The Effects of variations in the excitation parameters of blast waves on the high frequency response of circular rings

1974

Paul John Mirabella
University of Central Florida

Find similar works at: https://stars.library.ucf.edu/rtd

University of Central Florida Libraries http://library.ucf.edu

Part of the Engineering Commons

STARS Citation

https://stars.library.ucf.edu/rtd/118

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of STARS. For more information, please contact lee.dotson@ucf.edu.
THE EFFECTS OF VARIATIONS IN THE EXCITATION PARAMETERS OF BLAST WAVES ON THE HIGH FREQUENCY RESPONSE OF CIRCULAR RINGS

BY

PAUL JOHN MIRABELLA
B.S., Polytechnic Institute of Brooklyn, 1971

THESIS

In Partial Fulfillment of the Requirements for the Degree of Master of Science in Engineering Florida Technological University

Orlando, Florida
1974
THE EFFECTS OF VARIATIONS IN THE EXCITATION PARAMETERS OF BLAST WAVES ON THE HIGH FREQUENCY RESPONSE OF CIRCULAR RINGS

Paul John Mirabella

ABSTRACT

The high frequency response of a circular ring of rectangular cross section interior to a conical shell excited by a blast wave is examined. The ring supports a rigidly attached mass during excitation. It is hypothesized that the response is a function of the four excitation parameters which characterize the loading. These are: peak reflected pressure; characteristic time (pulse duration); wave engulfment time; and circumferential distribution. These parameters are varied over a range of interest in an effort to ascertain the structural sensitivity to such perturbations.

A series of tests sponsored by the Department of the Army and the Martin Marietta Corporation were conducted by the Stanford Research Institute on the missile. Data acquired during these tests will be used to verify and support the hypothesis. In addition analytic correlation is presented based on parametric studies performed on a simple two degree of freedom ring model excited by a local pressure forcing function.

The experimental data indicated a linear dependence of the structural response on variations in pressure, duration and engulfment. The analytic results indicated higher sensitivities by comparison, but supported the experimental results in general.

David L. Block
Director of Thesis
All pressures appearing in this text have been non-dimensionalized for reasons of National security. The values indicated are obtained by a ratio method from an undisclosed base pressure and appear in their present form to facilitate an understanding of the effects of increase or decreases (i.e., ± 30%, ± 40%, etc.) in that parameter.
TABLE OF CONTENTS

List of Symbols and Terms ........................................ v

I. EXPERIMENTAL PROGRAM ...................................... 1

   General
   Test Model and Charge Configurations
   Test Results - General
   Rationale for Utilization of Triaxial Envelope
    in Data Presentation

II. PARAMETRIC DEVIATIONS AND RESULTING RESPONSE EFFECTS .... 26

   The Effects of Variations in Peak Reflected Pressure
   The Effects of Variation in Duration
   The Effects of Variation in Engulfment Time
   The Effects of Variations in Circumferential Distribution

III. ANALYTICAL CORRELATION .................................... 62

   Structural Modeling - General
   Solutions to the Forced Ring Vibration for Excitation
    by Planar and Circular Blast Waves
   Planar Blast Wave Excitation
   Conclusions

IV. APPENDIX .................................................... 75

V. BIBLIOGRAPHY .................................................. 85
LIST OF SYMBOLS AND TERMS

\( P_r: \) Pressure; refers to peak reflected pressure recorded at the designated transducer stations in psi

\( I: \) Impulse; the integral of pressure with respect to time \( t_1 \) to \( t_2 \) such that \( t_1 \) corresponds to \( P_{r_{\text{max}}} \) and \( t_2 \) corresponds to \( P(t) = \frac{1}{10} P_{r_{\text{max}}} \)

\( P_{r_{\text{max}}}: \) Maximum peak reflected pressure measured at \( 0^\circ \) gage

Duration: The quotient of Impulse divided by peak pressure (characteristic time)

Engulfment Time: Arbitrarily the time difference in \( \mu \text{sec} \) between the response of the \( 0^\circ \) pressure transducer and the \( 90^\circ \) transducer multiplied by two.

Stand-off: Distance in feet from the ring to the explosive

Station 54: Refers to the location measured in inches aft of the nose

\( P_{op}: \) Overpressure

Side on Encounter: Blast wave traveling perpendicular to the center-line of the missile

Head on Encounter: Blast wave traveling parallel to missile centerline
I. EXPERIMENTAL PROGRAM

General

The purpose of the test program was to determine the response of a ring interior to a missile (conical shell structure) produced by a simulated blast wave entry. The various blast entry conditions (altitude, velocity, angle of attack) have associated blast excitation parameters (pressure, duration, engulfment and circumferential distribution). The objectives of this study are:

- To vary the blast entry parameters independently over a range of interest
- To measure and record the transient response of the ring
- To present the data in a manner facilitating the analysis and indicating the relative magnitude and sense (increasing-decreasing) of a variation
- To assess the sensitivity of the structural response to perturbations in the blast excitation parameters.

The actual encounter conditions (altitude, velocity, etc.) are classified SECRET and will not be presented or referenced in any part of this document. However, the associated excitation parameters are unclassified when examined independently. The two circumferential distributions examined are those produced by a side-on encounter and head-on encounter. In the former case a planar wave traveling perpendicular to the missile flight axis is swept across the missile. The forcing functions produced by these encounters are examined only on the windward half of the ring. Past experience and calibration measurements have shown the pressures on the leeward half of the ring to be negligible by comparison.
The resulting forcing function for the side-on encounter is described by an exponentially decaying, step pressure pulse whose load distribution as a function of \( \theta \) (independent of time) is:

\[
P(\theta) = (P_{\text{r max}} - P_{\text{op}}) \cos^2 \theta + P_{\text{op}} \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}
\]  

(1)

where \( P_{\text{r max}} \) is the peak reflected pressure measured at the windward ray, \( 0^\circ \), and \( P_{\text{op}} \) is the over-pressure measured at 90\(^\circ\). For the head-on encounter a uniform load distribution is generated.

The pressure pulse at the windward ray location of the exterior ring can be described as shown in Figure 1.

![Typical Pressure Pulse](image)

**FIGURE 1. Typical Pressure Pulse**

By integrating the pressure-time function above with respect to time the impulse is obtained. The quotient of the impulse divided by the peak pressure gives the value of the characteristic time, \( t_o \), commonly referred to as the duration. An approximation of this value may be obtained by constructing the tangent to the initial decay curve. The duration is varied over a range of 100 \( \mu \text{sec} \leq 350 \mu \text{sec} \).
The engulfment time is a measure of the velocity with which the blast front passes over the missile at the particular ring station of concern. Arbitrarily, the value assigned is the time lag between the response of the 0° gage and the 90° gage multiplied by two. Obviously this parameter is only applicable to loadings described by equation (1), since all head-on encounters would produce instantaneous engulfment. The range examined varied from 150 μsec ≤ engulfment ≤ 450 μsec.

