An Algorithm for Motion Estimation from the Tomographic Data

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ALGORITHM FOR MOTION ESTIMATION FROM THE TOMOGRAPHIC DATA

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Field of Classification Search
See application file for complete search history.

References Cited

U.S. PATENT DOCUMENTS

OTHER PUBLICATIONS


ABSTRACT

The methods and systems of the present invention is an algorithm which estimates motion inside objects that change during the scan. The algorithm is flexible and can be used for solving the misalignment correction problem and, more generally, for finding scan parameters that are not accurately known. The algorithm is based on Local Tomography so it is faster and is not limited to a source trajectory for which accurate and efficient inversion formulas exist.

7 Claims, 8 Drawing Sheets
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Rit, et al., Comparison of Analytic and Algebraic Methods for Motion-Compensated Cone-Beam CT Reconstruction of the Thorax, Medical Imaging, pp. 1513-1525, vol. 28, No. 10, abstract.


* cited by examiner
Fig. 6a

Fig. 6b

Fig. 6c

Fig. 6d

Fig. 6e

Fig. 6f
ALGORITHM FOR MOTION ESTIMATION FROM THE TOMOGRAPHIC DATA

This application claims the benefit of priority to U.S. Provisional Application No. 61/351,614 filed on Jun. 4, 2010, which is incorporated by reference in its entirety.

FIELD OF THE INVENTION

This invention relates to tomography and, in particular, to methods, systems and devices for tomography with motion estimation for reconstructing objects that change during the scan and misalignment correction for reconstructing objects when some scan parameters are not accurately known.

BACKGROUND AND PRIOR ART

Cardiac and, more generally, dynamic imaging is one of the top challenges facing modern computed tomography. When the object being scanned changes during data acquisition the classic tomographic reconstruction theory does not apply. In cardiac computed tomography there are two major groups of approaches for dealing with this issue. One is based on gating, i.e., selecting the computed tomography data which correspond to a fixed cardiac phase, and then using mostly that data for image reconstruction. The second approach, known as motion compensation, is based on incorporating a motion model into a reconstruction algorithm. Motion compensation algorithms are preferable, because they use all data and have the potential to provide good image quality with reduced x-ray dose. The main difficulty of using such algorithms is that the motion model needs to be known. There are motion estimation algorithms available, but significant research still needs to be done to improve efficiency, accuracy, and stability with respect to noise, flexibility, and the like.

The methods and systems of the present invention solve the problems associated with the prior art using a novel approach to motion estimation, which is based on local tomography (LT). The ultimate goal is a robust algorithm which can reconstruct objects that change during the scan. Since there is no formula that recovers the object \( f \) and motion function \( \varphi \) from the tomographic data, the most realistic approach to finding \( f \) and \( \varphi \) is via iterations. On the other hand, recovering both of them at the same time would result in an iterative problem of a prohibitively large size.

The best approach is to decouple the two tasks, motion estimation and motion compensation, as much as possible. Not all methods achieve this goal. For example, when finding \( \psi \) using registration, one uses the images of \( f \) at different times. In other words, finding \( \psi \) depends on the knowledge of \( f \). This has undesirable consequences. When motion is not known, \( f \) is reconstructed with significant artifacts, making subsequent registration unreliable and inaccurate. In contrast, LT is an ideal candidate for decoupling. LT does not reconstruct pointwise values of \( f \), but rather a gradient-like image of \( f \) with edges enhanced. Thus the only informative feature of LT is the location of edges.

SUMMARY OF THE INVENTION

A first objective is an algorithm for motion estimation from tomographic data that can be used for improved image reconstruction from CT data in the case when there is motion in the object (e.g., cardiac motion or breathing motion) during the scan.

A second objective is an algorithm for estimating scan parameters from tomographic data that can be used for correcting for the imperfections in the x-ray source trajectory or other scan parameters such as misalignment and the like.

The methods and systems of the present invention show that when any given edge is seen from the data from two or more source positions, then in the case of incorrectly known motion, the single edge “spreads” and becomes a double edge. As a result, the image looks more cluttered. A solution to the problems associated with the prior art is to iteratively improve the motion model so that image clutter is minimized.

The present invention provides an empirical measure of clutter, referred to as “edge entropy.” Note that the word “entropy” in the name is largely symbolic, since the present invention does not establish any properties that conventional entropy must possess. In the present invention motion estimation is completely independent of the knowledge of \( f \), and the desired decoupling is achieved. No knowledge of \( f \) is required. The only thing needed is that \( f \) possess a sufficient number of edges, which is true for practically all \( f \) occurring in medical imaging. The use of LT has other benefits as well.

1. LT is very fast. First, it does not require global filtering. Second, backprojection is greatly simplified, since there is no need to compute complicated weights that are mandatory for quasi-exact motion compensating inversion formulas. The weights compensate for variable length of illumination for every voxel in an image. Clearly, high reconstruction speed is critically important for iterative-based motion estimation.

