A Diffraction Model for Prediction of Radar Signal Attenuation by a Rocket Exhaust Plume

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A DIFFRACTION MODEL FOR PREDICTION OF RADAR SIGNAL ATTENUATION BY A ROCKET EXHAUST PLUME

BY

DOUGLAS HARRISON SPHAR
B.S.E., Florida Technological University, 1971

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering in the Graduate Studies Program of Florida Technological University

Orlando, Florida
1976
ABSTRACT

This report documents the development of a method of estimating the signal attenuation induced by a rocket exhaust plume. The method is applicable to the early system design phase of high energy solid propellant rockets that produce highly ionized exhaust plumes. The method is based on the premise that when a plume is highly ionized, observed signal levels can be explained by assuming the signal propagates around the plume. A simple diffraction at a straightedge model is developed and compared to measured data. The report also provides an overview of exhaust plume electromagnetics and surveys prediction techniques.
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CHAPTER I

INTRODUCTION

Tactical rockets that intercept moving targets are often remotely controlled by a radio command link. These remotely guided rockets find application in air-to-air, air-to-surface, and surface-to-air intercept missions. The command guided rocket generally operates in conjunction with a radar system that tracks both target and rocket in addition to transmitting guidance commands to the rocket. Tracking of the rocket is usually enhanced by a transponder in the rocket that transmits a signal upon reception of the command signal.

System requirements frequently dictate locating both rocket launcher and command radar on the same platform (e.g. aircraft, tracked vehicle, etc.). Unfortunately, the proximity of the rocket launcher to the command radar creates a situation where the radar and transponder must communicate through the rocket exhaust plume. The communication link designer will be furnished operational requirements such as maximum range, tracking accuracy, and probability of erroneous
detection of a command signal. To achieve these requirements, the link designer must recognize the potential link degradation incurred by the rocket exhaust plume - especially when link parameters such as transmitter power or receiver performance are specified.

The paper by Victor and Breil [1] documents a case where plume induced signal degradation was not accounted for in system design. The Navy's radio beam riding Terrier missile could not be controlled through the plume - a fact that forced an expensive system redesign effort. Of course, it is also possible to err on the side of conservatism and over-allocate exhaust plume losses. System cost is now driven up by superfluous capability; a severe penalty in the highly competitive aerospace market. It has become evident that the system designer needs a reasonably accurate tool for predicting rocket exhaust plume induced signal losses.

The purpose of this paper is to document the development of a diffraction technique for estimating signal attenuation caused by the exhaust plume of a tactical rocket. This model is applicable during the initial system design phase when design parameters such as propellant formulation are undefined; however,
utility is limited to rockets powered by high energy solid propellants. The paper will present an overview of exhaust plume electromagnetics, followed by a survey of prediction techniques. With this background in hand, the rationale for selecting an attenuation prediction technique based on signal diffraction will be discussed. The paper will conclude with the development and testing of a prediction model.
CHAPTER II

PLASMA FUNDAMENTALS

The development of techniques for predicting exhaust plume induced signal degradation requires some understanding of how the hot, highly ionized exhaust gases interact with electromagnetic signals. This chapter will provide an overview of RF attenuation by a plasma and relate this attenuation to the ionized state of the exhaust plume. Finally, construction of a simple electromagnetic model of the exhaust plume will be discussed.

RF Attenuation

The exhaust plasma contains neutral and charged particles. The charged particles mainly consist of electrons (negative charge) and positive ions. These charged particles maintain an equilibrium spacing due to their electric fields. A radar signal traversing the plasma drives the free electrons into oscillation about their equilibrium positions and the free electrons oscillate at the signal frequency and become dipole radiators. The incident radar signal is both
radiated in the direction of propagation and reflected opposite to it. Additionally, the oscillating electrons collide with heavier particles and energy is dissipated by momentum transfer. Thus, the incident radar signal is attenuated by scattering and absorption as it passes through the plume. A mathematical expression for this attenuation is developed by Geiger [2]. The key parameters for determining the RF attenuation induced by a plasma are the plasma frequency and the collision frequency.

Plasma frequency ($f_p$) is the natural frequency of electrons in the plasma and is expressed as:

$$f_p = \sqrt{\frac{e^2}{m_e} N_e} = 8970 \sqrt{N_e} \text{ Hertz}$$

where

- $f_p = \text{plasma frequency in hertz}$
- $e = \text{electron charge} = 4.8 \times 10^{-10} \text{ electrostatic units}$
- $m_e = \text{electron mass} = 9.1 \times 10^{-28} \text{ grams}$
- $N_e = \text{number of free electrons per cubic centimeter}$

Collision frequency ($\nu$) is the rate of occurrence of electrons colliding with heavier particles in the plasma. Collision frequency is given by:
(2) \[ \nu = \sqrt{\frac{8KT}{m_e}} \sum_i Q_i N_i \text{ collisions/second} \]

where

- \( \nu \) = collision frequency in collisions/second
- \( K \) = Boltzmann constant = \( 1.38 \times 10^{-16} \) ergs/degree K
- \( T \) = absolute temperature in degrees K
- \( m_e \) = electron mass = \( 9.1 \times 10^{-28} \) gram
- \( Q_i \) = collision cross section of \( i^{th} \) species
- \( N_i \) = density \( i^{th} \) species

Table 1 contains a tabulation of collision cross sections of chemical species that commonly result from propellant combustion. This table is from Victor's report on rocket plume attenuation [3].