On this basis the experimental program was divided into six test Series each examining a specific set of excitation parameters at five different pressures. Table I defines the test matrix.

### TABLE I

**TEST MATRIX**

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Pressure Range</th>
<th>Duration μsec</th>
<th>Engulfment μsec</th>
<th>Circumferential Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50 - 500</td>
<td>100</td>
<td>350</td>
<td>( \cos^2\theta )</td>
</tr>
<tr>
<td>2</td>
<td>50 - 500</td>
<td>300</td>
<td>350</td>
<td>( \cos^2\theta )</td>
</tr>
<tr>
<td>3</td>
<td>50 - 350</td>
<td>100</td>
<td>Instantaneous</td>
<td>Uniform</td>
</tr>
<tr>
<td>4</td>
<td>50 - 350</td>
<td>300</td>
<td>Instantaneous</td>
<td>Uniform</td>
</tr>
<tr>
<td>5</td>
<td>50 - 350</td>
<td>100</td>
<td>150</td>
<td>( \cos^2\theta )</td>
</tr>
<tr>
<td>6</td>
<td>50 - 350</td>
<td>300</td>
<td>150</td>
<td>( \cos^2\theta )</td>
</tr>
</tbody>
</table>

**Test Model and Charge Configurations**

The basic structure used in the program consisted of the Guidance and Control, Warhead, and Nose sections of the Sprint II Missile. In order to simulate the correct mass and center of gravity, engineering models were substituted for actual missile components. The ring examined throughout the program is located at Station 54 and supports the Missile Controller Set. (See Figure 2.)
The ring was instrumented with Endevco piezoelectric accelerometers (No. 2292) for measuring the shock transients. The system frequency response was approximately 20,000 Hz. Although the system was calibrated prior to testing, pressure transducers were also located on the structure to verify the forcing function. Figure 3 locates the triaxial accelerometers on the ring. A typical pressure-time history is shown in Figure 4.

The shock transients obtained were analyzed using a SPECTRAL DYNAMICS Shock Spectrum Analyzer. The shock spectra were used as a means of evaluating the structural response sensitivity. Typical acceleration time histories and shock spectra are shown in Figures 5 and 6 respectively.

Tests were performed by exposing the structure to explosive of arrays of various densities, standoff distances and charge configurations. Figure 7 depicts the various charge configurations necessary to achieve the perturbations in the blast excitation parameters. Figure 8 explains the numbering system used for test designation. Figures 9 - 14 depict the actual model and the corresponding explosive arrays. In addition photographic coverage was provided utilizing FASTEX filming. Examination of these films verified such criteria as blast wave planarity and engulfment times. Figure 15 depicts the photographic coverage.
FIGURE 3. Ring Accelerometer Locations
FIGURE 4. Typical Oscilloscope Pressure Record
FIGURE 5. Typical Oscillograph Records of Accelerometer Measurements

Records of Test S2.5-4C Pressure = 262 Impulse = 5115
FIGURE 6. Typical Shock Spectrum
Figure 7. Charge Configuration

Figure 8. Explanation of Test Designation

<table>
<thead>
<tr>
<th>S</th>
<th>1.5</th>
<th>4</th>
<th>SC</th>
</tr>
</thead>
</table>
| DESIGNED CHARGE CONFIGURATION*.
| DESIGNATES TEST NUMBER.
| DESIGNATES STAND-OFF DISTANCE IN FEET.
| DESIGNATES SPRINT VEHICLE (S) OR CALIBRATION MODEL (C12)

*NO LETTER → FLAT
C → CIRCULAR
SC → SEMI-CIRCULAR
FIGURE 10. PRIMACORD - 4.0' Standoff
FIGURE 11. Series III - DETASHEET - 1.0' Standoff Uniform Distribution
FIGURE 13. Series V - DETASHEET - 1.5' Semi-Circular
Test Results - General

This section qualitatively examines the effects of blast encounter excitation parameter variations on the structural response of the ring. The data acquired is presented in the form of comparative shock spectra such that the relative magnitude of the response variation is apparent. The sensitivity of the structural response to the blast parameter perturbations is plotted in Figures 32, 42 and 48. Due to the amount of data acquired during the program, all of the shock spectra will not be presented to avoid burdening the reader with superfluous information. However, all data will be incorporated in the appendix for reference.

Evaluations of the response effects were achieved by selecting tests having approximately equal values in three of the four blast excitation parameters and comparing the resulting ring shock spectra. For example, to obtain a data point in evaluation of duration, two tests would be selected as shown in Table II.

<table>
<thead>
<tr>
<th>Test No. Designation</th>
<th>Pressure</th>
<th>Engulfment (μsec)</th>
<th>Circumferential Distribution</th>
<th>Duration μsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.5-2</td>
<td>145</td>
<td>515</td>
<td>$\cos^2\theta$</td>
<td>120</td>
</tr>
<tr>
<td>S-4.0-2</td>
<td>148</td>
<td>545</td>
<td>$\cos^2\theta$</td>
<td>356</td>
</tr>
</tbody>
</table>

Common Values

In this manner comparisons were made for all excitation parameters provided the valid test data was available. Since the actual peak reflected pressures experienced during the various test series were not necessarily equal, four common reference pressures were selected for comparison. These pressures are 90, 150, 225 and 300... For each
comparison test data obtained at pressures within 5% of these reference values was used. Where data available was not within 5%, the available shock spectra was scaled linearly to match the reference pressure before comparisons were made.

Table III lists the matrix of test conditions as they were utilized in all comparisons. The values presented are nominal values associated with the particular series and are used as constants only to facilitate the comparison.

**TABLE III**

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Pressure (llSec)</th>
<th>Duration (μsec)</th>
<th>Engulfment (μsec)</th>
<th>Circumferential Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>118.2</td>
<td>430</td>
<td>cos²θ</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>354</td>
<td>450</td>
<td>cos²θ</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>103</td>
<td>Instantaneous</td>
<td>Uniform</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>232</td>
<td>Instantaneous</td>
<td>Uniform</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>135</td>
<td>130</td>
<td>cos²θ</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>353</td>
<td>160</td>
<td>cos²θ</td>
</tr>
<tr>
<td></td>
<td>225</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Those tests having anomalous data resulting from electrical disturbances and drastically asymmetric loadings (partial detonations) are not used for comparison. Throughout the report nominal values are used exclusively except where the actual test conditions accent a specific characteristic or phenomenon. These exceptions are appropriately noted.

