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Crossed-gratings volume hologram: backward reflection with high angular and spectral selectivity

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Abstract: A new type of reflective hologram with both high angular and spectral selectivity is suggested. With novel design of crossed gratings inside the volume hologram, unusual angular selectivity is achieved via coupling of four waves. Analytic solution with spectral and angular detuning from Bragg regime is derived. The compensation between these two detuning forms an interesting figure of saddle shape.

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References and links

1. O.M. Efimov, L.B. Glebov, L.N. Glebova, K.C. Richardson, and V.I. Smirnov, "High-Efficiency Bragg Gratings in Photothermorefractive Glass." *Appl. Optics, Optical Technology and Biomedical Optics (OT&BO)*, **38**, 619-627 (1999).
2. L.B. Glebov, V.I. Smirnov, C.M. Stickley, I.V. Ciapurin, "New approach to robust optics for HEL systems", In *Laser Weapons Technology III*, W.E. Thompson and P.H. Merritt, eds., *Proc. SPIE*, **4724**, 101-109 (2002).
3. L.B. Glebov, "Volume hologram recording in inorganic glasses," *Glass Science and Technology* **75** C1, 73-90 (2002).
4. D.K. Jacob, S.C. Dunn, M. G. Moharam, "Normally incident resonant grating reflection filters for efficient narrow-band spectral filtering of finite beams", *J. Opt. Soc. Am. B* **18**, 2109-2120 (2001).
5. C.-C. Tsai, L. B. Glebov, B. Ya. Zeldovich, "Adiabatic three-wave volume hologram: large efficiency independent on grating strength and polarization", *Opt. Lett.* **31**, 718-720 (2006)
6. J. Zhao, Pochi Yeh, M. Khoshnevisan, and I. McMichael, "Diffraction properties of vector synthetic volume index gratings", *J. Opt. Soc. Am. B* **17**, 898-903 (2000).
7. R. Alferness, S.K. Case, "Coupling in doubly exposed, thick holographic gratings", *J. Opt. Soc. Am.*, **65**, 730-739 (1975)
8. H. Kogelnik, "Coupled Wave Theory for Thick Hologram Gratings", *Bell Syst. Tech. J.*, **48**, 2909 (1969)
9. S. T. Peng, T. Tamir, and H. L. Bertoni, "Theory of periodic dielectric waveguides", *Trans. Microwave Theory Tech.*, *MTT-23*, 123-133 (1975).
10. M. G. Moharam and T. K. Gaylord, "Rigorous coupled-wave analysis of planar-grating diffraction," *J. Opt. Soc. Am.* **71**, 811-818 (1981)

1. Introduction

Bragg mirror is one of most widely used optical components in many applications. One of these applications is the reflection mirror in a laser cavity. Reflection hologram made of PTR glass as Bragg mirror has great advantage of resisting the damage under the working condition of high power laser beam [1-3]. The good spectral selectivity of reflection hologram indeed gives a sharp frequency spectrum in the output laser beam. However, the conventional Bragg mirror doesn't feature good angular selectivity. In this paper, we propose a new scheme based on double-recorded crossed gratings inside the volume hologram, which shows great improvement of angular selectivity.

A scheme considered earlier by Jacob *et al.* [4] uses surface hologram with the grooves placed at top of a planar waveguide. It selects beam with normal incident angle, which beam then propagates transversely inside the waveguide, and then is normally reflected back. Our scheme here is functionally analogous to the one from [4] but with reflection and transverse propagation at the same time. The low tolerance of surface gratings to strong laser power in their design will possibly be replaced by rather strong durability of holographic gratings.

For beams propagation inside the transmission hologram of double-recorded-gratings, some results have been shown in previous works [5] and [6]. The early work of Alferness [7] in detail explored the angular selectivity of transmission hologram of crossed gratings but not with reflection hologram. Here our new type of cross-gratings results a high reflectivity would considerably act as a reflection hologram. With respect to such reflection hologram, especially in the case of symmetric crossed-gratings, nontrivial behavior of four-wave coupling arises, which will be presented and analyzed in the following sections.

2. Four-waves coupling in the reflection hologram

Considering the derivation of the equations of coupled-wave theory [8-10], non-resonant terms are dropped off in the phase matching of Bragg wavevector. Along the same way, in our crossed-hologram, we can consider the wavevector matching condition of the crossed-gratings in Fig. 1(a), and have the beam propagation directions in the following diagram, Fig. 1(b).

Fig. 1. (a).Two crossed-gratings are to be recorded throughout the whole PTR glass. (b) Propagation of waves in two-crossed-grating hologram, where four waves arise and couple to each other. They are A_+ , A_- , B_+ , B_- , respectively, and A_+ is the incident wave.