2. LT uses only local data; hence it is not sensitive to data truncation.

3. LT is very flexible and can be used with practically any source trajectory.

Let us mention some other attractive features of the present invention. First, it is local in time. Motion estimation is done inside a reasonably short time window, e.g., not much longer than the length of a short scan. This eliminates the need for making the periodicity assumption as described in S. Bonnet, A. Koenig, S. Roux, P. Hugonnard, R. Guillemaud, and P. Grangeat, Dynamic X-ray computed tomography, Proc. IEEE, 91 (2003), pp. 1574-1587, which frequently holds only approximately. Second, the approach is fairly general and can be used for several types of motion, e.g., cardiac, breathing, etc. Finally, with simple modifications the approach can be applied to solving other practically important problems. As an example we show how to solve a misalignment correction problem for a distorted circular scan. A similar iterative algorithm, which is based on the Feldkamp inversion formula, is described in Y. Kyriakou, et al., Simultaneous misalignment correction for approximate circular cone-beam computed tomography, Phys. Med. Biol., 53 (2008), pp. 6267-6289.

Since the algorithm of the present invention is based on LT, it is faster and is not limited to a source trajectory for which accurate and efficient inversion formulas exist. As before, estimation of the unknown source trajectory is completely decoupled from finding \( f \), so for the latter purpose one can use any algorithm. For example, when the data are truncated, one might want to use an iterative reconstruction algorithm. If the two problems are coupled, an iterative algorithm for finding \( f \) inside an iterative algorithm for estimating the source trajectory is prohibitively slow.

Further objects and advantages of this invention will be apparent from the following detailed description of preferred embodiments which are illustrated schematically in the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1a shows \( x_1 x_2 \) cross-sections of the phantom at time corresponding to view 101.
FIG. 1b shows x₁, x₂-cross-sections of the phantom at time corresponding to view 900.
FIG. 1c shows x₁, x₂-cross-sections of the phantom at time corresponding to view 901.
FIG. 2a shows x₁, x₂-cross-sections of the phantom at time corresponding to view 101.
FIG. 2b shows x₁, x₂-cross-sections of the phantom at time corresponding to view 100.
FIG. 3a shows x₁, x₃-cross-sections of the phantom at time corresponding to view 900.
FIG. 3b shows x₁, x₃-cross-sections of the phantom at time corresponding to view 901.
FIG. 3c shows x₂, x₃-cross-sections of the phantom at time corresponding to view 100.
FIG. 4a shows a density plot of BF at the beginning of iterations when zero motion is assumed for x₁, x₂-cross-section through the center of the grid.
FIG. 4b shows images of the bright spots corresponding to density plots shown in FIG. 4a.
FIG. 4c shows a density plot of BF at the beginning of iterations when zero motion is assumed for x₁, x₃-cross-section through the center of the grid.
FIG. 4d shows images of the bright spots corresponding to density plots shown in FIG. 4c.
FIG. 4e shows a density plot of BF at the beginning of iterations when zero motion is assumed for x₂, x₃-cross-section through the center of the grid.
FIG. 4f shows images of the bright spots corresponding to density plots shown in FIG. 4e.
FIG. 4g shows the results of bright pixel detection using the LT function shown in FIG. 4f.
FIG. 5a shows the a density plot of BF at the end of iterations for cross-section x₁, x₂ through the center of the grid.
FIG. 5b shows images of bright spots corresponding to the density plot shown in FIG. 5a.
FIG. 5c shows the a density plot of BF at the end of iterations for cross-section x₁, x₃ through the center of the grid.
FIG. 5d shows images of bright spots corresponding to the density plot shown in FIG. 5c.
FIG. 5e shows the a density plot of BF at the end of iterations for cross-section x₂, x₃ through the center of the grid.
FIG. 5f shows images of bright spots corresponding to the density plot shown in FIG. 5e.
FIG. 6a shows a density plot of BF at the beginning of iterations when a pure circular trajectory is assumed for cross-section x₁, x₂ through the center of the grid.
FIG. 6b shows density plots of BF at the end of iterations corresponding to FIG. 6a.
FIG. 6c shows a density plot of BF at the beginning of iterations when a pure circular trajectory is assumed for cross-section x₁, x₃ through the center of the grid.
FIG. 6d shows density plots of BF at the end of iterations corresponding to FIG. 6c.
FIG. 6e shows a density plot of BF at the beginning of iterations when a pure circular trajectory is assumed for cross-section x₂, x₃ through the center of the grid.
FIG. 6f shows density plots of BF at the end of iterations corresponding to FIG. 6e.
FIG. 7a shows a density plot of BF at the beginning of iterations based on ∂²/∂q² for cross-section x₁, x₂ through the center of the grid.
FIG. 7b shows the results of bright pixel detection using the LT function shown in FIG. 7a.
FIG. 7c shows the results of bright pixel detection using the LT function based on the filter according to the present invention.
FIG. 7d shows a density plot of BF at the beginning of iterations based on ∂²/∂q² for cross-section x₁, x₃ through the center of the grid.
FIG. 7e shows the results of bright pixel detection using the LT function shown in FIG. 7d.
FIG. 7f shows the results of bright pixel detection using the LT function based on the filter according to the present invention.
FIG. 7g shows a density plot of BF at the beginning of iterations based on ∂²/∂q² for cross-section x₂, x₃ through the center of the grid.
FIG. 7h shows the results of bright pixel detection using the LT function shown in FIG. 7g.
FIG. 7i shows the results of bright pixel detection using the LT function based on the filter according to the present invention.
FIG. 8a shows a density plot of BF at the end of iterations based on the filter of present invention for cross-section x₁, x₂ through the center of the grid.
FIG. 8b shows the results of bright pixel detection using the LT function shown in FIG. 8a.
FIG. 8c shows the results of bright pixel detection using the LT function based on the filter according to the present invention.
FIG. 8d shows a density plot of BF at the end of iterations based on the filter of present invention for cross-section x₁, x₃ through the center of the grid.
FIG. 8e shows the results of bright pixel detection using the LT function shown in FIG. 8d.
FIG. 8f shows the results of bright pixel detection using the LT function based on the filter according to the present invention.
FIG. 8g shows a density plot of BF at the end of iterations based on the filter of present invention for cross-section x₂, x₃ through the center of the grid.
FIG. 8h shows the results of bright pixel detection using the LT function shown in FIG. 8g.
FIG. 8i shows the results of bright pixel detection using the LT function based on the filter according to the present invention.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

