Design of the Ultralight Two Place Gyroplane

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Martin. Hollmann

University of Central Florida
DESIGN OF THE ULTRALIGHT
TWO PLACE GYROPLANE

BY

MARTIN HOLLMANN
B.S., California State University at San Jose

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CHAPTER I

INTRODUCTION

It is the intent of this thesis to demonstrate the utilization of existing aerodynamic and structural analysis for establishing general and specific design guidelines to be used for the rapid selection of ultra-light two-place gyroplane parameters. A Computer Aided Gyroplane Synthesis, CAGRØS, program is set up and utilized in a parametric evaluation to determine the performance characteristics of a 1000 pound and a 1500 pound gross weight gyroplane. The 1000 pound gyroplane is considered representative of a stripped (no starter, generator, rotor prespin, radio, etc.) ultra-light two-place gyroplane. Whereas, the 1500 pound gyroplane is considered to be a fully equipped aircraft.

CAGRØS is also utilized to evaluate the performance characteristics of a specific 1050 pound gross weight gyroplane for which a detailed rotor blade system has been designed. Named the HA-27 rotor system, this rotor is analyzed in detail using a modified matrix method for obtaining spanwise moments and deflections similar to the method presented by Mayo. The method of Mayo is modified to include precone angle. This method is accepted as giving good results; see Mayo. Although the method requires a computer with appropriate subroutines capable of manipulating complex variable matrices and considerable time is needed for programming and making computer runs, considerable time is
saved in comparison to tabular methods which have been used extensively in rotor blade analysis.

A simplified matrix method similar to the method described by Mayo, except that a reduced number of blade stations are used, is presented in Appendix 3. This method can be used for calculating a steady airload structural root bending moment and the calculations can be carried out by hand and used for preliminary rotor blade sizing. The results thus obtained are compared with those using ten blade stations as presented in Appendix 4.
CHAPTER II

SUMMARY

It is shown that a combination of low disc loading (large rotor diameter) and low power loading (powerful engine) are required in the design of gyroplanes if good performance is to be expected. Low power loading is required for high speed, while a low disc loading is necessary for low forward speed. It is also shown that blade pitch setting and selection of a NACA 8-H-12 airfoil with a smooth blade contour will significantly affect performance. (A smooth blade contour is one having accurate leading-edge contours and smooth, rigid surfaces. A rough contour is one having deformable surfaces such as those on poorly built fabric covered blades with ribs insufficient to prevent blade surface distortion.) Changes in solidity ratio and blade twist were found to be less significant parameters.

A light-weight rotor which was designed by the author for a 1050 pound gross weight gyroplane and which is analyzed in detail in this thesis will meet the requirements set forth by the Federal Aviation Regulations, Part 27. It is shown that the blade stresses in this rotor are governed primarily by the centrifugal force and that bending moment stresses are small. However, if no built-in precone angle is used, the magnitude of the root bending moments may increase considerably.
A Goodman diagram established for the HA-27 rotor extrusion and a blade resonance check shows that no fatigue or resonance problems are to be expected within the operating range of the rotor. Should the blade be operated within resonance conditions large blade deflections and large alternating stresses can be expected which may be catastrophic to the functioning of the aircraft.

A simplified matrix method, as outlined in Appendix 3, can be used to approximate the root bending moment of a rigid or teetering rotor. However, for a detailed determination of the bending moments along the span of a blade or for the bending moment calculations of an articulated rotor blade, the method of Appendix 4 must be utilized.

Several claims are made by the author for which no verifying published data exists. It should be pointed out that these claims are based on the author’s six years of experience in designing, analyzing, building, and flying gyroplanes.
CHAPTER III
PARAMETRIC STUDY

To facilitate the parametric study a computer program titled CAGRØS, Computer Aided Gyroplane Synthesis, is set up. The input to CAGRØS constitutes a specific gyroplane design for which the design parameters must be known. No attempt to optimize the design is made. Instead, the performance characteristics are calculated for various design combinations and a discussion of how the variations of the input data affect performance is made. As shown in Fig. 1, the input data for a specific gyroplane design consists of:

- Gross weight; from 1000 to 1500 pounds
- Power loading; gross weight/maximum installed horsepower, pounds per horsepower
- Disc loading; gross weight/rotor disc area, pounds per square feet
- Solidity; rotor blade area/rotor disc area
- Blade pitch; angle between zero lift of blade section at hub and plane perpendicular to axis of no feathering, radians
- Blade twist; difference between hub and tip pitch angles; positive when tip angle is larger, radians
- Lift slope; slope of curve of section lift coefficient against section angle of attack, per radian
Drag coefficient; coefficients in power series expressing the section profile drag coefficient, $c_{d_0}$, as a function of blade element angle of attack, $\alpha$, ($c_{d_0} = \delta_o + \delta_1 \alpha + \delta_2 \alpha^2$, where $\delta_o$, $\delta_1$, and $\delta_2$ are the drag coefficients). The lift slope and drag coefficients are calculated for three airfoils per Bailey\(^4\) and tabulated below.

<table>
<thead>
<tr>
<th>Airfoil</th>
<th>Lift Slope</th>
<th>$\delta_o$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NACA 23012 (smooth)</td>
<td>5.73</td>
<td>0.0087</td>
<td>-0.0216</td>
<td>0.400</td>
</tr>
<tr>
<td>NACA 8-H-12(smooth)</td>
<td>6.418</td>
<td>0.0087</td>
<td>-0.0353</td>
<td>0.700</td>
</tr>
<tr>
<td>NACA 8-H-12(rough)</td>
<td>6.418</td>
<td>0.014</td>
<td>-0.0353</td>
<td>0.700</td>
</tr>
</tbody>
</table>

Data cards for rotor performance; a simplified method in which the theoretical expressions for thrust coefficient, flapping coefficient, torque coefficient, and the profile drag-life ratio were reduced to simple functions of the inflow factor, blade pitch angles, and the tip-speed ratio as determined by Gessow and Myers\(^5\) is used for determining the rotor performance. The coefficients of the inflow ratio and blade pitch terms are expressed as functions of the tip-speed ratio, the blade mass constant, and the tip-loss factor and tabulated for a mass constant equal to 15 and a tip-loss factor equal to 15 and a tip-loss factor equal to 0.97 for a series
of specific values of tip-speed ratio. These tabulated coefficients are read in by CAGRØS and the rotor performance calculated. Departure from a mass constant of 15 and a tip-loss factor of 0.97 has negligible effect on the values of the coefficients and the tabulated values may be used for calculating the aerodynamic characteristics of any conventional rotor according to Gessow and Myers.

In addition to CAGRØS output as shown in Fig. 1, the minimum airspeed, the maximum speed, the speed for best range, the speed for best endurance, and the speed for best rate of climb can be determined from the power available and power required versus airspeed curves as shown later.

**CAGRØS Calculations and Assumptions**

From Fig. 1 it can be seen that the calculations are separated into five basic independent groups: the rotor performance, engine thrust, vertical power-off rate of sink, airframe drag, and a weight-break-down. These calculations are now discussed.

**Rotor Performance**

A detailed discussion of rotor performance calculations is made by Gessow and Myers\textsuperscript{5} and Bailey\textsuperscript{4} and will not be given here. However, the basic assumptions are summarized below.

Although all calculations were made for a blade mass constant of 15 and a tip-loss factor of 0.97, negligible errors will result for
INPUT

- Gross weight
- Power loading
- Disc loading
- Solidity
- Blade pitch
- Blade twist
- Lift slope
- Drag coefficients
- Read in data card for rotor performance

CALCULATIONS

- Rotor performance
- Engine thrust
- Power-off rate of sink
- Airframe drag
- Weight-break-down

OUTPUT

- As a function of forward speed;
  - Power available
  - Power required
  - Rate of climb
  - Power-off rate of sink
  - Rotor speed
  - Rotor drag
  - Airframe drag
  - Total drag
- Rotor dimensions
- Airframe weight-break-down
- For power-off vertical autorotation;
  - Rotor speed
  - Rate of sink

Fig. 1. CAGRØS Flow Chart
other rotor systems. All blades are assumed to have a constant chord with linear or no twist. The series used to approximate the blade drag coefficient of the blade elements will seriously underestimate the drag coefficient at angles of attack near or beyond stall or at blade tip speeds approaching the speed of sound. However, for moderate values of thrust and tip-speed ratio, high values of the angle of attack at which stall occurs are either confined to parts of the rotor disc in which the square of the velocity of the air relative to the blade element is quite low or to very small areas. According to Gessow and Myers under such conditions the total contribution of these blade elements to the rotor thrust and torque is very small and the error in their estimation is negligible. It will be demonstrated later that, for a moderate tip-speed ratio of a typical gyroplane, the advancing blade tip-speed is below shock compressibility speed. An investigation of tip-speed ratio limits for blade compressibility effects was not made.

It should be noted that no change in tip-loss factor for various number of blades has been accounted for. This may be justified when noting in Fig. 4-4 in Gessow and Myers\(^5\) that for low thrust coefficients, typically equal to 0.002 for gyroplanes, the tip loss factor differs only slightly with the number of blades. The number of blades will not effect the performance calculations in that the solidity ratio is specified as an independent variable and used in the performance calculations.
Engine Thrust

CAGRØS utilizes four commercially available power plants ranging from 100 to 210 horsepower as shown in Table 2. Engine selection is made by determining the desired power from the power loading and gross weight and selecting an engine with the next highest power rating, if the corresponding propeller speed, propeller diameter, number of propeller blades, propeller weight, fuel weight, and accessories weight are acceptable. Engine thrust as a function of forward speed is calculated by the Hamilton Standard Method. This method uses basic airscrew theory in conjunction with empirical curves to correct for propeller parameters. The manner in which the Hamilton Standard Method is used and the assumptions made in CAGRØS are now summarized in the following equations.

The power coefficient is given as:

\[
 C_p = \frac{10^{11} \text{HP}}{2 N^3 D^5 \frac{p}{\rho_o}}
\]

where,

- \( HP \) = maximum installed horsepower
- \( N \) = maximum propeller speed, revolutions per minute
- \( D \) = propeller diameter, feet
- \( \frac{p}{\rho_o} \) = air density ratio, equal to one for sea level
## TABLE 2
CAGRØS ENGINES

<table>
<thead>
<tr>
<th>Engine</th>
<th>Continental 0-200-A</th>
<th>Franklin Sports 4B</th>
<th>Continental 0-200-A</th>
<th>Continental 0-200-A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum installed horsepower</td>
<td>100</td>
<td>130</td>
<td>140</td>
<td>210</td>
</tr>
<tr>
<td>Engine speed, rpm</td>
<td>2750</td>
<td>2800</td>
<td>2700</td>
<td>2800</td>
</tr>
<tr>
<td>Engine weight, lbs.</td>
<td>220</td>
<td>260</td>
<td>300</td>
<td>350</td>
</tr>
<tr>
<td>Propeller weight, lbs.</td>
<td>18</td>
<td>18</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>Propeller diameter, ft.</td>
<td>5.5</td>
<td>5.5</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Number of propeller blades</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Fuel weight lbs.</td>
<td>72</td>
<td>72</td>
<td>120</td>
<td>180</td>
</tr>
<tr>
<td>Accessories weight, lbs.</td>
<td>25</td>
<td>30</td>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>
The propeller activity factor is:

\[ A,F. = \frac{100,000}{16} \int_{0.2}^{1.0} \frac{b}{D} x^3 dx \]  

(2)

where,

- \( b \) = chord width of propeller, ft.
- \( x = r/R \)
- \( r \) = radius to propeller element, ft.
- \( R = D/2 \)

