{\partial Q^1}{\partial p_2}(p_1-c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C197}$$

$$\Rightarrow \frac{\frac{\partial Q^2}{\partial p_1}(p_2-c)}{\frac{\partial Q^1}{\partial p_1}}=\frac{\frac{\partial Q^1}{\partial p_2}(p_1-c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C198}$$

In terms of the accrual rate and participation rate, we can see that since A is fixed, we must have

$$A = \alpha G^{1^*} = \alpha G^{2^*} \tag{C199}$$

$$\Rightarrow \quad G^{1^*} = G^{2^*} \tag{C200}$$

Also, since $\theta_1 > 0$ and $\theta_2 > 0$, we must have

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} > 0 \quad \Rightarrow \alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \quad \text{and} \tag{C201}$$

$$\theta_2 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \quad \Rightarrow \alpha > \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}} \tag{C202}$$

$$\Rightarrow \alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}}\} \tag{C203}$$

Hence, the manufacturer can induce the first best solution only when the following conditions are simultaneously satisfied

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C204}$$

$$G^{1^*} = G^{2^*} \tag{C205}$$

$$\alpha > max\{\frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}}, \frac{1}{1 - \frac{\frac{\partial Q^2}{\partial e_2} Q^2}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}}}\} \tag{C206}$$

$\underline{\theta_1 > 0 \textbf{ and } \theta_2 = 0}$

The first order conditions for the retailers are given by

$$\frac{\partial LR1}{\partial p1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C207}$$

$$\frac{\partial LR1}{\partial e1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\left[\alpha\frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C208}$$

$$\frac{\partial LR1}{\partial \theta_1} = A - \alpha G^1 = 0 \tag{C209}$$

$$\frac{\partial LR2}{\partial p_2} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C210}$$

$$\frac{\partial LR2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C211}$$

$$\frac{\partial LR2}{\partial \theta_2} = A - \alpha G^2 \geq 0 \tag{C212}$$

Since $\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}$, retailer 1's price and effort choices are governed by (C207) and (C209) respectively. Retailer 2's price and effort are governed by (C210) and (C211) respectively. Also note that retailer 2 also ensures that (C212) is satisfied. Hence, the first order conditions can be expressed as

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C213}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C214}$$

$$A - \alpha G^1 = 0 \tag{C215}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1 - \alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C216}$$

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C217}$$

$$A - \alpha G^2 \geq 0 \tag{C218}$$

The first order conditions noted above can be expressed as

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{C219}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C220}$$

$$A - \alpha G^1 = 0 \tag{C221}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \alpha\frac{\partial G^2}{\partial e_2} \tag{C222}$$

$$A - \alpha G^2 \geq 0 \tag{C223}$$

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1} Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} \tag{C224}$$

We know that the first order conditions that induce the first best solution are given by

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = 0 \tag{C225}$$
$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial Q^1}{\partial p_2}(p_1 - c) + \frac{\partial Q^2}{\partial p_2}(p_2 - c) + Q^2 = 0 \tag{C226}$$
$$\frac{\partial \Pi}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = 0 \tag{C227}$$
$$\frac{\partial \Pi}{\partial e_2} = \frac{\partial Q^1}{\partial e_2}(p_1 - c) + \frac{\partial Q^2}{\partial e_2}(p_2 - c) - \frac{\partial G^2}{\partial e_2} = 0 \tag{C228}$$

Comparing (C225) and (C219), we can see that the manufacturer can induce R1 to charge first best price by setting the wholesale price such that

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) = 0 \tag{C229}$$
$$\Rightarrow \quad w = c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} \tag{C230}$$

Similarly, comparing (C226) and (C220), we can see that the manufacturer can induce R2 to charge first best price by setting the wholesale price such that

$$\frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) = 0 \tag{C231}$$
$$\Rightarrow \quad w = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C232}$$

Since the manufacturer charges a single wholesale price, we must have

$$c - \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} = c - \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C233}$$

$$\Rightarrow \frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C234}$$

Comparing (C228) and (C222), we can see that the manufacturer can induce R2 to exert first best effort levels by choosing the participation rate α such that

$$\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \alpha\frac{\partial G^2}{\partial e_2} = 0 \tag{C235}$$

$$\Rightarrow \quad \alpha = \frac{(p_1 - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{C236}$$

