Theory of Eddy Currents for Nondestructive Testing

1976

Craig Charles Biddle

University of Central Florida

Find similar works at: https://stars.library.ucf.edu/rtd

University of Central Florida Libraries http://library.ucf.edu

Part of the Engineering Commons

STARS Citation

https://stars.library.ucf.edu/rtd/201

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of STARS. For more information, please contact lee.dotson@ucf.edu.
THEORY OF EDDY CURRENTS
FOR NONDESTRUCTIVE TESTING

BY

CRAIG CHARLES BIDDLE
B.A., University of South Florida, 1968

RESEARCH REPORT

Submitted in partial fulfillment of the requirements
for the degree of Master of Engineering
in the Graduate Studies Program of
the College of Engineering
Florida Technological University

Orlando, Florida
1976
ABSTRACT

Theory of Eddy Currents for Nondestructive Testing

by

Craig Charles Biddle

Eddy current inspection methods are used extensively in industry for the nondestructive testing of a wide variety of materials and product applications.

The general theory of eddy current inspection is described. Equations defining the depth of penetration are derived from Maxwell's equations.

Signal analysis methods are described using coil impedance diagrams. The impedance diagrams for a number of inspection applications are presented. A discussion of the techniques for theoretical calculation of the impedance of a coil is described.

Typical coil configurations and instrumentation techniques are discussed.

Committee Chairman
ACKNOWLEDGEMENTS

I wish to acknowledge the invaluable assistance provided by my wife, Sharon, for her careful typing, editing, and proof reading of this report.

I am especially grateful to my wife and children for their moral support and personal sacrifices during the pursuit of my degree at Florida Technological University and the writing of this paper.

I wish to acknowledge the assistance of Dr. B. E. Mathews for his effort as my research report advisor.

I have benefitted immeasurably from the books and papers authored by H. L. Libby of Battelle-Northwest and C. V. Dodd of Oak Ridge National Laboratory.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACKNOWLEDGEMENTS</strong> ........................................... iii</td>
</tr>
<tr>
<td><strong>LIST OF FIGURES</strong> ............................................. v</td>
</tr>
<tr>
<td><strong>I. INTRODUCTION</strong> ............................................... 1</td>
</tr>
<tr>
<td><strong>II. ELECTROMAGNETIC THEORY OF EDDY CURRENT INSPECTION</strong> ........ 6</td>
</tr>
<tr>
<td><strong>III. EDDY CURRENT SIGNAL ANALYSIS</strong> .......................... 16</td>
</tr>
<tr>
<td>Impedance of a Long Sheath Current Encircling a Conductive Cylinder</td>
</tr>
<tr>
<td>The Impedance Functions of the Short Coil</td>
</tr>
<tr>
<td><strong>V. EDDY CURRENT COIL CONFIGURATIONS</strong> ....................... 55</td>
</tr>
<tr>
<td>Probe Coils</td>
</tr>
<tr>
<td>Encircling Coils</td>
</tr>
<tr>
<td>Inside Coils</td>
</tr>
<tr>
<td><strong>VI. EDDY CURRENT INSTRUMENTATION</strong> .......................... 69</td>
</tr>
<tr>
<td>Instrumentation Concepts</td>
</tr>
<tr>
<td><strong>LIST OF REFERENCES</strong> ......................................... 76</td>
</tr>
</tbody>
</table>

iv
LIST OF FIGURES

1. Coil's Magnetic Field in a Test Specimen ........................................... 3
2. Resultant Eddy Current Flow in a Specimen ......................................... 4
3. Eddy Current Flow Around a Crack in a Flat Specimen ............................ 4
4. Eddy Current Flow in a Cylinder ........................................................ 5
5. Relative Eddy Current Density (Jx/Jo) ................................................ 12
6. Phase Angle Lag .................................................................................. 14
7. Logarithmic Chart of the Major Eddy Current Parameters ......................... 15
8. Skin Depth Nomograph ........................................................................ 16
9. Phasor Diagram for Free Space ............................................................ 18
10. Phasor Diagram for a Coil over a Test Specimen .................................... 19
11. Phasor Diagram for Varying Secondary Coil Conductance at Three Coupling Efficiencies ......................................................... 20
12. Phasor Diagram for Various Coupling Efficiencies Showing Actual Values of $\omega L_s/R_s$ ................................................................. 21
13. Phasor Diagram for the Primary Coil .................................................... 22
14. Frequency Effects on a Secondary Coil ................................................ 22
15. Impedance Diagram Normalized with Respect to Primary Reactance ($\omega L_0$) ............................................................... 23
16. Impedance of a Long Solenoid Encircling a Conductive Cylinder ............. 25
17. Conductivity Loci ................................................................................ 27
18. Measurement of Conductivity Using Amplitude Sensitive Instrumentation (1 to 100% IACS) .......................................................... 27
19. Measurement of Conductivity Using Amplitude Sensitive Instrumentation (1-2.4%) ................................................................. 28
20. Conductivity Measurements for Alloy Identification ............................... 29
21. Impedance Diagram Showing Thickness Loci .......................................... 30
22. Measurement of Wall Thickness with Amplitude Sensitive Instrumentation .......................................................... 31
23. Impedance Diagram for a Probe Coil Adjacent to a Thin Plate ............... 32
24. Complex Impedance Plane of a Typical Eddy Current Probe Coil Adjacent to Thick Metal Plates .................................................. 34
25. Impedance Diagram Showing the Lift Off Loci for Nonconductive Coating Thickness Measurements .............................................. 35
27. Impedance Diagram for Subsurface Crack Distance from the Surface ...... 37
28. Impedance Diagram for Surface Crack Depth ........................................ 39
29. Impedance Diagram Showing the Effect of Permeability on Reactance ...... 40
<table>
<thead>
<tr>
<th>Number</th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.</td>
<td>Long Sheath Current in Cylindrical Coordinates</td>
<td>43</td>
</tr>
<tr>
<td>31.</td>
<td>Impedance Diagram Calculated for a Long Sheath Current</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Encircling a Cylindrical Nonmagnetic Bar</td>
<td></td>
</tr>
<tr>
<td>32.</td>
<td>Eddy Current Coil and Test Object Divided into a Cylindrical Coordinate Matrix</td>
<td>50</td>
</tr>
<tr>
<td>33.</td>
<td>A Coil Above Multiple Planar Conductors</td>
<td>52</td>
</tr>
<tr>
<td>34.</td>
<td>Magnetic Hysteresis for (a) Nonmagnetic, (b) Magnetic, and (c) Saturated Magnetic Materials</td>
<td>58</td>
</tr>
<tr>
<td>35.</td>
<td>Torroidal Gap Probe</td>
<td>59</td>
</tr>
<tr>
<td>36.</td>
<td>Pinched Field Focus</td>
<td>60</td>
</tr>
<tr>
<td>37.</td>
<td>Shielded Probe Coil</td>
<td>60</td>
</tr>
<tr>
<td>38.</td>
<td>Typical Spring Loaded Eddy Current Coil</td>
<td>62</td>
</tr>
<tr>
<td>39.</td>
<td>Differential Probe Consisting of Two Opposed Secondary Coils in a Single Housing</td>
<td>63</td>
</tr>
<tr>
<td>40.</td>
<td>Gap Probe</td>
<td>64</td>
</tr>
<tr>
<td>41.</td>
<td>Encircling Coils</td>
<td>65</td>
</tr>
<tr>
<td>42.</td>
<td>Differential Encircling Coil</td>
<td>66</td>
</tr>
<tr>
<td>43.</td>
<td>External Comparison Coil Arrangement</td>
<td>67</td>
</tr>
<tr>
<td>44.</td>
<td>Inside Coils (Bobbin)</td>
<td>68</td>
</tr>
<tr>
<td>45.</td>
<td>Eddy Current Instrument Block Diagram and Functions</td>
<td>69</td>
</tr>
<tr>
<td>46.</td>
<td>Single Leg AC Bridge</td>
<td>71</td>
</tr>
<tr>
<td>47.</td>
<td>Double Leg AC Bridge for Comparison and Differential Coil Configurations</td>
<td>72</td>
</tr>
<tr>
<td>48.</td>
<td>Fixed Generator Instrument with Phase Discrimination</td>
<td>74</td>
</tr>
<tr>
<td>49.</td>
<td>Turbine Blade Airfoil Inspection for Cracks:</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>(A) Recorder Output Unmodulated, (B) Recorder Output Modulated</td>
<td></td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The objective of this report is to introduce the theory of eddy current inspection techniques for nondestructive testing.

