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# Fluctuations of scattered waves: going beyond the ensemble average

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**Abstract:** The interaction between coherent waves and random media is a complicated, deterministic process that is usually examined upon ensemble averaging. The result of one realization of the interaction process depends on the specific disorder present in an experimentally controllable interaction volume. We show that this randomness can be quantified and structural information not apparent in the ensemble average can be obtained. We use the information entropy as a viable measure of randomness and we demonstrate that its rate of change provides means for discriminating between media with identical mean characteristics.

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A random medium is typically characterized by an ensemble of realizations of disorder. When waves interact with a random medium, each member of this ensemble, i.e. each particular realization of disorder, has its own pattern of fluctuations in the scattered wave. The interaction is a

non-self-averaging process, and the complicated features of the scattered waves are all rooted in the structural properties of the specific realization of randomness [1, 2, 3, 4, 5, 6]. In principle, some of this structural information can be recovered if (i) the phase coherence is maintained over the entire interaction, (ii) the process is not lossy, and (iii) the disorder does not vary in time. In practice, however, due to finite sizes and experimental noise, one always infringes at least on the second requirement. Furthermore, the information available is often too complex to process in a practical manner. Therefore, an average over an ensemble of realizations of disorder is usually taken to determine mean statistical properties. For instance, numerous speckle patterns resulting from different interactions must be averaged to learn about the global properties [7]. Unfortunately, this averaging inherently discards information specific to particular realizations as well as the variations from one realization to the next.

As such, the following question arises: can one learn anything about the stochastic process by examining a set of its individual realizations? In principle, having access to a number of samples should allow one to study the rate of convergence toward the ensemble average characteristics. Following the example of stochastic interaction between coherent waves and random media, we will demonstrate that the significant sample to sample fluctuations can be used to extract information not available in the ensemble average. Because the convergence of the statistical properties of moments is a general problem for numerous physical phenomena that are described as random processes, concepts similar to the one discussed here may be exploited in other situations.

As an example, let us consider the interaction of photons with a random medium characterized by a number density  $N_V$  of scattering centers and by the scattering cross-section  $\sigma$  describing the properties of a single scattering event. For each realization of disorder  $\alpha$ , the interaction will be defined by a specific distribution  $p_\alpha(s)$  of available photon path-lengths  $s$  through the medium. When an ensemble average is taken over many such realizations, the photon interaction will be described by a probability distribution function  $p(s) = \langle p_\alpha(s) \rangle = f(s, D)$  that has a universal behavior depending only on the normalized diffusion coefficient  $D \propto 1/(N_V \sigma)$  [8]. Note that all of the experimentally observable properties of the stochastic interaction can be described in terms of the probability distribution  $p(s)$  whose exact functional form  $f(s, D)$  may also depend on the particular geometry of an experiment. Clearly, there could be many dissimilar media with different  $N_V$  and  $\sigma$  that nevertheless display the same characteristics upon ensemble averaging. In practice, this ensemble can be acquired in different ways for dynamic or stationary systems, but the final result is the same: the number density and the scattering cross-section are being coupled through the diffusion coefficient, and only their product is accessible.

For a specific realization of the material disorder, the distribution of available path-lengths  $p_\alpha(s)$  will deviate from the one corresponding to the ensemble average:

$$p_\alpha(s) = f(s, D) [1 + \delta_\alpha(s, \xi_\alpha)]. \quad (1)$$

Because this deviation  $\delta_\alpha(s, \xi_\alpha)$  is specific to a particular realization of disorder, it depends on variables not present in the ensemble average. Specifically, this can be expressed through a configuration function  $\xi_\alpha$  describing the particular morphology of the given realization  $\alpha$ . In the example above, the function  $\xi_\alpha$  describes the locations of scattering centers available in the realization and depends only on the number density and not on the scattering cross-section. For media with continuously varying refractive indexes, one can still define a configuration function  $\xi_\alpha$  that is independent of the strength of the scattering. By examining the statistical properties of  $\delta_\alpha(s, \xi_\alpha)$ , one could infer information not available in the ensemble average.

