Third-order Optical Nonlinearities for Integrated Microwave Photonics Applications

2019

Marcin Malinowski
University of Central Florida

Find similar works at: https://stars.library.ucf.edu/etd

University of Central Florida Libraries http://library.ucf.edu

Part of the Electromagnetics and Photonics Commons, and the Optics Commons

STARS Citation


This Doctoral Dissertation (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Electronic Theses and Dissertations by an authorized administrator of STARS. For more information, please contact lee.dotson@ucf.edu.
THIRD-ORDER OPTICAL NONLINEARITIES FOR INTEGRATED MICROWAVE PHOTONICS APPLICATIONS

by

MARcin MALINOWSKI
M.S., University of Cambridge, 2014
B.A., University of Cambridge, 2013

A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Optics in the College of Optics and Photonics at the University of Central Florida Orlando, Florida

Summer Term
2019

Major Professor: Sasan Fathpour
The field of integrated photonics aims at compressing large and environmentally-sensitive optical systems to micron-sized circuits that can be mass-produced through existing semiconductor fabrication facilities. The integration of optical components on single chips is pivotal to the realization of miniature systems with high degree of complexity. Such novel photonic chips find abundant applications in optical communication, spectroscopy and signal processing. This work concentrates on harnessing nonlinear phenomena to this avail.

The first part of this dissertation discusses, both from component and system level, the development of a frequency comb source with a semiconductor mode-locked laser at its heart. New nonlinear devices for supercontinuum and second-harmonic generations are developed and their performance is assessed inside the system. Theoretical analysis of a hybrid approach with synchronously-pumped Kerr cavity is also provided. The second part of the dissertation investigates stimulated Brillouin scattering (SBS) in integrated photonics. A fully-tensorial open-source numerical tool is developed to study SBS in optical waveguides composed of crystalline materials, particularly silicon. SBS is demonstrated in an all-silicon optical platform.
To my parents.
I would like to thank my advisor, Dr. Sasan Fathpour, for giving me the freedom to pursue topics that I found interesting but were tangential to core research interests of the group. Under his guidance I have matured as a researcher and I have expanded my skills and knowledge in the field of optics.

I am also very grateful to Dr. Peter Delfyett with whom I was working on the DARPA DODOS project. For that period of time he effectively acted as my co-advisor providing valuable feedback and access to measurement equipment.

I also thank Dr. Demetrios Christodoulides for help with numerical problems in nonlinear optics.

I would like to thank Dr. Rodrigo Amezcua Correa for assistance in the construction and operation of the fiber mode-locked laser.

I would also like to express my appreciation to my current and former group members, Jeff Chiles for teaching me the ropes of the trade, Guillermo F. Camacho-Gonzalez and Amirmahdi Honardoost for all the fun volleyball games, Tracy Sjaardema for swing dancing with me and Ashutosh Rao, Saeed Khan and Kamal Khalil for valuable discussions in the office.

I want to also thank Ricardo Bustos Ramirez and Michael Plascak for all the long hours we spent on the DODOS project crammed in the same lab.

I would like to thank my Cambridge supervisor and a friend, Themis Mavrogordatos, for all the intellectual stimuli.

I would like to thank my high-school teachers, Krzysztof Kuśmierczyk, Anna Mazurkiewicz and Barbara Tarnowska whose zeal for teaching jump-started my career in science.
Finally, I would like to thank my parents for all the time and effort they have put into my upbringing without whose support I would not be able to achieve as much.
TABLE OF CONTENTS

LIST OF FIGURES . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . xi

LIST OF TABLES . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . xix

LIST OF ACRONYMS . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . xx

CHAPTER 1: TOWARDS INTEGRATED FREQUENCY COMBS . . . . . . . . . . . . 1

1.1 Abstract . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1

1.2 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1

1.3 Applications of miniature frequency combs . . . . . . . . . . . . . . . . . . . . 3

1.4 Stabilization . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5

1.5 Supercontinuum generation in integrated waveguides . . . . . . . . . . . . . . 10

1.6 Requirements for semiconductor lasers . . . . . . . . . . . . . . . . . . . . . . . . 13

1.7 Materials for nonlinear processes . . . . . . . . . . . . . . . . . . . . . . . . . . . 20

1.8 Future outlook . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27

1.9 Acknowledgements . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28

CHAPTER 2: EXPERIMENTAL METHODS . . . . . . . . . . . . . . . . . . . . . . . . 29
2.1 Chalcogenide processing ........................................ 29
2.2 Mode-locked laser .............................................. 34
2.3 Supercontinuum generation ...................................... 37
2.4 Thin-film PPLN .................................................. 39
2.5 CEO detection .................................................... 41

CHAPTER 3: SYNCHRONOUSLY PUMPED KERR CAVITY ............ 43
3.1 Abstract .......................................................... 43
3.2 Introduction ....................................................... 43
3.3 Pulsed input - analytical treatment .............................. 45
3.4 Numerical simulations ........................................... 50
3.5 Conclusions ....................................................... 53

CHAPTER 4: NUMERICAL TOOLS .................................. 54
4.1 Split-step algorithm .............................................. 54
4.2 Genetic algorithms for dispersion engineering .................... 55
4.3 TDFA model ....................................................... 58

CHAPTER 5: STIMULATED BRILLOUIN SCATTERING IN SILICON .... 63
LIST OF FIGURES

1.1 Frequency combs find applications in (i) low-noise microwave signal generation and processing, (ii) gears in atomic clocks that measure the precise frequency of atomic transitions, (iii) broadband light sources in spectroscopy. They can also be viewed as (iv) a source of densely-spaced optical channels for optical communications and (v) source of high peak-power pulses in time-of-flight lidar. ................................................................. 3

1.2 Carrier envelope offset ($f_0$) detection schemes. ......................................................... 9

1.3 a) Coupled-optoelectronic oscillator with InP MLL using multitone injection locking. The output pulses are pulse-picked, compressed and amplified to generate supercontinuum in chalcogenide (ChG) waveguides. The experimental data comes from [1]. b) Autocorrelation of pulses coming out of the chip. c) High extinction modulator is essential for pulse picking, otherwise the residual pulse train is amplified leading to a wide pedestal in the autocorrelation. d) Octave-spanning supercontinuum generated in ChG waveguides using the InP-based source as a seed. Abbreviations: TL - tunable laser, (HX)-MZ - (high-extinction) Mach-Zehnder modulator, MMI - multimode interferometer, EAM - electrooptic amplitude modulator, SA - saturable absorber, SOA - semiconductor optical amplifier, EDFA - Erbium-doped fiber amplifier, ISO - optical isolator, VC-PS voltage-controlled RF phase shifter, BT - bias tee, PID - proportional-integral-derivative controller, SMF - single-mode fiber. ................................................................. 17
1.4 a) The vision for a fully-integrated f-2f referencing chip and the performance of individual components. b) measured (red) octave spanning TM supercontinuum requiring only 26 pJ of pulse energy and the simulated spectrum (blue) together with the phase-matching condition (green) that predicts the position of dispersive waves, c) spectral splitters showing 30 dB extinction ratio at 2 µm, d) second-harmonic generation in thin-film PPLN.

1.5 a) Experimental setup for generating second harmonic from supercontinuum, in turn, generated in chalcogenide (ChG) waveguides. A Thulium-doped amplifier (TDFA) is inserted between the ChG chip and bulk periodically-poled lithium niobate (PPLN) to compensate for insertion losses. b) The measured spectra in the experiment: blue - supercontinuum generated from ChG, green - amplified supercontinuum, red - second harmonic. c) The same experiment performed with highly nonlinear fiber and thin-film lithium niobate.

2.1 (a) SEM of a waveguide facet (b) SEM of a waveguide (c) Microscope image of a fabricated ring resonator

2.2 A diagram for the notch filter with symbols used in the text.

2.3 Ring transmission measurement. (a) Multiple notches (b) Single notch with a fit to the theoretical equation.

2.4 Mode-locked laser

2.5 The autocorrelation and spectrum of the mode-locked laser before (red) and after (green) pulse compression in the custom EDFA.
2.6 Octave spanning supercontinuum generated from chalcogenide waveguides and the custom fiber mode-locked laser. .............................................. 37

2.7 (Red) Octave spanning supercontinuum generated using only 26 pJ of coupled power. (Blue) Simulated spectrum. (Green) Phase-matching condition for emission of dispersive waves that manifest themselves as peaks in the spectrum. ................................................................. 38

2.8 CW simulations of gain inside the TDFA in (a) and the experimental setup used for testing the PPLN in (b). (c) Spectra showing PPLN characterization. Supercontinuum generated in highly-nonlinear fiber (HNLF) in green, amplified portion of the spectrum in blue and the generated second harmonic in red. ................................................................. 40

2.9 CEO detection setup and a 7dB SNR signal in the insert. ......................... 42

3.1 (a) Excellent agreement between the matched asymptotics (MA) method and the numerical solution from the Newton-Raphson (NR) solver plotted for $\Delta = 12$, zero loss and $C_1 = 6$. (b) Solitons formed from Gaussian pulses, $\exp\left(-\left(\tau/\tau_p\right)^2\right)$, are the same as those from CW background provided that the loss and the detuning is the same ($\Delta = 4$), and the peak power of the pulse is equal to the CW background ($A_{in} = 2$). All solutions are stable, as confirmed using the split-step Fourier time evolution method. ......................... 47
3.2 (a) The path towards pulse compression (single soliton formation) closely follows that of the CW case. The plot shows the power in the cavity on a log scale and corresponding selected snapshots. Initially the pulses develop modulational instability (MI) for $\Delta = 0, t < 20$. After an abrupt increase in the detuning at $t = 20, \Delta = 2$, unstable breathing solitons appear that coalesce and break apart, at $\Delta = 4.16, t = 80$ they condense on a stable single soliton solution. Simulation performed for Gaussian input pulse, $2.0 \exp(- (\tau/10)^2)$.

(b) The same simulation parameters as in (a) are used except that 3rd order dispersion is added, $-0.12 \frac{\partial^3 A}{\partial \tau^3}$, this causes the soliton to emit a dispersive wave (Cherenkov radiation) that affects it's group velocity, the soliton drifts in the simulation in the moving frame of reference. For the single soliton state to be stable the input pulse has to also drift (the repetition of the input pulses has to be adjusted).

3.3 (a) Comparison of compressed pulses in the 100 ps ring cavity plotted together inside a 10 ps window. The Gaussian in all plots corresponds to width of $\tau_p = 1$ ps and MLL is a breathing mode-locked laser. The solitons formed inside the ring cavity are the same provided that the peak of the pulse is the same as the CW background. The oscillations present next to the solitons is the Chernekov radiation. (b) Octave spanning spectra of the pulses from (a) after coupling into the bus waveguide. Also shown is the case of supercontinuum generation in straight waveguides using Gaussian input pulses. For the supercontinuum to reach similar bandwidth as in the case of a cavity, the average power has to be 8 time higher.

4.1 NASA wire antenna optimized using evolutionary algorithms.
4.2 Top: The individuals of each generation are plotted on height vs width curve plane. The best individual in each generation is marked with a red cross. The algorithm is initially seeded with a random population that converges via evolution and mutation towards an optimal solution. Bottom: The desired dispersion (red) and the dispersion of the best individual (blue) from each generation. The desired curve corresponds to width 1500 nm and height 600 nm. GA solution is width of 1543 nm and height of 599 nm.

4.3 (a) Energy diagram for in-band pumping of Tm$^{+3}$ (b) Absorption and emission cross-sections of Nufern fiber SM-TSF-9/125.

4.4 (a) Simulation of small signal gain at 1990 nm for 1.1W, 1565 nm pump showing 24 dB of gain (b) Amplification of supercontinuum signal for second harmonic generation showing 20 dB of gain.

5.1 The error between the numerical and analytical eigenvalues plotted for Lagrange finite elements of different polynomial order. The convergence slopes for first, second, and third-order Lagrange elements are -0.77, -1.66 and -2.42, respectively.
5.2 (a) The eigenvalue of the fundamental longitudinal mode of a square chalcogenide waveguide embedded in thermal oxide calculated using the scalar acoustic Equation 5.6, the scalar elastic Equation 5.7 and the full-tensorial elastic Equation 5.3. The computation was performed on the same mesh for each waveguide width. The acoustic model is too simplistic for waveguides with tight confinement. However, the scalar elastic equation remains accurate; (b) The cross-section of the eigenmodes calculated for 0.5-µm-wide waveguides using the three models. The plot shows the Z-displacement normalized to unity.

5.3 A comparison of simulation time between the mode-solver in this work, and a fully-3D simulation in COMSOL\textsuperscript{TM}, confirming that the present solver is over 100 times faster. Additionally, the computational time increases faster with resolution for the 3D simulation than for the 2D SAFE method used in this work. The simulation was performed for chalcogenide waveguides embedded in oxide and backward scattering modes. In both cases, second-order Lagrange elements were used and 10 modes were calculated. COMSOL\textsuperscript{TM} was called using the LiveLink\textsuperscript{TM} Matlab interface for timing purposes. Resolution is understood as number of finite elements per facet. In the 3D simulation, Floquet boundary conditions were used and the size of elements in the z-direction was the same as in the xy plane.

5.4 (a) The fundamental $x$-shear mode for an SOI waveguide in air; (b) The fundamental $x$-shear mode for an SOI waveguide in air.
5.5 Comparison of bulk and boundary forces and resulting gain for forward and backward Brillouin scattering originating from the fundamental TE mode and acting on a silicon waveguide suspended in air. The coupling is computed for the fundamental $x$-shear mode from Figure 5.4a and different crystal orientations. Bulk electrostriction forces are in plotted in black, the boundary radiation pressure in blue and boundary electrostriction in red. In the case of back scattering in Figure (e) bulk electrostriction has a large imaginary component acting in the $z$ direction in addition to lateral components. The gain dependence on crystal orientation shows discontinuities due to mode merging as explained in detail in text. 78

5.6 Resonant elastic frequencies of an silicon waveguide, suspended in air, for various crystal orientations. For an arbitrary direction, the waveguide is not symmetric elastically, thus merging of the symmetric (S) and asymmetric modes (AS) is observed. 80

6.1 The dimensions of ASOP used in the SBS experiment 82

6.2 SBS experiment 83

6.3 Measured SBS response. 84

6.4 Insertion loss of ASOP waveguides with gratings. 84

7.1 The fundamental elastic mode and the fundamental TE mode at 2200 nm wavelength of the ASOP waveguides together with the $x$-component of the electrostriction force responsible for majority of the coupling between these modes. 86
7.2 Due to strong two photon absorption only the 2200 nm experiences measurable gain. ........................................ 87

7.3 The nonlinear and linear losses for the 1550 nm pump. For 0.5 W, the total losses correspond to 29 dB/cm, and at 125 mW the nonlinear losses are the same as the linear losses (1.7 dB/cm) ...................................... 87

7.4 Principle of RF filter design using two elastically coupled waveguides. . . . . 88

7.5 A proposed microwave filter design inside an characterization setup. Phase modulation from SBS is converted to intensity modulation in an on-chip interferometer in analogy to Mach-Zehnder modulators. . . . . . . . . . . . 89

7.6 A proposed microwave filter design inside an characterization setup. Phase modulation from SBS is converted to intensity modulation using a resonant microring structure. The microring provides additional improvement to sensitivity in a fashion similar to gain peaking in microring modulators. . . . . 89
**LIST OF TABLES**

1.1 On-chip CEO detection experiments .......................... 6

1.2 Passively mode-locked semiconductor lasers in comparison to Erbium-doped fiber oscillators. ........................................... 14

1.3 On-chip supercontinuum. The $n_2$ for Ge$_{23}$Sb$_7$S$_{70}$ and InGaP are inferred indirectly through measurement of $\gamma$ coefficient in waveguides. For SiN, AlN, and LiNbO$_3$ refer to table 1.1 with CEO detection experiments. ........... 20

1.4 Second harmonic generation in integrated waveguides. .................. 22

2.1 Two chemistries used for etching Chg. .......................... 31

5.1 Si Elastic Modes .................................................. 74
## LIST OF ACRONYMS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>APD</td>
<td>Avalanche photodiode</td>
</tr>
<tr>
<td>ASE</td>
<td>Amplified spontaneous emission</td>
</tr>
<tr>
<td>BCB</td>
<td>Benzocyclobutane</td>
</tr>
<tr>
<td>BOX</td>
<td>Burried oxide layer</td>
</tr>
<tr>
<td>CEO</td>
<td>Carrier envelope offset</td>
</tr>
<tr>
<td>ChG</td>
<td>Chalcogenide</td>
</tr>
<tr>
<td>COEO</td>
<td>Coupled optoelectronic oscillator</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous wave</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium doped fiber amplifier</td>
</tr>
<tr>
<td>ES</td>
<td>Electrostriction</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element method</td>
</tr>
<tr>
<td>FSR</td>
<td>Free spectral range</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full-width half-maximum</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic algorithm</td>
</tr>
</tbody>
</table>

xx
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>LL</td>
<td>Lugiato-Lefever equation</td>
</tr>
<tr>
<td>LN</td>
<td>Lithium niobate</td>
</tr>
<tr>
<td>LPCVD</td>
<td>Low pressure chemical vapor deposition</td>
</tr>
<tr>
<td>MLL</td>
<td>Mode-locked laser</td>
</tr>
<tr>
<td>MZ</td>
<td>Mach-Zehnder modulator</td>
</tr>
<tr>
<td>NLS</td>
<td>Nonlinear Schrödinger equation</td>
</tr>
<tr>
<td>NR</td>
<td>Newton-Raphson</td>
</tr>
<tr>
<td>PBS</td>
<td>Polarization beam splitter</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-integral-derivative controller</td>
</tr>
<tr>
<td>PPLN</td>
<td>Periodically-poled lithium niobate</td>
</tr>
<tr>
<td>OSA</td>
<td>Optical spectrum analyzer</td>
</tr>
<tr>
<td>RFSA</td>
<td>Radio frequency spectrum analyzer</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>RP</td>
<td>Radiation pressure</td>
</tr>
<tr>
<td>SAFE</td>
<td>Semianalytical finite element method</td>
</tr>
<tr>
<td>SBS</td>
<td>Stimulated Brillouin scattering</td>
</tr>
</tbody>
</table>
SEM Scanning electron microscope
SHG Second-harmonic generation
Si Silicon
SMF Single mode fiber
SNR Signal to noise ratio
SOI Silicon on insulator

TDFA Thulium doped fiber amplifier
TE Transverse electric
TL Tunable laser
TM Transverse magnetic

VCSEL Vertical cavity surface emitting laser
CHAPTER 1: TOWARDS INTEGRATED FREQUENCY COMBS

The contents of this chapter were submitted to Micromachines on May 14, 2019 as a review article under the MDPI open access license under the title: "Towards on-chip self-referenced supercontinuum sources", Marcin Malinowski, Ricardo Bustos-Ramirez, Jean-Etienne Tremblay, Guillermo F. Camacho-Gonzalez, Ming C. Wu, Peter J. Delfyett, and Sasan Fathpour

1.1 Abstract

Miniaturization of frequency-comb sources could open a host of potential applications in spectroscopy, biomedical monitoring, astronomy, microwave signal generation and distribution of precise time or frequency across networks. This review article places emphasis on an architecture with a semiconductor mode-locked laser at the heart of the system and subsequent supercontinuum generation and carrier-envelope offset detection and stabilization in nonlinear integrated optics.

1.2 Introduction

The field of integrated photonics aims at harnessing optical waves in submicron-scale devices and circuits, for applications such as transmitting information (communications) and gathering information about the environment (imaging, spectroscopy, etc.). The applications pertaining to the transmission of information include optical transceivers[2], interconnects for high-performance computing [3, 4], optical switches [5], and perhaps neural networks [6]. The sensing applications can be long-range, e.g., lidar [7], or short-range, e.g., absorption, Raman or florescence spectroscopy [8]. This includes spectroscopy of atomic vapors [9], which is essential in realizing a miniature atomic clock. Somewhere in between these two ranges is the quest to design an on-chip
frequency-stabilized comb source. The unprecedented frequency stability, coupled with the broad comb bandwidth, has had such an impact that two of its inventors, John L. Hall and Theodor W. Hänsch, are awarded the 2005 Nobel Prize in Physics for their "contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique".

This review paper summarizes efforts in developing an on-chip stabilized broadband supercontinuum source in the context of above sensing goals. The paper discusses in detail the applications, the physics of supercontinuum broadening and finally various integrated-photonic architectures and associated material choices. We concentrate specifically on supercontinuum from waveguides, because the attained spectrum is typically broader and flatter than the competing architecture based on microring resonators [10]. However, this comes at the cost of the need for on-chip narrow linewidth and high-power mode-locked laser (which is yet to be demonstrated), instead of continuous-wave (CW) pump sources needed for microrings. Irrespective of the specific implementation requirements of such a supercontinuum source, some of the challenges for developing such systems are common to the whole field of integrated photonics. For example, if silicon is chosen as the primary optical material, a.k.a., silicon photonics, there exist fundamental material limitations, such as two-photon absorption and the material’s indirect bandgap. The high propagation loss of the III/V compound semiconductor competitors - which can possess direct bandgaps, hence lasing - also renders them a less than ideal alternative. Just like in optical transceivers, the solution is ushered by heterogeneous integration of various materials for different optical functionalities [11, 12]. Material heterogeneity is therefore another common feature of the technologies reviewed in this paper.
Figure 1.1: Frequency combs find applications in (i) low-noise microwave signal generation and processing, (ii) gears in atomic clocks that measure the precise frequency of atomic transitions, (iii) broadband light sources in spectroscopy. They can also be viewed as (iv) a source of densely-spaced optical channels for optical communications and (v) source of high peak-power pulses in time-of-flight lidar.