It is assumed that all propellers have an activity factor equivalent to the activity factor of a constant five inch chord propeller with a diameter of 5.5 ft. such that from Eq. (2), \( A,F. = 118 \). From Fig. 23 of the Hamilton Standard Method\(^6\) for \( A,F. = 118 \), \( P_{AF} = 0.70 \). And from Fig. 24 of the same reference for a blade thickness ratio of 0.1, typical for wood propellers, \( P_h = 0.95 \). Now, for a two-bladed propeller,

\[ C_{p1} = C_{p} \cdot P_{AF} \cdot P_h = C_{p} \cdot 0.665 \]  

(3)

And for a three-bladed propeller,

\[ C_{p1} = C_{p} \cdot P_{AF} \cdot P_h \cdot 0.7 = C_{p} \cdot 0.466 \]  

(4)

The static thrust coefficient for a tip-speed of 900 feet per second, \( (\frac{C_T}{C_p})_{900} \), as a function of \( C_{p1} \) is given for a Clark-Y airfoil for different numbers of blades in Fig. 27 of the Hamilton Standard Method\(^6\). This relationship between \( (\frac{C_T}{C_p})_{900} \) and \( C_{p1} \) is simulated by determining an approximate-fit-quadratic equation from three datum points from Fig. 27 of the Hamilton Standard Method\(^6\). The three points were selected to fit between the two and three blade curves as shown
in Fig. 2. From Fig. 2,

\[
\left( \frac{C_T}{C_p} \right)_{900} = 265.42 \left( C_p \right)^2 - 56.99 C_p + 3.875
\]  

(5)

Fig. 2. Variation of Static Thrust Coefficient with Effective Power Coefficient. (Fig. 27 of the Hamilton Standard Method6)

For a tip-speed of 800 to 925 feet per second, \( S_T = 1.00 \) and for an assumed body to propeller diameter ratio of 0.65, \( S_B = 0.9 \). See Figures 29 and 31 respectively of the Hamilton Standard Method6.
The static thrust coefficient for the previously assumed tip-speed range and body configuration is,

\[ \frac{C_T}{C_p} = 0.9 \left( \frac{C_T}{C_p} \right)^9 \quad (6) \]

The static thrust, \( T_s \), is calculated from,

\[ T_s = \frac{C_T}{C_p} \cdot \frac{HP \cdot 33,000}{ND} \quad (7) \]

The thrust for forward flight is determined by calculating \( \sqrt{\frac{J}{C_p}} \).

Where \( J \) is equal to,

\[ J = \frac{88V}{ND} \quad (8) \]

where \( V \) is the forward speed of the aircraft in miles per hour.

From Equations (1) and (8),

\[ \sqrt{\frac{J}{C_p}} = 124.457 V \left( \frac{ND^3}{HP \cdot 10^{11}} \right)^{1/2} \quad (9) \]

The forward flight to static thrust ratio is given as a function of \( \sqrt{\frac{J}{C_p}} \) in Fig. 33 of the Hamilton Standard Method\(^6\). Again, an approximate-fit-quadratic curve is calculated for selected blade pitch angles (less than 14 degrees) from three coordinate points as shown in Fig. 3. The thrust ratio is calculated as,

\[ \frac{T}{T_s} = 0.00625 \left( \frac{J}{C_p} \right)^2 - 0.1525 \left( \frac{J}{C_p} \right) + 1.0 \quad (10) \]

The flight thrust is computed as,

\[ T = \frac{T}{T_s} \cdot T_s \quad (11) \]
Fig. 3. Variation of Propeller Thrust with Airplane Speed (Fig. 33 of the Hamilton Standard Method\textsuperscript{6})

Power-Off Rate of Sink

The equations used for calculating the vertical power-off rate of sink are taken from Nikolsky\textsuperscript{7} and are summarized below. A constant inflow distribution and sea level air density are assumed and an effective blade pitch angle, $\theta_e$, at 75 percent tip radius is calculated from the blade root pitch setting, $\theta_0$, and blade twist, $\theta_1$, as,

$$\theta_e = \theta_0 + 0.75 \theta_1$$  \hspace{1cm} (12)

$$c_1 = \frac{267.76 \text{ GWT}}{\sigma_0 R^4}$$  \hspace{1cm} (13)

$$c_5 = -0.25 (\delta_0 + \delta_1 \theta_e + \delta_2 \theta_e^2)$$  \hspace{1cm} (14)
\[ C_6 = \frac{a \theta_e}{3} - \frac{1}{3} (\delta_1 + 2 \delta_2 \theta_e) \]  
(15)

\[ C_7 = \frac{1}{2} (a - \delta_2) \]  
(16)

\[ \lambda = \frac{-C_6 + \sqrt{C_6^2 - 4C_5C_7}}{2C_7} \]  
(17)

where,

- \( GWT \) = gross weight, lbs.
- \( \sigma \) = solidity ratio
- \( R \) = tip radius, ft.

The rotor speed is,

\[ \Omega = \sqrt{C_1/(0.333 \theta_e + 0.50 \lambda)} \]  
(18)

And the rate of sink is,

\[ v_s = \frac{GWT (2 \bar{F} + 2)}{0.015 \bar{F} R^2} \]  
(19)

where,

\[ \bar{F} = \frac{GWT}{0.015 R^2 u^2} \]

\[ u = \lambda \Omega R \]

For the power-off rate of sink in forward flight, Eq. (19) is used. However, instead of using Eqs. (13) thru (18) for determining \( \lambda \) and \( \Omega \), these parameters are calculated from the forward flight condition.
Airframe Drag

Without wind tunnel data it is extremely difficult to predict an accurate drag coefficient, $C_D$, for a specific airframe. The literature shows that drag coefficients for standard shapes have been established. For example, an estimated drag coefficient of 0.5 can be expected for a sphere at the anticipated flight conditions according to Dommasch, Sherby, and Connolly. The rather blunt and nonstreamlined airframe and a drag coefficient of 0.55 was assumed for this airframe. The total airframe drag for all airframes is based on the frontal area, $A$. It is assumed that a heavier gyroplane's larger frontal area is offset by the ability to streamline its airframe such that the drag coefficient is reduced and the product of area and drag coefficient remains constant regardless of the size of the gyroplane.

A frontal area of 18.3 square feet is calculated for a 1000 pound gyroplane, and, together with an assumed drag coefficient of 0.55 and sea level air density, $\rho_0$, the total airframe drag is,

$$D = \frac{1}{2} \rho_0 V^2 C_D A = 0.012 V^2 \quad (20)$$

The total drag is the sum of the rotor and airframe drag.

Weight-Break-Down

The gross weight of a gyroplane is divided into the following weights:

- Airframe
- Engine
- Accessories
- Rotor
o Propeller-
o Fuel
o Pilot and passenger
o Oil

An empirical quadratic equation is selected to fit two coordinate points such that the airframe weight can be expressed as a function of the gross weight. The two data points are: an airframe weight of 270 pounds (corresponding to a gross weight of 1000 pounds); and an airframe weight of 480 pounds (corresponding to a gross weight of 2000 pounds).

The airframe weight thus determined is,

\[ \text{AFWT} = 0.30 \text{GWT} - 0.00003 \text{GWT}^2 \]  \hspace{1cm} (21)

The engine, accessories, propeller, and fuel weight are determined from the engine selection. The rotor weight is calculated from Eq. (22). Eq. (22) is determined for any nonspecified rotor by summing the moments generated by the centrifugal force and the aerodynamic lift acting on a constant chord uniform mass blade about the blade flapping hinge. A natural precone angle of two degrees is assumed.

\[ \text{RWT} = \frac{1685 \text{GWT} (0.15 \text{BN} + 0.7)}{R\Omega^2} \]  \hspace{1cm} (22)

where,

- \( \text{RWT} \) = rotor weight, lbs.
- \( \text{BN} \) = number of blades
- \( \Omega \) = rotor speed, radians per second
- \( R \) = rotor radius, ft.

This method for determining rotor weight is suggested by Young.
For the engines listed in CAGRØS, approximately 5.5 quarts of oil are needed. At 7.5 pounds per gallon for standard U.S. lubricating oil, the oil weight is 10.3 pounds. It should be noted that this weight is an approximation only. The quantity of oil in the oil cooler will vary from aircraft to aircraft and will not be included here.

The allowable weight for pilot and passenger is determined by subtracting all component and oil and fuel weights from the gross weight.

Finally, the power available is calculated from Eq. (23) and the power required is calculated from Eq. (24). The rate of climb is determined from the thrust available for climb, $TAFC$, which is equal to the difference between the flight thrust, $FLTT$, and total drag, $TDRAG$.

All calculations are made for standard air at sea level for which the air density is 0.002378 slugs per cubic foot.

The power available is,

$$PAV = FLTT \cdot \frac{VFWD}{550}$$  \hspace{1cm} (23)

where,

$VFWD$ = airspeed, feet per second

The power required is,

$$PREQ = TDRAG \cdot \frac{VFWD}{550}$$  \hspace{1cm} (24)

And the rate of climb is,

$$R_{FC} = 60 \cdot \frac{TAFC}{GWT} \cdot VFWD$$  \hspace{1cm} (25)
Results and Discussion

The two most important design variables affecting gyroplane performance are disc loading and power loading. The effects of these variables on the maximum and minimum speed, speed for best range, speed for best endurance, and rate of climb are investigated for a gyroplane weighing 1000 pounds and one weighing 1500 pounds in Figs. 4 thru 11. From these figures it is immediately recognized that the minimum speed (at the first intersection of the power available and power required curves) is primarily controlled by disc loading. Whereas the maximum speed (the last intersection of the power curves) is predominantly effected by power loading.

Decreasing the disc loading for the 1000 pounds aircraft shown in Fig. 4 from 2.4 to 1.0 gives a weight penalty of 47 pounds due to the increased rotor size and a 54 pound weight penalty accompanies the decrease in power loading from 10.0 to 7.69 due to a larger engine.

Low disc loading gives lower power-off rates of sink but also decreases the best endurance speed as represented in Fig. 24 by a line drawn thru the origin and tangent to the power required curve.

Significant increases in maximum rates of climb are obtained by decreasing disc loading and power loading as shown in Fig. 8 thru 11 for 1000 and 1500 pound gyroplanes. It is the author’s opinion that to achieve good performance, such as high rates of climb, it is first essential to select a low disc loading and secondly, a low power loading.

The lesser design parameters affecting performance are solidity ratio, blade pitch, blade twist, and airfoil selection. The power curves,
Fig. 4. Power Curves For a 1000 lb. Gyroplane With Various Disc Loadings

Fig. 5. Power Curves For a 1500 lb. Gyroplane With Various Disc Loadings
Fig. 6. Power Curves for a 1000 lb. Gyroplane with Various Power Loadings

Fig. 7. Power Curves for a 1500 lb. Gyroplane With Various Power Loadings
Fig. 8. Rate of Climb of a 1000 lb. Gyroplane At Various Disc Loadings

Fig. 9. Rate of Climb of a 1500 lb. Gyroplane At Various Disc Loadings
Fig. 10. Rate of Climb of a 1000 lb. Gyroplane at Various Power Loadings

Fig. 11. Rate of Climb of a 1500 lb. Gyroplane at Various Power Loadings
Fig. 12. Power Curves For a 1000 lb. Gyroplane With Various Solidity Ratios

- Gross Weight 1000 lbs.
- Disc Loading 1.8 lbs/sq.ft.
- Power Loading 10 lbs/hp
- No Blade Twist
- Blade Pitch 2°
- NACA 8-H-12 Airfoil
Fig. 13. Rate of Climb at Various Solidity Ratios

Fig. 14. Power Off Rate of Sink for Various Solidity Ratios
Gross Weight 1000 lbs.
Disc Loading 1.8 lbs/sq.ft.
Power Loading 10 lbs/hp
NACA 8-H-12 Airfoil (Smooth)
No Blade Twist
Solidity 0.035

Fig. 15. Power Curves For a 1000 lb. Gyroplane With Various Blade Pitch Settings
Fig. 16. Rate of Climb at Various Blade Pitch Angles

Fig. 17. Power Off Rate of Sink for Various Blade Pitch Angles
Fig. 18. Power Curves For a 1000 lb. Gyroplane With an Effective Blade Pitch of Two Degrees and Varying Blade Twist
Fig. 19. Rate of Climb for Various Blade Twist

Fig. 20. Power Off Rate of Sink for Various Blade Twist
Fig. 21. Power Curves For a 1000 lb. Gyroplane With Different Airfoils

- NACA 8-H-12 Smooth
- NACA 23012 Smooth
- NACA 8-H-12 Rough

- Power Required
- Power Available

Gross Weight 1000 lbs.
Disc Loading 1.8 lbs/sq.ft.
Power Loading 10 lbs/hp
No Blade Twist
Blade Pitch 2°
Solidity 0.035
Fig. 22. Rate of Climb for Various Airfoils

Fig. 23. Power Off Rate of Sink for Various Airfoils
rate of climb, and power-off rate of sink curves for variations in these parameters are also calculated by CAGRØS and evaluated from Figs. 12 thru 23 for a 1000 pound gyroplane with a disc loading of 1.8 pounds per square foot and a power loading of 10 pounds per horsepower. It was determined by the author that the trends demonstrated by the 1000 pound gyroplane also apply to the 1500 pound aircraft. Therefore, curves for the 1500 pound aircraft are not shown here.

