Linear Systems with Integral Constraints on Transient Step-Response

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LINEAR SYSTEMS WITH INTEGRAL CONSTRAINTS ON TRANSIENT STEP-RESPONSE

by

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A dissertation submitted in partial fulfilment of the requirements
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ABSTRACT

The topic of shaping and controlling transient responses of dynamic systems has important applications. Achieving a desired transient response is an essential design requirement for many control systems.

In this research, we discuss the impact on the transient response of linear systems when it is subjected to a set of integral constraints. The investigation is generalized in a theoretical framework. Formulation of three types of integral constraints is first discussed. The underlying goal of the problem is to shape the step response to generate a specific type of transient response which in turn satisfies the desired integral constraints. The problem is transformed to that of determining the specific structures of transfer functions that can satisfy these aforementioned constraints. Analytical results are established for a class of second order systems with an additional zero. Subsequently, the results are extended to higher order transfer functions and the desired characteristics that a general transfer function should have, to meet the three types of integral constraints, are derived. Next, the implementation requirements to ensure these dynamic characteristics with a given plant transfer function are addressed. In this regard, a control structure, employing combined feedforward and feedback actions, is proposed. Furthermore, necessary conditions to maintain the stability of the overall closed-loop system, generated by the proposed compensation, are established. Subsequent analysis that aims to satisfy the above-mentioned integral constraints in the presence of parametric uncertainties, is presented. In this regard, structured adaptive estimation strategies are proposed to deal with uncertainty. Implementation examples and simulation results are provided to validate
the approaches developed in this work. Further examples to demonstrate the analysis using practical applications are also presented. Here, the utilization of the methods proposed in this work, arises in the applications of designing decentralized control for power management of hybrid power systems.
To my dear father and mother. You are the greatest thing in my life and your love and support will never be forgotten.

To my angels, Anas and Mary. You keep my spirit alive!
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CHAPTER 1

INTRODUCTION

The topic of shaping and controlling transient responses of dynamic systems has not been widely investigated compared to conventional control issues. However, this area has important applications. Achieving a desired transient response is an essential design requirement for many control systems [FPEN18]. One of the early results in this regard was introduced in [Mul49], where the author explored the structure of transfer functions and the effect of poles and zeros location on transient response. Accordingly, in his work, the author proposed a method to determine the magnitudes and locations of the extrema that appear in the transient response of linear systems to a step input. Early investigations to evaluate the transient response of rational linear transfer functions were also proposed in the studies [Cor65, Lio66, AK65].

The author of the study [Hau96a] presented analysis in order to derive analytical formulas that characterize transient responses based on the numerator coefficients of transfer functions. Algorithms to specify the required number of the zeros, as well as their locations, were proposed to define an upper bound on the overshoot and the number of extrema of step responses. In this analysis, the utilized transfer function was assumed to have stable poles and a unity numerator with no zeros. The problem of extrema in linear step responses was also discussed in the study [Rac95]. Here, a sufficient condition was introduced to eliminate the overshoot of the response based on the corresponding pole-zero structure of a given stable transfer function. Furthermore, a class of $nth$ order transfer functions that have only poles and no zeros was considered in the study [KKB03] to propose a combined feedforward and feedback control procedure to shape transient
responses. In this regard, analytic formulations based on two independent design parameters were presented. Here, both parameters, namely the *characteristic ratio* and the *generalized time constant*, were defined in terms of the transfer function characteristic coefficients, and designed to be independently adjustable. The first parameter characterizes the system overshoot to a step input to achieve a small or no overshoot response, while the later parameter characterizes the speed of the response to achieve faster time response specifications. Consequently, the form of the closed loop desired transfer function can be obtained. Moreover, the authors of the study in [MB90a] proposed a zero placement-based technique to choose the optimal locations of the free zeros that minimize the overshoot of step responses. The effect of the transfer function zeros on transient response was also discussed in studies [EKCL93, SP11]. Other transient response control studies and research can be found in [Hau96b, Moo89, MB90b], and the references therein.

The emphasize of the aforementioned studies was directed toward characterizing the overshoot of step responses and achieving a desired response speed, i.e. system rise time. On the other hand, the focus of this research is to propose control strategies that can shape the transient response of linear systems. In this regard, predefined integral constraints, imposed on the step response of linear systems, are introduced to generate desired transients. Consequently, the appropriate response characteristics that allow the fulfillment of these integral constraints are investigated. In addition, the corresponding form of second and higher order transfer functions to achieve these proper characteristics are explored. Furthermore, control structures to transform a given plant response in order to obtain a compatible form of a transfer function, that fits a targeted constraint, are illustrated. Moreover, aspects related to system stability, robustness, and parametric uncertainty are addressed. Application examples to implement the proposed procedures
and control approaches are also considered. The remainder of this chapter provides a literature review about feedforward and feedback control techniques to be utilized in this research. Followed by a background review of energy management approaches of hybrid power systems as an application example. Next, the research problem formulation and statement is presented. Then, the dissertation motivation and contributions, and the dissertation organization are stated.

1.1 Feedforward and Feedback Techniques

Feedback control approaches have been widely employed to address conventional linear systems challenges such as reference tracking and disturbance rejection. In addition, feedback actions are essential to robustify the stability of closed-loop control structures. On the other hand, there are fewer researches about feedforward control approaches [BSLvdW03, GHZ05]. In literature, feedforward-based control methods can be traced back to 1960, see [Dob60, Shi63]. In addition, feedforward control actions are commonly structured by a combination with feedback techniques; thus, it is categorized as a two degree of freedom control scheme [YT99, Hor13]. Figure 1.1 depicts schematic examples of 2DOF control structure.

The utilization of the feedforward techniques can be mainly categorized into the applications of a tracking error elimination, the applications of disturbances effects compensation, and the applications of input shaping controllers to reduce system vibration. In addition, feedforward actions enhance tracking performance significantly. One of the main methods in this regard is the inversion-based feedforward which, in combination with feedback control, can provide a precise output tracking. Here, a common approach is the perfect tracking approach where the feedforward controller injects the inverse form of the exact
plant dynamic to the closed-loop system. Thus, all the poles and zeros of the feedback system will be canceled that produces a unity transfer function between the desired and the actual output [HT91]. However, in the presence of plant uncertainty, inversion-based feedforward controllers can negatively impact the output of the system and lead to significant tracking errors [CSD00,ZJ94]. This challenge was addressed by the study in [Dev02] which states the conditions that define the acceptable level of plant uncertainty where the inverse feedforward approach continues to provide an improved tracking performance; other related studies can be found in [ZJ94,HRM98,WZ09]. Another challenge is that not all the zeros are cancellable. Furthermore, the presence of non-minimum phase zeros in the feedback loop that would result in unstable tracking performance. Here, the Stable Pole Zero Cancelling approach (SPZC) and the Zero Phase Error Tracking approach (ZPET) can provide a remedy to these challenges [HT91,CTL⁺09]. The effect of adding
additional zeros to feedforward controllers in order for reducing the tracing errors was also discussed in the study [HT91]. Other modified inversion-based feedforward models can be found in [CTL^{+}09, BSLvdW03, GHZ05] and the references therein.

Feedforward control was also utilized by the authors of the study in [KKB03]. Here, the analysis was derived to shape the transient response of pure transfer functions that have all poles and no zeros. Thus, the feedforward action was designed to ensure that cancellation of the plant’s original minimum phase zeros and that there is no additional zeros to be introduced to the plant. Another combined feedforward and feedback based control was introduced in the study [YS08]. Here, the design of the feedback controller was determined using a pole-placement approach to achieve a desired performance and ensure the stability of the overall closed-loop system. In addition, the controlled plant was assumed to have a non-minimum phase zero, hence, the feedforward controller was designed to handle this challenge and yet minimize the tracking error. Various input shaping feedforward-based schemes were also proposed in the literature to eliminate or minimize system vibration [SP96, JU99]. The fundamental approach is to introduce an input shaper to the system by applying a sequence of impulses convoluted with a desired system command. Hence, the input shaper leads to generate a new, shaped, input to command the dynamic response of the system as desired; examples of desired command objectives are: eliminating residual vibration or maintaining actuator constraints [SCS97]. Different input shaper designs and techniques can be found in [AHR16, SS02, T94, BvD12, CYB16].

The stability of the overall closed-loop dynamic, produced by the combined feedforward and feedback controllers when imposed on a plant system, represents another challenge to be addressed. Here, in general, the controlled plant is considered to be originally stable. Nevertheless, the stability of the overall structure is not guaranteed. Hence, a stability investigation is required. In this context, the design of a proposed feedforward
or feedback controllers must guard the stability of the controlled plant dynamic and enhance the robustness of the overall system response. In addition, obtaining the necessary and sufficient conditions for the stability of the overall higher order closed-loop system represents a challenge and would require a full knowledge about the dynamic of the controlled plant. A key approach in this regard is the use of an integral action with a proper gain inside the feedback loop. Related theorems and discussions can be found in [Mor85, MG04, GMH85, MP94]. Final remark here is that the stability of a closed-loop system can be assessed based on the stability of its corresponding open loop system using the standard frequency techniques such as Bode-Plots and Root-Locus, refer to [FPEN18, Nis11].

1.2 Power Management of Hybrid Power Systems

Hybrid power systems are a combination of diversified modes of energy generation technologies, both renewable and non-renewable. Hybrid power systems augment flexibility of power generation sources by optimizing the utilization and balancing the strengths and shortages of each source [BW96, PBK09]. Examples are solid oxide fuel cells combined with gas turbines or wind turbines combined with solar dishes and a battery storage. A thorough review of such systems and other examples can be found in [BW97, NWS+11].

Strengthening a hybrid power system with an energy storage device could improve efficiency significantly. The integration of energy storage devices in hybrid power systems ensures maintaining an instantaneous power demand when it exceeds the maximum power

1 Section 1.2 contains material previously published in [MBD16, SD17]
supplied by the system [BW96]. Ultra-capacitor is dominantly used, solo or in combina-
tion with a lithium-ion battery, in order to augment hybrid power system performance and
improve load following capability. This is due to its ability to be charged or discharged
quickly without degradation or damaging the cell [CB02].

In order to integrate ultra-capacitor in hybrid power system, a control strategy and power
management algorithm are required to safeguard the storage device from being over-
charged or progressively discharged. Robust control strategies for a solid oxide fuel cell
system with an ultra-capacitor, based on nonlinear and H∞ control applications, were in-
troduced in [DNM10, AD12]. In the study [OUA06], design of power flow controllers of a
hybrid power system - which controls the load sharing between a wind turbine, a fuel cell,
and, an ultra-capacitor - was proposed.

The core of the aforementioned strategies is to have a central processor that receives
locally sensed information and controls all the components of the hybrid power system.
Such a scheme is referred to as a centralized power management control. Although a
centralized control approach is easier to be developed, its implementation can encounter
practical limitations such as system scalability. Another drawback is that any failure in
the central processor would crucially affect the functionality of the entire system [VG07].
In contrast, decentralization solves the issue of scalability and provides a remedy for the
practical limitations mentioned above. Research investigations of decentralized control
strategies of power systems can be found in [Bak08, UP05, SK11].

In prior work, addressed in the study [BW96], the authors proposed decentralized power
management for hybrid power systems. The system consists of a solid oxide fuel cell
as a power source connected in parallel to an ultra-capacitor storage device. While the
load demand is being met by the hybrid power system, the proposed power manage-
ment approach applies energy conservation principles to assure that the total energy of the ultra-capacitor remains constant after the charge/discharge cycles. Hence, individual controllers, $k_1$ and $k_2$, were developed for the power source and the storage device, respectively. The schematic is shown in Fig. 1.2. $C1$ is a unidirectional $dc/dc$ converter that passes the power from the power source to the load. In the meantime, the load is connected to the storage device through $C2$ which is a bidirectional $dc/dc$ converter to allow charge and discharge operations. Here, the converters $C1$ and $C2$ have fast responses, thus both are considered to be static energy conversion devices. Consequently, the following relation ensures maintaining the power demand $V_L i_L$ at every instant:

$$V_L i_L = \eta_1 V_{fc} i_{fc} + \eta_2 V_{uc} i_{uc}, \tag{1.1}$$

where $\eta_1$ is the efficiency of the converter $C1$. In addition, $\eta_2, \bar{\eta}_2$ are the efficiencies of the converter $C2$ during discharge and charge cycles of the ultra-capacitor, respectively. Hence, during the discharge cycle of the ultra-capacitor, i.e. $i_{uc} > 0$, Eqn.(1.1) becomes:

$$V_L i_L = \eta_1 V_{fc} i_{fc} + \eta_2 V_{uc} i_{uc} \tag{1.2}$$

Similarly, Eqn.(1.1) during the charge cycle of the ultra-capacitor, i.e. $i_{uc} < 0$, becomes:

$$V_L i_L = \eta_1 V_{fc} i_{fc} + \bar{\eta}_2 V_{uc} i_{uc} \tag{1.3}$$

The fundamental goal is to overcome the slow response behavior of the power source, i.e. the fuel cell system, in following the changing load demand; thus, prevent the fluctuation in delivered power. Hence, the ultra-capacitor should work as a buffer to deliver the power demand and absorb the extra energy provided by the power source. Accordingly, the
objective of the power management approach is to meet the power demand by the hybrid power system and simultaneously maintain the state of charge of the storage device at a constant level after each charge/discharge cycles. More precisely, the fuel cell system should provide enough energy to maintain the load demand and compensate the energy removed from the ultra-capacitor during the transient response. In addition, the energy of the ultra-capacitor should be preserved in order from being overcharged while the load demand is being met. Consequently, controller $k_1$ was designed to utilize the transient response history of the fuel cell system to anticipate the energy deficit that should be recovered by the ultra-capacitor, and accordingly, alter the output power of the power source i.e. the fuel cell system. In the meantime, the controller $k_2$ was designed to enable the ultra-capacitor to work as an energy buffer and track the load in the presence of fluctuating load demand. It worth to mention that the controller $K1$ measures only $V_{fc}, V_L, i_L$ and the fuel flow rate $\tilde{N}_f$ to command $C1$, while the controller $k2$ only uses the measurements of $V_{uc}$ and $i_{uc}$ to command $C2$. Thus, decentralization is confirmed.
The concept of this energy conservation-based power management scenario, presented in this section, is later utilized in Chapter 3 to demonstrate the feasibility of the integral constraints control analysis in practical applications.