Before initiating a dissertation on eddy current techniques it would be well to define nondestructive testing [1, 2, 3].

Nondestructive testing is an engineering discipline dedicated to the measurement of all aspects of materials and hardware homogeneity and conformance to established quality specifications. Nondestructive testing involves the examination of an article by a procedure that does not impair its service-ability. Nondestructive test techniques use practical applications of the principles of physics to measure hidden properties and to detect material discontinuities. Some common nondestructive testing methods include eddy currents, ultrasonics, radiography, magnetic methods, penetrants, holography, and infrared techniques. A detailed comparison of most nondestructive testing techniques, their merits, and their limitations has been published by Vary [4].

Eddy current nondestructive test methods are used extensively in industry for a wide variety of materials and product applications. Applications include [2, 5]:

1. Alloy type testing to insure use of specific materials for manufacturing a product
2. Electrical conductivity measurements
3. Identification of material heat treat conditions
4. Defect detection including location, identification and measurement of cracks, laps, porosity, inclusions etc.

5. Wall thickness measurements in tubing, complex structures sheet metal products, etc.

6. Coating thickness measurements including:
   a) Nonconductive coatings on conductive substrates
   b) Conductive coatings on nonconductive substrates
   c) Conductive coatings on substrates of higher or lower conductivity

7. Corrosion detection

8. Eddy current proximity probes

9. Counting devices

10. Measurements of liquid concentrations

11. Control of fluid levels

Eddy currents are primarily useful in applications requiring a surface or near surface inspection technique. Eddy current methods are also useful for measurement of parameters within a hostile environment including temperature extremes or background radiation.

Eddy current inspection methods depend upon the principles of electromagnetic induction. An eddy current flow is obtained by applying a time varying sinusoidal electric current to a test coil. The resultant sinusoidally varying magnetic field (figure 1) induces an electric current flow when placed in the vicinity of an electrically conductive material. The electric currents will flow in the conductive material in circular concentric paths,
hence the name eddy currents (figure 2). In accordance with Lenz's Law, the eddy currents produce a secondary magnetic field which interacts with the primary magnetic field. The resultant field (a vectorial addition of the primary and secondary magnetic field) induces a current through and a voltage across the coil. Variation in the eddy current magnitude or path will result in a perturbation in the magnetic field and a resultant perturbation in the test coil voltage.

![Magnetic field lines](image)

**Fig. 1.** Coil's magnetic field in a test specimen

In a homogeneous test object, eddy currents flow in a plane parallel with the surface. Eddy currents can be used to inspect for any irregularities or material inhomogeneities which alter or interfere with the eddy current magnitude, phase, or flow path. The magnitude and phase of these induced eddy currents depend upon the amplitude and frequency of the driving
Fig. 2. Resultant eddy current flow in a specimen

current, electrical conductivity, magnetic permeability, specimen geometry, coil positioning, and material homogeneity. The flow path is determined by the nature and orientation of a test object surface irregularity or material inhomogeneity (figures 3 and 4).

Fig. 3. Eddy current flow around a crack in a flat specimen
Fig. 4. Eddy current flow in a cylinder
II. ELECTROMAGNETIC THEORY OF EDDY CURRENT INSPECTION

High frequency eddy currents are primarily confined to the surface of an electrically conductive material. The phenomenon is referred to as the "skin effect".

In devising an eddy current inspection for a specific application, one must determine what depth of penetration is required. For example, when inspecting coating or wall thicknesses the depth of penetration of the majority of the eddy current energy should equal or exceed the thickness of the characteristic to be measured. Similarly, if inspecting for subsurface defects, the depth of penetration must be sufficient to provide a relatively high eddy current density at the potential defect depth. However, inspection for exceptionally shallow, tight cracks requires high current densities confined to the object surface.

Determination of the depth of penetration for a given application depends upon knowledge of the effect or variables governing eddy current flow and their depth of penetration. With a good grasp of the variables affecting eddy current inspection, the NDT engineer can preselect the optimum inspection parameters within the constraints or boundary conditions of the applications.

In this section the basic theory of eddy current depth of penetration will be investigated and equations defining the
principle parameters affecting the eddy current inspection will be derived.

**Eddy Current Depth of Penetration**

Electromagnetic derivations traditionally begin with a mathematical development from Maxwell's equations given here in differential form.

\[ \mathbf{\nabla} \cdot \mathbf{D} = \rho \]  \hspace{1cm} (1)
\[ \mathbf{\nabla} \cdot \mathbf{B} = 0 \]  \hspace{1cm} (2)
\[ \mathbf{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]  \hspace{1cm} (3)
\[ \mathbf{\nabla} \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \]  \hspace{1cm} (4)
\[ \mathbf{B} = \mu \mathbf{H} \]  \hspace{1cm} (5)
\[ \mathbf{D} = \epsilon \mathbf{E} \]  \hspace{1cm} (6)
\[ \mathbf{J} = \sigma \mathbf{E} \]  \hspace{1cm} (7)

Where

\( \mathbf{D} \) = electric flux density \( \text{(coulombs/meter}^2 \)\)
\( \mathbf{B} \) = magnetic flux density \( \text{(webers/meter}^2 \)\)
\( \mathbf{H} \) = magnetic field intensity \( \text{(amperes/meter} \)\)
\( \mathbf{J} \) = current density \( \text{(amperes/meter}^2 \)\)
\( \mathbf{E} \) = electric field intensity \( \text{(volts/meter} \)\)
\( \epsilon \) = electric permittivity \( \text{(farads/meter} \)\)
\( \mu \) = magnetic permittivity \( \text{(henry/meter} \)\)
\( \sigma \) = electric conductivity \( \text{(mho/meter} \)\)
\( \rho \) = free charge density \( \text{(coulombs/meter}^3 \)\)
Using Maxwell's equations, a relatively straightforward description of eddy current flow in a material can be derived [5]. Eddy current inspection techniques are applied to electrically conductive metals in which the free charge density (\( \rho \)) is zero.

For this derivation one traditionally assumes a homogeneous conductive region with a unidirectional sheet of current in the conductive \( Y-Z \) plane where \( 0 < x < \infty \) (representative of an infinitely large test object relative to the coil dimensions).

Combining Maxwell's equations (3), (6), and (7) eliminates current and electric flux densities giving

\[
\nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\partial \epsilon \mathbf{E}}{\partial t} \tag{8}
\]

\[
\nabla \times \mathbf{H} = [\sigma + \epsilon \frac{\partial}{\partial t}] \mathbf{E} \tag{9}
\]

Inspection of equations (4) and (9) suggests that \( \mathbf{E} \) can be eliminated by taking the curl of (9).

\[
\nabla \times \nabla \times \mathbf{H} = [\sigma + \epsilon \frac{\partial}{\partial t}] \nabla \times \mathbf{E} \tag{10}
\]

\[
\nabla \times \nabla \times \mathbf{H} = [\sigma + \epsilon \frac{\partial}{\partial t}] [-\mu \frac{\partial \mathbf{H}}{\partial t}] \tag{11}
\]

From vector calculus theory

\[
\nabla \times \nabla \times \mathbf{H} \equiv \nabla (\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} \tag{12}
\]

Now from equations (2) and (5)

\[
\nabla \cdot \mathbf{H} = 0 \tag{13}
\]

and

\[
\nabla \times \nabla \times \mathbf{H} = \nabla(0) - \nabla^2 \mathbf{H} \tag{14}
\]
Thus from equation (11)
\[-\nabla^2 \mathbf{H} = [\sigma + \varepsilon \frac{\partial}{\partial t}] [\mathbf{H}] - \mu \frac{\partial \mathbf{H}}{\partial t}\] (15)

or
\[-\nabla^2 \mathbf{H} = \mu \sigma \frac{\partial \mathbf{H}}{\partial t} + \mu \varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}\] (16)

Eddy currents are generated from sinusoidal current sources which in turn create sinusoidally varying magnetic fields by electromagnetic induction. Assuming no material inhomogeneities, steady state conditions will prevail.

