Modeling the Silicon Solar Cell as an Optical Detector

1977

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MODELING THE SILICON SOLAR
CELL AS AN OPTICAL DETECTOR

BY

LEO ALBERT MALLETTE
A.A., Florida Technological University, 1975
B.S.E., Florida Technological University, 1975

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the
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1977
MODELLING A SILICON SOLAR CELL AS AN OPTICAL DETECTOR

by

Leo A. Mallette

ABSTRACT

Solar cells have traditionally been used for direct sunlight to energy conversion, and there has been relatively little investigation into their use as a low data rate optical detector. This report summarizes the results of experimental work to model a silicon solar cell, and its response to a pulse of light. A lumped circuit model, and governing equations for each of the elements is developed. Experimental data on several cells are used to curve fit the governing equations. The parameters of interest are tested as a function of both temperature, and background illumination. Having derived a working model, using open circuit measurements, the behavior of the operational model can be predicted for several values of load resistance. The energy of the output pulse and the Fourier spectrum of the output of the cell are heuristically examined.
Solar cells have traditionally been used for direct sunlight to energy conversion, and there has been relatively little investigation into their use as a low data rate optical detector. This report summarizes the results of experimental work to model a silicon solar cell, and its response to a pulse of light. A lumped circuit model and governing equations for each of the elements are developed. The response to a pulse of light is seen to be an exponential decay, characterized by two parameters; \( K \), the maximum value, and \( \tau \), the decay time constant. Experimental data on several cells from Sensor Technology are used to curve fit the governing equations. The two parameters of interest are tested as a function of both temperature and background illumination. Having derived a working model, using open circuit (photovoltaic) measurements, the behavior of the operational model can be predicted for several values of load resistance. The decay time constant is found to be nearly constant over the temperature range \(-30^\circ C\) to \(110^\circ C\). As background illumination is varied, both the decay time constant, and the maximum value are seen to vary. The maximum value, \( K \) is approximately constant for small load resistances, and the decay time constant, \( \tau \), is approximately constant for the larger values of load resistance. The energy of the pulse, and the Fourier output of the cell are heuristically examined.
ACKNOWLEDGEMENT

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Special thanks are extended to Dr. Ronald Lee Phillips who, as the research report director, gave generously of his time and assistance.

Many thanks to the other members of the Committee-in-Charge, Drs. Ernest E. Erickson and Brian E. Petrasko, for their helpful advice, patience, and encouragement.

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<td>A</td>
<td>Diode cross section</td>
</tr>
<tr>
<td>B</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>C_d</td>
<td>Diffusion capacitance</td>
</tr>
<tr>
<td>C_j</td>
<td>Junction capacitance</td>
</tr>
<tr>
<td>C_T</td>
<td>Parallel combination of C_j and C_d</td>
</tr>
<tr>
<td>D_p</td>
<td>See ( I_p^2/D_p )</td>
</tr>
<tr>
<td>f_c</td>
<td>Cutoff frequency defined by ( \tau )</td>
</tr>
<tr>
<td>f</td>
<td>Frequency</td>
</tr>
<tr>
<td>h</td>
<td>Planck's constant (6.626 ( \times ) ( 10^{-34} ) J-s)</td>
</tr>
<tr>
<td>I_D</td>
<td>Diode current</td>
</tr>
<tr>
<td>I_o</td>
<td>Reverse saturation current</td>
</tr>
<tr>
<td>I_\lambda</td>
<td>Light generated current, internal to solar cell</td>
</tr>
<tr>
<td>K</td>
<td>Arbitrary constant, with or without subscript</td>
</tr>
<tr>
<td>k</td>
<td>Boltzmann constant (1.381 ( \times ) ( 10^{-23} ) J(^\circ)K)</td>
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<tr>
<td>k_d</td>
<td>Constant coefficient associated with C_d</td>
</tr>
<tr>
<td>k_j</td>
<td>Constant coefficient associated with C_j</td>
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<tr>
<td>L_p^2/D_p, ( \tau_p )</td>
<td>Lifetime of minority holes in n-region</td>
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<tr>
<td>n</td>
<td>Charge carrier density</td>
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<tr>
<td>p'(0)</td>
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<tr>
<td>Q</td>
<td>Minority carrier charge</td>
</tr>
<tr>
<td>q</td>
<td>Electronic charge (1.602 ( \times ) ( 10^{-19} ) C)</td>
</tr>
<tr>
<td>R_L</td>
<td>External load resistance</td>
</tr>
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\[ R_S \quad \text{Diode series resistance} \]
\[ R_{SH} \quad \text{Diode shunt resistance} \]
\[ R_T \quad \text{Parallel combination of } r, R_{SH}, \text{ and } (R_S + R_L) \]
\[ r \quad \text{Diode dynamic resistance} \]
\[ T \quad \text{Temperature in degrees Kelvin unless otherwise stated} \]
\[ V_D \quad \text{Diode Voltage} \]
\[ V_L \quad \text{Steady state value of cell output voltage} \]
\[ V_N \quad \text{Noise density in volts per Hertz} \]
\[ V_T \quad \text{Total voltage measured across load resistor} \]
\[ V_D \quad \text{Maximum AC value of } V_D \text{ across diode (above DC Value) due to impulse} \]
\[ V_L \quad \text{Transient value of cell output voltage} \]
\[ V_N \quad \text{Noise voltage used in the section on Noise Generation} \]
\[ W_C \quad \text{Energy in output impulse} \]
\[ \Omega \quad \text{Ohms, or solid angle of view} \]
\[ \lambda \quad \text{Wavelength} \]
\[ \tau \quad \text{Decay time constant } = RC \]
\[ \tau_p \quad \text{See } L_p^2/D_p \]
CHAPTER I

