The Design and Analysis of a Unique Broadband Underwater Acoustic Source

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THE DESIGN AND ANALYSIS OF A UNIQUE BROADBAND UNDERWATER ACOUSTIC SOURCE

By

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ABSTRACT

Requirements exist for a unique type of underwater acoustic source. The transducer is in the form of a linear array of discrete elements and is required to have a constant transmitting voltage response and carefully controlled directivity characteristics over a two octave bandwidth. A generalized model of a linear array of cylindrical piezoelectric ceramic acoustic radiators is developed and applied to the design of a prototype which operates over approximately one half of the required bandwidth. The prototype transducer was built and the measured results are compared with those predicted by the model. Recommendations are made for improving the performance of both the prototype and the array required to meet the full bandwidth specified.
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INTRODUCTION

Background

The United States Navy utilizes underwater electroacoustic transducers in applications ranging from SONAR to oceanographic research. These transducers are generally broken down into two categories; those which convert acoustical to electrical energy (acoustic receivers or hydrophones) and those which convert electrical to acoustical energy (acoustic sources or projectors). Although many analytical techniques are equally applicable to both types of instruments, in this case we will not consider hydrophones and will address only the design and analysis of a specific type of acoustic source.

Underwater electroacoustic projectors are used in various shapes and sizes and are configured in many array geometries. The geometry is generally determined by the intended application with spherical, cylindrical, and round, square, or rectangular piston radiators configured in spherical, cylindrical, planar, and linear arrays. Commonly used transduction mechanisms include; electrodynamic, natural piezoelectric crystals, piezoelectric ceramics, and magnetostrictive metal alloys. The piezoelectric ceramics enjoy the most widespread use due to low cost, ease of fabrication, transduction efficiency, and other factors and are the only transduction materials which will be considered here.
Piezoelectric projectors are normally specified in terms of the following parameters: transmitting voltage or current response (the output sound pressure level per volt or per ampere measured as a function of frequency) electroacoustic efficiency (the ratio of the output acoustical power to the input electrical power), impedance (as measured at the electrical leads with the transducer acoustically loaded), directivity (the output sound pressure relative to the pressure along a specified axis, measured as a function of orientation), and maximum operating depth. Design is complicated by the fact that most of these parameters are interdependent and may not be varied separately.

There is currently a need for an acoustic source with very unusual and rigid requirements. The next section will discuss those requirements and how they determine the basic design of the transducer.

Requirements

Directivity. Directivity is typically the most difficult parameter to control over a wide frequency range and is specified in terms of the major lobe beamwidth and the relative level of the minor lobes. The intended application for this transducer requires a torroidal beam pattern; that is, omnidirectional in the horizontal plane with a main beam width of 30° to 60° (at the 3 dB down points) in the vertical plane. The sidelobes are required to be a minimum of 15 dB below the maximum level of the
main beam. The directivity requirements are summarized in the drawing of Figure 1.

It is the directivity specifications of the subject transducer which determine its basic geometrical configuration. The requirements for omnidirectionality in one plane and a specified beamwidth in a perpendicular plane imply the geometry of a linear array with the longitudinal array axis perpendicular to the omnidirectional plane. Also implied by the omnidirectionality in the horizontal plane is the cylindrical symmetry of the individual radiating elements. So, based upon the directivity requirements, we can assume the subject transducer will be in the basic form of a linear array of radially poled cylindrical piezoelectric ceramic shells.

In order to meet the beamwidth requirement across the frequency range of operation, the effective array length will have to change as a function of frequency.

Transmitting response. The system incorporating the subject transducer requires an output sound pressure level of 190 dB (referenced to one micropascal measured at one meter) over the frequency range from 10 to 40 kilohertz. It is also required that the transmitting voltage response be constant (±1.5 dB) over the same frequency range.

The typical transmitting voltage response of a piezoelectric ceramic cylindrical shell with the ends and inner surface acoustically shielded (or a linear array of such elements) is
Figure 1. Summary of the directivity requirements
shown in Figure 2. The curve is obviously not flat, but instead has a +12 dB per octave slope at frequencies somewhat below the frequency of the first radial resonance. Obviously then, a simple linear array of cylindrical elements will not meet the transmitting voltage response requirements.

The requirement for the flat response demands that the distribution of the volume velocity generated by the array be such that the sum of the pressures in the far-field is a constant. Coupled with the directivity requirements that the array length be a function of frequency, this implies the biconical configuration shown in Figure 3. The elements are arranged symmetrically with respect to the array center according to the frequency of their radial resonance; that is, the largest, lowest frequency elements are at the ends of the array with the element size decreasing and the resonance frequency increasing symmetrically toward the array center. When the elements are driven electrically in parallel, the effective array length is a function of frequency with the array, in general, becoming shorter with increasing frequency.

