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ANALYSIS AND MODELING OF THE EDS MAGLEV SYSTEM BASED ON THE HALBACH PERMANENT MAGNET ARRAY

by

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ABSTRACT

Electro-dynamic suspension (EDS) Magnetic levitation (Maglev) with its advantage in maintenance, safety, efficiency, speed, and noise is regarded as a leading candidate for the next generation transportation / space launch assist system. The Halbach array due to its unique magnetic field feature has been widely used in various applications. The EDS system using Halbach arrays leads to the potential EDS system without super-conductor (SC) technology. In this thesis, the Halbach array magnetic field and the dynamics of a novel Halbach array EDS Maglev system were considered.

The practical Halbach array magnetic field was analyzed using both a Fourier series approach and the finite element method (FEM). In addition, the optimal Halbach array geometry was derived and analyzed. A novel active magnetic array was introduced and used in the Halbach array EDS Maglev configuration. Furthermore, since the system is self-regulated in lateral, roll, pitch, and yaw directions, the control was simplified and can be implemented electronically. The dynamic stability analysis and simulation results showed that the system is marginally stable and a control mechanism is needed for stability and ride comfort control. The six degree of freedom (DOF) dynamics, and the vehicle’s mass center offset effects on those dynamics were investigated with multiple passive and active magnetic forces. The results indicated that the vehicle’s mass center offset has a strong effect on the dynamics of the Maglev
system due to the uniqueness of the magnetic force and also that the mass center offset can cause Maglev oscillations at the take off stage. In order to guarantee the dynamic stability and ride comfort of the Maglev system, an optimized active damping and a linear quadratic regulator (LQR) control were developed. Finally, the simulation confirmed the effectiveness of the proposed multi-input and multi-output (MIMO) control designs.
To my Father and Mother, my wife Chen Ying,
son Dylan, and my baby to be born.
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CHAPTER ONE: INTRODUCTION

1.1. Background

Maglev has been investigated as a potential next generation transportation and space launch assistant system due to the advantages in high-speed, safety, reliability, low environmental impact and minimum maintenance. Several prototypes have been built [1, 2, 8, 29, 31]. The studies on electro-dynamic suspension (EDS) maglev systems in the past relied on very high current density magnets carried aboard the vehicles, which can be supplied only by superconductor (SC) magnets. Recently, an EDS system using Halbach arrays has demonstrated that the lift to drag ratio can reach 300:1 at typical operation speeds, which leads to the potential practical EDS system without SC [1, 2, 3].

1.2. Scope

This research considers a novel Halbach array EDS Maglev system. The Halbach array EDS Maglev has been studied primarily to verify that the levitation is possible [1, 2, 3]. The
optimized system design, stability analysis, and control remain open areas. Although much work has been done on EDS system stability studies, there is still a need to obtain a better understanding of how various factors may influence the stability of an EDS system over its entire speed range [37]. The purpose of this research is to

i) explore the Halbach array EDS system;

ii) propose a new system configuration;

iii) analyze the system stability and dynamics;

iv) investigate the suitable control approach;

v) design the proper controller.

The practical (non-ideal) Halbach array, which is the core component of this EDS system, is not able to generate the zero magnetic field intensity on the canceled side and the purely sinusoidal magnetic field intensity on the enhanced side. The quantitative analysis of the non-ideal magnetic field and optimization of geometry are basic steps to further investigate the non-purely sinusoidal and the non-zero magnetic field and design the system.

1.3. Literature Survey

This research involved the Maglev and incorporated diverse research areas; including Maglev system, EDS system dynamics, Maglev control approach, and Halbach array magnetic field analysis. An extensive literature survey was performed. Following is a relevant summary.
1.3.1. Maglev System

The latest Maglev projects summaries can be found in [26, 29, 30]. Beside the several typical Maglev systems [27, 41, 42] such as German Transrapid, the Japanese MLX, and Swissmetro, Yoshida presented under water marine maglev with linear induction motor (LIM) [80]. Morizane et al. [75] presented an EMS system, using LIM for both levitation and propulsion. Foster-Miller built a null flux coil demonstration system with PM. Although levitation works well for this small demonstration system, it is not practical for a full size real system without SC. In theory, null flux coil suspension is a very good design; the draw back is SC is needed [44, 45, 46, 47, 48, 49]. The Lawrence Livermore National Laboratory (LLNL) has built a demonstration system using Halbach arrays called Inductrack [1]. LLNL proposed a design modification base on two Halbach array face each other configuration also [2, 3]. Oleg V. Tozoni proposed new designs with self-regulation [38, 39, 40].

1.3.2. EDS Maglev Stability

The most recent and detailed research summary on EDS stabilities was performed by the Cai and Chen et al., a maglev research group at Argonne National Laboratories [37]. The dynamics of EDS become unstable above a certain critical speed since it produces negative damping that destabilizes the system. There are also many uncertainties associated with the track, and disturbances induced from operational environments. This group has published many papers on maglev stability. Suspension instabilities of EDS systems with three and five DOF
have been evaluated by computer simulation and experiment. The results showed that coupling effects among the six DOF played an important role and there were several potential instabilities [25, 88, 89]. Yamada et al. [90] built an experimental facility to test the dynamics of EDS maglev in 1973. The damping behavior of the system was observed at various operating speeds, and it was found that negative damping exists for linear velocities above a critical velocity. For a full-scale train traveling over a sheet guideway, these results extrapolated to negative damping for train speeds higher than ~60 km/hour. Iwamoto et al. [92] and Ohsaki [93] applied the inductance-modeling method to the study of maglev stability. Iwamoto predicted a negative damping coefficient for train speed over 50 m/sec and recommended using passive damping to achieve good ride quality. Nguyen et al. [94] described the design of a passive magnetic damper. Chu and Moon [96] demonstrated instabilities in a 2 DOF EDS Maglev model, showing limit cycle oscillations at operating speeds near the Maglev drag peak. Due to the small scale of their model, aerodynamics significantly affected their results. In other experiments, Moon reported results from a rotating wheel test facility for study of lateral, heave, roll, yaw, and pitch motions [95]. A yaw-roll instability was observed. Carabelli in [97, 98] presented a control system on a PMS repulsive levitation system with a six DOF vehicle mode. The passive repulsive system is stable along the vertical axis and in pitch and roll, while being unstable in lateral direction and yaw. However, even if repulsive systems are intrinsically stable for vertical, roll and pitch motions, the low damping supplied by the interaction may lead to the need of additional passive or active damping on all axes. Greene [99] studied the LSM control for EDS system with a two DOF model (heave-vertical motion and surge-horizontal motion). Zhao, [100] with a 10 DOF model, studied the EMS system on vertical motions (heave and pitch). Zhang [101], using
numerical methods, studied the dynamic behavior of a high-speed maglev system moving on flexible guideway. The vehicle was simplified as a body with the primary and the secondary suspension parts and had five DOF, which were heave, sway, roll, pitch, and yaw.

1.3.3. Maglev Control and Controller

There are several potential control options for a EDS maglev system, such as passive coil, controlled coil, hydraulic system, and dashpot to name a few [43]. Secondary mechanical suspensions were discussed by Abe and Tsunashima [84] for EDS systems. Atherton, Eastham, and Sturgess [82] proposed secondary magnetic damping using short-circuited aluminum coils coupled to the linear synchronous motor. Nagai, Mori, and Nakadai [81] built a small scale one DOF EDS maglev test fixture, with a resultant damping ratio without control of 0.5%. With active control, the damping ratio was increased to 20%. Boldea [85] performed an analysis where it is shown that an EDS system with active control of magnet currents can theoretically provide good ride comfort at 100 m/sec without a secondary suspension system. Modern control systems with control of multiple DOF for EDS systems was discussed by Nakadai, Nagai, Nonami, He, and Nishimura [86, 87]. Sakamoo presented a control design for an onboard superconducting magnet maglev system’s guidance with dynamic compensator [74]. Morizane et al. [75] presented a control design for both propulsion and levitation of an EMS system using LIM for both levitation and propulsion. Yoshida, K. and Fuji, performed an experiment with propulsion control for the PM LSM vehicle by a new direct torque control (DTC) method [76]. M. Chen et
al. in [77, 78] presented an adaptive precision positioning controller for a small EDS maglev. Bittar and Moura [34] presented both $H_2$ and $H_\infty$ controller for an EMS maglev system levitation control. Yoshida, K presented a decoupled levitation and thrust force control method in LIM for a under water marine maglev [80].

1.3.4. Halbach Array Magnetic Field and Geometry Optimization Analysis

There are several papers available about linear Halbach array [63, 79] magnetic field calculations. Single square permanent magnet (PM) generated magnetic flux density [57], the transfer relation [61] with magnetic vector potential [59], magnetic scalar potential [58] and Fourier series [119, 120] were used to calculate the magnetic field. These analyses results are not identical, the analyses steps are either complex or not given in details. There are several approaches to optimize the magnet array geometry, such as maximizing the ratio of force to the magnet weight per spatial wavelength [120], maximizing the average pressure produced by the array over the weight of magnet [122], and maximizing the flux square over the magnet weight [50].

1.4. Technical Approach and Major Contributions

A novel active magnet array was introduced and used in a new Maglev configuration. The proposed passive EDS system uses Halbach arrays for self-regulation and levitation and uses
the active magnet arrays for stability and ride comfort control with independent control of multiple levitation and guidance active arrays. The system is self-regulated in the lateral, roll, pitch, and yaw directions. The Maglev system control can be simplified due to these self-regulations. The system configuration, stiffness, dynamics, and optimized damping and LQR control with multiple passive and active magnetic forces were investigated. The analysis and simulation results showed that the system is marginally stable, the mass center offset can cause oscillations, and a control mechanism is needed. The optimized damping and LQR control are introduced and designed. With six DOF modeling and dynamic simulation, we have the analytical capability to predict the detailed behavior of a given design before it is tested in the field. The full six DOF maglev dynamic analysis results give us a better understanding of how various factors may influence the dynamics of an EDS system over its entire speed range. The simulation results verified the effectiveness of the active array damping and LQR control approaches for Halbach array EDS maglev system. FEM and Fourier series analysis approaches with Maxwell equations were utilized to analyze Halbach array magnetic field. The FEM and Fourier series results match quite well. The optimized Halbach array geometry was investigated by taking into consideration the filed harmonics.

1.5. Format of the Report

This proposal is organized as follows.

Chapter 1 contains the background, motivation, scope, literature survey, technical
approach, and major contributions of the research.

Chapter 2 presents an overview on Maglev system and technology introduction about Halbach array EDS Maglev system.

Chapter 3 introduces the magnetic field analysis theory, which includes Maxwell equation, FEM, Laplace’s equation, Fourier series, and magnetic field harmonic analysis.

Chapter 4 discusses practical four and eight piece Halbach array harmonic field analysis, and geometry optimization.

Chapter 5 presents the novel maglev system configuration, control mechanism, system modeling and stiffness analysis.

Chapter 6 focuses on the dynamic analysis and optimized damping and LQR control of Maglev system.

Chapter 7 presents summary of the research and points out the future research direction.

A reference list is given at end of this report.
CHAPTER TWO: MAGLEV OVERVIEW AND OPERATION PRINCIPLE

2.1. Introduction to Maglev

After the first commercial Maglev train was put into operation in Shanghai at the end of 2002, we have more reasons to believe that Maglev, this new technology with its advantages in speed, energy efficiency, noise and maintenance cost, will have a prospective future. Maglev train is only one of the main applications of Maglev technology. Maglev is also becoming attractive in many other applications such as vibration isolation system, magnetic bearings, and space launch assistant system.

The former president of IEE, John C. West said [33], “Electromagnetic suspension devices are intriguing and fascinating. There are many aspects open for research and development but they all are fundamentally interdisciplinary concepts of electro-magnetism, electronics, mechanical engineering, measurement and control.” Even today, these words remain true. There are lot of opportunities and challenges in Maglev research.
2.1.1. The Current Maglev Project

The United States’ first Maglev train of 1-mile length, at Old Dominion University in Norfolk, Va., is still being held up due to budget and technical problems. Expenditures to date are approximately $14 million. Additional funding of $5 million is needed from the federal government instead of the proposed $2 million [29].

The U.S. navy is investigating Maglev to launch aircraft from carriers. It converts stored energy to aircraft kinetic energy with an efficiency of 40–70 percent, compared to 5 percent for steam. Navy’s $373 million Electromagnetic Aircraft Launch System (EALS) project is in the building phase. Later this year (2003), prototype catapults based on linear synchronous motors will be tested at the naval facility [30].

Maglev projects in Pittsburgh received follow-on awards to consider environmental factors, total costs, and revenue projections over 4 years. The project expected award up to $950 million, about one-third of estimated costs. Pennsylvania plans a 76-km link joining Pittsburgh to its international airport and two other cities. Southern California Association of Governments (Los Angeles) awarded a $16 million contract to a team to assess four possible maglev corridors. In October 2002, the San Bernardino (Calif.) Associated Governments, approved funds for feasibility and pre-construction studies for a 433-km Anaheim-to-Las Vegas maglev line. Project supporters hope to begin construction in mid-2007 [30].

The German government budgeted 550 million euro (US $638 million) out of 1.6 billion for the Munich system to link to the nearby international airport, the country’s second busiest. The project is in the “legal planning process”. Construction is to begin around 2005–2006 [29].
A summarized Maglev high speed train projects is given in [29], and a selected Maglev projects is given in Table 2.1:

Table 2.1. A selected Maglev projects

<table>
<thead>
<tr>
<th>Location</th>
<th>In Operation</th>
<th>Awaiting Approval</th>
<th>Under Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai, China</td>
<td>Airport and subway in city</td>
<td>Munich, Germany</td>
<td>Cities and airport</td>
</tr>
<tr>
<td>Munich, Germany</td>
<td>Airport and subway in city</td>
<td>Pittsburgh</td>
<td>Old Dominion University</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>Cities and airport</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Old Dominion University</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance</td>
<td>30 km</td>
<td>37 km</td>
<td>76 km</td>
</tr>
</tbody>
</table>

**2.1.2. Space Launch Assistant System**

Access to space is becoming increasingly expensive. The space shuttle has cost about $4500 per kilogram to overcome Earth’s gravity and enter space orbit. A Shuttle mission typically costs more than $400 million per flight. During the past decade NASA has been studying ways of assisting the launch of space vehicles to reduce the cost. For the same reasons the rail industry has looked at Maglev. Launch assist system requires a low maintenance, inexpensive, environmentally clean, safe, and reliable system. NASA is pursuing a launch scheme that accelerates the craft horizontally along the ground using a power source external to the vehicle, thereby eliminating one stage of rockets. This approach would allow the first stage to be replaced with an alternative power source that is not mounted on the craft, therefore reducing cost and complexity of the launch system. By providing an initial velocity to the space vehicle it is possible to save over 20% of the onboard fuel. Also by lowering the amount of fuel, more
payloads can be added, the size of the vehicle can be reduced or a stronger more robust vehicle can be built. Once solid rocket boosters are ignited they can be extinguished by only complete consumption. The launch cannot be aborted once the solid rockets have been ignited-- the craft must launch, consume and jettison the solid rockets, and land at another location. Maglev launch assistant system would allow the craft to reach a speed at which all systems could be assessed under load, and the determination to complete or abort the launch could be made while still on the runway.

NASA has a program called the Advanced Space Transportation Program (ASTP) to develop technologies in the next 25 years that will improve safety and reliability by a factor of 10 000 while reducing the cost for space access by a factor of 100. Among these technologies is the area of launch assist. Magnetic levitation and propulsion are viewed as a safe, reliable, and inexpensive launch assist for sending payloads into orbit. NASA’s plan is to mature these technologies in the next 25 years to achieve goal of launching a full sized space vehicle for under $300 a kilogram.

NASA has contracted with three companies to initially produce magnetic levitation concepts; Foster–Miller (FM); Lawrence Livermore National Laboratory (LLNL); and PRT Advanced MagLev Systems. Each of these contracts was to show a small demonstration of their concepts at the conclusion of the first phase [8, 31]. Two of the prototypes with a total cost of up to half million dollars are with FSI currently.

NASA’s Maglev launch assist system is different from the Maglev train. However there are numerous similarities between them and lot of concepts and experience can be gained from the rail industry.
2.1.3. Maglev Technologies

There are three basic types of Maglev systems: Electro Dynamic Suspension (EDS) system, Electro Magnetic Suspension (EMS) system and Permanent Magnetic Suspension (Passive Magnetic Suspension, PMS) system. EDS is commonly known as "Repulsive levitation" and EMS is commonly known as "Attractive Levitation". PMS can be used with both attractive and repulsive configurations. Recently, PMS using Halbach array system has shown good characteristics such as low levitation speed and high levitation to drag ratio [1, 2, 3].

Among the existing and planed Maglev systems, the German Transrapid, Japanese HSST, Pittsburgh, Old Dominion University, Swissmetro, and British Birmingham are of the EMS type. Japanese MLU and Canadian maglev are of the superconductor (SC) EDS.

The difficulties of achieving stable suspension or levitation are highlighted by the nature of the forces in the case when an inverse square law relates force and distance. Earnshaw showed mathematically that it is impossible for a pole placed in a static field of force to have a position of stable equilibrium when an inverse square law operates. Braunbeck carried out a similar analysis specifically for unvarying magnetic and electric fields, and deduced that suspension or levitation is not possible in such fields when all materials present have relative permeability $\mu_r > 1$ or relative permittivity $\varepsilon_r > 1$, but that it is possible when materials of $\mu_r < 1$ or $\varepsilon_r < 1$ are introduced. It is impossible for stable suspension or levitation without diamagnetic materials ($\mu_r<1$) or superconducting materials ($\mu_r=0$) [32, 33]. Recently, this theory’s suitability for Maglev application is raised [38, 39, 40]. But it is beyond the paper’s scope.

EMS system is unstable and some form of control must be used to achieve stable
levitation [34]. Also, any malfunction of the servo control system would cause the vehicle to be disastrously attracted to the rails. This is an incurable defect of EMS [35]. EMS system does not need cooling system as EDS SC system and the levitation force can be generated and controlled at any speed. But the system is intrinsically unstable and strongly nonlinear, with a severely restricted equilibrium region, the suspension air gap for EMS system is typically about 1 to 2 cm, which makes it difficult to obtain closed loop stability [34].

The EDS system is inherently stable and does not need a complicated feedback control system comparing with EMS system. Although the EDS is inherently stable, the damping force is not large enough to suppress the vibrations excited by guideway irregularities and other disturbances [36, 37]. The dynamics of EDS become unstable above a certain critical speed since it produces negative damping that destabilizes the system [37].

The studies on EDS systems in the past relied on very high current density magnets carried aboard the vehicles. The current density can be supplied by only superconducting magnets. This requirement makes the system expensive and complex due to the fact that the high temperature superconductor is not technologically ready and the cryogenic system is a complex system. At high speed the damping forces are weak, and, at low speed the levitation force is weak and break force is strong. Thus, levitation is feasible only at high speed. PM can be used in repulsive systems to levitate vehicles, but the levitation height is quite small. Halbach array levitation system is a new research topic and a more detailed discussion will be given in later section.

Although much work has already been done, there is still the need to obtain a better understanding of how various factors may influence the stability of an EDS system over its entire
speed range. One would like to have the analytical capability to predict the detailed behavior of a given proposed design before it is tested in the field [37].

A comparison of the EMS, SC EDS and PM EDS systems are given in [33], and a comparison between SC EDS and Halbach array EDS systems are given in Table 2.2.

Table 2.2. Comparison of SC EDS and Halbach array EDS systems

<table>
<thead>
<tr>
<th>Magnet type</th>
<th>Advantages</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC EDS</td>
<td>Very strong magnet field can be generated. A large air gap is possible.</td>
<td>High temperature SC is not technology ready.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cryogenic system is a very complex system.</td>
</tr>
<tr>
<td>Halbach array</td>
<td>No complex cryogenic system is need. Vary reliable.</td>
<td>Magnetic field is not as strong as the SC magnet.</td>
</tr>
<tr>
<td>EDS</td>
<td></td>
<td>Currently only a small air gap is possible.</td>
</tr>
</tbody>
</table>

2.2. EDS System

It is the opinion of the author although the EDS system with SC or PM has some limitations; with the technology advancing on SC and PM materials EDS maglev system will have bright future.

There are several common SC EDS options [41, 42]. Two typical systems are listed here.
2.2.1. Continuous Sheet Suspension

The continuous sheet guideway employs one of the basic levitation methods for EDS maglev systems. The repulsive levitation force is generated by the interaction between the superconducting magnets aboard the vehicle and the eddy currents induced in the conducting sheet. A typical configuration is shown in Figure 2.1.

![Continuous Sheet Suspension Diagram]

Figure 2.1: Continuous sheet suspension

For a conductor wire of unit length, moves with a velocity \( v_0 \) normal to itself. The left and drags force are given by [32, 33, 43]

\[
F_L = \frac{\mu_0}{4\pi} \frac{v_0^2}{(v_0^2 + \xi^2)} \frac{I^2}{h} \\
F_d = \frac{\xi}{v_0} F_L
\]

(2.1)

where \( \xi \) is a characteristic velocity inversely proportional to the track thickness \( d \) and electrical
conductivity $\sigma$; $I$ is the constant coil current; $v_0$ is the speed of the vehicle, $\mu_0$ is the permeability of free space, and

$$\xi = \frac{2}{\mu_0 d \sigma}$$ (2.2)

It is found that increasing the levitation height and the length of the coil in the direction of motion of the coil improves the ratio $F_L/F_D$ but increasing the levitation height beyond 30 cm is not considered practical as the left force diminishes with an increase in the levitation height. The coil geometry does not have any influence on the force [32, 33, 43].

2.2.2. Null Flux Coil Suspension

The Japanese have succeeded in designing and testing several versions of EDS systems based on the null flux concept. A model picture is shown in Figure 2.2. Two arrays of null flux ground coils are mounted vertically on both guideway sidewalls. Both vertical suspension and horizontal guidance forces are generated by the interaction between the on board superconductor magnets (SCMs) and the null flux coils.
The figure "8" levitation coils are installed on the sidewalls of the guideway. When the on-board superconducting magnets pass several centimeters below the center of these coils at a high speed, an electric current is induced within the figure "8" coils, which act as electromagnets for the time being. The electromagnets forces push and pull the superconducting magnet upwards simultaneously, levitating the Maglev vehicle. The levitation coils facing each other are connected under the guideway, constituting a loop. When a running Maglev vehicle displaces laterally, an electric current is induced in the loop, resulting in repulsive forces acting on the levitation coils of the side near the car and attractive force acting on the levitation coils of the side farther apart from the car. Thus, a running car is always located at the center of the guideway [http://www.rtri.or.jp].