The sinusoidal magnetic field can be represented by the equation
\[\mathbf{H} = \hat{\mathbf{H}} e^{j\omega t}\] (17)

where \(\hat{\mathbf{H}}\) is the vector-phasor representation of maximum field amplitude.

Differentiating equating (17) yields
\[\frac{\partial \mathbf{H}}{\partial t} = j\omega \hat{\mathbf{H}} e^{j\omega t}\] (18)

Differentiating again
\[\frac{\partial^2 \mathbf{H}}{\partial t^2} = -\omega^2 \hat{\mathbf{H}} e^{j\omega t}\] (19)

Substituting into (16) gives
\[-\nabla^2 \hat{\mathbf{H}} = j\omega \sigma \hat{\mathbf{H}} - \omega^2 \mu \varepsilon \hat{\mathbf{H}}\] (20)

Rewriting equation (20)
\[-\nabla^2 \hat{\mathbf{H}} - \omega \mu (j\sigma - \omega \varepsilon) \hat{\mathbf{H}} = 0\] (21)

Selecting a constant for ease of solution of the pending differential equation
\[a^2 = \omega \mu (j\sigma - \omega \varepsilon)\] (22)
Then
\[ \vec{\nabla}^2 \vec{A} - a^2 \vec{A} = 0 \]  
(23)

Using the original assumption of the infinite conductor in a half space and assuming only a \( Z \) component of magnetic field parallel to the surface, equation (23) becomes

\[ \frac{d^2 \hat{A}_Z}{dx^2} + a^2 \hat{A}_Z = 0 \]  
(24)

Solving the differential equation gives

\[ \hat{A}_Z = Ae^{-ax} \]  
(25)

Rewriting (22)

\[ a^2 = \omega \mu \sigma (1 - j \frac{\omega \epsilon}{\sigma}) \]  
(26)

Conductivity \( (\sigma) \) for most metals range from approximately \( 5.7 \times 10^7 \Omega/m \) for copper through \( .09 \times 10^7 \Omega/m \) for nickel base alloys and down to \( .057 \times 10^7 \Omega/m \) for the titanium base alloys. Electric permittivity \( (\epsilon) \) for metals is approximately \( 9 \times 10^{-12} \) farads/meter. Thus, for frequencies used in eddy current inspection \( \sigma \gg \omega \epsilon \).

Rewriting (26) for conductive metals

\[ a = (j \omega \mu \sigma)^{\frac{1}{2}} \]  
(27)

\( a \) is known as the propagation constant of a plane wave in a conductor. One must understand the physical phenomenon accompanying the real and imaginary terms contained in \( \hat{A}_Z \) and \( a \).

\[ \hat{A}_Z = A\exp[-x(j \omega \mu \sigma)^{\frac{1}{2}}] \]  
(28)

\[ \hat{A}_Z = A\exp[-x(\frac{1+j}{\sqrt{2}})(\sqrt{\omega \mu \sigma})] \]  
(29)

\[ \hat{A}_Z = A\exp[-(\frac{\omega \mu \sigma}{2})^{\frac{1}{2}}x] \exp[-j(\frac{\omega \mu \sigma}{2})^{\frac{1}{2}}x] \]  
(30)

The real component of the exponential term describes the exponential
decay of the magnetic field as a function of depth within the material and, as will be shown, the exponential attenuation of eddy currents with depth.

The imaginary term describes a linear increase in phase angle lag from the surface with increasing penetration depth.

It is evident that $A$ is the value of $\hat{H}_z$ at the surface, $\hat{H}_z(0)$ thus

$$\hat{H}_z(x) = \hat{H}_z(0)\exp[-(1 + j) \left(\frac{\omega \mu \sigma}{2}\right)^{\frac{1}{2}}x]$$  \hspace{1cm} (31)$$

The relationship of $\hat{H}_z$ to eddy current density in the half space can now be calculated. Referring back to equation (3) and neglecting displacement current with respect to conduction ($a \gg \omega \epsilon$)

$$\nabla \times \vec{H} = \vec{J} + \frac{D}{\partial t} \hspace{1cm} (3)$$

$$\nabla \times \vec{H} = \vec{J} \hspace{1cm} (32)$$

$$\frac{d\hat{H}_z}{dx} = -\hat{J}_y \hspace{1cm} (33)$$

Substituting (33) into (30)

$$\hat{J}_y = -\frac{d}{dx} \hat{H}_z(0)\exp[-(1 + j) \left(\frac{\omega \mu \sigma}{2}\right)^{\frac{1}{2}}x]$$  \hspace{1cm} (34)$$

$$\hat{J}_y = -(1 + j) \left(\frac{\omega \mu \sigma}{2}\right)^{\frac{1}{2}}\hat{H}_z(0)\exp[-(1 + j) \left(\frac{\omega \mu \sigma}{2}\right)x]$$  \hspace{1cm} (35)$$

$\hat{J}_y$ at the surface is found by letting $x = 0$

$$\hat{J}_y(0) = -(1 + j) \left(\frac{\omega \mu \sigma}{2}\right)^{\frac{1}{2}}\hat{H}_z(0)$$  \hspace{1cm} (36)$$

$$\hat{J}_y(x) = \hat{J}_y(0)\exp[-(1 + j) \left(\frac{\omega \mu \sigma}{2}\right)^{\frac{1}{2}}x]$$  \hspace{1cm} (37)$$
It is convenient to define depth of penetration in terms of standard depth of penetration \( \delta \) defined as the depth at which \( J_y(x) \) decays to \( (1/e)J_y(0) \) or 36.8\% \( J_y(0) \). The standard depth of penetration is found by setting \( x = \delta \) and

\[
\delta \left( \frac{\omega \mu \sigma}{2} \right)^{\frac{1}{2}} = 1 \tag{38}
\]

yielding

\[
\delta = \left( \frac{2}{\omega \mu \sigma} \right)^{\frac{1}{2}} \tag{39}
\]

or

\[
\delta = \frac{1}{\sqrt{\pi \mu \sigma}} \tag{40}
\]

The significance of \( \delta \) can be seen in figure 5 where each standard depth reduces the magnitude of flow 63.2\%.

\[\text{Fig. 5. Relative eddy current density (Jx/Jo)}\]
Eddy current depth of penetration can now be seen to increase with decreasing frequency, electrical conductivity, and magnetic permeability. Referring to equation (37), eddy current density is seen to increase at the surface with increases in the frequency, conductivity, and permeability parameters.

The phase angle lag (separation angle) as measured relative to the surface is obtained from the imaginary term in equation (37)

\[ \theta = \tan^{-1} \left( \frac{\omega \mu \sigma}{2} \right) \]  

Using the standard depth of penetration from (38) phase angle lag becomes

\[ \theta = \frac{X}{\delta} \]  

The relationship of phase angle to depth of penetration is shown in figure 6.

Libby [5] has constructed charts showing the relationship of skin depth and the test variable frequency, conductivity, and permeability. Figure 7 is a rather complex overlay of logarithmic charts of resistivity and the product of a relative magnetic permeability and skin depth. Figure 8 is a nomograph used to more accurately calculate skin depth over a smaller range of parameters. Brown [6] at Battelle Memorial Institute, Pacific Northwest Laboratory, has developed a slide rule type calculator for convenient computation of eddy current testing parameters for nonmagnetic applications.
Fig. 6. Phase angle lag
Fig. 7. Logarithmic chart of the major eddy current parameters
Fig. 8. Skin depth nomograph
III. EDDY CURRENT SIGNAL ANALYSIS

The eddy current coil comprises the only link with the test object. The output signal is not generally usable in the form in which it is received. It is the function of the eddy current instrumentation systems to reduce the signal to a format which can be demodulated and processed into meaningful data about the test object.

To describe and interpret the coil signals, it will be necessary to understand and put into use the phase vector diagram. The phasor diagram eliminates the signal frequency information and describes a specific signal in terms of amplitude and phase. The following discussions are included to help clarify the principles required for the more complex systems to follow in later sections.