Electrical and mechanical requirements. The subject transducer is limited in size to a maximum length of 0.30 meters and a diameter of 0.15 meters. The transducer must operate to depths of 400 m or a hydrostatic pressure of 4 megapascals.

Electrical power available to the transducer is limited to approximately 400 watts, implying a required electroacoustic
Figure 2. The typical transmitting voltage response of a radially poled piezoelectric ceramic cylindrical shell with the ends and inner surface acoustically shielded.
Figure 3. The biconical array configuration implied by the transmitting voltage response and directivity requirements.
efficiency of at least twenty percent. Although not mandatory, a nominal input impedance of 100 ohms with a minimum change as a function of frequency is desirable.

Approach and Objectives

The subject transducer is a unique type of linear array and was analyzed by means of a generalized model. A short review of conventional linear arrays will provide the necessary background for development of the model.

A computer program was written to solve the generalized equations for the on-axis transmitting voltage response and the normalized far-field directivity patterns in the vertical (XZ) plane.

A simplified prototype of the subject array was designed, fabricated, and evaluated and the results compared with those predicted by the model.

Conclusions are drawn about the results and recommendations made.

The primary objectives of this thesis then are as follows:

1. To develop a model sufficiently descriptive of a generalized linear array of cylindrical acoustic radiators to allow for the design of a transducer with the previously described characteristics.

2. To design, fabricate, and test a prototype of the subject transducer.
3. To compare the results derived from the model with the measured results obtained from the prototype transducer.
LINEAR ARRAYS IN GENERAL

Equally Spaced Sources in Linear Arrays

When an acoustic source is very small when compared to the radiated wavelength in the medium it becomes an omnidirectional or isotropic source. Such radiators are normally referred to as simple or point sources and the analysis of their use in linear arrays has been well developed in acoustics and antenna theory [1-4]. The far-field pressure amplitude produced by an array of \( N \) such sources radiating with equal amplitude and phase, where the elements are spaced a distance \( d \) apart, is given by [1]

\[
P = p \frac{\sin \left( \frac{Nkd}{2} \sin \theta \right)}{\sin \left( \frac{kd}{2} \sin \theta \right)},
\]

where \( p \) is the pressure amplitude produced by each element, \( k = \frac{2\pi}{\lambda} \) and \( \lambda \) is the acoustic wave length in the medium, and \( \theta \) is the angle between the normal to the longitudinal axis of the array and the direction of the observation point. The geometry of the array is shown in Figure 4.

Since equation (1) represents a simple vector sum of the pressures produced by the individual elements, the normalized far-field directivity pattern of such an array is represented by
Figure 4. An equally spaced linear array of N point sources.
\[ P(\theta) = \frac{\sin \left( \frac{N \cdot kd}{2} \sin \theta \right)}{N \sin \left( \frac{kd}{2} \sin \theta \right)} \]  

(2)

The total width of the main beam at any one frequency is a function of the number of elements in the array and the element spacing. When the spacing is equal to or greater than the wavelength, additional major lobes with the same amplitude as the main beam appear and are commonly referred to as grating lobes.

If the overall length of the array remains fixed and the number of elements is allowed to increase indefinitely while maintaining uniform element spacing, the linear array of point sources approaches a continuous line. Defining \( Q \) as the pressure per unit length of the line and \( \lambda \) as the length of the line, the resultant pressure is

\[ P = \frac{Q \cdot \lambda \sin \left( \frac{k \cdot \lambda}{2} \sin \theta \right)}{k \cdot \lambda \sin \theta} \]  

(3)

Going one step further in approximating an array more like the subject transducer, the acoustic pressure at a point in the far-field produced by a line divided into an even number of equally spaced identical line segments can be determined. If \( s \) is the length of a line segment and the magnitude of the pressure from each segment is equation (3) with \( \lambda \) replaced by \( s \), the far-field pressure produced by the segmented line is [2].
\[ p = \frac{2Qs \sin \left( \frac{ks}{2} \sin \theta \right)}{ks \sin \theta} \left[ \cos u + \cos 3u + \ldots + \cos (N-1)u \right], \quad (4) \]

where \( u = \frac{kd}{2} \sin \theta \). It has already been shown that in order to avoid grating lobes the spacing must be less than \( \lambda \), obviously then, the line segments must also be less than \( \lambda \) in length. In fact, for a line segment length of \( \lambda/2 \) or less it can be shown that the directivity pattern is almost entirely dependent upon the \( \cos u \) term in equation (4).

