Design of High Efficiency Brushless Permanent Magnet Machines and Driver System

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DESIGN OF HIGH-EFFICIENCY BRUSHLESS PERMANENT-MAGNET MACHINES AND DRIVER SYSTEM

by

CHENGYUAN HE
M.S. Wright State University, 2013

A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the Department of Electrical and Computer Engineering in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

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ABSTRACT

In recent years, renewable green energy has increased the demand associated with the development of high efficiency electrical permanent magnet synchronous machinery. The electrical motors consume more than half of all electrical energy in the state according to the U.S. Department of Energy. Therefore, designing a high-efficiency energy conversion device and its control system becomes critical. Permanent magnet synchronous motor (PMSM) and Permanent mag-net synchronous generator (PMSG) can be such energy conversion device. This dissertation is concerned with the high-efficiency permanent magnet synchronous machinery, and their control system design.

The dissertation first talk about the design method of high-efficiency electrical machinery. The advantage of the design method is that it can increase the high load capacity at no cost of increasing the total machine size. A much smaller torque angle $\delta$ than that in the traditional design at relating load is selected, which is between about 2 degrees and about 10 degrees. The most important part of the design is to determine the air-gap and permanent magnet size. According to the design method relating to the much smaller torque angle, which will result in the larger air-gap size and larger magnet thickness. The larger air-gap size contributes to reducing the windage loss and noise level. The increase magnet thickness helps to avoid demagnetization. Both them contribute to high efficiency and high overload capability. Based on the design method, all the parameters will be related to the torque angle, working point of a permanent magnet, and the permanent magnet embrace, which is easier for the designer to make a new design.

The control methods of the permanent magnet machinery are introduced. There is two class of brush-less PM motor control: AC and DC. The design requirements are totally different, and this is related to the back-EMF waveform and the rotor-position sensing. For DC control, the back
EMF is trapezoidal and hall-sensor board is used to detect the switching positions, substantial field weakening is not required. For AC control, the back EMF is sinusoidal, and an encoder or resolver is used to get rotor position. In order to avoid to use encoder or resolver, sensor-less control such as sliding mode observer algorithm can be used.

A permanent magnet brushless DC motor for electric impact wrench and a Surface permanent magnet synchronous generator (SPMSG) have been designed with good performance and high-efficiency base on the design method. Using brush-less DC motor (BLDC) instead of brushed DC motor for electric impact wrench has been a world worldwide trend because of its high reliability, good control performance, small size, and environmental protection. Maxwell 2D model is built to optimize the design and the control board is designed using Altium Designer. Bother the motor and control board have been fabricated and tested to verify the design.

A 2kw high-efficiency alternator system and its control board system are also designed, analyzed and fabricated applying to the truck auxiliary power unit (APU). The alternator system has two stages. The first stage is that the alternator three-phase outputs are connected to the three-phase active rectifier to get 48V DC. An advanced Sliding model control (SMO) is used to get an alternator position. The buck is used for the second stage to get 14V DC output. The whole system efficiency is much higher than the traditional system using induction motor.
This thesis work is dedicated to my parents, Chunnan Zhu and Cuanghui He, who have always loved me unconditionally and taught me to work hard diligently. This work is also dedicated to my advisor Lei Wei and Thomas Wu for their constant support and encouragement during the challenges of my graduate life.
I would like to express my sincere gratitude to my advisors Lei Wei and Thomas Wu, for their continuous support of my Ph.D study and related research. Their guidance helped me in all the time of research and writing of this thesis. Moreover, I acknowledge support from advanced electrical machinery lab team.

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CHAPTER 1: INTRODUCTION

Permanent magnet (PM) motor and generator [17] [51] [5] [28] [83] [33] have been widely used in industry by demand for increased functionality, precision, increased energy efficiency and better utilization of primary energy sources. The development of ferrite magnets, Samarium-Cobalt magnets (in the late 1960s) and Neodymium-Iron-Boron magnets [59] are the most important driver of application of brushless PM machines. Soft iron materials [4] have also made significant contribution in the development of brush-less PM machines. Thin gauges, improved chemical composition, heat-treatment and improved core-plate insulation are used to reduce loose in electrical lamination steels. High-strength non-magnetic materials such as inconel and carbon-fiber make big contribution in the design of high-speed brush-less PM machines [99] [65] [3], where they are used in retaining sleeves that extend the range of operation to much higher speed. Due to the developments of power electric, the PM machines control systems [79] [22] [9] become more efficiency, reliability and cheaper, which permit a wider range of affordable application. These technical development made the PM machines become a trend in industry.

Brushless PM machines generally fall into two classification: AC and DC. Both AC and DC PM machines can be design as Interior permanent magnet (IPM) machine or Surface permanent magnet (SPM) machine. There are different requirements when designing them, and this is related to the back-Electromotive Force (EMF) waveform and rotor-position sensing. For DC PM machine [68] [71] [45] [69] [54], the back EMF is trapezoidal and low-resolution shaft-position sensor is used to synchronize the switching of the drive transistors with the rotor position, substantial field weakening is not required. For AC PM machine [73] [72] [27] [105] [7], the back EMF is sinusoidal, and encoder or resolver [92] [13] [1] is used to get rotor position. The following are the characteristics of DC PM machine operation:
• concentrated windings and full pitched;
• trapezoidal back EMF;
• hall sensor board is used to detect the correct switching position;
• higher power density;
• tolerating some torque ripple and no extra field weak required;
• suitable for power drives;

The following are the characteristic of AC PM machine operation:

• distributed and fractional-slot winding;
• sinusoidal back EMF;
• smooth operation and extended field weaken;
• shaft encoder or resolver to get the position;
• suitable for servo derives, electric vehicles drives requiring extra field-weaken capabilities at high speed.

SPM and IPM are two basic topologies for the rotor. For SPM, the direct- and quadrature-axes inductance are the same, there is essentially no reluctance torque generated by SPM machines, although flux-weakening mode is possible if appropriately designed. For IPM machines, they have q-axis saliency that comes from the shape of steel lamination, which called reluctance torque. The total torque is the combination of reluctance torque and magnet torque. The air-gap of IPM is usually designed smaller then that of a SPM, which tends to increase the flux per pole, the
reluctance torque and the inductance. If an extended field-weakening range is required, then the IPM should be used.

Permanent magnet synchronous machines have no brushes, no slip-rings, and no mechanical commutator, and they use permanent magnets to replace the rotor winding which leads to higher efficiency compared to other machines. Permanent magnet synchronous motor (PMSM) can maintain a high efficiency when its output power varies from 50-120% of the rated power. The following are the advantages of PMSM:

- high efficiency and power density;
- small output torque;
- reliable at high speed;
- advanced control topology;
- lighter weight and good heat performance.

Researchers try to develop new methods to increase the efficiency of PMSM, reduce the harmonic and torque ripple, making the machines have better and better performance.

For Electric tool application, brush-less DC motor (BLDC) is a most suitable choice. The motors should be tolerate some torque ripple and do not require extra field weakening at higher speed. The hall sensor board is used to get the correct current switching position. For example BLDC designed for the electric impact wrench requires high reliability, good control performance, small size, less bolting time, low cost and easy mass industrial production. An interior permanent magnet synchronous motor with concentrated winding has large torque to volume ration and high efficiency, so it is suitable for battery-powered impact wrench [84] [91] [10].
High-efficiency alternator system is commonly equipped with the heavy-duty truck. The three-phase outputs from the alternator are processed through a three-phase rectifier [107] [95] [47] to get the DC bus voltage. DC bus output will be used to charge the auxiliary power unit and battery whenever the truck is operating. There are two stages outputs in our alternator system: 48V DC and 14V DC. The 48V DC is directly got from the three-phase active rectifier, and 14V DC is obtained through a buck converter. The battery is charged by 48V DC, and some 14V devices are powered by 14V DC. High current is required if we want to charge the battery quickly, which require good heat dissipation performance of the converter board. The PCB structure using PCB board, aluminum board, and copper bars is designed to satisfy the requirement. The Sliding model control (SMO) method is used to get the alternator shaft position. The disadvantage of SMO can be avoided because the alternator doesn’t need to start by itself, otherwise, at the low speed, we need to use the open loop. The alternator is controlled by FOC [100] [96] [97] [26], and the DC output is connected to the buck converter. In order to get a fast pulse-by-pulse response and good over-current protection, the peak current control [30] [11] [12] [20] is applied to control buck converter.

Dissertation Organization

Chapter two will introduce the key design method of permanent magnet machinery. The motor dynamic modeling and control method will be presented in Chapter Three. Base on Chapter Two and Chapter Three, Chapter Four will present the Brush-less dc motors design and control system applying to the electric impact wrench. Chapter Five will focus on the high-efficiency alternator design and the control system, which is equipped to the heavy-duty trucks. In Chapter Six, the contribution of this dissertation and research conclusion will be highlighted.
CHAPTER 2: ADVANCED DESIGN METHODS OF BRUSH-LESS PERMANENT MAGNET MACHINES

Introduction

Permanent magnet synchronous motor (PMSM) is a new type of motor and has many advantages in comparing with the traditional machines with electromagnetic excitation. It has small inertia moment, large power density, high efficiency, and high reliability, therefore is very suitable for the high-performance application [75] [60]. Due to no excitation winding, PMSM significantly reduces the volume, weight, winding losses and electric heating and it saves energy. PMSM is gradually replacing dc motor and induction motor [110] [101] [102].

This chapter is concerned with key design methodology, which has to do with the configuration, the size of the stator, rotor, permanent magnet, winding, shaft, housing, etc. Some popular materials and their characteristics for machine design applications are also presented. Winding configuration and how to determine the coil pith and winding sequence will be discussed. The different types of magnetization [35] [36] of the Permanent magnet (PM) and performances will be presented. At last, the power losses of PMSM will be introduced.

Key Design Procedure of PMSM

When starting to design the PMSM, it is the several important decisions must be made at the early stage. The following list covers most aspects of most cases:

---

1The content in this chapter was partly published at ICEM, 2016 XXII International Conference [38], IECON 2017 [41] and journal Energies [40].
• decide the configuration of the PM machines and Its control;

• select numbers of phases, poles, and slots;

• estimate the volume of the machine;

• choose materials;

• design slot;

• design permanent magnet;

• design rotor, shaft, and housing;

• winding configuration.

Firstly, the slots, poles number, control topology and the materials will be decided; then the initial size of all parts of the machine will be obtained using our PMSM design method based on the rotating speed, power lever, maximum tolerance and cooling type. Finite element analysis (FEA) will be used to confirm the initial design and analyze electromagnetic variation and distribution, which can provide a detailed analysis of the performance of the design. The obtained result waveforms will be compared with some standard waveforms. The torque and flux density of the machine will also be analyzed. Maxwell 2D FEA will be repeated until optimize parameters are obtained. Comprehensive consideration will be taken when we do optimum design. Also, thermal analysis and mechanical analysis need to be done to make sure the mechanical satisfies the requirements.
Figure 2.1: 3D structure of the PMSM

Figure 2.1 shows a surface-mounted permanent magnet motor structure including a stator with winding, a rotor with magnets and a shaft.

Drive Configuration

There are two types of brush-less PM motor control: the square-wave drive and the sine-wave drive, which is related to the back-EMF waveform and the rotor position sensing, and their characteristics are summarized here.
Square-wave Drive

The motor should have a trapezoid EMF waveform, and this tends toward the use of concentrated windings. Hall-sensor board is used to detect the switching positions; substantial field weakening is not required. Chopping regulates the current. Phase advance can be used to increase speed but at the expense of torque ripple and power factor. The following are the characteristics of DC PM machine operation:

- concentrated windings and full pitched;
- trapezoidal back Electromotive Force (EMF);
- hall sensor board is used to detect the correct switching position;
- higher power density;
- tolerating some torque ripple and no extra field weakening required;
- suitable for power drives.

Sine-wave Drive

Ideally, the current and EMF of this kind of PM machines are both sinusoidal. Continuous position feedback using encoder or resolver is assumed, although sensor-less control is also available. It can be designed with distributed or concentrated winding with integral or fractional slots/pole. The following are the advantages of PMSM:

- high efficiency and power density;
- small output torque;
• reliable at high speed;
• advanced control topology;
• lighter weight and good heat performance.

Numbers of Slots and Poles

The combinations of poles number and slots number of a PM motor [14] [93] have a profound effect not only on the winding layout but also on the space-harmonics of the resulting ampere-conductor distribution and the peak value of cogging torque waveform, thus choosing poles and slots number are very critical for motor design. The least common multiplier (LCM) between the slot number and the pole number and the greatest common divider (GCD) between the slot and pole number are two factors, which can be used to find the combination of the number of poles and slots. The value of the LCM traduces the value of the first harmonic of the cogging torque, so the higher the LCM value is, the lower the cogging torque. The GCD value illustrates the balanced radial forces applied to the rotor, the smaller the GCD value is, the lesser the cogging torque will be.

Machine Volume

The stator size is the foundation of the design. We can get the general size of the motor from

\[ \frac{D^2 L}{T_m} = \varpi_0 T_m = \frac{P_r}{\omega} \]  (2.1)

Where \( D \) is stator bore diameter, \( L \) is stator core length, \( T_m \) is related mechanical torque, which is determined by the required output power \( P_r \) and the rotor rotational speed \( \omega \). \( \varpi_0 \) is a coefficient.
The value of $\varpi_0$ depends on the cooling method. Typically, for air cool, the value of $\varpi_0$ is around $5-7 \text{ in}^3/(\text{ft} \cdot \text{lb})$ for 10 hp output power or less; for water or other liquid cool methods, the value of $\varpi_0$ is around $2-5 \text{ in}^3/(\text{ft} \cdot \text{lb})$ for 10 hp output power or less. The better the cooling method, the smaller the motor size. Otherwise, the larger size is required for adequate heat dissipation. The proper size for the machine can be determined using Eqn. 2.1 based on the design specifications and thermal requirement. For initial design, assuming the diameter-to-axial-length ratio to be close to unity. The diameter of some high-pole-number machinery is much larger than the length, and it will make the machinery move from a long cylindrical shape to disk shape, which means high-pole-number machines’ diameter tend to be much higher compared with the axial length. The machinery will be more compact and high-efficiency if high-energy magnets such as Nd-Fe-B and SmCo are used. It is a crude size approximation for radial-flux machines over a wide power range. The flux per pole determines the stator yoke thickness; therefore, the thickness decreases as the pole number increase.

