Using Ebers-Moll Equations to Evaluate the Nonlinear Distortion in Bipolar Transistor Amplifiers

1977

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USING EBERS-MOLL EQUATIONS TO EVALUATE THE NONLINEAR DISTORTION IN BIPOLAR TRANSISTOR AMPLIFIERS

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RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering in the Graduate Studies Program of the College of Engineering of Florida Technological University

Orlando, Florida
1977
ABSTRACT

USING EBERS-MOLL EQUATIONS TO EVALUATE THE NONLINEAR DISTORTION IN BIPOLAR TRANSISTOR AMPLIFIERS
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The Ebers-Moll model, which is applicable to static or quasi-static conditions, is used as a basis for developing a simple method for the evaluation of harmonic distortion generated in bipolar transistor amplifiers. The Ebers-Moll equations are transformed into the desired forms using a Maclaurin Series expansion. A computer program is written to provide numerical results of the method, and these predictions are compared to measured distortion values.
ACKNOWLEDGEMENTS

I wish to acknowledge Dr. R. L. Walker for his assistance and advisement in reviewing this research report.

I also wish to extend thanks to my committee members Dr. E. E. Erickson and Dr. B. E. Mathews. I am especially grateful to Dr. B. E. Mathews for his assistance and encouragement throughout the entire school program.
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I. INTRODUCTION

Distortion In Transistor Amplifiers

In an ideal amplifier, the amplified output voltage is an enlarged version of the input voltage without any change in waveshape. However, in practical amplifiers there always exists some difference in the waveshapes of the amplified output voltage and the applied input voltage. This difference is called distortion. The distortion may be caused either by the transistor itself or by the associated circuit or by both. An important and common type of distortion is the so-called "non-linear distortion", which results from a non-linear relationship between the instantaneous values of the input voltage and the amplified output current. Such a relation is caused generally by the non-linearity of transistor characteristics.

In cases where distortion is present it is convenient to express the response of an amplifier using a periodic input waveform. A periodic input waveform may be expressed in terms of its Fourier Series components in the form:
\[ e_i = E_0 + \sum_{n=1}^{n=\infty} E_m \sin(n\omega t + \phi_n) \]  \hspace{1cm} (1.1)

where

- \( E_m \) = amplitude of \( n \)th harmonic in volts.
- \( E_0 \) = d.c. component of the input voltage (in volts).
- \( \phi_n \) = phase angle of the \( n \)th harmonic.
- \( \omega = 2\pi f \) = angular frequency of the input signal \( e_i \).
- \( f \) = frequency of the input signal \( e_i \).

If the amplifier is ideal, the amplitude of all Fourier terms which appear at the input are multiplied by a constant voltage gain \( A \), and the phase angles are increased by a quantity proportional to their frequencies. Thus, in an ideal amplifier, the amplified output waveform (a.c. component) may be expressed in the form:

\[ e_0 = A E_m \sin(\omega t + \psi + \phi_1) \]
\[ + A E_m \sin(2\omega t + 2\psi + \phi_2) \]
\[ + A E_m \sin(3\omega t + 3\psi + \phi_3) + \ldots \]  \hspace{1cm} (1.2)

Let \( \omega t + \psi = \omega t' \).

Thus,

\[ e_0 = A E_m \sin(\omega t' + \phi_1) \]
\[ + A \text{Em}_2 \sin(2\omega t' + \phi_2) \]
\[ + A \text{Em}_3 \sin(3\omega t' + \phi_3) + \ldots \]  \hspace{1cm} (1.3)

From Eq. (1.3) it can be seen that the amplified output voltage \( e_0 \) has the same waveshape as the applied input voltage \( e_i \), the only difference being an increased magnitude and a time delay of the wave by an amount \( \psi/\omega \), and consequently there is no distortion.

If the response of the amplifier differs from the above, distortion is present. Different types of distortion that may occur in an amplifier, either separately or simultaneously, are:

1. Amplitude distortion
2. Frequency distortion
3. Phase distortion.

1. Amplitude distortion

In the case of amplitude distortion the voltage gain \( A \) of the amplifier varies with the amplitude of the input wave, i.e., the amplifier output waveform has a non-linear relationship with the applied input voltage. An example of a characteristic which results in amplitude distortion is shown in Fig. 1.1.

2. Frequency distortion

Frequency distortion occurs when the voltage gain
Fig. 1.1. Input-output characteristics of an Ideal and Practical Amplifier.
varies with the frequency of the applied input voltage. Thus, the input voltage as given by Eq. (1.1), experiences a different amplification factor dependent on frequency.

Frequency distortion is primarily caused by the presence of reactive elements in the amplifier circuit. In general, both amplitude and frequency distortions may be present simultaneously and may cause the output waveform to differ materially from the input waveform.

3. Phase distortion

Previously it was stated that for an ideal amplifier the phase shift should be either zero or proportional to the frequency. If an amplifier has a phase shift which is not proportional to the frequency, phase distortion occurs. Phase distortion, like frequency distortion, generally results from the frequency dependence of the characteristics caused by reactive elements in the circuit associated with the amplifier.

Sources of Distortion

The sources of distortion in transistor amplifiers are listed below:

1. Non-constant spacing between constant-current curves, particularly along the load line.
2. Non-linear input resistance.
3. Too-large a signal so that clipping occurs from saturation or cut off.
4. Movement of the bias point with variation in temperature, which causes clipping of large-signal inputs.

In the grounded-base connection the constant-emitter current curves are normally nearly equally spaced so that distortion is small. The presence of distortion may be checked for any transistor amplifier by drawing the load line and making a plot of $I_C$ versus $I_E$. If the result is a straight line up to the saturation region, there is no distortion from non-equal spacing.

If the input resistance of the transistor amplifier is voltage- or current-dependent, distortion is present. In a transistor amplifier this will result in a distorted input current, and the output current will be an amplified version of this distorted wave.

Distortion caused by clipping occurs when the signal causes the amplifier to swing into saturation or into the cut-off region. Both saturation and cut-off will cause the output to be clipped. If the allowable power dissipation permits, the clipping may be remedied by moving the operating point farther from the origin (of the graph).
If the signal operates close to saturation or the cut-off region, distortion may be caused by the bias point shifting because of a change in ambient temperature. This shift may cause the signal to move into either the saturation or the cut-off region.

Characteristics of the grounded emitter amplifier are normally not as uniformly spaced as the grounded base. Thus, the grounded emitter has inherently more distortion. This may be corrected to a certain extent by choosing a proper source resistance. Since the grounded emitter input resistance is non-linear, as was the case for the grounded base amplifier, the source resistance is chosen to compensate for this non-linearity and also to correct the inherent distortion mentioned above. Since these two considerations are opposing, there is usually an optimum value of source resistance to provide minimum distortion. The clipping and bias stabilization aspects of the grounded emitter are identical to those of the grounded base.

**Graphical Method For Calculating Distortion**

The output current of a transistor amplifier may be expressed by a Fourier Series of the form:
\[ i_0 = I_{0A} + \sum_{k=1}^{\infty} \sqrt{2} I_{0k} \cos k\omega t \]

\[ = I_{0A} + \sqrt{2} (I_{01} \cos \omega t + I_{02} \cos 2\omega t...) \quad (1.4) \]

where:

\[ I_{0A} = \text{The average value, in amps, of } i_0 \text{ when a.c. signal is impressed at the input.} \]
\[ I_{0k} = \text{rms value of } k^{th} \text{ harmonic in amps.} \]

Assuming the final position of the a.c. load line is known, one can use the following procedure to determine the harmonics. Fig. 1.2 shows an output characteristic of a transistor. The intersections of the a.c. load line and the shifted a.c. load line (due to distortion) with the characteristic curve corresponding to the d.c. bias, determine the quiescent operating point Q and point P. Point A is the intersection of the shifted a.c. load line with the d.c. load line.