1.3 Applications of miniature frequency combs

The development of mode-locked laser was crucial to the proliferation of frequency combs and has spawned a multitude of applications [13], which are schematically depicted in figure 1.1. Among them, spectroscopy is of prime importance. The first spectroscopic experiments concentrated on atoms from the first group of the periodic table. For instance, the Cesium D$_1$ line (895 nm) was measured, thus its hyperfine constant could be calculated to high precision [14]. Further improvements were made by using self-referenced frequency combs and counter-propagating beams to avoid Doppler broadening [15]. In the initial experiments, only a single line of the frequency comb was used. However, the main advantage of the frequency comb is to use all the available frequency teeth. The next technology leap came with the invention of dual-comb spectroscopy (DCS) [16],
where two locked frequency combs, with slightly detuned repetition rates, are used. In DCS, one of the combs is transmitted through the sample and then beat against the other comb producing a radio-frequency (RF) comb. The phase and amplitude information of the probing comb is mapped into the RF comb and thus the sample’s absorption spectrum can be recorded. DCS is superior - in terms of resolution, acquisition speed, accuracy and signal-to-noise ratio (SNR) - to other methods, such as high-resolution virtually-imaged phased array (VIPA) disperser [17]. Since it does not require a spectrometer, i.e., a grating or an interferometer, it is a perfect candidate for an on-chip source [18]. It should be stressed that for applications in organic chemistry, it is the mid-infrared (mid-IR) region lying approximately between 4000-400 cm\(^{-1}\) or 2.5-25 \(\mu\)m is the most spectroscopically interesting window, and is referred to as the fingerprint region. Within this wavelength range, the rich number of rotational and vibrational excitations is sufficient for identification of organic molecules. Should a miniature frequency comb spectrometer be realized in this regime, one could envision a multitude on biomedical and environmental monitoring applications. However, there are limited laser sources beyond 3\(\mu\)m, hence alternative light generation methods, e.g., difference-frequency generation and optical-parametric oscillation, are required [19]. Furthermore, mid-IR detectors require cryogenic cooling, in order to limit thermal noise, and even so perform worse than their Si or InGaAs counterparts. Consequently, on-chip stabilization experiments have been primarily performed in the near-IR wavelengths, as summarized in Table 1.1. In this region, it is still possible to detect the overtones of vibrational resonances of certain molecules, e.g., methane (\(\text{CH}_4\)) [20]. Additionally, the near-IR range is used in astronomical spectrograms, where frequency combs are used for calibration [21], in order to detect Doppler shifts as small as 1 cm s\(^{-1}\). Miniaturization, would enable extraterrestrial applications for frequency-comb sources.

Intimately intertwined with the topic of atomic spectroscopy is the subject of atomic clocks. Here, a narrow-linewidth laser is tuned to an atomic transition locked to one of the optical comb teeth. The frequency comb serves as the clockwork that maps the optical frequencies to microwave fre-
quences that can be counted by electronics. Atomic clocks form the core of international time standard disseminated globally [22]. Simultaneously, this makes them an equally capable frequency standard that would benefit from miniaturization. As the data rates and the number of data channels grow, it becomes increasingly important to synchronize the frequency among devices on the same network [23]. Miniature atomic vapors cells with vertical-cavity surface-emitting lasers (VCSELs), locked to the atomic transition, and Rubidium (Rb) vapor cells integrated with silicon waveguides [9], have already been demonstrated [23]. The next step is integration with an on-chip frequency comb source that would link the optical and microwave frequencies. Miniature atomic clocks would greatly improve the resilience of receivers for global-positioning system (GPS) against jamming [24].

Finally, there are potential applications that stem from the ability to separate individual comb lines and alter their phase and amplitude. In this manner, it is possible to synthesize arbitrary optical pulses [25] and characterize them via multiheterodyne beat [26]. Pulse shaping in the optical domain can also be used to realize programmable and tunable filters for microwave signals encoded on the optical carrier [25]. The individual comb lines can equally well-function as separate channels for coherent terabit-per-second communication [27].

1.4 Stabilization

Frequency comb spectra are composed of series of equally-spaced lines, hence the name. The frequency comb has two free parameters, namely, the repetition rate or the spacing between the comb lines, \( f_r \), and the carrier-envelope offset (CEO) \( f_0 \), which is a measure of the phase slippage between the carrier frequency and the peak amplitude of a pulse [13], or alternatively the offset of the comb lines, with respect to zero frequency. Thus, the position of the comb lines is given by the simple relation \( f_n = f_0 + nf_r \), where \( n \) is an integer as depicted in diagrams 1.2. To
take full advantage of the frequency comb as an optical metrology tool, stabilizing both of these parameters through detection and a feedback loop back to the oscillator is demanded. Stabilization of the repetition rate is easier, as GHz-bandwidth photodetectors are readily available. Some of the repetition stabilization schemes are discussed in the section 5. Stabilization of the CEO is more challenging, because in absence of an external reference, it is impossible to measure the optical frequency precisely. Thereby, the optical frequencies ought to be mapped into the microwave domain, so that they can be measured electronically. This is accomplished by frequency broadening of the original spectrum, frequency multiplication of one or two portions of the spectrum and subsequent measurement of the heterodyne beat between the two.

Table 1.1: On-chip CEO detection experiments

<table>
<thead>
<tr>
<th>$\chi^{(3)}$ material</th>
<th>$\text{Si}_3\text{N}_4$</th>
<th>$\text{Si}_3\text{N}_4$</th>
<th>$\text{Si}_3\text{N}_4$</th>
<th>AlN</th>
<th>LiNbO$_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_2$ [m$^2$W$^{-1}$]</td>
<td>$2.5 \times 10^{-19}$</td>
<td>$2.5 \times 10^{-19}$</td>
<td>$2.5 \times 10^{-19}$</td>
<td>$2.3 \times 10^{-19}$</td>
<td>$1.8 \times 10^{-19}$</td>
</tr>
<tr>
<td>span</td>
<td>600-1700+ nm</td>
<td>520-1700+ nm</td>
<td>600-1900 nm</td>
<td>500-4000 nm</td>
<td>400-2400 nm</td>
</tr>
<tr>
<td>pump wavelength</td>
<td>1510 nm</td>
<td>1550 nm</td>
<td>1055 nm</td>
<td>1550 nm</td>
<td>1506 nm</td>
</tr>
<tr>
<td>pulse duration</td>
<td>200 fs</td>
<td>80 fs</td>
<td>64 fs</td>
<td>80 fs</td>
<td>160 fs</td>
</tr>
<tr>
<td>repetition rate</td>
<td>80 MHz</td>
<td>100 MHz</td>
<td>1 GHz</td>
<td>100 MHz</td>
<td>80 MHz</td>
</tr>
<tr>
<td>total insertion loss</td>
<td>7 dB</td>
<td>4 dB</td>
<td>8 dB</td>
<td>8 dB</td>
<td>8.5 dB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\chi^{(2)}$ material</th>
<th>strained $\text{Si}_3\text{N}_4$</th>
<th>NA</th>
<th>PPLN</th>
<th>AlN</th>
<th>LiNbO$_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>referencing scheme</td>
<td>f-2f</td>
<td>f-3f</td>
<td>f-2f</td>
<td>f-2f</td>
<td>f-2f</td>
</tr>
<tr>
<td>SHG / THG</td>
<td>770 nm</td>
<td>420 nm</td>
<td>680 nm</td>
<td>780 nm</td>
<td>800 nm</td>
</tr>
<tr>
<td>variable delay</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>CEO SNR</td>
<td>27 dB</td>
<td>23 dB</td>
<td>40 dB</td>
<td>37 dB</td>
<td>30 dB</td>
</tr>
<tr>
<td>reference</td>
<td>[28]</td>
<td>[29]</td>
<td>[30]</td>
<td>[31]</td>
<td>[32]</td>
</tr>
</tbody>
</table>
Different frequency-stabilization methods are introduced in the following. Their common challenge is that, in general, frequency broadening is a non-trivial task, as the requirements for broad bandwidth and coherence have to be met simultaneously. This aspect of frequency broadening is discussed in more detail in the section 4.

As shown in diagram 1.2a, the most common scheme is the so-called $f - 2f$ self-referencing technique, where a tooth from the long-wavelength portion, $f_0 + nf_r$ is frequency-doubled to $2f_0 + 2nf_r$ and beat against a tooth an octave apart $f_0 + 2nf_r$. In practice, the frequency doubling has a few nanometers of bandwidth, so actually multiple comb lines around these frequencies are used.

Another method is $f - 3f$ referencing, which requires two octaves of bandwidth. Here, frequency-doubling is replaced with frequency-tripling, which means that the same third-order nonlinear material, i.e., with a strong $\chi^3$ optical susceptibility, can be simultaneously used for supercontinuum generation and frequency multiplication. In this case, the beat frequency is actually $2f_0$, as evident in figure 1.2b. But this attribution does not make a difference for the controllers used in the electronic-feedback loops, because it just affects the proportionality factor.

Finally, there are various fractional schemes, such as $2f - 3f$ referencing that require shorter bandwidth, in this case $2/3$ of an octave, but also frequency multiplication of both sides of the supercontinuum spectrum. The frequency-tripling is done in two stages, through second-harmonic- and sum-frequency-generations, making the whole process inefficient. The $2f - 3f$ referencing technique is more prevalent in the case of microring resonators with limited bandwidth [33]. In one report [33], two CW lasers are locked to opposite ends of the spectrum to boost the power for $2f - 3f$ referencing.

Several CEO detection experiments involving integrated optical waveguides are collected in table 1.1. Due to the simplicity of the approach, there is particular allure of using a single, straight
waveguide for both frequency-doubling and supercontinuum generation. Materials such as AlN and LiNbO$_3$ (LN) or even strained Si$_3$N$_4$, possessing both $\chi(3)$ and $\chi(2)$ nonlinear responses, can be considered. However, this comes at the price of increased power consumption for two reasons. First, efficient frequency-doubling requires phase matching. In large waveguides used for supercontinuum generation, the phase-matching typically occurs for higher-order modes, as in experiments on LiNbO$_3$ and AlN suggest (table 1.1). This results in the CEO signal being generated from beating two different spatial modes, which limits the signal strength. Secondly, the frequency components used in $f - 2f$ referencing are separated by an octave, which means that they have substantially different group velocities. Therefore, as they exit the waveguide they are separated in space (or equivalently the arrival times at the detector are different). Thus, in $f - 2f$ referencing experiments, it is common to split the spectral components and compensate from the time delay in an interferometer, as depicted in figure 1.2d. This feature is not easily available in a single waveguide with simultaneous $\chi(3)$ and $\chi(2)$ nonliterary.

Due to above-mentioned reasons, the experiment with the highest CEO SNR and lowest power consumption in table 1.1 employs the architecture from figure 1.2d, together with periodically-poled lithium niobate (PPLN) device to achieve quasi-phase-matching to the preferred fundamental mode. To date, this has only been done in free-space optics, but progress on efforts for on-chip integration are discussed in section 6.
Figure 1.2: Carrier envelope offset ($f_0$) detection schemes.

Strides towards miniaturization have been made that extend beyond CEO detection. A self-referenced frequency comb source with a fiber oscillator and silicon nitride waveguides, consuming only 5 W of electrical power, was demonstrated [34]. In this work, power-efficient repetition rate control is achieved by resistive thermal fiber heater. In another report [35], the CEO signal of a semiconductor disk laser, operating at 1.8 GHz, is stabilized via supercontinuum generation in photonic crystal fiber. The whole system consumed 6 W of optical power. Replacing the fiber with integrated sil-
icon nitride waveguides reduced the optical power requirement to 160 mW for a similar system, operating at 1.6 GHz [36]. However, the semiconductor disc lasers still require an external cavity with active feedback to one of the mirrors to limit amplitude and phase noise.

There also exist alternative architectures for self-referenced on-chip frequency combs. They utilize the spontaneous formation of solitons in microring resonators from CW background. This approach benefits from using a CW laser, instead of a mode-locked laser. An on-chip optical synthesizer has been realized based on this approach [37]. However, the spectrum of frequency combs generated in microrings follows the sech envelope function and has a limited bandwidth. Thus, from the spectroscopy viewpoint, photonic waveguides are preferable.

In the future, hybrid approaches could be possible. An example is synchronous pumping of microrings with a mode-locked laser, which should further reduce the system power consumption [38, 39].

1.5 Supercontinuum generation in integrated waveguides

Supercontinuum generation has been thoroughly studied in the context of optical fibers [40]. Here, we review the most important findings that are relevant to obtaining a broadband, coherent supercontinuum in waveguides required for $f - 2f$ referencing. The master and accurate equation describing the dynamics of fs-range pulses is

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A + \sum_{k=2}^{i+1} \frac{i^k}{k!} \beta_k \frac{\partial^k A}{\partial T^k} + i \gamma (1 + i \tau_{\text{shock}}) \frac{\partial}{\partial T} (A(z, t) \int_{-\infty}^{+\infty} R(T') |A(z, T - T'|^2 dT' + i \Gamma_R(z, T)), \tag{1.1}$$

where $A$ is the pulse amplitude in $W^{-1/2}$, $\beta_i$ are dispersion coefficients from the Taylor expansion around the center frequency, $\omega_0$, and $\gamma$ is the nonlinear coefficient. $\tau_{\text{shock}} \approx 1/\omega_0$ is the shock
timescale, although there are more complicated expressions that account for the dispersion of the
effective area [40]. \( R(T) \) is the Raman response function and various semi-empirical models are
developed for it [41]. Finally, \( \Gamma_R(z,T) \) is the stochastic noise term arising from spontaneous
Raman scattering.

The dynamics of supercontinuum generation from ultrashort (100 fs range) pulse in the anomalous
dispersion region, \( \beta_2 < 0 \), are dominated by soliton fission, as opposed to self-phase modulation
in the normal dispersion case. The latter involves amplification of quantum noise via four-wave
mixing (FWM) processes, which muddle the coherence of the supercontinuum. Hence, in the
context of stabilization we concentrate on the anomalous dispersion regime.

Regarding the dynamics of supercontinuum generation, it is necessary to introduce the soliton
number, \( N \), defined as \( N^2 = \gamma P_0 T_0^2 / |\beta_2| \), where \( P_0 \) and \( T_0 \) refer to the peak power and duration of
the input pulse in equation 1.1. The first ejected soliton - which is the shortest and most energetic
- has a temporal width of \( T_0/(2N - 1) \) and a peak power of \( (2N - 1)^2/N^2 P_0 \) [42]. All solitons
formed during the fission process are perturbed by the higher order dispersion terms and Raman
scattering, but the perturbation of the first soliton dominates the spectrum. Under phase-matching
conditions,

\[
D = \sum_{k \geq 2} \frac{\beta_k}{k!} = \frac{1}{2} (2N - 1)^2 \left| \frac{\beta_2}{T_0^2} \right|,
\]

the soliton can transfer energy to a linear dispersive wave [42]. To a good approximation, the
right-hand side of the equation can be equated to zero. The dispersive waves are also referred to
as Cherenkov radiation [43], due to similarities with radiation emitted by charged particles travel-
ing through a dielectric medium. This equation forms the basis of dispersion engineering. Since
semiconductor processing offers tight control of the waveguide geometry, it is possible to control
the shape of supercontinuum spectrum in integrated waveguides (such an attribution is difficult to
accomplish in largely axially symmetrical fibers). For example, it is possible to introduce a slot
into the waveguide structure, effectively flattening the dispersion profile and increasing the bandwidth to two octaves [44]. This design leads the phase-matching condition from equation 1.2 to be satisfied at four different wavelengths [45] with relatively flat spectrum, that would be advantageous in spectroscopic applications. Dispersion engineering is especially important in microring resonators [46], where the envelope follows the sech-shape of solitons. A simulated spectrum of supercontinuum generated in chalcogenide waveguides together with experimental data from [47] is appended to figure 1.4b. The phase-matching condition, shown as a green curve, from equation 1.2 predicts the position of dispersive waves to great accuracy.

The second nonlinear effect that perturbs the solitons is Raman scattering. In Raman scattering, some of the energy of impeding photon is transferred to electronic oscillations of molecules. This means that the soliton spectrum is continuously shifted toward red wavelengths. The speed of the frequency drift scales as $d\omega_R/dz \sim |\beta_2|/T_0^4$ [48]. Importantly, spontaneous Raman scattering is an additional source of noise. It has been shown that this noise term leads to the degradation of coherence in the long wavelength portion of the supercontinuum, hence materials with weak Raman response are preferred [30].

Apart from octave-spanning or wider bandwidth, it is necessary to ensure that the generated supercontinuum is coherent in order to observe the CEO beat. Coherence is defined as [40]

$$\left| g_{12}^{(1)} (\lambda, t_1 - t_2) \right| = \left| \frac{\left< A_1^* (\lambda, t_1) A_2 (\lambda, t_2) \right>}{\sqrt{\left< |A_1 (\lambda, t_1)|^2 \right> \left< |A_2 (\lambda, t_2)|^2 \right>}} \right|,$$

where $A_1$ and $A_2$ refer to different pulse amplitudes in the pulse train separated by $t_1 - t_2$.

Coherence requires sub-100-fs pulses [40] and the highest degree of coherence is usually observed near the soliton fission point. Also, there is a number of noise sources that have adverse effect on
coherence. As mentioned beforehand, strong Raman effect also leads to undesired additional non-coherent photons generated through spontaneous Raman scattering. Additionally there is quantum noise (shot noise), which is usually dominant and cannot be eradicated [49]. Furthermore, there are additional sources of noise that come from the oscillator itself, such as phase (optical linewidth), timing jitter (RF linewidth), amplitude noise and technical noise from environmental changes[50]. As the optical-fiber-based oscillators usually do not produce sufficient power for CEO detection experiments, it is necessary to amplify their output, but this leads to additional amplified spontaneous emission (ASE) noise. It has been shown that ASE noise leads to variations in the group velocity of solitons and thus timing jitter [51].

A challenge for supercontinuum generation is that, as a nonlinear process, the above noise sources (ASE, Raman and shot noise) are not additive but are actually amplified through FWM processes [52]. Consequently, the FWM gain grows exponentially with pulse energy, whereas the supercontinuum bandwidth increases only linearly [53]. In other words, the minimal pulse energy that produces sufficient bandwidth would also be the point of maximum coherence.

1.6 Requirements for semiconductor lasers

To date, all CEO detection and stabilization experiments using integrated waveguides are performed with either fiber lasers [29], optical parametric oscillators (OPOs) [28] or pumped vertical external-cavity surface-emitting lasers (VECSEL) [54]. Just like mode-locked fiber lasers were crucial to the development of stabilized frequency combs [14], a semiconductor laser is necessary for on-chip counterparts.

Thus, it would be prudent to compare current state-of-the art semiconductor mode-locked lasers (MLL) with these prior alternatives. As noted in reference [50], the overall frequency comb per-
formance is much more sensitive to the noise inside the oscillator than the noise added through subsequent amplification. The reason is that the noise inside the cavity (ASE or length variations with temperature) translates into frequency shifts or broader linewidth, whereas the same noise sources outside the cavity would increase the noise floor on the detector that is used for CEO detection.

The comparison presented here is by no means a thorough review of semiconductor MLLs and the reader is directed to references [55, 56, 57]. Table 1.2 collects performance of representative lasers. The subsequent discussion compares them against Erbium-doped fiber oscillators used in frequency comb experiments [58, 34, 59, 60].

Table 1.2: Passively mode-locked semiconductor lasers in comparison to Erbium-doped fiber oscillators.