The phase angle of a magnetic flux and coil voltage is related to the test object, the variables within the object, and the eddy current coil.

If a coil of zero resistance is evaluated in free space, no test object interaction occurs and a secondary magnetic field will not be produced. In the absence of a secondary field, the primary field remains in phase with the excitation current and the induced coil voltage leads by $90^\circ$ (figure 9).

If an electrically conductive test object is within range of the magnetic field of an inductive coil, eddy currents will be induced within the object resulting in the formation of a
secondary magnetic field. The secondary magnetic field opposes the primary field to a degree proportional to conductivity, object geometry, and operating frequency. The resulting magnetic field produces an induced coil voltage which leads the current by 90° (figure 10). The coil voltage resulting from the test object is identified by $E_s$. $E_s$ is not directly measurable since it is included in $E_T$.

In the remainder of this section the eddy current systems used to explain eddy current signal analysis will consist of a primary coil used to generate the primary electromagnetic fields and a secondary coil which will be used to detect the secondary electromagnetic fields produced by the eddy currents.

An advantage to using impressed current in describing an electromagnetic induction system is that the resulting voltage drop is directly proportional to the electrical impedance of the circuit.
When the resistance and the inductance of the coil and associated circuit are experimentally varied, the voltage drop is represented by correspondingly varying phasors on an impedance diagram producing curves as shown in figure 11. Each curve represents the phasor diagram obtained for a specific value of coupling efficiency. Each smaller diameter semicircle represents progressively lower values of electromagnetic coupling efficiency. Coupling efficiency is inversely proportional to probe to specimen spacing or directly proportional to the fill factor, a measure introduced later in this section to describe the ratio of cylinder diameter to encircling diameter. The impedance at point A is the impedance of the primary coil if the secondary is open and thus independent of fill factor.

To measure the impedance of the circuit for various values of secondary circuit resistivity \( R_s \), coupling efficiency, 

\[ E_T = E_p + E_s \]
and coil inductance \( (L_s) \), lines representing the values of \( \omega L_s/R_s \) have been identified (Figure 12). The impedance of a circuit can be evaluated by reading the resistance and reactance corresponding to a specific value of \( \omega L_s/R_s \) for the coil and fill factor describing the test system. Figure 12 has been constructed for a single operating frequency, thus the lines are only a function of the ratio \( L_s/R_s \).

![Phasor diagram for varying secondary coil conductance at three coupling efficiencies (\( \eta \)).](image)

Fig. 11. Phasor diagram for varying secondary coil conductance at three coupling efficiencies (\( \eta \)).
By adjusting the frequency in a primary loop having no secondary loop, there will be a variation in reactance as shown in figure 13. As the figure indicates, no change in $R$ occurs, but reactance varies proportionally with frequency.

The effect of frequency on a secondary loop is shown in figure 14. As the frequency is increased, the value of $\omega L_s / R_s$ also increases.

The family of curves shown in figure 14 is often normalized with respect to $\omega L$ after first subtracting $R_1$, the primary resistance with the secondary loop open. Figure 15 represents
Fig. 13. Phasor diagram for the primary coil

Fig. 14. Frequency effects on a secondary coil
a universal phasor diagram obtained by normalizing the impedance diagram of figure 14.

Many impedance functions can be calculated from theory, but as boundary conditions become more complex, the solutions to the equations become correspondingly more difficult. The determination of test coil impedance functions for various test conditions and objects combined with actual measurement of these functions increases one's ability to predict system response and performance in a given test situation.

Some Eddy Current Applications and Impedance Diagrams

The variables affecting eddy current coil impedance diagram include: conductivity, frequency, coupling or lift off,
thickness, coil length, magnetic permeability, and material homogeneity. Typical impedance curves and explanations of their use and informational content will now be discussed.

**Inspection of a Cylindrical Bar Using a Long Encircling Coil**

The universally accepted impedance diagram for a long encircling coil with a cylindrical bar is shown in figure 16. The diagram is normalized with respect to the empty coil reactance. Only nonmagnetic bar stock shall be considered at this time.

The impedance plot has been made for two values of fill factor \( \eta = 1 \) and 0.81. Use of an impedance diagram for evaluating a potential application can be shown by referring to the region around point C. As the test bar conductivity decreases, the operating point A moves along locus CD. However, as the test bar radius increases the operating point moves along CB toward B. The operating point moves two different directions for these varied test parameters, thus the signals generated in each case have correspondingly varying phase angles. These effects can be identified and separated by using instrumentation techniques which will discriminate and measure phase angle. Inspection of the impedance diagram reveals that the angle between conductivity and fill factor or test bar radius (a) varies as a function of reference number (Ka). Where

\[
Ka = a\sqrt{\omega \mu \sigma}
\]  
(43)
Fig. 16. Impedance of a long solenoid encircling a conducting cylinder. Impedance values normalized with respect to empty coil reactance.

At point F the two loci have nearly the same phase angle; whereas at point H, representing the larger reference numbers, the difference in the phase angle approaches $45^\circ$. Changes in
the fill factor and test object conductivity are reflected in proportion to the variations seen on the normalization impedance diagram. Changes in test frequency are reflected in the actual impedance values, or output voltages of the unnormalized impedance curve.

Conductivity of a test object can be measured from the impedance diagram and two simple measurements. Measurement of conductivity proceeds as follows

\[ \sigma = \frac{(Ka)^2}{a} \]  

(44)

Frequency, radius, and permeability are usually known. By measuring the coil impedance with and without the test object permits the determination of the reference number (Ka). As can be seen in the impedance diagram, (Ka) is independent of fill factor. Now all quantities in (44) are known and conductivity may be calculated.

Impedance measurements have been made for a number of typical materials of varying conductivity using conventional finite length encircling coils. The resulting generalized conductivity loci are shown in figure 17.

Figures 18 and 19 are graphs obtained using an amplitude sensitive eddy current instrument to measure conductivities ranging from approximately 1% IACS to 100% IACS.

Figure 20 is included to further demonstrate the usefulness of conductivity for alloy type testing. The figure demonstrates the separativity of a number of common alloys.
Nickel base alloys
Stainless steel

Lead
Brass
Aluminum
Copper

Fig. 17. Conductivity loci

Aluminum
Copper

Cadmium
Platinum
Chromium
Lead

Commercially pure titanium
304 stainless steel

Waspalloy, Astroalloy, Inconel alloys
6-4 Titanium alloy
8-1-1 Titanium alloy

Fig. 18. Measurement of conductivity using amplitude sensitive instrumentation (1 to 100% IACS).
Fig. 19. Measurement of conductivity using amplitude sensitive instrumentation (1.0 to 2.4% IACS)
Fig. 20. Conductivity measurements for alloy identification
Wall Thickness Measurements

The impedance loci associated with varying wall thicknesses in cylindrically shaped tubing are shown in figure 21. Each thickness loci starts and ends on the conductivity curve. The end points correspond to zero wall thickness (empty coil impedance) and "infinite" thickness (impedance associated with material conductivity measurement). Figure 21 illustrates the amplitude and phase variations accompanying a wall thickness variation.

Fig. 21. Impedance diagram showing thickness loci

Figure 22 is a graph of a typical wall thickness measurement obtained using amplitude sensitive eddy current instruments.
Fig. 22. Measurement of wall thickness with amplitude sensitive instrumentation.

Probe Coil Impedance Curves

The probe coil and the encircling coil respond similarly to inspection variables. However, the probe coil is, in general, of smaller diameter, shorter, and perpendicular rather than concentric with the test object. As a result, the impedance plane diagrams may vary to a much greater extent than experienced with encircling coils. An experimentally generated impedance diagram for a typical low frequency (120KHz) probe coil is shown.
The broken ordinate axis results from the much larger reactance values encountered using probe coils as opposed to encircling coils. A probe coil having the same physical dimensions has a much lower coupling efficiency due largely to the orientation of the probe to the test object.