Before explaining the disclosed embodiments of the present invention in detail it is to be understood that the invention is not limited in its application to the details of the particular arrangements shown since the invention is capable of other embodiments. Also, the terminology used herein is for the purpose of description and not of limitation.

The methods of the present invention provide an algorithm for tomography in the motion contaminated case. The motion contaminated case occurs when the object being scanned is undergoing some transformation during the scan. Thus, the phrases “the object is undergoing a transformation” and “there is some motion in the object” have the same meaning in this invention.

It is shown that micro locally, away from some critical directions, LT is equivalent to a pseudo differential operator of order one. LT also produces nonlocal artifacts that have the same strength as useful singularities. When motion is not accurately known, singularities inside the object f being scanned spread in different directions. For example, a single edge can become a double edge. In the case of a double edge, the image of f looks cluttered. Based on this observation the present invention provides an algorithm for motion estimation using an empiric measure of image clutter, referred to
throughout the description as edge entropy. By minimizing edge entropy, the motion model is found. The algorithm is flexible and can also be used for solving the misalignment correction problem.

The following detailed description discloses cone-beam LT function $Bf$ and establishes its main properties and then explains the location and strength of the nonlocal artifacts. As opposed to LT in the static case, it is not possible to find the direction of differentiation, which reduces the strength of the artifact by one order in the scale of Sobolev spaces. A similar result for a different geometry was recently reported in E. T. Quinto, Electron microscope tomography, Conference talk at workshop on Mathematical Methods in Emerging Modalities of Medical Imaging. Banff International Research Station, Banff, Canada, 2009. Then, explicit formulas for the shift between the singularities in $Bf$ and $f$ in the case when motion is known incorrectly is obtained. A similar result is described in A. Katsevich, Improved cone beam local tomography, Inverse Problems, 22 (2006), pp. 627-643 which gives only an implicit relation, and the model used for describing changes in $f$ is different from the model used in the present invention. The novel motion estimation algorithm as well as a description of the motion model and definition of edge entropy is throughout the description as edge entropy. By minimizing edge entropy, the motion model is found. The algorithm is flexible and can also be used for solving the misalignment correction problem.

The present invention introduces the following LT function:

$$
\Theta(s,x): \{x \in U \} \rightarrow \mathbb{R}^{3}
$$

is a smooth function, and $\phi \in C_{0}^{\infty}(\mathbb{R} \times U)$. Note that Equation (5) reduces to Equation (2.2) of A. Katsevich, Improved cone beam local tomography, Inverse Problems, 22 (2006), pp. 627-643, if $\beta(s,x+q\Theta(s,x))$ is replaced with $\beta(q,x)$. Equation (2.2) of A. Katsevich, Improved cone beam local tomography, Inverse Problems, 22 (2006), pp. 627-643, was developed with the goal of reducing the global artifact inherent in cone-beam data inversion as much as possible. The additional flexibility provided by $\Theta$ is needed for increasing computational efficiency. A slight change in the direction of differentiation away from the optimal one may lead to a significant speed-up at the expense of only a slight increase in the global artifact. The function $\psi$ in Equation (5) determines the time interval, which is used for motion estimation. Define

$$
\Phi(x,y,s) = \Psi(z(s),z(s),z(s))\psi(z(s),z(s)) \psi(z(s),z(s))
$$

for a fixed $x \in U$ and $s \in \mathbb{E}$, $\Phi(x,y,s), t=0$, is the pre-image of the ray with vertex at $z(s)$ and passing through $\psi(z(s))$. For a fixed $x \in U$, $\Phi(x,y,s), t=0$, is a surface parameterized by and $t$. For convenience, this surface is denoted $\Phi_x$. Using Equation (3), we get

$$
\Phi_x(s,x,t) := \psi(x,y,s,t) \psi(y,x,t) \psi(x,y,s,t) \psi(y,x,t) \psi(x,y,s,t) \psi(y,x,t)
$$

which is smooth at the point $x$. However, the surface $\Phi_x$ is smooth at the point $x$, $s$, $t$ if the divergence of the two velocities is not parallel to the line segment. We say that $\Phi_x$ is smooth if it is smooth at any point $z \in \Phi_x$, $z \in X$.