Since $\theta_1 > 0$, we must have

$$\theta_1 = 1 - \frac{1}{\alpha} - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}} > 0 \tag{C237}$$

$$\Rightarrow \alpha > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{C238}$$

Hence, in order for equations (C238) and (C236) to be satisfied simultaneously, we must have

$$\frac{(p_1 - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{C239}$$

Next, the manufacturer can induce R1 to exert first best effort levels by offering the fixed accrual rate A such that

$$A - \alpha G^{1^*} = 0 \tag{C240}$$

$$\Rightarrow \quad A = \alpha G^{1^*} \tag{C241}$$

$$\Rightarrow \quad A = G^{1^*} \frac{(p_1 - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} \tag{C242}$$

where G^{1^*} is the cost of effort incurred to expend first best levels of cost and α is the participation rate (given by (C236)) that induces R2 to exert first best effort. Also, in order for the first best solution to be achieved, the participation rate α and the accrual rate A and the first best effort levels must satisfy the following inequality

$$A - \alpha G^{2^*} \geq 0 \tag{C243}$$

$$\Rightarrow \quad \alpha G^{1^*} - \alpha G^{2^*} \geq 0 \tag{C244}$$

$$\Rightarrow \quad G^{1^*} \geq G^{2^*} \tag{C245}$$

Hence, the first best solution can be achieved only when R1's cost of effort is greater than R2's cost of effort under first best conditions. Hence, the manufacturer can achieve the first best solution only when the following conditions are simultaneously satisfied

$$\frac{\frac{\partial Q^2}{\partial p_1}(p_2 - c)}{\frac{\partial Q^1}{\partial p_1}} = \frac{\frac{\partial Q^1}{\partial p_2}(p_1 - c)}{\frac{\partial Q^2}{\partial p_2}} \tag{C246}$$

$$\frac{(p_1 - c)(\frac{\partial Q^1}{\partial e_1}\frac{\partial Q^2}{\partial p_2} - \frac{\partial Q^1}{\partial p_2})}{\frac{\partial Q^2}{\partial p_2}\frac{\partial G^2}{\partial e_2}} > \frac{1}{1 - \frac{\frac{\partial Q^1}{\partial e_1}Q^1}{\frac{\partial Q^1}{\partial p_1}\frac{\partial G^1}{\partial e_1}}} \tag{C247}$$

$$G^{1^*} \geq G^{2^*} \tag{C248}$$

The solution to the case where $\theta_1 = 0$ and $\theta_2 > 0$ is symmetric to the solution described above.

Effort Levels are Chosen Such That the Total Cost Reimbursement is Greater than the Total Accrual

When the retailers choose effort such that $\alpha G^i > A$, the retailers are reimbursed A. Hence, the retailer's profits can be expressed as

$$\pi^1 = Q^1(p_1 - w) - F^1 - G^1 + A \tag{C249}$$

$$\pi^2 = Q^2(p_2 - w) - F^2 - G^2 + A \tag{C250}$$

The manufacturer's profit function is given by

$$\Pi = Q^1(w - c) + Q^2(w - c) + F^1 + F^2 - A - A \tag{C251}$$

The first order conditions for $R1$'s optimization problem are given by

$$\frac{\partial \pi^1}{\partial p_1} = \frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C252}$$

$$\frac{\partial \pi^1}{\partial e_1} = \frac{\partial Q^1}{\partial e_1}(p_1 - w) - \frac{\partial G^1}{\partial e_1} = 0 \tag{C253}$$

The first order conditions for $R2$'s optimization problem are similar to that of retailer 1 and are given by

$$\frac{\partial \pi^2}{\partial p_1} = \frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C254}$$

$$\frac{\partial \pi^2}{\partial e_1} = \frac{\partial Q^2}{\partial e_2}(p_2 - w) - \frac{\partial G^2}{\partial e_2} = 0 \tag{C255}$$

Notice that the first order conditions listed above are identical to the second best scenario. Hence, the manufacturer cannot achieve the first best solution. ■

