Multi-Scale Modeling of Viscoelastic Thin-Ply Composites

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MULTI-SCALE MODELING OF VISCOELASTIC THIN-PLY COMPOSITES

by

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B.S. University of Moratuwa, 2017

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Major Professor: Kawai Kwok
ABSTRACT

Thin ply composites have been under the microscope of researchers for around two decades since Kawabe introduced the thin ply technology, in the Industrial Technology Center in Fukui, Japan. With the advancement of the technological aspects of manufacturing carbon fiber composites the tow spreading has improved so much as to reduce the thickness of a conventional ply up to even six-fold. Over the years carbon fiber composites have gained interest in space structures for reinforcement purposes in antennas, as structural elements that store strain energy and support the deployment of deployable structures, and in reflector concepts. Now thin-ply composites are gaining attention for utilizing in small scale cube satellites in deployable structures. Since the deployable structures take larger space compared to the satellite, they have to be stowed in a smaller volume for logistic purposes. Therefore, achieving light weight structures and increasing the packing efficiency will improve the performance of the payload. With this perspective, thin-ply can make a significant improvement on deployable space structures. However, during stowage the deployable structures are subjected to stress relaxation due to viscoelastic effects reducing the strain energy, hindering the deployment and the stability of the structures. This study will focus on evaluating viscoelastic properties of thin-ply composites using a two-step homogenization process. A representative unit cell is extracted from the laminate and the geometry is constructed by using data extracted from micrographs. The tow is homogenized to a fiber filament model in the micro scale and the time dependent properties of the matrix are used to compute viscoelastic properties of the tow. Incorporating the properties of the viscoelastic tow to the unit cell model, the viscoelastic behavior is determined for the laminaes. The effects of laminate orientation, high curvatures and ply arrangements are studied for the laminaes considered in the study.
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CHAPTER 1: INTRODUCTION

1.1 Thin-Ply Composite and Application

Thin ply composites have been studied by researchers for around two decades since Kawabe [1] introduced the thin ply technology, in the Industrial Technology Center in Fukui, Japan. With the advancement of the technological aspects of manufacturing carbon fiber composites, the tow spreading has improved so much as to reduce the thickness of a conventional ply up to even six fold [2]. Carbon fiber composites even by conventional processing have improved some industrial standard for materials immensely substituting structural materials such as steel. Carbon fiber composites have a high stiffness to weight ratio, fatigue resistance, high temperature tolerance and low thermal expansion. This accounts to a clear advantage in aerospace, automobile, marine industries and recreational industries producing sporting goods.

The technique of tow spreading developed by the Industrial Fukui Center uses a flow of air to pull apart the tow and spread it. The process is known to be effective in terms of cost and the handling of fiber to not cause damage [3]. The tows are sent through a machine that contains an air duct and a vacuum. As depicted in Fig. 1.1a, the air flow is acting downwards sagging the fibers, relaxing them and attaining a tension free state. Fibers are not damaged due to the low velocity of air flow. As presented in of Fig. 1.1b, in the first stage due to the reduction of air speed around the filament compared to the air stream further away an increase of pressure is generated close to the filament sides. This introduces a force, pulling the tow apart and becomes progressive as it goes to the second stage and finally the tow is spread. The result is a spread tow that is subsequently winded on a spool. The reduction of this thickness has improved mechanical performance in laminate composites.
Over the years woven carbon fiber composites have gained interest in space structures for reinforcement purposes in antennas, as structural elements that store strain energy and support the deployment of deployable structures, and in reflector concepts. The use of composites in deployable structures have enabled the production of lightweight structural elements with expected mechanical performance. They are adopted in reflector concepts [4], [5], deployable booms in the Roll-Out-Solar array [6], solar sails [7], [8] and lightweight truss elements in other gossamer structures.
With the development of low orbit low cost small scale cube satellites for communication purposes and small interplanetary missions such as the Near Earth Asteroid Scout [9], the need for proper efficient materials is in demand. When considering the advantages of thin-ply composites, the capabilities of deployable space structures can be optimized. When considering low orbit satellite missions, the use of solar sails as a promising mechanism has been established. Solar sails can be of different scales with the small scale size suited for cube satellites with a solar sail of size up to 100 m$^2$. The solar sails are propelled by the momentum transfer that occurs due to the photons from the sun hitting the sail. This acts as a propellant for the payload or the satellite; therefore, carrying propellant fuel is avoided which can produce continuous thrust at no cost. This enables the satellite to reach non-Keplerian orbits that were not possible with conventional propellant techniques. But the thrust provided by the photon momentum transfer is very small, about 9.12$x_{10^{-6}}$ Pa, with a perfectly reflecting surface at 1 astronomical unit distance from the sun [8]. Therefore, the structures should be optimized for their weight in order to achieve a good performance. The use of thin-ply composites are promising materials to incorporate in structural elements which will help reduce this load. The NASA (National Aeronautics and Space Administration), ESA (European Space Agency), DLR (German Aerospace Center) as well as non-profit organizations like the Planetary Society have been working to employ the solar sail concept as an economical and a feasible solution for space missions. A cube-sat successfully launched by the Planetary Society known as the Light Sail 2 [10] on June 2019 has given promising insight to the future of solar sails. Another system developed by the NASA Langely Research Center is the Advance Composites-based Solar Sail System [8] (ACS3) for interplanetary missions. One of their 6U (10 cm x20 cm x30 cm) Cube-sats is depicted in Fig. 1.3. This form of the cub-sat has a solar sail that can be stowed in a 3U volume (20 cm x 10 cm x15 cm).
In addition to the reduction of load, the packing efficiency of deployable structures is a desired aspect. The deployable structure has to be large due to the area requirement for effective propulsion. For small scale satellites, the size can be 6U (10 cm x 20 cm x 30 cm) or even as small as 3U (10 cm x 10 cm x 10 cm) and the deployable structure should fit in an even smaller volume. For example, for a solar sail of area 100 m$^2$ which takes up a large volume in space has to be folded to fit a small volume. The situation is similar to the system illustrated in Fig. 1.3. One of the main stages between the launch and the final operating stages of the satellite is the deployment of the solar sail and ensuring the stability of the structure after deployment. For solar sails specifically, the deployment is supported by deployable booms. In addition to the deployment stage, the booms also act as structural elements to ensure the stability of the solar sail after deployment. The Fig. 1.3a, indicates the location where the booms are stored for launch. Four booms run diagonally starting from the payload and act as a skeleton for the solar sail. In general, the booms are rolled in spools.
for stowage. Several designs of booms are shown in Fig. 1.4 [7], [11].

In stowage, by reducing the radius of the pool, a smaller packaging volume can be obtained for the booms. The thickness of the boom is related to the strain produced in the material after folding, in the sense that higher strains are produced for greater thicknesses. For a given thickness, if the bending radii is reduced, then the boom is bent to higher curvatures increasing the strain. So if thin-ply composites are used, lower bending radii can be achieved increasing the packing efficiency. Several research works have been conducted on thin-ply composite booms to study their behavior and capabilities [12], [13].

1.2 Motivation and Aims

By winding the composite booms in spools, strain energy is stored which during deployment is expected to support the process. After unwinding the structure, the correct deployment of the booms in terms of shape is desirable to keep the structural integrity intact. Considering the timeline
of the projects involved from packaging of the cube-sat to the operation stage of the deployable structure, the stowage duration for the composite booms can be up to two years. Experiments conducted by the NASA Langley Research Center have observed that for a 45 mm tall boom section (a CTM boom with flattened height) after mechanical testing has lost a significant amount of strain energy due to stress relaxation [8]. This is present due to the time dependent or viscoelastic properties present in the polymeric epoxy resin in the composite material. In design perspective, this should be addressed as a noteworthy risk. Booms are wound on the spool with a flattened cross section as illustrated in Fig. 1.5. Fig. 1.5a shows the boom before folding and Fig. 1.5b shows the boom after winding.

![Figure 1.5: a) Unfolded boom, and b) Winded boom with a flattened cross section.](image)

The risk associated with viscoelastic effects is that after the stowage period, the recovery of the flattened boom to the original shape may be hindered. The Fig. 1.6 shows the schematic of a cross section of a composite boom. If the original cross section is not recovered the second moment of area about the z axis will be reduced, decreasing the buckling load. In addition to that, the booms are long slender elements and if they do not achieve a straight final shape and remain curved, the
tendency to buckle will increase. When a satellite is in motion however small, a lateral force will be applied onto the sail which will be converted to a compressive force at the booms, so it is crucial to ascertain such conditions will not cause failure in the sail.

![Boom cross section](image)

Figure 1.6: Boom cross section.

This study focuses on characterizing viscoelastic behavior of woven thin-ply composites using numerical models. Having an understanding about the behavior of the materials used for the composite booms would improve their design. This characterization is carried out by adopting a two step homogenization technique [14]. The first at the fiber filament scale determines the properties of the tow and the second studies the laminate composite. The flattened booms are bent to higher curvatures when winding on spools to attain high packing efficiency. The models are used to assess the effects of viscoelasticity on the laminates at high curvatures. The study has been extended to determine the properties in two different orientations of the laminate to verify the assumption of plate homogenization. In the manufacturing process when plies are stacked on one another, with respect to the position of each ply, the wave geometries of each ply may be situated in phase or out of phase or in a random configuration. The two extreme configurations of plies in-phase and out-of-phase have been studied here to determine their effects.
CHAPTER 2: LITERATURE REVIEW

This section represents the overview of the multi-scale modeling of composites and testing. First a review of the prediction of tow properties equivalent to that of unidirectional lamina is discussed. Then the development of determining viscoelastic properties for woven composites is presented. Finally, the experimental arrangements and developments on studying thin-ply composites are discussed.

2.1 Viscoelastic Modeling of Fiber Composites

Textile composite materials are desired over the unidirectional counterparts due to performance in intra and inter laminar strength. In this study, plain weave composites and laminate lay ups with unidirectional laminar are considered. The typical composition of a plain weave composite is given in Fig. 2.1.

Figure 2.1: The composition of fiber composites.
In the tow, bundles of fibers are held together by an epoxy matrix. The epoxy is a polymer material displaying time dependent properties by nature which introduces viscoelastic behavior in fiber composites. The space in between woven tows are also filled with the epoxy. The individual tows exhibit viscoelastic behavior due to the matrix binding the fiber filaments together which progresses to the laminate, combined with the matrix filling the gaps between tows.

In the context of numerical modeling, composites are studies in two different scales. The fiber filament and matrix combination of the tows is studied as a unidirectional fiber composite to evaluate the mechanical properties of the tow and incorporated into the model of the woven laminate. Determining the elastic components of unidirectional lamina can be done starting from the rule of mixtures [15], Hashin-Shtrikman bounds [16] or by approximate prediction methods such as the Mori-Tanaka method [17]. Sun and Vaidya [18] developed a finite element model for unidirectional lamina using conventional stress analysis of reference volume elements. Read (1950) [19] and Lee [20] developed mathematical methods of stress analysis for linear compressible viscoelastic materials by using Fourier transforms which enables which converts the time domain to a frequency domain. It is known that by using Fourier transform, static elasticity solutions can be used to predict the time dependent stresses of viscoelastic materials [21]. This is now regarded as the dynamic correspondence principle. Brinson and Knauss [22] utilized the dynamic correspondence principle, using complex material descriptions, with a finite element approach to determine viscoelastic properties of unidirectional composites. Brinson and Knauss extracted a unit cell (Fig 2.2a) from the unidirectional composite and a discretized quarter of the unit cell (Fig. 2.2b) was selected for the boundary value problem. The problem analysis was conducted by using Finite Element Analysis Program with modifications to the code.
Figure 2.2: a) Smallest repeating unit, and b) Quarter unit cell boundary value problem.

