Digital Computer Analysis of the Kinetic Response of a Thermal Pressurized Water Reactor Power Plant

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DIGITAL COMPUTER ANALYSIS OF THE KINETIC RESPONSE OF A THERMAL PRESSURIZED WATER REACTOR POWER PLANT

BY

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RESEARCH REPORT

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ABSTRACT

A digital computer model has been developed to simulate the kinetic response of a thermal pressurized water reactor power plant. The program is capable of predicting core power and plant temperature variations which result from disturbances due to routine demand changes, control rod movement, and fission product poison transients as well as from transients during plausible accident situations.

The development of the differential equations describing the influence of various primary plant components on the dynamics of the reactor have been heretofore documented in the literature. Finite difference equations facilitating the employment of a digital computer solution are fully derived.

Confidence in program validity is supported by the simulation of previously studied accidents and comparison with the safety analyses of a licensed nuclear power generating plant.

A source listing of the computer program is provided in the Appendix.
ACKNOWLEDGMENT

I would like to thank the members of my research committee, Dr. Donald B. Wall, Dr. Richard C. Rapson and Professor Minardi, for the advise and assistance they provided during the undertaking of this research effort.

The patience, support and understanding afforded me by my wife, Joyce, have been intangible but ever necessary elements in my research and are greatly appreciated.
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LIST OF VARIABLES

A_s = steam generator tube heat transfer surface area

C_c = coolant specific heat capacity at constant pressure

C_f = fuel specific heat capacity at constant pressure

C_m = moderator specific heat capacity at constant pressure

C_p = reactor inlet specific heat capacity at constant pressure

C_s = steam-water specific heat capacity at constant pressure

C = total delayed neutron precursor concentration

C' = total effective delayed neutron precursor concentration

C' = total fission equivalent delayed neutron precursor concentration

C_i = delayed neutron precursor concentration of ith group

C_i' = effective delayed neutron precursor concentration of ith group

C_i'' = fission equivalent delayed neutron precursor concentration of ith group

DRW = differential rod worth

f = thermal utilization

H = control rod height from core bottom

k_eff = effective multiplication factor

\lambda_i = slowing down time

\lambda^* = effective neutron lifetime

\lambda_T = neutron lifetime

\lambda_f = fast non-leakage probability
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\( T_h \) = reactor coolant hot leg plenum inlet temperature
\( T_m \) = moderator temperature
\( T_{rci} \) = reactor coolant cold leg plenum inlet temperature
\( T_{ref} \) = arbitrary reference temperature
\( T_{rhol} \) = reactor coolant hot leg plenum outlet temperature
\( T_s \) = steam generator secondary steam-water temperature
\( T_{sc} \) = steam generator coolant cold leg tube outlet temperature
\( T_{sco} \) = steam generator coolant cold leg plenum outlet temperature
\( T_{sg} \) = steam generator average coolant temperature
\( T_{sh} \) = steam generator coolant hot leg tube inlet temperature
\( T_{shi} \) = steam generator coolant hot leg tube inlet temperature
\( U_r \) = overall heat transfer coefficient of reactor core
\( U_s \) = overall heat transfer coefficient of steam generator
\( \dot{V} \) = coolant volumetric flow rate
\( V_r \) = reactor core volume of fueled region
\( V_{ri} \) = reactor inlet plenum volume
\( V_{ro} \) = reactor outlet plenum volume
\( V_{si} \) = steam generator inlet plenum volume
\( V_{so} \) = steam generator outlet plenum volume
\( \alpha_T \) = temperature coefficient of reactivity
\( \alpha_{Xe} \) = xenon coefficient of reactivity
\( \beta \) = delayed neutron fraction
\( \bar{\beta} \) = effective delayed neutron fraction
\( \beta_i \) = delayed neutron fraction of ith group
\( \bar{\beta}_i \) = effective delayed neutron fraction of \textit{i}th group

\( \Delta \rho \) = change in reactivity

\( \Delta t \) = iteration time interval

\( \Delta T \) = reactor core inlet-to-outlet temperature difference

\( \epsilon \) = fast fission factor

\( \eta \) = reproduction factor

\( \lambda \) = effective total delayed neutron precursor decay constant

\( \lambda_{\text{eff}} \) = effective total delayed neutron precursor decay constant

\( \lambda_i \) = decay constant of \textit{i}th group

\( \nu \) = neutron yield from fission

\( \rho \) = reactivity

\( \rho_f \) = fuel density

\( \rho_m \) = moderator density

\( \rho_R \) = control rod reactivity

\( \rho_T \) = temperature reactivity

\( \rho_{\text{Xe}} \) = xenon-135 reactivity

\( \Sigma_{f2} \) = macroscopic fission cross-section for thermal neutrons

\( \tau_d \) = half loop delay time

\( \tau_o \) = reactor core transit time

\( \phi_2 \) = thermal neutron flux

\textbf{Subscripts}

\( i \) = of \textit{i}th group delayed neutron precursors

\( n \) = property evaluated at node \( n \)
I. INTRODUCTION

An essential consideration in reactor system design is the investigation of the reactor power response resulting from external disturbances such as core temperature changes, control rod movement, fission product poisoning and power demand changes. Approaches to the problem have necessarily required the aid of machine methods due to the complexity introduced by the many elements contributing to the overall power behavior. The analog computer and control theory provided early support in the design analysis from conception to operation of the first power generating nuclear plants. Sophisticated but costly digital computer techniques are now used to simulate plausible reactor transients which must be considered prior to the construction, licensing and operation of present-day facilities.

The utility of a reactor plant model which is versatile, inexpensive, and which generally can be used with confidence, primarily lies in the preliminary design phase and personnel training aspects of a power plant. Such a simulator would not as such replace the highly complex digital reactor safety analyses which provide verification for safe plant operations during even the most adverse conditions and which are required for licensing by the Nuclear Regulatory Commission.1 It would offer a tolerable range of plant parameters for adequate stability, furnish
preliminary quantitative design data for the reactor and control systems, and serve as an instructional aid in the training of plant operators and supervisors.

A digital computer model rather than the analog type was chosen as a result of the many inherent advantages of digital computers in this area of application:

1. Quantitative rather than semi-quantitative solutions are available with optional plotting routines.

2. Reactor power level and other parameters may be allowed to vary over many decades without the loss of information. Scaling problems, whereby variables must be assigned values ordinarily between -100 volts and +100 volts, do not exist.

3. Large data reduction capabilities significantly reduce setup time.

4. Multiple and simultaneous variable readout is possible and limited only by the number of variables used in the program.

5. Digital computers with multi-language capabilities are much more readily available than their analog counterparts.

6. The digital computers provide a greater degree of accuracy.

7. Digital programming affords the user a much greater flexibility in quickly altering the program input data
to accommodate different problems.

8. The technological community is more familiar with and better equipped to utilize the digital computer than the analog variety.

The objective then is to develop a digital computer program that will perform a kinetic response simulation of a thermal pressurized water reactor power plant capable of handling a number of frequently encountered plant transients and adaptable to a variety of nuclear power generating sites. Since FORTRAN is available for nearly every technical computer and is the most widely used technical program language in the country, it shall be used in this model analysis.

Chapter II develops the reactor kinetics equations for a point reactor. Chapter III provides the derivation of the differential equations for the primary plant. Finite difference equations, corresponding to the differential equations obtained in Chapter III, are then developed in Chapter IV. Chapter V introduces the computer model, its limitations, and a discussion of its validity. Several of the more common plant transients are investigated in Chapter VI, which also suggests several areas in which application of the computer model might be useful.

The Appendix provides a source listing of the computer program and a sample output.
II. DEVELOPMENT OF THE POINT REACTOR KINETICS EQUATIONS

By necessity a dynamic analysis of an entire reactor plant must originate at the primary source of energy within the core itself. The fissioning process then is responsible for the production of power and can be described as a function of time and position inside the reactor. In order to achieve a basic understanding of the time-dependent solution to the problem rather than the much more complex behavior of a coupled spatial-temporal one, the reactor kinetics equations for a homogeneous reactor core will be sought, thus eliminating the spatial dependence from the discussion. Hence a technique of considering a single "point" within this homogeneous region, deriving the kinetics equations, and then integrating over the core volume will yield the desired result.

Since thermal pressurized water reactor power plants designed for electrical power generation use uranium-235 nearly exclusively as a nuclear fuel, the following description will involve this fissionable isotope only. The fissioning process involves the absorption of neutrons by the heavy nucleus of uranium-235, the resulting formation of a compound nucleus, uranium-236, at a relatively high excitation energy of about 6.4 Mev, and the decay of this compound nucleus by fission into usually two moderately heavy nuclei known as primary fission fragments. Since these heavy
nuclei contain more neutrons than stable nuclides with the same mass number, they release within the next $10^{-17}$ second, several energetic prompt neutrons and then are considered to be secondary fission fragments, which in turn may produce delayed neutrons as one mode of radioactive decay to the ground state during a time spanning from a fraction of a second to several minutes. Approximately 100 isotopes which decay by ejecting delayed neutrons are identified as delayed neutron precursors, and are divided into six groups according to their halflives. Ultimately, an average of 2.43 fission neutrons (prompt and delayed) are produced from a single thermal fissioning event.\(^3\)

Neutrons from sources other than this neutron-induced fission are also produced within the reactor core. These "source neutrons" may find their origin from the products of gamma-neutron, alpha-neutron, or cosmic interactions. They also may be the result of spontaneous fission, radioactive decay chains of uranium, or intentionally installed non-fission sources. Consequently, three types of neutrons (prompt fission, delayed fission, and source) must be accounted for in the reactor kinetics equations.

