Multicoupled Bandpass Filter Design Using a Multiple Feedback Configuration

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MULTICOUPLED BANDPASS FILTER DESIGN
USING A
MULTIPLE FEEDBACK CONFIGURATION

BY

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THESIS

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In this paper, two methods for the design of active analog feedback bandpass filter pairs are examined. A third method is presented that does not use extra amplifiers for summation nor multicoupling. This third method uses identical bandpass amplifiers and employs resistor summing to provide multicoupling. The name of this configuration is the "Dual Amplifier Bandpass Filter Employing Resistor Summing" (DABFERS). This configuration is economically attractive, has low sensitivity and better phase lag characteristics. In addition, third order prototypes are examined and a solution method for higher order prototypes is suggested.
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INTRODUCTION

The purpose of this paper is to present a design technique and a novel configuration for multicoupled active analog band-pass filter. This is derived by a straight-forward application of Mason's Loop rule to a polynomial prototype, then a popular single operational amplifier band-pass configuration is adapted to the configuration. The intent of this design is to offer an alternative to the two established multicoupled band-pass feedback design techniques. The first of these techniques has been called the "Leap Frog" (LF) circuit after its block diagram and signal flow graph appearance. This configuration was derived from a low-pass passive prototype because it was noted that passive filters had lower sensitivity than cascade filters. The reason for this lower sensitivity was because of the multicoupled nature of the passive filter. The second configuration was derived by the influence of state variable techniques as applied to the low-pass prototype polynomial. This second filter type is called the "Follow the Leader Feedback" (FLF) configuration. Research has shown that such multicoupling via the interconnection of biquadratic band-pass configuration yields reduced sensitivity. This improvement was obtained at the expense of design simplicity and alignability. The resulting low sensitivity was considered worth the increased design complexity and tuning difficulties. The scope of this work will be to demonstrate the
design techniques for all three configurations as applied to a second order low-pass prototype and indicating the steps required to implement higher order configurations. A method of simple second and first order cascade design is also presented.

The uniqueness of the DABFERS configuration is that no additional amplifiers are required to implement a multicoupled band-pass design as required in both of the above configurations. These additional amplifiers are used for summing the phase inversion functions in the existing design techniques. These tasks are implemented by open point resistor summing and appropriately phased feedback. The elimination of these excess amplifiers through the use of the DABFERS configuration is not only cost effective but offers the performance advantages of less Q enhancement and lower sensitivity for high quality factor, Q, configurations. In higher Q designs excess phase lag is a significant Q enhancement parameter. These additional amplifiers add phase lag to the overall loop. Thus, the elimination of these amplifiers reduces phase lag and allows a more stable high Q design.
SECOND ORDER DESIGN COMPARISON

The basic design techniques for both the LF and FLF configurations will now be reviewed and a typical design example for each configuration will be presented. These designs are presented for comparative purposes. Both of these designs will use a second order Butterworth low-pass prototype. Each design will use the popular band-pass circuit of T. Deliyanis\(^9\). This one operational amplifier circuit has been shown to be a useful biquadratic section of J. J. Friend\(^10\) and has been used extensively because of its simplicity and low sensitivity. The basic configuration is shown in Figure 1. In addition to the existing configurations, the DABFERS approach will also be implemented using this basic circuit. First, the LF design procedure will be presented and a design example. Second, the FLF design will be similarly summarized. Finally, the second order example will be implemented using the novel approach and a design procedure for that configuration formulated.

The LF design procedure may be summarized by the following five simple design steps.

1. Draw a passive ladder network that implements the normalized low-pass prototype polynomial.
2. Derive the branch-node equations for the ladder network.
3. Using the equations of step 2, draw the block diagram described by the branch-node equations.
Figure 1. A Good Single Stage Bandpass Filter
4. Perform a low-pass to band-pass transformation on each element of the block diagram.

5. Implement the transfer function of each transformed element of the block diagram using a band-pass configuration.

Consider the specific second order Butterworth prototype as described earlier. The transformed final circuit will have a second order single coefficient numerator and a fourth order denominator. Before proceeding, the low-pass to band-pass transformation required in step 4 of the design procedure will be derived.

Consider the following identity for the classic first order band-pass transfer function as follows:

\[
\frac{1}{p + 1} = \frac{s\omega_0 / Q_0}{s^2 + s\omega_0 / Q_0 + \omega_0^2}
\]

where \( p = \) low-pass prototype polynomial complex variable.

\( s = \) band-pass prototype polynomial complex variable.

\( \omega_0 = \) frequency scaling parameter in radians per second.

\( Q_0 = \) quality factor.

Equation number 1 may be rearranged to the following form.

\[
\frac{1}{p + 1} = \frac{1}{Q_0 \left( \frac{s}{\omega_0} + \frac{\omega_0}{s} \right) + 1}
\]

The now obvious transformation or variable substitution required then is,

\[
p = Q_0 \left( \frac{s}{\omega_0} + \frac{\omega_0}{s} \right).
\]
The quality factor $Q_0$ is defined as the center frequency, $\omega_0$, divided by a reference bandwidth. The reference bandwidth is typically chosen as a given first order attenuation reference. Unless otherwise specified, the half power or three decibel (dB) attenuation will be used. Let us now return to the LF design example at hand.
**LF DESIGN EXAMPLE**

Step 1: Draw a second order passive ladder network with normalized low-pass element values that will implement a Butterworth prototype polynomial. This low-pass ladder is shown in Figure 2 with the block diagram described by the branch-node equations.