Here four waves are appearing in the hologram, with A_+ , A_- as the incident and reflected waves, and B_+ , B_- as the excited intermediate waves. Due to the symmetry of our crossed-gratings, the waves A_+ and A_- propagate oppositely in the $\pm z$ -directions, while B_+ and B_- propagate oppositely in the $\pm x$ -directions. This specific wave coupling actually requires that the two gratings should be mutually perpendicular to each other and be oriented at 45° with respect to incident and reflected beams. To illustrate how it works, we should discuss the total field of TE polarized beam ($\mathbf{E} \propto \mathbf{e}_y$) within the Maxwell equations. When the recording exposure is weak, the modulation of refractive index δn is proportional to the linear sum of these two independent interference profiles. The refractive index thus can be written as,

$$n(\mathbf{r}) = n_{\text{background}} + c[I_1(\mathbf{r}) + I_2(\mathbf{r})] = n_0 + |\mu_1| \cos(\mathbf{Q}_1 \cdot \mathbf{r} + \varphi_1) + |\mu_2| \cos(\mathbf{Q}_2 \cdot \mathbf{r} + \varphi_2) \quad (1)$$

Here $I_{1,2}(\mathbf{r})$ are the respective intensity profiles of exposure during holographic recording, n_0 is the average index of refraction, μ_1, μ_2 are the values of grating modulation strength, and $\mathbf{Q}_1, \mathbf{Q}_2$ are the grating vectors. If the recording process is strongly nonlinear, for example with saturation as a function of exposure, additional “inter-modulation” gratings may arise, such as the ones with $\mathbf{Q}_3 = \mathbf{Q}_1 + \mathbf{Q}_2$ or $\mathbf{Q}_4 = \mathbf{Q}_1 - \mathbf{Q}_2$ etc. Since here we assume the exposure is sufficiently weak, no such inter-modulation gratings will appear. Therefore, the resultant Helmholtz wave equation is

$$\nabla^2 E + \frac{\omega^2}{c^2} \left[n_0^2 + n_0(\mu_1 e^{i\mathbf{Q}_1 \cdot \mathbf{r}} + \mu_1^* e^{-i\mathbf{Q}_1 \cdot \mathbf{r}} + \mu_2 e^{i\mathbf{Q}_2 \cdot \mathbf{r}} + \mu_2^* e^{-i\mathbf{Q}_2 \cdot \mathbf{r}}) \right] E = 0. \quad (2)$$

with the total field E ,

$$E = A_+ \exp[i \frac{\omega}{c} n_0 \mathbf{k}_A \cdot \mathbf{r}] + A_- \exp[-i \frac{\omega}{c} n_0 \mathbf{k}_A \cdot \mathbf{r}] + B_+ \exp[i \frac{\omega}{c} n_0 \mathbf{k}_B \cdot \mathbf{r}] + B_- \exp[-i \frac{\omega}{c} n_0 \mathbf{k}_B \cdot \mathbf{r}] \quad (3).$$

For volume hologram with thickness $L \ll \lambda/2\pi$, and very small modulation of refractive index, $|\delta n| \leq 10^{-3}$, the Slowly Varying Envelope Approximation (SVEA) is valid for interacting waves, for example, in the visible / near infrared range of λ and for $L \cong 2.5 \text{ mm}$. Substituting of Eq. (3) into Eq. (2) and using SVEA, we arrive to the following set of four equations including the amplitude absorption coefficient $\beta = n_i \omega / c$ [1/meter] (n_i is the imaginary part of refractive index), and the first order frequency detuning $\gamma = n_0 \delta \omega / c$, also [1/meter]:

$$\begin{aligned} \frac{\partial A_+}{\partial z} &= i\kappa_1 B_+ + i\kappa_2 B_- + (i\gamma - \beta)A_+, & \frac{\partial A_-}{\partial z} &= -i\kappa_2^* B_+ - i\kappa_1^* B_- - (i\gamma - \beta)A_-, \\ \frac{\partial B_+}{\partial x} &= i\kappa_1^* A_+ + i\kappa_2 A_- + (i\gamma - \beta)B_+, & \frac{\partial B_-}{\partial x} &= -i\kappa_2^* A_+ - i\kappa_1 A_- - (i\gamma - \beta)B_-. \end{aligned} \quad (4)$$