Most of the shock spectra presented in this report are triaxial envelopes. The only exceptions occur when data sufficient to construct triaxial envelopes does not exist for the tests being compared. In these cases, double and single axis plots are presented. The rationale for comparing environments on the basis of triaxial spectra is discussed subsequently.

In order to examine the effects of variations in the excitation parameters, the following comparisons of shock spectra are made:

1. Pressure: Within each series, pressure was varied from 50 to 300 (minimum). Thus by plotting the response for the four pressures (90, 150, 225, 300) and a given set of conditions (i.e. series), the effects of pressure may be displayed.

2. Duration: Comparison of Series I vs Series II; Series III vs Series IV; Series V vs Series VI

3. Engulfment: Comparison of Series I vs Series V; Series II vs Series VI

4. Distribution: Comparison of Series III vs Series V

It must be noted that testing is still in the development stage. Therefore, it is not possible to maintain common durations and engulfment times for all series.
In some cases experimental difficulties made comparisons impossible because more than one parameter was varied. Noteworthy examples are as follows:

1. In the case of uniform distribution (Series III and IV), the duration characteristic time was only varied for 103 μsec and 232 μsec (nominal). Thus, it is difficult to make conclusive statements regarding the overall effect of duration for such conditions.

2. Because a $\cos^2\theta$ distribution could not be achieved at 0 μsec engulfment time, the effects of circumferential distribution cannot be evaluated in a rigorous manner. However, information regarding certain trends at various frequencies can be obtained by comparing Series III and IV (uniform, instantaneous engulfment) with those tests in Series V and VI, respectively, that resulted in 70 - 100 μsec engulfment times with $\cos^2\theta$ loadings.

Rationale for Utilization of Triaxial Envelope In Data Presentation

In an effort to present the data in a manner facilitating the evaluation the method of enveloping the three orthogonal axes response is incorporated. Although a unidirectional study is more rigorous, a sample comparison demonstrates that there is no strong directional sensitivity and consequently little to be gained from such a treatment of the data. In addition, availability of data and instrumentation difficulties prohibited the exclusive use of the blast axis data for certain tests.
Figures 16, 17, and 18 depict the development of a triaxial envelope based on the unidirectional responses from tests S-1.0-1C, -3C and -4C. Figure 19 represents the effects of increasing peak reflected pressure based on the three envelopes developed. Thus the data presentation enables the fulfillment of one of the major objectives; an indication of the relative magnitude and sense of the variation.
FIGURE 16. Development of Triaxial Envelope - S-1.0-1C
FIGURE 17. Development of Triaxial Envelope - S-1.0-3C

PEAK ACCELERATION IN G'S

10^5

10^4

10^3

10^2

10^1

10^2

10^3

10^4

10^5

PRESSURE: 229
DURATION: 103 μsec (nominal)
ENVELOPE: 0 μsec
DISTRIBUTION: UNIFORM

TRIAXIAL ENVELOPE

Z-AXIS

Y-AXIS

X-AXIS

DAMPING RATIO = 0.03

FREQUENCY - Hz
FIGURE 18. Development of Triaxial Envelope - S-1.0-4C
FIGURE 19. Effects of Pressure - Triaxial Envelope
II. PARAMETRIC DEVIATIONS
AND RESULTING RESPONSE EFFECTS

The Effects of Variations in Peak Reflected Pressure

The development of the sensitivity of the response to perturbations in a parameter is dependent on the mathematical relationship between the transient acceleration and the parameter. An examination of the generalized equation of motion (Equation 2) for the ring under blast excitation suggests a linear variation of acceleration with peak reflected pressure measured at the windward ray ($P_{r_{\text{max}}}$).

$$\ddot{\xi} + 2\mu \dot{\xi} + \omega_n^2 \xi = \left( \frac{1}{M} \right) \left( \frac{1}{2\pi} \right) \int_0^{2\pi} F(\theta, t) \cos \theta d\theta$$

(2)

where $F(\theta, t) = [(P_{r_{\text{max}}} - P_{\text{op}}) \cos^2 \theta + P_{\text{op}}] e^{-t/t_0} T_n R$

For "side-on" blast loadings

$$= P_{r_{\text{max}}} e^{-t/t_0} T_n R$$

for uniform loading independent of $\theta$

$P_{\text{op}}$ = overpressure

$\xi$ = generalized coordinate

$\mu$ = damping coefficient

$R$ = radius of center line of ring

$T$ = thickness of the ring

$n = 0, 1, 2, 3, 4, 5 \ldots$ mode number

$\omega_n^1$ = natural undamped frequency of the ring

\[
\omega_n = \sqrt{\left(\frac{E}{\gamma}\right) \left(\frac{1}{AR^4}\right) \left(\frac{n^2(n^2-1)}{(n^2+1)}\right)} \quad n = 2, 3, 4 \ldots \quad (3a)
\]

\[
\omega_n = \sqrt{\left(\frac{E}{\gamma R^2}\right)} \quad n = 0 \quad (3b)
\]

\(\gamma\) = weight density

\(t_o\) = characteristic time

\(M\) = mass = \(2\pi \frac{\gamma}{g} A\)

\(A\) = cross-sectional area of the ring

In the case of head-on or uniform loading only the \(n = 0\) or extentional breathing mode is excited. Consequently the solution to Equation (2) (for this case) is of the form (Equation 4)

\[
\xi = e^{-\mu_0 t} (A \cos P_o t + B \sin P_o t) + C e^{-t/t_o} \quad (4)
\]

where \(P_o^2 = \omega_o^2 - \mu_o^2\)

Solving the particular equation and applying the initial conditions \(\xi(0) = \dot{\xi}(0) = 0\) yields a solution from which the acceleration may be generated by differentiation (Equation 5)

\[
\ddot{\xi} = P_{r,\text{max}} \left[ \frac{gT_o^2}{(1 - 2\mu_o t_o + (\omega_o t_o)^2)} \right] \left\{ (P_o^2 - 2\mu_o D P_o - \mu_o^2 \cos P_o t) - (D P_o^2 + 2\mu_o P_o - \mu_o^2 D) \sin P_o t + \frac{1}{t_o^\frac{1}{2}} e^{-t/t_o} \right\} \quad (5)
\]

where \(D = \frac{1}{P_o} \left( \frac{1}{t_o} - \mu_o \right)\)

Thus one would expected a linear variation of acceleration with pressure for uniform loading.

