An Action Research Study Involving Fifth-grade Students Learning Fractions Through A Situative Perspective With Story Problems

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AN ACTION RESEARCH STUDY INVOLVING FIFTH-GRADE STUDENTS
LEARNING FRACTIONS THROUGH A SITUATIVE PERSPECTIVE WITH
STORY PROBLEMS

by

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B.S. University of Central Florida, 1997

A thesis submitted in partial fulfillment of the requirements
for the degree of Master of Education
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ABSTRACT

The purpose of this action research study was to investigate the affects of teaching through a situative perspective with story problems on students’ understanding of fraction concepts and operations in my fifth-grade mathematics classroom. Students participated in twelve weeks of instruction. Data was collected in the form of pre and post tests, audiotaped and videotaped recordings of instructional sessions, and student work samples.

Data analysis revealed that my students constructed their own knowledge about various fraction concepts and operations because students engaged in discussions, after solving story problems, that developed, extended and restructured their knowledge. One example of this occurred after students had solved an equal-sharing problem. Two students came up with different answers and another student explained why both answers were equivalent. Student work samples and post test results indicated that the one student’s explanation was understood, adopted and extended by all the students in my class.

The data also revealed that students’ pictures typically represented the context and action of the story problems. For example, subtraction problems dealing with length were usually represented by number lines or horizontal rectangles with crossed-out markings to show the subtraction operation.

Throughout this research study, I discovered that my students were capable of learning from each other and solving problems for which they have no preconceived algorithm. I also learned that analyzing students’ work and listening to their discussions in ways that focused on their thinking, not their answers, provided me with information about what my students were grasping and not grasping.
This thesis is dedicated to God for through Him all things are possible; to my husband, Doug for praying for me, believing in me, and being my best friend; to my chairperson, Dr. Juli Dixon for guiding me and being an inspiration to me; to my committee for supporting me; to my wonderful students and parents for making this experience enjoyable and enlightening; to Lauren, for all the blonde moments; to Sammy for always knowing when I needed a break; and to my family and friends for supporting and encouraging me in all my endeavors, big and small.
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CHAPTER 1: INTRODUCTION

From a young age, children are interested in mathematical ideas. Through their experiences in everyday life, they gradually develop a rather complex set of informal ideas about numbers…and size, and many of these ideas are correct and robust… Students’ understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge… The kinds of experiences teachers provide clearly play a major role in determining the extent and quality of students’ learning (NCTM, 2000, p. 21).

Research suggests that students come to instruction with a bank of informal knowledge about fractions that they can access to solve story problems (Empson, 1999; Mack, 1990; 1995) and that “initial work with fractions should be based on the natural activities children experience with sharing” (Hatfield, Edwards, Bitter & Morrow, 2005, p. 267). Research also suggests that the teacher’s selection of tasks and how he or she guides interactions around tasks is central to the development of thinking. Students develop and deepen their knowledge through the social interactions and discussions that take place during instruction and teachers need to take into account this “sociocognitive or situative perspective” (Empson, 1999).

The practice of developing students’ conceptual understanding of fractions through a situative perspective with story problems provides a “potentially rich basis for fraction understanding that draws on children’s informally acquired resources and, through instruction, motivates those resources toward more sophisticated mathematical understanding” (Empson, 1999, p.286). Students are able to work like real mathematicians in that they are not limited to certain methods or procedures for finding solutions. Some students may draw pictures, others use mental mathematics, and some may use manipulatives. During and afterwards, teachers and students engage in discussions about various solution strategies and continually try to make sense of and interpret what each person said or did. Students begin to realize that their thinking
process is just as important as their answer. Using a situative perspective with story problems to teach fractions was the method I employed to foster and develop my students’ understanding of fractions concepts and operations.

Rationale

Last year, I was asked to write a reflection from a chapter in Liping Ma’s *Knowing and Teaching Elementary Mathematics* book. The chapter reported the results of her interviews with numerous American and Chinese teachers. One of the questions Ma (1999) asked was the following:

Imagine that you are teaching division with fractions. To make this meaningful for kids, something that many teachers try to do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story-problems to show the application of some particular piece of content. What would you say would be a good story or model for one and three-fourths divided by one-half? (p. 55).

Before reading the rest of the chapter, I decided to test my own understanding of division of fractions by coming up with a story or model of the question posed. I remember mentally computing the answer of three and one-half first, then taking a moment to picture what that would look like with a candy bar or rope. After several long minutes passed, I still had no idea how to manipulate the materials in my head so that I could start out with less and end up with more. I searched my memory for an example that I had used in class or seen in one of the textbooks, but came up empty-handed. Then I tried to remember an example that one of my mathematics teachers would have used during a class demonstration, but soon remembered that I was not taught using story problems or manipulatives. At that moment, I realized that I was teaching fractions in the same way that I had been taught. I was teaching my students how to manipulate the symbols in order to perform an algorithm for which they probably did not
understand. Kieren (1988) states that “premature formalism in rational number work leads to a person having technical symbolic knowledge which cannot be connected recursively to real situations” (p. 178). In other words, my knowledge of rote procedures was a stumbling block that kept me from really utilizing my understanding of fraction concepts and operations. Instead of helping me think logically and reason my way to a solution, the mathematics I learned stifled me because I put too much emphasis and confidence in an algorithm that I had memorized, but never really thought about. I will never forget that moment of realization, because it taught me that learning procedures without understanding both the rationale behind the procedure and a real-world situation that could be solved using the procedure was meaningless.

Ma’s question sparked in me the desire and determination to be a better teacher by helping my students understand the mathematics concepts and procedures they learn, and develop the ability to reason and use logic to solve problems. Ma’s question made me realize that connecting mathematics to real-life situations was crucial to understanding, and furthermore, that being able to reason and use logic to solve problems demonstrated a deeper understanding than being able to use a preconceived algorithm.

Since reading Ma’s book, I have embarked on a journey to make mathematics meaningful and relevant to my students’ lives and to teach for understanding instead of memorization. Recent studies (Empson, 1999; Huinker, 2002; 1998; Lappan & Bouck, 1998; Mack, 1998; & Sharp, Garofalo & Adams, 2002) indicate that using a situative perspective with story problems can help make fraction concepts and operations more meaningful to students. The research (Empson, 1999; Huinker, 2002; 1998; Lappan & Bouck, 1998; Mack, 1998; & Sharp, Garofalo & Adams, 2002) suggests that instruction geared toward giving students opportunities to solve and discuss real-world problems aids in the development of fraction operation sense which lays a
solid foundation for future work. Researchers (Empson, 1999; Huinker, 2002; 1998; & Sharp, Garofalo & Adams, 2002) have also found that many students are able to generate meaningful and flexible solution strategies and algorithms for fraction computations and problem-solving activities when given real-life problems to solve and discuss.

At the end of one study in which a situative perspective with story problems was employed, one teacher commented that her first-graders understood fractions better than the fifth-graders she had previously taught because instruction in the study was not geared toward learning symbolic procedures (Empson, 1995). “Since the first graders were not trying to remember procedures that had been taught, they had to use conceptual knowledge to devise solutions” (p.112). Surprisingly, many of the first graders in the study were able to solve addition-and-subtraction-of-fractions problems by using the strategies that made sense to them based on their fundamental understanding of fractions that was developed through class discussions and interactions.

As a result of both my preliminary findings and desire to make mathematics more meaningful for students, the intention of my study was to use a situative perspective with story problems during a fraction unit of to determine its affects on my students’ abilities to understand, develop and work fluently with fraction concepts and operations.

**Purpose of the Study**

The purpose of this study was to investigate the affects of teaching fraction concepts and operations using a situative perspective with story problems on my students’ abilities to understand fraction concepts and perform fraction operations.
Research Question #1

In what ways does teaching fractions through a situative perspective with story problems affect my students’ understanding of fraction concepts?

Research Question #2

In what ways does teaching fractions through a situative perspective with story problems affect my students’ understanding of fraction operations?

Definitions

- Concepts: For the purpose of this study, concepts include equivalence, partitioning, parts of a whole, equal parts and equal sharing.
- Operations: For the purposes of this study, operations include addition and subtraction.
- Rational Numbers: For the purposes of this study, rational numbers include fractions and mixed numbers.
- Symbols: “Entities that stand for or take the place of something else… standard written marks that stand for quantities and operations on quantities” (Hiebert, 1988, p. 334).
- Informal Knowledge: The term “informal knowledge” (Mack, 1990) has also been referred to as “prior knowledge” (Saxe, 1988), “intuitive knowledge” (Leinhardt, 1988), or “situated knowledge” (Brown et al., 1989). Informal knowledge is characterized by Leinhardt (1988)
as “applied, real-life, circumstantial knowledge, which can be either correct or incorrect” (p.120).

- Situative perspective: A situative perspective is a point of view that sees learning as a “socially organized activity” (Empson, 1999) and that social interaction is the way that learning takes place (Packer, 1993, p.262). The social interactions and organized activities include class discussions, student-to-student and student-to-teacher discussions, and activities that emerge from the discussions.

- Story problems: Story problems are word problems. Story problems include equal sharing situations.

**Significance of Study**

Rational number concepts are some of the most important mathematical ideas that students will face prior to entering secondary school (Behr, Wachsmuth, Post & Lesh, 1984; Charles & Nason, 2000). Yet, national assessment reports and research studies reveal that many students lack a conceptual understanding of fraction symbols and procedures (Behr, Wachsmuth, & Post, 1985; Behr et al., 1984; Mack, 1995; Mack, 1990; Post, 1981). A considerable body of literature indicates that many students do not understand the meaning underlying the symbolic representations and perform operations with little understanding (Behr, Wachsmuth, & Post, 1985; Behr et al., 1984; Mack, 1995; 1990; Post, 1981). “Students often do not consider numerators and denominators in relation to one another, but, rather, handle them as separate entities to be operated on independently” (Behr et al., 1984, p. 324). In addition, many students do not appear to possess a quantitative notion of rational numbers (Behr et al., 1984).
My review of research studies (Behr et al., 1984; Charles & Nason, 2000; Empson, 1999; Mack, 1990; & Pothier & Sawada, 1983) conducted on learning rational numbers has shown me that a variety of topics have been examined in an attempt to uncover the problem and resolve it. Most research that has taken place over the last twenty years has focused on models of cognition and products of understanding (Behr et al., 1984; Charles & Nason, 2000; Mack, 1990; Pothier & Sawada, 1983). In many cases, individual children were interviewed and observed solving rational number tasks. Empson (1999) concluded that “a major goal of these studies has been to identify and elucidate the relations between tasks and children’s ways of thinking about them in settings designed to minimize assistance and bring individual children’s ways of thinking to the fore” (p.3). These studies have yielded important insights, but they do not “take into account the socially organized ways of thinking in which children participate during instruction” (Empson, 1999; p. 1). Most of the data collected in those studies came from observing and interviewing individual students outside the regular classroom setting. As valuable as those studies were, I needed to find a methodology that would be suitable for studying individual student’s fraction understanding within the normal classroom setting and the effects of the classroom setting on students’ understanding. The situative perspective seemed to be the appropriate model for my study.

A burgeoning number of cognitive psychologists are calling for studies that document “students’ mathematical development in the social context of a classroom” (Bowers, 1999, p. 1). Romberg and Carpenter (1986) called for dynamic research designs that could obtain “the way meaning is constructed in classroom settings on specific mathematical tasks” (p. 868). These dynamic models needed to consider the “socially organized processes” that engage students and shape their thinking (Empson, 1999, p. 2). Over the last decade, several researchers (Bowers,
have conducted studies that focus on learning from this “situative point of view” (Empson, 1999, p. 1). Empson (1999) stated that:

A situative perspective on learning suggests that the structures and processes of understanding are initially visible in the structures and processes of socially organized activity. If this is the case, then we should see more mathematically sophisticated understanding, such as that involved in fraction knowledge, emerge first in the joint coordination of distributed resources in classroom interactions. Individually, children would show little prior evidence of such understanding. One would also expect that the resources learners contribute to joint activity would be extended, enhanced, restructured, and recontextualized in conjunction with socially provided structures and processes to result in greater fraction knowledge (p. 6).

Empson (1999) documented the ways that first-grade students’ discussions with each other and the teacher advanced the students’ fraction understanding. Empson collaborated with the first-grade teacher assigned to the class during the teacher’s weekly planning sessions, and together they decided and designed problem-solving centers and activities. The first-grade teacher had been part of the Cognitively Guided Instruction (CGI) study and was experienced at eliciting students’ mathematical thinking and basing instructional decisions on that thinking. After five weeks of study, Empson concluded that all of the seventeen students that participated in the study had furthered their fraction understanding and suggested that their growth was a result of “collective construction of key fraction structures and processes through classroom activity” (p. 6).

Empson’s study and methodology made me think that I could advance my students’ fraction thinking in similar ways. No other study had investigated the development of 5th grade students’ fraction understanding through a situative perspective with story problems. I wanted to discover if 5th grade students could have the same success as the 1st grade students. I was also curious about whether I could elicit students’ thinking and make instructional decisions based on that knowledge. Thus, the aim of my study was to investigate the effects of teaching fractions
using a situative perspective with story problems on my students’ understanding of fraction concepts and operations.
One of the most compelling issues facing mathematics teachers concerns how teachers can help students develop meaning for fraction concepts and operations they encounter in mathematics (Empson, 1995; Huinker, 1998; Mack, 1995). A considerable body of literature indicates that many students do not understand the meaning underlying the symbolic representations and perform operations with little understanding (Behr, Wachsmuth, & Post, 1985; Behr, Wachsmuth, Post & Lesh, 1984; Mack, 1995; 1990; Post, 1981). Many of the problems students grapple with in learning about fractions can be attributed to instructional practices that almost exclusively focus on memorizing rules and procedures for symbols that have no meaning to students and have no connection to students’ intuitive knowledge (Kerslake, 1986; Leinhardt, 1988; Mack, 1995; 1990).

Traditional mathematics instruction tends to ignore students’ bank of intuitive knowledge; thus students’ intuitive knowledge is not connected to the knowledge of formal mathematical procedures and symbols taught in school settings (Leinhardt, 1988, Mack, 1995; 1990). Students memorize rote procedures and are never encouraged to “impose meaning on the new material by attaching it to what is already known or altering what is already known” (Leinhardt, 1988, p. 122).

In the following section, I will discuss the consequences that result when students do not attach meaning to fraction concepts and operations and the teacher’s role in helping students construct their knowledge.
Assessing Students’ Fraction Knowledge

Rational number concepts are some of the most important mathematical ideas that students will face prior to entering secondary school (Behr, Wachsmuth, Post & Lesh, 1984; Charles & Nason, 2000). Yet, national assessment reports and research studies reveal that many students lack a conceptual understanding of fraction symbols and procedures (Behr, Wachsmuth, & Post, 1985; Behr, Wachsmuth, Post & Lesh, 1984; Mack, 1995; Mack, 1990; Post, 1981).