<table>
<thead>
<tr>
<th>Material</th>
<th>III-V/Si</th>
<th>III-V/Si</th>
<th>III-V/Si</th>
<th>InP</th>
<th>InP</th>
<th>InP</th>
<th>Er fiber (+EDFA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repetition rate</td>
<td>19 GHz</td>
<td>1 GHz</td>
<td>20 GHz</td>
<td>30 GHz</td>
<td>2.5 GHz</td>
<td>1 GHz</td>
<td>50-200 MHz</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>1.83 ps</td>
<td>15 ps</td>
<td>900 fs</td>
<td>900 fs</td>
<td>9.8 ps</td>
<td>70 ps</td>
<td>&lt;300 fs (&lt;100 fs)</td>
</tr>
<tr>
<td>Opt. bandwidth</td>
<td>-</td>
<td>12 nm</td>
<td>3 nm</td>
<td>15 nm</td>
<td>3 nm</td>
<td>5 nm</td>
<td>(~50 nm)</td>
</tr>
<tr>
<td>Band. threshold</td>
<td>-</td>
<td>-10 dB</td>
<td>-3 dB</td>
<td>-10 dB</td>
<td>-3 dB</td>
<td>-10 dB</td>
<td>-3 dB</td>
</tr>
<tr>
<td>Power</td>
<td>9 mW</td>
<td>-</td>
<td>1.8 mW</td>
<td>0.25 mW</td>
<td>80 uW</td>
<td>0.59 mW</td>
<td>~5 mW</td>
</tr>
<tr>
<td>3dB Optical lw</td>
<td>-</td>
<td>400 kHz</td>
<td>-</td>
<td>29 MHz</td>
<td>-</td>
<td>80 MHz</td>
<td>10s kHz</td>
</tr>
<tr>
<td>RF lw</td>
<td>6 kHz</td>
<td>0.9 kHz</td>
<td>1.1 kHz</td>
<td>500 kHz</td>
<td>6 kHz/61 kHz</td>
<td>1 MHz</td>
<td>-</td>
</tr>
<tr>
<td>RF lw threshold</td>
<td>-3 dB</td>
<td>-10 dB</td>
<td>-3 dB</td>
<td>-20 dB</td>
<td>-3 dB/-20 dB</td>
<td>-20 dB</td>
<td>-</td>
</tr>
<tr>
<td>Timing jitter</td>
<td>1.2 ps</td>
<td>-</td>
<td>-</td>
<td>4.5 ps</td>
<td>-</td>
<td>4.16 ps</td>
<td>&lt;2 fs</td>
</tr>
<tr>
<td>Int. range</td>
<td>0.1 MHz</td>
<td>-</td>
<td>-</td>
<td>100 Hz</td>
<td>10 kHz</td>
<td>-</td>
<td>20 kHz</td>
</tr>
<tr>
<td></td>
<td>100 MHz</td>
<td>-</td>
<td>-</td>
<td>30 MHz</td>
<td>10 MHz</td>
<td>80 MHz</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Reference</td>
<td>[61]</td>
<td>[57]</td>
<td>[55]</td>
<td>[62]</td>
<td>[63]</td>
<td>[64]</td>
<td>[58, 34, 59, 60]</td>
</tr>
</tbody>
</table>
With regards to the power consumption metric, nonlinear integrated waveguides typically require lower pulse energies than those in photonic-crystal fibers for supercontinuum generation. However, semiconductor-based MLL cavities are shorter and thus the repetition rates are higher than those in fiber cavities. If we take the most optimistic repetition rate value of 1 GHz from table 1.2, and the best InGaP waveguides from 1.4 (which require only 2 pJ pulses, and also assume no coupling loss), the average power requirement turns out to be only 2 mW. This power is perfectly reasonable, albeit the value can quickly grow in practice. For instance, a 10-GHz InP cavity and the chalcogenide waveguides discussed later on would consume 260 mW of optical power.

The next metric discussed is the pulse width. From the fiber-based comb literature [40], it is generally assumed that coherent supercontinuum generation requires 100 fs pulses, while the best mode-locked lasers from table 1.2 produce 900 fs pulses. However, this is not prohibitive, as it has been shown that some of the pulse compression task can be off-loaded to nonlinear waveguides. It is, for example, shown via simulations that 1 ps pulses can be compressed to 41 fs in 39-cm-long silicon nitride waveguides with ultralow propagation loss of 4.2 dB/m [65]. Similar ideas are proposed for optical fibers [66].

The subtlety lies in the fact that these two requirements are not independent. As seen in table 1.2, lasers with long cavities (low repetition rates) tend to produce pulses that span tens of picoseconds. This is because, in general, longer cavities lead to larger net accumulated dispersion, which leads to wider solitons [58], i.e., \( \tau_s = 2|\beta_2|/\gamma P_0 \) for \( A(T) = \sqrt{P_0/cosh(T/\tau_s)} \) in the basic form of equation 1.1. In this context, the path forward would probably involve further work on 1 GHz cavities with dispersion engineering or addition of gain flattering filters as in [62], in order to broaden the spectrum and reduce pulse width to picosecond level which is sufficient for further compression in nonlinear waveguides. The issues of quick gain saturation could be addressed via the breathing mode architecture [67], where the pulse is stretched before the gain section and compressed afterwards.
In the elastic-tape model of frequency combs [58, 50], there exists a fixed point in the frequency spectrum, \( f_r \), around which the comb teeth breath. In other words, the linewidth of individual comb teeth increases as we move away from \( f_r \) [50]. In the case of active/hybrid MLLs, the fixed point is located near the carrier frequency, while in passive MLLs it is close to the zero frequency [56]. In CEO detection experiments, the comb teeth that are beat against one another (after frequency multiplication) are on the extremes of the spectrum. Hence, the linewidth of the CEO is directly proportional to the linewidth of these comb teeth. Reducing the linewidth of the CEO improves its SNR and typically a value of 25 dB is required by the locking electronics for CEO stabilization. For comparison, the free-running CEO linewidth of Erbium-doped fiber lasers is in the range of 10-1000 kHz [58]. Thus, to compare the performance of MLL in the context of frequency comb generation, several key factors should ideally be known. They include the fixed point and its optical linewidth, as well as the spectral power density of RF phase noise, which dictates the RF linewidth and thus the magnitude of the breathing of the comb lines. However, the fixed point is rarely measured, so the optical linewidth near the carrier frequency is used as an alternative metric.

A clear distinction can be made in table 1.2, between the heterogeneously integrated III-V/Si MLL and monolithic InP lasers. The former have narrower optical linewidth (e.g., 400 kHz [57]) than the latter (e.g., tens of MHz [64, 62]). This is because the silicon cavities in III-V/Si have longer cavity photon lifetime than the monolithic, high-loss InP counterparts and thus wider optical linewidth through the Schawlow-Townes limit [68]. The narrowest optical linewidth of 400 kHz is still higher than typical values of tens of kHz in Erbium-doped fiber cavities, but it can be further improved with injection-locking techniques mentioned below.

The timing jitter of passive semiconductor MLLs is on the order of picoseconds, as seen in table 1.2, in contrast to the femtosecond timing jitter of passive fiber cavities. However, the situation is not as dire as the numbers might imply. The 1.8 GHz VECELS used in CEO stabilization experiments [54], which are closer in repetition rate to semiconductor MLLs, have an integrated
timing jitter of 60 fs (1 Hz-100 MHz) after stabilization to an external synthesizer. Therefore, in semiconductor MLLs some feedback system equalizing the repetition rate is necessary before the CEO signal can be detected.

Figure 1.3: a) Coupled-optoelectronic oscillator with InP MLL using multitone injection locking. The output pulses are pulse-picked, compressed and amplified to generate supercontinuum in chalcogenide (ChG) waveguides. The experimental data comes from [1]. b) Autocorrelation of pulses coming out of the chip. c) High extinction modulator is essential for pulse picking, otherwise the residual pulse train is amplified leading to a wide pedestal in the autocorrelation. d) Octave-spanning supercontinuum generated in ChG waveguides using the InP-based source as a seed. Abbreviations: TL - tunable laser, (HX)-MZ - (high-extinction) Mach-Zehnder modulator, MMI - multimode interferometer, EAM - electrooptic amplitude modulator, SA - saturable absorber, SOA - semiconductor optical amplifier, EDFA - Erbium-doped fiber amplifier, ISO - optical isolator, VC-PS voltage-controlled RF phase shifter, BT - bias tee, PID - proportional-integral-derivative controller, SMF - single-mode fiber.
To date, there is no single integrated passively mode-locked laser that can compete with the performance of fiber cavities simultaneously on all metrics. However, there are various methods to augment an oscillator to improve its performance. For example, hybrid mode-locking of cavity is shown to reduce the 10-dB RF linewidth from 0.9 kHz to 1 Hz [57]. In this case, hybrid mode-locking was accomplished by supplying an RF signal to the saturable absorber. We also note that the benefit of hybrid mode-locking is that it provides an access point for repetition rate control in a feedback loop in a fully self-referenced frequency comb source.

Another possibility is optical feedback, with on-chip external cavity [61]. However, the performance of the feedback is proportional to the cavity length, as it increases the memory of the system. In one report [61], the on-chip external cavity is only twice as long as the oscillator cavity and the 3-dB RF linewidth is 6 kHz. In contrast, in a report based on a much longer (22 m) fiber-based external cavity [69], the RF linewidth is only 192 Hz. However, fabricating equally long waveguides is challenging, due to higher propagation losses of integrated photonics and limited chip space.

Another scheme is to use multitone injection locking [70]. Multitone injection locking is an extension of the injection locking technique. It reduces the optical linewidth through the use of a narrow linewidth CW seed that is injected into the cavity [71]. The benefit of multitone injection locking is that it simultaneously reduces optical linewidth and also provides a means of controlling the repetition rate by varying the spacing of the tones. We also note that injection locking provides an alternative to the standard way of controlling $f_0$, via tuning the wavelength of the seed instead of modulation of the current injected into the gain section of the MLL [36].

An example of such an approach is shown in figure 1.3a. The cavity used in the experiment is a 10-GHz InP colliding pulse MLL [72]. Here, the multitone injection locking is part of a coupled opto-electronic oscillator loop (COEO). That is, the repetition rate of the MLL is detected on the
electroabsorption modulator (EAM), amplified and fed back into the Mach-Zehnder modulator, generating sidebands on the injection seed and forming a feedback loop in the system. The resonance condition is ensured by adding a phase-shifter in the loop. Another way to look at COEO is to notice that the MLL acts as a selective filter that amplifies RF modes of the cavity that overlap with the modes of the MLL. In this manner, the performance of the InP MLL is greatly improved. The free-running optical linewidth is reduced by a factor of 6000 to 100 kHz, limited by the optical linewidth of the injection seed, and the COEO loop reduces the RF linewidth by a factor of 70, yielding a 3-dB RF linewidth of 400 Hz [73]. The RF phase noise is further reduced by locking the COEO loop to an external reference, resulting in integrated timing jitter of 500 fs at 1 kHz. Unfortunately, again, the performance of COEO loop is proportional to the length of the optical delay. In this case, the optical delay line is 100 m long.

Such augmented cavity acts as a seed for supercontinuum generation in chalcogenide waveguides that are discussed in detail in section 6. The cavity by itself produces 2.5 ps pulses (see figure 1.3b), that are further amplified and compressed to 111 fs. We note that because the carrier dynamics in an EDFA is on the order of $\mu$s, every 500th pulse has to be picked by a high-extinction modulator. The high extinction modulator is essential, otherwise the residual 10 GHz pulse train is amplified, leading to a large pedestal in the autocorrelation trace as in figure 1.3c.

Finally, an octave-spanning supercontinuum is achieved, as presented in figure 1.3d [1]. It is anticipated that the pulse picking and multiple stages of amplification can be integrated on a single InP-based monolithic superchip.
1.7 Materials for nonlinear processes

Table 1.3: On-chip supercontinuum. The $n_2$ for Ge$_{23}$Sb$_7$S$_{70}$ and InGaP are inferred indirectly through measurement of $\gamma$ coefficient in waveguides. For SiN, AlN, and LiNbO$_3$ refer to table 1.1 with CEO detection experiments.

<table>
<thead>
<tr>
<th>$\chi^{(3)}$ material</th>
<th>Ge$_{23}$Sb$<em>7$S$</em>{70}$</th>
<th>As$_2$S$_3$</th>
<th>InGaP</th>
<th>Si</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_2$ [m$^2$W$^{-1}$]</td>
<td>$3.7 \times 10^{-18}$</td>
<td>$3.8 \times 10^{-18}$</td>
<td>$2.3 \times 10^{-17}$</td>
<td>$4.5 \times 10^{-18}$</td>
</tr>
<tr>
<td>span</td>
<td>1030-2080 nm</td>
<td>1200-1700+ nm</td>
<td>1000-2100 nm</td>
<td>1150-2400+ nm</td>
</tr>
<tr>
<td>pump wavelength</td>
<td>1550 nm</td>
<td>1550 nm</td>
<td>1550 nm</td>
<td>1900 nm</td>
</tr>
<tr>
<td>pulse duration</td>
<td>240 fs</td>
<td>610 fs</td>
<td>170 fs</td>
<td>50 fs</td>
</tr>
<tr>
<td>repetition rate</td>
<td>25 MHz</td>
<td>10 MHz</td>
<td>82 MHz</td>
<td>200 MHz</td>
</tr>
<tr>
<td>propagation loss</td>
<td>0.5 dB/cm</td>
<td>0.6 dB/cm</td>
<td>12 dB/cm</td>
<td>1.5 dB/cm</td>
</tr>
<tr>
<td>references</td>
<td>[74]</td>
<td>[75]</td>
<td>[76]</td>
<td>[77]</td>
</tr>
</tbody>
</table>

As mentioned in the introduction, despite significant success of silicon photonics, some of its optical properties have certain limitations. The material has a large third-order nonlinearity, $n_2 = (4.5 \pm 1.5) \times 10^{-18}$ m$^2$ W$^{-1}$ at 1.55 $\mu$m [78], but researchers have been unable to achieve an octave-spanning supercontinuum pumped at 1.55 $\mu$m. As noted in [79], where 3/10 of an octave was demonstrated with 100 fJ pulses at 1310 nm, the two-photon and associated free-carrier absorptions clamp the maximum power inside the waveguides and hence limit the spectral broadening. Nonetheless, the pump does not have to be completely outside the two-photon absorption region supercontinuum to achieve a full octave-span as in report [77]) where 1900 nm laser was used. The experimental details are provided in table 1.3.
However, once appropriate on-chip sources become available, silicon is an excellent material for supercontinuum generation in the mid-IR. An octave span is already demonstrated with only 5 pJ and 300 fs pulses at 2.5 $\mu$m wavelength [80]. The bandwidth could be further improved by removing the buried oxide layer, as in suspended-membrane air-clad silicon [81], silicon-germanium [82], or silicon-on-sapphire waveguides [83].

However, in the nearest future a near-IR frequency comb is the most viable path, and therefore the $\chi(3)$ limitations of silicon have to be addressed by heterogeneous integration with other materials. Additionally, silicon has a centrosymmetric lattice structure, hence does not possess an intrinsic second-order nonlinear optical susceptibility, $\chi(2)$, needed for frequency doubling.

When comparing the waveguides used for CEO detection supercontinuum generation, i.e., experiments from tables 1.1 and 1.3, respectively, it is evident that materials with a strong $\chi(3)$ response are preferred, as it can lead to lower pulse energies required for octave-spanning supercontinuum. This is in spite of the fact that the Kerr nonlinearity, $n_2$, scales as $1/E_g^4$ with bandgap, $E_g$, and is inherently tied to the two photon-absorption via Kramers-Kroning relationship [84]. This means that materials with high $n_2$ possess a bandgap that is relatively close to the pump. For example, the transparency range of Si$_3$N$_4$ is 0.4-4.6 $\mu$m, whereas for silicon the transmission window is 1.2-8 $\mu$m. Due to the proximity of the bandgap, the high $n_2$ materials are likely to suffer from two-photon absorption. Nevertheless, within the tables, the best performing material is InGaP, which has the highest nonlinearity, despite having the highest propagation losses of 12 dB/cm [76]. Octave span is achieved with only 2 pJ of pulse energy in short (2 mm long) waveguides, which is why the high loss is not prohibitive.
Table 1.4: Second harmonic generation in integrated waveguides.

<table>
<thead>
<tr>
<th></th>
<th>LiNbO$_3$</th>
<th>GaAs</th>
<th>AlN</th>
<th>GaN</th>
<th>strained Si$_3$N$_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\left[\frac{1}{2}\chi^{(2)}\right]$</td>
<td>30 pmV$^{-1}$</td>
<td>119 pmV$^{-1}$</td>
<td>1 pmV$^{-1}$</td>
<td>8 pmV$^{-1}$</td>
<td>eff. 0.02 pmV$^{-1}$</td>
</tr>
<tr>
<td>Efficiency</td>
<td>17 %W$^{-1}$</td>
<td>255% W$^{-1}$</td>
<td>2300% W$^{-1}$</td>
<td>0.015% W$^{-1}$</td>
<td>-</td>
</tr>
<tr>
<td>SHG mode order</td>
<td>fundamental</td>
<td>fundamental</td>
<td>5th</td>
<td>6th</td>
<td>6th</td>
</tr>
<tr>
<td>Architecture</td>
<td>waveguide</td>
<td>waveguide</td>
<td>microring</td>
<td>microring</td>
<td>microring</td>
</tr>
<tr>
<td>Reference</td>
<td>[85]</td>
<td>[86]</td>
<td>[87]</td>
<td>[88]</td>
<td>[89]</td>
</tr>
</tbody>
</table>

The second functionality, required for the $f - 2f$ referencing scheme, is efficient SHG. For a thorough review of SHG in heterogeneous integrated photonics, the reader is directed to [90] and for a discussion of appropriate figures of merit to [85]. Table 1.4 collects the most important demonstrations in integrated photonics. First, we note the the efficiency of SHG can be greatly improved through the power enhancement of resonant cavities such as microrings. Consequently, the highest efficiency from table 1.4, as expressed in %W$^{-1}$, is for low-loss AlN microrings, when compared to LN and GaAs waveguides, despite the AlN’s smaller $\chi^{(2)}$ nonlinearity. However, the bandwidth of these resonant structures is limited by their quality factor, $Q$, and thus they have limited applicability to the $f - 2f$ referencing schemes, because for SHG to occur one of the comb lines would have to be tuned to the microring resonance, and tuning would amount to having an already stabilized frequency comb source. Secondly, it is advantageous to use all comb lines that fall within the phase-matching bandwidth (usually couple of nanometers) for highest CEO SNR. Thus, in terms of architecture, straight waveguides may be a better choice.

Efficient SHG requires phase matching between the pump and the signal. Since the two are separated by an octave, in terms of frequency, this leads to the phase-matching condition being satisfied
for a higher-order mode at the signal wavelength as for Si₃N₄, GaN and AlN cases in table 1.4. This is suboptimal, as it compromises the mode-overlap between the pump and signal and thus overall conversion efficiency [90], as well as the mode-overlap between the signal short-wavelength part of supercontinuum (which is usually in the fundamental mode).

As shown in table 1.4, record efficiencies of 13,000% W⁻¹cm⁻² have been demonstrated in thin-film GaAs. [86]. This is possible due to GaAs’s highest $\chi^{(2)}$ response among the materials in table 1.4 and a large refractive index contrast leading to a small mode area. However, this has been achieved by modal-phase matching between fundamental modes of different polarizations, i.e., the pump being in the transverse-electric (TE) and the signal in the transverse-magnetic (TM mode). This attribution has potential hurdles in $f - 2f$ referencing scheme polarization rotators must be included, because the generated SHG would be in an orthogonal polarization state to the short-end of supercontinuum.

Finally, there is the thin-film equivalent of the commercially successful PPLN technology, where the phase matching is achieved through periodic-poling of the ferroelectric LN crystals [91, 92, 90]. Here, efficiencies of up to 4600 % W⁻¹cm⁻² have been recently demonstrated for 1550 nm pump [85] and functionality for 2 µm pump has also been shown [93, 94]. Similar to the above GaAs report, operation in the fundamental mode at both pump and signal wavelengths is feasible. An advantage of the above PPLN devices is that the pump and signal can have the same TE polarization modes, hence polarization rotation is not needed.
Figure 1.4: a) The vision for a fully-integrated f-2f referencing chip and the performance of individual components. b) measured (red) octave spanning TM supercontinuum requiring only 26 pJ of pulse energy and the simulated spectrum (blue) together with the phase-matching condition (green) that predicts the position of dispersive waves, c) spectral splitters showing 30 dB extinction ratio at 2 µm, d) second-harmonic generation in thin-film PPLN.

To summarize, the most optimal on-chip CEO detection scheme, for sufficient CEO signal with the lowest pulse energy, would incorporate a) strong $\chi^{(3)}$ material, b) integrated spectral splitters and delay lines, and c) a strong $\chi^{(2)}$ material with a device geometry leading to SHG generated in the fundamental mode and same polarization as the pump.

A sensible approach to this task is the integration of chalcogenide glass, e.g., Ge$_{23}$Sb$_7$S$_{70}$, with thin-film LN. As discussed earlier, LN is one of the prime contenders for efficient SHG and ChG was chosen as the $\chi^{(3)}$ material, because it is a glass that can be easily deposited on LN, without heating the substrate that would damage the thin-film LN.
Another material alternative, with strong $\chi(3)$ nonliterary, is silicon nitride. Low-loss material is typically achieved by low-pressure chemical-vapor deposition (LPCVD), which requires deposition temperature of 700°C. Hence, in this case, it is preferred to bond LN onto patterned silicon nitride layer to avoid thermal damage to the LN thin films [95].

The fabrication flow for integration of ChG and LN has already been demonstrated together with efficient mode-conversion from the ChG layer and rib-loaded LN with only 1.5 dB loss [96]. The strongest $\chi(2)$ response of LN is along the $z$ axis. Since the demonstrated thin-film PPLN devices utilize $y$-cut crystals, so the electrodes could be placed in the plane of the wafer, the pump field for the SHG process has to be orientated along the $z$-axis. This in turn requires the geometry of the ChG waveguide to support broadband TE supercontinuum generation, which is shown to be feasible [97].

The grand vision of the final device to be demonstrated is depicted in figure 1.4, together with the measured performance of discrete components. Figure 1.4b shows an octave-spanning transverse-magnetic (TM) supercontinuum generated with 240-fs-wide pulses, carrying 26 pJ of energy [74]. Figure 1.4c shows the performance of spectral splitters, as measured using a fiber-based supercontinuum source. Spectral separation is achieved through intermediate coupling to a higher-order mode (TE$_1$), which permits definition of the whole structure in a single lithography step and avoids tiny gaps between waveguides or sharp terminations [98].