The solidity ratio may be altered by either changing the number of blades or by increasing or decreasing the blade chord length. Whichever method is selected, the performance calculations of CAGRØS are not affected since the tip-loss factor is assumed constant. The effects of the solidity ratio on performance are shown in Figs. 12 thru 14. From these figures it is recognized that, for an increasing solidity ratio of 0.035 to 0.045, a decrease in minimum speed, an increase in maximum speed, and lower power-off rates of sink are realized. The weight penalty for increasing the chord length would be less than the 22.3 pounds calculated by CAGRØS for the 1000 pound gyroplane and the increase in performance may be worth the increase in rotor weight.

Most ultra-light gyroplane utilize hand starting of the rotor which requires that the blade pitch be set at a small angle. At larger pitch angles (3 to 4 degrees), the low rotor speed obtained by hand starting is insufficient to allow autorotation. Figs. 15 thru 17 show considerable performance gains by increasing blade pitch settings from one to three degrees. From practical experience in testing autorotating rotors the author has found that most rotors with blade pitch set at two degrees are easily started by hand and that many blades set three degrees cannot
be started by hand. It is, therefore, recommended that the highest pitch angle that will allow starting by hand be used for hand started rotors. If a prerotator is utilized, a blade pitch setting of three to four degrees can be used. At excessively high pitch angles the rotor will not autorotate. 

Now considering blade twist, for which a negative angle of twist indicates that the pitch of the blade at the tip is such as to decrease the angle of attack. All comparative data showing the effects of blade twist in Figs. 18 thru 20 are made for an effective blade pitch of two degrees. Figs. 18 thru 20 show that, just as in helicopter rotor blades, only a small advantage can be gained by twisting the blades into the airstream at the tip. Degraded performance can be expected for twisting the blades out of the airstream at the tip. If the blade twist can be readily built into the blade, a ten percent increase in maximum rate of climb can be expected. If twisting of the blades complicates their fabrication such as in standard metal blades, twisting of the blades cannot be justified.

An investigation of blade section and airfoil roughness is made in Figs. 21 thru 23. It is shown that a 147 percent increase in rate of climb can be expected for a smooth NACA 8-H-12 airfoil over the same rough airfoil. It is also seen that a 23 percent increase in rate of climb can be realized if a NACA 8-H-12 airfoil is utilized instead of a NACA 23012 airfoil. Also, for vertical power-off rate of sink the NACA 8-H-12 airfoil shows a slightly lower rate of sink. The results obtained from the airfoil roughness study clearly stress the importance of and the gains that can be achieved by the use of a smooth airfoil.
The utilization of CAGRØS to a specific detail design is now made. Before defining the detail design a word should be said about the rotor system and power plant selection for this aircraft which has been named, the "HA-2 Sportster." Prior to this investigation, a rotor system, the HA-27 rotor, and an engine, a Franklin Sports 4B, had been selected. The HA-27 rotor had been designed earlier and several sets have been built and flown on a variety of aircraft including the Boomerang, built and flown in San Diego, and the Cougar, built by Campbell Aircraft, Ltd., in England. To the author's knowledge, both aircraft are still flying at the time of this thesis. The Boomerang has logged over 20 hours, while the Cougar has logged over 40 hours with the HA-27 rotor. The engine had been purchased and installed in the HA-2 Sportster which will be flown by the author. Either justification for the selected design or improvements for the HA-2 Sportster are suggested herein.

For good performance, that is, a high forward speed, low minimum speed, good rate of climb, and comfortable power-off rate of sink a combination of low power loading and low disc loading are needed. For the HA-2 Sportster the combination of a 8.08 pounds per horsepower power loading and a 1.8 pounds per square foot disc loading assures this aircraft a maximum speed of 95 miles per hour, a minimum speed of 36 miles per hour, and a 65 miles per hour speed for best range, according to Fig. 24. A moderate maximum rate of climb of 610 feet per minute can be achieved at 63 miles per hour as shown in Fig. 25. Rates of climb of less than 500 feet per minute are not recommended for small aircraft.

It is shown in Fig. 26 that low speed performance is primarily determined by rotor drag and high speed performance is governed by air-
frame drag. If higher maximum speeds are desired for the HA-2 Sportster, the airframe drag would have to be reduced by streamlining. In the present design little attention is paid to streamlining.

A positive two degree blade pitch was selected for easy hand starting. A larger blade pitch angle, as seen in Fig. 16 for a 1000 pound aircraft, will increase performance. Therefore, a prerotator which will allow higher blade pitch angles is very desirable. For the sake of simplicity a prerotator has not been installed on the HA-2 Sportster at this time.

A NACA 8-H-12 smooth airfoil was selected. The blades were not twisted to facilitate the fabrication of the all metal HA-27 rotor blades. Appendix 2 shows the CAGRØS output for the HA-2 Sportster and its performance characteristics are summarized in Fig. 72.

A check on the advancing blade tip speed is made to assure against danger of loss in efficiency due to compressibility on the advancing blade tip. The analysis used in CAGRØS assumes no compressibility affects. Compressibility shock usually forms at 75 percent the speed of sound which, for standard air at sea level is 762 mph. The tip speed should, therefore, not exceed 572 mph. For the HA-2 Sportster flying at a maximum speed of 95 mph and with a corresponding rotor speed of 432 rpm, the advancing blade tip is traveling at 305 mph which is well below the critical speed.

A 27 foot diameter rotor was selected for the HA-2 Sportster to give a low disc loading and a 9.00 inch chord was selected to give a solidity ratio of 0.035. From Fig. 13 it is realized that a higher rate of climb can be achieved with a higher solidity ratio. A computer run with CAGRØS
Fig. 24. Power Curves For the HA-2 Sportster

Gross Weight 1050 lbs.
Power Loading 8.08 lbs/hp
Disc Loading 1.8 lbs/sq.ft.
NACA 8-H-12 Airfoil Smooth
No Blade Twist
Blade Pitch 2°
Fig. 25. Rate of Climb of the HA-2 Sportster

Fig. 26. Drag and Thrust for the HA-2 Sportster
Fig. 27. Specifications for the HA-2 Sportster

Engine: Franklin Sports 4B
- Power: 130 hp at 2800 rpm
- Power loading: 8.08 lb/hp

Weight Break Down:
- Airframe: 282 lbs
- Engine: 260 lbs
- Accessories: 30 lbs
- Rotor: 72 lbs
- Propeller: 18 lbs
- Empty Weight: 662 lbs
- Fuel, 12 gal.: 72 lbs
- Pilot and passenger: 306 lbs
- Disposable lead: 378 lbs
- Oil: 10 lbs
- Cross Weight: 1050 lbs

Rotor: HA-27 Rotor System
- Number of blades: 2
- Diameter: 27 ft.
- Chord: 9.00 in.
- Airfoil: NACA 8-H-12
- Solidity ratio: 0.035
- Blade pitch: plus 2 degrees
- Disc loading: 1.8 psf

Flight Performance:
- Maximum speed: 95 mph
- Minimum speed: 36 mph
- Speed for best range: 65 mph
- Speed for best endurance: 43 mph
- Maximum rate of climb at 63 mph: 610 fpm
- Vertical power off autorotation: 1860 fpm
- Range, no reserve: 128 miles
for a solidity ratio of 0.045 shows an increase of 70 feet per minute in maximum rate of climb, as shown in Fig. 25. Increasing the chord from 9.00 inches to 11.5 inches for a solidity of 0.045 would yield only a moderate increase in rotor weight. However, the increased performance would be worth the small additional weight.
CHAPTER IV

ROTOR STRUCTURAL DESIGN ANALYSIS

The design of the HA-27 rotor, Fig. 28, utilizes a single leading edge extrusion classified as a tension beam column which is subjected to the harmonic and steady loads of the rotor translates thru the air during forward flight. The harmonic loads produce cyclic bending moments and a large number alternating stress cycles in the blades. In the design of the HA-27 rotor safe-life design philosophy is utilized. Safe-life design is employed in single element structures typical of rotor blades. In safe-life structures sufficient structural metal is used, sometimes beyond the normal strength requirements, to assure the structural integrity and fatigue resistance properties necessary for the system to function for its intended service life.

The limit and ultimate stresses for 3.5g flapping normal to the plane of rotation and the 1.0g cyclic limit stresses are primarily investigated for maximum forward speed. The bending stresses in the plane of rotation are low. This is due, first, to the low in-plane bending moments of an autorotating rotor and second, to the large structural moment of inertia of the blade about the vertical axis. Because of their small magnitude, bending and torsional stresses in the plane of rotation are not investigated. The torsional stresses are shown to be small as follows.
Assuming that each airfoil section along the span of the blade is operating at the maximum moment coefficient of 0.01, see NACA TN 1998[10], the maximum torque at the root is found by summing the torques along the blade span:

\[ T = \frac{1}{2} \rho c m c_m \Omega^2 R^3 \int_{0}^{1} X^2 \, dx \]  \hspace{1cm} (26)

where,

\[ \rho = \text{mass density of air, slugs per cubic feet} \]
\[ c = \text{blade chord, ft.} \]
\[ c_m = \text{moment coefficient, per radian} \]
\[ \Omega = \text{rotor speed, radians per second} \]
\[ R = \text{rotor tip radius, ft.} \]
\[ X = r/R \]
\[ r = \text{radial distance to station X, ft.} \]

Substituting the maximum rotor speed, 83.26 radians per second, and the appropriate values into Eq. (26) and integrating, the root torque is 621.6 in.-lbs. The angle of twist for a unit blade section is given by Peery[11] as,

\[ \phi = \frac{T}{KG} \]

where for the HA-27 rotor leading edge extrusion,

\[ K = 0.0559 \]
\[ G = 4.1 \times 10^6 \text{ psi} \]

Substituting into Eq. (27), \( \phi = 0.0027 \) radians per inch.
For either of the extrusion flanges, $K = 0.0019$. Using Eq. (27), the torque required to twist one flange thru the angle $\phi$ is 21.0 in.-lbs. Again, from Peery, the maximum shear stress which will occur at the middle of the flanges is given as

$$f_s = \frac{T}{\alpha b t^2} \quad (28)$$

For the leading edge flange, which has a width, $b$, of 1.80 inches and a thickness, $t$, of 0.15 inches, $\alpha = 0.32$ according to Table 13.1 by Peery.\(^{11}\)

Substituting into Eq. (28), the maximum shear stress at the root for a flight limit load factor of 3.5g's is 1,620 psi. Although this shear stress is about one tenth of the magnitude of the tensile stress, the combined stress will be affected. However, the combined stress will not be investigated in this thesis.