The RF attenuation of a uniform plasma for a specific signal frequency can be expressed as a function of plasma frequency and signal frequency:

(3) \[ \alpha = (1.28 \times 10^{-7} f) \left\{ \frac{1 + \left( \frac{\nu}{2\pi f} \right)^2 - \left( \frac{f_p}{f} \right)^2}{1 + \left( \frac{\nu}{2\pi f} \right)^2} \right\} \]
TABLE 1 [3]
COLLISION CROSS SECTION (Q) OF VARIOUS SPECIES

<table>
<thead>
<tr>
<th>Species</th>
<th>Cross Section, m^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiF</td>
<td>6.9 x 10^{-18}</td>
</tr>
<tr>
<td>LiCl</td>
<td>6.0 x 10^{-18}</td>
</tr>
<tr>
<td>LiBi</td>
<td>5.0 x 10^{-18}</td>
</tr>
<tr>
<td>AlCl</td>
<td>4.0 x 10^{-18}</td>
</tr>
<tr>
<td>HF</td>
<td>6.0 x 10^{-19}</td>
</tr>
<tr>
<td>H_2O</td>
<td>5.0 x 10^{-19}</td>
</tr>
<tr>
<td>HCN</td>
<td>4.0 x 10^{-19}</td>
</tr>
<tr>
<td>AlCl_3</td>
<td>3.4 x 10^{-19}</td>
</tr>
<tr>
<td>AlF_3</td>
<td>3.4 x 10^{-19}</td>
</tr>
<tr>
<td>NH_3</td>
<td>3.0 x 10^{-19}</td>
</tr>
<tr>
<td>HCL</td>
<td>2.0 x 10^{-19}</td>
</tr>
<tr>
<td>H</td>
<td>1.4 x 10^{-19}</td>
</tr>
<tr>
<td>H_2</td>
<td>1.4 x 10^{-19}</td>
</tr>
<tr>
<td>CO</td>
<td>1.9 x 10^{-19}</td>
</tr>
<tr>
<td>HBr</td>
<td>9.0 x 10^{-20}</td>
</tr>
<tr>
<td>N_2</td>
<td>8.5 x 10^{-20}</td>
</tr>
<tr>
<td>O_2</td>
<td>6.0 x 10^{-20}</td>
</tr>
<tr>
<td>N_2O</td>
<td>5.6 x 10^{-20}</td>
</tr>
<tr>
<td>CH_4</td>
<td>2.8 x 10^{-20}</td>
</tr>
</tbody>
</table>
where

\[ f = \text{signal frequency, hertz} \]
\[ \alpha = \text{RF attenuation, decibels per meter} \]
\[ \nu = \text{collision frequency, collisions per second} \]

Equation (3) has been calculated for 1.0 gigahertz (GHz), 5.0 GHz, and 10 GHz as a function of electron density with collision frequency as a parameter. These calculations are graphically depicted in Figures 1 through 3. The FORTRAN computer program used for this calculation is contained in Appendix III.

The selected signal frequencies cover the commonly used radar microwave bands. Experimental data (the work of Victor [3] or Geiger [2], for example) show that collision frequency in the plasma region of an exhaust plume is typically in the range of \(10^{10}\) to \(10^{11}\) collisions per second, so these two collision frequencies are employed in the attenuation calculations.

Figures 1 through 3 indicate that for a given electron density and collision frequency, RF attenuation decreases as signal frequency increases. This fact will be important when the rationale for selecting an attenuation prediction technique is developed.
Figure 1. Plasma Attenuation at 1.0 Ghz

**Figure 1.** Plasma Attenuation at 1.0 Ghz
Figure 2. Plasma Attenuation at 5.0 Ghz
Figure 3. Plasma Attenuation at 10 Ghz
Thermal Ionization

Rocket plume induced radio frequency (RF) signal attenuation is primarily caused by the plasma portion of the plume. A plasma is an ionized gas characterized by a high density of free electrons. In a plasma, free electrons can be produced by shock ionization, chemical ionization, thermal ionization, or photo ionization; but in an exhaust plume, thermal ionization is the dominant mechanism [4].