Proposition 1.

Suppose $\Phi_x$ is smooth for all $x \in U$. The operator $B$ defined by Equation (5) extends to a map $\mathcal{B}(U) \rightarrow \mathcal{B}(U)$, and

$$
WF \mathcal{B}(f) \subset WF \mathcal{B}(f) \mathcal{B}(E(Cq)) E(Cq) \subset \{x \in \mathbb{R}^{3} \} \mathbb{R}^{3} U(1)
$$

where $\mathcal{B}(Cq)$ is the co-normal bundle of $\Phi_x$. In short, $g$ has an additional singularity at $x$ if $\Phi_x$ is tangent to $\text{supp} f$ at some point $y \in U$. The singularities of $Bf$, which coincide with those of $f$, are “useful” (from the point of view of practical applications of tomography), while the set $\mathcal{B}(x, C, \psi)$ represents the artifact.
Applying Equation (2) gives

Now follows from continuity. The proof of Equation (11) is where \( B^* \in C^0(U) \). The first assumption of the proposition neighborhood of zero, and \( t_1 \) is bounded away from zero, it is we get that the argument of \( f \) in Equation (14) becomes

\[ \phi(s, z; (s, t = 1)) \]

\[ \psi(s, x) \]

Since \( \Theta(s, x) \) is a smooth function, \( q \) is restricted to a small neighborhood of zero, and \( t_1 \) is bounded away from zero, it is clear that Equation (17) defines a smooth diffeomorphism \( z \mapsto x = X(s, t, q) \). i) Taking the derivative with respect to \( q \) outside the integral in Equation (14), (ii) interchanging the order of integration so that the integral with respect to \( x \) becomes the innermost one, and (iii) changing variables \( x \leftrightarrow z \) according to Equation (17), we get

\[ \phi(s, z; (s, t = 1)) = x + q\Theta(s, x) \]

\[ \psi(s, x) \]

where \( B^* \in C^0(U) \). The first assumption of the proposition now follows from continuity. The proof of Equation (11) is given below.

Next we compute the principal symbol of \( B \). Besides the smoothness of \( \phi_* \), the additional assumptions we make in this calculation are that (1) \( \Phi^0_*(s, x, t = 1) \) is never a zero vector, and (2) \( \Phi^0_*(s, x, t = 0) \) and \( \Phi^0_*(s, x, t = 1) \) are not parallel. Let us discuss these assumptions. Setting \( t = 1 \) in Equation (9) and Equation (10) gives \( \tilde{Y} = \psi(s, x) \) and

\[ \Phi^0_*(s, x, t = 1) = \]
where \( Q_1 \) is a symbol from the class \( S^1 \) in a conic neighborhood of \((x_0, \xi_0)\) (cf. (26)), and the asymptotics of \( Q_1 \) is given by

\[
Q_1(x, \xi) = \frac{Q_0(x, \xi, s(x, \xi), t(x, \xi))}{|\det(\xi \cdot \Phi'')|^\frac{1}{2}} + O(1), |\xi| \to \infty,
\]

where \( \xi \cdot \Phi'' \) is the matrix in Equation (36), and constant \( c \) incorporates some powers of \( 2\pi \) and \( i \). Combine Equation (33) and Equation (38):

\[
(\mathcal{B}f)(x) = \frac{1}{(2\pi)^2} \int f(\xi)Q_0(x, \xi)e^{-\xi \cdot \Phi'\xi}d\xi. \tag{39}
\]

where \( a(x, \xi) = \xi \cdot \Phi(x, s(x, \xi), t(x, \xi)) \). If \( \det(\xi^2 a(x, \xi)\xi^2) > 0 \) at \((x_0, \xi_0)\), then locally Equation (39) is a Fourier Integral Operator (FIO) associated with a canonical transformation, and the order of the operator equals one (see L. Hormander, The Analysis of Linear Partial Differential Operators. IV. Fourier Integral Operators, Springer-Verlag, Berlin, 1985, pp. 25, 26). In view of Equation (30) this means that the artifacts and useful singularities can be of the same strength in the scale of Sobolev spaces.

Incorrectly Known Motion.