### 1.3 Formulation of Integral Constraints

Consider a stable dynamical system \( y(s)/r(s) = \bar{G}(s) \) with unity DC gain, with step response as shown in Fig. 1.3.

Next we define three areas, which are depicted in Fig. 1.3, as follows,

\[
A_a = -\frac{1}{2} \int_{t_a}^{\infty} \left\{ 1 - \text{sgn}(r - y) \right\} (r - y) dt \Rightarrow A_a = \sum_{i=1}^{\infty} A_{a,i}, \quad (1.4)
\]

\[
A_b = \frac{1}{2} \int_{t_a}^{\infty} \left\{ 1 + \text{sgn}(r - y) \right\} (r - y) dt \Rightarrow A_b = \sum_{j=1}^{\infty} A_{b,j} \quad (1.5)
\]

and,

\[
A_c = \frac{1}{2} \int_{t_a}^{\infty} \left\{ 1 - \text{sgn}(r - y) \right\} r dt + \frac{1}{2} \int_{t_a}^{\infty} \left\{ 1 + \text{sgn}(r - y) \right\} y dt \quad (1.6)
\]

where \( \text{sgn}(.) \) is the signum function.

Roughly, \( A_a, A_b \) refer to areas above and below the step input and the area \( A_c \), shaded in Fig. 1.3, is common to both.
Step Response

Figure 1.3: Transient step response of \( y(s)/r(s) = \bar{G}(s) \)

We also note that,

\[
\int_{t_s}^{\infty} y \, dt = A_a + A_c
\]

(1.7)

\[
\int_{t_s}^{\infty} r \, dt = A_b + A_c.
\]

Next, we define the following parameter

\[
\eta = \frac{A_b}{A_a} = \frac{\sum_{j=1}^{\infty} A_{b,j}}{\sum_{i=1}^{\infty} A_{a,i}}
\]

(1.8)

and three types of integral constraints

\[
\eta = 1 \quad (I), \quad \eta < 1 \quad (II), \quad \eta > 1 \quad (III).
\]

(1.9)
The primary goal of this work is to determine *the types of stable, proper rational (PR) transfer functions* $\bar{G}(s)$ *that satisfy the constraints above with finite polynomial coefficients*. As we discuss later in the paper, such constraints arise in hybrid power systems.

From Eqn.(1.7) and Eqn.(1.8), constraint (I) in Eqn.(1.9) can be expressed as follows:

$$
\eta = 1 \Rightarrow A_b = A_a \tag{1.10}
$$

$$
\Rightarrow \int_{t_s}^{\infty} y \, dt = \int_{t_s}^{\infty} r \, dt \Rightarrow \int_{t_s}^{\infty} e \, dt = 0 \Rightarrow e_r = 0
$$

where, we define $e = r - y$ and $e_r = \int_{t_s}^{\infty} e \, dt$.

Similarly, constraints (II) and (III) reduce to

$$(\eta A_a - A_b) = 0 \Rightarrow \eta A_a + A_a - A_a = A_b + A_c - A_c$$

$$(\eta - 1)A_a + \int_{t_s}^{\infty} y \, dt = \int_{t_s}^{\infty} r \, dt$$

$$e_r = -A_a(1 - \eta). \tag{1.11}$$

Note that when $\eta = 1$, $e_r = 0$ from Eqn.(1.11) which is consistent with Eqn.(1.10). From Eqn.(1.10), we note that for a unit step input in $r$,

$$e_r = \lim_{s \to 0} s \left[ \frac{r(s)}{s} - \frac{y(s)}{s} \right] = \lim_{s \to 0} s[1 - \bar{G}(s)] \frac{1}{s^2}. \tag{1.12}$$

Since the term $1/s^2$ is a ramp signal in Laplace domain, therefore from Eqn.(1.12), $e_r$ is
equivalent to the unit ramp error of the system described by $\bar{G}(s)$. Satisfying the integral constraints of Eqn.(1.9) thus translates to imposing the following conditions on $e_r$,

$$
e_r = 0 \Leftrightarrow \eta = 1, \quad e_r < 0 \Leftrightarrow \eta < 1, \quad e_r > 0 \Leftrightarrow \eta > 1 \quad (1.13)$$

In addition, we note that constraint (I) of Eqn.(1.9) refers to a case of matched areas $A_a$ and $A_b$, and constraints (II) and (III) refer to mismatched areas. Finally, from Eqn.(1.11) it is evident that for the mismatched area scenarios, i.e. for constraints (II) and (III), the value of $e_r$ is determined not only by $\eta$ but the above areas $A_a = \sum_{i=1}^{\infty} A_{a,i}$ which is dependent on the transient characteristics of $\bar{G}(s)$. 

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1.4 Dissertation Contributions and Objectives

The objectives and contributions of this research can be summarized as follows:

1. Formulate three types of integral constraints that are employed to control the transient response of linear systems.

2. Define the proper structure of a general transfer function whose step response can be controlled using the three types of integral constraints to achieve a desired transient behavior.

3. Derive an analytic expression for second order systems with a zero that quantifies the relation between the imposed integral constraint and the corresponding zero location.

4. Propose control frameworks using combined feedforward and feedback actions to shape the step response of any given stable plant and thus generate a desired transient.

5. Propose adaptive estimation strategies to satisfy the three types of integral constraints in the presence of parametric uncertainties.

6. Validate the proposed methods and strategies in simulations.

7. Demonstrate the utilization of the proposed methods with practical applications.
1.5 Dissertation Overview

The literature review of the subjects related to the scope of this Dissertation is provided in Chapter 1. Then, the formulation of the research problem is illustrated. Followed by the list of the Dissertation contributions and objectives.

The rest of the Dissertation is arranged as follows: the proper structure of transfer functions the can satisfy the three types of integral constraints, formulated in this research, is addressed in Chapter 2. Analytical results are established for a class of second order systems and generalized to higher order transfer functions. In addition, the desired characteristics, a general transfer function should have to meet the three types of integral constraints, are derived. Furthermore, simulation results and examples are impeded to validate the analysis of each corresponding section of chapter 2. Subsequently, control frameworks to accomplish the strategies and approaches of this research are addressed in Chapter 3. The utilization of a combined feedforward and feedback structure that enables the closed-loop form of any given plant transfer function to satisfy the three types of integral constraints is demonstrated. Then, the necessary conditions for the stability of the aforementioned closed-loop structure are investigated. Followed by analysis that aims to satisfy the integral constraints in the presence of parametric uncertainties where adaptive estimation strategies are presented. Moreover, an analytic derivation to satisfy overshoot restriction withing integral constraint is presented. Practical application examples are also introduced in Chapter 3 while implementation examples and simulation are provided within their respective sections. Finally, Chapter 4 concludes the Dissertation with a summary of the main results and outcome of the conducted research and investigations. Chapter 4 also highlight the future path to extend the analysis of this research. Thereafter, the references are provided.
CHAPTER 2

STRUCTURE OF TRANSFER FUNCTIONS
WITH INTEGRAL CONSTRAINTS

2.1 Chapter Overview

In this chapter, the goal is to investigate which characteristics, transient response of a system should have in order to provide a behavior that can satisfy the three types of integral constraints from Eqn.(1.9). Hence, the proper structure of a general transfer function $\bar{G}(s)$, that allows shaping the response of the system in a form for which the difference between the above area $A_a$ and the below area $A_b$ equals a targeted value $\eta$, is investigated. Consequently, a class of second order systems that can satisfy the constraints from Eqn.(1.9) is first explored. Accordingly, the appropriate structure of second order transfer functions is defined. Analytical relations between the constraints from Eqn.(1.9), expressed in terms of $\eta$, with systems characteristics such as the damping ratio $\zeta$ and the natural frequency $\omega_n$, are addressed. Subsequently, generalization analysis to higher-order transfer functions is presented. Simulation results to validate the analysis of second and higher order responses are also provided.

1 Chapter 2 contains material previously published in [SD17, SD19]
2.2 Integral Constraints and Second Order Systems

The step response of first order systems do not exhibit the oscillatory behavior that is essential to produce areas above the step input (see Eqn. 1.4 and $A_{a,i}$ in Fig. 1.3). Thus, we begin by exploring the response of a pure second order system that has only poles with no zeros as shown below:

\[ \ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \omega_n^2 r(t) \]  
(2.1)

where $\zeta$ and $\omega_n$ are the damping ratio and the natural frequency, respectively. However, the standard second order system from Eqn.(2.1) cannot satisfy all the three types of constraints stated in Eqn.(1.9) as shown in the next calculations. Eqn.(2.2) shows the frequency domain representation of the second order system from Eqn.(2.1),

\[ \bar{G}(s) = \frac{y(s)}{r(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]  
(2.2)

Thus, the ramp error $e_r$ of $\bar{G}(s)$ from Eqn.(2.2) is:

\[ e_r = \lim_{s \to 0} s \left[ \frac{r(s)}{s} - \frac{y(s)}{s} \right] \]  
(2.3)

\[ = \lim_{s \to 0} s \left[ 1 - \bar{G}(s) \right] \frac{1}{s^2} = \frac{2\zeta}{\omega_n} \]

The result from Eqn.(2.3) shows that the ramp error $e_r$ of a pure second order dynamic system takes a positive value, thus it can only satisfy constraint (III) from Eqn.(1.9) i.e. the case of $e_r > 0 \iff \eta > 1$; refer to Eqn.(1.13). Note that the necessary condition of stability requires that the coefficient $2\zeta\omega_n$ to be positive. Consequently, and since $A_a > 0$, to satisfy constraint (I) in Eqn.(1.9) of the case $e_r = 0 \iff \eta = 1$, it is required to have a
system with \( \omega_n = \infty \), or undamped system, i.e. \( \zeta = 0 \). Such an undamped system, which exhibits a sinusoidal step response, is not appropriate for realistic applications such as power systems. In addition, for constraint (II) with the case of \( e_r < 0 \iff \eta < 1 \), Eqn.(1.8) requires that the \( \sum_{i=1}^{\infty} A_{a,i} \) be greater than \( \sum_{j=1}^{\infty} A_{b,j} \), \( \eta = \frac{\sum_{i=1}^{\infty} A_{b,i}}{\sum_{i=1}^{\infty} A_{a,i}} \). This can be achieved by having a system with a negative \( e_r \) which is not feasible in systems with a pure second order dynamic.

**Remark 1.** Consider the the under-damped step response of the system presented in Eqn.(2.1),

\[
y(t) = 1 - \exp^{-\sigma t} \left[ \cos \omega_d t + \frac{\sigma}{\omega_d} \sin \omega_d t \right]
\]

(2.4)

where \( \sigma = \zeta \omega_n \) and \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \). Referring to Fig. 1.3, let \( y \) be the unit step response from Eqn.(2.4), that is \( y(t) \). In addition, let \( r \) be a unit step input reference, i.e. \( r(t) = 1(t) \).