MODELING

Introduction

The photoelectric effect was first observed in 1839 by Becquerel. Over one hundred years later the effect was observed by Ohl in a silicon p-n junction. (US patent, filed 27 May 1941). Modern manufacturing techniques have made the silicon solar cell available at a reasonable price. Availability of the material, and price, has prompted their use in areas other than direct sunlight to voltage conversion. The feasibility of using solar cells as low data rate optical detectors has been investigated by relatively few researchers (Saltsman 1977, Witherell 1970). Characteristics of one variety of solar cells manufactured by Sensor Technology Inc. of Chatsworth, California, have been measured. This paper continues that work by modeling the solar cell with lumped components. The model is derived from experimental measurements and previous work with these cells. Open circuit measurements used temperature as well as background illumination as variables. The noise voltage is also measured and found to be white. The derived model is then used to predict the output voltage as a function of load resistance, temperature, background illumination, and diode voltage. The Fourier spectrum and the energy available in the pulse output is also predicted.
Solar Cell Model

The solar cell is essentially a p-n diode with part of its surface exposed to the environment. This can be modeled as a current source and a diode in parallel, as shown in Fig. 1-a. The model is inadequate for all but the simplest analyses. All diodes have an inherent series and shunt resistance as shown in Fig. 1-b. This model assumes that the resistances are lumped and not distributed throughout the material. This assumption is accurate for all practical models. Fig. 1-b is known as the DC model of a solar cell. Knowing the behavior of the diode as a function of voltage, or current, the output can be predicted using DC analysis techniques.

When the solar cell is to be used as a detector, we cannot assume there will be only DC currents. The very fact that a voltage, or current, is changing conveys information. Information that something has changed, and perturbed the system. This information is what we seek to detect and identify.

To identify this information, we need to know how the solar cell system responds to AC inputs. There is negligible inductance in a solar cell, but there are two very dominant capacitance terms, (a) the junction capacitance and (b) the diffusion capacitance, Fig. 1-c.
Fig. 1. Steps in derivation of circuit model of a solar cell
Junction Capacitance

For a specified bias voltage, there exists an equilibrium p-n junction depletion region, in which the sole charges are bound ionized donors or acceptors. Mobile holes and electrons tend to be swept out by the high electric field existing in the space charge region. The width of the depletion region increases (decreases) for an increased reverse (forward) bias. This bias increases (decreases) the number of bound donors or acceptors that are not neutralized by mobile electrons and holes, and thus increases (decreases) the fixed charge on either side of the junction. The incremental charge increase, \( dq_{sc} \), of ionized donors or acceptors, taken per incremental voltage change, \( dv \), can be defined (Navon 1975) as the junction capacitance.

\[
C_j = \frac{dq_{sc}}{dv} \text{ farads} \tag{1}
\]

This capacitance is not constant, and consequently cannot be defined as the ratio \( q/v \). This capacitance term has been modeled and curve-fitted for these diodes to the equation:

\[
C_j = k_j / (0.46 - V_D)^{1/2} \text{ farads} \tag{2}
\]

as referenced in appendix B.

Diffusion Capacitance

In a semiconductor material, light of a specific frequency*

*Photon of energy equal to \( h\nu \), and greater than the ionization energy.
will excite an electron (up) into the conduction band for n-type materials, or the photon will excite a hole (down) into the valance band for p-type material. A p-n semiconductor junction diode has a conduction-valance band diagram as shown in Fig. 2.

![Conduction-Valence band diagram of a p-n junction diode.](image)

In the diode, the electron-hole pairs (EHP) are generated in three regions:

1. p-region: free electrons will diffuse to the junction area and drift to the n-region

2. n-region: free holes will diffuse to the junction and drift to the p-region

3. Transition region (junction): holes and electrons will drift to the p- and n-regions respectively

In each case, it is the minority carrier which is moving, and generating a (photo induced) current in negative direction.

The movement causes a current $I_\lambda$ which is the basis for the photoelectric effect. This current is divided between the diode
and the output loop; $R_S$ and the load resistance $R_L$.

The added minority carriers cause a significant change in their concentration in the bulk region. The average useful lifetime of the minority carriers is $\tau_p$.

Prior to their reaching the transition region, and drifting to the opposite side, the diffusion of minority carriers result in a charge distribution being built up.

This constitutes a diffusion capacitance which is proportional to the amount of light striking the semiconductor surface. This capacitance is the factor that we wish to determine to fully complete the AC model of the silicon solar cell.

The diffusion capacitance $C_d$ is classically modeled to be proportional to the diode current $I_D$. As shown in appendix A, this is only the approximation used by most authors, and a better solution is:

$$C_d = k_d e^{\frac{qV_D}{\eta kT}} \text{ farads}$$

where:

$$k_d = \frac{qV_D}{\eta kT} \text{ farads}$$

From Fig. 1-c we see the last unknown is the diode behavior with different currents flowing. Since the diode I-V characteristics vary exponentially, the resistance of the diode is not expected to be linear. Many authors have derived the dynamic resistance of a diode, and have shown it to be:
Saltsman (1977) has measured the above variables, and the dynamic resistance $r$, is modeled to the equation:

$$r = \left( \frac{n k T}{q I_o} \right) e^{-qV_D/\eta k T} \text{ ohms}$$

(5)

Fig. 3 shows the final lumped circuit model which will be used as the primary model for the remainder of the work in this report. When measuring in the open circuit mode, the circuit simplifies further. The series resistance, $R_S$ can be neglected since there will be no voltage drop across it as pointed out in appendix B.

![AC circuit model of a silicon solar cell](image)

Fig. 3. AC circuit model of a silicon solar cell

**Open Circuit Measurements**

Given the AC circuit model of the preceding section; measurement of the time constant was accomplished as explained in appendix B. The final AC circuit model as measured, derived, or assumed from previous work is shown in Fig. 3, where:

- $I_\lambda$ is light-controlled current source (amps)
- $r = 37,000 \left( e^{-16.7V_D} \right) \text{ ohms}$
\[ R_{SH} = 30,000 \text{ ohms} \]

\[ C_J = \frac{5 \times 10^{-9}}{(0.46 - V_D) \cdot 5} \text{ farads} \quad (7) \]

\[ C_d = (2 \times 10^{-9}) (e^{16.7V_D}) \text{ farads} \]

\[ R_s = 5 \text{ ohms} \]

**Noise Generation**

Optical detectors, as well as all other electrical circuits, are plagued with an interference mechanism which is given the rather indefinite name of noise.

