Design Curves for Groundwater Mounds Beneath Recharging Basins

1978

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DESIGN CURVES FOR GROUNDWATER MOUNDS BENEATH RECHARGING BASINS

BY

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RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the graduate studies program of the College of Engineering of Florida Technological University

Orlando, Florida
1978
ABSTRACT

Design curves are developed to simplify solution of the Hantush (1967) equation for groundwater mound build-up under a rectangular recharge basin. Curves are developed by analyzing a large number of solutions to the theoretical equation. Two sets of curves are developed, one for build-up beneath the center of the basin, the second for build-up anywhere in the field. Center-point curves are presented for 3 basins with length to width ratios of 2 to 1, and were easily developed. The more generalized curves are developed for the limited case of a 200 ft by 100 ft basin and are complicated by the many parameters involved. They require a multi-step process for utilization.

A computer program, DELTAH, was developed to produce the required data by solving the Hantush equation. DELTAH may be the most valuable solution method as it can be applied to a variety of cases. A user's manual and example problem are included.
ACKNOWLEDGEMENT

I extend my gratitude and respect to:

Dr. J. P. Hartman for serving as my committee chairman and providing guidance and support for this research report, and for serving as my academic advisor at Florida Technological University.

Dr. W. M. McLellan and Dr. M. P. Wanielista for serving on my committee and reviewing this report.

Dr. Hartman, Dr. McLellan, Dr. Wanielista, and Dr. Y. A. Yousef who form the core of the Environmental Sciences Program at F.T.U.; its high quality is a reflection of their qualities as people, instructors, and engineers.
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I. INTRODUCTION

Artificial Recharge and Groundwater Mounds

The build-up of a groundwater mound is a phenomenon produced by the artificial recharge of a water table aquifer. Artificial recharge is typically accomplished through recharge basins and injection wells. The following descriptions are taken from an American Water Works Association (AWWA) Task Group Report (1963):

Artificial ground water recharge through recharge basins consists of the impoundment of surplus waters at selected locations where the permeability of the underlying soils permits the infiltration of the impounded waters into the water-bearing stratum of the subsoil.

Recharge basins are used for the following purposes:

1. To conserve surplus rain waters [storage of rainfall excess]
2. To supplement the quantity of ground water naturally available
3. To reclaim waste waters and make them part of the available ground water supply
4. To remove suspended matter by filtration

Other advantages ... are: (1) artificial drainage of storm water and waste water effluents without construction of long conduits to streams or tide waters; (2) reduction in size of conduits for storm water drainage because of impoundment of peak flows ...; (3) reduction of stream flood flows by impoundment.

The use of recharge systems is becoming more and more important as water demands become more intense and as water pollution
and stormwater control measures are instituted. An excellent example of the successful use of basin recharge is provided by Long Island, New York.

Originally, Long Island drew all of its water from wells and replenished the aquifer by returning the wastewater through cesspool systems. As development progressed with population growth and increased sewerage systems the wastewater which previously had provided the recharge was wasted to tide waters resulting in serious aquifer depletion and salt water intrusion (Baffa 1967). Between 1935 and 1964 more than 400 stormwater storage and recharge basins were placed into operation in Nassau County with 40 mgd annual recovery estimated for 1964 (Hunziker 1964).

Design and implementation of recharge basins requires an understanding of their effect on the water table surface for prediction of increased storage capacity, loading limitations, and possible flooding conditions. Several solutions to this groundwater mound problem have been developed.

Included among these are solutions by Glover (1960), Baumann (1965), Dagan (1967), and Hantush (1967) which are based on Dupuit-Forchheimer theory and utilize linearization techniques to solve the non-linear Boussinesq equation (Tseng 1973). More recent solutions by Tseng and Ragan (1973), Hunt (1971), Brock (1974), and Singh (1976) utilize finite difference techniques. Results of these studies indicate that the linearization methods are of limited accuracy since the Dupuit assumption fails in regions of steep
hydraulic gradients. Brock concluded that solutions using the Dupuit-Forchheimer assumptions closely approximated his results for shallow aquifers only ("as long as the initial saturated depth of the aquifer is less than about 0.6L in which L is the half-width of a strip basin").

Typically, solutions are presented for an infinite strip recharging basin and for the three-dimensional case of a circular recharging area. Glover and Hantush present solutions for the three-dimensional case of a rectangular recharge area. The Glover solution assumes a constant rate of rise for the phreatic surface and an absence of horizontal spreading in addition to the basic assumptions of an isotropic, homogeneous aquifer of infinite areal extent.

The Hantush solution becomes, therefore, the only sophisticated approach to the mound growth beneath a rectangular recharge basin. The Hantush solution claims to be applicable with deviations of less than 6%, as tested by viscous flow models, for water table rises of up to 50% of the initial depth of saturation.

Comparisons of the Hantush solution with actual field data collected by Haskell and Bianchi (1968) for build-up beneath the center-point of two square 2-acre ponds is presented in appendix A.

**Objectives**

The objective of this study was to simplify the use of the Hantush solution for groundwater mound build-up beneath a rectangular recharge area. The primary intention was to determine the
feasibility of developing design curves for percolation basins and to develop such curves if possible. As the number of variables involved is large (see eqs. 1-4) an analytical method of accomplishing this was immediately dismissed. It was hoped that an empirical method could be found by analyzing a large number of solutions.

A secondary objective was thereby introduced which was to develop a large number of solutions. The obvious response to this requirement was to write a computer program which would solve the equation. The program needed to be versatile to data input and able to loop over various parameters to provide output for varying conditions. The program in itself would provide one means of simplifying the use of the Hantush equation.
II. THE HANTUSH SOLUTION