Next, we evaluate \( e_r = \int_{t_s}^{\infty} e \, dt = \int_{t_s}^{\infty} r \, dt - \int_{t_s}^{\infty} y \, dt \), refer to the discussion related to Eqn.(1.10) and Eqn.(1.11),

\[
e_r = \int_{0}^{\infty} \exp^{-\sigma t} \left[ \cos \omega_d t + \frac{\sigma}{\omega_d} \sin \omega_d t \right] dt
\]

(2.5)

Solving the integral from Eqn.(2.5) yields:

\[
e_r = \left[ \frac{\sigma}{\omega_d^2 + \sigma^2} + \frac{\sigma}{\omega_d} \frac{\omega_d}{\omega_d^2 + \sigma^2} \right] \implies e_r = \frac{2\zeta}{\omega_n}
\]

(2.6)

The result from Eqn.(2.6) is consistent with that from Eqn.(2.3), and thus confirms the discussion related to Eqn.(1.10), Eqn.(1.11), and Eqn.(1.12), introduced in Section 1.3.
2.2.1 A Class of Second Order Transfer Functions with a Zero

In the previous section, it is shown that a system with a pure second order dynamic does not satisfy all the constraints that are stated in Eqn.(1.9). Increasing the order of the system represents an alternate path to be explored; however, as a result, the system will be more complicated. One of the effective methods to modify response characteristics of systems and at the same time preserve the order is having a minimum-phase zero in the system. The presence of a proper zero in the system can substantially enhance the steady-state response and guarantee minimum or zero error in following the input signal. Hence, the class of second order transfer functions that satisfies the integral constraints of Eqn.(1.9) is found to be:

\[
\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \omega_n^2 r(t) + \alpha \dot{r}(t)
\]

(2.7)

\[
\Rightarrow \bar{G}(s) = \frac{y(s)}{r(s)} = \frac{\alpha s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

Therefore, the ramp error \(e_r\) of the dynamic system from Eqn.(2.7), is obtained as:

\[
e_r = \lim_{s \to 0} s[1 - \bar{G}(s)] \frac{1}{s^2} = \frac{2\zeta\omega_n - \alpha}{\omega_n^2}
\]

(2.8)

In Eqn. (2.8), setting \(\alpha = 2\zeta\omega_n\) yields a zero ramp error \((e_r = 0)\), thus satisfying constraint (I) of \(\eta = 1\). Furthermore, when \(\alpha > 2\zeta\omega_n > 0\) and \(0 < \alpha < 2\zeta\omega_n\), \(e_r\) takes negative and positive values, respectively. Hence, constraint (II) and constraint (III) can also be satisfied. Thus all three constraint-types in Eqn.(1.9) can be satisfied by finite values of \(\alpha\), without affecting \(\zeta\) and \(\omega_n\). The analysis of specifying the proper value of \(\alpha\) for a given value of \(\eta\) is discussed in the next section.
2.2.2 Analysis to Relate $\alpha$ with $\eta$

From Eqn.(1.11) and Eqn.(2.8),

$$e_r = -A_a (1 - \eta) = \frac{2\zeta \omega_n - \alpha}{\omega_n^2}$$ \hspace{1cm} (2.9)

Hence, the relation between $\alpha$ and $\eta$ is,

$$\alpha = (1 - \eta) A_a \omega_n^2 + 2\zeta \omega_n$$ \hspace{1cm} (2.10)

It is noted that while Eqn.(2.10) represents a closed form solution of $\alpha$, an analytical form of $A_a$ when the dynamic response of the system converges to a steady-state is required. To this end, we state the proof of the following theorem.

**Theorem 1.** The step response of a stable second order system, having the structure of the following strictly proper transfer function:

$$\bar{G}(s) = \frac{\alpha s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2},$$

satisfies Eqn.(2.10) with the following expression for $A_a$,

$$A_a = \frac{A_{a,1}}{1 - \rho}, \quad A_{a,1} = \frac{\beta \omega_d}{\sigma^2 + \omega_d^2} \left[ e^{-\frac{\sigma - \phi}{\omega_d}} + e^{-\frac{\sigma + \phi}{\omega_d}} \right], \quad \rho = e^{-\frac{2\pi a}{\omega_d}}$$ \hspace{1cm} (2.11)

where, $A_a$ is defined in Eqn. (1.4), $\eta$ is defined in Eqn. (1.8), $\sigma = \zeta \omega_n$, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, $\beta = \sqrt{1 + (\sigma - \alpha)^2 / \omega_d^2}$, and $\phi = \cos^{-1} \left( \frac{\sigma - \alpha}{\omega_d}/\beta \right)$. 

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Figure 2.1: Geometric Progression of areas $A_a$

**Proof:** The step response of $\bar{G}(s) = \frac{\alpha s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ is:

$$y(t) = 1 - e^{-\sigma t} \left[ \cos \omega_d t + \frac{\sigma - \alpha}{\omega_d} \sin \omega_d t \right]$$

$$= 1 - \beta e^{-\sigma t} \sin (\omega_d t + \phi)$$

(2.12)

From, Fig. 2.1, and Eqn.(1.4) and Eqn.(2.12), $A_{a,i}$ can be expressed as

$$A_{a,i} = \int_{t_{2i-1}}^{t_{2i}} (y(t) - 1)dt, \quad y(t_{2i-1}) = y(t_{2i}) = 1$$

(2.13)

$$\Rightarrow A_{a,i} = -\beta \int_{t_{2i-1}}^{t_{2i}} e^{-\sigma t} \sin (\omega_d t + \phi)dt$$

where $t_{2i-1}$ and $t_{2i}$ represent the time-instants when $A_{a,i}$ begins and ends respectively.
The following expression is obtained for $A_{a,i}$ upon integrating:

\[
\frac{1}{\beta} A_{a,i} = \left[ \frac{1}{\sigma} e^{-\sigma t} \sin (\omega_d t + \phi) + \frac{\omega_d}{\sigma^2} \cos (\omega_d t + \phi) + \frac{\omega_d^2}{\sigma^2} A_{a,i} \right] \Bigg|_{t_{2i-1}}^{t_{2i}} \tag{2.14}
\]

which yields:

\[
A_{a,i} = \left[ \frac{-\beta}{\sigma^2 + \omega_d^2} e^{-\sigma t} \{ \sigma \sin (\omega_d t + \phi) + \omega_d \cos (\omega_d t + \phi) \} \right] \Bigg|_{t_{2i-1}}^{t_{2i}} \tag{2.15}
\]

To evaluate the integration intervals $t_n$, $n = 0, 1, 2, ...$, from Eqn.(2.12), we note that at instants $t_{n=2i-1}$ and $t_{n=2i}$, $\sin (\omega_d t + \phi) = 0$. These instants correspond to $t_1$, $t_2$, $t_3$, etc., in Fig. 2.1. Thus, $r(t) - y(t) = 0$ at

\[
(\omega_d t_n + \phi) = n\pi, \ n = 0, 1, 2, ...
\]

which leads to:

\[
t_n = \frac{n\pi - \phi}{\omega_d}, \ n = 1, 2, ..., \ t_n > 0 \tag{2.17}
\]

From Eqn.(2.15) and Eqn.(2.17), the areas $A_{a,i}$, $i = 1, 2, 3, ...$ can be obtained as:

\[
A_{a,i} = \frac{\beta \omega_d}{\sigma^2 + \omega_d^2} \left[ e^{-\sigma \frac{2i\pi - \phi}{\omega_d}} + e^{-\sigma \frac{2(i-1)\pi - \phi}{\omega_d}} \right] \tag{2.18}
\]

Referring to Fig. 2.1, one can conjecture that the areas of the shown response with respect to the reference line of a step signal follow a specific progression. Thus, the result
from Eqn.(2.18) is next utilized to determine the ratio between two successive areas \( A_{a,i} \), \( A_{a,i+1} \), as follows:

\[
\frac{A_{a,i+1}}{A_{a,i}} = \frac{\frac{e^{-\sigma \frac{2(i+1)\pi - \phi}{\omega d}} + e^{-\sigma \frac{2(i+1)\pi - \phi}{\omega d}}}{e^{-\sigma \frac{2i\pi - \phi}{\omega d}} + e^{-\sigma \frac{2(i+1)\pi - \phi}{\omega d}}}}{\frac{e^{-\sigma \frac{2i\pi - \phi}{\omega d}} + e^{-\sigma \frac{2i\pi - \phi}{\omega d}}}{e^{-\sigma \frac{2i\pi - \phi}{\omega d}}}} = e^{-\frac{2\sigma}{\omega d}}
\]

(2.19)

which yields:

\[
\rho = \frac{A_{a,i+1}}{A_{a,i}} = e^{-\sigma \frac{2\pi}{\omega d}} = e^{-\frac{2\pi}{\sqrt{1-\zeta^2}}} \quad i = 1, 2, 3, \ldots
\]

(2.20)

Equation (2.20) confirms that the ratio \( \rho \) is constant with \(|\rho| < 1\). Hence the areas \( A_{a,i} \) reduce in a geometric progression and the sum \( A_a = \sum_{i=1}^\infty A_{a,i} \) converges to,

\[
A_a = \sum_{i=1}^\infty A_{a,i} = \sum_{i=1}^\infty A_{a,1} \rho^{(i-1)} = \frac{A_{a,1}}{1-\rho}, \quad |\rho| < 1.
\]

(2.21)

where \( A_{a,1} \) can be found from Eqn.(2.18),

\[
A_{a,1} = \frac{\beta \omega_d}{\sigma^2 + \omega_d^2} \left[ e^{-\sigma \frac{2\pi - \phi}{\omega d}} + e^{-\sigma \frac{\pi - \phi}{\omega d}} \right]
\]

(2.22)

This completes the proof. ■

From Eqn.(2.10) and Eqn.(2.11), note that \( \alpha \) can not be explicitly expressed in terms of \( \eta \), \( \zeta \) and \( \omega_n \), since the dependence of \( A_{a,1} \) on \( \alpha \) is not simple enough to be inverted. However, the relation can be analyzed numerically to obtain/plot values of \( \eta \) for given values of \( \zeta \), \( \omega_n \) and \( \alpha \); refer to Fig. 2.5 and its related discussion in Section 2.2.4.
2.2.3 Derivation of a Non-Dimensional Expression for $\eta$

In this section, we express $\eta$ using non-dimensional parameters that can be utilized to achieve a desired integral constraint.

From Eqn.(2.10) and Eqn.(2.11) and since $\omega_n^2 = (\omega_d^2 + \sigma^2)$, we have

$$\alpha = 1 - \eta \frac{\beta \omega_d}{1 - \rho} \left[ e^{-\sigma \frac{2\pi - \phi}{\omega_d}} + e^{-\sigma \frac{\pi - \phi}{\omega_d}} \right] + 2\zeta \omega_n \quad (2.23)$$

Upon further simplification and recalling that $\sigma = \zeta \omega_n$ and $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, we have

$$\frac{\alpha}{\omega_n} = \frac{1 - \eta}{1 - e^{-\frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}}} \beta \sqrt{1 - \zeta^2} \left[ e^{-\frac{\zeta}{\sqrt{1 - \zeta^2}} (2\pi - \phi)} + e^{-\frac{\zeta}{\sqrt{1 - \zeta^2}} (\pi - \phi)} \right] + 2\zeta \quad (2.24)$$

Next, defining the non-dimensional frequency-ratio parameter $\kappa = \alpha / \omega_n$, $\beta$ and $\phi$ can be expressed as

$$\beta = \sqrt{1 + \frac{(\sigma - \alpha)^2}{\omega_n^2}} = \sqrt{1 + \frac{(\zeta - \kappa)^2}{1 - \zeta^2}}, \quad (2.25)$$

$$\phi = \cos^{-1} \left( \frac{(\zeta - \kappa)}{\sqrt{1 - \zeta^2}} / \sqrt{1 + \frac{(\zeta - \kappa)^2}{1 - \zeta^2}} \right)$$

and hence Eqn.(2.24) can be expressed as

$$\kappa = \frac{1 - \eta}{1 - e^{-\frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}}} \beta(\kappa, \zeta) \sqrt{1 - \zeta^2} \left[ e^{-\frac{\zeta}{\sqrt{1 - \zeta^2}} (2\pi - \phi(\kappa, \zeta))} + e^{-\frac{\zeta}{\sqrt{1 - \zeta^2}} (\pi - \phi(\kappa, \zeta))} \right] + 2\zeta \quad (2.26)$$
Equation (2.26) contains non-dimensional parameters $\kappa$, $\zeta$ and $\eta$ only. It represents the general relation between the integral constraints in Eqn. (1.9), expressed in terms of $\eta$, with the damping ratio $\zeta$ and frequency ratio $\kappa$.

It can be noted that the non-dimensional relation, expressed in Eqn. (2.26), requires knowing the exact damping ratio of the system (i.e. $\zeta$), in order to select a proper $\kappa$ that fits a desired constraint (i.e. achieves a desired $\eta$). The question that arises is how to determine $\kappa$ from Eqn. (2.26) if $\zeta$ is unknown. To this end, we first address the case of satisfying constraint (I) of $\eta = 1$. Subsequently, we discuss the case of satisfying constraint (II) and constraint (III) of $\eta \neq 1$.