Assuming the harmonics higher than the fourth are negligible, the output current can be written as:

\[ i_0 = I_{0A} + \sqrt{2} (I_{01} \cos \omega t + I_{02} \cos 2\omega t + I_{03} \cos 3\omega t + I_{04} \cos 4\omega t). \quad (1.5) \]
Expressions for the five unknown coefficients, \( I_{0A}', I_{01}', I_{02}', I_{03}', \) and \( I_{04} \) may be obtained by evaluating the above at five different points on the a.c. load line. The magnitude of the a.c. input signal is assumed to be \( 2\Delta x \) and the input bias to be \( x_c \). The input signal is then given by:

\[
x = x_c + 2\Delta x \cos \omega t
\]

if one chooses the origin of the time axis so that it coincides with the peak value of the input signal.

Choose the values of \( \omega t \) as \( 0, \pi/3, \pi/2, 2\pi/3, \) and \( \pi \); the corresponding values of \( x \) are: \( x_c + 2\Delta x, x_c + \Delta x, x_c, x_c - \Delta x, \) and \( x_c - 2\Delta x \). The corresponding values for \( i_0 \) will be designated by \( I_{0\text{max}}, I_{0\alpha}, I_{0p}, I_{0\beta}, \) and \( I_{0\text{min}}, \) as illustrated in Fig. (1.2).

Insert these values for \( \omega t \) and \( i_0 \) into Eq. (1.5); the following five simultaneous equations may be written:

\[
I_{0\text{max}} = I_{0A} + \sqrt{2}(I_{01} + I_{02} + I_{03} + I_{04})
\]
\[
I_{0\alpha} = I_{0A} + \sqrt{2}(\frac{1}{2}I_{01} - \frac{1}{2}I_{02} - \frac{1}{2}I_{03} - \frac{1}{2}I_{04})
\]
\[
I_{0p} = I_{0A} + \sqrt{2}(-I_{02} + I_{04})
\]
Fig. 1.2. The generalized output characteristics with (1) the d.c. load line, (2) the a.c. load line, (3) the a.c. load line showing the shift due to distortion.
\[ I_{0\beta} = I_{0A} + \sqrt{2}(-\frac{1}{2}I_{01} - \frac{1}{2}I_{02} + I_{03} - \frac{1}{2}I_{04}) \]
\[ I_{0\min} = I_{0A} + \sqrt{2}(-I_{01} + I_{02} - I_{03} + I_{04}) . \]

Solve these equations; the expressions for harmonic magnitudes are obtained as follows:

\[ I_{0A} = \frac{1}{6}(I_{0\max} + I_{0\min}) + \frac{1}{3}(I_{0\alpha} + I_{0\beta}) \] (1.6)

\[ \sqrt{2}I_{01} = \frac{1}{3}(I_{0\max} - I_{0\min}) + \frac{1}{3}(I_{0\alpha} - I_{0\beta}) \] (1.7)

\[ \sqrt{2}I_{02} = \frac{1}{4}(I_{0\max} - I_{0\min}) - \frac{1}{2}I_{0\nu} \] (1.8)

\[ \sqrt{2}I_{03} = \frac{1}{6}(I_{0\max} - I_{0\min}) - \frac{1}{3}(I_{0\alpha} - I_{0\beta}) \] (1.9)

\[ \sqrt{2}I_{04} = \frac{1}{12}(I_{0\max} + I_{0\min}) - \frac{1}{3}(I_{0\alpha} + I_{0\beta}) \]
\[ + \frac{1}{2}I_{0\nu} \] (1.10)

The various harmonics are usually given as a percentage of the magnitude of the fundamental term. Thus, the percentage harmonic for second, third, and fourth harmonics are defined by:

\[ D_2 = \left| \frac{I_{02}}{I_{01}} \right| \times 100\% \]
\[ D_3 = \left| \frac{I_{03}}{I_{01}} \right| \times 100\% \]

\[ D_4 = \left| \frac{I_{04}}{I_{01}} \right| \times 100\% . \]

**Ebers-Moll Equations**

The Ebers-Moll equations are for low frequency or d.c. operation of the transistor and are derived from fundamental consideration. For pnp transistors, the Ebers-Moll equations are given by equations 1.11 and 1.12:

\[ I_E = I_{ES} \left( e^{\frac{qV_{EB}}{KT}} - 1 \right) - \alpha_R I_{CS} \left( e^{\frac{qV_{CB}}{KT}} - 1 \right) \]

(1.11)

and

\[ I_C = -\alpha_F I_{ES} \left( e^{\frac{qV_{EB}}{KT}} - 1 \right) + I_{CS} \left( e^{\frac{qV_{CB}}{KT}} - 1 \right) \]

(1.12)

where:

\[ I_{ES} = \text{Emitter saturation current when collector is shorted to the base.} \]
\[ I_{CS} = \text{Collector saturation current when emitter is} \]
shorted to the base.

\[ \alpha_F = \text{Forward short-circuit current gain.} \]

\[ \alpha_R = \text{Reversed short-circuit current gain.} \]

The four constants depend on the diffusion constants, the diffusion lengths, the equilibrium minority carrier concentration, the area and the base width.

The assumptions made in deriving these equations are listed below:

1. The transistor is operated at low frequency, so the reactive elements can be neglected.
2. Components of the terminal currents which change the excess carriers stored in the transistor are neglected.
3. The dependence of the base-width on junction voltage is neglected, i.e., base-width is constant.
4. The transistor is subject to low-level injections only.
5. Ohmic voltage drops at the contacts and in the neutral regions are neglected.

Equations (1.11) and (1.12) apply to pnp transistors. The corresponding relationships for an npn tran-
sistor are given by:

\[ I_E = -I_{ES} \left( e^{-qV_{EB}/kT} - 1 \right) + \alpha R_{I_{CS}} I_{CS} \left( e^{-qV_{CB}/kT} - 1 \right) \]  
(1.13)

\[ I_C = \alpha F I_{ES} \left( e^{-qV_{EB}/kT} - 1 \right) - I_{CS} \left( e^{-qV_{CB}/kT} - 1 \right) \]  
(1.14)

The Ebers-Moll equations, which describe the large-signal \( V-I \) characteristics of the idealized transistor model, can be given a simple and useful interpretation in terms of a circuit model that uses two idealized exponential diodes. This model is shown in Fig. 1.3. The model has a simple interpretation in terms of the internal mechanisms of the transistor. The emitter and collector currents can each be resolved into two components. The component which flows into the idealized diode is the consequence of minority-carrier injection at that junction while the other component, which is provided by the current source, is the consequence of minority-carrier injection at the other junction and transport across the base. Thus, the first component of emitter current

\[ I_{ES} \left( e^{qV_{EB}/kT} - 1 \right) \] results from diode action at the emitter junction. The second current \(-\alpha R_{I_{CS}} I_{CS} \left( e^{qV_{CB}/kT} - 1 \right)\) is the consequence of diode action at the collector.
Fig. 1.3 (a). Ebers-Moll static model for a pnp transistor.

\[ I = I_S \left( e^{\frac{qV}{kT}} - 1 \right) \]

Fig. 1.3 (b). Idealized pn-junction Diode symbol.
junction and exists because a fraction of that diode current, $\alpha_{R_I R}$, is transported across the base to the emitter, where it contributes to the total emitter current.
II. STATEMENT OF THE PROBLEM

The graphical method, described in Chapter I, to evaluate the harmonic content in the output of a bipolar transistor amplifier is very laborious and cumbersome. In addition, to determine the a.c. component of the base current (common emitter connection) assumptions are necessary. For example, $V_{be}$ is assumed negligible. Thus introducing additional errors in the calculations. Secondly, graphical methods are normally inaccurate.