Here $\kappa_1 = \mu_1 \omega / 2c$, $\kappa_2 = \mu_2 \omega / 2c$. We further consider the effect of angular detuning by taking Fourier components of all the waves with respect to the normal incidence angle, i.e. $B_{\pm} \approx e^{iqx}$ with $q = \delta k_x = \delta \theta_{\text{air}} \omega / c$. Obviously from the last two equations in Eq. (4), this leads to,

$$B_+ = \frac{i\kappa_1^*}{iq - i\gamma + \beta} A_+ + \frac{i\kappa_2}{iq - i\gamma + \beta} A_-, \quad B_- = -\frac{i\kappa_2^*}{iq + i\gamma - \beta} A_+ - \frac{i\kappa_1}{iq + i\gamma - \beta} A_- \quad (5)$$

Next, we substitute Eq. (5) back into the first two equations in Eq. (4) to obtain the coupled-wave equations for A -waves only. For the case $\kappa_1 = \kappa_2 = \kappa = \kappa^*$, we get

$$\begin{aligned} \frac{\partial A_+}{\partial z} &= PA_+ + QA_-, & \frac{\partial A_-}{\partial z} &= -QA_+ - PA_-, \\ Q &= i\kappa^2 \left[(1/q - \gamma - i\beta) - (1/q + \gamma + i\beta) \right], & P &= -\beta + i\gamma + Q. \end{aligned} \quad (6)$$

While we have come to the coupled-wave Eqs. (6) for A -waves, originally there was no direct coupling between A_+ and A_- . If we carefully take a look at Eqs. (4), it is the intermediate waves $B_{\pm}$ are directly coupled with $A_{\pm}$. This means that two Bragg conditions should be satisfied simultaneously for each coupling in order to have good diffraction efficiency. And this is the main feature of our new design of hologram, which results in improvement of angular selectivity. In reality, for the finite size of incident beams, the angular detuning q actually plays a critical role in our scheme. As Eq. (7) indicates, since P

grows together with Q at small values of angular detuning q , the coupling strength between A_+ and A_- waves is mostly characterized by Q . Indeed, if we ignore the absorption and rewrite it as $Q = ik^2 [1/(q-\gamma) - 1/(q+\gamma)] = 2ik^2 \gamma / (q^2 - \gamma^2)$, then $Q \propto 1/q^2$, i.e. the coupling strength is inverse proportional to the square of angular detuning q . As a consequence, this relation illustrates why our new type of reflection hologram has very good angular selectivity.

3. Results of reflection efficiency with proper boundary condition

Now we seek the solution of Eq. (6) with proper boundary condition. The solution in terms of initial condition has the following matrix form:

$$\begin{bmatrix} A_+(z) \\ A_-(z) \end{bmatrix} = \exp\left(\hat{M} z\right) \begin{bmatrix} A_+(0) \\ A_-(0) \end{bmatrix}. \quad (8)$$

With definition $\rho = \sqrt{P^2 - Q^2}$, and $\hat{M} = \begin{bmatrix} P & Q \\ -Q & -P \end{bmatrix}$, the matrix elements are

$$\exp(\hat{M} z) = \begin{bmatrix} P \sinh(\rho z)/\rho + \cosh(\rho z) & Q \sinh(\rho z)/\rho \\ -Q \sinh(\rho z)/\rho & -P \sinh(\rho z)/\rho + \cosh(\rho z) \end{bmatrix} \quad (9)$$

Consider the boundary condition for $A_{\pm}(z)$ with incident beam $A_+(z=0) = 1$, and the reflected beam at the bottom is zero, $A_-(z=L) = 0$, here L being the thickness of hologram. Accordingly, the reflection and transmission efficiency coefficients are respectively defined as $\eta_{\text{ref}} = |A_-(0)/A_+(0)|^2$ and $\eta_{\text{tran}} = |A_+(L)/A_+(0)|^2$. After calculations, the results could be easily found,

$$\eta_{\text{ref}} = \left| \frac{A_-(0)}{A_+(0)} \right|^2 = \left| \frac{Q \sinh(\rho L)}{P \sinh(\rho L) + \rho \cosh(\rho L)} \right|^2 \quad (10)$$

$$\eta_{\text{tran}} = \left| \frac{A_+(L)}{A_+(0)} \right|^2 = \left| \frac{P \sinh(\rho L) + \cosh(\rho L) - \frac{Q^2 \sinh^2(\rho L)}{\rho P \sinh(\rho L) + \rho^2 \cosh(\rho L)}}{\rho} \right|^2 \quad (11)$$

From the above solution, we are ready to see some characteristics of our hologram. In Fig. 2, we compare both the spectral and angular selectivity of our crossed-gratings hologram with those of one-grating reflection hologram. We see that in Figs. 2(a) and 2(b), the maximum reflection efficiency are depicted as $\eta_{\text{ref}} = 0.8$ for reflection grating and $\eta_{\text{ref}} = 0.81$ for our crossed gratings for comparison. It shows both gratings all possess good spectral selectivity. At first glance, the diffraction efficiency curve of typical reflection hologram seems sharper. But the reflection hologram should pay the price in increasing its thickness, i.e. larger hologram strength to catch up the same reflection efficiency η_{ref} of our crossed hologram. The curve will broaden if the hologram strength decreases and the η_{ref} will also reduce dramatically.