Suppose that instead of the motion function \( \psi \) we know its approximation \( \psi_{\epsilon}(s, x) = \psi(s, x) + \epsilon \psi_{\epsilon}(s, x) \). In this case the function \( \Phi \) of Equation (7) is replaced by

\[
\Phi(x_0, s(x_0), t(x_0), \xi_0; \epsilon) = (\Phi_{\epsilon}(x, s, t) + (\Phi_{\epsilon}(x, s, t) - \Phi_{\epsilon_{0}}(x, s, t))). \tag{40}
\]

Consequently, the useful singularities of \( \mathcal{B}f \) no longer coincide with the singularities of \( f \). To find the shift between them we assume that the error in \( \psi_{\epsilon} \) is small, i.e., \( \epsilon \to 0 \), and find the first order approximation of the shift. If \( \epsilon = 0 \), the function \( \psi_{\epsilon}(s, x) \) is determined from Equation (35) with \( t(x, \xi) = 1 \). If \( \epsilon \neq 0 \), we have to solve

\[
\xi \cdot \Phi_{\epsilon}(x_0, s(x_0), t(x_0), \xi_0; \epsilon) = 0, \quad \Phi_{\epsilon}(x_0, s(x_0), t(x_0), \xi_0; \epsilon) = 0. \tag{41}
\]

Equations (41) mean that the singularity of \( f \) at \( x \) is mapped into the singularity of \( \mathcal{B}f \) at \( x_\epsilon \). Set \( x_\epsilon = x + \Delta x, s_\epsilon = s + \Delta s, t_\epsilon = 1 + \Delta t \). Expanding Equation (41) in the Taylor series around \( \epsilon = 0 \), using Equation (35), and keeping the first order terms in \( \epsilon \), gives

\[
\epsilon \cdot \Phi_{\epsilon}(x_\epsilon, s_\epsilon, t_\epsilon) = 0, \quad \epsilon \cdot \Phi_{\epsilon}(x_\epsilon, s_\epsilon, t_\epsilon) = 0. \tag{42}
\]

All the derivatives in Equation (42) are computed at \( \epsilon = 0 \), \( x = s(s(x_0), t(x_0), \xi_0) \), and \( t = 1 \). Since \( \Phi_0(x, x_0, t) = 0 \), then \( \Phi_{\epsilon_0} = 0 \). The first and third equations in Equation (42) yield:

\[
\Delta x = \epsilon \cdot \Phi_{\epsilon_0} \cdot \Delta s + \epsilon \cdot \Phi_{\epsilon_0} \cdot \Delta t = 0, \tag{43}
\]

when \( t = 1, \mathcal{B}f = \mathcal{I}d \), and

\[
\Phi_{\epsilon} = \mathcal{V}V_{\psi}(s(x_0, \xi_0)) \Delta \Phi_{\epsilon} = \mathcal{V}V_{\psi}(s(x, \xi)). \tag{44}
\]
Substituting into Equation (43) gives
\[ \Delta x = \nabla v(s, \theta(s, x))(\theta(x, s) - x) - \phi_0(x, s) + O(\epsilon^2), \]
Eq. (45)
\[ c = \frac{\Phi_{\theta}^0}{\Phi_{\theta}^0}. \]

By our assumption \( \xi \Phi_{\theta}^0 \neq 0 \), so \( \Delta x \) is indeed of order \( O(\epsilon) \), \( \epsilon \to 0 \).

End of Proof of Proposition 1.

Using a partition of unity we may suppose that \( WF(f) \) is a subset of a sufficiently small conic neighborhood of \( (s_0, t_0) \) \( \subset \mathbb{R}^2 \), and \( \Phi_{\theta}^0(\xi) \) (cf. Equations (26) and (33)) for \( (s, t) \) outside a sufficiently small neighborhood of \( (s_0, t_0) \). Initially we consider the case \( \epsilon = 1 \). First of all, from Equations (4) and (5), \( BF(x) \) is smooth near \( y_0 = \Phi(x_0, s_0, t_0) \). In view of the partition of unity, Equation (4) needs to be modified by including a cut-off function depending on \( \epsilon \). Passing to a sufficiently small neighborhood of \( y_0 = \Phi(x_0, s_0, t_0) \), we get that \( BF(x) \) is smooth near \( y_0 \) unless \( \xi \in 0 \), parallel to \( \Phi_x(x_0, s_0, t_0) \). If the two vectors are parallel, we multiply Equation (33) by \( \phi_0(x) e^{i\theta_0} \), where \( \Phi(x_0, s_0, t_0) \) and is supported in a neighborhood of \( y_0 \), and \( \eta_0 \in \mathcal{C}_0 \). Integrating with respect to \( x \) and using the standard argument (see e.g. Y. V. Fgorov and B.-W. Schulze, Pseudo-Differential Operators, Singularities, Applications, Birkhäuser, Basel, 1997, p. 114), we get that \( \Phi_{\theta}^0(\xi) \Phi_{\theta}^0(\xi)^* \) is smooth, defined on the interval \( [0, 1] \).

A Motion Estimation Algorithm.