The 3.5g's limit bending stress in the flapping direction is compared to the yielding allowable stress of the blade material to show that the blade will not yield during maximum limit flight loads and the 3.5g's ultimate bending stress in the flapping direction is compared to the ultimate allowable stress to show that the blade will not fail at ultimate loads. According to the Federal Aviation Regulations, Part 27\(^{3}\), the ultimate load is defined as 1.5 times the limit load.

The fatigue quality of the extrusion at the point of maximum stress which is found to be at the first bolt holes (Bolt 1, Fig. 28) is investigated by determining the magnitude of the alternating and steady stresses at a forward speed of 105 mph and a load factor of 1.0g. A stress concentration factor of 2.0 is assumed in the bolt area and this
Fig. 28. HA-27 Rotor Blade Design
factor is applied to the alternating stresses as suggested by Roark\textsuperscript{12} on page 41.

The maximum cyclic blade stress is plotted on a Goodman diagram.

A discussion of a simplified matrix method for obtaining the flapping bending stress at the hub of a rigid or teetering rotor for steady airloads such as loads experienced in hovering or vertical autorotative descent is presented in Appendix 3. This method includes the effects of built-in precone angle which has a significant effect on root bending moments. The method is derived from the method presented by Mayo\textsuperscript{1} and calculations can be quickly performed by hand for preliminary sizing only. For the detailed analysis of the HA-27 rotor the matrix method by Mayo is modified to include built-in precone angle as used in the HA-27 rotor. See Appendix 4.

Finally, the bending frequencies of the HA-27 rotor are compared to the harmonics of the rotor speed to show that the rotor operating speed is outside of resonance frequencies.

**Blade Load Conditions**

The maximum stresses in the rotor blade are determined from the first three load cases. A fatigue analysis is made from Load Case 4. Except as noted, all loads are for a 1050 pound aircraft and all load factors are per the Federal Aviation Regulations, Part 27\textsuperscript{3}.

Load Case 1. The rotor in the stopped position and the aircraft taxing over the roughest ground that may reasonably be expected in normal operation. The Federal Aviation Regulations, Part 27, do not
give a limit load factor for this case. However, since the airframe of the HA-2 Sportster is designed for a limit ground load frame of 3.0g's, a limit ground load factor of 3.0g's is used in determining the maximum ground bending moments in the HA-27 rotor.

Load Case 2. Vertical power-off autorotation. Limit flight load factor equal to 3.5g's.

Load Case 3. Maximum forward speed of 117.2 mph, a limit flight load factor of 3.5g's, and a corresponding rotor speed of 795.22 rpm.

Load Case 4. Maximum forward speed of 105.4 mph, a limit flight load factor of 1.0g and a corresponding rotor speed of 432.33 rpm.

It is recognized that the maximum forward speed of Load Case 3 and 4 are not inagreement with the maximum speed of 95 mph from Fig. 24. This discrepancy comes from making all performance and airload calculations for increments of tip-speed ratio, \( \mu = 0.05 \), per Gessow and Myers. Should smaller increments of \( \mu \) and therefore closer agreement in forward speed be desired, the equations in the appendix of NACA Report No. 716 should be utilized. This, however, requires considerable extra programming. A tip-speed ratio of 0.15 is used for Load Case 3, while \( \mu = 0.25 \) is used for Load Case 4.

**Blade Loads, Load Case 2**

For a flight limit load of 3.5 x 1050 pounds, CAGRØS is utilized to determine a rotor speed of 758.45 rpm, a sink speed of 57.9 feet per second, and an inflow ratio of 0.0174 for power-off vertical autorotation. Using the airloads calculated from Eq. (A15), Appendix 5, and Eqs. (A9) and (A11), Appendix 4, the structural bending moments and deflections are
determined as shown in Figs. 29 and 30 for a built-in precone angle, \( \beta = 0 \) and \( \beta = 2 \) degrees. It is recognized that the root bending moments are halved for \( \beta = 2 \) degrees and that \( \beta \) has no effect on bending moments outboard of blade station \( X = 0.35 \). For a very flexible blade such as the HA-27 rotor blade Fig. 30 shows that negligible decreases in blade deflection can be expected for \( \beta = 0 \).

The structural moment at station, \( X = 0.125 \) for \( \beta = 0 \) and for \( \beta = 2 \) degrees is calculated using the simplified procedure of Appendix 3. As shown in Fig. 29, good approximate agreement for root bending moments is obtained, while the bending moments outboard of \( X = 0.125 \) are in considerable error and have not been shown. The blade deflections for \( \beta = 2 \) degrees is shown in Fig. 30 and is also shown to be in considerable error. The simplified four blade element method should, therefore, be used with extreme caution and for preliminary sizing only.

**Blade Loads, Load Case 3**

Again, CAGRO is used to calculate rotor speed, inflow ratio, and forward speed for a limit flight load of \( 3.5 \times 1050 \) pounds and a maximum forward speed of 117.2 mph. The flapping coefficients are calculated from Gessow and Myers\(^5\) and together with the tip speed ratio, rotor speed, and inflow ratio are used in Eq. (A15), (A16), and (A17) of Appendix 5 to determine the steady, first harmonic, and second harmonic airloads respectively. Using the method described in Appendix 4 for the steady and harmonic airloads the structural moments are calculated for eight azimuth positions, \( \chi = 0, 45, 90, 135, 180, 225, \) and 315 degrees and each sum is recorded in Fig. 31. The maximum root
structural moments occur at $\chi = 270$ degrees as seen in Fig. 31. The harmonic components and the total moments are plotted in Fig. 32 as a function of blade station to show the detailed loads along the span of the blade.

Matrix $\Omega^2(F)$ is printed out by the bending moment program and the centrifugal force along the span of the blade, which is the first non-zero diagonal of the matrix, is recorded in Fig. 38.

**Blade Loads, Load Case 4**

The same procedure as used in the preceding section is used to determine the airloads and structural bending moments for a limit flight load of 1050 pounds at a forward speed of 105.4 mph. The airloads are recorded in Figs. 33 thru 35 and the structural bending moments are plotted as a function of them in Fig. 36.

As can be expected, a reversal of airflow, negative lift, is seen over the blade at the downwind blade position, $\chi = 270$ degrees at blade station $X = 0.25$ in Fig. 33.

The total airload plot, Fig. 35, clearly shows that a two cycles per revolution vibration can be expected from the airload. It can also be seen from Fig. 36 that the structural bending moment at the root lags the airload by approximately 90 degrees.

The centrifugal force along the blade span is plotted in Fig. 37.

**Stress Analysis**

The maximum stress in the blade at Bolt 1, $X = 0.11$, Fig. 28, is determined from the first three load cases. The maximum stress is
Fig. 29. Blade Bending Moments for Vertical Autorotation

Fig. 30. Blade Deflections For Power Off Vertical Autorotation

Point Calculated For Four Element Blade

Scale A

$\beta = 0^\circ$

$\beta = 2^\circ$

Scale B

$\beta = 2^\circ$

Limit Load $3.5 \cdot 1050$ lbs.
Rotor Speed 758.4 rpm
Fig. 31. Blade Bending Moments in Forward Flight, Limit Load = 3.5 \times 1050 \text{ lbs.}
Fig. 32. Blade Bending Moment Components at $\chi = 270^\circ$ Forward Flight
Limit Load 1050 lbs.
Forward Speed 105.4 mph
Rotor Speed 432.3 rpm

μ = 0.25

Fig. 33. Airload vs Azimuth Position in Forward Flight

Fig. 34. Airload vs Blade Station in Forward Flight
Fig. 35. Total Blade Airload vs Azimuth Position

Limit Load 1050 lbs.
Forward Speed 105.4 mph
Rotor Speed 432.3 rpm

μ = 0.25
Fig. 36. Blade Bending Moments in Forward Flight, Limit Load = 1050 lbs.
Fig. 37. Centrifugal Force For Forward Flight,
Limit Load = 3.5 x 1050 lbs.

Centrifugal Force, Kips

Blade Station X

Forward Speed 117.2 mph
Rotor Speed 795.2 rpm

$\mu = 0.15$

Fig. 38. Centrifugal Force For Forward Flight,
Limit Load = 1050 lbs.

Centrifugal Force, Kips

Blade Station X

Forward Speed 105.4 mph
Rotor Speed 432.3 rpm

$\mu = 0.25$
calculated as the maximum combined stress from the bending moment and centrifugal force for each case and is determined by,

\[ f_t = \frac{P_c}{A_b} + \frac{M_s Y}{I_e} \]  

where,

- \( f_t \) = tensile stress, psi
- \( P_c \) = centrifugal force, lbs.
- \( A_b \) = cross section area of blade less bolt holes, in.\(^2\)
- \( M_s \) = structural bending moment, in.-lbs.
- \( Y \) = distance from centroid to outer fibers of blade section, in.
- \( I_e \) = moment of inertia of cross section of extrusion less bolt holes, in.\(^4\)

The maximum stress at Bolt 1 for each load case is calculated from Eq. (29) and recorded in Table 3. From Table 3 it is seen that Load Case 3 generates the largest stress at Bolt 1 and the root attachment is analyzed for this load. The combined stress is used only in analyzing the stress in the extrusion at Bolt 1.

For the load distribution throughout the attach fitting only the centrifugal force is considered and equal strain is assumed. That is, the strains in all the components located in one bay are assumed equal. However, the strain will vary from one bay to the next. The strain in any one bay for any one of the fitting components is given as,

\[ \delta_i = \frac{P_i L_i E_i}{A_i I_i} \]
# TABLE 3

**COMBINED BENDING AND TENSILE STRESS IN THE EXTRUSION AT BOLT 1**

<table>
<thead>
<tr>
<th>Loadcase</th>
<th>P_c, lbs.</th>
<th>M_s, in.-lbs.</th>
<th>f_t From P_c</th>
<th>f_t From M_s</th>
<th>f_t Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4,695.6</td>
<td>0</td>
<td>26,636.2</td>
<td>26,636.2</td>
</tr>
<tr>
<td>2</td>
<td>32,622</td>
<td>667.0</td>
<td>27,763.4</td>
<td>2,818.2</td>
<td>20,581.6</td>
</tr>
<tr>
<td>3</td>
<td>36,000</td>
<td>250.0</td>
<td>30,638.3</td>
<td>1,056.3</td>
<td>31,694.6</td>
</tr>
<tr>
<td>4</td>
<td>10,700</td>
<td>225.0</td>
<td>9,106.4</td>
<td>950.7</td>
<td>10,057.1</td>
</tr>
</tbody>
</table>
where,

\[ P = \text{load, lbs.} \]
\[ L = \text{length of bay, inches} \]
\[ A = \text{mean cross section area of component, in.}^2 \]
\[ i = \text{number of component being investigated} \]

From Eq. (30) and with the equilibrium conditions the load in each component is determined and recorded in Fig. 39. The equilibrium condition states that the sum of the loads in all components in one bay is equal to the total load transmitted by the bay.

The detailed stress calculations of the HA-27 rotor blade are made in Appendix 7 using the material properties given in Appendix 6. A summary of the critical margins of safety is given in Table 4.