Magnetic Materials

Introduction

There are two types of materials: soft and hard material. Soft materials are easy to magnetize and demagnetize so that they can transfer or store magnetic energy in circuits with AC waveforms. Therefore, softer materials are widely used as stator and rotor materials. Hard materials are materials which retain their magnetism and are difficult to demagnetize and thus are used as permanent magnets in applications such as PMSM and Permanent magnet synchronous generator (PMSG). The following are the major parameters of magnetic materials:

- relative permeability $\mu_{rc}$;
- saturation magnetic flux density $B_s$;
- Curie temperature $T_c$ and operating temperature $T_w$;
- resistivity $\rho_c$;
- eddy-current and hysteresis losses per unit volume $P_v$;
- bandwidth $BW$.

PM motor’s stator and rotor are made of soft magnetic materials. The relative permeability $\mu_r$ of soft magnetic materials is from 1 to $10^5$. If the motor working temperature is below $100 \degree C$, steels will be chosen in the design. The saturated magnetic flux density of steels is 1.7 Tesla (T). The performance of steels will be reduced when temperature increase, so if machines require high working temperature, Hiperco50 (an iron-vanadium soft magnetic alloy) can be used as the soft magnetic material. The saturation magnetic flux density of Hiperco50 is 2.3T, and it has high DC maximum permeability, low DC coercive force, and low AC core loss.

Neodymium iron boron (NdFeB) and Samarium-cobalt (SmCo) are most popular permanent magnet materials. NdFeB is the strongest rare-earth material, but the Curie and operating temperature of NdFeB are slow, for example, the curl and operating temperature of NdFeB-28 are $310 \degree C$ and $150 \degree C$ respectively. The performance of it will weaken when the temperature increase. NdFeB will demagnetize when cure temperature reached. SmCo has much higher curl and operating temperature. The price of SmCo is high compared with NdFeB. If a PM motor needs to work under wide temperature, SmCo will be used in the design.
The magnetization curve [58] [80] [15] [56] is called the hysteresis. It describes the relationship between the magnetic flux density $B$ and the magnetic field density $H$. A ferromagnetic material remains magnetized after the external field is removed. It is only practically reversible in the process of repeated magnetization and demagnetization. We can see from Fig. 2.2, the $B − H$ curve is a multi-valued function, is nonlinear and exhibits saturation. The shape and size of the $B − H$ curve depend on the properties of the ferromagnetic materials and the magnitude of the applied field $H$. For hard material, it is difficult to move the magnetic wall, so the hysteresis loop is wide. For soft material, it is easy to move the magnetic wall, so the hysteresis loop is narrow. The $B − H$ loop area is equal to the hysteresis energy loss per volume for one cycle. When the magnitude of the amplitude of the AC component of magnetic flux density is increased, so is the area of the $B − H$ loop. It will increase hysteresis energy loss. The $B − H$ loop will become wider when the frequency increase due to eddy currents.
Before we design the stator slot size, we define slot pitch $\tau_s$ as:

$$\tau_s = \frac{\pi D}{N_s}$$  \hspace{1cm} (2.2)

where $N_s$ represents the number of the stator slots. Typically, we use the experienced equations to determine the stator slots size. Schematic structural view of the stator slot is shown in Figure 2.3.
Figure 2.3: 2D structure of the PMSM stator

\[
\begin{align*}
0.4\tau_s & \leq t_s \leq 0.6\tau_s \\
3t_s & \leq d_s \leq 7t_s \\
\text{where } b_s & \approx (0.1 - 0.5)b_s \\
\text{and } d_s & \approx (0.1 - 0.5)b_s
\end{align*}
\]

The following equation is used to determine the rated phase voltage:

\[
V_{\Phi,\text{rated}} = \sqrt{2}\pi f_e \hat{N}_a \Phi_{g,pk} \approx 4.44 f_e \hat{N}_a \Phi_{g,pk},
\]

where \( b_s = \tau_s - t_s \).
where \( f_e \) is electrical frequency, \( \Phi_{g,pk} = \frac{2B_{g,pk}Dl_i}{P} \) is the peak flux of air-gap, \( \hat{N}_a = k_w N_a / k_{ls} \) is the effective number of series turns per phase, \( k_w \) is winding factor, and number of series turns per phase \( N_a = Pq N_c / C \). If the number of parallel turns of armature winding \( C = 1 \), then \( N_a = Pq N_c \). We add assumed coefficient \( k_{ls} \) to compensate for the leakage flux. Finally, the number of turns per coil can be expressed as

\[
N_c = \frac{k_{ls} V_{\phi,\text{rated}} C}{2 \sqrt{2} \pi f_e q k_w B_{g,pk} Dl_i}.
\]

The phase current can be determined regarding current density and slot parameters as

\[
I_{p,\text{rated}} = \frac{\varphi_s \pi D r_s}{2 N_c N_s} J_s
\]

where \( J_s \) is the current density, \( N_s \) is the number of slots, \( r_s \) is the ratio of slot width and slot pitch, and \( \varphi_s \) is the slot pitch. We can use the current density equation to choose reasonable slot dimensions. It also provides some restriction of motor design, which is determined by cooling condition and thermal conductivity. The input power is related to phase voltage and phase current of the motor, which can be calculated as

\[
P_{\text{in}} = 3 V_{\phi,\text{rated}} I_{p,\text{rated}} \cos \theta.
\]

Stator Core Design

The flattened view of a motor is shown in Fig. 2.4.
It can be seen that the total flux through the yoke is equal to the flux in the air-gap vier a half pole pitch from the Fig 2.4. Therefore, the flux in the core can be calculated by integrating the air-gap flux.

\[
\Phi_{\text{core}} = \Phi_{\text{gap, per half pole pitch}}
\]

\[
= l_e \int_0^{\pi/N_p} B_g(\theta_a) r_{is} d\theta_a
\]

\[
= \frac{D}{2} l_e \frac{2}{P} \int_0^{\pi/N_p} B_{g,pk} \cos(\theta_{ae}) d\theta_{ae}
\]

\[
\approx \frac{D}{N_p} l_e B_{g,pk}
\]

where \( l_e \) is the stator effective length, \( N_p \) is number of poles. Then the core flux density can be calculated:
where \(d_y\) is yoke thickness. Assuming the flux density across one stator tooth is constant because the slot pitch is much smaller than pole pitch. The flux through one stator tooth is gained by integrating the air-gap flux over the whole slot pitch. Its value is approximately equal to the peak flux density.

\[
\Phi_{\text{tooth}} = l_{\text{eff}} \int_0^r B_g(\theta_{ae}r_i d\theta_{ae}) \approx B_{g,pk} \tau_s l_{\text{eff}}
\]  

(2.10)

where \(\tau_s\) is slot pitch. The relationship between back iron and tooth flux density can be got after some derivation

\[
\frac{B_{\text{core, pk}}}{B_{\text{tooth}}} = \frac{Dt_s}{Pd_y \tau_s}
\]  

(2.11)

Assuming \(t_s \approx 0.5\tau_s\), \(B_{\text{core}} \approx 0.8B_{\text{tooth}}\), the yoke thickness can be expressed as:

\[
d_y = \frac{D}{1.6N_p}
\]  

(2.12)

This equation has physical meaning, it means if we pick up the larger number of poles in the design, yoke thickness will be smaller, we don’t need that much yoke thickness to finish the design.

**Design of Air-gap and Permanent Magnet**

For a multi-pole surface mount rotor as shown in Fig. 2.5.
According to the magnetic circuit analysis:

\[ 2H_g + 2H_mD_m = 0, \]  

(2.13)

where \( H_g \) is the air gap magnetic field, \( g \) is the air gap size, \( H_m \) is the PM magnetic field and \( d_m \) is the magnetic thickness. We use magnetic flux density \( B \) to replace magnetic field \( H \), than the Eqn. 2.13 becomes \( gB_g + \mu_0H_md_m = 0 \), the relationship between the magnet thickness and air-gap size can be obtained:

\[ \frac{d_m}{g} = -\frac{B_m}{\mu_0H_m} \frac{A_m}{A_g}. \]  

(2.14)

When designing the magnet and air-gap, a working point for permanent magnetics should be selected. Firstly, we are going to get maximum energy point. \( B \) can be got from Fig. 2.5 easily:
\[ B = \frac{B_r}{H_c} (H + H_c), \quad (2.15) \]

letting \( B \) multiply \( H \), \( BH \) equation can be obtained as:

\[ BH = \frac{B_r}{H_c} (H + H_c) H. \quad (2.16) \]

In order to get maximum energy point, we get \( BH \) derivative:

\[ \frac{\partial (BH)}{\partial H} = 0, \quad (2.17) \]

after some derivation, the \((BH)_{max}\) can be got, which is located in \( B_m = \frac{B_r}{2}, H_m = -\frac{H_c}{2} \). It is in the middle of device line. It can be seen that there are two lines in Fig. 2.5: device line and load line. Device line equation can be easily got:

\[ B_m = \left( \frac{B_r}{H_c} \right) (H_m + H_c) = \mu_0 \mu_r + H_B, \quad (2.18) \]

\( P_c \) is called pemeance coefficient, which is equal to:

\[ P_c = \frac{d_m A_g}{g A_m} = \frac{A_g}{g} \frac{A_g}{A_m/d_m} \approx \frac{R_m}{R_g} \approx \frac{\rho_g}{\rho_m}. \quad (2.19) \]

From magnetic circuit:

\[ F_m + F_g = 0, \quad (2.20) \]

where \( F_m = H_m d_m \), and \( F_g = H_g g \), put them to Eqn. 2.20, than we can get \( H_g = -\frac{d_m}{g} H_m \), finally, we get:

\[ B_g = \mu_0 H_g = -\mu_0 \frac{d_m}{H_m}, \quad (2.21) \]

from \( \Phi = B_g A_g = B_m A_m \), we get \( B_m = B_g \frac{A_g}{A_m} \), replace \( B_g \) using Eqn. 2.21, the load line
equation can be obtained:

\[ B_m = -\mu_0 \frac{d_m}{A_g} A_m H_m = -P_c(\mu_0 H_m). \]  \hspace{1cm} (2.22)

We define \( B_m = \alpha_m B_r \), \( H_m = -(1 - \alpha_m) H_c \), where \( \alpha_m \) is called working point of permanent magnetics. We can get the maximum energy by choosing \( \alpha_m = 0.5 \). However, in order to avoid demagnetization and knee effect, we always choose working point higher than 0.5. Now we are going to determine the air-gap magnetic field from PM rotor based on the parallel magnetization magnet.

![Figure 2.6: The parallel magnetization waveform](image)

Fig. 2.6 shows the parallel magnetization waveform, after doing Fourier expansion, we can get:

\[ B_{rh} = \frac{2}{2\pi} \left[ \int_{-\rho_{PM}/2}^{\rho_{PM}/2} B_m \cos(h\theta_{ae})d\theta_{ae} + \int_{\pi-\rho_{PM}/2}^{\pi+\rho_{PM}/2} (-B_m) \cos(h\theta_{ae})d\theta_{ae} \right] = \frac{4}{\pi} \frac{\sin(h\rho_{PM} \pi)}{h} B_m \]  \hspace{1cm} (2.23)

where \( \rho_{PM} \) is the electrical angle of permanent magnet, can be calculated by \( \rho_{PM} = e_m \pi \), where \( e_m \) represents the embrace of the permanent magnet. Its value varies from 0.5 to 1. It affects the air-gap magnetic field, peak flux density of the teeth and yoke, cogging torque, etc. \( k_{ph} \) is pitch factor for the \( h^{th} \) harmonic, \( k_{ph} = \sin \left( \frac{h\rho_{PM}}{2} \right) \). The peak air gap magnetic field \( B_{r,pk} \) can be
approximately calculated by:

\[ B_{r, pk} \approx \frac{4}{\pi} \sin \left( \frac{\rho_{PM}}{2} \right) B_m. \]  

(2.24)

Typically, power angle \( \theta \) is approximate 0 at full-load for the design of PMSM. The torque angle \( \delta \) for PMSM is usually designed to be in the range of 15-30 degree. The phase diagram and relationship between torque angle and the output power are shown in Fig. 2.7.

![Diagram](image.png)

**Figure 2.7:** The phase diagram and relationship between torque angle and the output power for SPMSM

Normally, \( R_s \) is very small, we can neglect, then the phase A inducted back EMF \( E_A \approx V_\phi + jX_s I_A \). When selecting power factor is approximately to one. The peak value of the net magnetic
filed \( B_{g,pk} \) and peak winding magnetic field \( B_{a, pk} \) are defined:

\[
B_{g,pk} = B_{r,pk} \cos \delta B_{a,pk} = B_{r, pk} \sin \delta.
\] (2.25)

The output power \( P_{out} = 3V_\phi I_\phi \cos \theta \), after some derivation based on Fig. 2.7, we can get the output power equation related to the torque angle as shown below,

\[
P_{out} \approx \frac{3V_\phi I_\phi \sin \theta}{X_s},
\] (2.26)

where \( X_s \) is inductance phase winding. It is obviously shown that If we choose a smaller torque angle for the machinery at the related power in the design, the machinery will have more power handling capability and pull out torque. Our design method will be based on the much smaller torque angle compared with the traditional value, which will increase the magnets thickness and air-gap size. The larger air-gap size contributes to reducing the windage loss and noise level. The increase magnet thickness helps to avoid demagnetization.

If air-gap is small, the peak winding magnetic field can be calculated from:

\[
B_{a,peak} = \frac{4}{\pi} \frac{\mu_0}{\hat{g}_{total}} \frac{\hat{N}_a}{N_p} \sqrt{2} I_p, \text{rated},
\] (2.27)

We can get the initial total effective air-gap size based on Eqn. 2.27

\[
\hat{g}_{total} = \frac{4}{\pi} \frac{\mu_0}{g_{a,peak}} \frac{\hat{N}_a}{N_p} \sqrt{2} I_p, \text{rated},
\] (2.28)

where \( \hat{g}_{total} \) also can be calculated as:

\[
\hat{g}_{total} = k_c \hat{g}'_{total}
\] (2.29)
where $k_c$ is called cater’s coefficient and can be determined by:

$$k_c = \frac{\tau_s}{\tau_s - \frac{2b_{s0}}{\pi}} \left[ \arctan \frac{b_{s0}}{2g'_{total}} - \frac{g'_{total}}{b_{s0}} \ln \left( 1 + \left( \frac{b_{s0}}{2g'_{total}} \right)^2 \right) \right] \approx \frac{\tau_s}{\tau_s - \frac{b_{s0}^2}{5g'_{total} + b_{s0}}}$$

where $g'_{total} = g + \frac{d_m}{\mu_{rm}}$. We know $\frac{d_m}{g} \approx P_c$ and $P_c = \frac{\alpha_m}{1 - \alpha_m} \mu_{rm}$, after some derivation, we get:

$$\begin{cases} 
  g = (1 - \alpha_m) g'_{total} \\
  d_m = \alpha_m g'_{total} \mu_{rm}.
\end{cases}$$

(2.31)

If the designed air gap is large, which will needed to calculated the MMF from the permanent magnet and MMF generated from the air gap. The total MMF from the two parts can be added up and expressed by:

$$F_{total} = \frac{r_g B_{r,pk}}{\mu_0} \left[ \frac{1}{\mu_{rm}} \ln \left( \frac{r_a + d_m}{r_a} \right) + \ln \left( \frac{r_{i_s}}{r_a + d_m} \right) \right],$$

(2.32)

where $r_{i_s} = r_a + d_m + g_{eff}$, $r_g$ is the radius of the actual air gap, $r_a$ represents the inner radius of the rotor. According Eqn. 2.32, $g_{eff}$ can be calculated, then the actual air gap and magnet thickness can be obtained.