It is important to point out the assumptions and approximations utilized in this derivation of the reactor kinetics equations to provide an avenue by which the validity of the equations may be evaluated. The reactor core will be considered homogeneous throughout. The spatial dependence of both delayed and source neutrons will be the same as that of the prompt neutrons.
This implies that the secondary fission fragments that ultimately produce delayed neutrons do not travel appreciably from their point of birth, and that source neutron birth mechanisms have distributional characteristics similar to those of prompt neutrons. The slowing down time from birth to thermalization of all three types of neutrons will be considered negligible when compared to the thermal diffusion time from thermalization to absorption. This will make possible the utilization of a self-contained neutron cycle model which incorporates any fast fission effects within the same cycle rather than in subsequent ones. Finally, the fission rate density, $P$, will be considered to be separable into spatial and time dependent functions. This will allow a kinetic analysis to be made independent of spatial solutions.

To preclude the use of different multiplication factors for the three types of neutrons that would make the equations unnecessarily complex, a single effective multiplication factor, $k_{\text{eff}}$, will be applied to cycle processes for prompt, delayed and source neutrons. Were all neutrons born at the same energies, no preferential treatment would be afforded any neutron in a region during the slowing down process; however, delayed and source neutrons witness average birth energies below those of prompt neutrons. Therefore, the former types will undergo higher probabilities for thermalization as opposed to leakage or fast absorption since fewer collisions in the moderator are required for neutrons born at lower energies. Fewer collisions will result
in a smaller fermi age (a measure of the distance traveled while slowing down), and a smaller likelihood of leaking out or becoming absorbed while in the fast group (neutron energy greater than about 1 eV). An effective source rate density, \( S' \), and an effective delayed neutron precursor concentration of the \( i \)th group, \( C_i' \), are defined to facilitate the use of a single thermalization probability for an average fission neutron:

\[
S' = \frac{(\mathcal{L}_{FP})_{\text{source}}}{(\mathcal{L}_{FP})_{\text{average}}} \quad \text{II.1}
\]

\[
C_i' = \frac{(\mathcal{L}_{FP})^i}{(\mathcal{L}_{FP})_{\text{average}}} \quad \text{II.2}
\]

Since resonance absorption regions lie well below birth energies in general, all three types of neutrons experience essentially the same probability for resonance absorption. Equations II.1 and II.2 thus may be rewritten as:

\[
S' = S \frac{\mathcal{L}_F}{\mathcal{L}_{F_{\text{average}}}} \quad \text{II.3}
\]

\[
C_i' = C_i \frac{\mathcal{L}_F^i}{\mathcal{L}_F_{\text{average}}} \quad \text{II.4}
\]

Since 0.65% of the fission neutrons from the fissioning process of uranium-235 are delayed, the delayed neutron fraction, \( \beta \), equals 0.0065. Because delayed neutrons are preferentially treated while slowing down, their relative effectiveness in being
thermalized and later causing thermal fission is greater than the average fission neutron whose birth energy is slightly less than that of prompt neutrons. Consequently, an effective delayed neutron fraction for the ith group, $\bar{\beta}_i$, is defined to account for this effect:

$$\bar{\beta}_i = \beta_i \frac{\rho_F}{\rho_{\text{average}}}$$

The basic neutron lifecycle can be described as follows: Thermal fission at time $t$ produces both prompt neutrons and delayed neutron precursors. Due to previous fissioning events, an inventory of these precursors already exists with each group having a given probability of decay. Therefore, some precursors are decaying producing delayed neutrons also at time $t$. Finally source neutrons are introduced at the same time from perhaps several different non-fission reactions. All three types of neutrons will then undergo a slowing down process during which they may either experience fast leakage, fast absorption or thermalization. Some of the neutrons which are absorbed while in the fast group may cause fast fission. Since the assumption that the slowing down process takes negligible time relative to the thermal diffusion time, precursors and prompt neutrons that are products of this fast fission may be incorporated in the very same slowing down process. This initial amplification introduces the concept of the total fission rate density, $P$, uniting the effects of both fast and
thermal fission at time $t$. Once neutrons are thermalized, they may either be absorbed in fuel material, non-fuel material, or thermally leak out. Upon absorption in the uranium-235 fuel, an excited uranium-236 nucleus is formed and may either fission or decay to the ground state by gamma emission. The neutrons causing thermal fission have taken one neutron lifetime, $\lambda_T$, or approximately 100 microseconds, from birth to fission. At this point, the next cycle begins at time $(t+\lambda_T)$.

By describing these events by rates per unit volume, or rate densities, and providing expressions for each respective reaction, Figure 1 illustrates this neutron lifecycle process. All events in the lifecycle are reaction rate densities with units of events/cm$^3$-sec. The factor $k_{\text{eff}}$ is the six factor product, $\varepsilon_{\text{ref}} \sigma_f \bar{\lambda}_{\text{TH}}$. The abbreviation $\bar{\lambda}_F$ represents the average fission neutron fast non-leakage probability, $\bar{\lambda}_{F,\text{ave}}$. The slowing down time, $\lambda_1$, is taken as zero.

Indicated are two total fission rate densities, one at time $t$ and the second at time $(t+\lambda_T)$. The time rate of change of the fission rate density can then be determined by using these two expressions and the definition of a derivative of a continuous function $f(x)$:

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \tag{II.6}$$

Since the fission rate density as a function of time is continuous, the time rate of change of the fission rate density is expressed
Fig. 1. Neutron Lifecycle
as:

\[ \frac{d}{dt}P(t) = \lim_{\Delta t \to 0} \frac{P(t + \Delta t) - P(t)}{\Delta t} \]  \hspace{1cm} \text{II.7}

Using the relatively small value of the neutron lifetime, \( \lambda_T \), as the \( \Delta t \), this limit becomes:

\[ \frac{d}{dt}P(t) = \frac{P(t + \lambda_T) - P(t)}{\lambda_T} \]  \hspace{1cm} \text{II.8}

Substituting expressions for \( P(t + \lambda_T) \) and \( P(t) \) from the diagram into this equation and rearranging yields the first reactor kinetics equation:

\[ \lambda_T \frac{d}{dt}P(t) = k_{\text{eff}}(1-\bar{\beta})P(t) + k_{\text{eff}} \sum_{i=1}^{6} \overline{C}_i(t) + k_{\text{eff}} \overline{S}(t) - P(t) \]  \hspace{1cm} \text{II.9}

where \( \bar{\beta} = \sum_{i=1}^{6} \bar{\beta}_i \)

\[ \overline{C}_i(t) = \overline{C}_i(t)/\nu \]

\[ \overline{S}(t) = \overline{S}(t)/\nu \]

The definition of reactivity (\( \rho \)) is:

\[ \rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \]  \hspace{1cm} \text{II.10}

The effective neutron lifetime is defined as:

\[ \lambda^* = \frac{\lambda_T}{k_{\text{eff}}} \]  \hspace{1cm} \text{II.11}

By using these two definitions and by rearranging equation II.9, the first reactor kinetics equation can be written as:

\[ \lambda^* \frac{d}{dt}P(t) = (\rho - \bar{\beta})P + \sum_{i=1}^{6} \lambda_i \overline{C}_i + \overline{S} \]  \hspace{1cm} \text{II.12}
The second through seventh reactor kinetics equations may be written directly from the diagram as:

\[ \sum_{i=1}^{6} \dot{C}_i(t) = P(t)\nu - \sum_{i=1}^{6} \lambda_i C_i(t) \]  

or as:

\[ \dot{C}_i(t) = P(t)\nu_i - \lambda_i C_i(t) \quad i = 1, 2, 3, 4, 5, 6 \]

By applying the birth energy correction to the terms in equation II.14, the second through seventh kinetics equations then become:

\[ \dot{\overline{C}}_i = P\beta_i - \lambda_i \overline{C}_i \quad i = 1, 2, 3, 4, 5, 6 \]

It is frequently convenient to make use of a comparison between the time rate of change of the fission rate density and the fission rate density. Reactor period is defined as:

\[ T(t) = \frac{P(t)}{\dot{P}(t)} \]

and has units of seconds. Physically, a reactor period of one second describes a fission rate density which changes by a factor of e, or \(\sim 2.71828\), in one second. Lograte is defined as:

\[ \text{LGR}(t) = \frac{d}{dt} [\log_{10} P(t)] \]

with units of decades per minute (DPM). This is equivalent to:

\[ \text{LGR}(t) = \frac{26.06}{T(t)} \]

where the constant 26.06 has the units of DPM-seconds. Lograte is useful in indicating the rate at which the fission rate density is rising (falling) relative to the fission rate density at that time.
III. KINETIC MODEL OF THE REACTOR PLANT

Power Plant Model

A basic power plant is depicted in Figure 2 which illustrates the coupled primary and secondary loop configuration. Heat is transferred by the fissioning process in the reactor core by conduction and convection to the primary coolant flowing through channels. The heated coolant then is transported by forced convection to the steam generator which acts as a heat exchanger between the primary and secondary plants. Feedwater from the steam plant is fed into the steam generator, heated by the circulating coolant through tubes, and produces steam through evaporation. The steam is throttled and admitted to a turbine which in turn becomes the prime mover for an electrical generator. Exhaust steam from the turbine is first cooled by a condenser forming a saturated or subcooled liquid, and is then returned as feedwater to the steam generator through forced convection by the use of a feed pump. Therefore, coolant leaving the steam generator has transferred heat to the secondary plant and is returned to the reactor through a coolant pump for reheating.

This model assumes a single loop and a single steam generator. Were an actual plant to have two or more coolant loops, this model is valid providing all loops had the same mass flow rate, and all steam generators shared equal loads and experienced similar demand.
Fig. 2. Basic Power Plant
transients. If these conditions did not exist a multiloop analysis would be required should solutions for individual loop and steam generator temperatures be sought. However, if complete mixing existed inside the reactor inlet plenum, the transient kinetic response of the reactor core would be essentially the same for a given steam demand change for any single or multiloop arrangement with equally distributed loop mass flow rates. Since reactor power behavior will depend on the coolant loop transients, a primary plant model will now be proposed.

