An Analysis of the Effect of Vibration Sensitivity on Hydrophone Design

Fall 1978

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AN ANALYSIS OF
THE EFFECT OF VIBRATION
SENSITIVITY ON HYDROPHONE
DESIGN

By

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ABSTRACT

Hydrophones used in the ocean produce spurious outputs due to vibration sensitivity which can severely degrade measurement accuracy. Sources of these vibration inputs are ocean surface waves, flow turbulence, and induced mechanical vibration. The hydrophone response to these vibrations is a noise voltage output. This can lead to a signal-to-noise problem particularly when measurements of small sound pressure levels are to be made. The objective of this thesis is to analyze the vibration response of three typical piezoelectric hydrophone sensor elements configurations and give design methods and constraints for reducing the problem of vibration sensitivity to an acceptable level. The sensor element configurations analyzed are the radially polarized cylindrical shell, radially polarized spherical shell, and axially polarized cylindrical shell. The analysis is carried out due to two causes. An electromechanical analysis is given of the voltage sensitivity of each of the three sensor configurations to the inertial effect of acceleration inputs. The second effect analyzed is the voltage sensitivity of a pressure sensitive sensor element to the hydrostatic pressure amplitude caused by periodic vertical displacement of a hydrophone. Results of the analyses show that the radially polarized cylindrical and spherical shell configurations have zero acceleration sensitivity to inputs on the axes analyzed. An equation is derived for the axial
acceleration sensitivity of the axially polarized cylindrical shell in terms of the equivalent sound pressure. The analysis of hydrophone sensor response to periodic vertical displacements shows high voltage sensitivity to very small displacement amplitudes. Data is given for the maximum permissible vertical displacement amplitude to produce a 20 dB signal-to-noise ratio. Based on these analyses, design considerations are given to minimize hydrophone vibration sensitivity.
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INTRODUCTION

Hydrophones are often used at sea to calibrate other transducers and to make acoustic noise measurements. These standard hydrophones must have an acoustic sensitivity which is constant with time, frequency, temperature, and depth. Despite this stability, spurious outputs occur which severely degrade measurement accuracy. Sources of these unwanted outputs are electrical self-noise and that due to external inputs, such as hydrodynamic flow turbulence and induced vibration. The purpose of this thesis is to analyze the design problems associated with vibration inputs to standard hydrophones.

Vibration is often an inherent condition in the use of hydrophones at sea. When suspended from a ship or buoy, a hydrophone can experience large vertical periodic displacements due to surface waves, subsurface waves, or subsurface currents. Vibration can be induced due to flow turbulence and vortex shedding of electrical and support cables. The hydrophone response to these vibrations becomes a noise voltage output. A requirement for acoustic measurements at sea is the ability to detect and measure very low acoustic pressures [1,2]. These measurement conditions lead therefore to a low signal-to-noise ratio problem when the acoustic and vibration input frequencies are coincident. Also, due to vibration, the hydrophone sensor output voltage can exceed the dynamic range of the
integral preamplifier causing electrical blockage and damage of the electronically unprotected input stage.

The objective of this thesis is, therefore, to analyze the response to vibration of typical hydrophone sensor configurations and give design methods and constraints for reducing this measurement problem to an acceptable level.
SCOPE AND APPROACH

The configuration of a typical standard hydrophone, shown in Fig. 1, consists of a pressure sensitive sensor element which is suspended by a rubber mount inside a metal support frame. This assembly is either immersed in an acoustic coupling liquid which is contained by a molded rubber boot or encapsulated in a thermoplastic elastomer that has a good acoustic impedance match to that of water. Many standard hydrophones have an integral preamplifier whose function is to amplify the sensor output signal and provide a good match between the high output impedance of the sensor element and the low input impedance of the electrical cable. The sensor element is usually fabricated from the piezoelectric ceramic, lead zirconate-lead titanate.

Three common sensor element configurations used in standard hydrophones will be analyzed: (1) the radially polarized cylindrical shell, (2) the radially polarized spherical shell, and (3) the axially polarized cylindrical shell. The analyses of hydrophone sensor element vibration response presented is analyzed from two causes. A sensor element, when subject to vibration input, produces outputs due to two transduction effects. The first effect is the generation of inertial forces within the piezoelectric material which produces a voltage output that is proportional to the acceleration amplitude. The sensor element acts like an accelerometer
Fig. 1. Schematic of a typical standard hydrophone: (1) sensor element, (2) rubber mount, (3) metal support frame, (4) acoustic coupling fluid, (5) molded rubber boot, (6) preamplifier, (7) electrical cable.
which in this case is undesirable. The second effect is that, when subjected to vertical periodic displacements, a pressure sensitive sensor produces a voltage output that is proportional to the resultant hydrostatic pressure amplitude. This is due to the high hydrostatic pressure gradient with depth in the water.