Figure 2.3: Unidirectional tow model.

Kwok and Pellegrino [14] implemented a similar but a three dimensional finite element representative volume element model to predict the viscoelastic behavior of unidirectional tows. The
representative volume element selected for the analysis is given in Fig. 2.3. Boundary conditions have been applied to the faces of the volume element to impose strains on the model to evaluate the relaxation moduli of the viscoelastic tow by obtaining stress relaxation data. This study implements a similar representative volume element model to evaluate the properties for the viscoelastic tow. Wenbin Yu proposed a unified theory for constitutive modeling of composites to predict the viscoelastic properties of unidirectional composites [23]. The method first identifies a structure genome which is regarded as the smallest mathematical building block of a structure. Co-ordinate systems and displacement fields are defined for the structure at macro and original detailed states and the principle of minimum information loss is implemented to minimize the difference between transient strain energy of the original detailed structure and the structure at the macro scale.

The evaluation of properties of woven composites is a challenging problem due to the complex micro-structure of the woven tows. The prediction of viscoelastic properties of woven composites boomed with the use of correspondence principles [24]. Govindarajan [25] et al. used the 4-parameter model (a model with Maxwell and Voigt combination) extending the fiber undulation model developed by Ishikawa and Chou [26] for the evaluation of creep. He used Laplace Transform to derive the constitutive equations and the time dependent behavior of the composites was obtained through Laplace inverse transformation. Shrotiya and Sottos [27] developed existing elastic models to produce creep and relaxation data for woven composites. For the first model, the warp and fill fibers were separated into three different layers and studied. Here the warp bundle is divided into two parts and the fill bundle is sandwiched in between the averaged warp yarns forming a symmetric cross ply laminate (Fig. 2.4).
The yarn properties with the fiber filament and matrix combination have been calculated by using the self-consistent model [28] by Laplace transformation. The second model was done by using a curved beam model. The woven fabric was separated into transverse yarns and longitudinal yarns. The longitudinal yarns were modeled as a wavy layer and the transverse yarns and the matrix were modeled as a homogeneous material surrounding the wavy layer. Upadhyaya and Upadhyay [29] extended the 3-D micromechanical model proposed by Donadon et al. [30] for a woven composite, into the viscoelastic context. The model geometry is illustrated in Fig. 2.5. The wave geometry was defined using a sinusoidal shape functions and the classical lamination theory was utilized for each infinitesimal section in the unit cell to obtain the effective properties of the unit cell. Similar to previous work, Laplace transform was used to evaluate the properties of the unit cell for time dependent behavior.
Kwok and Pellegrino [14] created a finite element model to predict the viscoelastic properties of carbon fiber composites. The model is shown in Fig. 2.6. The tow was idealized (Fig. 2.6 a)) to contain a portion of the matrix and the tow. The arrangement was done so that the laminate volume fraction is consistent with the actual composite. The wave geometry was defined to follow a fourth root of a sin wave. Boundary conditions for the simulations were applied assuming periodic boundary conditions according to Kirchhoff-Love plate theory. The tow properties determined from the single fiber model (Fig. 2.3) were incorporated into the tow part of the model. Since finite element methods are used to determine the properties by using a two step homogenization, the assumptions made in other analytical models are avoided.
Xin Liu et al. [23] extended their work with mechanics of structure genome concept to formulate an MSG-based plate model to evaluate viscoelastic properties of woven composites. The model is a 3-D representative volume element similar to what one would model in a finite element package.

2.2 Experimental Development

The experimental aspects discussed in this study would be focused on the bending tests of composite materials. To assess the bending behavior of thin-ply composites (also regarded as high strain composites) the traditional flexural testing methods for beams, like the three-point or the four-point
bending tests cannot be used due to the incapability to facilitate the change in shape from the elastic deformations that undergo before the failure of these materials. Therefore, new testing methods have been under development to study the bending behavior of thin-ply composites.

One such test developed [31] is illustrated in Fig. 2.7. The specimen is held by tape in between two metal plates. The tapes are regarded to have a negligible stiffness assuming a prefect hinge once the two metal plates are moved closer together. The specimen is bent in a U shape. The test was recorded using a digital video camera and the failure position was obtained as a photograph to measure the failure curvatures. But the test method is prone to gravity induced horizontal lateral loads that causes shear distortions at large angles.

There have been developments with another test set up with a coupon compressed by monotonically driven plates [32], creating a U shaped specimen. The set up is illustrated in Fig.2.8. It was found that the U shape of the specimen during bending follows an elastica curve. This means that
pure moments do not act at the transition from the flat part to the curved part inducing transverse loading that compresses the mid section of the specimen. Even though the test method can be used to predict the failure curvatures of thin-ply composites the pure bending states cannot be studied using this test method.

Murphey et al. [33] constructed a new test method called the large deformation four point bending test to address some of the issues that were seen in the previous tests. The Fig. 2.9 illustrates the test set up for the method developed [34]. The test method is similar to the four point bending test with modifications to accommodate the bending deformations of thin specimens. There are two carts with two ball bearings that bend the specimen once a downwards displacement is applied on the carts from the crosshead. The carts move towards each other horizontally and rotate to apply this bending moment on the specimen. One issue with this test method is that there can be stress concentrations at the grips failing the sample at the grips instead at the mid section.

Figure 2.8: Platen test method.
As discussed by Fernandez [2], column bending test methods have been designed to address the issues of the three aforementioned test methods. This test method hybridizes both the platen and the four point bending for large deformation materials, attaining a uniform stress state on the bent coupon. The maximum bending moment is at the mid section of the coupon and reduces to about 80-90% of that towards the grips [2]. The fixtures (Fig. 2.10) rest on two pins and are free to rotate about the pin creating a hinge facilitated by bearings. At the unloaded state of the sample, the pin axis is located with an eccentricity to the vertical plane of the sample. Therefore, when a compression is applied it is transferred as a bending moment to the specimen. Initially the bending

Figure 2.9: a) The test set up of the bending test, and b) A specimen under bending.
action is sensitive to even the smallest vertical displacement and decays with further displacement. Similar to the platen test, the bending stresses are much higher compared to the axial stress on the coupon.

![Figure 2.10: Column Bending Test fixture.](image)

By assuming a constant curvature, kinematics can be derived to relate the vertical displacement to the curvature and subsequently to determine the moment along the specimen. It can be shown that the moment varies from a maximum from the specimen mid section to a minimum at the grips. But due to the weight of the fixtures, there are gravity induced loads. The free body diagram of the test set up is shown in Fig. 2.11. When moments are taken about each pin for equilibrium, it can be shown that due to the weight of the fixtures there are horizontal forces acting at the pin location on the fixtures. Since the forces are on opposite directions when equilibrium is considered, this imposes a shear force on the sample. When higher curvatures are applied the shear forces become more prominent since the moment generating from fixture weight increases as the pin approaches a horizontal position. Therefore, when the curvature is increased the sample is distorted due to shear creating a higher curvature on the lower part of the coupon compared to the upper region.
Figure 2.11: Free body diagram of the test set up.

Figure 2.12: Counter-weight balanced test fixture.
These effects have been avoided by the new counter-weight balanced bending test fixtures as discussed by Fernandez [2]. The shear deformation created by the gravity induced loading has been addressed by a modification to the fixture design. Instead of extending the fixture to one side from the pin axis there is a part of the fixture extending to the opposite side with identical geometry and mass. This simply counters the weight of the fixture on the other side shifting the center of gravity of the fixtures to the pin axis vertical. Therefore, there will not be any moment since the weight of the fixtures align with the vertical axis of the pin.
CHAPTER 3: THEORETICAL FRAMEWORK

This chapter presents the theoretical background that governs the analysis of the carbon fiber composites. First part of the chapter discusses the inherent viscoelastic behavior of the epoxy polymeric component by mathematical models and how time scale and temperature are interdependent. The second part of the chapter presents how time dependent properties are incorporated into the fiber filament matrix assembly in the micro-scale to find the effective viscoelastic properties of the tow. Then the viscoleastic plate model is introduced that describes the relaxation properties of the woven fiber composite.

3.1 Viscoelasticity

This section presents an overview of viscoelastic theory. Viscoelasticity relates to materials that exhibit characteristics of both solid and fluid materials. The time dependency of mechanical properties in polymeric materials comes from the structure of the molecules. To characterize the behavior of viscoelastic materials physically, two testing methods can be adopted, relaxation and creep testing (Fig. 3.1). Experimentally in relaxation, the stress is observed for a fixed applied strain ($\varepsilon_0$) on the specimen. For viscoelastic materials a reduction of stress with time is observed and for a thermoset polymer like epoxy, the stress approaches an equilibrium stress $\sigma_\infty$ at $t = \infty$. Creep is studied by applying a constant stress ($\sigma_0$) on the specimen and the strain is observed to be increasing with time and for a thermoset polymer, the strain approaches an equilibrium strain $\varepsilon_\infty$ at $t = \infty$ [35]. On the other hand, there are polymeric materials called thermoplastics, which have ever increasing strains in creep and stress approaching zero in relaxation. These materials are not a part of this study. In this study, the epoxy matrix is assumed to behave as a linear viscoelastic material. The relaxation modulus and the creep compliance corresponding to relaxation and creep
behaviors respectively can be expressed by the Eq. (3.1) and Eq (3.2).

\[ E(t) = \sigma(t)/\varepsilon_0 \]  

\[ D(t) = \varepsilon(t)/\sigma_0 \]  

Experimental methodologies can be used to study the behavior of viscoelastic materials as observed in Fig. (3.1). But for further analysis, the observed behavior must be formulated by using mathematical models. Early models that originated from Maxwell and Kelvin are now studied to characterize the viscoelastic behavior of materials. The solid part is treated as an elastic spring and the fluid part of the material is treated as a damper. With the development of these models, the generalized Maxwell model is now used to study the relaxation of viscoelastic materials. The
arrangement of the spring damper set up is given in Fig. (3.2).

![Generalized Maxwell Model](image)

**Figure 3.2: Generalized Maxwell Model.**

The generalized Maxwell model consists of an array of series connected spring and damper assembly. At the end is a free spring. When a strain is applied to this system each parallel branch will attain the same strain and every spring will be in full resisting behavior achieving the highest possible stiffness for the system, but with time depending on the damping power of each damper, the springs in each branch will relax reducing the stiffness of the system. In other words, initially the modulus of the system is highest but with time it decays due to the damping. A free spring is incorporated in the set up to simulate the approach of a thermoset polymer such, as epoxy, that achieves an equilibrium stress state. The elastic effect of the branches will diminish as the dampers relaxes them but ideally at $t = \infty$ the free spring will counter the stress on the system and achieve equilibrium stress. This system can be solved to obtain an expression for the modulus dependent
on time in the form of a Prony series as follows.

\[ E(t) = E_\infty + \sum_{k=1}^{n} E_k \exp \left( \frac{-t}{\rho_k} \right) \]  

(3.3)

where \( E_\infty \) is the modulus of the free spring which is called the long term modulus. \( E_k \) represents the modulus of each spring in each branch and \( \rho_k \) is the ratio of \( \mu_k / E_k \) in each branch. \( \rho_k \) are also known as the relaxation times of the Prony series. This result is derived for a homogeneous material.