**Reactor Plant Model**

Figure 3 shows the basic primary plant control loop model. Coolant flowing out of the steam generator tubes at temperature $T_{sc}$ enters an outlet plenum region and experiences a mixing effect which imposes a time delay. Coolant leaving this plenum at temperature $T_{sco}$ then undergoes a pure transport delay in the cold leg until it reaches the reactor vessel inlet plenum wherein another mixing delay occurs prior to entry into the reactor core region. Coolant leaving this inlet plenum at temperature $T_{c}$ flows through the core and is heated to the core outlet temperature $T_{h}$ via the fissioning process. Fission fragments which are primarily responsible for the heat from fission are prevented from entry into the coolant by cladding material surrounding the uranium-235 fueled region.

The average coolant (moderator) temperature $T_{av}$ and its time
Fig. 3. Primary Plant Control Loop
rate of change will effect a change in the reactivity through the
temperature coefficient of reactivity $\alpha_T$, and, in turn, a change
in the fission rate density as predicted by the reactor kinetics
equations already developed. Control rod position changes and
transient fission products, primarily xenon-135, will also affect
the core reactivity and thus the fission rate density. The
coolant exiting the core region enters the reactor vessel outlet
plenum in which mixing effects produce a time delay in the flow of
this coolant back to the steam generator. Another pure transport
delay arises in the hot leg between the outlet of the reactor
vessel outlet plenum and the steam generator inlet plenum. A
fourth mixing effect occurs in the steam generator inlet plenum.
Coolant flowing from the outlet of the inlet plenum then transfers
heat by conduction and convection through the steam generator tubes
to the steam-water mixture of the secondary plant in an evaporator
section. Finally, the cycle begins again as the coolant leaves
the steam generator tubes. In order to adequately describe the
changes in the fission rate density and various loop temperatures,
an analysis of each component of the primary plant will be
necessary.

Primary Component Thermal Analysis

Reactor Thermal System

Figure 4 illustrates a typical cross section of a simplified
reactor core configuration. The assumptions that the overall fuel
Fig. 4. Reactor Core Cross Section
element has a constant density and constant specific heat capacity will be made. A basic thermodynamic energy balance for an element of the core having a unit height and unit depth consists of a onedimensional heat flow description. The rate of accumulation of energy inside the fuel element per unit area will equal the heat rate into the fuel per unit area plus the heat generation rate within the fuel per unit area minus the heat rate out of the fuel per unit area:

\[
\rho_f \frac{\partial q}{\partial t} \frac{\partial T_f}{\partial t} = 0 + \dot{Q}_r'' t_f - 2U_r (T_f - T_m) \quad \text{III.1}
\]

By identifying the third term on the left-hand side of the equation as the specific heat capacity at constant pressure for the fuel and rearranging, the above heat balance reduces to a non-homogeneous first order, linear ordinary differential equation with constant coefficients for a constant flow rate:

\[
\frac{\partial T_f}{\partial t} + \frac{2U_r}{\rho_f c_f} T_f = \frac{2U_r}{\rho_f c_f} T_m + \frac{1}{\rho_f c_f} \dot{Q}_r'' \quad \text{III.2}
\]

Heat generated per unit volume by processes such as fission fragment decay heat normally accounting for less than 10 percent of the reactor power density is excluded from the above equation for simplification. By integrating this differential equation over the space coordinates of the core, average values of the fuel temperature \( T_f \) and moderator temperature \( T_m \) will yield the average fuel temperature \( \overline{T}_f \) and average moderator temperature \( T_{av} \), respectively. Furthermore, by integrating \( \dot{Q}_r'' \) over the core volume,
the reactor power \( \dot{Q}_r \) now appears in the equation:

\[
\frac{dT_f}{dt} + \frac{2U_r}{\rho_f c_f} T_f = \frac{2U_r}{\rho_f c_f} T_{av} + \frac{1}{\rho_f c_f V_r} \dot{Q}_r
\]  

III.3

A similar analysis of the channel coolant results in the following heat balance:

\[
\rho_m t_m \frac{\partial q}{\partial T_m} \frac{\partial T_m}{\partial t} = 2U_r (T_f - T_m) + 0 - \frac{\rho_m t_m}{\tau_0} \frac{\partial q}{\partial T_m} (T_h - T_c)
\]  

III.4

where \( \tau_0 \) equals the time for a unit volume of coolant to pass through the reactor core. By identifying the third term on the left hand side of the equation as the specific heat capacity at constant pressure for the moderator, integrating over the space coordinates of the reactor, a second non-homogeneous first order, linear ordinary differential equation with constant coefficients for a constant flow rate is obtained:

\[
\frac{dT_{av}}{dt} + \frac{2U_r}{\rho_m t_m c_m} T_{av} = \frac{2U_r}{\rho_m t_m c_m} T_f - \frac{1}{\tau_0} T_h - \frac{1}{\tau_0} T_c
\]  

III.5

A single second order, linear ordinary differential equation will result by combining equations III.3 and III.5, thereby eliminating the average fuel temperature \( \bar{T}_f \). Since the average moderator temperature \( T_{av} \) is the arithmetic average of \( T_h \) and \( T_c \), the resulting second order equation may be written in terms of these latter two dependent variables:

\[
\frac{d^2T_h}{dt^2} + k_1 \frac{dT_h}{dt} + k_2 T_h = -\frac{d^2T_c}{dt^2} + k_3 \frac{dT_c}{dt} + k_4 T_c + k_5 \dot{Q}_r
\]  

III.6
where

\[ k_1 = \frac{2U_r}{\rho m c m} + \frac{2}{\tau_o} + \frac{2U_r}{\rho f c f} \]

\[ k_2 = \frac{4U_r}{\rho f c f \tau_o} \]

\[ k_3 = \frac{2U_r}{\rho m c m} - \frac{2}{\tau_o} + \frac{2U_r}{\rho f c f} \]

\[ k_4 = k_2 \]

\[ k_5 = \frac{2U_r}{\rho m c m \rho f c f V} \]

Due to the volumetric and directional changes experienced by the coolant as it flows from the cold leg piping into the reactor vessel inlet plenum to the core inlet, the flow cannot be characterized as sheetlike. Mixing to a considerable degree takes place in the inlet and outlet plenums of both the reactor vessel and the steam generator. Consider the simplified diagram of a mixing volume representing the reactor inlet plenum as shown in Figure 5. Assuming 100% mixing, an energy balance is made equating the rate of accumulation of energy in the volume to the rate of energy in plus the rate of energy generated minus the rate of energy out of the volume:

\[ \frac{m_{ri}}{dt}[c_p(T_c - T_{ref})] = \dot{m}_c c_p(T_{rci} - T_{ref}) + 0 - \dot{m}_c c_p(T_c - T_{ref}) \]

Since the specific heat capacity at constant pressure can be considered constant and by recognizing that the mass flow rate
Temperature $T_C$
Specific heat capacity, $c_p$
Reactor inlet plenum mass, $m_{ri}$

---

Fig. 5. Simplified Reactor Inlet Plenum
equals the product of the coolant density (assumed constant) and the volumetric coolant flow rate, equation III.7 reduces to:

$$\frac{dT_c}{dt} + \frac{V}{V_{ri}} T_c = \frac{V}{V_{ri}} T_{rci}$$  \hspace{1cm} \text{III.8}$$

or

$$\frac{dT_c}{dt} = k_6(T_{rci} - T_c)$$  \hspace{1cm} \text{III.9}$$

where \( k_6 = \frac{V}{V_{ri}} \)

An analogous study of the mixing effects in the reactor vessel outlet plenum results in a similar differential equation:

$$\frac{dT_{rho}}{dt} = k_7(T_h - T_{rho})$$  \hspace{1cm} \text{III.10}$$

where \( k_7 = \frac{V}{V_{ro}} \)

Steam Generator Thermal System

The same basic thermodynamic techniques are now applied to the steam generator. Considering first the primary coolant side of the tubes and by referring to Figure 6, the rate of accumulation of energy inside the tubes will equal the heat rate in plus the heat generation rate minus the heat rate out:

$$0 = \dot{m}_c c_c(T_{sh} - T_{sc}) + 0 - U_s A_s (T_{sg} - T_s)$$  \hspace{1cm} \text{III.11}$$

By algebraic rearrangement, this equation becomes:

$$\dot{m}_c c_c(T_{sh} - T_{sc}) = U_s A_s (T_{sg} - T_s)$$  \hspace{1cm} \text{III.12}$$

In the steam-water region of Figure 6, the rate of accumulation of energy in the steam-water mixture equals the heat
Fig. 6. Steam Generator
rate into the mixture plus the heat generation rate minus the heat rate out of the mixture:

\[ m_s c_s \frac{dT_s}{dt} = U_s A_s (T_{sg} - T_s) - \dot{Q}_s \]

\[ \text{III.13} \]

Since the steam temperature \( T_s \) is common to both equations III.12 and III.13 and the average steam generator primary coolant temperature \( T_{sg} \) is the arithmetic average of \( T_{sh} \) and \( T_{sc} \), equations III.12 and III.13 may be combined resulting in a first order, linear differential equation in two dependent variables, \( T_{sh} \) and \( T_{sc} \):

\[ \frac{dT_{sc}}{dt} + k_{10} T_{sc} = k_{9} \frac{dT_{sh}}{dt} + k_{9} T_{sh} - k_{11} \dot{Q}_s \]

\[ \text{III.14} \]

where

\[ k_9 = \frac{1}{k_8} \]

\[ \frac{1}{2} - \frac{m_{cc}}{U_s A_s} \]

\[ \frac{1}{2} + \frac{m_{cc}}{U_s A_s} \]

\[ k_{10} = k_9 \]

\[ k_{11} = \frac{1}{m_s c_s \left( \frac{1}{2} + \frac{m_{cc}}{U_s A_s} \right)} \]

A plenum mixing analysis for the steam generator uses the same basic techniques as those used for the reactor vessel inlet plenum. The resulting equations are:
\[
\frac{dT_{sh}}{dt} = k_{l2}(T_{shi} - T_{sh})
\]

and

\[
\frac{dT_{sco}}{dt} = k_{l3}(T_{sc} - T_{sco})
\]

where

\[
k_{l2} = \frac{V}{V_{si}}
\]

\[
k_{l3} = \frac{V}{V_{sco}}
\]

Loop Transport Delays

The convective transfer of heat from the reactor to the steam generator may now be examined. The temperatures are represented as:

\[
T_{shi}(t) = T_{rho}(t - \tau_d)
\]

or the inlet coolant temperature to the steam generator assumptions the same form as the reactor coolant hot leg plenum outlet temperature \(T_{rho}\), although it attains this specific value after a fixed transport delay \(\tau_d\). Providing the coolant piping is well insulated to prevent heat losses to ambient, this becomes a good approximation. By a similar treatment the transport delay from the steam generator back to the inlet of the reactor vessel inlet plenum can be described by:

\[
T_{rci}(t) = T_{sco}(t - \tau_d)
\]

This representation for a pure cold leg delay provides reasonable
accuracy if the coolant piping is well insulated and if pump heating is ignored.