A repulsive force and an attractive force induced between the magnets are used to propel
the vehicle (superconducting magnet). The propulsion coils located on the sidewalls on both sides of the guideway are energized by a three-phase alternating current from a substation, creating a shifting magnetic field on the guideway. The on-board superconducting magnets are attracted and pushed by the shifting field, propelling the Maglev vehicle.

FM built a null flux coil demonstration system with PM. Even the levitation works well for this small demonstration system, it is not practical for a full size real system according to the scale law. In theory, Null Flux Coil Suspension is a very good design; the drawback is SC is needed. There are several research papers available on this topic [44, 45, 46, 47, 48, 49].

2.3. LLNL System

Almost all the research conducted on early systems since 1960 was not related to PMS. With the new permanent magnetic materials, high magnetic flux density can be achieved with PM. Especially, with the Halbach array configuration, practical EDS Maglev without using SC could be possible. Recently, LLNL has built a demonstration system using Halbach arrays, which open a new way for the PMS Maglev.

Halbach arrays produce a strong spatially periodic magnetic field on one surface of the arrays, while canceling the field on another surface. Two pictures of Halbach array are shown in Figure 2.3 and Figure 2.4. The Figure 2.4 is generated using Finial Element Analysis Method (FEM) with remanent flux density $B_r = 1.29$ Tesla, and magnetic permeability $\mu_r = 1.05 \mu_0$. Many researches have explored the Halbach array and its broad applications [2, 50, 51, 52, 53, 54, 55, 56].
Halbach array EDS system, using high-field permanent magnets Halbach arrays on the levitating cradle, moves above a "track" consisting of a close-packed array of shorted coils, which are interleaved with special drive coils. Relative motion between the Halbach arrays and the track coils induces currents in those coils. These currents levitate the cradle by interacting with the horizontal component of the magnetic field. At rest no levitation occurs, however as
soon as the cradle is in motion the moving magnet array will induce currents in the conductor array. At a speed greater than a few kilometers per hour, the levitation force will levitate the cradle. Due to the inductive loading of the circuits, self-inductance plus the effect of mutual inductance, the phase of the induced current is shifted by ninety degrees, thus maximizing the lift force, while minimizing the drag force. As a result, in a high-speed state, the drag power can be made to be a small fraction of the power required overcoming aerodynamic friction. In theory the lift to drag ratio increases linearly with increasing speed, which can reach to 300:1 at typical operation speeds.

The Halbach array levitation force is given by Equation 2.3 [1, 2, 3],

$$< F_z > = K_f \frac{B_{rh}^2 \{[1 - \exp(-kd_t)]^2 \sin(\pi/M) \}^2 w^2}{2kL},$$

where $k=2\pi/\lambda$ is the wave number, $\lambda$ is the wavelength of the array, $d_t$ is the thickness of the Halbach array. $M$ is the number of magnet bars per wavelength in the array, $B_{rh}$ is the remanence flux density of the permanent magnet material, $\Delta z$ is the distance between track and the surface of the array, $v$ is the traveling speed of array, $w$ is the width of the magnet, $L$ is the inductance (self plus mutual) of a coil, $R$ (ohms) is the coil resistance, and $K_f$ is a scale factor.

Figure 2.5 to Figure 2.8 are the pictures of LLNL Inductrack system main components [134].
LLNL’s Inductrack is unique in the levitation using Halbach array. The detailed discussions and calculations can be found in reference [1]. Its levitation and drag force are given by

$$< F_z >= \frac{B_0^2 w^2}{2kL} \frac{1}{1+(R/\omega L)^2} \exp(-2k\Delta z) \text{ [N]}$$  \hspace{1cm} (2.4)$$

$$< F_x >= \frac{B_0^2 w^2}{2kL} \frac{(R/\omega L)}{1+(R/\omega L)^2} \exp(-2k\Delta z) \text{ [N]}$$  \hspace{1cm} (2.5)$$

$B_0$ is the peak strength of the magnetic field at the surface of the Halbach array. For the track composed of close-packed shorted circuits in the form of rectangular “window frames” with a transverse width, \(w\) (m.),

The lift to drag ratio is

$$\frac{\text{Lift}}{\text{Drag}} = \frac{\omega L}{R} = \frac{2\pi \nu}{\lambda} \left[ \frac{L}{R} \right]$$  \hspace{1cm} (2.6)$$

The levitating efficiency: Newton of levitating force per watt of power dissipated in the track. The average power, \(<P>\), dissipated per circuit is given by the product \(\nu <F_x>\)

$$K = \frac{2\pi}{\lambda} \left[ \frac{L}{R} \right] \text{ [N/W]}$$  \hspace{1cm} (2.7)$$

From Equation 2.7, increasing \(L\) will make any desired levitation efficiency with the expense of the reducing the lifting force.

For velocities less than the critical speed, the drag force dominates. The drive coils must provide a force to exceed this drag force \(F_x\).
\[ F_s = \frac{v B_c^2 w^2}{2R} \frac{N_c}{\left( \frac{KvL}{R} \right)^2 + 1} \] (2.8)
CHAPTER THREE: MAGNETIC FIELD ANALYSIS THEORY

3.1. **Maxwell Equation**

3.1.1. Basic Vector Concept

We represent a vector $\vec{A}$ symbolically in terms of the vector addition of three mutually perpendicular vectors as follows:

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors in the $x$, $y$, $z$ directions, $A_x$, $A_y$ and $A_z$ are the magnitude projections of the vector $\vec{A}$ on the $x$, $y$ and $z$ axes, respectively.

The scalar product of the vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \cdot \vec{B}$. The quantity is by definition a scalar having the magnitude

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Note that $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The scalar product can also be represented as follows
\[ \vec{A} \cdot \vec{B} = AB \cos \theta \]

where \( A \) and \( B \) are the magnitudes of the vectors \( \vec{A} \) and \( \vec{B} \), respectively, and \( \theta \) is the angle between the two vectors.

The vector product of two vectors \( \vec{A} \) and \( \vec{B} \) is denoted by \( \vec{A} \times \vec{B} \) and is defined as

\[
\vec{A} \times \vec{B} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
A_x & A_y & A_z \\
B_x & B_y & B_z
\end{vmatrix}
\]

\[= \vec{i}(A_yB_z - A_zB_y) + \vec{j}(A_zB_x - A_xB_z) + \vec{k}(A_xB_y - A_yB_x)\]

The vector product is a vector. The magnitude can be expressed by

\[ \vec{A} \times \vec{B} = AB \sin \theta \]

The direction of the vector \( \vec{A} \times \vec{B} \) is perpendicular to the plane of the vectors \( \vec{A} \) and \( \vec{B} \), which is given by the right hand rule convention.

The gradient of a scalar function \( S \) is a vector whose magnitude is the directional derivative at the point and whose direction is the direction of the directional derivative at the point.

Consider a scalar \( S \), the value of which is dependent upon its position in space.

\[ S = S(x,y,z) \]

The ascendant of \( S \) is defined as

\[ \vec{A} = \vec{i} \frac{\partial S}{\partial x} + \vec{j} \frac{\partial S}{\partial y} + \vec{k} \frac{\partial S}{\partial z} \]

This represents a vector which has a direction normal to the equiscalar surface at a given
point \( x, y, z \) and points in the direction of ascending values of \( S \). The differential operation indicated above is given a special symbol \( \nabla \), defined by

\[
\nabla \equiv \hat{i} \frac{\partial S}{\partial x} + \hat{j} \frac{\partial S}{\partial y} + \hat{k} \frac{\partial S}{\partial z}
\]

The quantity will be referred to as the gradient of \( S \) (or grad \( S \)); i.e.,

\[
\text{Grad } S \equiv \nabla S
\]

If the vector \( \vec{A} \) is defined at each point \( x, y, z \) in a given region, then we say that a field of \( \vec{A} \) exists.

\[
\vec{A} = \vec{A}(x, y, z)
\]

\[
= \hat{i} A_x(x, y, z) + \hat{j} A_y(x, y, z) + \hat{k} A_z(x, y, z)
\]

which implies three functions of space.

The divergence of a vector is the limit of its surface integral per unit volume as the volume enclosed by the surface goes to zero. The divergence of such a vector field (Div \( \vec{A} \)) is defined as

\[
\text{Div } \vec{A} \equiv \nabla \cdot \vec{A}
\]

the term on the right being an abbreviation of

\[
\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}
\]

Div \( \vec{A} \) is a scalar.

\[
\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}
\]
The integral of the divergence of a vector $\vec{A}$ over a volume $V$ is equal to the surface integral of the normal component of the vector over the surface bounding $S$.

$$\int_V \nabla \cdot \vec{A} \, dV = \int_S \vec{n} \cdot \vec{A} \, dS$$

The curl of a vector is the limit of the ratio of the integral of its cross product with the outward drawn normal, over a closed surface, to the volume enclosed by the surface as the volume goes to zero.

The line integral of a vector around a closed curve is equal to the integral of the normal component of its curl over any surface bounded by the curve.

$$\oint_C \vec{A} \cdot dl = \int_S \text{Curl} \vec{A} \cdot \vec{n} \, da$$

The curl of a vector field $\vec{A}(x, y, z)$ is defined by

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y) + \hat{j}(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z) + \hat{k}(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x)$$

Thus, the curl of $\vec{A}$ is a vector having the three components in Cartesian coordinates.

Laplacian operator is defined by
\[ \nabla \cdot \nabla = \nabla^2 \]

\[ \nabla^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} \]

The curl of the gradient of any scalar field is zero

\[ \nabla \times (\nabla S) = 0 \]

The divergence of any curl is zero

\[ \nabla \cdot (\nabla \times \vec{A}) = 0 \]

\[ \nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \]

### 3.1.2. Maxwell Equation

The base for electromagnetic analysis is the four Maxwell equations, which were derived from earlier Biot-Savart law, Faraday’s law and Gauss’s law. In differential form these equations are given by (3.1) [51, 61, 111].

\[ \nabla \times \vec{E} = -\frac{\partial \mu_0 \vec{H}}{\partial t} \]

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \varepsilon_0 \vec{E}}{\partial t} \]  
(3.1)

\[ \nabla \cdot \varepsilon_0 \vec{E} = \rho \]

\[ \nabla \cdot \mu_0 \vec{H} = 0 \]

where \( \vec{E} \) is the electric field intensity, \( \vec{J} = \sigma \vec{E} \) electric current density, \( \rho \) is the charge density,
$H$ is the magnetic field intensity, for permanent magnet $B = \mu_0 H + \mu_0 M$, $M$ is magnetization vector, $B = \mu_0 \mu_h H$ is the magnet flux density, $\mu_0$ is the magnetic permeability of free space, $\mu_r$ is the relative magnetic permeability.

### 3.1.3. Magnet Vector Potential

Since the divergence of any curl is zero, Equation 3.2 can be derived from Equation 3.1 in magnetoquasistatic (MQS) approximation [51, 61, 111].

\[ \nabla \times H = J, \]
\[ \nabla \cdot (\mu_0 H) = 0 \]

\[ \mu_0 H = \nabla \times \vec{A} \]

and in MQS systems, for convenience we make $\nabla \cdot \vec{A} = 0$. The only other requirement placed on $\vec{A}$ is that

\[ \nabla \times (\mu_0 H) = \nabla \times \nabla \times \vec{A} = \mu_0 J \]

Using the identity

\[ \nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \]

the Poisson’s equation is given by

\[ \nabla^2 \vec{A} = -\mu_0 J \]
3.1.4. Magnet Scalar Potential

The vector potential \( A \) describes magnetic fields that possess curl wherever there is a current density \( J \). The curl of the magnetic induction is zero wherever the current density is zero. When this is the case, the magnetic induction in such regions can be written as the gradient of a scalar potential [51, 61, 111]:

\[
\nabla \times \mathbf{H} = 0
\]

\[
\mathbf{H} = -\nabla \varphi
\]

\[
\nabla \cdot (\mathbf{B}) = 0
\]

However, the divergence of \( B \) is also zero,

\[
\nabla \cdot B = -\mu_0 \nabla^2 \varphi = 0
\]

(3.4)

where \( \varphi \) is called the magnetic scalar potential, satisfies Laplace’s equation.

3.2. Laplace’s Equation

In Cartesian Coordinates, Laplace's equation is

\[
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0
\]

Normally the separable variable method is used to solve the equations. The separable solutions has the form

\[
\varphi = X(x)Z(z)
\]
Substituting into Laplace’s equation we obtain

\[ \frac{\partial^2 X}{\partial x^2} Z + \frac{\partial^2 Z}{\partial z^2} X = 0 \]

\[ \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = K \]

The form of the solution depends on the sign of the separation constant \( K \). \( K = \rho^2 \). The solution is

\[ \varphi = (a_1 e^{\rho z} + a_2 e^{-\rho z})(b_1 \sin \rho z + b_2 \cos \rho z) \]

In general, we need to sum over all values of \( \rho \) to obtain the most general solution.

\( K = -\rho^2 \) In this case the solution is

\[ \varphi = (a_1 e^{\rho z} + a_2 e^{-\rho z})(b_1 \sin \rho z + b_2 \cos \rho z) \]

\( K = 0 \) The solution is somewhat simpler in this case, with linear solutions for both \( X \) and \( Z \).

### 3.3. FEM

#### 3.3.1. Introduction of FEM

FEM, Finite element method, some times abbreviated as FEA for finite element analysis, was introduced in 1950 and is a powerful tool for solving any field potential related engineering problems using transform to an algebraic problem. In this research, FEM was used as a tool to
analyze the magnetic field. Here only related fundamental knowledge of FEM is briefly introduced.

The finite element method (FEM) is one of the most popular numerical methods for constructing approximate solutions to differential equations. The FEM is widely used by scientists and engineers. Originally developed by aircraft structural engineers, the method stands on solid mathematical footing for obtaining approximate solutions to variational boundary value problems. The method subdivides a problem domain into element domains, and assembles the contributions from each element to build a global approximation. A series linear algebraic equation is used to solve the boundary value problem (BVP) for a linear partial differential equation (PDE).

Once an appropriate BVP is formulated, there are two components involved in performing a finite element analysis; the division of the problem domain into a set of sub domains or finite elements, and the translation of the BVP into a series of linear algebraic equations that can be easily solved by computer.

Some notations of the FEM are:

Each sub domain is called an element; the union of the elements is called a mesh. Over each element, the solution (whether magnetic field, temperature, velocity, concentration, stresses, etc.) is approximated by a set of functions associated with degrees of freedom (DOF). These degrees of freedom are often geometrically associated with interior element points or element vertices called nodes.

The resulting governing equations for elements are assembled into a global set of linear equations.
There are several different means to develop the local finite element equations. The first approach is to use physical reasoning (as in matrix structural analysis). The other approach to developing the finite element equations is energy or virtual work method. These are usually motivated by the presence of an energy function, such as the total potential energy $F$. Using a calculus of variations, we may then consider stationary points of the functional (i.e. $\partial F = 0$). The disadvantage of this approach is that not all problems possess such a functional (as in non conservative systems). The third approach involves variational methods. These are general techniques in which integral statements of the governing equations are used. Examples of variational methods include Rayleigh-Ritz, Galerkin, Petrov-Galerkin, and Least-Squares, to name a few.

The energy function approach is used widely in FEM for the temperatures, stresses, and field analysis. Energy functional consists of all the energies associated with the particular finite element model. The FEM obtains the correct solution for any finite element model by minimizing the energy functional. The minimum of the functional is found by setting the derivative of the functional with respect to the unknown grid point value to zero. The fundamental equation of energy approach FEM is

$$\frac{\partial F}{\partial p} = 0$$

where $p$ is the unknown grid point value to be calculated, which is potential for magnetic filed analysis.

For magnetic systems, the energy functional $E$ can be given by
\[
F = \int_V \left[ \int_0^B {\vec{H} \cdot d\vec{B}} - \int_0^A {\vec{J} \cdot d\vec{A}} + j \frac{1}{2} \omega \sigma \vec{A}^2 \right] \, dV
\]

\[
= \int_V \left[ \frac{B^2}{2\mu} - \vec{J} \cdot \vec{A} + j \frac{1}{2} \omega \sigma \vec{A}^2 \right] \, dV
\]

In problems with PMs

\[
\vec{B} = \mu_0 \vec{H} + \mu_0 x \vec{H} + \vec{B}_r = \mu_0 \mu_r \vec{H} + \vec{B}_r
\]

the functional is

\[
F = \int_V \left[ \frac{B^2}{2\mu} - \frac{\vec{B}\vec{B}_r}{\mu} \vec{J} \cdot \vec{A} + j \frac{1}{2} \omega \sigma \vec{A}^2 \right] \, dV
\]

where \( \vec{B}_r \) is the remanent magnetic flux density. \( \omega \) is the angular frequency and \( \sigma \) is the electric conductivity. The first term on the right side is the magnetic stored energy, the second is the electric input energy and the third term is the losses due to induced currents. The energy functional \( F \) is minimized when

\[
\frac{\partial F}{\partial \vec{A}} = 0
\]

The two-dimensional sinusoidal time varying field can be described with the aid of the magnetic vector potential

\[
\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial \vec{A}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial \vec{A}}{\partial y} \right) = -\vec{J} + j \omega \sigma \vec{A}
\]

where the magnetic vector potential \( \vec{A} \) and excitation current density vector \( \vec{J} \) are directed out or into the flat model along the \( z \) axis, i. e.,

\[
\vec{A} = k \vec{A}_z
\]

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\[ \mathbf{J} = k \mathbf{J}_z \]

The magnetic flux density vector has two components in the \(x\)-\(y\) plane perpendicular to \(\mathbf{A}\) and \(\mathbf{J}\),

\[
\mathbf{B} = i \mathbf{B}_x + j \mathbf{B}_y = \nabla \times \mathbf{A} = i \frac{\partial A_x}{\partial y} - j \frac{\partial A_y}{\partial x}
\]

for magnet static field \(\omega = 0\).

\[
\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial \mathbf{A}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial \mathbf{A}}{\partial y} \right) = -\mathbf{j}
\]

Minimization of the magnetic energy functional over each mesh leads to a matrix equation, which has to solve for the magnetic vector potential \(\mathbf{A}\).

Figure 3.1: A part of a typical 2D mesh
Figure 3.1 shows part of a typical mesh, and the coordinate system for 2D problems. Each mesh has at least three vertices (nodes). The number of nodes corresponding to each mesh depends on the shape of the element and the shape function, which is used to model the potential within the mesh. Usually linear or second order shape functions are used. Following the linear shape function is used. The vector potential within each mesh is given by

\[ A = a_1 + a_2x + a_3y \]

\[ A = \sum_{k=l,m,n} A_k [a_k + b_kx + c_ky] \]

the matrix form values of \( A \) for the three nodes are given by

\[
\begin{bmatrix}
A_l \\
A_m \\
A_n
\end{bmatrix}
= 
\begin{bmatrix}
1 & x_l & y_l \\
1 & x_m & y_m \\
1 & x_n & y_n
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3
\end{bmatrix}
\]

and

\[ a_1 = \frac{1}{2\Delta} \left| \begin{array}{ccc} A_l & x_l & y_l \\ A_m & x_m & y_m \\ A_n & x_n & y_n \end{array} \right| \]

\[ a_2 = \frac{1}{2\Delta} \left| \begin{array}{ccc} 1 & A_l & y_l \\ 1 & A_m & y_m \\ 1 & A_n & y_n \end{array} \right| \]

\[ a_3 = \frac{1}{2\Delta} \left| \begin{array}{ccc} 1 & x_l & A_l \\ 1 & x_m & A_m \\ 1 & x_n & A_n \end{array} \right| \]
\[ \Delta = \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_m - x_l & y_m - y_l & 0 \\ x_n - x_l & y_n - y_l & 0 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & x_l & y_l \\ 1 & x_m & y_m \\ 1 & x_n & y_n \end{vmatrix} \]

where \( \Delta \) is the surface area of a triangle with nodes \( l, m, n \).

\[
A = \sum_{k=l,m,n} A_k \begin{bmatrix} a_k + b_k x + c_k y \end{bmatrix} = \begin{bmatrix} N_l & N_m & N_n \end{bmatrix} \begin{bmatrix} A_l \\ A_m \\ A_n \end{bmatrix}
\]

where \( N_k = (a_k + b_k x + c_k y)/(2\Delta) \) for \( k = l, m, n \)

the node point potentials \( A_k \) can be calculated by minimizing the energy, for the single triangular mesh case.

\[
F = \int_S \frac{\partial}{\partial A_k} \left[ \frac{B^2}{2\mu} - \vec{J} \cdot \vec{A} \right] dS = 0
\]

this minimization leads to the magnetic vector potential can be approximated by the following set of equations

\[
[S] = [A] [I]
\]

where \([S]\) is the global coefficient matrix, \([A]\) is the matrix of nodal magnetic vector potentials and \([I]\) is nodal currents (forcing functions) which are given by

\[
[S] = \frac{1}{4\mu\Delta} \begin{bmatrix} b_l b_l + c_l c_l & b_l b_m + c_l c_m & b_l b_n + c_l c_n \\ b_m b_l + c_m c_l & b_m b_m + c_m c_m & b_m b_n + c_m c_n \\ b_n b_l + c_n c_l & b_n b_m + c_n c_m & b_n b_n + c_n c_n \end{bmatrix}
\]

\[
[I] = J \frac{\Delta}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\]
the above equation solve for the potential $\bar{A}$ in a region containing the triangle with nodes $l$, $m$, and $n$. For practical problems with $K$ nodes, the preceding process is repeated for each element, obtaining the matrix $[S]$ with $K$ rows and columns, $[A]$ and $[I]$ are column matrices containing $K$ rows of complex terms. And the boundary conditions are incorporated into calculations.

### 3.3.2. Finite Element Method Software Package

The ease of FEM implementation is supported by the availability of good software packages available. Almost all software packages have three main components: pre-processor, processor (solver), and post-processor.