For the \(\cos^2\theta\) or side-on encounter, the solution to the generalized equation of motion is a Fourier Series summed over the number of
in-plane flexural modes considered. However all of the coefficients will contain a common $P_{r_{\text{max}}}$ term, producing a linear dependence. Figures 20 through 25 plot the values of peak transient acceleration with respect to pressure for the various test series against a solid line representing a linear least squares fit. Although a slight non-linearity is noted at pressures above 300, the approximation of a linear pressure-acceleration relationship is justified.

Figures 26 through 31 depict the shock response spectra variation with pressure. By digitizing this data and evaluating the percentage increase in response (peak acceleration) for a given increase in pressure, the sensitivity of the ring to pressure is ascertained. This analysis was conducted for frequencies corresponding to the first nine in-plane flexural bending modes computed from Equations 3a and 3b. Table IV lists the frequencies of interest. Since our system frequency response was limited to 20,000 Hz, $n = 9$ is the highest mode considered. The $n = 1$ or rigid body motion of the ring is disregarded because of the obvious physical constraints of the missile.
### TABLE IV

RING NATURAL FREQUENCIES

<table>
<thead>
<tr>
<th>Mode n=</th>
<th>Frequency ($\omega_n$) In Radians/sec</th>
<th>Frequency ($f_n$) In Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30366.</td>
<td>4832.89</td>
</tr>
<tr>
<td>2</td>
<td>3696.0</td>
<td>688.30</td>
</tr>
<tr>
<td>3</td>
<td>10455.</td>
<td>1663.96</td>
</tr>
<tr>
<td>4</td>
<td>20047.</td>
<td>3190.57</td>
</tr>
<tr>
<td>5</td>
<td>32421.</td>
<td>5159.96</td>
</tr>
<tr>
<td>6</td>
<td>47561.</td>
<td>7569.56</td>
</tr>
<tr>
<td>7</td>
<td>65461.</td>
<td>10418.44</td>
</tr>
<tr>
<td>8</td>
<td>86120.</td>
<td>13706.42</td>
</tr>
<tr>
<td>9</td>
<td>109540.</td>
<td>17433.83</td>
</tr>
</tbody>
</table>

where $\omega_n = 2\pi f_n$

After examination of the sensitivity at the frequencies listed in Table IV, the average sensitivity was obtained and is plotted in Figure 32. This result supports the postulation that deviations in structural response are linearly dependent upon pressure variation.
FIGURE 20. Series I

FIGURE 21. Series II
FIGURE 22. Series III

FIGURE 23. Series IV
ACCELERATION IN G'S X 10^3

PRESSURE

FIGURE 24. Series V
FIGURE 25. Series VI
FIGURE 26. Effects of Variation in Pressure - Series I
FIGURE 27. Effects of Variation in Pressure - Series II
FIGURE 28. Effects of Variation in Pressure - Series III

PEAK ACCELERATION IN G'S

10^5

DURATION = 103.25 μsec
ENSUPEMENT = 0 μsec
DISTRIBUTION = UNIFORM

SERIES III

DAMPING RATIO = 0.3

FREQUENCY - Hz

300 CENTERI
225
150 (x,y) ONLY
90
FIGURE 29. Effects of Variation in Pressure - Series IV
FIGURE 30. Effects of Variation in Pressure - Series V
FIGURE 31. Effects of Variation in Pressure - Series VI
FIGURE 32. Peak Pressure Deviation vs Ring Structural Response Deviation
The Effects of Variation in Duration

Figures 33 through 41 present a quantitative comparison of the shock response spectra for various durations, under various load conditions. These responses were subjected to an analysis similar to that detailed in the previous section. Having ascertained the sensitivity at the frequencies of interest (reference Table IV) for the various Distribution - Engagement load conditions, an average was computed. The resulting plot (Figure 42) indicates that structural response variation is linearly dependent on variations of duration, but that the slope of the function varies with loading and engulfment.
FIGURE 33. Effects of Variation in Duration - Series I vs Series II
FIGURE 34. Effects of Variation in Duration - Series I vs Series II
FIGURE 35. Effects of Variation in Duration - Series I vs Series II
FIGURE 36. Effects of Variation in Duration - Uniform Loading
FIGURE 37. Effects of Duration - Series III vs Series IV
FIGURE 38. Effects of Duration - Series III vs Series IV
FIGURE 39. Effects of Duration - Series V vs Series VI
FIGURE 40. Effects of Duration - Series V vs Series VI
FIGURE 41. Effects of Duration - Series I vs Series II
\[ \cos^2 \theta = \text{DISTRIBUTION} \]

450 \( \mu \text{SEC} = \text{ENGULFMENT} \)

**BASE DURATION = 100 \( \mu \text{SEC} \)**

\[ \cos^2 \theta = \text{DISTRIBUTION} \]

150 \( \mu \text{SEC} = \text{ENGULFMENT} \)

**BASE ACCELERATION = 1000g's**

**FIGURE 42. Duration Deviation vs Structural Response Deviation**
The Effects of Variation in Engulfment Time

Engulfment time was varied nominally from 450 \( \mu \text{sec} \), while maintaining approximately constant duration, pressure and circumferential distribution. Figures 43 through 47 graphically depict the resulting response variations.

An analysis of the response sensitivity (Figure 48) indicates that although the relationship is approximately linear, the structural response sensitivity was considerably less significant for engulfment time variation when compared with that of pressure or duration.
FIGURE 43. Effects of Engulfment Time - Series I vs V
FIGURE 44. Effects of Engulfment Time - Series I vs Series V
FIGURE 45. Effects of Variation in Engulfment - Series I vs Series V
FIGURE 46. Effects of Engulfment Time - Series II vs Series VI
FIGURE 47. Effects of Variation in Engulfment Time - Series II vs Series VI.
Figure 48. Engulfment Time Deviation vs Structural Response Deviation

Base engulfment time = 300 μsec

Base acceleration = 1000g's
The Effects of Variations in Circumferential Distribution

The experimental program utilized two peripheral load distributions simulating head-on and side-on missile blast sphere entries. The resulting distributions were uniform and \( P = (P_{\text{max}} - P_{\text{op}}) \cos^2 \theta + P_{\text{op}} \) respectively. Since little or no intentional perturbation was achieved about these loadings, only a comparison may be drawn. In addition the engulfment time variation was not entirely independent of distribution, thus the comparison is not based on loading variation alone. However, a comparative shock spectrum does demonstrate the influence of the breathing mode \((n = 0)\) excitation produced by the uniform loading. As Figure 49 indicates the Series III (uniform) loading characteristiclly has a substantially greater high frequency content. Nevertheless, a gross deviation from \( \cos^2 \theta \) is required to produce such a change.
FIGURE 49. Effects of Circumferential Distribution
III. ANALYTICAL CORRELATION