Reactivity Effects

This primary plant kinetics model considers reactivity variations with the average moderator temperature $T_{av}$, control rod position, and fission product poisons. The fission product poison, xenon-135, is by far the most important one of concern due to its extremely high thermal neutron absorption cross section of nearly three million barns, its relatively high effective steady state yield from fission and the variations in concentration that it experiences during short time intervals. Therefore this model will consider xenon-135 as the only major fission product poison of significance. Other nuclear parameter changes in the reactor core, such as fuel and burnable poison depletion, moderator pressure changes, and long term stable fission product poison buildup, will also affect core reactivity; however, these changes have much smaller effects on the core kinetic response. They are therefore omitted from the model.

Since the seven kinetics equations are directly linked with the core reactivity, a method by which the reactivity $\rho$ and its time derivative $\dot{\rho}$ at any given time must be provided. Since reactivity is a function of $T_{av}$, control rod position and xenon-135 concentration, the time rate of change of reactivity by application of the chain rule becomes:
\[
\frac{dp}{dt} = \frac{\partial p}{\partial T_{av}} \frac{dT_{av}}{dt} + \frac{\partial p}{\partial H} \frac{dH}{dt} + \frac{\partial p}{\partial N} \frac{dN}{dt} \tag{III.19}
\]

or, more simply:

\[
\dot{\rho} = \alpha_T \dot{T}_{av} + DRW \dot{H} + \alpha_{Xe} \dot{N} \tag{III.20}
\]

where

\[
\alpha_T = \text{temperature coefficient of reactivity} = \frac{\partial p}{\partial T_{av}}
\]

\[
DRW = \text{differential rod worth} = \frac{\partial p}{\partial H}
\]

\[
\alpha_{Xe} = \text{xenon coefficient of reactivity} = \frac{\partial p}{\partial N} \tag{III.21}
\]

By integrating equation III.20 from time zero to time \( t \) and rearranging, the reactivity as a function of time is expressed as:

\[
\rho(t) = \rho(0) + \int_{0}^{t} \left( \alpha_T \dot{T}_{av} + DRW \dot{H} + \alpha_{Xe} \dot{N} \right) dt
\]

This completes the analytical treatment of primary loop temperatures and nuclear parameters. To provide a complete cast of equations by which the solution to the kinetic response of reactor power may be determined, approximation techniques will be utilized in the following chapter.
IV. FINITE DIFFERENCE EQUATIONS FOR PRIMARY LOOP PARAMETERS

When grouped together, the foregoing primary loop differential equations constitute a system of linear differential equations. Conceivably, the system could be placed in fundamental form and then solved using matrix techniques. Alternately, the system could be combined to yield a single eleventh order linear differential equation which could possibly be solved using advanced analytic techniques. The digital computer, however, has presented itself as a more than adequate means by which problems of this genre may be solved using finite difference and iteration methods.

Therefore, this chapter shall reduce the previously developed equations to ones which will facilitate the employment of a digital computer approach.

The ten temperature variables will be considered first. A forward difference procedure, or Taylor series, is chosen to express the temperature $T_{sc_n}$ as:

$$T_{sc_{n+1}} = T_{sc_n} + \Delta t \dot{T}_{sc_n} + \frac{\Delta t^2}{2} \ddot{T}_{sc_n} + \ldots$$  \hspace{1cm} IV.1

where $\Delta t$ represents the iteration time interval. For small values of $\Delta t$, the terms on the right hand side of equation IV.1 in the full Taylor series expansion corresponding to the higher powers $\Delta t^3$, $\Delta t^4$, etc., will be very small and are neglected. Equation
III.14 provides a differential equation for the time rate of change of $T_{sc}$; therefore, in finite difference notation it may be written as:

$$
\dot{T}_{sc_{n+1}} = k_9 T_{sh_n} + k_9 T_{sh_n} - k_10 T_{sc_{n+1}} - k_{11} Q_{s_{n+1}}
$$

If $T_{sc}$ is approximated as a piecewise linear function between time node $n$ and time node $(n + 1)$, the Euler-Cauchy method approximates $\dot{T}_{sc_{n+1}}$ as:

$$
\ddot{T}_{sc_{n+1}} = \frac{T_{sc_{n+1}} - T_{sc_n}}{\Delta t}
$$

The temperature leaving the steam generator outlet plenum $T_{sc_{o}}$ may be approximated by the same method used in equation IV.1. The first derivative of this temperature is described by equation III.16, which represents the mixing effect in the plenum. Thus, $T_{sc_{o}}$ may be written as:

$$
\dot{T}_{sc_{o_{n+1}}} = k_13(T_{sc_{n+1}} - T_{sc_{o_{n+1}}})
$$

Finally, $\ddot{T}_{sc_{o}}$ is approximated by the Euler-Cauchy method as in equation IV.3.

The temperature entering the reactor vessel inlet plenum has experienced a pure transport delay in the cold leg piping and is expressed as:

$$
T_{rci_{n+1}} = T_{sc_{o_{(n+1)}}} - \frac{\dot{T}_{d}}{\Delta t}
$$

The temperature of the coolant entering the reactor core from
the inlet plenum and its second time derivative may be expressed by the same form as equations IV.1 and IV.3, respectively, while the first time derivative is written with indicial notation from the differential equation III.9:

$$\dot{T}_{c_{n+1}} = k_6(T_{rci_{n+1}} - T_{c_{n+1}}) \quad \text{IV.6}$$

The average moderator temperature in the core $T_{av}$ is simply the arithmetic average of the colder inlet and warmer outlet temperatures, $T_c$ and $T_h$, respectively. The value of $T_{h_{n+1}}$ will, however, depend on the reactor power at time node $(n+1)$, and has yet to be determined after application of the reactor kinetics equations. Therefore, the following approximation is used to establish a value for $T_{av}$:

$$T_{av_{n+1}} = \frac{T_{c_{n+1}} + T_{h_{n}}}{2} \quad \text{IV.7}$$

Reactivity feedback through the temperature coefficient of reactivity requires knowledge of the rate at which the plant is heating up or cooling down. Therefore, the first derivative of $T_{av}$ with time is:

$$\dot{T}_{av_{n+1}} = \frac{\dot{T}_{c_{n+1}} + \dot{T}_{h_{n}}}{2} \quad \text{IV.8}$$

The temperature of the coolant as it leaves the core is $T_h$, which is approximated by the same form as equation IV.1. Since no equation has been developed explicitly equating the first derivative of $T_h$ with time to other variables or their time derivatives, an
Euler-Cauchy approximation is utilized:

\[ \ddot{T}_h = \dot{T}_h + \Delta t \ddot{T}_h \] \hspace{1cm} IV.9

Equation III.6 provides a means of evaluating the second time derivative of \( T_h \) and is written in indicial notation as:

\[ \ddot{T}_h = -k_1 \dot{T}_h - k_2 T_h - T_c - k_3 T_{c+1} + k_4 T_{c+1} + k_5 Q \] \hspace{1cm} IV.10

The values of the terms \( T_{\rho\rho} \) and \( \ddot{T}_{\rho\rho} \) are evaluated by finite difference equations of the form of equations IV.1 and IV.3, respectively. The first time derivative of \( T_{\rho\rho} \) is written by referring to equation III.10:

\[ \dot{T}_{\rho\rho} = k_\gamma (T_h - T_{\rho\rho}) \] \hspace{1cm} IV.11

Accounting for the pure transport delay in the hot leg, equation III.17 shows the relationship between the temperature entering the steam generator inlet plenum to the temperature leaving the reactor vessel outlet plenum, and is written for digital application as:

\[ T_{sh} = \dot{T}_{\rho\rho} - \frac{r_d}{\Delta t} \] \hspace{1cm} IV.12

The temperature entering the tube region of the steam generator \( T_{sh} \) and its second time derivative may be approximated in the same manner as the temperature \( T_{sc} \) and \( \ddot{T}_{sc} \). Equation III.15 accounts for the mixing effects within this steam generator inlet plenum and is written for iterative purposes as:
\[ \dot{T}_{sh_{n+1}} = k_{12}(T_{sh_{n+1}} - T_{sh_{n+1}}) \]  \hspace{1cm} \text{IV.13} \\

The average temperature of the coolant in the tubes of the steam generator is simply the arithmetic mean of \( T_{sh} \) and \( T_{sc} \) and is expressed as:

\[ T_{sg_{n+1}} = \frac{T_{sh_{n+1}} + T_{sc_{n+1}}}{2} \]  \hspace{1cm} \text{IV.14} \\