Thermal ionization of a gas component is expressed by the Saha relation [7]:

\[
\log_{10} \left( \frac{x^2}{1 - x^2} \right) = -5041 \left( \frac{I}{T} \right) + \frac{5}{2} \log_{10}(T) - 6.49 + \log_{10} \left[ \frac{g_e g_i}{g_a} \right]
\]

where

- \(x\) = fraction of the ionizable component that is ionized
- \(p\) = partial pressure of the ionizable component (atmospheres)
- \(I\) = ionization potential of the ionizable component (electron volts)
- \(T\) = temperature (degrees Kelvin)
- \(g_e\) = weight factor for the electron (=2)
- \(g_i\) = weight factor for the ion
- \(g_a\) = weight factor for the atom
and the weight factor is \(2j - 1\), where \(j\) is the principal quantum number of the ground state for the particular atom or ion. Expression (4) relates the fractional ionization of a propellant combustion component to temperature and ionization potential. Solid propellants are comprised of compounds of oxygen, hydrogen, and carbon; however, small amounts of sodium and potassium are present as contaminants.

Carbon has an ionization potential of 11.3 electron volts (ev), whereas sodium and potassium have ionization potentials of 5.1 ev and 4.3 ev respectively [5]. The low ionization potentials of sodium and potassium cause them to produce large quantities of free electrons at the temperatures that exist in rocket exhaust plumes. Indeed, research [4,6,7] has demonstrated that sodium and potassium impurities are the primary producers of free electrons in the exhaust plume plasma. Unfortunately, the cost of eliminating these impurities is prohibitive [6]; especially since sodium chloride is used in the manufacture of ammonium perchlorate - a solid propellant oxidizer.

Figure 4 is an adaptation of solutions of (4) that appear in the Balwanz report [7]. Figure 4 indicates an electron density range that can occur in a rocket exhaust plume. The data are based on pressures
Figure 4. Plasma Density Range for A Typical Exhaust Plume
of one to twelve atmospheres, the typical pressure range in a plume, and a 100 part-per-million (ppm) sodium or potassium concentration. This figure will be of use in Chapter VI where the thermal ionization properties of sodium are used in defining a diffracting surface within the plume.

**Electromagnetic Model of the Exhaust Plume**

The previous section presented an expression for attenuation per meter plume length for a uniform plasma. Unfortunately, an exhaust plume plasma is not uniform throughout the plume length. Very high free electron concentrations exist at the motor nozzle exit and at the outer plume boundary where unburned propellant products mix with the airstream and ignite. Analysis of signal propagation through a plume would prove intractable if the plume were treated as a continuous medium; hence analysis is usually facilitated by dividing the plume into regions of constant electrical property. Another approach used by Aerochem [8] is to represent the plume as an array of points and determine the electron density and collision frequency at each point.

Figure 5 depicts a plume that has been divided into regions of constant electrical property (electron
Figure 5. Exhaust Plume Electromagnetic Propagation Modes
density or conductivity are commonly used). This figure also contains a ray-tracing depiction of the various signal propagation modes that could be associated with an exhaust plume. These propagation modes, singularly or in a group, form the basis of attenuation prediction techniques that will be discussed in Chapter III.
CHAPTER III

SURVEY OF ATTENUATION PREDICTION TECHNIQUES

Ideally, the interaction of the exhaust plume with communication signals would be established by determining the electrical properties of the plume and then solving Maxwell's equations for the system at hand. Unfortunately, this approach exceeds the scope and power of existing engineering minds and computer facilities. Consequently, all attenuation prediction techniques involve making appropriate simplifications to the plume electrical structure and to RF propagation modes. This chapter will discuss these simplifications and survey attenuation prediction techniques that employ them.

Through-the-Plume Approaches

The classical approach to predicting exhaust plume signal attenuation is to assume that observed effects are due to signal propagation through the plume and ascribe losses to absorption, refraction, and scattering. The first step in this approach is to determine the electromagnetic model of the plume. Plume electrical
properties can be analytically derived by the use of thermochemical gas dynamics computer programs such as the Aerochem program [8].

The next step is to "propagate" signal rays at various angles through the plume. The most elementary approach is illustrated in Figure 6 where the ray path is a straight line through the plume. Using equation (3), the attenuation per unit length is found for each plume region and the ray path attenuation for each region is obtained by multiplying the region ray length by the unit attenuation. The total attenuation along a ray path is obtained by a summation of each region traversed by the ray. This process is repeated for each ray thereby yielding a characterization of plume attenuation as a function of look angle. The method described above is commonly called the "line-of-sight" method.

The "ray-tracing" method also illustrated in Figure 6 is a more elaborate through-the-plume prediction method. As with the line-of-sight method, attenuation due to absorption is obtained for each plume region traversed by a signal ray; however, refraction and internal reflection occur at the boundaries of the plume regions. The rays appear to bend as they pass through the plume and in addition to absorption losses,
Figure 6. Prediction Techniques Using "Through-the-plume" Propagation
some models allocate signal losses to scattering caused by internal reflections.

The Victor report on plume-signal interference [3] documents examples of both line-of-sight and ray-tracing prediction models that are coded for running on a computer. Through-the-plume prediction models have been successful in predicting attenuation induced by the plumes of liquid propellant rocket motors [1,3,6]. In fact, at one time it was thought that the signal attenuation induced by any rocket exhaust plume could be completely characterized by sophisticated application of ray-tracing techniques. When high energy solid propellants appeared on the scene, through-the-plume prediction techniques failed completely. These methods that had predicted liquid propellant attenuation to within a few decibels were now found to err by as much as 40 to 100 decibels [1,3,9]. Thus, the utility of the ray-tracing and line-of-sight methods is limited to liquid propellant rockets. The next section will address the development of a prediction technique that is applicable to high energy solid propellants.