**Shading of equally spaced linear arrays.** For the case of a continuous line source, the level of the first side lobe is 13.3 dB below the level of the main beam. Although the side lobe levels of discrete element arrays are generally high, when the array elements are densely packed (in terms of \( \lambda \)) they closely approximate the continuous line. Obviously, our requirement for side lobe levels of at least 15 dB below the level of the main beam cannot be met by either simple array.

The far-field sum of the pressures from an array may be varied by varying the relative amplitudes and/or phases of the individual elements. This is a common technique referred to as shading and is used to control the relative level of the side lobes and the width of the main beam. Amplitude shading is the most common approach used with the element amplitudes, in general, decreasing symmetrically from the center to the ends of
the array. Several schemes for determining the element amplitude coefficients necessary for a given sidelobe level have been developed and applied. Binominal shading and Dolph-Chebyshev [5-6] shading are probably the most commonly used techniques in acoustics although there are many others available. Binominal shading, where the element amplitudes are proportional to the coefficients in a binominal expansion, provides the narrowest main beam for the condition of no sidelobes. In Dolph-Chebyshev shading the element amplitude coefficients for an N element array are equated to the coefficients of a Chebyshev polynomial of order \((N-1)\). This method optimizes the patterns in the sense that for a specified sidelobe level the narrowest possible main beam is obtained. Neither technique is applicable in this case.

Unequally Spaced Sources in Linear Arrays [7-9]

The nonuniformly spaced discrete element array is in many ways analogous to the amplitude shaded equally spaced linear array. However, due to the design constraints of most applications and the more simple analysis techniques required, amplitude shading of an equally spaced array is generally the preferred method.

The nonuniform array does, however, offer some advantages [7]; for example, the number of elements required for a given beamwidth and frequency can be reduced, and the bandwidth of
operation may be extended over several octaves without interference from grating lobes. These advantages can be obtained at the expense of an increase in the sidelobe level and a decrease in the maximum gain of the array. Unlike the equally spaced array, the directivity pattern of the nonuniform array is characterized by a narrow main beam followed by a region of very low sidelobes (the clean sweep region) which in turn is followed by a region of relatively high sidelobes (the plateau region).

The primary disadvantages of nonuniform arrays are the difficulty in analysis they pose for the transducer designer and that they are best applied when an array is to contain an extraordinarily large number of elements.

Cylindrical Elements and Diffraction

If the individual elements can no longer be considered small when compared to the acoustic wavelength in the water, the array analysis becomes somewhat more complicated. The first complication arises because the elements are no longer omnidirectional; that is, each element now has a non-unity normalized directivity pattern. If we consider the element as a continuous line source of length \( \ell \), a technique frequently used in antenna theory may be applied to find the far-field amplitude pattern of the source [10]. Associated with any antenna are two important quantities, the far-field amplitude pattern, \( F(u) \), and the aperture distribution function, \( g(z) \). The two quantities are related by
the Fourier transform, that is

\[
F(u) = \frac{1}{2\pi} \int_{-\pi}^{\pi} g(z)e^{jzu} \, dz = \frac{1}{2\pi} \int_{-\pi}^{\pi} g(z)(\cos zu + jsinz) \, dz \tag{5}
\]

where \( u = \frac{\theta}{\lambda} \) and \( z = 2\pi x/\lambda \) for \(-\lambda/2 \leq x \leq \lambda/2\). If the cylinder is assumed to radiate with equal amplitude and phase along its length, the aperture distribution function will be a constant \((g(z) = 1)\). Since the distribution function is symmetrical about the origin (the geometrical center of the line) the sine term in the integrand of equation (5) may be dropped and the expression for the amplitude pattern becomes

\[
F(u) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos zu \, dz. \tag{6}
\]

Carrying out the integration,

\[
F(u) = \frac{\sin\left(\frac{\pi \lambda}{\lambda} \sin \theta\right)}{\frac{\pi \lambda}{\lambda} \sin \theta} \tag{7}
\]

which is the expression obtained for the normalized directivity pattern of a continuous line source of length \( \lambda \) (the first term in equation (4)) [11].

A second complication comes about because the elements being
of finite size means the effect of the physical presence of the transducer on the radiated pressure distribution cannot be ignored. A parameter frequently found in the analysis of electroacoustic systems is the diffraction constant, \( D \), which was more descriptively defined by Bobber [12] as being not only a function of diffraction but of anything that alters the ratio of the average blocked pressure to the free-field pressure (the concept of blocked pressure is used to describe the pressure at a hydrophone when the device is acousto-mechanically inactive). Bobber extended the definition to include the case of a sound source by defining \( D \) as the ratio of the pressure produced by a transducer at a point in a free-field to the pressure produced at the same point by a simple source with the same volume velocity. Although the diffraction constant is defined differently, the acoustic reciprocity theorem [11] holds and the diffraction constant is the same whether the transducer is receiving or transmitting. The diffraction constants for several simple transducer shapes can be determined in closed form and are a commonly used design tool [13].