Proof of Proposition 4

When the manufacturer offers a cooperative advertising contract that includes a participation rate as well as variable accruals, the manufacturer can use the participation rate to influence one retailer's efforts while using the accruals to influence the other retailer's efforts. Assuming $R1$'s efforts are influenced using the accrual rate, $R1$'s first order conditions are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 + \theta_1(\delta w \frac{\partial Q^1}{\partial p_1}) = 0 \tag{C256}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} + \theta_1\left[\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C257}$$

$$\delta w Q^1 - \alpha G^1 = 0 \tag{C258}$$

As derived in appendix B, the comparative statics for $R1$'s effort and price are given by

- $\frac{\partial p_1}{\partial w} > 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$,
- $\frac{\partial e_1}{\partial w} > 0$ when $\Delta^6 > 0$,
- $\frac{\partial p_1}{\partial \alpha} < 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$,
- $\frac{\partial e_1}{\partial \alpha} < 0$ when $\Delta^5 < 0$,
- $\frac{\partial p_1}{\partial \delta} < 0$ when $\Delta^1 < 0$,
- $\frac{\partial e_1}{\partial \delta} > 0$ when $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$.

where

$$\begin{aligned}\Delta^1 = &-\frac{\partial Q^1}{\partial e_1}(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1}) w Q^1 + \theta_1 w \frac{\partial Q^1}{\partial p_1}(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1})^2 + \\ &\delta w \frac{\partial Q^1}{\partial p_1}\left[w Q^1 (\frac{\partial^2 Q^1}{\partial e_1^2}(p_1 - (1-\theta_1\delta)w) - [1 - \alpha(1-\theta_1)]\frac{\partial^2 G^1}{\partial e_1^2}) - \theta_1 w \frac{\partial Q^1}{\partial e_1}(\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1})\right] < 0\end{aligned} \tag{C259}$$

$$\Delta^5 = -\left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right] G^1(\delta w\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \delta w\frac{\partial Q^1}{\partial p_1}\left[\delta w\frac{\partial Q^1}{\partial p_1}(1-\theta_1)\frac{\partial G^1}{\partial e_1} + G^1\frac{\partial Q^1}{\partial e_1}\right], \tag{C260}$$

$$\Delta^6 = +(1-\theta_1\delta)\frac{\partial Q^1}{\partial p_1}\delta w\frac{\partial Q^1}{\partial p_1}(\delta w\frac{\partial Q^1}{\partial p_1} - \alpha\frac{\partial G^1}{\partial e_1}) + \left[2\frac{\partial Q^1}{\partial p_1} + \frac{\partial^2 Q^1}{\partial p_1^2}(p_1 - (1-\theta_1\delta)w)\right]\delta Q^1(\delta q\frac{\partial Q^1}{\partial e_1} - \alpha\frac{\partial G^1}{\partial e_1})$$
$$\delta w\frac{\partial Q^1}{\partial p_1}\left[-\delta w\frac{\partial Q^1}{\partial p_1}(1-\theta_1\delta)\frac{\partial Q^1}{\partial e_1} - \delta Q^1\frac{\partial Q^1}{\partial e_1}\right]. \tag{C261}$$

Since $R2$'s efforts are influenced by the participation rate, the first order conditions are given by

$$\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C262}$$
$$\frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C263}$$

Notice that these first order conditions are identical to the case where the cooperative advertising contract only includes a participation rate. Hence, the comparative statics of $R2$'s price and effort are given by

$$\frac{\partial p_2}{\partial w} > 0, \frac{\partial e_2}{\partial w} < 0, \frac{\partial p_2}{\partial \alpha} > 0, \frac{\partial e_2}{\partial \alpha} > 0, \frac{\partial p_2}{\partial \delta} = 0 \text{ and} \frac{\partial e_2}{\partial \delta} = 0. \tag{C264}$$