Step 2: Derive the branch-node equations for the low-pass prototype ladder network. These are:

\[
\begin{align*}
(V_1 - V_2) Y_i(p) &= I_1 \\
I_1 Z_i(p) &= V_2
\end{align*}
\]

(4)

Where \( V_i \) = the node voltages and \( i \) is the node number, \( I_i \) = branch current and \( i \) is the branch number, \( Y_i(p) \) = shunt ladder element admittance, \( Z_i(p) \) = shunt ladder element impedance.

Step 3: Using the node and branch equations from step 2 above, draw a block diagram of the low-pass ladder prototype as described by the equations. This diagram is shown in Figure 2.

Step 4: Perform a low-pass to band-pass transformation on each element in the block diagram using the transformation described by the variable substitution of equation (3). In the elementary form, the integrators of the low-pass prototype are replaced by oscillators. In the prototype, \( Z_1(p) \) is equal to
Figure 2. Low-Pass Prototype (a) Ladder Network With Normalized Elements (b) The Block Diagram That Describes the Branch-Node Equations
$Y_2(p)$. The required equations are given by:

$$Z_1(p) = Y_2(p) = \frac{1}{\sqrt{2}p + 1},$$  \hspace{1cm} (5)

and the variable substitution yields,

$$Z_1(S) = Y_2(S) = \frac{S\omega_0/Q_0\sqrt{2}}{S^2 + S\omega_0/\sqrt{2} + \omega_0^2}. \hspace{1cm} (6)$$

The resultant block diagram to be implemented is shown in Figure 3.

**Step 5:** Implement the transfer function for each transformed block in the diagram. At each summing node of the diagram an amplifier must be added for correct phase summation of the signals. In this case, the configuration shown in Figure 1 is used and a single summing amplifier is required. The summing amplifier is configured such that the midband voltage gain of the filter is unity. If the diagram of Figure 3 were strictly adhered to, the midband voltage gain would have been at negative six dB. The schematic of the configuration is shown in Figure 4. Note that three amplifiers are required.
FLF DESIGN EXAMPLE

The FLF configuration is derived from a state variable model of the low-pass prototype polynomial. In the specific model used for this prototype, each state is fed back to a single summing junction. Each state "follows" the lead or output state. Hence, the name "follow the leader" feedback. The basic design for this example may be summarized by the following simple steps.

Step 1: Determine the coefficients of the low-pass prototype and draw the signal flow graph or block diagram of the low-pass prototype.

Step 2: Perform a low pass to band-pass conversion by the variable substitution as derived earlier on each integrator of the block diagram.

Step 3: Implement the transfer functions of the block diagram using a band-pass configuration capable of infinite Q for each integrator.

The design example for a second order Butterworth prototype proceeds from the normalized polynomial to the block diagram and finally to the schematic. Consider each of the above steps in detail.

Step 1: Write the equation for the normalized prototype and derive the coefficients from the form:

\[ G(p) = \frac{b_0}{\sum_{i=0}^{n} a_i p^i} \] (7)
Figure 3. The Complete Block Diagram with the Bandpass Transfer Functions
Figure 4. LF Schematic
For the second order example, equation (7) reduces to,

\[ G(p) = \frac{b_0}{a_2p^2 + a_1p + a_0} = \frac{1}{p^2 + \sqrt{2}p + 1}. \tag{8} \]

Equating coefficients in equation (8) yields:

\[ b_0 = 1, \]
\[ a_2 = 1, \]
\[ a_1 = \sqrt{2}, \]
\[ a_0 = 1. \]

The above coefficients may now be used in a state variable model represented by a signal flow graph. A simple development of this model will now be presented.

Define a transfer function as follows: \( T(p) \) is the transfer function of a simple input, \( v_{in}(t) \), single output, \( v_{o}(t) \), differential system. \( T(p) \) is defined as the ratio of the LaPlace transforms of the input and output. Thus,

\[ T(p) = \frac{L[v_{o}(t)]}{L[v_{in}(t)]}. \tag{9} \]

The LaPlace operator is defined as

\[ L[v(t)] = V(p) = \int_{0}^{\infty} v(t)e^{-pt}dt \tag{10} \]

If transfer functions are limited to the linear time invariant case, then the transfer function may be represented by the ratio of two polynomials of the form:
For minimum phase transfer functions $a_i$ and $b_i$ are real positive constants. There is no loss of generality in giving $a_n$ a value of unity. Such an action allows one to rewrite equation (11) as follows:

$$T(p) = \frac{\sum_{i=0}^{n-1} b_i p^i}{\sum_{i=0}^{n} a_i p^i}.$$

Equation (12) is recognizable as a special case of Mason's loop rule. Recall the case where all feedback loops are touching and all forward paths touch the feedback loops. For this case, Mason's loop rule is 11:

$$T(p) = \frac{\sum_{\text{forward paths}} b_i p^{i-n}}{1 - \sum_{i=0}^{n-1} (-a_i) p^{i-n}}.$$

More than one signal flow graph may be drawn subject to the touching restrictions above. The follow the leader feedback is only one case where all feedback loops sum at an input node and all forward paths sum at an output node. This structure is shown in Figure 5.