The motion estimation algorithm is based on LT. In the case of static objects, the discontinuities (or, edges) of \( f \) and \( BF \) generally coincide (see e.g., D. Finch, L. R. Laug, and G. Hultman, Microlocal analysis of the x-ray transform and with sources on a curve, in Inside Out: Inverse Problems and Applications, G. Uhlmann, ed., Cambridge University Press, Cambridge, UK, 2003, pp. 193-218; A. Katsevich, Improved cone beam local tomography, Inverse Problems, 22 (2006), pp. 627-643; A. K. Louis and P. Maass, Contour reconstruction in 3-D X-ray CT, IEEE Trans. Med. Imaging, 12 (1993), pp. 764-769) except for the added singularities or artifacts in \( BF \) and the singularities of \( f \) that are invisible from the data. As mentioned above, if there is some uncompensated motion in \( f \), the edges of \( f \) and \( BF \) no longer coincide. Practically this means that if motion is not known (or, is known incorrectly), edges in the reconstructed image spread out. A single edge produces multiple edges at several locations. Consequently, the reconstructed image looks cluttered or random. We can use a measure of randomness in the reconstructed image \( BF \) to gauge whether our motion model is accurate or not. In what follows we call this measure "edge entropy". Using this idea, we summarize the proposed motion estimation algorithm as follows:

1. Assume some motion model;
2. Perform motion-compensated LT image reconstruction using current motion model;
3. Compute edge entropy of the LT image;
4. If edge entropy is low (i.e., the edges have not spread too much), stop. If edge entropy is high, change the motion model and go to step (2).

A similar idea was used in Y. Kyriakou, R. M. Lapp, L. Hilebrand, D. Ertel, and W. Kalender, Simultaneous misalignment correction for approximate circular cone-beam computed tomography, Phys. Med. Biol., 53 (2008), pp. 6267-6289, for misalignment correction in circular cone-beam CT. The main novelty of our approach is that we use LT instead of global Feldkamp type (FDK) reconstruction. On one hand, the use of LT allows us to significantly speed up the iterations. On the other hand, many tools that work with conventional images (most notably, image entropy) do not work with LT images, so we had to develop alternative tools from scratch. The following paragraphs describe the key steps of the algorithm in more detail.

Motion Model.

Let \( [s_0, t_0] \) be a parameter/time window, which is used for motion estimation. The center point \( s_0 = (s_1, s_2, s_3) \) is taken as reference time. The primary purpose of the algorithm of the present invention is to perform local (in time) motion estimation, thus the width of the window \( S = s_s - s_e \) is usually rather short. In our experiments \( S \) is typically less than one gantry rotation. Let \( D = U \) denote the region where motion takes place. We assume that \( D \) is a rectangle, i.e.,
\[ D = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 1.0 \leq x_1 \leq 2.0, 1.0 \leq x_2 \leq 2.0, 1.0 \leq x_3 \leq 2.0 \}. \]

To represent motion, we consider a regular grid over \( D \). The grid planes are
\[ \Delta x_r = \Delta x_l = \Delta x_3 = 0.5 \Delta x_{1,0}, k = 1, 2, 3, \]
Eq. (46)
where \( \Delta x_r = (R_r - L_r)(N_r + 1) \) is the step-size along the \( r \)-th axis. Thus, grid in Equation (46) has \( N_1 \times N_2 \times N_3 \) interior nodes, and for each direction \( r \) there are \( N_r + 1 \) points \( x_r = L_r, L_r + \Delta x_r, \ldots, L_r + \Delta x_r(N_r + 1) \). Because of motion, the grid planes deform over time:
\[ x_r = x_r(\Delta x_l, \Delta x_r, \Delta x_3), k = 1, 2, 3, \]
Eq. (47)
Each line in Equation (47) defines a separate surface, which corresponds to a deformation of one of the original planes Equation (46). We assume that motion equals zero at the boundary of \( D \), so the boundary grid planes (i.e., those given by \( x_r = \text{constant}, r = 1, 2, 3 \)) do not deform. In Equation (47), the function \( f \) is smooth, defined on the interval \([0, 1]\), and equals zero at both endpoints of the interval. Since the time window \([s_0, t_0]\) is sufficiently short, we assume that the functions \( a_{rs}(s) \) are linear:
\[ a_{rs}(s) = a_{rs}(0) + s a_{rs}(1), k = 1, 2, 3, \]
Eq. (48)
Note that substituting \( s = s_0 \) into Equation (47) gives the rectangular grid of Equation (46). Equations (47) and (48) allow us to describe motion of every point in \( D \). To determine where to deform them according to Equation (47), and then find the point of intersection of the three resulting surfaces. Location of all other pixels is computed using trilinear interpolation.

Edge Entropy.

Suppose \( BF \) is computed on a regular grid \( (x_1, x_2, x_3) \), \( 1.0 \leq x_1 \leq 2.0, 1.0 \leq x_2 \leq 2.0, 1.0 \leq x_3 \leq 2.0 \), which covers \( D \). Suppose, for simplicity, that the step-size of the grid is the same along every axis and equals \( \Delta x \). Nodes of the grid are denoted \( x_r = (x_1, x_2, x_3) \), where \( 1 \leq i < N_r \leq 1 \). Of course, this grid should be much more dense than the one in Equation (46) \( (M_r > N_r) \). We also need a shifted grid with nodes \( \tilde{x}_r = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \), where \( \tilde{x}_r = x_r + \Delta x / 2 \), \( 1 \leq i < M_r - 1 \), \( k = 1, 2, 3 \). Introduce the distance function:
\[ \delta_{ij} = \max |i_1 - i_2, i_2 - i_3, i_3 - i_4|. \]
Eq. (49)

Calculation of edge entropy consists of several steps. Let parameter \( k, 0 < k < 1 \), be fixed.
1. Using finite differences, compute the norm of the gradient at the nodes of the shifted grid \( \nabla(Bf(\tilde{x}_i)) \).
2. Compute the empirical histogram of the norm of the gradient.
3. Using the histogram, estimate the value \( M \) such that \( \nabla(Bf(x)) \leq M \) for 100% of the points (such points are called "bright");
4. By running a sliding window over the image compute the total number of points whose distance (in the sense of Equation (49)) to the closest bright point equals either 2, 3, or 4;
5. Divide this number by the total number of nodes in the grid and multiply by 100 (to get percents). The result is the edge entropy of the image \( Bf \).