### Rotor Size

Total rotor diameter including magnet is:

$$D_r = D - 2g.$$  

(2.33)
Rotor inner diameter can be determined by:

\[ D_i = D_r - 2d_m. \] (2.34)

Double layer lap Winding Scheme

There are numerous winding schemes in a PM motor such as single-layer concerting lap winding, single-layer distributing winding, two-layer concerting lap winding, two-layer distributing winding, etc. The two-layer winding scheme is widely used in different kinds of PM machinery. The number of the conductors per coil \( N_c \) is determined by using:

\[ N_c = \frac{1.1V_{\phi,rated}}{2\sqrt{2\pi f_e q k_w B_{g,pk} D L}} \] (2.35)

Where \( V_{\phi,rated} \) represents the rated phase voltage, \( f_e \) is the electrical frequency, \( q \) is the number of the winding groups per pole, \( k_w \) describes the winding factor including pitch factor, distribution factor and winding skew factor. Similarly, the effective turns per phase is determined by:

\[ N_{eff} = \frac{N_p q N_c K_w}{1.1} \] (2.36)

Where \( N_p \) is the number of poles. Next, we will introduce a method step by step to show how to determine two-layer winding coil pitch and connection.

Step 1. Find the nominal coil span( \( S_c \) ) in slots

- If Slot numbers \( (N_s) / \) Pole numbers \( (N_p) \) is an integer \( S_c = \frac{S}{N_p} - 1 \)
- If \( N_s/N_p \) is not an integer \( S_c = max(fix(\frac{N_s}{N_p}), 1) \).
Step 2. Calculate the relative electric angle of in slots of all coils

The relative electrical angle (expressed in the range of $-180^\circ$ to $180^\circ$) of the $k$th slot is:

$$\theta_{\text{slot}}(k) = \text{mod}\left[(k - 1)\gamma + 180^\circ, 360^\circ\right] - 180^\circ.$$

The relative electrical angle of in slot of the $k$th coil is also:

$$\theta_{\text{coil}}(k) = \text{mod}\left[(k - 1)\gamma + 180^\circ, 360^\circ\right] - 180^\circ.$$

Step 3. Readjust in slot angles if their magnitude are greater than $90^\circ$. It means that if the magnitude of coil angles are greater than $90^\circ$, we need to reverse the coil direction; thereby changing the angle by $180^\circ$.

Step 4. Picking up $(N_s/\text{phase number}(m))$ coils for Phase A. For the angles calculated in the last step, we pick up those $(S/m)$ close to $0^\circ$ as phase A coils.

Step 5. Calculate the slot offsets to other wind phases. From $\text{mod}[S_{off}\gamma, 360^\circ]$ and $\gamma = \frac{N_p 360^\circ}{2}$, we can get:

$$S_{off} = 2\frac{N_s}{mP}(1 + mn) = 2q(1 + mn)$$

where $n$ is an integer value that makes $S_{off}$ also an integer. Phase B will start from slot ($\text{mod}(S_{off}, S) + 1$).

Step 6. Check to find out whether the winding is valid. The winding is valid if all slots contain two coil sides each. If it is not valid, go back to Step 4 and pick up coil for Phase A.

Power losses

Power losses [109][44][2][29] [108] are the key point to know the motor performance because they determine the machine’s efficiency and temperature-rise. There are three types of losses: Copper loss, Core loss, and Mechanical loss. Mechanical loss contains windage and friction loss typically
is very small and can be ignored compared to the other two types of losses. The copper losses are generally the largest component of power loss in brush-less PM motors. They can be calculated by:

\[ P_{\text{copper}} = m I^2_{p,\text{rated}} R_s \]  

(2.37)

where \( m \) is the number of phases; \( I^2_{p,\text{rated}} \) is the RMS phase current and \( R_s \) is the phase resistance and can be determined by \( R_s = \rho_{\text{copper}} \frac{I}{s} \), \( \rho_{\text{copper}} \) represents the resistivity of the copper and can be calculated as \( \rho = \rho_{20}[1 + \alpha(T - 20)] \) ohm-m, where \( \rho_{20} = 1.724 \times 10^{-8} \) ohm-m is the resistivity at \( 20^\circ C \); and \( \alpha \) is the temperature coefficient of resistivity. The winding temperature affects the resistivity of the windings. Normally, the winding temperature increase \( 50^\circ C \), the resistivity of the windings rise by \( 20\% \), the winding temperature increases \( 135^\circ C \), the resistivity of the windings rise by \( 53\% \), the \( I^2_{p,\text{rated}} R_s \) losses increase the same if the current remains the same. A useful formula can be derived from the resistivity of the copper equation to scale the resistance from one temperature to another:

\[ R_{s2} = \frac{234.5 + T_2}{234.5 + T_1} \times R_{s1} \]  

(2.38)

The effect of eddy-current in the stator conductors must be taken in to account when the machines run in high speed or machines operating with an inverter having a high switch frequency. The density of the current flowing in a conductor tends to a thin skin on its surface, which make the individual strands of wire small compared to the skin-depth. In general, the proximity effects occur in neighbution conductors cause a time-varying magnetic field and induces a circulating current inside the windings, usually with the same slot. Litz wire is helpful in spurring effect proximately.

Core losses are generally the second largest component of power loss in the brush-less PM motor, which can be subcategories as the hysteresis and eddy current losses. The hysteresis loss in the energy used to align and rotate magnetic domains. The loss per \( BH \) cycle is proportional to the
enclosed loop area, suggesting that the mean power loss due to hysteresis is proportional to the frequency $f$. We can use C.P.Steinmetz formula to determine the hysteresis loss:

$$P_h = K_h f B_{pk}^n [W/\text{kg}]$$  \hspace{1cm} (2.39)

where $K_h$ is the coefficient of the hysteresis loss.

Induced current causes the eddy-current loss by an alternating magnetic field induces a voltage. This voltage generates circulating currents in a conducting core, called eddy currents. Whenever there is a change in a magnetic field, an eddy current is induced. The higher the resistivity of the soft materials of the stator and rotor, the lower the eddy-current loss. The eddy-current loss can be determined using the classical formula:

$$P_e = K_e B_{pk}^2 f^2 [W/\text{kg}]$$  \hspace{1cm} (2.40)

where $K_e$ is the coefficient of eddy-current loss and can be calculated from an idealized theory as:

$$K_e = \frac{\pi^2 t_{lam}^2 \sigma}{6 \rho_m}$$  \hspace{1cm} (2.41)

where $t_{lam}$ is the lamination thickness, $\sigma$ is the conductivity, and $\rho_m$ is the mass density. From Eqn. 2.40 and Eqn. 2.41, we can know that the eddy-current loss can be reduced in two ways: using a high-resistivity material and using laminations. Using a high-resistivity material will increase the skin depth, which will make the distribution of magnetic flux density more uniform. Dividing the core into a large number of thin slices that are electrically insulated from each other by an oxide film. These thin insulated sheets, called lamination. The thinner lamination steels, the lower value of $K_e$; for example, if $t_{lam}$ is reduced from 0.5mm to 0.35mm, $K_e$ decrease by half. The total core
loss we can calculated by:

\[ P_c = K_h f B_{pk}^n + C_e f^2 B_{pk}^2 \]  

(2.42)

Summary

This section talks about the design method of high-efficiency electrical machinery. The advantage of the design method is that it can increase the high load capacity at no cost of increasing the total machine size. A much smaller torque angle than that in the traditional design at relating load is selected, which is between about 2 degrees and about 10 degrees. The most important part of the design is to determine the air-gap and permanent magnet size. According to the design method relating to the much smaller torque angle, which will result in the larger air-gap size and larger magnet thickness. The larger air-gap size contributes to reducing the windage loss and noise level. The increase magnet thickness helps to avoid demagnetization. Both them contribute to high efficiency and high overload capability. Based on the design method, all the parameters will be related to the torque angle, working point of a permanent magnet, and the permanent magnet embrace, which is easier for the designer to make a new design.
CHAPTER 3: PERMANENT MAGNET MACHINERY CONTROL

Introduction

Typically, there are two types of the diver for the PMSM: square-wave drive and sin-wave drive. In the square-wave system, the back EMF is a flat-topped waveform, and the ampere-conductor distribution of the stator ideally remains constant and fixed in space for a predetermined commutation interval while the magnet rotates past it. Hall sensor board is used to detect the correct current switching position. The transistors turn on or off based on the commutation stable. In the sine-wave system, the back EMF is sinusoidal, and the ampere-conductor distribution of the stator rotates at the synchronous speed fixed by the frequency and the number of poles. The torque is produced by the interaction between the stator and rotor magnetic field. The angle between the rotor magnetic field and stator magnetic field should be controlled well to get the maximum torque performance. For PMSM, we need to control the magnetic field generated from the stator winding. It is complicated to control motor on a three-phase reference frame directly, so we need to transfer the three-phase \(abc\) quantities into \(dq\) quantities. The critical point of this transfer is the rotor position that is a significant factor of the all PMSM control system. People usually use the encoder or resolver to get the rotor position information. We also can use sensor-less control based on Sliding model control (SMO) to estimate rotor position.

\(dq\) Theory of Permanent Magnet Synchronous Machines

To simply the PMSM equation and control the motor, people always to use \(dq0\) frame to model and analyze PMSM. It divides the PMSM armature quantities into two rotating components, one aligned with the field-winding axis (the direct-axis component), and one in quadrature with the
field-winding axis (the quadrature-axis component), which is called Park’s transformation.

Figure 3.1: $d - q$ axis on synchronous machine

Letting $S$ represent a stator quantity to be transferred and $\theta_{me} = \frac{N_p}{2} \theta_m$. This conversion comes through the $K$ matrix.

\[
S_{dqo} = KS_{abc}, \quad (3.1)
\]
and its inverse format is

\[ S_{abc} = K^{-1} S_{dqo}. \]  \hspace{1cm} (3.2)

The \( K \) and \( K^{-1} \) matrix in the MIT’s notation can be expressed as:

\[
K = \frac{2}{3} \begin{bmatrix}
\cos(\theta_{me}) & \cos(\theta_{me} - 2\pi/3) & \cos(\theta_{me} - 2\pi/3) \\
-\sin(\theta_{me}) & -\sin(\theta_{me} - 2\pi/3) & -\sin(\theta_{me} - 2\pi/3) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\]  \hspace{1cm} (3.3)

\[
K^{-1} = \frac{2}{3} \begin{bmatrix}
\cos(\theta_{me}) & -\sin(\theta_{me}) & 1 \\
\cos(\theta_{me} - 2\pi/3) & -\sin(\theta_{me} - 2\pi/3) & 1 \\
\cos(\theta_{me} + 2\pi/3) & -\sin(\theta_{me} + 2\pi/3) & 1
\end{bmatrix}
\]  \hspace{1cm} (3.4)

In the Purdue’s notation system, \( \theta_r = \theta_{me} + \frac{\pi}{2} \) is used to replace \( \theta_{me} \). The \( K \) and \( K^{-1} \) matrix in the Purdue’s notation can be expressed as:

\[
K = \frac{2}{3} \begin{bmatrix}
\sin(\theta_r) & \cos(\theta_{me} - 2\pi/3) & \sin(\theta_{me} + 2\pi/3) \\
\cos(\theta_r) & \cos(\theta_{me} - 2\pi/3) & \cos(\theta_r + 2\pi/3) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\]  \hspace{1cm} (3.5)

\[
K^{-1} = \frac{2}{3} \begin{bmatrix}
\sin(\theta_r) & \cos(\theta_r) & 1 \\
\sin(\theta_r - 2\pi/3) & \cos(\theta_r - 2\pi/3) & 1 \\
\sin(\theta_r + 2\pi/3) & \cos(\theta_r + 2\pi/3) & 1
\end{bmatrix}
\]  \hspace{1cm} (3.6)

In our system, all the derivations will be based on MIT’s notation system. Here dq means direct and
quadrature. Direct axis is aligned with the rotor’s pole. Quadrature axis refers to the axis whose electrical angle is orthogonal to the electric angle of direct axis. A third component $S_0$ is called the zero-sequence component, which is also included. Under balanced-three-phase conditions, there is no zero-sequence component.