As mentioned, second harmonic was demonstrated on thin-film PPLN, reviews of which can be found elsewhere [90, 85]. The first PPLN device on silicon substrates showed only 3 dB difference between the signal and the pump [91] and the recorded spectrum is shown in figure 1.4d. We note that this early experiment was performed using a pulsed source and therefore there is a significant contribution from sum-frequency signal generation in the signal. Highly efficient devices, under CW pumping, were later reported [85].
Lastly, figure 1.5 shows an experiment used to detect second-harmonic generated from supercontinuum from integrated waveguides. The long end of the supercontinuum is amplified using a custom-made Thulium-doped fiber amplifier (TDFA). The pump wavelength at 1984 nm is amplified by 20 dB to compensate for the high, 12 dB, coupling losses of the ChG chip. This is sufficient to observe a SHG signal at 992 nm at -71 dBm, that falls within the short end of the supercontinuum [99]. A similar experiment with a highly nonlinear fiber and thin film PPLN was also performed and is shown in figure 1.5c. It is anticipated that integration would reduce the coupling losses rendering the TDFA unnecessary.

Figure 1.5: a) Experimental setup for generating second harmonic from supercontinuum, in turn, generated in chalcogenide (ChG) waveguides. A Thulium-doped amplifier (TDFA) is inserted between the ChG chip and bulk periodically-poled lithium niobate (PPLN) to compensate for insertion losses. b) The measured spectra in the experiment: blue - supercontinuum generated from ChG, green - amplified supercontinuum, red - second harmonic. c) The same experiment performed with highly nonlinear fiber and thin-film lithium niobate.
Great progress has been made towards realizing on-chip frequency-stabilized supercontinuum-based comb sources. The integrated nonlinear components outperform the bulk counterparts, due to their smaller effective mode areas that increase the strength of interaction. The $\chi^{(3)}$ waveguides require less power than optical fibers, in order to produce an octave-spanning supercontinuum, and in some cases the supercontinuum extends far enough to even enable $f - 3f$ referencing. Some materials, like AlN, exhibit both $\chi^{(3)}$ and $\chi^{(2)}$ responses, allowing the detection of the CEO signal straight out of the waveguides. Record-high second-harmonic generation efficiencies have been demonstrated in thin-film PPLN and GaAs. The missing component is a semiconductor mode-locked laser that would serve as the heart of an on-chip frequency comb source. In this case, heterogeneous integration of III-V gain media with long low-loss cavities, and possibly further stages of amplification and pulse compression to achieve low noise, sub-ps, 1-GHz oscillators seems to be the path forward. Subpicosecond pulses would be sufficient, provided that the nonlinear waveguides could shoulder compression to 100 fs pulses. Injection locking and hybrid-mode locking could be used to reduce the optical and RF linewidth of passive cavities. In the nearest future, an stabilized frequency combs in the near-IR is the next milestone, with potential applications in microwave synthesis, miniature atomic clocks, astronomy and lidar. In the distant future, the technology could be extended to encompass the mid-IR range, where spectroscopic identification of organic molecules is possible. This is where a flat-, broadband- and stabilized-supercontinuum source would shine and open plethora of possibilities for inexpensive biomedical diagnostics and environmental monitoring.
1.9 Acknowledgements

Large portion work presented here was carried out under the DARPA DODOS program. The views, opinions, and/or findings expressed are those of the authors and should not be interpreted as representing the official views or policies of the Department of Defense or the U.S. Government.
CHAPTER 2: EXPERIMENTAL METHODS

2.1 Chalcogenide processing

Some excerpts of this section were published in "Low-loss, submicron chalcogenide integrated photonics with chlorine plasma etching", Jeff Chiles, Marcin Malinowski, Ashutosh Rao, Spencer Novak, Kathleen Richardson, and Sasan Fathpour, Appl. Phys. Lett. 106, 111110 (2015);

There are three main techniques which have been employed for patterning submicron ChG waveguides, namely, liftoff patterning, nanoimprinting, and dry etching. Liftoff patterning has only demonstrated low effective propagation losses for microdisk modes (which are different from that of conventional waveguides due to the smaller interaction with sidewalls) and for larger, shallow ridge waveguides. Nanoimprinting and dry etching have produced the lowest losses for submicron optical waveguides in the range of 1.6–2 dB/cm at 1550 nm [100, 101]. Although submicron features can be obtained at a low cost using nanoimprinting, the intimate physical contact required between the stamp and substrate can be hindered by particle contamination or other surface irregularities, limiting device yield.

Dry etching can produce features with steep sidewall angles, allowing very small gaps and grating periods to be achieved. In addition, it places no restrictions on the distribution or height of features that can be produced, making it suitable for a variety of applications. Although low propagation losses around 0.2 from 0.05 to 0.20 dB/cm have been achieved for wider ridge waveguides with widths of 3-5 μm, [102, 103, 104] there is still substantial opportunity for loss reduction in submicron dry-etched chalcogenide waveguides.

The waveguides described here were obtained by a novel dry etching method, featuring com-
pact waveguide dimensions and excellent device yield. Previously, studies of dry-etched ChG waveguides have focused mostly on fluorine-based etching recipes based on SF$_6$, CHF$_3$, or CF$_4$. [104, 105] However, no published work has explored chlorine-based plasma etching of Ge-Sb-S/Se glasses for waveguiding applications. Chlorine (Cl$_2$) plasma is highly reactive and readily forms volatile compounds with germanium and antimony, making it a strong candidate for highly selective etching. Fabrication started with standard solvent cleaning of 100-mm diameter silicon wafers with 2 m of thermal oxide grown on the surface. Electron-beam deposition was used to produce the ChG films on the surface of the silicon dioxide. This deposition method has been previously reported, [106] and is further investigated here under different deposition conditions. The deposition was performed at a rate of about 1 nm/s, starting with one 1 cm$^2$ cylindrical rods of bulk glass with the composition Ge$_{23}$Sb$_7$S$_{70}$. Bulk Ge$_{23}$Sb$_7$S$_{70}$ glass was fabricated by traditional melt quenching techniques [106, 107] A film approximately 630-nm-thick was deposited on the surface of the wafer. Compositional analysis with energy dispersive X-ray spectroscopy (EDX) using a JEOL 733 Super Probe was used to compare composition of the starting bulk glass and as-deposited films. Error on the measurement was approximately ±1 at%. The bulk glass had a composition of Ge 22 at%, Sb 7 at%, and S 71 at%. The deposited film composition was Ge 22 at%, Sb 11 at%, S 67 at%. The film composition is slightly non-stoichiometric compared to the bulk glass, which is likely an effect of the varying volatilization rates of the elements, commonly seen in ChG film deposition, and observed in previous studies of thermally evaporated Ge$_{23}$Sb$_7$S$_{70}$ [107, 108] The film refractive index was measured to be 2.22 at a wavelength of 1550 nm using the prism-coupling method. Waveguide patterning was performed using electron-beam lithography with ZEP 520A photoresist. After resist development, the line edge roughness of the pattern was reduced by reflowing it on a hot plate at 150°C for 5 minutes. The final waveguide width after reflow was found to be 750 nm (starting with 700 nm in the design).
Table 2.1: Two chemistries used for etching Chg.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Gases</th>
<th>Gas flow (sccm)</th>
<th>Bias, ICP</th>
<th>Selectivity ChG:ZEP</th>
<th>Pressure (mT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>CHF$_3$, Ar</td>
<td>5, 20</td>
<td>50, 500</td>
<td>5:1</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>Cl$_2$, Ar</td>
<td>2, 40</td>
<td>50, 100</td>
<td>4:1</td>
<td>5</td>
</tr>
</tbody>
</table>

Plasma etching was performed using a Unaxis Shuttleline inductively-coupled plasma reactive ion etcher (ICP-RIE). Two different etching recipes based on CHF$_3$ and Cl$_2$ were investigated in order to determine the best performance (see table 2.1). Both samples A and B were etched by approximately 290 nm at a temperature of 20°C. Sample A had a thickness of 600 nm, and Sample B was 650 nm thick. After the etching, the samples were coated with a benzocyclobutene (BCB) polymer cladding to protect the surfaces from contamination and to reduce scattering from the sidewalls. The curing of the polymer was performed in a nitrogen oven at a maximum temperature of 220°C. BCB has negligible (0.04 dB/cm) losses in the near-infrared. A scanning-electron-microscope (SEM) image of a cleaved waveguide facet is shown in figure 2.1a. The simulated waveguide from this geometry is single-mode, and its mode size is not significantly affected by the 50 nm thickness difference between samples A and B. The best device on sample B, which was Cl$_2$-etched, showed a propagation loss of 0.84 dB/cm, corresponding to an intrinsic (unloaded) Q-factor of 450,000. The corresponding values on the best device on sample A are loss of 1.34 dB/cm and an intrinsic Q of 288,000. Consult the measurements section for details.
The linear Q measurements provide a robust way of measuring waveguide losses. The diagram of a ring resonator with a single bus waveguide (notch filter) is shown in figure 2.2.

The relations between field components is given by the matrix equation [109]

\[
\begin{bmatrix}
E_{t1} \\
E_{t2}
\end{bmatrix} = \begin{bmatrix}
t & \kappa \\
-\kappa^* & t^*
\end{bmatrix} \times \begin{bmatrix}
E_{i1} \\
E_{i2}
\end{bmatrix},
\]

(2.1)

where \(|t|^2 + |\kappa|^2 = 1\) from coupled mode theory. For simplicity set \(E_{i1} = 1\). Taking into account losses in the microring \(E_{i2} = a \exp(j\theta)E_{i2}\), where \(a = \exp(-\alpha L/2)\), \(\alpha\) being the ring loss and \(L\) it’s circumstance. In that case the ring transmission is given by

\[
|E_{t1}|^2 = \frac{a^2 + |t|^2 - 2a|t| \cos(\theta + \phi_t)}{1 + a^2|t|^2 - 2a|t| \cos(\theta + \phi_t)},
\]

(2.2)

where extra phase comes from \(t = |t| \exp(j\phi_t)\). The issue with this equation is that it is symmetric
with respect to \( a \) and \( t \). To disambiguate between the two loss terms, ring resonators are fabricated with a sweep of coupling gaps for a constant radius. A least squares fit is used to find the two coefficients. In a sweep the coefficient that does not vary is the intrinsic loss of the waveguides. The loss is extracted from \( a \). An example of a fit is provided in figure 2.3.

![Figure 2.2: A diagram for the notch filter with symbols used in the text.](image)

![Figure 2.3: Ring transmission measurement. (a) Multiple notches (b) Single notch with a fit to the theoretical equation.](image)
2.2 Mode-locked laser

The nonlinear measurements performed as part of this dissertation required a pulse source. 100-fs pulses are necessary for coherent supercontinuum generation in $\chi(3)$ devices, whereas the $\chi(2)$, periodically-poled lithium niobate benefits both from the high peak power and the large bandwidth because the exact phase-matching wavelength is subject to fabrication intolerances.

In principle, pulses can be generated via gain switching but for such method the pulse duration is on the order of picoseconds. One example in electrically switched VCSELs [110]. An alternative method would be synchronously pumped dye lasers [111]. However these methods do not ensure coherence between subsequent pulses.

A preferred method for coherent pulse generation is mode-locking. There exists a large body of literature dedicated to mode-locking with two general methods of active and passive mode-locking and also hybrid methods. Active mode-locking is a technique of gain modulation at the resonance wavelength of the laser cavity [112].

The passive mode-locking requires a saturable absorber, a device that exhibits intensity dependent transmission, examples include semiconductors [113] and components utilizing the nonlinear Kerr effect [114]. The requirements of the supercontinuum measurements call for a fs-source, with MHz repetition rate to ensure high peak power but low average power and ease of operation. These are met by fiber mode-locked laser (MLL) that relies on nonlinear-polarization rotation as means of passive mode-locking [115].
A diagram of the constructed MLL laser is provided in fig 2.4 together with a picture of the actual setup. The salient features of the oscillator circuit include and a polarization dependent isolator in form of a polarization beam splitter (PBS) and a pair of quarter- and half- wavelength plates be before and after the PBS that act as polarization controllers. This setup acts as the fast saturable absorber. To understand the operation of the MLL it is necessary to mention cross-phase modulation. Two coupled Nonlinear-Schrodinger equations describing the propagation of the two orthogonal amplitudes $A_x, A_y$ are [116]

$$\frac{dA_x}{dz} + \frac{\alpha}{2}A_x + i\frac{\beta_2}{2} \frac{d^2A_x}{dt^2} = i\gamma(|A_x|^2 + 2|A_y|^2)A_x$$

$$\frac{dA_y}{dz} + \frac{\alpha}{2}A_y + i\frac{\beta_2}{2} \frac{d^2A_y}{dt^2} = i\gamma(|A_y|^2 + 2|A_x|^2)A_y,$$

other symbols are explained in the introduction. If losses and dispersion are neglected it is possible to show that for constant intensity the phase difference acquired is between the two polarizations.
is

\[ \phi_x - \phi_y = \gamma(|A_x|^2 - |A_y|^2)z. \]  

(2.4)

A conclusion to take from 2.4 is that the phase rotation of the field will depend on the intensity. Now returning to the MLL consider what happens to a linear pulse when it exits the PBS. The impending polarization controller induces and elliptical polarization on the pulse. As the pulse propagates through the fiber various parts of the pulse acquire different phase shifts depending on the local intensity due to self- and cross- phase modulation. The second polarization controller is adjusted in such a manner that the polarization at the center of the pulse will be passed through the PBS but the wings will be rejected. Thus effectively, the pulse wings will be absorbed and the pulse width will decrease. Since the pulse is attenuated from both sides, the setup acts as a fast saturable absorber. The process repeats itself on each roundtrip until there is equilibrium between gain, dispersion and attenuation.

Figure 2.5: The autocorrelation and spectrum of the mode-locked laser before (red) and after (green) pulse compression in the custom EDFA.
The oscillator circuit by itself generates 40 mW of average power which is insufficient for our purposes. The output of the MLL is collected using a collimator and fed to a custom build EDFA that amplifies the signal. The narrowest pulses were generated at 600mA of injection current to the pump diode of the EDFA. This corresponds to 120fs pulses and 100mW of optical power. The autocorrelation and spectrum is provided in figure 2.5.

2.3 Supercontinuum generation

There were multiple iterations of the chalcogenide waveguides before octave spanning supercontinuum was generated. In this section we mention two milestones. The first octave spanning spectrum from figure 2.6 used as the source the custom fiber mode-locked laser described in detail in the previous section.

![Figure 2.6: Octave spanning supercontinuum generated from chalcogenide waveguides and the custom fiber mode-locked laser](image)

The second spectrum was generated using a commercial source, 240 fs at 1550 nm, 25 MHz and
only 26 pJ of coupled power. The chalcogenide waveguides were 1200 nm wide, with the rib height of 910 nm and the slab height of 240 nm. A PECVD oxide cladding was used to reduce propagation losses to 0.56 dB/cm and 100 nm oxide was deposited on the facets as an anti-reflective coating. The insertion loss of the sample was measured to be 7 dB. The second spectrum is plotted in figure 2.7 together with the phase-matching condition for dispersive waves.

Figure 2.7: (Red) Octave spanning supercontinuum generated using only 26 pJ of coupled power. (Blue) Simulated spectrum. (Green) Phase-matching condition for emission of dispersive waves that manifest themselves as peaks in the spectrum.

The solid green line corresponds to

\[ D = \sum_{k \geq 2} \frac{\beta_k}{k!} = 0 \quad (2.5) \]

and the dashed line to

\[ \frac{1}{2} (2N - 1)^2 \frac{|\beta_2|}{T_0^2} \quad (2.6) \]
the symbols are the same as in chapter 1. The crossing point between these two lines is the phase-matching condition between the highest order soliton that is formed from the input pulse and the linear waves in the system. The transfer of energy from the soliton to these linear waves is called Cherenkov radiation and manifests itself as peaks in the supercontinuum spectrum. These peaks are useful in $f$-$2f$ referencing scheme because they increase signal strength at critical points in the spectrum.

2.4 Thin-film PPLN

A necessary component of a future on-chip $f$-$2f$ referencing scheme is periodically poled lithium niobate (PPLN). This section describes the fabrication and characterization of thin-film PPLN. To ensure phase-matching which is essential for efficient second harmonic generation, periodic poling was used. Periodic poling uses the ferroelectric property of lithium niobate crystals to flip the crystal domains along the Z-axis, thus producing a grating that compensates for the phase-mismatch between the pump and second harmonic.

The PPLN devices were fabricated from 300 mm X-cut LiNbO$_3$ films bonded on oxidized silicon wafers. 100 nm gold electrodes were fabricated with 40% duty cycle and 3.65 $\mu$m period using metallization and lift-off. The gold electrodes were placed parallel to the Z-axis with a 7 $\mu$m gap, to minimize optical losses in the devices. Poling was performed using short electrical pulses generating 400 kV/cm field field inside the crystal. Next, chalcogenide waveguide was defined on-top of the thin film lithium niobate to provide confinement of the optical field. The chalcogenide was deposited using thermal evaporation. 200 nm thickness of chalcogenide was used to maximize the pump overlap in lithium niobnated. COMSOL$TM$ simulations show an overlap of 50%. Finally, 1 $\mu$m thick SiO$_2$ cladding is deposited using plasma-enhanced chemical vapor deposition (PECVD).
Figure 2.8: CW simulations of gain inside the TDFA in (a) and the experimental setup used for testing the PPLN in (b). (c) Spectra showing PPLN characterization. Supercontinuum generated in highly-nonlinear fiber (HNLF) in green, amplified portion of the spectrum in blue and the generated second harmonic in red.

Due to a lack of appropriate tunable sources in the wavelength of interest supercontinuum source was used as a pump. Supercontinuum was generated in a highly nonlinear fiber and than later amplified in a thulium fiber amplifier to compensate for the high (30dB) insertion loss of the chip. The thulium doped fiber was pumped at 1565 nm with 3W of power supplied through an tunable laser amplified with an EDFA. The schematic of the experimental setup is shown in figure 2.8 together with a CW simulation of the gain provided by the EDFA. The expected CW gain was 29 dB.
dB, whereas the experimentally observed gain of the supercontinuum was around 20 dB as depicted in figure 2.8c. The amplified supercontinuum, shown in blue was coupled into the fabricated PPLN device described above. The resulting SHG is shown in red in same figure. The peak of the SHG signal is evident at 937.5 nm corresponding to the peak TDFA gain of 1875 nm.

2.5 CEO detection

The principle of CEO detection can be summarized as follows. Since a supercontinuum is is a train of pulses in the time domain it is a set of equally spaced comb lines in the frequency domain as dictated by Fourier transform, $f_n = f_{ceo} + n \times f_{rep}$. An octave spanning supercontinuum contains a fundamental tone $f_{fund} = f_{ceo} + n \times f_{rep}$ at around $2\mu m$ and its second harmonic $f_{octave} = f_{ceo} + 2n \times f_{rep}$ at $1\mu m$. If the fundamental is frequency doubled and beat against the harmonic, the beat signal is given by $2f_{fund} - f_{octave} = f_{ceo}$, which is also called the carrier envelope offset. Since the $1\mu m$ and $2\mu m$ components travel at different group velocities, observation of a strong beat signal requires an introduction of optical delay. This is best done in a Michelson interferometer as in figure 2.9. The light out of the chip is collected using an aspheric lens and than guided into the interferometer. There the $1\mu m$ and $2\mu m$ components are split using a dichroic mirror, $2\mu m$ is delayed and recombined with the $1\mu m$. The beam is fed into a commercial bulk PPLN with a temperature oven where second harmonic generation happens. The beat signal is detected on a silicon APD proceeded by a 10 nm bandpass filter which increases the SNR of the signal.
Figure 2.9: CEO detection setup and a 7dB SNR signal in the insert.
CHAPTER 3: SYNCHRONOUSLY PUMPED KERR CAVITY

The contents of this chapter were published in "Optical frequency comb generation by pulsed pumping", Marcin Malinowski, Ashutosh Rao, Peter Delfyett, and Sasan Fathpour, APL Photonics 2, 066101 (2017); licensed under a Creative Commons Attribution (CC BY) license.

3.1 Abstract

A synchronously-pumped Kerr cavity is proposed and studied for power-efficient frequency comb generation in optical microring resonators. The system is modeled using the Lugiato-Lefever equation. Analytical solutions are provided for an ideal case, and extended by numerical methods to account for optical loss and higher orders of dispersion. It is shown that the average power requirement is reduced by the duty cycle of the pulse with respect to conventional continuous-wave-pumped microrings and it is significantly lower than pulsed pumping of straight waveguides.

3.2 Introduction

Frequency combs are composed of a series of short and high-power pulses in the time domain that translate into a broad comb in the frequency domain. This duality translates into a host of applications for optical combs [117, 118]. For instance, the broad spectrum facilitates spectroscopy in regimes from the ultraviolet (UV) to the infrared (IR) [119], wherein high peak power of the pulses is necessary for precision distance measurements to overcome noise [120]. Atomic clocks, where combs are used to count the optical cycles, is another important application [121]. In conjunction with the quest for miniaturization, there has been a strive towards reducing the footprint of frequency optical comb sources using continuous-wave (CW) pump sources. The first step was the
demonstration of frequency comb generation in whispering-gallery mode resonators [122, 123], followed by a demonstration of on-chip frequency comb generation on silicon nitride [124]. The transition from smooth laser-ablated structures to etched waveguides, however, came at the expense of higher threshold power (e.g. $\approx 1.3 \text{ W}$) [124]. Additionally, the conversion efficiency for generation of frequency combs is low and on the order of 1 to 2% for CW pumping [125].