Results from one national assessment (Post, 1981) reveal that 30% of America’s 13-year-olds found the sum of $\frac{1}{2} + \frac{1}{3}$ by adding the numerators together and adding the denominators together. Results also show that more than 75% of America’s 13-year-olds failed to successfully estimate $\frac{12}{13} + \frac{7}{8}$ given the choices of 1, 2, 19, 21, and I don’t know. More than 50% of the 13-year-olds in the sample chose either 19 or 21 as their estimate. The students that answered 19 or 21 did so because they added the numerators or denominators to determine the solution. The students in both cases had no idea what the symbols for one half, one third, twelve thirteenths or seven eighths really meant, nor did the students have any idea how to add fractions (Post, 1981). The invalid procedures that students employed to solve the addition and estimation problems indicate a major gap in their fundamental understanding of fractions (Post, 1981).

Behr, Wachsmuth & Post (1985) also found that students lacked a conceptual understanding of fraction symbols and procedures. Behr, Wachsmuth & Post’s (1985) study was designed to measure fifth grade students’ understanding of fraction size. Instruction was developed to make extensive use of manipulative aids in hopes of helping students grasp the relative size of fraction symbols, concepts, and procedures. Students were interviewed individually during and after 20 weeks of instruction and asked questions pertaining to “order
and equivalence, translations within and between modes of representation, the concept of unit, the
quantitative notion of rational number, and the ability to apply rational number concepts in
problem situations” (Behr, Wachsmuth, Post & Lesh, 1984, p. 325). One type of interview
question students were asked to solve involved selecting two fractions that, when added together,
would be close to the whole number one, but not exactly one. Two students’ responses are given
below to illustrate the misconceptions that many students grappled with during and after
instruction.

One student named Ted chose to add 5/6 and 4/7 because he thought that 9/13 came close
to one whole. When Ted was asked to explain his selection, he stated that he added the
numerators to get 9 and the denominators to get 13. Like Ted, Jeannie added the numerators and
denominators to find her answer. She chose to add 6/7 and 3/1. Jeannie justified her answer by
stating that 9/8 was one more than a whole. Behr, Washsmuth, & Post (1985) wrote the
following:

Given the amount of instructional time, the degree of special attention, the extensive use
of manipulative aids, and the amount of time devoted to developing rational number
concepts, it is surprising that 20 of the 41 responses (49%) given by these students in the
middle of Grade 5- those responses in the categories of mental computation based on an
incorrect algorithm, gross estimate, and other- reflected a process of fraction addition that
was based on the incorrect algorithm of adding numerators and denominators or on some
other procedure reflecting little or no comprehension of fraction addition or rational
number size. (p. 128)

In other words, almost half of the students failed to demonstrate a conceptual framework
for fraction size, symbols, estimation and arithmetic. Most of the struggling students in both
studies thought of the numerators and denominators as separate entities, with no relation to one
another, and did not truly understand the fraction quantities (Behr, Wachsmuth & Post, 1985;
Post, 1981). Otherwise, students might have looked at 5/6 and thought of that fraction as close to
one whole, and seen $4/7$ as a little more than a half, and realized that adding those fractions would result in an answer close to one and one half.

In two separate studies, Mack (1990, 1995) also found that students were missing a conceptual understanding of fraction symbols and procedures. When students were asked to tell Mack which fraction was bigger, $1/8$ or $1/6$, most of the students answered $1/8$. Students justified their answers by stating that $8$ is bigger than $6$. In addition to incorrectly answering the previous question, students failed to identify and name fraction units depicted symbolically and pictorially (Mack, 1990). For example, the students were asked how much was shaded and were shown a picture of one whole circle shaded in and another circle equally partitioned into four parts, with one part shaded. Most students thought the answer was $5/8$, instead of one and one fourth (Mack, 1990).

The research (Behr, Wachsmuth, & Post, 1985; Behr, Wachsmuth, Post & Lesh, 1984; Mack, 1995; Mack, 1990; Post, 1981) suggests that students struggle to understand rational number concepts. Students in the mentioned studies failed to demonstrate a firm understanding of fraction symbols, quantities, arithmetic, and estimation (Behr, Wachsmuth, & Post, 1985; Mack, 1990; Post, 1981). Students often used invalid strategies to help them solve rational number questions, which in turn led them to incorrect answers. The research literature (Behr, Wachsmuth, & Post, 1985; Behr, Wachsmuth, Post & Lesh, 1984; Mack, 1995; Mack, 1990; Mack, 1990; Post, 1981) clearly demonstrates that students need to build firmer rational number foundations, especially in the area of fraction size and symbols.
Assessing Teachers’ Fraction Knowledge

However, school-aged pupils are not the only people that are struggling to understand rational number concepts. Research (Ma, 1999; NCTM, 2000; Stigler & Hiebert, 1999) suggests that a teacher’s understanding of the big ideas of mathematics and the relationships among the big ideas affects his or her decisions and classroom practices, and thus, students’ learning.

Liping Ma (1999) found that a sample of experienced United States (U.S.) teachers lacked a conceptual understanding of fraction symbols, concepts, and procedures. The following is a question posed by Ma (1999) to 23 U.S. teachers:

Imagine that you are teaching division with fractions. To make this meaningful for kids, something that many teachers try to do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story-problems to show the application of some particular piece of content. What would you say would be a good story or model for $1\frac{3}{4}$ divided by $\frac{1}{2}$? (pp. 55).

Twenty-one of the 23 U.S. teachers computed an answer in response to Ma’s question, and only nine (43%) gave a correct and complete answer of $3\frac{1}{2}$. The nine correct teachers gave the standard step-by-step procedure taught in school. Of the twelve incorrect teachers, two were not completely correct because they did not simplify $14/4$ and turn it back into a mixed number. Nine teachers gave responses that showed they either mixed together fragments of different fraction algorithms or they used completely incorrect algorithms. For example, one teacher explained the following strategy: I know to convert $1\frac{3}{4}$ to $7/4$, then you make the denominators the same, so $\frac{1}{2}$ equals $2/4$, and then you cross multiply $7/4$ and $2/4$ to get $28/8$. The remaining teachers looked at the problem for a while and then stated that they simply did not know how to calculate the answer.

In addition to incorrectly calculating the problem, many U.S. teachers also failed to successfully represent the division of fractions problem accurately. Only one of the 23 U.S.
Sixteen teachers provided word problems with misconceptions and 6 could not make-up a word problem. The teachers demonstrated a variety of misconceptions about the meaning of division by fractions.

Almost half of the U.S. teachers created stories that involved dividing equally between two or in half, which corresponds to dividing by 2, not ½. For example, when a person says they have 4 pies that will be evenly shared between two people or cut in half, the four is divided by 2, not ½. Many U.S. teachers in the study did not recognize this discrepancy. Six teachers confounded dividing by ½ with multiplying by ½, which revealed gaps in the teachers’ conceptions of multiplication with fractions. For example, if a person wants to know how much butter Johnny would get if he took 2/3 of two sticks of butter, the person would multiply 2 by 2/3 and get 1 1/3 sticks of butter. The problem above represents multiplication by fractions, not division by fractions. Two teachers seemed to think that dividing by ½, dividing by 2, and multiplying by ½ were all the same types of exercises. For example, Tr. Bernadette stated the following: “Dividing the one and three-fourths into the half…you would have all this whole, you would have the three fourths here. And then you want only half of the whole” (Ma, 1999, p.65). Notice that in her first statement, she divided the 1 ¾ into half, confusing division by ½ with division by 2; and in her very next sentence, Tr. Bernadette confused division by ½ with multiplication by ½. Two teachers were unable to create a word problem, but they did not confuse the division of ½ with something else. For example, Mr. Fred stated:

Dividing something by one half and so I confused myself with the two, thinking it meant dividing by two, but it doesn’t… It means something totally different… Well, for me what makes it difficult is not being able to envision it, what it represents in the real world. I can’t really think of what dividing by a half means (Ma, 1999, p.66).
The one teacher that created a conceptually correct word problem struggled to create a problem that was pedagogically correct because her solution resulted in $3 \frac{1}{2}$ students. In real life, a fraction of a child is not possible.

Ma’s (1999) research provides several insights and questions to consider. One insight is that even experienced educators who are considered experts do not appear to possess a thorough understanding of the mathematics curriculum taught in elementary school. Another insight is that even expert teachers do not appear to have ever attached the fraction symbols and procedures they encountered in elementary school to real-life concepts, otherwise, more teachers would have been able to correctly answer the division of fractions problem and come up with a story to represent it.

**A Situative Perspective with Story Problems**

“There has been a long standing frustration with both the teaching and learning about rational numbers” (Kieren, 1980, p. 69). Historically, learning and teaching have been viewed as two separate events (Hiebert & Wearne, 1988). However, recently the research perspectives have evolved and aligned themselves closer together. Researchers involved in the study of teaching understand that students’ cognitions throughout instruction need to be examined directly in order to truly understand the way instruction affects achievement (Empson, 1999; Hiebert & Wearne, 1988; Mack, 1990). “Students’ cognitive processes are viewed as mediators between the teaching processes and its products” (Hiebert & Wearne, 1988, p.105). Conversely, researchers studying learning also understand the importance of research designs that incorporate studying explicit instruction on students’ cognition and changes in cognition (Hiebert & Wearne, 1988). With the
advances in research designs and methodologies, much has been learned about students’
cognitive processes during instruction and how instruction affects students’ cognitive processes
(Empson, 1999; Mack, 1995; 1990).

These research studies have highlighted the benefits of a type of instructional approach
that utilizes a situative perspective with story problems that is more in sync with the natural ways
Teaching through a situative perspective with story problems acknowledges that students’ have a
bank of intuitive knowledge that they bring to instruction (Mack, 1990). Teaching through a
situative perspective with story problems encourages and facilitates students’ use of their
intuitive knowledge by focusing instruction around the use of story problems that are set in real-
life situations. As students encounter real-life story problems, they draw upon their intuitive
knowledge to solve the problems. The teacher acts as a facilitator and helps students connect
their intuitive knowledge to the mathematical concepts, symbols and procedures involved. Thus,
teaching fractions through a situative perspective with story problems focuses on students’
cognitive processes during instruction and instruction’s affect on those processes.

For example, Mack (1990) studied sixth grade students’ abilities to utilize their intuitive
knowledge about fractions to give meaning to fraction symbols and procedures by using story
problems, concrete materials, and problems represented symbolically. At the beginning of
instruction, Mack found that students’ intuitive knowledge was disconnected from their
knowledge of fraction symbols and procedures. The following is an illustration of the gap:

Mack (1990) asked each student a question like “Suppose you have two pizzas of the
same size, and you cut one of them into six equal-sized pieces and you cut the other one into
eight equal-sized pieces. If you get one piece from each pizza, which one do you get more
from?” (p. 21). Every student replied that they would receive more from the pizza cut in six pieces. Mack also presented each student with a symbolic representation of the same problem either just before or just after the previous problem. The symbolically represented problem was similar to: “Tell me which fraction is bigger, 1/6 or 1/8?” (p.22). Only one of the eight students recognized and responded that 1/6 was bigger.

The research (Mack, 1990) documented similar cases in which students intuitive knowledge about fractions was originally only accessed for problems posed in the context of real-life situations but not applied to problems posed using symbolic and concrete representations. In an attempt to bridge the gap in students’ thinking, instruction frequently vacillated between problems posed symbolically and in contexts of real-life situations. In general, students were first presented with story problems in the context of real-life situations and then presented problems represented symbolically that were closely related to the story problems. “When appropriate problems were used, all eight students attempted to use their informal knowledge of fractions to construct meaningful algorithms for problems presented symbolically” (p.23). For instance, all eight students were able to successfully connect and apply their intuitive knowledge to subtraction-of-fraction problems posed symbolically and in real-life contexts. In addition, students were able to convert improper fractions and mixed numerals as they solved story problems and symbolic problems involving subtraction and regrouping.

These results support Hiebert’s (1988) proposal that intuitive knowledge can serve as the starting point for the development of understanding of mathematical symbols and procedures in school settings. These results also support the idea that teaching fractions through a situative perspective with story problems is an effective vehicle through which the gap between students’ intuitive knowledge and knowledge of mathematical symbols and procedures is shortened.
Another researcher (Empson, 1995) adds support for teaching fractions through a situative perspective with story problems and using students’ intuitive knowledge to give meaning to fraction concepts.

Empson (1995) documented a first grade teacher’s five-week unit on fractions in which a situative perspective with equal sharing problems was used. The teacher working with Empson in the study believed that “first graders might be able to use their intuitive knowledge of equal sharing to solve story problems about equal-sharing situations to produce fractional quantities” (Empson, p. 110). In addition, the teacher believed her students could devise their own strategies for solving the equal-sharing problems. The teacher encouraged her students to record their answers by drawing pictures or writing sentences using invented spelling. Regarding manipulatives, the teacher made the following decision:

Although the teachers planned on making counters and letting children use paper and pencil to solve problems, a decision was made not to introduce fraction manipulatives, such as preformed plastic pieces that fit together to form circles. When children use preformed manipulatives to represent fractions, they may not realize the significance of all the pieces’ being of the same size… rather than letting manipulatives dictate the situation, children’s thinking should dictate the use of manipulatives and other tools for thinking (p. 110).

The following problem was presented before formal instruction began and solved by fourteen of the seventeen students in the teacher’s class: “Four children want to share ten cupcakes so that each child gets the same amount. Show how much one child can have” (Empson, 1995, p. 110). After five weeks of instruction, sixteen of the seventeen students could successfully solve equal-sharing situations like the one above as well as situations that required partitions other than repetitive halving. Even more noteworthy, approximately half the students were able to solve novel story problems using their new knowledge of fractions. In addition, ten students were able to successfully solve a subtraction problem involving fractions. At the conclusion of the study, Empson (1995) stated the following:
Readers may be wondering how first graders could solve problems that involved addition and subtraction of fractions without prior instruction. The key lies in students’ solving and discussing equal-sharing problems, which can develop an understanding of equivalence. Equal-sharing problems fostered a sound conceptual foundation for understanding how fractional quantities are related by prompting children to reflect on different ways to partition the same amounts. Since the first graders were not trying to remember procedures that had been taught, they had to use conceptual knowledge to devise solutions” (p. 112).

Empson’s research supported the view that learning through a situative perspective with story problems advanced students’ understanding of fractions. This occurred because students shared their thinking which contributed to a shared understanding that was extended, deepened, and restructured in ways that promoted greater knowledge of fractions.
CHAPTER THREE: METHODOLOGY

Action research provides teachers with a systematic way to study the effects of their instructional practices on student learning (Mills, 2003). My action research was designed to describe the effects of teaching through a situative perspective with story problems on students’ understanding and performance with fraction concepts, operations and procedures. My first goal was to find out how my implementation of a situative perspective with story problems affected students’ understanding of fraction concepts. My second goal was to find out how my implementation of a situative perspective with story problems affected students’ understanding of fraction operations.

Design of Study

The methodology used in this study was qualitative in nature. I used this approach because qualitative studies allow the researcher to go through a “series of steps and iterations: gathering data, examining data, comparing prior data to newer data, and developing new data to gather… so that the researcher’s emerging hunches or thoughts become the focus for the next data collection period” (Gay & Airasian, 2003, p.228). The characteristics of qualitative research allowed me to both effectively study my practices and provide for my students’ changing needs in the mathematics classroom.
School Setting

This action research took place in a PK-5 public elementary school located in central Florida. There were 673 students enrolled at my school during the 2004-2005 school year. The population consisted of 349 Hispanic students, 215 Caucasian students, 63 African American students, 15 Asian students, and 31 other students. There were 213 Language Enriched Pupils (LEP) and 121 Exceptional Student Education (ESE) students. My school qualified for Title I status because more than 75% of our population qualified for free and reduced lunch. In all, 517 students qualified for free and reduced lunch.