In this work the object is to investigate a method which will predict with reasonable accuracy the harmonics produced for a given input voltage. The basis for this work will be the Ebers-Moll equations.

To use these equations the four parameters $\alpha_F$, $\alpha_R$, $I_{ES}$ and $I_{CS}$ (known as the forward short-circuit current-gain, reverse short-circuit current-gain, emitter saturation current when emitter is shorted to the base, respectively) must be known.

These four parameters are normally measured from the transistor characteristic curves or taken from the average data supplied by the manufacturer.
III. DERIVATION OF THE METHOD

A. Derivation of Basic Equation

Repeating the Ebers-moll equations given by Eqs. (1.11) and (1.12), one obtains:

\[ i_E = I_{ES} (e^{qV_{EB}/KT} - 1) - \alpha R_{IC} (e^{qV_{CB}/KT} - 1) \]  
\[ (3.1) \]

\[ i_C = -\alpha F I_{ES} (e^{qV_{EB}/KT} - 1) + I_{CS} (e^{qV_{CB}/KT} - 1) . \]
\[ (3.2) \]

For normal operation of an amplifier, the emitter-base junction is forward biased and the collector-base junction is reversed biased:

\[ V_{CB} = (V_{CE} - V_{BE}) < -0.1 \text{ volt} . \]

and at room temperature \( 4KT/q = 0.1 \text{ volt} \).

Thus, the term \( e^{qV_{CB}/KT} \) in Eqs. (3.1) and (3.2) can be considered negligibly small compared to unity. Hence Eqs. (3.1) and (3.2) reduce to:
\[ i_E = I_{ES} \left( e^{\frac{qV_{EB}}{KT}} - 1 \right) + \alpha R ICs \]  
(3.3)

\[ i_C = -\alpha F_{ES} \left( e^{\frac{qV_{EB}}{KT}} - 1 \right) - I_{CS} \]  
(3.4)

From this point on, only the collector current equation (Eq. 3.4) will be studied, and the corresponding harmonic evaluated.

Refer to Fig. 3.1; we use Kirchhoff’s voltage law to obtain

\[ v_{BE} = v_{BB} + v_S - R'_{S}i_B \]

where

\[ R'_S \triangleq R_S + R_B \]

\[ v_{EB} = v_{BB} - v_S + R'_{S}i_B \]  
(3.5)

Substituting Eq. (3.5) and \( q/KT \triangleq L \) in Eq. (3.4), one obtains:

\[ i_C = -\alpha F_{ES} \left[ e^{L(v_{BB} - v_S + R'_{S}i_B)} - 1 \right] - I_{CS} \]  
(3.6)

Now for a transistor working in the normal active region the following equation holds:
Fig. 3.1. Basic amplifier circuit.

Fig. 3.2. Equivalent Ebers-Moll model of Fig. 3.1.
\[ i_C = \frac{\alpha_F}{1 - \alpha_F} i_B - \frac{I_{CO}}{1 - \alpha_F} \]

where \[ I_{CO} = I_{CS} (1 - \alpha_F \alpha_R). \]

Hence:

\[ i_B = \frac{i_C (1 - \alpha_F) + I_{CO}}{\alpha_F} \]

Substituting the above in Eq. (3.6) yields:

\[ i_C = -\alpha_F I_{ES} \left[ e^{L(v_{BB} - v_S) + \frac{R_S i_C (1 - \alpha_F)}{\alpha_F}} + \frac{I_{CO} R_S'}{\alpha_F} - 1 \right] - I_{CS} \]

Define now:

\[ k_1 \triangleq -\alpha_F I_{ES} \]

\[ k_2 \triangleq L v_{BB} + \frac{L I_{CO}}{\alpha_F} R_S' \]

and

\[ k_3 \triangleq \frac{L R_S' (1 - \alpha_F)}{\alpha_F}. \]
One obtains

\[ i_C = k_1 e^{(k_2 - L v_S + k_3 i_C)} - k_1 - I_{CS}. \]  

(3.10)

**B. Harmonic Equation For Collector Current**

To determine the harmonic content, Eq. (3.10) will be expanded using Maclaurin Series Analysis. This analysis is explained in Appendix A. In this work only the first three harmonics will be considered, as the previous work has shown that the higher order terms are negligible.

The factor \( e^{(k_2 - L v_S + k_3 i_C)} \) can be expanded as:

\[
\begin{align*}
\left( k_2 - L v_S + k_3 i_C \right) e^{(k_2 - L v_S + k_3 i_C)} &= 1 + (k_2 - L v_S + k_3 i_C) \\
&\quad + \frac{(k_2 - L v_S + k_3 i_C)^2}{2} \\
&\quad + \frac{(k_2 - L v_S + k_3 i_C)^3}{6} \\
&\quad = 1 + (k_2 - L v_S + k_3 i_C) \\
&\quad + \frac{k_2^2 + L^2 v_S^2 + k_3^2 i_C^2 - 2 L k_2 v_S i_C}{2}.
\end{align*}
\]
\[
-2Lk_3v_Si_C + 2k_2k_3i_C - \frac{k^2_2}{2} - \frac{L^2v_S^3}{6} + \frac{k_3^3i_C}{3}
+ \frac{3k^2_2(-Lv_S + k_3i_C) + 3L^2v_S^2(k_2 + k_3i_C)}{6}
+ \frac{3k^2_2i_C^2(k_2 - Lv_S) - 6k_2Lv_sk_3i_C}{6}
\]

substituting this in Eq. (3.10), one obtains:

\[
i_C = k_1 \left[ 1 + k_2 - Lv_S + k_3i_C + \frac{k^3_2}{6}
+ \frac{k^2_2}{2} + \frac{L^2v_S^2}{2} + \frac{k_3^3i_C}{2} - Lk_2v_S
\right.

- Lk_3v_Si_C + k_2k_3i_C + \frac{k^3_2}{6}

- \frac{L^3v_S^3}{6} + \frac{k_3^3i_C}{6} - \frac{1}{6}Lk_2v_S + \frac{4}{6}k_2k_3i_C

+ \frac{L^2k_2v_S^2}{2} + \frac{L^2k_3v_S^2i_C}{2} + \frac{k^2_2k_3i_C}{2}

+ \frac{Lk_3v_Si_C^2}{2} - k_2Lv_sk_3i_C \right] - k_1 - i_{CS}
\]

or, in terms of the descending powers of i_C:
\[
\frac{k_1^3}{6} + \left( \frac{k_1^2}{2} + \frac{k_1^2 L v_S}{2} - \frac{k_1 k_2 L v_S}{2} \right) i_c^2
\]

\[+ (k_1 k_3 - L k_1 k_3 v_S + k_1 k_2 k_3)
\]

\[+ \frac{k_1 k_3^2}{2} + L^2 \frac{k_1 k_3 v_S^2}{2} - k_1 k_2 k_3 v_S^2 - 1 \]

\[+ (k_1 k_2 - L k_1 v_S + \frac{k_1 k_2^2}{2} + \frac{L^2 v_S^2 k_1}{2})
\]

\[- L k_1 k_2 v_S + \frac{k_1 k_3^3}{6} - \frac{k_1 L^3 v_S^3}{6} - \frac{k_1 L v_S^2}{2}
\]

\[+ \frac{k_1 L^2 k_2 v_S^2}{2} - \frac{i_{CS}}{2} = 0 \quad (3.11)
\]

In order to use the Maclaurin Series it is necessary to differentiate \(i_c\) successively with respect to \(v_S\), as indicated in Appendix A. The technique is as follows:

Differentiating Eq. \((3.11)\) with respect to \(v_S\) yields:

\[
\frac{k_1 k_3^3}{6} \frac{d i_c}{d v_S} + \frac{k_1 k_2^2}{2} \frac{d i_c}{d v_S} + \frac{k_1 k_2 k_3^3}{2} \frac{d i_c}{d v_S}
\]
\[
- \frac{k_1 L k_2^2}{2} \left[ v_S^2 \frac{d i_c}{d v_S} + i_c^2 \right]
+ (k_1 k_3 + k_1 k_2 k_3 + \frac{k_1^2 k_3}{2} - 1) \frac{d i_c}{d v_S}
+ (-Lk_1 k_3) (v_S \frac{d i_c}{d v_S} + i_c)
+ \frac{L^2 k_1 k_3}{2} (v_S^2 \frac{d i_c}{d v_S} + 2i_c v_S)
+ (-k_1 k_2 Lk_3) (v_S \frac{d i_c}{d v_S} + i_c)
+ (-Lk_1) + \frac{L^2 k_1}{2} 2v_S - Lk_1 k_2 - \frac{k_1 L^3}{6} 3v_S^2
- \frac{k_1 L k_2^2}{2} + k_1 L^2 k_2 v_S = 0
\] (3.12)

At the operating point, \(v_S=0\), \(i_c=I_C\) (in this work the operating point will be established). So to obtain the value of the derivative \(\frac{d i_c}{d v_S}\) at the operating point, i.e., \(\frac{d i_c}{d v_S}\bigg|_{v_S=0}\), substituting \(v_S=0\) and \(i_c=I_C\) in Eq. (3.12);
\[
\frac{k_1 k_2^3}{2} I_C^2 \left. \frac{dI_C}{dv_S} \right|_{v_S=0} + \left. \frac{dI_C}{dv_S} \right|_{v_S=0}
\]

\[
(k_1 k_2^2 + k_1 k_2 k_3^2) I_C - \frac{k_1 k_3^2}{2} I_C^2 L
\]

\[
+ (k_1 k_3 + k_1 k_2 k_3 + \frac{k_1 k_2 k_3}{2} - 1) \left. \frac{dI_C}{dv_S} \right|_{v_S=0}
\]

\[
+ (-Lk_1 k_3) I_C + (-k_1 k_2 k_3 L) I_C + (-Lk_1)
\]

\[
- Lk_1 k_2 - \frac{k_1 L k_3^2}{2} = 0
\]

Substituting \( \left. \frac{dI_C}{dv_S} \right|_{v_S=0} = A_1 \), and collecting terms containing \( A_1 \), we get:

\[
A_1 \left[ \frac{k_1 k_3^2 L}{2} I_C^2 + (k_1 k_2^2 + k_1 k_2 k_3^2) I_C^2 \right. \\
+ (k_1 k_3 + k_1 k_2 k_3 + \frac{k_1 k_2 k_3}{2} - 1) \]

\[
= \left\{ \frac{k_1 L k_3^2}{2} I_C^2 + Lk_1 k_3 I_C + k_1 k_2 k_3 L I_C \right\}
\]
\[ + Lk_1 + Lk_1 k_2 + \frac{Lk_1 k_2^2}{2} \]

or

\[
A_1 = \frac{\left\{ \frac{k_1 k_3^2 L}{2} I_C^2 + Lk_1 k_2 I_C + k_1 k_2^2 k_3 I_C \right\}}{\left\{ \frac{k_1 k_3^2}{2} I_C^2 + (k_1 k_3^2 + k_1 k_2 k_3^2) I_C \right\}} + (k_1 k_3^2 + k_1 k_2 k_3^2 + \frac{k_1 k_2^2 k_3}{2} - 1)
\]

(3.13)

where

\[ A_1 = \left. \frac{dI_C}{dv_S} \right|_{v_S=0} = \text{a constant coefficient of the Maclaurin Series expansion as indicated in Appendix A.} \]

\[ I_C = \text{Collector current (d.c.) at operating point,} \]
\[ k_1, k_2, k_3 \text{ are constants given by Eqs. (3.7), (3.8), and (3.9).} \]

Successive differentiation must be taken of Eq. (3.12) to get \( A_2 \), the harmonic coefficient of the Maclaurin Series expansion.

Differentiating once again yields,

\[ \frac{k_1 k_3}{2} \left[ \frac{d^2 I_C}{dv_S^2} + \frac{dI_C}{dv_S} \frac{dI_C}{dv_S} \right] + (k_1 k_3^3 + k_1 k_2 k_3^2) \]
\[
\left[ i_C \frac{d^2i_C}{dv_S^2} + \frac{di_C}{dv_S} \frac{di_C}{dv_S} \right] - \frac{k_1Lk_2^2}{2} \left[ v_S^2i_C \frac{d^2i_C}{dv_S^2} \right]
\]

\[
+ v_S \frac{di_C}{dv_S} 2 \frac{di_C}{dv_S} + 2i_C \frac{di_C}{dv_S} + 2i_C \frac{di_C}{dv_S} \]

\[
+ (k_1k_3 + k_12k_3^2 \frac{k_1k_2^2}{2} - 1) \frac{d^2i_C}{dv_S^2} + (-Lk_1k_3)
\]

\[
\left[ v_S \frac{d^2i_C}{dv_S^2} + \frac{di_C}{dv_S} + \frac{di_C}{dv_S} \right] + \frac{L^2k_1k_2}{2} \left[ v_S^2 \frac{d^2i_C}{dv_S^2} \right]
\]

\[
+ \frac{di_C}{dv_S} 2v_S + i_C^2 + 2v_S \frac{di_C}{dv_S} \right] + (-k_1k_2k_3L)
\]

\[
(v_S \frac{d^2i_C}{dv_S^2} + \frac{di_C}{dv_S} + \frac{di_C}{dv_S}) + L^2k_1 - \frac{k_1L^3}{2} 2v_S
\]

\[
+ k_1L^2k_2 = 0
\]

Evaluating at the operating point, i.e., when \(v_S=0\),

\[
i_C = I_C \text{ and substituting } \frac{d^2i_C}{dv_S^2} \bigg|_{v_S=0} = A_2 \text{ yields,}
\]

\[
\frac{k_1k_2^3}{2} (I_C^2A_2 + A_1^2I_C) + (k_1k_3^2 + k_1k_2k_3^2) (I_CA_2 + A_1^2)
\]
- \frac{k_1 L k_3^2}{2} (2I_C A_1 + 2I_C A_1) + (k_1 k_3 + k_1 k_2 k_3)
+ \frac{k_1 k_2 k_3}{2} (2I_C) + (-Lk_1 k_3) (A_1 + A_1)
+ L^2 k_1 + k_1 k_2 L^2 = 0.

Rearranging yields the final equation for A_2:

\[
A_2 = \frac{-(A_1^2 I_C k_1 k_3^2) - (k_1 k_3^2 + k_1 k_2 k_3) A_1^2
+ Lk_1 k_3^2 A_1 - L^2 k_1 k_3 I_C + k_1 k_2 k_3^2 A_1}{\frac{k_1 k_3^2}{2} I_C^2 + (k_1 k_3^2 + k_1 k_2 k_3) I_C
+ k_1 Lk_3^2 I_C A_1
- L^2 k_1 - k_1 k_2 L^2
+ (k_1 k_3 + k_1 k_2 k_3 + \frac{k_1 k_2 k_3}{2} - 1)}
\]

(3.15)
where:

\[ A_2 = \frac{d^2 i_C}{dv^2_S} \Bigg|_{v_S=0} \]

is a coefficient of the Maclaurin Series expansion as indicated in Appendix A, and \( i_C \) = Collector current (d.c.) at the operating point.