The remaining finite difference equations involving temperatures, the forms of which have already been specified, are provided here for purposes of completeness:

\[ T_{sc_{n+1}} = T_{sc_{n}} + \Delta t \dot{T}_{sc_{n}} + \frac{\Delta t^2}{2} \ddot{T}_{sc_{n}} \]  \hspace{1cm} \text{IV.15} \\
\[ \ddot{T}_{sc_{n+1}} = \frac{T_{sc_{n+1}} - T_{sc_{n}}}{\Delta t} \]  \hspace{1cm} \text{IV.16} \\
\[ T_{c_{n+1}} = T_{c_{n}} + \Delta t \dot{T}_{c_{n}} + \frac{\Delta t^2}{2} \ddot{T}_{c_{n}} \]  \hspace{1cm} \text{IV.17} \\
\[ \ddot{T}_{c_{n+1}} = \frac{T_{c_{n+1}} - T_{c_{n}}}{\Delta t} \]  \hspace{1cm} \text{IV.18} \\
\[ T_{h_{n+1}} = T_{h_{n}} + \Delta t \dot{T}_{h_{n}} + \frac{\Delta t^2}{2} \ddot{T}_{h_{n}} \]  \hspace{1cm} \text{IV.19} \\
\[ T_{rho_{n+1}} = T_{rho_{n}} + \Delta t \dot{T}_{rho_{n}} + \frac{\Delta t^2}{2} \ddot{T}_{rho_{n}} \]  \hspace{1cm} \text{IV.20} \\
\[ \ddot{T}_{rho_{n+1}} = \frac{T_{rho_{n+1}} - T_{rho_{n}}}{\Delta t} \]  \hspace{1cm} \text{IV.21} \\
\[ T_{sh_{n+1}} = T_{sh_{n}} + \Delta t \dot{T}_{sh_{n}} + \frac{\Delta t^2}{2} \ddot{T}_{sh_{n}} \]  \hspace{1cm} \text{IV.22}
At this point all loop temperatures and their derivatives of interest have been specified in a finite difference format. The primary dependent variables in the seven reactor kinetics equations, namely the fission rate density and the fission equivalent precursor concentrations for the six groups, must be specified. Using a Taylor series expansion approximation, the fission equivalent precursor concentration of the ith group is:

\[
\bar{c}_{i,n+1} = \bar{c}_{i,n} + \Delta t \frac{\dot{c}_i}{c_i} + \frac{\Delta t^2}{2} \frac{\ddot{c}_i}{c_i}
\]

The second through seventh kinetics equations are used to evaluate the first time derivative of \(\bar{c}_i\) as:

\[
\dot{\bar{c}}_{i,n+1} = \beta_i P_n - \lambda_i \bar{c}_{i,n+1}
\]

The Euler-Cauchy method is utilized in approximating the second time derivative of \(\bar{c}_i\). The resulting equation becomes:

\[
\ddot{\bar{c}}_{i,n+1} = \frac{\dot{c}_{i,n+1} - \dot{c}_{i,n}}{\Delta t}
\]

The fission rate density can be described by a truncated Taylor series expansion as:

\[
P_{n+1} = P_n + \Delta t \dot{P}_n + \frac{\Delta t^2}{2} \ddot{P}_n
\]

The right and left hand sides of equation II.12 are divided by the effective neutron lifetime \(\lambda^*\), placed in indicial notation, and may then be written as:
\[
\dot{P}_{n+1} = \frac{1}{\lambda^*_{n+1}} \sum_{i=1}^{6} \lambda_i \bar{C}_i - \left( \overline{\beta} - \rho_{n+1} \right) P_{n+1} + \frac{\overline{S}}{\lambda^*_{n+1}} \quad \text{IV.28}
\]

To approximate the second time derivative of the fission rate density, the Euler-Cauchy method is used as follows:

\[
\ddot{P}_{n+1} = \frac{\dot{P}_{n+1} - \dot{P}_n}{\Delta t} \quad \text{IV.29}
\]

To complete the cast of equations required for a digital computer analysis, several other parameters must be described. Any steam plant demand changes will be treated as linear functions of time. Therefore:

\[
\dot{Q}_{s,n+1} = \dot{Q}_{s,n} + \Delta t \ddot{Q}_{s,n} \quad \text{IV.30}
\]

Likewise, changes in control rod position with time will be considered to have an effect on reactivity (all other parameters held constant) which also is a function that varies linearly with time. Consequently,

\[
\rho_{R,n+1} = \rho_{R,n} + \Delta t \dot{\rho}_{R,n} \quad \text{IV.31}
\]

Transients in fission product poison concentration (i.e. xenon-135) will also be considered to vary linearly with time. This is generally a good approximation since we seek a solution to the reactor power over a period of approximately ten minutes whereas a transient in xenon-135 concentration due to a previously altered power level will take nearly fifty hours to reach a steady state condition. The reactivity change due to a time varying xenon-135 concentration will be represented as:
\[ \rho_{\text{Xe}}_{n+1} = \rho_{\text{Xe}}_n + \Delta t \dot{\rho}_{\text{Xe}} \]  

It cannot be assumed that the average moderator temperature in the core will change linearly with time. Therefore, the actual heatup or cooldown rate must be considered in evaluating the time rate of change of reactivity due to temperature (all other parameters held constant). Hence, the reactivity change due to a time varying \( T_{\text{av}} \) becomes:

\[ \rho_{T_{n+1}} = \rho_{T_n} + \alpha_{T_{T_{\text{av}}}} \]

Combining all three of the above effects, the reactivity of the core at the time node \((n+1)\) appears as:

\[ \rho_{n+1} = \rho_n + \alpha_{T_{T_{\text{av}}}} + \Delta t \dot{\rho}_R + \Delta t \dot{\rho}_{\text{Xe}} \]  

The effective neutron lifetime \( \tau^* \) is expressed in terms of the neutron lifetime \( \tau \) modified by the multiplying ability of the reactor, and may be expressed as:

\[ \tau^*_{n+1} = \tau (1 - \rho_{n+1}) \]  

Reactor power and the average fission rate density are directly related in that a single fissioning of uranium-235 will yield 193 MeV of recoverable energy \( ^{10} \) and that the average power density when integrated over the volume of the core equals the product of the average power density and the core volume. Hence,

\[ Q_{r_{n+1}} = P_{n+1} V_r \cdot \text{(mass-energy conversion factor)} \]  

Chapter V will next organize these finite difference equations in a logic format to simulate various plant disturbances.
and the resulting kinetic response of the reactor.
V. COMPUTER MODEL AND VALIDITY

The computer model has been designed so as to be capable of simulating the plant kinetics of most of the operating conditions that are likely to occur. It is able to reliably calculate the power response in all four ranges, namely the power range (full power to $10^{-2}$ of full power), the period range ($10^{-2}$ to $10^{-6}$ of full power), the counter range ($10^{-6}$ to $10^{-11}$ of full power), and the source range (below $10^{-11}$ of full power). Additionally, the model is equipped to analyze transients in secondary steam demand, motion of control rods, and changes in fission product poison concentrations in the power range, individually or collectively. A reactor shutdown by scrambling of control rods or by the normally sequenced insertion of rods may be examined without any modification to the program as it currently exists. Alternately, a reactor startup can be simulated with the addition of only a few program steps to detail the sequence and duration of control rod shims. Accidents involving the inadvertent addition of positive reactivity can be analyzed.

There are several limitations of the plant model which are worth noting. Were a situation to be encountered such that the throttle valve setting to a turbinegenerator in the secondary plant be controlled effecting a non-linear variation of the steam demand with time, the resulting changes in steam temperature and pressure
in the steam generator must be considered in order that the steam transient be accurately detailed. Also the present model uses an equivalent single loop. If a primary plant configuration were such that loop flow rates were different, a multiloop analysis would be indicated involving a much more complex study. Most power plants utilize a pressurizer which maintains the primary plant pressure relatively constant by providing a surge volume. If a plant had no such pressure accommodating component, reactivity effects and core heat transfer coefficients would have to be reevaluated. The plant model also does not consider any metallurgical changes which might take place in the fueled region of the core during an accident transient. Flow through the reactor is considered constant. Equations arrived at in the development of this plant model presupposed a single flow rate, and would have to be modified if a multiflow capability were utilized. Convective heat transfer characteristics are considered fixed. If core channel boiling or channel flowrate changes were contemplated during operations, a much more extensive analysis would be required to more accurately predict actual heat transfer mechanisms.

With only slight modification involving the addition or deletion of only a few program steps, the versatility of the current program may be increased. Since differential rod worth is normally a strong function of control rod axial position and that control rod speed may vary from group to group, subroutines accounting for these variations might be employed were they
considered necessary. The temperature coefficient of reactivity is considered a constant, the value of which may be varied on initial data entry. In reality, this coefficient is functionally dependent on the temperature of both the fueled region and the moderator, as well as on the position of control rods and xenon-135 concentration. Again, special subroutines could easily provide for these variations.

Should a reactivity addition accident be simulated with this program, normal protective measures used for safeguarding the reactor are not presently reflected in the program. The addition of conditional statements, which would initiate these protective measures should any unsafe conditions develop such as overpower or excessive temperatures, are easily made to provide for a higher level of sophistication. Finally, the inclusion of a decay heat factor as part of the total reactor power would better describe the actual thermal output of the core after a protective shutdown occurred by the scrambling of control rods. As the program currently exists, the reactor power resulting from such a scram would mirror only the immediate fission energy change taking place as seen by the nuclear detectors.

**Program Flow Diagram**

The program flow chart is illustrated by Figure 7. Statements for inputting data comprise the first section of the program. The input data consists of both plant related data and physical
Fig. 7. Program Flow Chart
constants independent of any specific plant design. Plant related data includes heat transfer coefficients, material properties and dimensions of both the reactor and steam generator, coolant flow rate, initial (steady state) average moderator temperature, and initial reactor power. Parameters which may be varied to simulate a given plant transient include the temperature coefficient of reactivity, the neutron lifetime, the effective delayed neutron fraction, and predetermined transient schemes for steam demand, control rod motion, and xenon-135 concentration. The program is normally initialized as one in steady state although this is not a requirement and may in fact start from some point in a previous transient. Physical constants include delayed neutron precursor decay constants, and necessary dimensional conversion factors. Program control constants are also provided as inputed data so as to facilitate a greater flexibility with which to modify program runs.