Conclusions from these analyses, and experience with the design and use of these sensor configurations, are presented to allow the designer to minimize hydrophone vibration response.
ANALYSES OF RESPONSE TO ACCELERATION

When a piezoelectric hydrophone sensor element is subject to vibration, inertial forces generated within the material produce a voltage output that is proportional to the acceleration amplitude. In order to determine the voltage output of each sensor configuration due to acceleration, an electromechanical analysis is developed. The following assumptions are made to expedite the analyses, and simplify the final equations:

- Each configuration is a linear system.
- Damping is neglected.
- The cylindrical shell configurations are analyzed assuming a lumped parameter model.
- The acceleration, and therefore displacement, is sinusoidal and in a direction parallel to the axis of symmetry.
- The cylindrical and spherical shells analyzed are thin walled, that is the inertial stress and forces are constant across any wall cross section.

It will be seen that the assumption of neglecting damping in the analyses of the radially polarized cylindrical and spherical shell configurations will have no effect on the resulting acceleration sensitivity. In the analysis of the axially polarized cylindrical shell, the simplifying assumption of neglecting damping is justified because it has the least effect on the steady state solution, and this
assumption greatly simplifies the resulting relationships between the mass and compliance parameters.

Radially Polarized Cylindrical Shell

This hydrophone sensor configuration is shown in Fig. 2. It consists, typically, of a radially polarized cylindrical shell made of piezoelectric ceramic. The interior of the cylindrical shell contains air and is acoustically shielded by rigid end caps hermetically sealed at each end. This assembly is suspended by a rubber mount inside a support frame. The cylindrical shell has electrodes on its inside and outside surfaces. The rubber mount has three purposes, to provide vibration isolation to the sensor, to accurately position the sensor, and to electrically insulate the outer electrode from the metal support frame.

A variation in this mounting method was documented by O'Neill [3] in which two piezoelectric cylindrical shells connected in parallel were attached to either side of a support platform.

Fig. 3 shows the lumped mechanical parameter model associated with this sensor configuration. The acceleration response is determined as a function of these parameters which are under limited control by the designer.

If $F_3$ is assumed to be applied uniformly to the wall of the cylindrical shell in a plane perpendicular to the cylindrical axis at its midsection, then

$$m_d = m_e + m_c/6.$$
Fig. 2. Schematic of radially polarized cylindrical shell sensor element. Interior contains air and exterior is surrounded by acoustic coupling fluid.
Fig. 3. Lumped mechanical parameters of radially polarized cylindrical shell sensor element: $m_d$ is combined inertial mass of one end cap and half of cylindrical shell; $m_m$ is mass of rubber mount; $C_m$ and $C_c$ are axial mechanical compliances of the mount and half cylindrical shell, respectively; $z_1$, $z_2$, $z_3$ are displacement amplitudes of the top end cap, bottom end cap, and center of cylindrical shell, respectively; $z_0$ is the system input displacement of the frame; $F_1$, $F_2$, and $F_3$ are the force amplitudes on the masses and compliances as shown.
The term $m_c/6$ comes from the equivalent mass of a spring being $1/3$ of its total mass or in this case, $(1/3)(m_c/2)$. In this equation, $m_e$ is the mass of an end cap and $m_c$ is the mass of the cylindrical shell. The mass $m_m$ is the lumped mass of the rubber mount and any added inertial mass.

Writing the equations of equilibrium for the inertial forces parallel to the $z$ axis where $z_o$ is the input displacement amplitude, one has

$$F_1 - m_d \ddot{z}_1 = 0, \quad (1)$$

$$-F_2 - m_d \ddot{z}_2 = 0, \quad (2)$$

and

$$F_3 + F_2 - F_1 - m_m \ddot{z}_3 = 0. \quad (3)$$

Equations for the same forces in terms of the compliances are

$$-F_1 + (z_3 - z_1)/C_c = 0, \quad (4)$$

$$F_2 + (z_3 - z_2)/C_c = 0, \quad (5)$$

and

$$F_3 + (z_3 - z_0)/C_m = 0. \quad (6)$$

Solving Eqs. (4), (5), and (6) for the forces, one obtains

$$F_1 = (z_3 - z_1)/C_c, \quad (7)$$

$$F_2 = (z_2 - z_3)/C_c, \quad (8)$$

and

$$F_3 = (z_0 - z_3)/C_m. \quad (9)$$
Let $\omega = 2\pi f$, where $f$ is the frequency in Hz of $z_0$, and let

$$z_1 = -\omega^2 z_1, \quad z_2 = -\omega^2 z_2, \quad \text{and} \quad z_3 = -\omega^2 z_3.$$  

Solving Eqs. (1), (2), (3), (7), (8), and (9) simultaneously gives the equations for the displacement amplitudes

$$z_3 = \frac{z_0}{1 - (\omega/\omega_r)^2 + \frac{2 CM}{C_c} \left[ 1 - \frac{1}{1 - (\omega/\omega_r)^2} \right]}$$  

and

$$z_1 = z_2 = z_3 \frac{1}{1 - (\omega/\omega_r)^2}.$$  

Eq. (10) and (11) because of identical masses $m_d$ and compliances $C_c$ contain only two resonance frequencies even though this model has three degrees of freedom.