Classroom Setting

There were 21 heterogeneously mixed ability students, 11 boys and 10 girls in my classroom. Of those 21 students, there were 12 Hispanics, 6 Caucasians, 1 Asian, 1 African American, and 1 other. Two students qualified for the gifted program, one student received Specific Learning Disabilities (SLD) services, five students were English as a Second Language (ESOL), and one student had been fully mainstreamed into my classroom from the Exceptional Student Education (ESE) program.

Procedures

After receiving IRB (Internal Review Board) approval (Appendix A) and principal approval (Appendix B), I sent home parent consent forms (Appendix C). Twenty of the twenty-
one students in my class returned the parent consent forms with parental permission to participate in my study. Then the twenty students completed student assent forms (Appendix D). Data collection began after all the documentation had been completed.

Mathematics instruction usually lasted between 45-60 minutes on Mondays, Tuesdays, Thursdays, and Fridays for twelve weeks. Instruction time was never shorter than 45 minutes, but sometimes a session would extend past 60 minutes. The extended sessions tended to occur more in the beginning of the twelve weeks, than the middle or end.

Four of the students that participated in the study were only able to attend the first 45 minutes worth of instruction each day because they were part of mandatory reading intervention groups. On several occasions, the four students missed sharing opportunities that may or may not have affected their learning.

The unit of study for this research was Fractions. The Sunshine State Mathematics Standards for Fractions is found in Strand A: Number Sense, Concepts, and Operations. The benchmarks covered in Strand A that guided my curriculum in the study were:

MA.A.1.2.1: The student… associates verbal names, written word names, and standard numerals with whole numbers, commonly used fractions, decimals, and percents.

MA.A. 1.2.2: The student understands the relative size of whole numbers, commonly used fractions, decimals, and percents.

MA.A.1.2.3: The student understands concrete and symbolic representations of whole numbers, fractions, decimals, and percents in real-world situations.

MA.A.1.2.4: The student understands that numbers can be represented in a variety of equivalent forms using whole numbers, decimals, fractions, and percents.

MA.A.3.2.1: The student understands and explains the effects of addition, subtraction, and multiplication on whole numbers, decimals, and fractions including mixed numbers.

MA.A.3.2.2: The student selects the appropriate operation to solve specific problems involving addition, subtraction, and multiplication of whole numbers, decimals, and fractions.
MA.A.3.2.3: The student adds, subtracts, and multiplies whole numbers, decimals, and fractions, including mixed numbers… to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil and calculator.

MA.A.4.2.1: The student uses and justifies different estimation strategies in a real-world problem situation and determines the reasonableness of results of calculation in a given situation.

MA.A.5.2.1: The student understands and applies basic number theory concepts, including primes, composites, factors, and multiples.

Description of Instruction

I began the unit on fractions by administering a pretest titled Initial Assessment Questions (Appendix F). Then I used the data to plan instruction. During the first lesson, and for every subsequent lesson thereafter, I encouraged my students to solve problems using pictures, diagrams, mental mathematics, fraction circles, counters and/or any other method of their choice. However, I did not introduce students to fraction circles or other preformed plastic pieces, nor did I plan tasks around the use of manipulatives. I made this decision based on my understanding of Empson’s (1995) research described in Chapter Two. I also told my students that they were allowed to work with or help other students at their table, but each student needed to be able to explain how he or she got his or her answer. Students were told that I was more interested in how they got their answers than the answers themselves.

I always created or re-worded four-to-ten story problems that had the fraction content embedded in them that I wanted to explore. Then I would print enough copies for each student or I would write the story problems on chart paper. Most of the students were able to complete four questions and some of the more advanced students were able to complete all of the questions.
During the 45-minute lessons, we usually had a sufficient amount of time for students to share their solution strategies for two-to-four problems. Then for the remaining 15-minutes, the students who were able to complete more than four problems would share their solution strategies to the remaining problems.


At the beginning of each lesson, I would review a key strategy, idea or misconception that surfaced during the discussion on the previous day. That key strategy, idea, or misconception would serve as the hook for grabbing students’ attention. After several minutes of discussion about that strategy, idea or misconception, students would be given a story problem to solve that was directly related to the discussion. Students would be given time to solve the problem independently or with other students at their table. After most students were finished, we would discuss the problem, strategies, and questions. Students were asked to describe in detail what steps they took to solve problems and why. If students drew diagrams or pictures, they were asked to reproduce their drawings on the white board. Students that mentally computed problems explained their strategies orally. After their explanations were complete, I wrote their method on the board for others to see.

After wrapping up the discussion, I gave the students time to work on the remaining questions. I walked around, listened to discussions, made notes of interesting strategies, and stopped to help students in need. I also asked questions for clarification, explicitness or insight.
In some cases, I had to start a dialogue between students at a table in order for any discourse to occur among peers. When necessary, I provided pieces of information that might help a student overcome an obstacle or get a jump start. Students were encouraged to take risks and follow their hunches, explore, share failures and successes, and to question their peers. Some problems required students to explain their solution strategies in writing.

At the end of the study, I administered the post test which consisted of the same questions as the Initial Assessment Questions. I compared the pre test results to the post test results. Two students that participated in the study moved before completing the post test. Therefore the post test results only report the findings of 18 students compared to 20 on the pretest.

**Instruments of Data Collection**

Qualitative methods for data collection included the Initial Assessment Questions (Mack, 1985), student work samples, and videotaped and audiotaped sessions of students working in groups. All data were collected within the boundaries of my classroom and care was taken to prevent data collection from interfering with classroom activities. In addition, students’ names were changed in order to assure privacy and confidentiality.

**Initial Assessment Questions**

Mack (1985) used the Initial Assessment Questions (IAQ) in her doctoral dissertation and gave me permission to use her IAQ in my action research study. The IAQ was used as a pretest to gauge my students’ level of knowledge regarding fractions before instruction began and as a
posttest to gauge my students’ level of knowledge after instruction ended. A copy of the IAQ can be found under the heading: Appendix F.

**Student Work Samples**

Throughout the study, students solved fraction problems posed in context. Students were asked to show all their work on the paper I provided them daily. I collected each student’s work each day in order to gather valuable information about students’ understanding of the material presented.

**Videotaping and Audiotaping**

The use of audiotaping provided another source of data collection for me when I was observing and assisting other students in class. The audiotaping focused on one group during a class session. Videotaping was conducted on selected groups of students that I had permission to keep from physical education, music or art class. The videotaped sessions lasted between 25-35 minutes and consisted of three-four students. These students sat at the same table during the regular school day, and were accustomed to working together during mathematics class. Students were given story problems to solve that the class had worked on previously. Students were allowed to work together or individually, and they had access to pencil and paper and fraction circles. During the videotaped sessions, I interacted with the students as needed in order to promote understanding of the content.
Methods of Data Analysis

My action research was designed to investigate the affects of teaching fractions through a situative perspective with story problems on students’ understanding of (1) fraction concepts, and (2) fraction operations. I considered it necessary that my findings be accurate and trustworthy. “Teachers who engage in action research can take steps to ensure that their data are trustworthy through triangulation” (Feldman & Minstre, 2000, p.437). Triangulation of findings on students’ understanding and performance regarding fraction concepts, operations and procedures was provided through several different overlapping data sources and methods including: Pre and post assessments, audiotaping, videotaping, and student work samples.

The pre and post assessments and student work samples were read and reread in order to note persistent themes or common threads (Gay & Airason, 2003, p.231). The audiotapes and videotapes were played and replayed in order to note persistent themes and common threads. Only portions of the audiotapes and videotapes that I used were transcribed.

Limitations

Several limitations were present during this action research project. One limitation was that there was only one group of twenty fifth-grade students involved in my study. Another limitation was that story problems for the fraction concepts, operations and procedures I taught were not abundant in textbooks or publications, which meant that I was mostly responsible for creating the story-problems. The third limitation was that four students left after 45 minutes of instruction and may have missed critical concepts discussed and/or may have caused me to cease
discussions prematurely so the students would not miss anything. The fifth limitation involved the stress and damage to the residents of Central Florida caused by the four hurricanes that hit in the Fall 2004. Some of my students’ homes were condemned because of mold and structural damage. There is no way to accurately measure those affects on my students. After every hurricane, our superintendent asked us to spend time listening, supporting, and talking with our students about the disasters. In addition, every time school resumed, we had to review the school and classroom’s policies and procedures. All of these factors limited the time spent on academics and pushed back the typical timeframe of the curriculum.

Summary

In Chapter Three, I described the Methods utilized in my action research study, specifically the design, procedures, methods of data collection and data analysis, and limitations. Interpretation of the data will be discussed in Chapter Four, Data Analysis. In Chapter Four, I will present the findings of my research as related to the effects of teaching fractions through a situative perspective with story problems on students’ understanding and academic performance and the teaching implications. Does teaching fractions through a situative perspective with story problems increase students’ level of understanding of fraction concepts and operations?
CHAPTER FOUR: RESULTS

“Data analysis is an attempt by the teacher researcher to summarize the data that have been collected in a dependable, accurate, reliable, and correct manner” (Mills, 2003, p.104). Data analysis involves examining the information collected and finding common threads or patterns within each data source (Mills, 2003). Throughout my study, I examined the data collected and searched for emerging patterns around the following themes: (1) students’ understanding of fraction concepts; and (2) students’ understanding of fraction operations.

Students’ Understanding of Fraction Concepts

Students in this study were expected to solve many problems for which they had no preconceived algorithm or method. The context of the problems I presented during the first two weeks of the study appeared to help students solve the problems because most of the contexts were familiar to them. These familiar contexts included everyday events like sharing candy bars, and many students had experience solving these types of problems.

Almost all of the students used pictures to solve these problems initially. During class discussions, students used their pictures to justify their thinking and solution strategies. These pictures and class discussions seemed to help students develop and deepen their conceptual understanding of fractions. Therefore, the process of answering my first research question became difficult because the affect of teaching fractions through a situative perspective with story problems on students’ conceptual understanding of fractions appeared to be closely tied to the affect of students’ pictures and discussions on their conceptual understanding. In many cases,
class discussions that took place after solving story problems became the source of learning. As the discussions developed, the story problems took a back seat to the concept that I was trying to elicit or develop. Thus, it may appear to the reader that I am talking more about pictures and discussions. Though this may be true, I believe that the story problems set the stage and facilitated the discussions and pictures.

In the following section, I will attempt to answer my first research question: How does teaching fractions through a situative perspective with story problems affect my students’ abilities to develop a conceptual understanding of fraction concepts. The following themes emerged as I sought to answer this question: (1) Developing students’ understanding of equivalence; (2) Developing students’ understanding of one whole; and (3) The Role of Pictures and Class Discussions. These themes will be discussed below.

**Development of Students’ Understanding of Equivalent Fractions for One-Half**

I spent the first two lessons developing and eliciting students’ ideas about equivalence through problems that required my students to solve and discuss equal-sharing situations involving two, three, four, six and eight sharers. I started fraction instruction by giving students equal-sharing problems that required halving and repeated halving such as: “There are 22 muffins in the oven for 4 bakers. The bakers want everyone to get the same amount of muffins. Show how many muffins each baker will get.”

Most students solved this type of problem by drawing pictures of the four bakers (sometimes represented by stick figures, boxes, etc) and the 22 muffins (sometimes represented by circles, tally marks, etc.). Students drew lines from the muffins to the bakers, representing
they had dealt out the muffins one by one to the four people until two remained. At that point, students had to figure out what to do with the two remaining muffins. Nearly all of the students partitioned the two remaining muffins in half, and gave a half to each of the four bakers for a total of five and one-half (see Figure 1).

One student partitioned the two remaining muffins into fourths, and gave each baker one-fourth from each muffin for a total of five and two-fourths (see Figure 2). Three students tried dividing 22 by 4, but only one knew what to do with the remainder. Thomas told me that he divided 22 by four and got two as a remainder. Then he mentally pictured cutting the two left over muffins in half and got a total of two and one-half. Two of the three students did not know what to do with the remainder, so those students drew pictures like the ones described above and solved the problem using pictures (see Figure 3). Ashley told me that “it was easier to do it that way than to figure out the remainder.”
Figure 2: This method of giving each baker one-fourth was used by one student.

Figure 3: First, this student tried solving by division. Then she drew pictures because she didn’t know what to do with the remainder.

One way that students’ concept of equivalent fractions for one-half was developed was through the discussions students engaged in after solving equal-sharing problems. Throughout the study, I purposely called on students that had solved the same problem in different ways and came up with similar or different answers. The following discussion occurred during the first lesson of the study. Students had solved the following problem and were sharing their strategies with the class: “Four children want to share ten cupcakes so that each child gets the same amount. Show how much each child will receive.” Britney was telling the class how she shared the two left over cupcakes.
Britney: “Well, there are two cupcakes left so we have to split them in half. And so each kid gets two and a half.” Britney proceeds to point to her drawings of the cupcakes split in half and the words two and a half.

Mrs. Allen: “Did anyone solve this problem in a different way?”

Julia: “I did. I did the same thing as Britney at first and gave each child two cupcakes, which left me with two cupcakes. So I knew there were four children, so I split each cupcake into four parts and gave one-fourth of one cupcake to each child and one-fourth of the other cupcake to each child, which gave me a total of two and two-fourths, which is the same thing as two and one-half.”

Mrs. Allen: “Wait a minute. How can two-fourths be the same as one-half? Somebody explain that to me.”

Kathy: “Well, I kinda see what she’s saying. It’s like… well, if you shade in those two parts (pointing at the fourths) and give them to one child and then… I’m not sure.”

Mrs. Allen: “Anybody else?”

Malcolm: “Yes, I can explain. I learned this at my old school. Its’ like this. One-half equals two-fourths because one is half of two and two is half of four. So anytime the top is half of the bottom, it’s half. Can I show you on the board?”

Mrs. Allen: “Using pictures only.”

Malcolm: Draws two circles on the board, partitions one in half and the other into fourths. Then he turns and faces the class. “Okay, so one kid gets one-
half.” He turns to shade in one-half, then turns and faces the class. “That’s one-half. Now watch and learn.” (He turns and shades in two-fourths so that it looks exactly like the one-half). “That’s two-fourths. They are equal. Any questions?”

Mrs. Allen: “Does anybody disagree?”

Fred: “I get it now. He’s right. One-half does equal two-fourths, cause look, you can do it a different way. Can I come up to the board?”

Mrs. Allen: “Sure.”

Fred: He picks up the marker and puts a horizontal diameter through the circle that Malcolm had partitioned in to two equal parts, making it a circle with four equal parts. “See, now there are four pieces and two are shaded.”

As the discussion progressed, most of the students came to a shared understanding of fractions that were equivalent to one-half. Malcolm came up to the board again and showed the class an adding strategy he used to find equivalent fractions that he had learned in fourth grade at his other school. The strategy was to add the top number by itself, or twice and if it equaled the bottom number, the fraction equaled one-half.