Continuing the procedure of differentiation with respect to \( v_S \) yields Eq. (3.16):

\[
\frac{1}{2} k_1 k_2^2 \left[ i_C^2 \frac{d^3 i_C}{dv^3_S} + \frac{d^2 i_C}{dv^2_S} 2i_C \frac{di_C}{dv_S} + (\frac{di_C}{dv_S})^2 2 \frac{di_C}{dv_S} \\
+ 2i_C \frac{d^2 i_C}{dv^2_S} 3 \frac{di_C}{dv_S} \right] + (k_1 k_2^2 + k_1 k_2 k_3)
\]

\[
\left[ i_C \frac{d^3 i_C}{dv^3_S} + \frac{d^2 i_C}{dv^2_S} \frac{di_C}{dv_S} + 2 \frac{di_C}{dv_S} \frac{d^2 i_C}{dv^2_S} \right]
\]

\[
-\frac{k_1 L k_2^2}{2} \left[ v_S 2i_C \frac{d^3 i_C}{dv^3_S} + 2i_C \frac{d^2 i_C}{dv^2_S} + v_S \frac{d^2 i_C}{dv^2_S} \frac{di_C}{dv_S} \\
+ 2v_S^2 \frac{di_C}{dv_S} \frac{d^2 i_C}{dv^2_S} + (\frac{di_C}{dv_S})^2 2 + 4i_C \frac{d^2 i_C}{dv^2_S} \\
+ 4 \frac{di_C}{dv_S} \frac{di_C}{dv_S} \right] \frac{d^3 i_C}{dv^3_S} (k_1 k_3 + k_1 k_2 k_3)
\]
\[
\begin{align*}
&+ \frac{k_1 k_2 k_3^2}{2} - 1) + (-Lk_1 k_3) \left[ v_S \frac{d^3 i_C}{dv_S^3} + \frac{d^2 i_C}{dv_S^2} + 2 \frac{d^2 i_C}{dv_S} \right] \\
&+ \frac{L^2 k_1 k_3}{2} \left[ v_S \frac{d^3 i_C}{dv_S^3} + \frac{d^2 i_C}{dv_S^2} \right] 2v_S + 4v_S \frac{d^2 i_C}{dv_S} + 4 \frac{di_C}{dv_S} + 2 \frac{di_C}{dv_S} \\
&+ (-k_1 k_2 k_3 L) v_S \frac{d^3 i_C}{dv_S^3} + \frac{d^2 i_C}{dv_S^2} + 2 \frac{d^2 i_C}{dv_S} - k_1 L^3 = 0 \\
\end{align*}
\]  

(3.16)

Substituting \( \frac{d^3 i_C}{dv_S^3} = A_3 \), \( v_S = 0 \) (at the operating point),
\( i_C = I_C \) and rearranging yields Eq. (3.17).

\[
\begin{align*}
- k_1 k_3^3 (3I_C A_1 A_2 + A_1^3) & - (k_1 k_3^2 + k_1 k_2 k_3^2) \left( 3A_1 A_2 \right) \\
+ 3k_1 k_3^2 L (I_C A_2 + A_1^2) & + 3Lk_1 k_3 A_2 - 3A_1 L^2 k_1 k_3 \\
+ 3A_2 k_1 k_2 k_3 L & + kL^3 \\
A_3 &= \frac{k_1 k_3}{2} I_C^2 + (k_1 k_3^2 + k_1 k_2 k_3^2) I_C + (k_1 k_3 + k_1 k_2 k_3 \\
&+ \frac{k_1 k_2 k_3}{2} - 1) \\
\end{align*}
\]  

(3.17)
Proceeding for $A_4 = \left. \frac{d^4 i_c}{dv_S^4} \right|_{v_S=0}$, the Maclaurin Series Coeff.,

\[
\frac{k_1k_3^3}{2} \left[ i_c^2 \frac{d^4 i_c}{dv_S^4} + \frac{d^3 i_c}{dv_S^3} 2i_c \frac{di_c}{dv_S} + 2 \frac{d^2 i_c}{dv_S^2} i_c \frac{d^2 i_c}{dv_S^2} \\
+ 2 \frac{d^2 i_c}{dv_S} \frac{di_c}{dv_S} \frac{di_c}{dv_S} + 2i_c \frac{di_c}{dv_S} \frac{d^3 i_c}{dv_S^3} + 6(\frac{di_c}{dv_S})^2 \\
\frac{d^2 i_c}{dv_S^2} + 4i_c \frac{d^2 i_c}{dv_S^2} \frac{d^2 i_c}{dv_S^2} + 4i_c \frac{di_c}{dv_S} \frac{d^3 i_c}{dv_S^3} \\
+ 4 \frac{d^2 i_c}{dv_S^2} \frac{di_c}{dv_S} \frac{di_c}{dv_S} \left] + (k_1k_3^2 + k_1k_2^2k_3^2) \right.
\]

\[
\left( i_c^2 \frac{d^4 i_c}{dv_S^4} + \frac{d^3 i_c}{dv_S^3} \frac{di_c}{dv_S} + \frac{d^2 i_c}{dv_S^2} \frac{d^2 i_c}{dv_S^2} + \frac{di_c}{dv_S} \frac{d^3 i_c}{dv_S^3} \\
+ 2 \frac{di_c}{dv_S} \frac{d^3 i_c}{dv_S^3} + 2 \frac{d^2 i_c}{dv_S^2} \frac{d^2 i_c}{dv_S^2} - \frac{k_1Lk_3^3}{2} \left[v_S^2 i_c \frac{d^4 i_c}{dv_S^4} \\
+ v_S^3 \frac{d^3 i_c}{dv_S^3} \frac{di_c}{dv_S} + 2i_c \frac{d^3 i_c}{dv_S^3} + 6i_c \frac{d^3 i_c}{dv_S^3} \right]
\]
\[
+ 6 \frac{d^2 i_C}{dv_S^2} \frac{d i_C}{dv_S} + 6v_S \frac{d^2 i_C}{dv_S^2} \frac{d^2 i_C}{dv_S^2} + 6v_S \frac{d i_C}{dv_S} \frac{d^3 i_C}{dv_S^3} \\
+ 6 \frac{d i_C}{dv_S} \frac{d^2 i_C}{dv_S^2} + 12 \frac{d i_C}{dv_S} \frac{d^2 i_C}{dv_S^2} + (k_1 k_3 + k_1 k_2 k_3) \\
+ \frac{k_1 k_2 k_3}{2} - 1 \right) \frac{d^4 i_C}{dv_S^4} + (-L k_1 k_3) \left[ \frac{d^4 i_C}{dv_S^4} + \frac{d^3 i_C}{dv_S^3} \right] + \frac{d^3 i_C}{dv_S^3} \\
+ 3 \frac{d^3 i_C}{dv_S^3} + \frac{L^2 k_1 k_3}{2} \left[ v_S^2 \frac{d^4 i_C}{dv_S^4} + \frac{d^3 i_C}{dv_S^3} \right. 2v_S + \frac{d^2 i_C}{dv_S^2} 6 \\
+ 6v_S \frac{d^3 i_C}{dv_S^3} + 6 \frac{d^2 i_C}{dv_S^2} \right] + (-k_1 k_2 k_3) \\
(v_S \frac{d^4 i_C}{dv_S^4} + \frac{d^3 i_C}{dv_S^3} + \frac{d^3 i_C}{dv_S^3}) = 0
\]

