Techniques for Determination of Complex Permittivity of Dielectric Materials at Microwave Frequencies

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TECHNIQUES FOR DETERMINATION OF COMPLEX PERMITTIVITY OF DIELECTRIC MATERIALS AT MICROWAVE FREQUENCIES

BY

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CHAPTER I

INTRODUCTION

Dielectric Properties and Measurement Methods

The measurement of dielectric properties of materials may be accomplished in many different ways. The range of parameter values of various materials, however, is very large and not all the available methods are suitable for each material. Often, approximations must be made in order to obtain reasonable solutions. For example, a copper loss of a guide may be neglected when measurements of a high loss dielectric material is made. For a low loss material, on the other hand, the losses may be neglected when the phase constant is computed. Dielectric properties of gases or aerosols are very difficult to measure. The difficulty lies mostly in obtaining a homogeneous medium within a well defined electromagnetic boundary. The homogeneity of the material is required for reliable results and a well defined electromagnetic boundary is necessary for accurate calculations.

In this paper several methods of dielectric properties measurements are reviewed. These methods were obtained from technical literature, and in their
presentation the emphasis is placed on theory rather than on practical results. Besides these methods, the description of a new, but experimentally proven, method is given. Although this method was tested with solid dielectrics, it is particularly applicable for measurement of properties of aerosols.

**Definition of Complex Permittivity**

The Coulomb's law states that a force between two point charges is proportional to the product of magnitudes of the two charges and inversely proportional to the square of the distance between the two charges. The proportionality constant is

$$\frac{1}{4\pi\varepsilon},$$

where $\varepsilon$ which is called the permittivity. In free space, in mks units, $\varepsilon$ equals $8.854 \times 10^{-12}$ farads/meter and the force is in newtons. Electric force of one newton acting on a charge of one coulomb is defined as an electric field of magnitude of one volt per meter.

Dielectric material containing polar molecules (having a permanent dipole moment) inserted into the electric field will change this field inside the dielectric. When an electric field is applied to polar molecules, each dipole experiences a torque tending to align its dipole moment parallel to the electric field. The
field will also act on the molecule as to stretch the molecule, thus increasing its dipole moment. Since the field inside the molecule aligns itself opposite to the external field, the electric field inside the dielectric will be reduced. Because of reduction of the electric force in a dielectric the proportionality constant $\epsilon$ is increased. The $\epsilon$ in free space is usually designated as $\epsilon_0$, and the ratio of $\epsilon/\epsilon_0$ is called a dielectric constant of the material.

In a static case, or if the electric field variation is slow enough as not to produce losses resulting from the motion of molecules, the $\epsilon$ is real, which effectively means that the alignment of dipole moments with the electric field is proportional to the electric field. The rate of displacement of charges, which is called a "displacement current," is proportional to the rate of change of the electric field in respect to time.

In a lossy dielectric there will be a component of the displacement current which will be in phase with the electric field, and the product of this component and the electric field will represent the power loss in the dielectric. Thus, in a lossy dielectric $\epsilon$ is a complex number and can be expressed as:

$$\epsilon = (\epsilon' - j\epsilon'')$$ (Von Hippel 1962, p. 4).
For time varying fields the Maxwell equations state that in a dielectric medium

\[ \nabla \times H = \frac{\partial D}{\partial t} = i_d \]

where \( i_d \) represents the displacement current density. Since

\[ \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \]

then for a sinusoidal case where

\[ E = |E| \ e^{-j\omega t} \]
\[ i_d = -j\omega \varepsilon |E| e^{-j\omega t} \]

For \( \varepsilon \) having two components, in and out of phase with the electric field

\[ \varepsilon = \varepsilon' - j\varepsilon'' \]
\[ i_d = -j\omega (\varepsilon' - j\varepsilon'') E = -\omega (\varepsilon'' + j\varepsilon') E \]

The ratio

\[ \frac{\varepsilon''}{\varepsilon} \]

is called the power factor and

\[ \frac{\varepsilon''}{\varepsilon'} \]
is called a loss tangent.
CHAPTER II

MEASUREMENTS BASED ON INTERFERENCE EFFECT

Shorted Waveguide Method

A shorted waveguide method is the simplest to implement method for the measurement of dielectric properties of materials. Since there is only little variation in this method for coaxial lines or for rectangular waveguides, and since the coaxial line can be used over wider frequency band than the rectangular guide, use of this method with coaxial line is of advantage.

The coaxial line method requires a shorted coaxial line with a standing wave detector and a signal source. The short in the coaxial line should be movable. The dielectric sample should be machined to a slip fit with the slotted line.