Consider the time-domain representation of the second order system from Eqn. (2.7). In order to satisfy constraint (I), $\kappa = \alpha / \omega_n = 2\zeta \rightarrow \eta = 1$. The damping ratio $\zeta$ is unknown, and its estimate is denoted by $\hat{\zeta}$. For the sake of simplicity, and without loss of generality, the system is assumed to have a natural frequency $\omega_n = 1 \text{ rad/s}$. Hence, the parameter $\kappa \equiv \alpha$ is also estimated. Denoting the estimate of $\kappa$ by $\hat{\kappa}$, the estimated $\hat{\kappa} = 2\hat{\zeta}$. Hence, when $\zeta$ is unknown, Eqn. (2.7) takes the form

$$\ddot{y} + 2\zeta \dot{y} + y = r(t) + \hat{\kappa} \dot{r}(t), \quad \hat{\kappa} = 2\hat{\zeta}$$

(2.27)

In Eqn. (2.27), consider $r(t)$ to be a unit ramp, i.e. $\dot{r}(t) = 1$. The dynamic equation of the ramp error $e = r - y$, derived from Eqn. (2.27), is expressed as:

$$\ddot{e} + 2\zeta \dot{e} + e = 2(\zeta - \hat{\zeta})$$

(2.28)
Subsequently, the integral adaptation law

\[ \dot{\zeta} = \gamma e \]  

(2.29)

ensures that \( e_r = \lim_{t \to \infty} e(t) = 0 \Rightarrow \eta \to 1 \) as the system reaches steady-state, with proper choice of \( \gamma \). The range of the adaptation gain \( \gamma \) that guarantees stability of the overall system is derived using Routh’s stability criterion as \( 0 < \gamma < 2\zeta_{\text{min}} \), where \( 0 < \zeta_{\text{min}} < \zeta < \zeta_{\text{max}} \leq 1 \).

When satisfying constraints (II) and/or constraint (III) is desired, i.e. the desired \( \eta \neq 1 \), knowing the damping ratio \( \zeta \) is still required to select a proper \( \kappa \) for a desired value of \( \eta \), (Eqn.2.26). However, in comparison to \( \kappa = 2\zeta \) for constraint (I) of \( \eta = 1 \), the relation from Eqn.(2.26) for constraint (II) and constraint (III) is significantly more complicated. Also, while \( \eta = 1 \Leftrightarrow \lim_{t \to \infty} e = e_r = 0 \), the corresponding relation between desired \( \eta \) and \( e_r \) is more complicated and nonlinear for \( \eta \neq 1 \), (Eqn.2.9). For the case of \( \eta \neq 1 \) increasing \( \kappa \) decreases \( \eta \) in a nonlinear but monotonic manner and vice versa. Hence, updating the value of \( \kappa \) in the right direction leads to achieve a desired \( \eta \). Therefore, the instantaneous value of \( \eta \) is needed to be calculated online and utilized to update the value of \( \kappa \) until the desired \( \eta \) is achieved. We refer to this approach by the numerical measurement approach. In this context, the summation of the above areas \( \sum_i A_{a,i} \), the summation of the below areas \( \sum_j A_{b,j} \), and the instantaneous \( \eta \), denoted by \( \eta_m \), are calculated using Eqn.(1.4), Eqn.(1.5), and Eqn.(1.8) as:

\[
\eta_m(t) = \frac{\sum_j A_b}{\sum_i A_a} = \frac{\frac{1}{2} \int_{t_s}^t \{1 + \text{sgn}(r-y)\}(r-y)dt}{\frac{1}{2} \int_{t_s}^t \{1 - \text{sgn}(r-y)\}(r-y)dt}
\]

(2.30)
where \( sgn(.) \) is the signum function. Consequently, the calculated value of \( \eta_m \) is utilized to update the estimate \( \kappa \) as:

\[
e = (\eta_m - \eta_d), \quad \dot{\kappa} = \tau e = \tau \int_0^t e \, dt + \kappa_0
\](2.31)

where \( \eta_d \) is the desired \( \eta \), \( \kappa_0 \) is the initial estimate of \( \kappa \), and \( \tau > 0 \) represents the integrator gain. Thus Eqn.(2.7) would now take the form

\[
\ddot{y} + 2\zeta \dot{y} + y = r(t) + \dot{\kappa} \hat{r}(t), \quad \dot{\kappa} = \tau \int_0^t e \, dt + \kappa_0, \quad \omega_n = 1 \text{ rad/s}
\](2.32)

Here, Eqn.(2.26) can be utilized to specify the right direction at which \( \hat{\kappa} \) is updated. In this regard, the ramp error \( e_r \) when \( \hat{\kappa} \rightarrow \kappa > 2\zeta \) takes negative values that fit constraint (II) while \( \hat{\kappa} \rightarrow \kappa < 2\zeta \) provides positive values of the ramp error \( e_r \) that meet constraint (III). It is also noted that the feasibility of the numerical measurement approach to handle constraint (II) and constraint (III) is not limited to second order systems but applicable to higher order transfer functions as well. The mechanism by which the value of \( \kappa, \alpha \) is updated, is demonstrated in Chapter 3.
2.2.4 Simulation Results

The discussion from the previous section shows that a proper value of $\alpha$ is necessary in order for second order transfer functions to satisfy the constraints from Eqn.(1.9). In this section, simulation plots are provided to demonstrate the feasibility of the analysis and results, related to second order transfer functions, introduced in the previous sections.

The non-dimensional relation between $\kappa$ and $\eta$, from Eqn.(2.26), is plotted in Fig. 2.2 for various values of $\zeta$. Note that a higher value of $\kappa$ indicates a more dominant zero of $\bar{G}(s)$ in Theorem 1. Figure 2.2 indicates that as the damping ratio increases, $\kappa$ needs to be higher to achieve the same $\eta$. This is expected since, with the increase in $\zeta$ the dominance of the zero (located at $-\omega_n^2/\alpha$) must increase to provide the required overshoot to produce $A_a$. This observation is also depicted in Fig. 2.3 using different values of $\zeta$ and $\kappa$ where the second order system from Eqn.(2.7) is considered.
In Fig. 2.3(a), the damping ratio is set to be $\zeta = 0.3$. On the other hand, in Fig. 2.3(b), the damping ratio is set to be $\zeta = 0.7$ which provides low oscillation behavior; thus cycling is minimized which is desired in realistic applications such as power systems. The responses with $\kappa = 0.6$ and $\kappa = 1.4$ are associated with constraint (I) of $\eta = 1$ for $\zeta = 0.3$ and $\zeta = 0.7$, respectively. On the other hand, the responses with $\kappa = 0.93$ and $\kappa = 1.54$ correspond to constraint (II) with $\eta = 0.75$. Similarly, the responses with $\kappa = 0.15$ and
\( \kappa = 1.21 \) correspond to constraint (III) with \( \eta = 1.5 \). Hence, the observations related to Eqn.(2.26) and Fig. 2.2 are confirmed. The results from Fig. 2.3 show that, for the same value of \( \zeta \), increases \( \kappa \) leads to increase the areas \( A_{a,i} \) and decreases the areas \( A_{b,j} \). In addition, increasing the damping ratio from \( \zeta = 0.3 \) to \( \zeta = 0.7 \) necessitates selecting a higher value of \( \kappa \) to achieve the same \( \eta \). This is confirmed for all the three types of integral constraints from Eqn.(1.9).

Figure 2.4 shows the simulation output of the transfer function:

\[
\tilde{G}(s) = \frac{\alpha s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \frac{0.6s + 1}{s^2 + 0.6s + 1} \tag{2.33}
\]

The objective is to satisfy constraint (I), i.e. \( \eta = 1 \). The damping ratio is assumed to be unknown. Hence, an initial, estimated, value of the damping ratio is chosen to be \( \hat{\zeta} = 0.1 \). Figure 2.4(a) compares two step responses of the transfer function from Eqn.(2.33). The response \( \hat{y}(t) \) represents the response where the damping ratio is estimated and updated to calculate the proper value of \( \hat{\kappa} = \hat{\alpha}/\omega_n \), \( \omega_n = 1 \) rad/s. On the other hand, the response \( y(t) \) represents the case where the damping ratio \( \zeta = 0.3 \) is exactly known. It can be seen from the results that the estimation approach from Eqn.(2.28), and Eqn.(2.29), works as designed and accurately estimates the unknown damping ratio. The value of \( \hat{\zeta} \) from Fig. 2.4(a) converges to the actual \( \zeta = 0.3 \). In addition, the feasibility of the estimation approach is confirmed from the results presented in Fig. 2.4(b) where the ratio \( \eta \) is calculated for both cases, the estimated case \( \hat{\eta} \) and the exact case \( \eta \). The estimation of the correct value of \( \kappa \) to maintain \( \eta = 1 \) is also confirmed. The steady state value of \( \hat{\kappa} \) from Fig. 2.4(b) coincides with the exact value of \( \kappa = 0.6 \).
Figure 2.4: Estimation of $\zeta$ and $\kappa$ for $\eta = 1$
In Fig. 2.5, we plot $\eta$ as well as $e_r$ see (Eqn.2.8) vs. $\alpha$ for the case $\zeta = 0.6$, $\omega_n = 1$ rad/s.

In this context, the effect of $\alpha$ on the transient characteristics of the system response is demonstrated in Fig. 2.6 which shows the responses of the dynamic system stated in Eqn.(2.7). Here, the damping ratio is set at $\zeta = 0.6$. The value of $\alpha = 1.2$ yields $e_r = 0$, hence satisfies constraint (I) of $\eta = 1$. On the other hand, the values $\alpha = 2$ and $\alpha = 0.5$ correspond to constraints (II) and (III) of $\eta \neq 1$, respectively. It is noted that changing the value of $\alpha$ impacts the overshoot of the step response that influences the areas $A_{a,i}$ and $A_{b,j}$ in Fig. 2.6(a). Accordingly, increasing the value of $\alpha$ proportionally affects the areas $A_{a,i}$, thus decreases the parameter $\eta$ and reduces the steady-state ramp error $e_r$ from positive to negative value, as evident in Fig. 2.6(b).
Figure 2.6: Response of a Second Order System with Different Values of $\alpha$
2.3 Integral Constraints and Higher Order Responses

In this section, we extend the discussion to higher order transfer functions and explore conditions under which the constraints of Eqn.(1.9) are satisfied.

2.3.1 Characteristics of Transfer Function

Consider a general unity gain, strictly proper, and rational transfer function,

$$\tilde{G}(s) = \frac{N(s)}{D(s)} = \frac{a(s) + a_1 s + g_o}{b(s) + b_1 s + g_o}$$  \hspace{1cm} (2.34)

where $a(s)$ and $b(s)$ are the higher order terms of $G(s)$, and the order of $b(s)$ is higher than the order of $a(s)$. The steady-state ramp error $e_r$, as defined in Eqn.(1.12), is

$$e_r = \lim_{s \to 0} s [1 - \tilde{G}(s)] \frac{1}{s^2} = \frac{b_1 - a_1}{g_o}$$  \hspace{1cm} (2.35)

To this end, we prove the following Lemma.

**Lemma 1.** For any stable linear system with a unity DC gain, the ramp error $e_r$ can be predicted by taking the derivative of the closed-loop transfer function with respect to $s$ and setting it to zero.

$$e_r = \lim_{s \to 0} \frac{d}{ds} \tilde{G}(s)$$  \hspace{1cm} (2.36)

**Proof:** The result from Lemma 1 can be easily proven by considering the general unity
gain and strictly proper rational transfer function stated in Eqn.(2.34), \( \bar{G}(s) = \frac{N(s)}{D(s)} = \frac{a(s) + a_1 s + g_o}{b(s) + b_1 s + g_o} \). Thus, the ramp error \( e_r \) (from Eqn 2.35) is \( e_r = \frac{b_1 - a_1}{g_o} \).

On the meantime, the derivative of \( \bar{G}(s) \) can be expressed as:

\[
\frac{d}{ds} \left( \frac{N(s)}{D(s)} \right) = \left[ \frac{[a(s)b_1 + b(s)a_1](s - 1) + g_o [a(s) - b(s) + b_1 - a_1]}{[b(s) + b_1 s + g_o]^2} \right]
\]

(2.37)

Hence,

\[
\lim_{s \to 0} \frac{d}{ds} (\bar{G}(s)) = \frac{b_1 - a_1}{g_o}
\]

(2.38)

which is identical to the result stated in Eqn.(2.35). This completes the proof. ■

The result from Eqn.(2.35) shows that the the ramp error \( e_r \) of a general transfer function is determined only by the parameters \( b_1, a_1 \) and \( g_o \). Note that, compared to Eqn.(2.8), in Eqn.(2.35) \( b_1 \equiv 2\zeta \omega_n, a_1 \equiv \alpha, \) and \( g_o \equiv \omega^2_n \). Hence, complete knowledge of the transfer function is not required. Accordingly, \( a_1 = b_1 \) yields \( e_r = 0 \), thus satisfying constraint (I) of Eqn.(1.13), corresponding to \( \eta = 1 \). In the same manner, when \( a_1 > b_1 \) and \( a_1 < b_1 \), the ramp error is \( e_r < 0 \) and \( e_r > 0 \) and thus constraint (II) and constraint (III) are met, respectively. In this case however an analytic relation similar to that of Eqn.(2.10), relating \( \alpha \) with a desired \( \eta \neq 1 \), may not be derivable. Consequently, the numerical measurement approach may be used in such cases following the steps given next.