The figure shows the basic single frequency conductivity curve and thickness loci seen earlier in discussions describing
encircling coil impedance diagrams. The dotted lines represent probe to object spacing, typically referred to as lift off effect. As would be expected, removing the probe from intimate contact with the test object generates an impedance shift which tends toward "empty coil" impedance values.

The angle between the lift off loci and the thickness loci is referred to as the separation angle. Separation angle becomes important when impedance analysis is used to minimize the effects of lift off when measuring such variables as conductivity and wall thickness.

For example, to measure wall thickness, while minimizing lift off interference, a separation angle of approximately 90° should be chosen, if possible. If the angle approaches 180° representing parallel loci, the eddy current instrumentation will not distinguish between the two variables. Thus the only method of measurement would depend upon an inspection system design capable of totally eliminating the lift off variable. On the other hand, measurements of conductivity will be easiest where the separation angle is 180°. This condition, shown in figure 24, provides for the inclusion of both lift off and specimen thickness as easily detected variables distinguishable from conductivity variations yet indistinguishable from each other.
Fig. 24. Complex impedance plane of a typical eddy current probe coil adjacent to thick metal plates.

Nonconductive Coating Thickness Measurements

Measurement of a nonconductive coating on a conductive substrate is basically synonymous with putting a nonconductive spacer between the coil and a test specimen to increase lift off. Thus measurement of nonconductive coatings is to measure lift off. Figure 25 illustrates, in more detail than previously, the typical lift off impedance functions for a variety of base metal conductivities.

Figure 26 is a graph of a typical amplitude sensitive eddy current instrument used to measure nonconductive coating
Fig. 25. Impedance diagram showing the lift off loci for nonconductive coating thickness measurements.
Fig. 26. Nonconductive coating thickness measurement (lift off) using amplitude sensitive instrumentation.
The effect of material inhomogeneities such as subsurface cracks on the impedance diagram can be seen in figure 27.

Fig. 27. Impedance diagram for subsurface crack distance from the surface.

The operating point (B) in the curve represents a material having a conductivity similar to stainless steel. The solid loci emanating from B represents impedance variations due to varying sizes of subsurface cracks. The dotted line represents the lift off effects at the operating point. The subsurface crack loci BC, BD, BE, and BF, respectively, represent four
individual subsurface cracks of decreasing depth below the surface. As crack depth increases, the impedance variations decrease while phase angle lag increases. This phenomenon results largely from the effects of skin effect whereby eddy current density decreases as a function of depth of penetration. As can be seen, line BFEDC shows increasing subsurface crack depth from F to C. The impedance variation due to the presence of a subsurface crack occur as the probe is scanned over the defect.

Surface cracks respond quite a bit differently than the subsurface crack. Figure 28 illustrates a typical impedance curve for four cracks, BC, BD, BE, and BF, having increasingly greater depths. An immediate observation is that phase angle lag increases with crack depth as does the phasor magnitude. This effect is distinctly identifiable permitting the investigator to distinguish between surface and subsurface defects. It should also be noted that phase measurements are highly useful for applications requiring the clear separation of indication due to lift off and crack detection.

An excellent paper defining methods of optimizing defect detection by eddy current methods has been recently published by Dodd et al [7].

Magnetic Permeability Effects

Eddy currents are induced within an article by altering its magnetic flux density. A material having a high permeability ($\mu > 1$) will provide a greater flux density in the presence of a
magnetic field than a material of relatively low permeability. This is due to the alignment of the magnetic domains within the ferromagnetic material.

The increased magnetic flux density in a ferromagnetic material results in an increased induced voltage and effective coil impedance. The increased impedance results from the constructive interaction of the primary and secondary magnetic fields causing an effective energy storage and an increased reactive
component in the impedance diagram. In the case of a non-ferromagnetic material ($\mu_r > 1$) the secondary field tends to cancel the primary field resulting in an energy loss and a reduced reactance component. In either case, increasing conductivity increases energy loss resulting in reduced reactance components on the impedance curves.

Figure 29 is a conductivity impedance diagram for three values of relative permeability. As can easily be seen, increasing permeability increases the reactive impedance component proportionally.

![Impedance diagram showing the effect of permeability on reactance.](image)
Relative magnetic permeability commonly exceeds $\mu_r = 100$ resulting in a significant increase in eddy current coil impedance. Thus the effect of a varying permeability can easily overshadow a measurement of another parameter such as conductivity or wall thickness. The deliterious effects of magnetic permeability may be compensated for by subjecting the ferromagnetic material to a saturating magnetic field.
IV. EDDY CURRENT IMPEDANCE AND VECTOR

POTENTIAL CALCULATIONS

The impedance functions discussed in the preceding section were obtained primarily from experimental data. Theoretical calculation of the impedance function is complex and often dependent upon the availability of a computer, knowledgable application of numerical techniques, and application of a strong background in electromagnetic theory.

This section demonstrates the methods used to analytically calculate eddy current coil impedance functions.

**Impedance of a Long Sheath Current Encircling A Conductive Cylinder**

The long sheath current encircling a conductive cylinder (figure 30) does not simulate the typical eddy current coil, however, by permitting one to ignore end or edge effects, the long sheath coil is the simplest to evaluate analytically. The impedance solution for the long coil, as derived by Libby [5], provides excellent perspective to the general character of test coil impedance functions as well as the complexity of their analytical solutions.

Assume the medium and cylinder within the coil are isotropic, linear, and homogeneous. Further assume a steady state sinusoidal excitation current.
Fig. 30. Long sheath current in cylindrical coordinates

The magnetic and electric fields are sinusoidal due to the sinusoidal excitation current. The solution proceeds as in section II, equations (17) through (21).

Since the solution is for a long cylindrical sheath, one must convert equation (21) to cylindrical coordinates and assume that \( \frac{\partial H}{\partial \theta} = 0 \) and \( \frac{\partial H}{\partial z} = 0 \).

Thus
\[
\nabla^2 \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 H}{\partial \theta^2} + \frac{\partial^2 H}{\partial z^2} \tag{45}
\]

Now assuming \( \vec{H} \) has a component in the \( z \) direction only
\[
\nabla^2 \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H_z}{\partial r} \right) \tag{46}
\]
Combining (21) and (47) gives
\[
\frac{\partial^2 \hat{H}_Z}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{H}_Z}{\partial r} = j\omega (\sigma + j\omega) \hat{H}_Z
\]  
(48)

Now absorbing the electromagnetic constants from the right hand side of equation (48) let
\[
\varphi^2 = \omega \mu (\sigma + j\omega)
\]  
(49)

\[
\frac{\partial^2 \hat{H}_Z}{\partial r^2} + \frac{1}{r} \frac{\partial \hat{H}_Z}{\partial r} - j\varphi^2 \hat{H}_Z = 0
\]  
(50)

The solution to this differential equation is described in McLachlan [8, p. 136].

\[
\hat{H}_Z = C_1 J_0 (\varphi r \sigma) + C_2 K_0 (\varphi r \sigma)
\]  
(51)

Where \(J_0(\varphi r)\) is a Bessel function of the first kind of the zero order; \(K_0(\varphi r)\) is a modified Bessel function of the second order and of zero order.

The Bessel function \(K_0(\varphi Z^{1/2})\) increases without limit as \(Z \rightarrow 0\). This is not realizable within the boundary condition since \(Hz \rightarrow \infty\) at the center of the cylinder. Thus \(K_0\) must be equal to zero.

Then
\[
\hat{H}_Z = C_1 J_0 (\varphi r \sigma)
\]  
(52)

Now using Kevin Bessel real and imaginary functions
\[
\hat{H}_Z = C_1 [\text{ber}(\varphi r) + j\text{bei}(\varphi r)]
\]  
(53)

Solving for \(C_1\), let \(\hat{H}_Z = \hat{H}_Zo\) and \(r = a\)
\[
C_1 = \frac{\hat{H}_Zo}{\text{ber}(\varphi a + j\text{bei}(\varphi a)}
\]  
(54)

The total magnetic flux within the coil is the sum of the flux
within the cylinder, and the annulus between the cylinder and coil.