Noise is "Interfering and unwanted currents of voltages in an electrical device or system" (McGraw-Hill 1974). Due to internal resistance, physical size, exposure to the environment, and the quantum mechanism of operation, solar cells are subject to nearly every known noise identified to date! The five noises of interest are as follows:

1. Johnson noise, also called Nyquist, or thermal noise

\[ V_n = \sqrt{\frac{4kT}{R_B}} \quad (8) \]

2. Temperature noise

\[ V_n \propto \frac{\Delta T^2}{K_T + 4\pi f C_n} \quad (9) \]

where \( \Delta T^2 \) = mean square temperature fluctuations

- \( K_T \) = Thermal conductance
- \( C_n \) = Heat capacity
3. Modulation noise, also known as excess, or 1/f noise

\[ v_N = R I_D \left( \frac{B}{A_d} \right)^{1/2} \left( \frac{1}{f} \right)^n \]  

(10)

where \( d \) = cell thickness and \( n \) varies from .8 to 2

4. Generation-Recombination (G-R) noise

\[ v_N = RI_D \left( \frac{2 \tau_0 B}{n(1+4\pi^2f^2\tau^2)} \right) \]  

(11)

where \( n \) = charge carrier density

This noise arises from the statistical variations in the rate of generation (recombination) of particles in the solar cell. The variation can be caused by charge carrier-phonon interactions or by the random arrival of photons from the background illumination. For the case where the later is dominant, the noise is also called photon, radiation, or background noise.

5. Shot noise

\[ v_N = R (2q I_D B)^{1/2} \]  

(12)

Shot noise is due to the discrete nature of the current. The total current is comprised of current pulses produced by the individual electron-hole pairs being generated.

Identification of the individual noises would be difficult, if not impossible. As described in appendix C, the noise was measured to be constant (white) over the range of the wave analyzer (20 Hz. to 50 KHz.). The output noise density was found to be:

\[ v_N = 2 \times 10^{-7} \text{ volt/hertz} \]  

(13)

The major noise problem when measuring \( \tau \) was G-R noise. This was
caused because the source of background illumination was being modulated with a 150μV, 60 Hertz signal, and several of its harmonics.

**Thermal Characteristics**

To predict the thermal characteristics of the solar cell operating in the short circuit mode, tests were conducted of the solar cell in the open circuit mode. Measurements were taken as outlined in appendix D. The circuit of Fig. 3 is still used for the model. For the thermal characteristic measurements, a constant current, and therefore a constant voltage, are assumed. As before, the series resistance $R_S$ is neglected due to the nearly infinite load resistance. The shunt resistance is constant (Millman 1972). The dynamic resistance has been shown to be:

$$r = \frac{nkT}{qI_0} e^{-qV/\eta kT} \text{ ohms}$$  \hspace{1cm} (14)

For these tests, $I_0$ cannot be considered to be constant, and as Millman (1972) shows:

$$I_0 = KT^{1.5} e^{-(V_{GO}/q/\eta kT)} \text{ amps}$$  \hspace{1cm} (15)

where $K = \text{constant}$

$V_{GO} = 1.21 \text{ volts}$

and $qV_{GO}$ is the forbidden band gap energy. The origin of this $V_{GO}$ term is rather nebulous. Sze (1969) points out that the energy band gap of Silicon is not constant, but varies with temperature *, as shown in Fig. 4.

---

*Pankove (1971) curve fits the temperature dependence of Si to the equation $E_g(T) = E_g(o) -(7.021 T^2)/(T + 1108)$ volts. Where $E_g(o)$ is the voltage at the zero degree Kelvin intercept (=1.1557 eV)."
At room temperature, the temperature dependence is linear, and if this straight line approximation is extended to the $0^\circ K$ axis, $E_g(0)$ is 1.21 volts (the $V_{GO}$ of above).

Combining equations 14 and 15 yields:

$$ r = \frac{k}{qKT^5} \left( \exp \left[ \frac{q}{\eta kT} (1.21-V_D) \right] \right) \text{ ohms} \quad (17) $$

or

$$ r = \frac{K_1}{T^5} \left( \exp \left[ \frac{q}{\eta kT} (1.21-V_D) \right] \right) \text{ ohms} \quad (18) $$

The numerical value of $K_1$ is chosen to be $1.074 \times 10^{-3}$. This value of $K_1$ equates equation 17, evaluated at room temperature ($300^\circ K$) to the equation for dynamic resistance found in equation 6.
\[
\frac{K_1}{T^{0.5}} \left( \frac{q}{kT} \right) \left( 1.21-V_D \right)^{-5} = 37000 \exp(-16.7V_D) \quad \text{ohms}
\]

\[T=300^\circ\text{K} \quad V_D = 0 \quad V_D = 0\]

Millman (1972) shows that the junction capacitance is proportional to:

\[
\epsilon A \quad \text{W}
\]

where:

\[W < (V_{B1} - V_D)^5\]

Sze (1969) shows that:

\[V_{B1} = \frac{kT}{q} \ln \left( \frac{P_{p0}}{P_{n0}} \right) \quad \text{volts}\]

At room temperature, Saltsman (1977) curve fitted the junction capacitance for these diodes at room temperature, to:

\[C_j = 5 \times 10^{-9} / (0.46 - V_D)^5 \quad \text{farads}\]

Combining equations 19, 21, and 22 we find the temperature dependence of the junction capacitance to be:

\[C_j = \frac{5 \times 10^{-9}}{(1.533 \times 10^{-3} T - V_D)^{0.5}} \quad \text{farads}\]

Appendix A derived the diffusion capacitance to be:

\[C_d = \frac{\tau_p q I_0}{\eta k T} \left( e^{qV_D/\eta kT} - 1 \right) \quad \text{farads}\]

where \(\tau_p\) is the minority hole lifetime. Combining this, equation 17 and the \(I_0\) temperature dependence, we find:

\[C_d = \tau_p/r \quad \text{farads}\]

*See also Fig. 11, for a pictorial representation of \(qV_{B1}\)
As above, the open circuit decay time constant is:

\[ \tau = \frac{\tau}{R_{SH}}(C_j + C_d) \text{ seconds} \]  