An initial value of \( a_1 \) is first considered (assuming \( a_1 \) to be tunable). Next, the instantaneous value of \( \eta \), namely \( \eta_m \), is calculated online using Eqn.(2.30). Subsequently, the result of \( e_r \) from Eqn.(2.35) is utilized to specify the right direction at which the value of \( a_1 \) should be updated to achieve the desired \( \eta \). Here, the parameter \( a_1 \) is updated in a similar manner to the approach shown in Eqn.(2.31).
A similar discussion about parameter uncertainty as discussed for unknown $\zeta$ in Section 2.2.3, is valid here. If the parameters $b_1$ and/or $a_1$ are unknown but the system dynamic allows a tunable $\hat{\alpha}$ such that

$$
\bar{G}(s) = \frac{[N(s) + \hat{\alpha}s]}{D(s)},
$$

(2.39)

then

$$
e_r = \frac{(b_1 - a_1) - \hat{\alpha}}{g_o}
$$

(2.40)

To meet constraint (I), selecting an accurate $\hat{\alpha}$ that satisfies $[(b_1 - a_1) - \hat{\alpha}] = 0 \rightarrow e_r = 0$ and $\eta = 1$, is needed. Hence, exact knowledge of $(b_1 - a_1)$ is required. Therefore, in the event when $(b_1 - a_1)$ is unknown, an estimation can be implemented to update $\hat{\alpha}(t)$. Define $\alpha = (b_1 - a_1)$ which is unknown. The parameter $g_o$ is assumed to be known. The ramp error $e_r$ is then

$$
e_r = \lim_{t \to \infty} \frac{\hat{\alpha}(t) - \alpha}{g_o}
$$

(2.41)

and we equivalently want

$$
e_r = \lim_{t \to \infty} \frac{\hat{\alpha}(t) - \alpha}{g_o} \rightarrow 0
$$

(2.42)

Subsequently, we design the update law:

$$
\dot{\hat{\alpha}} = g_o \gamma e(t), \quad \gamma > 0, \quad e(t) = r(t) - y(t)
$$

(2.43)

For the case where constraint (II) or constraint (III) is desired to be met, the numerical measurement approach can be utilized to update the value of $\hat{\alpha}$ in the right direction to obtain the desired $\eta_d$. Here, the instantaneous $\eta_m$ and $\eta_d$ are evaluated in a similar manner to the steps shown in Eqn.(2.30) and Eqn.(2.31).
2.3.2 Simulation Results

Simulation results to validate the analysis related to higher order transfer functions are presented next. Figure 2.7 shows the response output of the transfer function:

$$\bar{G}(s) = \frac{s^4 + 3s^3 + 6s^2 + a_1 s + 2}{s^5 + 2s^4 + 6s^3 + 5s^2 + 6s + 2} \quad (2.44)$$

Figure 2.7: Higher Order Response with Different Values of $\alpha$
Here, increasing the value of $a_1$ decreases $\eta$ in a nonlinear but monotonic manner and vice versa. The step response associated with $a_1 = 6$ represents the case where constraint (I) is being met. Similarly, the values of $a_1 = 8$ and $a_1 = 3$ satisfy constraint (II) and constraint (III), respectively. The corresponding ramp responses, shown in Fig. 2.7(b), show the steady-state $e_r$ and their respective $\eta$ values.

The numerical measurement approach is also simulated with the following 4th order transfer function:

$$\bar{G}(s) = \frac{(N(s) + \alpha s)}{D(s)} = \frac{[(a_1 \equiv 2) + \alpha]s + 1}{s^4 + 7s^3 + 5s^2 + (b_1 \equiv 3)s + 1} \quad (2.45)$$

In this case, a set of simulations that considers constraint (I), is first implemented. Here, in order for the dynamic of the system from Eqn.(2.45) to satisfy constraint (I) of $\eta = 1$, $[(b_1 - a_1) - \hat{\alpha}] \to 0$. However, the parameter $a_1$ is assumed unknown. Consequently, the numerical measurement approach is utilized to calculate the instantaneous value of $\eta_m$ and estimate the proper value of $\hat{\alpha} \to \alpha = (b_1 - a_1)$ in order to achieve a desired $\eta_d = 1$. Accordingly, the steady value of $\hat{\alpha}$ is obtained to be 1 which is identical to the actual value of $\alpha$ in the event $a_1$ would be known. The results are shown in Fig. 2.8. In Fig. 2.8(a), the step response of Eqn.(2.45) with the estimate $\hat{\alpha}$ is denoted $\hat{y}(t)$, while the true step response of Eqn.(2.45) with the actual $\alpha$ is denoted $y(t)$. Moreover, the corresponding results of $\eta_d$ and $\hat{\alpha}$ for $\hat{y}(t)$, and $\eta$ and $\alpha$ for $y(t)$, are shown in Fig. 2.8(b) and Fig. 2.8(c).
Figure 2.8: Numerical Measurement Approach with Constraint (I) of $\eta = 1$
Another set of simulation is implemented to address the case of maintaining a desired \( \eta \neq 1 \) using the numerical measurement approach. Hence, a desired \( \eta_d = 0.7 \) is assumed. Subsequently, two simulation scenarios are considered as shown next. In the first scenario, the parameters \( a_1 \) and \( b_1 \) of the numerator and denominator of \( \bar{G}(s) \) in Eqn.(2.45), respectively, are assumed unknown. Hence, \( (b_1 - a_1) \) is unknown. Consequently, the instantaneous value of \( \eta_m \) and the estimate \( \hat{\alpha} \) are evaluated to achieve \( \eta_d \). Here, Eqn.(2.35) is utilized to update the value of \( \hat{\alpha} \) in the direction that ensures \( \alpha > (b_1 - a_1) \), thus \( e_r < 0 \) that fits constraint (II) of \( (\eta = 0.7) < 1 \). Accordingly, the steady value of \( \hat{\alpha} \) is obtained to be 2.6. Then the second simulation scenario is implemented to validate the obtained value of \( \hat{\alpha} = 2.6 \). Hence, the response of the transfer function from Eqn.(2.45), is simulated where \( \alpha \) is set to be 2.6. Accordingly, the corresponding ratio \( \eta \) is obtained as \( \eta = 0.70 \). Thus, the feasibility of the numerical measurement approach is confirmed for the case of \( \eta \neq 1 \).

Figure 2.9 shows the simulation results of the previous two scenarios correspond to constraint (II) with \( \eta_d = 0.7 \). Figure 2.9(a) shows the step responses from Eqn.(2.45) using the numerical measurement approach, denoted as \( \hat{y}(t) \), and the true \( y(t) \) in absence of uncertainty. Moreover, Fig. 2.9(b) and Fig. 2.9(c) show the corresponding results of \( \eta_d \) and \( \hat{\alpha} \) for \( \hat{y}(t) \), and \( \eta \) and \( \alpha \) for \( y(t) \).
Figure 2.9: Numerical Measurement Approach with Constraint (I) of $\eta \neq 1$
CHAPTER 3

INTEGRAL CONSTRAINTS FULFILLMENT AND IMPLEMENTATION

3.1 Chapter Overview

In Chapter 3, we discuss control frameworks to satisfy the three types of integral constraints from Eqn. (1.9). In this regard, a combined feedforward and feedback technique is proposed to transform any given linear-system plant to a proper closed-loop structure that ensures the integral constraints to be met. Here, the proposed structure only requires the plant to be stable and with a finite positive DC gain. Hence, complete knowledge of the plant transfer function is not needed. Subsequently, the necessary conditions to maintain the stability of the aforementioned closed-loop transfer function is investigated. Furthermore, structured adaptive estimation strategies are introduced to address robustness to parameters uncertainties. Simulation results to validate the proposed control structures and strategies are presented. Demonstration using practical application examples is also illustrated.

1 Chapter 3 contains material previously published in [SD17, SD19]
3.2 Combined Feedforward and Feedback Actions

3.2.1 Compensation Structure

The constraints, defined in Eqn.(1.9), necessitate a transfer function to exhibit certain characteristics as discussed in the previous sections of Chapter 2. To this end, the class of transfer functions, considered in the analysis, is assumed to have a dynamic model that allows a tunable parameter, $\alpha$. Here, $\alpha$ is modified to achieve a proper ramp error $e_r$ that fits a desired constraint, i.e. a desired $\eta$. Next, the following question arises: how these dynamic characteristics can be accomplished if a given plant transfer function does not possess a system dynamic that allows $e_r$ to be modulated through a tunable $\alpha$. Consequently, a transformation structure using combined feedforward and feedback actions is proposed, which is shown in Fig. 3.1.

$$\alpha \quad \frac{\lambda}{s} \quad L(s)$$

$G(s) = \frac{y(s)}{r(s)}$  
$\bar{G}(s) = \frac{y(s)}{r(s)}$  
$y(s)$  
$r(s)$  
$\alpha$  
$\lambda/s$  
$L(s)$  
$G(s)$

Figure 3.1: Schematic of Combined Feedforward and Feedback Structure
Referring to Fig. 3.1, we state and prove the following lemma.

**Lemma 2.** Consider a proper rational (PR) plant $G(s)$ with a finite positive DC gain $G(0)$. The combined feedforward and feedback structure shown in Fig. 3.1, with $\lambda > 0$, enables the closed-loop system

$$\bar{G}(s) = \frac{y(s)}{r(s)} = \frac{G(s)[\alpha s + \lambda]}{s + \lambda G(s)} \quad (3.1)$$

to satisfy the integral constraints in Eqn.(1.9) with finite values of $\alpha$, provided $\bar{G}(s)$ is stable.

**Proof:** The result from Lemma 2 can be easily proven by noting that, if $\bar{G}(s)$ is stable, the steady state step error $e_s$ will be zero, since

$$e_s = \lim_{s \to 0} s \left[1 - \bar{G}(s)\right] \frac{1}{s} = \lim_{s \to 0} \frac{s[1 - \alpha G(s)]}{s + \lambda G(s)} = \lim_{s \to 0} \frac{s[1 - \alpha G(0)]}{\lambda G(0)} = 0 \quad (3.2)$$

The steady state ramp error then will be,

$$e_r = \lim_{s \to 0} s \left[1 - \bar{G}(s)\right] \frac{1}{s^2} = \lim_{s \to 0} \frac{[1 - \alpha G(0)]}{\lambda G(0)} = \frac{1}{\lambda} \left[G^{-1}(0) - \alpha\right] \quad (3.3)$$

Thus, when $\alpha = G^{-1}(0)$, we have $\eta = 1$ in Eqn.(1.9). In addition, $\alpha > G^{-1}(0)$, and $\alpha < G^{-1}(0)$ produces $\eta < 1$ and $\eta > 1$, respectively. This completes the proof. ■.
Based on the discussion above, we state the following remark.

**Remark 2.** The feedforward gain $\alpha$ can be generalized to a transfer function $F(s)$, and the feedback control $\lambda/s$ can be generalized to a transfer function $D(s)$, for stabilization and to improve bandwidth, robustness, etc.

![Figure 3.2: Numerical Estimation of the Zero Location for $\eta \neq 1$](image)

It is noted that the combined feedforward and feedback structure shown in Fig. 3.1, reduces the necessary information about the plant transfer function to that of knowing only the D.C. gain. In this context, the result from Lemma 2, shows that using $\alpha = G^{-1}(0)$ satisfies constraint (I). More precisely, if $G(s)$ has a unity D.C gain then $\alpha = 1$ achieves $\eta = 1$. In addition, the result from Lemma 2, shows that the ramp error $e_r$ can take a negative and a positive value, hence handling constraints (II) and constraint (III) respectively. However, for constraint (II) and constraint (III) where $\eta \neq 1$, calculating the instantaneous value of $\eta \rightarrow \eta_m$ is necessary to update $\alpha$ until the desired $\eta$ is achieved. This is because for a general transfer function $G(s)$, from Eqn.(1.11), we see that $e_r$ cannot be directly
calculated for a desired $\eta$. Since $A_a$ requires complete knowledge of $\bar{G}(s)$. Note that an exception to this limitation occurs in the case of second order systems with a zero, as shown in Theorem 1. For the general $\bar{G}(s)$, if $\alpha > G^{-1}(0)$, then $e_r < 0$, and thus fits constraint (II). On the other hand, using $\alpha < G^{-1}(0)$ produces $e_r > 0$ that meets constraint (III). Consequently, the numerical measurement approach with the expression of $\eta_m$ from Eqn.(2.30) is utilized to adaptively update an initial estimate of $\alpha$, namely $\hat{\alpha}$, until a desired $\eta_d$ is fulfilled. The mechanism is shown in Fig. 3.2 and the adaptive approach is formulated as,

$$e_\eta = (\eta_m - \eta_d), \quad \dot{\hat{\alpha}} = \nu e_\eta \rightarrow \hat{\alpha} = \nu \int_0^t e_\eta dt + \hat{\alpha}_0, \quad \nu > 0. \quad (3.4)$$

3.2.2 Implementation Examples

The proposed feedforward and feedback structure is utilized to transform a first order plant with a known D.C gain of the form:

$$G(s) = \frac{1}{s + 1.2} \quad (3.5)$$

where the D.C. gain, $G(0) = 1/1.2$. According to Fig. 3.1 and Lemma 2, the corresponding closed-loop transfer function would be,

$$G(s) = \frac{y(s)}{r(s)} = \frac{\alpha s + 1}{s^2 + 1.2s + 1} \quad (3.6)$$

where we choose $\lambda = 1$. Here, to generate the oscillatory behavior, the plant dynamic is transformed to a second order system through the integral action. In addition, a minimum
phase zero is introduced to the system to shape the overshoot of the response based on the value of $\alpha$. Consequently, to place a zero that satisfies constraint (I) of $\eta = 1$, then $\alpha = G^{-1}(0) = 1.2$. Similarly, a zero that satisfies constraint (II) or constraint (III), can be added by choosing $\alpha > G^{-1}(0)$ or $\alpha < G^{-1}(0)$. We consider two sample cases where $\alpha = 2$, and $\alpha = 0.6$.