**Diffraction Approach**

The advent of high energy solid propellants for rockets forced a rethinking of the problem of predicting
plume-induced signal degradation. Through-the-plume prediction techniques do not predict the observed signal phenomena. Generally, the observed attenuation is considerably lower than predicted and the frequency dependency of the observed attenuation is not in accordance with plasma theory. When the Titan III-C solid booster rocket was first flown, it was noted that attenuation of VHF (200 mhz) telemetry signals was substantially less than predicted and was even less than the attenuation of C-band (5000 mhz) radar signals. Classical propagation through the plasma theory, as manifested in equation (3), predicts that signal attenuation decreases as signal frequency increases. Just the opposite was occurring with the Titan III-C plume.

Working with the Titan III-C plume, Golden, Taylor, and Vicente postulated that observed effects could be explained if it was assumed that the signal traveled around the plume, rather than through it. They presented an argument that the Titan exhaust plume was so highly ionized that it became opaque to RF signals. They further argued that the signal propagation mode was one of diffraction over the opaque plume.

Golden et al. verified the diffraction hypothesis in a laboratory experiment. A scale model Titan III-C,
complete with exhaust plume simulated by an opaque object, was set up in an anechoic chamber and radiation patterns were measured. Scaled frequencies were utilized, as were several types of reflecting and absorbing scale plumes. The measured patterns exhibited an aspect angle dependency that was similar to the intensity pattern produced by Fresnel diffraction around a strip.

The next step was to construct a two dimensional diffraction model of the plume. Using photographic data, the RF opaque region of the plume was defined as lying within the highly luminous portion of the plume. Plotting this data and the rocket to scale, lines were drawn from the rocket antenna tangent to the luminous region of the plume as shown in Figure 7. In the plume model, the RF opaque plume was replaced by an infinite width opaque strip as indicated in Figure 7. For this model, the ratio \( I \) of the far field intensity to the unobstructed intensity was defined as:

\[
I = \left| A(V_1) + A(V_2) \right|^2
\]

where \( A(V_1) \) and \( A(V_2) \) are Fresnel integral relations that are functions of \( \psi_1, \psi_2, a, \) and wavelength.

This diffraction model was tested against measured flight data [9] and a sample of this data appears in Figure 8. It is seen that the diffraction prediction
Figure 7. Two Dimensional Diffraction Model of Exhaust Plume [4]
Figure 8. Predicted Versus Measured Titan III-C Plume Attenuation [4]
closely agrees with the measured VHF flight data, whereas a line-of-sight prediction over-predicts by tens of decibels.

Consideration of the work by Golden et al. [9] has prompted selection of a diffraction approach for developing an attenuation prediction technique applicable to high energy solid propellant rockets. The development of this technique is described in the next section of this report.
CHAPTER IV

ESTIMATION OF THE DIFFRACTION SURFACE

The primary objective of this report was stated as the development of a simple to apply, but reasonably accurate predictor of rocket exhaust plume induced signal attenuation. With this in mind, a method of determining the RF opaque region of the plume was sought that does not involve elaborate thermochemical techniques such as the Aerochem computer program [8]. Use of programs such as the Aerochem requires considerable gas and aerodynamics expertise. Also, during the early system planning phase, propellant formulation and combustion species data are usually not available.

This report proposes to define the RF opaque region of the plume as lying within a specific contour of constant temperature. Simple computer programs exist that generate temperature and pressure contours within a plume as a function of motor combustion chamber temperature and pressure and nozzle geometry. The Martin Marietta Plume Program [10] is an example. The problem now is to associate a specific temperature contour with
a plasma attenuation that is sufficiently high as to represent a diffracting surface. Victor [3] has conducted experiments that show line-of-sight prediction models work with plumes exhibiting maximum electron densities of less than $10^{10}$ electrons/cm$^3$. High energy solid propellants (propellants containing greater than 5 percent aluminum) have electron densities considerably higher than this.

Based on the experience of Victor, this report defines the RF opaque region of the plume as lying within the temperature contour that represents an electron density of $10^{10}$ electrons/cm$^3$. Reference is made to Figure 4, which graphs electron density as a function of temperature. It is seen that $10^{10}$ electrons/cm$^3$ corresponds to an ionization-to-temperature ratio of $3 \times 10^{-3}$ to $3.3 \times 10^{-3}$. Using an average ionization potential of 4.7 ev for sodium and potassium and entering Figure 4 at $3.15 \times 10^{-3}$ on the abscissa, a temperature of 1500 degrees Kelvin ($^0$K) is obtained. In this report, the 1500$^0$ K plume contour is defined as the surface of the RF opaque region around which diffraction occurs.
CHAPTER V

TWO-DIMENSIONAL DIFFRACTION MODEL

This chapter will discuss the development of a simple plume attenuation prediction model that is based on diffraction by a straight-edge. The model predicts attenuation as a function of variables that are generally available to the system planner. Predicted attenuation is compared to measured flight data. Attenuation predicted by a line-of-sight technique is also furnished for comparison. A computer coded version of the model and detailed mathematical development are included in appendices.