A reference has been made to a differential system representation of the polynomial. In this representation of the filter a set of differential equations is written in a matrix form as follows 12:
\[
\dot{x} = \begin{bmatrix}
0 & 1 & 0 & \ldots \\
0 & 0 & 1 & 0 & \ldots \\
\vdots & & \vdots \\
-a & -a & 1 & \ldots & a_{n-1}
\end{bmatrix}
\begin{bmatrix}
x \\
x \\
\vdots \\
1
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
\vdots \\
1
\end{bmatrix}
\begin{bmatrix}
in(t)
\end{bmatrix}
\]  

(14)

where \( \bar{x} \) is a column vector and

\( x_i \) = states or integrator outputs.

The above represents the input matrix, an additional equation is needed to represent the output.

Step 1

\[
G(p) = \frac{1}{p^2 + a_1 p + a_0} = \frac{1}{p^2 + \sqrt{2} p + 1}
\]
Step 1

\[ G(p) = \frac{1}{p + a_1p + a_o} = \frac{1}{p^2 + \sqrt{2}p + 1} \]

Step 2

\[ V_{in}(p) \]

\[ V_o(p) \]

Step 3

\[ V_{in}(s) \]

\[ V_o(s) \]

Figure 5. FLF Design Steps
For the all pole case all \( b_i \) except \( b_0 \) are identically zero. Hence, \( x_1 \) may be taken as the output through a gain path of magnitude \( b_0 \). This completes the development for obtaining the low-pass prototype structure for the FLF filter.

Step 2: Using equation (3), the low-pass to band-pass configuration is obtained. For the design example infinite Q conversions are used. Actually, a sensitivity advantage may be found by using finite Q transformations and work has been done in this area\(^{13}\). However, for this work the infinite Q case is sufficient to demonstrate the technique. Thus, the integrators of the structure obtained in step 2 are simply replaced by oscillators to complete this step and the results are shown in Figure 5.

Step 3: The final step results in a schematic representation of the structure as obtained in steps 2 and 3 above. For the purpose of continuity, the same configuration for implementation is used for FLF as for LF. The resulting schematic is shown in Figure 6. A single input summing amplifier is used.
NOVEL APPROACH: THE FEEDBACK BAND-PASS PAIR

The novel approach starts with the assumption that the two stages presented in the design examples above may be multicoupled without the use of an additional amplifier. The method of multicoupling desired may be indicated by observing that equation (12) may be implemented by more than one structure. This alternate structure need only obey Mason's loop rule restriction that all feedback loops be touching and all of the forward paths touch the feedback loops. In this case, the feedback loops touch and the forward paths all feedback loops at the output. This structure is shown in Figure 7. For the second order case, a technique must be found for negative feedback from the output to the input stage. If one examines the single stage analysis of the band-pass filter as shown in Figure 1, then it becomes obvious that the non-inverting terminal may be used as the feedback path. This analysis will now be performed to provide the reader with the facts to reach this conclusion.

A close examination of the single stage is required. First, redraw the circuit shown in Figure 1 such that the passive tee network is represented by a single block. The single block may be analyzed using the "A" chain matrix as defined below.

The A matrix is defined for a two terminal pair network as shown in Figure 8 by the equations:
Figure 6. FLF Schematic
\[ I_{in}(t) = U \]

Figure 7. Block Diagram of a General Polynomial Implemented by Mason's Loop Rule
\[ V_1 = a_{11} V_2 + a_{12} I_2 \]  \hspace{1cm} (16)

\[ I_1 = a_{21} V_2 + a_{22} I_2 \]  \hspace{1cm} (17)

or

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix}
= \bar{A}
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]  \hspace{1cm} (18)

Where \( V_1 \) = input voltage transform

\( I_1 \) = input current transform

\( V_2 \) = output voltage transform

\( I_2 \) = output current transform

\( \bar{A} \) = chain matrix

and

\[
\bar{A} = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\]  \hspace{1cm} (19)

This matrix may be used to find the transconductive currents \( I_A(s) \) and \( I_B(s) \) as shown in the figure. If the amplifier is assumed to be ideal, then only the transconductive paths described by the \( a_{12} \) path through the passive network. This is true because the operational amplifier forces the voltage \( V_2 \) to zero by removing the \( V_2 \) node currents via \( R_2 \). If \( V_2 \) is zero then equation (16) reduces to:
Figure 8(a). A General Two Terminal Pairs

Figure 8(b). A Simple Version of Figure 1.
\[
\frac{I_2}{V_1} = (a)_{12}^{-1}
\] (20)

\[V_2 = 0\]

Superposition will be used to find \(I_A\) and \(I_B\) through the network.