The steps involved in pre-processor module may include,

- Define the problem’s computational in 2D or 3D, normally start with 2D for preliminary design and analysis then moving to 3D.
- Drawing the geometric outline of the models
- Assign materials properties for each region of geometric models.
- Connect the voltage or current source if there is any.
- Assign the boundary and edge constrains for the geometric models
- Discrete the region into elements, choose proper shape, size and generate the mesh. Lot of software package has the ability to adaptively setting the size in different region. Normally start with small number of cell for preliminary design and analysis then moving to high-resolution cell.
Using solver get the results, then do post processing.

### 3.4. Fourier Series

A periodic power signal \( x(t) \) with a period \( T_0 \) can be represented by an exponential Fourier series of the form

\[
x(t) = \sum_{n=-\infty}^{\infty} C_x(nf_0) e^{(j2\pi fn t)}; -\infty < t < +\infty
\]

\[
C_x(nf_0) = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) e^{(-j2\pi fn t)} dt
\]

\[
f_0 = \frac{1}{T_0}
\]

for real valued signal \( x(t) \) [62]

\[
x(t) = A_0 + 2 \sum_{n=1}^{\infty} A_n \cos(2\pi fn_0 t) + 2 \sum_{n=1}^{\infty} B_n \sin(2\pi fn_0 t); -\infty < t < +\infty
\]

\[
A_n = \frac{1}{T_0} \int_{0}^{T_0} x(t) \cos(2\pi fn_0 t) dt; (n = 0, 1, 2, \ldots)
\]

\[
B_n = \frac{1}{T_0} \int_{0}^{T_0} x(t) \sin(2\pi fn_0 t) dt; (n = 1, 2, \ldots)
\]

For the real even signal the transform has only cosine components.
\[ B_n = 0 \]

\[ x(t) = A_0 + 2 \sum_{n=1}^{\infty} A_n \cos(2\pi f_0 t) \]

\[ A_n = \frac{1}{T_0} \int_{0}^{T_0} x(t) \cos(2\pi f_0 t) dt \]

For real odd signal the transform result has only sine components.

\[ A_n = 0 \]

\[ x(t) = 2 \sum_{n=1}^{\infty} B_n \sin(2\pi f_0 t) \]

\[ B_n = \frac{1}{T_0} \int_{0}^{T_0} x(t) \sin(2\pi f_0 t) dt \]

3.5. Magnetic Field Harmonic Analysis Theory

The Halbach array is used widely in particle accelerators, magnet bearings, linear motors, and PMSM [56, 60], and has been used as the core component in Maglev designs [1, 3]. A summary of the analysis and applications of Halbach array can be found in [55, 58].

The ideal linear Halbach array has sine and cosine magnetization in the vertical and horizontal directions, respectively, resulting in no magnetic field on one side and an enhanced, pure sinusoidal magnetic field on another side [131]. Figure 3.2 (a) illustrates an ideal Halbach array. However, The ideal Halbach array is impractical to fabricate. Instead, an array of
rectangular or square permanent magnets is used. Practical (non ideal) Halbach arrays with four and eight piece are shown at Figure 3.2 (b). The non-ideal Halbach array is not able to generate the zero magnetic field intensity on the canceled side and the pure sinusoidal magnetic field intensity on the enhanced side.

![Figure 3.2: Halbach array, (a). Ideal; (b). Practical four and eight piece](image)

The purely sinusoidal magnetic field and zero magnetic field are desired for numerous applications. For example, without a pure sinusoidal magnetic field in the linear synchronous motor force ripples and noise may occur. The zero magnetic field is desired on the maglev train’s passenger side for minimal magnetic interference. Unfortunately, with the practical linear Halbach array, the magnetic field does not have purely sinusoidal magnetic field on one side and zero field on the opposite side. The quantitative analysis of the non-ideal magnetic field is a
basic step to further investigate the non-zero and non-purely sinusoidal field effect. Linear Halbach array magnetic field calculations have been done by several authors using different approaches. Single square PM generated magnetic flux density [57], the transfer relation [61] with magnetic vector potential [59], magnetic scalar potential [58] and Fourier series [119, 120] have been used. These results are not focused on the harmonic component of magnetic field. The calculations, especially for detailed harmonic components, are either quite complex or not given in detail.

Figure 3.3 shows the geometry of a permanent magnet sheet with a thickness of $d_t$.

Figure 3.3. The geometry of a magnet sheet and coordinate definitions

For the ideal Halbach array, the magnetizations are given by
\[ m_x = m_0 \sin(kx) \quad (3.5) \]
\[ m_z = m_0 \cos(kx) \quad (3.6) \]

For magnet field with no transport currents

\[ \nabla \times \mathbf{H} = 0 \quad (3.7) \]
\[ \mathbf{H} = -\nabla \varphi \quad (3.8) \]

For a permanent magnet

\[ \mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} \quad (3.9) \]
\[ \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \quad (3.10) \]
\[ \nabla^2 \varphi = \nabla \cdot \mathbf{M}, \quad (3.11) \]

where \( \mathbf{H} \) is the magnetic field intensity, \( \mathbf{B} \) is the magnetic flux density, \( \mu_0 \) is the magnetic permeability of free space, \( \mathbf{M} \) is the magnetization vector, and \( \varphi \) is called the magnetic scalar potential. For materials which are magnetically linear, \( \mathbf{M} = \mathbf{B}/\mu_0 \). \( \mathbf{B} \) is the remanence of PM.

Inside permanent magnet

\[ \nabla \cdot \mathbf{M} = \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} + \frac{\partial m_z}{\partial z} \]
\[ = m_0 k \cos(kx) \quad (3.12) \]
\[ \nabla^2 \varphi = mk \cos(kx) \quad (3.13) \]

Outside permanent magnet

\[ \nabla^2 \varphi = 0 \quad (3.14) \]

Equation 3.13 and Equation 3.14 are solved using the boundary conditions. The potentials are diminished at infinity and continue at the boundary. The flux density of the normal
component is also continuous. The potentials are given by Equation 3.15 [125,131].

\[ \varphi_{canceled} = 0 \]

\[ \varphi_{inside} = \frac{m_0}{k} (e^{kz} - 1) \cos(kx) \]  

(3.15)

\[ \varphi_{enhanced} = \frac{m_0}{k} (1 - e^{kld}) e^{kz} \cos(kx) \]

With \( \mathbf{H} = -\nabla \varphi \), the magnetic field intensities are given by

\[ \mathbf{H}_{canceled} = 0 \]

\[ \mathbf{H}_{inside} = -\left( i \frac{\partial S}{\partial x} + j \frac{\partial S}{\partial y} + k \frac{\partial S}{\partial z} \right) \varphi \]  

(3.16)

\[ \mathbf{H}_{enhanced} = -\left( i \frac{\partial S}{\partial x} + k \frac{\partial S}{\partial z} \right) \varphi \]

\[ = -i m_0 (1 - e^{kld}) e^{kz} \sin(kx) - k m_0 (1 - e^{kld}) e^{kz} \cos(kx). \]

Suppose there is another ideal array with a different spatial period and wave number. Let the \( k_n = 2\pi n/l = n k \). The new array has the following magnetization values,

\[ m_{xn} = m_0 \sin(k_n x) \]  

(3.17)

\[ m_{zn} = m_0 \cos(k_n x) \]  

(3.18)

For this new array, the magnetic potentials are given by

\[ \varphi_{canceled} = 0 \]

\[ \varphi_{inside} = \frac{m_0}{k_n} (e^{k_n z} - 1) \cos(k_n x) \]  

(3.19)
\[ \varphi_{\text{enhanced}} = \frac{m_{n0}}{k_n} (1 - e^{k_nz}) e^{k_nx} \cos(k_n x) \]

and the magnetic fields are given by

\[ \mathbf{H}_{\text{canceled}} = 0 \]

\[ \mathbf{H}_{\text{inside}} = -\left( i \frac{\partial S}{\partial x} + j \frac{\partial S}{\partial y} + k \frac{\partial S}{\partial z} \right) \varphi \]  

\[ \mathbf{H}_{\text{enhanced}} = -\left( i \frac{\partial S}{\partial x} + k \frac{\partial S}{\partial z} \right) \varphi = -i m_{n0} (1 - e^{k_nz}) e^{k_nx} \sin(k_n x) \]

\[ -\vec{k} m_{n0} (1 - e^{k_nz}) e^{k_nx} \cos(k_n x) \]

If there is a pair magnetization of \( m_x \) and \( m_z \), these can be decomposed into the sine and cosine pairs as follows,

\[ m_x = \sum_i m_{x_i} = \sum_i m_{0_i} f_n \sin(k_n x) = \sum_i m_{0_i} f_n \sin(nk x) \]  

\[ m_z = \sum_i m_{z_i} = \sum_i m_{0_i} f_n \cos(k_n x) = \sum_i m_{0_i} f_n \cos(nk x) \]

\[ m = \vec{i} m_x + \vec{k} m_z = \vec{i} \sum_i m_{x_i} + \vec{k} \sum_i m_{z_i} \]

\[ = \sum_i [\vec{i} m_{0_i} f_n \sin(k_n x) + \vec{k} m_{0_i} f_n \cos(k_n x)] \]

\[ = \sum_i [m_{0_i} f_n (\vec{i} \sin(k_n x) + \vec{k} \cos(k_n x))] \]  

\[ = \sum_i [m_{0_i} f_n (\vec{i} \sin(k_n x) + \vec{k} \cos(k_n x))] \]

According to the superposition principle, the potentials are given by

\[ \varphi_{\text{canceled}} = 0 \]
\[
\phi_{\text{inside}} = \sum_i \frac{m_0 f_n}{nk} (e^{nkz} - 1) \cos(nkx)
\]  

(3.24)

\[
\phi_{\text{enhanced}} = \sum_i \frac{m_0 f_n}{nk} \left(1 - e^{nkz}\right) e^{nkz} \cos(nkx)
\]

and the magnetic field intensities are given by

\[
H_{\text{canceled}} = 0
\]

\[
H_{\text{inside}} = -(\hat{i} \frac{\partial S}{\partial x} + \hat{j} \frac{\partial S}{\partial y} + \hat{k} \frac{\partial S}{\partial z}) \phi
\]

(3.25)

\[
H_{\text{enhanced}} = \sum_i \left\{-i m_0 f_n \left(1 - e^{nkz}\right) e^{nkz} \sin(nkx)\right\}
\]

- \left\{-\hat{k} m_0 f_n \left(1 - e^{nkz}\right) e^{nkz} \cos(nkx)\right\}.

The key point is that if we can decompose the magnetization into vertical and horizontal components. These two components can be grouped as sine and cosine pairs. The sine and cosine components in each pair have the same amplitude and Fourier series frequency. The resulting magnetic field will be the superposition of the enhanced and canceled fields of different frequencies.
CHAPTER FOUR: HALBACH ARRAY FIELD ANALYSIS AND GEOMETRY OPTIMIZATION

This chapter presents the four and eight piece Halbach array analysis and geometry optimization for Maglev application with consideration of the non-ideal array magnetic field harmonics. The field analysis results, using scalar potential and Fourier series, are confirmed by FEM. The geometric optimization is based on the maximization of the ratio of square of flux to the unit area magnet weight.

4.1. Four Piece Halbach Array Analysis

4.1.1. Mathematic Modeling

For the practical four-piece Halbach array, with \( \lambda = 4d_s \), \( k_1 = 2\pi/\lambda \), and thickness of \( d_t \). The magnetizations are shown in Figure 4.1. Let \( m_{z4}(x) \) and \( m_{x4}(x) \) be the vertical and horizontal components of the magnetization respectively. The magnetizations can be written as
\[ m_z(x) = m_{zd}(x) = m_0 \Pi(4x/l)^* \left\{ \sum_{n=-\infty}^{\infty} \left[ \delta(x-n l) - \delta(x-(2n-1) l/2) \right] \right\} \] \hspace{1cm} (4.1)

\[ m_x(x) = m_{xd}(x) = m_0 \Pi(4x/l)^* \left\{ \sum_{n=-\infty}^{\infty} \delta(x-n l - l/4) \right\} - \delta(x-(2n-1) l/2 - l/4) \}. \hspace{1cm} (4.2)\]

Figure 4.1: The magnetization of four-piece Halbach array, (a) vertical; (b) horizontal

The Fourier series of \(m_{x4}(x)\) and \(m_{z4}(x)\) are given by

\[ m_{x4}(x) = \sum_{n=1}^{\infty} B_{n4} \sin(2\pi f_4 x) \]

\[ = \sum_{n=1}^{\infty} - \frac{4}{\pi n} \left\{ \sin \left( \frac{\pi n}{2} \right) \sin \left( \frac{\pi n}{4} \right) \cos(\pi n) \right\} \sin(2\pi f_4 x) \] \hspace{1cm} (4.3)
\[m_{z4}(x) = \sum_{n=1}^{\infty} A_{n4} \cos(2\pi f_o t)\]
\[= \sum_{n=1}^{\infty} -\frac{4}{\pi n} \left\{ \sin\left(\frac{\pi n}{2}\right) \cos\left(\frac{\pi n}{4}\right) \cos(\pi n) \right\} \cos(2\pi f_o x).\]  

(4.4)

From Equation 4.3 and Equation 4.4, together with previous chapter conclusion, it is easy to see that the practical Halbach array is not an ideal Halbach array. Its magnetic field is not zero on the canceled side and is not purely sinusoidal on the enhanced side. If we fix the vertical components and find the theoretical horizontal components to form the ideal Halbach array, the magnetization pairs are

\[m_{z4}(x) = \sum_{n=1}^{\infty} -\frac{4}{\pi n} \left\{ \sin\left(\frac{\pi n}{2}\right) \cos\left(\frac{\pi n}{4}\right) \cos(\pi n) \right\} \sin(2\pi f_o x)\]  

(4.5)

\[m_{z4}(x) = \sum_{n=1}^{\infty} -\frac{4}{\pi n} \left\{ \sin\left(\frac{\pi n}{2}\right) \cos\left(\frac{\pi n}{4}\right) \cos(\pi n) \right\} \cos(2\pi f_o x),\]  

(4.6)

The waveforms of Equation 4.5 and Equation 4.6 are given by Figure 4.2.
The magnet with magnetization as Figure 4.2 (b) is impractical to fabricate. If we fix the horizontal components and form the ideal Halbach array, the magnetization pairs are given by Equation 4.7 and Equation 4.8, and the waveforms are given by Figure 4.3.

\[
\begin{align*}
    m_{x4}(x) &= \sum_{n=1}^{\infty} \frac{4}{\pi n} \left\{ -\sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi n}{4}\right)\cos(nm) \right\} \sin(2\pi f_t x) \\
    m_{z4}(x) &= \sum_{n=1}^{\infty} \frac{4}{\pi n} \left\{ \sin\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi n}{4}\right)\cos(nm) \right\} \cos(2\pi f_0 x).
\end{align*}
\]

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Figure 4.3: The theoretical paired vertical magnetization (b) with fixed horizontal magnetization (a) to form one side magnet array for one period.

The magnet with magnetization as Figure 4.3 (b) is impractical to fabricate also.

From Equation 4.3 and Equation 4.4, $A_{n4} = B_{n4} = 0$, for $n = 0, 2i$; $A_{n4} = B_{n4}$, for $n=4i+1$, resulting magnetic field on enhanced side; and $A_{n4} = -B_{n4}$, for $n = 4i - 1$, resulting magnetic field on canceled side, where $A_{n4}$ is the $n^{th}$ harmonic amplitude of $m_z(x)$, and $B_{n4}$ is the $n^{th}$ harmonic amplitude.
harmonic amplitude of \( m_x(x) \). Let \( \Delta z \) be the distance from the observation point to the surface of permanent magnet. The field on the up (canceled) side will have the 3\(^{rd} \), 7\(^{th} \), 11\(^{th} \), and 15\(^{th} \) harmonics. The field intensity is given by

\[
H_{\text{canceled}} = \sum_{n=3,7,11,15,...(4i-1)} \left\{ i m_0 A_{n4} (1 - e^{-nk_i d}) e^{-nk_i \Delta z} \sin(nk_i x) \right\}
\]

\[
+ km_0 A_{n4} (1 - e^{-nk_i d}) e^{-nk_i \Delta z} \cos(nk_i x) \right\} \tag{4.9}
\]

The field on the bottom (enhanced) side will have the fundamental, 5\(^{th} \), 9\(^{th} \), and 13\(^{th} \) harmonics. The magnetic field intensity is given by

\[
H_{\text{enhanced}} = \sum_{n=1,5,9,13,...(4i+1)} \left\{ i m_0 A_{n4} (1 - e^{-nk_i d}) e^{-nk_i \Delta z} \sin(nk_i x) \right\}
\]

\[
+ km_0 A_{n4} (1 - e^{-nk_i d}) e^{-nk_i \Delta z} \cos(nk_i x) \right\} . \tag{4.10}
\]

From Equation 4.3 and Equation 4.4, the coefficients up to the 15\(^{th} \) harmonic are given in Table 4.1. These harmonic pairs can be grouped into four different classes.

1. Both vertical and horizontal coefficients are positive which generate the desired magnetic field, with a zero field on the canceled side and an enhanced field on other side. This group includes the Fundamental frequency and the 9\(^{th} \) harmonic.

2. Vertical coefficients are positive and horizontal coefficients are negative generating an undesirable magnetic field with a canceled field on the enhanced side and an enhanced field on the canceled side. This group includes the 3\(^{rd} \) and the 11\(^{th} \) harmonics.

3. Both vertical and horizontal coefficients are negative generating a magnetic field with the canceled field on the canceled side and enhanced field on the enhanced side. This group includes the 5\(^{th} \) and 13\(^{th} \) harmonics.
4. Vertical coefficients are negative and horizontal coefficients are positive generating the
desirable magnetic field having a canceled field on the enhanced side and an enhanced field on
the canceled side. This group includes the $7^{\text{th}}$ and the $15^{\text{th}}$ harmonics.

Table 4.1. Coefficients of four piece practical Halbach array

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>0.9003</td>
<td>0.3001</td>
<td>-0.1801</td>
<td>-0.1286</td>
<td>0.1</td>
<td>0.0818</td>
<td>-0.0693</td>
<td>-0.06</td>
</tr>
<tr>
<td>H</td>
<td>0.9003</td>
<td>-0.3001</td>
<td>-0.1801</td>
<td>0.1286</td>
<td>0.1</td>
<td>-0.0818</td>
<td>-0.0693</td>
<td>0.06</td>
</tr>
</tbody>
</table>

4.1.2. Results Discussion and Comparison with FEM

The permanent magnet with $B_r$ over 1.3 Tesla is available in the market. $B_r=1.29$ Tesla is
used in the calculation.

Figure 4.4 through Figure 4.7 illustrate the four-piece magnetic field flux density
harmonics for both vertical and horizontal components on the enhanced and canceled sides for
two distances ($0.1d$ and $0.5d$) with the square block case $d_4 = d_{i4} = d$.
Figure 4.4. Four-piece array magnetic field harmonic components (Enhanced side) at distance 0.1 $d$ for one period

Figure 4.5: Four-piece array magnetic field harmonic components (Canceled side) at distance 0.1 $d$ for one period
Figure 4.6: Four-piece array magnetic field harmonics components (Enhanced side) at distance 
0.5 \(d\) for one period

Figure 4.7: Four-piece array magnetic field harmonics components (Canceled side) at distant 0.5 \(d\)
for one period
The vertical and horizontal components of array magnetic field flux density have similar characteristics, the higher order harmonic components will decrease faster as the distances between the observation point and the magnet surface increases on both the enhanced and canceled sides. Now, we will give an analysis for the horizontal component only. The vertical component may be analyzed in a similar way.

Figure 4.8 and Figure 4.9 illustrate the total magnetic field’s horizontal components on the enhanced and on the canceled sides respectively, for four different distances (0.1\(d\), 0.25\(d\), 0.5\(d\) and 0.7\(d\)).

Figure 4.8: The four-piece array magnetic field (Horizontal component) on the enhanced side at different distance for one period
Figure 4.9: The four-piece array magnetic field (Horizontal component) on the canceled side at different distance for one period

For the enhanced side with the increasing of the distance to the magnet surface, the total magnetic field approaches a purely sinusoidal pattern as the higher order harmonics decrease faster with the increase in the distance between the observation point and the magnet surface. Beyond 0.5 \( d \), the magnetic field intensity is almost purely sinusoidal. For Maglev at low speed with low levitation height, the higher order magnetic field harmonics should be taken into consideration.

Figure 4.10 and Figure 4.11 compare the total magnetic field’s horizontal components of the FEM and Fourier series results at 0.1 \( d \) for both the enhanced and canceled sides. The FEM result has some sharp jumps at several points as illustrated in Figure 4.10 and Figure 4.11. The largest one is located at position 65 on Figure 4.11. The reason is that the FEM mesh size is not
small enough (The total number of mesh is 8672). With new FEM software, the resolution can be increased and the FEM results will eventually be same as the Fourier series result.

Figure 4.10: The magnet field comparison between FEM and Fourier Harmonic calculation for the magnetic field Horizontal component on enhanced side at 0.1 d for one period
Figure 4.11: The magnet field comparison between FEM and Fourier approach for the magnet field Horizontal component at canceled side at 0.1 d for one period

The FEM and Fourier series results compare quite well considering that the Fourier series calculation is based on an infinite length array and FEM uses one middle period of 10-block array (two and half period).