Transformation of the stator winding voltage equations can be presented by:

$$V_{abc} = R_s i_{abc} + \frac{d}{dt}\lambda_{abc}, \quad (3.7)$$

after some derivation, we can get:

$$V_{dq0} = R_s i_{dq0} + \frac{d}{dt}\lambda_{dq0} + K\left(\frac{d}{dt}K^{-1}\right)\lambda_{dq0}, \quad (3.8)$$

where $R_s = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix}$. In order to get the $V_{dq0}$, we derive the derivation of $K^{-1}$,

$$\frac{d}{dt}K^{-1} = -\omega_{me} \begin{bmatrix} \sin(\theta_{me}) & \cos(\theta_{me}) & 0 \\ \sin(\theta_{me} - 2\pi/3) & \cos(\theta_{me} - 2\pi/3) & 0 \\ \sin(\theta_{me} + 2\pi/3) & \cos(\theta_{me} + 2\pi/3) & 0 \end{bmatrix}, \quad (3.9)$$

then, we can get:

$$K\left(\frac{dK^{-1}}{dt}\right) = \begin{bmatrix} 0 & -\omega_{me} & 0 \\ \omega_{me} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (3.10)$$
Finally, the $V_{dq0}$ can be obtained:

$$
\begin{bmatrix}
V_d \\
V_q \\
V_0
\end{bmatrix}
= \begin{bmatrix}
R_d i_d + \frac{d}{dt} \lambda_d - \lambda_q \omega_{me} \\
R_s i_q + \frac{d}{dt} \lambda_q - \lambda_d \omega_{me} \\
R_s i_0 + \frac{d}{dt} \lambda_0
\end{bmatrix}.
$$

(3.11)

For salient pole rotor, the inductance can be approximately expressed as:

$$
L_{abc} = \begin{bmatrix}
L_{aa} & L_{ab} & L_{ac} \\
L_{ba} & L_{bb} & L_{bc} \\
L_{ca} & L_{cb} & L_{cc}
\end{bmatrix}.
$$

(3.12)

The self-inductance can be expressed as:

$$
\begin{align*}
L_{aa} &= L_{ls} + L_1 + L_2 \cos(2\theta_{me}) \\
L_{bb} &= L_{ls} + L_1 + L_2 \cos(2(\theta_{me} - \frac{2\pi}{3})) \\
L_{cc} &= L_{ls} + L_1 + L_2 \cos(2(\theta_{me} + \frac{2\pi}{3}))
\end{align*}
$$

(3.13)

where $L_{ls}$ is the winding leakage inductance, $L_1$ is the inductance corresponding to the constant component of the air-gap permeance and $L_2$ is the magnitude of the inductance which corresponds to the component of air-gap permeance which varies with rotor angle. The mutual-inductance can be expressed as:

$$
\begin{align*}
L_{ab} &= L_{ab} = -\frac{1}{2}L_1 + L_2 \cos(2\theta_{me} - \frac{2\pi}{3}) \\
L_{bc} &= L_{CB} = -\frac{1}{2}L_1 + L_2 \cos(2\theta_{me}) \\
L_{ac} &= L_{ca} = -\frac{1}{2}L_1 + L_2 \cos(2\theta_{me} + \frac{2\pi}{3})
\end{align*}
$$

(3.14)
Therefor, we can get the following inductance matrix in $dq0$ frame:

$$L_{dq0} = KL_{abc}K^{-1} \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix},$$

(3.15)

where $L_d = L_{ls} + L_{md}$, $L_q = L_{ls} + L_{mq}$, $L_0 = L_{ls}$, and $L_{md} = \frac{3}{2}(L_A + L_B)$, $L_{mq} = \frac{3}{2}(L_A - L_B)$.

The flux-linkage of PMSM in the $abc$ frame is:

$$\lambda_{abc} = L_{abc}\dot{i}_{abc} + \lambda_{PMabc},$$

(3.16)

where $\lambda_{PMabc}$ is the permanent magnet flux-linkage in the $abc$ frame, the matrix is equal to:

$$\lambda_{PMabc} = \lambda_{PM} \begin{bmatrix} \cos(\theta_{me}) \\ \cos(\theta_{me} - \frac{2\pi}{3}) \\ \cos(\theta_{me} - \frac{2\pi}{3}) \end{bmatrix}.$$

(3.17)

The flux-linkage of PMSM in the $abc$ frame can be transformed into $dq0$ frame as:

$$\lambda_{dq0} = L_{dq0}\dot{i}_{dq0} + \lambda_{PMdq0},$$

(3.18)

where $\lambda_{PMdq0} = K\lambda_{PMabc} = \begin{bmatrix} \lambda_{PM} \\ 0 \\ 0 \end{bmatrix}$, $\lambda_d = L_d\dot{i}_d + \lambda_{PM}$, $\lambda_q = L_q\dot{i}_q$, and $\lambda_0 = L_{ls}\dot{i}_0$. 

34
For the linear model $\frac{d\lambda_{PMabc}}{dt} = V$ and $\lambda_{PMabc} = \lambda_{PM}$, we can get:

$$\frac{di_{dq0}}{dt} = L_{dq0}^{-1}V.$$ (3.19)

Finally, the dynamical equation in terms of current can be obtained as:

$$\frac{di_d}{dt} = \frac{1}{L_d}(v_d - R_s i_d + \omega_{me} L_q i_q)$$
$$\frac{iq}{dt} = \frac{1}{L_q}(v_q - R_s i_q + \omega_{me} L_d i_d - \omega_{me} \lambda_{PM})$$
$$\frac{i_0}{dt} = \frac{1}{L_0}(v_0 - R_s i_0).$$ (3.20)

If the winding is Y connected, $i_0$ is equal to zero, only need to consider $i_d$ and $i_q$.

Electrical instantaneous input power on stator can also be expressed through dq0 theory,

$$P_{m} = \frac{3}{2} R_s (i_d^2 + i_q^2 + i_0^2) + \frac{3}{2} (i_d^2 \frac{d\lambda_d}{dt} + i_q^2 \frac{d\lambda_q}{dt} + 2 i_0^2 \frac{d\lambda_0}{dt}) + \frac{3}{2} P_{mecg} (\lambda_d i_q - \lambda_q i_d),$$ (3.21)

where $\frac{3}{2} R_s (i_d^2 + i_q^2 + i_0^2)$ is copper loss, $\frac{3}{2} (i_d^2 \frac{d\lambda_d}{dt} + i_q^2 \frac{d\lambda_q}{dt} + 2 i_0^2 \frac{d\lambda_0}{dt})$ is magnetic power in windings, and $\frac{3}{2} P_{mecg} (\lambda_d i_q - \lambda_q i_d)$ is mechanical power. We use $P_{mecg}$ to express the mechanical power, so the electromagnetic torque on rotor can be expressed by:

$$T_e = \frac{P_{mecg}}{\omega_m} = \frac{3}{2} \frac{P}{2} (\lambda_d i_q - \lambda_q i_d),$$ (3.22)

putting $\lambda_d$ and $\lambda_q$ into electromagnetic torque equation, then we can get

$$T_e = \frac{P_{mecg}}{\omega_m} = \frac{3}{2} \frac{N_p}{2} (\lambda_{PM} i_q + (L_d - L_q) i_q i_d) = K_T i_q,$$ (3.23)
where $K_T = \frac{3N_p}{2}(\lambda_{PM} i_q + (L_d - L_q)i_d)$ is called torque constant. For round rotor machine ($L_d = L_q$), the torque constant $K_T = \frac{3N_p}{\lambda_{PM} i_q}$.

After the $dq$ frame analysis, we can describe the motor’s non-linear behavior based on the dynamic equation. Four dynamic equations are used to built the PMSM or PMSM modeling.

$$\frac{d i_d}{dt} = \frac{1}{L_d} (v_d - R_s i_d + \omega_m L_q i_q)$$ (3.24)

$$\frac{d i_q}{dt} = \frac{1}{L_q} (v_q - R_s i_q + \omega_m L_d i_d - \omega_m \lambda_{PM})$$ (3.25)

$$\frac{d \omega_m}{dt} = \frac{1}{J} (T_e - T_L - c\omega_m)$$ (3.26)

$$\frac{d \theta_m}{dt} = \omega_m$$ (3.27)

where $v_d$ is input voltage on $d$ axis, $v_q$ is input voltage on $q$ axis, $T_L$ is load torque, $J$ is initial moment of rotor, and $c$ is coefficient of friction.

The following Fig shows the motor modeling built by MATLAB based on the dynamical equation.
Figure 3.2: Motor modeling
Space Vector Pulse Width Modulation

Space Vector Pulse Width Modulation (SVPWM) is popular for controlling motor drivers or three-phase rectifier because it offers good utilization of the DC-link voltage, low current ripple, and reduced switching losses compared to conventional PWM modulation.

As we can see from Fig. 3.3, there are 8 possible switching states, for which two of them are zero switching states and six of them are active switching states. They can be represented by active vector \((V_1 - V_6)\) and zero vector \((V_0)\). Active vectors are stationary and not rotating, zero vectors are placed in the axis origin.

![Figure 3.3: Three phase switching states](image-url)

<table>
<thead>
<tr>
<th>Switching State</th>
<th>On-state Switch</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1 1 1]</td>
<td>(S_1, S_2, S_3)</td>
</tr>
<tr>
<td>[0 0 0]</td>
<td>(S_2, S_3, S_6)</td>
</tr>
<tr>
<td>[1 0 0]</td>
<td>(S_1, S_4, S_5)</td>
</tr>
<tr>
<td>[1 1 0]</td>
<td>(S_1, S_2, S_3)</td>
</tr>
<tr>
<td>[0 1 0]</td>
<td>(S_2, S_3, S_5)</td>
</tr>
<tr>
<td>[0 1 1]</td>
<td>(S_2, S_3, S_6)</td>
</tr>
<tr>
<td>[0 0 1]</td>
<td>(S_2, S_4, S_5)</td>
</tr>
<tr>
<td>[1 0 1]</td>
<td>(S_1, S_4, S_5)</td>
</tr>
</tbody>
</table>
Figure 3.4: Space voltage vectors in different sectors

Assuming the reference voltage space vector $\vec{V}_{\text{ref}}$ falls between two adjacent base vectors in sector $I$ as shown in the below picture.
According volt-second balancing, we can get the following equation.

\[
\begin{align*}
\overrightarrow{V_{\text{ref}}} T_s &= \overrightarrow{V_1} T_a + \overrightarrow{V_2} T_b + \overrightarrow{V_0} T_0, \\
T_s &= T_a + T_b + T_c
\end{align*}
\]  

(3.28)

where \(T_a, T_b\) and \(T_0\) are dwell times for \(\overrightarrow{V_1}, \overrightarrow{V_2}\) and \(\overrightarrow{V_0}\), \(T_s\) is sampling period, space vector \(\overrightarrow{V_{\text{ref}}} = V_{\text{ref}} e^{j\theta}\), \(\overrightarrow{V_1} = \frac{2}{3} V_d\), \(\overrightarrow{V_2} = V_d e^{j\frac{\pi}{3}}\) and \(\overrightarrow{V_0} = 0\), then we can get the real part and imag part of the reference vector,

\[
\begin{align*}
\overrightarrow{V_{\text{ref}}} T_s &= \overrightarrow{V_1} T_a + \overrightarrow{V_2} T_b + \overrightarrow{V_0} T_0, \\
T_s &= T_a + T_b + T_c
\end{align*}
\]  

(3.29)

Figure 3.5: Approximation of an arbitrary voltage space vector using base voltage
After some derivation we can get three time durations

\[
\begin{align*}
T_a &= \frac{\sqrt{3} T_s V_{ref}}{V_d} \sin(\frac{\pi}{3} - \theta), \\
T_b &= \frac{\sqrt{3} T_s V_{ref}}{V_d} \sin(\theta), \\
T_0 &= T_s - T_a - T_b
\end{align*}
\] (3.30)

where \(0 \leq \theta \leq \pi/3\). These equations mean that an arbitrary space vector within the triangle defined by the two adjacent base vectors, between which the expected vector is located, can be represented by the sum of these two vectors. This is realized by timely activating the two vectors combined with zero vectors sequentially. If the switching process is fast enough, meaning the period \(T_s\) is short, the approximation can precisely represent the reference vector.

Figure 3.6: Duty time for each sector
As we can see from Fig. 3.6, there are seven switching states for each sector within one cycle. It always starts and ends with a zero vector. This also means that there is no additional switching state needed when changing the sector. The uneven numbers travel counterclockwise in each sector, and the even segments go clockwise.

**Shaft Position**

The shaft position is very important for field orientation control. Optical encoder and resolver are normally used to get the rotor position. They are stalled on the shaft and expensive, which will require the designer to make the driver board bigger and limit the PMSM application. Based on these disadvantage, the sensor-less control technique based on the SMO is necessary to replace the resolver and encoder. The SMO control is a strategy of variable structure control. The stability of the system entirely depends on the sliding surface. In order to make sure the system error can be controlled and stable, the gain factor should be choose big enough. Normally, a discontinuous sign function is used as a switching function, which caused the chattering issue. In order to reduce the chattering problem, sigmoid function is selected as a switching function to build the SMO model.

The PMSM model in the Alpha-beta reference frame can be expressed as:

\[
\begin{align*}
L_s \frac{d i_\alpha}{dt} &= -R_s i_\alpha - e_\alpha + u_\alpha \\
L_s \frac{d i_\beta}{dt} &= -R_s i_\beta - e_\beta + u_\beta
\end{align*}
\]

(3.31)

where \(i_\alpha, i_\beta, u_\alpha, u_\beta\) are the phase currents, phase voltages, and back EMF in the Alpha-beta frame, respectively, \(R_s\) is the stator phase resistance and \(L_s\) is the stator phase inductance.

The electromotive force in Alpha-beta frame is related to the flux linkage, electrical velocity and
electrical rotor position, which can be calculated as:

\[
\begin{align*}
    e_\alpha &= -\psi_f \omega_r \sin \theta_m, \\
    e_\beta &= -\psi_f \omega_r \cos \theta_m,
\end{align*}
\]

where \(\psi_f\) is the flux linkage of the PMSM, \(\omega_r\) is the electrical angular velocity of the shaft, and \(\theta_m\) is the electrical rotor rotating angle. The back EMF equations contain the rotor position information, that means if we can know the PMSM back EMF, the rotor position can be obtained.