The \(A\) matrix allows simple matrix multiplication to obtain the description required for the network. Let \(I_A\) be defined as the current arising from the input voltage \(V_{in}\) then matrix \(A\) is found from:

\[
A_{in} = \begin{pmatrix}
1 & R_1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
C_3 + C_1S & 1
\end{pmatrix}
\begin{pmatrix}
1 & 1/C_2S \\
0 & 1
\end{pmatrix}
\]

(21)

multiplying to find \(A_{in}^{12}\) of \(A_{in}\) results in:

\[
I_A(S) = \frac{V_{in}(S)^2 \frac{C_3S}{R_1 + R_3}}{1 + \frac{R_3}{R_1} \frac{R_3}{(C_3 + C_1)S}}
\]

(22)

The feedback transconductance for \(I_B\) is found by grounding the input and finding the feedback current from the output.

\[
A = \begin{pmatrix}
1 & 1/C_1S \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
C_1 + C_3 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 1/C_2S \\
0 & 1
\end{pmatrix}
\]

(23)

then
\[
I_B(S) = \frac{R_1 R_3}{R_1 + R_3} \cdot \frac{C_1 C_2 S^2}{1 + \frac{R_1 R_3}{R_1 + R_3} (C_1 + C_2)S}
\]

The two currents \(I_A\) and \(I_B\) sum in the feedback resistor \(R_2\) to generate the output voltage.

\[
V_o(S) = -R_2(I_A(S) + I_B(S)).
\]  \hspace{1cm} (25)

Inserting equations (21) and (24) into equation (25) and rearranging yields:

\[
V_o(S) = \frac{-R_2 C_2 S}{R_1 + R_3} \cdot \frac{R_3}{1 + \frac{R_1}{} R_3 (C_1 + C_2)S^2} + 1.
\]  \hspace{1cm} (26)

Equation (26) is the transfer function of Figure 1 with the exception that there is no positive feedback. This is equivalent to assuming that \(R_5\) has a resistance of zero ohms. If \(R_5\) is not zero, then positive feedback exists at the noninverting terminal of the operational amplifier. To simplify the analysis of this positive feedback case, the gain from the noninverting input to the output will be derived. This will be accomplished by first considering the general case for a noninverting operational amplifier with any passive feedback network that may be described by a \(\bar{A}\) matrix. The schematic of this case of shown in Figure 9. If the operational amplifier is assumed to be ideal, then \(V_o(S)\) will be of such a value that \(V_f'(S)\) is equal to \(V_f(S)\). Furthermore, there will be no current into the \(V_f'(S)\) node which reduces the gain equation to simply finding the \(A_{11}\) term of the \(A\) matrix because the \(I_2\) term is zero. Thus
The Feedback Bridged Tee

Bridged Tee For Two Port Reduction Theorem

Figure 9.
\[
\frac{V_o(S)}{V_{f}(S)} = a_{11f}
\]

(27)

where \(a_{11f}\) is a term in the feedback matrix \(A_f\).

In order to find \(a_{11f}\), the matrix \(A_f\) must be found. A useful two port reduction theorem may be used to derive the \(A_f\) matrix. This theorem is 14

\[
\bar{A}_f = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{\alpha_{11}Z + \alpha_{12}}{Z + \alpha_{12}} & \frac{Z\alpha_{12}}{Z + \alpha_{12}} \\
Z\alpha_{21} + \alpha_{11} + \alpha_{22} + 2Z & \frac{Z\alpha_{22}Z + \alpha_{12}}{Z + \alpha_{12}}
\end{bmatrix}
\]

(28)

The figure for the above equation is shown in Figure 9c. For the case of the bridged tee \(Z\) is equivalent to \(R_2\) and the tee is composed of \(C_1\), \(C_2\), \(R_1\) and \(R_3\). The submatrix for the tee network is described by the matrix equation,

\[
\begin{bmatrix}
\alpha_{11} & \alpha_{12} \\
\alpha_{21} & \alpha_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{(R_1 \parallel R_3)C_1S + 1}{(R_1 \parallel R_3)C_1S} & \frac{1 + (R_1 \parallel R_3)(C_1 + C_2)S}{(R_1 \parallel R_3)C_1C_2S^2} \\
\frac{1}{R_1 \parallel R_3} & \frac{1 + (R_1 \parallel R_3)(C_1 + C_2)S}{(R_1 \parallel R_3)C_1C_2S^2}
\end{bmatrix}
\]

The above may be inserted into equation (29) and manipulated to yield:

\[
\frac{V_o(S)}{V_{in}(S)} = a_{11f} = \frac{R_1 \parallel R_2 R_3 C_1 C_2 S^2 + (R_1 \parallel R_3)(C_1 + C_2) + R_2 C_2 S + 1}{R_1 \parallel R R C C S^2 + (R \parallel R)(C + C)S + 1}
\]

(29)
where
\[ R_1 R_3 / R_3 = \frac{R_1 R_3}{R_1 + R_3} \]

Assume that the amplifier's input impedance is sufficiently large that the positive feedback network is not loaded. Under this approximation the feedback voltage, \( V_f(S) \), is related to the output voltage by:

\[ V_f(S) \frac{1 + R/R_4}{1 + R/R_5} = V_o(S). \quad (30) \]

Now writing the node equations for Figure 1 yields:

\[
\begin{align*}
V(S) - V(E) &= \frac{V(S)}{E} + \frac{(V(S) - V(S)) C S}{R_1 E} + \frac{(V(S) - V(S)) C S}{R_3 E} + \frac{(V(S) - V(S)) C S}{R_2 E} \\
(V(S) - V(S)) C S &= \frac{2}{R_2} \quad (31) \\
\end{align*}
\]

Where \( V_E(S) \) = transform node voltage at the junction of \( R_1, R_3, C_1 \) and \( C_2 \).