In relaxation, the stress can be determined for a constant application of strain given the terms in the Prony series describing the modulus are known. Computing the stress output for a variable strain can be performed by using the Boltzmann superposition integral introduced by Boltzmann.

\[ \sigma(t) = \int_0^t E(t - \tau) \frac{d\varepsilon}{d\tau} d\tau \]  

(3.4)

Polymers are found to have a relation between time and temperature which is explained by the Time-Temperature-Superposition-Principle. As the temperature of a polymer material is raised, the material gets softer yielding a lower initial modulus compared to the initial modulus of lower temperatures. Therefore, relaxation tests performed at different temperatures will have different initial modulus values and hence yielding curves with offsets from one another. The Time-Temperature-Superposition-Principle states that the curves can be shifted through time to fit to a single curve with respect to a reference temperature. The shifting amount is called the temperature shift factor. This allows a master curve to be built at a desired reference temperature with the ability of shifting this master curve to obtain behavior at different temperatures.

For a thermorheologically simple material the shift factor relating one temperature \( (T) \) to the ref-
erence temperature $T_0$ is given by the ratio of the relaxation times (Eq.( 3.5)). This indicates that all the relaxation times are shifted by the same factor for a given temperature.

$$a_T = \frac{\rho(T)}{\rho(T_0)} \quad (3.5)$$

For temperatures above the glass transition temperature ($T_g$), the shift factor can be fitted to the William-Landel-Ferry empirical formula developed by M.L. Williams, R.F. Landel and J.D. Ferry [36].

$$\log(a_T) = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)} \quad (3.6)$$

Below the glass transition temperature, the Arrhenius activation energy equation [35] can be used to calculate the shift factors, given by

$$\log_{10}a_T = -\frac{E_a}{2.303R}\left(\frac{1}{T} - \frac{1}{T_0}\right) \quad (3.7)$$

### 3.2 Viscoelastic Plate Model

This section will introduce the viscoelastic plate model that describes the constitutive model of thin-ply composites. The Boltzmann superposition principal is introduced for a homogeneous material in Eq.( 3.4). The fiber matrix assembly will introduce anisotropic viscoelastic properties
to the composite. The Eq. (3.4) can be extended for anisotropy as expressed in Eq. (3.8).

$$\sigma_i(t) = \int_0^t C_{ij}(t - \tau) \frac{d\varepsilon_j}{d\tau} d\tau$$  \hspace{1cm} (3.8)

where $\sigma$ and $\varepsilon$ are stress and strain presented according to the Voigt notation, $t$ is time, and $C$ is the relaxation modulus tensor in the form of a 6-by-6 relaxation modulus tensor. Similar to elasticity theory, the matrix is symmetric. All the entries represent modulus components expressed in terms of Prony series given by Eq. (3.9). Here, $C_{ij,\infty}$ are the long-term moduli, $C_{ij,k}$ are Prony coefficients, $\rho_k$ are relaxation times at the reference temperature $T_0$, and $a_T$ is the temperature shift factor. Each entry in the modulus tensor are sorted to have the same set of relaxation times. For this study, a thermorheologically simple material is assumed implying that the same shift factor is valid for all the relaxation times. The formulation of the relaxation modulus tensor is paramount when the fiber filament and matrix effective properties are computed for the viscoelastic tow of the composites.

$$C_{ij} = C_{ij,\infty} + \sum_{k=1}^{n} C_{ij,k} \exp \left( \frac{-t}{a_T\rho_k} \right)$$  \hspace{1cm} (3.9)

The composite is treated as a thin plate with viscoelastic behavior that can be formulated according to Kirchhoff plate assumptions. The transverse normal is assumed to be rigid in its direction and remains straight. Furthermore, the transverse normal is regarded to remain normal to the mid-plane after deformation. The strain field of the plate can be expressed in terms of in-plane strains and the out of plane curvatures of the mid-plane as follows.

$$\varepsilon_1 = \bar{\varepsilon}_1 + x_3 \bar{k}_1, \quad \varepsilon_2 = \bar{\varepsilon}_2 + x_3 \bar{k}_2, \quad \varepsilon_6 = \bar{\varepsilon}_6 + x_3 \bar{k}_3$$  \hspace{1cm} (3.10)
where $\varepsilon$ and $\kappa$ are the in-plane strains and the out-of-plane curvatures of the plate mid-plane respectively. $\varepsilon_4 = \varepsilon_5 = 0$ is derived with the assumption that the transverse normal will remain perpendicular to the mid-plane. $\varepsilon_3 = 0$ is considered since the transverse normal is rigid. The force and moment resultants are calculated by integrating the stresses over the plate thickness $h$,

$$
\mathcal{N}_1 = \int_h \sigma_1 dx_3, \quad \mathcal{N}_2 = \int_h \sigma_2 dx_3, \quad \mathcal{N}_3 = \int_h \sigma_6 dx_3
$$

$$
\mathcal{M}_1 = \int_h x_3 \sigma_1 dx_3, \quad \mathcal{M}_2 = \int_h x_3 \sigma_2 dx_3, \quad \mathcal{M}_3 = \int_h x_3 \sigma_6 dx_3
$$

The viscoelastic plate model is formulated by substituting the 3D strains on to the constitutive relationship given in Eq. (3.8). The force and moment results are derived as follows,

$$
\mathcal{N}_i(t) = \int_0^t A_{ij}(t-\tau) \frac{d\varepsilon_j(\tau)}{d\tau} d\tau + \int_0^t B_{ij}(t-\tau) \frac{d\kappa_j(\tau)}{d\tau} d\tau
$$

$$
\mathcal{M}_i(t) = \int_0^t B_{ij}(t-\tau) \frac{d\varepsilon_j(\tau)}{d\tau} d\tau + \int_0^t D_{ij}(t-\tau) \frac{d\kappa_j(\tau)}{d\tau} d\tau
$$

The $A$, $B$ and $D$ terms follow the same characteristics as defined in the classical laminate theory [37] but composes of time dependent behavior. $A$ matrix is the extensional relaxation stiffness matrix which reflects the in-plane stiffness properties of the composite. The $B$ matrix is the extensional-bending coupling relaxation stiffness denoting the coupling between in-plane and out-
of-plane properties and the $D$ is the bending relaxation stiffness accounting for the out-of-plane characteristics. Each entry of these three matrices can be represented by Prony series as

$$A_{ij} = A_{ij,\infty} + \sum_{k=1}^{n} A_{ij,k} \exp \left( \frac{t}{\rho_k} \right)$$  \hspace{1cm} (3.16)$$

$$B_{ij} = B_{ij,\infty} + \sum_{k=1}^{n} B_{ij,k} \exp \left( \frac{t}{\rho_k} \right)$$  \hspace{1cm} (3.17)$$

$$D_{ij} = D_{ij,\infty} + \sum_{k=1}^{n} D_{ij,k} \exp \left( \frac{t}{\rho_k} \right)$$  \hspace{1cm} (3.18)$$
CHAPTER 4: MODELING APPROACH

This chapter presents the modeling approach of the study conducted with three major steps. Three thin ply plain weave composites were studied in this research; two plain weaves each with 3 and 4 plies and a laminate 3-ply with two woven plies and a unidirectional ply. Throughout this thesis, the laminate 3-ply would refer to the composite containing the unidirectional ply. First, the geometric properties for the models are extracted by studying micrograph images. Secondly, the modeling is carried out in a textile modeling software with the parameters that were acquired from the micrograph image analysis. Finally, a two step homogenization was adopted for the numerical analysis. The steps are discussed in detail in this chapter.

4.1 Material Properties

The woven thin ply laminates in this study are all fabricated with M30S carbon fiber and PMT-F7 epoxy and the unidirectional ply is made of IM7 fibers and PMT-F7 epoxy. The laminate behavior depends both on the elastic properties of the fibers and the viscoelastic properties of the epoxy matrix. The modulus of elongation for the M30S fiber was acquired from Toray Industries [38] and the modulus of elongation for IM7 was obtained from Hexcel [39]. The availability of elastic properties of the fibers for the other parameters were not available, so they were estimated with respect to literature [40]. The material properties are tabulated in Table 4.1. Epoxy is an isotropic viscoelastic material with behavior obeying the homogeneous model discussed in Section 3.1. The material properties were extracted from literature [41]. The relaxation modulus is plotted against time in Fig.(4.1). The Prony series consists of fourteen relaxation terms with the behavior extending to $10^{14}$ seconds and the master curve is presented for a reference temperature of 30°C.
### Table 4.1: Elastic Properties of M30S

<table>
<thead>
<tr>
<th>Tow Properties</th>
<th>M30S Fibers</th>
<th>IM7 Fibers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Stiffness $E_1 (N/mm^2)$</td>
<td>294000</td>
<td>276000</td>
</tr>
<tr>
<td>Transverse Stiffness $E_2 (N/mm^2)$</td>
<td>29148</td>
<td>27363</td>
</tr>
<tr>
<td>Shear Stiffness $G_{12}=G_{13} (N/mm^2)$</td>
<td>11310</td>
<td>10617</td>
</tr>
<tr>
<td>In-plane Shear Stiffness $G_{23} (N/mm^2)$</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_{12} = \nu_{13}$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_{23}$</td>
<td>0.46</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Figure 4.1: Uniaxial relaxation modulus of PMT-F7 epoxy at 30°C.
### Table 4.2: Prony Series of PMT-F7

<table>
<thead>
<tr>
<th>k</th>
<th>$\rho_k$</th>
<th>$E_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>-</td>
<td>149.5</td>
</tr>
<tr>
<td>1</td>
<td>1.89E+01</td>
<td>74.75</td>
</tr>
<tr>
<td>2</td>
<td>1.00E+02</td>
<td>194.35</td>
</tr>
<tr>
<td>3</td>
<td>1.00E+03</td>
<td>254.15</td>
</tr>
<tr>
<td>4</td>
<td>2.00E+04</td>
<td>110.63</td>
</tr>
<tr>
<td>5</td>
<td>1.00E+05</td>
<td>158.47</td>
</tr>
<tr>
<td>6</td>
<td>1.95E+06</td>
<td>92.69</td>
</tr>
<tr>
<td>7</td>
<td>1.77E+07</td>
<td>71.76</td>
</tr>
<tr>
<td>8</td>
<td>1.74E+08</td>
<td>299</td>
</tr>
<tr>
<td>9</td>
<td>1.38E+09</td>
<td>299</td>
</tr>
<tr>
<td>10</td>
<td>1.00E+10</td>
<td>299</td>
</tr>
<tr>
<td>11</td>
<td>1.00E+11</td>
<td>299</td>
</tr>
<tr>
<td>12</td>
<td>1.00E+12</td>
<td>299</td>
</tr>
<tr>
<td>13</td>
<td>1.00E+13</td>
<td>299</td>
</tr>
<tr>
<td>14</td>
<td>1.00E+14</td>
<td>89.7</td>
</tr>
</tbody>
</table>

### 4.2 Geometric Parameters

This section describes the acquiring of material properties in order to construct the geometric models and the formulation of the models by using TexGen [42], a textile modeling software. The geometric properties were acquired from micrographs. Before analyzing the micrographs, the
geometric parameters that are required must be known. A representative unit cell is extracted from the plain weave composite for the numerical model. $\Delta L$ sets the boundaries for the unit cell, which is also the wave length of the weave. The representative unit cell is shown in Fig. 4.2.