The shorted guide method is based on measurement of the interference between the forward traveling wave \( E^+ \), and the reflected from the short, wave \( E^- \). Since both \( E^+ \) and \( E^- \) terminate on charges, at points where they are equal in sign the currents subtract (the charges have the same polarity and flow in opposite directions), and the electric fields add up because of equal sign charges.
terminating them.

Any place on the line (Johnson 1950, p. 94) the total electric field \( E \) and the current \( I \) equals to:

\[
E = E^+ + E^- \quad I = I^+ - I^- = E^+/Z_0 - E^-/Z_0
\]

\[
Z = E^+ + E^- = \frac{E^+ + E^-}{I^+ - I^-} = \frac{Z_0 E^+ + E^-}{E^+/Z_0 - E^-/Z_0}
\]

where \( Z \) is the impedance at any point on the line and \( Z_0 \) is the characteristic impedance of the line. All the quantities are complex. The propagating wave changes phase by the factor \( e^{-j\phi x} \) and the magnitude by \( e^{-\gamma x} \). The total variation of the wave propagating in the \( x \) direction is \( e^{-(\alpha+j\phi)x} = e^{-\gamma x} \), where \( x \) increases towards the load. If for convenience the plane of the load is used as the \( x = 0 \) reference and \( x \) increases towards the generator, the wave traveling towards the load will change by the factor \( e^{\gamma x} \) and the wave traveling towards the generator (reflected wave) will change by the factor \( e^{-\gamma x} \). If \( E^+ \) and \( E^- \) refer to the plane of the load, and \( x \) increases towards the generator, then at the load:

\[
E = Z_r I = E^+ + E^- \quad \text{and} \quad Z_0 I = E^+ - E^-
\]

where \( Z_r \) is the load impedance. In terms of \( Z_0 \) and \( Z_r \) the expressions for \( E^+ \) and \( E^- \) are:
\[ E^+ = (I/2) \left( Z_r + Z_0 \right) \quad \text{and} \quad E^- = (I/2) \left( Z_r - Z_0 \right); \]

then if

\[ Z_r = Z_0 \frac{E^+ + E^-}{E^+ - E^-} \]

and \( Z \) at any point on the line is

\[ Z = Z_0 \frac{E^+ e^{\gamma x} + E^- e^{-\gamma x}}{E^+ e^{\gamma x} - E^- e^{-\gamma x}} \]

then

\[ \frac{Z}{Z_0} = \frac{Z_r + Z_0 \tanh \gamma x}{Z_0 + Z_r \tanh \gamma x} \quad (\text{Johnson 1950, p. 105}). \]

A. Von Hippel (1962, pp. 58-59) suggests measurements with the dielectric sample terminated in a short and open circuit (Figure 1). If the sample is terminated in a short circuit, then

\[ Z_t/Z_0 = \tanh \gamma_d x \]

or

\[ Z_{od} = Z_t/\tanh \gamma_d x \]

and if the termination is an open circuit then

\[ Z_c/Z_{od} = \coth \gamma_d x \quad \text{or} \quad Z_{od} = Z_c/\coth \gamma_d x. \]

In this case \( Z_t \), the short circuit impedance, and \( Z_c \),
Figure 1. Dielectric Sample Terminated in
(A) Short Circuit (B) Open Circuit
the open circuit impedance, refer to impedance in the
dielectric and $Z_{od}$ refers to impedance of a perfectly
matched dielectric coaxial line. The open circuit ter-
mination of the dielectric is obtained by moving the
short terminating the slotted line by a one-fourth wave-
length away from the sample.

The impedance $Z_{od}$ of the dielectric filled line can
be calculated:

$$Z_{od}^2 = Z_t Z_c.$$  

The measurement of $Z_t$ and $Z_c$, the interface impedances
between the air filled portion of the slotted line and
the sample, can be made by measuring the minimum voltage
and its position on the slotted line. In terms of the
slotted line measurements the impedance $Z(0)$ at the air-
sample interface ($Z_t$ or $Z_c$) can be expressed:

$$
Z(0) = \frac{H-j\tan2\pi d/\lambda_0}{Z_0} = \frac{H-j\tan2\pi d/\lambda_0}{1-jH\tan2\pi d/\lambda_0}
$$

where $H$ is the inverse VSWR, $d$ is the distance from the
sample interface to the first voltage minimum and $\lambda_0$ is
the free space wavelength (Von Hippel 1961, p. 66). Con-
sidering that

$$
\frac{Z_{od}}{Z_0} = \sqrt{\frac{\varepsilon_0}{\varepsilon_d}}
$$

for non-magnetic materials then
\[
\frac{Z_{od}}{Z_0} \tanh \gamma_d x = \frac{Z_t}{Z_0}
\]

and

\[
\frac{\tanh \gamma_d x}{\sqrt{\varepsilon_d}} = \frac{Z_t}{Z_0} \frac{1}{\sqrt{\varepsilon_0}}
\]

and

\[
\frac{\coth \gamma_d x}{\sqrt{\varepsilon_d}} = \frac{Z_c}{Z_0} \frac{1}{\sqrt{\varepsilon_0}}.
\]