The gain of the amplifier may be taken into account by noting that

\[ V_2(S) = V_3(S) - V_3(S)/A(S) \quad (33) \]

and

\[ V_3(S) = V_f(S) \]

Combining equations (29) through (33) yields the closed loop gain equation for Figure 1. This equation includes the open loop gain but
neglects the input impedance effects. The transfer function for Figure 1 is:

\[ \frac{V_o(S)}{V_i(S)} = \frac{-R_2 C_2 S(1 + \frac{R_5}{R_4}) (\frac{R_3}{R_1 + R_3})}{D(S) + \frac{1 + \frac{R}{R}}{A(S)}} \]  

(34)

where

\[ D_1(S) = \frac{R R_3}{1 + R_3} R C C_2 S^2 + \left( \frac{R R_3}{1 + R_3} C_1 + C_2 - \frac{R_2}{R_4} C_2 \right) \frac{S + 1}{D_2(S) = \frac{R R_3}{1 + R_3} R C C_2 S^2 + \left( \frac{R R_3}{1 + R_3} C_1 + C_2 + R C_2 \right) \frac{S + 1}{A(S) = \text{open loop of the operational amplifier.}} \]

For amplifiers that use dominant pole compensation the first order approximation for the open loop gain is given by:

\[ \frac{A_o}{\tau_A S + 1} \]  

(35)

where \( A_o \) = open loop gain

\( \tau_A \) = dominant pole time constant.

For filters whose center frequency is much larger than \( 1/\tau_A \) a useful approximation is:

\[ A(S) \approx \frac{\omega_{co}}{S} \]  

(36)

Where \( \omega_{co} \) = crossover frequency \( \sim \frac{A_o}{\tau_A} \).

In general, \( R_4 \) is much greater than \( R_5 \) and \( A(S) \) is much greater than one at the frequency of interest. Inserting these approximations into equation (34) yields
The above equation may be rewritten as

$$V_o(S) = \frac{-R_2 C_2 S(1 + R_5/R_4)(R_3/(R_1 + R_3))}{(R_1 + R_3)\omega_c} S^3 + \left(\frac{R_1 R_2 R_3 C_1 C_2}{R_1 + R_3} + \frac{R_1 R_3 (C_1 + C_2)}{(R_1 + R_3)\omega_c} + \frac{R_2 C_2}{\omega_c}\right) S^2 + \left(\frac{R_1 R_3 (C_1 + C_2)}{R_1 + R_3} - \frac{R_5 R_2 C_2}{R_4}\right) S + 1. \quad (37)$$

The above equation may be rewritten as

$$V_o(S) = \frac{-R_2 C_2 S(1 + R_5/R_4)(R_3/(R_1 + R_3))}{(R_1 + R_3)\omega_c} S^3 + \left(\frac{R_1 R_2 R_3 C_1 C_2}{R_1 + R_3} + \frac{R_1 R_3 (C_1 + C_2)}{(R_1 + R_3)\omega_c} - \frac{R_5 R_2 C_2}{R_4}\right) S + 1$$

$$+ S^2(1 + \frac{R_5}{R_4})\frac{R_2 C_2}{\omega_c}. \quad (38)$$

For center frequencies much less than $\omega_c$ then equation (38) becomes:

$$V_o(S) = \frac{-R_2 C_2 S(1 + R_5/R_4)(R_3/(R_1 + R_3))}{(R_1 + R_3)\omega_c} S^3 + \left(\frac{R_1 R_2 R_3 C_1 C_2}{R_1 + R_3} + \frac{R_1 R_3 (C_1 + C_2)}{(R_1 + R_3)\omega_c} - \frac{R_5 R_2 C_2}{R_4}\right) S + 1 \quad (39)$$

The above equation may be equated to the general bandpass equation:

$$V_o(S) = \frac{K_{so} S \omega_c}{\omega_c^2 + \frac{S}{\omega_c} + 1} \quad (40)$$

Where $K_{so}$ = mid-band gain

$$\omega_c = \text{center frequency} = 2\pi f_o$$

$$Q_o = \text{quality factor}$$

Equating coefficients in equations (39) and (40) yields the following:
\[ \omega_o = \left( \frac{R_1 + R_3}{R_1 R_2 R_3 C_1 C_2} \right)^{1/2} \] 
\[ Q_o = \left( \frac{R_1 R_2 R_3 C_1 C_2}{R_1 + R_3} \right)^{1/2} \]

\[ \frac{1}{1 + \left( \frac{R_1 + R_3}{R_1 R_2 R_3 C_1 C_2} \right)^{1/2}} \]

\[ \frac{1}{\left( \frac{R_1 R_3 (C_1 + C_2)}{R_1 + R_3} - \frac{R_5 R_2 C_2}{R_4} \right)} \]  