The equations developed by Hantush (1967) estimate the rise and fall of groundwater mounds in response to uniform vertical recharge. The aquifer is assumed to be infinite in areal extent, homogeneous, and isotropic. Solutions for both circular and rectangular recharging areas were developed. The solution for a rectangular recharging area as depicted in figure 1 is as follows (Walton 1970):

\[ h^2 - h_1^2 = \frac{wbt}{15S_y} \left[ W* \left( 1.37(b + x) \sqrt{\frac{S_y}{Tt}} \right) + 1.37(a + y) \sqrt{\frac{S_y}{Tt}} \\ W* \left( 1.37(b + x) \sqrt{\frac{S_y}{Tt}} \right) + 1.37(a - y) \sqrt{\frac{S_y}{Tt}} \\ W* \left( 1.37(b - x) \sqrt{\frac{S_y}{Tt}} \right) + 1.37(a + y) \sqrt{\frac{S_y}{Tt}} \\ W* \left( 1.37(b - x) \sqrt{\frac{S_y}{Tt}} \right) + 1.37(a - y) \sqrt{\frac{S_y}{Tt}} \right] \] (1)

where:
- \( h_1 \) = initial height of water table above aquiclude, in feet
- \( h \) = height of water table above aquiclude with recharge, in feet
- \( w \) = recharge rate, in gpd per ft²
- \( \bar{b} \) = 0.5(\( h_1 + h \)), in feet
- \( t \) = time after recharge starts, in days
- \( S_y \) = specific yield of aquifer, fraction
- \( b \) = one-half width of recharge area, in feet
Figure 1. Diagrammatic Representation of a Water-table Aquifer Artificially Recharged from a Rectangular Area. (Hantush 1967)
x, y = coordinates of observation point in relation to center of recharge area, in feet

T = coefficient of transmissibility, in gpd/ft

a = one-half length of recharge area, in feet

The function of $W^*$ is defined by

$$W^*(\alpha, \beta) = \int_0^1 \text{erf} \left( \frac{\alpha}{\sqrt{T}} \right) \text{erf} \left( \frac{\beta}{\sqrt{T}} \right) d\tau$$  \hspace{1cm} (2a)

Values of $W^*$ are presented in appendix B for the range of $\alpha$ and/or $\beta$ between 0.02 and 3.00. For $\alpha$ and/or $\beta$ outside this range the following relationships hold:

if $\alpha$ or $\beta$ negative

$W^*(-\alpha, \beta) = W^*(\alpha, -\beta) = -W^*(\alpha, \beta)$

$W^*(-\alpha, -\beta) = W^*(\alpha, \beta)$

$W^*(\alpha, \beta) = W^*(\beta, \alpha)$  \hspace{1cm} (2b)

if $\alpha$ or $\beta$ = 0

$W^*(0, \beta) = W^*(\alpha, 0) = 0$  \hspace{1cm} (2c)

if $\alpha \geq 3$

$W^*(\alpha, \beta) \approx 1 - 4i^2 \text{erfc} (\beta)$  \hspace{1cm} (2d)

if $\beta \geq 3$

$W^*(\alpha, \beta) \approx 1 - 4i^2 \text{erfc} (\alpha)$  \hspace{1cm} (2e)

if $\alpha \geq 3$ and $\beta \geq 3$

$W^*(\alpha, \beta) \approx 1$  \hspace{1cm} (2f)

if $\alpha^2 + \beta^2 \leq 0.10$

$W^*(\alpha, \beta) \approx \frac{4\alpha\beta}{\pi} \left[ 3 + W(\alpha^2 + \beta^2) \right] \left[ -\frac{\alpha}{\beta} \tan^{-1} \frac{\beta}{\alpha} + \frac{\beta}{\alpha} \tan^{-1} \frac{\alpha}{\beta} \right]$  \hspace{1cm} (2f)

in which $W(x)$ is the well function

$$W(x) = \int_x^\infty \frac{\exp(-t)}{t} \, dt$$  \hspace{1cm} (3)

The decay of the water table is given by

$$h^2 - h_0^2 = Z(x, y, t) - Z(x, y, t - t_0)$$  \hspace{1cm} (4)

in which $Z$ is the right side of equation 1 and $t_0$ is the time of interest after recharge ceases.
Theoretical Development of the Equation

The Hantush solution develops out of the differential equation of groundwater motion based on conservation of matter:

\[ -\left[ \frac{T \partial v_x}{\partial x} + \frac{T \partial v_y}{\partial y} + \frac{T \partial v_z}{\partial z} \right] + \frac{\partial F}{\partial T} = \rho S_s \frac{\partial \rho}{\partial T} \]  

(5)

in which \( v_x, v_y, \) and \( v_z \) are the fluid velocity components, \( \rho \) is the fluid density, \( F(x,y,z,t) \) is the rate at which water is being released within the field per unit time per unit volume, \( S_y \) is the specific yield, \( S_s \) is the specific storage, \( \phi(x,y,z,t) \) is the piezometric or hydraulic head.

Figure 2 is a diagrammatic representation of flow in a vertically replenished aquifer for a general case (Hantush 1964). Integration of equation 2 with respect to \( z' \) assuming \( \rho \) to be constant, followed by substitution of appropriately derived expressions for \( v_z \), followed by application of the rule of differentiation under the integral sign to interchange the differentiation and integration operations results in the following:

\[ \left( \frac{\partial}{\partial x} \right) \left[ \frac{\partial (b \phi) / \partial x - \phi (H)}{\partial x} \right] + \left( \frac{\partial}{\partial y} \right) \left[ \frac{\partial (b \phi) / \partial y - \phi (H)}{\partial y} \right] + \frac{(u - w)}{K} + \frac{b F}{K} = \left( \frac{S_y}{K} \right) \frac{\partial H}{\partial t} \]

\[ + \left( \frac{S_s}{K} \right) \frac{\partial (b \phi)}{\partial t} - \phi (H) \frac{\partial H}{\partial t} \]  

(6)
Figure 2. Diagrammatic Representation of Flow in a Vertically Replenished Aquifer. (Hantush 1964)
where:

\[
\phi(f) = \phi(x,y,f,t)
\]

\[
\phi(H) = \phi(x,y,H,t)
\]

\[
\overline{F}(x,y,t) = \text{the average rate at which water is added (hypothetically generated) per unit time per unit volume to the flow}
\]

\[
\overline{\phi}(x,y,t) = \text{the average head over the thickness } b = H - f
\]

that is:

\[
\overline{F} = \frac{1}{b} \int_{f}^{H} Fdz \quad \text{and} \quad \overline{\phi} = \frac{1}{b} \int_{f}^{H} \phi dz
\]

By comparison of figures 1 and 2 the following substitutions can be made to develop the solution to the groundwater mound problem:

\[
u = 0 \quad (\text{no percolation at the lower boundary})
\]

\[
f(x,y) = 0 \quad (\text{horizontal, or near so, lower boundary})
\]

\[
H = h
\]

\[
b = H - f = H = h
\]

\[
\overline{\phi}(H) = \overline{\phi}(h) = 0
\]

\[
\overline{\phi} = \frac{1}{b} \int_{f}^{H} zdz = \frac{1}{h} \int_{0}^{h} zdz = \frac{h}{2}
\]

\[
F(z,y,z,t) = 0
\]

These substitutions result in the following expression:

\[
\frac{\partial^{2}}{\partial x^{2}} \left( \frac{h^{2}}{2} \right) + \frac{\partial^{2}}{\partial y^{2}} \left( \frac{h^{2}}{2} \right) - \frac{w}{K} = \frac{S_{y}}{K} \frac{\partial H}{\partial t} + \frac{S_{s}}{K} \frac{\partial}{\partial t} \left( \frac{h^{2}}{2} \right)
\]

(7)

in which \( K \) is the hydraulic conductivity.