Substituting \( \frac{d^4 i_C}{dv_S^4} = A_4 \), \( v_S = 0 \) and \( i_C = I_C \) (at the operating point) and rearranging the terms yields Eq. (3.18):
$$- k_1 k_3^3 \left( 4 I_c A_1 A_3 + 3 I_c A_2^2 + 6 A_1^2 A_2 \right) - (k_1 k_3^2 + k_1 k_2 k_3^2)$$

$$\left( 4 A_1 A_3 + 3 A_2^2 \right) + 4 k_1 L k_3^2 \left( I_c A_3 + 3 A_1 A_2 \right) + 4 A_3 \left( L k_1 k_3 \right)$$

$$- 6 A_2 L^2 k_1 k_3 + (k_1 k_2 k_3 L) 4 A_3$$

$$A_4 = \frac{k_1 k_3^2}{2 I_c^2 + I_c \left( k_1 k_3^2 + k_1 k_2 k_3^2 \right) + (k_1 k_3 + k_1 k_2 k_3)}$$

$$+ \frac{k_1 k_2 k_3}{2} - 1$$

(3.18)

Now repeating the Eq. (A.4) from the Appendix A,

$$i_c = (A_1 v_m + \frac{A_2 v_m^3}{8}) \cos \omega t + \left( \frac{A_2 v_m^2}{4} + \frac{A_4 v_m^4}{48} \right)$$

$$\cos 2 \omega t + \left( \frac{A_3 v_m^3}{24} \right) \cos 3 \omega t + \left( \frac{v_m^2 A_2}{4} + \frac{A_4 v_m^4}{64} \right)$$

(3.19)

where $v_m$ = peak-value of the input signal. The values of the coefficients $A_1$, $A_2$, $A_3$, and $A_4$ are evaluated from the known values of the Ebers-Moll parameters ($\alpha_F$, $\alpha_R$, $I_{ES}$ and $I_{CS}$); the harmonics can be determined from Eq. (3.19).
IV. NUMERICAL EXAMPLE

To illustrate the method and to provide an estimate of the accuracy of the work, an experiment was performed using the circuit shown in Figure 4.1. To check the derived method, experimentally measured values of $V_{BB}$, $R$, $\beta$, $I_{ES}$, and $I_{CS}$ were substituted in Equation (3.19) for the collector current, and a program was run on the computer. The complete program is shown in Appendix B. A pnp type 2N1234 transistor was used in the experiment, for which the values of $\beta$, $I_{ES}$, and $I_{CS}$ were found experimentally as follows.

As explained in Chapter III, for the normal active region collector current $I_C$ is given by:

$$I_C = \frac{\beta}{1 - \beta} I_B - \frac{I_{CO}}{1 - \beta}$$

and by definition,

$$\left. \frac{\Delta I_C}{\Delta I_B} \right|_{V_{GE} = \text{const.} < 0} = \frac{\beta}{1 - \beta} \Rightarrow \beta$$

(4.1)
The basic amplifier circuit.

Fig. 4.1. The basic amplifier circuit.
Thus, $\beta_F$ can be found by noting the average spacing of the common-emitter output curves in the normal region. Subsequently, $\alpha_F$ can be calculated from Equation (4.1).

The average spacing of the common-emitter output curves in the normal active region was found using a curve tracer. After several measurements were taken the average value shown below was computed and was used in the experiment.

$$\beta_F = 11.5$$

Thus,

$$\alpha_F = \frac{\beta_F}{1 + \beta_F} = 0.92$$

Similarly $\alpha_R$ can be determined by noting the average spacing of the common-emitter output curves in the inverse region. Thus,

$$\left. \frac{\Delta I_C}{-\Delta I_B} \right|_{V_{CE} = \text{const.} > 0}^{\text{inverse-gain region}} = \frac{1}{1 - \alpha_R}$$

But here, for a more accurate result, the emitter and collector leads were interchanged (collector grounded), and by noting the average spacing of output curves in the
normal region of the inverted connection, the value of $\beta_R$ was found as:

$$\frac{\Delta I_C}{\Delta I_B} = \beta_R = \frac{\alpha_R}{1 - \alpha_R} = 1.5$$

From which

$$\alpha_R = \frac{\beta_R}{1 + \beta_R} = \frac{1.5}{2.5} = 0.6$$

The values of the saturation current parameters, $I_{ES}$ and $I_{CS}$, can be determined most accurately by direct measurement on the transistor. The technique is to let $V_{EB}$ approach zero, which reduces the Ebers-Moll equation to:

$$I_C = I_{CS} \left( e^{\frac{qV_{CB}}{KT}} - 1 \right)$$

The values of $I_C$ for different values of $V_{CB}$ were recorded. Then $I_{CS}$ was determined by plotting a graph of $\log I_C$ versus $V_{CB}$ for $V_{CB} > 0.1$, and by extrapolating back to zero $V_{CB}$.

To determine the corresponding reverse parameter $I_{CS'}$, on setting $V_{CB}=0$ the emitter current becomes:
As mentioned in chapter III, the four parameters satisfy the following equation.

\[ I_E = I_{ES} \left( e^{\frac{qV_{BE}}{KT}} - 1 \right) \]

\[ \alpha_F I_{ES} = \alpha_R I_{CS} \quad (4.2) \]

Consequently, we need to know only either \( I_{CS} \) or \( I_{ES} \). The experiment was performed to determine \( I_{CS} \). Then a graph (Fig. 4.1) was plotted and the value of \( I_{CS} \) was calculated from the intercept of the ordinate. Then using the Equation (4.2), \( I_{ES} \) was calculated.

Therefore:

\[ I_{ES} = 8.93 \times 10^{-9} \text{ A} \]

\[ I_{CS} = 13.69 \times 10^{-9} \text{ A} \]

\[ I_{CO} = I_{CS}(1 - \alpha_F \alpha_R) = 6.13 \times 10^{-9} \text{ A} \]

The experimental results, which were obtained using Hewlett-Packard Model 302A wave analyzer (Fig. 4.2) are shown in Table 4.1(a) and 4.1(b). The results using
Fig. 4.1. The graph log of $I_C$ versus $V_{CB}$. 

\[ I_{GS} \left( \frac{10^{-6}}{10^{-6}} \right) = -1.87 \]

or

\[ I_C \left( \frac{10^{-6}}{10^{-6}} \right) = 0.01369 \]

or

\[ I_{GS} = 13.69 \times 10^{-9} \text{Amp.} \]
Fig. 4.2. Harmonic Distortion Measurement.
<table>
<thead>
<tr>
<th>Harmonic</th>
<th>$V_{ce1}$ (d.c.)</th>
<th>$V_{ce2}$ in volts</th>
<th>$V_{ce3}$ in volts</th>
<th>$V_{ce1}$ in percentages of $V_{ce1}$</th>
<th>$V_{ce2}$ in percentages of $V_{ce1}$</th>
<th>$V_{ce3}$ in percentages of $V_{ce1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Harmonic</td>
<td>$V_s$ in volts</td>
<td>28.0</td>
<td>1.2783</td>
<td>28.15</td>
<td>9212</td>
<td>1.2783</td>
</tr>
<tr>
<td>2nd Harmonic</td>
<td>$V_{ce1}$ in rms volts</td>
<td>11.6</td>
<td>0.58</td>
<td>0.73%</td>
<td>4.96%</td>
<td>0.22</td>
</tr>
<tr>
<td>3rd Harmonic</td>
<td>$V_{ce2}$ in per-rms volts</td>
<td>0.9212</td>
<td>0.084</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**REMARKS**

- $V_s$: Supply voltage
- $V_{ce1}$, $V_{ce2}$, $V_{ce3}$: Collector-emitter voltages for specific harmonics.
TABLE 4.1 (b)