Structural Modeling - General

The analytical study of the blast excitation parameter variations was conducted by driving a simple ring model with an aerodynamic forcing function code. The code simulated the environment resulting from missile/blast wave encounters by providing a pressure-time history at various locations, \( \theta \), on the exterior windward half of the ring. These pressure distribution profiles, at given intervals of time comprise the forcing function. The ring model numerically computes the transient acceleration-time history, by evaluating the solution to the generalized equation of motion (Equation 2) for the in-plane flexural bending vibration modes \( n = 0 \) \( n = 2, 3, \ldots, 9 \). Torsional vibration and flexural vibration normal to the plane of the ring are of secondary importance due to the restrictive boundary conditions of the missile on the ring and are, therefore, neglected. The computed acceleration time history then serves as the input to a shock spectrum response code. In the latter analyses the acceleration-time history is utilized as the base excitation of a one degree of freedom system. The peak accelerations of this system for a given value of damping and a range of frequencies comprises the shock spectra. Having the analytical response in the same form as the experimental data now permits a direct comparison. Evaluating the percent response variation for a given percent change in a specific blast excitation parameter, the structural sensitivity is analytically ascertained.
In order to provide a ring model which reasonably approximated the acquired response data from the tests, it was necessary to impose a feedback loop in the development of the code. By varying the values of modal damping, spectral shaping was achieved. Such shaping iterations continued until a reasonable match was achieved. Figure 50 depicts an example of the spectra generated analytically using input data (forcing functions) similar to those used in Test Series III. Figure 51 provides a flow chart to facilitate the understanding of the method of analysis. The subsequent sections deal with the solution to the forced vibration of the ring utilized by the ring model.

Solutions to the Forced Ring Vibration for Excitation by Planar and Circular Blast Waves

Uniform Distribution (Circular Blast Wave)

A purely uniform excitation would excite only the breathing (extentional) mode and the solution is generated as follows:

Equation of motion: \[ \ddot{\xi} + 2\mu_o \dot{\xi} + \omega_o^2 \xi = \frac{gT}{2\pi\gamma A} P_{r_{\text{max}}} e^{-t/t_o} \] (2) ref.

Initial Conditions \[ \xi (0) = \dot{\xi} (0) = 0 \]

Solution: \[ \xi = e^{-\mu_o t} (A \cos (P_o t) + B \sin P_o t) + C e^{-t/t_o} \] (4) ref.

where \[ P_o^2 = \omega_o^2 - \mu_o^2 \quad \omega_o = \sqrt{\frac{E}{\gamma R^2}} \]

Solution to the particular equation yields

\[ C = \frac{gT \ t_o^2 P_{r_{\text{max}}}}{[1 - 2\mu_o t_o + (\omega_o t_o)^2]} \] (6)

Applying the Initial Conditions

\[ A = -C \quad B = \frac{C}{P_o} \left( \frac{1}{t_o} - \mu \right) \] (7)
FIGURE 50. Analytically Produced Shock Spectra
AERODYNAMIC FORCING FUNCTION CODE

PRESSURE DISTRIBUTION PROFILE

RING MODEL

ACCELERATION TIME HISTORY

BASE

SHOCK RESPONSE CODE

PEAK ACCELERATION VS FREQUENCY

TEST DATA

COMPARISON

GOOD

SENSITIVITY ANALYSIS

FIGURE 51. Analytical Model Flow Chart
thus

\[ \xi = C \ e^{-\mu_0 t} \left[ \left( \frac{1}{P_0 t_0} - \frac{\mu_0}{t_0} \right) \sin P_0 t - \cos P_0 t \right] + C \ e^{-t/t_0} \]  

\[ \xi = C \left[ (P_o^2 - 2\mu_o D P_o - \mu_o^2) \cos P_0 t - (D P_o^2 + 2\mu_o P_0 - \mu_o^2 D) \sin P_0 t + \frac{1}{t_0^2} e^{-t/t_0} \right] \]  

where \( D = \frac{1}{P_0} \left( \frac{1}{t_0} - \mu_0 \right) \)

**Planar Blast Wave Excitation**

For the side-on encounter condition the forcing function is given by

\[ F(S,t) = \int_{-\pi/2}^{\pi/2} P(\theta) e^{-t/t_0} \cos \theta \ TRd\theta \]  

where \( P(\theta) = (P_{\text{max}} - P_{op}) \cos^2 \theta + P_{op} \) (1) ref.

The complex transient solution will then be expressed as the summation of Fourier series resulting from the generalized forces in modes \( n = 0 \) through \( n = 9 \) (\( n = 1 \), rigid body motion, excluded). The Generalized Masses are given by :

**TABLE V**

<table>
<thead>
<tr>
<th>Generalized Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
</tr>
<tr>
<td>( n = )</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
</tbody>
</table>
let \( P_{\text{r max}} - P_{\text{op}} = A \)
\[ P_{\text{op}} = B \]

Equation (10) becomes

\[
\frac{RT e^{-t/\tau_0}}{GM_n} \int_{-\pi/2}^{\pi/2} (A \cos^2 \theta + B) \cos n\theta \, d\theta
\]

From symmetry the limits can be changed to 0 \( \int_{0}^{\pi/2} \).

Integrating distributively we note for all modes \( 0 \leq n \leq 9 \)

\[
2 \int_{0}^{\pi/2} B \cos n\theta \, d\theta = \frac{2B}{n} \sin n\theta \bigg|_{0}^{\pi/2} = \frac{2B}{n}
\]

Examining the \( n = 2 \) mode

\[
2 \int_{0}^{\pi/2} A \cos^2 \theta \cos 2\theta = 2A \int_{0}^{\pi/2} (\cos^4 \theta - \cos^2 \theta \sin^2 \theta) \, d\theta
\]

\[
2A \int_{0}^{\pi/2} \cos^4 \theta \, d\theta = 2A \left[ \frac{1}{4} \cos^3 \theta \sin \theta + \frac{3}{4} \int_{0}^{\pi/2} \cos^2 \theta \, d\theta \right]_{0}^{\pi/2}
\]

\[
= 2A \left[ \frac{1}{4} \cos^3 \theta \sin \theta + \frac{3}{4} \left( \frac{1}{2} \sin \theta \cos \theta + \frac{1}{2} \theta \right) \right]_{0}^{\pi/2}
\]

\[
= 2A \left( \frac{3}{4} \cdot \frac{\pi}{4} \right) = \frac{3\pi A}{8}
\]

\[
-2A \int_{0}^{\pi/2} \cos^2 \theta \sin^2 \theta \, d\theta = + \frac{1}{32} \sin^4 \theta - \frac{\theta}{8} \bigg|_{0}^{\pi/2}
\]