**Fatigue Check on Extrusion**

There are many unknown factors governing fatigue and most critical aircraft structures are fatigue tested to failure to determine the number of cycles and the magnitude of the applied steady and alternating loads required for failure. Such testing is quite expensive and will not be done on the HA-27 rotor. Instead, a generally accepted Goodman diagram as suggested by Roark\(^{12}\) for aluminum is constructed to determine if the critically stressed area at Bolts 3 in the extrusion will sustain one g limit flight loads at maximum forward speed without failing in fatigue. In the Goodman diagram the mean stress, \( f_m \), is plotted on the abscissa and the alternating stress, \( f_a \), is plotted on the ordinate axis in a rectangular coordinate system such that a relationship between these variables may be investigated. According to Roark, the endurance limit
**Fig. 39. Blade Root Fitting Loads**

<table>
<thead>
<tr>
<th>Component</th>
<th>Load in Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension Strap</td>
<td></td>
</tr>
<tr>
<td>Relief Strap</td>
<td></td>
</tr>
<tr>
<td>Blade Extrusion</td>
<td></td>
</tr>
<tr>
<td>Hub</td>
<td></td>
</tr>
<tr>
<td>Relief Strap</td>
<td></td>
</tr>
<tr>
<td>Blade Extrusion</td>
<td></td>
</tr>
<tr>
<td>Tension Strap</td>
<td></td>
</tr>
<tr>
<td>Bolt Shear</td>
<td></td>
</tr>
<tr>
<td>Blade and Relief Strap</td>
<td></td>
</tr>
<tr>
<td>Bolt Shear</td>
<td></td>
</tr>
<tr>
<td>Blade and Relief Strap</td>
<td></td>
</tr>
<tr>
<td>Bolt Shear</td>
<td></td>
</tr>
<tr>
<td>Tension Straps and Hub</td>
<td></td>
</tr>
<tr>
<td>Bolt Shear</td>
<td></td>
</tr>
<tr>
<td>Tension Straps and Hub</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** All Loads in Pounds
<table>
<thead>
<tr>
<th>PART</th>
<th>CRITICAL STRESS LOCATION</th>
<th>TYPE OF STRESS</th>
<th>MARGIN OF SAFETY</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extrusion</td>
<td>Bolt 3</td>
<td>Bolt Bearing</td>
<td>+ 0.005 Yield</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+ 0.019 Ultimate</td>
<td></td>
</tr>
<tr>
<td>Relief Strap</td>
<td>Section in Bay 1-2</td>
<td>Tensile</td>
<td>+0.27 Yield</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+0.51 Ultimate</td>
<td></td>
</tr>
<tr>
<td>Tension Straps</td>
<td>Net Section At Bolts 4</td>
<td>Tensile</td>
<td>+0.09 Yield</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>+0.29 Ultimate</td>
<td></td>
</tr>
<tr>
<td>Bolts</td>
<td>Bolts 3</td>
<td>Shear</td>
<td>+0.10 Ultimate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bolts 8</td>
<td>Shear</td>
<td>-0.06 Ultimate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bolts 9</td>
<td>Shear</td>
<td>+0.70 Ultimate</td>
<td></td>
</tr>
<tr>
<td>Rivet</td>
<td>MS20426AD4 Rivet in Rib</td>
<td>Shear</td>
<td>+0.39 Ultimate</td>
<td></td>
</tr>
</tbody>
</table>
of aluminum is usually taken as one third the ultimate tensile strength. The endurance limit, that is, the maximum stress level which can be reversed an indefinitely large number of times without producing fracture, is marked on the ordinate axis and the ultimate tensile stress is measured along the abscissa as shown for the HA-27 extrusion in Fig. 40. According to the Goodman theory, the region above and to the right of the area of a straight line connecting the endurance limit and ultimate tensile stress represents the region of failure. Whereas, the region below and to the left of the line represents the area in which the part will not fail for an indefinite number of cycles.

The alternating bending moments at Bolts 3, \( X = 0.097 \), are found by interpolating between \( X = 0.0625 \) and \( X = 0.15 \) in Fig. 36. The maximum alternating bending moments are -90 and 225 in.-lbs. The centrifugal force is a steady load found from Fig. 38 as 10,700 lbs. The total stress in the extrusion is found from Eq. (29). The maximum stress will be generated by a positive moment. Therefore, the bottom outermost fibers which are in tension in the extrusion are investigated for fatigue. The stresses are calculated as,

\[
\begin{align*}
    f_{\text{max}} &= 9,744 \text{ psi} \\
    f_{\text{min}} &= 8,975 \text{ psi} \\
    f_{m} &= \frac{1}{2} (f_{\text{max}} + f_{\text{min}}) = 9360 \text{ psi} \\
    f_{a} &= \frac{1}{2} (f_{\text{max}} - f_{\text{min}}) \times 2 = 769 \text{ psi}
\end{align*}
\]

For a conservative analysis a stress concentration factor of two was assumed for the alternating stress. The endurance strength of the extrusion is 21,333 psi. The Goodman diagram is now constructed as shown.
in Fig. 40 and it can be seen that the maximum alternating stress lies well within the no-fail zone.

![Goodman Diagram for the Blade Extrusion Rotor Resonance](image)

**Rotor Resonance**

For the safe operation of the HA-27 rotor system is is imperative that the rotor will be flown out of rotor resonance conditions. Rotor resonance occurs when a rotor speed harmonic coincides with one of the natural bending frequencies of the rotor. Under such conditions the blades may experience large deflections which may cause the blade to strike the rudder or propeller and large bending stress may predominate. Rotor resonance can be catastrophic. The rotor's first unsymmetric and first symmetric bending modes are calculated and compared with the harmonics of rotor speed to assure that no resonance conditions prevail. The method by Anerson is used to calculate the stopped natural frequencies. The actual natural frequencies were determined by placing accelerometers on the rotor and forcing the blade to vibrate. As can be seen in Fig. 41, the calculated and measured frequencies are in close
agreement. The method presented by Yntema\textsuperscript{14} is used to determine the rotating blade frequencies. The results are plotted as solid lines and the rotor harmonics are plotted as dashed lines in Fig. 41. Resonance occurs where the solid lines intersect the rotor harmonics and it is shown that no resonance problems are to be expected for the HA-27 rotor. A total of more than 60 hours of actual flight time on the HA-27 rotor have also indicated that no resonance problems exist.
Fig. 41. HA-27 Rotor Blade Resonance Characteristics For Various Harmonics
CHAPTER V
CONCLUSION

The Parametric Study clearly shows that a combination of low disc and low power loading are required if a good performing gyroplane is desired. The low power loading with moderate streamlining assures an aircraft of high maximum speed and the low disc loading assures a low minimum speed and a low power-off rate of sink. A combination low disc and low power loading assures moderate rates of climb. Furthermore it is shown that:

- Changes in solidity ratio have a moderate effect on minimum and maximum speed and rate of climb.
- Large blade pitch settings increase overall gyroplane performance significantly as shown in Figs. 15 thru 17.
- Only moderate increase in performance can be achieved in twisting the blades such that the blade tip element is operating at a lower angle of pitch. Whereas, increasing blade pitch at the tip shows a degradation of performance over non-twisted blades.
- Figs. 21 thru 23 show that considerable increase in gyroplane performance can be achieved by selecting the proper airfoil and designing a smooth blade.

From the Rotor Structural Design Analysis it is seen that the HA-27 rotor (a low weight, 62.5 pounds, rotor designed for a 1050 pound gross weight gyroplane) is designed to meet the Federal Aviation Regulations,
Part 27$^3$. An approximate method for calculating a structural root bending moment as given in Appendix 3 can be used for preliminary structural sizing. It is also shown that the stresses due to the bending moments are low in comparison to the stresses generated by the centrifugal force. This is especially true if a built-in precone angle is utilized such as in the HA-27 rotor. Fatigue and resonance are not expected to be a problem. A total of more than 60 hours of trouble free flight with the HA-27 rotor have substantiated this claim.
APPENDIX I

SYMBOLS

For Chapter III, Parametric Study

A = flat-plate area of airframe, ft.²
A.F. = propeller activity factor
AFWT = airframe weight, lbs.
a = rotor blade; slope of section lift coefficient against section angle of attack, per radian
BN = number of rotor blades
b = propeller chord width, ft.
CAGROS = Computer Aided Gyroplane Synthesis program
Cₙ = airframe drag coefficient
Cₙ₀ = rotor blade section profile drag coefficient
Cₚ = propeller power coefficient
Cʹₚ = propeller power coefficient with standard values of A.F., thickness ratio, and number of blades.

\[ \frac{C_T}{C_p} \] = propeller static thrust coefficient with given condition of tip-speed and body interference.

\[ \left( \frac{C_T}{C_p} \right)_{900} \] = propeller static thrust coefficient with standard value of tip-speed of 900 feet per second and body interference

D = propeller diameter, ft.
D.L. = disc loading, pounds per square foot
\( \overline{F} \) = rotor thrust coefficient
FLTT = flight thrust, lbs.
GWT  = gross weight of aircraft, lbs.
HP  = maximum installed horsepower
J  = \( \frac{88V}{ND} \)
N  = propeller speed at maximum rated horsepower, rpm
P_{AF}  = correction to propeller power coefficient due to activity factor
PREQ  = power required, HP
P_{h}  = correction to propeller power coefficient due to thickness ratio
P.L.  = power loading, pounds per horsepower
R  = propeller or rotor tip radius, ft.
ROC  = rate of climb, feet per seconds
r  = radius to propeller or rotor blade element, ft.
RWT  = rotor weight, lbs.
S_{B}  = correction to static thrust coefficient due to body interference
S_{T}  = correction to static thrust coefficient due to tip-speed
T  = propeller thrust in forward flight, lbs.
TAFC  = thrust available for climb, lbs.
TDRA  = total aircraft drag, lbs.
T_{s}  = static propeller thrust, lbs.
u  = rotor inflow velocity, feet per second
V  = forward speed of aircraft, mph
VFWD  = forward speed of aircraft, feet per second
V_{s}  = rate of sink, feet per second
X  = \( \frac{r}{R} \)
\( x \) = blade element angle of attack, radians

\( S_0, S_1, S_2 \) = coefficients in a power series expressing the section profile drag coefficient as a function of blade element angle of attack

\( \theta_e \) = effective rotor blade pitch angle, radians

\( \theta_0 \) = rotor blade root pitch, radians

\( \theta_l \) = rotor blade twist; positive when tip angle is larger, radians

\( \lambda \) = rotor blade inflow ratio

\( \mu \) = rotor blade tip-speed ratio

\( \rho \) = air density, slugs per cubic foot

\( \rho_o \) = sea level air density, slugs per cubic foot

\( \sigma \) = rotor solidity ratio

\( \Omega \) = rotor speed, radians per second

For Chapter IV, Rotor Structural Design Analysis

\( A \) = mean cross section area of components, in.\(^2\)

\( A_b \) = blade cross section area less bolt holes, in.\(^2\)

\( B \) = tip loss factor

\( b \) = blade width, ft.

\( c \) = blade chord, ft.