Thus

\[ \phi_T = 2\pi \mu \left[ C_1 \int_a^b \text{ber}(\varphi r) + j\text{bei}(\varphi r) \right] \, dr + 2\pi \mu H_0 \int_a^b r \, dr \]  

(55)

From McLachlan [8]

\[ \int Z \text{ber}(Z) \, dZ = Z \text{ber}'(Z) \]  

(56)

and

\[ \int Z \text{bei}(Z) \, dZ = -Z \text{bei}'(Z) \]  

(57)

where

\[ \frac{d}{dz} \left[ \text{Jo}(z) \right] = \text{ber}'(z) + j\text{bei}'(z) \]  

(58)

(55) then becomes

\[ \phi_T = 2\pi \mu C_1 \int_a^b \frac{r}{\varphi} \left[ \text{ber}'(\varphi r) - j\text{ber}'(\varphi r) \right] + \pi \mu H_0 (b^2 - a^2) \]  

(59)

Substituting the value of \( C_1 \) from (54) yields

\[ \phi_T = 2\pi \mu H_0 \int_a^b \frac{r}{\varphi} \left[ \frac{\text{ber}'(\varphi r) - j\text{ber}'(\varphi r)}{\text{ber}(\varphi r) + j\text{bei}(\varphi r)} \right] + \pi \mu H_0 (b^2 - a^2) \]  

(60)

The induced voltage can now be calculated by using Faraday’s Law

\[ e_i = -\frac{d\phi}{dt} \]  

(61)

\[ e_i = -2\pi \mu \int_a^b \frac{r}{\varphi} \left[ \frac{\text{ber}'(\varphi r) - j\text{ber}'(\varphi r)}{\text{ber}(\varphi r) + j\text{bei}(\varphi r)} \right] - \pi \mu \int_a^b \frac{dH_0}{dt} (b^2 - a^2) \]  

(62)

\( H_0 \) being sinusoidal, i.e., \( H_0 = H_0 e^{j\omega t} \)

\[ \frac{dH_0}{dt} = j\omega H_0 \]  

(63)

\[ e_i = -2\pi j\omega H_0 \int_a^b \frac{r}{\varphi} \left[ \frac{\text{ber}'(\varphi r) - j\text{ber}'(\varphi r)}{\text{ber}(\varphi r) + j\text{bei}(\varphi r)} \right] - j \pi \mu H_0 (b^2 - a^2) \]  

(64)

\[ e_i = -\pi \mu H_0 \left[ \frac{2a}{\varphi} \frac{\text{ber}'(\varphi r) + j\text{bei}'(\varphi r)}{\text{ber}(\varphi r) + j\text{bei}(\varphi r)} \right] + j(b^2 - a^2) \]  

(65)
The impedance of coil neglective of internal coil resistance is

\[ Z = \frac{e_i}{I} \]  

(66)

From Amperes Current Law

\[ \int \vec{H} \cdot d\vec{I} = I \]  

(67)

For the case of a long coil of sheath current \( I_s \),

\[ H_0 = I_s \]  

(68)

therefore

\[ Z = \frac{e_i}{H_0} \]  

(69)

and

\[ z = \pi \mu \omega \left[ \left( \frac{2a}{\varphi} \right) \left( \frac{\text{ber}'(\varphi) + j\text{bei}'(\varphi)}{\text{ber}(\varphi) + j\text{bei}(\varphi)} \right) \right] + j(b^2 - a^2) \]  

(70)

The empty coil impedance is often used for normalization impedance plots and is obtained by permitting \( \varphi \to 0 \).

\[ Z_e = j\omega \pi b^2 \]  

(71)

since

\[ L_0 = \mu \pi b^2 \]  

(72)

\[ Z_e = j\omega L_0 \]  

(73)

The normalized impedance \( \frac{Z}{\omega L_0} \) is obtained by dividing \( Z \) by \( \omega \pi b^2 \)

\[ \frac{Z}{\omega L_0} = \left( \frac{a^2}{b^2} \left( \frac{2}{\varphi a} \left[ \frac{\text{ber}'(\varphi a) + j\text{bei}'(\varphi a)}{\text{ber}(\varphi a) + j\text{bei}(\varphi a)} \right] + j \left( \frac{b^2 - a^2}{b^2} \right) \right) \right) \]  

(74)

where

\[ \varphi = \left( \omega \mu \sigma + j\omega^2 \mu \epsilon \right)^{\frac{1}{2}} \]  

(75)

for metals ranging from a few tenths of a per cent conductivity on the IACS scale to 100% IACS and frequencies from \( 10^5 \) to \( 10^7 \) Hz.

\( \omega \mu \sigma >> \omega \mu \epsilon \) therefore
Most investigators have plotted coil impedance encircling the conducting cylinder as a family of curves and a function of fill factor and reference number.

Reference number

\[ \varphi a = a \sqrt{\omega \mu \sigma} \]  

(77)

and fill factor

\[ \eta = \frac{a^2}{b^2} \]  

(78)

Figure 31 is a plot of the long coil impedance diagram assuming a fill factor equal to \((1.0)^2\) and \((.9)^2\).

Note that the wave number and fill factor include all significant variables. Computation of these factors permits quick reference to an appropriate impedance plot to obtain the impedance of the coil/cylinder system without having to perform the derivation using the Bessel Functions.

Eddy current coils are made of multiple windings forming a helix around the test object, inside a test object or around a ferrite core. Thus the sheath current assumption is impractical for most applications.

The impedance of a long coil having \(n\) turns must now be calculated.

Current through an \(n\) turn coil

\[ I_n = \frac{I_s}{n} \]  

(79)

(80)

The induced emf will be \(n\) times that induced by the sheath current.
Fig. 31. Impedance diagram calculated for a long sheath current encircling a cylindrical nonmagnetic bar.

The coil impedance \( Z_n \) is now

\[
Z_n = \frac{en_i}{I_n} = \frac{neiA}{I_s} = \frac{n^2ei}{I_s} = n^2Z_s
\]

Where \( Z_s \) is the impedance of the coil calculated using a sheath current.

The Impedance Functions of the Short Coil

The calculation of the impedance of a short coil requires numerical techniques coupled with a computer. The short coil is used extensively in eddy current inspection. Dodd et al [9 - 17] have developed solutions for various probe and encircling coils using a series of computer programs. By applying these or similar
programs one may investigate the effect of material parameters such as wall thickness, coatings, defects, etc., on the impedance functions of a coil. From these programs, Dodd has demonstrated the ability to optimize the design of a coil to obtain maximum sensitivity to the specific material parameters of interest.

**Eddy Current Coil Analysis Using Computer Techniques**

Eddy current coils have traditionally been designed by experience and experimental observation. Each new design of coil modification has required experimental demonstration.

Recent advances in eddy current electromagnetic theory have significantly improved the ability to analyze and design coils and instrument systems. In 1964 Burrows [18] developed a technique to accurately predict the effect of a material defect on an electromagnetic field. In 1967 Dodd [10] devised solutions to the electromagnetic field equations.

Two techniques have been devised to analytically determine the value of an electromagnetic field. Both methods solve for the vector potential which is used to calculate induced voltage, coil impedance, eddy current density, magnetic field, dissipated power density, and eddy current force density.

The techniques devised are (1) relaxation technique (iteration) and (2) boundary value solutions.

**Relaxation techniques**

The relaxation technique divides the entire problem into
a matrix of points in either two or three dimensions \([9, 10, 11]\) as sketched in figure 32.

![Image of eddy current coil and test object divided into a cylindrical coordinate matrix](image)

**Fig. 32.** Eddy current coil and test object divided into a cylindrical coordinate matrix.

A finite difference equation then is derived to describe the vector potential at each point in terms of the neighboring points. The difference equation is described as an approximation of the differential equation and approaches the differential equation as the matrix spacing shrinks to zero.

Assuming cylindrical coordinates, as in figure 32, the vector potential equation for a point \((r, z)\) can be denoted by
Using computers, the entire matrix of points can be calculated giving the vector potential in terms of the potential at neighboring points. Although this technique is exceptionally versatile, large amounts of storage and run time are required of the computer.