\[
= - \frac{\pi A}{8}
\]

\[
2A \int_{0}^{\pi/2} A \cos^2 \theta \cos 2\theta = \frac{3\pi A}{8} - \frac{\pi A}{8} = \frac{\pi A}{4}
\]
thus for $n = 2$ we have
\[
\frac{GF_2}{GM_2} = R e^{-t/t_o} \left[ \frac{\pi A}{4} + B \right]
\]

similarly for $n = 3$
\[
2A \int_0^{\pi/2} \cos^2 \theta \cos 3\theta \, d\theta = 2A \int_0^{\pi/2} \cos^2 \theta \left[ 4 \cos^3 \theta - 3 \cos \theta \right] \, d\theta
\]
\[
= 8A \int_0^{\pi/2} \cos^5 \theta \, d\theta - 6A \int_0^{\pi/2} \cos^3 \theta \, d\theta
\]
\[
= \frac{64}{15} A - 4A = \frac{4}{15} A
\]

thus for $n = 3$
\[
\frac{GF_3}{GM_3} = R e^{-t/t_o} \left( \frac{10}{9} \pi A Y \right) \left[ \frac{4A + 10}{15} \right]
\]
\[
= \frac{3Te^{-t/t_o}}{25 \pi A Y} \left[ 2P_{\text{max}} + 5P_{\text{op}} \right]
\]
The process of integration is continued over the modes considered until we can express the generalized equation as
\[
\sum_{n=0}^{9} \left( \ddot{\xi} + 2\mu_n \dot{\xi} + \omega_n^2 \xi \right) = \sum_{n=0}^{9} \frac{F_n}{GM_n}
\]
(Neglecting $n = 1$ rigid body motion)

Solving these equations for the accelerations and summing yields a time dependent solution which approximates the complex transient response of the ring. By utilizing this acceleration time history as the base excitation for a single degree of freedom system the acceleration shock spectra is obtained. The natural frequency of the system,
\[
\omega_n = \sqrt{\frac{K}{M}},
\]
is varied such that $100 \text{ Hz} \leq (f_n = \frac{\omega_n}{2\pi}) \leq 20,000 \text{ Hz}$. 
Having ascertained the various spectra the sensitivity of the ring model response to the various parameter perturbations is established for the frequencies listed in Table IV. These sensitivities were averaged. Figure 52 depicts the results of this analysis for deviations in peak pressure, duration and engulfment time.

Figures 53, 54, and 55 compare the analytical results with the experimental data presented in Section I.

Conclusions

The high frequency response of the circular ring considered was found to be dependent on the blast excitation parameters of the findings of this paper indicates:

1. The mathematical relationship between peak reflected pressure and transient acceleration is slightly non-linear, however, it may be approximated by a linear curve fit within an acceptable error margin.

2. Analysis of the structural sensitivity of the ring to variations in pressure and duration indicated a linear relationship. Such behavior was verified experimentally, however, the structural sensitivity to these parameters was less severe than the analysis predicted.

3. Variations in engulfment time appeared to be of secondary importance in comparison to the response deviations produced by pressure and duration perturbations.

4. Although the response of the ring was shown to be a function of circumferential distribution the explosive testing prohibited a conclusive examination of this parameter.
FIGURE 52. Parameter Variations vs Resulting Structural Response Variations In Ring Model
FIGURE 53. Pressure Deviation vs Response Deviation
FIGURE 54. Percent Duration vs Structural Response Deviation
Figure 55. Engulfment Time Variation vs Structural Deviation
5. Gross deviations from a $\cos^2 \theta$ distribution to a uniform loading produced an increase in ring acceleration high frequency content. This was attributed to the excitation of the $n = 0$ or extentional breathing mode.

Inherent non-linearities in the parameter-response relationships are the result of losses due to local slippage of the attached mass with respect to the ring. The investigation of this phenomenon and any other resulting from the multi-layer shell structure is beyond the scope and intent of this report. However, the analytical modeling enabled one to gain incite as to the magnitude and sense of the expected parameter variations.
IV. APPENDIX

Blast Excitation Parameter Data

SERIES I

Pressure Range . . . . . . . 50-500
Duration . . . . . . . . . . 100 μsec
Engulfment Time . . . . . . 350 μsec
Distribution . . . . . . . . cos²θ

TABLE VI

SERIES I - BLAST EXCITATION PARAMETER DATA

Recorded Values:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0º (K-Taps)</th>
<th>Impulse K-Taps</th>
<th>Duration μsec</th>
<th>Engulfment μsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.5-1</td>
<td>92</td>
<td>.850</td>
<td>129</td>
<td>545</td>
</tr>
<tr>
<td>S-1.5-2</td>
<td>145</td>
<td>1.250</td>
<td>120</td>
<td>515</td>
</tr>
<tr>
<td>S-1.5-3</td>
<td>230</td>
<td>2.060</td>
<td>125</td>
<td>435</td>
</tr>
<tr>
<td>S-1.5-4</td>
<td>340</td>
<td>2.650</td>
<td>112</td>
<td>354</td>
</tr>
<tr>
<td>S-1.5-5</td>
<td>630</td>
<td>4.750</td>
<td>105</td>
<td>300</td>
</tr>
<tr>
<td>S-1.5-6</td>
<td>80</td>
<td>.730</td>
<td>130</td>
<td>608</td>
</tr>
</tbody>
</table>
SERIES II

Pressure Range ........ 50-500

Duration ............. 300 μsec

Engulfment Time ....... 350 μsec

Distribution ........... $\cos^2 \theta$

TABLE VII

SERIES II - BLAST EXCITATION PARAMETER DATA

Recorded Values:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Impulse K-Taps</th>
<th>Duration μsec</th>
<th>Engulfment μsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-4.0-1</td>
<td>90</td>
<td>2.060</td>
<td>326</td>
<td>625</td>
</tr>
<tr>
<td>S-4.0-2</td>
<td>148</td>
<td>2.630</td>
<td>356</td>
<td>545</td>
</tr>
<tr>
<td>S-4.0-3</td>
<td>210</td>
<td>5.230</td>
<td>362</td>
<td>388</td>
</tr>
<tr>
<td>S-4.0-4</td>
<td>310</td>
<td>7.060</td>
<td>353</td>
<td>364</td>
</tr>
<tr>
<td>S-4.0-5</td>
<td>560</td>
<td>14.500</td>
<td>375</td>
<td>336</td>
</tr>
</tbody>
</table>
SERIES III

Pressure Range .............. 50-350

Duration .............. 100 µsec

Nominal Values:
Engulfment Time .............. Instantaneous
Distribution .............. Uniform