To overcome this power-efficiency shortcoming, we have previously proposed pulsed pumping in a fully integrated on-chip frequency comb source [126]. The notion of a synchronously-pumped Kerr cavity was first theoretically explored in fibers [127]. The system has drawn particular interest due to its chaotic and bistable behavior [128]. Synchronous pumping with sinusoidal input, coming from a beat signal of two CW sources, was first suggested in [129] as a means of frequency comb generation. Furthermore, wide Gaussian pulses with sinusoidal modulation are proposed for storing solitons in optical memory devices [130]. A synchronously-pumped fiber Kerr cavity has been studied in optical fibers for observing spontaneous symmetry breaking [131]. Soliton formation in a 10 GHz fiber cavity pumped by picosecond pulses has recently been demonstrated experimentally [132].

In this letter, synchronous pulsed pumping is studied for the first time in the context of octave-spanning optical comb generation in integrated microring resonators. Microrings offer the possibility of a compact, chip-size stable frequency comb source. The Lugiato-Lefever model is employed here [133], which can be shown to be equivalent to the coupled-mode equations formalism [133]. We show that for broadband frequency comb generation, essential for f-2f referencing, the large dispersion slope, $(\beta_3)$, of the microring resonators results in generation of Cherenkov radiation and a change in the solitons group velocity that has to be compensated by adjusting the repetition rate of the driving pulse train. Such an effect is not observed in the aforementioned narrow-band synchronously-pumped fiber-based combs [132], due to lower dispersion in fibers. The power efficiency of the proposed approach is explored and design and modeling guidelines
are developed.

The paper is structured as follows. We first present an analytical expression for a special lossless case with a soliton pulse pump, \text{sech}(\tau), to develop basic understanding of the system behavior. Next, losses are included for a sweep of Gaussian input pulses where the pulse width is varied. Next, we add a third order dispersion term and discuss the effects of Cherenkov radiation. Finally, an experimentally feasible 10-GHz ring architecture pumped by a mode-locked laser is simulated and compared against both CW pumping of microrings, as well as pulsed pumping of straight waveguides.

3.3 Pulsed input - analytical treatment

The Lugiato-Lefever (LL) equation [133] is a mean-field model of microresonator behavior, where the effect of coupling is averaged over each roundtrip. The two equations from Ikeda map approach governing pulse propagation and periodic boundary conditions are reduced to one making it computationally more efficient [134]. The full LL equation is

\begin{equation}
\begin{aligned}
t_r \frac{\partial A}{\partial t} = & -\frac{\alpha + \theta}{2} - i\delta_0 + iL \sum_{k>2} \frac{\beta_k}{k!} (i \frac{\partial}{\partial \tau})^k A + iL\eta|A|^2 A + \sqrt{\theta} A_m, \\
\end{aligned}
\end{equation}

where \( t \) is the slow time of the cavity (duration of multiple roundtrips), \( \tau \) is the fast time in the moving frame of reference, \( \alpha \) is the integrated loss, \( \theta \) is the coupling loss, \( L \) is the length of the cavity, \( t_r \) is the round trip time, \( \delta_0 \) is the detuning or the phase difference between driving field and the cavity resonance, \( \eta \) is the Kerr non-linearity coefficient \( (= \frac{2\pi n_2}{\lambda_p A_{eff}}) \), where \( A_m \) is the envelope of the pump field and \( A \) is the envelope of the field in the cavity.

In the next section, only the second-order anomalous dispersion (\( \beta_2 \)) is retained and the normalized
version of the same equation is used [135], i.e.,

$$\frac{\partial A}{\partial t} = \left[-1 - i\Delta + i \frac{\partial^2}{\partial \tau^2} + i|A|^2\right]A + A_{in}(\tau). \quad (3.2)$$

The normalization is achieved via the following transformations: $t \to (\theta+\alpha)t/2t_r$, $A \to \sqrt{2L\eta/(\alpha + \theta)}A$, $\tau \to \sqrt{(\alpha + \theta)/L}\beta_2|\tau|$, $\Delta = 2\delta_0/(\alpha + \theta)$ and $A_{in} \to \sqrt{8L\theta\eta/(\alpha + \theta)^3}A_{in}$.

First, we note that there exists an analytical solution to the steady state [129], lossless LL equation

$$[-i\Delta + i \frac{\partial^2}{\partial \tau^2} + i|A|^2]A + A_{in}(\tau) = 0, \quad (3.3)$$

when the input and the field inside the cavity are given by

$$A_{in}(\tau) = \frac{C_1}{\cosh(C_2\tau)}, \quad A(\tau) = \frac{C_3}{\cosh(C_2\tau)}, \quad (3.4)$$

with the coefficients related by $|C_3| = \sqrt{2}C_2$ and $\Delta C_3 - 1/2|C_3|^2 C_3 + iC_1 = 0$. Physical intuition suggests that if the input pulse is wide, it can be locally approximated as a constant background for which the stable solutions are known. Mathematically, this concept is encompassed by the method of matched asymptotics. For $\tau \ll 1$, $A_{in}$ is constant, i.e., $A_{in} = C_1$, and the stable solution to (3.3) is known [136]. Far away from the center of the pulse $\tau = 0$, the solution is just the soliton obtained from (3.4). The field inside the cavity is the sum of these two minus the overlap at $\tau = \infty$.

Following this prescription, the compressed pulse is given by

$$A(\tau) = \frac{C_3}{\cosh(C_2\tau)} + \frac{2A_0 \sinh^2\alpha}{1 - \cosh \alpha \cosh(B\tau)}, \quad (3.5)$$
where the coefficients are related implicitly via $\alpha$ as

$$C_1 = i \sqrt{\frac{4\Delta^3 \cosh^2 \alpha}{(1 + 2 \cosh^2 \alpha)^{3/2}}} \quad A_0 = \sqrt{\frac{\Delta}{(1 + 2 \cosh^2 \alpha)}},$$

(3.6)

and $B = \sqrt{2} A_0 \sinh \alpha$.

The relation for coefficients of eq. (3.4) is a third-order polynomial, thus in principle there can be three solutions of $C_3$, the smallest of which (widest pulses) was found to be stable. Also, two solutions are available for a constant pump power (CW) case, referred to as $\psi_+$ and $\psi_-$ in [136]. $\psi_-$ is stable and has been used in the construction of the solutions presented in (3.5). Excellent agreement between the above approximate analytical solution and the numerical solutions, produced by the Newton-Raphson method, is evident in Fig. 3.1.

![Figure 3.1](image)

Figure 3.1: (a) Excellent agreement between the matched asymptotics (MA) method and the numerical solution from the Newton-Raphson (NR) solver plotted for $\Delta = 12$, zero loss and $C_1 = 6$. (b) Solitons formed from Gaussian pulses, $\exp - \left(\frac{\tau}{\tau_p}\right)^2$, are the same as those from CW background provided that the loss and the detuning is the same ($\Delta = 4$), and the peak power of the pulse is equal to the CW background ($A_{in} = 2$). All solutions are stable, as confirmed using the split-step Fourier time evolution method.

Since the output solitons formed from pulses have the same shape as those formed from a CW
pump, it can be inferred that when the locally flat approximation holds for input pulses, the average power requirement for a pulse pump will be a fraction of the CW power, that is the duty cycle of the pulse with respect to CW. This is an important, yet simple, finding of this work for comb generation in microring structures.

So far for the analytical treatment, loss was not considered in eq. (3.2). In the next section we retain it and use Gaussian pulses, \( \exp(-\tau/\tau_p)^2 \), for the input field. A computationally efficient method of investigating the steady state solutions utilizes a Newton-Raphson (NR) solver [137] and a continuation method [136]. The single soliton state for the CW case, generated by the split-step Fourier method time-evolution code [138], is used as the initial guess for a NR solver. We wish to find a steady-state solution to the case of pulse pumping while knowing the solution to the CW case. Thus, we use a NR solver to smoothly transform the solution from the extreme case \( \tau_p = \infty \) i.e. CW case to that of the desired input pulse width. The peak power and the detuning has to remain constant in this transformation. The steady state solutions are plotted in Fig. 3.1. The stability of the solutions is confirmed by time-evolving them in the split-step Fourier method code with initial amplitude noise (max amplitude \( 10^{-7} \)). The noise is added to seed any potential instabilities. It is evident that the solitons formed from pulses have the same functional form as those formed from the CW case. The observation remains valid for relatively short pulses \( \tau_p = 3 \) in comparison to solitons with a full-width at half maximun (FWHM) of 1.25 and for different pump powers and detunings, as long as the undesired spontaneous symmetry breaking does not occur. The temporal symmetry breaking manifests itself as the soliton forming on one side of the input pulse [131].

We also confirmed that soliton formation (pulse compression) can be achieved starting from Gaussian pulses themselves. The time evolution of the LL equation is studied using the split-step Fourier method.
Figure 3.2: (a) The path towards pulse compression (single soliton formation) closely follows that of the CW case. The plot shows the power in the cavity on a log scale and corresponding selected snapshots. Initially the pulses develop modulational instability (MI) for $\Delta = 0, t < 20$. After an abrupt increase in the detuning at $t = 20, \Delta = 2$, unstable breathing solitons appear that coalesce and break apart, at $\Delta = 4.16, t = 80$ they condense on a stable single soliton solution. Simulation performed for Gaussian input pulse, $2.0 \exp(-{(\tau/10)}^2)$. (b) The same simulation parameters as in (a) are used except that 3rd order dispersion is added, $-0.12 \frac{\partial^3 A}{\partial \tau^3}$, this causes the soliton to emit a dispersive wave (Cherenkov radiation) that affects its group velocity, the soliton drifts in the simulation in the moving frame of reference. For the single soliton state to be stable the input pulse has to also drift (the repetition of the input pulses has to be adjusted).
The detuning is changed non-adiabatically to induce various phases. First, as in the CW case [138],
the modulation instability develops on top of the input pulses. Next after and abrupt increase of
detuning, unstable breathing solitons appear that coalesce and break apart. Finally, the detuning is
adjusted to obtain a stable single soliton phase. An example of such evolution is shown in Fig.
3.2a. The compression can be also achieved via increasing the input pulse power in abrupt steps.

Next, we add a third order dispersion term. To accentuate the effects the term is arbitrarily chosen
as $-0.12 \frac{\partial^3 A}{\partial \tau^3}$. The additional dispersion term perturbs the solitons and results in an emission of
a dispersive wave, referred to as Cherenkov radiation [137, 139]. The dispersive wave creates an
additional peak in the spectrum, but more importantly leads to a temporal drift of solitons in the
moving reference frame. Since the drift velocity is constant, this would experimentally correspond
to a change of the spacing of comb lines. It also means that, in the simulations, the pump pulse
has to drift with the solitons to form stable solutions, as demonstrated in Fig. 3.2b. If the pump is
allowed to drift with the solitons, all the above-discussed observations regarding soliton formation
remain valid.

3.4 Numerical simulations

Finally, realistic simulations on a specific material system for a microring cavity are presented here
utilizing the full Lugiato-Lefever equation (3.1). The physical parameters are borrowed from our
recent experimental results on chalcogenide glass ($\text{Ge}_{23}\text{Sb}_{7}\text{S}_{70}$) [140], as well as further disper-
sion and nonlinear index measurements we have performed on the material. The platform’s loss
has been steadily decreasing [140, 141]. Accordingly, the following values are assumed: 2 mm
radius corresponding to 10 GHz FSR, $\gamma = 1.92 \text{ W}^{-1} \text{ m}^{-1}$, propagation loss of 0.2 dB/cm, critical
coupling, and $\beta_2 = -6.9 \times 10^{-26} \text{ s}^2 \text{ m}^{-1}$. It is noted that chalcogenide glasses have higher non-
linear coefficient than silicon nitride which has been used for frequency comb generation [124].
The higher nonlinearity of the material can compensate for the higher losses, as verified by our models. Supercontinuum generation on this platform has been demonstrated experimentally by our collaborators and us[142].

Regardless of the material system choice, the power requirement is reduced by the duty cycle of the pulse with respect to the CW case. The simulated field inside the cavity and the corresponding spectra are shown in Fig. 3.3. For an octave spanning spectrum generated in a 10 GHz system, the CW power requirement in the bus waveguide was found to be 10 W, which is experimentally hard to achieve. It is reduced to a manageable average power of 130 mW (1.3%) for Gaussian input pulses ($\tau_p = 1$ ps), and 200 mW (2%) for more realistic breathing mode-locked lasers that could be integrated on the same chip [143, 144].

Figure 3.3: (a) Comparison of compressed pulses in the 100 ps ring cavity plotted together inside a 10 ps window. The Gaussian in all plots corresponds to width of $\tau_p = 1$ ps and MLL is a breathing mode-locked laser. The solitons formed inside the ring cavity are the same provided that the peak of the pulse is the same as the CW background. The oscillations present next to the solitons is the Chernekov radiation. (b) Octave spanning spectra of the pulses from (a) after coupling into the bus waveguide. Also shown is the case of supercontinuum generation in straight waveguides using Gaussian input pulses. For the supercontinuum to reach similar bandwidth as in the case of a cavity, the average power has to be 8 time higher.
It is known that pulse compression can be also achieved in straight waveguides [145], thus a comparison of the power efficiency with this approach would be useful. As summarized in Fig. 3.3, for Gaussian pulses with $\tau_p = 1$ ps, the required peak power is 80 W in order to attain similar comb bandwidths of the microring cases. This power is 8 times higher than the above synchronously-pumped microring cavity. The optimal propagation distance for which the spectrum is the broadest was found to be 30 cm, which demonstrates that, on top of better power efficiency, the rings provide compactness for the same performance but at the expense of increased complexity in the system architecture, as discussed above. The spectrum of the supercontinuum from straight waveguides is flatter, but the waveguide dispersion was designed such that Cherenkov radiation boosts the power of the microring spectra at the short (1223 nm) and long (2456 nm) side. This is essential for f-2f referencing.

Pulsed pumping of a microring cavity is more complicated than using CW pump, nevertheless we believe that it is experimentally feasible. Such a system has been demonstrated in fiber cavities [139] and it could be transferred to microring resonators, as outlined in the following. For precise control of the cavity detuning, part of the MLL beam could be tapped off and injected into the laser in the counterpropagating direction. Locking would be achieved by monitoring the coupled power. The low power of the counterpropagating beam would ensure locking of the carrier frequency to the closest CW resonance, thus detuning of the main beam could be precisely controlled using for example a phase modulator. Consequently, the soliton formation process of Fig. 3.2a would be experimentally possible. Either the MLL or the cavity could be controlled thermally to achieve matching of the FSRs. Furthermore, Ref. [132] demonstrates that for a 10-GHz fiber cavity, the soliton remains locked to the driving pulse train over 10s of kHz, thus exact FSR matching may not be necessary. The important finding of the present work is that in microrings, which are much more dispersive than fibers, the soliton emits a dispersive wave that results in solitons with different group velocity than the CW light case. This difference in FSRs of the soliton and the cold cavity
is constant and can be extracted from the simulations and will have to be taken into account in the actual experiment. Finally, we note that using a MLL is conceptually similar to using a CW light modulated at the repetition rate of the cavity [146], which supports the feasibility of the proposed scheme. Also, Ref. [146] has shown that parametric seeding has an added benefit of suppressing undesirable non-equidistant combs.

3.5 Conclusions

In conclusion, synchronously-pumped Kerr cavity is proposed for octave spanning frequency comb generation in integrated microrings and its power efficiency benefits are analytically and numerically modeled. It is demonstrated that for pulsed input, solitons remain the fundamental solution of the system, and hence the power requirement is reduced by the duty cycle of the pulse with respect to CW pumping of the same rings. When compared with straight waveguides, the proposed approach benefits from the power enhancement of the microring and hence again lower power requirement.

This research was developed with funding from the Defense Advanced Research Projects Agency (DARPA). The views, opinions and/or findings expressed are those of the authors and should not be interpreted as representing the official views or policies of the Department of Defense or the U.S. Government. We also thank Meer Nazmus Sakib for performing chalcogenide depositions, Seyfollah Toroghi for dispersion measurement and Akbar Ali Syed for z-scan measurements of $n_2$. 

53
CHAPTER 4: NUMERICAL TOOLS

4.1 Split-step algorithm

The Nonlinear Schrödinger equation (NLS) equation is usually solved using the symmetric split Fourier algorithm [147]. The NLS terms can be subdivided into two categories,

\[ \frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A, \quad (4.1) \]

where

\[ \hat{D} = -\frac{\alpha}{2} + i \sum_{k \geq 2} \frac{\beta_k}{k!} (i \frac{\partial}{\partial t})^k \]

is the linear part and the \( \hat{N} = i\gamma |A|^2 \) is the nonlinear part.

We note that formally

\[ A(z + h, t) = \exp((\hat{D} + \hat{N})h)A(z, t) \quad (4.3) \]

We can split the step and apply the operators independently

\[ A(z + h, t) = \exp(\hat{D}h) \exp(\hat{N}h)A(z, t). \quad (4.4) \]

In this case the error is \( O(h^2) \) and is due to the non-commutativity of the two operators.

\[ \exp(\hat{D} + \hat{N}) = \exp(\hat{D} + \hat{N} + \frac{1}{2} [\hat{D}, \hat{N}]...) \quad (4.5) \]

\( \hat{D} \) is evaluated in the frequency domain

\[ \hat{D}(t) = FT^{-1} \hat{D}(\omega)FT \quad (4.6) \]
and $\hat{N}$ in the time domain. It is possible further refine the process by evaluating $\hat{N}$ in the middle of the step

$$A(z + h, t) = \exp(\hat{D}h/2) \exp(\hat{N}h) \exp(\hat{D}h/2)A(z, t).$$

(4.7)

This method has $O(h^3)$ error. To increase the accuracy of the simulations the step is adjusted adaptively. The step adaptation is based on the relative error between a step of $2h$ and two steps of $h$ for

$$\epsilon = \frac{||A_c - A_f||}{||A_c||}.$$  

(4.8)

where c,f refer to coarse and fine steps. Finally the method is improved by noting that

$$\frac{4}{3}A_f - \frac{1}{3}A_c = O(h^4),$$

(4.9)

thus adding the the coarse and fine steps decrease the order of the error and the calculation time is not wasted. The source code for solving the Lugiato-Lefever equation (NLS with periodic boundary conditions) is provided in the appendix.

### 4.2 Genetic algorithms for dispersion engineering

Genetic algorithms for electromagnetic optimization algorithms have been developed in the 80’s [148]. An elegant application is NASA’s wire X-band antenna of the ST5 mission [149] shown in picture 4.1.
Genetic algorithms are very intuitive because they mimic biological evolutionary processes. The core elements of the GA algorithm in the context of dispersion engineering [150] are summarized below.

**Population** The algorithm operates on a population of individuals. In the case of dispersion engineering, the population corresponds to waveguides of different dimensions. It is paramount that the initial distribution of individuals is random and uniform for the GA to explore all possible solutions and not to prematurely converge on a suboptimal solution. The initial population size is related to the number of parameters that are to be optimized. 50 was found to be experimentally optimal for 3 geometrical degrees of freedom.

**Chromosomes** The physical dimensions of the waveguides have to be encoded into genes. Due to the binary nature of storing data in modern PC binary encoding is the preferred method of choice. Dispersion engineering is an example of a constrained optimization problem that is an upper and a lower bound of various dimensions in known. Given an upper value of a parameter $P_{\text{max}}$ and a lower one $P_{\text{min}}$, the mapping between a N bit string and physical values is trivial

$$P = P_{\text{min}} + \frac{M}{2^{N-1}}(P_{\text{max}} - P_{\text{min}}),$$

for a string of value M.
Termination The GA, just like biological evolution, does not have an encoded stopping criterion, thus one has to be artificially supplied. An example would an diversity of the population, for waveguides the spread of dimensions in the population.

Fitness function This is the central feature of the GA. A fitness function is a means of evaluating the suitability of a given individual. This is the most computationally intensive part of the algorithm because evaluating the fitness of a single individual corresponds to a full dispersion simulation. Furthermore, the fitness function has to return a single number but nothing is known a priori about what dispersion curves a feasible. Knowing, that a flat anomalous dispersion curve with spanning large bandwidth is desired is is sensible to define an artificial desired curve and use sum of squares of distances between points of the simulated and desired dispersion curves as the fitness function.

Scaling The fitness function defined above will have a large spread of values, that might lead to overly preferential selection of certain individuals and hence premature convergence. It is better to sort individuals according to their fitness function, and scale it’s value in relation to the position of the individual in the line.

Reproduction Once the individuals have been ranked a small proportion (10%), called the elite, is advanced to the next generation. Next children are generated by from the previous generation by mutation and crossover. For crossover parents are choosen proportional the their relative fitness. Cross-over points are randomly chosen and the the genes between two chromosomes are exchanged.
Figure 4.2: Top: The individuals of each generation are plotted on height vs width curve plane. The best individual in each generation is marked with a red cross. The algorithm is initially seeded with a random population that converges via evolution and mutation towards an optimal solution. Bottom: The desired dispersion (red) and the dispersion of the best individual (blue) from each generation. The desired curve corresponds to width 1500 nm and height 600 nm. GA solution is width of 1543 nm and height of 599 nm.

The GA algorithm was implemented in Matlab and COMSOL was interfaced using LiveLink™ for Matlab. An example of optimization of a channel waveguide is shown if 4.2. The initially random population quickly converges on the optimal solution. Figure 4.2 shows the population spread for each generation, together with the dispersion curve for the best (blue) individual plotted with the desired curve (red).