(26)

Combining the appropriate equations:

\[ \tau = \left\{ \frac{K_1}{T^{0.5}} \exp \left[ \frac{q}{\eta kT} (1.21 - V_D) \right] \right\} \times \left\{ \frac{5 \times 10^{-9}}{(1.533 \times 10^{-3} T - V_D)^{0.5}} + \frac{K_1}{T^{0.5}} \exp \frac{q}{\eta kT} (1.21 - V_D) \right\} \text{ seconds} \]  

(27)

The value of the minority hole lifetime \( \tau_p \) is on the order of one \( \mu s \)\(^*\) (Navon 1975). The individual value of \( \tau_p \) is chosen to give a good fit to the experimental results. For \( \tau_p = 1 \mu s \), Fig. 5 plots \( \tau \) vs temperature for various values of shunt resistance, \( R_{SH} \). Typical values found for the diffusion capacitance were found to be:

\[ C_d = \frac{6.6 \times 10^{-13}}{T^{5}} \exp \left( \frac{5010}{T} \right) \]  

(28)

which correlates very well with the values found in the section on open circuit measurements.

The work to this point indicates the solar cell model of Fig. 3 is an accurate mathematical representation of the physical mechanism found in the solar cell. Using this "good" model, the following sections attempt to predict the operation of the solar cell.
cell for various values of load resistance $R_L$.

Fig. 5. Variation of open circuit decay time constant with temperature for several values of shunt resistance (calculated).
CHAPTER II

OPERATION

Operational Model

The term operational model is used in lieu of short-circuit model because the load resistance $R_L$ is not allowed to be zero. Having shown the open circuit solar cell model of Fig. 3 to be accurate, we can proceed to predict the characteristics of the silicon solar cell when operated into a resistive load. The equivalent circuit model for the solar cell is shown in Fig. 6. For very large values of $R_L$, the model reduces to the open circuit model of Fig. 3. For lower values of $R_L$, the output resistance loop will begin to affect the total resistance, and the amount of current that is flowing through the diode. This current difference will change the dynamic resistance of the diode and the value of the diffusion capacitance. As above, the current can be defined in terms of the voltage across the diode, $V_D$. The total capacitance $C_T$, is the same as equation B-6. The total resistance is now the parallel combination of $R_{SH}$, $r$, and $(R_S + R_L)$.

$$R_T = \frac{37000 e^{-16.7V_D}}{1 + \frac{37000(R_S + R_L + R_{SH})}{R_{SH}(R_S + R_L)} e^{-1.67V_D}} \text{ Ohms} \quad (29)$$
Fig. 6. Equivalent model for the operational model of the silicon solar cell.

Variation With Diode Voltage

The operational time constant $\tau$ is the product of the above resistance, $R_T$, and the sum of the two capacitances.

$$C_T = \frac{5 \times 10^{-9}}{(.46 - V_D) \cdot 5} + 2 \times 10^{-10} e^{16.7V_D} \text{ F.}$$

(30)

Fig. 7 shows the variation of $\tau$ with diode voltage $V_D$. As shown by Saltsman (1977), the diode voltage is proportional to the incident power. The constant of proportionality was stated to be $R_{V\lambda}$, the voltage spectral responsivity. $R_{V\lambda}$ was measured, and results of tests on a typical cell are shown in Fig. 8. The value of $R_{V\lambda}$ was found to be 83 volts/watt. As Fig. 7 shows, the variation of $\tau$ is nearly constant for all values of $V_D$, and is therefore nearly constant for all intensities of background illumination. The lower values of $R_L$ give the greatest variation in $\tau$, which may lead to signal processing problems in later stages of the system. Fig. 9

*Note, this value for $R_{V\lambda}$ is for the operational model with $R_L$ specified to be 317 ohms.
shows the variation of $\tau$ as a function of incident light. This data was derived from the above information.

Fig. 7. Variation of $\tau$ with diode voltage for several values of load resistance, $R_L$. 

DC Diode voltage, $V_D$ (volts)
Fig. 8. Variation of $V_D$ with Incident power (measured).
Fig. 9. Variation of operational circuit time constant with load resistance, for several values of incident power (calculated).
Combining the above equations into equation 31 yields:

\[ \tau = \left[ \frac{\frac{1.074 \times 10^{-4}}{T^{0.5}} \exp \frac{q}{n k T} (1.21 - V_D)}{1 + \frac{1.074 \times 10^{-4} \left( R_S + R_L + R_{SH} \right)}{R_{SH} \left( R_S + R_L \right) T^{0.5}} \exp \frac{q}{n k T} (1.21 - V_D)} \right] \times \left[ \frac{5 \times 10^{-9}}{(1.533 \times 10^{-3} T - V_D)^{0.5}} + \frac{2 \times 10^{-6}}{1.074 \times 10^{-4} T^{0.5}} \exp \frac{q}{n k T} (1.21 - V_D) \right] \quad (sec) \quad (35) \]

For \( R_{SH} = 30,000 \) ohms, \( \tau \) is plotted in Fig. 10 as a function of temperature \( T \) (°C) and load resistance \( R_L \). It is seen from Fig. 10 that the time constant \( \tau \) is approximately constant over the entire range of interest. The following section will assume the system is operating over a range of temperatures where \( \tau \) does not vary appreciably.

**Power Available in Pulse**

The data accumulated to this point has provided an excellent model for the silicon solar cell. The previous section has shown the dependence of \( \tau \) on diode voltage \( V_D \), and on temperature. The solar cell is assumed to be operated over a narrow temperature range and therefore \( \tau \) will not vary significantly with temperature. Appendix B indicated that the response to an impulse of light would be exponential as follows:

\[ V_L = K \exp \left( -t/\tau \right), \quad t > 0 \quad (36) \]
Fig. 10. Variation of decay time constant with temperature for several values of load resistance (Calculated)

Fig. 11 illustrates the concept. The two variables which uniquely define equation 36 are \( K \) and \( \tau \). The relationship between \( \tau \) and \( V_D \) was defined in the previous section. The numerical value of \( K \) can be shown to be proportional to \( V_D \) by the following equation.

\[
K = \frac{R_L}{R_S + R_L} \frac{V_D}{V_D_{\text{max}}}. \tag{37}
\]

The equation is seen to be the diode voltage fed to a voltage divider network comprised of \( R_S \) and \( R_L \).