**Design of SMO**

The sigmoid function SMO model in Alpha-beta frame is built as:

\[
\begin{align*}
    \frac{d\hat{i}_\alpha}{dt} &= H\hat{i}_\alpha + u_\alpha - kF(\hat{i}_\alpha - i_\alpha), \\
    \frac{d\hat{i}_\beta}{dt} &= H\hat{i}_\beta + u_\beta - kF(\hat{i}_\beta - i_\beta)
\end{align*}
\]

where \(H = -\frac{R_s}{L_\alpha}, [\hat{i}_\alpha \hat{i}_\beta]^T\) is the desired Alpha-beta frame current value and \([i_\alpha i_\beta]^T\) is the measured current value in the Alpha-beta frame. The sigmoid switching function is represented as:

\[
F(x) = \frac{s}{\alpha + |s|}.
\]
Stability Analysis

The sliding surface of the sigmoid SMO model is chosen as:

\[
S(X) = \begin{bmatrix} s_\alpha & s_\beta \end{bmatrix}^T = \begin{bmatrix} \hat{i}_\alpha - i_\alpha & \hat{i}_\beta - i_\beta \end{bmatrix}^T.
\]  

(3.35)

The estimation errors should become zero when estimation errors of the SMO mode reach the sliding surface. The Lyapunov function is selected to verify the stability of the sigmoid function SMO model.

\[
\Gamma = \frac{1}{2} S(X)^T S(X) + \frac{1}{2}(\hat{R}_s - R_s)^2,
\]  

(3.36)

where \(\frac{1}{2}(\hat{R}_s - R_s)^2\) is used to estimate the stator resistance which is a variable parameter. The stability condition of the SMO is as fellow:

\[
\dot{\Gamma} = S(X)^T \dot{X} + (\hat{R}_s - R_s) \dot{\hat{R}}_s \leq 0.
\]  

(3.37)

The sliding condition is obtained by subtracting Eqn. 3.31 from Eqn. 3.33 as

\[
\dot{\Gamma} = \begin{bmatrix} \hat{i}_\alpha & \hat{i}_\beta \end{bmatrix} \begin{bmatrix} (\hat{H} - H) \hat{\alpha} + H(\hat{\alpha} - i_\alpha) + \frac{1}{L_s} [e_\alpha - kH(\hat{i}_\alpha)] \\ (\hat{H} - H) \hat{\beta} + H(\hat{\beta} - i_\beta) + \frac{1}{L_s} [e_\beta - kH(\hat{i}_\beta)] \end{bmatrix} + \hat{R}_s \dot{\hat{R}}_s \leq 0,
\]  

(3.38)

where \(H = -\frac{\hat{R}_s}{L_s}, \hat{H} = -\frac{\hat{R}_s}{L_s}, \) and \(\hat{R}_s = \hat{R}_s - R_s\). To satisfy the condition \(\dot{\Gamma} \leq 0\), Eqn. 3.38 is decomposed into two equations as follows:

\[
\begin{bmatrix} \hat{i}_\alpha & \hat{i}_\beta \end{bmatrix} \begin{bmatrix} (\hat{H} - H) \hat{\alpha} \\ (\hat{H} - H) \hat{\beta} \end{bmatrix} + \hat{R}_s \dot{\hat{R}}_s = 0,
\]  

(3.39)
\[
\begin{bmatrix}
\ddot{x} \\
i_\alpha \\
i_\beta
\end{bmatrix}
\begin{bmatrix}
H(\dot{\alpha} - i_\alpha) + \frac{1}{L_s}[e_\alpha - kH(i_\alpha)] \\
H\dot{\beta} + A(\dot{\beta} - i_\beta) + \frac{1}{L_s}[e_\beta - kH(i_\beta)]
\end{bmatrix} = 0. \quad (3.40)
\]

The estimation of the stator resistance can be obtained from Eqn. 3.39

\[
\dot{\hat{R}}_s = \frac{1}{L_s}(\ddot{i}_\alpha \cdot \dot{i}_\alpha + \ddot{i}_\beta \cdot \dot{i}_\beta). \quad (3.41)
\]

Using this estimated value of stator resistance in the current control can improve the stability of the system. In order to keep the SMO stable, the observer gains should satisfy the inequality condition found in Eqn. 3.38. As a result

\[
k > \max(|e_\alpha|, |e_\beta|). \quad (3.42)
\]

Therefore, if \(k\) is selected is large enough, which can make suer the stability of this kind of sliding motion. The back EMF can be obtained once the system reaches the sliding surface.

\[
\begin{bmatrix}
e_\alpha \\
e_\beta
\end{bmatrix} = \begin{bmatrix}
kF(\dot{i}_\alpha - i_\alpha) \\
kF(\dot{i}_\beta - i_\beta)
\end{bmatrix}. \quad (3.43)
\]

**Position and Velocity Estimation of the Rotor**

The equivalent back EMF still contains the high-frequency components. A first-order low-pass filter can be used for filtering, but it will cause phase delay. In order to get the real-time angular velocity information without using the low-pass filter and phase compensation part, we built an observer to extract the back EMF signal. The changer rate of the shaft angular velocity is far lower than that of stator current, then we can assume the shaft angular velocity \(\dot{\omega}_r = 0\). The back EMF
model of PMSM can be express as:

\[
\begin{aligned}
    \frac{de_\alpha}{dt} &= -\omega_r e_\beta \\
    \frac{de_\beta}{dt} &= \omega_r e_\alpha
\end{aligned}
\]  

(3.44)

A back EMF is constructed

\[
\begin{aligned}
    \frac{de_\alpha}{dt} &= -\hat{\omega}_r \hat{e}_\beta - p(\hat{e}_\alpha - e_\alpha) \\
    \frac{de_\beta}{dt} &= -\hat{\omega}_r e_\alpha - p(\hat{e}_\beta - e_\beta) \\
    \frac{de_\omega}{dt} &= (\hat{e}_\alpha - e_\alpha)\hat{e}_\beta - (\hat{e}_\beta - e_\beta)\hat{e}_\alpha
\end{aligned}
\]  

(3.45)

where \( p \) is the gain of observe, whose value is greater than zero. By subtracting Eqn. 3.44 from Eqn. 3.45 and doing further consolidation, the error equation of the observer is determined as

\[
\begin{aligned}
    \frac{de_\alpha}{dt} &= -\tilde{\omega}_r \tilde{e}_\beta - \omega_r \hat{e}_\beta - p\tilde{e}_\alpha \\
    \frac{de_\beta}{dt} &= -\tilde{\omega}_r \hat{e}_\alpha + \omega_r \tilde{e}_\alpha - p\tilde{e}_\beta \\
    \frac{de_\omega}{dt} &= \hat{e}_\alpha \hat{e}_\beta - \hat{e}_\beta \hat{e}_\alpha
\end{aligned}
\]  

(3.46)

The Lyapunov function is defined to verify the stability of the observer.

\[
\Gamma = \tilde{e}_\alpha^2 + \tilde{e}_\beta^2 + \tilde{\omega}_r^2
\]  

(3.47)

Differentiating the Eqn. 3.47

\[
\dot{\Gamma} = \tilde{e}_\alpha \dot{e}_\alpha + \tilde{e}_\beta \dot{e}_\beta + \tilde{\omega}_r \dot{\omega}_r
\]  

(3.48)
Substituting Eqn. 3.46 into the above equation yields

\[
\dot{\Gamma} = -p(\dot{e}_\alpha^2 + \dot{e}_\beta^2) \leq 0. \tag{3.49}
\]

We can see from Eqn. 3.49 that the back EMF observer is asymptotically stable. The position signal can be obtained through the observer without phase delay.

\[
\dot{\theta}_m = -\arctan\left(\frac{\dot{e}_\alpha}{\dot{e}_\beta}\right) \tag{3.50}
\]

**Sensor-less Field Oriented Control of PMSM**

As we know, the rotor of PMSM has a constant flux magnitude because of permanent magnets and the stator winding of PMSM create a rotating electromagnetic field when it is being energized. The rotating magnetic field can be controlled by controlling the stator currents. In order to get better dynamic performance, it is necessary to decouple the torque generation and the magnetization functions in PMSM, which is called Field Oriented Control (FOC). Doing park transformation based the shaft angle that is obtained using the advanced SMO, \(i_d\) and \(i_q\) can be obtained. The \(i_d\) is related to the flux component and \(i_q\) is related to the torque component, so we only need to control the \(dq\) current. In order to get the maximum torque, the referenced d axis current \(i_{dref}\) should be set to 0 for surface permanent magnet motor. We should give a negative referenced d axis current \(i_{dref}\) when flux weakening operation is required. The output of the speed regulator will be used as the torque command \(i_{qref}\). We will use this control for our alternator control system. For the Wrench BLDC motor, hall sensor board is used to help the motor start at the begin, then the sensor-less FOC will be used to control the motor. The sensor-less field oriented control of PMSM is shown below.
In this chapter, the dynamic modeling and control methods of the permanent magnet machinery are introduced. The shaft position is key for the PMSM control based on FOC. In order to avoid using resolver or shaft, a sigmoid function SMO is built to reduce chattering issue. This kind of SMO is verified using Lyapunov function and it shows that the large factor $k$ value can ensure the system stable. We also built an observer to replace the low-pass filter to extract the back EMF signal, which can avoid the phase delay. The observer also was verified by Lyapunov function.
CHAPTER 4: PERMANENT MAGNET BRUSH-LESS DC MOTOR AND MECHANICAL STRUCTURE DESIGN FOR THE ELECTRIC IMPACT WRENCH SYSTEM

Introduction

Brushless dc motors are popular in a wide range of industrial applications, such as computer peripherals, servo control systems and electrical tools due to their robustness, simplicity, large torque to volume ratio and high-efficiency[46] [41] [88] [25] [64] [6] [57] [55]. Interior permanent magnet brushless DC (IPMBLDC ) electric motor [53] [62] [98] [70] [81] [89] [87] [40] is an important category of these motors, constructed with the permanent magnets inserted into the steel rotor core and does not need to be glued such as in surface mounted permanent magnet motors. The leakage path of interior magnet motors usually includes a saturable magnetic bridge and the web, which will make the coefficient of flux leakage variable. In the previous chapter, we have described the key points of how to design a PM motor. In this chapter, we apply the methodology to the design of a IPMBLDC [24] [21] [78] [104] [52] [42] [76] for the electric impact wrench. The differences between designing a IPM motor and SPM motor is that we need to assume a flux leakage coefficient in the initial design and built an advanced equivalent magnetic circuit of IPMBLDC to calculate the modify the coefficient of flux.

Using an electric impact wrench instead of the traditional wrench has been a worldwide trend. Car and other automatic machines purchases number are rapidly increasing every year indicating that there will be a big market for an electric impact wrench. Brush-less DC motor design for the electric impact wrench requires high reliability, good control performance, small size, losing

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1This chapter has been published at journal Energies [40].
bolting time, low cost and easy industrial mass production. The motor connects to planetary gear reducer with a transmission ration to get high output torque. The speed of impact wrench will be reduced and the bolting time is related to motor speed, main pressure spring as well as shock block. High pull out torque is required to get high load capability and improve motor lifetime.

A permanent interior magnet synchronous motor (IPMSM) with concentrated windings is chosen in the design. Because IPMSM usually has the large torque to volume ratio and high efficiency. The motor of electric impact should be able to tolerate some torque ripple and does not require extra field weakening at higher speed. Based on these requirements, we choose the dc control drive for the brush-less DC motor (BLDC) design. Hall sensor board was designed and fabricated to detect the correct current switching position at the low speed to help the motor start, then advanced SMO is used to control the BLDC.

Figure 4.1: Simple block of BLDC control topology
Assumed flux leakage coefficient and selected working point of a permanent magnet were used in the initial design. An advanced equivalent magnetic circuit was developed to verify the total flux leakage and the quiescent operating position based on initial design parameters. Key design method points are considered and analyzed. Thermal analysis is given to simulate the temperature rise of all parts of the motor. The new impact wrench mechanical structure is designed, and its working principle analyzed. An electromagnetic field analysis based on MATLAB and MAXWELL 2D FEM was used in the design to verify the equivalent magnetic circuit and optimize the IPMBLDC parameters. Experimental results are obtained to confirm the configuration. The electrical and mechanical models are combined and provides an analytical IPMBLDC design method. We also show an innovative and reasonable mechanical dynamical calculation method for the impact wrench system.
Magnetic Bridge and Rib

The magnetic bridge (Fig. 4.3) affects the leakage coefficient of the interior permanent magnet motor [76] as shown in Eqn. 4.1. Flux density around the magnetic bridge is very high, which results in low permeability and high reluctance, thus magnetic flux leakage is small. If we would like to obtain stronger magnetism isolating effect, the size of the magnetic bridge should be smaller, but the mechanical strength will be reduced when the motor runs at high speed. Comprehensive consideration should be taken when choosing the size of the magnetic bridge.

\[
\Phi_0 = \frac{\alpha_m B_r A_m}{k_{ls}}
\]

(4.1)

where \( \Phi_0 \) is no-load main flux, \( k_{ls} \) is leakage coefficient, \( \alpha_m \) is working point of a permanent magnet, \( B_r \) is residual magnetization density and \( A_m \) is cross-sectional area providing the magnetic flux per pole.

Figure 4.3: Magnet and flux guide dimensions
Pull Out Torque

Pull-out torque is the largest torque that a motor can operate under synchronism in case of momentary overload with. It can be calculated by traditional power angle equation ignoring the effects of the stator resistance of the torque.

\[ T = \frac{1}{2} m N_p \frac{e_{PM} U}{L_d} \sin(\delta) + \frac{U^2}{2} \left( \frac{1}{L_q} - \frac{1}{L_d} \right) \sin(2\delta) \]  
(4.2)

where \( m \) is the number of phase, \( \omega_s \) is the angular frequency of the stator current, \( L_d \) and \( L_q \) are the direct axis inductance and quadrature axis inductance, \( e_{PM} \) is the permanent magnet flux linkage induced back EMF and \( \delta \) is the torque angle.

\[ e_{PM} = \frac{\xi_1 N_a \omega_s \phi_\sigma}{\sqrt{2}} \]  
(4.3)

where \( \xi_1 \) is the fundamental winding factor, \( N_a \) is the number of series turns of each phase and \( \phi_\sigma \) is air gap magnetic flux.

\[ \phi_\sigma = \frac{H_c d_m}{\mu_r \mu_0 + R_{\sigma} + R_{Fe}} \]  
(4.4)

where \( H_c \) is the magnet coercive field strength, \( d_m \) is the magnetic thickness, \( \mu_r \) is the magnet relative permeability, \( \mu_0 \) is the vacuum permeability and \( R_{Fe} \) is the iron reluctance.

\[ L_{md} = \frac{m}{2} \frac{4}{\pi} \alpha \mu_0 \frac{\tau_p}{2pg_{eff} + d_m} \ell (\xi_1 N)^2 \]  
(4.5)

where \( m \) is phase number, \( \alpha \) is the arithmetic average of the flux density distribution in one pole area, \( \tau_p \) is the pole pith, \( g_{eff} \) is the effective air gap length (excluding the magnets), \( \ell \) is the stack electromagnetic length. We can know from Eqn. 4.2 that the first part of Eqn. 4.2 is the magnetic torque, and the second part represents the reluctance torque due to saliency. The inductance in
q-direction is higher than in d-direction because of inverse saliency. For surface-mounted PMSM, direct axis inductance is equal to the quadrature axis inductance, so there is no saliency, which means the torque will proportional to the load angle.

When the thickness of the magnets increases, also the back-EMF increase as we can see from Eqn. 4.3 and Eqn. 4.4, while the direct-axis inductance decrease as shown in Eqn. 4.5, which result in increased torque production capability of the PMSM. The pull-out torque increases and the rated torque is obtained at lower load angle.

Typically, we choose the power factor angle equal to 0 at the full-load for PMSM. If we want to get a high pull-out angle in the design, we can select the torque angle as our reference to design the motor, the smaller torque angle we choose, the more significant pull-out torque we will get when the full-load torque is known.