Actually, the directivity of a transducer and its diffraction constant are related by the radiation resistance. The relationship is given by

\[
D^2 = \frac{R_A}{R_0} R_0
\]
where $R_A$ is the radiation resistance of the transducer, $R_0$ is the radiation resistance of a simple source, and $R_\theta$ is the directivity factor.
ANALYSIS OF THE SUBJECT TRANSDUCER

General Discussion

In terms of what has already been discussed about linear arrays, the desired transducer will be an unequally spaced, amplitude and phase shaded linear array. The elements will be unequally spaced primarily because of the requirement to tailor the volume velocity distribution to produce the flat transmitting voltage response; that is, different element lengths are required for each element pair based upon their radial resonance frequency. Since the elements are connected electrically in parallel, the same voltage amplitude is present across each of the elements independent of the frequency. The array will, however, be amplitude shaded for two reasons. First, at any one frequency there is a relative difference in element amplitude due to their different frequencies of radial resonance and secondly, the "adjusted" volume velocity distribution necessary for the required transmitting voltage response is, of course, a form of amplitude shading. It should be obvious that the amplitude shading required by the transmitting response and the directivity are not necessarily the same. In fact, the two requirements are for the most part contradictory. The array will be phase shaded because, for a given frequency, the response
curves of the individual element pairs are at different points relative to their resonance frequencies; in other words, there is a phase difference between the surface velocities of the element pairs.

The biconical configuration is essential to the directivity requirement in that the array will appear to effectively shorten as a function of increasing frequency. The key, of course, is controlling the response of each element pair at frequencies above resonance. One simplifying alternative would be to electrically series tune each element pair to its frequency of radial resonance. However, to do so would severely limit the bandwidth obtained from each pair and would result in a requirement for many more closely spaced elements, both physically and in frequency. In this configuration the very small diameter thin walled cylinders required for the highest frequencies could not produce the required sound pressure level.

The biconical configuration also results in a reverse shading or negative tapering of the array. For example, at the resonance frequency of the largest, lowest frequency elements the amplitude is highest at the ends of the array and decreases toward the center. As might be expected from the earlier discussion, this condition leads to a narrower main beam and higher sidelobe levels [7]. This may be partially overcome by judiciously choosing the center to center spacing of the elements.
In summary, the primary design parameters (all of which are interrelated) are: the effective array length (as a function of frequency), the center to center spacing of the element pairs, the radiating area of the individual element pairs, and, of course, the relative positions (in frequency) of the radial resonance frequencies of the elements.

Analysis

For the sake of simplicity, we will first consider the case of the far-field pressure generated by an array of two point sources radiating in phase. The generated pressure will be of the form

$$P_{1,2}(\theta) = A_1 e^{i\theta_1} + A_2 e^{i\theta_2},$$

where $A_1$ and $A_2$ are the respective pressure amplitudes and $\theta_1$ and $\theta_2$ represent the phase of the two points with respect to the geometrical center of the array. Looking only at the real pressure amplitude;

$$P_{1,2}(\theta) = \left[ (A_1 \cos \theta_1 + A_2 \cos \theta_2)^2 + (A_1 \sin \theta_1 + A_2 \sin \theta_2)^2 \right]^\frac{1}{2},$$

or

$$2P_{1,2}(\theta) = A_1^2 + A_2^2 + 2A_1A_2 \cos(\theta_1 - \theta_2),$$

where $A_1$ and $A_2$ are the respective pressure amplitudes and $\theta_1$ and $\theta_2$ represent the phase of the two points with respect to the geometrical center of the array. Looking only at the real pressure amplitude;
where the argument of the cosine term, $\theta_1 - \theta_2$, represents the total phase difference between the two points. From the geometry of Figure 5, it is obvious that $\theta_1 = kx$ and $\theta_2 = -kx$, where $k = \frac{2\pi}{\lambda}$ and $\lambda$ is the acoustic wavelength in water. Using the figure, it is a simple matter to solve for $x$ to yield $\theta_1 = \frac{kd}{2} \sin \theta$ and $\theta_2 = -\frac{kd}{2} \sin \theta$.

If we now consider the two sources to be cylindrical elements of finite but unequal radii, the phase relationship between the two is shown in Figure 6. Now the total phase difference between the sources due to their separation in space is $k(x-r)$, where $x$ and $r$ may be found from the figure to be $d \sin \theta$ and $(a_2 - a_1) \cos \theta$ respectively. If the driving function (the electrical signal) is applied to the two elements in parallel, another phase difference becomes apparent.