Numerical Experiments.

The original phantom is a superposition of seven balls as shown in FIGS. 1-3. FIGS. 1a, 1b, and 1c show \( x_1x_2 \)-cross-sections of the phantom at a time corresponding to the second derivative of the cone beam data along detector trajectory versus.

The time corresponding to \( s_0 \) is shown at the nodes of the shifted grid.

The changes in density were not tracked due to motion using the following approach.

Step 1. Let \( a_k, k=1, \ldots, N_k \), \( k=1, 2, 3 \), be the current best estimate of the motion parameters. Let some \( \Delta a=0 \) be given.

Pick one of the \( D_a \)'s. Let \( a_{k1}, \ldots, a_{kN_k} \) be the six parameters affecting the chosen region. Compute \( 3^N \) subimages \( B(x) \), \( x \in \Omega \), corresponding to the sets \( a_{k1}, \ldots, a_{kN_k} \), where each \( a_k \equiv \pm a_k \) equals either \( a_k \) or \( a_k=\Delta a+\pm a_k \). Store all the subimages on the disk, and repeat for all \( D_a \).

Step 2. Run the loop over all \( 3^{N_1} \times 3^{N_2} \times 3^{N_3} \) sets \( a_{k1}, \ldots, a_{kN_k} \), \( k=1, 2, 3 \), where each \( a_k \) equals either \( a_k \) or \( a_k=\Delta a \), or \( a_k+\pm a_k \). This is done by reading the appropriate subimages from the disk and combining them into a single image of \( B(x) \), \( x \in \Omega \). Then compute edge entropy for the obtained image. From the \( 3^{N_1} \times 3^{N_2} \times 3^{N_3} \) sets of parameters find the one which produces the image with the smallest entropy.

Steps 1 and 2 constitute a single iteration. The initial values of \( a_k \) are chosen to be zero (which is the no motion assumption). The value of \( \Delta a \) is chosen from some a priori considerations. After the end of each iteration, the optimal set of parameters identified at Step 2 is passed on to Step 1. Also, the value of \( \Delta a \) is decreased. In our experiments we used \( \Delta a_{\text{final}}=10 \), \( \Delta a_{\text{init}}=0.75 \Delta a_{\text{final}} \) and three iterations were performed.

Results of experiments are shown in FIGS. 4 and 5. FIG. 4 shows the initial image of \( Bf \) computed under the (incorrect) assumption of no motion. FIG. 5 shows the final image of \( Bf \) computed for the motion model, that was determined by the algorithm. In these experiments we used \( N_1=N_2=N_3=4 \) (cf. (47)). We used \( \kappa=0.0125 \) to compute edge entropy. At the beginning of iterations (FIG. 4) the value of entropy is 9.81%, and at the end -8.40% (FIG. 5). To illustrate another application of the present invention we use it for solving a bias correction problem in the case of a distorted circular scan. Suppose that under the ideal circumstances the source trajectory is a circle. Suppose that because of mechanical instabilities the actual trajectory is a distorted circle given by

Here \( c_{x_{0i}}, i=1, 2, \ldots, N_i \), are unknown and are to be determined from the tomographic data. Suppose, for simplicity, that the detector always contains the \( x_3 \)-axis, and the center of the sphere is at the origin. Consequently, the procedure outlined earlier applies here as well (with the optimization of the "motion model" replaced by the optimization of the "trajectory model"). In our numerical experiment we used the same seven-ball phantom as before (only it is not moving now), and took \( N=5 \) with \( c_{x_0}=c_{y_0}=c_{z_0}=5 \). Other parameters: sources to isocenter distance \( R \) and size of reconstruction grid are the same as in the first experiment. As initial approximation all \( c_i \)'s were taken to be zero. Optimization was done using the Nelder-Mead simplex algorithm, see J. Nelder and R. Mead, A simplex method for function minimization, Comput. J., 7 (1965), pp. 308-313. At the end of iterations the computed constants are \( 3.58, -5.87, 4.81, -5.36, 4.52 \). Initial entropy was equal 20.1%, and at the end of iterations it was 8.44%. See FIG. 6 for the density plots of \( Bf \).