\( c_m \) = blade element pitching moment coefficient, per radian

\( E \) = tensile modulus

\( F \) = strength, psi

\( F_{tu} \) = ultimate tensile strength, psi

\( F_{ty} \) = yield tensile strength, psi

\( F_{su} \) = ultimate shear strength, psi
\[F_{bru} = \text{ultimate bearing strength, psi}\]
\[F_{bry} = \text{yield bearing strength, psi}\]
\[f = \text{stress, psi}\]
\[f_a = \text{alternating stress, psi}\]
\[f_m = \text{mean stress, psi}\]
\[f_{max} = \text{maximum stress in fatigue analysis, psi}\]
\[f_{min} = \text{minimum stress in fatigue analysis, psi}\]
\[f_t = \text{tensile stress, psi}\]
\[f_s = \text{shear stress, psi}\]
\[f_{br} = \text{bearing stress, psi}\]
\[G = \text{shearing moulus of elasticity, psi}\]
\[g = \text{one gravitational acceleration}\]
\[I = \text{blade section moment of inertia about neutral horizontal axis, in.}^4\]
\[I_e = \text{extrustion moment of inertia about neutral horizontal axis, in.}^4\]
\[I_l = \frac{1}{3}lmR^2, \text{blade moment of inertia, slugs-ft.}^2\]
\[M_s = \text{structural bending moment, in.-lbs.}\]
\[M.S. = \text{margin of safety}\]
\[R = \text{rotor tip radius, ft.}\]
\[r = \text{radial distance to blade element, ft.}\]
\[P = \text{load, lbs.}\]
\[P_f = \text{ultimate single shear strength, lbs.}\]
\[P_a = \text{airload, lbs.}\]
\[P_c = \text{centrifugal force, lbs.}\]
\( T \) = torsional load, ft.-lbs.
\( X \) = \( r/R \)
\( Y \) = vertical distance from blade section neutral axis to outer fibers, inches
\( WT \) = weight, lbs.
\( \beta \) = coning angle, radians
\( \gamma \) = \( \frac{c \cdot \rho \cdot aR^4}{I_1} \), blade mass constant
\( \Theta \) = blade pitch angle, radians
\( \lambda \) = inflow ratio
\( \mu \) = tip-speed ratio
\( \rho \) = mass density of air, slugs per cubic foot
\( \sigma \) = blade solidity ratio
\( \phi \) = blade structural twist due to torsion, radians per inch
\( \psi \) = blade azimuth angle measured in direction of blade rotating from downwind position, radians
\( \Omega \) = rotor speed, radians per second
### APPENDIX II

#### CAGRO'S OUTPUT

<table>
<thead>
<tr>
<th>ENGINE</th>
<th>FRANKLIN SPORTS 4B ***</th>
</tr>
</thead>
<tbody>
<tr>
<td>POWER LOADING</td>
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</tr>
<tr>
<td>HORSE POWER</td>
<td>130.00</td>
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<tr>
<td>ENGINE SPEED</td>
<td>2800.00</td>
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#### WEIGHT BREAK DOWN ***

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<th>Component</th>
<th>Weight (lbs)</th>
</tr>
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<tbody>
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<td>ENGINE</td>
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<tr>
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<tr>
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<td>PROPELLER</td>
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<td>EMPTY</td>
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<tr>
<td>FUEL</td>
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<tr>
<td>PILOT AND PASS</td>
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<tr>
<td>DISPOSABLE LOAD</td>
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<td>GROSS WT</td>
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#### ROTOR PARAMETERS ***

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<th>Parameter</th>
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<tr>
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<td>DISC LOADING</td>
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#### FLIGHT PERFORMANCE ***

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<th>FWD SPEED</th>
<th>POWER REQUIRED</th>
<th>POWER AVAIL</th>
<th>RATE OF CLIMB</th>
<th>RATE OF SINK</th>
<th>ROTOR SPEED</th>
<th>PROP THRUST</th>
<th>TOTAL DRAG</th>
<th>ROTOR DRAG</th>
<th>AIRFRAME DRAG</th>
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<tbody>
<tr>
<td>MPH</td>
<td>HP</td>
<td>HP</td>
<td>FPM</td>
<td>FPM</td>
<td>RPM</td>
<td>LPS</td>
<td>LPS</td>
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**RANGE**: 128.80 MILES

**VERTICAL P-O AUTHORIZATION ***

| VSINK | 30.95 | OMEGA | 42.44 | AMDA | 017372 |

72
APPENDIX III

A SIMPLIFIED MATRIX METHOD FOR CALCULATING ROOT BENDING MOMENT FOR A CANTILEVERED OR TEETERING ROTOR BLADE FOR STEADY AIRLOADS

The following method is derived by Mayo [1]. The method is modified to include built in precone angle.

A cantilevered rotor blade as shown in Fig. Al is divided into four equal segments along its span and the airload, \( L_n \), and the centrifugal force \( \Omega^2 F_n \) acting upon a blade element are assumed to be acting at the center of each blade element.

The structural moment at each of the inboard three blade stations can be expressed as the sum of the moments outboard of the station being investigated. The structural moment at station \( X = \)}
0.625 is,

\[ M_{s0.625} = -\Omega^2 F_{0.875} (Z_{0.875} - Z_{0.625}) + \Delta rL_{0.875} \]

The structural moment at station \( X = 0.375 \) is,

\[ M_{s0.375} = -\Omega^2 F_{0.875} (Z_{0.875} - Z_{0.375}) - \Omega^2 F_{0.625} (Z_{0.625} - Z_{0.375}) + 2\Delta rL_{0.875} + \Delta rL_{0.625} \]  

(A1)

The structural moment at station \( X = 0.125 \) is,

\[ M_{s0.125} = -\Omega^2 F_{0.875} (Z_{0.875} - Z_{0.125}) - \Omega^2 F_{0.625} (Z_{0.625} - Z_{0.125}) - \Omega^2 F_{0.375} (Z_{0.375} - Z_{0.125}) + 3\Delta rL_{0.875} + 2\Delta rL_{0.625} + \Delta rL_{0.375} \]

where,

\[ \Delta r = \frac{R}{4} \]

\( Z_n \) = deflection at \( X = n \)

Equation (A1) can be written in matrix notation as,

\[ \{M_s\} = -\Omega^2 [F]\{Z\} + \{R\}\{L\} \]  

(A2)

where,

\[ \{M_s\} = \begin{bmatrix} M_{s0.625} \\ M_{s0.375} \\ M_{s0.125} \end{bmatrix} \]
\[ n^2[F] = n^2 \begin{bmatrix} F_4 & -F_4 & 0 & 0 \\ F_4 & F_3 & 4 & -\Sigma F_n \\ F_4 & F_3 & -\Sigma F_n \\ n=3 \\ F_4 & F_3 & F_2 & 4 \\ n=2 \end{bmatrix} \]

For which \( F_n \) is equal to the product of the mass of the blade element \( n \) and the distance from the axis of rotation to where the load \( F_n \) is acting.

\[ F_n = m r \]

\[ \{Z\} = \begin{bmatrix} Z_{0.875} \\ Z_{0.625} \\ Z_{0.375} \\ Z_{0.125} \end{bmatrix} \]

\[ \{L\} = \begin{bmatrix} L_{0.875} \\ L_{0.625} \\ L_{0.375} \\ L_{0.125} \end{bmatrix} \]

\[ \{R\} = \begin{bmatrix} \Delta r & 0 & 0 & 0 \\ 2\Delta r & \Delta r & 0 & 0 \\ 3\Delta r & 2\Delta r & \Delta r & 0 \end{bmatrix} \]

From engineering mechanics the deflection \( \{Z\} \) can be expressed in terms of the structural moment \( \{M_s\} \) and rigid blade displacement \( \{B\} \) as,
\[ \{Z\} = [ZM] \{M_s\} + \{B\} \quad \text{(A3)} \]

where,

For a rotor with a constant stiffness distribution,

\[
[ZM] = \frac{\Delta r^2}{EI} \begin{bmatrix}
1 & 2 & 3 \\
\frac{1}{8} & 1 & 2 \\
0 & \frac{1}{8} & 1 \\
0 & 0 & \frac{1}{8}
\end{bmatrix}
\]

\( EI = \text{blade bending stiffness, lb-in.}^2 \)

\( r \) = radial distance to the center of element \( x = n \)

Now substituting Eq. (A3) into Eq. (A2) and multiplying we have,

\[ \{M\} = -\Omega^2 [F][ZM]\{M_s\} + [R]\{L\} - \Omega^2[F]\{B\} \quad \text{(A4)} \]

Rearranging terms it is readily seen that,

\[ \{M_s\} = [\{1.0\} + \Omega^2 [F][ZM]]^{-1}[\{R\}\{L\} - \Omega^2[F]\{B\}] \quad \text{(A5)} \]

Using Eq. (A5) a relatively quick conservative approximation of the structural moment at station \( X \) can be made.
APPENDIX IV

MODIFIED MATRIX EQUATIONS FOR DETERMINING THE BENDING MOMENTS AND DEFLECTIONS FOR ROTOR BLADES IN FORWARD FLIGHT

The structural damping for metal rotor blades is extremely small and can, according to Mayo\(^1\), be neglected for most practical purposes. The structural damping is not included in the equations presented herein. However, should the structural damping be desired, the reader is referred to Mayo\(^1\) which includes structural damping.

For Rigid Rotor Blades Including the Effects of Built In Precone Angle

The method for determining the matrix equations is the same as outlined in Appendix III. However, the blade is divided into twelve elements along the span and the airload matrix is replaced by the total vertical loads which includes airdynamic damping and vertical inertia loads. From Mayo\(^1\) the total vertical load is expressed as:

\[
\{L\} = \{L_a\} + i\omega[A] [Z] + \omega^2[MZ] \{Z\} \tag{A6}
\]

where,
$$\{L_a\} = \begin{bmatrix}
L_{0.95} \\
L_{0.85} \\
L_{0.75} \\
L_{0.65} \\
L_{0.55} \\
L_{0.45} \\
L_{0.35} \\
L_{0.25} \\
L_{0.15} \\
L_{0.0625} \\
L_{0.05} \\
L_{0.0125}
\end{bmatrix}$$

\[ i = \sqrt{-1} \]

\( \Omega \) = angular rotor speed, radians/sec.

\( \omega \) = angular frequency, radians/sec.

for steady loads, \( \omega = 0 \)

for 1st harmonic, \( \omega = \Omega \)

for 2nd harmonic, \( \omega = 2\Omega \)
\[
\begin{bmatrix}
  c_{10}r_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & c_{9}r_{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & c_{8}r_{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & c_{7}r_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & c_{6}r_{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & c_{5}r_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & c_{4}r_{4} & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{3}r_{3} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{2}r_{2} & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{1}r_{1} & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{0}r_{0} & 0 \\
\end{bmatrix}
\]

\( [A] = K \)

\( \{Z\} = \{ z_{0.95}, z_{0.85}, z_{0.75}, z_{0.65}, z_{0.55}, z_{0.45}, z_{0.35}, z_{0.25}, z_{0.15}, z_{0.0625}, z_{0.05}, z_{0.0125} \} \)
\[
[MZ] = \\
\begin{bmatrix}
M_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2M_{10} & M_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3M_{10} & 2M_9 & M_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4M_{10} & 3M_9 & 2M_8 & M_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5M_{10} & 4M_9 & 3M_8 & 2M_7 & M_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6M_{10} & 5M_9 & 4M_8 & 3M_7 & 2M_6 & M_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
7M_{10} & 6M_9 & 5M_8 & 4M_7 & 3M_6 & 2M_5 & M_4 & 0 & 0 & 0 & 0 & 0 \\
8M_{10} & 7M_9 & 6M_8 & 5M_7 & 4M_6 & 3M_5 & 2M_4 & M_3 & 0 & 0 & 0 & 0 \\
8.87M_{10} & 7.87M_9 & 6.87M_8 & 5.87M_7 & 4.87M_6 & 3.87M_5 & 2.87M_4 & 1.87M_3 & 0.87M_2 & 0 & 0 & 0 \\
9.37M_{10} & 8.37M_9 & 7.37M_8 & 6.37M_7 & 5.37M_6 & 4.37M_5 & 3.37M_4 & 2.37M_3 & 1.37M_2 & 0 & 0.37M_1 & 0 \\
\end{bmatrix}
\]
\[ K = -\frac{1}{2} \rho a \Delta r \]

\[ C = \text{mean chord of blade element, } 2\text{ in.} \]

\[ \rho = \text{mass density of air, } \frac{1\text{b-sec.}}{4\text{ in.}} \]

\[ \alpha = \text{blade element lift slope, per radian} \]

\[ \Delta r = 0.10R, \text{ in.} \]

\[ M_n = \text{mass of blade element, } \frac{1\text{b-sec.}^2}{4\text{ in.}} \]