(41)
Figure 10. Two Pole Band-Pass Filter
\[
K_0 = \frac{R_3}{R_1 + R_3} \cdot \frac{R_2 C_2}{R_5} \cdot \frac{1 + R_5/R_4}{1 + R_2 C_2} + \frac{R_1 + R_3}{R_4}
\]

The most reasonable design procedure is to design the circuit without positive feedback (i.e., \( R_5 = 0 \)) such that the variance of the center frequency is not dominated by the tolerance of the crossover frequency, \( \omega_{co} \). Note that element normalizations may be used to yield the design equations for the quality factor without positive feedback, \( \hat{Q} \), to the design quality factor \( Q_0 \). The temperature variation for a typical operational amplifier is about thirty percent over the full military range. The sensitivity of \( \omega_o \) with respect to \( \omega_{co} \) is given by

\[
\frac{\Delta \omega_o}{\omega_{co}} = \frac{\omega_{co}}{2 \omega_o} \cdot \frac{\Delta \omega_o}{2 \omega_o} = \frac{\omega_{co}}{R_1 + R_3} \cdot \frac{R_3 C_1}{R_1 + R_3} + \frac{1}{\omega_{co}}
\]

where \( S^{\omega_o}_{\omega_{co}} \) is the sensitivity of \( \omega_o \) with respect to \( \omega_{co} \).

The allowable variation of the center frequency and the known variation of \( \omega_{co} \) may be used to determine the allowable \( \hat{Q} \) from the approximation

\[
\frac{1}{2 \hat{Q} \omega_o} = \frac{R_3 C_1}{R_1 + R_3}
\]

which yields

\[
\hat{Q} = \frac{\omega_{co}}{\omega_o} \left( \frac{1}{\omega_o} \right)^{-2} \]

(44)
as the maximum allowable $Q$ for a given center frequency and crossover frequency in terms of the circuit's center frequency sensitivity.

The design for the selection of equal capacitors may be formulated as follows:

Select $C_1 = C_2$ then calculate

$$\frac{R_1 R_3}{R_1 + R_3} = \frac{1}{2\omega_0 C_1}$$

(45)

$$R_2 = \frac{2Q}{\omega_0 C_1}$$

(46)

and

$$R_5 = \frac{1}{2Q} \left( \frac{1}{Q} - \frac{1}{Q_0} \right) R_4$$

(47)

Where $R_5$ or $R_4$ is selected based on parasitics and amplifier loading.

If two circuits as shown in Figure 1 are to be operated as a feedback pair in a multicoupled mode, then a feedback path must be found. Implicative in equations (13) and (23) is the circuit shown in Figure 10. Now it will be shown that this circuit can be represented by a second order lowpass polynomial in the prototype domain. This development will use the ideal gain approximation for the amplifiers. Now consider the transfer function between the various observable test points. These transfer functions will be summed in a block diagram.

These transfer functions are:

$$\frac{V_{TPI}(S)}{V_o(S)} = \left( \frac{R_5 \parallel R_4}{R_6 + R_5 \parallel R_4} \right) \cdot \left( \right)$$

$$V_{in}(S) = 0$$
\[
\frac{V_{\text{TPI}}(S)}{V_{\text{in}}(S)} = \frac{-R_2 C_2 S}{(R_1 || R_3) R_2 C_1 C_2 S^2 + ((R_1 || R_3) (C_1 + C_2) S + (R_2 C_2) S + 1)} \bigg( \frac{R_3}{R_1 + R_3} \bigg) \left(1 + \frac{R_5 || R_6}{R_4}\right) + \frac{(R_1 || R_3) (C_1 + C_2) S - \frac{R_5 || R_6 || R_2 C_2}{R_4}}{S + 1}
\]

\[
V_o(S) = 0
\]

\[
\frac{V_o(S)}{V_{\text{TPI}}(S)} = \frac{-R_8 C_3 S}{(R_7 || R_9) R_8 C_3 C_4 S^2 + ((R_7 || R_9) (C_3 + C_4) - \frac{R_5 || R_6 || R_2 C_2}{R_4}}{S + 1}
\bigg( \frac{R_3}{R_7 + R_9} \bigg) \left(1 + \frac{R_{11}}{R_{11}}\right)
\]

For ease in manipulation, use the following substitutions:

\[
\frac{V_{\text{TPI}}(S)}{V_o(S)} = \frac{\beta \left( \frac{\omega^2}{\omega_1^2} + \tau S + 1 \right)}{\omega^2 + \omega_1 + 1}
\]

\[
V_{\text{in}}(S) = 0
\]

\[
\frac{V_{\text{TPI}}(S)}{V_{\text{in}}(S)} = \frac{-K_i S / \omega_1}{S^2 + \omega_1 + 1}
\]

\[
V_o(S) = 0
\]
\[
\frac{V_o(S)}{V_{TPI}(S)} = \frac{-K_2S/\omega_2Q_2}{S^2 + S + 1}
\]

There is no loss in generality by making \(\omega_1 = \omega_2 = \omega_0\) and allowing \(Q_1 = Q_2\). Now using the variable substitution described in equation (3), the equations (48) may be written as follows.

\[
\frac{V_{TPI}(p)}{V_o(p)} = \frac{\beta(p + \tau_nQ_o)}{p + Q_o/Q_1}
\]