Model Development

The two-dimensional diffraction model discussed in Chapter III (see Figure 7) is not in a form that is convenient for system planning work. The relation for diffraction at a straight edge will be defined so that signal attenuation can be determined as a function of signal frequency and center-line aspect angle. In this paper, centerline aspect angle is defined as the angle between the rocket axial centerline and a line from the
origin of the rocket antenna coordinate system to the origin of the tracking radar antenna coordinate system.

The diffraction model of Figure 7 is simplified by assuming that the plume is so highly attenuative that essentially zero power is diffracted over the lower half strip. This changes the problem from one of diffraction by an infinite width strip to one of diffraction by an infinite width straight-edge. Figure 9 schematically depicts the geometry of the proposed attenuation model. The angle $\theta_o$ and the location of the straight edge are obtained by drawing a line from the rocket antenna tangent to the 1500°K plume temperature contour. The angle $\theta_1$ will be shown to be equivalent to the centerline aspect angle.

Using the relation for diffraction by a straight-edge, the ratio of source power to received power is [11]:

\[
\frac{I}{I_o} = \frac{1}{2} \left\{ \left[ C(V) - \frac{1}{2} \right]^2 + \left[ S(V) - \frac{1}{2} \right]^2 \right\}
\]

where $C(V)$ and $S(V)$ are Fresnel integrals and:

\[
V = \sqrt{\frac{2a}{\lambda} \left( \theta_o - \theta_1 \right)^2}
\]

where $\lambda$ is the signal wavelength. A detailed development of the above relations is found in Appendix I.
Figure 9. Simplified Diffraction Prediction Model

\( \lambda = \text{Wavelength} \)
Figure 9 and Equation (5) and Equation (6) represent the two-dimensional diffraction model for predicting rocket plume induced signal attenuation. A FORTRAN coded version of this model is found in Appendix II.

Comparison to Measured Data

This section compares the two-dimensional diffraction attenuation prediction to signal attenuation measured during a rocket flight. An attenuation prediction by the line-of-sight method is performed and compared to the diffraction prediction and the measured data. Measured attenuation data that can be published in this report are severely limited by proprietary and other considerations. However, plume temperature profiles and measured in-flight attenuation for a small test rocket were obtained from Reference 5. This same source also includes a plume electron density profile calculation for the test rocket. The electron density profile will be employed in the line-of-sight attenuation prediction.

The test rocket featured a 5 inch diameter motor nozzle exit and the rear facing antenna was located on the rocket base, 4.75 inches from the nozzle centerline. The two data measurement (at centerline aspect angles of +10 degrees and -10 degrees) occurred at an altitude
of 35,000 feet and while the rocket velocity was 2500 feet/second. Of course, signal level was measured throughout the flight, but plume temperature and electron density calculations are only available for this one instance in the flight. The measured signal level has been normalized to give plume attenuation by factoring out range and antenna pattern losses. The normalized data do contain errors due to uncertainties in determination of range, rocket flight attitude, and antenna pattern.

Input variables to the prediction model are determined from Figure 10. The location of the diffracting straight-edge is determined by drawing a line from the rocket antenna location tangent to the $1500^\circ$K temperature contour. The variables of Equation (6) are:

\[
\begin{align*}
\theta_o &= 7.59 \text{ degrees} \\
a &= 1.20 \text{ feet} \\
\lambda &= 0.24 \text{ feet} \\
\theta_1 &= -16 \text{ degrees to } +16 \text{ degrees}
\end{align*}
\]

These values were used as inputs to the FORTRAN code of Appendix II. The total input to this code was:

\[
\begin{align*}
A &= 1.2 \\
XLAM &= 2.4 \\
TZD &= 7.59 \\
TSD &= -16.
\end{align*}
\]
Figure 10. Calculated Plume Temperature Contours
TFD = 16.
TID = .25

A computer produced plot of the diffraction prediction and the two points of measured data appear in Figure 12.

A plot of signal attenuation predicted by the line-of-sight approach also appears in Figure 12. This line-of-sight prediction was obtained through use of the plume electron density contours of Figure 11. Straight line rays originating at the antenna were drawn through the plume at the angles indicated in Figure 11. The length of each ray segment in a specific electron density region was calculated next. Using Figure 2, the attenuation for a ray segment was obtained by multiplying the segment length by the unit attenuation associated with the region electron density. A collision frequency of $10^{11}$ collisions per second was assumed. The total attenuation along a line-of-sight ray was obtained by summing the attenuation contribution of each ray segment.