A graph of \( K \) versus \( V_D \) would yield a few straight lines emanating from the origin, and would provide little useful information. A more useful graph is shown in Fig. 12. This curve plots \( k \),
Each point, on the individual load resistor lines, is a calculation for a specific $V_D$. The $V_D = 0$ volt point is the highest (or leftmost) point, and increasing values of $V_D$ are to the right and/or down.

Incident Power

Fig. 11. Model of incident light pulse, and exponential response.

*The "and/or" is due to the 500 ohm and 1000 ohm curves which tend to oscillate at high values of $V_D$. 
Fig. 12. Plot of $K$ and $\tau$, for several values of $R_L$. Individual points are diode voltage $V_D$, from zero to .44 volts. The $V_D = 0$ volts is the far left point.
Fig. 12 shows that for a specified load resistance, both $K$ and $\tau$ vary as a function of $V_D$. Near the two extremities (10 and 1000 ohms), either $K$ is approximately constant, or $\tau$ is approximately constant.

The exponential curve of Fig. 11-b is the AC voltage across the load resistor $R_L$. The power at any time $t$, can be calculated using the equation:

$$\text{Power} = \frac{V^2}{R}$$

(38)

Combining equations 37 and 38, we obtain the power as a function of time:

$$\text{Power} = \frac{K^2}{R_L} \exp\left(\frac{-2t}{\tau}\right), \ t > 0$$

(39)

Equation 39 defines the power obtainable from the pulse at any time $t$. Knowing the power is useful, but not in itself. A more useful relation would be the total energy in the pulse. The energy is obtained by integrating the power from minus infinity to plus infinity. In practice, there is a specified time frame in which the integration takes place. The lower limit of integration is taken to be the beginning of the pulse ($t=0$), and the upper limit is taken to be much greater than five time constants (or $t = \infty$). Applying the above to equation 39, we obtain the energy output of the solar cell:

$$W_C = \frac{K^2 \tau}{2R_L}$$

(40)
It has been shown that both the time constant, and the coefficient \( K \), are functions of diode voltage \( V_D \). The value of diode voltage has been shown to be proportional to the incident power, via the constant \( R_{V_D} \). Fig. 13 portrays a visual display of the energy in the pulse for various values of load resistance. As can be seen, the energy is very low and varies of over several magnitudes from the no light condition \((V_D = 0)\) to the built in junction potential, \( V_{BI} \) \((V_D = .46 \text{ volt})\).

**Fourier Analysis of Output**

The output voltage, taken across the solar cell can be broken down into three parts:

1. \( V_L \), the DC voltage due to the background illumination
2. \( v_L \), the transient voltage, due to an impulse, and
3. \( V_N \), the inevitable noise density which is present in all electrical devices. \( V_N \) has been defined to be a noise density in units of volts per hertz. Therefore, to get a voltage \( V_N \) must be multiplied by the effective bandwidth of the system. As defined by Saltsman (1977), the 3 db bandwidth is the cutoff frequency \( f_c \):

\[
B = f_c = \frac{1}{2\pi T}
\]  
(41)

The total voltage \( V_T \), across the load resistor can be defined to be:

\[
V_T = V_L + v_L + B V_N
\]  
(42)

The total output voltage can be analyzed via Fourier transform techniques. Using the linearity theorem and assuming there is only one pulse incident on the solar cell, we find the Fourier transforms of
Fig. 13. Variation of energy output with diode voltage for several values of load resistance (calculated).
the individual voltages in question are:

\[ F(V_L) = \delta(f) \]  
\[ F(BV_N) = \text{Constant} \]  
\[ F(V_L) = \frac{\tau}{1 + j2\pi f \tau} \]

The amplitude of the Fourier transform of \( v_L \) can be shown to be:

\[ \text{AMPL } F(v_L) = \frac{\tau(1 + (2\pi f \tau)^2)}{1 + (2\pi f \tau)^2} \cdot 5 \]

Summing equations 43, 44 and 46 will produce a frequency domain picture of solar cell output voltage.

The sketch of this frequency domain representation is given in Fig. 14. For clarity, only one side of the two sided frequency spectrum is shown. The impulse at zero hertz corresponds to the \( V_L \) term, and the broken line corresponds to the noise superimposed on the signal. As can be seen either visually, or by examining the equations; the major part of the signal is in the frequency range:

\[ 0 < f < f_c \]

The majority of the noise is also found in this range. The noise is shown in the broken line manner to prevent the reader from thinking that the signal is more easily detected with more noise. Rather, the noise amplitude may (or may not) stay constant, but the phase may vary from \(-\pi\) to \(+\pi\). At any particular frequency, the noise will randomly add or subtract various amplitudes from the signal. This tends to destroy the information carried in the signal and most types of signal processing can do nothing to correct this situation. Attempting to filter the noise, in the frequency domain, leads to
Fig. 14. One sided Fourier spectrum of total voltage across load resistor (calculated).

three possible problems:

1. The signal is also being filtered

2. The 3 db bandwidth of the system is equal to $f_c$, which is a function of $\tau$, which is a function of $V_D$, which is a function of the incident power. Therefore the bandwidth varies as a function of background light.

3. "Bandlimiting and timelimiting are mutually incompatible" (Carlson 1975). The latter may not be a problem, because this is a low data rate system. One pulse may be all that needs to be detected. Work in this area could be carried on for many pages, but due to the nature of this report, will not be pursued here.

A third type of limiting is aperture limiting, in which the
effective viewing angle is reduced. This should, theoretically, also reduce noise pickup from directions which are not of interest. As appendix E shows, the solar cell has a 130 degree beamwidth. This corresponds to a solid viewing angle of 2.58 steradians. Of the total $4\pi$ steradians of viewing angle available*, the solar cell is "looking" at over 20 percent of it! This entire aperture may be needed, since the direction of the pulse may not be known.