4.2. Eight Piece Halbach Array Analysis

4.2.1. Mathematic Modeling

The eight-piece magnet array is also used in Maglev design [2]. The magnetizations are
shown at Figure 4.12, and can be written as

\[ m_z(x) = m_{z8} \Pi(8t/l) \sum [\frac{\sqrt{2}}{2} \delta(t-nl) - \frac{\sqrt{2}}{2} \delta(t-nl-1l/8) + \frac{\sqrt{2}}{2} \delta(t-nl-2l/8) - \frac{\sqrt{2}}{2} \delta(t-nl-3l/8) - \frac{\sqrt{2}}{2} \delta(t-nl-4l/8)] \] (4.11)

\[ m_x(x) = m_{x8} \Pi(8t/l) \sum [\frac{\sqrt{2}}{2} \delta(t-nl-1l/8) + \frac{\sqrt{2}}{2} \delta(t-nl-2l/8) + \frac{\sqrt{2}}{2} \delta(t-nl-3l/8) - \delta(t-nl-4l/8) - \delta(t-nl-5l/8) - \delta(t-nl-6l/8) + \frac{\sqrt{2}}{2} \delta(t-nl-7l/8)] \] (4.12)
As with the four piece array, these magnetizations can be represented by Fourier series.
\[ m_{28}(x) = \sum_{n=0}^{\infty} A_{n8} \cos(2\pi f_0 x) \]
\[ = \sum_{n=1}^{\infty} \frac{2}{\pi n} \left\{ (2 - \sqrt{2}) \cos\left(\frac{3\pi n}{8}\right) + \sqrt{2} \cos\left(\frac{\pi n}{8}\right) \right\} \sin\left(\frac{\pi n}{2}\right) \cos(2\pi f_0 x) \]
\[ (4.13), (4.14) \]

\[ m_{38}(x) = \sum_{n=0}^{\infty} B_{n8} \sin(2\pi f_0 x) \]
\[ = \sum_{n=1}^{\infty} \frac{2}{\pi n} \left\{ (2 - \sqrt{2}) \sin\left(\frac{\pi n}{8}\right) + \sqrt{2} \sin\left(\frac{3\pi n}{8}\right) \right\} \sin\left(\frac{\pi n}{2}\right) \sin(2\pi f_0 x) \]

\[ n= 0, 2i, 8i+3, 8i-3, A_{n8} = B_{n8} = 0 ; n= 8i+1, A_{n8} = B_{n8} ; n= 8i-1, A_{n8} = -B_{n8} \]

From Equation 4.13, the coefficients up to the 15\(^{th}\) harmonic are given in Table 4.2.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>7</th>
<th>9</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>0.9745</td>
<td>-0.1392</td>
<td>0.1083</td>
<td>-0.065</td>
</tr>
<tr>
<td>H</td>
<td>0.9745</td>
<td>0.1392</td>
<td>0.1083</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Let \( \Delta z \) be the distance from the observation point to the surface of permanent magnet. The field on the up (canceled) side will have the 7\(^{th}\), and 15\(^{th}\) harmonics. The field is given by

\[ H_{canceled} = \sum_{n=7,15, \ldots (8i-1), \ldots} t m_0 A_{n8} (1 - e^{-nk_d}) e^{-nk[\Delta z]} \sin(nk_1 x) \]
\[ + \tilde{t} m_0 A_{n8} (1 - e^{-nk_d}) e^{-nk[\Delta z]} \cos(nk_1 x) \]
\[ (4.15) \]

The field on the bottom (enhanced) side will have the fundamental, and 9\(^{th}\) harmonics.
The field is given by

\[ H_{\text{enhanced}} = \sum_{n=1,9,\ldots(8i+1),-1} i m_0 A_{n8} (1-e^{-n k x d}) e^{-n k_1 |x|} \sin(n k_1 x) \]

\[ + k m_0 A_{n8} (1-e^{-n k x d}) e^{-n k_1 |x|} \cos(n k_1 x) \}

(4.16)

4.2.2. Results Analysis

Figure 4.13 through Figure 4.16 illustrate the eight piece magnetic field flux density harmonics for both vertical and horizontal components on the enhanced and canceled sides for two distances (0.1d and 0.5d) with the square block case \( d_8 = d_{i8} = d \).

Figure 4.13: The eight piece array magnetic field harmonic components (Enhanced side) at distance 0.1d for one period
Figure 4.14: The eight piece array magnetic field harmonic components (Canceled side) at distance $0.1 \, d$ for one period

Figure 4.15: The eight piece array magnetic field harmonic components (Enhanced side) at distance $0.5 \, d$ for one period
Figure 4.16: The eight piece array magnetic field harmonic components (Canced side) at distance 0.5 \(d\) for one period.

Figure 4.17: The eight-piece array magnet field Horizontal component on enhanced side at 0.1, 0.25, 0.5 and 0.7\(d\) for one period.
Figure 4.18: The eight-piece array magnet field Horizontal component on canceled side at 0.1, 0.25, 0.5 and 0.7\(d\) for one period.

Same as the four piece array, for the enhanced side with the increasing of the distance to the magnet surface, the total magnetic field approaches a pure sinusoidal pattern as the higher order harmonics decrease faster with the increase in the distance between the observation point and the magnet surface. Beyond 0.5 \(d\), the magnetic field intensity is almost purely sinusoidal.

### 4.3. Four Piece Halbach Array and Eight Piece Halbach Array Comparison

Figure 4.19 and 4.20 show the magnet field horizontal component of four and eight piece array on both enhanced and canceled sides, at different to magnet surface distances 0.1, 0.25, 0.5
and 0.7d for one period with the square block case $d_4 = d_{14} = d$ and $d_8 = d_{18} = d$

Figure 4.19: The four and eight piece array magnet field Horizontal component on enhanced side at 0.1, 0.25, 0.5 and 0.7d for one period.

Figure 4.20: The four and eight piece array magnet field Horizontal component on canceled side at 0.1, 0.25, 0.5 and 0.7d for one period
Even the scales for four piece and eight piece array in x axis may be different for Figure 4.19, we can still get following conclusions. For enhanced side field, if both four and eight piece arrays are built with same size square block permanent magnet module. The magnetic field flux density of eight piece array is not greater than four piece array for all the to surface distance. The eight piece generates high magnetic field at 0.5$d$ and 0.7$d$, and has more distortion and weaker field at 0.25$d$ and 0.1$d$ than the four piece array.

The terms \(1 - e^{-nk_1d_4}\) and \(e^{-nk_1|\Delta|}\) of Equation 4.9, Equation 4.10, Equation 4.15, and Equation 4.16 show the reason. For fundamental component with \(d_4 = d_{44} = d_8 = d_{88} = d\),

\[
A_{n8}(1 - e^{-nk_18d_4})e^{-nk_1|\Delta|} > A_{n4}(1 - e^{-nk_14d_4})e^{-nk_14|\Delta|}
\]  

(4.17)

with \(|\Delta|>0.3775d\)

It is not true that the eight piece array is better than four piece array in all cases.

4.4. Halbach Array Geometry Optimization

There are several approaches to optimize the magnet array geometry, such as maximizing the ratio of force to the magnet weight per spatial wavelength [120] or maximizing the average pressure produced by the array over the weight of magnet [122]. In this thesis we maximize the flux square over the magnet weight, which as indicated by [50] is a well posed problem with no artificial constraints. The following assumptions are used in the subsequent analyses.
1) the magnet array is larger enough to consider as infinite in length;

2) the edge effect is not included in this research;

3) the optimization will focus on the enhanced side;

4) and optimization is based on $|\Delta z|_{nomin,al}$, the high speed nominal levitation height.

As we showed in the previous analysis, the array magnetic field has harmonic component. The optimization should take them into consideration. The approach we used is that first we focus on the fundamental component, find the optimized geometry, then investigate the harmonics effect under this optimized geometry.

The spatial harmonic component flux density amplitude at enhanced side for both four and eight piece are given by

$$B_{n4} = B_r A_{n4} (1 - e^{-nk_{d}l}) e^{-nk_{d}m_{l}} = B_r A_{n4} (1 - e^{-\frac{2m_{l}d_i}{\lambda_k}}) e^{-\frac{2m_{l}k_{d}d_i}{\lambda_k}}$$

(4.18)

$$B_{n8} = B_r A_{n8} (1 - e^{-nk_{d}l}) e^{-nk_{d}m_{l}} = B_r A_{n8} (1 - e^{-\frac{2m_{l}d_i}{\lambda_k}}) e^{-\frac{2m_{l}k_{d}d_i}{\lambda_k}}$$

(4.19)

The optimization has several parameters that need to be determined. The magnetic horizontal block length, the magnet vertical block length, the magnet thickness, the array magnet wavelength. It is clear that due to the natural of Halbach array magnet the array horizontal and vertical block length should be equal in order to minimize the high order magnetic field harmonics.

The fundamental spatial component flux density amplitude at enhanced side for both four and eight piece are given by

$$B_{14} = B_r A_{14} (1 - e^{-k_{d}l}) e^{-k_{d}m_{l}} = K_f A_{14} (1 - e^{-\frac{2m_{l}l_{h}}{\lambda_k}}) e^{-\frac{2\pi k_{d}l_{h}}{\lambda_k}}$$

(4.20)
\[ B_{18} = B_r A_{18} (1 - e^{-k d_t}) e^{-k_3 |\lambda \varepsilon|} = K_{f8} (1 - e^{-\frac{2\pi d_t}{\lambda_4}}) e^{-\frac{2\pi}{\lambda_8} |\lambda \varepsilon|}. \tag{4.21} \]

It is obvious that the \( d_t \) is desired to increase in order to maximize magnet field. But the magnet weight and cost will be increased too. The optimization should be constrained by the per unit area magnet weight. The array magnet thickness \( d_t \) and the wavelength \( \lambda \) or the magnet block length \( d \), should be optimized to get the maximized fundamental spatial frequency flux square over the per unit area magnet weight. The optimization index is given by

\[ \text{Opt}_f = \frac{B_1^2}{W_{mag}} = \frac{B_1^2}{\rho d_t}, \tag{4.22} \]

where \( \rho \) is the magnet density of unit thickness and unit area.

For both four and eight piece array, the fundamental spatial component equations are given by:

\[ \text{Opt}_{f4} = \frac{B_{14}^2}{W_{mag}} = \frac{K_{f4}^2}{\rho d_{t4}} (1 - e^{-k_4 d_{t4}})^2 e^{-2k_4 |\lambda \varepsilon|} \]
\[ = \frac{K_{f4}^2}{\rho d_{t4}} (1 - e^{-\frac{2\pi d_{t4}}{\lambda_4}})^2 e^{\frac{4\pi}{\lambda_4} |\lambda \varepsilon|} \tag{4.23} \]

\[ \text{Opt}_{f8} = \frac{B_{18}^2}{W_{mag}} = \frac{K_{f8}^2}{\rho d_{t8}} (1 - e^{-k_8 d_{t8}})^2 e^{-2k_8 |\lambda \varepsilon|} \]
\[ = \frac{K_{f8}^2}{\rho d_{t8}} (1 - e^{-\frac{2\pi d_{t8}}{\lambda_8}})^2 e^{\frac{4\pi}{\lambda_8} |\lambda \varepsilon|} \tag{4.24} \]

with \( d_4 = \frac{\lambda_4}{4}, d_8 = \frac{\lambda_8}{8}, k_{41} = \frac{2\pi}{\lambda_4}, k_{42} = \frac{\pi}{2d_4}, k_{81} = \frac{2\pi}{\lambda_8}, k_{82} = \frac{\pi}{4d_8} \). The two parameters need
to be optimized are $\lambda$ (or $k$, or $d$) and $d_t$. For both four and eight piece array using one equation to determine $\lambda$ and $d_t$.

$$Opt_f = \frac{K_{fa}}{d_t} \left(1 - e^{-\frac{2\pi d_t}{\lambda}}\right)^2 e^{-\frac{4\pi}{\lambda} |\Delta z|}. \quad (4.25)$$

Let $d_t = a_1 |\Delta z|$, $\lambda = a_2 |\Delta z|$, and $a_1/a_2 = a_3$,

then

$$Opt = \frac{1}{a_1 |\Delta z|} \left(1 - e^{-\frac{2\pi a_1}{a_2} |\Delta z|}\right)^2 e^{-\frac{4\pi}{a_1} |\Delta z|} \quad (4.26)$$

the Opt function can be plot as Figure 4.21, which has a maximized point.
The maximization point can be solved with partial differential equations

\[
\frac{\partial}{\partial a_1} \frac{Opt}{a_1} = \frac{\partial}{\partial a_2} \frac{Opt}{a_2} = 0
\]  

(4.27)
find the \( a_1 \) and \( a_2 \)

\[
\frac{\partial \text{Opt}}{\partial a_1} = \frac{e^{-\frac{4\pi}{a_1}}}{|\Delta z|} \left\{ \frac{1}{a_1^2} (1 - e^{-\frac{2\pi a_1}{a_1}})^2 + \frac{2}{a_1} (1 - e^{-\frac{2\pi a_1}{a_1}}) * (e^{-\frac{2\pi a_1}{a_2}}) * \left( \frac{2\pi}{a_2} \right) \right\}
\]

\[
\frac{\partial \text{Opt}}{\partial a_2} = \frac{e^{-\frac{4\pi}{a_1}}}{|\Delta z| a_1} \left\{ e^{-\frac{4\pi}{a_2}} * \left( \frac{4\pi}{a_2^2} \right) * (1 - e^{-\frac{2\pi a_1}{a_2}})^2 + e^{-\frac{4\pi}{a_2}} 2 * (1 - e^{-\frac{2\pi a_1}{a_2}}) * (-e^{-\frac{2\pi a_1}{a_2}}) * \left( \frac{2\pi a_1}{a_2^2} \right) \right\}
\]

solve them the results are

\[
a_1 = \frac{4}{5} \pi
\]

\[
a_2 = 4\pi
\]

(4.28)

\[
a_3 = \frac{1}{5}
\]

The optimized geometry is a function of levitation nominal height.

For four piece and eight piece case the optimized geometries are

\[
d_{44} = \frac{4}{5} \pi \ | \Delta z \ |_{\text{nominal}}
\]

(4.29)

\[
d_4 = \pi \ | \Delta z \ |_{\text{nominal}}
\]

and

\[
d_{84} = \frac{4}{5} \pi \ | \Delta z \ |_{\text{nominal}}
\]

(4.30)

\[
d_8 = \frac{\pi}{2} \ | \Delta z \ |_{\text{nominal}}
\]

The index is indication of optimized factors, and the harmonic amplitude at this levitation
The actual value of optimized flux density

\[ B_{14} = 0.5078 \text{ Tesla} \]

\[ B_{54} = 0.0192 \text{ Tesla} \]

\[ B_{94} = 0.0014 \text{ Tesla} \]

\[ B_{18} = 0.5497 \text{ Tesla} \]

\[ B_{98} = 0.0016 \text{ Tesla} \]

the largest one is the 4 piece 5\(^{th}\) harmonic which is only 3.78\% of the fundamental component.

For both four and eight piece, the fundamental spatial component equation has similar format.

\[ \text{Opt}_{f4} = \frac{K_{f4}^2}{\rho d_{i4}} (1 - e^{-\frac{2\pi d_{i4}}{\lambda_{f4}}} e^{-\frac{4\pi |\alpha|}{\lambda_{f4}}}) \]

\[ \text{Opt}_{f8} = \frac{K_{f8}^2}{\rho d_{i8}} (1 - e^{-\frac{2\pi d_{i8}}{\lambda_{f8}}} e^{-\frac{4\pi |\alpha|}{\lambda_{f8}}}) \]

The ratio of the optimization factors is

\[ \frac{\text{Opt}_{f4}}{\text{Opt}_{f8}} = \frac{K_{f4}^2}{K_{f8}^2} = \frac{0.9003^2}{0.9745^2} = 0.8553 \]

The optimization index of optimized the eight piece is about 15\% larger than that of the
four piece.

To build the optimized array, two module types are needed for four piece array and three module types are needed for the eight-piece array. To build the square block array, the four piece array need only one module type and the eight piece array needs two module types. Due to these fabrication and cost reasons, the square Halbach array may be preferred. The optimization index has only one variable with $d_e = d$.

\[
Opt_{f4} = \frac{K_{f4}^2}{\rho d_4} (1 - e^{-\frac{\pi}{5}})^2 e^{\frac{\pi}{d_4} |\delta|} \\
\]

\[
Opt_{f8} = \frac{K_{f8}^2}{\rho d_8} (1 - e^{-\frac{\pi}{5}})^2 e^{\frac{\pi}{2d_8} |\delta|} \\
\]

One example of with 0.1 meter levitation height case is given by Figure 4.22.
Figure 4.22: The optimization index of square block array; (a) eight piece array, (b) four piece array.
The optimization can be performed for both four and eight piece array using one equation to determine $d$.

\[ Opt_4 = \frac{1}{d_4} (1 - e^{-\frac{\pi}{2}})^2 e^{-\frac{\pi}{2d_4} |\Delta z|} \]  

(4.39)

\[ Opt_8 = \frac{1}{d_8} (1 - e^{-\frac{\pi}{4}})^2 e^{-\frac{\pi}{2d_8} |\Delta z|} \]  

(4.40)

The maximization can be performed by using same procedure as rectangular array used.

The results are

\[ d_4 = \frac{\pi |\Delta z|}{2} \]  

(4.41)

\[ d_8 = \frac{\pi |\Delta z|}{2} \]  

for four piece and eight piece respectively.

The harmonic amplitude at this levitation height with optimized geometry are

\[ B_{n4} = B_r A_{n4} (1 - e^{-\frac{\pi n}{2}}) e^{-\frac{n}{2}} \]  

(4.42)

\[ B_{n8} = B_r A_{n8} (1 - e^{-\frac{\pi n}{4}}) e^{-\frac{n}{2}} \]  

(4.43)

The actual value of optimized flux density

\[ B_{14} = 0.5623 \text{ Tesla} \]

\[ B_{54} = 0.0192 \text{ Tesla} \]

\[ B_{94} = 0.0014 \text{ Tesla} \]  

(4.44)

78
\[ B_{s8} = 0.418 \text{ Tesla} \]

\[ B_{s8} = 0.0016 \text{ Tesla} \]

The largest harmonic is the four piece 5th harmonic, which is only 3.42% of the fundamental component. The optimization indexes are given by

\[ Opt_{f4} = \frac{K_{f4}^2}{\rho d_4} \left(1 - e^{-\frac{\pi}{27}} \right)^2 e^{\frac{\pi}{d_4} |\Delta z|} = \frac{K_{f4}^2}{\rho \pi |\Delta z|} (1 - e^{-\frac{\pi}{27}})^2 e^{-1} \] (4.45)

\[ Opt_{f8} = \frac{K_{f8}^2}{\rho d_8} \left(1 - e^{-\frac{\pi}{27}} \right)^2 e^{\frac{\pi}{2d_8} |\Delta z|} = \frac{2K_{f8}^2}{\rho \pi |\Delta z|} (1 - e^{-\frac{\pi}{4}})^2 e^{-1} \] (4.46)

The ratio of optimization indexes is

\[ \frac{Opt_{f4}}{Opt_{f8}} = \frac{K_{f4}^2 (1 - e^{-\frac{\pi}{27}})^2}{2K_{f8}^2 (1 - e^{-\frac{\pi}{4}})^2} = 0.9046 \] (4.47)

The optimization index of eight piece is about 10% larger than that of four piece case.

### 4.5. Summary

For linear Halbach array magnetic field harmonics analysis, the Fourier series method is accurate. Compared with FEM, a closed form for individual harmonic components can be found. Moreover, the results are not constrained by the mesh size as in the FEM method. The results show that the higher order harmonics will decrease faster as the harmonic order increases with a corresponding increase in the distance between the observation point and the permanent magnet surface. For Maglev applications, where the levitation height is small at low speeds, the higher
order magnetic field harmonic component effect may need been taken into consideration. With increasing the speed, the higher order harmonics will decrease exponentially with increasing distance between the observation point and the permanent magnet surface. The higher order harmonic component effect can be neglected, as this distance is greater than 0.5 $d$.

For eight piece array the magnet field intensity is not always greater than four piece array even with same size block magnet. The optimized geometry is dependent only on the nominal levitation height. Eight piece is better than four piece array for optimized geometry for both square and rectangular block array (rectangular block 15% less weight; square block 10% less weight). For optimized geometry the largest high order magnetic field harmonic intensity is about only 4 % of fundamental component, which may be neglected normally.
CHAPTER FIVE: DESIGN AND MODELING OF AN EDS MAGLEV

In this chapter, a novel active magnet array is introduced and investigated in a Maglev configuration. The system configuration, static stability, and magnet array force analysis are presented. The proposed passive EDS system uses Halbach array for self-regulation and levitation, uses the active magnet array for stability and ride comfort control with independent control of the vertical and lateral dynamics of the suspension system. It is self-regulation not only in lateral, but also in roll, yaw, and pitch movement.

5.1. A Novel Maglev System

5.1.1. Introduction

High-speed maglev projects, which are currently attracting attention, are German Transrapid (air gap < 12 mm, the Japanese MLX (air gap > 80mm), and Swissmetro (air gap 20mm). The air gaps tolerances are less than 2.5 mm for Swissmetro and Transrapid and 6 mm for MLX [42]. Forces such as gravity, lateral wind pressure, centrifugal forces on curves etc.,
except for gravity, may act unexpectedly and vary considerably in strength. With such small air
gaps, high speeds and unexpected forces, Vehicle control and stability must be addressed.
Various control and system configurations have been proposed [1, 2, 3, 35, 43]. There are several
potential control options for Maglev EDS system, such as passive coil, controlled coil, hydraulic
system, and dashpot to name a few [43]. The proposed system, with self-regulating force and
active magnet control arrays, is a new and suitable solution for the PMS EDS system.

The proposed system has separate levitation and guidance arrays in an orthogonal
arrangement. This configuration allows independent control of vertical and lateral dynamics of
the suspension system. With a symmetric configuration, the lateral arrays act as the null flux
system to keep the system in the equilibrium position. This configuration self-regulates in the
lateral, roll, pitch, and yaw directions.

5.1.2. Proposed System Configuration

The proposed system is shown in Figure 5.1
The system consists of levitation and guidance Halbach arrays and active magnet arrays, which are used to self-regulate and control movement in the roll, pitch, yaw, and lateral directions. The levitation and guidance arrays are arranged in an orthogonal configuration. The levitation arrays in the front and rear of system provide self-regulation in the pitch direction. The left and right levitation arrays provide self-regulation in the roll direction. The front and rear lateral arrays provide self-regulation in the yaw direction, and the left and right lateral arrays self-regulate both lateral and roll directions. As shown in next section, these self-regulations lead to system being static stable in the roll, pitch, yaw, and lateral directions. The maglev system control can be simplified due to these self-regulations. The vehicle travel speed is controlled separately according to a desired trajectory. The simulation results, given in following section, show the levitation height oscillations without any external disturbances and control. For
levitation and guidance control, active magnetic arrays are introduced to improve the controllability of the vehicle in the levitation, lateral, roll, pitch, and yaw directions. With the active magnet array, control is implemented electronically with the advantages in faster response and easier implementation compared to mechanical and hydraulic systems. The linear permanent magnet synchronous motor (LPMSM) using Halbach arrays is proposed for propulsion. The LPMSM coil will be driven with three-phase sine wave current.