Substituting Eq. (A6) into Eq. (A4) and utilizing Eq. (A3) to remove \{Z\} we have,

\[ \{M\} = [R] \{L_a\} - [X] [ZM] \{M\} - [X] \{\beta\} \tag{A7} \]

where,

\[ [X] = \Omega^2 [F] - \omega^2 [MZ] - i \Omega \omega [R] [A] \tag{A8} \]

\[
\{M_s\} = \begin{bmatrix}
M_s^{0.85} \\
M_s^{0.75} \\
M_s^{0.65} \\
M_s^{0.55} \\
M_s^{0.45} \\
M_s^{0.35} \\
M_s^{0.25} \\
M_s^{0.15} \\
M_s^{0.0625} \\
M_s^{0.0125}
\end{bmatrix}
\]
\[
[R] = \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\
8.87 & 7.87 & 6.87 & 5.87 & 4.87 & 3.87 & 2.87 & 1.87 & 0.87 & 0 & 0 & 0 \\
9.37 & 8.37 & 7.37 & 6.37 & 5.37 & 4.37 & 3.37 & 2.37 & 1.37 & 0 & 0.37 & 0 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
\frac{1}{EI_{10}} & \frac{2}{EI_{19}} & \frac{3}{EI_{18}} & \frac{4}{EI_{17}} & \frac{5}{EI_{16}} & \frac{6}{EI_{15}} & \frac{7}{EI_{14}} & \frac{8}{EI_{13}} & \frac{27}{4EI_{12}} & \frac{19}{8EI_{1}} \\
0 & \frac{1}{EI_{9}} & \frac{2}{EI_{8}} & \frac{3}{EI_{7}} & \frac{4}{EI_{6}} & \frac{5}{EI_{5}} & \frac{6}{EI_{4}} & \frac{7}{EI_{3}} & \frac{24}{4EI_{2}} & \frac{17}{8EI_{1}} \\
0 & 0 & \frac{1}{EI_{8}} & \frac{2}{EI_{7}} & \frac{3}{EI_{6}} & \frac{4}{EI_{5}} & \frac{5}{EI_{4}} & \frac{6}{EI_{3}} & \frac{21}{4EI_{2}} & \frac{15}{8EI_{1}} \\
0 & 0 & 0 & \frac{1}{EI_{7}} & \frac{2}{EI_{6}} & \frac{3}{EI_{5}} & \frac{4}{EI_{4}} & \frac{5}{EI_{3}} & \frac{18}{4EI_{2}} & \frac{13}{8EI_{1}} \\
0 & 0 & 0 & 0 & \frac{1}{EI_{6}} & \frac{2}{EI_{5}} & \frac{3}{EI_{4}} & \frac{4}{EI_{3}} & \frac{15}{4EI_{2}} & \frac{11}{8EI_{1}} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{EI_{5}} & \frac{2}{EI_{4}} & \frac{3}{EI_{3}} & \frac{12}{4EI_{2}} & \frac{9}{8EI_{1}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{EI_{4}} & \frac{2}{EI_{3}} & \frac{9}{4EI_{2}} & \frac{7}{8EI_{1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{EI_{3}} & \frac{6}{4EI_{2}} & \frac{5}{8EI_{1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4EI_{2}} & \frac{3}{8EI_{1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{32EI_{2}} & \frac{5}{32EI_{1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8EI_{1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{32EI_{1}}
\end{bmatrix}
\]
\[ F_n = M_n r_n \]

\[
[F] =
\]

\[
\begin{bmatrix}
F_{10} & -F_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & -\Sigma F & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & -\Sigma F & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & -\Sigma F & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & -\Sigma F & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & -\Sigma F & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & -\Sigma F & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & -\Sigma F & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & F_2 & -\Sigma F \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & F_2 & 0 \\
\end{bmatrix}
\]
Eq. (A7) is solved for \( \{ M_s \} \) to give,

\[
\{ M_s \} = \left( [1.0] + [X] [ZM] \right)^{-1} \{ [R] \{ L_o \} - [X] \{ \beta \} \}
\] (A9)

For the steady airloads Eq. (A9) reduces to,

\[
\{ M_s \} = \left( [1.0] + \Omega^2 [F] [ZM] \right) \{ [R] \{ L_o \} - \Omega^2 [F] \{ \beta \} \}
\] (A10)

where \( \{ L_o \} \) is the steady airload.

The deflection due to the steady airload is,

\[
\{ Z \} = [ZM] \{ M_s \} + \{ \beta \}
\] (A11)

Now for all harmonic loads Eq. (A9) reduces to,

\[
\{ M_s \} = \left( [1.0] + [X][ZM] \right) \{ [R] \{ L_i \} + i\Omega [R] [A] \{ \beta \} \}
\] (A12)

where \( \{ L_i \} \) are the harmonic airloads for \( i = 1,2, \ldots \).

The blade deflections for the harmonic loads are calculated from,

\[
\{ Z \} = [ZM] \{ M_s \}
\] (A13)
The total bending moments are determined by using Eq. (A10) to calculate the steady load bending moments and Eq. (A12) to compute the harmonic load bending moments and summing the bending moments thus obtained. The total deflections are found in the same manner using Eq. (A11) and (A12).

For an Articulated Rotor Blade

For a hinged rotor blade the equations and matrices are the same as those of Mayo. For the sake of completeness, these equations and matrices are repeated in this report.

The structural bending moment is given as,

\[
\{M_s\} = \begin{bmatrix} 1 \cdot 0 & \{0\} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} X \end{bmatrix} \begin{bmatrix} [ZM] \{x-h\} \end{bmatrix} \begin{bmatrix} R \{L_1\} \end{bmatrix} \]  

(A14)

where, \{L_1\} are the steady and harmonic airloads for \( i = 0, 1, 2, \ldots \).

Notice that \( \beta \) enters the left hand, the solution side of Eq. (A14) and that unlike the built in precone angle of Eq. (A8), \( \beta \) is the natural precone angle.

All matrices are the same as defined in Eq. (A8) except for the following altered matrices.
\[ [F] = \]

\[
\begin{bmatrix}
F_{10} & -F_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & -\Sigma F \quad n=9^n & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & -\Sigma F \quad n=8^n & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & -\Sigma F \quad n=7^n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & -\Sigma F \quad n=6^n & 0 & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & -\Sigma F \quad n=5^n & 0 & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & -\Sigma F \quad n=4^n & 0 & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & -\Sigma F \quad n=3^n & 0 & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & F_2 & -\Sigma F \quad n=2^n & 0 & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & F_2 & F_1 & -\Sigma F \quad n=1^n & 0 \\
F_{10} & F_9 & F_8 & F_7 & F_6 & F_5 & F_4 & F_3 & F_2 & 0 & F_1 & 0
\end{bmatrix}
\]
\[
[[ZM]\{r-h\}] = \\
\begin{bmatrix}
\frac{\Delta r^2}{EI_{10}} & \frac{2\Delta r^2}{EI_{9}} & \frac{3\Delta r^2}{EI_{8}} & \frac{4\Delta r^2}{EI_{7}} & \frac{5\Delta r^2}{EI_{6}} & \frac{6\Delta r^2}{EI_{5}} & \frac{7\Delta r^2}{EI_{4}} & \frac{8\Delta r^2}{EI_{3}} & \frac{27\Delta r^2}{4E1_2} & \frac{19\Delta r^2}{8E1_1} \\
0 & \frac{\Delta r^2}{EI_{9}} & \frac{2\Delta r^2}{EI_{8}} & \frac{3\Delta r^2}{EI_{7}} & \frac{4\Delta r^2}{EI_{6}} & \frac{5\Delta r^2}{EI_{5}} & \frac{6\Delta r^2}{EI_{4}} & \frac{7\Delta r^2}{EI_{3}} & \frac{24\Delta r^2}{4E1_2} & \frac{17\Delta r^2}{8E1_1} \\
0 & 0 & \frac{\Delta r^2}{EI_{8}} & \frac{2\Delta r^2}{EI_{7}} & \frac{3\Delta r^2}{EI_{6}} & \frac{4\Delta r^2}{EI_{5}} & \frac{5\Delta r^2}{EI_{4}} & \frac{6\Delta r^2}{EI_{3}} & \frac{21\Delta r^2}{4E1_2} & \frac{15\Delta r^2}{8E1_1} \\
0 & 0 & 0 & \frac{\Delta r^2}{EI_{7}} & \frac{2\Delta r^2}{EI_{6}} & \frac{3\Delta r^2}{EI_{5}} & \frac{4\Delta r^2}{EI_{4}} & \frac{5\Delta r^2}{EI_{3}} & \frac{18\Delta r^2}{4E1_2} & \frac{13\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & \frac{\Delta r^2}{EI_{6}} & \frac{2\Delta r^2}{EI_{5}} & \frac{3\Delta r^2}{EI_{4}} & \frac{4\Delta r^2}{EI_{3}} & \frac{15\Delta r^2}{4E1_2} & \frac{11\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & 0 & \frac{\Delta r^2}{EI_{5}} & \frac{2\Delta r^2}{EI_{4}} & \frac{3\Delta r^2}{EI_{3}} & \frac{12\Delta r^2}{4E1_2} & \frac{9\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta r^2}{EI_{4}} & \frac{2\Delta r^2}{EI_{3}} & \frac{9\Delta r^2}{4E1_2} & \frac{7\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta r^2}{EI_{3}} & \frac{6\Delta r^2}{4E1_2} & \frac{5\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3\Delta r^2}{4E1_2} & \frac{3\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3\Delta r^2}{32E1_2} & \frac{5\Delta r^2}{32E1_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta r^2}{8E1_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta r^2}{32E1_1}
\end{bmatrix}
\]

where,

\( h = \) perpendicular distance from axis of rotation to flapping hinge, in.
$$[M_Z] = \begin{bmatrix} 
M_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2M_{10} & M_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3M_{10} & 2M_9 & M_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4M_{10} & 3M_9 & 2M_8 & M_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5M_{10} & 4M_9 & 3M_8 & 2M_7 & M_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6M_{10} & 5M_9 & 4M_8 & 3M_7 & 2M_6 & M_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
7M_{10} & 6M_9 & 5M_8 & 4M_7 & 3M_6 & 2M_5 & M_4 & 0 & 0 & 0 & 0 & 0 \\
8M_{10} & 7M_9 & 6M_8 & 5M_7 & 4M_6 & 3M_5 & 2M_4 & M_3 & 0 & 0 & 0 & 0 \\
8.87M_{10} & 7.87M_9 & 6.87M_8 & 5.87M_7 & 4.87M_6 & 3.87M_5 & 2.87M_4 & 1.87M_3 & 0.87M_2 & 0 & 0 & 0 \\
9.37M_{10} & 8.37M_9 & 7.37M_8 & 6.37M_7 & 5.37M_6 & 4.37M_5 & 3.37M_4 & 2.37M_3 & 1.37M_2 & 0 & 0.37M_1 & 0 \\
9.5 M_{10} & 8.5 M_9 & 7.5 M_8 & 6.5 M_7 & 5.5 M_6 & 4.5 M_5 & 3.5 M_4 & 2.5 M_3 & 1.5 M_2 & 0 & 0.5 M_1 & 0 
\end{bmatrix}$$
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>0</td>
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<td>0</td>
</tr>
</tbody>
</table>

\[ [R] = \]

\[
\begin{array}{cccccccc}
1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
The blade deflections are given by Eq. (A13) and the total blade bending moments for a hinged rotor blade are found by calculating the moments for the steady loads and each harmonic load using Eq. (A14) and superimposing these moments. The total deflections are found in the same manner.