When the manufacturer uses a cooperative advertising contract that includes a fixed accrual in addition to the participation rate, the manufacturer can influence one retailer's efforts by using the participation rate and the other retailer's efforts using the accrual. Assuming $R1$'s efforts are influenced by the accrual and $R2$'s efforts are influenced by the participation rate, the first order conditions for $R1$ are given by

$$\frac{\partial Q^1}{\partial p_1}(p_1 - w) + Q^1 = 0 \tag{C265}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - w) - (1-\alpha)\frac{\partial G^1}{\partial e_1} - \theta_1\left[\alpha\frac{\partial G^1}{\partial e_1}\right] = 0 \tag{C266}$$

$$A - \alpha G^1 = 0 \tag{C267}$$

As shown in appendix B, the comparative statics of $R1$'s price and effort are given by

$$\frac{\partial p_1}{\partial w} > 0, \frac{\partial e_1}{\partial w} = 0, \frac{\partial p_1}{\partial \alpha} < 0, \frac{\partial e_1}{\partial \alpha} < 0, \frac{\partial p_1}{\partial A} > 0 \text{ and} \frac{\partial e_1}{\partial A} > 0, \tag{C268}$$

Next, $R2$'s first order conditions are given by

$$\frac{\partial Q^2}{\partial p_2}(p_2 - w) + Q^2 = 0 \tag{C269}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - w) - (1-\alpha)\frac{\partial G^2}{\partial e_2} = 0 \tag{C270}$$

Notice that these first order conditions are identical to the case where the cooperative advertising contract only includes a participation rate. Hence, the comparative statics of $R2$'s price and effort are given by

$$\frac{\partial p_2}{\partial w} > 0, \frac{\partial e_2}{\partial w} < 0, \frac{\partial p_2}{\partial \alpha} > 0, \frac{\partial e_2}{\partial \alpha} > 0, \frac{\partial p_2}{\partial A} = 0 \text{ and} \frac{\partial e_2}{\partial A} = 0. \tag{C271}$$

As we can see from the comparative statics, while the impact of the accrual on $R1$'s retail price is positive when the accrual is fixed, variable accruals have a negative impact on retail price. Also notice that the impact of the wholesale price on $R1$'s efforts is negative for fixed accruals while it is positive for variable accruals. These differences form the basis of the comparison between the two contracts.

Recall that the Lagrangian for the manufacturer's optimization problem under the single participation rate contact is given by

$$\begin{aligned} L =& Q^1(w-c) + Q^2(w-c) + F^1 + F^2 - \alpha G^1 - \alpha G^2 + \\ & \lambda_1 \left[Q^1(p_1 - w) - (1-\alpha)G^1 - F^1 \right] + \lambda_2 \left[Q^2(p_2 - w) - (1-\alpha)G^2 - F^2 \right] + \\ & \mu_1 \left[\frac{\partial Q^1}{\partial p_1} p_1 + Q^1 - \frac{\partial Q^1}{\partial p_1} w \right] + \mu_2 \left[\frac{\partial Q^2}{\partial p_2} p_2 + Q^2 - \frac{\partial Q^2}{\partial p_2} w \right] + \\ & \eta_1 \left[\frac{\partial Q^1}{\partial e_1} p_1 - \frac{\partial Q^1}{\partial e_1} w - (1-\alpha)\frac{\partial G^1}{\partial e_1} \right] + \eta_2 \left[\frac{\partial Q^2}{\partial e_2} p_2 - \frac{\partial Q^2}{\partial e_2} w - (1-\alpha)\frac{\partial G^2}{\partial e_2} \right] \end{aligned} \tag{C272}$$

If the demand parameters are such that the marginal change in the Lagrangian with respect to an increase in price is less than the marginal change in the Lagrangian with respect to an increase in effort when first best prices are charged by the retailers (i.e., $\frac{\partial L}{\partial p_i}|_{p_i^*} < \frac{\partial L}{\partial ei}|_{p_i^*}$), then under the participation rate contract, the manufacturer has an incentive to induce the retailers to exert prices greater than first best prices. Since the manufacturer benefits from an increase in effort, retail efforts are induced to be higher. Since higher efforts lead to higher prices, the manufacturer may lose revenues due to the increase in price. Since the benefit from increasing the effort dominates the loss from the increased price (since $\frac{\partial L}{\partial p_i}|_{p_i^*} < \frac{\partial L}{\partial ei}|_{p_i^*}$