Taking \( S_{y} >> S_{s} \), and \( h_{1} \) to be a valid solution for the case of \( w = 0 \), and substituting into equation 7 results in:
which when subtracted from the original equation (7) yields:

\[
\frac{\partial^2 h_1^2}{\partial x^2} + \frac{\partial^2 h_1^2}{\partial y^2} = \frac{2S_y}{K} \frac{\partial h_1}{\partial t}
\]  

(8)

After redefining \( w \) to be positive in the downward direction, defining the term \( \bar{b} = \frac{1}{4}(h + h_1) \) to be a constant of linearization, and creating the step function \( g(x, y) \) (This function defines the recharge rate to be 0 outside of the recharging area boundaries) equation 9 becomes:

\[
\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} + \frac{2w}{K} g(x, y) = \frac{S_y}{K \bar{b}} \frac{\partial Z}{\partial t}
\]  

(10)

in which:

\[
Z = h^2 - h_1^2
\]

\[
g(x, y) = g_1(y) \quad 0 < x < b
\]

\[
= 0 \quad x > b
\]

\[
g_1(y) = 1 \quad 0 < y < a
\]

\[
= 0 \quad y > a
\]

\[
\bar{b} = \frac{1}{4}(h_1 + h)
\]

The boundary conditions for this case are:

\[
Z(x, y, 0) = 0
\]

\[
\frac{\partial Z}{\partial x} (0, y, t) = \frac{\partial Z}{\partial y} (x, 0, t) = 0
\]

\[
\frac{\partial Z}{\partial x} (\omega, y, t) = \frac{\partial Z}{\partial y} (x, \omega, t) = 0
\]
The solution to this system of equations is given by equations 1-4. It was accomplished by Hantush (1967) using successive applications of both Laplace and Fourier transforms and inverse transforms. The function $W^{*}(\alpha, \beta)$ was integrated by parts and the results tabulated. This table is presented in appendix B.
III. DEVELOPED SOLUTION METHODS

The DELTAH Computer Program

DELTAH is a Fortran computer program written by the author which models the Hantush solution for build-up of a groundwater mound beneath a rectangular recharge area, as presented in section II. The model solves these equations in exactly the same arithmetic and algebraic form in which they appear there. Two linear interpolations are performed in extracting the values of $W^*(x,\beta)$ from the table of $W^*$ values in appendix B.

The DELTAH program is written such that all of the parameters can be varied. Nested DO loops are programmed for transmissibility, specific yield, time, recharge rate, and point coordinates $x$ and $y$. Any combination of these parameters may be varied during the same run. The basin size is fixed, and any number of data sets, each referring to a selected basin, can be run at the same time.

The output provides a topographic map of build-up for each set of time, recharge, and formation parameters. Line solutions along the axes of the basin or single point (any desired point) solutions can also be obtained. To obtain solutions along a diagonal traverse, the entire topographic map must be printed. By symmetry, the points $(x,y)$, $(-x,-y)$, $(-x,y)$, and $(x,-y)$ are all equivalent with respect to build-up. With $x$ and $y$ oriented as
shown in figure 1, the program develops the map for the southeast quadrant, which is an image of that for the other quadrants.

A user's manual including input format and descriptions of variables and an example problem with developed input data is included in appendix C.

**Design Curves for Mound Build-up Beneath the Center of the Basin**

Maximum groundwater build-up occurs beneath the center-point of the percolation basin. This value can be used as an indicator for surface flooding or as a maximum value from which to estimate the elevation of the mound surface elsewhere in the field. The build-up ($\Delta h$) is a function of transmissibility ($T$), time ($t$), recharge rate ($w$), specific yield ($S_y$), and the basin dimensions. It follows that for a specified basin, a family of curves, each for a specified $S_y$ and $T$, relating the normalized build-up, $\Delta h/w$, to time will perform the desired function.

A set of three such curves is developed for rectangular basins with length to width ratios of 2 to 1. These are presented as figures 3, 4, and 5. The basin sizes selected are 200 ft x 100 ft ($20,000$ sq ft $= \frac{1}{4}$ acre), 400 ft x 200 ft ($40,000$ sq ft $= 2$ acres), and 800 ft x 400 ft ($160,000$ sq ft $= 8$ acres). As a logarithmic scale is used for these graphs, one graph of build-up vs time is presented as figure 6 to illustrate the shape of the curve in arithmetic coordinates.

Similar sets of curves can be developed using DELTAH for other
Figure 3. Mound Build-up per ft/gpd/ft$^2$ of Recharge Beneath the Basin Center for Stated Transmissibility (gpd/ft) and Specific Yield: Basin = 200ft by 100ft.
Figure 4. Mound Build-up per ft/gpd/ft² of Recharge Beneath the Basin Center for Stated Transmissibility (gpd/ft) and Specific Yield: Basin = 400ft by 200ft.
Figure 5. Mound Build-up per ft/gpd/ft² of Recharge Beneath the Basin Center for Stated Transmissibility (gpd/ft) and Specific Yield: Basin = 600ft by 400ft.
Figure 6. Groundwater Mound Build-up vs Time, for Stated Transmissibilities (gpd/ft) at Center of 200 ft by 100 ft Seepage Basin, $S_y = 0.25$. 
rectangular shapes thereby increasing their usefulness as a design reference.

**Design Curves for Mound Build-up Beneath Any Point in the Field**

**Solution Approach and Development**

To extend the solution from a fixed point to one for any point in the field, the location parameters, x and y, must be introduced. This greatly increases the complexity of the solution.

The problem was approached by observing the geometry of the solution. The topographic output of the DELTAH program showed the iso-build-up contours to be elliptical in shape (ellipses were not rigorously established) and concentric about the projected center of the basin.