I asked students to prove Malcolm’ strategy with a drawing or to come up with their own strategy for finding fractions that were equivalent to one-half. As students worked, I heard some of them repeat Malcolm’s word, “one is half of two, and two is half of four.” I also heard students saying, “it’s easy, just take any number and divide it by two,” and “it’s half because you can add the top number twice to get the bottom number,” (see Figure 4) and “just multiply the top by two.” Several students came up with halves for odd numbers such as “2 ½ is half of 5,”
(see Figure 5) and “12 ½ is half of 25.” Altogether, the students generated over 19 different equivalent fractions for one-half during the first lesson of the study.

![Image](image1.png)

*Figure 4: One student describes her method for knowing what is equivalent to one-half.*

![Image](image2.png)

*Figure 5: One student shows that two and one-half is half of five by comparing it to one-half.*

The concept of equivalent fractions for one-half seemed to develop in one day. Several of the statements made by Malcolm on the first day of instruction were continually repeated by students during class discussions and on the post assessment questions, which suggests that his explanation made sense to students and was meaningful. About four weeks into the study, I was videotaping a group of students working on equal-sharing problems. The following question had been posed and students were discussing their strategies: “Four squirrels wanted to share six berries equally. Tell how many berries each squirrel will get.” Danielle explained her thinking and gave the answer one and two-fourths. Then Kathy explained her thinking and gave the answer one and one-half.
Mrs. Allen: “Okay, now Kathy got one and one-half and Danielle got one and two-fourths, can you both be right?”

Kathy: “Yes.”

Mrs. Allen: “Why?”

Kathy: “Two is half of four and one is half of two and they’re both a half.”

Mrs. Allen: Looking at Danielle. “Is that true?”

Kathy: Interjecting without noticing that I was asking Danielle the question. “They each get a whole and a half.”

Mrs. Allen: Looking at Kathy, then Danielle. “They each get a whole and a half. Danielle, is two-fourths a half?”

Danielle: “Yes.”

Mrs. Allen: “Can you show me what two-fourths looks like?”

Danielle: “Two-fourths. In fractions?”

Mrs. Allen: “However you want to show me.”

Danielle: Nods, then begins drawing a circle, cuts it into four parts, and shades in the two top parts to make it look like half.

Mrs. Allen: “Is that the same thing as one-half?”

Danielle: Nods yes.

Mrs. Allen: “Okay, good.”

Results from the pre and post assessment also indicate that students’ concept of equivalent fractions for one-half was firmly established during the study. The assessment questions required students to answer one open-ended question about one-half and one matching question (see Appendix F, Question 24b and 27a). Neither of the two questions was set in
context. Pre assessment results reveal that 3 out of 20 students could correctly answer both of the questions about equivalent fractions for one-half prior to instruction compared to post assessment results that reveal 18 out of 18 students could answer both of the questions about equivalent fractions for one-half correctly after instruction. Both questions asked students to explain why the fractions were equivalent. Several responses included statements that were made during the first lesson such as: “2 is half of 4 so 2/4 equals ½,” and “3 is half of 6 and 1 is half of 2 so they are both halves.”

In conclusion, pre and post assessments, class discussions, and student work samples indicate that students’ concept of equivalent fractions for one-half was firmly established during the study. Equal-sharing situations involving two, four, six and eight sharers provided many opportunities for students to discuss their answers and come to a shared understanding of equivalence. The statement that was repeated by students most often was “one is half of two and two is half of four.” Throughout the study, I heard students replace the “two is half of four” with the numbers they were working with at the time. Other strategies for finding equivalent fractions were also discussed and used by students. Those strategies included dividing the fraction by two, adding the numerator twice to see if it equaled the denominator, and multiplying the numerator by two to see if it equaled the denominator. Though instruction began with the use of problems set in context, students were able to transfer their understanding of equivalence regarding one-half to problems that were not set in context.
Development of Students’ Conceptual Understanding of One Whole

The data reveal that the concept of one whole was also firmly developed by all of my students during the study and that most students recognized and could name several equivalent fractions for one whole. The concept of one whole appeared to develop through discussions and pictures that stemmed from the story problems students were solving. These problems involved adding fractional quantities together. The concept of one-half also appeared to affect students’ understanding of one whole. For example, students were given the following problem during a videotaped session: “Lester found 2/4 cup of sugar in the cupboard and ½ cup of sugar in the refrigerator. Show how much sugar Lester found in all.”

Most students told me that they solved this problem by drawing two tall rectangles. One rectangle was cut into two equal parts and the other was cut into four equal parts. Students shaded in one of the two parts in the first rectangle and then two out of the four parts in the second rectangle. Then students mentally combined the quantities of the two rectangles together and found that one whole rectangle was shaded. Figure 6 shows that one student drew the cups described in the context of the problem to resolve the situation. The following discussion occurred after students solved this problem.

Figure 6: Four-fourths equals one whole.
Mrs. Allen: “Fred, how did you come up with 4/4 or 1 whole?”

Fred: “Well because 2/4 is the same thing as one half and one half plus one half equals one whole or four fourths. I know that because if I draw a rectangle and cut it into 4 parts and shade in all 4 parts, I have the whole rectangle or 4/4.”

Mrs. Allen: “Danielle, you have eight-eighths equals one whole. Can you and Fred both be correct?”

Danielle: “Yes, because eight over eight is one whole and so is four over four. They’re both one whole because you have all the parts.”

Additional student work samples provide evidence that students understood the concept of one whole and equivalent fractions for one whole as well as one-half. Figure 7 illustrates one students’ understanding of the relationship between improper fractions and mixed numbers and shows that the context of the problem may have led her to use a clock-like representation.

Figure 7: Six-fourths equals one whole and one-half.

The pre and post assessment results also indicate that most students developed and deepened their understanding of one whole and equivalent fractions for one whole. The pre assessment results reveal that only 3 out of 20 students could name at least one equivalent fraction for one whole prior to instruction compared to 15 out of 18 students after instruction (see
Appendix F, Question 24a). Most students named at least three equivalent fractions for one whole.

The three students that did not name an equivalent fraction for one whole on the post assessment question 24a surprised me because two of them named equivalent fractions for one whole on different post assessment questions and never appeared to struggle with that concept before. Malcolm said he “forgot” which did not make sense, because on question 5a of the post assessment (see Appendix F), Malcolm listed 10 equivalent fractions for one whole (See Figure 8). Danielle wrote $\frac{5}{7}$ and $\frac{1}{2}$ for question 24a, but on questions 17 and 18, she wrote an equivalent fraction for one whole (see Figure 9). I also recorded Danielle explaining why eight out of eight equaled one whole and wrote about that discussion earlier in this section. The only explanation I can conjure to explain these discrepancies is that the context of the questions may not have given enough information for students to demonstrate their understanding of one whole.

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{Figure8.png}
\caption{Malcolm writes 10 different equivalent fractions for one whole.}
\end{figure}

In summary, the data suggest that all of my students developed an understanding of one whole and equivalent fractions for one whole. Story problems provided many opportunities for students to explore and investigate the relationship between one whole and equivalent fractions such as four-fourths or eight-eighths. Three students did not demonstrate their understanding on the post test question that specifically asked for equivalent fractions for one whole, but demonstrated their understanding on other post assessment questions that indirectly dealt with...
equivalent fractions for one whole. The three students also demonstrated their understanding through work samples and class discussions.

Figure 9: Danielle wrote an equivalent fraction for one whole on both questions.

The Role of Pictures and Class Discussions

Many students drew pictures to solve story problems and then justified their thinking by explaining what they did to the pictures. The use of both words and pictures seemed to help students communicate their ideas in ways that other students understood. The discussions and pictures also appeared to help develop and deepen students’ conceptual understanding of fractions. Evidence to support this theme can be found in the example of Malcolm and one-half. Another example that supports this theme can be found in the following illustration.
My students initially learned about the importance of size in determining equivalence and order through one student’s pictures and words. The following question was given to students during the seventh lesson: “Six foxes catch four rabbits. Each fox gets the same amount. Show how much each fox gets.”

My students solved this problem in three different ways. Eight students solved this problem by partitioning the rabbits into thirds and giving each fox one-third of two rabbits for a total of two-thirds (see Figure 10). Four students solved this problem by partitioning the rabbits into sixths and giving each fox one-sixth of each rabbit for a total of four-sixths (see Figure 11). Four students solved this problem by giving each fox one-half and one-sixth. The students began by partitioning three rabbits in half, so that each fox received one-half. Then they partitioned the last rabbit into sixths, so that each fox received one-sixth (see Figure 12). Out of 19 students present that day, 16 students used an appropriate solution strategy to solve this problem.

Figure 10: All four rabbits are cut into thirds.

Figure 11: All four rabbits are cut into sixths.
During the discussion about this problem, one student explained why the answers were equivalent. He justified his thinking through words and pictures.

Mrs. Allen: “Let’s just look at Ashley and Julia’s pictures. Ashley’s foxes get two-thirds, but Julia’s get four-sixths. How can this be?”

Thomas: “Well, they’re the same because two-thirds are just bigger pieces and four-sixths are smaller pieces, but you get more of them. It’s like, it’s like this.

If you cut a circle into three pieces”

Mrs. Allen: “Thomas, go to the board and show us.”

Thomas: Goes to the board and draws a circle. He cuts the circle into three parts and shades in two. “This is two thirds. But if I cut each piece in half, then I have six pieces and four are shaded in. The foxes get the same amount, but their pieces are smaller.”

Thomas’s words and pictures appeared to help other students understand the importance of size in finding equivalence. Instead of students merely looking at the number of pieces each sharer received, students began to look at the size of the pieces. Figure 13, 14, and 15 suggest that Thomas’s explanation and picture was a source of learning for other students. The explanation written in Figure 13 is similar to the one Thomas used. Figure 14 indicates that this student was looking at the size of the parts, not just the quantity and Figure 15 reveals that this student was able to recognize that two small parts were equivalent to one large part.
During the sixth week of the study, one student explained why one-fifth was equivalent to two-tenths using an explanation that was similar to Thomas’s.

James: “Can I go to the board?”

Mrs. Allen: “Yes.”
James: Draws a circle, partitions it into five equal parts and shades in one. “This is one-fifth, but it’s also two-tenths because I can cut each piece in half.” He halves each fifth so that there are ten parts. “See, they’re equal. The pieces in one-fifth are bigger, but you only get one. The pieces in two-tenths are smaller, but you get two.”

Another example that illustrates the way that class discussions and pictures worked together to help students develop their conceptual understanding involved a student named Julia. Julia knew how to find equivalent fractions by multiplying. She shared her procedure with the class during the same lesson that Thomas shared his method.

Julia: “Well, I learned that you can multiply. Like I can multiply two times two and get four and two times three and get six. So I know that two-thirds is equal to four-sixths because you can multiply.”

Mrs. Allen: “Do you know why you can multiply?”

Julia: “Not really. I just know.”

Mrs. Allen: “Let me show you why. It has to do with Thomas’s explanation. Take for instance this circle that Thomas drew. It’s cut into three parts. Well, what happened to each part when you made it into six parts?”

Harry: “Each piece got smaller.”

Mrs. Allen: “Okay, how much smaller?”

Janet: “Like half as big as before.”

Mrs. Allen: “Yes, each piece became two pieces because each piece was cut in half. So that piece became two and that piece became two, so that there are now six pieces instead of three. That’s why
you can multiply two-thirds by two to get four-sixths. Because each piece becomes two pieces when you go from three to six pieces.”

I knew that some students would try to use Julia’s strategy for finding equivalent fractions. Initially, when students used multiplication to find equivalent fractions, I required them to draw pictures to support their findings (see Figures 16-18).

**Figure 16: Pictures model multiplication.**

![Figure 16](image1)

1. Joey has six marbles. Two are blue, three are black and the rest are green. Marsha has three marbles. One is blue, one is black and the rest are green. What is the fraction of blue marbles Joey has? What is the fraction of blue marbles Marsha has?

2. Compare fractions using <, >, or =. Explain in words.

   \[
   \frac{2}{6} = \frac{1}{3}
   \]

   B/c three x two = six \neq one x two = two.

**Figure 17: Multiplying to find equivalent fractions.**

![Figure 17](image2)
Eventually, students began multiplying mentally. When students mentally computed the answers, I asked them to explain why they could multiply. Students typically used pictures to explain. As students used pictures to explain the rationale, other ways of finding equivalent fractions emerged.

During the third week of the study, Mike noticed that division could also be used to find equivalent fractions (see Figure 19). Mike explained that erasing the lines was like dividing. I asked him to show the class what he meant. He went to the board and drew a circle with eight parts. He shaded in two parts and told the class that he had two-eighths. Then he drew a circle with four parts, shaded in one and said he had one-fourth.
Mike: “Now I can erase this line (pointing to the line in two-eighths that separated the shaded parts) and then this line (skips two lines and erases), and this line (skips another two lines and erases), and this line (skips two lines and erases).”

Mrs. Allen: “Does anybody understand what Mike just did?”

Ashley: “Yes, he’s just making the pieces bigger.”

Fred: “Yeah, it’s like, I was just thinking about that. I was just thinking about, well can I come to the board and show you what I was thinking about?”

Mrs. Allen: Nods yes.

Fred went to the board and demonstrated the exact same thing as Mike, except Fred used two-sixths and one-third. Even though the method of division was explained and several students said they understood the procedure (see Figure 20), most of my students preferred using multiplication to find equivalent fractions. In the work samples and student discussion, I did not find many examples of students using division, but I did find a lot of examples of students using multiplication (see Figures 16-20).

2. Jo, Fred, and Pat walk home from school. Jo walks ½ mile. Fred walks 2/3 mile, and Pat walks 3/12 mile. Put the distances each person walks home from school in order from greatest to least. Explain how you decided.

Figure 20: This student divided and multiplied to compare fractions.
The pre and post assessment data suggest that most students understood that the size of the parts mattered just as much as the quantity of parts. On the pre assessment, 3 students correctly stated that one-fourth was equivalent to two-eighths on question 21 (see Appendix F). On the post assessment, 14 students correctly answered question 21.

Another example that illustrates the partnership between pictures and discussions working together to help students formulate their thinking about fraction concepts and operations occurred after Thomas and Julia’s explanation. Four students wrote one-half and one-sixth as their answer to the fox problem. Two students wanted to know if their answer was correct, and what one-half and two-sixth equaled.

Gus: “Mrs. Allen, I want to learn how to add fractions when the denominator is different.”

Mrs. Allen: “Gus, I don’t want to go there right now. Your answer is fine just as it is. The foxes did get one-half and one-sixth.”

Chase: “Mrs. Allen, is the answer two-eighths?”

Mrs. Allen: “Well, let’s think about that. Go to the board and draw a picture of one-half and another picture of two-eighths.”

Chase: Goes to the board, draws two circles. One is partitioned into two parts and the other is partitioned into eight parts.

Mrs. Allen: “Now shade in one-half and two-eighths. The one-half is just a portion of what the foxes got. They still got another sixth, but let’s just focus on the half for now. Could your answer of two-eighths be correct?”

Chase: “No.”

Mrs. Allen: “Why not?”
Chase: “Because, um, one-half is bigger than two-eighths, so I don’t think it would make sense.”

Mrs. Allen: “So should we add the bottom numbers, also called the denominators, together.”

Chase: “I don’t think so.”

Mrs. Allen: “Does anybody have any comments?”