THE VALUES OF THE HARMONICS OBTAINED USING WAVE ANALYZER

<table>
<thead>
<tr>
<th>$V_s$ (rms volts)</th>
<th>$I_c$ (d.c.)</th>
<th>1$^{st}$ Harmonic (peak value)</th>
<th>2$^{nd}$ Harmonic (peak value)</th>
<th>3$^{rd}$ Harmonic (peak value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9212</td>
<td>2.10 mA</td>
<td>1.10 mA</td>
<td>0.054 mA</td>
<td>0.008 mA</td>
</tr>
<tr>
<td>1.2783</td>
<td>2.09 mA</td>
<td>1.60 mA</td>
<td>0.11 mA</td>
<td>0.022 mA</td>
</tr>
</tbody>
</table>
the measured parameters in the computer program are shown in Table 4.2. A comparison of the harmonic values is given in Table 4.3.
### Table 4.2

<table>
<thead>
<tr>
<th>( V_s ) in peak (d.c.)</th>
<th>( I_C ) in Amp</th>
<th>1(^{st}) Harmonic ( I_C1 ) in mA</th>
<th>2(^{nd}) Harmonic in mA in per- (Peak centage value) of ( I_{C1} )</th>
<th>3(^{rd}) Harmonic in mA in per- (Peak centage value) of ( I_{C1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9212 1.3026 2.39 1.27</td>
<td>0.064 5.06%</td>
<td>0.0064 0.51%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2783 1.8090 2.39 1.71</td>
<td>0.130 7.48%</td>
<td>0.017 1.09%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 4.3
COMPARISON OF THE RESULTS OF THE EXPERIMENTAL AND COMPUTER METHOD

<table>
<thead>
<tr>
<th>$V_s$ in rms volts</th>
<th>Harmonic Component</th>
<th>Experimental Method</th>
<th>Computer Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9212</td>
<td>$I_C$ (d.c.)</td>
<td>2.100 mA</td>
<td>2.3941 mA</td>
</tr>
<tr>
<td></td>
<td>1st Harmonic</td>
<td>1.100 mA</td>
<td>1.2700 mA</td>
</tr>
<tr>
<td></td>
<td>2nd Harmonic</td>
<td>0.054 mA</td>
<td>0.0640 mA</td>
</tr>
<tr>
<td></td>
<td>3rd Harmonic</td>
<td>0.008 mA</td>
<td>0.0064 mA</td>
</tr>
<tr>
<td>1.2783</td>
<td>$I_C$ (d.c.)</td>
<td>2.09 mA</td>
<td>2.3941 mA</td>
</tr>
<tr>
<td></td>
<td>1st Harmonic</td>
<td>1.60 mA</td>
<td>1.707 mA</td>
</tr>
<tr>
<td></td>
<td>2nd Harmonic</td>
<td>0.11 mA</td>
<td>0.13 mA</td>
</tr>
<tr>
<td></td>
<td>3rd Harmonic</td>
<td>0.022 mA</td>
<td>0.017 mA</td>
</tr>
</tbody>
</table>
V. ESTIMATED SOURCE OF ERRORS

Experimental Method: The Hewlett-Packard Model 302A wave analyzer used for determining the harmonics has a measurement error of less than 1%. The sensitivity of the instrument is from 30 \( \mu \text{V} \) to 300 volts. Since the harmonic values obtained, by using the Ebers-Moll equation, and the computer program, are not expected to approach the sort of accuracy that the experimental results have, these results (experimental) will be considered the standard for comparison.

Computer Method: Accuracy of this method depends on (1) the validity of the Ebers-Moll model, (2) accuracy of the ac-dc voltmeter (used for measuring the voltages), and (3) the validity of the assumptions and approximations made in the derivation of the expression for harmonic evaluation.

As previously stated in the derivation of the Ebers-Moll equations, the effect of the space-charge-layer widening upon \( \alpha_F \), \( \alpha_R \), \( I_{ES} \) and \( I_{CS} \) has been ignored. Secondly, \( \alpha_F \) and \( \alpha_R \) were considered to be independent of current in the model. In actual practice this is not the case. So these assumptions will cause some error in the results. Experience has shown that these errors are of
the order of magnitude of 5%.

In deriving the computer method two approximations were made. (1) \( V_{CB} < 0.1 \) volt, where for the most part, the magnitude of this voltage is much greater than 0.1 volt. This error can be considered small. (2) The expansion of \( \exp(K_2 - L V_S + K_3 i_C) \) was terminated after the 4th term. Under certain circumstances, this could introduce considerable error; however, previous experimental work has shown that if the bias and load networks are linear resistors, the harmonics higher than the 3rd do not contribute significant changes in the waveshape. In summary, based on the literature and experience with this method, errors of the order of 10% can be expected routinely.
VI. SUMMARY AND CONCLUSION

The basic objective of this work was to obtain a method for evaluating the harmonic distortion generated in the bipolar transistor amplifiers. Ebers-Moll equations were used as the basic equations for developing this method. These equations were then transformed into a desired form using the Maclaurin Series expansion.

By this method the value of harmonics that will be produced for a given applied input signal, knowing the values of $\alpha_F$, $\alpha_R$, $I_{ES}$ and $I_{CS}$ (the four parameters of the Ebers-Moll equations), can be determined.

For this work with a 2N1234 transistor, the four parameters were measured experimentally, and harmonics were predicted using the derived method. The computer program used is shown in Appendix B. An experiment was performed, and harmonics were measured applying the two input signals.

All the results of the two methods (computer and experimental) were compared as shown in table 4.3.

In summary, considering the experimental errors and the assumptions made to derive the computer method, the results are in excellent agreement.
APPENDIX A

If there is a function \( Z = f(x) \), it can be expanded using Maclaurin Series, as follows:

\[
f(x) = f(0) + xf'(0) + \frac{x^2 f''(0)}{2!} + \ldots + \frac{x^n f^{(n)}(0)}{n!},
\]

where all the derivatives are taken at point \( x = 0 \).

If there is a function \( i_C = f(v_S) \), (Eq. 3.10), we can expand this function about the operating point \( (v_S = 0, i_C = I_C) \):

\[
i_C = f(0) + v_S \left. \frac{di_C}{dv_S} \right|_{v_S=0} + \frac{1}{2} v_S \left. \frac{d^2i_C}{dv_S^2} \right|_{v_S=0} + \frac{1}{6} v_S \left. \frac{d^3i_C}{dv_S^3} \right|_{v_S=0}
\]

but \( i_C = I_C + i_c (d.c. \ component + a.c. \ component) \) but \( I_C = f(0) \) in Eq. (A.3). So Eq. (A.3) reduces to
\[
\begin{align*}
    i_c \text{ (a.c. component)} &= v_S \left. \frac{di_c}{dv_s} \right|_{v_S=0} + \frac{1}{2} v_S^2 \left. \frac{d^2i_c}{dv_s^2} \right|_{v_S=0} \\
    &\quad + \frac{1}{24} v_S^4 \left. \frac{d^4i_c}{dv_s^4} \right|_{v_S=0}.
\end{align*}
\]

Substituting \( A_1, A_2, A_3 \) and \( A_4 \) for \( \frac{di_c}{dv_s}, \frac{d^2i_c}{dv_s^2}, \frac{d^3i_c}{dv_s^3}, \frac{d^4i_c}{dv_s^4} \), respectively, one obtains:

\[
i_c = v_S A_1 + \frac{1}{2} v_S^2 A_2 + \frac{1}{6} v_S^3 A_3 + \frac{1}{24} v_S^4 A_4
\]

when only four terms are retained.