Figure 3.3: Step Response of $G(s)$, and $\bar{G}(s)$ with Different Values of $\alpha$

The step response of $G(s)$, without the compensation, is plotted in Fig. 3.3. In addition, three step responses of $\bar{G}(s)$, with the compensation, are plotted to demonstrate the three constraints from Eqn.(1.9). The corresponding results of $\eta$ are calculated numerically using the numerical measurement approach associated with Eqn.(2.30).
Figure 3.4: Analytic Calculation of $\eta$ using $\bar{G}(s)$ from Eqn.(3.6)

It is noted that the form of $\bar{G}(s)$ from Eqn.(3.6) resembles the transfer function stated in Eqn.(2.7) where $\omega_n^2 = 1$ and $2\zeta\omega_n = 1.2$. Accordingly, the analytical results derived in Theorem 1 and Eqn.(2.10) are feasible here. Consequently, the online calculated values of $\eta$ are validated using the ($\eta$ vs. $\alpha$) relation associated with $\zeta = 0.6$, refer to Fig. 2.5. This validation is visualized in Fig. 3.4 which confirms the consistency of the analytical and numerical approaches of $\eta$ calculation.
Another set of simulation is shown in Fig. 3.5. The implemented transfer function \( G(s) \) and the corresponding transformed structure \( \bar{G}(s) \) are:

\[
G(s) = \frac{2s + 4}{s^2 + 3s + 6} \implies \bar{G}(s) = \frac{2\alpha s^2 + (4\alpha + 2)s + 4}{s^3 + 3s^2 + 8s + 4}, \quad \lambda = 1
\]  

\( (3.7) \)

![Figure 3.5: Step Response of Eqn.(3.7), and Corresponding \( \eta \) with Different \( \alpha \)](image)
Hence, the D.C gain $G(0) = 4/6$ is used to define three values for $\alpha$. These are $\alpha = G^{-1}(0) = 1.5$, $\alpha > G^{-1}(0) = 2$, and $\alpha < G^{-1}(0) = 1.25$ which are associated with constraint (I) of $\eta = 1$, constraint (II) of $\eta < 1$, and constraint (III) of $\eta > 1$, respectively. Fig. 3.5(a) shows the step responses of the transfer functions $G(s)$ and $\bar{G}(s)$. The corresponding results of $\eta$ are obtained numerically using Eqn.(2.30) as shown in Fig. 3.5(b).

The results, presented in this section, confirm the feasibility of the proposed feedforward and feedback approach to produce a transformed plant dynamic whose structure can fulfill constraint (I), constraint (II), and constraint (III). The finding that a proper zero can be added based on the D.C gain of the plant, is also confirmed. In addition, the behavior of the simulated results is consistent with the effect occurs when an additional zero is introduced to the system. Here, the added zero influences the overshoot of the response, thus shaping $A_a$ and $A_b$ in order to achieve a desired $\eta$. Note that the mechanism of the numerical management approach of calculating $\eta$ online, presented in Fig. 3.2, is applicable to achieve a desired $\eta \neq 1$ under parameters uncertainty as well; this is demonstrated in Section 3.3.
3.2.3 Stability Investigation

It is noted that the plant transfer function $G(s)$, utilized in the foregoing discussions, is considered to be strictly stable. In this section, the stability of the closed-loop transfer function $\bar{G}(s)$, generated by the combined feedforward and feedback compensation structure, is investigated. From Eqn.(3.1) and Lemma 2, it can be observed that the feedforward action, represented by $\alpha$, does not affect the roots of the characteristic equation of $\bar{G}(s)$, thus, the stability discussion is reduced to that of an integral controllability problem where an open loop stable system is controlled by an integral action. To this end, a plant transfer function $G(s)$ is called integral controllable if a positive range of integral gains $\lambda > 0$ can be utilized to ensure the stability of $\bar{G}(s)$. In this regard, we state the following theorem [MG04], using Fig. 3.1,

Theorem 2. Assume that $G(s)$ is a proper rational and stable transfer function. There exists a $\lambda > 0$ such that the closed-loop system $\bar{G}(s)$ is stable if and only if $G(0) > 0$.

Proof: See [MG04].

Theorem 2 can be directly illustrated for SISO system by examine the characteristic equation of the closed-loop transfer function $\bar{G}(s)$, which is

$$s + \lambda G(s) = 0 \quad \Rightarrow \quad sD(s) + \lambda N(s) = 0, \quad G(s) = \frac{N(s)}{D(s)} \quad (3.8)$$

where $G(s)$ is the plant transfer function. Upon expanding $G(s)$, we have

$$G(s) = \frac{n_ms^m + n_{m-1}s^{m-1} + \ldots + n_1s + n_0}{d_qs^q + d_{q-1}s^{q-1} + \ldots + d_1s + d_0}, \quad q > m \quad (3.9)$$
Hence, the characteristics equation of $\bar{G}(s)$ becomes:

$$d_q s^{q+1} + d_{q-1} s^q + \ldots + (d_{m-1} + \lambda n_m) s^m$$

$$+(d_{m-2} + \lambda n_{m-1}) s^{m-1} + \ldots + (d_0 + \lambda n_1) s + \lambda n_0 = 0$$

(3.10)

Note that in order for $\bar{G}(s)$ to be stable then all the coefficients of the characteristic equation in Eqn.(3.10) have to be positive. Also, $G(s)$ is given to be strictly stable. Consequently, in order for a gain $\lambda > 0$ to exist then $n_0 > 0$, and thus $G(0) > 0$.

The result from Theorem 2 indicates that a plant $G(s)$ with a negative D.C. gain $G(0) < 0$ can not satisfy the three types of integral constraints from Eqn.(1.9). Hence, the necessary condition for $\bar{G}(s)$ to be stable is to have a plant with a positive D.C gain $G(0) > 0$. Furthermore, for a second order $G(s)$, a gain of $\lambda > 0$ is sufficient. On the other hand, there is restriction on the range of $\lambda > 0$ within which a higher order $\bar{G}(s)$ can be stable.

Here, the Routh’s stability criterion can be utilized to specify the proper range of $\lambda$ that is required to ensure the stability of $\bar{G}(s)$ with higher order transfer functions. However, it requires more detailed knowledge of the characteristic equation of $G(s)$. In general, it is preferable if stability can be concluded without specific knowledge of the characteristics equation; refer to the discussion about the D.C. gain in Section 3.2.
A more definitive stability result is given in Lemma 3.

**Lemma 3.** Consider a strictly stable and proper rational (PR) plant $G(s)$ in a finite unity feedback loop with an integral control $\frac{1}{s}$. Then, the characteristic equation of the closed-loop system,

$$1 + \frac{\lambda}{s}G(s) = 0$$

(3.11)

will be stable for a range of positive values of $\lambda$, $0 < \lambda < \lambda_{\text{max}}$, if and only if $G(s)$ has none or an even number of zeros in the open right half plane (RHP).

**Proof:** To prove this result, we will take the root locus approach to stability analysis. Without loss of generality, assume that the numerator and denominator polynomials of $G(s)$ to be monic. Since $G(s)$ is strictly stable, all poles of $G(s)$ are on the open left half plane (LHP). Thus, the term $\frac{1}{s}$ adds a single pole to the loop gain transfer function at the origin (i.e. multiplicity 1). Hence, as $\lambda$ increases, the single branch of root locus emanating from $s = 0$ will lie either along positive real axis, or along negative real axis. Essentially, this is equivalent to the statement that the angle of departure from $s = 0$ will be $\phi_{\text{dep}} = 0^\circ$ or $\phi_{\text{dep}} = 180^\circ$, i.e.

$$\phi_{\text{dep}} = \sum \psi_i - \sum \phi_i - 180^\circ$$

(3.12)

Consider $\sum \phi_i$ which represents the sum of all the angles subtended by the origin from each pole of $G(s)$. Since all poles of $G(s)$ are in the open LHP, therefore $\sum \phi_i = 0^\circ$. The angle formed by complex conjugate poles will cancel when considered in pair.

Next, let us split $\sum \psi_i$ which represents the sum of all angles subtended from each zero to the origin as $\sum \psi_i = (\sum \psi_i)_{\text{LHP}} + (\sum \psi_i)_{\text{RHP}}$, where $(\sum \psi_i)_{\text{LHP}}$ and $(\sum \psi_i)_{\text{RHP}}$ are the angles formed by LHP and RHP zeros, respectively. Now, $(\sum \psi_i)_{\text{LHP}} = 0^\circ$ for the same
reason as \((\sum \psi_i = 0^\circ)\). For \((\sum \psi_i)_{\text{RHP}}\), it can be verified that

\[
(\sum \psi_i)_{\text{RHP}} = \begin{cases} 
0^\circ & \text{0 or even RHP zeros} \\
180^\circ & \text{odd RHP zeros}
\end{cases}
\] (3.13)

Thus,

\[
(\phi)_{\text{dep}} = \begin{cases} 
-180^\circ & \text{0 or even RHP zeros} \\
0^\circ & \text{odd RHP zeros}
\end{cases}
\] (3.14)

The other branches of the root locus, that cross over to the RHP, will do so only for some finite \(\lambda_k > 0\), since all poles lie in the open LHP. Let the smallest crossover value be \(\lambda_{\text{max}} = \arg\min_{s = j\omega}(\frac{\lambda_k}{s})\). Hence, closed-loop system will be stable for \(0 < \lambda < \lambda_{\text{max}}\). On the other hand, when \(\phi_{\text{dep}} = 0^\circ\), there will be roots of characteristic equation in the RHP for \(\lambda > 0\), This proves both the necessary and sufficient conditions for stability. ■.

### 3.3 Satisfying Integral Constraints Under Uncertainty

The combined feedforward and feedback structure, proposed in Section 3.2, illustrates that a precise location of the added zero is essential to satisfy a desired \(\eta\) associated with constraints (I), constraint (II), or constraint (III). Furthermore, it requires an exact determination of the D.C gain of the plant i.e. \(G(0)\); although full knowledge of the transfer function is not needed. This is because the closed-loop transfer function

\[
\bar{G}(s) = \frac{y(s)}{r(s)} = \frac{G(s)[\frac{\lambda}{s} + \alpha]}{1 + \frac{\lambda}{s}G(s)}
\] (3.15)

requires \(\alpha\) according to Eqn.(3.3) to satisfy \(\eta = 1, \eta < 1\), or \(\eta > 1\).
Then, \( \alpha = G^{-1}(0) \) satisfies constraint (I) of \( \eta = 1 \), while \( \alpha > G^{-1}(0) \) and \( \alpha < G^{-1}(0) \) satisfies constraint (II) of \( \eta < 1 \) and Constraint (III) of \( \eta > 1 \), respectively. However, let us assume that the D.C gain of the plant is not accurately known. To address this issue we propose online adaptation of the zero location to ensure consistent performance in the presence of this parametric uncertainty. We first address constraint (I) of \( \eta = 1 \). Subsequently, we discuss an approach to handle constraints (II) and constraint (III) corresponding to \( \eta \neq 1 \).

### 3.3.1 Adaptive Zero Placement: \( \eta = 1 \)

Consider the generic form of \( \bar{G}(s) \) given above and stated in Eqn.(3.15). The dynamic equation of the error \( e = r(s) - y(s) \) can be expressed as:

\[
e(s) = \frac{r(s)}{1 + G(s)\left[\frac{\lambda}{s} + \hat{\alpha}\right]} \tag{3.16}
\]

We next state and prove the following Lemma.

**Lemma 4.** Consider the closed-loop system in Fig. 3.6, where an adaptive estimation law

\[
\dot{\hat{\alpha}} = \gamma \int e(t), \quad \gamma > 0 \tag{3.17}
\]

is used to determine \( \alpha \). The stability of this combined feedforward, feedback, and adaptively estimated system to a step input, is determined by the characteristic equation

\[
1 + \frac{G(s)}{s^2}[\gamma + \lambda s] = 0, \quad \gamma, \lambda > 0 \tag{3.18}
\]
Figure 3.6: Constraint (I) with Adaptive Zero Placement

**Proof:** From the expression of $e(s)$ in Eqn.(3.16), and upon incorporating $\hat{\alpha}(s) = \frac{\gamma}{s^2} e(s)$, from Eqn.(3.17), we have

$$e(s)[1 + \frac{G(s)}{s^2}(\lambda s + \gamma)] = r(s)$$

(3.19)

Thus, the characteristic equation would be,

$$1 + \frac{G(s)}{s^2}[\lambda s + \gamma] = 0$$

(3.20)

This completes the proof. ■.