The diffraction prediction agrees with the measured attenuation to within 3 decibels (db) at $+10$ degrees aspect angle and to within 6 db at $-10$ degrees aspect angle. The line-of-sight prediction underpredicts attenuation by 7 db at $+10$ degrees and overpredicts by 60 db at $-10$ degrees. At intermediate angles,
Figure 11. Calculated Electron Density Contours
Figure 12. Comparison of Attenuation Predictions to Measured Data
line-of-sight prediction goes off scale at 100 db attenuation - a fact not born out by the flight data. During the flight, measured attenuation ranged between 1.5 db and 22 db.

It is seen that the diffraction attenuation prediction model predicts the general range and trend of attenuation that occurred during the flight, whereas the line-of-sight model predicted a signal black-out over a range of aspect angles. The correlation between the measured data and the diffraction prediction is considered good - especially in view of the aforementioned errors inherent in normalizing the measured attenuation.
CHAPTER VI

CONCLUSIONS

Various exhaust plume attenuation prediction techniques were surveyed with a view toward development of a simple to apply, reasonably accurate plume attenuation prediction model. Applicability of the model was specified as being limited to high energy solid propellant rockets that produce very highly ionized plumes. With the above criteria in mind, a prediction model was developed that utilized signal diffraction around the exhaust plume.

The diffraction model was tested against signal attenuation measured during a rocket flight and was seen to predict the magnitude and trend of the measured data. An attenuation prediction using the line-of-sight method was performed and grossly overpredicted the observed attenuation.

The attenuation prediction model using diffraction provides a reasonable estimate of the signal attenuation induced by the exhaust plume of a high energy rocket. For this reason, the model can be of value in
specifying signal margin requirements in the design of radar to rocket communication links.
APPENDIX I

DERIVATION OF ATTENUATION PREDICTION MODEL

Problem Statement

The problem is to define the argument of the Fresnel integrals in (5) of the main text so that it is a function of the variables in Figure 9 of the main text. That is, show that:

\[ v = \sqrt{\frac{2a (\theta_0 - \theta_1)^2}{\lambda}} \]

Pages 129 through 135 of Fowles [11] are the reference for this development.

Derivation

Figure 13 depicts the geometry employed by Fowles in developing the relation for Fresnel diffraction by a straight-edge. The objective here is to eliminate the distance between the opaque sheet and the tracking radar (r and h) from the formulation.
Figure 13. Geometry of Fresnel Diffraction By A Straight-Edge
Using Figure 13, Fowles defines the diffracted signal seen at the detector \( p \) as:

\[
U_p = \frac{U_s}{1+i} \left\{ \left[ C(V) - \frac{1}{2} \right] + i \left[ S(V) - \frac{1}{2} \right] \right\}
\]

where

\[
V = R \sqrt{\frac{2(h + \frac{h^3}{h^1})}{hh^1}}
\]

and \( C(V) \) and \( S(V) \) are the Fresnel integrals

\[
C(V) = \int_0^V \cos \left( \frac{\pi \theta^2}{2} \right) d\theta
\]

\[
S(V) = \int_0^V \sin \left( \frac{\pi \theta^2}{2} \right) d\theta
\]

The Fresnel integrals are not solvable in closed form but are tabulated [12] or solved numerically (Appendix II).

The ratio of source power to received power is:

\[
\frac{I_p}{I_o} = \left| \frac{U_p}{V_0} \right|^2 = \frac{1}{2} \left\{ \left[ C(V) - \frac{1}{2} \right]^2 + \left[ S(V) - \frac{1}{2} \right]^2 \right\}
\]

To eliminate distance \( r \) and \( h \) from (7), the problem is reconfigured as shown in Figure 14. The key variables of Figure 14 are:

\( \theta_1 \) = Centerline aspect angle

\( \theta_o \) = Angle subtended by opaque straight-edge

\( a \) = Distance from antenna to straight-edge

\( b \) = Distance from straight-edge to tracking radar
Figure 14. Geometry of Attenuation Prediction Model
Inspection shows that Figure 14 also contains the variables of Figure 13.

Since \( h \gg h^1, \ r \gg r^1 \), and both \( r \) and \( h \) are much larger than \( R \), then a reasonable simplification is to state that \( r = h \) and \( \theta_1 = \theta_1^1 \). Figure 14 also shows that \( b = h \) is a reasonable approximation. A further assumption is made by stating that \( a = h^1 \).

Equation (7) may now be rewritten:

\[
(9) \quad V = R \sqrt{\frac{2(a + b)}{\lambda ab}}
\]

but since \( b \gg a \), then \( a + b = b \). Thus (9) can be written:

\[
(10) \quad V = R \sqrt{\frac{2b}{\lambda ab}} = R \sqrt{\frac{2}{a}}
\]

From Figure (14) it is seen that:

\[
R = h^1 \tan \psi
\]

But \( \psi = (\theta_0 - \theta_1) \), hence \( R = h^1 \tan (\theta_1^1 - \theta_1) \). As will be demonstrated later, negligible error is introduced by making the final assumption that \( h^1 = a \). Thus the desired relation for \( V \) is achieved:
(11) \[ V = a(\theta_o - \theta_1)\sqrt{\frac{2}{a\lambda}} = \sqrt{\frac{2a(\theta_o - \theta_1)^2}{\lambda}} \]

which is the same as equation (6) in the main text.