Size of the Permanent Magnet

After we get the air-gap size, we can use Eqn. 4.6, Eqn. 4.7 based on to determine the size of the permanent magnet. Assumed flux leakage coefficient and selected permanent magnet working point are used in the equations.

\[ h_m = \frac{K_s K_{\alpha_m} g}{(1 - \alpha_m) k_{l_s}} \]  \hspace{1cm} (4.6)

\[ b_m = \frac{2 k_{l_s} B_{g,pk} \tau_1}{\pi \alpha_m B_r K_\phi} \]  \hspace{1cm} (4.7)

where \( h_m \) is magnet length, \( b_m \) is magnet width, \( g \) is air gap length, \( K_s \) is motor saturation factor with values ranging from 1.05 to 1.3, \( K_{\alpha_m} \) is rotor structure factor whose value range is between 0.7 and 1.2, \( B_{g,pk} \) is peak value of air gap fundamental wave, \( K_\phi \) is air gap flux waveform factor, which
is related to the pole arc coefficient. The air gap flux waveform of an ideal BLDC is a square wave, so the value of pole arc coefficient should be big enough with reference to [94]. For IPMBLDC, we can calculate the pole arc coefficient approximately using

$$\alpha_p \approx \frac{b}{\tau_p} \quad (4.8)$$

where $b$ is pole shoe arc length, and $\tau_p$ is pole pitch

Improved Magnet Circuit Model of BLDC

A commonly used half of BLDC configurations is shown in flowing picture.

![Figure 4.4: Half of BLDC configuration](image)

The route of main flux loop goes through the magnet, rotor yoke, air gap, stator tooth, and stator yoke. Taking the flux linkage of the magnet to magnet and magnet end flux linkage into consideration, the improved equivalent magnetic circuits of BLDC, which is composed of a half-pole pair for which the symmetry is considered, as shown in Fig. 4.5 (a).
Where $R_{sy}$, $R_{st}$, $R_{g}$, $R_{rya}$, $R_{ryb}$, $R_{\sigma}$, $R_{mo}$, $R_{ml}$, $R_{mm}$ are the reluctances of stator yoke, stator tooth, air-gap, rotor yoke above the magnet, rotor yoke below the magnet, assembly gap between magnets and laminations, the magnet, the magnet end flux leakage, magnet to magnet flux leakage, respectively.

Fig. 4.5 (a) can be simplified as shown in Fig. 4.5 (b). $R_z$ is the total reluctances of air-gap, stator tooth, rotor tooth, stator yoke, rotor yoke above and below the magnet, which can be calculated as

$$R_z = R_{sy}/4 + R_{st} + R_{g} + R_{rya} + R_{ryb}.$$  

(4.9)
The magnet end flux leakage reluctances can be expressed as

\[ R_\sigma = \frac{d_\sigma}{\mu_0 A_\sigma}, \quad (4.10) \]

where \( d_\sigma \) is the distance between the magnet and the duct, \( A_\sigma \) is the cross-sectional area of the air-gap between the magnet and the duct. The magnet reluctances is equal to

\[ R_{mo} = \frac{h_m}{\mu_0 \mu_r A_m}, \quad (4.11) \]

where \( h_m \) is the length of the permanent magnet, \( A_m \) is the cross-sectional area of magnet. \( R_0 \) is sum of magnet end flux leakage reluctances and magnet reluctance. Thus,

\[ R_0 = R_\sigma + R_{mo}. \quad (4.12) \]

To calculate the rationality of the point of the operation and the coefficient of the flux leakage, we need to analyze the equivalent of the magnet circuit, estimate the main magnetic circuit as shown in the Fig. 11, analyze the magnet end flux leakage \( \Phi_{ml} \) and the magnet to magnet flux leakage \( \Phi_{mm} \). We assume the bridges and webs are saturated, which can be replaced by a flux-source. We also assume the magnet web flux density is 1.8 Tesla. Therefore, we can calculate the magnet end flux leakage \( \Phi_{mi} \). The flux density of the magnet depth can be limited to 2 Tesla. Therefore, we can calculate the magnet to magnet flux leakage \( \Phi_{mm} \). Comparing the calculated total flux leakage value to the value of the assumption, some structural parameters will be adjusted based on errors using computer-aided tools. We do the same to the operating point \( \alpha_m \). All these repeated calculation will be done with the aid of Matlab.
A flowchart of the procedure of the calculation can be obtained shown in Fig. 4.6. Some magnetic circuit calculation equations are listed in Table. 4.1.

Figure 4.6: The flowchart of the procedure of the design calculation
Table 4.1: Magnetic circuit calculation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-load main flux</td>
<td>( \Phi_0 = \frac{B_m B_r A_m}{k \Phi_0} )</td>
</tr>
<tr>
<td>Average air-gap flux density</td>
<td>( B_g = \frac{\Phi_0}{\alpha_p \tau L_e} )</td>
</tr>
<tr>
<td>Air-gap MMF</td>
<td>( F_g = \frac{2B_a (d_x + k_c)}{\mu_0} )</td>
</tr>
<tr>
<td>Total MMF</td>
<td>( \Sigma F = F_g + F_{st} + F_{sy} + F_{ry} )</td>
</tr>
<tr>
<td>Main magnetic permeability</td>
<td>( \lambda_\delta = \frac{\Phi_\delta}{\Sigma F} )</td>
</tr>
<tr>
<td>Per-unit value of the main magnetic permeability</td>
<td>( \xi_\delta = \frac{2\xi_\delta h_m}{\mu_0 \mu_r A_m} )</td>
</tr>
<tr>
<td>Magnet to magnet flux leakage</td>
<td>( \Phi_{mm} = \frac{B_{w1} w_1 L_e}{2} )</td>
</tr>
<tr>
<td>Magnet end flux leakage</td>
<td>( \Phi_{ml} = B_{w2} w_2 L_e )</td>
</tr>
<tr>
<td>Total flux leakage</td>
<td>( \Phi_\sigma = \Phi_{mm} + \Phi_{ml} )</td>
</tr>
<tr>
<td>Flux leakage coefficient</td>
<td>( k_{ls} = \frac{\Phi_\delta \Phi_\sigma}{\Phi_\delta + \Phi_\sigma} )</td>
</tr>
<tr>
<td>Magnet operating point</td>
<td>( \alpha_m = \frac{k_{ls} \xi_\delta}{k_{ls} \xi_\delta + 1} )</td>
</tr>
</tbody>
</table>

**The Design and Calculation of Impact Wrench**

**Working Principle**

An electric impact wrench includes a motor, planetary gear, main pressure spring and shock block. The new mechanical structure makes the planetary gear retarding mechanism as the main transmission mechanism, which can guarantee small volume, lightweight, simple structure, high torque and power, and simple control requirement of IPMBLDC. The motor output force is transmitted by the planetary reducer to the mandrel, and then by the ball, driven by the main pressure spring to make the shock block rotate. Shock block uses its two convex claws to impact shock rod. The impact rod drives the bolt through the sleeve under the action of impact force. When the resistance torque of the bolt exceeds the torque transmitted by the main spring to the impact head, the impact head is retracted along the v-groove of the mandrel under the restriction of the ball, resulting in impact shock block and shock rod convex shoulder tripping. The shock block will continue to rotate under the motor driven at this time. The pawl crosses the shoulder and produces an additional angular
velocity under the main pressure spring, which pushes the pawl against the shoulder and generates an impact torque. The torque is then passed through the sleeve to the bolt or nut, which will make the bolt or nut rotate by an angle. The cycle of shock will continue until the completion of the bolt loading and unloading works. The mechanical structure is shown in Fig. 4.7.

![Figure 4.7: Mechanical structure of impact wrench](image)

*Planetary Gear Ration Calculations and Design*

Our planetary gear for electrical impact wrench is made of one sun gear, one ring, and three planet gears. The sun gear works as the active part, three planet gears are the followers, and the ring is fixed to the housing. The simple planetary gear mechanism is shown in Fig. 4.8.
According to the theory of machines and mechanisms, we know:

$$i_{13}^H = \frac{n_1^H}{n_3} = \frac{n_1 - n_H}{n_3 - n_H} = -\frac{z_3}{z_1} \quad (4.13)$$

where $n_1, n_2, n_3$ are the speed of sun gear, ring, and planet gear, respectively, $z_1$ is tooth number of sun gear and $z_3$ is tooth number of ring.

Since the ring is fixed to the housing, its speed is 0, we can get the planetary gear transmit ratio.

$$i_r = i_{1H} = \frac{n_1}{n_H} = 1 + \frac{z_3}{z_1} \quad (4.14)$$

After we know the required planetary gear transmit ratio. We can select the tooth number for the sun gear, ring, and planet gear based on Eqn. 4.14 for our design.
The Main Compression Spring Design

The main parameters of pressure spring of impact wrench are shown in Table. 4.2. According to dynamic principles, spring index $C_s = \frac{d_c}{D_c}$; spring constant $k = \frac{GD_c}{8C_s^2N_2}$, where $G$ is shear modulus of elasticity; the minimum load on spring is $F_1 = kS_1$; the maximum load on spring is $F_2 = kS_2$; the average load on spring is $F_a = (F_1 + F_2)/2$; the resistance torque of the spring to the mandrel $M_F = F_a r_o \tan \beta$. For our electric impact wrench, we need to make sure the torque from the motor to the mandrel is less than the resistance torque of the spring to the mandrel, this is the special requirement for our design. We can follow the mechanical design handbooks to do the compression spring design step by step so we are not going to introduce the procedures in detail. The basic design process is shown in Fig. 4.9.

---

![Diagram of spring design process](image-url)

Figure 4.9: The basic spring design procedure
Table 4.2: Parameters of pressure spring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>60Si2MnA</td>
</tr>
<tr>
<td>Installed length</td>
<td>60mm</td>
</tr>
<tr>
<td>Minimum amount of elastic deformation $S_1$</td>
<td>5mm</td>
</tr>
<tr>
<td>Maximum amount of elastic deformation $S_2$</td>
<td>14mm</td>
</tr>
<tr>
<td>Impact stroke length $h$</td>
<td>9mm</td>
</tr>
<tr>
<td>Angle of spiral $\beta$</td>
<td>22°</td>
</tr>
<tr>
<td>Out diameter of a coil $D_c$</td>
<td>71mm</td>
</tr>
<tr>
<td>Inner diameter of a coil $d_c$</td>
<td>60mm</td>
</tr>
<tr>
<td>Total number of winding $N_1$</td>
<td>6</td>
</tr>
<tr>
<td>Number of active winding $N_2$</td>
<td>3</td>
</tr>
</tbody>
</table>

**Shock Block Dynamic Calculation and Design**

The shock block shape is shown in Fig. 4.10 (a). Some experience equations are listed below.

\[
\begin{align*}
    l_1 & \approx (1 - 2.5)l_2 \\
    l_3 & \approx (0.3 - 0.5)l_s \\
    d_1 & \approx (1.2 - 1.5)d_2 \\
    D_o & \approx (1.2 - 1.5)d_1
\end{align*}
\]  

(4.15)

For convenient calculations, we simplify it to two steel tubes and two fan-shape claws as shown in Fig. 4.10 (b). The shock block quality is

\[
m = \frac{p\pi l_1(D_o^2 - d_1^2)}{4} + \frac{p\pi l_2(D_o^2 - d_2^2)}{4} + \frac{\alpha p\pi (D_o^2 - d_2^2)(l_s - l_1 - l_2)}{180 \times 4}.
\]

(4.16)
When it is rotating, the steel tube is like the hollow cylinder rotating around the rotation center. The fan-shaped claw is equivalent to a symmetrical fan rotating around the rotation center. We can obtain the moment of inertia of the shock block based on the theory of machines and mechanisms

\[
J = \frac{m_1 + m_2 + m_3}{8} D_0^2 + \frac{m_1}{8} d_1^2 + \frac{m_2}{8} d_2^2 + \frac{m_3}{8} d_3^2.
\]  

(4.17) The absolute angular velocity of the shock block before the impact consists of the average angular velocity of the mandrel and the additional angular velocity of the shock block, that is \( \omega_0 = \omega_t + \omega_a \).
The average angular velocity of the mandrel $\omega_t = \frac{2\pi n}{(60i)}$. The output power of the IPMBLDC is stored in the form of a compressing spring. The stored energy by the compression spring releases into two parts. One part is converted to the kinetic energy of the downward moment of the shock block, and the other part is converted to the kinetic energy of the shock block rotation. According to energy conservation law, we have

$$F_a h = \frac{J\omega_a^2}{2} + \frac{m(Jr \tan \beta)^2}{2}$$

(4.18)

after some derivations, we can obtain the additional angular velocity of the shock block

$$\omega_a = \sqrt{\frac{2F_m h}{J + m(r \tan \beta)^2}}.$$  

(4.19)

We performed the design based on the desired value $\omega_a$. After we pick up coefficients for each parts of length and diameters according to Eqn. 4.15, we put Eqn. 4.16 and Eqn. 4.17 to Eqn. 4.19. Matlab will be numerically calculate the relationship between $D_o$ and $l_s$. Normally we pick up $D_o$ values according to our IPMBLDC housing diameter; than we can use the relationship between $D_o$ and $l_s$ to get the value of $l_s$. Finally, we can obtain values of the all parameters.

Impacting shock rod, sleeve, and bolts, in essence, is an elastic collision process of shock block around the rotating center. During the elastic collision, the energy will be transferred. The efficiency of energy transmission is

$$\eta = \frac{\Delta E_1}{E_1},$$

(4.20)

where $\Delta E_1$ is the energy difference before and after impact, $E_1$ is energy before impact. Assuming the collision is elastic, the recovery coefficient is 1. According to collision theory, we can obtain

$$\eta = \frac{4JJ'}{J + J'} = \frac{4a}{1 + a}^2$$

(4.21)
where $J'$ is the converted inertia of the impact system, $a = J/J'$. In the process of disassembly of bolts, $J'$ changes all the time. Therefore $a$ is also variable. Assuming the change range from $a_1$ to $a_2$, then we can get the average theory impact efficiency. In our design, the range of $a$ is from 0 to 18.

\[
\hat{\eta} = \int_{a_1}^{a_2} \frac{4a}{(a+1)^2} da \\
(4.22)
\]

Finally, we get

\[
\hat{\eta} = 4\left[\frac{1}{a_2 - a_1} \ln \frac{a_2 + 1}{a_1 + 1} - \frac{1}{(a_1 + 1)(a_2 + 1)}\right].
\]

(4.23)

If the required tightening torque is $T'$, we can obtain the time of tighten a bolt $t_b = T'/E\hat{\eta}$.

Performance Analysis

**Optimization and Simulation**

To verify the magnetic circuit model and the design parameters, the 2-D Finite element analysis has been used.