The characteristic equation of Eqn.(3.18) shows addition of a zero at $s = -\frac{\gamma}{\lambda}$ due to adaptation. Based on Lemma 4, if we consider a basic first order plant,

$$G(s) = \frac{a}{\tau s + 1}$$

(3.21)
then from Eqn.(3.18), the characteristic equation of the system $\bar{G}(s)$ with an adaptively estimated $\alpha$ would be:

$$1 + \frac{a}{s^2(\tau s + 1)}[\gamma + \lambda s] = 0$$

(3.22)

which simplifies to

$$\tau s^3 + s^2 + a\lambda s + a\gamma = 0$$

(3.23)

The stability of Eqn.(3.23) can be investigated using the Routh-Hurwitz Criterion [FPEN18], yielding

$$\gamma < \frac{\lambda}{\tau}$$

(3.24)

and a conservative estimate of $\gamma$ would be

$$\gamma < \frac{\lambda}{\tau_{max}}, \quad 0 < \tau < \tau_{max}$$

(3.25)

Similar stability analysis can be conducted for specific classes of transfer functions $G(s)$, such as second order, third order, etc.

Remark 3. As $\gamma \to 0^+$, the zero introduced by adaptation, at $s = -\frac{\lambda}{\tau}$ moves toward the origin of the complex plane. For $\bar{\gamma} = 0$, the characteristic equation reduces to

$$1 + \frac{\lambda}{s}G(s) = 0$$

(3.26)

same as that in Eqn.(3.11) with no adaptation. Lemma 3 gives a necessary and sufficient condition under which there exists a range of $\lambda > 0$ where Eqn.(3.26) is stable. Therefore, by extension, we can say that by choosing a small $\gamma$, such that the zero at $-\frac{\lambda}{\tau}$ is dominant, we can improve stability of the combined system of Fig. 3.6.
Figure 3.6 demonstrates the mechanism of the adaptive estimation of the zero location to satisfy constraint (I). This approach is also implemented for validation as shown in Fig. 3.7.

Figure 3.7: Adaptive Zero Placement for \( \eta = 1 \)
Here, the D.C. gain of the plant transfer function, stated in Eqn.(3.28), is assumed to be uncertain. Accordingly, the value of \(\hat{\alpha}\) is adaptively estimated to simulate the corresponding transfer function \(\bar{G}(s)\).

\[
G(s) = \frac{8}{s^2 + 4s + 6} \tag{3.27}
\]

\[
\Rightarrow \bar{G}(s) = \frac{8\hat{\alpha}s + 8}{s^3 + 4s^2 + 6^2 + 8}, \quad \lambda = 1 \tag{3.28}
\]

Figure 3.7(a) compares two step responses of \(G(s)\) from Eqn.(3.28). The first response \(\hat{y}(t)\) represents the adapted response where \(\hat{\alpha}\) is estimated and adaptively updated until a desired \(\eta = 1\) is achieved. On the other hand, the response \(y(t)\) represents the case where \(G(0)\) would be exactly known, i.e. \(\alpha = G^{-1}(0) = 0.75\). Figure. 3.7(b) presents the output results of the exact \(\eta\), and the estimated \(\hat{\eta}\) where the feasibility to satisfy constraint (I) of \(\eta = 1\), given uncertain \(G(0)\), is confirmed. Accordingly, it is confirmed that the estimated value of \(\hat{\alpha}\) properly converges to the presupposed value of the exact \(\alpha\), i.e. \(\hat{\alpha} \to \alpha = 0.75\). Consequently, the adaptive estimation of the zero location approach works as designed and accurately estimates the uncertain \(\alpha\).

### 3.3.2 Adaptive Zero Placement: \(\eta \neq 1\)

In this section, we address the case of satisfying constraint (II) and constraint (III) under system uncertainty. Here, to achieve a desired \(\eta \neq 1\), an accurate D.C. gain is still required to select a proper \(\alpha\) that places an accurate zero. However, in comparison to \(\alpha = G^{-1}(0)\) for constraint (I) of \(\eta = 1\), the relation between \(\alpha\) and \(\eta \neq 1\) is nonlinear and more compli-
cated. Furthermore, in the event of uncertain \( G(0) \), \( \eta = 1 \leftrightarrow \lim_{t \to \infty} e = e_r = 0 \). However, the corresponding relation between desired \( \eta \neq 1 \) and \( e_r \) requires a numerical calculation of the instantaneous value of \( \eta \rightarrow \eta_m \) to update \( \alpha \) until the desired \( \eta \) is achieved. Consequently, the numerical measurement approach, demonstrated in Fig. 3.2 with the steps related to Eqn.(2.30) and Eqn.(3.4), is applicable to adaptively estimate the zero location that is proper to satisfy a desired \( \eta \neq 1 \).

Figure 3.8 shows a simulation scenario with the following 4th order plant transfer function:

\[
G(s) = \frac{s^3 + 4s^2 + 7s + 4}{s^4 + 5s^3 + 8s^2 + 9s + 4}
\]  

(3.29)

In this scenario, a desired \( \eta_d = 0.75 \) is considered. Subsequently, the instantaneous value of \( \eta_m \) is calculated online. Accordingly, \( \hat{\alpha} \) is evaluated as demonstrated above until the desired \( \eta_d \) is achieved. As a result, the proper \( \hat{\alpha} \) is obtained to be 1.4843. It is noted that the obtained \( \hat{\alpha} = 1.4843 > (G^{-1} = 1) \), and thus it is consistent with the requirement of constraint (II). In addition, it is indicated that knowing the exact D.C gain of the plant is not essential for this process. Yet, it helps selecting an appropriate initial value for \( \hat{\alpha} \). Figure 3.8(a) shows the step response of \( \bar{G}(s) \), denoted \( \hat{y} \). While Fig. 3.8(b) shows the corresponding results of \( \hat{\eta} \) and \( \hat{\alpha} \). In addition, a validation simulation is implemented where the combined feedforward and feedback structure is used to introduce a zero to the plant \( G(s) \). Here \( \alpha \) is set to be 1.4843 to examine the output result of \( \eta \). Accordingly, the corresponding \( \eta \) is calculated as \( \eta = 0.75 \), thus validates the numerical result of \( \hat{\alpha} \). The step response of \( y \) is shown in Fig. 3.8(a), while the corresponding results of \( \eta \) and \( \alpha \) are shown in Fig. 3.8(b).
Figure 3.8: Numerical Estimation of the zero Location for $\eta < 1$
The plant transfer function from Eqn.(3.29) is also simulated to satisfy constraint (III) with a desired \( \eta_d = 1.25 \). The results are shown in Fig. 3.9. The estimate \( \hat{\alpha} \) is obtained to be 0.722 which is contestant with constraint (III) requirement; \( (\hat{\alpha} = 0.722) < (G^{-1} = 1) \). In addition, the results of Fig. 3.9 are validated by placing a zero at the corresponding location of \( \hat{\alpha} = \alpha = 0.722 \). Accordingly, the corresponding \( \eta \) is calculated as \( \eta = 1.25 \).

The step responses correspond to \( \alpha \) and \( \hat{\alpha} \) are shown in Fig. 3.9(a). Figure 3.9(b), on the other hand, shows the respective results of \( \eta \) and \( \alpha \), and \( \hat{\eta} \) and \( \hat{\alpha} \).

![Figure 3.9: Numerical Estimation of the zero Location for \( \eta > 1 \)](image-url)
The combined feedforward and feedback structure of Fig. 3.1 is a 2 DOF controller with two tunable parameters $\alpha$ and $\lambda$. The parameter $\alpha$ is used to satisfy the integral constraint and $\lambda$ is used to ensure stability as discussed in Section 3.1. We have established that $\alpha$ takes unique values, independent of $\lambda$, in satisfying specific integral constraints. However, closed-loop stability is generalized by a range of $\lambda$.

In this section, we explore how the range of stabilizing $\lambda$ values can be utilized for enhancing robustness. In particular, we address how an additional constraint, in the form of a maximum allowable overshoot, can be met by proper choice of $\lambda$ while satisfying the imposed integral constraint. Such a restriction would be practical in hybrid power systems and self-driving vehicles. In the former case, energy storage devices such as batteries and ultra-capacitors have current-limits which directly translates to overshoot. In the latter case, acceleration limits and quality of driving would impose restrictions on speed fluctuations. We took two approaches for this analysis. Firstly, we analyze the problem from a frequency domain perspective. The second approach will be in the time domain.

### 3.4.1 Robustness Through Frequency Domain Principles

In this section we provide an overview of the frequency domain approach. Specific steps would depend on the plant $G(s)$ are considered. For a general plant $G(s)$ in Fig. 3.1, the crossover frequency $\omega_c$ of the loop-gain $\left[\frac{\lambda G(s)}{s}\right]$ would be given by,

$$\left|\frac{\lambda G(j\omega_c)}{\omega_c}\right| = 1 \Rightarrow \left|G(j\omega_c)\right| = \frac{\omega_c}{\lambda}$$  \hspace{1cm} (3.30)
Therefore, the Phase Margin, $\theta_{PM}$ is

$$\theta_{PM} = 180^\circ + \frac{\lambda}{j\omega_c} G(j\omega_c) = 90^\circ + \langle G(j\omega_c) \rangle \quad (3.31)$$

For curtailing the maximum overshoot, $\theta_{PM}$ must be high. Say $0 < \theta_{PM,des} < 90^\circ$ is the desired phase margin. Then,

$$-90^\circ < \langle G(j\omega_c) \rangle = \theta_{PM,des} - 90^\circ < 0^\circ \quad (3.32)$$

A method of determining $\lambda$ would be to solve Eqn.(3.32) to determine $\omega_c$ and then using Eqn.(3.30) to determine $\lambda$. To do so, the specific expression of $G(s)$ may not be needed. For instance, consider $G(s)$ to have the frequency response shown in Fig. 3.10.

The desired phase margin $\theta_{PM,des}$ and the corresponding desired $\langle G(j\omega_c) \rangle$ and desired $\omega_c$ for $L(s)$ as shown in Fig. 3.10. Note that the bandwidth of the compensated system will be limited by that of $G(s)$, i.e. by $\omega_{BW}$. hence, a desired $\omega_c < \omega_{BW}$ can be obtained by approximating $|G(j\omega_c)| \approx 1$ for $\omega < \omega_{BW}$, leading to $\lambda = \omega_c$ (from Eqn.3.30 ) as an initial choice of $\lambda$ from where iteration can be started.
3.4.2 Robustness Through Time Domain Analysis

For simple $G(s)$, a direct time-domain analysis can be applied to impose an upper bound on the overshoot. Consider $G(s) = \frac{a}{\tau s + 1}$. Here,

$$
G(s) = \frac{y(s)}{r(s)} = \frac{G(s)\left[\frac{1}{s} + \alpha\right]}{1 + \frac{1}{s}G(s)} = \frac{a\alpha s + a\lambda}{\tau s^2 + s + a\lambda}
$$

(3.33)
we rewrite Eqn.(3.33) as

\[ \tilde{G}(s) = \frac{\tilde{\alpha} s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]  

(3.34)

where \( \omega_n^2 = \frac{a\lambda}{\tau} \), \( 2\zeta \omega_n = \frac{1}{\tau} \), and \( \tilde{\alpha} = \frac{a\alpha}{\tau} \). Thus, by changing \( \lambda \), we vary \( \omega_n \) and \( \zeta \). The solution of Eqn.(3.34) to zero initial conditions and a unit step input is

\[ y(t) = 1 - \beta e^{-\sigma t} \sin(\omega_d t + \phi) \]  

(3.35)

with \( \sigma = \zeta \omega_n \), \( \omega_d = \omega_n \sqrt{1 - \zeta^2} \), \( \beta = \sqrt{1 + (\sigma - \tilde{\alpha})^2/\omega_d^2} \), and \( \phi = \cos^{-1}\left(\frac{\sigma - \tilde{\alpha}}{\omega_d}/\beta\right) \). To determine the point of maximum overshoot, we impose

\[ \frac{dy}{dt} = \beta e^{-\sigma t} \left[ \sqrt{\sigma^2 + \omega_d^2} \sin(\omega_d t + \phi - \psi) \right] = 0 \]  

(3.36)

where

\[ \sin \psi = \frac{\omega_d}{\sqrt{\sigma^2 + \omega_d^2}} \quad 0^\circ < \psi < 90^\circ \]  

(3.37)

which implies

\[ \omega_d t^* + \phi - \psi = 0 \quad \text{or} \quad \pi \]  

(3.38)

Here \( t^* \) takes the smallest positive value that cause \( \frac{dy}{dt} = 0 \) and \( y > 1 \). Since \( \beta \) and \( e^{-\sigma t} \) are positive, then \( \sin(\omega_d t^* + \phi) < 0 \) for \( y \) to be greater than 1. In addition, since \( 0^\circ < \psi < 90^\circ \), at \( t = t^* \), then

\[ \sin(\omega_d t^* + \phi) = \sin(\pi + \psi) = -\sin \psi \]  

(3.39)
Hence, the overshoot at $t = t^*$ is

$$M = y(t^*) - 1 = \beta e^{-\sigma t^*} \sin \psi$$

(3.40)

and

$$t^* = \frac{\pi + \psi - \phi}{\omega_d}$$

(3.41)

Upon substituting for $t^*$ from Eqn.(3.41), $\psi$ from Eqn.(3.37), noting $\sigma = \zeta \omega_n$, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, $\beta = \sqrt{1 + (\sigma - \bar{\alpha})^2/\omega_d^2}$ and defining the dimensionless parameter $\bar{\kappa} = \frac{\bar{\alpha}}{\omega_n}$, $M$ reduces to

$$M = \sqrt{1 - \zeta^2} + (\zeta - \bar{\kappa})^2 e^{-\frac{\zeta}{\sqrt{1 - \zeta^2}}(\pi + \psi - \phi)}$$

(3.42)

with $\psi = \sin^{-1}(\sqrt{1 - \zeta^2})$ and $\phi = \cos^{-1}\left[\frac{(\zeta - \bar{\kappa})}{\sqrt{(1 - \zeta^2)^2 + (\zeta - \bar{\kappa})^2}}\right]$. A plot of $M$ vs. $0 \leq \zeta \leq 1$ for different values of $\bar{\kappa}$ is given in Fig. 3.11.