4.3 TDFA model

The Thulium doped fiber amplifier (TDFA) was developed to assist in second harmonic generation experiments to compensate for coupling losses between chalcogenide and thin-film PPLN samples.
The numerical model of in-band pumped TDFA is described in detail below. In-band pumping was used because high power EDFA were readily available and the 1565 nm wavelength was chosen because it lies on the outer edge of the EDFA amplification bandwidth but at the same time falls inside the absorption band of Tm$^{+3}$ (see figure 4.4b).

![Energy diagram for in-band pumping of Tm$^{+3}$](image)

Figure 4.3: (a) Energy diagram for in-band pumping of Tm$^{+3}$ (b) Absorption and emission cross-sections of Nufern fiber SM-TSF-9/125.

The rate equation for in-band pumping from $^3$H$_6$ level to $^3$F$_4$ is given by [151, 152]

$$\frac{dN_2(z,t)}{dt} = w_{p12}N_1(z,t) - w_{p21}N_2(z,t) - \frac{N_2(z,t)}{\tau_2} - w_{s21}N_2(z,t) + w_{s12}N_1(z,t), \quad (4.10)$$

where the relationship between the population level of $^3$F$_4$ ($N_2$) and the population level of $^3$F$_4$ ($N_1$) is given by

$$N_1(z,t) = N_T - N_2(z,t), \quad (4.11)$$

where $N_T$ is the concentration of Tm$^{+3}$ atoms. $\tau_2$ is the spontaneous lifetime of $^3$F$_4$ level. The
The pumping rate from $^3\text{H}_6$ to $^3\text{F}_4$ is given by

$$w_{p12} = \frac{\lambda_p \Gamma_p}{\hbar c A_{\text{core}}} \sigma_a (\lambda_p) \left[ p_p^+ (z) + p_p^- (z) \right].$$  \hspace{1cm} (4.12)

The de-excitation rate of $^3\text{F}_4$ is

$$w_{p21} = \frac{\lambda_p \Gamma_p}{\hbar c A_{\text{core}}} \sigma_e (\lambda_p) \left[ p_p^+ (z) + p_p^- (z) \right].$$  \hspace{1cm} (4.13)

The stimulated emission rate from $^3\text{F}_4$ to $^3\text{H}_6$ is

$$w_{s12} = \frac{\lambda_s \Gamma_s}{\hbar c A_{\text{core}}} \sigma_a (\lambda_s) \left[ p_s (z) + ASE_f(z) + ASE_b(z) \right]$$  \hspace{1cm} (4.14)

and the corresponding stimulated absorption rate is

$$w_{s21} = \frac{\lambda_s \Gamma_s}{\hbar c A_{\text{core}}} \sigma_e (\lambda_s) \left[ p_s (z) + ASE_f(z) + ASE_b(z) \right].$$  \hspace{1cm} (4.15)

The symbols used in the equations are $\hbar$ - Plank constant, $c$ - speed of light, $\gamma_p$ and $\gamma_s$ - the pump and signal wavelengths respectively, $\Gamma_p$ and $\Gamma_s$ are the pump and signal confinement factors, $A_{\text{core}}$ is the area of the fiber core, $\sigma_a$ and $\sigma_e$ correspond to absorption and emission cross-sections.

The pump power $p_p$ evolves along the fiber length as

$$\frac{dp_p^\pm}{dz} = \pm p_p^\pm (z) \left[ \Gamma_p (\sigma_e (\lambda_p) N_2(z) - \sigma_a (\lambda_p) N_1(z)) - \alpha_p \right]$$  \hspace{1cm} (4.16)

whereas the signal power, $p_s$, evolves as

$$\frac{dp_s}{dz} = p_s (z) \left[ \Gamma_s (\sigma_e (\lambda_s) N_2(z) - \sigma_a (\lambda_s) N_1(z)) - \alpha_s \right].$$  \hspace{1cm} (4.17)
with $\alpha_p$ and $\alpha_s$ being the attenuation constants. The forward amplified spontaneous emission (ASE) is given by

$$\frac{dASE_f}{dz} = ASE_f(z) [\Gamma_s (\sigma_e (\lambda_s) N_2(z) - \sigma_a (\lambda_s) N_1(z)) - \alpha_s] + 2\sigma_e (\lambda_s) N_2(z) \frac{hc^2}{\lambda_s^3} \lambda \ (4.18)$$

and the backward ASE is given by

$$\frac{dASE_b}{dz} = -ASE_b(z) [\Gamma_s (\sigma_e (\lambda_s) N_2(z) - \sigma_a (\lambda_s) N_1(z)) - \alpha_s] - 2\sigma_e (\lambda_s) N_2(z) \frac{hc^2}{\lambda_s^3} \lambda \ (4.19)$$

where $\Delta \lambda$ is the bandwidth of amplified spontaneous emission. These coupled differential equations are solved by the relaxation method, that is by first propagating them forward with $ASE_f = 0$ and using the calculated boundary values at the end of TDFA the equations are propagated backwards, this time setting $ASE_b = 0$. The new boundary conditions at the beginning of the TDFA are used as the seed for the next calculations but with $ASE_f = 0$ again set to zero. The process is repeated until convergence is observed. The source code is provided in the Appendix.

Figure 4.4: (a) Simulation of small signal gain at 1990 nm for 1.1W, 1565 nm pump showing 24 dB of gain (b) Amplification of supercontinuum signal for second harmonic generation showing 20 dB of gain.
Figure 4.4 shows a small signal simulation of the TDFA gain for 1.1W, 1565 nm pump showing 24 dB of gain and the experimentally measured value of 20 dB. In the experiment the source is pulsed (20 MHz supercontinuum from chalcogenide waveguides), nevertheless the small signal CW model is valid because the TDFA is not saturated by the pulses.
CHAPTER 5: STIMULATED BRILLOUIN SCATTERING IN SILICON


5.1 Abstract

A thorough study of elastic waves in waveguides, taking into account the full tensorial nature of the stiffness tensor, is presented in the context of stimulated Brillouin scattering. Various approximations of the elastic wave equation used in the stimulated Brillouin scattering literature are implemented and their validity and applicability are discussed. The developed elastic wave mode-solver is also coupled with an electromagnetic counterpart to study the influence of elastic anisotropies on Brillouin gain.

5.2 Introduction

Stimulated Brillouin scattering (SBS) has been extensively studied in optical fibers, albeit it is generally considered a nuisance for the long-haul optical communication application [153]. Recently, there has been renewed interest in SBS in order to harness it in integrated photonic devices [154, 155, 156, 157]. The narrow bandwidth of the SBS response leads to a host of applications in microwave photonics [158], such as the construction of tunable bandpass [159] and notch [160] filters, phase shifters [161] and microwave synthesizers [162]. Also, long phonon lifetime has been used to store optical pulses in the acoustic domain [163]. SBS has been, furthermore, utilized in wavelength-selective amplifiers [164] and narrow-linewidth lasers [165]. Cascaded SBS process
has been proposed as a multiwavelength source for optical communications [166].

Unlike silica, the common material in optical fibers, most of the materials used in integrated photonics are not isotropic. A prime example is silicon with a cubic lattice structure [167], on which large Brillouin gain has been demonstrated [155]. To fully explore the whole parameter space of integrated acoustooptic devices, knowledge of all acoustic modes is necessary, especially given that the acoustic and optical modes can be tailored independently, while retaining coupling between the two classes of modes as in the case of a Brillouin laser on silicon membranes [168].

The finite-element method (FEM) is a versatile modeling choice for implementing an elastic-wave mode-solver that takes into account the tensorial stiffness of materials. The method has been originally developed in the 1960s and 70s to model the problems of structural mechanics [169], including the elastic-wave equation [170], sometimes also called the seismic wave equation. Unlike the finite-difference method, FEM can easily handle complex geometries. Among other modeling problems, FEM has also been used to find the acoustic modes of structures in the ultrasound and the GHz ranges. Examples are the scalar pressure model in chalcogenide waveguides [171], the scalar elastic model for SBS in fibers [172], isotropic beams [173], and the full-viscoelastic response of waveguides [174] at ultrasound frequencies. The mode-solvers fall under the general scheme of the semi-analytical finite element method (SAFE), which reduces the three-dimensional (3D) wave-propagation problems to 2D variants, by assuming periodicity in the direction of propagation [170]. In this paper, we develop and implement the SAFE method for fully-tensorial elastic-wave equation, described in Section III, using an open-source element-solver, called FEniCS [175].

In Section IV on isotropic materials, we perform a thorough review of various approximations of the elastic-wave equation in the SBS literature and discuss their validity in the context of integrated waveguides. As an example, a material system consisting of a chalcogenide glass, As$_2$S$_3$ embedded in thermal oxide is studied [176]. So far, the consensus for such isotropic materials, and
following the optical fiber literature [177], has been to assume that the displacement in waveguides follows the scalar acoustic-wave model [171]. However, we find that this model overestimates the eigenmodes of the system by as much as 0.5 GHz in submicron-sized waveguides. In addition, the acoustic-wave equation assumes close to plane-wave propagation. A better approximation is provided in reference [172], which we later refer to as the scalar elastic equation. In both cases, nonetheless, these models do not appropriately capture the mode profile near the material interfaces, when compared with the most general model used in this work.

In Section V on anisotropic materials, we couple the elastic mode-solver with an electromagnetic counterpart to explore the effect of elastic anisotropies on Brillouin gain, in addition to the variation of the mechanical resonance frequency. Silicon is chosen as an example, which is ubiquitous in integrated photonics and provides high Brillouin gain in suspended structures [155, 156]. Here, researchers have resorted to full-3D simulations with the Floquet boundary conditions [178, 179] usually implemented in the commercial FEM solver, COMSOL\textsuperscript{TM}. Following the micro-electromechanical systems (MEMS) literature [167], some authors have chosen to use the simplified isotropic model for silicon [180, 181]. While qualitatively acceptable, this assumption can lead to large discrepancies quantitatively. We show that in extreme cases, where the waveguide is aligned along the [100] or the [110] crystalline axis, the difference in eigenfrequencies can be as high as 0.8 GHz, which highlights the need for a fully-tensorial formulation. This is directly translated to the Brillouin gain, which scales with the inverse of the mechanical resonance-frequency squared. For back-scattering, we show that, for an arbitrary crystal orientation in silicon, the elastic modes do not need to have the same symmetry as the optical modes, which greatly affects the Brillouin coupling coefficient. While, in principle, the anisotropic behavior of silicon can be simulated in COMSOL\textsuperscript{TM}, the use of the SAFE method in this work leads to substantial - over two orders of magnitude - computational-time improvement. Furthermore, since the SAFE method reduces the equations to a 2D problem, it requires less computational memory than the full-3D simulation,
without neglecting any physical modeling features. Finally, it should be noted that recently a mode-solver for isotropic materials utilizing the finite-difference method has been presented [182], but our work provides faster convergence through the use of higher-order finite elements.

5.3 Governing Equations

The constitutive equation of motion for an elastic medium is [169]

\[
\nabla \cdot \sigma = \rho \frac{\partial^2 u}{\partial t^2},
\]

where \( u \) is the displacement vector, \( \rho \) is the density and the \( \sigma \) is the stress. Stress is linearly related to strain, \( \epsilon \), via the stiffness tensor, \( C \), as follows:

\[
\sigma = C : \epsilon \quad \text{and} \quad \epsilon = \frac{1}{2} [\nabla u + (\nabla u)^T].
\]

To obtain the equations describing the modal distribution, the Fourier transforms of \( t \) and \( z \) are employed. We also introduce the transverse gradient operator \( \nabla_T \) such that \( \nabla = \nabla_T + iq_b \hat{z} \), where \( q_b \) is the acoustic-wave propagation constant. Afterwards, the elastic wave equation reads [170, 183]

\[
\nabla_T \cdot C : \frac{1}{2} [\nabla_T u + (\nabla_T u)^T] + iq_b \hat{z} \cdot C : \frac{1}{2} [\nabla_T u + (\nabla_T u)^T] + \\
\nabla_T \cdot C : \frac{1}{2} [iq_b u + (iq_b u)^T] + iq_b \hat{z} \cdot C : \frac{1}{2} [iq_b u + (iq_b u)^T] = \\
- \rho \omega^2 u.
\]
To obtain the weak form of Equation 5.3, we multiply it by a test function \( v^\ast \), integrate and apply the Green’s theorem in analogy to the 3D case [184], to obtain

\[
- \int_\Omega \varepsilon_T(v^\ast) : C : \varepsilon_T(u) \, dx + \int_\Omega \varepsilon_Z(v^\ast) : C : \varepsilon_T(u) \, dx \\
+ \int_\Omega \varepsilon_T(v^\ast) : C : \varepsilon_Z(u) \, dx + \int_\Omega \varepsilon_Z(v^\ast) : C : \varepsilon_Z(u) \, dx \\
+ \int_{\partial\Omega} v^\ast \cdot [\hat{n} \cdot C : \varepsilon_T(u)] \, dx = - \int_\Omega \rho \omega^2 v^\ast \cdot u \, dx
\]

where the symmetry of the stiffness tensor \( C_{ijkl} = C_{jikl} \), the notations \( \varepsilon_T(u) = \frac{1}{2} [\nabla_T u + (\nabla_T u)^T] \), as well as \( \varepsilon_Z(u) = \frac{1}{2} [iq_i u + (iq_i u)^T] \), have been utilized. Also, \( \Omega \) denotes the whole space, \( \partial\Omega \) the boundary and \( \hat{n} \) the normal to that boundary. Finding the elastic modes is equivalent to finding the eigenvectors of Equation 5.4, where \( \omega^2 \) is the eigenvalue.

5.4 Implementation

5.4.1 Description

FEniCS is an open-source FEM solver that automates significant portion of the finite-element assembly. The package contains the unified form language (UFL) [185], a domain-specific language for declaration of variational forms with syntax that follows mathematical notation. The core of our code consists of Equation 5.4, implemented in UFL. The notable difference is that UFL does not support complex numbers, thus Equation 5.4 is split into the real and imaginary parts that form a linear system of coupled equations. The Dolfin package [175] provides a high-level interface to various linear algebra packages that are need for efficient solution of the eigenvalue problem. The FIAT package [186] enables quick testing of different finite elements. For the tensorial elastic mode-solver, we have used the Lagrange finite elements and found them to be stable.
When the eigenvectors are expressed in terms of the basis functions of the finite elements, \( u = u_i \phi_i \) in Equation 5.4 becomes a generalized eigenvalue problem of the form \( S(\phi_i, \phi_j)u_i = -M_{ij}\omega^2 u_i \), where the eigenvalues are weighted by the mass matrix, \( M \), in our case being the material density. After splitting the complex coefficients into the real and imaginary parts, the weak form (Equation 5.4) forces the stiffness matrix, \( S \), to be symmetric, which ensures that \( \omega \) is real. The stiffness matrix is sparse, since \( S(\phi_i, \phi_j) \) is zero for nonadjacent finite elements. While solving the eigenvalue problem, we are interested only in a few modes close to the fundamental mode, therefore there is no need to find all the eigenvectors. Given these requirements, the appropriate linear algebra package is recognized to be SLEPc [187] with the Krylov-Shur algorithm and a spectral shift-and-invert preconditioner. SLEPc enables the parallel computation of eigenvalues, which is necessary for higher resolution. On a personal computer, with 16 GB of memory, we were able to compute eigenmodes for meshes up to 180 second-order Lagrange elements per facet.

Both the stress and the strain tensors are symmetric, therefore to minimize memory usage we use the Voigt notation [169], where the two tensors are mapped onto six-dimensional vectors, i.e., the stress-strain relation becomes

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{xz} \\
\sigma_{xy}
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66}
\end{bmatrix} \begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
2\epsilon_{yz} \\
2\epsilon_{xz} \\
2\epsilon_{xy}
\end{bmatrix}. \tag{5.5}
\]

Better numerical performance is obtained by using small numerical values, so dimensions of the parameters used in the solver are \([\rho] = g/cm^3 = 10^{-12} \text{ g}/\mu m^3\), \([C] = \text{GPa} = 10^{-12} \text{ g}/(\text{ns}^2\mu m)\), \([x, u] = \mu m\), which result in eigenvalues \([\omega] = \text{GHz} \) after cancellation of prefactors. It is fi-
nally reminded that the mode-solver finds two modes for each eigenvalue, corresponding to waves traveling in opposite directions [170].

5.4.2 Testing

In the case of homogeneous isotropic materials, Equation 5.1 can be rewritten as $-\rho \omega^2 \mathbf{u} = (2\mu + \lambda) \nabla(\nabla \cdot \mathbf{u}) - \mu \nabla \times \nabla \times \mathbf{u}$. Conveniently, for shear waves $\nabla \cdot \mathbf{u} = 0$ and the problem reduces to the Maxwell’s eigenvalue problem. If facets at $\pm a$ are clamped, then one of the solutions is $u_y = \cos(k_x x) \exp(iq_b z)$, with $u_x, u_z = 0$ and for the given boundary conditions $k_x = 2\pi/a$.

We use this fundamental mode to test the accuracy of the solver against a known solution. For the numerical simulations, we choose a $2\mu\text{m} \times 5\mu\text{m}$ waveguide of thermal oxide and $q_b = 11.67\mu\text{m}^{-1}$.

Figure 5.1: The error between the numerical and analytical eigenvalues plotted for Lagrange finite elements of different polynomial order. The convergence slopes for first, second, and third-order Lagrange elements are -0.77, -1.66 and -2.42, respectively.

The error is estimated as $|\omega_{\text{analytical}}^2 - \omega_{\text{numerical}}^2|$. This error is plotted against the resolution of the mesh for Lagrange elements of different polynomial orders [188] in Fig. 5.1. As expected,
for third-order Lagrange elements, the convergence rate exceeds the second-order finite-difference scheme \[182\].

5.5 Isotropic materials: chalcogenides

As the first example, the material parameters of a chalcogenide glass composition, As\(_2\)S\(_3\), embedded in thermal oxide are implemented \[176\]. Both materials are isotropic, hence the non-zero components of their stiffness tensor elements are \(c_{11} = c_{22} = c_{33} = 2\mu + \lambda\), \(c_{44} = c_{55} = c_{66} = \mu\). Also, the non-diagonal elements \(c_{12}, c_{13}, c_{23} = \lambda\) together with their mirrored components. For thermal oxide, \(\mu = 29.9\) GPa, \(\lambda = 15.4\) GPa and \(\rho = 2.2\) g/cm\(^3\) \[189\], while As\(_2\)S\(_3\) is expectedly softer with \(\mu = 6.2\) GPa, \(\lambda = 9.78\) GPa and \(\rho = 3.2\) g/cm\(^3\) \[176, 190\].

In the below comparison, the focus is on the fundamental longitudinal mode, \(u \approx u_z\). Following the optical fiber literature \[177\], the consensus so far has been to assume that the displacement in waveguides follows the scalar acoustic model \[171\]

\[ v_i^2 \nabla^2_T \tilde{\rho} - q_b^2 v_i^2 \tilde{\rho} = -\omega^2 \tilde{\rho}, \]  

(5.6)

where the longitudinal velocity is related by \(\rho v_i^2 = 2\mu + \lambda\) to the stiffness constants, and \(\tilde{\rho}\) refers to the change of material density. Since the displacement is only in the \(z\) direction, \(\tilde{\rho}\) is proportional to \(\rho u_z\), which implies that Equation 5.6 can be rewritten as \((2\mu + \lambda)\nabla^2_T u_z - q_b^2 (2\mu + \lambda) u_z = -\rho \omega^2 u_z\).

The eigenvalues of this and preceding equation are indeed identical. The issue with the acoustic model, however, is that it was originally developed for fluids and therefore neglects shear waves, or components of thereof. A better approach is to start with Equation 5 and enforce \(u = u_z\), upon which we arrive at

\[ \mu \nabla^2_T u_z - q_b^2 (2\mu + \lambda) u_z = -\rho \omega^2 u_z, \]

(5.7)
which is known as the scalar elastic equation and has been previously used to model SBS in fibers [172].

![Graph](image)

**Figure 5.2:** (a) The eigenvalue of the fundamental longitudinal mode of a square chalcogenide waveguide embedded in thermal oxide calculated using the scalar acoustic Equation 5.6, the scalar elastic Equation 5.7 and the full-tensorial elastic Equation 5.3. The computation was performed on the same mesh for each waveguide width. The acoustic model is too simplistic for waveguides with tight confinement. However, the scalar elastic equation remains accurate; (b) The cross-section of the eigenmodes calculated for 0.5-μm-wide waveguides using the three models. The plot shows the Z-displacement normalized to unity.

Figure 5.2a presents the fundamental longitudinal eigenvalues of chalcogenide waveguides, based on the three models (namely, Equations 5.3, 5.6 and 5.7) on the same mesh with second-order Lagrange elements. In all cases, the acoustic propagation constant is fixed to $q_b = 18.23 \, \mu m^{-1}$, hence the eigenfrequency increases with mode confinement. The figure clearly shows that the
scalar acoustic model is too simplistic for integrated devices with a discrepancy of 550 MHz for waveguides that are 0.5 \( \mu \text{m} \) wide. However, the scalar elastic equation remains accurate to within 100 MHz of the tensorial model for waveguides wider than 1 \( \mu \text{m} \). The modal cross-sections normalized to unity are shown in Figure 5.2b, which shows a large discrepancy between the scalar models and the tensorial model near the material interfaces.