Test of Assumptions

The assumptions and simplifications leading to (11) will be tested by solving for \( V \) by means of both equations (7) and (11); then the results will be compared.
Test Case 1 - Aspect Angle Along Shadow Edge

Given

\[ a = 50 \text{ feet} \]
\[ \theta_0 = 5 \text{ degrees} = 0.0873 \text{ radian} \]
\[ \theta_l = 5 \text{ degrees} \]
\[ r = 5000 \text{ feet} \]
\[ \lambda = 0.984 \text{ feet} \]

Then

\[ R = 0 \]
\[ r^1 = 50.2 \text{ feet} = h^1 \]
\[ h = 5000 \text{ feet} \]
Using equation (7)

\[ V = R \sqrt[\lambda hh^1]{\frac{2(h + h^1)}{\lambda hh^1}} \]

\[ V = (0) \sqrt[.984][5000 (50.2)]{2 (5000 + 5000)} \]

\[ V = 0 \]

Using equation (11)

\[ V = \sqrt{\frac{2a(\theta_0 - \theta_1)^2}{\lambda}} \]

\[ V = \sqrt{(2)(50)(.0873 - .0873)^2}{.984} \]

\[ V = 0 \]
Test Case 2 - Aspect Angle in Geometric Shadow

Given

\[ a = 50 \text{ feet} \]
\[ \Theta_0 = 5 \text{ degrees} \]
\[ \Theta_1 = -5 \text{ degrees} \]
\[ r = 5000 \text{ feet} \]
\[ \lambda = 0.984 \text{ feet} \]

Then

\[ h = 4999 \text{ feet} \]
\[ R = 8.63 \text{ feet} \]
\[ h_1 = 49.4 \text{ feet} \]
\[ b = 4981 \text{ feet} \]
\[ \Theta_1 = 4.90 \text{ degrees} \]
\[ r_1 = 50.2 \text{ feet} \]
\[ \lambda = 9.90 \text{ degrees} \]
Using equation (7) \[ V = 8.63 \sqrt{\frac{2(4999 + 49.4)}{(0.984)(4999)(49.4)}} \] Using equation (11) \[ V = \sqrt{(2)(50)(0.0783 \cdot 0.0783)^2} \cdot 0.984 \]

\[ V = 1.76 \quad V = 1.76 \]

The above results indicate that the assumptions and simplifications leading to Equation (11) are reasonable and proper.
APPENDIX II

A COMPUTER CODE FOR SIGNAL ATTENUATION PREDICTION

This FORTRAN Code calculates the predicted exhaust plume induced signal attenuation as a function of centerline aspect angle. This code is based on Equation (5) and Equation (6) in the main text. The input variables are in accordance with Figure 9 in the main text and are:

- **A** = Distance from rocket antenna to opaque sheet (feet)
- **XLAM** = Signal wavelength (feet)
- **TZD** = Angle subtended by opaque sheet, $\theta_0$ (degrees)
- **TSD** = Initial aspect angle (degrees)
- **TFD** = Final aspect angle (degrees)
- **TID** = Angle increment (degrees)

The program calculates attenuation for aspect angles that are between TSD and TFD, inclusive. Angles below the rocket centerline are negative. Sample program input/output data are provided.
INPUT FORMAT

Card 1

<table>
<thead>
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<th>Columns</th>
<th>List</th>
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<tbody>
<tr>
<td>0 - 10</td>
<td>A</td>
</tr>
<tr>
<td>11 - 20</td>
<td>XLAM</td>
</tr>
<tr>
<td>21 - 30</td>
<td>TZD</td>
</tr>
<tr>
<td>31 - 40</td>
<td>TSD</td>
</tr>
<tr>
<td>41 - 50</td>
<td>TFD</td>
</tr>
<tr>
<td>51 - 60</td>
<td>TID</td>
</tr>
</tbody>
</table>

Card 2 (if measured data is to be plotted)

| 0 - 10 | NDAT |

Cards 3 to NDAT (if measured data is to be plotted)

| 0 - 10 | X (Measured aspect angle) |
| 11 - 20 | Y (Measured attenuation) |
CENUS 2-D DIFFRACTION CODE

DOUBLE PRECISION DLOG10, DSGRT

NEG = 0

C

IOP IS USED IN PLOT SUBROUTINE
IOP = 0

C

SET I PLOT = 1 TO MAKE A PLOT
IPLOT = 1

C

SET IDAT = 1 TO PLOT MEASURED DATA
IDAT = 1

DTR = 0.0174532925

HF = 0.5

READ(5,9000) A, XLAM, Tzd, Tsd, Tfd, Tid

9000 FORMAT(8F10.0)

WRITE(6,9005) A, XLAM, Tzd, Tsd, Tfd, Tid

9005 FORMAT( 6H1A= '1PE10.3/ 6H XLAM=,E10.3/ 6H Tzd= ,E10.3/

*6H Tsd= ,E10.3/ 6H Tfd= ,E10.3/ 6H Tid= ,E10.3///

*44H PREDICTED ATTENUATION USING 2-D DIFFRACTION///

*26H ASPECT ANGLE ATTENUATION/)