If

$$\omega_r = \left( \frac{1}{(C_c m_d)} \right)^{\frac{1}{2}}$$

and

$$\omega_r = \left( \frac{1}{(C_c m_m)} \right)^{\frac{1}{2}}$$

then Eqs. (10) and (11) become

$$z_3 = \frac{z_0}{1 - (\omega/\omega_r)^2 + \frac{2 CM}{C_c} \left[ 1 - \frac{1}{1 - (\omega/\omega_r)^2} \right]}$$  

and

$$z_1 = z_2 = z_3 \frac{1}{1 - (\omega/\omega_r)^2}.$$  

The voltage sensitivity of this sensor configuration due to acceleration can then be derived. Forces of amplitudes $F_1$ and $F_2$ acting on the cylindrical shell generate a voltage amplitude which is expressed by

$$e = \frac{\Delta Z}{C_E} (F_1 + F_2),$$
where $d_{31}$ is the piezoelectric charge coefficient and $C_E$ is the electrical capacitance. Substituting Eqs. (7) and (8) into this equation gives the expression

$$e = \frac{d_{31}}{C_E C_C} (z_2 - z_1). \quad (14)$$

Substituting Eq. (13) into Eq. (14) gives the result

$$e = 0. \quad (15)$$

Radially Polarized Spherical Shell

This sensor configuration is shown in Fig. 4. It consists, typically, of two thin walled, radially polarized, piezoelectric hemispheres hermetically bonded together to form a spherical shell. The hemispheres are connected electrically in parallel and have electrodes covering the inside and outside surfaces. The assembly is suspended by a rubber mount inside a support frame which serves the same purpose as for the cylindrical shell sensor configuration.

This analysis is based on the theory by Timoshenko [4] with the following additional assumptions:

. A distributed parameter model is assumed.
. There are no shear forces in the shell wall.
. There is no bending of the shell wall.

The equations for the shell bounded by the spherical sector shown in Fig. 5 are

$$2\pi a c N_1(\phi) \sin \phi + F_z(\phi) = 0 \quad (16)$$
Fig. 4. Schematic of radially polarized spherical shell sensor element. Interior contains air and exterior is surrounded by acoustic coupling fluid.
Fig. 5. Schematic of inertial forces in the radially polarized spherical shell: $N_1(\phi)$ is the tangential component of the inertial force per unit length of shell wall acting on the edge of the spherical segment having included angle $2\phi$ and chord mean radius $a_c$; $N_2(\phi)$ is the tangential component of the inertial force per unit length of shell wall acting perpendicular to $N_1(\phi)$; $a$ is the mean radius of the spherical shell.
and \[ \frac{N_1(\phi) + N_2(\phi)}{a} = -F_z(\phi), \tag{17} \]

where \( F_z(\phi) \) is the amplitude in the z direction of all the inertial force acting on the spherical segment of included angle \( 2\phi \), and \( F_s(\phi) \) is the inertial force amplitude per unit area of the same spherical segment in the radial direction, both as a function of \( \phi \). Deriving the equation for \( F_z(\phi) \) from the geometry gives

\[ F_z(\phi) = 2m \int_0^\phi m_s \omega^2 z a_c d\phi. \tag{18} \]

Since \( a_c = a \sin \phi \), \( \tag{19} \)

\( m_s \) is the mass per unit area of the spherical shell, \( z \) is the displacement amplitude, and \( \omega \) is the same as before, Eq. (18) becomes for the upper hemisphere in Fig. 5.

\[ F_z(\phi) = 2ma^2m_s \omega^2 z \int_0^\phi \sin \phi d\phi. \]

Integrating this expression gives

\[ F_z(\phi) = 2ma^2m_s \omega^2 z(1-\cos \phi). \tag{20} \]

Substituting Eqs. (19) and (20) into Eq. (16) gives

\[ N_1(\phi) = -\frac{am_s \omega^2 z}{1+\cos \phi}. \tag{21} \]

Deriving the equation for \( F_s(\phi) \) from the geometry of the upper hemisphere yields the result
Substituting Eqs. (21) and (22) in to Eq. (17) gives

\[ N_2(\phi) = a m_s \omega^2 z \left( \frac{1}{1+\cos\phi} - \cos\phi \right). \]  \hspace{1cm} (23)