The possibility that geometric similarity would produce a useful parameter was tested by using the ratio of the length of the major half-axis, j, to that of the minor half-axis, n, as a measure of eccentricity. (Eccentricity is rigorously defined as the ratio of c to j where c is the focal distance [Protter and Morrey 1964].) As the eccentricity did not remain constant as the distance from the center of the basin, y, increased, similarity failed.

The possibility that the shape of the contour ellipse, (j/n), varied in exactly the same manner for all basins of similar geometry was quickly checked by comparing a sequence of eccentricity vs y/a relationships for three different sized basins. (Note that for a traverse along the major axis, y = j.) This also failed.

The next attempt was based on using the eccentricity of the
ellipse to relate the build-up at any point \((x,y)\) to that at a specific point at which it is known. A 200 ft x 100 ft basin was selected and the specific yield was set at 0.25. Given a family of graphs of build-up vs \(y\) \((x=0)\), each for a specified transmissibility, the build-up at point \((x,y)\) can be determined if a second point \((0,y')\), such that both points lie on the same contour ellipse, is known. The equation of an ellipse whose foci lie on the \(y\) axis is:

\[
\frac{x^2}{2} + \frac{y^2}{2} = 1
\]

Recalling the plottable relationship \(j/n\) vs \(y\) provided the second "equation" for the problem with 2 unknowns.

Since the function \(y/n = f(y)\) is not explicitly defined, the equations cannot be solved algebraically. Fortunately a trial and error method is simplified by the fact that given a point on an ellipse with a known range of "eccentricity", a reasonable first estimate of \(j\) can be made. The problem is then solved by getting \(j/n\) from a graph of \(j/n\) vs \(j\) and solving the equation of an ellipse for \(n\). The initial assumption of \(j\) is then checked by comparing:

\[
? \quad j_{\text{assumed}} = (j/n)_{\text{assumed graph}} \times n_{\text{calculated}}
\]

The process is then repeated until satisfactory agreement is reached. This was therefore accepted as a workable mechanism for relating the build-up at point \((x,y)\) to that at some point along the major axis.

For design purposes graphs of build-up along the major axis and graphs of \(j/n\) vs \(j\) (or \(y\)) are required for useful ranges of
time, transmissibility, specific yield, and basin dimensions.

Continuing with the case of fixed basin size and specific yield, a set of eccentricity charts was prepared for nine different combinations of $T$ and $t$. One such graph is shown in figure 7 for $T = 1000$ gpd/ft and $t = 30$ days. The nine curves all exhibit considerably similar shape and are relatively parallel. This indicated that they might be mappable, one onto another, by a linear function such as:

\[(j/n) = m(j/n) + b_{\text{int}}\]

where $m$ = the constant of slope

$b_{\text{int}}$ = the constant of intercept

The curve for $T = 1000$ gpd/ft and $t = 30$ days was selected as a reference and linear regression was performed to determine the linear constants relating the other curves to it. Values of $j/n(T_1,t_1)$ were taken at intervals of $y = 10$ ft over the range $40$ ft $\leq y \leq 190$ ft. The linear correlations were excellent; results are shown in table 1.

The nine sets of $(m_1,b_1)$ were then plotted in figure 8. Two families of curves were developed; $m$ vs $T$ for various $t$, and $b_{\text{int}}$ vs $T$ for various $t$, with $T$ as a coaxial ordinate. These curves are reasonably consistent and provide a means of obtaining $m$ and $b_{\text{int}}$ for all combinations of $T$ and $t$ within the ranges plotted. This in turn provides a means for determining $j/n$ for the same range of parameters from the reference curve of $j/n$ vs $y$ for $T = 1000$ gpd/ft and $t = 30$ days.
Figure 7. Eccentricity vs Major Axis Length of Topographic Ellipses for Standard Basin: Basin = 200ft. by 100ft., Transmissibility = 1000 gpd/ft, Time = 30 days.
<table>
<thead>
<tr>
<th>Transmissibility gpd/ft</th>
<th>Linear Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 days</td>
</tr>
<tr>
<td>500</td>
<td>1.500(^a)</td>
</tr>
<tr>
<td></td>
<td>-0.432(^b)</td>
</tr>
<tr>
<td></td>
<td>0.995(^c)</td>
</tr>
<tr>
<td>1000</td>
<td>1.170</td>
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<td>2000</td>
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<td>0.998</td>
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<tr>
<td>3000</td>
<td>1.030</td>
</tr>
<tr>
<td></td>
<td>-0.040</td>
</tr>
<tr>
<td></td>
<td>0.999</td>
</tr>
</tbody>
</table>

\(^a\)Slope, m

\(^b\)Intercept, \(b_{int}\)

\(^c\)Correlation constant, \(r\)

\(^d\)Non-conforming data - unexplained
Figure 8. Linear Constants for Eccentricity Correlation vs Transmissibility for Stated Times (days)  
Basin = 200 ft. by 100 ft.
The graphs of build-up along the major axis, shown in figure 9, were easily developed directly from the DELTAH topographic output.

The Hantush solution by this design curve method is therefore generalized for all cases of a 200 ft x 100 ft basin and specific yield of 0.25. Efforts to generalize the solution over other basin sizes and specific yields were terminated due to time constraints.

It must be emphasized here that similar data sets can be developed for a variety of basin sizes, and, that a reasonable possibility exists that some relationship could be developed relating them to one another. Thus, a standard basin would be developable similar to the standard eccentricity curve presented previously (figures 7 and 8).

Use of the Design Curves

The method for using the design curves thus far developed is presented in the following sequence of operations. The procedure assumes a specified basin size and specific yield. (Only one set of curves is developed at this time, although any number of others could be developed if desired.)