The active magnet array, with similar characteristics to the Halbach array, has an enhanced nearly sinusoidal magnetic field on one side (Figure 5.2). The active array may be built with air or ferromagnetic core coils.

Figure 5.2: (a) Sketch of an active array; (b) The magnetic field of an active array using FEM.
The active magnetic arrays can be arranged in several different ways. Non aligned with the passive array with the direct control and adjusting magnetic force by adjusting the active array control current to enhance or decrease the magnetic force as shown by Figure 5.3 (a). Aligned with the magnet array with the control and adjusting magnetic field by adjusting the active array control current to enhance or decrease the magnetic force generated on the shared coil of active and passive array as showed by Figure 5.3 (b). The lateral and levitation arrays can be arranged with same ways. For the arrangement of aligned with the Halbach array with direct control of the magnetic field by changing the active array coil current, Figure 5.3 (b), the total magnetic flux density $B$ can be controlled through changing the active array current, $I_0$. If the maximum controllable range of active array magnetic flux density $B_{rA}$ is 10% of the Halbach array magnetic flux density $B_{rh}$, the total magnetic force ranges from 0.83 to 1.2 times the force due to $B_{rh}$ alone. We prefer the Figure 5.3 (a) or Figure 5.3 (b) to Figure 5.3 (c), the reason is that they have the symmetry character between front and rear, which has an advantage in the dynamic control. For. Figure 5.3 (a) or Figure 5.3 (b), the different is between the superposition of the magnetic force directly and the superposition of magnetic flux density directly.
Figure 5.3: Possible array arrangement top view.
5.2. Six DOF Dynamics Modeling

5.2.1. Modeling and Stiffness Analysis

First let define some quantities as shown in Figure 5.4.

\[ \eta_1 = \begin{bmatrix} x & y & z \end{bmatrix}^T \] Inertial Position

\[ \eta_2 = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T \] Inertial Orientation

\[ v_1 = \begin{bmatrix} u & v & w \end{bmatrix}^T \] Body-fixed Linear Velocity

\[ v_2 = \begin{bmatrix} p & q & r \end{bmatrix}^T \] Body-fixed Angular Velocity

Figure 5.4: Body-fixed and inertial coordinate systems
\[ \tau_1 = \begin{pmatrix} F_x & F_y & F_z \end{pmatrix}^T \text{ External Forces} \]

\[ \tau_2 = \begin{pmatrix} M_K & M_M & M_N \end{pmatrix}^T \text{ External Moments} \]

The Halbach array levitation force is given by \([1, 2, 3]\)

\[
< F_z >= K_f \frac{B_{rh}^2 \left\{ \left[ 1 - \exp(-kd_h) \right] \sin(\pi/M) \right\}^2 w^2}{2kL} \cdot \frac{1}{1 + (R/kvL)^2} \exp(-2k\Delta z), \tag{5.1}
\]

where \( k = 2\pi/\lambda \) is the wave number, \( \lambda \) is the wavelength of the array, \( d_h \) is the thickness of the Halbach array. \( M \) is the number of magnet bars per wavelength in the array, \( B_{rh} \) is the remanence of the permanent magnet material, \( \Delta z \) is the distance between track and the surface of the array, \( v \) is the traveling speed of vehicle, and \( K_f \) is a scale factor.

The following assumptions are used in the subsequent analyses.

1) The displacement from equilibrium is small.;

2) The forces generated by the magnetic arrays are even within every single Halbach array;

3) The couplings among arrays are neglected;

4) The vehicle and track are rigid bodies;

5) The center of mass is in the vehicle’s geometric center along \( x-y \) directions;

6) Only Equation (5.1) is used as the magnetic force.

Figure 5.5 and Figure 5.6 show the magnet array arrangements. \( F_{1\text{LevitationH}}, F_{2\text{LevitationH}}, F_{3\text{LevitationH}} \) and \( F_{4\text{LevitationH}} \) are the forces generated by the four levitation arrays. These forces are
equal to $F_{b\text{Levitation}}$ in the equilibrium position, $F_{b\text{Levitation}} = \frac{1}{4} mg$, where \( m \) is the mass of the vehicle. The coordinate system is defined as Figure 5.5 with \( x \) as traveling and \( y \) as lateral direction.

![Diagram](image-url)

Figure 5.5: (a) Levitation array arrangements, (b) Lateral array arrangements

\[ F_1\text{Lateral}, \ F_2\text{Lateral}, \ F_3\text{Lateral}, \text{ and } F_4\text{Lateral} \] are the forces generated by the four lateral arrays in the equilibrium position, which are equal to $F_{b\text{Lateral}}$.

Driving Force

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\[ F_{\text{travling}} = F_x \]

Levitation Force

\[ F_{\text{levitation}} = F_z = Mg - [F_{1\text{Levitation}} + F_{2\text{Levitation}} + F_{3\text{Levitation}} + F_{4\text{Levitation}}] \]

Lateral force

\[ F_{\text{lateral}} = F_y = F_{1\text{Lateral}} - F_{2\text{Lateral}} + F_{3\text{Lateral}} - F_{4\text{Lateral}} \]

External Moments

\[ M_K = [F_{1\text{Levitation}} - F_{2\text{Levitation}} + F_{3\text{Levitation}} - F_{4\text{Levitation}}] \times L_y \]
\[ M_M = [-F_{1\text{Levitation}} - F_{2\text{Levitation}} + F_{3\text{Levitation}} + F_{4\text{Levitation}}] \times L_x \]
\[ M_N = [F_{1\text{Lateral}} - F_{2\text{Lateral}} - F_{3\text{Lateral}} + F_{4\text{Lateral}}] \times L_y \]

Above analysis is based on the equilibrium position. Following the system static stiffens is analyzed for lateral, roll, pitch, and yaw directions.

5.2.1.1. Lateral

Consider the vehicle is in a non-equilibrium position with displacement \( \Delta y \). The offset is \( \Delta y \) for array 1 and 3, and \(-\Delta y\) for array 2 and 4 laterally. The lateral arrays will generate the lateral restoration force,

\[ F_{\text{Lateral}} = -F_{1\text{Lateral}} - F_{3\text{Lateral}} + F_{2\text{Lateral}} + F_{4\text{Lateral}} \]

\[ = 2 \frac{B^2 rh \left[ \frac{\pi/M}{\pi/M} \right]^2 w^2}{2kL} \]
\[
\frac{1}{1 + (R/\omega L)^2} [\exp(-2k(y_0 - \Delta y)) - \exp(-2k(y_0 + \Delta y))] 
\approx 8F_{bLateral} H k \Delta y .
\] (5.2)

The lateral stiffness is

\[
\frac{\partial}{\partial (\Delta y)} F_{bLateral} H = 8F_{bLateral} H k ,
\] (5.3)

which is always positive, meaning that the lateral restoration force will always increase as the lateral displacement increases.

### 5.2.1.2. Roll

The roll restoration moment is contributed by both levitation array and guidance arrays. If the levitation array pair has displacement of \(-\Delta z\) on the lift (-y axis) side for array 2 and 4, and \(\Delta z\) on the right (+y axis) side for array 1 and 3, \(L_y\) is the distance between center of each levitation array to the center of the vehicle in y direction, the roll angle \(\Phi = \Delta z / L_y\). The restoration moment, generated by levitation arrays, is given by

\[
<T_{roll\_Levitation1} > = L_y * (F_{1Leviation1} + F_{3Leviation1} - F_{2Leviation1} - F_{4Leviation1})
\]

with
\[ < F_{\text{lateral}}^{\text{Levitation1}} >= F_{2}^{\text{Levitation1}} + F_{4}^{\text{Levitation1}} \]

\[ B_{rh} \left[ \left( 1 - \exp(-kd) \right) \frac{\sin(\pi/M)}{\pi/M} \right]^2 w^2 \approx \frac{1}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k(z - \Delta z)) \]

\[ \approx 2F_{b}^{\text{Levitation1}} \left[ 1 - 2kL_y \phi + 2(kL_y)^2 \phi^2 \right] \] (5.4)

\[ < F_{\text{rub}^{\text{Levitation1}}} >= F_{3}^{\text{Levitation1}} + F_{5}^{\text{Levitation1}} \]

\[ B_{rh} \left[ \left( 1 - \exp(-kd) \right) \frac{\sin(\pi/M)}{\pi/M} \right]^2 w^2 \approx \frac{1}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k(z - \Delta z)) \]

\[ \approx 2F_{b}^{\text{Levitation1}} \left[ 1 + 2kL_y \phi + 2(kL_y)^2 \phi^2 \right] \] (5.5)

\[ < T_{\text{roll, Levitation1}} >= 2F_{b}^{\text{Levitation1}} * 4kL_y \phi \] (5.6)

the lateral array will generated restore force, the related position change is \( \Delta y \approx L_z \sin(\phi) = \phi L_z \)

where \( L_z \) is the lateral array center to the center of the vehicle in the \( z \) direction.

\[ < F_{\text{rub}^{\text{Lateral1}}} >= < F_{\text{lateral1}} > + < F_{\text{lateral2}} > \]

\[ B_{rh} \left[ \left( 1 - \exp(-kd) \right) \frac{\sin(\pi/M)}{\pi/M} \right]^2 w^2 \approx \frac{1}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k(y_0 - \Delta y)) \] (5.7)

\[ \approx 2F_{b}^{\text{Lateral1}} \left[ 1 + 2kL_z \phi + 2(kL_z)^2 \phi^2 \right] \]

\[ < F_{\text{rub}^{\text{Lateral1}}} >= < F_{\text{lateral1}} > + < F_{\text{lateral2}} > \]

\[ B_{rh} \left[ \left( 1 - \exp(-kd) \right) \frac{\sin(\pi/M)}{\pi/M} \right]^2 w^2 \approx \frac{1}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k(y_0 + \Delta y)) \]
\begin{align}
\varphi \approx 2F_{\text{bias}}^* \left[ 1 - 2kL_z\varphi + 2(kL_z)^2 \varphi^2 \right] 
\end{align}

\begin{align}
<T_{\text{roll lateral}}^H > = 8F_{\text{y lateral}}kL_z^2\varphi 
\end{align}

The roll stiffness is

\begin{align}
\frac{\partial <T_{\text{roll}}^H >}{\partial (\varphi)} = 8F_{\text{y lateral}}kL_z^2 + 8F_{\text{bias}}kL_y^2, 
\end{align}

which is always positive, meaning that the roll restoring moment always increases as the roll displacement increases.

5.2.1.3. Pitch

![Diagram of levitation array arrangements analysis coordinator]

If the levitation array pair is in a non-equilibrium position with displacement \(-\Delta z\) for front array 1 and 2, \(\Delta z\) for array 3 and 4; and \(L_x\) is the distance between center of each levitation array and the center of the vehicle in \(x\) direction; the pitch angle \(\theta \approx \Delta z / L_x\). If \(\theta\) is very small, we can
neglect the higher order items.

\[ < F_{\text{fubLevitationH}} > = F_{1\text{LevitationH}} + F_{2\text{LevitationH}} \]

\[ B_{rh} \frac{[1 - \exp(-kd)] \sin(\pi/M) \cdot w^2}{\pi/M} \approx \frac{1}{2kL} \exp(-2k(z + \Delta z)) \]  

(5.11)

\[ \approx 2F_{b\text{LevitationH}} \left[ 1 - 2kL_x \theta + 2(kL_x)^2 \theta^2 \right] \]

\[ < F_{\text{bubLevitationH}} > = F_{3\text{LevitationH}} + F_{4\text{LevitationH}} \]

\[ B_{rh} \frac{[1 - \exp(-kd)] \sin(\pi/M) \cdot w^2}{\pi/M} \approx \frac{1}{2kL} \exp(-2k(z - \Delta z)) \]  

(5.12)

\[ \approx 2F_{b\text{LevitationH}} \left[ 1 + 2kL_x \theta + 2(kL_x)^2 \theta^2 \right] \]

Where \( F_{\text{fubLevitationH}} \), \( F_{\text{bubLevitationH}} \) are the front and rear levitation array group generated force under unbalanced position.

\[ < T_{\text{pitchH}} > = 8F_{b\text{LevitationH}} \cdot kL_x^2 \theta \]  

(5.13)

The pitch stiffness

\[ \frac{\partial < T_{\text{pitchH}} >}{\partial (\theta)} = 8F_{b\text{LevitationH}} \cdot kL_x^2 \]

(5.14)

which is always positive, meaning that the pitch restoration moment always increases as the pitch displacement increases.
5.2.1.4. **Yaw**

The yaw restore moment is generated by lateral arrays. Supposing the lateral array pairs are in non-equilibrium position with displacement of $+\Delta y$ for front array 1 and 2, and $-\Delta y$ for rear array 3 and 4. $L_{sla}$ is the distance between the centers of lateral array to the center of vehicle in $x$ direction. The yaw angle is $\psi = \Delta y / L_{sla}$.

\[
<F_1_{lateral\text{H}}>_H = <F_2_{lateral\text{H}}>_H
\]

\[
B_{rk} \left[\left(1 - \exp(-kd)\right)\frac{\sin(\pi/M)}{\pi/M}\right]^2 w^2 \frac{1}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k(y_0 + \Delta y))
\]

\[
\approx 2F_{blateralH} \left[ 1 - 2kL_x \Psi + 2(kL_x)^2 \Psi^2 \right]
\]

\[
<F_1_{lateral\text{R}}>_R = <F_2_{lateral\text{R}}>_R
\]

\[
B_{rk} \left[\left(1 - \exp(-kd)\right)\frac{\sin(\pi/M)}{\pi/M}\right]^2 w^2 \frac{1}{2kL} \frac{1}{1 + (R/\omega L)^2} \exp(-2k(y_0 - \Delta y))
\]

\[
\approx 2F_{blateralH} \left[ 1 + 2kL_x \Psi + 2(kL_x)^2 \Psi^2 \right]
\]

\[
<T_{yaw\_lateral\text{H}}>_H = 8F_{blateralH} * L_x^2 k \Psi
\]

The yaw stiffness

\[
\frac{\partial <T_{yaw\_lateral\text{H}}>_H}{\partial(\Psi)} = 8F_{blateralH} * L_{sla}^2 k
\]

is always positive, meaning that the yaw restore moment always increases as the yaw displacement increases.
The above analyses and calculations are based on separated pitch angle $\theta = \Delta z / L_x$, the roll angle $\Phi = \Delta z / L_y$, yaw angle $\psi = \Delta y / L_{xla}$, and lateral displacement $\Delta y$. These displacements are very small in real system, and coupling can be neglected in real situations. The above analyses and calculations are valid in combined cases. Because all stiffness parameters in lateral, roll, pitch, and yaw are positive, and there is no coupling among them, the system is stable in equilibrium position [110] or statically stable [123].

### 5.2.2. Six DOF Dynamic Force Modeling

The above force and stability analyses are focused on individual DOF of 6 DOF at static state. For a complete system dynamic analysis, a simulation with all 6 DOF is required.

The following assumptions are used in the subsequent analyses.

1) The displacement from equilibrium is small;
2) The forces generated by the magnetic arrays are even within every single Halbach array;
3) The couplings among arrays are neglected;
4) The vehicle and track are rigid bodies;
5) The center of mass is in the vehicle’s geometric center along x-y directions;
6) The propulsion force and motion are independent of the other five DOF;
7) The propulsion and levitation motions are decoupled from the other 4 DOF. The levitation height, traveling position and speed can be measured and used for
controlling the other 4 DOF;

8) Equation (5.1) is used as the only magnetic force.

For vehicle fixed coordinate and track reference coordinate have same origin. This origin is chosen as the mass center of vehicle. The levitation and lateral array magnet positions are given in Table 5.1.

Table 5.1. Magnet array balance position

<table>
<thead>
<tr>
<th>Halbach Array</th>
<th>Levitation Array</th>
<th>Lateral Array</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>1</td>
<td>$L_{xHLav}$</td>
<td>$L_{yHLav}$</td>
</tr>
<tr>
<td>2</td>
<td>$-L_{xHLav}$</td>
<td>$-L_{yHLav}$</td>
</tr>
<tr>
<td>3</td>
<td>$-L_{xHLav}$</td>
<td>$L_{yHLav}$</td>
</tr>
<tr>
<td>4</td>
<td>$-L_{xHLav}$</td>
<td>$-L_{yHLav}$</td>
</tr>
</tbody>
</table>

If vehicle with small rotational angle displacement roll, pitch, yaw, ($\phi, \theta, \varphi$), and translation displacement surge (traveling), sway (lateral), heave (levitation), ($\Delta x_t, \Delta y_t, \Delta z_t$), the distance of each magnetic array to the track will change accordingly due to these rotational and translational displacements. The magnetic force is the function of the distance between the magnet and track coils with considering both the original distance and the effect of the offset. For levitation array, in $z$ direction, simply adding offset. For guidance array, in $y$ direction, the array 1 and array 3, adding the offset; array 2 and array 4, subtracting the offset. The offset is the difference of the new position and original position.

The new position due to the rotational motion can be calculated with Equation 5.19. The
translation displacements are taken into consideration directly.

\[
V_{i \_ref} = [DCM]^T V_{b \_ref}
\]

\[
V_{b \_ref} = [DCM] V_{i \_ref}
\]

where \( DCM \) is Direction Cosine Matrix (\( DCM \))

\[
[DCM]^T = J_1(\eta_2)
\]

\[
= \begin{bmatrix}
\cos \theta \cos \varphi & -\cos \phi \sin \varphi & \cos \varphi \sin \theta \sin \phi & \sin \phi \sin \varphi & \cos \varphi \cos \theta \cos \phi \\
\cos \theta \sin \varphi & \cos \phi \cos \varphi & \sin \varphi \sin \theta \sin \phi & -\sin \phi \cos \varphi & \sin \varphi \cos \theta \cos \phi \\
-\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi & -\sin \theta & \cos \theta \cos \phi \\
\end{bmatrix}
\]

With above new distance of each magnetic array to the track, the magnetic force at unbalance position equation for each magnetic array can be calculated using Equation (5.1). For small displacement the force equations are given by following equations.

\[
<F_{1\_b\_LevitationH} >= \frac{B_{rh}}{2kL} \left\{ \left[ 1 - \exp(-kd) \right] \frac{\sin(\pi/M)}{\pi/M} \right\} w^2 \left[ \frac{1}{1 + (R/\omega L)^2} \exp(-2k(z + \Delta z)) \right]
\]

\[
\approx F_{b\_LevitationH} \left[ 1 - 2k\Delta z + 2(2k\Delta z)^2 \right]
\]

\[
<F_{1\_b\_LevitationH} >\approx F_{b\_LevitationH} \left[ 1 - 2k(-\theta L_{xHLelv} + \phi L_{yHLelv} + \Delta Z_0 - \Delta z_i) \right]
\]

\[
<F_{2\_b\_LevitationH} >\approx F_{b\_LevitationH} \left[ 1 - 2k(-\theta L_{xHLelv} - \phi L_{yHLelv} + \Delta Z_0 - \Delta z_i) \right]
\]

\[
<F_{3\_b\_LevitationH} >\approx F_{b\_LevitationH} \left[ 1 - 2k(\theta L_{xHLelv} + \phi L_{yHLelv} + \Delta Z_0 - \Delta z_i) \right]
\]

\[
<F_{4\_b\_LevitationH} >\approx F_{b\_LevitationH} \left[ 1 - 2k(\theta L_{xHLelv} - \phi L_{yHLelv} + \Delta Z_0 - \Delta z_i) \right]
\]
\[
< F_{1b\text{ateral}H} > \approx F_{b\text{Latera}lH} * \left[ (1 - 2k(\phi L_{xHLat} - \phi L_{zHLat} + \Delta y_i + \Delta Y_0) \right] \\
< F_{2b\text{ateral}H} > \approx F_{b\text{Latera}lH} * \left[ (1 - 2k(\phi L_{xHLat} - \phi L_{zHLat} - \Delta y_i + \Delta Y_0) \right] \\
< F_{3b\text{ateral}H} > \approx F_{b\text{Latera}lH} * \left[ (1 - 2k(-\phi L_{xHLat} - \phi L_{zHLat} + \Delta y_i + \Delta Y_0) \right] \\
< F_{4b\text{ateral}H} > \approx F_{b\text{Latera}lH} * \left[ (1 - 2k(-\phi L_{xHLat} - \phi L_{zHLat} - \Delta y_i + \Delta Y_0) \right] \\
\]

Where \( \Delta z \) is translational offset, \( \Delta Z_0 \) is static distance, \( \Delta y \) is translational offset, and \( \Delta Y_0 \) is static distance.