![Figure 4.2: The representative unit cell.](image)

The tow width, tow cross sectional shape, the laminate thickness and the weave geometry are the important properties that are required to model the geometry of the composite. The ImageJ software [43] was used for the measurement and analysis of the micrographs. A typical micrograph of a cross section of a 4-ply plain weave composite is shown in Fig 4.3. The micrograph images used for the analysis were provided by the NASA Langley Research Center. The width $w$ of the tow was measured by measuring the length of a line segment drawn across the two end points of the tow. Several micrographs were assessed and the average value was obtained for the tow width. The spacing $s$ between consecutive tows was measured by using a similar method and the wave length which defines the boundaries of the representative unit cells was calculated by using the
The laminate thickness was determined by measuring the length of a vertically drawn line segment across the thickness of the 4-ply. One eighth of the value of the thickness was regarded as the maximum thickness of the tow and this was verified by measuring the individual thicknesses of tows and comparing the values. A polygon was traced around the tow by drawing line segments and the area of the polygon was measured to determine the area of the tow. These parameters are sufficient to replicate the composite geometry in a computer generated model, but in an analysis perspective, volume fraction is one of the most crucial parameters. Several micrographs of the tows were analyzed for this purpose. There are two types of volume fractions, tow volume fraction and the laminate volume fraction. The volume fraction of the tows represents the volume of the fiber present in the tow with respect to the tow volume. The laminate volume fraction is the volume of fiber present in the laminate with respect to the volume of the laminate. It should be noted that there is matrix dispersed in the laminate to fill the space in between tows, so the laminate volume fraction is always less than the tow volume fraction. To find the tow volume fraction, an area of interest was selected bounded by a rectangle as shown in Fig. 4.4 containing a certain number of
fibers inside the boundary.

Figure 4.4: The cross section of a tow.

The analogy that the area fraction should be equivalent to the volume fraction was regarded in this analysis. The total area of the fibers present in the bounded rectangle was calculated by multiplying the number of fibers inside the region by the area of one fiber. The ratio of the area of the fibers to the area of the rectangle was considered as the volume fraction of the fibers in a tow. The average value was found to be 0.62. The laminate fiber volume fraction can be calculated by using mass conservation.

\[
V_f = \frac{\rho_m W_f}{\rho_m W_f + \rho_f W_m}
\]  

(4.2)

where the \(\rho_m\), \(\rho_f\) and \(W_m\), \(W_f\) are the densities and areal weights of matrix and fiber respectively. The areal weight \(W_f\) of one ply of M30S fiber is known to be 60 g/m\(^2\) from the manufacturer and the areal weight of epoxy \(W_m\) was calculated to be about 34 g/m\(^2\) after measuring the mass
of cured samples of the 4-ply plain weave composite. The densities $\rho_m$ of the matrix and $\rho_f$ of the fiber are 1.22 g/cm$^3$ and 1.73 g/cm$^3$ respectively. The laminate volume fraction that was obtained from this analysis is 0.56 but depending on the variations of the samples measured this value was found the vary roughly between 0.53 and 0.56. The most accurate methodology to evaluate the laminate volume fraction would be to measure a laminate sample and then wash away the matrix by using chemicals and weighing the sample again to recognize the amount of matrix that was present in the sample selected. Here a more accurate areal weight for the epoxy can be determined. The geometric parameters determined for the 3-ply and the 4-ply are arranged in Table 4.3. The geometric analysis was conducted for the 4-ply and the same tow properties and the weave geometry were assumed for the 3-ply as well.

Table 4.3: Geometric parameters of 3 and 4 ply plain weave laminates

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>3-Ply</th>
<th>4-Ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weave length</td>
<td>6.674 mm</td>
<td>6.674 mm</td>
</tr>
<tr>
<td>Laminate thickness</td>
<td>0.171 mm</td>
<td>0.228 mm</td>
</tr>
<tr>
<td>Maximum tow thickness</td>
<td>0.0286 mm</td>
<td>0.0286 mm</td>
</tr>
<tr>
<td>Tow width</td>
<td>3.164 mm</td>
<td>3.164 mm</td>
</tr>
<tr>
<td>Tow area</td>
<td>0.0802 mm$^2$</td>
<td>0.0802 mm$^2$</td>
</tr>
<tr>
<td>Fiber volume fraction of tow</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>Fiber volume fraction of laminate</td>
<td>0.56</td>
<td>0.56</td>
</tr>
</tbody>
</table>

The micrographs that were studied belonged to a specific batch manufactured in the NASA Langley Research Center. However, laminates of the same 3-ply and 4-ply configurations can have variations in thickness. This variation can arise from a variation in the maximum tow thickness. Al-
though an exact claim cannot be made, it was seen that for the laminates considered under this study, the thicknesses have variations of roughly 10% with respect to the thickness given in Table 4.3. The implications on the variation of thickness is discussed in Chapter 5. Due to the unavailability of micrographs for each of the samples that were manufactured from different batches, the same tow and weave geometries were used and the same volume fractions were assumed for a considered thickness.

The geometric properties of the laminate 3-ply are displayed in Table 4.4. The thickness of the woven plies was estimated based on a different laminate obtained from the NASA Langely Research Center. For the other geometric properties, the same parameters were assumed for the weave and the tow from the 4-ply data. The total laminate thickness was obtained from Roccor [44]. The thickness of the unidirectional ply was calculated by reducing the thickness of the woven plies from the total thickness of the laminate 3-ply.

<table>
<thead>
<tr>
<th>Geometric Properties</th>
<th>Laminate 3-ply</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Woven plies</td>
<td>Unidirectional ply</td>
</tr>
<tr>
<td>Weave length</td>
<td>6.674 mm</td>
<td>-</td>
</tr>
<tr>
<td>Ply thickness</td>
<td>0.063 mm</td>
<td>0.06 mm</td>
</tr>
<tr>
<td>Maximum tow thickness</td>
<td>0.0315 mm</td>
<td>-</td>
</tr>
<tr>
<td>Tow width</td>
<td>3.164 mm</td>
<td>-</td>
</tr>
<tr>
<td>Tow area</td>
<td>0.0802 mm²</td>
<td>-</td>
</tr>
<tr>
<td>Fiber volume fraction of tow</td>
<td>0.62</td>
<td>0.629</td>
</tr>
<tr>
<td>Fiber volume fraction of laminate</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>
4.3 Micromechanical Models

This section describes the implementation of the numerical models used to determine the viscoelastic properties of the laminates. The numerical analysis to determine the laminate viscoelastic properties were carried out by using a two step homogenization [14]. There are two scales present in the composition of the laminates. The fiber filament and matrix assembly inside the tow is at a micron scale since the fibers have diameters typically between $5\mu m$ and $7\mu m$. When modeling the unit cell geometry, the scale is in the order of millimeters and the tows are modeled as a single continuum and do not represent the structure of individual fiber filaments. Therefore, the effective properties for the fiber filament and matrix assembly inside the tows need to be computed to employ in the unit cell model. For this purpose, as an intermediate step, a tow fiber model has been used to evaluate the tow properties, hence the two step homogenization. There are two types of plain weave laminates studied in this thesis, 3-ply and 4-ply. The numerical models were developed considering two aspects. The first one is the laminate orientation. In the study, two different orientations were considered. In a plain weave, the tows are woven in two perpendicular directions namely $0^\circ$ and $90^\circ$. Numerical models were developed for both woven plain weave lay ups (3-ply and 4-ply) by selecting unit cells denoting the laminates at $0^\circ$ orientation as well as $45^\circ$ directions as illustrated in Fig. 4.5.

The two unit cells extracted are shown in Fig. 4.6. Two different orientations were considered to study the effect of the orientation on the laminate properties. The unit cell model is evaluated assuming a homogenized Kirchhoff plate behavior. In the past for plain weave composites, researchers have evaluated both elastic and viscoelastic properties considering only the $0^\circ$ orientation [14] [40] [45]. The properties extracted from these numerical models are implemented in deployable boom numerical models where the co-ordinate transformation of these properties are allowed. To investigate this further, two orientations have been considered in this study.
Figure 4.5: The two orientations of study for the plain weave, 0° and 45°.

Figure 4.6: a) 0° oriented laminate, and b) 45° oriented laminate
The second aspect taken into consideration for the plain weave woven laminates is the ply configuration. As observed from the micrographs, when the plies are stacked in the manufacturing process there can be relative horizontal translations between plies as opposed to an expected perfect alignment of plies. The stacking sequence appears to be random as depicted in the micrographs in Fig. 4.7.

![Figure 4.7: The stacking sequences of plies.](image)

There can be two perfect or extreme alignments where the parallel fibers through thickness are in-phase and out-of-phase in terms of the waviness of the tows. The two conditions are illustrated in Fig. 4.8. These different ply configurations were studied for the 4-ply only.

![Figure 4.8: a) Plies-in-phase configuration, and b) Plies-out-of-phase configuration.](image)

The laminate 3-ply was formulated with the two woven plies above and below the unidirectional ply aligning to an out-of-phase configuration. The unit cell was extracted such that the unidi-
rectional ply is at a 0° orientation which dictates the orientation of the woven plies to be at a 45° orientation.

### 4.3.1 Tow Fiber Model

The unit cell model as described in Section 4.3 has tows made of a single solid geometry but the properties of the viscoelastic tow must be known. The fiber matrix composition has viscoelastic properties that should be defined in the unit cell model. The tow, regardless of its waviness in reality is a unidirectional fiber arrangement with transversely isotropic properties. Here the tow is homogenized to a unit fiber model with matrix surrounded in the shape of a square and this assumption invites a square fiber arrangement in the tow. There are other possible arrangements for the fibers in the tow such as hexagonal, diamond or even randomly arranged fibers, but the dependency of the mechanical properties of these arrangements are found to be insignificant for a constant volume fraction [14]. The tow fiber model is shown in Fig. 4.9.

Figure 4.9: Unidirectional tow model.
First the methodology is described for the M30S fibers. The M30S fibers have a diameter of $5 \, \mu m$ and the dimensions of the cube unit cell of $5.627 \, \mu m$ is determined by the tow volume fraction of 0.62 and the fiber diameter. The geometry and the analysis were both formulated and carried out by using Abaqus finite element software [46]. There are six reference points corresponding to each surface. Fig. 4.9 displays three reference points $X_1, Y_1$ and $Z_1$ and the points $X_2, Y_2$ and $Z_2$ are located on the opposite faces respectively. The viscoelastic properties were defined for the matrix and elastic properties were defined for the fiber. The mesh geometry is illustrated in Fig. 4.10.