1. Determine the "eccentricity", \(j/n\), for the standard case of \(T = 1000\) gpd/ft and \(t = 30\) days:

   a. Estimate \(j\) from the coordinates \((x,y)\)

   The radius of a circle through the point \((x,y)\) will be < \(j\), therefore:

   \[\sqrt{x^2 + y^2} < j\]

   (Also, \(j\) will be larger than either \(x\) or \(y\))
Figure 9. Groundwater Mound Build-up per ft/gpd/ft² of Recharge vs Distance. Basin = 200 ft by 100 ft.
Consider the eccentricity range from figure 7 of 1.1 to 1.6

b. Get j/n from figure 7

2. Solve the following form of the equation of an ellipse for n:

\[ n = \left( \frac{x^2 + y^2}{(j/n)^2} \right)^{\frac{1}{2}} \]  

(a)

3. Check the initial assumption of j:

\[ j_{\text{assumed}} \neq j_{\text{graph}} \times n_{\text{calculated}} \]  

(b)

4. If initial assumption is not acceptable, go to la

5. Adjust j/n to actual T and t, as follows:

a. Get m and b_{int} from figure 8

b. Compute j/n by equation 11

\[ (j/n)_{(T_1,t_1)} = m(j/n)_{(T_2,t_2)} + b_{int} \]

6. Solve equations a and b for n and j using point location values for x and y

7. Get build-up per unit recharge rate (\( \Delta h/w \)) from figure 9

8. Multiply by w to get total build-up

Example Problem

An old sewer pipe is presently situated 5 ft above a water table aquifer. It is located in the vicinity of a proposed 200 ft x 100 ft percolation pond for waste cooling water. The pipe intersects the projections of the basin axes at the points (0,200) and (150,0) (see figure 10). The City will consider early upgrading of the pipe if the elevated water table will cause significant infiltration. The basin will only be used during the months of May through August inclusive. It is expected to be full continuously
Figure 10. Plan and Cross-section of Water Table Aquifer, Recharge Basin, and Sewer Pipe.
throughout that period. Water in excess of the percolation capacity of the basin will be discharged. Soil studies provide the following data:

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability</td>
<td>130 gpd/ft$^2$</td>
</tr>
<tr>
<td>Specific yield</td>
<td>0.22</td>
</tr>
<tr>
<td>Recharge rate</td>
<td>1.5-2.5 gpd/ft$^2$</td>
</tr>
<tr>
<td>Present depth of water table</td>
<td>12 ft</td>
</tr>
<tr>
<td>Present height of water table</td>
<td>19 ft</td>
</tr>
</tbody>
</table>

This data is similar to that found by Haskell and Bianchi (1968) for pond 1 as described in appendix A. The soil was an alluvial fan deposited by streams with a surface cover of Panoche clay loam. The clay loam is responsible for the very low rate of recharge as compared to the aquifer permeability.

Solution

Compute transmissibility:

$$T = K \times h_i = 130 \times 19 = 2500 \text{ gpd/ft}$$

Consider the most severe case, $w = 2.5 \text{ gpd/ft}^2$ and $t = 120$ days. Assume maximum build-up will occur for that portion of the pipe closest to the center of the basin. This point has the coordinates (75,100) (point A, figure 10). Apply the design curve methodology to the above data:

1a. Estimate $j = 150 \text{ ft}$

1b. From figure 7: $j/n = 1.18$

2. By equation a: $n = 118$

3. By equation b: $j = 1.18 \times 118 = 139 \neq 150$
4. Go to step 1

1a. Re-estimate $j = 140$

1b. $j/n = 1.2$

2. $n = 118$

3. $j = 1.2 \times 118 = 142 \approx 140$
   Select $j = 141$ and $j/n = 1.2$

4. Go to step 5

5a. From figure 8 select $m = 0.95$ and $b_{\text{int}} = 0.04$
   Note the insensitivity of these constants at large $T$
   and $t$

5b. By equation 11:

$$
\left. \frac{j}{n} \right|_{(2500, 120)} = (0.95 \times 1.2) + 0.04 = 1.18
$$

6. By equation a: $n = 118$
by equation b: $j = 141$
   Note that this was obvious due to the invariance of $j/n$
   for this example

7. From figure 9 (see point A) visually interpolate and
   select $\Delta h/w = 2 \text{ ft/gpd/ft}^2$

8. Total build-up, $\Delta h = 2 \times 1.5 = 3.0 \text{ ft}$

These results indicate that for recharge rates in the
neighborhood of 2.5 gpd/ft$^2$ a high probability exists that infiltra-
tion of groundwater into the sewer pipe can occur toward the end
of the recharge period. Infiltration probably will not occur if
lower recharge rates prevail. Additional applications of this
method can be used to estimate the time duration over which infil-
tration may occur and the length of pipe which is exposed.

The maximum build-up beneath the center point is estimated
from figure 3 to be 3.2 ft per gpd/ft$^2$ of recharge. For this
problem that is 8 ft. Note that this mound growth could conceivably intersect the bottom of the basin resulting in a flooded condition for which the Hantush solution is not applicable.

Recommendations

To be of optimum usefulness the design curves need to be further generalized over specific yield and basin sizes. Generalization over the specific yield can be accomplished in a manner similar to that used for the center-point only case. The build-up vs $y$ curves can be developed for coupled specifications of $T$ and $S_y$ (see figures 3, 4, 5). Generalization over basin sizes can be accomplished by developing several sets of curves similar to the set presented here for a 200 ft x 100 ft basin, each set for a different size rectangular basin. The possibility for further reduction of the number of charts required to contain the above amount of information may exist, and relationships to do this can be studied once the data has been developed.

It must be noted here that a considerable amount of data reduction is required to produce a 3-curve design set similar to the one presented here. Production of a series of these sets require proportionately more effort. In view of the fact that the design curve method as developed here is not an overly simple one, although less rigorous than hand calculation, further effort along these lines should be considered carefully.
IV. CONCLUSION

This attempt to develop design curves can at best be considered of moderate success. Due to the large number of parameters involved no simple solution was developed. Rather, the design curve method developed here is a multi-step operation and still remains limited to a fixed basin size.

Center-point build-up vs time curves can be used to predict maximum build-up and as a basis for crude estimates of build-up elsewhere in the field. These curves can be easily developed by the DELTAH computer program.

DELTAH represents the most useful tool developed by this research. Data can easily be entered to provide calculated build-up for any desired conditions within the fixed limitations of the basic assumptions of the original equation (homogeneous, isotropic aquifer of infinite areal extent under uniform recharge.)

It is believed that further generalization of the design curves could be obtained by reduction of a considerable amount of additional data. Such an attempt would need justification based on the overall utility of the curves, effort required to use them, and the accuracy of the Hantush solution. New methods using finite difference techniques are theoretically more accurate and less limited by basic assumptions. Whether or not this theoretical advantage is actually obtained in real applications will determine
if these more complicated and difficult solutions will replace the Dupuit-Forchheimer types such as the Hantush solution. Even so the Hantush equation is the only approach as determined by this study to address the three dimensional recharge and associated build-up from a rectangular basin.