TAOL = 2,.*A/XLAM

FREQ = ,984/XLAM

TZR = Tzd*DTR

10 CONTINUE

IF(Tsd,GT,TFD) GO TO 45

THt = Tsd*DTR

TWT = TZR - THt

USQ = TAOL*TWT**2

X = DSGRT(USQ)

IF(TWT,LT,0) X = -X

CALL FRENEL(X,C,S)

R = HF*((C-HF)*(C-HF) + (S-HF)*(S-HF))

RDB = 10,.*DLOG10(R)

WRITE(6,9015) Tsd, RDB

9015 FORMAT(1X,2F12,2)

IF(IPLOT,EQ,0) GO TO 40

C

SCALE DATA FOR PLOTTER
X = TSD*,25
Y = -RDB*,3
IF(Y,LE,0) NEG = 1
IF(NEG) 21,21,20
20 Y = 0,0
21 CONTINUE
ZETA0 = -TSD
ATTEN = -RDB
CALL PLOTER(ZETA0,ATTEN,IOP,FREQ)
40 TSD = TSD + TID
GO TO 10
45 CONTINUE

C INPUT MEASURED DATA FOR PLOTTER
IF(IPLLOT,EQ,0) GO TO 90
IF(IDAT,EQ,0) GO TO 50
IOP = 2
READ(5,9010) NDAT

DO 50 L = 1,NDAT
READ(5,9020) X,Y
ZETA0 = X
ATTEN = Y
X = X*,25
Y = Y*,3
CALL PLOTER(ZETA0,ATTEN,IOP,FREQ)
50 CONTINUE
IOP = 4
CALL PLOTER(ZETA0,ATTEN,IOP,FREQ)
90 CONTINUE
9010 FORMAT(I10)
9020 FORMAT(2F10,0)
END
SUBROUTINE FRENEL(X,C,S)
SV = X
X = ABS(X)
F = (1, +, 926*X) / (2, +, 1,792*X + 3,104*X**2)
G = 1 / (2, +, 4,142*X + 3,492*X**2 + 6,67*X**3)
U = 3,14159*X**272,
C = 5 + F*SIN(U) - G*COS(U)
S = 5 - F*COS(U) - G*SIN(U)
IF(SV) 10,20,20
10 C = -C
S = -S
20 RETURN
END
<table>
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<tr>
<th>Aspect Angle</th>
<th>Attenuation</th>
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<td>4.75</td>
<td>1.04</td>
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<tr>
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<td>1.26</td>
</tr>
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</table>
APPENDIX III

A COMPUTER CODE FOR CALCULATING PLASMA ATTENUATION

This FORTRAN code calculates the RF signal attenuation of a uniform plasma. Equation (3) of the main text is utilized. Attenuation is calculated for three frequencies (1, 5, and 10 ghz) as a function of electron density ($10^8$, $10^9$, $10^{10}$, $10^{11}$, and $10^{12}$) and collision frequency ($10^{10}$ and $10^{11}$). This program generated Figures 1, 2 and 3 in the main body of this report.
THIS PROGRAM CALCULATES ATTENUATION AS A FUNCTION OF ELECTRON DENSITY WITH COLLISION FREQUENCY AND SIGNAL FREQUENCY AS PARAMETERS.

REAL NE, NU, NUF
DOUBLE PRECISION ALPHA, AA, ATN, NUFSQ, DSORT, FPF
WRITE(6, 200)

200 FORMAT(1H1//)
PI = 3.1415926
PI2 = 2.*PI
NU = 1.E10
DO 10 I = 1, 2
FREQ = 1.E09
DO 20 J = 1, 3
COEF = 1.28E-07 * FREQ
NUF = NU/(PI2 * FREQ)
NUFSQ = NUF * NUF
NE = 1.E08
DO 30 K = 1, 5
FP = 8974*SORT(NE)
FPF = (FP/FREQ) * (FP/FREQ)
ALPHA = 1. + NUFSQ - FPF
AA = DSORT(ALPHA*ALPHA + NUFSQ*FPF*2)
ATN = DSORT((AA-ALPHA)/(1. + NUFSQ))
ATNDB = COEF*ATN
PRINT 9000, NU, FREQ, NE, ATNDB
9000 FORMAT(1X, 3E14.7, F10.3)
NE = NE*10.

30 CONTINUE
IF(J, EQ, 1) FREQ = 5.E09
IF(J, EQ, 2) FREQ = 10.E09

20 CONTINUE
NU = NU*10.
10 CONTINUE
END
LIST OF REFERENCES