After this data has been entered into memory, a series of computations are made to evaluate constants peculiar to the plant being investigated. To insure a steady state condition exists and to establish the initial temperature distribution, all temperatures in the cold leg and hot leg are set equal to $T_{\text{sc}}$ and $T_{\text{rho}}$, respectively. The computer is then directed to record by printout some of the entered data, computed constants, and loop temperatures for future reference and comparison purposes.

Four conditional statements then update the problem in time
and control the transient schemes of steam demand, control rod position, and xenon-135 concentration. If the program has reached a predetermined time limit, the computer is instructed to terminate execution. If the program continues at this point, cold leg temperatures between the steam generator and reactor, namely $T_{sc}$, $T_{sco}$, $T_{rcl}$, and $T_c$, along with their time derivatives are computed. The average moderator temperature $T_{av}$ and its first time derivative are evaluated and used as an input to determine the new core reactivity. Fission equivalent precursor concentrations and their time derivatives for each of the six groups are evaluated. The effective neutron lifetime which is dependent on the new value of reactivity is calculated.

The fission rate density and its derivatives are then computed. An effective delayed neutron precursor decay constant $\lambda_{eff}$, which would describe the fraction of all precursors decaying per unit time if all six groups were considered as a single group, is determined for informational purposes should a single group model be later pursued. The fission equivalent production rate density, $\beta P$, and the fission equivalent activity of all delayed neutron precursors, $\lambda \bar{U}$, are calculated to support such a single group study. From the fission rate density at this new nodal point, the reactor power is evaluated in both megawatts and percent of full power. Inversely proportional to reactor period, lograte is computed by using $P$ and $P$.

Temperatures in the hot leg between the reactor and steam
generator are considered next. The temperatures $T_h$, $T_{rho}$, $T_{shi}$, and $T_{sh}$ along with their time derivatives are evaluated. At this point instructions are given to the computer pertaining to the number of complete cycles over which iteration steps are conducted prior to printing parameters of interest. If the end of the time interval is reached, values of time, steam demand, reactor power, lograte, reactivity, and average moderator temperature are printed. Then the next series of cycle iterations are performed followed again by a printing of output parameters. When a satisfactory tabulation of data as determined by a preselected program stop time has been made, program execution is terminated.

The Appendix provides a source listing and sample program output for this digital computer analysis.

**Model Validity**

In order that a proper assessment of the validity of this digital model be presented, several considerations must be examined. First, if the primary plant differential equations have been developed based on generally accepted mathematical techniques applied to representative descriptions of plant components, a sound foundation results. A strict, closed form analytical approach, an analog computer treatment, or a digital computer analysis are then the three possible methods available for solution to this integrated problem. Since a digital study has been used here, the manner in which the differential equations are reduced for computer
application must be evaluated. If these finite difference techniques have shown success in previous dynamic simulations, a high degree of confidence should then apply to this model. Finally, comparison of this model solution with either experimental data or an ultra-sophisticated digital study to the same physical problem provides the ultimate test upon which the validity of this approach may be assessed.

The differential equations developed in Chapter III are essentially identical to those presented by Schultz, Glasstone and Sesonske, Tyror and Vaughan, and Grace in their reactor system and inherent stability analyses, with a few noteworthy exceptions. The motivation for these analyses was the eventual incorporation of the resulting differential equations into an analog computer arrangement to investigate the system stability using Nyquist's criterion, log magnitude and phase diagrams, and analog transient response curves. Adequate stability by these investigators was achieved by considering only inlet plenum mixing effects in the reactor vessel and steam generator. This model includes the damping effects of both inlet and outlet plenum mixing. Another simplification used in the analog studies involved the use of a simple exponential form loop transport delay time due to its inherent suitability for analog design. The use of digital iterative techniques in this model quite easily facilitates a pure transport delay which more closely approximates actual plant behavior. As a result, the differential equations presented here
lend credibility to the model.

The finite difference equations which utilized the Taylor series expansion and Euler-Cauchy methods are standard techniques used in numerical analyses. Solution stability was achieved by using the proper time step intervals in the iterations, and by closely modeling the program after actual plant component and system design offering various primary loop damping effects.

Further verification of model validity was made by a direct comparison of the model predictions of various nuclear and plant parameters to those resulting from reactor safety analysis studies of the Kewaunee Nuclear Power Plant. These studies provide the primary bases upon which this plant, operated by the Wisconsin Public Service Corporation, is licensed by the Nuclear Regulatory Commission. Rather than testing the model by evaluating the transient response to routine operations which would be expected to produce relatively slow temporal changes, accident situations involving rapidly changing load demands and control rod positions were analyzed by use of the model and then compared to the Kewaunee safety studies.

The Kewaunee Plant is a thermal pressurized water reactor power plant with a nominal reactor power output of 1650 MWt and an equivalent gross electrical output of 560 MWe. The primary plant utilizes a single reactor, two reactor coolant loops each of which employing a single steam generator and reactor coolant pump. Primary system nominal pressure is 2250 psia with a coolant flow
rate through the reactor of $68.2 \times 10^6 \text{ lb/hr (178,000 gpm)}$.

Average moderator temperature in the core is $569 \, ^\circ\text{F}$ with an average rise in the core of $66.4 \, ^\circ\text{F}$ at nominal power. The fuel is comprised of uranium-235 enriched to approximately $3\%$ at the beginning of core life in the form of sintered $\text{UO}_2$ and is fabricated into 21,659 fuel rods cladded with zircaloy. The equivalent core diameter is 96.5 inches while the core height of the active fuel region is 144 inches. There are 528 rod cluster control assemblies (RCCA) utilizing a cadmium-indium-silver neutron absorber material cladded with Type 304 stainless steel. Light water ($\text{H}_2\text{O}$) is used as the coolant, moderator, and reflector.

Automatic and shutdown control is by means of inherent stability through the temperature coefficient of reactivity, the RCCA’s and by a boron feed and bleed process. A pressurizer designed to provide a surge volume and a means of automatic pressure control is connected to the hot leg upstream of one loop’s steam generator. Hence, the Kewaunee Plant provides a suitable means of comparison for the digital plant model.

Three accidents were analyzed by this model and compared with the Kewaunee safety analysis transient curves. An uncontrolled RCCA withdrawal at power resulting in both a rapid $(8.2 \times 10^{-4}\Delta\rho/\text{sec})$ and slow $(0.3 \times 10^{-4}\Delta\rho/\text{sec})$ reactivity variation with time was first considered. Then an excessive load increase accident was evaluated.

The uncontrolled RCCA withdrawal at power with a reactivity
insertion rate of $8.2 \times 10^{-4} \Delta \rho/$sec results in an immediate response in the fission rate density throughout the core with an attendant increase in core heat flux. Assuming the steam generator pressure does not reach a relief or safety valve setpoint, the heat extracted from the steam generator remains constant and as a result a net increase in the reactor coolant temperature is seen. The protection analysis for this plant responds by actuating a reactor trip (scram), which is a gravity insertion of the RCCA's, rapidly shutting down the reactor and preventing damage to the core from boiling in the coolant. This trip occurs approximately 1.2 seconds after the start of the accident.

The safety analysis response curves for reactor power and average moderator temperature provided in the Final Safety Analysis Report of the Kewaunee Plant offer a convenient means by which a model comparison may be made during the interval prior to the reactor trip. Figure 8 illustrates resulting curves and the initial values which have been conservatively chosen by the design study and which are 102% for reactor power and 571.3 °F for the average moderator temperature. It is observed that significant agreement exists between the computer model and the safety analysis even within the relatively small parameter bands chosen for illustration.17

A second uncontrolled RCCA withdrawal at power with a much smaller reactivity insertion rate of $0.3 \times 10^{-4} \Delta \rho/$sec and a significantly longer time transient has been chosen for comparison.
Fig. 8. Uncontrolled Rod Withdrawal ($8.2 \times 10^{-4} \Delta \rho / \text{sec}$)
The protection system of the Kewaunee Plant initiates a reactor trip on overtemperature $\Delta T$ after approximately $44$ seconds. Results of the computer simulation and the Kewaunee safety analysis are shown in Figure 9. The maximum errors between the model and the safety analysis observed during the time interval are $1.4\%$ in reactor power and $3.5\,^\circ F$ in average moderator temperature, both of which are certainly acceptable results considering the fact that spatial effects within the three-region core have not been considered and that mixing within plenum volumes were considered to be $100\%$.\(^{18}\)

A third accident, an excessive load increase, involves the rapid increase in steam generator steam flow that causes a power mismatch between the reactor core power and the steam generator load demand. The Kewaunee safety analysis uses a detailed digital simulation of the plant including core kinetics, reactor coolant system, and the steam and feedwater systems in order to show proper safeguards against possible reactor core damage and adverse environmental impact. An analysis of this accident was conducted using this computer model with the same initial conditions of average moderator temperature of $571.3\,^\circ F$, reactor power of $102\%$ of nominal power ($1650$ MWt), and with a temperature coefficient of reactivity of $-4.0 \times 10^{-4}/^\circ F$. Results of both this model and the Kewaunee analysis are shown in Figure 10.\(^{19}\)

It is noted that the difference between peak reactor power values is approximately $1.7\%$ while the initial relative minimum
Fig. 9. Uncontrolled Rod Withdrawal ($0.3 \times 10^{-4}\Delta \rho /\text{sec}$)
Fig. 10. Excessive Load Increase
and final steady state values of the average moderator temperature differ by about 1.4°F. In both cases the transient exists for 130 seconds and experiences similar times to peaks and dips in both reactor power and temperature. Therefore, with a simpler, less time-consuming and costly program, acceptable results are obtained. The intention here is not to offer a digital modelling technique to supplant the very extensive and sophisticated digital simulation used in the safety analyses, but rather to provide a program which could be utilized, for example, to conduct several inexpensive preliminary studies to investigate the effects of varying parameters associated with plant components and reactor kinetics on overall plant control.