**Summary**

Based on experimental data, taken on a set of silicon solar cells, an experimental model was developed, using non-distributed parameters. Typical values for the circuit elements were chosen, and used to predict the operation of the solar cell for various values of load resistance. The value of the load resistance was varied from 10 ohms to 1 kilohm. Given an incident pulse of light, the output was shown to be an exponential decay. The three factors of interest in the pulse are its maximum value, decay time, and the energy in the pulse, which can be found from the first two. The decay time constant was shown to be approximately constant for normal variations in environmental temperature. The maximum value, and decay time constant vary widely with background illumination. The high values of load resistance tend to favor a constant decay time, whereas the lower values favor a constant maximum voltage.

---

*The surface area of a sphere is $4\pi R^2$. So, to "see" the entire sphere, the solid viewing angle is $4\pi$ steradians.
For low light levels, the larger resistance values gave a higher energy output than smaller resistances. As the background light level is increased, and the diode voltage approaches the built in junction potential, the energy output curves reach a maximum and decrease rapidly. The decay time constant also defines the 3 db bandwidth of the system. The dependence of the time constant on illumination implied the dependence of the signal processing bandwidth on background illumination. Detector noise was shown to be white, extremely small. The major source of noise would be introduced externally. Low frequency man made noise, as well as electro-magnetic interference are possible topics of future study.
APPENDIX A

DERIVATION OF DIFFUSION CAPACITANCE

As indicated in the text of this report, the solar cell is not biased externally. The only form of energy is the light generated current $I_\lambda$. In general, the diode in the solar cell model is forward biased by the light dependent current source. This forward bias introduces "an injected charge redistribution in the n-region which can be described by a capacitance known as a diffusion capacitance" (Navon 1975). The reason that we consider only the n-region is that the p-region is heavily doped compared to the n-region. This implies that the diode current is comprised almost entirely of holes moving from the n-side to the p-side.

Diffusion capacitance is defined as the "rate of change of stored minority carrier charge with the voltage across a semiconductor" (Lapedes 1974)

\[ C_d = \frac{dQ}{dV_D} \text{ farads} \quad (A-1) \]

where $Q$ is defined as

\[ Q = AqL p'(0) \quad (A-2) \]

$A$ is the diode cross section, $q$ is the electronic charge, $L$ is one hole diffusion length of the space-charge region, and $p'(0)$ is the injected concentration of holes into the n-region of the semiconductor material.

\[ p'(0) = \frac{IL}{qD_p} \quad (A-3) \]
Hence

\[ C_d = A q I_p \frac{d}{dV_D} \rho''(0) \]  \hspace{1cm} (A-4)

Combining equations A-3 and A-4

\[ C_d = \frac{A L_p}{D_p} \rho \frac{dI_D}{dV_D} \]  \hspace{1cm} (A-5)

where \( L_p^2/D_p \) is equal to the lifetime of minority holes in the n-region \((\tau_p)\), and \( I_D \) is the familiar diode equation:

\[ I_D = I_0 \exp \left[ \frac{(qV_D/\eta kT)-1}{(qV_D/\eta kT)} \right] \]  \hspace{1cm} (A-6)

The derivative with respect to diode voltage yields

\[ \frac{dI_D}{dV_D} = \frac{qI_0}{\eta kT} \exp \left( \frac{qV_D}{\eta kT} \right) \]  \hspace{1cm} (A-7)

which is the reciprocal of the dynamic resistance of the diode.

Combining equations A-5 and A-7 we obtain the diffusion capacitance.

\[ C_d = \frac{\tau}{P} \frac{qI_0}{\eta kT} \exp \left( \frac{qV_D}{\eta kT} \right) \]  \hspace{1cm} (A-8)

Most authors assume the diode will be operated in a forward biased mode, where the diode voltage is at least a few tenths of a volt. In this case, subtracting off a small error term,

\[ \tau p q I_o/\eta kT \]  \hspace{1cm} (A-9)

would yield a convenient equation for the diffusion capacitance which is proportional to the diode current, \( I_D \). For the range indicated \((V_D > .3 \text{ volts})\) this assumption causes an error of .66 percent. For greater values of \( V_D \), the error is much less. When using the silicon solar cell as an optical detector, the diode voltage \( V_D \).
is never greater than the built-in potential of approximately one-half volt, and for most instances will be significantly less. Hence, the approximation is not valid and the diffusion capacitance must be modeled by equation A-8.

For purposes of modeling the solar cell, the diffusion capacitance can be written as

\[ C_d = k_d e^{16.7V_D} \]  
(A-10)

where

\[ k_d = \tau_p I_o / \eta k T \]  
(A-11)

Appendix B will demonstrate how \( k_d \) is determined.
APPENDIX B

TIME CONSTANT MEASUREMENTS

The general model of the solar cell has been developed over many years, and various tests have yielded data on specific components of the model.

Until recently there has not been a need to determine the value of the diffusion capacitance. As pointed out in the text, solar cells have not been modeled as optical detectors. The equipment shown in Fig. 15 was assembled to measure the decay time, and therefore the diffusion capacitance could be calculated. Figs. 30, 31, and 32 in appendix F, are pictures of the experimental configuration. The argon laser-variable beamsplitter combination provided an adjustable source of "background" illumination for the solar cell, and the incident light generated a current $I_{\lambda}$, which remained constant with time. The model for the solar cell reduces to the DC model of Fig. 16. The light current generates a voltage across the resistances $r$, $R_{SH}$, and $(R_S + R_L)$. For the purposes of this test, the solar cell was operated in the open circuit, or photovoltaic, mode. The load resistance $R_L$ is equal to the input impedance of the oscilloscope. This value is typically one megohm.
Fig. 15. Equipment lay-out to test variation of $\tau$ with background illumination.
Fig. 16. DC model of solar cell without external bias.