Since the elements do not have the same mean radii, their radial resonances will occur at different frequencies and their surface velocities will therefore differ in phase. If we let the phase of the surface velocity with respect to the driving function be $\phi_1$ and $\phi_2$ respectively, equation (11) for the square of the generated far-field pressure becomes

$$P_{1,2}^2(\theta) =$$

$$A_1^2 + A_2^2 + 2A_1A_2 \cos \{\phi_1 - \phi_2 - k [d \sin \theta - (a_2 - a_1) \cos \theta]\}. \quad (12)$$

The square of the on-axis ($\theta=0$) pressure reduces to
Figure 5. An array of two point sources separated by a distance $d$. 
Figure 6. An array of two cylindrical sources of unequal radii separated by a distance $d$. 
and the normalized directivity pattern becomes

$$P_{1,2}(\theta) = \frac{P_{1,2}(\theta)}{P_{1,2}(0)} = \left\{ \frac{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2 - \psi_{1,2})}{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2 + k(a_2 - a_1))]} \right\}^{1/2},$$ (14)

where $\psi_{1,2} = k[d\sin\theta - (a_2 - a_1)\cos\theta].$

The elements are now of finite dimensions and can no longer be considered as point sources, that is, there is a non-unity directivity factor associated with each element. The pressure amplitudes of the individual elements as a function of $\theta$ now can be represented by

$$B_1 = A_1 \cdot \sin \left[ \frac{k\lambda_1}{2} \sin\theta \right] \quad \text{and} \quad B_2 = A_2 \cdot \sin \left[ \frac{k\lambda_2}{2} \sin\theta \right],$$

where the second term in each expression is the normalized directivity pattern for a continuous line source of length $\lambda_1$ and $\lambda_2$ respectively. This can be shown to be simply an application of the product theorem [14] which, as applied to this case, states; if the point sources in the two element array are replaced by finite sources, the resulting directivity pattern
for the array will be the product of the directivity patterns of the finite elements and the two element point source array. To illustrate we can rewrite equation (14) as

\[
\begin{split}
P_{1,2}(\theta) &= \left( \frac{B_1^2 + B_2^2 + 2B_1B_2\cos(\phi_1 - \phi_2 - \psi_{1,2})}{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2 + k(a_2 - a_1))} \right)^{\frac{1}{2}} \tag{15}
\end{split}
\]

and let the elements be of equal size, equation (15) then reduces to

\[
P_{1,2}(\theta) = \frac{\sin \left( \frac{kL}{2} \sin \theta \right)}{\sin \left( \frac{kL}{2\sin \theta} \right)} \cdot \frac{\sin \left( kdsin \theta \right)}{2 \sin \left( \frac{kdsin \theta}{2} \right)}, \tag{16}
\]

which is the product of the directivity patterns of a continuous line element of length \( L \) with that of an array of two point sources separated by a distance \( d \).

This overall approach may be generalized to include \( N \) arbitrarily dimensioned and spaced finite elements and equation (12) may be written as

\[
P_{N}(\theta) = \sum_{n=1}^{N} \sum_{m=1}^{N} B_n B_m \cos(\phi_n - \phi_m - \psi_{n,m}),
\]

where

\[
B_n = A_n \frac{\sin \left( \frac{kL_n}{2\sin \theta} \right)}{\sin \left( \frac{kL_n}{2\sin \theta} \right)}, \quad B_m = A_m \frac{\sin \left( \frac{kL_m}{2\sin \theta} \right)}{\sin \left( \frac{kL_m}{2\sin \theta} \right)}, \tag{17}
\]
\[ \psi_{n,m} = k[d_{n,m}\sin \theta - (a_m - a_n)\cos \theta], \text{ and } d_{n,m} \text{ (or } d_{m,n} \text{) is the spacing between the } n^{th} \text{ and } m^{th} \text{ elements.} \]

In generalized form, the square of the on-axis pressure is written as

\[ P_N^2(0) = \sum_{n=1}^{N} \sum_{m=1}^{N} A_n A_m \cos[\phi_n - \phi_m + k(a_m - a_n)]. \quad (18) \]

and the directivity pattern becomes

\[ P_N(0) = \left( \sum_{n=1}^{N} \sum_{m=1}^{N} B_n B_m \cos(\phi_n - \phi_m - \psi_{n,m}) \right)^{\frac{1}{2}} \left( \sum_{n=1}^{N} \sum_{m=1}^{N} A_n A_m \cos[\phi_n - \phi_m + k(a_m - a_n)] \right) \quad (19) \]