**Boundary value solutions**

A number of coil configurations can be analyzed from boundary value problems by using orthogonal functions along the boundaries [12, 13, 14]. Dodd et al [15, 16, 17] have programmed the solutions to several problems on the computer. The applications solved include coils between multiple conductive planes and coils with concentric cores. The material conductivity, permeability, thickness, and quantity of conductors are varied at will using the computerized techniques. In addition, the effects of small defects can be simulated.

The types of solutions which have been derived for the vector potential are of the form shown in equation (83). $A(r,z)$ is the vector potential in the $n$th plane for a coil above a conductor as shown in figure 33.
\[ A(r,z) = \frac{1}{2} n_c \mu I \int \frac{e^{-a(1-1)}}{a^3 V_{22}} \left[ 1 - e^{-a(1-1)} \right] J(R_2, R_1) \cdot \]

\[ \left[ V_{12}(n,1)e^{-anz} + V_{22}(n,1)e^{-anz} \right] J_1(ar) \, d 

(83) \]

Fig. 33. A coil above multiple planar conductors

Other new variables are

- \( n_c \) = turns density
- \( I \) = current per turn
- \( V_{22} \), \( V_{12}(n,1) \), and \( V_{22}(n,1) \) = transformation matrices
- \( J_1(ar) \) = Bessel function of first kind and first order
- \( a \) = Separation "constant"
\[ J(R_2, R_1) = \int_{aR_1} x J_1(x) \, dx \]  \hspace{1cm} (84)

Equation (83) can be solved on a small digital computer and need be solved only one time for a problem as opposed to \(10^6\) or \(10^7\) times for the relaxation method.

Upon calculating the vector potential, the eddy current variables of principle interest: induced voltage, impedance, current density, and magnetic field strength, can easily be calculated as shown in the following equations.

The induced voltage in a coil of wire can be obtained from the following equations

\[ V = \int \frac{dA}{dt} \cdot d\ell \]  \hspace{1cm} (85)

\[ V = j\omega A \cdot d\ell \]  \hspace{1cm} (86)

In the case of coaxial coils (transmitting and receiving) the solution has axial symmetry

\[ V = j\omega 2\pi n \frac{\pi r^2}{\text{area}} \int r A \, dr \, dz \]  \hspace{1cm} (87)

where \( n \) is the number of turns in the receiving coil.

The solution for a typical coil of rectangular cross-sectional area is

\[ V = \frac{j\omega 2m}{(r_2 - r_1)(r_2 - r_1)} \int_1^{r_2} \int_{r_1}^{r_2} A \, dr \, dz \]  \hspace{1cm} (88)
The impedance of the coil described in equation (88) is

\[
Z = \frac{V}{j\omega} = \frac{2\pi}{(r_2 - r_1)(r_2 - r_1)} \int \int rAdrdz
\]

(89)

Eddy current density for sinusoidal currents is given by the equation

\[
\bar{J} = -j\omega\bar{A}
\]

(90)

The magnetic field is expressed by

\[
\bar{B} = \bar{V} \times \bar{A}
\]

(91)
V. EDDY CURRENT COIL CONFIGURATIONS

The eddy current coil is the simplest component in an eddy current inspection system. Although simple in form and construction, the electromagnetics of the coil are complex and in most practical applications nearly impossible to model analytically without the use of a computer.

The electromagnetic behavior of a coil in an inspection application becomes complex due to the orientation between the eddy current coil and the nonsymmetrical test object and due to the complex eddy current paths which may flow in that object. Further complexity results from system inductances and stray capacitance. Often neglected, yet also of importance is the eddy current power dissipation resulting from internal resistance and unwanted eddy currents induced in the coil rather than the test object.

The eddy current coil comprises the only link between the material under investigation and the examining instrument. Eddy current coils assume a wide range of configurations designed to provide maximum electromagnetic efficiency for a wide variety of test article configurations and inspection requirements.

Many eddy current systems use a single coil to provide both the magnetic excitation field and to detect the resultant electromagnetic field. However, separate sensing coils may be used to monitor the resultant magnetic fields produced by the interaction
of the excitation field and the test article. Use of separate sensing coils permits increased system flexibility by allowing maximization of excitation and detection parameters. Commercial eddy current inspection systems use both the single and multiple coil eddy current probes and coils.

Design of an eddy current inspection coil is a marriage of theoretical techniques for characterization of coil properties and experimental optimization on actual specimens and/or parts representative of the inspection requirement. Experimental design techniques aided by theory are required because of the limited analytical treatments available for coil configurations, particularly in the field of interaction with a test object. The effect of defects on the impedance of an eddy current coil, is in most cases, much too involved to determine analytically thus necessitating the direct evaluation in an actual test or simulated test environment.

The eddy current coil configuration required for a given inspection is governed by the geometry, material, defect type and orientation, material parameter, and environment. Coils may be shielded from external electromagnetic fields or from interaction with extraneous areas within a test object by using magnetic or electrically conducting materials. These shields permit shaping the eddy current fields produced which frequently enhances sensitivity and/or resolution. Probe coils used for measurement of coating thickness, wall thicknesses or defect detection are frequently wound on magnetic cores.

Frequently eddy current testing reveals what may appear as
instrumentation drift. This can often be proven to be related to the stability of the test coil itself. Temperature variations in the test coil and test object are frequently produced by the eddy current $I^2R$ losses which heat the coil and/or test specimens. Dimensional variations caused by the thermal expansion of the heated coil may produce coil inductance fluctuations causing "drift".

A common difficulty occurring when magnetic or slightly magnetic materials are eddy current inspected is response variations due to magnetic permeability fluctuation which occur naturally within some materials. If permeability variations do not contribute to the detectibility of the particular material parameter being investigated, the effects of the variations must be eliminated to produce meaningful inspection data.

Elimination of permeability effects can be accomplished by forcing the material into magnetic saturation. Figure 34 (a) and (b) demonstrate the magnetic hysteresis loops for a magnetic and nonmagnetic material. In magnetic saturation magnetic permeability becomes constant as evidenced by the horizontal lines at the top and bottom of figure 34 (c).

Magnetic saturation may be generated through the use of small permanent magnets or electromagnets built into the eddy current probe or by passing the article through a large external magnetic field immediately prior to or during the eddy current test.
Fig. 34. Magnetic hysteresis for (a) nonmagnetic, (b) magnetic, and (c) saturated magnetic materials.

**Probe Coils**

The eddy current probe or pancake coil is the most common of modern coils used for the inspection of irregular shaped objects, flat surfaces, large diameter cylindrical or spherical surfaces, thin sheets, coatings, and other applications.

The probe coil consists of one or more sets of windings frequently wound on a cylindrical ferrite core. Single coil
Windings are typical with many of the general purpose eddy current instruments and are naturally the easiest to wind. Double or multiple coils consist of an excitation coil and one or more sensing coils. Large diameter coils with a relatively small number of turns are usually used for excitation. Smaller diameter windings with many more turns are used as sensing coils.

The probe coil is highly versatile and may be wound to produce widely varying performance characteristics for any particular application. Increased resolution and sensitivity may be achieved through focusing as shown in figures 35 and 36.

![Torroidal ferrite core](image)

**Fig. 35.** Torroidal gap probe. Typical applications include edge inspection of thin sheets or turbine blade airfoils.

Reducing edge effect or the influence from surrounding material may be accomplished using shields consisting of magnetic or electrically conductive materials as shown in figure 37.
Fig. 36. Pinched field focus. Pinched fields may be obtained by winding two opposing coils.

Fig. 37. Shielded probe coil
Defect resolution is inversely proportional to coil diameter. The higher the resolution desired, the smaller the coil diameter required. However, irrespective of the calculated standard depth of penetration derived in equation (39), the maximum depth of penetration will not exceed two to four coil diameters. Thus, an exceptionally small diameter coil will not provide the eddy current depth of penetration as calculated. The depth limitation is due to the limited range of the coil's magnetic field which is a function of coil diameter, length, shielding, focusing, etc.

Maximum defect sensitivity is achieved by orienting the coil with the test specimen so as to produce maximum signal amplitude. Optimum sensitivity is achieved with the eddy current flow perpendicular to the orientation of the defect.