Figure 5.3: A comparison of simulation time between the mode-solver in this work, and a fully-3D simulation in COMSOL\textsuperscript{TM}, confirming that the present solver is over 100 times faster. Additionally, the computational time increases faster with resolution for the 3D simulation than for the 2D SAFE method used in this work. The simulation was performed for chalcogenide waveguides embedded in oxide and backward scattering modes. In both cases, second-order Lagrange elements were used and 10 modes were calculated. COMSOL\textsuperscript{TM} was called using the LiveLink\textsuperscript{TM} Matlab interface for timing purposes. Resolution is understood as number of finite elements per facet. In the 3D simulation, Floquet boundary conditions were used and the size of elements in the \( z \)-direction was the same as in the \( xy \) plane.

Based on the detailed model presented later in section V B, we also calculate the Brillouin gain for chalcogenide waveguides. The physical parameters used in the simulation are as follows:
photoelastic constants of $p_{11} = 0.25$ $p_{12} = 0.24$ for As$_2$S$_3$, and $p_{11} = 0.12$ $p_{12} = 0.27$ for SiO$_2$ [191], waveguide dimensions of 4 $\mu$m by 0.85 $\mu$m and experimental $Q_{mech} = 226$ [176]. The phase-matching for back-scattering of the fundamental transverse-electric (TE) mode at 1544 nm dictates $q_b = 18.37$ $\mu$m$^{-1}$. Using the mode profile calculated using our solver, we obtain a gain coefficient of 322 W$^{-1}$m$^{-1}$, which agrees well with the scalar formulation, 321 W$^{-1}$m$^{-1}$, and the experimentally measured value of 311 W$^{-1}$m$^{-1}$ [176]. We note that for such large waveguides, the coupling is dominated by electrostriction and the radiation pressure is negligible, that is less than 1 W$^{-1}$m$^{-1}$.

Finally, the chalcogenide glass material system is used to highlight the computational benefits of using the SAFE method against a fully-3D simulation in COMSOL$^TM$ with Floquet boundary conditions. Figure 5.3 summarizes how this work provides two orders of magnitude decrease in computational time.

5.6 Anisotropic materials: silicon

5.6.1 Resonance frequency

Due to its high refractive index, it is anticipated that silicon should provide high Brillouin gain. Until recently, observing the gain has been elusive, because the acoustic velocity in the silica layers, typically used for cladding the waveguides, is lower than the acoustic velocity of silicon. In other words, the acoustic is leaky in standard silicon-on-insulator (SOI) waveguides. The solution to this problem is to use a suspended membrane [155] or a waveguide supported by a nanoscale strut [156]. It should be stressed that, due to the enhancement coming from radiation pressure, the strongest Brillouin gain in silicon arises from forward-propagating modes [180].

Herein, the solutions of our solver are compared against the 3D finite-element solution of COMSOL$^TM$. 

73
A rectangular silicon waveguide (450 nm × 230 nm) suspended in air is utilized, which means that \( \int_{\partial \Omega} dx = 0 \) in Equation 5.4. Also, the phase matching condition requires that the elastic wavevector be equal to the difference between the optical wavevectors, but to a good approximation the phonons oscillate in the \( xy \)-plane, thus we can look for modes with \( q_b = 0 \). Due to the symmetries of the stiffness tensor of silicon, a cubic material, the following identities hold true: \( c_{11} = c_{22} = c_{33} = 164 \) GPa, \( c_{44} = c_{55} = c_{66} = 79 \) GPa and the non-diagonal \( c_{12} = c_{13} = c_{23} = c_{21} = c_{31} = c_{32} = 64 \) GPa, while the rest of elements of the stiffness tensor are zero [156]. The density of silicon is 2.328 g/cm\(^3\).

The anisotropic nature of silicon necessitates the use of a fully-tensorial description of elasticity, as implemented in this work. As mentioned before and following the MEMS literature [167], some authors have chosen to use the simplified isotropic model for silicon [180]. While it is qualitatively fair, the isotropic model can lead to large discrepancies in quantitative values. In extreme cases, when the waveguide is aligned along the [100] or the [110] crystal directions, the differences in eigenfrequencies is as much as 0.8 GHz, as shown in Table 5.1. The modal profile is included for
completeness in Fig. 5.4a. Similar elastic anisotropy is expected from other cubic materials, such as germanium [192].

Figure 5.4: (a) The fundamental $x$-shear mode for an SOI waveguide in air; (b) The fundamental $x$-shear mode for an SOI waveguide in air.

5.6.2 Brillouin Forces and Gain

The presented elastic-wave mode-solver can be combined with an electromagnetic mode-solver, implemented in *FEniCS* and described in detail in [188], to calculate the forces acting on the optical waveguide. The electromagnetic mode-solver is based on minimizing the functional [188]
\[ F(E) = \int_{\Omega} \frac{1}{\mu_r} (\nabla_T \times E_T) \cdot (\nabla_T \times E_T) - k_0^2 \epsilon_r E_T \cdot E_T + \]
\[ \gamma^2 \left[ \frac{1}{\mu_r} \left( \nabla_T E_{z,\gamma} + E_T \right) \cdot \left( \nabla_T E_{z,\gamma} + E_T \right) - k_0^2 \epsilon_r E_z \cdot E_z \right] dx, \]

which comes from the wave equation with the \( E = (E_T + \gamma E_{z,\gamma}) \exp(-\gamma z) \) ansatz. Since for propagating modes, \( \gamma \) is imaginary, the transverse field is real and the \( z \)-field is imaginary. Also, \( \epsilon_r \) and \( \mu_r \) are assumed to be real and scalar.

There are three types of forces that contribute to the Brillouin gain [181]. First, there is bulk electrostriction, which is the dominant force in optical fibers. Given that there are two fields present in waveguides, described by \( E = \frac{1}{2} (E_p \exp(ik_p z - \omega_p t) + E_s \exp(ik_s z - \omega_s t) + c.c., \) the phase-matched component of the stress, \( \sigma_{ij}^{ES} \exp(iq_hz - \omega_{mech} t), \) is given by

\[ \sigma_{ij}^{ES} = -\frac{1}{4} \epsilon_0 n^4 p_{ijkl} (E_p k E_s^* + E_p E_{sk}^*), \]

where \( p \) is the electrostriction tensor and the force is

\[ f^{ES} = -\nabla_T \cdot \sigma^{ES} - iq_b \hat{z} \cdot \sigma^{ES}. \]

Second, there is a boundary electrostriction force at the interface of dielectrics 1 and 2, given by

\[ f_i^{BES} = (\sigma_{1ij}^{ES} - \sigma_{2ij}^{ES}) n_j, \]

where \( n_j \) is a normal pointing from material 1 to material 2. The third force, arising from radiation pressure, also appears only on the boundary and can be computed from Maxwell’s stress tensor.
Again, retaining only the phase-matched components, 
\[ T_{ij} \exp(\imath q_b z - \omega_{mech} t), \]
with
\[
T_{ij} = \frac{1}{2} \epsilon_0 \epsilon_r \left( (E_{pi}^* E_{sj}^* + E_{pj}^* E_{si}^*) - \delta_{ij} (E_{pk}^* E_{sk}^*) \right),
\]
the corresponding force from radiation pressure is given by
\[
f_i^{RP} = (T_{2ij} - T_{2ij}) n_j.
\]
Distribution of the forces and the shape of the elastic modes is necessary for the computation of the Brillouin gain coefficient. Since Brillouin coupling coefficient can be related to the photon generation rate through particle flux conservation [180] and the phonon generation rate is proportional to the power generated by optical forces, \( \int \Omega f \cdot \partial_t u^* dx \), the gain coefficient is expressed as [181]
\[
G = \frac{\omega_{opt} Q_{mech}}{4 P_{mech} P_s P_p} \left| \int \Omega f \cdot u^* dx \right|^2,
\]
where \( \omega_{opt} \) and \( \omega_{mech} \) refer to optical and mechanical frequencies of propagating waves, \( Q_{mech} \) is the mechanical quality-factor of the mode, \( P_{s,p} = \epsilon_0 \int \Omega E_s,p \cdot \epsilon_r E_{s,p}^* dx \) are the optical powers of the pump scattered field and \( P_{mech} = \frac{1}{2} \omega_{mech}^2 \int \Omega \rho u \cdot u^* dx \) is the mechanical power. The forces in the numerator are added coherently, i.e., \( f = f^{ES} + f^{RP} \).
Figure 5.5: Comparison of bulk and boundary forces and resulting gain for forward and backward Brillouin scattering originating from the fundamental TE mode and acting on a silicon waveguide suspended in air. The coupling is computed for the fundamental \(x\)-shear mode from Figure 5.4a and different crystal orientations. Bulk electrostriction forces are plotted in black, the boundary radiation pressure in blue and boundary electrostriction in red. In the case of back scattering in Figure (e) bulk electrostriction has a large imaginary component acting in the \(z\) direction in addition to lateral components. The gain dependence on crystal orientation shows discontinuities due to mode merging as explained in detail in text.

The simulations in Fig. 5.5 are performed for a rectangular silicon waveguide (450 nm \(\times\) 230 nm), suspended in air, and for the fundamental TE mode at the 1550 nm wavelength. We have assumed the photoelastic constants of \(p_{11} = -0.09\), \(p_{12} = 0.017\), \(p_{44} = -0.051\) and \(Q_{\text{mech}} = 249\) [156]. It is important to distinguish two cases. For the forward Brillouin scattering, \(E_s = E_p\) and \(q_b \approx 0\), since the frequency shift is small in comparison to the optical carrier frequency. In this case (Figs. 5.5a and 5.5b), all forces are real. The boundary electrostriction (ES) is weaker (11% when com-
paring the maxima) and in opposite direction to the radiation pressure (RP). For the [110] direction of propagation the dominant $x$-components of bulk electrostriction ($1684 \text{ W m}^{-1}$) and radiation pressure on the boundary ($151 \text{ W m}^{-1}$) add up constructively resulting in a total gain of $3099 \text{ W m}^{-1}$. This value is in good agreement with the experimental direct characterization of the gain yielding $3218 \text{ W m}^{-1}$ and indirect characterization through cross-phase modulation giving $3055 \text{ W m}^{-1}$ [156]. The total simulated gain for the [100] direction is $3896 \text{ W m}^{-1}$. This is to be expected from the $1/\omega_{\text{mech}}^2$ dependence of gain in Equation 6.1 and the aforementioned difference of $0.8 \text{ GHz}$ in resonance frequency between the two directions of propagation. Evidently, using the described fully-tensorial model is critical in modeling SBS in devices constructed from anisotropic materials.

For the backward Brillouin scattering, $E_s = E^*_p$ and $q_b = 2k_0n_{eff}$. In the software, the employed sign convention is negative wavevector, $-\gamma$, for the pump and positive, $+\gamma$, for the scattered field, which also implies positive wavevector, $+q_b$, for the elastic wave. In Fig. 5.5d, the boundary electrostriction (ES) is weaker (29% when comparing the maxima) and in opposite direction to the radiation pressure (RP). Since the dominant $x$-component of radiation pressure in Fig. 5.5d acts in antiphase to the bulk electrostriction in Fig. 5.5e, the total Brillouin gain is smaller than the electrostriction component alone, as shown in Fig. 5.5f. The value of total gain for the [100] direction is $380 \text{ W m}^{-1}$.

The features of backward SBS is even more complex than forward SBS and is best explained with the aid of a dispersion diagram from Fig. 5.6, where the resonant frequencies are plotted versus the crystal orientation with respect to the direction of propagation. For the gain coupling calculation, we follow the fundamental symmetric mode marked with a blue "S" in Fig. 5.6 at $24.513 \text{ GHz}$ and $Q_{\text{mech}} = 249$. We note that, in general, the elastic mode is not symmetric for cubic materials such as silicon [193], unlike the optical mode. Thus, the initially symmetric mode acquires a degree of asymmetry as the crystal orientation is tilted, to the point that at $3^\circ$, the symmetric- and
the antisymmetric-mode branches merge. This is the origin of the first discontinuity in the gain diagram in Fig. 5.5f. The next discontinuity at 42° occurs when the common branch splits again into the symmetric and antisymmetric modes. The optical mode is symmetric, thereby the optical force distribution is symmetric, so the coupling to the asymmetric modes is null. Therefore, the Brillouin gain drops as the crystal orientation is rotated away from the [100] or [110] directions, since the elastic modes acquire a degree of asymmetry.

Figure 5.6: Resonant elastic frequencies of an silicon waveguide, suspended in air, for various crystal orientations. For an arbitrary direction, the waveguide is not symmetric elastically, thus merging of the symmetric (S) and asymmetric modes (AS) is observed.

5.7 Conclusions

In summary, we have presented a fully-tensorial elastic-wave mode-solver and discussed its implementation in the open-source finite element solver, FEniCS. The source code is available at
https://github.com/MarcinJM/PySBS. The use of 2D SAFE method in this work leads to computational times smaller by two orders of magnitude than the commercial 3D finite-element solver, COMSOL\textsuperscript{TM}. We have also performed a thorough review and comparison of a multitude of approximations used in simulating the elastic modes in the SBS literature and compared them against the present fully-tensorial model. In some cases, the discrepancy in the calculated eigenvalues was found to be as large as 0.8 GHz. We have shown that for silicon with arbitrary crystal orientations, the elastic modes do not need to have the same symmetry as the optical modes, which greatly affects the Brillouin coupling coefficient. The modeling tool is made publicly available to other researchers and is expected to be useful in understanding and tailoring SBS in integrated devices.
CHAPTER 6: STIMULATED BRILLOUIN SCATTERING IN THE ALL-SILICON OPTICAL PLATFORM

For large waveguides (wider than 1 µm) the electrostriction is the dominant force in opto-elastic coupling. In this case the Brillouin gain is given by

\[ g_B = \frac{4 \cdot \pi \cdot n^8 \cdot p_{12}^2}{\lambda_p^3 \cdot c \cdot \rho \cdot v_B \cdot \Delta v_B \cdot \eta}, \]  

(6.1)

where \( n \) is the refractive index, \( p_{12} \) - the electrostriction coefficient, \( \lambda_p \) is the pump wavelength, \( c \) is the speed of light, \( \rho \) is material density, \( v_B \) and \( \Delta v_B \) are the elastic resonances frequency and its linewidth, respectively, and \( \eta \) is the elasto-optic overlap.

The careful reader will notice the strong, \( n^8 \), dependence of the Brillouin gain coefficient. This relationship implies that materials with high refractive index, such as silicon are a perfect candidate for building integrated devices that utilize SBS for microwave signal processing. Regrettably, the standard and mature SOI technology is not conducive for SBS experiments. The BOX layer that provides refractive index contrast in SOI, does not give the acoustic impedance contrast that is needed by elastic waves. Actually, the situation is reversed and the elastic waves leak into the
The solution to this predicament is to use the all-silicon optical platform (ASOP), originally developed for mid-IR applications [194]. ASOP is manufactured from two wafers, an initial silicon wafer with patterned trench layer and an SOI wafer that is bonded from top, with the thin 220 nm Si layer bonded to the handle wafer. Than the top of the SOI layer is etched down and the BOX layer is removed with HF. This results in a thin silicon membrane suspended above predefined trenches. The membrane is further patterned and a 120 nm ridge is formed on top. To ensure confinement of the elastic waves additional lithography step is required to defined gaps at the sides of the waveguide. The dimensions are summarized in figure 6.1.

![SBS experiment diagram](image)

**Figure 6.2: SBS experiment**

For the SBS experiment the 3 cm long sample was cleaved at both ends leading to 30 dB total insertion loss. To detect the SBS resonance in the forward scattering configuration a 1550 nm laser was used together with a Mach-Zehnder (MZ) amplitude modulator. A signal from a VNA was swept and than amplified in an RF amplifier and fed to the MZ modulator. This lead to a creation of side-tones on the original CW signal as in figure 6.3. This signal was further amplified in a 10 W EDFA. As originally symmetric signal was swept in frequency, a resonance was observed
at 4.30 GHz, which agrees well with the simulated value of 4.24 GHz. At this point, provided that the power is large enough, the observed spectrum develops asymmetry because the higher energy photons (short wavelengths) scatter to the creating a photon and a photon that is lower in wavelength. The process saturates due to two-photon absorption in the waveguides as shown in figure 6.3. The observed asymmetry was only 2 dB, which corresponds to SBS loss and gain. This value was limited by relatively large, 1.7 dB/cm propagation losses of the sample shown in figure 6.4.

Figure 6.3: Measured SBS response. As the EDFA power is increased the sidebands develop asymmetry due to stimulated Brillouin scattering. The process saturates due to two-photon absorption.

Figure 6.4: Insertion loss of ASOP waveguides with gratings.
CHAPTER 7: CONCLUSIONS AND FUTURE WORK

7.1 Beyond two-photon absorption in silicon

In the previous chapter SBS was demonstrated in the ASOP platform. The relatively weak response of the the waveguides is due to a combination of high linear (1.7 dB/cm) and nonlinear losses. To quantify these statements, numerical simulations are performed. The geometry of the waveguides used in the simulation is similar to the ones fabricated and measured in the previous chapter that is 1 µm wide, partially etched (80 nm) waveguide from a 220 nm thick membrane that is 3 µm wide. Again, the waveguide is suspended in air through thin tethers, thus in simulations free boundary condition is assumed. The other physical constants used in the simulation are the non-zero coefficients of the stiffness tensor $c_{11} = c_{22} = c_{33} = 164$ GPa, $c_{44} = c_{55} = c_{66} = 79$ GPa and the non-diagonal $c_{12} = c_{13} = c_{23} = c_{21} = c_{31} = c_{32} = 64$ GPa, the density of 2.328 g/cm$^3$, photoelastic constants of $p_{11} = -0.09$, $p_{12} = 0.017$, $p_{44} = -0.051$ and $Q_{mech} = 680$ [155]. For nonlinear losses we assume a steady state equation of

$$\frac{dP}{dz} = -\alpha P - \beta P^2 - \gamma P^3,$$

(7.1)

with linear losses, $\alpha$ of 1.7 dB/cm, two-photon absorption coefficient of $\beta = 48$ W$^{-1}$m$^{-1}$, resulting in free-carrier-induced losses of $\gamma = 2550$ W$^{-2}$m$^{-1}$ [155].
Figure 7.1: The fundamental elastic mode and the fundamental TE mode at 2200 nm wavelength of the ASOP waveguides together with the x-component of the electrostriction force responsible for majority of the coupling between these modes.

We distinguish two cases a 1550 nm pump subject to nonlinear losses from equation 7.1 and a 2200 nm pump, that is just outside the two-photon absorption region and thus $\beta$ and $\gamma$ are set to zero. The coupling is calculated between the fundamental elastic mode (4.23 GHz) and the fundamental TE optical mode. The modes are plotted in figure 7.1. The gain for the 1550 nm pump was calculated to be 779 W$^{-1}$m$^{-1}$ at 1.55 $\mu$m, whereas the corresponding value for the 2200 nm pump was found to be 368 W$^{-1}$m$^{-1}$. These values are used to simulate the evolution of the signal assuming a 0.5W coupled power in the waveguide. As evident from the plot in figure 7.2, only the 2200 nm pump experiences measurable gain of 9 dB in 5 cm of propagation length.
Figure 7.2: Due to strong two photon absorption only the 2200 nm experiences measurable gain.

Figure 7.3: The nonlinear and linear losses for the 1550 nm pump. For 0.5 W, the total losses correspond to 29 dB/cm, and at 125 mW the nonlinear losses are the same as the linear losses (1.7 dB/cm)

This can be explained by the strong nonlinear losses in the system as shown in figure 7.3. For 0.5 W pump and 1550 nm, the total nonlinear loss at the beginning of the waveguide is a staggering 29 dB/cm which quickly attenuates the pump in the first few mm of propagation.
7.2 Filter design

Figure 7.4: Principle of RF filter design using two elastically coupled waveguides.

As demonstrated in [195] SBS in elastically coupled waveguides can be used to realize narrow band RF filters. The principle can be explained as follows. First, the broadband RF signal is encoded onto the optical carrier using intensity modulation, creating two symmetric sidebands around the main CW tone. This modulated signal is coupled into the Write waveguide in diagram 7.4. The portion of the RF signal, that corresponds to the Brillouin resonance frequency, $\omega_{SBS}$, excites the elastic mode of the whole membrane through electrostriction. The membrane oscillations lead to refractive index modulations (phase modulation) in the Read waveguide. Another CW tone can be coupled into the Read waveguide to read off the phase-modulation. We note that only the RF frequency that corresponds to $\omega_{SBS}$, is coupled and thus effectively, the system acts as a narrowband, $\Delta \omega_{SBS}$, RF filter. $\Delta \omega_{SBS}$ is typically 30 MHz.

In reference [195], the phase-modulation is converted to intensity modulation by the use of a narrowband Bragg grating that rejects one of the side-bands created by phase-modulation, thus they no longer cancel (normally they are in antiphase), and a beat signal can be observed on a photodiode. This comes at a penalty of 3 dB, as some of the signal is rejected. Thus the first
trivial improvement would be to construct a Mach-Zehnder out of the second waveguide that would perform an on-chip phase to intensity transformation. The proposed experiment is shown in figure 7.5.

![Figure 7.5](image)

Figure 7.5: A proposed microwave filter design inside an characterization setup. Phase modulation from SBS is converted to intensity modulation in an on-chip interferometer in analogy to Mach-Zehnder modulators.

The next proposed improvement is include the read waveguide in a ring cavity, with FSR corresponding to $\omega_{SBS}$. This system would be similar to a silicon microring modulator driven on resonance and we could use the phenomena of gain peaking [196] to increase the sensitivity of the system and reduce the optical power requirement. The schematic for this experiment is shown in figure 7.6.