Therefore

\[
\varepsilon_d = \frac{1}{\varepsilon_0} \left( \frac{Z_t}{Z_0} \right) \left( \frac{Z_c}{Z_0} \right).
\]

In the above calculation it is assumed that the losses of the slotted line filled with air dielectric are negligible in comparison to the dielectric losses.

**Lumped Capacitance Method**

Measurement of a small sample of a dielectric material can be made by terminating an air dielectric coaxial line with this sample (Stuchly and Rzepecka 1974) (Figure 2). As in the previous section

\[
\frac{Z(0)}{Z_0} = \frac{H - j \tan \frac{2\pi d}{\lambda_0}}{1 - j H \tan \frac{2\pi d}{\lambda_0}}
\]
Figure 2. Coaxial Line Terminated in Dielectric Sample
(A) Configuration  (B) Equivalent Circuit
can be computed from the position of the closest voltage
minimum to the sample, and from the VSWR. Knowing
\(Z(0)/Z_0\) one can compute the reflection coefficient \(\Gamma\), where

\[
\Gamma = \frac{1 - Z_0/\phi(0)}{1 + Z_0/\phi(0)} = \frac{1 - j\omega C_0 \phi_0 \epsilon_r}{1 + j\omega C_0 \phi_0 \epsilon_r} = |\Gamma| e^{-j\Theta}
\]

\(Z(0) = 1/j\omega C_0 \epsilon_r\) and \(C_0\) and \(\epsilon_r\) are the capacitance of the
empty capacitor and relative dielectric constant respect-
ively. The permittivity of a dielectric can be determined
by first measuring the reactance of the capacitor with and
without the dielectric. Then the product of \(Z(0)/Z_0\) and
\(Z_0/Z(0)_d\) can be computed:

\[
(Z[0]/Z_0) (Z_0/Z[0]_d) = Z(0)/Z(0)_d = \frac{j\omega C_0 \epsilon_r}{j\omega C_0} = \epsilon_r
\]

where \(Z(0)_o\) is the reactance of an empty capacitor and
\(Z(0)_d\) is the reactance of dielectric filled capacitor.

Complex permittivity of the material can be com-
puted directly from the reflection coefficient. The real
and imaginary parts of the permittivity are:

\[
\epsilon'_r = \frac{2 |\Gamma| \sin \Theta}{\omega C_0 Z_0 (|\Gamma|^2 + 2 |\Gamma| \cos \Theta + 1)}
\]
\[
\varepsilon''_r = \frac{1 - |\Gamma|^2}{W_C \omega \varepsilon_0 (|\Gamma|^2 + 2 |\Gamma| \cos \theta + 1)}
\]

\[
\Theta = \arctan \frac{2 \varepsilon'_r (Z_0 / Z(0)) d}{1 - (Z_0 / Z(0)) d^2 (\varepsilon'_r^2 + \varepsilon''_r^2)}
\]

Computation of dielectric properties of the sample directly from the reflection coefficient requires calculations of the free space capacitance from the geometry of the sample. However, the errors due to fringing of the electric fields may be quite high in this type of measurement, and the uncertainty caused by ambiguous parameters should be investigated. Calculation of the complex components of permittivity by using the above method shows that the minimum error in the calculation of \( \varepsilon' \) occurs when

\[
z(0) d / Z_0 = \left[ 4 \varepsilon'_r \varepsilon''_r (\Delta |\Gamma| / |\Gamma|) + (\varepsilon'_r^2 - \varepsilon''_r^2)^2 \right]^{1/4}
\]

and the minimum of \( \varepsilon'' \) occurs when

\[
z(0) d / Z_0 = \left[ 4 \varepsilon'_r \varepsilon''_r (\Delta \theta / \Delta |\Gamma|) + (\varepsilon'_r^2 - \varepsilon''_r^2)^2 \right]^{1/4}
\]

where \( \Delta |\Gamma| \) and \( \Delta \theta \) are the uncertainties in the modulus and the phase of the input reflection coefficient.