The total resistance $R_T$ is

$$R_T = \left(1 + \frac{1}{R_S} + \frac{1}{R_L} + \frac{1}{R_{SH}} + \frac{1}{r}\right)^{-1} \text{ ohms} \quad (B-1)$$

where

- $R_S = 5$ ohms
- $R_L = 10^6$ ohms
- $R_{SH} = 30,000$ ohms
- $r = 37000 \, e^{-16.7V_D}$

The $1/(R_S + R_L)$ term is seen to be negligible due to the high value of $R_L$, and $R_T$ reduces to

$$R_T = \frac{37000 \, e^{-16.7V_D}}{1 + \frac{37000 \, e^{-16.7V_D}}{R_{SH}}} \text{ ohms} \quad (B-2)$$

The maximum error introduced by assuming this term to be negligible occurs at $V_D = 0$ volts. This error is 1.6 percent, and reduces exponentially with increased diode voltage. The AC model of the solar cell is shown in Fig. 17.
The junction capacitance has been previously modeled and curve-fitted to the following equation (Saltsman 1977).

\[ C_j = \frac{k_j}{(V_{BI} - V_D)\gamma} \text{ farads} \]  

where \( k_j \) is a constant \( = 5 \times 10^{-9} \text{ FV}^5 \).

\( V_{BI} \) built-in junction potential \( = .46 \text{ volt (Saltsman 1977)} \)

\( \gamma \) constant that is a function of junction doping \( \approx .5 \)

The exponent of .5 indicates that the transition from the p- to the n-region is fairly abrupt, as opposed to a linearly graded junction (= .33).

The diffusion capacitance was shown in appendix A, to be of the form

\[ C_d = k_d e^{16.7V} \]  

where \( k_d \) is a constant to be determined.

As seen in Fig. 18, the two capacitances are in parallel, and the total capacitance \( C_T \) is equal to the algebraic sum of the two.
Combining equations B-3, B-4, and B-5,

\[ C_T = C_j + C_d \tag{B-5} \]

The pulse laser shown in Fig. 15 and Fig. 31 generates a very weak pulse of infrared light at 9043 angstroms. The pulse length is on the order of one microsecond, and as will be shown, the decay time \( \tau \) is at least one order of magnitude greater. The pulse of light generates a pulse of current from the \( I_\lambda \) current generator, and the solar cell model reduces to a simple circuit analysis problem. Circuit theory tells us that a circuit composed of resistors and capacitors has an impulse response of the form:

\[ v_L = A E^{-t/\tau} \tag{B-7} \]

where \( \tau \) is the product of the total resistance, and the total capacitance,

\[ \tau = R_T C_T \text{ seconds} \tag{B-8} \]

For the case of the solar cell model (Fig. 17):

\[ \tau = \frac{37000 \exp(16.7V_D)}{1 + \frac{37000}{R_{SH}} \exp(-16.7V_D)} \left[ \frac{5 \times 10^{-9}}{(.46-V_D)^{.5}} + k_d \exp(16.7V_D) \right] \tag{B-9} \]

The decay time \( \tau \) versus temperature is plotted in Fig. 20 for several values of \( R_{SH} \), holding \( k_d \) constant at \( 2 \times 10^{-10} \). Photographs of typical measurements are shown in Figs. 34 and 35, in appendix F.
The decay time constant $\tau$ versus $V_D$ is plotted in Fig. 19 for $R_{SH} = 30,000$ ohms, and for various values of $k_d$. Several cells were tested and found to vary significantly from one cell to another. Typical values for the different cells are shown in Fig. 18.

A typical value for $C_d$ is:

$$C_d = 2 \times 10^{-10} \exp \left( \frac{16.7V}{D} \right)$$  \hspace{1cm} (B-10)

<table>
<thead>
<tr>
<th>CELL</th>
<th>$k_d$ (Farads)</th>
<th>$R_{SH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.5 \times 10^{-10}$</td>
<td>45 $\Omega$</td>
</tr>
<tr>
<td>2</td>
<td>$5.0 \times 10^{-10}$</td>
<td>25 $\Omega$</td>
</tr>
<tr>
<td>3</td>
<td>$1.0 \times 10^{-10}$</td>
<td>45 $\Omega$</td>
</tr>
<tr>
<td>4</td>
<td>$2.0 \times 10^{-10}$</td>
<td>10 $\Omega$</td>
</tr>
<tr>
<td>5</td>
<td>$1.0 \times 10^{-10}$</td>
<td>12 $\Omega$</td>
</tr>
<tr>
<td>6</td>
<td>$1.5 \times 10^{-10}$</td>
<td>10 $\Omega$</td>
</tr>
<tr>
<td>7</td>
<td>$2.0 \times 10^{-10}$</td>
<td>10 $\Omega$</td>
</tr>
</tbody>
</table>

Fig. 18. Curve fitted values for various cells under test.
Fig. 19. Variation of open circuit time constant with diode voltage for several values of $k_d$, holding $R_{SR} = 30 \, k\Omega$ (calculated).
Fig. 20. Variation of open circuit time constant with diode voltage for several values of shunt resistance, holding $k_d$ constant at $2 \times 10^{-10}$ (calculated).
APPENDIX C

NOISE MEASUREMENTS

The experimental configuration shown in Fig. 21 was used to measure the noise output of the solar cell as a function of frequency. The argon laser was used to provide the background illumination. Oscilloscope number one gave the output DC voltage, and was kept constant by adjusting the intensity of the laser light with the beamsplitter. The Hewlett-Packard 302 A wave analyzer* is essentially a very narrow band receiver with a visual indication of the intensity of the input signal. The 3 dB bandwidth is nominally six hertz, the output being measured in millivolts, and observed on oscilloscope number two. The results for several cells are shown in Fig. 22 and Fig. 23. For comparison purposes, the noise voltage of a United Detector Technologies p-i-n .125 P photocell is also included.

The noise is seen to be white over the entire range of measurements. Modulation of the laser light intensity (e.g., poor filtering) generates the 60 Hertz interference. The harmonics of the 60 Hertz are also believed to be generated by the laser power supply. The two major components of the noise are the 60 Hertz variation and its sixth harmonic. This can be seen either in the

*The specifications for the HP 302 A are given in Fig. 24.
frequency domain (Fig. 23), or in the time domain (Fig. 33 of appendix F).
Fig. 21. Equipment lay-out to test solar cell noise properties.
Fig. 22. Noise density variation with frequency (measured).