For a Teetering Rotor System

Blade bending moments for the teetering rotor system are determined by utilizing Eq. (A10) and (A12) to calculate the steady load bending moments and the even harmonic load bending moments respectively and by using Eq. (A14) to calculate the odd harmonic load bending moments. The bending moments thus obtained are again superimposed to determine the total blade bending moments. Eq. (A11) is used to calculate the steady load blade deflections and Eq. (A13) is used to calculate all harmonic blade deflections. Again, the total deflections are determined by summing the deflections obtained from the steady load and for each harmonic.
APPENDIX V

AIRLOAD EQUATIONS

All airloads equations are taken from Young. The steady load airloads are,

$$\frac{1}{2} \rho a c \Omega^2 R^2 \Delta r (\theta X^2 + \lambda X + \frac{\mu^2 \theta}{2} + \frac{\mu^2 b^2}{4})$$

(A15)

where,

\[
\begin{align*}
\theta &= \text{blade pitch, radians} \\
X &= \text{blade station, } \frac{r}{R} \\
r &= \text{radial distance to blade station } X, \text{ in.} \\
R &= \text{radius of rotor disc, in.} \\
\lambda &= \text{inflow ratio} \\
a &= \text{flapping coefficients in the expression for } b^n \\
\chi &= \text{blade azimuth angle measured from the downwind position in direction of rotation, radians} \\
\mu &= \text{tip speed ratio} \\
\Delta r &= \text{length of blade element, in.}
\end{align*}
\]

The first harmonic airloads are,

$$\frac{1}{2} \rho a c \Omega^2 R^2 \Delta r (-a_1 X^2 + 2\mu \theta X + \frac{\mu^2 a_1}{4} - \frac{\mu^2 b_2 X}{2})$$

$$\sin \chi + (b_1 X^2 - \mu a_0 X + \frac{\mu^2 b_1}{4} - \frac{\mu^2 a_2}{4} X) \cos \chi$$

(A16)

The second harmonic airloads are,

$$\frac{1}{2} \rho a c \Omega^2 R^2 \Delta r (-2a_2 X^2 \mu b_1 X - \frac{\mu^2 a_0}{2}) \sin 2\chi +$$

$$\left(2b_2 X^2 + \mu a_1 X - \frac{\mu^2 \theta}{2}\right) \cos 2\chi$$

(A17)
APPENDIX VI

MATERIAL PROPERTIES

All properties are A values per MIL-HDBK 515 and Bruhn17.

For the 2024-T8511 aluminum extrusion:

\[
\begin{align*}
F_{tu} &= 64 \text{ KSI} \\
F_{ty} &= 56 \text{ KSI} \\
F_{su} &= 35 \text{ KSI} \\
F_{bru} &= 123 \text{ KSI} \\
F_{bry} &= 93 \text{ KSI}
\end{align*}
\]

\[E = 10.8 \text{ MSI}\]

For the 2024-T351 aluminum bar:

\[
\begin{align*}
F_{tu} &= 62 \text{ KSI} \\
F_{ty} &= 40 \text{ KSI} \\
F_{su} &= 37 \text{ KSI} \\
F_{bru} &= 118 \text{ KSI} \\
F_{bry} &= 64 \text{ KSI}
\end{align*}
\]

\[E = 10.5 \text{ MSI}\]

For the 2024-T3 Alclad:

\[
\begin{align*}
F_{tu} &= 60 \text{ KSI} \\
F_{ty} &= 45 \text{ KSI} \\
F_{su} &= 38 \text{ KSI} \\
F_{bru} &= 114 \text{ KSI} \\
F_{bry} &= 73 \text{ KSI}
\end{align*}
\]

\[E = 10.5 \text{ MSI}\]
AN4 Bolt  $P_f = 3,680$ lbs.
AN5 Bolt  $P_f = 5,750$ lbs.
AN10 Bolt  $P_f = 23,000$ lbs.

MS20426AD4 Rivet in 0.040 thick sheet, $P_f = 386$ lbs.
MS20470AD3 Rivet in 0.025 thick sheet, $P_f = 211$ lbs.

Where,

- $F_{tu}$ = ultimate tensile strength
- $F_{ty}$ = yield tensile strength
- $F_{su}$ = ultimate shear strength
- $F_{bru}$ = ultimate bearing strength
- $F_{bry}$ = yield bearing strength
- $E$ = tensile modulus
- $P_f$ = ultimate single shear strength
APPENDIX VII

STRESS CALCULATIONS

Stress Check on Extrusion

Net Section at Bolt 1:

From Load Case 3,

\[ f_t = 31,694.6 \text{ psi} \]

For yielding a fitting factor of 1.15 is used per Federal Aviation Regulations, Part 27\(^3\), in the following equation to find the margin of safety.

\[
M.S. = \frac{F_{ty}}{1.15 \cdot f_t} - 1
\]  \hspace{1cm} (A18)

Substituting the appropriate values into Eq. (A18) for yield,

\[
M.S. = \frac{56,000}{1.15 \cdot 31,694.6} - 1 = +0.53 \text{ yield}
\]

For ultimate loads a safety factor of 1.5 is used instead of a fitting factor such that,

\[
M.S. = \frac{F_{tu}}{1.5 \cdot f_t} - 1
\]  \hspace{1cm} (A19)

Substituting the appropriate values into Eq. (A19),

\[
M.S. = \frac{64,000}{1.5 \cdot 31,694.6} - 1 = +0.34 \text{ ultimate}
\]

Net Section at Bolts 3:

From Figure 39,

\[
f_t = \frac{32,052}{1.110} = 28,876 \text{ psi}
\]
From Eq. (A18),
\[
M.S. = \frac{56,000}{1.15 \cdot 28,876} - 1 = +0.68 \text{ yield}
\]

From Eq. (A19),
\[
M.S. = \frac{64,000}{1.5 \cdot 28,876} - 1 = +0.47 \text{ ultimate}
\]

The bolt in bearing margins of safety in the extrusion are calculated using Eq. (A18) and (A19) with $F_{ty}$ replaced by $F_{bry}$, $F_{tu}$ replaced by $F_{bru}$, and $f_t$ replaced by $f_{br}$, $f_{br}$ is the applied bearing stress.

Bearing at Bolt 1:
\[
f_{br} = \frac{2,089}{0.065} = 32,139 \text{ psi}
\]

Bearing at Bolts 3:
\[
f_{br} = \frac{15,287}{0.19} = 80,458 \text{ psi}
\]

Bearing at Bolts 8:
\[
f_{br} = \frac{10,146}{0.19} = 53,400 \text{ psi}
\]

The maximum bearing stress occurs at Bolts 3 and from Eq. (A18) and (A19) the margins of safety are,
\[
M.S. = \frac{93,000}{1.15 \cdot 80,458} - 1 = +0.005 \text{ yield}
\]
\[
M.S. = \frac{123,000}{1.5 \cdot 80,458} - 1 = +0.019 \text{ ultimate}
\]

### Stress Check on Relief Strap

Cross Section at Bay 1-2:

From Figure 39,
\[
f_t = \frac{2,089}{0.0765} = 27,307 \text{ psi}
\]

Cross Section at Bay 2-3:
\[
f_t = \frac{3,948}{0.153} = 25,804 \text{ psi}
\]
Net Section at Bolts 3:

\[ f_t = \frac{10,326}{0.415} = 24,882 \text{ psi} \]

The margins of safety for the maximum stress occurring in the cross section at Bay 1-2 are from Eq. (A18) and (A19) respectively,

\[
\text{M.S.} = \frac{40,000}{1.15 \cdot 27,307} - 1 = +0.27 \text{ yield}
\]

\[
\text{M.S.} = \frac{62,000}{1.5 \cdot 27,307} - 1 = +0.51 \text{ ultimate}
\]

**Stress Check in Tension Straps**

Net Section at Bolts 3:

From Figure 38,

\[ f_t = \frac{8,909}{0.30} = 29,697 \text{ psi} \]

Net Section at Bolts 4:

\[ f_t = \frac{13,379}{0.42} = 31,855 \text{ psi} \]

Net Section at Bolts 5:

\[ f_t = \frac{15,303}{0.81} = 18,893 \text{ psi} \]

Net Section at Bolts 6:

\[ f_t = \frac{19,074}{0.94} = 20,292 \text{ psi} \]

Net Section at Bolts 7:

\[ f_t = \frac{19,605}{1.32} = 14,852 \text{ psi} \]

Net Section at Bolts 8:

\[ f_t = \frac{36,000}{1.874} = 19,210 \text{ psi} \]

Net Section at Bolt 9:

\[ f_t = \frac{36,000}{1.90} = 18,948 \text{ psi} \]
From Eq. (A18) and (A19) respectively the margins of safety for the maximum stress in the net section at Bolts 4 are,

\[ \text{M.S.} = \frac{40,000}{1.15 \cdot 31,855} - 1 = +0.09 \text{ yield} \]

\[ \text{M.S.} = \frac{62,000}{1.5 \cdot 31,855} - 1 = +0.29 \text{ ultimate} \]

The maximum bearing stresses are now calculated as follows.

At Bolts 3:

\[ f_{br} = \frac{8.909}{0.18} = 49,495 \text{ psi} \]

At Bolts 8:

\[ f_{br} = \frac{16.395}{0.626} = 26,190 \text{ psi} \]

At Bolt 9:

\[ f_{br} = \frac{18,000}{0.625} = 28,800 \text{ psi} \]

From Eq. (A18) and (A19) the margins of safety for the maximum bearing stress are,

\[ \text{M.S.} = \frac{64,000}{1.15 \cdot 49,495} - 1 = +0.12 \text{ yield} \]

\[ \text{M.S.} = \frac{118,000}{1.5 \cdot 49,495} - 1 = +0.59 \text{ ultimate} \]

**Bolt Shear Check**

Eq. (A19) is used to calculate the margins of safety for ultimate bolt shearing loads only. In Eq. (A19), \( f_t \) is replaced by \( p \) and \( F_{tu} \) is replaced by \( P_s \). From Fig. 39 it is seen that the maximum shear for the AN4 bolts occur from Bolts 3. Therefore,

\[ p = \frac{8.909}{4} = 2,227 \text{ lbs.} \]

\[ \text{M.S.} = \frac{3,680}{1.5 \cdot 2,227} - 1 = +0.10 \text{ ultimate} \]
For the AN5 bolts, Bolts 8:

\[
P = \frac{16,395}{4} = 4,099 \text{ lbs.}
\]

\[
M.S. = \frac{5,750}{1.5 \cdot 4,099} - 1 = -0.06 \text{ ultimate}
\]

For the AN10 bolt, Bolt 9:

\[
P = \frac{18,000}{2} = 9,000 \text{ lbs.}
\]

\[
M.S. = \frac{23,000}{1.5 \cdot 9,000} - 1 = +0.70 \text{ ultimate}
\]

It should be noted that the negative margin of safety for the AN5 bolts does not necessarily mean that the bolts will fail above the ultimate load in a multi-fastener fitting. Instead, it is acceptable to say that the bolts will yield and deform at \( P_f = 5,750 \) lbs. such that the adjacent bolts, Bolts 7, will pick up the additional load.

**Check on Rib Attachment to Extrusion**

The ribs are designed to transfer the airload on the trailing edge to the leading edge extrusion. A triangular airload distribution is assumed as shown in Fig. A2 and the airload is reacted as a couple of the MS20426AD4 rivets attaching the ribs to the extrusion.

**Airload distribution**

\[
P_a = \text{trailing edge load, lbs.}
\]

\[
 MS20426AD4 \text{ rivets}
\]

\[
\text{rib spacing} = 6.9 \text{ in.}
\]

**Fig. A2. Rib Load**
The maximum airload occurs for Load Case 3, at $\chi = 0$, and at $X = 0.85$
and is calculated as $20 \times 6.9 = 138$ lbs.

The airload over the trailing edge is calculated by multiplying the total
airload by the ratio of the areas under the airload distribution.

$$P_a = \frac{138 \times 6.25^2}{9^2} = 666 \text{ lbs.}$$

$$P = \frac{666 \times 0.08}{0.750} = 184.6 \text{ lbs.}$$

Using Eq. (A19) the margin of safety is,

$$\text{M.S.} = \frac{386}{1.5 \times 184.6} - 1 = +0.39 \text{ ultimate}$$
LIST OF REFERENCES


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Schaefer, Raymond F., and Smith, Hamilton A., "Aerodynamic Characteristics of the NACA 8-
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