TABLE VIII
SERIES III - BLAST EXCITATION PARAMETER DATA

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure Ave.</th>
<th>Impulse Ave. K-Taps</th>
<th>Duration µsec</th>
<th>Engulfment µsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.0-1C</td>
<td>87</td>
<td>.732</td>
<td>122</td>
<td>Instantaneous</td>
</tr>
<tr>
<td>S-1.0-2C</td>
<td>129</td>
<td>.950</td>
<td>106.5</td>
<td>Instantaneous</td>
</tr>
<tr>
<td>S-1.0-3C</td>
<td>229</td>
<td>1.573</td>
<td>99.5</td>
<td>Instantaneous</td>
</tr>
<tr>
<td>S-1.0-4C</td>
<td>326</td>
<td>2.113</td>
<td>94.0</td>
<td>Instantaneous</td>
</tr>
</tbody>
</table>
SERIES IV

Pressure Range ............. 50-350

Duration ................. 300 μsec

Engulfment Time ............ Instantaneous

Distribution ............... Uniform

TABLE IX

SERIES IV - BLAST EXCITATION PARAMETER DATA

Recorded Values:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure Ave.</th>
<th>Impulse Ave. K-Taps</th>
<th>Duration μsec</th>
<th>Engulfment μsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-2.5-1C</td>
<td>64</td>
<td>1.0650</td>
<td>258</td>
<td>Instantaneous</td>
</tr>
<tr>
<td>S-2.5-2C</td>
<td>110</td>
<td>1.4850</td>
<td>196</td>
<td>Instantaneous</td>
</tr>
<tr>
<td>S-2.5-3C</td>
<td>147</td>
<td>2.6610</td>
<td>263</td>
<td>Instantaneous</td>
</tr>
<tr>
<td>S-2.5-4C</td>
<td>308</td>
<td>5.1150</td>
<td>240</td>
<td>Instantaneous</td>
</tr>
</tbody>
</table>
SERIES V

Pressure Range . . . . . . . . . . 50-350

Duration . . . . . . . . . . . . . 100 μsec

Engulfment Time . . . . . . . . 150 μsec

Distribution . . . . . . . . . . . .cos²θ

TABLE X
SERIES V - BLAST EXCITATION PARAMETER DATA

Recorded Values:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Impulse K-Taps</th>
<th>Duration μsec</th>
<th>Engulfment μsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.5-1SC</td>
<td>45</td>
<td>.433</td>
<td>139</td>
<td>0</td>
</tr>
<tr>
<td>S-1.5-2SC</td>
<td>42 (85)*</td>
<td>.970</td>
<td>165</td>
<td>70</td>
</tr>
<tr>
<td>S-1.5-3SC</td>
<td>110</td>
<td>1.12</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>S-1.5-4SC</td>
<td>250</td>
<td>1.700</td>
<td>98.5</td>
<td>130</td>
</tr>
</tbody>
</table>

*Value Interpolated from Circumferential Distribution
SERIES VI

Pressure Range ........... 50-350
Duration ............... 300 µsec
Engulfment Time ........... 150 µsec
Distribution ........... \( \cos^2 \theta \)

TABLE XI
SERIES VI - BLAST EXCITATION PARAMETER DATA

Recorded Values:

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Impulse K-Taps</th>
<th>Duration µsec</th>
<th>Engulfment µsec</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-4.0-1SC</td>
<td>42 (70)*</td>
<td>1.2</td>
<td>250</td>
<td>150</td>
</tr>
<tr>
<td>S-4.0-2SC</td>
<td>61 (72)*</td>
<td>1.2</td>
<td>250</td>
<td>130</td>
</tr>
<tr>
<td>S-4.0-3SC</td>
<td>90</td>
<td>1.98</td>
<td>302</td>
<td>160</td>
</tr>
<tr>
<td>S-4.0-4SC</td>
<td>172</td>
<td>5.240</td>
<td>418</td>
<td>240</td>
</tr>
<tr>
<td>S-4.0-5SC</td>
<td>270</td>
<td>10.350</td>
<td>545</td>
<td>160</td>
</tr>
</tbody>
</table>

*Value Interpolated from Circumferential Distribution
### Table XII

**Summary of Blast Excitation Parameter Data**

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Test No.</th>
<th>Pressure</th>
<th>Duration in μsec</th>
<th>Engulfment Time μsec</th>
<th>Circumferential Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Actual</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>S-1.5-1</td>
<td>92</td>
<td>129</td>
<td>545</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.5-2</td>
<td>145</td>
<td>120</td>
<td>515</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.5-3</td>
<td>230</td>
<td>125</td>
<td>435</td>
<td>430</td>
</tr>
<tr>
<td></td>
<td>S-1.5-4</td>
<td>340</td>
<td>112</td>
<td>354</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.5-5</td>
<td>630</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>S-4.0-1</td>
<td>90</td>
<td>326</td>
<td>625</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-4.0-2</td>
<td>148</td>
<td>356</td>
<td>545</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-4.0-3</td>
<td>210</td>
<td>362</td>
<td>388</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>S-4.0-4</td>
<td>310</td>
<td>353</td>
<td>364</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-4.0-5</td>
<td>560</td>
<td>375</td>
<td>336</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>S-1.0-1C</td>
<td>87</td>
<td>122</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.0-2C</td>
<td>129</td>
<td>106</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.0-3C</td>
<td>229</td>
<td>91</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S-1.0-4C</td>
<td>326</td>
<td>94</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>S-2.5-1C</td>
<td>60</td>
<td>228</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-2.5-2C</td>
<td>110</td>
<td>196</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-2.5-3C</td>
<td>147</td>
<td>263</td>
<td>231.7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>S-2.5-4C</td>
<td>308</td>
<td>240</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>S-1.5-1SC</td>
<td>45</td>
<td>139</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.5-2SC</td>
<td>90</td>
<td>155</td>
<td>70</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>S-1.5-3SC</td>
<td>110</td>
<td>148</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-1.5-4SC</td>
<td>250</td>
<td>98</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-4.0-1SC</td>
<td>72</td>
<td>250</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>S-4.0-2SC</td>
<td>70</td>
<td>250</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-4.0-3SC</td>
<td>90</td>
<td>302</td>
<td>160</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td>S-4.0-4SC</td>
<td>172</td>
<td>418</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-4.0-5SC</td>
<td>270</td>
<td>545</td>
<td>160</td>
<td></td>
</tr>
</tbody>
</table>

**Equations**

\[
P = (P_{\text{max}} - P_{\text{op}}) \cos^2 \theta + P_{\text{op}}
\]
Summary - Explosive Data