In the case of noise-free (or low noise) data, the algorithm based on LT as outlined above works well. If data are sufficiently noisy, the proposed scheme is unstable. Indeed, to compute the LT function we differentiate the data two times.
Then, to find bright pixels, we differentiate the image one more time. To make the algorithm more robust observe that instead of $\partial^2 /\partial x^2$ in Equation (5) we can use almost any even convolution kernel which preserves singular support. It was found empirically that convolving the tomographic data along data rows with the kernel, whose frequency characteristic is given by $|\hat{h}(\omega)|^2$ produces good results. In Fig. 7, left two columns, we see the LT images and the results of bright pixel detection using the filter $\partial^2 /\partial x^2$ in Equation (5) in the case of (erroneous assumption) of zero motion in the data. The right column of Fig. 7 shows the results of bright pixel detection using the new filter. As is seen, the latter is much less noisy than the former. In Fig. 8 we see the reconstructions at the end of iterative motion estimation. Left column shows the intermediate “local” tomography image, middle column shows the results of bright pixel detection. Note that these images are computed from the noisy data in the process of iterations and correspond to the motion model with the least edge entropy. Even though the filter is no longer local, we keep using the name “local tomography” to emphasize the fact that we still do not reconstruct images values (as opposed to conventional “global tomography”). To better illustrate our results we computed the traditional LT function from the noise-free data using the computed motion model, see the right panel. It is clearly seen from these images that the edges are now much less spread than they were at the beginning of iterations.

Discussion.

In this patent application, LT in the motion contaminated case was studied. It is shown that microlocally, away from some critical directions, LT is equivalent to a pseudo-differential operator of order one. LT also produces non-local artifacts that are of the same strength as useful singularities. As opposed to the static case, here it is not possible to choose the direction of differentiation to reduce the strength of the artifact by one order in the scale of Sobolev spaces. On the other hand, if motion is sufficiently small, it is expected that choosing $\Theta$ as in Equation (31) which is analogous to what was done in A. Katsevich, Improved cone beam local tomography, Inverse Problems, 22 (2006), pp. 627-643 (compare Equation (32) with Equation (2.11) in Katsevich (2006)), may help reduce the artifacts. Then we consider the case when motion is not accurately known. It is shown that when a singularity is seen from two different source positions, it spreads in different directions. A simple edge becomes a double edge. Based on this observation we propose an algorithm for motion estimation. The algorithm is quite flexible and is used for solving the misalignment correction problem.

While the invention has been described, disclosed, illustrated and shown in various terms of certain embodiments or modifications which it has presumed in practice, the scope of the invention is not intended to be, nor should it be deemed to be, limited thereby and such other modifications or embodiments as may be suggested by the teachings herein are particularly reserved especially as they fall within the breadth and scope of the claims here appended.

We claim:

1. A motion estimation method for tomography comprising the steps of:
(a) collecting tomographic data of an object which undergoes a transformation during a scan;
(b) reconstructing an image of the original object, wherein information about the object useful for the motion estimation and contained in the image pertains only to one or more edges of the object, whereas the edges are defined as sharp spatial features inside the object;
(c) using the reconstructed image from step (b) for estimating a transformation of the object during the scan; and
(d) repeating steps (b) and (c) two or more times for improving the estimate of the transformation, wherein steps (b) and (c) are repeated until a number of edges with a predetermined characteristic are below a preselected threshold.

2. The method of claim 1, wherein step (b) of reconstructing an image of the object comprises the step of:
using x-rays only passing through a small neighborhood corresponding to each pixel for applying filtering and backprojection to data from the x-rays passing through the neighborhood, wherein a small neighborhood is defined as a region which is smaller than the smallest cross-section of the object.

3. The method of claim 1, wherein using step (c) includes the step of:
analyzing a spatial distribution of the one or more edges in the reconstructed image as a substep.

4. A motion estimation method for tomography comprising the steps of:
(a) collecting tomographic data of an object which undergoes a transformation during a scan;
(b) reconstructing an image of the original object, wherein information about the object useful for the motion estimation and contained in the image pertains only to one or more edges of the object, whereas the edges are defined as sharp spatial features inside the object;
(c) using the reconstructed image from step (b) for estimating a transformation of the object during the scan, which includes the step of:
analyzing a spatial distribution of the one or more edges in the reconstructed image as a substep, wherein the step of analyzing the spatial distribution of the edges in the reconstructed image includes the step of:
using a prevalence of double edges in the reconstructed image for the motion estimation, wherein double edges are defined as edges located close to each other; and
(d) repeating steps (b) and (c) two or more times for improving the estimate of the transformation, wherein steps (b) and (c) are repeated until a number of edges with a predetermined characteristic are below a preselected threshold.

5. The method of claim 4, wherein step (b) includes the step of:
assuming a selected model of the transformation that the object is undergoing and using the selected model for the reconstruction.

6. The method of claim 5, further comprising the step of:
making the conclusion that the assumed model is not an accurate model of the said transformation when a large number of double edges are detected; and
making the conclusion that the assumed model is an accurate model of the said transformation when a small number of double edges are detected.

7. The method of claim 5, wherein step (b) of reconstructing an image of the object includes the step of: using only x-rays passing through a small neighborhood corresponding to each pixel, wherein a small neighborhood is defined as a region which is smaller than the smallest cross-section of the object.