For \( v_S = v_m \cos \omega t \)

\[
i_c = A_1 v_m \cos \omega t + \frac{1}{2} v_m^2 A_2 \cos^2 \omega t + \frac{1}{6} v_m^3 A_3 \cos^3 \omega t
\]

\[+ \frac{1}{24} v_m^4 A_4 \cos^4 \omega t
\]

where

\[
\cos^2 \omega t = \frac{1 + \cos 2\omega t}{2}
\]
\[
\cos^3 \omega t = \cos \frac{3\omega t}{4} + 3 \cos \omega t
\]

\[
\cos^4 \omega t = \cos 4\omega t + 8 \cos^2 \omega t - 1
\]

\[
= \cos 4\omega t + 4 \cos 2\omega t + 3
\]

Substituting this value in the above equation for \(i_c\) yields:

\[
i_c = A_1 v_m \cos \omega t + \frac{A_2 v_m^2}{4} + \frac{A_2 v_m^2}{4} \cos 2\omega t + \frac{A_3 v_m^3}{24} \cos 3\omega t
\]

\[
+ \frac{A_2 v_m^2}{3} \cos \omega t + \frac{A_2 v_m^2}{24} 2 \cos \omega t + \frac{A_2 v_m^4}{24x8} \cos 4\omega t
\]

\[
+ \frac{A_4 v_m^4}{24x8} 4 \cos 2\omega t + \frac{A_4 v_m^4}{24x8} 3.
\]

Neglecting the fourth and higher order terms (terms containing \(\cos 4\omega t\) and higher) yields:

\[
i_c = (A_1 v_m + A_3 \frac{v_m^3}{8}) \cos \omega t + (\frac{A_2 v_m^2}{4} + \frac{A_4 v_m^4}{48}) \cos 2\omega t
\]

\[
+ \left( \frac{A_2 v_m^2}{24} \right) \cos 3\omega t + \left[ \frac{v_m^2 A_2}{4} + \frac{A_4 v_m^4}{64} \right]
\]

\(\text{(A.4)}\)
Coefficients of $\cos \omega t$, $\cos 2\omega t$ and $\cos 3\omega t$ in the parentheses represent fundamental, second, and third harmonics respectively. The last term (in the brackets) is called the rectified a.c. component. Hence, the first, second and third harmonics are given by:

$$i_{c1} = A_1 v_m + \frac{A_3 v_m^3}{8}$$

$$i_{c2} = \frac{A_2 v_m^2}{4} + \frac{A_4 v_m^4}{48}$$

$$i_{c3} = \frac{A_3 v_m^3}{24}$$

Rectified component $= \frac{v_m^2 A_2}{4} + \frac{A_4 v_m^4}{64}$. 
APPENDIX B

In the programming data were taken as follows:

$A_B L = L = \frac{q}{kT} = 38.46 \text{ volts}^{-1}$ (assumed)

$V_{BB} = 2.535 \text{ volts (Fig. 4.1)}$

$R'S = R_B + R_S = 10600 \text{ ohms (Fig. 4.1)}$

$A_F = F = 0.92$ (page 37)

$E_S = I_{ES} = 8.929 \times 10^{-9} \text{ amperes (page 39)}$

$C_O = I_{CO} = 6.133 \times 10^{-9} \text{ amperes (page 39)}$

$C_S = I_{CS} = 13.69 \times 10^{-9} \text{ amperes (page 39)}$

$2.394 = I_C = \text{ was obtained from eq. (3.10) for } V_s = 0.$
A PROGRAM FOR EVALUATION OF HARMONIC DISTORSION IN TRANSISTOR AMPLIFIER

DIMENSION AL(4),VM(2)
DATA ABL,VBB,RS,AF,ESI,COI,CSI/38.4615,2.535,10600.0,0.92,8.919E-9,6.133E-9,13.69E-9/
VM(1)=1.3026
VM(2)=1.8090
T1=-AF*ESI
T2=ABL*VBB+ABL*COI*RS/AF
T3=ABL*RS*(1.0-AF)/AF
P=3.0*(1.0+T2)/T3
Q=6.0*(T1*T3+T1*T2*T3+0.5*T1*T2*T2*T3-1.0)/(T1*(T3**3))
R=6.0*(T1*T3+T1*T2*T3+0.5*T1*T2*T2*T2*0.16666-CSI)/(T1*(T3**3.1))
A=Q-(P/P3.0)
B=(2.0*(P**3.0)-9.0*P*Q=27.0*R)/27.0
S1=B/2.0
S2=((B*B)/4.0)+((A**3.0)/27.0)
A1=(S1+SQRT(S2))**(1.0/3.0)
B1=(((-1.0)*(S1-SQRT(S2))**(1.0/3.0))*(-1.0)
QC=(A1+B1-(P/3.0))
AN1=T1*ABL*(T3*T3*QC*QC/2.0+(1.0+T2)*(1.0+T3*QC)+T2/2.0)
AD1=QC*T1*T3*T3*(T3*QC/2.0+(1.0+T2))+(T1*T3)*(1.0+T2+T2*T2/2.0)-1.0
AL(1)=ANI/ADI
AN2=T1*T3*T3*AL(1)*(2.0*ABL*QC-QC*T3*AL(1)-AL(1)*(1.0+T2))+(ABL*T1*T3)*(2.0*AL(1)-ABL*QC+2.0*T2*AL(1))
-ABL*ABL*T1*T1*T2*ABL
AL(2)=AN2/AD1
AN3=T1*T3*T3*(3.0*ABL*(QC*AL(2)+AL(1)*AL(1)-T3*(3.0*QC*AL(2)*AL(1)+AL(1)*AL(1)*AL(1))-3.0*AL(1)*AL(2)*AL(2)*(1.0+T2))+3.0*ABL*T1*T3*(AL(2)-AL(1)*ABL+AL(2)*T2)+T1*(ABL**3.0))
AL(3)=AN3/AD1
AN4=4.0*T1*ABL*T3*T3*(QC*AL(3)+3.0*AL(1)*AL(2))-T1*(T3**3.0)*AL(3)*QC*AL(1)*4.0+6.0*AL(1)*AL(1)*AL(2)+AL(2)+3.0*QC*AL(2)*AL(2)-T1*T3*T3*(1.0+T2)*(4.0*AL(1)*AL(3)+3.0*AL(2)*AL(2))+ABL*T1*T3*(4.0*AL(3)-6.0*ABL*AL(2)+4.0*AL(3)*T2)
AL(4)=AN4/AD1
DO5J=1.2
VMJ=VM(J)
AIC=1.0*AL(1)*VM(J)+AL(3)*((VM(J)**3.0)/8.0)
BIC=(AL(2)*VM(J))/(VM(J))**(4.0.0)+AL(4)*((VM(J)**4.0)/48.0)
CIC=AL(3)*((VM(J))**3.0)/24.0
DIC=(((VM(J))**2.0)*AL(2))/4.0+(AL(4)*((VM(J))**4.0)/64.0)-2.394
5 CONTINUE
WRITE (6,991) VMJ,AIC,BIC,CIC,DIC
991 FORMAT(///,15X,'VS = ',F6.4///,15X,'1ST HARM. =
',E10.4,'),'',2ND HARM. = ',E10.4,')'),3RD HARM. =
',E10.4///,15X,'IC (D.C.) = ',F6.4///,15X,'VS =
',F6.4///,15X,'1ST HARM. = ',E10.4,')'),2ND HARM. =
',E10.4,')'),3RD HARM. = ',E10.4///,15X,'IC (D.C.)
= ',F6.4)
STOP
END

$ENTRY

VS = 1.3026

1ST HARM. = 0.1265E-02,
2ND HARM. = 0.1277E-03,
3RD HARM. = 0.1718E-04

IC (D.C.) = 2.3941

VS = 1.3026

1ST HARM. = 0.1707E-02,
2ND HARM. = 0.1277E-03,
3RD HARM. = 0.1718E-04

IC (D.C.) = 2.3941
LIST OF REFERENCES