\(V_{in}(p) = 0\)

\[
\frac{V_{TPI}(p)}{V_{in}(p)} = \frac{-K_1/Q_o/Q_1}{p + Q_o/Q_1}
\]

\(V_o(p) = 0\)

\[
\frac{V_o(p)}{V_{TPI}(p)} = \frac{-K_2Q_o/Q_2}{p + Q_o/Q_2}
\]

Refer to Figure 11 and implement the above equations in a single lowpass prototype equation as follows:

\[
\frac{V_o(p)}{V_{in}(p)} = \frac{K_1K_2Q_o^2/Q_1Q_2}{p^2 + \left(\frac{Q_o}{Q_1} + \frac{Q_o}{Q_2} + \frac{\beta K_2 Q_o}{Q_2}\right) + Q_o^2} + \frac{K_2\beta_n Q_o^2}{Q_2}
\]

The general form of the above expression is a second order lowpass transfer function in the prototype, \(p\), domain. The general equation of such a transfer function is
\[ \frac{V_o(p)}{V_{in}(p)} = \frac{K_o \mathcal{S}_o^2}{p^2 + \alpha \mathcal{S}_o p + \mathcal{S}_o^2} \]  \hspace{1cm} (52)

where

\[ \Omega_o = Q_0 \left( \frac{1}{Q_1 Q_2} + \frac{K_2 \beta \tau_n}{Q_2} \right)^{1/2} \]  \hspace{1cm} (53)

\[ \alpha = \frac{1}{Q_1} + \frac{1}{Q_2} + \beta \frac{K_2}{Q_2} \]  \hspace{1cm} (54)

\[ K_o = \frac{K_1 K_2}{1 + K_2 \beta \tau_n Q_1} \]  \hspace{1cm} (55)

Recall that \( Q_1 = Q_2 \); in addition \( \tau_n \) is determined by the design requirements of the individual stage as given in equation (44). Now multiply equation (53) by equation (54) and recalling that \( Q_1 = Q_2 \)

\[ \beta K_2 = \frac{Q_o \alpha Q_o}{Q_o} - 2 \]  \hspace{1cm} (56)

Substituting equation (56) into equation (53) and squaring yields an equation that may be solved for \( Q_1 \) and \( Q_2 \) as given by

\[ Q_1 = Q_2 = \frac{Q_o}{\Omega_o \alpha - \Omega_o^2} \left( 1 + \sqrt{1 - \frac{\Omega_o}{Q_o \tau_n} + \frac{\Omega_o^2}{Q_o^2 \tau_n^2}} \right) \]  \hspace{1cm} (57)

The design steps for the configuration can now be enumerated as follows:
1. Estimate \( \hat{Q} \) based on sensitivity from (44)

2. Calculate \( \tau_n \) from
\[
\tau_n = \frac{1}{Q} + 2Q
\]

3. Calculate \( Q_1 \) and \( Q_2 \) using (57)

4. Calculate \( K_2 \) using (56)

5. Select \( K_1 \) and \( K_2 \) using (55)

6. Evaluate \( \beta \)

7. Design the individual stages

Note that for high \( Q \) designs, the above equations simplify. As \( Q_o \) increases so does \( \hat{Q} \) and \( \tau \). In addition \( \alpha \) and \( \Omega_o \) are typically near unity. Thus the term under the radical is small compared to one. Under this approximation, the binomial expansion may be used on equation (57). The expansion to be used on (57) is:
\[
\sqrt{1 + X} = 1 + \frac{1}{2} X - \frac{1}{8} X^2 + \cdots
\]

(58)

Putting (58) into (59)
\[
Q_1 = Q_2 = \frac{2Q_o}{\Omega_o - \frac{\Omega_o^2}{\tau_n Q_o}} \left( 1 - \frac{1}{4 \tau_n Q_o} \left( \frac{\Omega_o - \frac{\Omega_o^2}{\tau_n Q_o}}{\Omega_o} \right) \right)
\]

(59)

Even for low \( Q \)'s an additional approximation can be made if \( \alpha \) is not small.
\[
Q_1 = Q_2 = \frac{2Q_o}{\alpha \Omega_o}
\]

(60)

Recall that for butterworth prototypes \( \Omega_o = 1 \) and
Thus for lower order filters the above technique would seem applicable. For higher order filters the open loop quality factor becomes too large and other methods besides cascaded pairs must be used to obtain the desired response. The schematic of a third order multicoupled system is shown in Figure 11 and its signal flow graph is given in Figure 12. The design of this system is simplified by allowing $A_1$ to be an oscillator. However, lower sensitivity may be obtained by using the following coefficient set.

$$\Omega_0^3 = \beta_1 K_3 \frac{Q_o}{Q_1} \tau_n + (\beta_2 K_2 K_3 + 1) \frac{Q_0^3}{Q_1^2 Q_3}$$  \hspace{1cm} (61)$$

$$a_1 \Omega_0^2 = K_3 \beta_1 \frac{Q_0}{Q_1} + \tau_n + \frac{Q_2}{Q_1 Q_2} + \frac{Q_2}{Q_1 Q_3} + \frac{Q_2}{Q_2 Q_3}.$$  \hspace{1cm} (62)$$

$$a_2 \Omega_0 = K_3 \beta_1 \frac{Q_2}{Q_o} + \frac{Q_o}{Q_1} + \frac{Q_o}{Q_2} + \frac{Q_o}{Q_3}$$