Based on the generally accepted development of the system of primary plant differential equations, the well-recognized use of finite difference approximations to facilitate the use of digital methods, and the general agreement of the model predictions with the safety analysis results of an actual thermal pressurized water reactor power plant, it is concluded that the model be considered valid.
VI. APPLICATIONS

Now that the digital program has been developed, made reasonably versatile for application to essentially any thermal pressurized water reactor plant, and has demonstrated that it can be used with some degree of confidence up to and including rapid plant transients, its utility as a design and training tool will be discussed.

To provide several examples of how the program might be used in a parametric design analysis, three commonly encountered plant evolutions will be examined. Although many parameters such as loop size, vessel plenum volumes, heat transfer coefficients, effective delayed neutron fraction, and effective neutron lifetime, could be varied, it would be especially instructive to investigate the effects of varying the parameter which is primarily responsible for the negative feedback in the system. The temperature coefficient of reactivity, therefore, is the parameter which accounts for the inherent stability, the capability of automatic reactor power control by steam demand in most plants, and the essential factor in reducing the severity of many overpower accidents.$^{20}$

The first transient that will be considered is one whereby with the reactor initially critical with the power level in steady state and in the power range at 20% of nominal power, a ramp increase of the secondary steam load is made at one percent per
second, to a final value of 80% of nominal power. Eleven values of the temperature coefficient ranging from $+2.0 \times 10^{-4}/^\circ F$ through zero to $-15.0 \times 10^{-4}/^\circ F$ were used for an imaginary plant having representative time constants. As expected, for a positive temperature coefficient which presents a positive feedback mechanism, the program very quickly reflected these regenerative effects and printed out progressively decreasing average moderator temperature and reactor power. Also, for a zero value of the coefficient, reactor power did not change at all with the increasing steam demand, although $T_{av}$ along with other plant temperatures continuously fell indicating that the coolant was accommodating the increase in secondary load by a loss of its internal energy.

Depicted in Figure 11 are the response curves for three values of the coefficient along with the attendant steam demand. The final steady state values of reactor power and steam demand were found to equal 80% of nominal power, while the average moderator temperature returned to its original value with the reactor left critical once again. It is noted that the transient associated with the smallest magnitude of the coefficient produces a sluggish response with a relatively large overshoot and subsequent undershoot. A preliminary plant study such as this indicating an overshoot of more than 40% over nominal power would suggest that an overpower trip would result due to an imprudent choice for the coefficient's value, and that redesign would most probably be in
Steam Demand

- Reactor Power, $\alpha_T = -0.0005/^\circ F$
- Reactor Power, $\alpha_T = -0.0002/^\circ F$
- Reactor Power, $\alpha_T = -0.0005/^\circ F$

Fig. 11. Ramp Increase in Steam Demand
A design plot of the power overshoot versus $|\alpha_T|$ such as the one represented by Figure 12 generated by running the program for several values of the coefficient, would be beneficial in selecting an optimum value for the coefficient with other design constraints factored into the decision process.

The second transient considers, again, a reactor initially critical, in steady state, with the power level in the power range, but now at 80% of nominal power. A ramp reduction of the secondary steam demand is made at one percent per second to a final value of 20% of nominal power. A full range of eleven values of the coefficient were again used for the non-specific design. Positive values of the temperature coefficient showed plant instability while a zero value resulted in no change in reactor power level but continuously increasing core and loop temperatures due to the decrease in secondary demand.

Figure 13 provides plots of power versus time for steam demand and reactor power for three different feedback conditions. Observed again is the sluggishness of the response associated with the small magnitude of the coefficient. Of interest is a comparison between the response curves for $\alpha_T = -0.0002/\text{°F}$. For the downpower case reactor power drops more slowly than it rises in the uppower situation due to the more gradual production of neutrons from the longer-lived delayed neutron precursor decay preventing the fission rate density from falling more rapidly. In the uppower case, the shorter-lived precursors seek the higher equilibrium values much
Fig. 12. Power Overshoot for Ramp Increase in Steam Demand
Steam Demand
Reactor Power, $\alpha_T = -0.00005/\degree F$
Reactor Power, $\alpha_T = -0.0002/\degree F$
Reactor Power, $\alpha_T = -0.0005/\degree F$

Fig. 13. Ramp Decrease in Steam Demand
more rapidly and are in a greater relative abundance than the longer-lived ones referenced to a steady state condition, therefore having a greater effect on the fission rate density. The final steady state value of reactor power and steam demand is 20\% of nominal power while the average moderator temperature returns to its original value prior to the steam transient for all negative values of the temperature coefficient. Figure 14 provides a plot of the power undershoot versus $|\alpha_T|$ and displays a smaller degree of power undershooting when contrasted with the uppower case of Figure 12. Although the undershoot is smaller than the uppower overshoot, the duration of the power mismatch is much longer for the downpower evolution.

A ten second control rod withdrawal from a critical, steady state condition in the power range with initial power level at 50\% of nominal power is the third transient to be used to illustrate the application of the plant model. The rate of reactivity insertion due to the movement of control rods during the ten seconds is $+2.0 \times 10^{-4} \Delta \rho$/sec and is representative of the product of differential rod worth and control rod bank speed for many pressurized water reactors.

Figure 15 represents a plot of steam demand and reactor power versus time for three values of the temperature coefficient. Were the protection system of a reactor plant to be such that a reactor shutdown trip occurred whenever the power level exceeded 100\% of nominal power, the selection of a temperature coefficient equal to
Fig. 14. Power Undershoot for Ramp Decrease in Steam Demand
Steam Demand
Reactors Power, $a_T = -0.00005/°F$
Reactors Power, $a_T = -0.0002/°F$
Reactors Power, $a_T = -0.0005/°F$

Fig. 15. Ten Second Rod Withdrawal
-0.00005/°F obviously would be a poor choice as this would result in a shutdown. It is seen from the figure that a smaller magnitude of the coefficient yields a more sluggish response with a larger overshoot and undershoot. Steady state was essentially established in 76 seconds with $\alpha_T = -0.0005/°F$ whereas 300 seconds were required with $\alpha_T = -0.00005/°F$. The average moderator temperatures at equilibrium were 4.0 °F and 40 °F higher, respectively, than prior to the rod shim. If the protection system initiates a shutdown from an overtemperature condition, this would contribute to the disadvantages of designing a plant with an inadequately small magnitude for the temperature coefficient of reactivity.

The fourth transient considers a ten second control rod drivedown from a critical, steady state condition in the power range with initial power level at 50% of nominal power. The reactivity insertion rate due to control rods is $-2.0 \times 10^{-4} \Delta \rho/\text{sec}$ during the ten seconds. Figure 16 provides plots of steam demand and reactor power versus time for three different values of the temperature coefficient of reactivity. A slower response with a larger overshoot and undershoot results for the smallest magnitude of the coefficient as seen in the previous three transients. It is observed that the degree of undershoot for the drivedown is smaller than the degree of overshoot for the control rod withdrawal case, again due to the action of delayed neutron precursor decay.

A final transient is presented to illustrate the reactor kinetic response to a rapidly varying fission product poison
Steam Demand

Reactor Power, $\alpha_T = -0.0005/\text{F}$

Reactor Power, $\alpha_T = -0.0002/\text{F}$

Reactor Power, $\alpha_T = -0.0005/\text{F}$

Fig. 16. Ten Second Rod Drivedown
concentration. The fission product poison which most greatly affects the dynamic behavior of the reactor is xenon-135. This isotope will undergo its greatest variation in concentration immediately after a reactor startup to 100% of nominal power commenced approximately ten hours following a shutdown from steady state conditions at full power late in core life. With the startup occurring at this point in time, the xenon-135 poison will experience its greatest rate of burnout due to the absorption of thermal neutrons consequently decreasing its concentration. By using the isotopic balance equation for xenon-135 under these conditions and using typical fission cross-section, flux and physical constant values, the reactivity insertion rate which results is \( +0.122 \times 10^{-4} \Delta \rho / \text{sec} \). The reactor power transient for three values of the temperature coefficient is shown in Figure 17. This addition of positive reactivity with time effects a plant heatup due to the power mismatch between steam demand and reactor power if control rods are not repositioned to offset this reactivity change. These heatup rates were found to be 14.16 °F/minute, 3.66 °F/minute, and 1.46 °F/minute for \( \alpha_T = -0.00005/\text{°F} \), \( \alpha_T = -0.0002/\text{°F} \), and \( \alpha_T = -0.0005/\text{°F} \), respectively. Many pressurized water reactor plants have heatup limitations due to the thermal stresses developed in the reactor pressure vessel. Hence, the relatively large heatup rate of 14.64 °F/minute might prove to be unacceptable in preliminary design analyses. An additional problem seen here is the large power overshoot of 11.4% over nominal power, a condition
Fig. 17. Xenon Burnout on Startup
which could conceivably lead to a reactor shutdown due to overpower.

The computer program may find specific application in several areas of interest. Preliminary plant design investigations could utilize the program in the areas of routine plant evolutions, protective function simulation, and plausible accident studies. Routine evolutions might involve reactor startups and shutdowns, chemical poisoning, uppower and downpower steam transients, control rod and chemical shims, and various plant tests. Protective functions such as reactor scrams, partial rod insertions, and chemical injection shutdowns could be simulated successfully. A number of conceivable plant accidents could be evaluated including uncontrolled rod withdrawal from a subcritical condition, rod bank misalignment, chemical control system malfunction, steam line ruptures, and excessive or loss of feedwater system heat removal. Much more detailed and expensive studies using sophisticated programs could then follow in order that plant protective features could be shown to permit safe operation for licensing purposes.

Parametric studies might be made with the use of this program in the design stage of plant development to provide data upon which component and system sizing could be estimated prior to a final determination using much more costly digital techniques. The computer model would be useful as a personnel training device in the areas of nuclear reactor theory and plant operations by providing an immediately accessible and inexpensive program which could simulate an entire series of plant evolutions for the
instruction of plant operators and supervisors. Although present state-of-the-art hand-held programmable calculators do not have quite the programming and storage capacity for such a program, with only moderately improved memory capabilities they will soon be suitable for accommodating this digital analysis. Liquid sodium cooled reactor plants are certainly candidates for application of this plant model and program.