**Table 4.3: Calculated results and FEA results**

<table>
<thead>
<tr>
<th>initial design parameters</th>
<th>calculated results</th>
<th>FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{ls}$</td>
<td>$\alpha_m$</td>
<td>$w_1$</td>
</tr>
<tr>
<td>1.15</td>
<td>0.85</td>
<td>1</td>
</tr>
<tr>
<td>1.15</td>
<td>0.85</td>
<td>1.1</td>
</tr>
<tr>
<td>1.15</td>
<td>0.85</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 4.3 shows the primary design results, calculated results and the FEA results for changing
the magnetic bridge width. We assume the flux density of the bridge is limited to 1.8 Tesla and the flux density of the rib is limited to 2 Tesla. From the table, we can see that the 2-D finite element analysis verifies our calculation and design parameters.

For our IPMBLDC, we evaluate and optimize the motor using ANSYS Maxwell. The dimensions of the IPMBLDC are obtained as shown in Table. 4.4.

<table>
<thead>
<tr>
<th>Table 4.4: IPMBLDC dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of the slots/poles</td>
</tr>
<tr>
<td>Stator outer diameter/mm</td>
</tr>
<tr>
<td>Stator inner diameter/mm</td>
</tr>
<tr>
<td>Rotor inner diameter/mm</td>
</tr>
<tr>
<td>Air-gap length/mm</td>
</tr>
<tr>
<td>Magnet width/mm</td>
</tr>
<tr>
<td>Magnet length/mm</td>
</tr>
<tr>
<td>Rotor bridge depth/mm</td>
</tr>
<tr>
<td>Rotor magnet web/mm</td>
</tr>
</tbody>
</table>

When we design the motor. The motor must meet the following flux density constraints: (1) Stator tooth flux density lower than 2T; (2) Stator yoke flux density lower that 1.5T; (3) Rotor yoke flux density lower than 1.5T. Through the finite element simulation analysis, the flux distribution of the final designed structure is shown in Fig. 4.11. From the Maxwell 2D simulation results, we can see that the tooth average flux density is $1.62T$, the yoke average flux density is $1.41T$, and the average air-gap flux density is $0.585T$. The web flux density is around $1.76T$, and rib flux density is around $1.95T$. The web flux density is close to the value we assumed. All the results satisfy the requirements.
Cogging Torque and No Load Back-EMF

Cogging torque is the consequence of the interaction between the rotor-mounted permanent magnet field and the stator teeth. It will produce a pulsating torque that does not contribute to the net effective torque. The waveform of the cogging torque for the IPMBLDC at rated speed is shown in Fig. 4.12. We know from Fig. 4.12 that the value of cogging torque is around 0.014Nm, which is about 6.3% ratio of the rated load torque. In order to get high torque, we use concentrated windings, so the value of cogging torque is reasonable for 6-slots, 4-pole motor. For the impact wrench application, there are no critical requirements for cogging torque.
The no-load back-EMF simulation analysis of the IPMBLDC is given in Fig. 4.13. The line to line EMF has a 60° flat-top with delta connection.
Thermal Analysis and Cooling

In order to avoid demagnetization, the magnet’s temperature needs to be kept under control. To preserve the life of the insulation and bearing, excessive heating of the surrounding and injury caused by touch hot surface, the temperature rise of the winding and frame should be kept below a level. In this paper, a fan is used for cooling and power MOSFETs are soldered on aluminum board. We can see from the Fig 4.14 that the maximum temperature appearing in the rotor is around 62 °C after one hour, and the temperature remains stable. Actually, the motor of electric impact wrench is not expected to be in continuous operation, and therefore that maximum temperature will never be reached.

Figure 4.14: Thermal analysis resulting using MotorSolve
The prototype is shown in Fig. 4.15. The induced back EMF of the IPMBLDC motor and hall sensor position of the IPMBLDC control board at low speed are shown in Fig. 4.16. The ampere-conductor distribution of the stator remains constant and fixed in space for a predetermined commutation interval while the magnet rotates past it, producing a linear variation in phase flux-linkage and from it a flat-topped EMF waveform.
Figure 4.16: Oscilloscope trace of phase a inducted EMF and current waveform and signals form hall sensor board at low speed

We can see the flux density of air gap and Fast Fourier transform analysis of flux density of air gap at no load in Fig. 4.17 (a) and Fig. 4.17 (b) respectively. Even harmonics are canceled, only odd harmonics exist, which indicates the harmonics of air gap flux density distribution is good.
Fig. 4.17: (a) The flux density of air gap (b) The FFT analysis of air gap

Fig. 4.18 (a) shows the testing and simulated results of IPMBLDC speed vs torque and current vs torque. We use the fixed torque wrench to load the bolt, then use the impact wrench to unload the bolt. As the torque increases, the motor speed decreases. The simulated speed of IPMBLDC at no load is around 2780 rpm, and the tested speed is a litter lower; the simulated speed decreases to around 1602 rpm, and the tested speed decreased to around 1375 rpm at maximum torque. The simulated current increased to 19A, and the tested current increased to 21A at the maximum torque as we can see from the Fig. 4.18 (b).
Figure 4.18: (a) Speed Vs torque (b) Torque Vs current
Fig. 4.19 shows the whole system of electrical wrench simulation efficiency is around 72% and testing efficiency is around 67% at full load. When we do the simulation, we neglect some mechanical transmit losses, that is why the testing efficiency is 5% lower than simulation efficiency. The testing power loss includes the motor power losses, shock block system losses and some other mechanical losses.

![Efficiency Vs torque](image)

**Figure 4.19: Efficiency Vs torque**
Summary

In this chapter, an analytical method to design the IPMBLDC motor and a new mechanical transmission structure for electrical impact wrench step by step was introduced. The improved magnetic circuit model has been established to calculate the coefficient of the flux leakage and working point of a permanent magnet. Smaller torque is chosen to get the high pull-out torque. The design has been optimized and verified using MAXWELL 2D analysis based on the finite element method and MotorSlove packages. The motor has also been fabricated and can satisfy all the design requirements for electric impact wrench application. The new impact mechanical structure and working principle, the planetary gear reduce transmission ratio formula, as well as a dynamic model of main pressure spring and shock block in the impact process are also elaborated. The whole system of the impact wrench was fabricated as well.
CHAPTER 5: DESIGN AND ANALYSIS A HIGH EFFICIENCY PERMANENT MAGNET ALTERNATOR AND CONTROL SYSTEM FOR THE TRUCK AUXILIARY UNITS

Introduction

Auxiliary power units (APUs) are normally equipped with the heavy-duty truck as shown in Fig 5.1 to supply power to air conditioners, heaters, electronics appliances and so on.

Figure 5.1: Auxiliary power units equipped with the heavy-duty truck

In this chapter, a 2kw high-efficiency alternator [66] [74] [90] [86] [41] and its control board system are designed, analyzed applying to the truck auxiliary power unit (APU) [106] [19] [48]

1The content in this chapter was partly published at IECON 2017 [41].
to charge the battery and supply the 12V DC Power units. High-efficiency alternator [67] [77] is required for this application. The surface permanent magnet alternator [34] [31] is designed and fabricated based on the previous chapter theory because the surface permanent magnet alternator has less leakage and higher power density compared with interior permanent magnet alternator [39] [38]. The working point of a permanent magnet [85] [16] [43] should be selected when doing the design. The maximum power is reached when the working point of permanent is 0.5. In order to avoid demagnetization of the magnet [23] [37] [32], the working point should be selected higher than 0.5. A much smaller torque angle $\delta$ [82] [63] [111] than that in traditional design at relating load is used, which is between about 2 degrees and about 10 degrees. The design can utilize a smaller torque angle to get high overload capacity, which will increase the magnet thickness and air-gap size. The larger air-gap helps to reduce the wind-age loss and noise level, while the increased magnet thickness contributes to avoiding demagnetization. The relationship among the torque angle $\delta$, working point, and the permanent magnet embrace [18] [112] [49] will be analyzed. The control system has two outputs: 48V DC and 14V DC. The alternator three phase outputs are connected to the there phase active rectifier to get 48V DC. SMO is used to get alternator position. Then, a buck converter with peak current control is applied to get 14V DC. The peak current control has advantage of a fast pulse-by-pulse short circuit and over-current load protection, which enhance the converter reliability. The control topology is shown below:
Copper loss [103] [50] [61] is the most significant of all the losses in low and medium speed electric machines. Reducing the copper loss is the key to build a highly efficient machine. To build a high-efficiency machine, we choose lower current density copper wires that have large wire cross-section, which reduce the copper loss and improve the efficiency. This also makes thermal management easier and avoids to use active cooling methodologies (such as fan, liquid cooling or spray cooling). Small torque angle is selected in design at the rated power and speed, which will result in large air age size, increased thickness of permanent magnets and high pull out torque. The machine will get high load capability, long lifetime, less mechanical noise and is not easy to demagnetization.

Analysis of Mathematic Model

From magnet thickness and air-gap equation, we can know that air-gap magnetic flux density value $B_{rh}$ is a key design factor. The working point of a permanent magnet, permanent magnet embrace value affects the value of $B_{rh}$. The torque angle $\delta$ determines the peak value of the net magnetic
flux density and the peak winding magnetic flux density and it is also related to the pull-out torque and over-load capacity. The smaller the torque angle, the higher the pull-out torque and over-load capacity. The permanent magnet embrace is also related to the cogging torque and efficiency. After selecting the values of the working point and torque angle, the permanent magnet embrace will play a role in the design. Normally we selected the value of magnet embrace is around 0.8. Fig. 5.3 and Fig. 5.4 show how the embrace affects the magnet thickness, air-gap values based on chapter two design equation. The plots are based on a 2Kw, 6000rpm generator with 0.8 working point and selected 10-degree torque angle. We can see that when the value of embrace increase the magnet thickness and air-gap thickness values decrease because the increased embraced value will increase the $B_{a,pk}$ and $B_{g,pk}$ values.

Figure 5.3: Embrace values Vs magnet thickness values
A alternator with 0.8 working point, 10-degree torque angle, and 0.75 embrace value was designed and fabricated based on the design method. The magnet thickness value is selected to be $6 \text{mm}$,
and air-gap length is chosen to be 2\textit{mm} after the optimization. The air-gap thickness is slightly less than the simulation value 2.15\textit{mm} considering the fabrication process. After the analysis, the dimensions of the alternator are obtained as shown in Table 5.1. The alternator windings diagrammatic drawing is shown in Fig. 5.5.

Figure 5.5: Alternator windings connection

ANSYS Maxwell 2D is used to evaluate and optimize our design. Parameters such as slot size and shape, magnet thickness, air-gap size and yoke thickness, and so on have been optimized using the software. Fig 5.6 shows the distribution of magnetic flux density. It can be seen that the tooth flux density is limited to 1.1T and the maximum flux density of the yoke is around 1.45T. Fig. 5.7 shows the calculated efficiency of the alternator versus RMS phase current. It can be seen that the motor has a very high efficiency (around 97%) at the rated power.
Figure 5.6: Flux density in the alternator

Figure 5.7: Efficiency Vs. RMS phase current
Fig. 5.8 shows flux linkage is good sinusoidal shape, which indicates the magnetic does not saturate with high excitation. Thus, the distortion of EMF should not appear across the zero point. Besides the amplitude of A, B and C phases flux linkage are almost the same, which can be explained as that the fringing effect is minimal. Fig. 5.9 shows the cogging torque versus time. It shows that the cogging torque is low. The cogging torque peak value is around 0.015 N.

Figure 5.8: Phase flux Vs time
Figure 5.9: Cogging torque Vs time

Thermal Analyze

The housing is designed and the thermal performance of each part of the generator are analyzed as shown in Fig. 5.10. The peak temperature inside the generator is around 342K at the full load, which is below 350K and therefore the active cooling methodologies are not needed.
Simulation Control Topology of The Whole Alternator System

The MATLAB simulation model of the whole system control topology was built. It contains two stages: three phase active rectifier and buck converter. The SMO is used to get the rotor position for the first stage and the field oriented control is applied to the alternator control. The 48V is transferred from the alternator three phase outputs using the three phase active rectifier. The buck converter is designed as second stage to get 14V DC to charger the 14V units. The whole control topology is shown below.
Figure 5.11: The simulation model of the whole alternator control system
The Fig. 5.12 shows the estimated rotor position and the actual rotor position. It can be clearly seen that the rotor position from SMO can match the actual rotor position.

Figure 5.12: The estimated rotor position and the actual rotor position

The correct rotor position information is the key of the control topology, then the 48V can be obtained from field oriented control, which is shown below.
The peaking current control is applied to buck converter to get the 14V DC. The Fig. 5.14 shows the simulation result from the MATLAB.
The flux density of the air-gap and no-load air-gap flux density Four transform analysis are shown in Fig. 5.15. It indicates that the even harmonics are canceled, only odd harmonics exist.
Figure 5.15: (a) The flux density of air-gap and different order harmonic waveforms showing with different color; (b) The fast Fourier transformation (FFT) analysis of air-gap.

Alternator and Control Board Prototype

The alternator prototypes are shown in Fig. 5.16. We use lamination technique to reduce the core loss. The lamination factor is 0.92.
In order to get perfect heat dissipation performance and increasing load capability and lifetime, we soldered the MOSFETS on the aluminum board for high power and use the copper bars to connect the PCB board and aluminum board.
Figure 5.17: PCB layout of the alternator control board
The fabricated alternator control board is shown in Fig. 5.18. The DSPF28335 is selected as CPU and the DSPF28335 minimum board is connected to the PCB main board. The high frequency and large current inductor also has been designed and fabricated for the buck converter using area production method.

Figure 5.18: The fabricated alternator whole system control board

Alternator Whole System Testing

The whole system control process is shown in Fig. 5.19, the testing was did based on it.
Figure 5.19: The whole system control process

A permanent magnet motor is used to drive the alternator. The alternator three phase outputs are connected to the control board to get DC output. The resistors are connected in parallel as loads as shown in Fig. 5.20.
We can see the three phase currents in Fig. 5.21. The waveforms are not perfect sine wave due to harmonics.

Finally, the efficiency of the alternator under different loads has been tested as shown in Fig. 5.22,
which indicates the design of alternator has high efficiency. The efficiency is around 95% at the rated load, which is 2% lower than the simulation values. Because when we do the simulation, we ignore some factors effect.