![Figure 7.6](image)

Figure 7.6: A proposed microwave filter design inside an characterization setup. Phase modulation from SBS is converted to intensity modulation using a resonant microring structure. The microring provides additional improvement to sensitivity in a fashion similar to gain peaking in microring modulators.
The topics covered in this dissertation encompass the broad world of nonlinear optics with the intention of harnessing them for signal processing. In the first portion of the thesis novel nonlinear devices were developed for frequency stabilization of an on-chip frequency comb source. The devices included chalcogenide waveguides for supercontinuum generation and thin-film periodically poled lithium niobate for frequency doubling. The functioning of these components was tested in various system level configurations, octave spanning supercontinuum was generated first from a custom fiber mode-locked laser and then from the InP semiconductor laser, second harmonic was generated from thin-film PPLN and fiber supercontinuum source and then from on-chip supercontinuum and bulk PPLN. These demonstrations required the development of a custom fiber mode-locked laser and a custom Thulium-doped fiber amplifier. Theoretical work was carried out on hybrid architecture using a semiconductor mode-locked laser and a microring in synchronously pumped configuration that is promising due to high power efficiency.

In the second portion of the dissertation stimulated Brillouin scattering in silicon was studied. An open-source code was developed to calculate the elastic and optical modes of the system and the electrostrictive and boundary forces responsible for coupling between these modes. Initial demonstration of stimulated Brillouin scattering in the all-silicon optical platform was carried out.
% Marcin Malinowski
% simulating an in band pumped TDFA

clear;
close all;

%%%% physical parameters in a structure %%%%%
c0 = 299792458; % m / s
h0 = 6.62607004e−34; % m2 kg / s
params.Nt = 1.37e25; % thulium concentration , m−3
params.tau2 = 250e−6; % lifetime of 3F4 level
params.lambda_p1 = 1565e−9; % m pump wavelength
params.lambda_sig = 1990e−9; % m signal wavelength
params.sigma_a_p = 1.8e−25; % m2 laser absorption cross section
  at lambda 1
params.sigma_e_p = 0.06e−25; % m2 laser emission cross section at
  lambda 1
params.sigma_a_sig = 0.26e−25; % m2 absorption cross section signal
params.sigma_e_sig = 2.88e−25; % m2 emission cross section signal
params.A = 6.36e−11; % m2 area cross section of the core
params.gamma_p = 0.7; % pump confinement factor
params.gamma_s = 0.7; % signal confinement factor
params.alpha_p = 2.5e−2; % m−1 intrinsic absorption at lambda 1
params.alpha_s = 3.9e−2; % m−1 intrinsic absorption for the
  signal
params.delta_lambda = 100e−9; % bandwidth of ASE at 2 um
L = 5.77; % length of the TDFA
P_pump = 1.5; % pump power [W]
P_sig = 10^−7.0; % signal power [W]
N_grid = 100;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% relaxation algorithm for solving the TDFA equations

DL = L / N_grid;
p_vec = zeros(N_grid+1,4);
error_threshold = 1e−5;

% first iteration

ASE_b_guess = 0.0;
p_vec(1,:) = [P_pump, P_sig, 0.0, ASE_b_guess];
TDFA_eq = @(z,p) TDFA_model(z,p, params);

% forward propagation
for i = 1:N_grid
    [z,p] = ode45(TDFA_eq,[0.0 dL],p_vec(i,:)); % pump, signal, ASE_f, ASE_b
    n_sol = length(p);
    p_vec(i+1,:) = p(n_sol,:);
    clear p z;
end

P_sig_test = p_vec(N_grid+1,2);
error = 1.0;

% propagate back and forth till error threshold is reached
while error>error_threshold
    % backward propagation
    p_vec(N_grid+1,4) = 0.0; % set ASE_b(z=L) = 0.0
    for i = 1:N_grid
        [z,p] = ode45(TDFA_eq,[0.0 -dL],p_vec(N_grid+2-i,:)); % pump, signal, ASE_f, ASE_b
        n_sol = length(p);
        p_vec(N_grid+1-i,:) = p(n_sol,:);
        clear p z;
    end

    % forward propagation
    p_vec(1,1) = P_pump;
    p_vec(1,2) = P_sig;
    p_vec(1,3) = 0.0; % set ASE_f(z=0) = 0.0
    for i = 1:N_grid
        [z,p] = ode45(TDFA_eq,[0.0 dL],p_vec(i,:)); % pump, signal, ASE_f, ASE_b
        n_sol = length(p);
        p_vec(i+1,:) = p(n_sol,:);
        clear p z;
    end

    error = abs(P_sig_test - p_vec(N_grid+1,2));
    P_sig_test = p_vec(N_grid+1,2);
% In band pumped TDFA

function dpdz = TDFA_model(z,p, params)
% TDFA model for in band pumping
% includes ASE, only forward pumping
% notation
p(1) = forward pump power
p(2) = signal power
p(3) = ASE forward
p(4) = ASE backward
dpdz = zeros(4,1);

persistent Nt tau2 lambda_p1 lambda_sig sigma_a_p;
persistent sigma_e_p sigma_a_sig sigma_e_sig A gamma_p;
persistent gamma_s alpha_p alpha_s delta_lambda;
persistent h0 c0;

c0 = 299792458; % m / s
h0 = 6.62607004e−34; % m2 kg / s

Nt = params.Nt; % thulium concentration, m−3
tau2 = params.tau2; % lifetime of 3F4 level
lambda_p1 = params.lambda_p1; % m pump wavelength
lambda_sig = params.lambda_sig; % m signal wavelength
sigma_a_p = params.sigma_a_p; % m2 laser absorption cross section at lambda 1
sigma_e_p = params.sigma_e_p; % m2 laser emission cross section at lambda 1
\begin{verbatim}
sigma_a_sig = params.sigma_a_sig; % m2 absorption cross section signal 1900 nm
sigma_e_sig = params.sigma_e_sig; % m2 emission cross section signal 1900 nm
A = params.A; % m^2 area cross section of the core
gamma_p = params.gamma_p; % pump confinement factor
gamma_s = params.gamma_s; % signal confinement factor
alpha_p = params.alpha_p; % m^{-1} intrinsic absorption at lambda 1
alpha_s = params.alpha_s; % m^{-1} intrinsic absorption for the signal
delta_lambda = params.delta_lambda;

w_p12 = lambda_p1*gamma_p/(h0*c0*A)*sigma_a_p*p(1);
w_p21 = lambda_p1*gamma_p/(h0*c0*A)*sigma_e_p*p(1);
w_s12 = lambda_sig*gamma_s/(h0*c0*A)*sigma_a_sig*(p(2) + p(3) + p(4));
w_s21 = lambda_sig*gamma_s/(h0*c0*A)*sigma_e_sig*(p(2) + p(3) + p(4));

% assume steady state for the populations
N2 = (w_p12-w_s12)*Nt./(w_p12 + w_p21 + 1.0/tau2 + w_s21 - w_s12);
N1 = Nt - N2;

dpdz(1) = p(1)*(gamma_p*(sigma_e_p*N2 - sigma_a_p*N1)-alpha_p); % forward pump power
dpdz(2) = p(2)*(gamma_s*(sigma_e_sig*N2 - sigma_a_sig*N1)-alpha_s); % signal power
dpdz(3) = p(3)*(gamma_s*(sigma_e_sig*N2 - sigma_a_sig*N1)-alpha_s) + ...
                2*sigma_e_sig*N2*h0*c0*c0/lambda_sig^3*delta_lambda; % forward ASE

dpdz(4) = -p(4)*(gamma_s*(sigma_e_sig*N2 - sigma_a_sig*N1)-alpha_s) - ...
                2*sigma_e_sig*N2*h0*c0*c0/lambda_sig^3*delta_lambda; % backward ASE
\end{verbatim}

\textit{Lugiato-Lefever solver}

1 % this code solves the driven nonlinear schrodinger equation
2 % the parameters are non dimentional
3 % solved in the moving frame of reference
4 % solved using the split-fourier method and an adaptive step

95
% the source term is linearized i.e. dEdt = S dz
% Marcin Malinowski

% solving normalized Lugiato-Lefever equation
% dEdt = (-alpha - i*delta)E + i|E|^2E + i d^2E/dt + S
%

% % % % % % % % % % % % % % % % % simulation parameters % % % % % % % % % % % % % % % % %

close all; % close all figures
clear;

delta_init = 0.0;
delta_final = 4.0;
Tmax = 20;
end_slow_time = 100;
noise_lvl = 1.0e-14;
detuning_array = [0.0, 2, 4.16];
detuning_time = [0.0, 20.00, 80.0];

% % % % % % % % % % % % % % % % % numerical parameters % % % % % % % % % % % % % % % % %

nt = 2^8; % number of points
max_num_steps = 1e7;
initial_dtau = end_slow_time/1.0e3;
tolerance = 1e-5;
maximum_dtau = end_slow_time/100; % safety net
dT = 2*Tmax/nt;
T = (-nt/2.0:nt/2.0-1)*dT;
omega = pi/Tmax*[(0:nt/2.0-1) (-nt/2.0: -1)];
snapshot_time = (0:0.001:1)*end_slow_time;

% % % % % % % % % % % % % % % % % define input field % % % % % % % % % % % % % % % % %
E_in = 2.0*exp(-(T/8.0).^2);

% % % % % % % % % % % % % % % % % initialize main loop % % % % % % % % % % % % % % % % %
%rng(’default’);
%rng(0); % seed always the same random number
random_sequence1 = rand([nt,1]);
%rng(1);
random_sequence2 = rand([nt,1]);
A0 = zeros(nt,1) + noise_lvl.*(random_sequence1 + 1.0i*random_sequence2); %initial field distribution
init_vect = zeros(nt,1);
AA_const = complex(init_vect,1);
f_temp = complex(init_vect,1);
temp = complex(init_vect,1);
AA_coarse = complex(init_vect,1);
AA_fine = complex(init_vect,1);
AA_snapshot = zeros(nt,length(snapshot_time));
dtau = initial_dtau;
dispersion = -1i*omega.^2;
AA_const = A0;

%adaptive step size, error analysis
relative_error_plot = zeros();
dtau_changes = zeros();
position_changes = zeros();
check_stationary = zeros();
F = zeros();
counter = 1;
snapshot_counter = 1;
tau = 0.0; %distance
delta = 0.0;

while tau < end_slow_time
    for i = 1:length(detuning_array)
        if detuning_time(i) < tau
            delta = detuning_array(i);
        end
    end

% for stationary analysis
old_AA_const = AA_const;
old_dtau = dtau;

%take snapshots
if tau > snapshot_time(snapshot_counter)
    AA_snapshot(:, snapshot_counter) = AA_const;
end
snapshot_counter = snapshot_counter + 1;
end

%perform error analysis
%step once using current dau
linear_operator = exp((dispersion - 1.0 - 1i*delta)*0.5*dtau);
nonlinear_phase = 1.0i*dtau;
f_temp = ifft(AA const).*linear_operator;  \%linear step
AA coarse = fft(f_temp);
temp = AA coarse.*exp(abs(AA coarse).^2.*nonlinear_phase)+
E_in*dtau; \%nonlinear step evaluated half way through
f_temp = ifft(temp).*linear_operator;  \%again with 0.5 dau
AA coarse = fft(f_temp);

%step twice using dtau/2
dtau = dtau/2.0;
linear_operator = exp(((dispersion - 1.0 - 1i*delta)*0.5*dtau);
nonlinear_phase = 1.0i*dtau;
step 1
f_temp = ifft(AA const).*linear_operator;  \%linear step
AA fine = fft(f_temp);
temp = AA fine.*exp(abs(AA fine).^2.*nonlinear_phase)+E_in*
dtau;
\%we can save time by applying linear operator with dtau only
\%once
linear_operator = exp(((dispersion - 1.0 - 1i*delta)*dtau);
\%step 2
f_temp = ifft(temp).*linear_operator;  \%linear step
AA fine = fft(f_temp);
temp = AA fine.*exp(abs(AA fine).^2.*nonlinear_phase)+E_in*
dtau;
linear_operator = exp(((dispersion - 1.0 - 1i*delta)*0.5*dtau);
\%again 0.5 dz
f_temp = ifft(temp).*linear_operator;  \%linear step
AA fine = fft(f_temp);
\%calculate and store error
error = sum((abs(AA fine-AA coarse)))/sum(abs(AA coarse));
relative_error_plot(counter) = error;

if error < 0.5*tolerance %error is too small
  %perform the next step with dtau*2.0
  if 2.0*dtau < maximum_dtau
    tau = tau +2.0*dtau; %was decreased for error analysis
    AA_const = 4.0/3.0*AA_fine - 1.0/3.0*AA_coarse; % more accurate value
    dtau = 4.0*dtau; %use larger step
dtau_changes(counter) = dtau;
position_changes(counter) = tau;
else
  dtau = 2.0*dtau;
display( 'Warning! dtau reached maximum_dtau!');
dtau_changes(counter) = dtau;
position_changes(counter) = tau;
tau = tau + dtau;
AA_const = 4.0/3.0*AA_fine - 1.0/3.0*AA_coarse; % more accurate value
end

counter = counter +1;

elseif error > 2.0*tolerance %error is too large
  %perform the next step with dtau/2.0
  tau = tau +2.0*dtau; %was decreased for error analysis
  AA_const = 4.0/3.0*AA_fine - 1.0/3.0*AA_coarse; % more accurate value
  %keep the current dtau/2.0 from last step
dtau_changes(counter) = dtau;
position_changes(counter) = tau;
counter = counter+1;
else
  dtau = 2.0*dtau; %go back to original dtau
dtau_changes(counter) = dtau;
position_changes(counter) = tau;
tau = tau + dtau; %was decreased for error analysis
AA_const = 4.0/3.0*AA_fine - 1.0/3.0*AA_coarse; % more accurate value
end

counter = counter +1;
if counter > max_num_steps
    display('Exceeded the maximum number of steps. Terminating !')
    break;
end

%check if the solution satisfies the stationary NLS \( \frac{\partial A}{\partial \tau} = 0 \)
check_stationary(counter) = sum((AA_const - old_AA_const).^2) / old_dtau;
F(counter) = ...
    sum(abs((-1.0 -i*delta + fft(dispersion.*ifft(AA_const)) +i*abs(AA_const).^2).*AA_const + E_in));
end

fprintf('The end length of the simulation is %f . \n', tau);

figure();
surfl(abs(AA_snapshot))
shading interp; colormap(hot); rotate3d on;
APPENDIX B: COPYRIGHT
This Agreement between CREOL, University of Central Florida -- Marcin Malinowski ("You") and AIP Publishing ("AIP Publishing") consists of your license details and the terms and conditions provided by AIP Publishing and Copyright Clearance Center.

License Number 4595450869869
License date May 24, 2019
Licensed Content Publisher AIP Publishing
Licensed Content Publication Applied Physics Letters
Licensed Content Title Low-loss, submicron chalcogenide integrated photonics with chlorine plasma etching
Licensed Content Author Jeff Chiles, Marcin Malinowski, Ashutosh Rao, et al
Licensed Content Date Mar 16, 2015
Licensed Content Volume 106
Licensed Content Issue 11
Type of Use Thesis/Dissertation
Requestor type Author (original article)
Format Electronic
Portion Excerpt (> 800 words)
Will you be translating? No
Title of your thesis / dissertation THIRD-ORDER OPTICAL NONLINEARITIES FOR INTEGRATED MICROWAVE PHOTONICS APPLICATIONS
Expected completion date Jun 2019
Estimated size (number of pages) 150
Requestor Location CREOL, University of Central Florida
4304 Scorpius St
ORLANDO, FL 32816
United States
Attn: CREOL, University of Central Florida

Total 0.00 USD

Terms and Conditions

AIP Publishing -- Terms and Conditions: Permissions Uses

AIP Publishing hereby grants to you the non-exclusive right and license to use and/or distribute the Material according to the use specified in your order, on a one-time basis, for the specified term, with a maximum distribution equal to the number that you have ordered. Any links or other content accompanying the Material
are not the subject of this license.

1. You agree to include the following copyright and permission notice with the reproduction of the Material: “Reprinted from [FULL CITATION], with the permission of AIP Publishing.” For an article, the credit line and permission notice must be printed on the first page of the article or book chapter. For photographs, covers, or tables, the notice may appear with the Material, in a footnote, or in the reference list.

2. If you have licensed reuse of a figure, photograph, cover, or table, it is your responsibility to ensure that the material is original to AIP Publishing and does not contain the copyright of another entity, and that the copyright notice of the figure, photograph, cover, or table does not indicate that it was reprinted by AIP Publishing, with permission, from another source. Under no circumstances does AIP Publishing purport or intend to grant permission to reuse material to which it does not hold appropriate rights. You may not alter or modify the Material in any manner. You may translate the Material into another language only if you have licensed translation rights. You may not use the Material for promotional purposes.

3. The foregoing license shall not take effect unless and until AIP Publishing or its agent, Copyright Clearance Center, receives the Payment in accordance with Copyright Clearance Center Billing and Payment Terms and Conditions, which are incorporated herein by reference.

4. AIP Publishing or Copyright Clearance Center may, within two business days of granting this license, revoke the license for any reason whatsoever, with a full refund payable to you. Should you violate the terms of this license at any time, AIP Publishing, or Copyright Clearance Center may revoke the license with no refund to you. Notice of such revocation will be made using the contact information provided by you. Failure to receive such notice will not nullify the revocation.

5. AIP Publishing makes no representations or warranties with respect to the Material. You agree to indemnify and hold harmless AIP Publishing, and their officers, directors, employees or agents from and against any and all claims arising out of your use of the Material other than as specifically authorized herein.

6. The permission granted herein is personal to you and is not transferable or assignable without the prior written permission of AIP Publishing. This license may not be amended except in a writing signed by the party to be charged.

7. If purchase orders, acknowledgments or check endorsements are issued on any forms containing terms and conditions which are inconsistent with these provisions, such inconsistent terms and conditions shall be of no force and effect. This document, including the CCC Billing and Payment Terms and Conditions, shall be the entire agreement between the parties relating to the subject matter hereof.

This Agreement shall be governed by and construed in accordance with the laws of the State of New York. Both parties hereby submit to the jurisdiction of the courts of New York County for purposes of resolving any disputes that may arise hereunder.

V1.2

Questions? customercare@copyright.com or +1-855-239-3415 (toll free in the US) or +1-978-646-2777.
Fully-Tensorial Modeling of Stimulated Brillouin Scattering in Photonic Waveguides

Marcin Malinowski

Quantum Electronics, IEEE Journal of

IEEE

June 2019

Thesis / Dissertation Reuse

The IEEE does not require individuals working on a thesis to obtain a formal reuse license, however, you may print out this statement to be used as a permission grant:

Requirements to be followed when using any portion (e.g., figure, graph, table, or textual material) of an IEEE copyrighted paper in a thesis:

1) In the case of textual material (e.g., using short quotes or referring to the work within these papers) users must give full credit to the original source (author, paper, publication) followed by the IEEE copyright line © 2011 IEEE.
2) In the case of illustrations or tabular material, we require that the copyright line © [Year of original publication] IEEE appear prominently with each reprinted figure and/or table.
3) If a substantial portion of the original paper is to be used, and if you are not the senior author, also obtain the senior author's approval.

Requirements to be followed when using an entire IEEE copyrighted paper in a thesis:

1) The following IEEE copyright/credit notice should be placed prominently in the references: © [year of original publication] IEEE. Reprinted, with permission, from [author names, paper title, IEEE publication title, and month/year of publication]
2) Only the accepted version of an IEEE copyrighted paper can be used when posting the paper or your thesis on-line.
3) In placing the thesis on the author's university website, please display the following message in a prominent place on the website: In reference to IEEE copyrighted material which is used with permission in this thesis, the IEEE does not endorse any of [university/educational entity's name goes here]'s products or services. Internal or personal use of this material is permitted. If interested in reprinting/republishing IEEE copyrighted material for advertising or promotional purposes or for creating new collective works for resale or redistribution, please go to http://www.ieee.org/publications_standards/publications/rights/rights_link.html to learn how to obtain a License from RightsLink.

If applicable, University Microfilms and/or ProQuest Library, or the Archives of Canada may supply single copies of the dissertation.
LIST OF REFERENCES


[15] Jason E Stalnaker, Vela Mbele, Vladislav Gerginov, Tara M Fortier, Scott A Diddams, Leo Hollberg, and Carol E Tanner. Femtosecond frequency comb measurement of abso-


[54] Nayara Jornod, Kutan Gürel, Valentin J Wittwer, Pierre Brochard, Sargis Hakobyan, Stéphane Schilt, Dominik Waldburger, Ursula Keller, and Thomas Südmeyer. Carrier-


[75] Michael R.E. Lamont, Barry Luther-Davies, Duk-Yong Choi, Steve Madden, and Benjamin J. Eggleton. Supercontinuum generation in dispersion engineered highly nonlinear
\( \gamma = 10 \ \text{w/m} \) as2s3 chalcogenide planar waveguide. *Opt. Express*, 16(19):14938–14944, Sep 2008.


115


[159] Adam Byrnes, Ravi Pant, Enbang Li, Duk-Yong Choi, Christopher G Poulton, Shanhui Fan, Steve Madden, Barry Luther-Davies, and Benjamin J Eggleton. Photonic chip based


[193] Xiao-Xing Su, Xiao-Shuang Li, Yue-Sheng Wang, and Heow Pueh Lee. Theoretical study on the stimulated brillouin scattering in a sub-wavelength anisotropic waveguide: acousto-