Analysis of the two above equations shows that the measurement error does not strongly depend on selection of
optimum capacitance of the capacitor. Consequently, a properly chosen capacitor can be used over relatively broad frequency band.
CHAPTER III

MEASUREMENTS BASED ON RESONANT EFFECT

Resonant Cavity Method

A measurement of dielectric properties can be made by filling a resonant cavity with the dielectric and measuring cavity impedance versus the frequency. Consider a rectangular waveguide cavity, fed by a waveguide. The cavity is coupled to the waveguide through a small slot in the shorting plate separating the cavity from the waveguide. The impedance of the cavity, looking from the waveguide, can be represented as a RLC loop coupled to the source by a mutual inductance M, with a small inductive reactance $X_1$ in series (Figure 3). At integral number of half wavelengths from the short the impedance of the cavity can be measured by use of a slotted line. At a frequency at which the cavity is detuned, the impedance $Z_a$ at the shorting plate, differs from zero by $jX_1$; therefore

$$Z_a = Z_x = jX_1$$

at detuned conditions (Ginzton 1957, pp. 395-399). At resonance the impedance of the cavity, equivalent to an
Figure 3. Equivalent Circuit of a Cavity Coupled to a Signal Generator
inductance coupled loop, seen from the waveguide is

\[ Z_{\text{loop}} = \frac{\omega^2 M^2}{R + j[\omega L - 1/\omega C]} \]

and

\[ Z_a = Z_x + Z_{\text{loop}} = jX_1 + \frac{\omega^2 M^2}{R + j[\omega L - 1/\omega C]} \]

Dividing by the impedance of the waveguide \( Z_o \)

\[ \frac{Z_a}{Z_o} = \frac{jX_1}{Z_o} + \frac{T}{1 + j2Q_o \omega} \]

where

\[ T = \frac{\omega^2 M^2}{Z_o R} ; \quad \delta = \frac{\omega - \omega_o}{\omega} ; \quad Q_o = \frac{\omega L}{R} \]

At frequencies close to the resonant frequency, for a very high \( Q \) and small coupling \( M \), the term \( X_1 \) can be neglected. The impedance, at the input to the cavity, at frequencies close to the resonant frequency becomes

\[ \frac{Z_a}{Z_o} = \frac{T}{1 + j2Q_o \omega} \]

In a rectangular waveguide filled with low loss dielectric, the guide wavelength \( \lambda_g \) can be expressed as:
where \( \lambda_o \) is the free space wavelength, \( \varepsilon_r \) is the relative dielectric constant and \( \lambda_c \) equals double the waveguide width. The \( \varepsilon_r \) can be calculated by measuring the resonant frequency of free space filled cavity and dielectric filled cavity. Then

\[
\lambda^2_{g1} = \lambda^2_{g2} = \frac{(\lambda_{01})^2}{1 - (\lambda_{01}/\lambda_c)^2} = \frac{(\lambda_{02})^2}{\varepsilon_r - (\lambda_{02}/\lambda_c)^2}
\]

and

\[
\varepsilon_r = \left(\frac{\lambda_{02}}{\lambda_{01}}\right)^2 \left[1 - \left(\frac{\lambda_{01}}{\lambda_c}\right)^2\right] + \left(\frac{\lambda_{02}}{\lambda_c}\right)^2
\]

where \( \lambda_{01} \) and \( \lambda_{02} \) are the resonant wavelengths of air filled and dielectric filled guide respectively. The resonant point can be found from Smith chart plot of the impedance. It will occur at the frequency when the impedance curve crosses the zero reactance line.

To find the loss tangent one must first find the Q of the cavity. Consider the input impedance equation:
\[
\frac{Z_a}{Z_0} = \frac{T}{1 + j2Q \sigma}
\]

(Ginzton 1957, pp. 407-410).

At resonance the impedance will be strictly resistive and on a Smith chart \(Z_a/Z_0\) will lie on the zero reactance line. Half power points can be found by plotting the impedance curve. They will occur where the

\[
\frac{Z_a}{Z_0} = \frac{T}{1 + j1},
\]

and consequently

\[Q = \frac{1}{2\sigma} \text{ or} \]

\[Q = \frac{\omega_0}{\omega_1 - \omega_2}
\]

where \(\omega_1\) and \(\omega_2\) are the half power frequencies. The loss tangent can be found from the \(Q\) of the cavity

\[Q = \frac{\omega_0 U}{W},
\]

where \(U\) is the maximum energy storage and \(W\) is average power. Since for low loss dielectric \(\varepsilon \approx \text{Re} \varepsilon\), \(U\) may be calculated