The mesh is made out of 7680 linear wedge elements of type C3D6 that form a circle at the center of the fiber and the rest consists of 254880 linear hexahedral elements of type C3D8. The degrees of freedom of the faces were constrained to each corresponding reference point on the surface by

![Figure 4.10: Meshed tow fiber model.](image-url)
means of *coupling constrains* in Abaqus. The constitutive model describing the material model for the viscoelastic tow is given in Eq. (3.9) where $C$ is the 6-by-6 relaxation modulus tensor that represents the anisotropic viscoelastic properties of the tow.

$$C = \begin{bmatrix}
    C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
    C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
    C_{13} & C_{23} & C_{22} & 0 & 0 & 0 \\
    0 & 0 & 0 & C_{44} & 0 & 0 \\
    0 & 0 & 0 & 0 & C_{55} & 0 \\
    0 & 0 & 0 & 0 & 0 & C_{55}
\end{bmatrix} \tag{4.3}$$

where $C_{44}$ can be determined by $C_{22}$ and $C_{23}$ by transverse isotropy as

$$C_{44} = \frac{C_{22} - C_{23}}{2} \tag{4.4}$$

After this, $C_{11}$, $C_{12}$, $C_{13}$, $C_{23}$ and $C_{55}$ are the five independent coefficients that can be computed using three different simulations in Abaqus. The unknowns are related to the relaxation behavior of the tow model. So the basic idea is to apply a constant strain onto the tow fiber model and observe the relaxation of the stress in different directions. For example, to evaluate $C_{11}$ a displacement corresponding to a strain of 0.1% is applied on the reference point $X1$ which is one of the reference points situated on the faces perpendicular to the 1-axis. The $X2$ reference point is constrained not to have any displacement in the 1-direction. This applies a strain on the tow model in the 1-direction. The strain is applied in a small time step of $10^{-5}$ seconds. The selection of small time step is to minimize the effects of viscoelasticity during the step where the strain is applied. The
strain was held in position for a time period of $10^{14}$ seconds which characterize the viscoelastic behavior of the tow for the time period that has the available viscoelastic properties of the epoxy. The degrees of freedom of the faces that are parallel to the fiber filament were constrained to the reference points to not move in a perpendicular direction to their own plane. This ensures a zero strain in 2 and 3 directions. The stress ($\sigma(t)$) in the 1-direction was computed by using the force on the reference point $X1$ averaged over the area of the corresponding face. The relaxation of the stress over time was computed and the modulus was obtained after dividing by the strain. This simplified calculation is possible since the strains in 2 and 3 directions are zero. By using the same simulation, $C_{12}$ and $C_{13}$ can be determined by considering the relaxation data in 2 and 3 directions.

The extracted data were fitted into a Prony series with relaxation times that are equivalent to the PMT-F7 epoxy by using non-linear curve fitting method in Matlab [47].

The same methodology was adapted for the IM7 fibers. IM7 unidirectional ply has a volume fraction of 0.629 and the fibers with a diameter of 5 $\mu m$. These parameters set the dimensions of the cubic numerical model to be 5.81 $\mu m$. The mesh was created in a identical way to that of the M30S fiber model and contains 269204 linear hexahedral elements of type C3D8 and 7874 linear wedge elements of type C3D6. The wedge elements were used to fill the center of the fiber for smooth transition to hexahedral elements. The simulations were carried out similarly to the M30S fiber model and the relaxation modulus tensor was formulated.

### 4.3.2 Laminate Model

The geometry of the unit cell was built in an open source textile modeling software TexGen [42]. TexGen allows the user to produce woven geometries, manipulate the details through their python coding interface, and mesh the geometry and export an input file that can be read by Abaqus. First, the geometry pertaining to the 0/90 plain weave was modeled in TexGen. The unit cell consists
of four yarns two running in the $0^\circ$ orientation and two running in the $90^\circ$ orientation. Due to the variety of capabilities found in the python scripting linked to TexGen, coding was used to produce the required geometry. Each yarn has to be modeled separately by specifying the coordinates of the yarns and the entered coordinates are fit to a spline curve automatically by TexGen. The coordinates of a yarn are entered to a specific selected portion from the weave (length equaling to a wave length) and the modeled portion is repeated to obtain a large scale woven composite. The typical defined shape of a yarn is shown in Fig. 4.11.

![Figure 4.11: The geometry of the defined yarn.](image)

Seven points were defined to set the shape of the yarn. The points 1, 4 and 7 are always defined to form the geometry which is how TexGen creates the shape in the user interface. Four extra nodes were selected such that micro adjustments can be made to the yarn wave. If only three coordinates were used, depending on the cross sectional shape and the yarn spacing the yarns may overlap. Therefore when meshing, the overlapped portions are cut off from the geometry reducing the volume of the yarns causing a reduction in the laminate volume fraction. Each yarn has to be defined separately to form the unit cell. Then each yarn can be repeated to form a large scale plain weave pattern. For example, the yarn illustrated in Fig. 4.11 can be repeated in x and y axes in both
negative and positive directions to construct an array of yarns. Similarly, the other three yarns in the other orientations can be repeated accordingly to produce the plain weave pattern. This woven geometry can be multiplied and stacked in the vertical direction to imitate the lay up conditions in the actual composites. The boundaries for the unit cell are set up by defining a cuboid with geometric planes. When the plane is defined, it should be arranged in a way such that the direction of the normal vector to the plane is pointed towards the region of interest. TexGen will keep the region where the normal vector points to and ignores the region to the opposite side of the plane. The space in between the yarns and the cuboid is filled with matrix. The unit cells of the 3 and 4 ply laminates at the 0° orientation are presented in Fig. 4.12.

For the rotation of the created geometry, there is a built in function in TexGen to rotate the plain weave about an axis of choice. The point of rotation is at (0, 0, 0). Therefore, one must fix the matrix domain and rotate the laminate. The simplest way to do this is to arrange the yarn locations so that the unit cell after rotation matches the fixed domain. The geometric models of the 45° rotated 3 and 4 ply are presented in Fig. 4.13.
The laminate 3-ply has to be constructed such that a unidirectional ply is inserted in between two woven plies. The unidirectional ply was created in the shape of a cuboid equaling the dimensions of the unit cell. The geometry was selected such that the boundaries of the unit cell lie at $0^\circ$ and $90^\circ$ to the direction of the unidirectional fibers. The geometry is illustrated in Fig. 4.14.
The cross sectional shape is defined in the form of a super ellipse [48] given by Eq. (4.5) and Eq. (4.6). Selecting $n$ was an iterative process due to complications that occur in meshing.

\[ x = a \sin \theta \]  

(4.5)

\[ y = b \cos^n \theta \]  

(4.6)

If the value of $n$ is determined so that the area of the super ellipse [49] matches the average area of the cured tow, the material loss during meshing is not taken into account. It was found out that this loss can be significant and that an area greater than the actual geometry must be specified in the model cross sectional area. Due to the material loss, the yarn volume is reduced underestimating the laminate volume fraction. There are three factors contributing to obtaining the correct volume fraction. One is the cross sectional area before meshing which should be set at a higher value compared to the actual area since meshing takes away material on the boundary. If this is not sufficient, the merge tolerance parameter can be adjusted when meshing. This parameter sets the limit for the minimum value of separation that any two yarns can maintain in the geometry. If the separation is below the tolerance limit, the two yarns are merged together. They will act as separate sections with different fiber orientations but will share the same nodes at the boundary after contact due to merging. The regions that are directly affected by this parameter are the yarn cross over locations. This is a useful parameter that increases the yarn thickness despite of the loss due to meshing. The other parameter is the mesh size. Although this parameter does not have a significant influence it can be used to fine tune the final yarn volumes obtained. These parameters were adjusted to obtain the actual laminate volume fraction. It was an iterative process to mesh and
export the model to Abaqus finite element package where the volume of the yarns was obtained to
determine the laminate volume fraction. So after this process the value of $n$ was found to be best
suited at 4.

The meshed geometric model was exported to Abaqus for numerical analysis. After meshing,
TexGen identifies each yarn as separate sections including the matrix surrounding the yarns. With
the generation of the mesh in a format that Abaqus can read (input file), TexGen also generates a file
with fiber orientations for each element in the mesh. When the input file was imported, the element
orientations were also imported in to Abaqus. Since the yarns and surrounding matrix could be
separated in terms of sections in Abaqus, the relevant material properties could be defined in yarns
and matrix. The matrix made of PMT-F7 has the same properties as mentioned in Section 4.1. The
model that describes the viscoelastic characterization of the tows has been computed by using the
tow fiber model in Section 4.3.1. The typical meshed geometry of a unit cell is shown in Fig 4.15.

![Finite element mesh of 3-ply laminate.](image)

Figure 4.15: Finite element mesh of 3-ply laminate.
The finite element meshes of other model were obtained in a similar manner. The unit cell of the laminate was assumed to behave in agreement with Kirchhoff-Love plate theory as discussed in Section 3.2. The application of periodic boundary conditions were facilitated with the Kirchhoff plate equations. There are four reference points \((X_1, Y_1, X_2, Y_2)\) located at the periphery of the unit cell. The degrees of freedom of the faces are constrained to the unit cell for the purpose of applying boundary conditions, so that any translation or rotation applied to the reference points were imposed on the unit cell. The relative translations and rotations between opposite faces were imposed with equations given by

\[
\Delta u_1^1 = \varepsilon_1 \Delta L \\
\Delta u_2^1 = \frac{1}{2} \varepsilon_3 \Delta L \\
\Delta u_3^1 = -\frac{1}{2} \kappa_{3x2} \Delta L \\
\Delta \theta_1^1 = -\frac{1}{2} \kappa_3 \Delta L \\
\Delta \theta_2^1 = \kappa_1 \Delta L \\
\Delta \theta_3^1 = 0
\]
\[ \Delta u^2_1 = \frac{1}{2} \varepsilon_3 \Delta L \]  
(4.13)

\[ \Delta u^2_2 = \varepsilon_2 \Delta L \]  
(4.14)

\[ \Delta u^2_3 = -\frac{1}{2} \kappa_3 x_1 \Delta L \]  
(4.15)

\[ \Delta \theta^2_2 = \frac{1}{2} \kappa_3 \Delta L \]  
(4.16)

\[ \Delta \theta^2_1 = -\kappa_2 \Delta L \]  
(4.17)

\[ \Delta \theta^2_3 = 0 \]  
(4.18)

where \( u \) and \( \theta \) are translations and rotations of the reference points. The subscripts in \( u \) denote the direction of translation and the subscripts in \( \theta \) denote the axis of rotation. The superscript 1 represents the reference points \( X_1 \) and \( X_2 \) and similarly superscript 2 represents the reference points \( Y_1 \) and \( Y_2 \). \( \Delta L \) represents the dimensions of the unit cell. Translations and rotations were applied to the reference points so that in-plane strains and out-of-plane curvatures can be imposed on the unit cell. The simulations were carried out with a similar logic as the tow fiber model discussed in Section 4.3.1. For example, consider the computation of \( A_{11} \). The same methodology was used as described by Pellagrino and Kueh [50]. A strain of \( \varepsilon_1 = 0.001 \) is imposed on the unit cell while keeping all the other strains and curvatures zero (\( \varepsilon_2 = \varepsilon_3 = \kappa_1 = \kappa_2 = \kappa_3 = 0 \)). To impose this in the unit cell, a displacement was applied on the reference point \( X_2 \) while keeping the \( X_1 \) point fixed that translates to a strain of 0.001. The faces perpendicular to the 2-direction are constrained not to move in the 2 direction. Rotations were not allowed on any face. The strain was applied.
over a period of $10^{-5}$ seconds and held in position for $10^{14}$ seconds. The force can be read from the reference point $X_2$ and normalized to the length of the unit cell. $A_{11}$ at a selected time can be calculated by incorporating the normalized force and the strain imposed on the model. $A_{11}$ when plotted against the time gives the master curve at 30°C since the viscoelastic properties of the matrix used for the simulations is at 30°C. This can be fitted to a Prony series, by using the non-linear curve fitting capability in Matlab, to define the property $A_{11}$ in a mathematical form. The same simulation can be used to evaluate $A_{21}, A_{61}, B_{11}, B_{21}, B_{61}$. By performing four independent simulations, the ABD relaxation matrix can be fully populated.