In general, the design analysis and follow-on personnel training associated with thermal pressurized water reactor power plants can be undertaken more economically and with perhaps a better appreciation for the dynamic behavior of the plant.
VII. CONCLUSIONS AND RECOMMENDATIONS

The proposed objectives of Chapter I have been satisfied in that a computer program has been developed to simulate the kinetic response of a thermal pressurized water reactor power plant. The validity of the model has been verified by comparison with past efforts of other investigators and by comparison with safety analysis studies of a licensed nuclear power generating plant. Typically encountered primary plant transients have been discussed with the aid of the model program. Predictions of many other plant evolutions may now be made with some degree of confidence in a wide variety of applications.

The versatility of the computer program may be extended depending on the specific accuracy requirements and economical constraints. Recommended are the following modifications and improvements which will provide increased accuracy in future work:

1. Pressure effects on reactor power through the pressure coefficient of reactivity should be incorporated. A dynamic model of the pressurizer with automatic control schemes would be required.

2. The functional dependence of the temperature coefficient of reactivity with the average moderator temperature should be employed as a subroutine.

3. The fuel (doppler) coefficient of reactivity could be
used where fuel temperatures have a significant effect on power plant kinetics.

4. The capability of multiflow conditions via coolant pump speed changes could be provided.

5. The provision for multiloop analysis with appropriate reactor inlet plenum mixing effects accounted for should be made.

6. Radioactive decay heating effects as an addition to the immediate fission heating should be considered to better approximate the thermal output of the reactor core. Where significant, the effects of coolant pump heat and ambient losses should be factored into the loop analysis.

7. A major modification would be the development and use of spatial power density distributional effects coupled with core interpass coolant flow. Channel and fuel plate temperature variations due to boiling could then be integrated into the model.
APPENDIX

SOURCE LISTING AND SAMPLE OUTPUT
THermal Pressurized Water Reactor Power Plant Computer Program

Steam Demand Change 20 K-BAK, Temperature Coefficient = 0.002/DEG F

Input Section

READ(5,*)UR,PM,TH,CM,TD,RE,TF,CF,VR,VST,VSD
READ(5,*)DVC,VRT,VR0,MC,CC,US,AS,TA,TSG,DEL,T,SH,CS
READ(5,*)AT,FT,TR
READ(5,*)DT,IP,D3,DA,D4,DA,CON4,CON2,CON3
READ(5,*)XT,TP,TS,TP,TF,TP
READ(5,*)DOS,DDS,DRR,DXF,DUR,D
READ(5,*)S
READ(5,*)TSC,TSC,TSC,TSC,TSC,TSC,TSC,TSC,TSC,TSC,TSC
IDRT,DR,RC1,DC2,DC3,DC4,DC5,DC6,DC1,DC2,DC3

Compute Initial Plant Parameters and Temperature Distribution

$1=2*UR/(RH*TM*CM)+2/TH+2*UR/(RF*TF*CF)
$2=4*UR/(RF*TF*CF*TH)
S3=1-4/TH
S4=2
S5=2*UR/(RH*TM*CM*RF*CF*VR)*CON2
S6=DVC/VRT
S7=DVC/VR0
S8=(DNC*CC/(US*AS)-0.5)/(DNC*CC/(US*AS)+0.5)
S9=(DNC*CC/(SM*CS))/(DNC*CC/(US*AS)+0.5)
S10=1/(1/(2*DNC*CC)+1/(US*AS))/((SM*CS)
S11=1/(1/(SM*CS))/(DNC*CC/(US*AS)+0.5))*CON2
S12=DVC/VST
S13=DVC/VSO
A1=0.0297&R
A2=0.21905&R
A3=1.0995&R
A4=3.95&R
A5=11.5&R
A6=0.04203&R
C1=0.05&R/N
C2=R20&R/N
C3=R3&R/N
C4=R4&R/N
C5=0.5&R/N
C6=R6&R/N
TSC=TA-VDEL*TP*OR/T
TSC0=TSC
TRC1=TSC
TC=TSO
TH=TAV+DFLT\$\&\%R/
TRH0=TH
TSH1=TH
TSH2=TH
TSH01=TSO
TSH02=TSO
TSH03=TSO
TSH04=TSO
TSH05=TSO
TSH06=TSO
TSH07=TSO
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TRH08=TRH0
TRH09=TRH0
TRH10=TRH0
TRH11=TRH0
TRH12=TRH0
TRH13=TRH0
TRH14=TRH0
TRH15=TRH0
TRH16=TRH0
TRH17=TRH0
TRH18=TRH0
TRH19=TRH0
TRH20=TRH0
WRITE (6,1) UP\$\&\%R,TH,C4,TH,EF,CF,UF,VT,USN,
10VC,VR1,VR0,OMC,CC,US,AS,TAV,TSO,DFLT,TH,STCS,
1AT,ET,EX,TI,TP,TS,TP,TXF,TO05,OO05,OR70,00XF,004F
1S1•S2•S3•S4•S5•S6•S7•S8•S9•S10•S11•S12•S13.
1R1•R2•R3•R4•R5•R6•C1•C2•C3•C4•C5•C6.

Determine programmed reactivity and steam changes

C C C

3 I=0
5 T=T+XT
10 IF(I-TS)10,10,20
20 DQS=DO5+DQS*XT
30 IF(T-TR)40,40,30
40 IF(T-TXE)40,40,50
50 NXE=0
60 IF(T-TP)70,70,1000

Solve cold leg temperature equations

70 TSC=TSC+XT#TSC+XT#TSC+(XT#TSC)NMTC
DTSCA=TSC
DTSCA=S1#TSH#S2#TSC-S1#TSC-S1#TSC
DTSCA(DTSC-DTSCA)/XT
TSC0=TSC0+XT#TSC0+(XT#TSC0)NMTC
DTSC0=D1#TSC0
DTSC0=S1#(TSC-TSC0)
DTSC0=(DTSC0-DTSC0)/XT
TSC0=TSC0
TSC0=TSC0
TSC0=TSC0
TSC0=TSC0
TSC0=TSC0
TSC0=TSC0
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TSC0=TSC0

TC=TC+XT#TSC+(XT#TSC0)NMTC
\[ DTC = NTC \]
\[ DTC = S + (TPCI - TC) \]
\[ DNTC = (\text{DTC} - \text{NTCA})/\pi \]
\[ TAB = (TC+TH)/\pi \]

**EVALUATE CORE REACTIVITY**

\[ NTA = (\text{DTC} + \text{NTH})/2 \]
\[ DRT = A + T \times A \]
\[ DRT = A + NRT + NDXF \]
\[ A = A + \pi * XT \]

**SOLVE PRECURSOR EQUATIONS**

\[ C1 = C1 + XT \times NC1 + (XT*\pi^2/2) \times NNC1 \]
\[ C2 = C2 + XT \times NC2 + (XT*\pi^2/2) \times NNC2 \]
\[ C3 = C3 + XT \times NC3 + (XT*\pi^2/2) \times NNC3 \]
\[ C4 = C4 + XT \times NC4 + (XT*\pi^2/2) \times NNC4 \]
\[ C5 = C5 + XT \times NC5 + (XT*\pi^2/2) \times NNC5 \]
\[ C6 = C6 + XT \times NC6 + (XT*\pi^2/2) \times NNC6 \]
\[ DC1 = C1 \times R \times N1 \times C1 \]
\[ DC2 = C2 \times R \times N2 \times C2 \]
\[ DC3 = C3 \times R \times N3 \times C3 \]
\[ DC4 = C4 \times R \times N4 \times C4 \]
\[ DC5 = C5 \times R \times N5 \times C5 \]
\[ DC6 = C6 \times R \times N6 \times C6 \]
\[ DC1 = (DC1 - DC1A)/XT \]
\[ DC2 = (DC2 - DC2A)/XT \]
\[ DC3 = (DC3 - DC3A)/XT \]
\[ DC4 = (DC4 - DC4A)/XT \]
\[ DC5 = (DC5 - DC5A)/XT \]
\[ DC6 = (DC6 - DC6A)/XT \]

**COMPUTE FISSION RATE DENSITY TERMS**

\[ FL = ET \times (1 - R) \]
\[ P = P \times XT \times N + (XT*\pi^2/2) \times NNP \]
\[ NP = A \]
\[ S = N1 \times C1 + N2 \times C2 + N3 \times C3 + N4 \times C4 + N5 \times C5 + N6 \times C6 \]
\[ R = N \times P \]
\[ DEFF = SNC / (C1 + C2 + C3 + C4 + C5 + C6) \]
\[ DP = SNC / FL = (1 - P) \times FL \times P \times S / FL \]
\[ \text{NR} = \left( \frac{\text{NP} - \text{NPQ}}{\text{XT}} \right) \]
\[ \text{RLD} = 26.05747 \times \text{ND/P} \]

**CALCULATE REFRACTIVE POWER**

\[ \text{NPQ} = \frac{\text{P} \times \text{VP} \times \text{CON3}}{} \]
\[ \text{NPQ} = 2.95074 \times \text{CON3} \times \text{DPA} \]
\[ \text{NPQ} = 2.95074 \times \text{DP10} \times \text{DPA} \]

**SOLVE HOT LEG TEMPERATURE EQUATIONS**

\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
\[ \text{TH} = \text{TH} + \text{TM} + \text{XT} \times (\text{XT} + 2) \times \text{DTH} \]
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FOOTNOTES


3John R. Lamarsh, Nuclear Reactor Theory (Reading, MA.: Addison-Wesley, 1972), pp. 84-99.


9Ibid., p. 662.


12Schultz, Control of Nuclear Reactors and Power Plants, pp. 137-153.

13Glasstone, Nuclear Reactor Engineering, pp. 302-304.


18. Ibid.


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