Gus: “I don’t think you should add the denominators together, because in that problem you just showed us, uh, let me start again. If the foxes get the half that Chase drew plus some more, then how could their pieces be smaller?” Pointing to the two-eighths.

Ashley: “Yeah, that doesn’t make sense, cause when you add, it’s suppose to get bigger, not smaller. So, you probably shouldn’t add the bottom numbers, but how do you do it then?”

The pictures and discussion helped students realize that adding the denominators was not a viable strategy. In all of the student work samples that I collected after this discussion, I did not find one occurrence of students adding the denominators. This does not mean that students did not write incorrect solutions. It just means that students were not using the method that involved adding the denominators. I believe it is important to point out that the context of the fox story problem seemed to give students the freedom to solve it in a variety of viable ways. The variety of solution strategies offered the class the opportunity to discuss different concepts that may have helped students develop and deepen their understanding of fraction concepts and operations including, but not limited to: equivalence; equal-parts; equal-sharing; partitioning; and adding fractions.
The pre and post assessment data supports the idea that the pictures and discussion about adding denominators helped students understand the inappropriateness of that method. On the pre assessment question 32 that required students to determine if the answer to an addition problem was correct, nine students wrote that the answer was correct (see Figure 21), ten students did not respond and one student wrote that the answer was incorrect because “you don’t add the bottom number.” On the post assessment, 11 students wrote that the answer was incorrect (see Figure 22-23) and 7 students did not write a response.

Figure 21: Adding the denominators before instruction.

Figure 22: The denominators can not be added was discovered during instruction.

Figure 23: The answer is incorrect because the bottom numbers do not get added.
On the pre assessment questions 29a, which required students to add fractions with unlike denominators, none of my students responded (see Appendix F). On the post assessment question 29a, 11 students correctly solved the problem, 1 student did not respond, and 6 students wrote incorrect answers. Two of the six made computational errors but used a correct addition method. The other four students used an inappropriate addition method that I never addressed during the study. One of the methods, used by two students, involved adding the numerators and placing the sum over the biggest denominator in the problem. Figure 24 illustrates this inappropriate addition method. The second inappropriate method, used by one student, involved using one of the numerators as the denominator. The third inappropriate method, used by one student, involved adding one of the numerators to one of the denominators, and then placing the sum over the other denominator.

![Figure 24: An incorrect adding method.](image)

In conclusion, class discussions and pictures appeared to help most students develop their understanding of fraction concepts and operations. Some of the evidence that supported this conclusion came from the class discussions involving Malcolm teaching the class about one-half, Thomas teaching the class about the importance of the size of the parts, and Chase and Gus discussing why the denominators should not be added.
Most class discussions appeared to be sources of learning for my students, but the data suggest that one class discussion had an adverse effect on several students’ conceptual understanding of fractions. The main reason that the class discussion had an adverse effect deals with the fact that one student’s rationale was adopted by other students. The student who discussed his rationale understood the various aspects of his rationale and could use it at appropriate times, but some of the students who adopted the rationale did not completely understand all aspects of the rationale and used it at inappropriate times. Evidence to support this theme came mostly from problems that required students to order and/or compare fractions with unlike denominators. Figure 25 shows the problem that students were required to solve. The following is part of the discussion that took place after students had solved this problem involving two sharers and three sharers.

Mrs. Allen: “The problem asks you to compare one-half and one-third. Tell me how you solved this problem Mike.”
Mike: “Okay, well, I know that, um, okay. Here’s how I thought about it. It’s not like I had to multiply or anything, I just knew because it’s like this. If you have the same number on top, like they both have a one on top, then you just look at the bottom numbers and the smallest number means it has the bigger sized pieces. So that’s how I did it.”

Mrs. Allen: “That’s a great way to think about it Ty. I think you’re using your brain and thinking about the size of the pieces. That’s what I want you to do. Why do any multiplying if you can just think about the pieces.”

Mike: “Yeah, I learned that in fourth-grade. It’s so easy. You just look at the bottom number and that tells you which pieces are bigger.”

Our conversation about the denominator continued and I had students illustrate Mike’s theory by using pictures. I began by giving students problems with fractions that had the same numerators. For example: two-fourths, two-eighths, and two-tenths or one-third, one-fourth, and one-sixth. We discussed the fact that all of the numerators were the same and that was why we could look only at the denominators. Other students began to comment about the relationship between the number of sharers and the sizes of the parts.

Britney: “It’s like a pattern. The more people you are sharing with, the smaller your piece gets.”

Eric: “Yeah, the less people, the more you get, but the more people, the less you get.”

Then I went on to show why that strategy did not always work by giving students examples of fractions with numerators that were different. However, the data suggest that some students were introduced to this rationale before they were conceptually ready to understand and
apply it. These students inappropriately used Mike, Britney and Eric’s rationale, before they fully understood the rationale. Since they did not fully understand the rationale, but used it anyway, they were unable to check the reasonableness of their answer. Fortunately, these students did not always use Mike, Britney and Eric’s rationale. This is best illustrated in Figures 26-28.

Figure 26 indicates that this student used Mike’s rationale to answer the first question, but did not use Mike’s rationale to answer the next problem. Figure 26 also indicates that this student did not recognize the similarities among the two questions and/or that she did not check the reasonableness of her answers. The work samples in Figures 27-28 also indicate that Mike, Britney and Eric’s rationales were being used by students that did not fully understand the rationales. Both work samples were taken from the same student on the same day. They show that this student was able to correctly compare one-half to two-thirds on question 4, but was unable to correctly order one-half, two-thirds and two-fifths several questions later.

Figure 26: One student compares fractions by looking only at the number of sharers.

Figure 27: The student realizes that two-thirds is bigger than one-half.
The pre and post assessment data also support the idea that students were introduced to a rationale before they were conceptually ready and that they used the rationale without fully understanding it. On the pre assessment question 21, none of the students justified their answer using Mike’s rationale. However, on the post assessment question 21, three out of four students that answered question 21 incorrectly, justified their answer in a way that was similar to Mike’s rationale (see Figure 29).

In conclusion, one class discussion appeared to have an adverse effect on several students’ abilities to develop meaningful and appropriate rationales for comparing and ordering fractions. The discussion had an adverse effect because several students adopted another student’s rationale without really understanding the rationale. Thus, when they attempted to apply the adopted rationale, they did so in inappropriate ways, and could not check the
reasonableness of their answer. Fortunately, those students did not completely depend on Mike’s rationale.

An Adverse Effect of Drawing Pictures

Though most students’ pictures appeared to help them learn about fraction concepts and solve problems, several students’ pictures did not seem to help them learn about the concepts and/or solve problems. One reason was that these students did not develop a method of partitioning regions that was easy to read and/or was accurate. Instead of drawing pictures with well-defined partitions as in Figures 10-11, these students sometimes drew pictures with marks that were illegible and unclear. The partitions in Figure 30 are difficult to read and may have caused this student to miscount the shaded region. The answers in Figure 31 reveal this student had a difficult time counting the parts in her pictures.

Figure 30: Vivian’s partitions are not well-defined
These students also drew pictures that were inaccurate representations of the fractional quantity. In Figure 32, the pictures for one-half and two-fifths are partitioned so that they appear to be equivalent fractions. In Figure 33, the student’s pictures of fourths and eighths do not appear to help him recognize that two-eighths is equal to one-fourth. Two of the students’ inabilitys to learn how to partition accurately during the study, caused them to struggle with adding fractions with unlike denominators in the latter phases of the study. The pictures of one-half and one-third in Figure 34 appear to be equivalent. The pictures of one-third and two-fourths in Figure 35 also appear to be equivalent,
Figure 33: The fourths and eighths are not drawn in a way that reveals they are equivalent.

Figure 34: The picture of one-third appears to be equivalent to one-half.

Figure 35: The picture of one-third appears to be equivalent to one-fourth.

In conclusion, some students’ pictures may have hindered their abilities to form accurate concepts of equivalence. Some of the pictures were difficult to read, so students miscounted. Some of the pictures made equivalent fractions appear not equivalent, and some pictures made different fractional quantities appear equivalent. The ability to recognize the difference between two quantities did not seem to be developed in these students and affected their abilities to add fractions with unlike denominators.
Students’ Understanding of Fraction Operations

The main fraction operations I focused on during the study were addition of fractions with like and unlike denominators and subtraction of fractions with like denominators with regrouping. The data suggest that the context of problems may have affected the way students represented the fractions. The data also suggest that the context of problems may have affected the ways students modeled problems. In addition, the data suggest that students’ conceptual understanding affected their ability to perform operations on fractions. Students that developed a firm conceptual understanding of fractions as wholes, mixed numbers, improper fractions and equivalent fractions appeared to be more capable of adding and subtracting fractions and inventing their own algorithms.

Representations are Closely Related to the Context

I found that many students solved addition and subtraction problems by drawing pictures that closely resembled the context mentioned in the problem. For example, when cups were mentioned in the problems, most students drew vertical rectangles that resembled tall cups. Figure 36-38 illustrate this finding.
In subtraction problems with the word long in the context, most students drew horizontal rectangles or number lines. The horizontal rectangles and number lines drawn in Figures 39-40 illustrate this finding. In addition, all of the students that drew pictures on the post assessment question 30, used a horizontal rectangle or number line (see Figures 41-42).
Eric purchased a poster board that was 1 yard long, but he only needed $\frac{3}{5}$ of the yard. How much poster board would he have left? Explain using pictures.

$$\frac{1}{2} \text{ of a yard left.}$$

Figure 40: Students usually drew a horizontal rectangle when the word long was in the context.

30. Suppose you have a board 3 yards long. You cut off a piece that is $\frac{7}{8}$ yard long to make a shelf. How much of the original board do you have left?

Figure 41: A number line is used to model the length.

Figure 42: A horizontal rectangle models length.

However, the context of problems and pictures drawn were not always closely related. Figures 43-44 show that sometimes student used circles when the context described cups. Figure 45 shows that one student drew circles when the context described length and Figure 46 shows that one student drew a number line when the context described height. These discrepancies did not appear to affect students’ abilities to add or subtract fractions.
2. Lester found 2/4 cup of sugar in the cupboard and ½ cup of sugar in the refrigerator. Show how much sugar Lester found in all?

![Figure 43: The context does not match the pictures.](image)

1. Lucy had used 2/3 cup of milk to make pancakes and 2/5 cup of milk to make muffins. Show how many cups of milk she used in all.

![Figure 44: Cups are described, but circles are drawn.](image)

Pete's dog was 2 feet long and his cat was 1 ¼ feet long. How much longer was Pete's dog than his cat? Explain using pictures.

![Figure 45: Length is described, but circles are drawn.](image)

3. A six year-old girl was 3 1/2 feet tall and her brother was 5 1/4 feet tall. How much taller was the brother than the sister? Explain.

![Figure 46: Height is described, but length is drawn.](image)
In conclusion, most students drew pictures that closely resembled the context described in the problems. Cups were usually represented by vertical rectangles, while length was represented by horizontal rectangles or number lines.

**Representations Model the Operation**

Many of the pictures drawn to solve problems modeled the actual operation that took place in the context of the problem. This could be seen in the ways that students solved addition problems. One way students modeled addition was by using several pictures. For example, some students would draw a picture for each quantity mentioned in the context. Then, they would either draw a third picture that showed the combined quantities (see Figure 37), fill-in one of the two pictures to show the sum (Figure 36), or mentally combine the quantities (see Figures 38 and 43).

Figures 36, 38 and 43 also demonstrate the operation of finding a common denominator in order to add fractions with unlike denominators. The student that drew the pictures in Figure 36 told me that she drew one-half and one-fourth in different cups first. Then she realized that one-half was equivalent to two-fourths, so she added the lines in the one-half cup in order to make fourths. After she made the one-half into two-fourths, she added one-fourth to that cup and found the answer. The picture in Figure 47 also models the procedure required to add fractions with unlike denominators. I watched the student draw one-half first and then cut the halves in half to make fourths. Then he shaded in one-fourth and wrote his answer.

Another way students modeled addition was by using only one picture at a time. For example, some students began the problem by drawing one quantity first. Then, instead of
drawing another picture for the second quantity, those students added to the remaining part(s) of the first whole. If all the parts in the whole were filled completely, students drew another whole and shaded in the amount not already added. Figure 48 illustrates the way one student added three-fourths plus three-fourths by adding three-fourths plus one-fourth, then two-fourths.

Figure 47: This student used one picture to add one-half and one-fourth.

Figure 48: One whole shaded is shaded in completely before adding another whole

I pointed out the efficiency in just drawing one picture and filling up the whole before drawing another picture. I allowed students to solve the problems any way they felt most comfortable, but I let them know that I thought filling up one picture was a better way because it really showed the addition part of the operation. Another thing I encouraged students to do was to write their answers as both mixed numbers and improper fractions (see Figures 44 and 48). I wanted them to be comfortable with expressing their answers as mixed numbers and improper fractions, and with recognizing that both answers were the same.
The operation of subtraction could also be seen in the ways that students modeled problems. The way most students modeled subtraction was by drawing the largest quantity first. Some students would partition each whole according to the denominator (see Figures 39-41). Other students would only partition the wholes that they needed to use (see Figures 42 and 46). Then, the students would subtract the minuend by shading in or crossing out the quantity to be subtracted. Finally, students would mentally or physically calculate the remaining parts (see Figures 46 and 49).

![Figure 49: Using pictures to subtract](image)

Another way that several students modeled subtraction was by drawing two pictures, one for each quantity described in the context of the problem. Then students would mentally or physically calculate the difference (see Figure 45 and 50). However, this method of subtracting by drawing pictures posed a problem for one student.

![Figure 50: Both quantities are drawn.](image)
Danielle was given the problem: $3 \frac{1}{4} - 1 \frac{1}{4}$. Before she drew pictures, she made the mistake of subtracting one-fourth from three-fourths. Initially, she got the answer of two and two-fourths. I asked her to prove her answer using pictures. She drew three wholes broken into fourths and one more whole with only one-fourth shaded in. Then she crossed out one of the wholes. Next, she drew three-fourths.

Mrs. Allen: “Why did you draw that three-fourths?”
Danielle: “To see what I had to subtract.”

Mrs. Allen: “Can you take that picture and subtract it from any of the other pictures?”
Danielle: “No.”

Mrs. Allen: “Why not?”
Danielle: “Because this one only has one-fourth.”

Mrs. Allen: “Well, can you subtract that one-fourth and then take the rest from somewhere else?”
Danielle: “Yeah.” Crosses out the one-fourth and stops to think.

Mrs. Allen: “How many fourths do you still have to subtract?”
Danielle: “Three.”

Mrs. Allen: “How many did you already subtract?”
Danielle: “One.”

Mrs. Allen: “So how many do you still have to subtract?”
Danielle: “Four?”

Mrs. Allen: “Let’s start again. You drew three and one-fourth. You crossed out one whole because you’re subtracting one and three-fourths. Then you crossed
Danielle: “Three, no, two, two.”

Mrs. Allen: “Why two?”

Danielle: “Because I already took away one, so I have to take away two more so that will be three-fourths.”

Mrs. Allen: “So where can you take the other two from?”

Danielle: “Here.” She begins to cross out fourths that are not shaded in and do not make up the three and one-fourth.

Mrs. Allen: “Are those part of the three and one-fourth?”

Danielle: “Oh, no.”