Due to the rotation motion, the moments generated by magnet forces are not resulted from single type of magnetic array. The transformation to body reference frame with \( DCM \) is needed.

\[
\vec{F}_{b\_\text{ref}} = [DCM] \bullet \vec{F}_{i\_\text{ref}} \tag{5.30}
\]

The levitation array generates levitation force \( F_{Levzi} \), which has \( z \) competent only in initial reference frame. In order to calculate the moment in body reference frame, the force needs to be transformed to body reference frame. As indicated by Equation 5.31, the levitation force generated by levitation array has all three components \( (F_{z\text{Levxb}}, F_{z\text{Levyb}}, \text{and } F_{z\text{Levzb}}) \) in body reference frame.
\[
\begin{bmatrix}
F_{\text{LevA}} \\
F_{\text{LevB}} \\
F_{\text{LevC}}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \theta \cos \varphi & \cos \theta \sin \varphi & - \sin \theta \\
- \cos \varphi \sin \varphi + \cos \varphi \sin \theta \sin \varphi & \cos \varphi \cos \varphi + \sin \varphi \sin \theta \sin \varphi & \cos \varphi \sin \theta \sin \varphi \\
\sin \varphi \sin \varphi + \cos \varphi \sin \theta \cos \varphi & - \sin \varphi \cos \varphi + \sin \varphi \sin \theta \cos \varphi & \cos \varphi \sin \theta \cos \varphi
\end{bmatrix}
\begin{bmatrix}
0 \\
F_{\text{Levi}}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\tilde{i} (-\sin \theta F_z) \\
\tilde{j} (\cos \theta \sin \theta F_z) \\
\tilde{k} (\cos \theta \cos \theta F_z)
\end{bmatrix}
\]

(5.31)

For guidance array the guidance force, the \( F_{\text{Latvi}} \), is in initial frame with \( y \) competent only, and has all three components in body reference frame (\( F_{y\text{Latxb}} \), \( F_{y\text{Latyb}} \) and \( F_{y\text{Latzb}} \)).

\[
\begin{bmatrix}
F_{y\text{Latxb}} \\
F_{y\text{Latyb}} \\
F_{y\text{Latzb}}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \theta \cos \varphi & \cos \theta \sin \varphi & - \sin \theta \\
- \cos \varphi \sin \varphi + \cos \varphi \sin \theta \sin \varphi & \cos \varphi \cos \varphi + \sin \varphi \sin \theta \sin \varphi & \cos \varphi \sin \theta \sin \varphi \\
\sin \varphi \sin \varphi + \cos \varphi \sin \theta \cos \varphi & - \sin \varphi \cos \varphi + \sin \varphi \sin \theta \cos \varphi & \cos \varphi \sin \theta \cos \varphi
\end{bmatrix}
\begin{bmatrix}
F_{\text{Layi}} \\
0 \\
0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\tilde{i} \cos \theta \sin \varphi \ F_{\text{Latvi}} \\
\tilde{j} (\cos \varphi \cos \varphi + \sin \varphi \sin \theta \sin \varphi) \ F_{\text{Latvi}} \\
\tilde{k} (-\sin \varphi \cos \varphi + \sin \varphi \sin \theta \cos \varphi) \ F_{\text{Latvi}}
\end{bmatrix}
\]

(5.32)

Following above procedures for all the four levitation and four guidance arrays, the force and moment equations are given by

\[
F_y = (F_{\text{1bilateralH}} - F_{\text{2bilateralH}} + F_{\text{3bilateralH}} - F_{\text{4bilateralH}}) \ (\cos \varphi \cos \varphi + \sin \varphi \sin \theta \sin \varphi) - [F_{\text{1bLevitationH}} \\
+ F_{\text{2bLevitationH}} + F_{\text{3bLevitationH}} + F_{\text{4bLevitationH}}] \cos \theta \sin \varphi + Mg \cos \theta \sin \varphi
\]
\[
F_z = Mg \cos \theta \cos \phi - [F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} + F_{3u\text{Levitation}H} + F_{4u\text{Levitation}H}] \cos \theta \cos \phi + \\
[F_{1u\text{Lateral}H} - F_{2u\text{Lateral}H} + F_{3u\text{Lateral}H} - F_{4u\text{Lateral}H}] (-\sin \phi \cos \varphi + \sin \varphi \sin \theta \cos \phi)
\]

\[
M_K = [-F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} - F_{3u\text{Levitation}H} + F_{4u\text{Levitation}H}] \cos \theta \cos \phi \cdot L_y + [ -F_{1u\text{Lateral}H} + F_{2u\text{Lateral}H} - F_{3u\text{Lateral}H} + F_{4u\text{Lateral}H}] (-\sin \phi \cos \varphi + \sin \varphi \sin \theta \cos \phi) \cdot L_{y\text{lat}} + [ -F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} - F_{3u\text{Levitation}H} + F_{4u\text{Levitation}H}] \cos \theta \sin \phi \cdot L_z + [ -F_{1u\text{Lateral}H} + F_{2u\text{Lateral}H} - F_{3u\text{Lateral}H} + F_{4u\text{Lateral}H}] \cos \theta \cos \phi \cdot L_{z\text{lat}}
\]

\[
M_M = [F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} - F_{3u\text{Levitation}H} - F_{4u\text{Levitation}H}] \cos \theta \cos \phi \cdot L_x + [ -F_{1u\text{Lateral}H} + F_{2u\text{Lateral}H} - F_{3u\text{Lateral}H} + F_{4u\text{Lateral}H}] (-\sin \phi \cos \varphi + \sin \varphi \sin \theta \cos \phi) \cdot L_{x\text{lat}} - [ -F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} - F_{3u\text{Levitation}H} + F_{4u\text{Levitation}H}] \sin \theta \cdot L_x + [ -F_{1u\text{Lateral}H} + F_{2u\text{Lateral}H} - F_{3u\text{Lateral}H} + F_{4u\text{Lateral}H}] \cos \theta \sin \phi \cdot L_{x\text{la}}
\]

\[
M_N = [F_{1u\text{Lateral}H} - F_{2u\text{Lateral}H} - F_{3u\text{Lateral}H} + F_{4u\text{Lateral}H}] (\cos \phi \cos \varphi + \sin \varphi \sin \theta \sin \phi) \cdot L_{x\text{la}} + \\
[F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} - F_{3u\text{Levitation}H} - F_{4u\text{Levitation}H}] \cos \theta \sin \phi \cdot L_x - [ -F_{1u\text{Levitation}H} + F_{2u\text{Levitation}H} - F_{3u\text{Levitation}H} + F_{4u\text{Levitation}H}] \sin \theta \cdot L_y + [ -F_{1u\text{Lateral}H} + F_{2u\text{Lateral}H} - F_{3u\text{Lateral}H} + F_{4u\text{Lateral}H}] \cos \theta \sin \phi \cdot L_{y\text{la}}
\]

\[
(5.33) - (5.37)
\]

where \(L_x\) is the distance between center of each levitation array and the center of the vehicle in \(x\) direction; \(L_y\) is the distance between center of each levitation array to the center of the vehicle in \(y\) direction; \(L_z\) is the distance between center of the levitation array and center of vehicle in the \(z\) direction; \(L_{x\text{la}}\) is the distance between the center of lateral array to the center of vehicle in \(x\) direction. \(L_{y\text{la}}\) is the distance between the center of lateral array to the center of vehicle in \(y\) direction.
direction; $L_{zla}$ is the distance between center of the lateral array and center of vehicle in the $z$ direction.
CHAPTER SIX: MAGLEV DYNAMIC SIMULATION AND CONTROL

The stability analysis and dynamic simulation of a Halbach array Maglev EDS system with the novel active array are presented in this chapter. The simulations are conducted under different conditions, such as with and without control and with and without mass center offset. The analysis and simulation results show that the system is marginally stable in levitation, lateral, roll, pitch and yaw directions. Unlike ordinary vehicles, the offset of the mass center of vehicle has a strong effect on the dynamics of the Maglev system due to the uniqueness of the magnetic force. The mass center offset causes oscillations in all directions at the take off stage. In order to guarantee the Maglev system dynamic stability, the active damping and LQR control were developed. The simulation verified the effectiveness of the proposed control designs.

6.1. Six DOF Dynamic Analysis Theory

The equations of motion for rigid body, expressed in the body-fixed reference coordinate system, can be written as [65, 66, 126, 137]

\[ M_{RB} \ddot{v} + C_{RB}(v)v = \tau_{RB} . \]  

(6.1)
where $M_{RB}$ is a matrix of inertial and mass terms and $C_{RB}$ is a matrix of centrifugal and Coriolis terms. They are given by

\[
M_{RB} = \begin{bmatrix}
    m & 0 & 0 & 0 & m_{zG} & -m_{yG} \\
    0 & m & 0 & -m_{zG} & 0 & m_{xG} \\
    0 & 0 & m & m_{yG} & -m_{xG} & 0 \\
    0 & -m_{zG} & m_{yG} & I_{xx} & -I_{xy} & -I_{xz} \\
    m_{zG} & 0 & -m_{xG} & -I_{xy} & I_{yy} & -I_{yz} \\
    -m_{yG} & m_{xG} & 0 & -I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\]

and

\[
C_{RB}(v) = \begin{bmatrix}
    0 & 0 & 0 & m_{zG}v + m_{yG}q & mw - m_{xG}q & -mv - mxG \\
    0 & 0 & 0 & -mw - myGp & m_{zG}v + mxGp & mu - mxG \\
    0 & 0 & 0 & mv - mzGp & -mu - mzGq & mxGp + myGq \\
    -m_{zG}v - m_{yG}q & m_{yG}p & mzGp & -I_{xG} + I_{xy} & -I_{yG} - I_{xy} & I_{zG} + I_{yz} \\
    m_{zG}v - m_{xG}p & mzGp & I_{xG} + I_{xG} & -I_{yG} - I_{xy} & I_{zG} + I_{yz} & -I_{zG} - I_{zG} \\
    m_{xG}v & m_{yG}r & -mxGp - myGq & -I_{xG} - I_{xy} & I_{xG} + I_{xy} & I_{zG} - I_{yz} \end{bmatrix}
\]

(6.2)

$v = \{ u, v, w, p, q, r \}^T$ is the vector of translational and rotational velocities of the vehicle with respect to the vehicle body-fixed reference frame. $
\tau_{RB} = \{ F_x, F_y, F_z, M_K, M_M, M_N \}^T$ is the vector that represents all external forces and moments applied to the carriage.

Equation 6.1 consists of two parts, translational and rotational, which can be rewritten as

\[
F = m \left( \dot{v_1} + v_2 \times v_1 + v_2 \times v_G + v_2 \times (v_2 \times v_G) \right)
\]

(6.3)
\[ M = I_0 \dot{v}_2 + v_2 \times (I_0 v_2) + m \, r_G \times (\dot{v}_1 + v_2 \times v_1) \]  

where \( I_0 \) is the inertial tensor as defined at the origin of the body-fixed coordinate system, and \( r_G \) is the vector from origin of the body-fixed frame to the body center of gravity, and is defined as:

\[ r_G = \begin{bmatrix} x_G \\ y_G \\ z_G \end{bmatrix}^T. \]

The Equation 6.3 and Equation 6.4 can be expanded as:

\[ F_x = m[\ddot{u} - vr + wq - x_G (q^2 + r^2) + y_G (pq - \dot{r}) + z_G (pr + \dot{q})] \]  

\[ F_y = m[\ddot{v} - wp + ur - y_G (r^2 + p^2) + z_G (qr - \dot{p}) + x_G (qp + \dot{r})] \]  

\[ F_z = m[\ddot{w} - uq + vp - z_G (p^2 + q^2) + x_G (rp - \dot{q}) + y_G (rq + \dot{p})] \]

\[ M_K = I_{xx} \ddot{p} + (I_{zz} - I_{yy})qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \]  

\[ + m[ y_G (\dot{w} - uq + vp) - z_G (\dot{v} - wp + ur)] \]  

\[ M_M = I_{yy} \ddot{q} + (I_{xx} - I_{zz})rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{xz} + (qp - \dot{r})I_{yz} \]  

\[ + m[ z_G (\dot{w} - vr + wq) - x_G (\dot{w} - uq + vp)] \]  

\[ M_N = I_{zz} \ddot{r} + (I_{yy} - I_{xx})pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{yx} + (rq - \dot{p})I_{xy} \]  

\[ + m[ x_G (\dot{v} - wp + ur) - y_G (\dot{u} - vr + wq)] \]

If the vehicle is symmetric around both the x-z and y-z planes, it is implied that \( I_{xy} = I_{xz} = I_{yz} = 0 \). This reduces the rigid body inertia tensor to:

\[ I_0 = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}. \]
The Equation 6.6 to Equation 6.11 can be rewritten as

\[ F_x = m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] \quad (6.13) \]

\[ F_y = m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] \quad (6.14) \]

\[ F_z = m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] \quad (6.15) \]

\[ M_K = I_{xx} \dot{p} + (I_{zz} - I_{yy}) qr + m [y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] \quad (6.16) \]

\[ M_M = I_{yy} \dot{q} + (I_{xx} - I_{zz}) rp + m [z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] \quad (6.17) \]

\[ M_N = I_{zz} \dot{r} + (I_{yy} - I_{xx}) pq + m [x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] \quad (6.18) \]

If the center of mass of the vehicle is at the origin of the body-fixed reference frame, \(x_G = y_G = z_G = 0\), and the vehicle fixed coordinate frame and track reference coordinate have same origin.

The equations Equation 6.13 to Equation 6.18 are further simplified as

\[ F_x = m[\dot{u} - vr + wq] \quad (6.19) \]

\[ F_y = m[\dot{v} - wp + ur] \quad (6.20) \]

\[ F_z = m[\dot{w} - uq + vp] \quad (6.21) \]

\[ M_K = I_{xx} \dot{p} + (I_{zz} - I_{yy}) qr \quad (6.22) \]

\[ M_M = I_{yy} \dot{q} + (I_{xx} - I_{zz}) rp \quad (6.23) \]

\[ M_N = I_{zz} \dot{r} + (I_{yy} - I_{xx}) pq \quad (6.24) \]

With these six equations, Equation 6.19 to Equation 6.24, we can simulate the translational and rotational dynamics using the Simulink®. The results are \(v_t = \begin{bmatrix} u & v & w \end{bmatrix}^T\).
(Body-fixed linear velocity) and $\mathbf{v}_2 = \{p \quad q \quad r\}^T$ (Body-fixed angular velocity).

The following coordinate transform relates translational velocities between body-fixed and inertial coordinates:

$$
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} = \mathbf{J}_1(\eta_2) \begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}, \quad \dot{\eta}_1 = \mathbf{J}_1(\eta_2) \mathbf{v}_1
$$

(6.25)

where $\eta_1 = \{x \quad y \quad z\}^T$ (Inertial Position) and $\eta_2 = \{\phi \quad \theta \quad \psi\}^T$ (Inertial Orientation) are the final dynamic simulation results.

$$
\mathbf{J}_1(\eta_2) = \begin{bmatrix}
  \cos \theta \cos \phi & -\cos \phi \sin \phi + \cos \phi \sin \theta \sin \phi & \sin \phi \sin \phi + \cos \phi \sin \theta \cos \phi \\
  \cos \theta \sin \phi & \cos \phi \cos \phi + \sin \phi \sin \phi \sin \phi & -\sin \phi \cos \phi + \sin \phi \sin \phi \cos \phi \\
  -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
$$

$J_1(\eta_2)$ is an orthogonal matrix, $J_1^{-1}(\eta_2) = J_1^T(\eta_2) = [DCM]$

The coordinate transform relates rotational velocities between body-fixed and inertial reference coordinates by:

$$
\dot{\eta}_2 = \mathbf{J}_2(\eta_2) \mathbf{v}_2
$$

(6.26)

and

$$
\begin{bmatrix}
  \phi \\
  \theta \\
  \varphi
\end{bmatrix} = \mathbf{J}_2(\eta_2) \begin{bmatrix}
  p \\
  q \\
  r
\end{bmatrix} = \begin{bmatrix}
  p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \\
  q \cos \phi - r \sin \phi \\
  r \cos \phi + q \sin \phi
\end{bmatrix}
$$

(6.28)
\[
\begin{bmatrix}
   p \\
   q \\
   r
\end{bmatrix} = J_2^{-1}(\eta_2)
\begin{bmatrix}
   \phi \\
   \theta \\
   \varphi
\end{bmatrix} =
\begin{bmatrix}
   \phi - \varphi \sin \theta \\
   \theta \cos \phi + \varphi \cos \theta \sin \phi \\
   -\theta \sin \phi + \varphi \cos \theta \cos \phi
\end{bmatrix}
\]

\[ (6.29) \]

\[
\begin{bmatrix}
   \dot{p} \\
   \dot{q} \\
   \dot{r}
\end{bmatrix} =
\begin{bmatrix}
   \ddot{\phi} - \ddot{\varphi} \sin \theta - \ddot{\varphi} \dot{\theta} \cos \theta \\
   \dot{\theta} \cos \phi - \dot{\phi} \sin \phi + \dot{\varphi} \cos \theta \sin \phi + \dot{\varphi} \cos \theta \cos \phi - \ddot{\varphi} \dot{\theta} \sin \theta \sin \phi \\
   -\dot{\theta} \sin \phi - \dot{\phi} \cos \phi + \dot{\varphi} \cos \theta \sin \phi - \dot{\varphi} \cos \theta \cos \phi - \ddot{\varphi} \dot{\theta} \sin \theta \cos \phi
\end{bmatrix}
\]

\[ (6.30) \]

6.2. Levitation, Lateral Dynamic and Control

6.2.1. Levitation Dynamic and Control

It is clearly indicated that a repulsively levitated vehicle has the vertical oscillations. The maglev may be directly analyzed with classical linear vibration theory used in aircraft dynamic analysis [95, 120].

For the vehicle levitated above the track coils, the magnetic levitation force \( F \) is a function of the distance between the magnetic array and levitation coils, \( d \). The force can be expanded in a Taylor series in the perturbed small displacement variables about the equilibrium-nominal position.
and \( F(d_0) = mg \), \( m \) is the mass of the vehicle, and \( g \) is the gravity acceleration rate of earth.

\[
F(d) = F(d_0) + \frac{\partial F}{\partial d} \bigg|_{d=d_0} (d - d_0)
\]

(6.32)

for oscillatory motion \( e^{j\omega t} \) the frequency is given by solve above equation

\[
\omega = \sqrt{2gk} = \sqrt{4g\pi / \lambda}
\]

(6.35)

\[
f = \frac{\omega}{2\pi}
\]

Figure 6.1 and Figure 6.2 show the simulation result with \( \lambda = 0.13m \), the frequency is about 5 Hz, which agrees with the calculation result, 4.89 Hz.

Using Simulink\textsuperscript{®}, the levitation simulation under disturbances is performed with constant vehicle acceleration rate for a period of 15 seconds, then maintaining a constant speed. The disturbances used in the simulation consist of four force pulses. The first one is a six Newton disturbance force pulse at time 20 to 20.5 second on the levitation array 1. The second one is a six Newton force pulse at time 23 to 23.5 second on the lateral array 1. The third one is a six Newton disturbance force pulse at time 25 to 25.5 second on the lateral array 2. The fourth one is a six Newton disturbance force pulse at time 27 to 27.5 second on the levitation array 2.
Figure 6.1: Levitation simulation (a) disturbance force, (b) traveling speed, (c) levitation height

The simulation results confirm that the levitation height oscillates and a control mechanism is needed to damp the oscillations. The active magnetic array is considered as a suitable approach to damp these oscillations. Similarly, active magnetic arrays can be used for the roll, pitch, yaw, and lateral control.
Comparing the levitation equation at equilibrium position with typical second order system equation, Equation 6.36.

\[ \ddot{z} + 2\zeta \omega_n \dot{z} + \omega_n^2 z = f_L(t) \]  

(6.36)

where \( \zeta \) is damping ratio, \( \omega_n \) is the natural frequency, \( f_L(t) \) is the force function. The levitation equation shows that the system is an undamped sinusoidal system.

To adding damping, a control force \( f_L(t) = -2\zeta \omega_n \dot{z} \) can introduced with suitable value of damping ratio \( \zeta \). If \( 1 > \zeta > 0 \), under damped exponentially decaying motion; If \( \zeta = 1 \), critically damped exponentially decaying motion; If \( \zeta > 1 \), over damped exponentially decaying motion. Figure 6.3 to Figure 6.5 show the simulation results under different damping ratio.

Figure 6.2: Levitation simulation, (a) disturbance force; (b) traveling speed; (c) levitation height (Zoom in)
Figure 6.3: Levitation simulation (a) disturbance force, (b) traveling speed, (c) levitation height

(with damping ratio $\zeta = 0.1$)

Figure 6.4: Levitation simulation, (a) disturbance force; (b) traveling speed; (c) levitation height

(with damping ratio $\zeta = 1$, critical damping)
These simulation results show that the damping control is quite effective for eliminating levitation oscillation. The active array damping control current can be derived according to the coil character. Let us suppose that the active array magnetic coil block current $i$ and the coil magnetic flux density $B$ has a relationship

$$B = B(i) \tag{6.37}$$

The active array damping control force $f_L(t)$ can be generated by control the active control current $i$
\[
f_k(t) = K_f \frac{B(i)^2 \left[ \left( 1 - \exp(-kd_b) \right) \sin(\pi/M) \right]^2 w^2}{2kL} \cdot \frac{1}{1 + (R/k \nu L)^2} \exp(-2k\Delta z)
\]

\[
= K_{fa}(v, \Delta z) \cdot B(i)^2
\]

\[
= -2\zeta \omega_n z
\]

where

\[
K_{fa}(v, \Delta z) = K_f \frac{\left[ \left( 1 - \exp(-kd_b) \right) \sin(\pi/M) \right]^2 w^2}{2kL} \cdot \frac{1}{1 + (R/k \nu L)^2} \exp(-2k\Delta z)
\]

\[
B(i)^2 = \frac{-2\zeta \omega_n z}{K_{fa}(v, \Delta z)}
\]

\[
i = B^{-1}\left( -2\zeta \omega_n z \right)
\]

The infinite length solenoid flux density with \( N \) turn and current \( I \) is given by

\[
B = \mu_0 NI
\]

To make things simple, it is reasonable to assume that the control current \( i \) and magnetic flux density \( B \) of the active control array block has a relationship

\[
B = B(i) = k_{fac} I
\]

The Equation 6.41 can be simplified to

\[
i = \sqrt{\frac{-2\zeta \omega_n z}{k_{fac} K_{fa}(v, \Delta z)}}
\]
6.2.2. Lateral Dynamic and Control

For the vehicle at the equilibrium nominal position, the magnetic guidance force $F$ is balanced by the force generated by both sides of the arrays on the vehicle. These forces are a function of the distance $d$ between the magnetic array and track coils at a given traveling speed. The force can be expanded into a Taylor series of the perturbed displacement variables about the equilibrium-nominal position with neglecting the higher order terms.

$$F(d) = F_{13}(d_0) - F_{24}(d_0) + \frac{\partial(F_{13}(d) - F_{24}(d))}{\partial d} \bigg|_{d=d_0} (d - d_0)$$

(6.45)

$$F_{13}(d_0) = F_{24}(d_0)$$

$$= 2K_f \frac{B_r^2 \left[1 - \exp(-kd_h)\right] \sin(\pi/M) w_{lat}^2}{2kL} \left[\frac{1}{1 + (R/k \nu L)^2} \exp(-2kd_0)\right]$$

(6.46)

substitute Equation 6.46 into Equation 6.45

$$F(d) = 2K_f \frac{B_r^2 \left[1 - \exp(-kd_h)\right] \sin(\pi/M) w_{lat}^2}{2kL} \left[\frac{1}{1 + (R/k \nu L)^2} \exp(-2kd_0)\right]$$

$$\ast \left[\exp(-2k(d_0 + \Delta d)) - \exp(-2k(d_0 - \Delta d))\right]$$

$$= -4kF_{13}(d_0) \Delta d$$

(6.47)

where $F_{13}$ and $F_{24}$ are the lateral forces generated by guidance array 1, 3 and array 2, 4 respectively.