If \( m_s = \rho_o t, \)

where \( \rho_o \) is the mass density of the piezoelectric ceramic and \( t \) is the shell wall thickness, Eqs. (21) and (23) become

\[ N_1(\phi) = -\frac{a \rho_o t \omega^2 z}{1+\cos\phi}, \] \hspace{1cm} (24)

and

\[ N_2(\phi) = a \rho_o t \omega^2 z \left( \frac{1}{1+\cos\phi} - \cos\phi \right). \] \hspace{1cm} (25)

The voltage output amplitude of the upper hemisphere due to acceleration in the z direction as a function of \( \phi \) is then

\[ e(\phi) = g_{31} [N_1(\phi)+N_2(\phi)], \] \hspace{1cm} (26)

where \( g_{31} \) is the piezoelectric voltage coefficient. Substituting Eqs. (24) and (25) into Eq. (26) gives

\[ e(\phi) = -a \rho_o t g_{31} \omega^2 z \cos\phi. \] \hspace{1cm} (27)

The total voltage amplitude generated on the upper hemisphere can be found by integrating Eq. (27),

\[ e_u = -a \rho_o t g_{31} \omega^2 z \int_0^{\pi/2} \cos\phi d\phi, \]
For the lower hemisphere of Fig. 5, the amplitude of \( F_z(\phi) \) will be in the opposite direction to that of the upper hemisphere giving, from Eq. (20), the result

\[
F_z(\phi) = -2\pi a^2 m_s \omega^2 z (1 - \cos\phi). \tag{29}
\]

Also, the amplitude of \( F_s(\phi) \) will be opposite in direction for the lower hemisphere giving from Eq. (22) that

\[
F_s(\phi) = -m_s \omega^2 z \cos\phi. \tag{30}
\]

Substituting Eqs. (29) and (30) into Eqs. (16) and (17) gives

\[
N_1(\phi) = \frac{\alpha_0 t \omega^2 z}{1 + \cos\phi}
\]

and

\[
N_2(\phi) = -\alpha_0 t \omega^2 z \left( \frac{1}{1 + \cos\phi} - \cos\phi \right).
\]

Substituting these equations into Eq. (26) and integrating to find the voltage output amplitude of the lower hemisphere, one obtains

\[
e_u = -\alpha_0 t g_3 \omega^2 z. \tag{28}
\]

Since the two hemispheres are connected in parallel, the resulting voltage output of the sensor configuration is

\[
e_0 = e_u + e_u = 0. \tag{31}
\]
Axially Polarized Cylindrical Shell

This hydrophone sensor configuration is shown in Fig. 6. It consists of an axially polarized piezoelectric cylindrical shell with a piston end cap cemented to each end. This assembly is sealed concentrically by O-rings inside an air filled tubular housing made of aluminum oxide material. The O-rings provide a high compliance suspension to the inner assembly and seal it inside the housing. The dielectric housing electrically insulates the piston end caps which act as electrodes for the piezoelectric tube. Also, the housing acoustically shields the inner assembly so that only the outside faces of the piston end caps are exposed to sound pressure. The sensor element is suspended by a rubber mount inside a support frame at its midpoint.

Figure 7 shows the lumped mechanical parameter model associated with this sensor configuration. This analysis is based on the simplifying assumption that the assembly consisting of the piston end caps and piezoelectric tube is a rigid body of mass $m_d$. This assumption is valid because the mechanical compliance of this assembly is much lower than $C_m$ and $C_r$. Therefore, the derived acceleration response is valid only at frequencies well below electroacoustic resonance of this assembly.

Writing the equations of equilibrium for the inertial forces parallel to the $z$ axis, one has

$$F_{r1} + F_{r2} - m_d \ddot{z}_e = 0,$$  \hspace{1cm} (32)
Fig. 6. Schematic of axially polarized cylindrical shell sensor element. Interior of ceramic tubular housing contains air and exterior is surrounded by acoustic coupling liquid.
Fig. 7. Lumped mechanical parameters of axially polarized cylindrical shell sensor element: $m_d$ is combined mass of piston end caps and piezoelectric cylindrical shell; $m_h$ is mass of ceramic outer housing; $C_m$ and $C_r$ are the axial mechanical compliances of the rubber mount and O-rings, respectively; $z_e$ and $z_h$ are the displacement amplitudes of the piston end caps and ceramic outer housing, respectively; $z_0$ is the system input displacement of the support frame; $F_{r1}$, $F_{r2}$, and $F_h$ are the force amplitudes on the masses and compliances as shown.
and  
\[-F_{r1} - F_{r2} + F_h - m_h \ddot{z}_h = 0. \tag{33}\]

Equations for the same forces in terms of the compliances are  
\[-F_{r1} + (z_h - z_e)/C_r = 0, \tag{34}\]
\[-F_{r2} + (z_h - z_e)/C_r = 0, \tag{35}\]
and  
\[F_h + (z_h - z_o)/C_m = 0. \tag{36}\]