Comparisons with actual field data are an indication that this approach can reasonably predict mound build-up beneath the center-point. Errors due to non-homogeneity of aquifers and difficulty in determining formation constants may be more significant than theoretical inaccuracies.
APPENDIX A

TABLE OF $W^*$ VALUES

The table of $W^*(\alpha, \beta)$ values as developed by Hantush (1967) is presented on the following three pages.
<table>
<thead>
<tr>
<th>KETAO</th>
<th>0.0</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.10</th>
<th>0.14</th>
<th>0.16</th>
<th>0.22</th>
<th>0.26</th>
<th>0.30</th>
<th>0.34</th>
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W* (ALFA, ETA) FROM HANTUSH
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<th>( \alpha_3 )</th>
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<tr>
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<td>0.18</td>
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</tbody>
</table>

Notes: This table represents the relationships between different variables, likely in a scientific or mathematical context. Each row contains values for one variable, and each column represents a different variable. The values seem to be incremental, suggesting a pattern or progression in the data.
APPENDIX B

COMPARISON OF HANTUSH SOLUTION WITH FIELD DATA

Haskell and Bianchi (1968) have collected field data for mound build-up vs time beneath the center-points of two square 2 acre basins. The parameters as they determined them are presented in table 2.

### TABLE 2

**PHYSICAL CONSTANTS FOR FIELD EXAMPLES**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pond 1</th>
<th>Pond 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basin Width</td>
<td>295 ft</td>
<td>295 ft</td>
</tr>
<tr>
<td>Saturated Depth</td>
<td>16 ft</td>
<td>80 ft</td>
</tr>
<tr>
<td>Recharge Rate</td>
<td>0.32 ft/day</td>
<td>0.35 ft/day</td>
</tr>
<tr>
<td>Specific Yield</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Hydraulic Conductivity</td>
<td>104 ft/day</td>
<td>26 ft/day</td>
</tr>
</tbody>
</table>

The DELTAH program was used to develop similar data for both of these ponds. Results are compared in figures C-1 and C-2. It can be seen that the Hantush solution is very accurate in predicting the response for pond 1, whereas it is about 27% too low for pond 2. Additional data was developed for pond 2 using a specific yield of 0.05 to illustrate the sensitivity of the solution to input data. The difference is reduced to 14% by this change.
Figure 11. Comparison of Hantush Solution to Field Data for Pond 1.
Figure 12. Comparison of Hantush Solution to Field Data (Haskell and Biench, 1968) for Pond 2.

- Calculated for \( S_y = 0.05 \)
- Calculated for \( S_y = 0.12 \)

Field Data

Symbols:
- ○ for \( S_y = 0.05 \)
- □ for \( S_y = 0.12 \)

Time, \( t \), days

Legend:
- Field Data

Calculated for \( S_y = 0.05 \)
Calculated for \( S_y = 0.12 \)
Although the accuracy of the measured parameters in representing the aquifer can be cited as a possible cause for the difference, it must be noted that Haskell and Bianchi developed a satisfactory approximation using an equivalent-radius-circular-basin technique.
APPENDIX C

USER'S MANUAL FOR THE DELTAH COMPUTER PROGRAM

The DELTAH computer program models the Hantush solution (1967) for groundwater mound build-up beneath a rectangular recharge area. Descriptions of the program and of the Hantush solution are included in the text of this report.

Data Input

The data input information is arranged as follows:

<table>
<thead>
<tr>
<th>Card Number</th>
<th>Card Name</th>
<th>(Format)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>ENTRY</td>
<td>Description of entry</td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td>Additional comments where required</td>
</tr>
</tbody>
</table>

The Hantush W* table is contained on the first 140 data cards. This data is an implicit part of the program and is never changed. Data card number 1 is therefore defined as the first card succeeding these cards.

For each variable parameter, P, the user selects a starting value, Pl; an incremental value, DELP; and the number of times the increment is to be applied, NP. For cases in which a fixed value of P is to be used, NP is set at 1, and DELP at 0 (or blank).
1. **Print Indicator for W* Table (II)**

   1  JPRINT  The W* table will be printed if JPRINT = 1. If printing is not desired, enter blank card.

2. **Heading Card (20A4)**

   1-80  TITLE  This card will be reprinted exactly as is and can be used to describe the problem represented by data cards 3-10.

3. **Fixed Variables (3F10.2)**

   1-10  LE  Full length of the recharge area (feet).
   11-20  WI  Full width of the recharge area (feet).
   21-30  HI  Initial height of the aquifer (feet).

   The aquifer thickness or height is not utilized in the program, therefore this field may be left blank (HI = 0). It is provided for future program modification such as imputing the hydraulic conductivity or changing the reference elevation of the output data.

4. **Transmissibility (F10.2, I2, 8X, F10.2)**

   1-10  Tl  Transmissibility (gpd/ft) or first T of iteration
   11-12  NT  Number of iterations desired (integer). If T = Tl only, NT = 01
   13-20  DELT  Blank field.
   21-30  Desired increment between iterations (gpd/ft). May be left blank if Tl is fixed.

5. **Specific Yield (F10.2, I2, 8X, F10.2)**

   1-10  SYl  Specific yield or first SY of iteration (-).
   11-12  NSY  Number of iterations desired (integer). If SY = SYl only, NSY = 01
   13-20  DELSY  Blank field.
   21-30  Increment. Leave blank if SYl is fixed.
6. Recharge Rate (F10.2, I2, 8X, F10.2)

1-10  Q1  Recharge rate (gpd/ft²) or first Q (w) of iteration.
11-12  NQ  Number of iterations (integer). If Q = Q1 only, NQ = 01.
13-20  DELQ  Increment. Leave blank if Q1 is fixed.

7. Time (F10.2, I2, 8X, F10.2)

1-10  D1  Time after recharge begins (days) or first D of iteration.
11-12  ND  Number of iterations (integer). If D = D1 only, ND = 01.
13-20  DELD  Increment (days). Leave blank if D1 is fixed.

The time parameter is treated specially so that any iteration initiated at t = 0 will have a solution for t = 1 day. For example: if an iteration at 10 day increments is desired beginning with t = 0 and ending at t = 50, the user enters; D1 = 0, ND = 6, and DELD = 10. Solutions will be developed for t = 1, 10, 20, 30, 40 and 50 days.