Before these expressions can be used to compute the transmitting voltage response or the directivity patterns in the vertical plane, the pressure amplitudes (the \( A \) coefficients) for the individual cylindrical elements must be determined. In general, the far-field pressure from each element can be represented by [12]

\[ p = \left\{ \frac{U^2 R_A R_0 \rho_0 c}{4\pi r^2} \right\}^{\frac{3}{2}}, \quad (20) \]

where \( U \) is the root-mean-square (RMS) volume velocity of the radiator, \( R_A \) is the acoustic resistance acting on the radiator,
R₀ is the directivity factor, ρ₀c is the product of the density and speed of sound in the medium (characteristic impedance of the medium), and r is the distance to the measurement point. The diffraction constant for such a source may be written as [12]

\[ D = \left( \frac{R_A R_B 4\pi}{k^2 \rho_0 c} \right)^{1/2}, \]  

(21)

which may be solved for R₀ and substituted into equation (20). Following the substitution, the expression for the pressure becomes

\[ p = \frac{UDk\rho_0 c}{4\pi r}. \]  

(22)

Using the definition of the volume velocity (U=u·S) and the surface area of the cylinder (S = 2παl), the pressure may be expressed as

\[ p = \frac{u(\rho_0 c) Dk\alpha l}{2r}, \]  

(23)

where u is the RMS velocity of the radiating surface, α is the radius of the cylinder, and l is the cylinder's length. Now it is only necessary to find an expression for the surface velocity in order to compute the far-field pressure produced by each element.

The radial velocity of the cylindrical elements will be of
the form

\[ v = u \cos(\omega t - \phi), \]  

where the velocity amplitude, \( u \), is the ratio of the RMS force to the mechanical impedance of the ceramic cylinder and \( \phi \) is the phase difference between the driving function and the velocity. The mechanical impedance and phase angle are defined conventionally as

\[ Z_m = R_m + jX_m \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{X_m}{R_m}\right) \]

where \( R_m \) and \( X_m \) are the mechanical resistance and reactance respectively. By using the definition of the mechanical quality factor, \( Q \), and the fact that

\[ X_m = \omega M - \frac{1}{\omega C_m}, \]

the magnitude of the impedance is found to be

\[ |Z_m| = \left(\frac{\omega_q^2 + Q^2 \left(\frac{\omega}{\omega_0} - 1\right)^2}{\omega C_m Q} \right)^{\frac{1}{2}}, \]  

where \( M \) and \( C_m \) are the mass and mechanical compliance, \( \omega \) is the frequency in radians per second, \( \omega_0 \) is the radial resonance frequency in radians per second, and
Similarly, the phase angle can be found to be

$$\phi = \tan^{-1} \left( \frac{Q \left( \frac{\omega^2}{\omega_0^2} - 1 \right)}{\frac{\omega}{\omega_0}} \right).$$

(26)

The RMS force produced by the piezoelectric ceramic is given by

$$F = \frac{2\pi \varepsilon d_{31} V}{S_{11}^E},$$

(27)

d_{31} is the piezoelectric strain constant where the subscripts denote the axes of the applied electric field and induced strain respectively, \(S_{11}^E\) is the reciprocal elastic modulus at constant electric field (\(S_{11}^E\) is the reciprocal of Young's modulus), and V is the RMS value of the applied voltage. If we now take the ratio of the magnitude of the force to the magnitude of the mechanical impedance, we get

$$|u| = \frac{2\pi \varepsilon d_{31} V \omega C_m Q}{S_{11}^E \left( \frac{\omega^2}{\omega_0^2} + Q^2 \left( \frac{\omega^2}{\omega_0^2} - 1 \right) \right)^{\frac{1}{2}}}$$

(28)

for the magnitude of the velocity. Substituting \(C_m = \frac{a s_{11}^E}{2 \pi f t}\) [15] along with the above expression and the expression for \(\tan \phi\) into equation (23) and performing the algebraic manipulation, we get
for the pressure generated by the individual elements. This equation may be substituted into equation (17) for the $A$ coefficients and both sides of the result divided by $V^2$ to yield the far-field pressure per volt from the generalized array. The result is

\[
\left| \frac{P_n(\theta)}{V} \right| = \frac{\omega d_{31} \sqrt{\rho/S_{11}^E}}{2\pi c} \left( \sum_{n=1}^{N} \sum_{m=1}^{N} R_n u_n \cos(\phi_n - \phi_m - \psi_{n,m}) \right)^{1/2},
\]

where $R_n = D_n a_n \omega_n \cos \phi_n$, $R_m = D_m a_m \omega_m \cos \psi_{m}$, $u_n = \frac{k_n}{2} n \sin \theta$, and $u_m = \frac{k_m}{2} m \sin \theta$. The on-axis sum of the pressures per volt (the transmitting voltage response) reduces to

\[
\left| \frac{P_n(0)}{V} \right| = \frac{\omega d_{31} \sqrt{\rho/S_{11}^E}}{2\pi c} \left( \sum_{n=1}^{N} \sum_{m=1}^{N} R_n R_m \cos \left[ \phi_n - \phi_m + k(a_m - a_n) \right] \right)^{1/2},
\]
and the directivity pattern of the $N$ element line array becomes

$$
P_N(\theta) = \sum_{N} \left\{ \sum_{n=1}^{N} \frac{R_n \sin \theta_n}{u_n} \sum_{m=1}^{N} \frac{R_m \sin \theta_m \cos(\phi_n - \phi_m - \psi_{n,m})}{u_m} \right\}^{1/2}.
$$