Solving Eqs. (34), (35), and (36) for the forces  
\[F_{r1} = (z_h - z_e)/C_r, \tag{37}\]
\[F_{r2} = (z_h - z_e)/C_r, \tag{38}\]
and  
\[F_h = (z_h - z_o)/C_m. \tag{39} (39)\]

Letting  
\[\ddot{z}_e = -\omega^2 z_e, \dot{z}_h = -\omega^2 z_h (\omega = 2\pi f \text{ as before}, \text{ and solving Eqs.} \ (32), (33), (37), (38), \text{ and } (39) \text{ simultaneously results in the expression}\]

\[z_e = \frac{z_o}{(1-C_r m_d \omega^2/2)(1-C_m m_h \omega^2)-C_m m_d \omega^2} \tag{40}\]

The voltage sensitivity to acceleration of this sensor configuration can then be derived. Force amplitudes $F_{r1}$ and $F_{r2}$ acting on the ends of the piezoelectric cylindrical shell generate a voltage amplitude, where  
\[2F_{r1} = 2F_{r2} = 2F_r = m_d \ddot{z}_e.\]
This voltage amplitude, due to inertial force \( m_d \ddot{z}_e \), is

\[
e_a = 2F_r d_{33}/C_E, \tag{41}
\]

where \( d_{33} \) is the piezoelectric charge coefficient and \( C_E \) is the electrical capacitance. From Eqs. (37) and (38), one has the result

\[
F_r = (z_h - z_e)/C_r. \tag{42}
\]

Substituting Eq. (42) into Eq. (41) gives

\[
e_a = 2d_{33}(z_h - z_e)/C_E C_r. \tag{43}
\]

Eqs. (32), (37), and (38) can be solved again simultaneously for \((z_h - z_e)/C_r\) giving

\[
(z_h - z_e)/C_r = m_d \omega^2 z_e/2.
\]

Substituting this into Eq. (43), one obtains

\[
e_a = d_{33} m_d \omega^2 z_e/C_E.
\]

Substituting Eq. (40) for \( z_e \) in this expression gives

\[
e_a = \frac{d_{33} m_d \omega^2 z_0/C_E}{(1-C_r m_d \omega^2/2)(1-C_m m_d \omega^2)-C_m m_d \omega^2}. \tag{44}
\]

The voltage sensitivity of this sensor configuration to acceleration inputs on the z axis in mks units of \( V/m/sec^2 \) is

\[
(M_e)_a = \left| \frac{e_a}{\omega^2 z_0} \right| = \left| \frac{e_a}{\omega^2 z_0} \right|.
\]
Substituting Eq. (44) in this equation results in the expression

\[ (M_e)_a = \left| \frac{d_{33}m_d/C_E}{(1-C_r m_d \omega^2/2)(1-C_m m_h \omega^2) - C_m m_d \omega^2} \right| . \]

This equation can be normalized by the substitutions

\[ \omega_{\lambda 1} = \left[ \frac{1}{(C_m m_d)} \right]^{1/2}, \quad \omega_{\lambda 2} = \left[ \frac{2}{(C_r m_d)} \right]^{1/2}, \quad \omega_{\lambda 3} = \left[ \frac{1}{(C_m m_h)} \right]^{1/2}, \]

where \( \omega \) is in rad/sec, giving

\[ (M_e')_a = \left| \frac{d_{33}m_d/C_E}{(\omega/\omega_{\lambda 1})^2 - \left[ 1 - (\omega/\omega_{\lambda 2})^2 \right]\left[ 1 - (\omega/\omega_{\lambda 3})^2 \right]} \right| . \quad (45) \]

The sensitivity parameter given by Eq. (45) does not take into account the acoustic sensitivity of the hydrophone sensor configuration. It also does not provide a useful basis for comparison of various hydrophone sensor configurations. It has been proposed [5,6] that acceleration response be expressed by the ratio of the acceleration voltage sensitivity to the acoustic free field voltage sensitivity. This ratio can be derived theoretically, and also be measured. To evaluate this parameter, the acoustic free field voltage sensitivity of this hydrophone sensor configuration well below electroacoustic resonance is

\[ (M_e)_h = r_a d_{33} g/\varepsilon_{33}, \quad (46) \]

where \( (M_e)_h \) is in \( V/\mu Pa \), \( r_a \) is the ratio of the area of a piston end cap to the cross sectional area of the piezoelectric tube, \( g \) is the length of the piezoelectric tube, and \( \varepsilon_{33} \) is the piezoelectric
dielectric constant. Dividing Eq. (45) by Eq. (46) results in the acceleration sensitivity in terms of the equivalent sound pressure amplitude

$$ (M_p)_a = \frac{\varepsilon_3 m_d / r^2 c_E}{(\omega/\omega_1)^2 - [1 - (\omega/\omega_2)^2][1 - (\omega/\omega_3)^2]} \quad (47) $$