The unit cell models that belong to the 4-ply plain weave laminate (0° and 45°) were used to study the effect of high curvatures on the material. The models were imposed curvatures of $\kappa_1 = 0.001 \text{mm}^{-1}$, $\kappa_1 = 0.1 \text{mm}^{-1}$ and $\kappa_1 = 0.2 \text{mm}^{-1}$ and variation of bending stiffness was observed as the models relaxed over time. All the simulations were carried out by non-linear geometric analysis.

4.3.2.1 Numerical Integration

Abaqus does not have a model to handle anisotropic viscoelastic materials, so a user material model was introduced to Abaqus finite element software. The integration to determine the stress should be carried out by numerical means [51]. Eqn.( 3.8), Eqn.( 3.9) can be combined together to produce the Eq.( 4.19). The expression inside the integral of Eqn. 4.19 at time $t_n$ is regarded as $h_i^{b_n}$ and will be the term to be evaluated.

$$\sigma_i^t = C_{ij,\infty} \varepsilon_i^t + \sum_{k=1}^n \int_0^t C_{ij,k} \exp\left(-\frac{t - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau$$

(4.19)
\[ h_{i}^{n} = \int_{0}^{t_{n}} C_{ij,k} \exp\left(-\frac{t_{n} - \tau}{\rho_{k}}\right) \frac{d\varepsilon_{j}}{d\tau} d\tau \] (4.20)

Now consider the time interval \([t_{n}, t_{n+1}]\) and a step time of \(\Delta t = t_{n+1} - t_{n}\). The expression for time \(t_{n+1}\) can be written by

\[ h_{i}^{n+1} = \int_{0}^{t_{n+1}} C_{ij,k} \exp\left(-\frac{t_{n+1} - \tau}{\rho_{k}}\right) \frac{d\varepsilon_{j}}{d\tau} d\tau \] (4.21)

The deformation history can be separated into two integrates from 0 to \(t_{n}\) and \(t_{n}\) to \(t_{n+1}\).

\[ h_{i}^{n+1} = \int_{0}^{t_{n}} C_{ij,k} \exp\left(-\frac{t_{n+1} - \tau}{\rho_{k}}\right) \frac{d\varepsilon_{j}}{d\tau} d\tau + \int_{t_{n}}^{t_{n+1}} C_{ij,k} \exp\left(-\frac{t_{n+1} - \tau}{\rho_{k}}\right) \frac{d\varepsilon_{j}}{d\tau} d\tau \] (4.22)

Eqn. 4.22 can be rewritten with the substitution \(t_{n+1} = t_{n} + \Delta t\) for the time period 0 to \(t_{n}\)

\[ h_{i}^{n+1} = \int_{0}^{t_{n}} C_{ij,k} \exp\left(-\frac{t_{n} + \Delta t - \tau}{\rho_{k}}\right) \frac{d\varepsilon_{j}}{d\tau} d\tau + \int_{t_{n}}^{t_{n+1}} C_{ij,k} \exp\left(-\frac{t_{n+1} - \tau}{\rho_{k}}\right) \frac{d\varepsilon_{j}}{d\tau} d\tau \] (4.23)

The exponential function can be removed from its multiplicative form as follows

\[ \exp\left(-\frac{t_{n} + \Delta t - \tau}{\rho_{k}}\right) = \exp\left(-\frac{\Delta t}{\rho_{k}}\right) \exp\left(-\frac{t_{n} - \tau}{\rho_{i}}\right) \] (4.24)

By simplification, the expression in Eq. (4.23) leads to the following form since the exponential
with the $\Delta t$ can be removed from the integral since it’s a constant time step

$$
\int_{t_n}^{t_{n+1}} \mathcal{C}_{ij,k} \exp\left(-\frac{t_n - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau + \int_{t_n}^{t_{n+1}} \mathcal{C}_{ij,k} \exp\left(-\frac{t_n + 1 - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau
$$

(4.25)

By using Eqn. 4.20 and Eqn. 4.25

$$
\int_{t_n}^{t_{n+1}} \mathcal{C}_{ij,k} \exp\left(-\frac{t_n - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau + \int_{t_n}^{t_{n+1}} \mathcal{C}_{ij,k} \exp\left(-\frac{t_n + 1 - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau
$$

(4.26)

The integral in Eqn. (4.26) can be simplified by finite difference method

$$
\frac{d\varepsilon_j}{d\tau} = \lim_{\Delta t \to 0} \left( \frac{\Delta \varepsilon_j}{\Delta t} \right) = \lim_{\Delta t \to 0} \left( \frac{\varepsilon_j^{t+\Delta t} - \varepsilon_j^t}{\Delta t} \right)
$$

(4.27)

Eqn. 4.27 and Eqn. 4.26 yields

$$
\int_{t_n}^{t_{n+1}} \mathcal{C}_{ij,k} \exp\left(-\frac{t_n - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau + \int_{t_n}^{t_{n+1}} \mathcal{C}_{ij,k} \exp\left(-\frac{t_n + 1 - \tau}{\rho_k}\right) \frac{d\varepsilon_j}{d\tau} d\tau
$$

(4.28)

With respect to Eqn. 4.28 the stress can be found by going through an iterative process

$$
\sigma_i^{t_{n+1}} = C_{ij,\infty} \varepsilon_i^{t_{n+1}} + \sum_{k=1}^{n} \left( \exp\left(-\frac{\Delta t}{\rho_k}\right) h_i^n + C_{ij,k} \rho_k \frac{1 - \exp\left(-\frac{\Delta t}{\rho_k}\right)}{\Delta t} \left[\varepsilon_i^{t+\Delta t} - \varepsilon_i^t\right] \right)
$$

(4.29)

Defining the material Jacobian matrix, in other words, the instantaneous modulus is a requirement in Abaqus/Standard UMAT. Although an exact Jacobian does not need to be defined, a suitable
Jacobian depending on the type of problem should be defined to ensure the convergence of solution. For small deformations or even large deformation problems with incompressible materials, the Jacobian matrix can be defined by $\frac{\Delta \sigma}{\Delta \varepsilon}$ where $\Delta \sigma$ is a small increment in Cauchy stress and $\Delta \varepsilon$ is a small increase in strain. The Jacobian can be defined by the following equation

$$\frac{\Delta \sigma}{\Delta \varepsilon} = C_{ij,\infty} + \sum_{k=1}^{n} C_{ij,k} \rho_k \left(1 - exp\left(-\frac{\Delta t}{\rho_k}\right)\right) \Delta t$$ \hspace{1cm} (4.30)

4.4 Experimental Studies

The Column Bending Test (CBT) Method was used in this study to determine the bending response of thin-ply composites. CBT is a relatively new method of testing [2] that has been introduced to perform bending tests for thin-ply laminates. The general test set up is given in Fig. 4.16. The test set up consists of two arms connected to the MTS system (mechanical testing system) where a U shaped clevis is connected to each arm. At the end of each U shaped clevis is a pin connecting the two ends of the U. There are two fixtures sitting on the pins that are free to rotate about the pin axis. The coupon (Fig 4.17) is clamped to the fixtures in the vertical orientation as displayed in the Fig. 4.16a. The arms are moved closer together applying a compression on the test coupon. The design of the fixtures where the pins are located with a slight offset to the vertical coupon (at the unloaded state) allows the compression to be transferred to a bending effect applying a curvature on the coupon.
Figure 4.16: a) Coupon in unloaded state, and b) Loaded Coupon.

Figure 4.17: Sample test coupons.
The schematic of the undeformed and the deformed shape is given in Fig. 4.18.

Figure 4.18: Schematic of the CBT

Geometrically, the vertical displacement of the arms can be related to the curvature achieved by the coupon with a constant curvature assumption. $\theta$, $L$ (fixture arm length), $r_0$ and $s$ (gage length) are known from the undeformed configuration [2].

\[
\delta = s [1 - \frac{2}{\phi} \sin\left(\frac{\phi}{2}\right)] + 2L [\cos(\theta) - \cos(\theta + \frac{\phi}{2})]
\]

(4.31)
\[ \kappa = \frac{\phi}{s} \] (4.32)

where

\[ \sin(\theta) = \frac{r_0}{L} \] (4.33)

The bending moment (Eq.( 4.35)) acting on the coupon center can be found by using the compression force \( P \) applied onto the coupon and the moment arm length \( r \) (Eq.( 4.34)).

\[ r = \frac{s}{\phi} \left[ 1 - \cos\left(\frac{\theta}{2}\right) \right] + L \sin(\theta + \frac{\phi}{2}) \] (4.34)

\[ M = Pr \] (4.35)

The test is conducted by first applying a predetermined vertical displacement to achieve the desired curvature for the sample. The displacement is given with a constant displacement rate of about 0.1 mm⁻¹. After the desired curvature is achieved, the coupon is held at the curvature for 1-2 hours depending on the requirement. The test can be repeated to different temperatures and shifted to obtain the mater-curve for a reference temperature.
CHAPTER 5: RESULTS

This chapter presents the results obtained from the numerical simulations. In total there are six numerical models to evaluate the properties of three laminates two of woven plies 3-ply and 4-ply and the laminate 3-ply inclusive of the unidirectional lamina. There are two models for each 3-ply and 4-ply at $0^\circ$ and $45^\circ$ orientation and one for the plies out-of-phase configuration for the 4-ply in $0^\circ$ orientation. Finally, one model for the laminate 3-ply with the woven ply at a $45^\circ$ orientation. The intermediate tow fiber model was simulated to obtain the tow properties of the woven geometry. High curvature bending for the 4-ply has been performed for the $0^\circ$ and $45^\circ$ orientations and the results are presented.

5.1 Unidirectional Tow Model

The relaxation modulus tensor was found for the unidirectional tow models with three independent simulations run with the geometric non-linear option. The stress relaxation data obtained were used to compute the modulus entries of the relaxation modulus tensor and were fitted to Prony series by carrying out non-linear curve fit in Matlab. The fitted data are plotted in Fig. 5.1 and Fig. 5.2 for M30S fiber tow and the IM7 fiber tow respectively. The master curves produced are for a temperature of $30^\circ$C. It must be noted that the term $C_{11}$ does not show significant relaxation compared to the rest of the terms because the modulus $C_{11}$ is measured in the direction of the fibers where the elastic properties are prominent.
Figure 5.1: Tow relaxation moduli for the M30S fiber tow.