Similar to the levitation case, from the equation of motion

$$m \Delta d = -4kF_{13}(d_0) \Delta d$$

(6.48)
For oscillatory motion \( e^{j\omega t} \) the frequency is given by solving the above equation

\[
\omega = \sqrt{\frac{4kF_1(d_0)}{m}} \tag{6.49}
\]

The frequency is not simple as levitation natural frequency. It is a function of the magnetic force, which is a function of traveling speed.

Figure 6.6: Lateral dynamic simulation, (a) disturbance force; (b) traveling speed; (c) lateral position
Figure 6.7: Lateral dynamic simulation, (a) disturbance force; (b) traveling speed; (c) lateral position (Zoom in)

The natural frequency of lateral is about 8 Hz.

With same procedure, similar as the levitation damping control, the damping control can be added for lateral motion. Figure 6.8 through Figure 6.11 show the simulation results with different damping ratios (under damping, critical damping, over damping).
Figure 6.8: Lateral simulation, (a) disturbance force (b) traveling speed (c) lateral position (with damping ratio $\zeta = 0.01$)

Figure 6.9: Lateral simulation, (a) disturbance force (b) traveling speed (c) lateral position (with damping ratio $\zeta = 0.1$)
Figure 6.10: Lateral simulation, (a) disturbance force; (b) traveling speed; (c) lateral position
(with damping ratio $\zeta = 1$, critical damping)

Figure 6.11: Lateral simulation, (a) disturbance force; (b) traveling speed; (c) lateral position
(with damping ratio $\zeta = 2$)
6.2.3. Levitation and Lateral Dynamic Simulation Under Different Speed

As indicated by previous section, the speed has effect on the dynamic oscillation. The Figure 6.12 through Figure 6.14 show the simulation results with different traveling speeds.

Figure 6.12: Levitation and lateral simulation final speed 65 m/s (a) disturbance force, (b) traveling speed, (c) lateral position, (d) lateral position (Zoom in), (e) levitation position, (f) levitation position (Zoom in)

The levitation oscillation frequency is about 5 Hz, and lateral oscillation frequency is about 8.6 Hz.
The levitation oscillation frequency is about 5 Hz, and lateral oscillation frequency is about 8 Hz.
The levitation oscillation frequency is about 5 Hz, and lateral oscillation frequency is about 8 Hz.

The above simulation results show for the given vehicle the levitation oscillation frequency is almost constant for different traveling speed. But lateral oscillation frequency is changed with different traveling speed. The oscillation frequency will increase a little bit under the higher speed.
6.3. Six DOF Dynamic Simulation

Together with the dynamic equations, Equation 6.2 and Equation 6.3, and force, moment equations, Equation 5.33 to Equation 5.37, the translational and rotational dynamics of vehicle simulation under disturbances was performed using the Simulink®. The simulation under disturbances is performed with constant vehicle acceleration rate for a period of 1.5 seconds, then maintaining a constant speed. The disturbances used in the simulation consist of four force pulses. The first one was a disturbance force pulse at time 2 to 2.2 second on the levitation array 1, Lev 1. The second one was a disturbance force pulse at time 4 to 4.2 second on the lateral array 1, Lat 1. The third one was a disturbance force pulse at time 6 to 6.2 second on the lateral array 2, Lat 2. The fourth one was a disturbance force pulse at time 8 to 8.2 second on the levitation array 2, Lev 2.

Figure 6.15 through 6.17 show the dynamics of the vehicle under those four disturbance force pulses, with amplitude of six Newton. Figure 6.15 shows the translational position. Figure 6.16 shows the translational speed. Figure 6.17 shows the rotation speed.

The traveling direction speed is controlled separately according to a desired trajectory, so the traveling direction the position is also according to the desired trajectory. The lateral position has no oscillations before the disturbance force pulse, small amplitude oscillation after the first levitation disturbance force Lev 1, a oscillation pulse during the Lat 1 disturbance pulse, and a continuous oscillation after the lateral disturbance pulse. Which is in agreement with the analysis results. Without any disturbance the lateral will be kept in equilibrium position, the levitation
disturbance can be coupled into the lateral oscillation, the lateral oscillation will not go out of control due to the self regulation, and will not die out due to a lack of damping. The levitation position has small oscillation as soon as the vehicle take off the track and continues oscillation even after the levitation disturbance. The reason is the same as in the lateral position case.

Figure 6.15: The position simulation final with speed 24 m/s (a) disturbance force, (b) traveling (surge) (c) lateral (sway), (d) levitation (heave)
Figure 6.16: The velocity simulation final with speed 24 m/s (a) disturbance force, (b) traveling (surge) (c) lateral (sway), (d) levitation (heave).

Figure 6.17: The Euler angles simulation final speed 24 m/s (a) disturbance force, (b) roll angle, (c) pitch angle, (d) yaw angle.
The bigger disturbances result big oscillations, and may even lead to the vehicle touch the track. Figure 18 through Figure 23 show the simulation results with two additional amplitude disturbance forces, 15 Newton and 25 Newton.

Figure 6.18: The position simulation final with speed 24 m/s, 15 Newton (a) disturbance force (b) traveling (c) lateral (d) levitation
Figure 6.19: The velocity simulation final with speed 24 m/s, 15 Newton (a) disturbance force, (b) traveling (c) lateral (d) levitation

Figure 6.20: The Euler angles simulation final speed 24 m/s, 15 Newton (a) disturbance force, (b) roll angle, (c) pitch angle, (d) yaw angle
Figure 6.21: The position simulation final with speed 24 m/s, 25 Newton (a) disturbance force, (b) traveling (c) lateral (d) levitation

Figure 6.22: The velocity simulation final with speed 24 m/s, 25 Newton (a) disturbance force, (b) traveling (c) lateral (d) levitation
Without any disturbances, the rotational angles will be kept in equilibrium position, and the levitation disturbance can be coupled into the yaw oscillation, the lateral disturbance can be coupled into the roll and pitch oscillations. The rotational oscillations will not go out of control due to self regulation, and will not die out due to lack of the damping also. The 25 Newton disturbances result in the vehicle touching the track, which is a disaster for system.

The vehicle’s mass center may have a small offset. For ordinary vehicle a small offset in center of mass may not be a big issue. But due to the uniqueness of the Maglev system, it has a strong effect on vehicle dynamics and may lead to the vehicle touching the track. Figure 6.24 shows the drift of the vehicle’s mass center with an offset of $\Delta x_{\text{uneven}}$ and $\Delta y_{\text{uneven}}$ in $x$ and $y$ axles.
Figure 6.24: The drift of the uneven load mass center

With the vehicle’s mass center having an offset of \( \Delta x_{\text{uneven}} \) and \( \Delta y_{\text{uneven}} \) in \( x \) and \( y \) axles respectively, the balanced forces of the four levitation arrays are

\[
F_{\text{levitaion_{uneven}}} = \frac{L_y - \Delta y_{\text{uneven}}}{L_y + \Delta y_{\text{uneven}}} \times \frac{2L_y}{L_y - \Delta y_{\text{uneven}}} + \frac{L_y + \Delta y_{\text{uneven}}}{L_y + \Delta y_{\text{uneven}}} \times \frac{mg}{L_x - \Delta x_{\text{uneven}} + \Delta x_{\text{uneven}}}
\]

\[
F_{2\text{levitaion_{uneven}}} = \frac{2L_y}{L_y - \Delta y_{\text{uneven}}} + \frac{L_y + \Delta y_{\text{uneven}}}{L_y + \Delta y_{\text{uneven}}} \times \frac{mg}{L_x - \Delta x_{\text{uneven}} + \Delta x_{\text{uneven}}}
\]
\[
F_{\text{levitation uneven}} = \frac{L_y + \Delta y_{\text{uneven}}}{L_y - \Delta y_{\text{uneven}}} \times \frac{L_x - \Delta x_{\text{uneven}}}{L_x + \Delta x_{\text{uneven}}} \times mg
\]

\[
F_{\text{levitation uneven}} = \frac{L_x - \Delta x_{\text{uneven}}}{L_x + \Delta x_{\text{uneven}}} \times \frac{2L_y}{L_y - \Delta y_{\text{uneven}}} + \frac{L_y + \Delta y_{\text{uneven}}}{L_y - \Delta y_{\text{uneven}}} \times \frac{L_x - \Delta x_{\text{uneven}}}{L_x + \Delta x_{\text{uneven}}} + \frac{L_x - \Delta x_{\text{uneven}}}{L_x + \Delta x_{\text{uneven}}} + \frac{L_y - \Delta y_{\text{uneven}}}{L_y + \Delta y_{\text{uneven}}} \times \frac{L_x - \Delta x_{\text{uneven}}}{L_x + \Delta x_{\text{uneven}}} + \frac{L_x - \Delta x_{\text{uneven}}}{L_x + \Delta x_{\text{uneven}}}
\]

Figure 6.25 through Figure 6.36 show the dynamic simulation results with vehicle’s mass center offset $\Delta x_{\text{uneven}} = 0.001\text{m}$, $\Delta y_{\text{uneven}} = 0.002\text{m}$, and $\Delta x_{\text{uneven}} = 0.054\text{m}$, $\Delta y_{\text{uneven}} = 0.02\text{m}$ under final speed 24m/s and 64m/s demonstrating the vehicle dynamics with mass center offset.

Figure 6.25: The translational position ($\Delta x_{\text{uneven}} = 0.001\text{m}$, $\Delta y_{\text{uneven}} = 0.002\text{m}$, 24m/s)
Figure 6.26: Translational velocity ($\Delta x_{\text{uneven}} = 0.001\text{m}$, $\Delta y_{\text{uneven}} = 0.002\text{m}$, 24m/s)

Figure 6.27: Rotational Euler angle ($\Delta x_{\text{uneven}} = 0.001\text{m}$, $\Delta y_{\text{uneven}} = 0.002\text{m}$, 24m/s)
Figure 6.28: The translational position (\(\Delta x_{\text{uneven}} = 0.054\text{m}, \Delta y_{\text{uneven}} = 0.02\text{m}, 24\text{m/s}\))

Figure 6.29: Translational velocity (\(\Delta x_{\text{uneven}} = 0.054\text{m}, \Delta y_{\text{uneven}} = 0.02\text{m}, 24\text{m/s}\))
Figure 6.30: Rotational speed ($\Delta x_{uneven} = 0.054m$ and $\Delta y_{uneven} = 0.02m, 24m/s$)

Figure 6.31: The translational position ($\Delta x_{uneven} = 0.001m$, $\Delta y_{uneven} = 0.002m, 64m/s$)
Figure 6.32: Translational velocity \((\Delta x_{uneven} = 0.001m, \Delta y_{uneven} = 0.002m, 64m/s)\)

Figure 6.33: Rotational speed \((\Delta x_{uneven} = 0.001m, \Delta y_{uneven} = 0.002m, 64m/s)\)
Figure 6.34: The translational position ($\Delta x_{uneven} = 0.054\text{m}$, $\Delta y_{uneven} = 0.02\text{m}$, $64\text{m/s}$)

Figure 6.35: Translational velocity ($\Delta x_{uneven} = 0.054\text{m}$, $\Delta y_{uneven} = 0.02\text{m}$, $64\text{m/s}$)
Comparing with the non mass center offset case, the Maglev system has small oscillations even without the external disturbances in roll, pitch, yaw, lateral, and levitation directions. Larger mass center offset leads stronger oscillations in translational motion sway (lateral, $y$), heave (levitation, $z$), and rotational motion roll $\phi$, pitch $\theta$, and yaw $\varphi$. The larger is the acceleration rate, the larger is the oscillation amplitude.

### 6.4. Optimized Damping Control and Simulation

As we have seen the vehicle dynamic damping is not large enough to damp the oscillations in translational motion sway (lateral, $y$), heave (levitation, $z$), and rotational motion roll $\phi$, pitch $\theta$, and yaw $\varphi$. The larger is the acceleration rate, the larger is the oscillation amplitude.
roll $\phi$, pitch $\theta$, and yaw $\varphi$, a control mechanism is desired to add damping. The system has independent propulsion control system, which is not considered in this report. For small displacements around the nominal position, the high order terms are negligible compared to the principal terms [32]. The simplified vehicle dynamics are given by [32, 77, 78, 136]

\[
\mathbf{M}_{RB0} \ddot{a}_0 = \mathbf{\tau}_{RB0}
\]

where $\mathbf{\tau}_{RB0} = \{F_y \ F_z \ M_K \ M_M \ M_N \}^T$

\[
\mathbf{M}_{RB0} = \begin{bmatrix}
m & 0 & 0 & 0 & 0 \\
0 & m & 0 & 0 & 0 \\
0 & 0 & I_{xx} & 0 & 0 \\
0 & 0 & 0 & I_{yy} & 0 \\
0 & 0 & 0 & 0 & I_{zz}
\end{bmatrix}
\]

$a_0 = [\Delta y, \Delta z, \phi, \theta, \varphi]^T$

Substituting the passive Halbach array only forces and moments into Equation 6.50, with neglecting the high order terms we get equations.

\[
\mathbf{\tau}_{RB} = \mathbf{\tau}_0 = \begin{bmatrix}
-8F_{hLateral}k_{\Delta y} \\
-2mgk_{\Delta z} \\
-8F_{hLevitation}k_{L_y^2}\phi \\
-8F_{hLevitation}k_{L_z^2}\theta \\
-8F_{hLateral}k_{L_y^2}\varphi
\end{bmatrix}
\]

(6.51)

\[
\mathbf{M}_{RB0} \ddot{a}_0 = \mathbf{\tau}_0 = -K_{\text{st}}0 a_0
\]

(6.52)

where
where $\Delta z$ is the vehicle displacement from equilibrium position in levitation direction, and $\Delta y$ is the vehicle displacement laterally from equilibrium position.

From Equation 6.52, it is clear that the dynamic has no damping terms. This confirmed the simulation results of oscillation in these five DOF. A control mechanism is needed to stabilize the system. The system equation with control is given by

$$M_{RR0} \ddot{a}_0 + K_{sr0} a_0 + u = 0 \quad (6.53)$$

where $u$ is the control input with are give as

$$u = [u_y, u_z, u_{\phi}, u_{\theta}, u_{\psi}] \quad (6.54)$$

The control mechanism proposed in this paper is to add damping control for these five DOF.

For the system configuration of Figure 5.1, the control forces are generated by four levitation and four guidance active arrays, which are named as $F_{1\text{LevitationA}}, F_{2\text{LevitationA}}, F_{3\text{LevitationA}}, F_{4\text{LevitationA}}, F_{1\text{LateralA}}, F_{2\text{LateralA}}, F_{3\text{LateralA}},$ and $F_{4\text{LateralA}}$. Comparing the equation with desired damping and control function and using the force and moment at equilibrium equation. The control force equations are given as

$$K_{sr0} = \begin{bmatrix}
8F_{b\text{LateralA}} k & 0 & 0 & 0 & 0 \\
0 & 2mgk & 0 & 0 & 0 \\
0 & 0 & 8F_{b\text{LevitationA}} kL_y^2 & 0 & 0 \\
0 & 0 & 0 & 8F_{b\text{LevitationA}} kL_x^2 & 0 \\
0 & 0 & 0 & 0 & 8F_{b\text{LateralA}} L_{slat}^2 k
\end{bmatrix}$$
\[ u_y = (F_{1\text{LateralA}} - F_{2\text{LateralA}} + F_{3\text{LateralA}} - F_{4\text{LateralA}}) = K_y \dot{A}_y \]

\[ u_z = (F_{1\text{LevitationA}} + F_{2\text{LevitationA}} + F_{3\text{LevitationA}} + F_{4\text{LevitationA}}) = K_z \dot{A}_z \]

\[ u_\phi = L_y * (-F_{1\text{LevitationA}} + F_{2\text{LevitationA}} - F_{3\text{LevitationA}} + F_{4\text{LevitationA}}) = K_\phi \dot{\phi} \]

\[ u_\theta = L_x * (F_{1\text{LevitationA}} + F_{2\text{LevitationA}} - F_{3\text{LevitationA}} - F_{4\text{LevitationA}}) = K_\theta \dot{\theta} \quad (6.55)-(6.59) \]

\[ u_\varphi = L_{\text{slat}} * (F_{1\text{LateralA}} - F_{2\text{LateralA}} - F_{3\text{LateralA}} + F_{4\text{LateralA}}) = K_\varphi \varphi \]

where \( K_d = [K_y \quad K_z \quad K_\phi \quad K_\theta \quad K_\varphi]^{T} \) is the damping control factors. The Equation 6.55 to Equation 6.59 can be grouped into

\[ u_z = F_{1\text{LevitationA}} + F_{2\text{LevitationA}} + F_{3\text{LevitationA}} + F_{4\text{LevitationA}} \]

\[ u_\phi = L_y * (-F_{1\text{LevitationA}} + F_{2\text{LevitationA}} - F_{3\text{LevitationA}} + F_{4\text{LevitationA}}) \]

\[ u_\theta = L_x * (F_{1\text{LevitationA}} + F_{2\text{LevitationA}} - F_{3\text{LevitationA}} - F_{4\text{LevitationA}}) \quad (6.60)-(6.62) \]

and

\[ u_y = (F_{1\text{LateralA}} - F_{2\text{LateralA}} + F_{3\text{LateralA}} - F_{4\text{LateralA}}) \]

\[ u_\varphi = L_{\text{slat}} * (F_{1\text{LateralA}} - F_{2\text{LateralA}} - F_{3\text{LateralA}} + F_{4\text{LateralA}}) \quad (6.63)-(6.64) \]

These are two groups of equations with more variables than the equations. To solve these equations some constraints can be introduced given the practical situation to get optimized solutions. For example, to minimize the sum of control forces square by \( \min\{\sum F^2\} \), or to minimize the number of the devices by \( \min\{\text{control force number}\} \).
Following we use both options to illustrate the effectiveness of the active array damping control design. First one, minimizing number of the device by \( \min\{\text{control force number}\} \). From Equation 6.60 to Equation 6.62 with \( F_{1\text{Levitation}} = 0 \), we get

\[
F_{2\text{Levitation}} = \frac{1}{2} \left( u_z + \frac{u_\theta}{L_x} \right)
\]

\[
F_{3\text{Levitation}} = \frac{1}{2} \left( u_z - \frac{u_\theta}{L_y} \right)
\]

\[
F_{4\text{Levitation}} = \frac{1}{2} \left( u_\phi - \frac{u_\theta}{L_x} \right)
\]

From Equation 6.63 to Equation 6.64 with \( F_{2\text{Lateral}} = F_{4\text{Lateral}} = 0 \), we get

\[
F_{1\text{Lateral}} = \frac{1}{2} \left( \frac{u_\phi}{L_{\text{slat}}} + u_y \right)
\]

\[
F_{3\text{Lateral}} = \frac{1}{2} \left( u_y - \frac{u_\phi}{L_{\text{slat}}} \right)
\]
The simulation results given by Figure 6.38 and Figure 6.39, which show the effectiveness of the active damping control and optimized design.

The second approach of minimizing the mean square of control forces by \( \min \left\{ \sum F^2 \right\} \) can be solved with Lagrange multiplier optimization method [126]. For Equation 6.60 through Equation 6.62, The Lagrangian is given by

\[
Lag = F^2_{1,\text{Levitation}} + F^2_{2,\text{Levitation}} + F^2_{3,\text{Levitation}} + F^2_{4,\text{Levitation}} \\
+ \lambda_1 (F_{1,\text{Levitation}} + F_{2,\text{Levitation}} + F_{3,\text{Levitation}} + F_{4,\text{Levitation}} - u_z) \\
+ \lambda_2 (-F_{1,\text{Levitation}} + F_{2,\text{Levitation}} - F_{3,\text{Levitation}} + F_{4,\text{Levitation}} - \frac{u_\phi}{L_y}) \\
+ \lambda_3 (F_{1,\text{Levitation}} + F_{2,\text{Levitation}} - F_{3,\text{Levitation}} - F_{4,\text{Levitation}} - \frac{u_\theta}{L_\phi})
\]  

(6.70)

The solutions are given as.

\[
F_{1,\text{Levitation}} = -\frac{\lambda_1 - \lambda_2 + \lambda_3}{2} = -\frac{u_z}{4} + \frac{u_\phi}{4L_y} - \frac{u_\theta}{4L_\phi}
\]

(6.71)-(6.74)

\[
F_{2,\text{Levitation}} = -\frac{\lambda_1 + \lambda_2 + \lambda_3}{2} = -\frac{u_z}{4} - \frac{u_\phi}{4L_y} - \frac{u_\theta}{4L_\phi}
\]

\[
F_{3,\text{Levitation}} = -\frac{\lambda_1 - \lambda_2 - \lambda_3}{2} = -\frac{u_z}{4} + \frac{u_\phi}{4L_y} + \frac{u_\theta}{4L_\phi}
\]
For Equation 6.63 through Equation 6.64, The Lagrangian is given by

\[
\text{Lag} = F_{1Lateral}^2 + F_{2Lateral}^2 + F_{3Lateral}^2 + F_{4Lateral}^2 + \lambda_1 (F_{1Lateral} - F_{2Lateral} + F_{3Lateral} - F_{4Lateral} - u_y)
\]

\[
+ \lambda_2 (F_{1Lateral} + F_{2Lateral} - F_{3Lateral} + F_{4Lateral} - u_y) - \frac{u_\phi}{L_{sLat}}
\]

(6.75)

The solutions are given as.

\[
F_{1Lateral} = -\frac{\lambda_1 + \lambda_2}{2} = -\frac{u_y}{4} - \frac{u_\phi}{4L_{sLat}}
\]

\[
F_{2Lateral} = \frac{\lambda_1 + \lambda_2}{2} = \frac{u_y}{4} + \frac{u_\phi}{4L_{sLat}}
\]

\[
F_{3Lateral} = -\frac{\lambda_1 - \lambda_2}{2} = -\frac{u_y}{4} + \frac{u_\phi}{4L_{sLat}}
\]

(6.76)-(6.79)

\[
F_{4Lateral} = -\frac{\lambda_1 + \lambda_2}{2} = \frac{u_y}{4} - \frac{u_\phi}{4L_{sLat}}
\]

There are only 5 active arrays being used with the first approach. The force needed for each active array will be smaller for the second approach. No matter which optimization approach is used, the results will satisfy the Equation 6.55 through Equation 6.59 and simulation results are exactly same under same damping factors $K_\phi, K_z, K_\phi, K_y$, and $K_\phi$.