8. Point Location Coordinate, X (F10.2, I2, 8X, F10.2)

1-10  XL  X coordinate of point of interest (feet) or first X of iteration.
11-12  NX  Number of iterations (integer). If X = XL only, NX = 01.
13-20  DELX  Increment (feet). Leave blank if XL is fixed.

9. Point Location Coordinate, Y (F10.2, I2, 8X, F10.2)

1-10  Y1  Y coordinate of point of interest (feet) or first Y of iteration.
11-12  NY  Number of iterations (integer). If Y = Y1 only, NY = 01.
13-20  DELEY  Increment (feet). Leave blank if Y1 is fixed.
At this point the program will solve the problem represented on cards 2-10 and will then return to card 2 to read in the next data set. This will continue until all data sets have been processed.

Example Problem

The DELTAH program was used to solve the example problem which was presented in section III. Complete topographic map solutions over the region of interest were developed for recharge rates of 1.5 and 2.5 gpd/ft\(^2\), and for time intervals of 30 days over the 4 months of operation. A sample Fortran Coding Sheet illustrating the input data is presented as figure 13. The results are plotted in figure 14 (see figure 10) for the 2.5 gpd/ft\(^2\) recharge rate case.
RECALL: The Hantush table is always used.

Card 1. Print Indicator, JERINT.
Blank card

Card 2. Heading Card, Title.
EXAMPLE PROBLEM SEWER PIPE INFILTRATION.

Card 3. Fixed parameters, IE, WI, HI.
200. 100. 19.

Card 4. Transmissibility, Tl, NT, DELT.
2500. 01

Card 5. Specific yield, SY, NSY, DELSY.
0.22 01

Card 6. Recharge rate, QL, NQ, DELQ.
1.5 02 1.0

Card 7. Time, DL, ND, DELD.
0. 05 30.

Card 8. X Coordinate, XI, NX, DELX.
0. 10 18.

Card 9. Y Coordinate, Y1, NY, DELY.
0. 10 24.

Card 10. Page index, IPAGE.
2

Figure 13. Fortran Coding Sheet
Figure 14. Build-up Along Trend of Sewer Pipe for Monthly Time Intervals.
These results indicate that maximum build-up of 5.4 ft can be expected at the point (75,100) along the trend line of pipe. At the end of 120 days of recharge the length of pipe exposed to infiltration will be 137 ft. Infiltration will begin at $t = 90$ days at the point (90,80) (beginning of August).

The results using the design curves compare well with those from DELTAH. The design curve results predicted maximum build-up along the pipe trend of 5 ft for 2.5 gpd/ft$^2$ recharge and 3 ft for 1.5 gpd/ft$^2$ recharge; DELTAH, 5.4 and 3.3 ft. The difference might be explained by the extensive visual interpolation required in the use of figure 9 due to the lack of curves for times greater than 50 days and transmissibility of 2500 gpd/ft.

Program Listing

The program listing for DELTAH is reproduced on the following pages.
DIMENSION DELTAH (20,20), DELHXY (2,2), XX(20), YY(20), TITLE(20)
REAL INDEX, LE
COMMON W(36,36), INDEX(36), X, Y, AM, BM, ABK, SIGN

C HANTUSH'S TABLE FOR n* IS READ IN AND WILL BE REPRINTED IF JPRINT IS = +1
DU 15 I=1,36
w(1,1) = 0.0
w(1,1) = 0.0
15 CONTINUE
INDEX(I) = 0.0
READ (5,2000) (INDEX(I), I=1,36), ((W(J,K), K=1,36), J=1,36), J=1,36)
1JPRINT
IF (JPRINT .EQ. 0) GO TO 100
DU 80 I=1,3
IG = I
IF (I .GT. 1) IG = IG+12
IGG = IG+12
WRITE (6,1000) (INDEX(K), K=IG,IG+12)
1000 FORMAT('I',150, 'n*(ALFA,BETA) FROM HANTUSH//5X, 'ALFA', 5X,
113(F4.2, 5X)/1X, 'BETA'
DU 70 J=1,35
WRITE (6,1100) INDEX(J)
WRITE (6,1300) (W(J,K), K=1,36)
1300 FORMAT('0', F4.2)
1100 FORMAT ('+', F4.2)
1300 FORMAT ('+', 13(F6.4, 3X))
70 CONTINUE
60 CONTINUE

C READ FIELD DATA, AND OBSERVATION TIME AND LOCATION DATA
100 READ (5,4000, END=850) TITLE, LE, NI, HI, TI, NT, DELT, SY1, NSY, DELSY,
1 NY, DELG, UX, NY, DELO, X1, NX, DELX, Y1, NY, DELY, IPAGE
4000 FORMAT (20A4/ 3F10.2/ 6L F10.2, I2, 8X, F10.2/) , 11)
WRITE (6,1500) TITLE
1500 FORMAT ('1', ///////////// 20X, 20A4 )
AM = LE/2
BM = NI/2

1PAGE AND NPAGE ARE USED TO ALLOT AN INTEGRAL NUMBER OF OUTPUT SETS
TO A PAGE
NPAGE=IPAGE

THE DUMMY VARIABLE Z IS USED TO PREVENT MIXED MODE ARITHMETIC
DO 101 I = 1, NI
Z = I
T=I+(Z-1)*DELT
110 DU 111 J = 1, ND
Z = J