Eventually, Danielle came to subtract from the four-fourths left, but not without my guidance. One reason that she needed my guidance was because she made careless mistakes. One of the problems that Danielle struggled to solve because of a careless mistake was: $3 - \frac{2}{5}$.

Danielle drew three wholes. Two of them were cut into fifths, but the last whole was cut into fourths. After she drew the wholes, she drew a picture of two-fifths. After about a minute, Danielle looked up at me and asked me for help.

Mrs. Allen: “What do these pictures represent?” (pointing to the three wholes).

Danielle: “The three wholes.”

Mrs. Allen: “Why did you cut these into fifths and this one into thirds?”

Danielle: Shrugs. “I don’t.”

Mrs. Allen: “Why did you cut these into fifths?”
Danielle: “Cause there are five fifths in a whole, and that’s one whole, two wholes and three wholes.”

Mrs. Allen: “How many fifths are there in three wholes?”

Danielle: Counts her pictures, “14/5.”

Mrs. Allen: “Are those fifths?” pointing at the whole cut into fourths.

Danielle: “No.”

Mrs. Allen: “Should that one be cut into fifths?”

Danielle: Pauses to think about it.

Danielle did not see the problem with her whole that was cut into fourths. She did not attempt to change her picture and I thought it would be better to let her learn from the mistake. I let her continue solving the problem. After she subtracted the two-fifths, she counted the remaining fifths and fourths and said the answer was twelve-fifths.

Mrs. Allen: “Danielle, don’t look at your pictures just yet. Tell me how many fifths are in one whole?”

Danielle: “Five”

Mrs. Allen: “Okay, so five-fifths are in one whole. How many fifths in two wholes?”

Danielle: “Ten.”

Mrs. Allen: “Good, now how many fifths in three wholes?”

Danielle: “Fifteen.”

Mrs. Allen: “So, if you have fifteen-fifths and you subtract two-fifths from that, what will you get?”

Danielle: “Thirteen-fifths.”
twelve-fifths and not thirteen-fifths?”

Danielle: Counts her picture and shrugs.

Mrs. Allen: “Danielle, are they all cut into fifths?”

Danielle: Shakes her head yes.

Mrs. Allen: “Count the pieces in that one.”

Danielle: “No.”

Mrs. Allen: “You’re telling me that there are thirteen-fifths left over, but your pictures
aren’t all fifths, so how can that be right?”

Danielle: “Oh, I need to cut it one more time so they’ll all be fifths.”

Using pictures to model the operation of addition of fractions with unlike denominators
also created problems for Danielle, as well as three other students. Danielle and Rachel had
difficulty modeling the operation of addition because their pictures did not accurately represent
the fractional quantities (see Figures 34-35). These figures illustrate the way that different
fractional quantities were drawn and made to look like they were equivalent.

The other two girls were able to partition the fractional quantities in accurate ways, but
never fully grasped what to do with the quantities. Figure 51 shows that Kathy was able to draw
one-third and one-half in appropriate ways. However, she did not appear to recognize that both
quantities could be partitioned to make sixths. In Figure 52, Kathy’s pictures model the
operation, but she does not appear to realize that one-fifth was equal to two-tenths. Instead of
realizing that each fifth equaled two-tenths, she changed three-fifths into three-tenths.
Through repeated practice and my guidance, these four girls were able to subtract fractions with regrouping, but they never fully grasped the concepts and operations of equivalent fractions and addition with unlike denominators.

The pre and post assessment results indicated that students learned to add fractions with like denominators. On the pre assessment question 28, only 7 students wrote a correct answer for the addition of fractions with like denominators. Four students wrote an answer that indicated they added the numerators and denominators. On the pre assessment question 29b that required students to add fractions with like denominators, only one student wrote a correct answer. On the post assessment all 18 students correctly answered questions 28 and 29b (see Appendix F).

The pre and post assessment results also indicated that over half of the students learned to add fractions with unlike denominators. On the pre assessment questions 29a and 29c (see Appendix F), none of the students correctly solved either question. On the post assessment question 29a, 11 students correctly solved the problem, 1 student did not respond, and 6 students
wrote incorrect answers. Two of the six made computational errors but used a correct addition method. The other four students used an inappropriate addition method that I never addressed during the study. Two of the four students added the numerators together to get five. Then they picked the bigger denominator. Figure 24 illustrates this inappropriate addition method. The third student added the numerators together to get five. Then she used one of the numerators as the denominator. Her answer was five-halves. The fourth student wrote one and two-fourths. I was not able to determine his thinking process. On post assessment question 29c, twelve students correctly answered the problem, three students did not respond, and three students used inappropriate strategies, similar to those described above.

In addition, pre and post assessment results reveal that most of the students were able to subtract fractions with like denominators and regrouping after instruction. On the pre assessment, only two students attempted to answer question 30, but their answers were incorrect. None of the students attempted to answer questions 31a and 31b. On the post assessment, all 18 students correctly answered question 30, 17 of the 18 students correctly answered question 31a, and 14 of the 18 students correctly answered question 31b (see Appendix F). One student answered question 31a incorrectly because she switched the fractions around so that the greater fraction was on top. The second student left that answer blank. The third student added instead of subtracting and the fourth student drew the correct picture, but miscounted the remaining quantity.

In conclusion, students’ pictures tended to model the operation described in the context of the problem. Two main models were used for addition. One model involved drawing both addends. The other model involved drawing one picture and adding to the picture. Likewise, two main models were used for subtraction. One model involved drawing the largest quantity and
crossing out the minuend. This model was used by most students. The other model involved
drawing pictures of both quantities.

However several students were not able to use their pictures to help them solve the
problems. Two of the students’ pictures did not contain accurate partitions. The other two
students’ pictures contained accurate partitions, but they were unable to make additional
partitions that enabled them to add equal-sized parts.

**Students Invent Algorithms**

As students became proficient at subtracting fractions with regrouping using pictures,
several students noticed that they could solve the problems without drawing pictures. These
students told me they did not need pictures, because they could just use the numbers. They
pleaded with me to let them use the numbers instead of pictures.

Gus: “Mrs. Allen, can I stop drawing pictures. I can do this in my head.”

Thomas: “Yeah, please, please. This is so easy.”

James: “We’ll show our work.”

Mrs. Allen: “As long as you show your work and you can explain what you did, I
don’t see why you can’t use your own algorithms.”

Figure 53 shows one students’ algorithm for subtracting fractions with regrouping. The
problem given was 1 3/5 – 4/5.
Another student “invented” the same procedure that was described in Figure 53. Figure 54 shows that this student could work with the numbers without drawing the pictures. His work demonstrates his ability to convert mixed numbers into improper fractions, find equivalent fractions, and simplify fractions. His work sample also reveals that he understands how to check his work.

A different, but similar algorithm was also “invented” and used by several students in my class. This algorithm involved changing the whole number into an improper fraction based on the denominator of the minuend (see Figure 55). This student changed four wholes into twenty-
fifths and then he subtracted three-fifths. After subtracting, he changed the improper fraction into a mixed number.

![Image: Try these: 4 - 3/5 = 17/5 or 3 2/5.](image)

*Figure 55: Four wholes are changed to twenty-fifths.*

Most students were also able to invent algorithms for adding fractions with unlike denominators. While students were solving problems that involved adding halves and fourths or halves and thirds (see Figures 36, 38 and 43), they realized that they could use Julia’s multiplication strategy or Mike’s division strategy (both strategies were discussed earlier) to find equivalent fractions with the same denominators. While students were solving a problem that required them to add one-fourth and one-eighth, one student made the connection between his pictures and the procedure for adding fractions with unlike denominators.

Chase: “Mrs. Allen, I know an easier way to add these.”

Mrs. Allen: “What is it?”

Chase: “Look, here (pointing at his picture) you see one-fourth is equal to two-eighths. So, you can just turn one-fourth into two eighths and then add four-eighths.”

Mrs. Allen: “You’re absolutely right, but I’m not sure how that’s easier.”
Chase: “Okay, well say you have to add one-fourth and three-eighths. Well, you can just multiply one-fourth by two to get two-eighths, then you can add that to three-eighths.”

Mrs. Allen: “How did you figure that out?”

Chase: “I don’t know. I think I just figured it out because I saw a pattern. Every time the bottom numbers were different, I couldn’t just add them together, I had to find a number they both could go into before I could add them.”

Another student explained how she figured out the “easier” way to add fractions with unlike denominators. Britney was solving a problem that required her to add two-thirds and five-sixths.

Britney: “I know that if I draw pictures, I’m going to have to break the thirds into sixths so that I can add them together, so I just thought, what times three is six and I know that it’s two.”

Britney went on to explain that two-thirds would become four-sixths and the answer would be one and three-sixths or one and one-half. Britney used her mental pictures and understanding of multiplication to find an equivalent fraction with the same denominator as the other addend. Other students also began using their mental images of the pictures they drew and their knowledge of multiplication and/or division to solve addition problems.

The picture in Figure 56 suggests that this student mentally changed one-half into three-sixths, and one-third into two-sixths. Then he combined the quantities. Figure 57 reveals that this student found a common denominator first. Then she changed the fractions using multiplication and finally was able to add. Figure 58 demonstrates the student’s ability to
mentally find equivalent fractions so she could add. First, she changed four-eighths into two-fourths by dividing. Then, she added two-fourths and one-fourth to find three-fourths.

Figure 56: Finding common denominators helps this student add the fractions.

Show how you solved this problem: \( \frac{2}{4} + \frac{3}{6} = \)

\[
\frac{2}{6} \times \frac{2}{2} = \frac{6}{12}
\]

Figure 57: This student multiplies both fractions so their denominators are alike.

3. Jarvis, Victor and Connor bought a pizza that was cut into eight equal slices. Jarvis ate four-eighths of the pizza, Victor ate one-fourth of the pizza and Connor ate the rest. How much of the pizza did Connor eat? Explain.

\[ \frac{4}{8} \text{ equals } \frac{1}{2} \text{ so I changed it to } \frac{1}{2} \text{ and added the other } \frac{1}{4} \text{ with the } \frac{1}{2} \text{ and got } \frac{3}{4} \text{ so I just knew for Connor which is two slices of pizza.} \]

Figure 58: This student changes four-eighths to two-fourths then adds.

However, four of my students struggled to invent algorithms for the addition operation based on their pictures. Two of those students learned the procedure for adding fractions from another student or a parent, but they performed the procedure in a way that demonstrated a lack of understanding. For example, students were given the following addition problem: “Stacy
found 2/4 cup of sugar in the cupboard and ½ cup of sugar in the refrigerator. Show how much sugar Stacy found in all.” Most students immediately knew the answer without drawing pictures or using algorithms.

Vivian: “The answer is one because two-fourths equals one-half and one-half plus one-half equals one whole.”

Yet, Danielle was not able to use her pictures or understanding of equivalent fractions to solve this problem. Instead, she solved this problem by converting both fractions into eighths. Figure 59 illustrates her solution strategy. First, she drew pictures, but did not appear to realize that one-half was equivalent to two-fourths. Then, she thought of a common denominator, which was not the least common denominator, and converted both fractions into eighths. Then she added the fractions and solved the problem. I was surprised that she was not able to use her pictures or understanding of two-fourths being equivalent to one-half to solve this problem because I started the lesson by asking students to name and draw pictures of fractions that were equivalent to one-half and she was able to accomplish this task easily.

Figure 59: Performing operations unnecessarily

Mrs. Allen: “Tell me all the ways we can show one-half.”

Rachel: “Draw a picture with two parts and one is shaded in.”

Mrs. Allen: “How would we write that?”
Rachel:  “One over two.”

Mrs. Allen:  Writes \( \frac{1}{2} \) on board. “How about another way?”

Danielle:  “Draw a picture with four parts and shade in two. That would be two out of four or two-fourths.”

Even though I tried to help Danielle use her pictures and knowledge of equivalent fractions to think through the problem and solve it without using all the conversions, she continued to convert the addends to equivalent fractions using multiplication. I ended up encouraging her to use that method. Sometimes she had a hard time finding a common denominator and other times she did not trust her conversions. She would end up erasing her work and confusing herself to the point that she would say, “I need help” or “I don’t get it.” Figure 60 illustrates the four different ways she tried to solve the problem. In two of the pictures, Danielle does not appear to see that she can combine the quantities, so she tries to multiply. None of the ways are completely done, showing her lack of confidence in any of the ways and her inability to think through the problem.

![Figure 60: A variety of methods tried, but none completed](image)

Another student struggled to invent an algorithm based on her pictures (see Figures 51-52). One day while I was checking the students’ homework, I noticed that Kathy was using a preconceived algorithm that involved listing multiples, selecting a common multiple and then
multiplying. The multiple she selected was not the least common multiple and I knew that she probably did not know why the procedure worked. When I questioned her about her procedure, Kathy told me that her dad taught her how to add fractions with unlike denominators.

Kathy: “I was having a hard time doing my homework last night, so my mom told my dad to help me and he taught me this way that I get and I like it.”

Mrs. Allen: “Do you understand why one-eighth is equal to three-twenty fourths?”

Kathy: “Kinda, but I don’t know. I tried it with the pictures, but I was getting so confused.”

Mrs. Allen: “Okay, draw a picture with eight parts.”

Kathy: Draws a circle with eight parts.

Mrs. Allen: “Now, I need you to make those eight parts into twenty-four parts. So how many pieces will each part need to be cut into to get twenty-four parts?”

Kathy: Thinks for a while. “What times eight equals 24? Three. So if each part has three pieces, three, six, nine…”

Mrs. Allen: “So the reason you can multiply the eight and the one by three is because that’s how many parts each eighth is broken into to get to twenty-four.”

Two other students struggled to add fractions with unlike denominators for the same reason as Kathy. The two students struggled to use their pictures and they never learned the procedure for finding common multiples. Since I did not want them to perform a procedure without understanding the rationale behind it, I never taught it to them. I encouraged them to draw pictures over and over again, hoping that it would develop eventually.

In conclusion, most of the students were able to work flexibly with fractions and develop meaningful algorithms for subtracting fractions with regrouping and adding fractions with unlike
denominators. These algorithms were developed after students had worked to solve problems through pictures. As students became proficient at drawing pictures and adding or subtracting quantities from their pictures, they were able to generalize their procedures into algorithms that could be done mentally or on paper. One of the addition algorithms involved finding equivalent fractions by multiplication. The other addition algorithm involved dividing to find equivalent fractions. One of the subtraction algorithms involved regrouping one whole. The other subtraction procedure involved regrouping all the wholes.

However, several students were unable to make generalizations or develop procedures that closely resembled their pictures. Two of the students were told how to find common multiples and how to change the fractions so that the fractions had common denominators, but they did not appear to understand the rationale behind the algorithm. The other two students never learned how to find common multiples and had trouble using their pictures to add fractions with unlike denominators.

Making Connections between Fractions and Percents

I found that the students who were able to find equivalent fractions and add fractions with unlike denominators, were also able to convert fractions into percents. In fact, their ability to add and subtract fractions, as well as their ability to find equivalent fractions, continued to improve while they were learning to convert fractions into percents. Teaching through a situative perspective with story problems was instrumental in helping my students develop an understanding of the relationship between fractions and percents.
I began instruction with problems such as: “There are 20 questions on the spelling test. Each question is worth the same amount of points. If a student gets all 20 questions correct, that student’s grade will be 100 percent. What will a student’s percentage be if he gets 19 out of 20, or 19/20, correct on the test?”