The simulation results are given by Figure 6.38 and Figure 6.39, which show the
effectiveness of the design.

Figure 6.38: The position simulation final with speed 24 m/s (a) disturbance force (b) traveling (c) lateral (d) levitation

Figure 6.39: The Euler angles simulation final speed 24 m/s (a) disturbance force (b) roll angle (c) pitch angle (d) yaw angle
The damping control has the advantage of simple and the requirement for implementation is not very challenging, which results in only the time different for damping out oscillation due to the damping ratio inaccurate, and has no effect on the stability.

If more accurate control is needed, we recommend the LQR control, which is one kind of optimized control and widely used in missile and airplane control.

6.5. Optimized LQR Control and Simulation

LQR is a linear optimal control with quadratic performance indices [132, 133]. The advantages of linear optimal control are [132].

1. Nearly all linear optimal control problems have readily computable solutions.

2. Linear optimal control results may be applied to nonlinear systems operating on a small signal basis. It is quite suitable for the Maglev system stability control due to the natural of the Maglev small displacement in translational motion sway (lateral, $y$), heave (levitation, $z$), and rotational motion roll $\phi$, pitch $\theta$ and yaw $\psi$.

3. Linear optimal control designs where the plant states are measurable turn out to possess a number of properties, other than simply optimality of a quadratic index. These properties include good gain margin and phase margin, and good tolerance of nonlinearities. Hence, linear optimal design methods are in some ways applicable to nonlinear systems.

For system
\[ \dot{x} = Ax + Bu \] (6.80)

where \( x \) = state vector (\( n \) vector), \( u \) = control vector (\( r \) vector), \( A \) = constant matrix (\( n \) by \( n \) matrix), \( B \) = constant matrix (\( n \) by \( r \) matrix).

The performance index is given by

\[ J = \int_{0}^{\infty} (x^* Q x + u^* R u) dt \] (6.81)

where \( x^* \) is the complex conjugate of the transpose of matrix \( x \) and the control vector is given by

\[ u(t) = -K x(t) \] (6.82)

where \( Q \) is the weighting matrix on the states (\( n \) by \( n \)), \( R \) is a positive scalar and yields a matrix of optimal gains \( K \) for the state feedback. The optimization of the cost function gives the optimal control signal \( u \). The optimal configuration is shown in Figure 6.40.

![Figure 6.40: Linear Quadratic Optimal Control Block Diagram](image-url)
\[ K = R^{-1}B^*P \] (6.83)

where \( P = P^T \), is the unique positive definite solution of the algebraic Riccati equation

\[ A^*P + PA - PBP^{-1}B^*P + Q = 0 \] (6.84)

The weighting matrix \( Q = I \) and \( R = I \) are presented in this paper to demonstrate the effectiveness of the design.

The Equation 6.53 consists of five independent second order systems for translational motion sway (lateral, \( y \)), heave (levitation, \( z \)), and rotational motion roll \( \phi \), pitch \( \theta \), and yaw \( \varphi \).

The controller can be designed separately according to the LQR optimized control theory. Following we will give the design one by one.

Let define \( x_{1y} = \Delta Y, x_{2y} = \Delta \dot{Y}, K_{st0y} = 8F_{blateral}\lambda t k \). The lateral system equation is given by

\[ \dot{x}_y = A_y x_y + B_y u_y \] (6.85)

where

\[ \dot{x}_y = [x_{1y} \ x_{2y}] \]

\[ A_y = \begin{bmatrix} 0 & 1 \\ -\frac{K_{st0y}}{m} & 0 \end{bmatrix} \]

\[ B_y = \begin{bmatrix} 0 \\ -\frac{1}{m} \end{bmatrix} \]

substituting these into Equation 6.81 we get equation
\[
\begin{bmatrix}
-K_{x_{0y}}m/ p_{12} & -K_{x_{0y}}m/ p_{11} \\
-K_{y_{0y}}m/ p_{22} & K_{y_{0y}}m/ p_{12}
\end{bmatrix}
\begin{bmatrix}
-K_{x_{0y}}m/ p_{12} & p_{11} \\
-K_{y_{0y}}m/ p_{22} & p_{12}
\end{bmatrix}
\begin{bmatrix}
1/ m^2 p_{12}^2 & 1/ m^2 p_{11}^2 \\
1/ m^2 p_{12} p_{22} & 1/ m^2 p_{22}^2
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
= 0
\]

(6.86)

solving it and select the positive values, the results are given as

\[
p_{12} = m(\sqrt{K_{x_{0y}}^2 + 1} - 1)
\]

\[
p_{11} = (\frac{1}{m})^2 p_{12} + \frac{K_{x_{0y}}}{m} p_{22} = (\sqrt{1 + 2m(\sqrt{K_{y_{0y}}^2 + 1} - 1)(\sqrt{K_{y_{0y}}^2 + 1} - 1) + K_{x_{0y}}^2} - 1)
\]

\[
p_{22} = m(\sqrt{1 + 2p_{12}}) = m(\sqrt{1 + 2m(\sqrt{K_{x_{0y}}^2 + 1} - 1)})
\]

(6.87)

substituting these into Equation 6.98, we get the optimized control gain matrix K,

\[
K = R^{-1} B^* P
\]

\[
= \begin{bmatrix}
0 & \frac{1}{m} \\
\frac{1}{m} & p_{12}
\end{bmatrix}
\begin{bmatrix}
p_{11} & p_{12} \\
p_{12} & p_{22}
\end{bmatrix}
\]

(6.88)

\[
= \frac{1}{m} \begin{bmatrix}
p_{12} & p_{22}
\end{bmatrix} \sqrt{K_{x_{0y}}^2 + 1} - 1 \sqrt{1 + 2m(\sqrt{K_{y_{0y}}^2 + 1} - 1)}
\]

and the optimized control is

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\[ u_y = Kx \]

\[
= \left[ \sqrt{K_{st0y}^2 + 1 - 1} + \sqrt{1 + 2m\left( \sqrt{K_{st0y}^2 + 1 - 1} \right)} \right] \Delta Y \Delta \dot{Y} \tag{6.89}
\]

\[
= \left[ \sqrt{K_{st0y}^2 + 1 - 1} \Delta Y + \sqrt{1 + 2m\left( \sqrt{K_{st0y}^2 + 1 - 1} \right)} \Delta Y \right]
\]

Similarly, we can get the optimized control for \( u_z, u_{\phi}, u_{\psi}, \) and \( u_{\theta}. \)

\[ u_z = \left[ \sqrt{K_{st0z}^2 + 1 - 1} \Delta Z + \sqrt{1 + 2m\left( \sqrt{K_{st0z}^2 + 1 - 1} \right)} \Delta Z \right] \tag{6.90} \]

\[ u_{\phi} = \left[ \sqrt{K_{st0\phi}^2 + 1 - 1} \phi + \sqrt{1 + 2I_{ss} \left( \sqrt{K_{st0\phi}^2 + 1 - 1} \right)} \phi \right] \tag{6.91} \]

\[ u_{\theta} = \left[ \sqrt{K_{st0\theta}^2 + 1 - 1} \theta + \sqrt{1 + 2I_{sy} \left( \sqrt{K_{st0\theta}^2 + 1 - 1} \right)} \theta \right] \tag{6.92} \]

\[ u_{\psi} = \left[ \sqrt{K_{st0\psi}^2 + 1 - 1} \psi + \sqrt{1 + 2I_{sz} \left( \sqrt{K_{st0\psi}^2 + 1 - 1} \right)} \psi \right] \tag{6.93} \]

where \( K_{st0z} = 2mgk, \quad K_{st0\phi} = 8F_{b\text{Levitation}}kL_y^2, \quad K_{st0\theta} = 8F_{b\text{Levitation}}kL_x^2, \) and \( K_{st0\psi} = 8F_{b\text{Lateral}}^kL_{\text{xlat}}^2. \)

With the same procedure as the optimized damping control to minimize number of the device using equations Equation 6.65 through Equation 6.69, with \( F_{1\text{Levitation}} = 0 \) and \( F_{2\text{Lateral}} = F_{4\text{Lateral}} = 0, \) or to minimize the mean square of control forces using Equation 6.71 through Equation 6.74 and Equation 6.76 through Equation 6.79, the force and moment are implemented by optimized approach and the simulation results are showed in Figure 6.41 through Figure 6.43.
Figure 6.41: With LQR control, the position simulation final with speed 24 m/s (a) disturbance force (b) traveling (c) lateral (d) levitation

Figure 6.42: With LQR control, the velocity simulation with final speed 24 m/s (a) disturbance force (b) traveling (c) lateral (d) levitation
Figure 6.43: With LQR control, the Euler angles simulation final speed 24 m/s (a) disturbance force (b) roll angle (c) pitch angle (d) yaw angle.

The damping control and LQR control simulation results for mass center offset case are showed in Figure 6.44 through Figure 6.49, and Figure 6.50 through 6.55 respectively.
Figure 6.44: With damping control, the translational position ($\Delta x_{\text{uneven}} = 0.001\text{m}$, $\Delta y_{\text{uneven}} = 0.002\text{m}$, 24m/s) (a) disturbance force (b) traveling (c) lateral (d) levitation

Figure 6.45: With damping control, the translational velocity ($\Delta x_{\text{uneven}} = 0.001\text{m}$, $\Delta y_{\text{uneven}} = 0.002\text{m}$, 24m/s) (b) traveling (c) lateral (d) levitation
Figure 6.46: With damping control, the Euler angles ($\Delta r_{uneven} = 0.001\text{m}$, $\Delta \gamma_{uneven}=0.002\text{m}$, $24\text{m/s}$) (a) disturbance force (b) roll (c) pitch (d) yaw

Figure 6.47: With damping control, the translational position ($\Delta r_{uneven} = 0.054\text{m}$, $\Delta \gamma_{uneven}=0.02\text{m}$, $24\text{m/s}$) (a) disturbance force (b) traveling (c) lateral (d) levitation
Figure 6.48: With damping control, the translational velocity ($\Delta x_{\text{uneven}} = 0.054\text{m}, \Delta y_{\text{uneven}} = 0.02\text{m}, 24\text{m/s}$) (b) traveling (c) lateral (d) levitation

Figure 6.49: With damping control, the Euler angles ($\Delta x_{\text{uneven}} = 0.054\text{m}, \Delta y_{\text{uneven}} = 0.02\text{m}, 24\text{m/s}$) (a) disturbance force (b) roll (c) pitch (d) yaw
Figure 6.50: With LQR control, the translational position ($\Delta x_{uneven} = 0.001m$, $\Delta y_{uneven} = 0.002m$, 24m/s) (a) disturbance force (b) traveling (c) lateral (d) levitation

Figure 6.51: With LQR control, the translational velocity ($\Delta x_{uneven} = 0.001m$, $\Delta y_{uneven} = 0.002m$, 24m/s) (b) traveling (c) lateral (d) levitation

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Figure 6.52: With LQR control, the Euler angles ($\Delta x_{uneven} = 0.001m$, $\Delta y_{uneven} = 0.002m$, 24m/s)

(a) disturbance force (b) roll angle (c) pitch angle (d) yaw angle

Figure 6.53: With LQR control, the translational position ($\Delta x_{uneven} = 0.054m$, $\Delta y_{uneven} = 0.02m$, 24m/s) (a) disturbance force (b) traveling (c) lateral (d) levitation
Figure 6.54: With LQR control, the translational velocity ($\Delta x_{\text{uneven}} = 0.054m$, $\Delta y_{\text{uneven}} = 0.02m$, 24m/s) (b) traveling (c) lateral (d) levitation

Figure 6.55: With LQR control, the Euler angles ($\Delta x_{\text{uneven}} = 0.054m$, $\Delta y_{\text{uneven}} = 0.02m$, 24m/s) (a) disturbance force (b) roll angle (c) pitch angle (d) yaw angle

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The simulation results show that this marginally stable system has oscillations in levitation, lateral, roll, pitch and yaw directions under disturbances even without mass center offset. With mass center offset the dynamics have oscillations even without external disturbances, which has a big effect on the vehicle take off dynamics. The active array damping control and LQR control are designed. The simulation results confirmed that the active array control could provide the required dynamic control for levitation, lateral, roll, pitch, and yaw directions. With the active control, the Maglev system is dynamically stable. The analysis and simulation results will be used as the guidance for further theory and experimental research.
Table 6.1 The Maglev simulation system parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The peak strength of the magnetic field at the surface of the Halbach array</td>
<td>$B_0=0.9$ Tesla</td>
</tr>
<tr>
<td>The array wave length</td>
<td>$\lambda=0.13$ Meter</td>
</tr>
<tr>
<td>The Magnet array width</td>
<td>$w=0.1$ Meter</td>
</tr>
<tr>
<td>The coil resistance of each turn</td>
<td>$R=1.5,\text{m}\Omega$</td>
</tr>
<tr>
<td>The coil inductance of each turn</td>
<td>$L=2.6,\mu\text{H}$</td>
</tr>
<tr>
<td>The mass of the vehicle</td>
<td>$M=9.3,\text{kg}$</td>
</tr>
<tr>
<td>The mass center to levitation array center distance in $X$ direction</td>
<td>$L_x=0.27$ Meter</td>
</tr>
<tr>
<td>The mass center to levitation array center distance in $Y$ direction</td>
<td>$L_y=0.1$ Meter</td>
</tr>
<tr>
<td>The mass center to levitation array center distance in $Z$ direction</td>
<td>$L_z=-0.1$ Meter</td>
</tr>
<tr>
<td>Levitation coil center position in $Z$ direction</td>
<td>$L_{zcoil}=0.095-d_t,\text{Meter}$</td>
</tr>
<tr>
<td>The distance between the levitation coil center to levitation array surface in $Z$ direction</td>
<td>0.005 Meter</td>
</tr>
<tr>
<td>The mass center to lateral array center distance in $X$ direction</td>
<td>$L_{xla}=0.17$ Meter</td>
</tr>
<tr>
<td>The mass center to lateral array center distance in $Y$ direction</td>
<td>$L_{yla}=0.2$ Meter</td>
</tr>
<tr>
<td>The mass center to lateral array center distance in $Z$ direction</td>
<td>$L_{zla}=0$</td>
</tr>
<tr>
<td>Lateral coil center position in $Y$ direction</td>
<td>$L_{ycoil}=0.19-d_t,\text{Meter}$</td>
</tr>
<tr>
<td>The distance between the lateral coil center to the surface of lateral array in $Y$ direction</td>
<td>0.01 Meter</td>
</tr>
<tr>
<td>The thickness of the Halbach array</td>
<td>$d_t,\text{Meter}$</td>
</tr>
</tbody>
</table>
CHAPTER SEVEN: CONCLUSION

In this research, a new Halbach array EDS Maglev system with novel active array control mechanism has been investigated. The system uses Halbach arrays for self-regulation and levitation and uses active magnet arrays for stability and ride comfort control, with independent control of multiple levitation and guidance active arrays. The system is self-regulated in the lateral, roll, pitch, and yaw directions. The Maglev system control can be simplified due to these self-regulations. The system configuration, static and dynamic stability, and optimized control were investigated. The dynamic analysis and simulation results showed the system to be marginally stable and a control mechanism is necessary. With mass center offset the system was found to have oscillations even without external disturbances, which has an effect on the vehicle take off dynamics. The optimized damping and LQR control is introduced and designed. The simulation confirmed the effectiveness of the MIMO control designs. A Fourier series and FEM analysis approach with Maxwell equations were utilized to analyze Halbach array magnetic field harmonics. The FEM and Fourier series results match quite well. Based on the magnetic filed analysis, the optimized geometry of Halbach array was discussed.

Although this analysis was focused on a Halbach array EDS system, the techniques and results developed in this research can be utilized for further research and other applications. For example, the active magnet array may be used in magnet bearings and in other Maglev system;
Magnetic field and geometry analysis results may be applied to design and analyze other Halbach array related applications. With six DOF modeling and dynamic simulation, we have the analytical capability to predict the detailed behavior of a given design before it is tested in the field. The full six DOF Maglev dynamic analysis results give us a better understanding of how various factors may influence the stability of an EDS system over its entire speed range.

Based on the work done in this dissertation, there are several directions in which the continued research on Halbach array Maglev system may proceed. The future research may be as how to design and implement the active array in practical and optimized way. The propulsion for Maglev needs to be investigated to find the optimized propulsion system. All the propulsion related issues such as design, stability analysis, control, and coupling between propulsion with other five DOF need to be investigated. The Halbach array geometry optimization was based on the nominal levitation height for space launch assistant Maglev, the optimized geometry may need some modification. So far, the control design was based on ideal cases assuming that the states are measurable and without taking the noise into consideration, further research may be based on the current research and also include noise. The Maglev stability is a fundamental topic, as mentioned in the introduction; even Earnshaw theory’s suitability for Maglev applications is raised [40]. The negative damping is another interesting topic [37], especially for the symmetric guidance array. There are many open areas in Maglev research. For engineering, the research is only a beginning stage. It will be very helpful to cooperate the theoretical research with practical engineering system. The field experiments are highly desired to verify and contribute to the theory analysis.
APPENDIX

HALBACH ARRAY LEVITATION FORCE

One of the most important equations in this thesis is the levitation force equation. The derivation was given in [1].

As the Halbach array moves above the coils, the magnetic field cuts through the upper conductors of the coil, the time-variation in magnetic field acts as a voltage source in each closed loop of wire. The effective circuit of this wire is an inductor $L$ and resistor $R$ in series. The standard circuit theory applies.

$$V = L \frac{dI}{dt} + RI = (\phi_0 \sin(\omega t))' = \omega \phi_0 \cos(\omega t)$$

(A.1)

where $V$ is the induced voltage, $I$ is the induced current, $L$ is the inductance (self plus mutual) of a circuit, and $R$ is its resistance, and $\phi_0$ is the peak flux linked by the circuit. Equation A.1 can be rewritten as

$$\frac{dI}{dt} + \frac{R}{L} I = \frac{\omega \phi_0}{L} \cos(\omega t)$$

(A.2)
The steady-state solution of Equation A.2 is:

\[
I(t) = \frac{\phi_0}{L} \left[ \frac{1}{1 + (R/\omega L)^2} \left( \sin(\omega t) + (R/\omega L)\cos(\omega t) \right) \right]
\]  \hspace{1cm} (A.3)

where excitation frequency \( \omega \) of the circuit is \( \omega = k \nu \) and \( k = 2 \pi / \lambda \), \( \nu \) is the array velocity, and \( \lambda \) is the array wavelength.

The approximation Halbach array magnetic field flux densities are given as

\[
B_x = B_r (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \sin(kx) \exp(-k\Delta z)
\]  \hspace{1cm} (A.4)

\[
B_z = B_r (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \cos(kx) \exp(-k\Delta z)
\]  \hspace{1cm} (A.5)

where \( \Delta z \) is the distance between the magnet array to the coil, \( d \) is the thickness of the Halbach array, \( M \) is the number of magnet bars per wavelength in the array, and \( B_r \) is the remanence of the permanent magnet material.

The approximation induced flux is given by

\[
\phi = B_r (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \frac{w \exp(-2k\Delta z) \sin(kx) [1 - \exp(-kh)]}{k}
\]  \hspace{1cm} (A.6)

\[
\approx B_r (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \frac{w \exp(-2k\Delta z) \sin(kx)}{k}
\]

where \( h \) is the distance between the lower and upper legs of the coil.

\[
\phi_0 \approx \frac{wB_r}{k} (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \exp(-2k\Delta z)
\]  \hspace{1cm} (A.7)

Inserting Equation A.7 into Equation A.3, the induced current is given as
\[ I(t) = \frac{wB_z}{kL} (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \exp(-2k\Delta z) \]
\[ \star \left[ \frac{1}{1 + (R/\omega L)^2} \right] [\sin(\omega t) + (R/\omega L) \cos(\omega t)] \]  
\hspace{1cm} (A.8)

The levitation force is given by
\[ F_z = I(t) \star B_z \star w \]  
\hspace{1cm} (A.9)

Averaging Equation A.9 over the wavelength the average levitation force is given by:
\[ < F_z >= K_f \frac{B_r^2 \left[ (1 - \exp(-kd)) \frac{\sin(\pi/M)}{\pi/M} \right] ^2 w^2}{2kL} \]
\[ \star \frac{1}{1 + (R/\kappa L)^2} \exp(-2k\Delta z) \]  
\hspace{1cm} (A.10)
LIST OF REFERENCES


[8] M.T. Caprio and R. C. Zowarka, MAGLEV Launch Assist Technology Demonstrator: Task 1, Center for Electromechanics The University of Texas at Austin.


[10] Syed A. Nasar and I. Boldea. , Linear electric motors: theory, design, and practical


[27] M. Ono, S. Koga, Ohtsuki, H, “Japan's superconducting Maglev train,” *Instrumentation*


[55] Z. Zhu and D. Howe, “Halbach permanent magnet machines and applications: a review,”


172


no. 2, 1974.


[109] Zhong Li; Jin Bae Park; Young Hoon Joo; Bo Zhang; Guanrong Chen, “Bifurcations and chaos in a permanent-magnet synchronous motor,” Circuits and Systems I: Fundamental

176


[118] Zhang, Rong, “Multivariable Robust Control Of Nonlinear Systems With Application To An Electro-Hydraulic Powertrain,” Ph.D. Thesis, University Of Illinois At Urbana-
Champaign, 2002.


[128] S. Sivrioglu, K. Nonami “Active permanent magnet support for a superconducting magnetic-bearing flywheel rotor,” Applied Superconductivity, IEEE Transactions on,