A probe may be used with a variety of mechanically or hand held devices and are often housed in special adapters to control probe to part spacing (lift off), contact pressure, and angle incidence. Frequently the probe is molded to conform to the geometry of the part(s) to be inspected.

A typical hand held probe with spring loading is shown in figure 38.

The variety of probe housing which may be built are as numerous as the applications to which eddy current inspection is applied. Frequently, probes are embedded in epoxy molds and shaped to fit hardware contours for maximum sensitivity of inspection of a specific zone with minimum lift off effect.
As indicated earlier, there are a wide variety of eddy current probes. The simplest being the absolute solenoid coil arrangement containing either a single coil for excitation and sensing or multiple coils with individual excitation and sensing windings. The absolute probe is excellent for measuring most material or dimensional characteristics within the realm of eddy current inspection. However, the absolute coil does have limitations including simultaneous sensitivity to multiple eddy current parameters. Unless this sensitivity to multiple parameters can be compensated for by instrumentation and/or phasor relationships, erroneous interpretation of the eddy current data may result.

Solutions to this problem include the use of differential coil or a comparitor coil.
Differential Probe

The differential coil shown in figure 39 consists of two or more separate sensing coils aligned side by side. The signals from each coil are balanced using a bridge circuit to produce a null or zero signal over a homogeneous material. As the probe is scanned over a surface the output of each coil is compared using the bridge which produces the zero output over material of uniform condition. When one coil is presented to a differing condition an output voltage is generated. Thus the differential probe is ideal for discrimination between slowly varying test conditions and more rapid transitions such as crack detection or sharp geometrical variations.

![Diagram of differential probe]

Fig. 39. Differential probe consisting of two opposed secondary coils in a single housing.
Torroidal and Gap Probes

These coils are designed using a magnetic material shaped specifically to control the orientation and intensity of the magnetic field. The probes are frequently built from a torroidal ferrite core material which after winding has been machined to produce a narrow slit or gap (figure 35).

The gap creates a high magnetic field density over a very small area of the specimen. A variation of the gap probe is shown in figure 40.

Both coils are exceptionally sensitive to very small defects. As should be expected, the coils are also sensitive to irrelevent material or surface finish conditions thus creating a higher than average "noise" level.
The small effective field area produced by the gap probe is excellent for defect inspections on irregular shapes.

The magnetic flux vector is rotated $90^\circ$ in relation to the test specimen when using the gap probe. Thus the direction of eddy current is also rotated.

**Encircling Coils**

The encircling coil, in figure 41, is typically used in applications where a high rate of inspection is desired on long symmetrical shapes such as tubing. Other applications of the encircling coil include bulk inspection of smaller hardware for nondestructive alloy identification, conductivity measurements, etc.

![Encircling coils diagram](image)

**Fig. 41.** Encircling coils
Encircling coils have a wide variety of configurations including absolute, differential, self comparative, and external comparative. Each of these configurations, although typically cylindrically wound, may be wound rectangularly.

The differential encircling coil sketched in figure 42 operates by the same principles described for the differential probe coil. The self comparative probe is essentially a differential coil.

![Fig. 42. Differential encircling coil](image)

An external comparison probe consists of two sets of sensing coils as shown in figure 43.

The first sensing coil (test) forms one branch of the sensing bridge circuit similar to a differential coil. However, the second (reference) coil is positioned externally. A reference master of
similar size, shape, and material as the parts to be inspected is inserted into the reference coil. The bridge circuit then in nulled on an identical reference master in the test coil. Any material passing through the test coil produces an eddy current indication if dissimilar in properties to the reference master.

Inside Coils

Inside coils, figure 44, are wound and operate similar to the encircling coil. The prime difference being that the coils are usually wound on a nonconductive bobbin. Inspection is performed by inserting the coils into the inside diameter of the object(s) being inspected.
Fig. 44. Inside coils (bobbin)
VI. EDDY CURRENT INSTRUMENTATION

A wide variety of eddy current instruments are available for application in nondestructive testing which range from the relatively simple instruments to highly complex systems. All eddy current instruments perform at least five principle functions: excitation, modulation, signal preparation, demodulation and analysis, and signal display or response. A block diagram of an eddy current instrument showing the principle functions is given in figure 45.

**Fig. 45.** Eddy current instrument block diagram and functions

**Instrumentation Concepts**

Eddy current generators are usually single frequency sine wave oscillators with associated amplifiers. Multifrequency systems having two or more frequencies singularly or simultaneously are used in many applications. Pulsed generators for eddy current
inspection have been receiving considerable attention.

Signal modulation is a function of material properties and is developed through the interaction of the coil's magnetic field and the test object. Test coil design is the major parameter controlling the type and degree of modulation to be obtained from the test object. Numerous coil configurations are available. Common coil configurations have been discussed in section V.

Signal preparation consists of balancing and compensation using bridges, filters, and shapers. Balancing and compensation circuits bias the input signal to obtain an amplitude level within the linear operating range of the system amplifiers. A common bridge circuit used in numerous eddy current instruments from the most simple to highly complex systems is the AC bridge shown in figure 46. The AC bridge uses the eddy current coil as one leg of the bridge. The bridge is balanced for the typical operating condition and geometry. As the coil scans the object variations in material homogeneity or physical dimensions alter test coil impedance and thus unbalance the bridge generating a signal proportional to the bridge unbalance and test object condition.

A second bridge design consists of an AC bridge with comparison or differential coil configurations making up two of the bridge legs as shown in figure 47. In this application the test objects and/or reference masters are balanced in the bridge circuit to produce a null signal. Replacing the test object with a variant sample will generate a bridge unbalance and indication
signal.

Another circuit frequently encountered in eddy current instruments uses the test coil as the inductive component controlling the oscillator excitation frequency. As varying test parameters are detected, fluctuations in coil impedance will produce oscillator frequency, phase, and/or amplitude shifts. These variations can then be demodulated and processed through display circuits for inspection.

Fig. 46. Single leg AC bridge

A simple circuit used occasionally in amplitude sensitive applications is a fixed amplitude sine wave current generator. Variations in test coil impedance produce corresponding fluctuations in output signal amplitude which are amplified, demodulated, and
processed. A more complicated and versatile variation of the above instrument system is shown in figure 48. The fixed oscillator also supplies the excitation signal to a phase adjust circuit. The output of the sensing coil is subtracted from the fixed signal, amplified, and filtered. The phase detector is used to adjust the phase discrimination angle.

Eddy current instrument systems are typically classified as one of three major detector types.

1. Null balance with amplitude detector which is used primarily as an impedance analysis method for measurement of the magnitude of the induced eddy current field.

2. Null balance with phase detectors for measuring net changes in the time phase of the induced eddy current field with
respect to a test coil voltage, as well as measuring the magnitude changes.

3. Modulation analysis used to measure the rate of change of phase or amplitude of the eddy current field in instances where the test article is in motion with respect to the test coil. Modulation analysis is frequently accomplished by adding electronic filters between the bridge circuit and the indicator or recording devise. The filters are usually selected to pass only high frequency variations in the eddy current response such as cracks and other similar defects. Variations due to geometrical inconsistencies are usually filtered out. Figure 49 illustrates results of the modulation method of analysis applied to turbine blade airfoil inspection using a standard single coil probe. This method, however, may be applied to any coil configuration.

During an eddy current inspection, one significant problem which continually occurs, particularly in hand held applications, is that of varying coil to part spacing called "lift off". In mechanically controlled scanning systems, lift off can also be a significant variable due to object geometrical irregularities or motional instabilities. High volume mechanically controlled inspections frequently require noncontact probes (to eliminate undesirable wear) which also tend to experience susceptibility to lift off effects. Use of teflon tape and plastic shims between a contact probe and a specimen does reduce lift off variations, to some extent, where this technique is acceptable.
To reduce the effects of lift off variations, phase shift circuits have been incorporated in most modern eddy current instruments particularly those designed for defect detection applications. Instrument systems having inadequate compensation characteristics frequently are unable to distinguish between a defect and a momentary lift off induced fluctuation having amplitude characteristics of a shallow surface crack.
Fig. 49. Turbine blade airfoil inspection for cracks: (A) recorder output unmodulated, (B) recorder output modulated.
LIST OF REFERENCES