Many students solved this problem by figuring out “what number times twenty equals 100?” James figured out that five times 20 equaled 100, and then he multiplied 19 times five and got 95%. Thomas figured out the same thing, but he subtracted five from 100. He told me that he knew each question equaled five points, so if he subtracted the one wrong question which equaled five points, then he’d have his answer.

Initially, I found that many students could only solve problems set in familiar contexts, such as grades or tests. During another lesson, in which I did not present the problem in context, Fred struggled to find a strategy to solve the problem.

Mrs. Allen: “Fred, what percent is 1/5?”

Fred: “I don’t know. I know that percent is out of 100, but I don’t remember how to figure it out.”

Britney: “Fred, just think about it as a spelling test. If we had five questions on the spelling test, and you only got two right, what would be your grade?”

Fred: “Oh, a very bad grade. It would be, let me see. Every question is worth, um, 20 points, so I would get a 40% on the test.”

I also used quarters as an example of how to figure out the percent for fourths. I presented students with problems such as: I bought four marbles, three of which were blue. What percent of my marbles were blue?” Most students solved this problem using multiplication, but I wanted them to think about fourths in terms of quarters.
Mrs. Allen: “What other ways, besides drawing pictures and multiplying, could this problem be solved or thought about?” Class was silent, thinking about my question. “Okay, what about quarters?”

Thomas: “Oh, you could think about how many quarters equal a dollar. There’s four quarters in a dollar, so if you have three quarters, you have 75 cents, cause each quarter is worth 25 cents.”

On subsequent lessons, students often found the percentage for a problem that involved fourths by thinking about it in terms of quarters. The following problem was presented: Lester bought four things at the store. He bought an apple, banana, orange and milk. What percentage of the things he bought were not fruits?

Janet: “I figured out this problem by thinking about quarters. There are four quarters in a dollar, and Lester bought four things so each thing is worth 25 percent. There’s only one thing that wasn’t fruit, so that was worth 25%.”

I also showed students how to use pictures to figure out percent. I drew a circle and cut it into twenty pieces.

Mrs. Allen: “How many pieces would each part need to be cut into in order for this circle to equal 100 parts?”

Darla: “Um, I think each part would have to be cut into five pieces, because five times twenty equals 100.”

Then I shaded in 19 of the twenty parts. Next, I cut each twentieth into a hundredth by adding five more parts into each of the 20 parts. By doing this, I illustrated that 19/20 equaled 95 out of 100 or 95%. Every time we discussed a problem like this, I drew the circle and showed the
students the relationship between the percent and fraction. During one discussion about a
problem that described a boy painting his patio and showed a picture that had three-tenths of the
patio shaded in, I found that most students used multiplication to solve the problem (see Figure
61). I also wanted students to prove their answers by drawing me a picture.

![Figure 61: Converting fractions to percents using multiplication](image)

Figure 62 illustrates the way that one student proved his answer. Malcolm noticed that
there were three out of ten squares shaded, so he added ten more squares and shaded three. He
repeated this adding ten and shading in three until he reached 100 squares, with 30 squares
shaded in.

James drew a giant circle and cut it into ten parts. Then he cut one of the tenths into ten
more pieces to illustrate that if he kept going, he would eventually have 100 tiny parts, with 20
shaded in. Janet and Thomas divided 100 by 10 and found that every square was worth 10%.
Figure 63 illustrates the way Thomas attached a percent to each square.
The four students that struggled to find equivalent fractions and add fractions with unlike denominators could solve some of the percentage problems, but only if they were set in a familiar context with familiar numbers, such as tenths and fourths, but not because they understood the operation. They were able to convert fractions to percents because of multiplication short-cuts students had learned earlier in the year. During one problem, Vivian was able to figure out that 6/10 was equal to 60%.

Vivian: “I know that you can add a zero to the 10 and make it 100 and add a zero to the 6 and make it 60.”

Mrs. Allen: “Do you know why you can add a zero to both?”

Vivian: “No.”

Figure 62: For every ten, three are shaded in

Figure 63: Attaching a percent to each square
Mrs. Allen: “Can you draw me a picture to show me why?”

Vivian: “No. I don’t know why you can do that, I just know you can.”

I did not slow instruction down or create enough practice problems for students that struggled with converting fractions into percents. As a result, I did not find that they could convert fractions to percents with understanding, efficiency and consistency. During one lesson, Rachel tried to find the percentage for 15/20. For every 20 circles she drew, she colored in 15. This process took her over 15 minutes, and when she finished, she miscounted and came up with the wrong answer. She didn’t put her 20 circles in rows, they were altogether, so she couldn’t even see that five rows of 20, with 15 shaded in each row would be the same as multiplying 15 times five.

In conclusion, making connections between everyday events and calculating percents appeared to help my students understand the process of converting fractions into percents. Most students used their understanding and experience with spelling tests to find the percentage. Students also thought of percents in terms of quarters. Eventually, most students were able to convert fractions into percents through multiplication and/or pictures, even when problems were not presented in context. Several students did not firmly develop their understanding or ability to convert fractions into percents. Some of their methods were inefficient and others were done without understanding. These students’ inability to find equivalent fractions seemed to affect their abilities to develop computational fluency in other areas such as finding percents and adding fractions with unlike denominators.
Summary

In conclusion, the data suggest that teaching through a situative perspective with story problems was a vehicle that provided students with mathematical concepts to discuss and solve through pictures. The discussions and pictures that formulated during and after students solved story problems helped many students construct their knowledge about fraction concepts and operations. Some of the discussions and pictures helped students understand the importance of the size of the parts. Some of the discussions and pictures helped students realize the folly in adding denominators, and some of the discussions and pictures helped students develop their ideas about one-half.

The data also suggests that the discussions and pictures appeared to help many students develop flexible solution strategies. Some of the strategies students used to find equivalent fractions for one-half included: adding the numerator twice, dividing the denominator by two, and using the rationale of one is half of two and two is half of four. Some of the strategies students used to subtract fractions included: regrouping one of the wholes, regrouping all of the wholes, and adding up from the minuend. Some of the strategies students used to convert fractions into percents included: multiplying the denominator by a number that would make it equal to 100, dividing 100 by the denominator, and thinking of quarters.

In addition, the data revealed that the context of problems appeared to affect students’ representations. Most of the pictures students drew closely resembled the objects in the context. Problems involving cups were typically represented by vertical rectangles. Problems involving length were usually represented by horizontal rectangles or number lines.
Furthermore, the data indicate that the pictures drawn appeared to closely model the operation described in the story problem. Most addition problems included pictures of the object before and after the quantities were combined. Some addition problems just showed one picture of the combined quantities. Most subtraction problems started with drawings of the largest quantity. Students would cross out or shade in the minuend to show the action of the operation. Other subtraction problems included drawings of both quantities.

Overall, the data indicate that teaching through a situative perspective with story problems had a positive affect on my students’ abilities to understand fraction concepts and perform fraction operations. All the students in my study demonstrated the ability to recognize and name equivalent fractions for one-half and one whole. All the students also demonstrated the ability to add fractions with like denominators. In addition, all the students demonstrated the ability to subtract fractions with like denominators and regrouping. Fourteen out of 18 students demonstrated the ability to compare fractions. Fourteen students demonstrated the ability to consistently add fractions with unlike denominators and 14 students demonstrated the ability to convert fractions into percents.

In the next chapter, I will discuss the results of this study and draw conclusions about my findings. In addition, I will share recommendations to enhance quality of instruction and students’ learning in the future.
CHAPTER 5: CONCLUSIONS

The purpose of this study was to investigate the effects of teaching fractions through a situative perspective with story problems on students’ understanding of fraction concepts and their abilities to perform fraction operations with understanding. The 12-week study was conducted within my fifth-grade classroom and involved twenty students. The themes that emerged from the data stemmed from the concepts and operations we were studying. The concepts and operations included: equivalence; addition-of-fractions; subtraction-of-fractions; and converting fractions into percents.

Conclusions

In my study, I found that teaching fractions through a situative perspective with story problems helped students’ develop a conceptual understanding of fractions that was consistent with other research studies (Empson, 1999; Huinker, 1998; Mack, 1990; Sharp, Garofalo & Adams, 2002) because the problems furnished “a context for students to use concrete objects and diagrams to explore and make sense of fraction concepts” (Huinker, 1998, p.170). My teaching methods positively affected my students’ understanding of fraction concepts, which is supported by the fact that all of my students recognized and were able to find equivalent fractions for one-half and one whole using an invented solution strategy that was meaningful to each individual student. In addition, 80% of my students were able to convert fractions into percents using an invented solution strategy that was meaningful to each individual student. All of my students were able to justify their solution strategies in ways that revealed their thinking, reasoning and
understanding of the concepts. Finally, all of my students demonstrated flexibility in their choices of and abilities to use a variety of models and representations to solve problems.

My students constructed their understanding of fraction concepts by solving realistic problems and discussing their solution strategies. Instead of learning fractions through memorization and practicing rote procedures, my students learned that fractions make sense and fraction topics are connected. This can be seen in the fact that 80% of my students were able to add fractions with unlike denominators and subtract fractions with regrouping using invented algorithms that were closely related to their understanding of equivalence. In addition, nearly all of my students were able to avoid making common errors such as adding the denominators and switching around the fractions when subtracting which suggests that my students did not have a distorted belief that mathematics is “a collection of mysterious and often magical rules and procedures that must be memorized and practiced” (Burns, 1994, p. 472).

The results of this study made me acutely aware that students’ conceptual knowledge provided the base for them to develop their procedural knowledge. I discovered that students whose conceptual understandings were fragile struggled to develop procedural knowledge. Although I was tempted to introduce these students to formal algorithms, I did not because “teachers should be cautious to neither rush the child’s invention of an algorithm nor to push the child toward a preconceived algorithm” (Sharp, Garofalo & Adams, 2002, p. 26). Van de Walle (2004) states that

Premature attention to rules for fraction computation has a number of serious drawbacks. None of the rules helps students think about the operations and what they mean… students have no means for assessing their results to see if they make sense. Surface mastery of rules in the short term is quickly lost (p.265).
Van de Walle’s words were exemplified by the girl in my study whose father taught her a formal procedure for finding equivalent fractions which she became proficient at, even though she did not know why the procedure worked.

This study also made me appreciate the important place pictures and/or diagrams have in learning mathematics, especially fractions.

Children’s uses of pictures enable them to understand and resolve situations and perform procedures they might otherwise find beyond their grasps. Pictures also allow crucial issues, such as the importance of common denominators, to become clear at a conceptual level (Sharp, Garofalo & Adams, 2002, p.27).

Many students’ first solution strategy involved drawing pictures. After repeatedly using pictures as a means for solving problems, most students were able to make generalizations about fractions that gave students the ability to form mental images and invent algorithms. “Students who understand the structure of numbers and the relationships among numbers can work with them flexibly” (NCTM, 2000, p.149).

Finally, students in the study contributed valuable insights during discussions that served as “a source of learning” (NCTM, 2000, p.145) for every student in the class. Malcolm’s statement about how to determine if a fraction was equivalent to one-half was repeated throughout the entire study by various students. Individual student’s justifications and strategies also broadened and enhanced the learning environment for everyone. This occurred because students’ ideas were valued and seen as a source of learning. NCTM Principles and Standards (2000) states that

The teacher establishes the model for classroom discussion, making explicit what counts as a convincing mathematical argument. The teacher also lays the groundwork for students to be respectful listeners, valuing and learning from one another’s ideas even when they disagree with them (p.146).
I believed that my students were mathematicians and their discussions would help them to see themselves as mathematicians. Their discussions proved to be intellectually stimulating and provided opportunities for students to help each other make sense of the mathematics.

**Discussion**

This study provided me with a portrait of one group of fifth-graders’ ability to learn fraction concepts and operations through a situative perspective with story problems. I found that the careful selection of problems was a key factor in helping students develop and deepen their understanding of fraction concepts and operations. When I designed instruction based on students’ thinking and work samples, my students were able to use their conceptual knowledge in ways that developed, deepened and expanded their procedural knowledge. When I did not base instruction on students’ thinking and work samples, I found that my students were either bored or frustrated. I came to understand that one of the most important jobs a teacher performs is carefully selecting problems, based on informal and formal assessments, which build upon students’ knowledge in increasingly sophisticated ways. The increased ability to perform that job has direct affects on students’ learning.

I also found that my role as facilitator had direct affects on students’ learning. When I thought about the different ways my students would solve problems and how those ways fit into what I wanted them to learn, I was more prepared to guide discussions so that students learned the fraction concepts and/or operations that I planned for the lesson. For example, during the first several lessons of the study, I wanted my students to learn about equivalent fractions for one-half. I purposely presented problems that I thought my students would solve in two different
ways, and then called on students that had one-half and two-fourths. When I discovered that one student had extended knowledge about equivalent fractions, I allowed the discussion to focus on his knowledge so that other students would benefit. Knowing who to call on, what to ask, and what to do with the knowledge shared is an art that must be developed by a facilitator if he or she wants to give his or her students’ the greatest opportunities to learn. I believe that my ability to facilitate discussions in ways that gave students opportunities to construct their knowledge was crucial to the success of this study.

I will continue to use story problems when teaching fraction concepts and operations, and I will continually strive to select problems that build upon my students’ knowledge in ways that deepen it. I believe using a situative perspective with story problems is a powerful way to teach fractions because it promotes problem-solving and affords students the opportunity to engage in mathematics in deep and meaningful ways, much like a real mathematician because topics are integrated, not isolated and fragmented. “This integration of fraction topics seems to occur naturally in the context of problem situations” (Huinker, 1998, p. 174). Lastly, I believe that the teacher’s primary responsibility during instruction is to elicit students’ thinking and create an atmosphere where students learn from each other through the discussions and problems presented.

Recommendations

The reflection process that I have gone through during this study has led me to believe that certain changes to my implementation of teaching fractions through a situative perspective with story problems could make its effectiveness even greater. One of the benefits of teaching
fractions through a situative perspective with story problems, particularly equal sharing situations, is that it allows students the freedom to solve problems in a variety of ways. However, I think that students can get comfortable using only one method, the method that works for them, and they can stop analyzing the efficiency and effectiveness of other methods. Next year, I will make a point of getting students to analyze the various solution strategies in search of the most efficient and effective methods, as I believe this analysis will deepen students’ understanding of the mathematics and the purposes for common algorithms.

Another change I recommend centers around the sequencing or pacing of instruction. Being that this was the first time I used equal-sharing situations and that the discussions about the various solution strategies can take a fair amount of time, I did not always plan enough time for discussions and/or I may have presented too many problems during one lesson. In the future, I will be careful to limit the number of problems during one lesson in order to allot an adequate amount of time for groups to discuss their strategies so that no one is hurried or rushed.

A third recommendation involves teacher preparedness. I believe that the amount of time I spent analyzing students’ work helped me understand where my students’ knowledge was fragmented and where it was firmly woven together. However, I did not always spend time analyzing the work samples. Sometimes, I planned instruction on the basis of what a handful of students discussed during the lesson. In the future, I need to be more cognizant of whom or what I am not hearing and/or seeing. I need to identify students that do not understand a basic concept early on and help them to understand the concept before they fall too far behind. This type of preparation will greatly impact the learning environment.