This equation can be simplified further by the equations

$$ r_a = \frac{a_p^2}{(a_2^2 - a_1^2)} $$

and

$$ c_E = \frac{\pi \varepsilon_3 (a_2^2 - a_1^2)}{\lambda}, $$

where $a_p$ is the radius of the piston end caps, $a_2$ and $a_1$ are the outside and inside radii, respectively, of the piezoelectric cylindrical shell, and the other parameters are as given previously. Substituting for these parameters in Eq. (47) and simplifying gives the result

$$ (M_p)_a = \frac{m_d / m_p a_p^2}{(\omega/\omega_1)^2 - [1 - (\omega/\omega_2)^2][1 - (\omega/\omega_3)^2]} \quad (48) $$

which is expressed in units of $\mu$Pa/m/sec$^2$ or $\mu$Pa/g. If Eq. (48) is normalized so that it is only a function of frequency $\omega$, we get

$$ \pi (M_p)_a a_p^2 = \frac{1}{(\omega/\omega_1)^2 - [1 - (\omega/\omega_2)^2][1 - (\omega/\omega_3)^2]} \quad (49) $$

To show how $(M_p)_a$ behaves as a function of frequency, Eq. (49) is
is plotted in Fig. 8 for typical values of $\omega_{1}, \omega_{2},$ and $\omega_{3}$.

Since the right side of Eq. (49) is dimensionless and has a wide range of magnitudes, it is plotted in decibel units as indicated. Since damping was neglected in this analysis, Eq. (48) goes to infinity where the denominator becomes zero, as can be seen in Fig. 8. Setting the denominator of Eq. (48) to zero and solving for $\omega$ gives the roots, where $\omega_{5} > \omega_{4}$,

$$
\omega_{r4,5}^2 = \frac{1}{2} \left\{ \frac{(\omega_{2}\omega_{3})^2}{\omega_{1}} + \omega_{2}^2 + \omega_{3}^2 \right\} \pm \left[ \left( \frac{\omega_{2}^2 \omega_{3}^2}{\omega_{1}^2} + \omega_{2}^2 + \omega_{3}^2 \right)^{2} - 4 \left( \omega_{2}\omega_{3} \right)^2 \right]^{1/2}.
$$
Plot of Eq. (49) showing normalized acceleration sensitivity in dB vs. frequency for an axially polarized cylindrical shell sensor element with the typical sensor parameters:

\[ \omega_{k1} = 1040 \text{ rad/sec} \]
\[ \omega_{k2} = 4220 \text{ rad/sec} \]
\[ \omega_{k3} = 1190 \text{ rad/sec} \]
ANALYSIS OF RESPONSE
TO VERTICAL DISPLACEMENTS

A pressure sensitive hydrophone sensor element, when subject to vertical vibration in water, produces a voltage output due to the resultant periodic hydrostatic pressure change. This is because in water there is a pressure gradient, with depth, of $10^{10}$ $\text{Pa/m}$ (this number does not vary more than 4% for fresh or salt water). A sinusoidal displacement amplitude with vertical amplitude $z$ will result an equivalent sound pressure of

$$p = 10^{10} z.$$  \hspace{1cm} (50)

This is the sound pressure amplitude that the sensor element will detect even if no sound exists in the water. If the vertical acceleration amplitude component $\ddot{z}$ is known, then

$$z = \frac{\ddot{z}}{\omega^2},$$  \hspace{1cm} (51)

and substituting this into Eq. (50) gives an equivalent sound pressure of

$$p = 10^{10} \frac{\ddot{z}}{\omega^2}.$$  \hspace{1cm} (52)

If the free field voltage sensitivity of the hydrophone sensor element is

$$(M_e)_h = \frac{e}{p},$$  \hspace{1cm} (53)
where \( e \) is the open circuit voltage output for sound pressure \( p \), then the voltage sensitivity to a vertical displacement amplitude \( z \) is

\[
(M_e)_d = (e/p)(p/z) = e/z. \tag{54}
\]

Substituting Eqs. (50) and (53) into this equation results in

\[
(M_e)_d = 10^{10}(M_e)_h. \tag{55}
\]

These equations can be used to calculate the maximum allowable pressure amplitude due to vertical displacement of a hydrophone for accurate acoustic measurements. Assume the output of the hydrophone sensor element is the sum of the voltages generated by the sound pressure amplitude and the pressure amplitude due to a vertical displacement in phase and at the same frequency. If this second voltage is considered a noise voltage and a minimum signal to noise ratio of 20 dB (20 \( \log 10 \)) is desired, then from Eqs. (53) and (54),

\[
(M_e)_h \frac{p}{(M_e)_d} z_a \geq 10,
\]

where \( z_a \) is the maximum allowable vertical displacement amplitude. Substituting Eq. (55) into this equation gives

\[
z_a \leq 10^{-11} p. \tag{56}
\]

To express this equation in more common units, solve for \( p \) in the equation

\[
SPL = 20 \log (p/p_o). \tag{57}
\]
where SPL is the sound pressure level in dB and $p_0$ is a reference sound pressure of 1 μPa. This gives, by substituting the expression for $p$ from Eq. (57) into Eq. (56), the result

$$z_a < 10^{(SPL/20)-11}.$$  \hspace{1cm} (58)

Figure 9 is a plot of Eq. (58). It shows, for a 20 dB signal to noise ratio, the maximum allowable rms displacement versus a typical range of sound pressure levels measured in the ocean when the signal and noise frequencies are coincident and in phase.