Two observations about the general behavior of  $\delta_\alpha(s, \xi_\alpha)$  are worth making. First, as the length  $s$  of the path increases, more and more different trajectories of the same length are

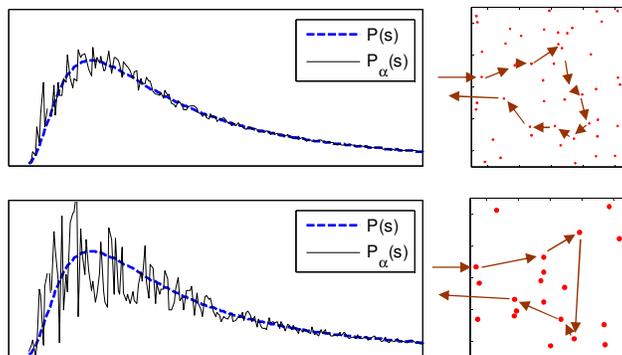


Fig. 1. Sketch of path-length distributions for two media with identical mean properties (same  $D$ ). The two media consist of scatterers of different cross-sections  $\sigma$  and different number densities  $N_V$  and are examined over the same range of path-lengths  $s$ . The medium with smaller number density provides fewer possible paths of given length  $s$  resulting in larger fluctuations of  $p_\alpha(s)$ .

possible through the medium, and  $p_\alpha(s)$  approaches the value corresponding to the weight of trajectories of length  $s$  in the ensemble average. In other words, in terms of the variable  $s$ , the function  $\delta_\alpha(s, \xi_\alpha)$  represents a nonstationary random process. Second, because upon ensemble averaging a scattering region will exist at any position, this random function is of zero mean,  $\langle \delta_\alpha(s, \xi_\alpha) \rangle = 0$ . However, because of the implicit dependence on the density of scattering regions, it is expected that higher order statistics of  $\delta_\alpha(s, \xi_\alpha)$  can be used to reveal characteristics of the wave-matter interaction not included in the value of  $D$ .

This concept is illustrated schematically in Fig. 1 where the path-length distributions corresponding to two different media are sketched over similar ranges of  $s$ . The two media consist of scattering centers having different cross-sections but also different number densities such that, upon ensemble average, they are described by the same diffusion coefficient. Clearly, when compared to all potential trajectories, there are fewer available paths of given length  $s$  through the medium with less scattering centers. Consequently, the path-length distribution  $p_\alpha(s)$  deviates more significantly from the ensemble average  $p(s) = \langle p_\alpha(s) \rangle$ . A measure of the nonstationary fluctuations in  $p_\alpha(s)$  should discriminate between the two media, as we will show in the following.

Evidently, the random function  $p_\alpha(s)$  displays not only fluctuations in  $s$  but also differences from one material realization  $\alpha$  to another. There are many ways in which the two-dimensional statistical characteristics of  $p_\alpha(s)$  can be quantified. Of course, a simple averaging over  $\alpha$  will provide a path-length distribution  $p(s) = f(s, D)$  which corresponds to the ensemble average. For a single realization  $\alpha$  on the other hand, higher order moments of the fluctuations in  $p_\alpha(s)$  can be evaluated. Even though  $p_\alpha(s)$  is nonstationary in  $s$ , one can still calculate simple estimators such as, for instance, the variance  $V_\alpha(\xi_\alpha) = \int \delta_\alpha^2(s, \xi_\alpha) ds - |\int \delta_\alpha(s, \xi_\alpha) ds|^2$  of the fluctuations along  $s$ . However, this simple estimate is inadequate because  $\delta_\alpha(s, \xi_\alpha)$  is a zero-mean random function and, consequently, a unique and meaningful normalization is difficult to define.

As the deviation  $\delta_\alpha(s, \xi_\alpha)$  from the ensemble average can be regarded as a form of disorder, we can choose to examine its variance in terms of the Shannon information entropy [9]:

$$H_\alpha(s_1, s_2) = - \int_{s_1}^{s_2} \frac{\delta_\alpha^2(s, \xi_\alpha)}{\int_{s_1}^{s_2} \delta_\alpha^2(s, \xi_\alpha) ds} \log \left( \frac{\delta_\alpha^2(s, \xi_\alpha)}{\int_{s_1}^{s_2} \delta_\alpha^2(s, \xi_\alpha) ds} \right) ds. \quad (2)$$

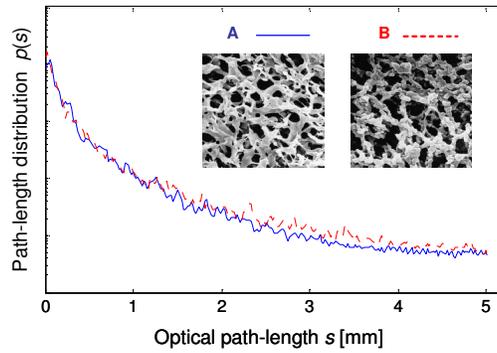


Fig. 2. The averaged backscattered intensities for medium A (blue solid line) and medium B (red dashed line). The insets show typical micrographs of the materials examined.