Figure 5.22: Three phase RMS current Vs efficiency
Summary

A truck alternator and its control system applying to APU system has been successfully design, fabricated and tested. 36-slot, 4-pole topology with surface-mounted permanent magnets and a double layer lap winding scheme is selected in our design. Motor structure consideration, key points of design and design procedures are introduced in detail. The design has been optimized and verified using MAXWELL 2D analysis based on the finite element method. We also created the housing and did the thermal analysis for our alternator. The heat dissipation performance of our alternator is pretty good, and no fan is required in our design. The sigmoid function SMO sensor-less control method is applied to the control system. Ti DSP 28335 MCU is selected as control CPU. The control PCB board is designed and fabricated. The Whole alternator system has also been completely fabricated. The simulation and test results show the whole system performers pretty well.
CHAPTER 6: CONCLUSION

The dissertation focuses on the high-efficiency electrical machinery design and control. The design method was introduced step by step. The dynamic modeling of the permanent magnet motor was built and analyzed. The advantage of the design method is that it can increase the high load capacity at no cost of increasing the total machine size. A much smaller torque angle than that in the traditional design at relating load is selected, which is between about 2 degrees and about 10 degrees. According to the design method relating to the much smaller torque angle, which will increase air-gap size and larger magnet thickness. The windage loss and noise level will be reduced because of larger air-gap size. The increased magnet thickness is contributed to avoid demagnetization. The larger air-gap sized and increased magnet thickness contribute to increase efficiency and overload capability. Based on the design method, all the parameters will be related to the torque angle, working point of a permanent magnet, and the permanent magnet embrace, which is easier for the designer to make a new design.

An interior permanent magnet brush-less DC electric motor (IPMBLDC) for a kind of electric impact wrench used for loading and unloading car bolts is designed and fabricated based on the design method. This kind of motor works on the discontinuous model. High pull-out torque and small size is required. Based on the design method, that two requirements can be satisfied when doing the BLDC design. Maxwell 2D FEM model is built to simulate and optimize the design. Thermal analysis is given to simulate the temperature rise of all parts of the motor. The new impact wrench mechanical structure is also designed. It provides an analytical whole system design method for the impact wrench system, which can be used for the other functional electric tools whole system design.

An advanced Sliding Mode Observer (SMO) is designed for the alternator system, which can get
the rotor position more accurate. The high-efficiency alternator is designed and fabricated based on the design method. The alternator system is applied to the truck auxiliary power unit (APU), which has two outputs (48V and 14V). All the Matlab simulation model, MAXWELL 2D model, and PCB board are done. The whole system efficiency is much higher than the traditional system using induction motor.
List of Symbols

\( A_m \)  \hspace{1em} \text{Cross section area of the magnets.}

\( A_g \)  \hspace{1em} \text{Cross section area of the air-gap.}

\( b \)  \hspace{1em} \text{Pole shoe arc length.}

\( b_m \)  \hspace{1em} \text{Magnet width.}

\( B_{\text{core, pk}} \)  \hspace{1em} \text{Core peak flux density.}

\( B_{g, \text{peak}} \)  \hspace{1em} \text{Air-gap peak flux density.}

\( B_{\text{tooth}} \)  \hspace{1em} \text{Tooth flux density.}

\( B_r \)  \hspace{1em} \text{Remanent flux-density.}

\( B_{a, \text{peak}} \)  \hspace{1em} \text{Peak winding magnetic density.}

\( c \)  \hspace{1em} \text{Coefficient of firection.}

\( C_s \)  \hspace{1em} \text{Spring index.}

\( d_y \)  \hspace{1em} \text{Yoke thickness.}

\( d_m \)  \hspace{1em} \text{The magnetic thickness.}

\( D \)  \hspace{1em} \text{Stator bore inner diameter.}

\( D_r \)  \hspace{1em} \text{Total rotor diameter including magnet.}

\( D_i \)  \hspace{1em} \text{Rotor inner diameter.}

\( D_c, d_c \)  \hspace{1em} \text{Out and inner diameter of a shock block coil.}

\( e_m \)  \hspace{1em} \text{Embrace of the permanent magnet.}
$e_{PM}$ The permanent magnet flux linkage induced back EMF.

e_{\alpha}, e_{\beta} Back EMF in the stationary frame.

$\tilde{e}_{\alpha}, \tilde{e}_{\alpha}$ Back EMF error in $\alpha, \beta$ frame.

$E$ Energy produced by shock block every minute.

$\Delta E_1$ Energy difference before and after impact.

$E_1$ Storing energy before impact.

$f_e$ Electrical frequency of phase voltage.

$F_m$ Permanent magnet MMF.

$F_g$ Air-gap MMF.

$F_{total}$ The total MMF.

$F_g, F_{st}$ MMF of air-gap and stator tooth.

$F_{sy}, F_{ry}$ MMF of stator yoke and rotor yoke.

$F_1, F_2, F_3$ The minimum, maximum, and average load on the spring.

$g$ The actual air-gap size.

$\hat{g}_{total}$ Initial total effective air-gap size.

$g'_{total}$ Effective air-gap size with magnet thickness.

$G$ Shear modulus of elasticity.

$h_m$ Magnet length.

$H_g$ The air-gap magnetic field.
\( H_m \) \hspace{1cm} \text{The PM magnetic field.}

\( H_c \) \hspace{1cm} \text{Magnet Coercive field strength.}

\( i_{\alpha}, i_{\beta} \) \hspace{1cm} \text{Phase current in the stationary frame.}

\( \hat{i}_{\alpha}, \hat{i}_{\beta} \) \hspace{1cm} \text{Estimated current in the stationary reference frame.}

\( i_r \) \hspace{1cm} \text{Planetary gear ratio.}

\( I_{p,\text{rated}} \) \hspace{1cm} \text{Rated phase current.}

\( I_{abc} \) \hspace{1cm} \text{Stator current in the \( abc \) frame.}

\( I_{dq0} \) \hspace{1cm} \text{Stator current in the \( dq \) frame.}

\( i_{dref} \) \hspace{1cm} \text{Referenced \( d \) axis current.}

\( i_{qref} \) \hspace{1cm} \text{Referenced \( q \) axis current.}

\( J \) \hspace{1cm} \text{Initial moment of the rotor.}

\( J_s \) \hspace{1cm} \text{Current density.}

\( J' \) \hspace{1cm} \text{Converted inertia of the impact system.}

\( k \) \hspace{1cm} \text{Spring constant.}

\( k_w \) \hspace{1cm} \text{The winding factor.}

\( k_c \) \hspace{1cm} \text{Carter’s coefficient.}

\( k_{ls} \) \hspace{1cm} \text{The leakage flux coefficient.}

\( k_i \) \hspace{1cm} \text{Insulation coefficient.}

\( k_{ph} \) \hspace{1cm} \text{Pitch factor for the \( h^{th} \) harmonic.}

\( k \) \hspace{1cm} \text{spring constant.}
$K_s$  Motor saturation factor.

$K_\alpha$  Rotor structure factor.

$K_\phi$  Air-gap flux waveform factor.

$K_h$  Coefficient of the hysteresis loss.

$l_e$  Stator effective length including fringing due to ducts.

$L$  Stator core length (not including air ducts and fringing).

$L_{aa}, L_{bb}, L_{cc}$  Stator self-inductances.

$L_{ab}, L_{bc}, L_{ac}$  Stator-to-stator mutual inductances.

$L_{1f}$  Leakage inductance of field winding.

$L_1$  Inductance corresponding to the constant component of the air-gap permeance.

$L_2$  Inductance component corresponding to the component of air-gap permeance which varies with rotor angle.

$L_s$  The stator phase inductance.

$L_{im}$  Impact stroke length.

$m$  Shock block quality.

$m_1, m_2$  One of the two different steel tubes quality.

$m_3$  Two fan-shape claws quality.

$M_F$  Resistance torque of the spring to the mandrel.

$n_1, n_2, n_3$  Speed of sun gear, ring, and plant gear.
\( \hat{N} \) The effective number of series turns per phase.

\( N_s \) The number of stator slots.

\( N_c \) The number of turns per coil.

\( N_a \) The number of series turns per phase.

\( N_p \) Number of poles.

\( N_1, N_2 \) Total numbers and active numbers of pressure spring winding.

\( P_r \) Required output power.

\( P_{mece} \) Mechanical power.

\( P_c \) Permeance coefficient.

\( P_{copper} \) Copper loss.

\( P_h \) Hysteresis loss.

\( P_e \) Eddy-current loss.

\( P_c \) The total core loss.

\( q \) The number of the winding groups per pole.

\( r_a \) The inner radius of the rotor.

\( r_s \) Ratio of slot width and slot pitch.

\( r_g \) The radius of the actual air-gap.

\( r_{is} \) The total length of inner radius, magnet thickness and effective air-gap size without magnet thickness.

\( r_o \) Ball to spindle center distance.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>The stator phase resistance.</td>
</tr>
<tr>
<td>$R_m$</td>
<td>Permanent magnet reluctance per pole.</td>
</tr>
<tr>
<td>$R_g$</td>
<td>Air-gap reluctance per pole.</td>
</tr>
<tr>
<td>$R_{ml}$</td>
<td>Reluctances of the magnet end flux leakage.</td>
</tr>
<tr>
<td>$R_{mm}$</td>
<td>Reluctances of the magnet to magnet flux leakage.</td>
</tr>
<tr>
<td>$R_\sigma$</td>
<td>Magnet end flux leakage reluctance.</td>
</tr>
<tr>
<td>$R_{sy}$</td>
<td>Reluctances of stator yoke.</td>
</tr>
<tr>
<td>$R_{st}$</td>
<td>Reluctances of stator tooth.</td>
</tr>
<tr>
<td>$R_g$</td>
<td>Reluctances of air-gap.</td>
</tr>
<tr>
<td>$R_{nya}$</td>
<td>Reluctances of rotor yoke above the magnet.</td>
</tr>
<tr>
<td>$R_{ryb}$</td>
<td>Reluctances of rotor yoke below the magnet.</td>
</tr>
<tr>
<td>$R_z$</td>
<td>Total reluctance of air-gap.</td>
</tr>
<tr>
<td>$R_{mo}$</td>
<td>Magnet reluctances.</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Sum of magnet end flux leakage reluctances and magnet reluctance.</td>
</tr>
<tr>
<td>$\hat{R}_s$</td>
<td>The stator estimated phase resistance in the stationary reference frame.</td>
</tr>
<tr>
<td>$S_{off}$</td>
<td>Slot offsets to other winding phases.</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Nominal coil span in slots.</td>
</tr>
<tr>
<td>$S$</td>
<td>A stator quantity to be transformed (current, voltage, or flux).</td>
</tr>
<tr>
<td>$S_1, S_2$</td>
<td>Minimum and maximum amount of elastic deformation.</td>
</tr>
<tr>
<td>$t_{lam}$</td>
<td>Lamination thickness.</td>
</tr>
</tbody>
</table>


\( \hat{\Gamma} \)  

\( \delta \)  

\( \eta \)  

\( \hat{\eta} \)  

\( \theta \)  

\( \theta_a \)  

\( t_b \) Time of tighten a bolt. 

\( T_m \) Related mechanical torque. 

\( T_L \) Load torque. 

\( T_a, T_b, T_c \) Dwell time. 

\( u_\alpha, u_\beta \) Phase voltage in the stationary frame. 

\( V_\Phi \) Phase voltage. 

\( V_{\Phi,\text{rated}} \) Rated phase voltage. 

\( V_{abc} \) Stator winding voltage in the \( abc \) frame. 

\( V_{dq0} \) Stator winding voltage in the \( dq \) frame. 

\( z_1 \) Tooth numbers of sun gear. 

\( z_3 \) Tooth numbers of ring. 

\( \alpha_p \) Pole arc coefficient. 

\( \alpha_m \) Working point of permanent magnet. 

\( \dot{\Gamma} \) The stability condition function. 

\( \delta \) Torque angle. 

\( \eta \) Efficiency of energy transmission. 

\( \hat{\eta} \) Average theory impact efficiency. 

\( \theta \) Power angle. 

\( \theta_a \) Phase A reference angle.
\[ \theta_{\text{slot}}(k) \] The relative electrical angle (expressed in the range of \([-180^\circ, 180^\circ]\) of the \(k\)th slot.

\[ \theta_{\text{coil}}(k) \] The relative electrical angle of in slot of the \(k\)th coil.

\( \theta_m \) Rotor electrical angle.

\( \theta_{me} \) Poles/2 times the rotor electrical angle.

\( \tilde{\theta}_m \) Position signal obtained through observer without phase delay.

\( \lambda_{abc} \) The current flux-linkage in the \(abc\) frame.

\( \lambda_{dq0} \) The current flux-linkage in the \(dq\) frame.

\( \lambda_{PMabc} \) The permanent magnet flux-linkage in the \(abc\) frame.

\( \lambda_{dqabc} \) The permanent magnet flux-linkage in the \(dq\) frame.

\( \lambda_\delta \) Main magnetic permeability.

\( \mu_0 \) Permeability of air.

\( \mu_r \) Relative permeability.

\( \rho_{\text{copper}} \) The resistivity of the copper.

\( \rho_{PM} \) Magnet electrical angle.

\( \rho_m \) The mass density.

\( \sigma \) The conductivity.

\( \tau_1 \) Polar distance.

\( \tau_p \) Pole pitch.

\( \tau_s \) Slot pitch.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_{core}$</td>
<td>Air-gap flux.</td>
</tr>
<tr>
<td>$\Phi_{g,pk}$</td>
<td>The peak flux of air-gap.</td>
</tr>
<tr>
<td>$\Phi_{tooth}$</td>
<td>One stator tooth flux.</td>
</tr>
<tr>
<td>$\Phi_{\sigma}$</td>
<td>Total flux leakage.</td>
</tr>
<tr>
<td>$\Phi_{ml}$</td>
<td>Magnet end flux leakage.</td>
</tr>
<tr>
<td>$\Phi_{mm}$</td>
<td>Magnet to magnet flux leakage.</td>
</tr>
<tr>
<td>$\xi_{\delta}$</td>
<td>Per-unit value of the main magnetic permeability.</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>Flux linkage of the Permanetn magnet synchronous motor (PMSM).</td>
</tr>
<tr>
<td>$\varpi_0$</td>
<td>Cooling coefficient.</td>
</tr>
<tr>
<td>$\varphi_s$</td>
<td>Slot pitch.</td>
</tr>
<tr>
<td>$\omega$</td>
<td>The rotor rotational speed.</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Electrical angular velocity of the rotor.</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Absolute angular velocity of the shock block.</td>
</tr>
<tr>
<td>$\omega_t$</td>
<td>Average angular velocity of the mandrel.</td>
</tr>
<tr>
<td>$\omega_a$</td>
<td>Additional angular velocity of the shock block.</td>
</tr>
<tr>
<td>$\omega_r^*$</td>
<td>Error of motor angular velocity.</td>
</tr>
</tbody>
</table>
LIST OF REFERENCES