\[U = \varepsilon/2 \int \int \int |E|^2 \, dx \, dy \, dz\]
\[
\omega = \frac{1}{2} \iiint \frac{\mathbf{E} \cdot \mathbf{E}}{\varepsilon} \, dx \, dy \, dz
\]

then

\[
Q = \frac{\varepsilon'}{\varepsilon''} = \frac{\omega \varepsilon'}{\sigma} = \frac{1}{\text{loss tangent}},
\]

which follows from Maxwell equations for lossy dielectrics

\[
\nabla \times \mathbf{H} = j\omega \mathbf{E} = j\omega (\varepsilon' - j\varepsilon'') \mathbf{E} = \left(j\omega \varepsilon' + \sigma\right) \mathbf{E} = j\omega (\varepsilon' - j\frac{\sigma}{\omega}) \mathbf{E}
\]

where \(\sigma\) is the conductivity of the dielectric.

Another useful relationship is given by A. R. Von Hippel (1962, p. 81) for low loss dielectrics:

\[
\frac{\omega_1 - \omega_2}{\omega} = 2 \frac{\omega}{\sigma}
\]

where the propagation constant

\[
\gamma = \alpha + j\beta.
\]

For Transverse Electromagnetic cavities \(2(\omega/\sigma) \approx \text{loss tangent}\). In the calculation of the Q of cavity filled with dielectric it is assumed that the copper losses are negligible.
Perturbation Method

This method permits measuring of dielectric properties of materials having high dielectric constant and high loss factor. The perturbation method is based on Slater Perturbation Theorem expressed mathematically

\[ \omega^2 = \omega_0^2 \left[ 1 + \int (H^2 - E^2) \, dv \right] \]

where \( \omega_0 \) and \( \omega \) are resonant frequencies without and with dielectric sample and \( H \) and \( E \) are the normalized magnetic and electric fields, integrated over the perturbed volume (Slater 1969, p. 81). This equation can be expressed for practical purposes:

\[ \mathcal{S} = k \int \frac{(\omega H^2 - \varepsilon E^2)}{4U} \, dv \]

where \( k \) depends on the shape of the dielectric sample, the integral is over the sample volume, \( U \) is the maximum energy stored in the resonator and

\[ \mathcal{S} = \frac{\omega - \omega_0}{\omega} \] (Ginzton 1957, p. 439).

With a small flat circular sample, placed in a maximum electric field of a resonator this formula can be approximated by:
\[ \mathcal{J} = \frac{-\int \Delta \varepsilon \mathbf{E}^2 \, dv}{4U} = \frac{-V_s \Delta \varepsilon \mathbf{E}^2}{4U} \]

where

\[ \Delta \varepsilon = \varepsilon' - \varepsilon_0 \]

and \( \varepsilon \) is the dielectric constant of the sample and \( V_s \) is the volume of the sample. If the measurement is made in rectangular waveguide, half wavelength cavity then

\[ \mathcal{J} = \frac{-V_s \Delta \varepsilon \mathbf{E}^2}{4\varepsilon_0 abd\mathbf{E}^2/8} = \frac{-\Delta \varepsilon}{\varepsilon_0} \frac{V_s}{\mathcal{V} \frac{g}{V}} \]

where \( V \) is the volume of the cavity. Thus

\[ \frac{\Delta \varepsilon}{\varepsilon_0} \]

can be found:

\[ \frac{\Delta \varepsilon}{\varepsilon_0} = -\mathcal{J} \frac{V_g}{2V_s} \]

and

\[ \frac{\varepsilon'}{\varepsilon_0} = \frac{\mathcal{J} V_g}{2V_s} + 1 = k' \]

where \( k' \) is the relative dielectric constant.
The loss tangent of the dielectric sample can be found from the $Q_0$ and $Q_d$, where $Q_0$ is the $Q$ of the guide without a dielectric sample and $Q_d$ is the $Q$ of the same guide with dielectric sample inserted, and with the copper losses neglected. Then, the $Q_1$, the loaded $Q$ of the guide, can be calculated:

$$\frac{1}{Q_1} = \frac{1}{Q_0} + \frac{1}{Q_d}$$

$$Q_d = \frac{Q_1 - Q_0}{Q_1Q_0}.$$ 

Considering that

$$Q_d = \omega U / W_d$$

and

$$Q_s = \omega U_s / W_d$$

where $U$ is the maximum electric energy storage in the resonator, $U_s$ is the maximum electric energy storage in the dielectric sample and $W_d$ is the average loss in the dielectric sample. For a half wave, rectangular guide resonator with volume $v$:

$$U = \frac{\varepsilon_0 \omega E^2}{8},$$
and for the dielectric sample having volume $v$ placed in the center of the guide in the uniform maximum electric field $E$

$$U_s = \frac{\varepsilon'_s v E^2}{2}$$

then

$$\frac{Q_s}{Q_d} = \frac{U_s}{U} = \frac{4\varepsilon'_s v}{\varepsilon_0 v} = 4 k'(v/V).$$