In Eq. (2), we define this finite scale entropy to account for realistic situations of any measurement that extends over a finite range  $[s_1, s_2]$ . Furthermore, the finite scale entropy can be normalized to its maximum allowable value for the entire range  $S = s_2 - s_1$  as

$$h_\alpha(s_1, s_2) = -H_\alpha(s_1, s_2) / \log\left(\frac{1}{S}\right). \quad (3)$$

Of course, the normalized entropy  $h_\alpha(s_1, s_2)$  will still vary from realization to realization and one can further build its average  $\overline{h_\alpha(s_1, s_2)}$  over the number of realizations available. Being constructed in terms of the specific fluctuations of each realization  $\alpha$ , this average is a comprehensive measure of the overall fluctuations in  $\delta_\alpha(s, \xi_\alpha)$ .

We conducted an experiment to analyze the fluctuations in realizations of  $p_\alpha(s)$ . The distribution of photon path-lengths through different multiply-scattering media was measured interferometrically using the procedure of optical path-length spectroscopy (OPS)[10, 11]. Using radiation with a short coherence length and an envelope detection of the interferometric signal, OPS provides a direct measure of scattering contributions with specified pathlengths. Measuring the magnitude of the envelope allows us to analyze differences in fluctuations of the signal from two different media. The OPS measurements are based on fiber optic arrangements that permit different modalities for injecting light into and collecting reflected light from a scattering medium. The configuration can be monostatic, where the same fiber acts as both the source and detector, or bistatic when the injection and detection points are separated by an adjustable distance  $\Delta$  allowing for an experimental control over the volume of interaction. In the frame of diffusion theory for lossless media, the path-length distribution  $p(s, \Delta)$  corresponding to the ensemble average can be evaluated to be

$$p(s, \Delta) \sim \exp\left(-\frac{z_e^2 + \Delta^2}{4sD}\right) D^{-\frac{3}{2}} s^{-\frac{5}{2}}, \quad (4)$$

where  $z_e$  is the so-called extrapolation length [10].

We examined two different highly diffusive media that have approximately identical average properties. These random dielectrics are non-absorbing polymer networks and have average pore sizes of  $0.45 \mu\text{m}$  and  $1.2 \mu\text{m}$ . Upon ensemble averaging, both are characterized by the same value of the transport mean free path of  $10 \mu\text{m}$ .

Path-length distributions averaged over ten different realizations of these random media are shown in Fig. 2 together with their corresponding scanning electron micrographs. Even though

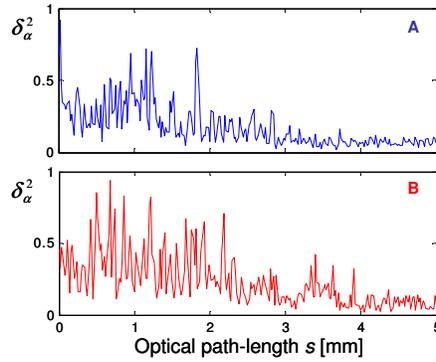


Fig. 3. Typical mean square fluctuations  $\delta_{\alpha}^2(s, \xi_{\alpha})$  of path-length distributions for media A and B shown in Fig.2.

their structural morphologies are obviously different, the similar behavior of  $p(s)$  is a clear indication that, on average, the two media are being described by the same diffusion coefficient. On the other hand, the fluctuations from the average are rather dissimilar as can be seen in Fig. 3 where we plot the typical mean square of the fluctuations  $\delta_{\alpha}^2(s, \xi_{\alpha})$  corresponding to the two media. In general, medium A exhibits smaller deviations from the average which can be interpreted as a larger number of scattering trajectories available for each  $s$ . Note also that the fluctuations in  $\delta_{\alpha}^2(s, \xi_{\alpha})$  decrease for larger values of  $s$  because these random processes are nonstationary as discussed above.