Figure 5.2: Tow relaxation moduli for IM7 fiber tow.
5.2 Laminate Models

The 4-ply laminate was studied with three models. Two at 0° and 45° orientation with a plies-in-phase configuration and the third one with a plies-out-of-phase configuration. The analyses were carried out in geometric non-linear regime. The model was validated by using data that were available from the NASA Langely Research Center [52]. The Column Bending Test method had been used to obtain the test data. The analyses were performed replicating the boundary conditions applied on the CBT method. The unit cell was clamped with two opposite faces fixed to two reference points rigidly by using coupling constrains in Abaqus. The reference points were given two rotations to obtain a curvature of $k_1 = 0.1 \text{mm}^{-1}$. The same faces were constrained not to attain curvatures in the transverse direction to replicate the fixed condition at the clamps. The other two faces perpendicular to the clamped faces were left without applying any boundary conditions. The results are compared in Fig. 5.3 and Fig. 5.4. The comparison is illustrated for a temperature of 30°C.

![Figure 5.3: Test data comparison with model output for 0° oriented 4-ply.](image)

60
The tests have been conducted with four different samples. The average thickness of 0.24 \textit{mm} of the samples was applied in the unit cell. The thickness measurements are shown in Table 5.1. The other geometric parameters were assumed from the previous micrographs since the type of fiber is same as M30S toughened with PMT-F7 epoxy resin. Due to the uncertainty of the true laminate volume fraction (since a matrix chemical wash away has not being performed to obtain the actual volume fraction of the samples) on these composites, a range for the volume fraction was assumed and two models were built expecting to capture two bounds. For one model, the volume fraction 0.56 resulting from the mass conservation was used. A lower volume fraction of 0.527 was used for the other model. Fine tuning of volume fraction to an exact desired value is difficult to achieve due to the iterative process followed. The sensitivity of the modification of the parameters to the volume fraction is not fully understood, so achieving an exact volume fraction is not always possible. It can be seen by the comparison that both the models overestimate bending.
resistance predicted by the column bending test.

An average thickness of 0.2623 *mm* was used for the 4-ply model oriented at 45°. The sample data are given in Table. 5.1. Again two volume fractions were considered, 0.56 coming from the mass conservation, and a lower value of 0.533. It can be seen from the comparison that the numerical model with 0.533 volume fraction agrees with the test results more, compared to the model with 0.56 volume fraction.

Table 5.1: Test coupon data

<table>
<thead>
<tr>
<th>0° oriented coupons</th>
<th>45° oriented coupons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>Thickness (mm)</td>
</tr>
<tr>
<td>SP1</td>
<td>0.2406</td>
</tr>
<tr>
<td>SP2</td>
<td>0.2337</td>
</tr>
<tr>
<td>SP3</td>
<td>0.2359</td>
</tr>
<tr>
<td>SP4</td>
<td>0.2417</td>
</tr>
</tbody>
</table>

The relaxation ABD matrix was computed for both 0° and 45° oriented laminates of the 4-ply. The curves are plotted in Fig. 5.5, Fig. 5.6, Fig. 5.7 and Fig. 5.8 for a temperature of 30°C. A model for the fibers out-of-phase was constructed for the 4-ply 0° oriented laminate and the ABD relaxation matrix is plotted in Fig. 5.9 and Fig. 5.10.
Figure 5.5: Relaxation A matrix for the plies in-phase configuration for the 4-ply 0° oriented model.

Figure 5.6: Relaxation A matrix for the plies in-phase configuration for the 4-ply 45° oriented model.
Figure 5.7: Relaxation D matrix for the plies in-phase configuration for the 4-ply 0° oriented model.

Figure 5.8: Relaxation D matrix for the plies in-phase configuration for the 4-ply 45° oriented model.
Figure 5.9: Relaxation A matrix for the plies out-of-phase configuration for the 4-ply 0° oriented model.

Figure 5.10: Relaxation D matrix for the plies out-of-phase configuration for 4-ply the 0° oriented model.
Bending tests were carried out for the 3-ply plain weave composite by using the column bending test fixtures. The original 3-ply panel was provided by the NASA Langely Research center. The samples were cut for a width of 1 inch and a height of 1.5 inches. A typical gauge length ranging from 10 mm to 15 mm was used in the tests.

The relaxation curves obtained for the temperatures 22°C, 40°C, 50°C and 70°C are given in Fig. 5.11. The data was shifted to obtain a master curve at 40°C and is displayed in Fig. 5.12 and the shift factors are plotted in Fig. 5.13. The temperatures of the conducted tests are below the glass transition temperature of the epoxy PMT-F7. The curing temperature of epoxy is typically about 350°F [53] and the glass transition temperature is expected to fall around the curing temperature. So, the shift factors were fitted to the expression developed from the Arrhenius activation energy equation 3.7. The fitted curve was used to extract the shift factor at temperature 30°C and convert the master curve obtained from the test data to 30°C for comparison with the numerical simulation results. The results are plotted in Fig. 5.14.
Figure 5.11: Relaxation curve for 3-ply plain weave.

Figure 5.12: Relaxation curve shifted to temperature 40°C.
Fig 5.13: Shift factors relative to temperature of 40°C.

Fig 5.14: The comparison with test data at 30°C.

Fig 5.15, Fig. 5.16 present the in-plane and out-of-plane relaxation matrices for 0° orientation for the 3-ply plain weave composite for a tows in-phase configuration with the thickness averaged to
be 0.1912 mm.

Figure 5.15: Relaxation A matrix for the plies in-phase configuration for the 3-ply 0° oriented model.

Figure 5.16: Relaxation D matrix for the plies in-phase configuration for 3-ply the 0° oriented model.
Fig. 5.17 and Fig. 5.18 represent the in-plane and out-of-plane relaxation matrices for 45° orientation for the 3-ply plain weave composite for a tows in-phase configuration. The thickness measured from samples was averaged to be 0.202 mm.

Figure 5.17: Relaxation A matrix for the plies in-phase configuration for the 3-ply 45° oriented model.

Figure 5.18: Relaxation D matrix for the plies out-of-phase configuration for 3-ply the 45° oriented model.
Fig 5.19 and Fig 5.20 represent the relaxation of $D_{11}$ subjected to high curvatures for both $0^\circ$ and $45^\circ$ oriented 4-ply.

Figure 5.19: $D_{11}$ computed to $\kappa_{0.001}$, $\kappa_{0.1}$ and $\kappa_{0.2}$ $0^\circ$ oriented 4-ply.

Figure 5.20: $D_{11}$ computed to $\kappa_{0.001}$, $\kappa_{0.1}$ and $\kappa_{0.2}$ $0^\circ$ oriented 4-ply.
The observed stress relaxation is plotted in Fig. 5.21 for the $0^\circ$ oriented 4-ply by considering the stresses in 1-direction. The plots are for a curvature of $0.001 \text{ mm}^{-1}$. Fig. 5.21a shows the distribution of stresses at the beginning of the holding step which is after bending to a curvature of $0.001 \text{ mm}^{-1}$. Fig. 5.21b illustrates the distribution of stresses at the end of the holding time step (at time $t=10^{14} \text{s}$). For better visualization of the stresses in the tows, the surrounding matrix filling the space between tows has been hidden from view.

Figure 5.21: Bending of 4-ply laminate, in-phase configuration oriented at $0^\circ$ a) Stress distribution in 1-direction at $t=0\text{s}$, and b) Stress distribution in 1-direction at $t=10^{14}\text{s}$.

A twisting moment was observed when an axial strain is imposed on the 4-ply that is in the plies-in-phase configuration. The simulations were extended to three different models with varying number of unit cells of the plies in-phase configuration to understand any variation that would come from the boundary effects. The data is given in Table 5.2
Table 5.2: Twisting Moment for an axial strain of $\varepsilon_1=0.001$

<table>
<thead>
<tr>
<th>4-Ply 0 Oriented</th>
<th>Number of unit cells</th>
<th>Resultant twisting moment (N/mm)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>M12</td>
</tr>
<tr>
<td>In-phase</td>
<td>One</td>
<td>0.271</td>
</tr>
<tr>
<td></td>
<td>Two</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td>Three</td>
<td>0.281</td>
</tr>
<tr>
<td>Out-of-phase</td>
<td>One</td>
<td>1.895x10^{-4}</td>
</tr>
</tbody>
</table>

The capabilities of the textile modeling methodology was extended to the laminate 3-ply containing a unidirectional lamina. The results obtained for the ABD matrix are given in Fig. 5.22 and Fig. 5.23.

Figure 5.22: Relaxation A matrix for the laminate 3-ply in the $0^\circ$ orientation of the unidirectional ply.
Figure 5.23: Relaxation D matrix for the laminate for 3-ply in the 0° orientation of unidirectional ply.
CHAPTER 6: DISCUSSION

This chapter discusses the results in detail, the reasoning behind the findings and the conclusions that can be drawn from them. The future work that will be conducted are also discussed.

6.1 Discussion

Numerical models were developed with a two step homogenization approach to evaluate the viscoelastic properties of thin-ply composites. The Fig. 5.3 and Fig. 5.4 shows the comparison of the 4-ply numerical model with the CBT results provided by the NASA Langley Research Center [52]. The numerical model findings overestimate the resistance to bending that is measured from the Column Bending Test. Although cannot be claimed exactly, there can be a few reasons for this discrepancy. The elastic properties for the fiber except the longitudinal modulus were all estimated by literature since the actual data were not available. With the manufacturing of new batches for the 4-ply, either due to manufacturing uncertainties or the variation of M30S fiber tow sizes, the thickness of the cured laminate can vary. Due to the unavailability of micrographs for tested specimens, all the geometric parameters except the thickness of the laminate and tow were assumed from the previously available micrographs including the tow volume fraction. These uncertainties can contribute to the differences in the comparison.

The Fig. 5.5 shows the relaxation A-matrix for the 4-ply tows in-phase configuration. The term $A_{12}$ initially reduces showing relaxation behavior but after a certain period of time it starts to display an increasing trend. This behavior was found out to be unique for the tows in-phase configuration. In contrast, when the $A_{12}$ is determined for the plies out-of-phase configuration it shows the expected relaxation behavior from viscoelasticity. This also accompanies another contrast between the two
models, that is the coupling between axial and twisting components. It was observed that when an axial strain is imposed on the model with tows in-phase, a twisting moment is generated on the unit cell (twisting moments are given in Table. 5.2). Three different models were created with one, two and three unit cells to confirm that it was not a consequence of the boundary effects due to small dimensions of the one unit cell model. It could be concluded that the three models generated similar behavior for the imposed axial strain. The variation in the twisting moment is due to the differences in meshing. As the number of unit cells was increased, the mesh size was adjusted to avoid the models becoming computationally heavy since the requirement was to verify the generation of the twisting moment only. When the 4-ply model with the out-of-phase ply arrangement was analyzed it was noted that the twisting moment generated is negligible. The entry in the ABD relaxation matrix that’s under investigation here is the term $B_{16}$. For the models with the tows in-phase configuration, $B_{16}$ dwells in the order of hundreds while for the unit cell model with the tows out-of-phase configuration, $B_{16}$ is less than unity. The reason for this contrasting behavior of $A_{12}$ and $B_{16}$ can be observed by the deformed shape of the unit cells (Fig 6.1). The deformed shape is magnified in both the models to the same scale for clarity.