*See also Fig. 23.
Fig. 23. Noise density variation at discrete frequencies of 60 hertz and several harmonics (measured).
Frequency range:  
20 Hz to 50 kHz.

Frequency accuracy:  
+ (1% + 5 Hz).

Frequency resolution:  
10 Hz per division.

Amplitude ranges:  
30 uV to 300 V in full scale in 15 ranges.

Amplitude accuracy:  
+ 5% of full scale.

Internal level calibrator:  
Amplitude accuracy: ± 2%.
Amplitude: 1 V full scale.
Frequency: 5 kHz ± 1 kHz.

Dynamic range:  
IM and harmonic distortion products: > 75 dB below 0 dB reference level.
Residual responses: > 75 dB below 0 dB reference level.

Selectivity:  
0.1 dB rejection: > 2 Hz bandwidth.
3 dB rejection: 6.0 Hz bandwidth ± 10%.
60 dB rejection: 60 Hz bandwidth ± 10%.
80 dB rejection: < 140 Hz bandwidth.

Input impedance:  
Resistance:  
30 mV to 1 V max input ranges: 100 kilohms.
3 V to 300 V max input ranges: 1 megohm.
Capacitance:  
30 mV to 1 V max input ranges: < 100 pF.
3 V to 300 V max input ranges: < 30 pF.

Fig. 24. Specifications for Hewlett Packard 302A wave analyzer.

APPENDIX D

TEMPERATURE MEASUREMENTS

The need for an alternate method to measure the diffusion capacitance led to measurements of $\tau$ as a function of temperature, at a constant current. A constant current that could be easily controlled was found to be no current, e.g., no background lighting. The experimental bench of Fig. 25 was set up.

![Equipment configuration for temperature measurements.](image)

Fig. 25. Equipment configuration for temperature measurements.

The individual solar cells were mounted on an aluminum block shown in Fig. 26 and in appendix F. The aluminum block was cooled to less than minus 30 degrees C with CO$_2$. As the temperature of the block increased to room temperature, measurements of $\tau$ were taken. Following this test, the block was heated to 100 degrees C with a heat gun. Similarly, as the block cooled, time constant measurements were taken. The graph in Fig. 27 shows measured curves for some of the cells.
Fig. 26. Aluminum block used to mount solar cells.
Fig. 27. Variation of $\tau$ with temperature for three cells (measured).
APPENDIX E

VIEWING ANGLE MEASUREMENTS

As mentioned in the text of this report, one method of reducing noise is to limit the aperture viewing angle. This reduces the received noise but also limits the field of view from which the information (in our case, a pulse) can be captured. As will be shown, the half power points are about $130^\circ$ in both the azimuth and elevation planes.

Measurements were taken by mounting the solar cell on the block of Fig. 26, and the block was mounted on a tripod. The block was mounted so that as the block was rotated, the solar cell would not be translated with respect to an intense beam of light. The DC voltage was recorded, and plotted data is shown in Fig. 28 and Fig. 29.

Due to physical symmetry of the solar cell, the half power beamwidth can be assumed to be (approximately) circularly symmetric about the normal. With this assumption, the solid viewing angle ($\Omega$) of the detector can be defined (U.S. Department of the Navy 1965) to be:

$$\Omega = \pi \sin^2(\theta/2)$$  \hspace{1cm} (E-1)

Using the value of $\theta$ equal to $130$ degrees, the detector solid angle is:

$$\Omega = 2.58 \text{ steradians}$$
The use of knowing the solid angle is to predict the effective aperture at some radius $R$. The relationship for the above is:

$$A = \Omega R^2$$  \hfill (E-2)

Another use of the solid angle when dealing with a point source radiator is:

Intensity = Power/Solid angle

$$\text{Intensity} = \frac{\text{Power}}{\text{Solid angle}}$$  \hfill (E-3)

---

Fig. 28. Variation of open circuit diode voltage due to rotation in the azimuth plane. Zero degrees is normal to the solar cell. Normalized to unity.
Fig. 29. Variation of open circuit diode voltage due to tilting the solar cell forward. Zero degrees is normal to the solar cell. Normalized to unity.
Several photographs were taken during the course of this study. It is believed that these pictures will help visualize the problems, and solutions, which may not be apparent in the text of this report. The photographs carry a short caption which is expanded in detail below.

Fig. 30. The experimental configuration used to take measurements of the open circuit decay time constant. Equipment, from left to right is; argon laser, mirror, variable beamsplitter, screen cage, pulse laser.

Fig. 31. Front view of infrared pulse laser, built by Electromagnetic Sciences, Inc., of Atlanta, Georgia. The output pulse is approximately triangular, and of one microsecond duration, operating at a wavelength of 9043 Angstroms (Plan and. . . 1973). The 50% mirror and beamsplitter can also be seen.

Fig. 32. View of the screen cage, and aluminum block. The screen cage was built of quarter inch mesh hardware cloth, on a frame of one-by-twos. The aluminum block is described in appendix D.

Fig. 33. Oscilloscope trace of the output of a typical solar cell with strong background illumination. The 360 Hertz signal and others are seen to be superimposed on the even stronger
60 Hertz signal. The picture was taken at f 1.8, 1/30 second, scope sync to line, 1 mV/div. vertical, 5 ms/div horizontal, and input coupling to AC.

Fig. 34. Output of the solar cell due to a pulse of light, without background illumination. Scope sync to internal, 1 mv/div vertical, 50 microsec/div horizontal, and input coupling to AC.

Fig. 35. Exponential decay with background illumination, similar to Fig. 34. The background illumination is seen to decrease the decay time, and add a great deal of noise to the system. For this case, the diode voltage was 100 mV, scope sync to internal, 1 mV/div vertical, 50 microsec/div horizontal, and input coupling to AC, the same settings as in Fig. 34.
Fig. 30. Experimental configuration.

Fig. 31. Pulse laser.
Fig. 32. Screen cage and block.

Fig. 33. 60 Hertz modulation of laser light.
Fig. 34. Open circuit decay without background light.

Fig. 35. Open circuit decay with background light.
LITERATURE CITED