(32)

It should be noted that the cylindrical elements have been assumed to vibrate only in the radial mode and that the mutual effects between radiating elements have been neglected, or at least to this point lumped into one unknown parameter, the diffraction constant.

If the mutual coupling effects between elements are ignored altogether, the precise diffraction constant for the biconical configuration is still unknown and would be extremely difficult to determine. However, expressions do exist for the diffraction constants of a thin cylindrical ring and a long cylinder. By direct integration, Henriquez found the diffraction constants for these two configurations to be [13]

$$
D_{\text{ring}} = J_0(ka)
$$

(33)

and

$$
D_{\text{cylinder}} = \frac{2}{\pi ka} \left[ J_1^2(ka) + N_1^2(ka) \right]^{-1/2},
$$

(34)

where $J_0$ and $J_1$ are the zeroth and first order Bessel functions.
and $N_1$ is the first order Neumann function. Considered individually, the array elements are probably best described as thin rings. In the array, however, adjacent elements act as a tapered cylindrical baffle (at least geometrically) and the long cylinder configuration is perhaps more descriptive. Equation (34) will be used as an approximation to the diffraction constants of the array elements in equations (31) and (32).

A program was written to carry out the calculations indicated in equations (31) and (32) on a PDP11-45 digital computer. User input is required to specify the piezoelectric material type (the piezoelectric constant, the reciprocal elastic modulus, and the material density), the element dimensions (outside diameter, wall thickness, and length), and the center to center spacing of the elements in the array. The user has the option of selecting the program output as either the transmitting voltage response over a specified frequency range or the directivity patterns at specified discrete frequencies. The data may be displayed in tabulated form on a terminal or plotted on an X-Y plotter.
THE PROTOTYPE ARRAY

The Design

In order to shorten the necessary fabrication time and to provide a relatively simple first test of the model, a prototype array was designed to operate over approximately half of the total required bandwidth. The array contains four elements (two element pairs) and operates from 15 kHz to 30 kHz. An iterative procedure using the computer program to compute the transmitting voltage response was utilized to determine the element sizes. The element diameters and wall thicknesses were varied within the constraints of available ceramic sizes and the lengths varied to obtain the desired flatness of response. While maintaining the same ratio of element lengths and remaining within the constraints of the required sound pressure level and electroacoustic efficiency, the elements were then shortened in order to minimize the element spacing in the array. Using this procedure, the array shown in Figure 7 was derived. The center to center spacing of the elements is not simply one half of the sum of the lengths of adjacent elements because of the space occupied by mounting hardware.

The generalized model describes an array where the elements are radially vibrating cylindrical radiators free from any mechanical or acoustical coupling effects. We have already chosen
Figure 7. Dimensions of the prototype array for the frequency range from 15 to 30 kHz.
to ignore any mutual acoustical effects to simplify the analysis, but care must be taken in the design to assure that the elements are not mechanically coupled through the mounting structure. In order to mechanically decouple each element from its mounting, glass loaded polycarbonate end rings are fastened to each end of the ceramic cylinders with a thin film of relatively compliant potting compound. Since the elements are to be air backed, the end rings seal against a center mounting spindle by using elastomer O-ring seals. The spindles are in turn mounted to a single central shaft in the transducer by a cast ring of compliant potting compound. A single element and its mounting configuration are shown in the exploded view of Figure 8.

Cylindrical stainless steel end plates are fastened to each end of the central mounting shaft and the transducer sealed in an elastomer cylindrical boot and filled with castor oil. The complete transducer is shown in the drawing of Figure 9.

Discussion of Results

The acoustical characteristics of the prototype array were measured in the frequency range from 15 kHz to 30 kHz and to hydrostatic pressures of 4 megapascals. Measurements made included; transmitting voltage response, directivity patterns in the horizontal (XY) and vertical (XZ) planes, and the linearity of output sound pressure level as a function of driving voltage. All of the measurements show very little
Figure 8. An exploded view of a single prototype element and its mounting structure.
Figure 9. The assembled prototype transducer.
change (less than ±0.5 dB) as a function of hydrostatic pressure and the horizontal directivity patterns are omnidirectional (±1.0 dB). The transducer is linear as a function of driving voltage within ±0.5 dB.