![Deformed shape](image)

Figure 6.1: Deformed shape of 0$^\circ$ oriented 4-ply for an axial strain $\varepsilon_1=0.001$, and a) Plies in-phase arrangement b) Plies out-of-phase arrangement.
Fig. 6.1 a) shows the deformed shape of the plies in-phase model. It is apparent that the tows have deformed increasing the waviness in the transverse direction. This deformation causes the unit cell to generate a twisting moment since the relative displacements of the opposite faces are constrained in the 3-direction. Furthermore with time, it was seen that the waviness increases in the transverse direction after a certain period of time which applies a contracting force on the fixed transverse boundary increasing the force in the transverse direction. It can be claimed that in fact, this behavior is the reason for the $A_{12}$ to increase after a certain period of time. It is likely that this rearranging of tows commence once the epoxy has softened enough the tows are free to move and deform reflecting unusual behavior in the $A_{12}$ term in the A-matrix. No such deformations are seen in the plies out-of-phase configuration and as a result, no twisting moments nor any unusual behavior in $A_{12}$ is seen.

The 4-ply laminate was subjected to high curvatures to study the behavior as well. The Fig. 5.19 and Fig. 5.20 depict the bending stiffness plotted against time for three different curvatures. Focusing on Fig. 5.19 it can be seen that effects are different as the curvature is increased. Plot with $\kappa_1 = 0.001 \text{ mm}^{-1}$ shows the regular relaxing behavior at small curvatures but as the curvature is increased to $0.1 \text{ mm}^{-1}$ after a certain period of time, it starts to reduce drastically. As the curvature is increased to $0.2 \text{ mm}^{-1}$ the time spent (30 decades for the $0.2 \text{ mm}^{-1}$ curvature) to trigger the accelerated reduction of bending stiffness is less compared to $0.1 \text{ mm}^{-1}$ curvature. The deformed shapes for the 4-ply $0^\circ$ oriented unit cell are given in Fig. 6.2 for the curvature of $0.2 \text{ mm}^{-1}$. From Fig. 6.2a, it can be seen that the shape is still held in place but as the matrix softens the stresses on the system are transferred to the tows and after a certain period of time, the tows are unable to resist the stresses and buckles under pressure. It should be noted that the tows have failed in the mid region where the fiber density is lowest across the thickness. In the mid section of the unit cell, the transverse tows relative to the bending direction are not present, therefore the tow volume in the region is reduced to half compared with the regions with both longitudinal and transverse
yarns. The situation studied here is an extreme case of plies in-phase where all the parallel tows are in-phase but in reality the plies are seen to have small relative displacements with each other reducing the probability of forming a region with low fiber density as in the numerical model. This reduces the possibility of expecting such behavior in reality at the same time frame as the numerical model but mechanical testing must be performed to study this further.

![Figure 6.2: Deformed shape of 0° oriented 4-ply](image)

(a) Mid way through the holding step, and (b) At the end of the holding step.

The plots drawn for the high curvatures are for a working temperature of 30°C but if higher temperatures are present in the operating environment the curve can be shifted leftward causing early failure. By using the shift factors available with the NASA Langley Research Center data, the master curves for curvatures 0.1 mm\(^{-1}\) and 0.2 mm\(^{-1}\) in Fig. 5.19 were shifted to a temperature of 80°C and the converted curve is plotted in Fig 6.3. As illustrated in the plot, the triggering of bending stiffness decline can start early at higher temperatures. For the curvature 0.2 mm\(^{-1}\), the decline of stiffness start around 4 months to 3 years for 80°C as opposed to the 30 decade wait at 30°C.
Figure 6.3: $D_{11}$ curves shifted to a temperature of 80$^\circ$C for curvatures 0.1 mm$^{-1}$ and 0.2 mm$^{-1}$.

6.2 Future Work

The work done for this thesis could be further investigated and improved. The comparison of the numerical results with the test data for the 3-ply are not in a good agreement in this study. It is important to locate the source of this discrepancy which would be beneficial for other models as well. The column bending test for more samples at different temperatures can be conducted to characterize the viscoelastic behavior of the 3-ply laminate to obtain a better understanding with respect to laminate thickness and its effect on the bending resistance.

The ply configurations considered here are at the extremes of plies in-phase and plies out-of-phase but in reality, the translations of plies with respect to each other are random. Since it was observed that the configuration has an effect on the laminate behavior it would be beneficial to consider random arrangements so the behavior can be captured close to reality.
In addition to this, a reduction of bending stiffness was observed for the laminate bent to high curvatures. The buckling of the tows were seen in the mid section of the unit cell where the fiber density is lowest and again the probability of having this low fiber density regions are low due to random translations. The model can be improved to address this as well.
APPENDIX A: TEXGEN PYTHON SCRIPT
# Create a textile
Textile = CTextile()

wl = 6.674
width = 3.164
x = (wl - 2 * width) / 4
n = 0.4

p = wl / 4  # a quarter of the weave length
a = 0.015  # amplitude
c = 0.14  # middle nodes
h = 0.014  # offset of the middle nodes
ys = 2 * x + (wl - 4 * x) / 2  # yarn length
hf = 0
R = 150

# Defining the yarns

# Yarn 1
yarn = CYarn()
yarn.AddNode(CNode(XYZ(-p + hf, p + hf, a)))
yarn.AddNode(CNode(XYZ(-c + hf, p + hf, h)))
yarn.AddNode(CNode(XYZ(c + hf, p + hf, -h)))
yarn.AddNode(CNode(XYZ(p + hf, p + hf, -a)))
yarn.AddNode(CNode(XYZ(2 * p - c + hf, p + hf, -h)))
yarn.AddNode(CNode(XYZ(2 * p + c + hf, p + hf, h)))
yarn.AddNode(CNode(XYZ(3 * p + hf, p + hf, a)))

yarn.AssignInterpolation(CInterpolationBezier())  # yarn geometry
yarn.AssignSection(CYarnSectionConstant(CSectionPowerEllipse((wl - 4 * x) / 2, 2 * a, n, 0)))  # defining the tow cross section
yarn.SetResolution(R)
yarn.AddRepeat(XYZ(wl, 0, 0)) #adding repeats
yarn.AddRepeat(XYZ(0, wl, 0)) #adding repeats
Textile.AddYarn(yarn)

#Yarn2
yarn=CYarn()
yarn.AddNode(CNode(XYZ(-p+hf, 3*p+hf, -a)))
yarn.AddNode(CNode(XYZ(-c+hf, 3*p+hf, -h)))
yarn.AddNode(CNode(XYZ(c+hf, 3*p+hf, h)))
yarn.AddNode(CNode(XYZ(p+hf, 3*p+hf, a)))
yarn.AddNode(CNode(XYZ(2*p-c+hf, 3*p+hf, h)))
yarn.AddNode(CNode(XYZ(2*p+c+hf, 3*p+hf, -h)))
yarn.AddNode(CNode(XYZ(3*p+hf, 3*p+hf, -a)))
yarn.AssignInterpolation(CInterpolationBezier())
yarn.AssignSection(CYarnSectionConstant(CSectionPowerEllipse(((wl-4*x)/2, 2*a, n, 0))))
yarn.SetResolution(R)
yarn.AddRepeat(XYZ(wl, 0, 0))
yarn.AddRepeat(XYZ(0, wl, 0))
Textile.AddYarn(yarn)

#Yarn3
yarn=CYarn()
yarn.AddNode(CNode(XYZ(p+hf, -p+hf, -a)))
yarn.AddNode(CNode(XYZ(p+hf, -c+hf, -h)))
yarn.AddNode(CNode(XYZ(p+hf, c+hf, h)))
yarn.AddNode(CNode(XYZ(p+hf, p+hf, a)))
yarn.AddNode(CNode(XYZ(p+hf, 2*p-c+hf, h)))
yarn.AddNode(CNode( XYZ(p+hf, 2*p+c+hf, -h) ))

yarn.AddNode(CNode( XYZ(p+hf,3*p+hf, -a) ))

yarn.AssignInterpolation(CInterpolationBezier())

yarn.AssignSection(CYarnSectionConstant(CSectionPowerEllipse((wl-4*x)/2 ,
2*a, n, 0)))

yarn.SetResolution(R)

yarn.AddRepeat(XYZ( 0, wl, 0 ))

yarn.AddRepeat(XYZ(wl, 0, 0 ))

Textile.AddYarn(yarn)

# Yarn4

yarn=CYarn()

yarn.AddNode(CNode( XYZ(3*p+hf, -p+hf, a) ))

yarn.AddNode(CNode( XYZ(3*p+hf, -c+hf, h) ))

yarn.AddNode(CNode( XYZ(3*p+hf, c+hf, -h) ))

yarn.AddNode(CNode( XYZ(3*p+hf, p+hf, -a) ))

yarn.AddNode(CNode( XYZ(3*p+hf, 2*p-c+hf, -h) ))

yarn.AddNode(CNode( XYZ(3*p+hf, 2*p+c+hf, h) ))

yarn.AddNode(CNode( XYZ(3*p+hf,3*p+hf, a) ))

yarn.AssignInterpolation(CInterpolationBezier())

yarn.AssignSection(CYarnSectionConstant(CSectionPowerEllipse((wl-4*x)/2 ,
2*a, n, 0)))

yarn.SetResolution(R)

yarn.AddRepeat(XYZ( 0, wl, 0 ))

yarn.AddRepeat(XYZ(wl, 0, 0 ))

Textile.AddYarn(yarn)

Textile.AssignDomain(CDomainPlanes(XYZ(-ys, -ys, -2*a), XYZ(ys, ys, 2*a)))
# assigning domain

90

91

92 AddTextile("Plainweave_single_Ply", Textile)
APPENDIX B: PRONY SERIES
Table B.1: The relaxation modulus tensor for the M30S carbon fiber toughened with PMT-F7 epoxy

<table>
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<tr>
<th>k</th>
<th>$\rho_k$</th>
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<th>$C_{21,k}$</th>
<th>$C_{22,k}$</th>
<th>$C_{23,k}$</th>
<th>$C_{44,k}$</th>
<th>$C_{55,k}$</th>
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Table B.2: The relaxation modulus tensor for the IM7 carbon fiber thoughned with PMT-F7 epoxy

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The thickness used in the models for 3-ply at 0° orientation is 0.1912 mm.

Table B.3: ABD relaxation matrix of 0° oriented 3-ply

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<th>A_{33,k}</th>
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The thickness used in the models for 3-ply at 45° orientation is 0.202 mm.

Table B.4: ABD relaxation matrix of 45° oriented 3-ply.

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<th>( A_{33,k} )</th>
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<th>( D_{12,k} )</th>
<th>( D_{33,k} )</th>
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<td>0.1214</td>
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<td>0.1459</td>
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The thickness used in the models for 4-ply at 0° orientation is 0.228 mm.

Table B.5: ABD relaxation matrix of 0° oriented 4-ply.

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The thickness used in the models for 4-ply at 45° orientation is 0.228 mm.

Table B.6: ABD relaxation matrix of 45° oriented 4-ply.

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<th>D_{12,k}</th>
<th>D_{33,k}</th>
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Table B.7: Relaxation A matrix of (0° oriented UD) laminate 3-ply.

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N/mm
Table B.8: Relaxation D matrix of (0° oriented UD) laminate 3-ply.

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