Figure 3.11: Plot of $M$ vs. $\zeta$ for Different Values of $\bar{\kappa}$
To satisfy $\eta = 1$, $\bar{\alpha}$ must be chosen as $\bar{\alpha} = 2\zeta \omega_n$. Then, $\bar{\kappa} = 2\zeta$ and $M$ in Eqn.(3.42) reduces to

$$M = e^{-\frac{2\zeta \psi}{(\sqrt{1-\zeta^2})}}, \quad \psi = \sin^{-1}(\sqrt{1-\zeta^2}) \quad (3.43)$$

Note from Eqn.(3.34) that here $M$ is still a function of $\lambda$. This is because

$$\zeta = \frac{1}{2\omega_n \tau} = \frac{1}{2\sqrt{\frac{a \lambda}{\tau} \tau}} = \frac{1}{2\sqrt{a \lambda \tau}} \quad (3.44)$$

A plot of $M$ vs. for different values of $\lambda$ is given in Fig. 3.12 where the parameters $a = 1$ and $\tau = 2$ are chosen.

![Figure 3.12: Plot of $M$ vs. $\lambda$ for $\bar{\kappa} = 2\zeta$](image-url)
3.5 Application Example: Decentralized Power Management of Hybrid Power Systems

In this section, a practical application scenario of decentralized power management for hybrid power systems is discussed. As reviewed in Chapter 1, augmenting hybrid power systems with energy storage devices, ensures maintaining the power demand of the load and improves the loadfollowing response of the system. However, it requires a control strategy to safeguard the storage device from being overcharged or progressively discharged. To this end, It is noted that for the case of power systems, the parameter $\eta$ represents the system efficiency. Consequently, the implementation is limited to satisfying constraint (I) and constraint (II) from Eqn.(1.9) with $\eta \leq 1$. The system is assumed to consist of a primary power source $PPS$ connected in parallel to an $ESD$. Here, the energy storage works as a buffer to deliver the power demand and absorb the extra energy provided by the power source; hence, delivered power fluctuation is avoided. Accordingly, the objective of this power management scheme is to meet the power demand by the hybrid power system and simultaneously maintain the state of charge of the storage device at a constant level i.e. after each charge/discharge cycles. More precisely, the power source should provide enough energy to maintain the load demand and compensate the energy removed from the storage device during the transient response. In addition, the energy of the storage device should be preserved in order from being overcharged while the load demand is being met. Consequently, two individual controllers for the power source and the energy storage device can be developed as follows: controller-1 utilizes the transient response history of the power source of the hybrid power system to anticipate the energy deficit that is recovered by the energy storage device, and accordingly, alters the primary source’s output power. In the meantime, controller-2 enables the storage device to work as an energy buffer and track the load in the presence of fluctuating load demand.
A schematic of this decentralized power management control scenario is shown in Fig. 3.13. Refer to Fig. 1.3, consider the response \( r = V_l i_l \) to represent the step change in load demand that should be supplied by the hybrid power system. Similarly, consider \( y = \eta_{ps} V_{ps} i_{ps} \) to represent the corresponding response of the primary power source in load-following mode. The interval \( t_s \leq t \leq t_f \) represents the transient region as the response \( y \) continues tracking the changing load demand \( r \), matching it at a steady state, \( t > t_f \). Hence, during the transient interval, there is a deficit between the load demand \( r \) and the primary source’s delivered power \( y \), denoted by \( \Delta P \).

\[
\Delta P \triangleq (V_l i_l - \eta_{ps} V_{ps} i_{ps})
\]

(3.45)

where \( V_l i_l \) is the load demand, \( V_{ps} i_{ps} \) is the delivered power, and \( \eta_{ps} \) is the efficiency of the power source i.e. \((PPS)\).
According to Eqn.(3.45) and Fig. 1.3, the below area $A_{b,1}$ over the interval $t_s \leq t \leq t_1$, represents the gap between the provided energy from the power source and the load demand. Thus, it can be defined as:

$$ A_{b,1} = \int_{t_s}^{t_1} \triangle P \, dt \tag{3.46} $$

$$ = \int_{t_s}^{t_1} (V_l i_l - \eta_{ps} V_{ps} i_{ps}) \, dt \equiv \int_{t_s}^{t_1} (\eta_{sd} V_{sd} i_{sd}) \, dt $$

where $V_{sd} i_{sd}$ represents the energy provided by the storage device, and $\eta_{sd}$ is the efficiency during the discharging cycle. Similarly, the above area $A_{a,1}$ over the interval $t_1 \leq t \leq t_2$, represents the extra energy to be supplied by the power source in order to maintain the load demand and charge the storage device. Hence, it can be defined as:

$$ A_{a,1} = \int_{t_1}^{t_2} -\triangle P \, dt \tag{3.47} $$

$$ = \int_{t_1}^{t_2} (V_l i_l - \eta_{ps} V_{ps} i_{ps}) \, dt \equiv \int_{t_1}^{t_2} (\bar{\eta}_{sd} V_{sd} i_{sd}) \, dt $$

where $V_{sd} i_{sd}$ represents the extra energy provided by the power source, and $\bar{\eta}_{sd}$ is the efficiency during the charging cycle. Consequently, the area $\sum_{j=1}^{\infty} A_{b,j}$ represents the total amount of the energy to be removed from the storage device i.e. discharge, while the area $\sum_{i=1}^{\infty} A_{a,i}$ represents the total amount of the energy to be returned to the storage device i.e. charge. Hence, the condition:

$$ \lim_{t \to \infty} \left[ \sum_{j=1}^{\infty} A_{b,j} - \eta \sum_{i=1}^{\infty} A_{a,i} \right] \to 0, \tag{3.48} $$

will ensure that the energy of the storage device is preserved while the load demand is being met. The presence of system losses of the power source and the storage device.
during the charge/discharge cycles are captured through the efficiency $\eta$. In this regard, $\eta = 1$ represents the ideal case where there are no losses in the system, thus it is equivalent to satisfying constraint (I) of Eqn.(1.9). On the other hand, in the presence of losses, the efficiency is represented by $\eta < 1$ that is equivalent to satisfying constraint (II) from Eqn.(1.9).

The concept of the decentralized power management scenario, presented in this section, was initially proposed in [MBD16]. Detailed analysis of the theoretical framework was introduced and discussed. In addition, experimental results for validation were presented. The experimental setup is shown in Fig. 3.14. A real-time emulation of a Solid Oxide Fuel Cell (SOFC) mathematical model is utilized as the primary power source of the hybrid power system. Furthermore, an Ultra-Capacitor was employed as the energy storage device. Experimental Results are shown in Fig. 3.15.

![Experimental Setup of the Decentralized Power Management Approach](image)

Figure 3.14: Experimental Setup of the Decentralized Power Management Approach [MBD16]
Figure 3.15: Experimental Results of the Decentralized Power Management Approach
[MBD16]
Figure 3.15(a) shows a repetitive step change in load demand from 10A to 20A while Fig. 3.15(d) shows the corresponding fuel utilization where the steady state target $U_{ss}$ was set to 80%. Figure 3.15(b) and Fig. 3.15(e) show the current and voltage of the emulated SOFC power source, $i_{fc}$ and $V_{fc}$ respectively. In addition, Fig. 3.15(f) shows the voltage of the Ultra-Capacitor ($V_{uc}$) with and without the dissipation control. Here, a resistor was utilized to dissipate the extra energy when the Ultra-Capacitor voltage surpasses a certain threshold; refer to Fig. 3.14. Figure 3.15(c) shows the current of the Ultra-Capacitor ($i_{uc}$). Here, it can be seen that, at steady-state, $i_{uc} = 0$ which indicates that, at steady-state, the load demand is completely provided by the power source. Furthermore, Fig. 3.15(g) shows the response of the power source in load-following mode. Here, the deficit in the supplied power by the power source, and the contribution of the Ultra-Capacitor to compensate for that deficit in order to meet the load demand, are both depicted. Moreover, Fig. 3.15(g) visualizes the above area $A_{a,1}$ and the below area $A_{b,1}$. It can be noted that $A_{a,1} > A_{b,1}$ because of the extra power provided by the primary power source to compensate for the losses in the system $\eta < 1$. 
CHAPTER 4

CONCLUSIONS

In this research, a theoretical approach for shaping and controlling transient step response of linear systems was proposed. First, the problem was formulated using three types of integral constraints that when are imposed on step response, can lead to generating a specific type of transient behavior which in turn satisfies the integral constraints. Subsequently, the specific class of general transfer functions that can satisfy these aforementioned integral constraints, was determined through analysis and validated through simulations. The characteristics of this specific form of transfer functions were then utilized to propose a compensation structure that can enable a given plant to be transformed into the form which is amenable to satisfy the integral constraints. Thereafter, adaptive approaches to address the robustness of the proposed compensation structure under parameters uncertainties were demonstrated through analysis and simulations. A practical application example where these integral constraints-based control approaches arise was also illustrated.

It was shown that the transient of a linear step-response can be controlled based on predefined integral constraints if an appropriate transfer function structure is fulfilled. Furthermore, the formulation of the three types of integral constraints was translated to that of imposing conditions on the ramp error of linear systems. It was proven that satisfying these constraints is infeasible for the first order and second order systems with no additional zero. Nevertheless, second-order transfer functions were shown to satisfy the integral constraints if a suitably placed zero is added to the system. In addition, for a class of second order systems with a zero, an analytic expression that relates the imposed in-
Integral constraint and the corresponding location of the zero, was derived. In this regard, a Theorem that quantifies the summation of the overshoots areas of the step response when it converges to a steady-state was stated and proved. Moreover, a frequency ratio was defined to express the integral constraints using a non-dimensional relation that is based on the damping ratio of the system. In addition to second-order responses, the investigation was extended to higher-order transfer functions. In this context, it was shown that the ramp error can be modulated by only considering the coefficients of the low order terms of the transfer function (i.e. $s^1$ and $s^0$). Hence, the higher-order terms of the transfer function are not essential for satisfying the integral constraints. Thereafter, a control framework employing combined feedforward and feedback actions was proposed. Here, the compensation structure enables any strictly stable and proper rational plant with a positive D.C. gain to be transformed into the appropriate closed-loop form that can satisfy the three types of integral constraints. In this regard, the proposed feedforward and feedback structure reduces the necessary information about the plant transfer function to that of knowing only the D.C. gain.

The overall closed-loop system, proposed in this work, was also tested for stability. In this context, the necessary conditions for stability were investigated. The emphasis was to provide recommendations by which the stability of the closed-loop system, generated by the feedforward and feedback compensation actions, can be concluded without requiring specific knowledge of the corresponding characteristic equation. In this regard, it was observed that the feedforward action does not influence the closed-loop characteristic equation. In addition, a Theorem was stated to prove that there exists a positive integral gain where the closed-loop system can be stabilized. The stability discussion was also extended to include non-minimum phase zeros. Hence, a Lemma was stated and proved to show that a range of positive integral gains can also exist for a class of plant trans-
fer functions with non or an even number of non-minimum phase zeros. Thereafter, the investigation was driven to address satisfying the constraints under parametric uncertainties. Accordingly, it was shown that the location of the proper zero, required to satisfy an integral constraint, can be adaptively estimated in the presence of uncertainties. Here, a structured adaptive estimation strategy based on an added parameter adaptation law was demonstrated for second-order and higher-order systems. The adaptation strategy was then generalized to handle a general plant transfer function with uncertain D.C. gain. Furthermore, a numerical measurement approach that quantifies the relation between the imposed integral constraint and the corresponding zero location was illustrated. Finally, a practical application scenario related to energy management of hybrid power systems was presented. A further extension to address nonlinear systems is considered in our ongoing research.
LIST OF REFERENCES