), the manufacturer may induce retail prices to be greater than first best prices. Notice that $\frac{\partial L}{\partial ei}$ is increasing in the own effort effects ($\frac{\partial Q^1}{\partial e_1}$ and $\frac{\partial Q^2}{\partial e_2}$) and in the free-riding or cross effort effects ($\frac{\partial Q^1}{\partial e_2}$, $\frac{\partial Q^2}{\partial e_1}$). Also, as the cost of effort ($\frac{\partial^2 G^1}{\partial e_1^2}$, $\frac{\partial^2 G^2}{\partial e_2^2}$) decreases, $\frac{\partial L}{\partial ei}$ increases. Hence, when the cost of effort is low or when the incentive to free ride is high or when the effect of own advertising on demand is high, the manufacturer may prefer to induce the retailers to exert prices that are greater than first best prices.

Since the first order conditions under the participation rate contract are given by the following equations

$$\frac{\partial Q^1}{\partial p_1}(p_1 - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) + Q^1 = \frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) \tag{C273}$$

$$\frac{\partial Q^2}{\partial p_2}(p_2 - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) + Q^2 = \frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) \tag{C274}$$

$$\frac{\partial Q^1}{\partial e_1}(p_1 - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c) - \frac{\partial G^1}{\partial e_1} = \left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} \tag{C275}$$

$$\frac{\partial Q^2}{\partial e_2}(p_2 - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c) - \frac{\partial G^2}{\partial e_2} = \left(\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c)\right) - \alpha\frac{\partial G^2}{\partial e_2} \tag{C276}$$

we can see that the conditions that must hold simultaneously for retail prices to be greater than first best prices and efforts to be less than first best efforts are given by

$$\frac{\partial Q^1}{\partial p_1}(w - c) + \frac{\partial Q^2}{\partial p_1}(p_2 - c) < 0 \tag{C277}$$

$$\frac{\partial Q^2}{\partial p_2}(w - c) + \frac{\partial Q^1}{\partial p_2}(p_1 - c) < 0 \tag{C278}$$

$$\left(\frac{\partial Q^1}{\partial e_1}(w - c) + \frac{\partial Q^2}{\partial e_1}(p_2 - c)\right) - \alpha\frac{\partial G^1}{\partial e_1} > 0 \tag{C279}$$

$$\left(\frac{\partial Q^2}{\partial e_2}(w - c) + \frac{\partial Q^1}{\partial e_2}(p_1 - c)\right) - \alpha\frac{\partial G^2}{\partial e_2} > 0 \tag{C280}$$

When inequalities (C277)-(C280) are simultaneously satisfied, we will have $p_1^\alpha > p_1^{fb}$, $p_2^\alpha > p_2^{fb}$,$e_1^\alpha < e_1^{fb}$ and $e_2^\alpha < e_2^{fb}$. Assuming these conditions hold, we now investigate whether the use of a participation rate and variable accrual rate contract can improve the manufacturer's profits by a greater amount than a contract that includes a participation rate and a fixed accrual rate. As shown in earlier sections, we know that the fixed accrual rate A has no impact on prices ($\frac{\partial p_1}{\partial A} = 0$ and $\frac{\partial p_2}{\partial A} = 0$). On the other hand, as the variable participation rate δ is increased, the retail prices of R1 decreases(when $\Delta^1 < 0$) and the price charged by R2 is not affected. Note that the impact of the other variables (w and the participation rate α) are identical for the two contracts. Since the contract with variable accrual rate would increase effort expended by one retailer as well as decrease that retailers price simultaneously, this contract will be able to move retail prices and efforts closer to the first best solution compared to what the fixed accrual rate contract can achieve. Hence, when inequalities (C277)-(C280), $\delta w \frac{\partial Q^1}{\partial e_1} - \alpha \frac{\partial G^1}{\partial e_1} < 0$, $\Delta^1 < 0$, $\Delta^5 < 0$ and $\Delta^6 > 0$ hold simultaneously (at the participation rate contract), the addition of the variable accrual rate to the contract can result in greater profits to the manufacturer compared to the addition of the fixed accrual rate to the single participation rate contract. ■

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