Here the open loop $Q'$s may be made equal and the solution carried out as in the second order case. This method of multicoupling may be extended to higher orders where the coefficient equations are solved by iterative solutions on a computer.
Figure 12. The Signal Flow Graph of the Lowpass Prototype for the Sixth Order Bandpass Filter Shown in Figure 11.
NOVEL APPROACH: CASCADE OPERATION

In lower Q circuits, it is sometimes desired to have open loop cascade of first and second order sections. This method evolves from the simple factoring of the prototype polynomial into first and second order polynomial for odd ordered prototypes and second order polynomials for even ordered polynomials. For the all pole case the prototype polynomial is given by:

$$T(p) = \frac{1}{\sum_{i=0}^{n} a_i p_i}$$

(63)

Which can be factored for $T(p)$ even to,

$$T(p) = \frac{1}{\prod_{i=1}^{n/2} \left( p^2 + \alpha_i \Omega_i p + \Omega_i^2 \right)}$$

(64)

and for $T(p)$ odd to

$$T(p) = \frac{1}{(p + \Omega_o) \prod_{i=1}^{(n-1)/2} \left( p + \alpha_i \Omega_i p + \Omega_i^2 \right)}$$

(65)

The above prototypes may be implemented in the bandpass domain by simply cascading feedback bandpass pairs and a single bandpass filter.
For even polynomials only second order prototype sections are needed; thus, only bandpass pairs are required. For odd polynomials one first order section is needed in addition to a product of second order sections; thus a single design bandpass filter is required in addition to bandpass pairs.
SUMMARY

The second order prototype bandpass filter implementations of two popular multicoupled filters have been examined and a third method, the DABFERS, has been derived which does not use an amplifier for inversion or summing. A design technique for the DABFERS configuration has been developed and the solution for a third order system has been indicated. Methods for higher order implementation have also been suggested. Actual design data is presented in the Appendix.

The DABFERS configuration is economically attractive because it uses fewer parts. This is particularly true in designs that require two pole compensation of the operational amplifiers, where up to seven parts can be saved. The DABFERS configuration has no phase lag associated with its multicoupling function which yields less Q enhancement in high Q designs. The DABFERS configuration uses identical open loop bandpass stages which facilitates fabrication and alignment. The DABFERS configuration is competitive with FLF and LF with respect to sensitivity. Finally, the design procedure presented here allows considerable gain to be implemented in the filter.
Consider the following design example, the following are specified.

\[ \omega_0 = 1 \]
\[ \alpha = 1.1 \]
\[ f_o = 2.46 \text{kHz} \]
\[ k_o = 1.37 \]

Step 1 - if 741's are used, \( c_o = 2 \times 10^6 \) rad/Sec
then for 1% tolerance use,
\[ Q = 1.414 \] thus
\[ S_{\omega_0} = 3.1 \times 10^{-3} \]
which is much less than one percent.

Step 2 -
\[ \tau_n = 3.52 \]

Step 3 -
\[ Q_1 = Q_2 = 4.37 \]

Step 4 -
\[ K_2\beta = .201 \]

Step 5 -
\[ K_1 = K_2 = 2.37 \]

Step 6 -
\[ \beta = 8.47 \times 10^{-2} \]

Step 7 -
Select \( C = .0047 \) ufd as a standard and convenient value.
Now using equations (45), (46), and (47) the individual stage values may be calculated.
\[
\frac{R_1 R_3}{R_1 + R_3} = 3.38 \, k\Omega
\]

\[R_2 = 56 \, k\Omega\]

Let \( R_4 = R_2 \) then

\[R_5 = 3.86 \, k\Omega\]

Similarly,

\[
\begin{align*}
R_1 &= R_7 & R_5 &= R_{11} \\
R_2 &= R_8 & R_4 &= R_{10} \\
R_3 &= R_9
\end{align*}
\]

Using \( K_1 = 2.37 \) then \( R_1 = 27 \, k\Omega \) and \( R_3 = 3.86 \, k\Omega \).

Finally,

\[R_6 \approx R_5/\beta = 39 \, k\Omega\]

The circuit shown in figure 10 has been implemented using lid and chip technology and is shown in the color photograph on the last page. A magnitude plot of the design example is shown on the following page.

In the example plot, the gain equation used is:

\[
\text{Gain}_{dB} = 20 \log \left[ \frac{-k_o \frac{\omega^2}{\omega_o^2}}{\sqrt{\left(1 - (2 + 1/Q_o^2) \frac{\omega^2}{\omega_o^2} + \frac{\omega^4}{\omega_o^4}\right) + \left(\frac{\omega}{\omega_o} - \frac{3}{\omega_o^3}\right)^2}} \right]
\]
4.63
2.73
-5.27

Gain in dB

log f in kHz

dB Gain Vrs Log Frequency
Plot of the Design Example
FOOTNOTES


12. Ibid., p. 267-269.