If the maximum permissible vertical acceleration amplitude is desired, substituting Eq. (51) into Eq. (58) gives

$$\ddot{z}_a < \omega^2 10^{(SPL/20)-11}.$$  \hspace{1cm} (59)

Therefore, by multiplying the displacement values obtained from Eq. (58) or Fig. 9 by $\omega^2$, the maximum allowable vertical acceleration amplitude can be calculated as a function of frequency.

Hydrophones used for ambient noise measurements at sea are often subject to large vertical displacements due to surface wave conditions which predominate in a continuous spectrum of .03 to 1 Hz. Acoustic measurements in this frequency range include typical sound pressure levels as low as 121 dB re 1 μPa at 1 Hz [7], which from Eq. (58), results in a .011 mm rms maximum allowable displacement amplitude. This displacement amplitude would be exceeded by a hydrophone suspended from floating structures where surface wave heights of 0.3 m or larger result from the predominant sea states
Fig. 9. Plot of Eq. (58) showing maximum hydrophone displacement amplitude vs. sound pressure level to achieve 20 dB signal-to-noise ratio when signal and noise frequencies are coincident and in phase.
of number 2 or greater. These displacement inputs could conceivably be isolated from a hydrophone by employing a mechanical isolation mount. The critical parameter of the mount would be its mechanical compliance which can be calculated from the equation

$$f_0 = \frac{1}{2\pi} \left( \frac{1}{C_m m_h} \right)^{1/2},$$

(60)

where $f_0$ is mount natural frequency, $C_m$ is the mount compliance, and $m_h$ is the hydrophone mass. From reference [8], the required natural frequency $f_0$ to achieve a 90% reduction in displacement, with a damping ratio of 0.5, would be $1/10$ of the lowest operating frequency. In this case, the lowest operating frequency is .03 Hz, so $f_0$ would be .003 Hz. For a typical value of $m_h$ of 2 kg, Eq. (60) gives a mount compliance of 1410 m/N (2.47 x $10^5$ in/lbf).

Excessive sensor voltage outputs caused by periodic vertical displacements in the .03 to 1 Hz frequency range can prevent acoustic measurements above 1 Hz by causing electrical blockage of the hydrophone preamplifier. Substituting Eq. (50) into Eq. (53) and solving for $z$ gives

$$z_b = 10^{-10} e_m / (M_e)_h.$$

(61)

Here the parameters are $z_b$, the maximum allowable rms hydrophone displacement amplitude, $e_b$, the sensor output voltage causing the onset of electrical blockage, and $(M_e)_h$, as defined before. Values of these parameters for a typical noise measuring hydrophone are $(M_e)_h = -183$ dB re 1 V/µPa and $e_b = 0.5$ V, giving $z_b = 71$ mm rms.
CONCLUSIONS AND RECOMMENDATIONS

The radially polarized cylindrical and spherical shell sensor configurations have zero voltage sensitivity to acceleration inputs on the axis analyzed at any frequency, as shown by Eqs. (15) and (31). Little damping is inherent in the piezoelectric ceramic sensor material and has been realistically neglected in the analysis. The rubber sensor mount and acoustic coupling fluid provide high damping but neglecting this does not affect the acceleration sensitivity analyzed because of the voltage cancellation effect inherent in these configurations. These sensor configurations are sensitive to acceleration inputs in directions other than that analyzed. Therefore, hydrophone designs employing these sensor types should be oriented so that vibration inputs are confined to the axis of zero acceleration sensitivity. This can be achieved by the orientation of the sensor element within the hydrophone and the hydrophone mounting orientation in the system.

The analysis of the axially polarized cylindrical shell sensor configuration shows that its acceleration sensitivity in any direction must be a design consideration. Examination of Eq. (48) shows that minimizing parameter \( m_d \) and maximizing parameter \( a_p \) lowers the acceleration sensitivity \( (M_p')_a \) at all frequencies. The magnitude of \( m_d \) can be minimized independently of the other parameters by fabricating the piston end caps of a low density material with a
low electrical conductivity. However, maximizing the magnitude of $a_p$ results in an increase in the magnitude of $m_d$ for a constant piston end cap thickness. Decreasing the piston end cap thickness, while reducing parameter $m_d$ and/or allowing an increase in parameter $a_p$, decreases its lowest mechanical resonant frequency, which must occur above the usable hydrophone frequency band.