The fourth recommendation involves teacher content knowledge. Due to the nature of equal sharing situations and other story problems, a variety of fraction concepts can be addressed
in a single lesson. This can cause the lesson to spin out of control or in directions that were unforeseen. It can also cause students to become frustrated. In the future, I plan on knowing the purpose for each question I present and the possible extensions or connections to other fraction topics that exist within each question. This will help me keep the lesson focused, while connecting the various but related fraction concepts, and it will help me plan questions that build upon my students’ understanding in ways that deepen it. Knowing how students might respond and bring up other topics will help me point out the connections between lessons and concepts, which will deepen my students’ understanding.

The fifth recommendation involves the use of manipulative aids and planned partitioning activities. Although most of my students were able to draw pictures that helped them to solve problems and make connections, several students’ pictures did not help them. In the future, I think it would be best to introduce those students to manipulative aids. I also think planned partitioning and paper folding activities would benefit the students that struggle to draw accurate pictures. I recommend manipulative aids and planned partitioning and paper folding activities because I believe I have gained a more complete understanding of the ways that the concepts of partitioning and equivalence are prerequisite skills for a majority of other fraction topics and fraction-related topics such as comparing and ordering fractions, adding fractions with unlike denominators and converting fractions into percents. I also believe that I more fully understand what Empson (1995) meant by not introducing her students to manipulatives until their thinking dictated it. I think I have learned to recognize when students need manipulatives and planned partitioning and paper folding activities.

The final recommendation involves communicating more with parents about the rationale behind why I do not teach students common algorithms. I assume that most parents were taught
to perform operations on fractions with little understanding. They were not encouraged to attach meaning to the operation or algorithm. So when their children ask for help, the parents pass along an algorithm for operating on fractions, but do not pass along the rationale behind the procedure. Thus, students’ conceptual and procedural understanding is not firmly developed and may be damaged by the introduction of a procedure before they were ready. Communicating with parents about the rationale behind letting their students invent the algorithms might help parents understand why their children are not learning the way they learned. This communication might also encourage parents that want to teach their children algorithms, to also teach their children the rationale behind the procedures.

In the future, I plan on studying the affects of teaching fractions through a situative perspective with story problems on students’ abilities to retain their knowledge. I will administer a post assessment four and eight weeks after the study has ended to assess what knowledge students retain. In addition, I plan on studying the affects of teaching fractions through a situative perspective with story problems on students’ understanding and abilities to multiply and divide fractions. I believe that this method can help students understand the operations of multiplication and division of fractions, and it can help students find or create meaningful ways to perform these operations.
APPENDIX A: UCF IRB LETTER OF APPROVAL
June 21, 2004

Mrs. Colleen Allen

Dear Mrs. Allen:

With reference to your protocol entitled, “Making Fraction Symbols and Procedures Meaningful for Fifth Grade Students by Building on Their Informal Knowledge,” I am enclosing for your records the approved, expedited document of the UCFIRB Form you had submitted to our office.

Please be advised that this approval is given for one year. Should there be any addendums or administrative changes to the already approved protocol, they must also be submitted to the Board. Changes should not be initiated until written IRB approval is received. Adverse events should be reported to the IRB as they occur. Further, should there be a need to extend this protocol, a renewal form must be submitted for approval at least one month prior to the anniversary date of the most recent approval and is the responsibility of the investigator (UCF).

Should you have any questions, please do not hesitate to call me at 823-2901.

Please accept our best wishes for the success of your endeavors.

Cordially,

Barbara Ward, CIM
Institutional Review Board (IRB)

Copies: Dr. Juli K. Doxon, Associate Professor, College of Education
IRB File
APPENDIX B: LETTER OF PRINCIPAL’S APPROVAL
May 24, 2004

To Whom It May Concern:

I, Mrs. Lorrie Butler, am aware that Mrs. Colleen Allen will be performing an action research thesis based on data collected from her own classroom in the 2004-2005 school year. I give her my permission to collect data on her students' ability to use and build upon their informal knowledge of fractions to make meaning of fraction symbols and procedures.

I am fully aware that she is obtaining her Master's degree in math and science education through the Lockheed Martin/UCF Academy for Math and Science and understand that all data collected will be used solely for the purpose of her thesis document. Please contact me at [email protected] with any questions or concerns.

Sincerely,

[Signature]

Lorrie Butler
Principal
Dear Parent/Guardian:

I am your child’s fifth grade teacher and a graduate student at the University of Central Florida under the supervision of faculty member, Dr. Juli K. Dixon. I am conducting research in my classroom from November 2004- May 2005. The purpose of the research study is to examine my practice of using students’ informal knowledge about fractions to help them make meaningful connections to the fraction symbols and procedures. The results of this study will help me better understand the ways in which I can help students learn fraction concepts best and allow me to design instructional practices accordingly.

Students in the study will receive the same instruction as students who do not participate. Students participating in the study will be interviewed by me, and I will collect work samples from them to use in my research. With your permission, your child will be videotaped and audiotaped during instruction and interviews. All tapes will be transcribed, and at the end of the study the tapes will be erased by me. Although the children will be asked to write their names on the work samples for matching purposes, their identity will be kept confidential and will not be used in the report of the research. I will replace their names with fictitious names. Results will be reported in the form of individual data and group data. Participation or nonparticipation in this study will not affect the children’s grades or placement in any programs.

You and your child have the right to withdraw consent for your child’s participation at any time without consequence. There are no known risks to the participants. No compensation is offered for participation. Group results of this study will be available in June upon request. If you have any questions about this research project, please contact me at (407) 672-3120 or my faculty supervision, Dr. Juli K. Dixon, at (407) 823-4140. Questions and concerns about research participants’ rights may be directed to the UCFIRB office, University of Central Florida Office of Research, Orlando Tech Center, 12443 Research Parkway, Suite 207, Orlando, FL 32826. The hours of operation are 8:00 am until 5:00 pm, Monday through Friday except on University of Central Florida official holidays. The phone number is (407) 823-2901.

Sincerely,

Colleen Allen

I have read the procedure described above and I voluntarily give my consent for my child ____________________________, to participate in Mrs. Colleen Allen’s action research study.

<table>
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APPENDIX D: CHILD ASSENT SCRIPT
Child Assent Script

My name is Mrs. Allen and I am your teacher and a graduate student at the University of Central Florida. I am studying the ways that students use and build upon their everyday knowledge of fractions to make meaning of fraction symbols and procedures. During my study, I will ask you to take a pre-test and post-test. I will also ask you to take part in video/audio taped recordings of you and other classmates working on fraction assignments. I may ask you to take part in audio/video taped interview sessions with me. During the interviews, I will ask you questions about fractions, and maybe teach you about fractions as well. Only my research team and I will have access to the audio/video tapes and tests. If you choose to take part, you may stop at any time and you will not have to answer any questions you do not want to answer. Taking part in this study will not affect your grades. Would you like to participate?

I, _________________________________ would like to participate in Mrs. Allen’s action research.

I, _________________________________ would not like to participate in Mrs. Allen’s action research.
APPENDIX E: NANCY MACK’S LETTER OF PERMISSION
Allen, Colleen M.

From: Nancy Mack
To: Allen, Colleen M.
Cc: Subject: RE: fraction research
Attachments:

Dear Colleen,

It is fine if you use my assessment questions in your study. Actually, several of my questions were derived, or taken from the work of Hiebert & Wearne, Behr and colleagues, Kieren, and Kerslake. I will put the questions in the mail tomorrow along with the reference list from my dissertation. I will highlight the references that were used for the interview questions.

Have a nice day!

Nancy Mack

Nancy K.Mack, Ph.D

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"Allen, Colleen M." < > 06/01/04 4:42 PM

Dr. Mack,

Next week is fine. May I use your assessment questions in my study?

Thank you,

Colleen Allen
Allen, Colleen M.

From: Nancy Mack
To: Allen, Colleen M.
Cc: 
Subject: RE: fraction research
Attachments:

Dear Colleen,

I would be happy to send you copies of my interview questions. Since I originally created these on a MAC 512, which I no longer have, it would be easier for me to send you a copy of the questions by U.S. mail rather than electronically. Would you please provide me with an address for where to send the materials?

Regarding validity and reliability measures for the interview questions. Due to the nature of the study I conducted, it was not appropriate to determine validity and reliability measures for the questions. I started out getting these measures, but as my study took form and progressed, it became apparent that measures such as these were not appropriate for an exploratory, or hypothesis generating study, which mine turned out to be.

I would be happy to look at your IRB and provide any assistance that I can. You can send me a copy electronically if you like, or through U.S. mail at the address in the signature below.

May I ask, what institution are you at and with whom are you working? I am just curious and I am unable to determine this from your messages.

Enjoy today!

Nancy Mack

Nancy K. Mack, Ph.D.

>>> "Allen, Colleen M." < 6/1/2004 1:34:47 PM
>>> 
>>> Dr. Mack,

Thank you so much for emailing me back so soon!!!! I am trying to semi-duplicate your study on using student's informal knowledge to help students make meaningful connections to fraction symbols and procedures. I will be using my entire class of fifth graders though, whereas I think you studied six sixth graders. I would like to see your interview questions and any validity or reliability measures you did on the questions you used. Is that possible?
APPENDIX F: INITIAL ASSESSMENT QUESTIONS
LEARNING FRACTIONS WITH UNDERSTANDING: EIGHT CLINICAL STUDIES

by

NANCY KATHERINE MACK

A thesis submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
(Curriculum & Instruction)

at the

UNIVERSITY OF WISCONSIN - MADISON
1987
INITIAL ASSESSMENT QUESTIONS

General Information
1. What do you like most about math? Why?

2a. What have you learned about fractions so far in school?

2b. Can you look at a picture and tell what fraction is shaded?

2c. Can you draw a picture of a fraction?

2d. Can you multiply and divide fractions?

3. Do you like learning about fractions? Why or why not?

4. Why do you think we need to learn about fractions in school?

Fractions are Numbers
5a. You've learned a lot about whole numbers: 1, 2, 3, 4, 5, 6, etc.; can you think of any ways that fractions are like whole numbers?

5b. Can you think of any ways that fractions are different from whole numbers?

6. What would you tell a friend, who didn't know, what a fraction is?

7a. How would you show a friend what a fraction is?

7b. How would you show this same fraction in another way?

8a. How many numbers are there between 0 and 1? Tell me some of them.
8b. How many numbers are there between 3 and 4? Tell me some of them.

8c. How do you know there are numbers between 0 and 1 and between 3 and 4?

9a. Where should you put the number 5 on this line? Why?

9b. Where should you put the number \( \frac{3}{4} \)? Why?

10. Suppose that you ate this much of a pizza, how much of the pizza did you eat? Why?

11a. How would you show me what the fraction five-eighths looks like?

11b. Show me in another way.

12. Tell me what you think the answer to this problem is close to without working the problem: \( \frac{7}{8} + \frac{5}{6} = \)? Why?

Partitioning

13. Suppose you make two pizzas of the same size, one is a sausage pizza and the other is a cheese pizza. You cut the cheese pizza into eight equal sized pieces, and you cut the sausage pizza into six equal sized pieces. You eat one piece from each pizza. Which pizza did you eat more of? Why?

14. Show me by drawing a picture.

15. Is \( 1 \) \( \frac{1}{2} \) bigger than \( \frac{1}{4} \), smaller than \( \frac{1}{4} \), or equal to \( \frac{1}{4} \)? Why?

16. How much of this rectangle is shaded? Why?
17. How much of this picture is shaded? Why?

18. Suppose you have some cake left over from a party. You have the shaded portion in the picture left over. How much cake do you have left over? Why?

19. Which of these three fractions is the smallest: 3/5, 1/4, 3/10? Why? Which is the biggest? Why?

20. (Use both fraction circles and a picture). Suppose I told you that you could have 2/3 of these cookies, show me how many cookies you would get.

Equivalence

21. Would you rather have 2/3 of your favorite candy bar, or 1/4 of your favorite candy bar? Why?
22. What would you tell a friend, who didn't know, what equivalent fractions are?

23. When do you use equivalent fractions, besides just finding them? Why?

24a. Tell me some fractions that are equivalent to one? Why?

24b. Tell me some fractions that are equivalent to ½? Why?

25c. Tell me some fractions that are equivalent to ¾? Why?

26a. Suppose I have this pizza and told you that you could have the part that is shaded. How much of the pizza would you get? Why?

26b. Tell me another name for how much you would get.

27a. Can you find some fractions in this group that are the same?

27b. Why do you say they're the same?

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**Meaning of Addition and Subtraction**

28. Suppose you need 3/8 cup of white sugar and 3/8 cup of brown sugar to make some cookies, how much sugar do you need?
29a. Try these: \( \frac{2}{3} + \frac{3}{4} = \)

29b. \( \frac{3}{8} - \frac{2}{8} = \)

29c. \( \frac{1}{10} + \frac{3}{5} = \)

30. Suppose you have a board 3 yards long. You cut off a piece that is 7/8 yard long to make a shelf. How much of the original board do you have left?

31a. Try these: \( 4 - \frac{3}{5} = \)

31b. \( 2 \frac{1}{3} - 1 \frac{2}{3} = \)

31c. Show or explain how you solved these problems.

32. I worked this problem: \( \frac{2}{5} + \frac{1}{4} \) and got 5/9 as my answer. Is my answer correct? Why or Why not?
Equal-Sharing Problems

1. Eight children want to share 12 sticks of gum so that everyone gets the same amount. Show how much gum each child will receive.

2. There are 22 muffins in the oven for 4 bakers. The bakers want everyone to get the same amount of muffins. Show how many muffins each baker will get.

3. Three children want to share seven pancakes so that everyone gets an equal amount. Show how many pancakes each child gets.

4. Six children have eight sandwiches. If the children shard the sandwiches evenly, how much will each child receive?

Addition of Fractions

1. Lucy had used 2/3 cup of milk to make pancakes and 2/3 cup of milk to make muffins. Show how many cups of milk she used in all.

2. Mr. Allen spent ¾ of an hour emailing his friends and ¾ of an hour playing online video games. Show how many hours Mr. Allen spent on the internet in all.

3. Fred used 1 4/5 yd of fabric to make a pillowcase and 3/5 yd of fabric to make baby booties. Show how much fabric Fred used in all.
1. Samantha drank $\frac{1}{2}$ cup of milk before breakfast and $\frac{1}{2}$ cup of milk after breakfast. Show how many cups of milk Samantha drank in all.

2. Lester found $\frac{2}{4}$ cup of sugar in the cupboard and $\frac{1}{2}$ cup of sugar in the refrigerator. Show how much sugar Lester found in all?

3. Show how you solved this problem: $\frac{1}{2} + \frac{1}{2} =$

4. Show how you solved this problem: $\frac{2}{4} + \frac{3}{6} =$

5. Mr. Fitz had one-half cup of coffee in the morning and $\frac{1}{4}$ cup of coffee in the afternoon. Show how many cups of coffee Mr. Fitz had in all?

6. The white mouse ate $\frac{1}{2}$ of the cheese that was lying on the counter and the black mouse ate $\frac{1}{3}$ of the cheese that was lying on the counter. Show how much cheese was eaten altogether.
REFERENCES