The $Q_s$ can be calculated

$$Q_s = Q_d (4 k'[v/V]) = \varepsilon'/\varepsilon'' = 1/\text{loss tangent} \quad \text{(Ramo and Whinnery 1965, p. 543).}$$
CHAPTER IV

TWO COUPLER METHOD

The dielectric properties of a material can be determined by using two couplers feeding a waveguide filled with this material. Consider two Bethe-hole couplers with 30 db coupling factor and having over 25 db directivity (Montgomery 1947, pp. 858-866). If these couplers are fed from one source, through a power divider, attenuator, and a phase shifter (Figure 4), then the phase and the magnitude of the power coupled from each coupler can be adjusted so that a complete cancellation of power in the secondary guide is achieved. The phase shift between the power divider and an arbitrary point \( b \) inside the secondary guide, between coupling hole 2 and the crystal detector, can be expressed as:

\[
\theta_1 = \psi_1 + \phi + c_1 + \psi_a + \psi_b \\
\theta_2 = \psi_2 + \phi + c_2 + \psi_2 + \psi_b
\]

where subscripts indicate which coupler's path is considered, \( \psi \) is the phase shift introduced by the primary guide, \( \phi \) is the phase shift through the phase shifter, C
Figure 4. Two Coupler Method for Measurement of Dielectric Properties
is the phase shift through the coupling hole, \( \varphi \) is the phase shift through the attenuator, \( \phi \) is the phase constant through the secondary guide, \( a \) is the distance between the holes, and \( b \) is the distance from coupling hole 2 to an arbitrary point in the secondary guide.

If we make the two arms symmetrical, and the coupling holes and the respective components in each arm identical, then:

\[
\theta_1 - \theta_2 = \Delta \theta + \Delta \phi + \phi a
\]

where \( \Delta \theta \) and \( \Delta \phi \) are the phase differences introduced by the relative position changes of the phase shifters and the attenuators. If the values of the components from only coupler 1 arm are changed, then define

\[
\theta = \Delta \theta_1 + \Delta \phi_1 + \phi a = \theta + \phi + \phi a
\]

The attenuation (in nepers) from the power divider into the secondary guide, to the same point \( b \) is, through arm 1

\[
A_1 = P_1 + \phi a + \phi b
\]

and through arm 2

\[
A_2 = P_2 + \phi b
\]
where $P_1$ and $P_2$ are the attenuations in arms 1 and 2 respectively, and $\Delta$ is the attenuation constant of the waveguide. The difference in attenuation is

$$A_1 - A_2 = P_1 - P_2 + \Delta a.$$ 

If $P_1$ is changed by an amount $\Delta P$ such that

$$P_1 + \Delta P = P_2$$

then

$$A_1 - A_2 = \Delta P + \Delta a.$$ 

In order to produce a complete cancellation of electric field in the secondary waveguide at point b, it is necessary that

$$\theta_1 - \theta_2 = 0 + \phi + \phi a = 0$$

$$A_1 - A_2 = \Delta P + \Delta a = 0$$

The real part of permittivity can be found from measurements of phase shift between the two holes caused by variation of guide wavelength at several frequencies. Providing that permittivity change at these frequencies is negligible and that $\varepsilon \approx \varepsilon'$, the distance $a$ between the two coupling holes can be written in terms of various
wavelengths:

$$a = n \lambda_{g1} = \frac{n \lambda_{01}}{\sqrt{\varepsilon - (\lambda_{01}/\lambda_c)^2}}$$

$$a = (n + A) \lambda_{g2} = \frac{(n + A) \lambda_{02}}{\sqrt{\varepsilon - (\lambda_{02}/\lambda_c)^2}}$$

$$a = (n + B) \lambda_{g3} = \frac{(n + B) \lambda_{03}}{\sqrt{\varepsilon - (\lambda_{03}/\lambda_c)^2}}$$

where $\lambda_{g1}$, $\lambda_{g2}$, and $\lambda_{g3}$ are the waveguide wavelengths, and $\lambda_{01}$, $\lambda_{02}$, and $\lambda_{03}$ are the corresponding free space wavelengths, $n$ is the number of wavelengths between the two holes, and $A$ and $B$ represent the increase in the number of wavelengths caused by the change of frequency. Substituting:

$$(\lambda_{01}/\lambda_c)^2 = S^2$$

$$\lambda_{02} = c \lambda_{01}$$

$$\lambda_{03} = d \lambda_{01}$$

we can set up two equations with two unknowns $\varepsilon$ and $n$:

$$\frac{n \lambda_{01}}{\sqrt{\varepsilon - S^2}} = \frac{(n + A) \lambda_{01}}{\sqrt{\varepsilon - (cS)^2}} = \frac{(n + B) \lambda_{01}}{\sqrt{\varepsilon - (dS)^2}}$$
The phase shifts A and B are obtained from the change in \( \varnothing \) and which are the changes in phase required to compensate for the change in wavelength.

The calculations described above require the knowledge of phase shift introduced by the attenuator. If the permittivity of a solid dielectric is to be determined, a precise null is required to eliminate the possible effects of the dielectric-to-air boundary. If, however, measurements of properties of a homogenous gas or an aerosol are made, it is possible to obtain the phase constant by observing two symmetrical points around the minimum, and calculating the null point in between.

Measurement of the dielectric properties of an aerosol can be made using equipment shown in Figure 5. Waveguide containing two coupling holes is connected to a duct, thus establishing a closed path through which the tested medium is circulated. The waveguide is terminated at both ends in a good load. At the end toward which the wave is propagating (the power propagating toward the other end is reduced by the directivity) a probe is inserted. It is used to measure the resultant of the power coupled from both couplers. The terminating loads are triangular resistive cards placed in the center of the guide, in the direction of propagation in the E plane. The loads and
Figure 5. Measurement of Properties of Aerosol
and the probe should be matched in order to minimize any reflections in the guide. The matching can be done in a guide filled with air since the permittivities of air and aerosol are not expected to differ substantially. A very thin dielectric window should be placed in each coupling hole in order to prevent leakage of the medium into the couplers. Without a window, accumulation of powder inside the coupling hole may be possible, and this could change the coupling factor. It is also desirable that the medium flows rapidly through the guide and that the guide is positioned vertically in order to prevent powder accumulation on the guide surfaces.

The measurement of attenuation can be accomplished in several ways. If a solid, dielectric slab is involved, the easiest way is first to obtain a deep null in a guide filled with air. The sample is then inserted into the guide and the attenuators are adjusted to compensate for the attenuation introduced by the sample. The loss tangent can be calculated from the following equation

\[ \frac{\varepsilon''}{\varepsilon'} = \frac{2\pi}{\lambda} \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} \]

where the attenuation constant \( \lambda \) is the attenuation in nepers per meter, \( \lambda \) is the wavelength in the dielectric in
meters, and $\lambda_c$ equals twice the width of the guide (Ramo and Whinnery 1965, p. 426). In the measurement of attenuation of gases or aerosols, adjustment of the attenuators with the medium removed from the waveguide can be avoided if a waveguide switch is used. The attenuation and the phase of coupler 2 is first adjusted for a deep null. Thus the attenuation introduced in the dielectric is compensated by the attenuation in the coupler 2 circuit. The inputs to coupler 1 and coupler 2 are then switched so that the wave coupled into the secondary guide from coupler 1 is attenuated by twice the attenuation in the dielectric. The null is obtained by introducing into the circuit of coupler 2 attenuation equal to twice the attenuation of the dielectric. Calculation of the loss tangent can be accomplished by using the same equation as used in the former method (Ramo and Whinnery 1965, p. 426).
CHAPTER V

EXPERIMENTS AND RESULTS

Validity of the two coupler method was proven experimentally. Although the equipment used in the experiment did not have the required accuracy for precision measurements, it was sufficient to prove the feasibility of this method. The equipment was set up as shown in Figure 6. A sufficiently symmetrical microwave circuit could not be built because of insufficient funds, and therefore the phase measurements were done by comparing minima positions of the guide filled with air and with dielectric. Properties of nylon and plexiglass were measured. For the phase constant calculations, dielectric losses of the materials were neglected. The following procedure was used in determining of the phase constant of the dielectrics:

1. In air filled waveguide the line stretcher was adjusted for a minimum at the detector probe. The position of the line stretcher was noted at several frequencies.

2. Dielectric slab was placed inside the waveguide and, for each frequency in step 1, the phase was adjusted for the minimum at the detector probe. The position of the line stretcher, requiring minimum displacement from the
Figure 6. Equipment Used for Measurements
position in step 1, was noted for each of the frequencies in step 1.