**TABLE XIII**

SERIES I* - EXPLOSIVE DATA

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Charge Density lbs/ft²</th>
<th>Thickness Inches</th>
<th>Width Inches</th>
<th>Spacing Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.5-1</td>
<td>92</td>
<td>.004</td>
<td>.015</td>
<td>.067</td>
<td>1.5</td>
</tr>
<tr>
<td>S-1.5-2</td>
<td>145</td>
<td>.007</td>
<td>.015</td>
<td>.079</td>
<td>1.0</td>
</tr>
<tr>
<td>S-1.5-3</td>
<td>220</td>
<td>.011</td>
<td>.015</td>
<td>.123</td>
<td>1.0</td>
</tr>
<tr>
<td>S-1.5-4</td>
<td>340</td>
<td>.017</td>
<td>.015</td>
<td>.190</td>
<td>1.0</td>
</tr>
<tr>
<td>S-1.5-5</td>
<td>630</td>
<td>.023</td>
<td>.015</td>
<td>.258</td>
<td>1.0</td>
</tr>
<tr>
<td>S-1.5-6</td>
<td>80</td>
<td>.004</td>
<td>.015</td>
<td>.067</td>
<td>1.5</td>
</tr>
</tbody>
</table>

*All array dimensions 10' x 4' - DETASHEET Explosive

**TABLE XIV**

SERIES II - EXPLOSIVE DATA

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Charge Density lbs/ft²</th>
<th>PRIMACORD gr/ft</th>
<th>Spacing Inches</th>
<th>Array Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-4.0-1</td>
<td>90</td>
<td>.0085</td>
<td>18</td>
<td>3.6</td>
<td>12' x 8'</td>
</tr>
<tr>
<td>S-4.0-2</td>
<td>148</td>
<td>.0160</td>
<td>25</td>
<td>2.68</td>
<td>12' x 8'</td>
</tr>
<tr>
<td>S-4.0-3</td>
<td>210</td>
<td>.0270</td>
<td>50</td>
<td>3.18</td>
<td>12' x 8'</td>
</tr>
<tr>
<td>S-4.0-4</td>
<td>310</td>
<td>.0450</td>
<td>50</td>
<td>1.90</td>
<td>12' x 8'</td>
</tr>
<tr>
<td>S-4.0-5</td>
<td>560</td>
<td>.0600</td>
<td>50</td>
<td>1.43</td>
<td>12' x 8'</td>
</tr>
</tbody>
</table>
### TABLE XV

**SERIES III* - EXPLOSIVE DATA**

<table>
<thead>
<tr>
<th>Test Test No.</th>
<th>Pressure Ave.</th>
<th>Charge Density lbs/ft²</th>
<th>Thickness Inches</th>
<th>Width Inches</th>
<th>Spacing Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.0-1C</td>
<td>87</td>
<td>.0016</td>
<td>.015</td>
<td>.044</td>
<td>2.42</td>
</tr>
<tr>
<td>S-1.0-2C</td>
<td>129</td>
<td>.0026</td>
<td>.015</td>
<td>.060</td>
<td>2.04</td>
</tr>
<tr>
<td>S-1.0-3C</td>
<td>229</td>
<td>.0039</td>
<td>.015</td>
<td>.069</td>
<td>1.55</td>
</tr>
<tr>
<td>S-1.0-4C</td>
<td>326</td>
<td>.0072</td>
<td>.015</td>
<td>.098</td>
<td>1.55</td>
</tr>
</tbody>
</table>

*All DETASHEET Explosive

### TABLE XVI

**SERIES IV - EXPLOSIVE DATA**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure Ave.</th>
<th>Charge Density lbs/ft²</th>
<th>Thickness Inches</th>
<th>Width Inches</th>
<th>Spacing Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-2.5-1C</td>
<td>60</td>
<td>.00195</td>
<td>.015</td>
<td>.100</td>
<td>4.5</td>
</tr>
<tr>
<td>S-2.5-2C</td>
<td>110</td>
<td>.0032</td>
<td>.015</td>
<td>.165</td>
<td>4.5</td>
</tr>
<tr>
<td>S-2.5-3C</td>
<td>147</td>
<td>.0047</td>
<td>.015</td>
<td>.230</td>
<td>4.5</td>
</tr>
<tr>
<td>S-2.5-4C</td>
<td>308</td>
<td>.0070</td>
<td>.015</td>
<td>.480</td>
<td>4.5</td>
</tr>
</tbody>
</table>
### TABLE XVII

**SERIES V - EXPLOSIVE DATA**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Charge Density lbs/ft²</th>
<th>Thickness Inches</th>
<th>Width Inches</th>
<th>Spacing Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-1.5-1SC</td>
<td>45</td>
<td>.0011</td>
<td>.015</td>
<td>.050</td>
<td>4.0</td>
</tr>
<tr>
<td>S-1.5-2SC</td>
<td>42 (85)*</td>
<td>.0020</td>
<td>.015</td>
<td>.055</td>
<td>2.5</td>
</tr>
<tr>
<td>S-1.5-3SC</td>
<td>110</td>
<td>.0035</td>
<td>.015</td>
<td>.068</td>
<td>1.75</td>
</tr>
<tr>
<td>S-1.5-4SC</td>
<td>250</td>
<td>.0054</td>
<td>.015</td>
<td>.110</td>
<td>1.75</td>
</tr>
</tbody>
</table>

*Value Interpolated from Circumferential Distribution

### TABLE XVIII

**SERIES VI - EXPLOSIVE DATA**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Pressure At 0°</th>
<th>Charge Density lbs/ft²</th>
<th>PRIMACORD gr/ft</th>
<th>Spacing Inches</th>
<th>Height Feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-4.0-1SC</td>
<td>42 (72)*</td>
<td>.0027</td>
<td>18</td>
<td>11.33</td>
<td>12</td>
</tr>
<tr>
<td>S-4.0-2SC</td>
<td>61 (70)*</td>
<td>.0027</td>
<td>18</td>
<td>11.33</td>
<td>12</td>
</tr>
<tr>
<td>S-4.0-3SC</td>
<td>90</td>
<td>.0040</td>
<td>18</td>
<td>7.73</td>
<td>12</td>
</tr>
<tr>
<td>S-4.0-4SC</td>
<td>172</td>
<td>.0058</td>
<td>25</td>
<td>7.39</td>
<td>12</td>
</tr>
<tr>
<td>S-4.0-5SC</td>
<td>270</td>
<td>.0083</td>
<td>25</td>
<td>5.15</td>
<td>12</td>
</tr>
</tbody>
</table>

*Value Interpolated from Circumferential Distribution
V. BIBLIOGRAPHY