The array easily produces the required on-axis sound pressure level. At an output sound pressure level of 190 dB (referenced to one micropascal and measured at a distance of one meter) the electric field on the smallest, thinnest, wall ceramic is 1000 V/cm. or approximately one half of the voltage it can safely withstand.

The transmitting voltage response as predicted by equation (31) is compared to the measured response in Figure 10. The two response curves agree to within 1.5 dB or less and are flat (±1.5 dB) over the design frequency range of 15 kHz to 30 kHz. The close agreement between the curves indicates that the error introduced by neglecting mutual radiation effects between the elements is small, at least for the case of the prototype transducer.

The predicted (equation (32)) vertical directivity patterns are compared to the measured patterns at 15 kHz, 20 kHz, and 30 kHz in Figures 11(a), (b) and (c) respectively. Only one half of the full vertical patterns are shown in the figures because the other half is simply its mirror image. Some general conclusions can be reached about the measured patterns in terms of the specifications; the width of the main beam at
Figure 10. Comparison of the measured transmitting voltage response of the prototype array with that predicted by the model.
Figure 11(a). Comparison of the measured vertical (XZ) directivity pattern at 15 kHz with that predicted by the model.
Figure 11(b). Comparison of the measured vertical (XZ) directivity pattern at 20 kHz with that predicted by the model.
Figure 11(c). Comparison of the measured vertical (XZ) directivity pattern at 30 kHz with that predicted by the model.
the 3 dB down points is marginally acceptable but the sidelobe levels are too high, particularly at the lower frequencies. Some general observations may also be made about the accuracy of the model; while the width of the main beam is reasonably well predicted, the fine structure of the pattern (the nulls and sidelobe levels) is not. The prediction of the width of the main beam is reasonable probably because the diffraction constant used (equation (34)) and the normalized directivity pattern assumed for the individual elements in the model (equation (7)) are accurate for the narrow angular confines of the main beam. As the observation point becomes further off the axis of the main beam the unaccounted for effects from the adjacent elements and the housing end plates alter the effective diffraction constants and the normalized directivity patterns of the individual elements. This is probably best illustrated by examining the measured directivity pattern at 15 kHz. The sidelobes of the pattern look very similar to what one would expect from the larger two end elements spaced 0.8λ apart; that is, the effect of the center elements is less than predicted by the model.
CONCLUSIONS AND RECOMMENDATIONS

The primary objectives of this thesis have essentially been met:

(1) The model developed is sufficiently descriptive of a generalized array to allow for an iterative procedure for the determination of the required element sizes and array geometry.

(2) Based upon the model predictions, a prototype transducer was designed, fabricated, and underwent acoustic tests.

The directivity characteristics of the prototype array can be brought to within the specifications over the 15 kHz to 30 kHz frequency range very easily; that is, the array can simply be shortened to an effective length of approximately λ/2. This is a feasible approach in the case of the prototype because we can reduce the ceramic volume by almost one half and still maintain a safe driving voltage level. If the lengths of the element pairs are scaled from the present configuration, the flatness of the transmitting voltage response will be maintained but the output level per volt will, of course, be lower. Reducing the
volume of piezoelectric ceramic will also lower the electro-acoustic efficiency although some of the loss may be regained by parallel tuning the array with the appropriate inductor.

Model predictions indicate that an array designed for the full required frequency range of 10 kHz to 40 kHz will require eight elements (four element pairs). The safe driving voltage limit for such an array is determined by the addition of a higher frequency, even thinner walled pair of elements. Lowering the driving voltage limit has the effect of lengthening the lower frequency elements and therefore the total array. Obviously then, shortening the full array to a length of approximately $\lambda/2$, as recommended for the prototype, will probably not be feasible.

Another approach, however, does seem feasible for controlling the directivity characteristics of the full array. The requirements for the flat transmitting voltage response and the broad main beam and low sidelobes are contradictory; that is, the flat response requires that the element amplitudes be greatest at the ends of the array and decrease toward the center, the beamwidth and sidelobe requirements imply just the opposite. If the response requirements can be relaxed from flat to a constant positive slope as a function of increasing frequency (for example, +6 dB per octave) the directivity requirements can probably be met. This is true because adding a positive slope to the response is the same as applying a linear taper
(linear shading) to the array. The output sound pressure from the array could still be made constant as a function of frequency by applying the reverse slope (-6 dB per octave) to the amplitude of the driving voltage. The computer model could still be used to determine the element sizes required for the desired slope and to optimize the array length in terms of the directivity and efficiency requirements.
LIST OF REFERENCES