Fig. 2. The reflection efficiency η for conventional reflection hologram with thickness $L = 2.5 \cdot 10^{-3} \text{ m}$ and for our crossed hologram with thickness $L = 1 \cdot 10^{-3} \text{ m}$, for wavelength $\lambda_{\text{vac}} = 1.06 \cdot 10^{-6} \text{ m}$, refraction index $n_0 = 1.5$, amplitude absorption $\beta = 10 \text{ m}^{-1}$, and the grating amplitude κ varies from 0 to 1000 m^{-1} at normal incidence. (a) Conventional reflection hologram, thickness $L = 2.5 \cdot 10^{-3} \text{ m}$, with no angular detuning, $\Delta\theta = 0$, $\Delta\lambda$ from $-3.58 \cdot 10^{-10} \text{ m}$ to $3.58 \cdot 10^{-10} \text{ m}$, $\Delta\lambda_{\text{HWHM}} = 1.48 \cdot 10^{-10} \text{ m}$ (b) Our crossed hologram, thickness $L = 10^{-3} \text{ m}$, with other parameters same as (a), $\Delta\lambda_{\text{HWHM}} = 2.34 \cdot 10^{-10} \text{ m}$; (c) Reflection hologram, thickness $L = 2.5 \cdot 10^{-3} \text{ m}$, with no spectral detuning $\Delta\lambda = 0$, the angular detuning $\Delta\theta_{\text{air}}$ from -3° to 3° , $\Delta\theta_{\text{HWHM}} = 1.44^\circ$ (d) Our crossed hologram, thickness $L = 2.5 \cdot 10^{-3} \text{ m}$, the angular detuning $\Delta\theta_{\text{air}}$ from -0.03° to 0.03° , $\Delta\theta_{\text{HWHM}} = 0.0028^\circ$; (e) The compensation between spectral detuning and angular detuning for reflection hologram with thickness $L = 2 \cdot 10^{-3} \text{ m}$, $\Delta\lambda$ from $-3.58 \cdot 10^{-10} \text{ m}$ to $3.58 \cdot 10^{-10} \text{ m}$ and $\Delta\theta_{\text{air}}$ from -3° to 3° (f) The compensation for our crossed hologram with parameters same as (e) except that $\Delta\theta_{\text{air}}$ is from -0.03° to 0.03° .

In Figs. 2(c) and 2(d) we keep the same maximum reflection efficiency and other parameters as in Figs. 2(a) and 2(b) except there is no spectral detuning, which is replaced by angular detuning. We can see that our crossed grating has a much better angular selectivity. The reflection grating has a small but pretty flat region of high diffraction efficiency. In comparison, our angular selectivity is very sharp. Although people may argue that this crossed hologram has a longer interacting length, since the $B_{\pm}$ waves are propagating horizontally. This interacting thickness would be diminished as long as we include the absorption in our calculation.

We also explored the case that both spectral and angular detunings are present. From our Fig. 2(e) and 2(f), it indicates the compensation between spectral and angular detuning in our crossed-grating are more in a symmetric manner. As deviated from the central region, this compensation gradually fades away. On the contrary, this is not the case for reflection hologram. We can see that, along the top bent curve in the two diagonal regions, significant compensation is observed and continuously keeps the effect. Thus it is more unpleasant to obtain the unexpected higher reflection efficiency for typical reflection hologram.

4. Conclusion

In conclusion, we have suggested a new type of crossed-reflection hologram. With four-wave interaction inside the volume, the incident and reflected wave indirectly coupled to each other through two excited intermediate waves, which propagate along the transverse direction with respect to the incident wave. Our analysis shows that the coupling behavior is inverse square to the detuning angle. Therefore, extraordinary high angular selectivity is achieved by our crossed-reflection hologram. With regard to the poor beam quality of high divergence angle of semiconductor lasers, our hologram may be used to improve this divergence due to its high angular selectivity as used for the mirror of external laser cavity. This work is supported by DARPA under the contract number H0011-06-1-0010.