3. The displacement of the line stretcher in step 2 was plotted versus the frequency (Figure 7).

4. Calculations were made of how much the position of the line stretcher has to be changed from a position required for a null at the detector when the guide is filled with the air, to a null position when the guide is filled with dielectric. The calculations were made for a range of relative dielectric constants from 2.4 to 3.1, for each frequency in step 1. The selection of the range of relative dielectric constant was made using information from tables in Von Hippel (1961, pp. 323-334).

5. The results of step 4 were plotted in Figure 7.

The calculations of step 4 were made as follows:

1. The number of wavelengths between the two coupling holes was calculated, for air and for other assumed dielectric constants, by dividing the distance a by guide wavelength $\lambda_g$, where $\lambda_g$ was computed:

$$\lambda_g = \frac{\lambda_o}{\sqrt{\varepsilon/\varepsilon_o - (\lambda_o/\lambda_c)^2}}$$

where $\lambda_o$ is the free space wavelength and $\varepsilon/\varepsilon_o$ is the relative dielectric constant.
Figure 7. Displacement of Line Stretcher Versus Frequency
2. $\Delta n$, the difference between the number of wavelengths between the coupling holes of air filled waveguide and dielectric filled waveguide was calculated. In order to obtain the value for minimum adjustment required for a null at the detector, the integral in $\Delta n$ was subtracted from $\Delta n$, thus leaving the fraction $\Delta'n$, the required adjustment of the line stretcher expressed in number of wavelengths.

3. The distance between the two null positions of line stretcher expressed in cm was obtained by multiplying $\Delta'n$ by free space wavelength. Then $\Delta'n\lambda_0$ represents the required adjustment of line stretcher (in cm).

As shown in Figure 7, the relative dielectric constant of plexiglass was measured to be about 2.6 and of nylon it was about 2.95. These results agree with tables in Von Hippel (1961, pp. 323 and 334).

The measurement of attenuation was made by inserting a set of fixed attenuators into the two coupler circuit, and by adjusting the line stretcher until a null in an air filled guide was obtained. Next, the dielectric slab was placed in the guide. A resistive card was placed in a slot of a slotted section in the coupler 2 circuit. The card and the line stretcher were adjusted until a null was obtained at the detector. Coupler 1 was
then disconnected and the line terminated in a load. The attenuation of the resistive card was determined by noting the increase of power detected after the resistive card was removed from the slotted section. This increase of power equaled to the attenuation of the dielectric. The loss tangent was computed using the formula given in previous section. The loss tangent of plexiglass was measured to be 0.0060, which is in agreement with Von Hippel (1961, p. 334). The loss tangent of nylon was measured to be 0.0084, which is about 30 percent lower than shown in Von Hippel (1961, p. 323). Both measurements were made at 4 GHz. The apparent reason for the error in the measurements of loss tangent of nylon was that higher order modes were possible in the particular guide used for the measurements, if this guide was filled with material having relative dielectric constant higher than 2.9. At 4 GHz the higher order modes could not be supported in this guide filled with plexiglass, having the relative dielectric constant of 2.6.
CHAPTER VI

CONCLUSIONS

The results obtained with the two coupler method prove its feasibility. Much more accurate results could be expected if the equipment used for this experiment was more precise. The main problem was in the initial design, where the utilized waveguide (WR-187) was not the optimum choice for the available frequency range. The redesign was infeasible because of the scope of the effort. Lack of a good variable attenuator also limited the accuracy of the measurement. From the results it is evident that existence of higher order modes did not affect the phase constant measurements. This can be explained by the fact that these modes have a very high phase velocity near their cut-off frequency, and therefore the electric field variation produced by these modes was insignificant within the guide wavelength of the principle mode. In order to measure the attenuation constant a Traveling Wave Amplifier had to be used. This amplifier introduced errors because of harmonics generation. The attenuation measurement of plexiglass was fairly accurate, while the results for nylon were considerably off. The apparent reason for the errors in these measurements can be attributed to the
fact that the attenuation measurements were made only at 4 GHz, and at this frequency the WR-187 waveguide, filled with nylon, can support the TE\textsubscript{20} mode.

The applicability of the two coupler method for measurement of dielectric properties of aerosols is apparent. The homogeneity of the medium can be preserved by controlling the velocity of the stream of aerosol with the blower. A very fast velocity will prevent powder deposits on the walls of the guide. The waveguide electromagnetic boundaries are well mathematically defined and permit accurate calculations. Two identical dielectric windows can be inserted in each coupling hole to prevent leakage from the couplers into the secondary guide. Properly implemented, this method provides a good tool for measurements not possible with other conventional methods.


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