C
IF (Z.EQ.1 .AND. DELD.EQ.1 .AND. D1.EQ.0) CONTINUE
D=D1+(Z-1)*DELD
IF (Z.EQ.1 .AND. D1.EQ.0) D = 1
120 DU 121 K = 1, NSY
Z = K
SY = SY1+(Z-1)*DELSY
130 DU 131 L = 1, NYS
Z = L
Q = Q1+(Z-1)*DELO
140 DU 141 M = 1, NX
Z = M
X = X1+(Z-1)*DELX
150 DU 151 N = 1, NY
Z = N
Y = Y1+(Z-1)*DELY
160 HK = (Q*M) / (SU*SY)
ABK = 1.37*SQRT(SY/(1*D))
C
XX, M, MM, MMM ARE ALL DUMMY VARIABLES WHICH CONNECT X WITH DELTHA(X,Y)
C
YY, N, NN, NNN ARE ALL DUMMY VARIABLES WHICH CONNECT Y WITH DELTHA(X,Y)
C
MMM = M
NNN = N
C
ARRAYS XX AND YY ARE TO BE USED AS POINT LOCATION INDICES IN OUTPUT
XX(MM) = X
YY(NNN) = Y
C
DELTHA IS THE DESIRED GROUNDWATER MOUND BUILDUP DUE TO SEEPAGE. IT IS THE SUM OF FOUR TERMS, DELHXY(IH,JH). DELHXY IS A FUNCTION OF HANTUSH'S W*, HEREIN NAMED WSIR, WHICH IS DEVELOPED IN THE SUBPROGRAM FUNCTION WSIR(I,J)
C
DELTHA(MMM,NNN) = 0
170 DU 171 IH = 1, 2
DU 171 JH = 1, 2
SIGN = 1
DELHXY(IH,JH) = WSIR(IH,JH)*HK*SIGN
DELTHA(MMM,NNN) = DELTHA(MMM,NNN)+DELHXY(IH,JH)
IF (IH.EQ.2 .AND. JH.EQ.2 .AND. DELTHA(MMM,NNN).LT.0 )
171 CONTINUE
151 CONTINUE
141 CONTINUE
C
FIELD PARAMETERS AND CAPTIONS ARE WRITTEN
IF (NPAGE.EQ.1) WRITE (6,2900)
2900 FFORMAT ('I')
WRITE (6,3000) LE, MI, I, SY, Q, HI
3000 FFORMAT ('O',130,'FIELD PARAMETERS'/T40,'SEEPAGE BASIN LENGTH (FT)',
, T69,'LE',I80,F6.2/T40,'DRAINAGE BASIN WIDTH (FT)', T69,'NI',T80,F
26.2/140, 'TRANSMISSIBILITY (GPD/FT)', I70, '1', I78, F8.2/
3140, 'SPECIFIC YIELD (-)', I69, 'SY', T80, F6.2/
4140, 'RECHARGE RATE (GPD/FT/FT)', I70, 'Q', I60, F6.2/
5140, 'INITIAL HEIGHT OF GND (FT)', I69, 'HI', T80, F6.2/)
WRITE (6,5000) D,U
5000 FORMAT (130, 'OBSERVATION PARAMETERS'/
140, 'TIME (DAYS)', I70, 'D', T80, F5.2, 10X, E10.3/)
1XY=1
   IF (NX .GT. 10) 1XY=2
   DU 17  MJ=1, IXY
   KX=10*(MJ-1)+1
   LX=NX
   IF ( M(j .EQ. 1) .AND. (NX .GT. 10) ) LX = 10
WRITE (6,7000)(XX(IJ), IJ=KX,LX)
7000 FORMAT (130, 'GROUNDWATER BUILDUP MOUND HEIGHT (FT) "DELTAH(X,Y)"'/
1714, 'Y', 6X, 'X', 3X, 10(F5.1, 5X)//)
C C GROUNDWATER BUILDUP MATRIX, DELTAH, IS PRINTED
DU 137  NN = 1, NNN
WRITE (6,9000)(YY(NN), (DELTAH(MM,NN), MM=KX,LX)
9000 FORMAT (0), T10, F5.1, 10X, 10(F6.3,4X))
13/ CONTINUE
17 CONTINUE
   IF ( NPAGE .EQ. IPAGE ) NPAGE = 0
   NPAGE = NPAGE + 1
131 CONTINUE
121 CONTINUE
111 CONTINUE
101 CONTINUE
END STOP
FUNCTION WSTR(I, J)
COMMUN N(36,36), INDEX(36), X, Y, AM, BM, ABH, SIGN
DIMENSION A(2), B(2)
REAL INDEX
PI = 3.14159
C DETERMINE THE SIGN OF WSTR
A(1) = ABH*(BM+X)
A(2) = ABH*(BM-X)
B(1) = ABH*(AM+Y)
B(2) = ABH*(AM-Y)

IF ((A(1)*B(J)) .LE. 0 ) SIGN = -1.0
ALFA = AMS(A(I))
BETA = ABS(B(J))

C CHECK FOR VALUES OF ALFA AND/OR BETA NUT IN TABLE
IF ( ALFA .EQ. 0.00 .AND. BETA .EQ. 0.00 ) GO TO 500
IF ( ALFA .GE. 3. .AND. BETA .GE. 3. ) GO TO 499
RX=100
IF ( BETA .GE. 5. ) RX=ALFA
IF ( ALFA .GE. 3. ) RX=BETA

U = RX
IF ( RX .NE. 100 ) GO TO 498
ALBE1 = ALFA**2 + BETA**2
U = ALBE1
IF ( ALBE1 .LE. 0.1 ) GO TO 497

C LOCATE ALFA AND BETA ON INDEX OF HANTUSH'S W* TABLE
200 L1 = -1
M1 = -1
D0 401 K = 1,36
IF (L1 .NE. -1) GO TO 350
IF (ALFA .GT. INDEX(K) ) GO TO 350
L2 = K
L1 = K-1
350 IF (M1 .NE. -1) GO TO 400
IF (BETA .GT. INDEX(K) ) GO TO 400
M2 = K
M1 = K-1
400 IF (L1 .NE. -1 .AND. M1 .NE. -1) GO TO 402
401 CONTINUE
C DOUBLE INTERPOLATION TO DETERMINE TRUE VALUE OF W*, WSTR
402 PCB = (INDEX(M2)-BETA)/(INDEX(M2)-INDEX(M1))
WBUP = W(L1,M2)-PCB*(W(L1,M2)-W(L1,M1))
WBUL = W(L2,M2)-PCB*(W(L2,M2)-W(L2,M1))
PCA = (INDEX(L2)-ALFA)/(INDEX(L2)-INDEX(L1))
WSTR = WBUL-PCA*WBUP

V1 N
GU TU 501

C WELL(A) IS THE WELL FUNCTION - NUMERICAL EQUIVALENT USED
497 WELL = -.5722 -ALOG(U) + U -U**2/4 +U**3/18 -U**4/96 + U**5/600
1-U**6/4320
WSTR = 4*ALFA*BETA/PI* ( 3 + WELL -(ALFA/BETA*ATAN2(BETA, ALFA) +
 1*BETA/ALFA*ATAN2(ALFA, BETA)))
GU TU 501

C FERGC2 IS THE SECOND INTEGRAL OF THE CUNVERSE ERROR FUNCTION
498 FERGC2 = 0.25* ( ERFL(T) -2*(EXP(-T**2)/PI -ERFC(T)*T) )
WSTR = 1 - 4*FERGC2
GU TU 501
499 WSTR = 1
GU TU 501
500 WSTR = 0
501 RETURN
END