Examination of Eq. (49) and Fig. 8 for the axially polarized cylindrical shell configuration shows that its acceleration sensitivity versus frequency passes through two resonances, $\omega_{k4}$ and $\omega_{k5}$ ($\omega_{k5}\omega_{k4}$) before declining at the rate of -24 dB per octave. If the actual damping were considered in this analysis the acceleration sensitivity would be much lower at $\omega_{k4}$ and $\omega_{k5}$, and higher on either side of these resonance frequencies, due to the highly damped rubber mount and O-ring suspensions. It is not apparent from Fig. 8 that the electroacoustic resonance frequency of the axially polled cylindrical shell sensor element is much higher than frequency $\omega_{k5}$. This is due to the frequency of electroacoustic resonance being inversely proportional to the square root of the low axial compliance of the piezoelectric cylindrical shell. Although Eqs. (48) and (49) are not valid near electroacoustic resonance, the acceleration sensitivity would be low at this resonance frequency due to its rapid decline above frequency $\omega_{k5}$. Note finally that increasing the mass $m_d$ increases frequencies $\omega_{k1}$, $\omega_{k2}$, and $\omega_{k3}$ with a resulting increase in resonance frequencies $\omega_{k4}$ and $\omega_{k5}$.

Consideration of the analysis of hydrophone response to
periodic vertical displacements results in several conclusions. Equation (58) shows that the maximum vertical displacement amplitude that can be permitted to achieve at least a 20 dB signal-to-noise ratio is independent of frequency and hydrophone acoustic sensitivity. The impossibility of achieving the required displacement isolation over the frequency range of .03 to 1 Hz occurring in acoustic measurements at sea is pointed out by the extremely high isolation mount compliance calculated from Eq. (60) in the analysis. A mount having a compliance of 1410 m/N would be impossible to construct. Therefore a hydrophone must be removed from the effect of surface wave motion in order to make accurate acoustic measurements due to the extreme sensitivity to vertical displacements shown in Fig. 8. However, displacement isolation can be achieved at higher frequencies by a properly designed mechanical isolation mount. This can be seen by examination of Eq. (60) which shows that the required isolation mount compliance is inversely proportional to the square of its resonance frequency.

The effect of an excessive sensor voltage output causing electrical blockage of the hydrophone preamplifier can be predicted by inspection of Eq. (61). The maximum displacement amplitude is proportional to the blocking voltage and inversely proportional to the hydrophone sensor acoustic sensitivity. The small magnitude of $z_b$ calculated for a typical noise measuring hydrophone in the analysis points out the severe constraint placed on this parameter. Protection against excess sensor voltage can be provided by diode
protection of the preamplifier input [9]. Protection can also be achieved by designing the preamplifier gain characteristics to give a -3 dB break frequency that is as far as possible above the frequency where the excess sensor voltage output occurs. Attenuation of this voltage is achieved as frequency decreases at the rate of -6 dB per octave below the break frequency.

It is recommended that the problem of hydrophone vibration sensitivity be minimized by the following design considerations. First an analysis of the system in which the hydrophone is to be used should be made. The critical parameters to be determined are the hydrophone displacement or acceleration input amplitudes and their corresponding frequencies, the latter of which is the most easy to predict.

If the vibration input frequencies are within the acoustic measurement frequency range then, if possible, one should determine if vibration amplitudes exist which could compromise acoustic measurement accuracy. This can be predicted by the use of Eqs. (48) and (58), and vibration testing under simulated conditions. It must be kept in mind that vibration induced noise voltage is the sum of the voltage caused by acceleration added to that caused by vertical displacement. The vibration sensitivity of the hydrophone can then be reduced by two methods. First the hydrophone should be configured so that the predominant vibration inputs are confined to the sensor axis having the smallest acceleration sensitivity. Second, one should provide for the mechanical isolation of the
hydrophone from vibration inputs if the excitation frequencies are high enough that a practical isolation mount can be constructed (Eq. (60)). If the vibration input frequencies are not within the acoustic measurement frequency range but will cause excessive hydrophone sensor voltage outputs (Eq. (61)) then one method of dealing with the problem is attenuation of the sensor output by appropriate preamplifier design.

In summary, vibration is an environmentally induced noise problem that must be considered in the design of standard hydrophones used to make measurements in the ocean. The acceleration amplitude to which any sensor configuration is exposed is proportional to the frequency of vibration squared, while sensitivity to vertical displacement amplitude is independent of frequency. Also, hydrophones used in the ocean are usually subjected to large vertical displacements at frequencies in the infrasonic and low audio range. Therefore, noise outputs due to vibration inputs to a hydrophone are dominated by the effect of vertical displacement sensitivity at low frequencies and acceleration sensitivity at higher frequencies.
LIST OF REFERENCES