Let us now examine in more detail the situation where the scale of available path-lengths is varied. In practice, this amounts to controlling the size of the interaction volume which can be implemented in the bistatic OPS measurements as suggested in the inset of Fig. 4. By increasing the source-detector separation  $\Delta$  the interaction volume is enlarged while enforcing a minimum path-length. According to our notation in Eq. (2), this amounts to setting the lower path-length limit at  $s_1 = \Delta$  and the upper one at  $s_2 = \Delta + S$ . Here  $S$  denotes the value of the total span of path-lengths available in the measurement;  $S$  is constant in our experiments. Subsequent normalization and averaging over different realizations was performed following the procedure outlined by Eq. (3). In Fig. 4 we present the values of the normalized scale dependent entropy  $\overline{h_{\alpha}(\Delta)}$  averaged over ten realizations of disorder for both media examined.

As can be seen for both media, when the interaction volume increases, the entropy increases as expected because in all realizations  $\alpha$ ,  $\delta_{\alpha}(s, \xi_{\alpha})$  is a nonstationary process, and its fluctuations decrease at larger  $s$ . The absolute values and the rate of increase for  $\overline{h_{\alpha}(\Delta)}$  however are medium specific.

Two main observations are in place. First, we notice the higher values of the entropy for medium A. This is the result of a higher number density of scattering centers which determines a larger number of possible optical paths having a given length  $s$ . Therefore, there are smaller fluctuations in  $\delta_{\alpha}^2(s, \xi_{\alpha})$  as discussed before and, consequently, the entropy tends toward its value corresponding to an infinite number of possible trajectories of length  $s$ .

The second observation relates to the different rates of entropy increase as suggested by the dashed lines in Fig. 4. This behavior can be understood by realizing that a certain path-length  $s$  can be reached through a different number  $m$  of scattering events. For independent scattering, the joint distribution  $p(s, m)$  of such a process is Poissonian and the cumulative probability of scattering orders up to  $M$  that contribute to paths of length  $s$  is described, in average, by a universal cumulative distribution function  $p(s, M)$  [12]. This cumulative distribution increases fast for low values of  $M$  and tends to saturate for higher scattering orders. In one realization where

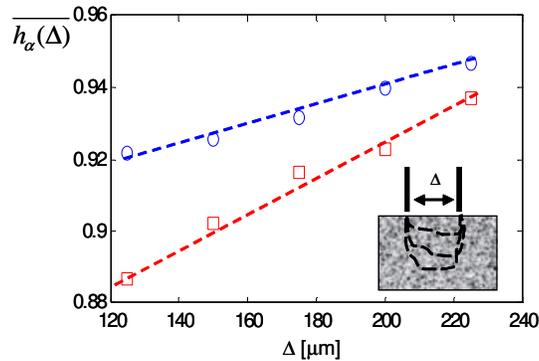


Fig. 4. Average normalized entropy  $\overline{h_\alpha(\Delta)}$  for medium A (blue circles) and medium B (red boxes) for increasing volumes of interaction in a bistatic configuration as depicted in the inset.

the interaction volume is finite, the maximum scattering order  $M$  contributing to a certain  $s$  is essentially determined by the number density of available scattering centers. Thus, processes involving different number densities will in fact experience different regions of the cumulative distribution function. For the sparser medium B, a change in  $M$  results in a faster increase of the corresponding values of  $p(s, M)$  and, consequently, a faster decrease in the possible fluctuations. Because the entropy is a measure of magnitude of these fluctuations, it follows that the medium B should be characterized by a faster rate of entropy increase as can be seen in Fig. 4. As a result, in spite of being described in average by the same diffusion coefficient  $D$ , the two media can be discriminated based on their corresponding densities of scattering regions. This information was not available in the ensemble average.

In conclusion, we have demonstrated a new way of analyzing the fluctuations of scattered waves resulting from the interaction between coherent fields and disordered media. In general, the complexity of such deterministic processes can be reduced only through ensemble averaging at the expense of available information. We have demonstrated that analyzing individual members of the ensemble of interactions provides means to extract information beyond that available in the ensemble average.

The deviation of an individual path-length distribution from the ensemble average is a non-stationary random process which also varies from one realization to another. There are different ways to analyze such a random function. Here we have shown that specific properties of the random medium's morphology can be evidenced by using the scale dependent entropy associated with the variance of path-length fluctuations. This is of particular interest in practice where the volume of interaction can be easily controlled from macroscopic to microscopic scales [13].

Finally, the procedure outlined here is quite general and may find other applications for discriminating between physical processes with identical mean parameters. One example may be the possibility to extract information from nonstationary "noise" and a limited number of samples in the study of neurotransmitter receptors [14].