Aggregate Production Planning Techniques and Comparison

Fariborz Mazaheri
University of Central Florida
AGGREGATE PRODUCTION PLANNING
TECHNIQUES AND COMPARISON

BY

FARIBORZ MAZAHERI
B.S.I.E., Arya-Mehr University of Technology, 1976

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering in the Graduate Studies Program
Florida Technological University

Orlando, Florida
1977
ABSTRACT

Aggregate production planning models are of the greatest importance to operations management, since these plans enable management to utilize the major resources at its command. In this report the structure of the aggregate planning problem and a number of different approaches are reviewed and presented. Approaches are classified in three categories: a) workforce smoothing models, b) production smoothing models, c) production and workforce models. The models are compared with respect to the cost structure, parameters estimation, forecast requirement, decision variables, computability and optimization techniques.
ACKNOWLEDGEMENTS

I would like to express my appreciation to those persons whom without their assistance this research report could not have been completed. I would like to express my sincere gratitude to my chair­man, Dr. Benjamin Lin, for his excellent advice, encouragement, and helpful criticisms. I would also like to acknowledge my graduate com­mittee, Dr. Harold Klee and Ms. Karen Ehlert whose comments and sug­gestions led to the successful completion of this research report.
# TABLE OF CONTENTS

List of Tables .................................................. v
List of Figures................................................... vi

Chapter

I. INTRODUCTION....................................................... 1

II. NATURE OF AGGREGATE PRODUCTION PLANNING.................. 3

III. APPROACHES TO AGGREGATE PRODUCTION PLANNING........... 14

3.1 Introduction

3.2 Non-quantitative Models

3.3 Quantitative Models

3.3.1 Workforce smoothing models and comparison

3.3.2 Production smoothing models and comparison

3.3.3 Production and workforce models with comparison

IV. APPLICATIONS OF QUANTITATIVE MODELS...................... 46

V. SUMMARY AND CONCLUSIONS...................................... 53

REFERENCES..................................................... 55
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cost Items Involved in Changing Production Levels (Workforce and Rate).</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Comparison of Single-Item Workforce Scheduling Models.</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>Comparison of Multi-Item Workforce Scheduling Models.</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>Comparison of Single-Item Production Smoothing Models.</td>
<td>39</td>
</tr>
<tr>
<td>5</td>
<td>Multi-Item Production Smoothing Model</td>
<td>41</td>
</tr>
<tr>
<td>6</td>
<td>Comparison of Production and Workforce Scheduling Models.</td>
<td>45</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Classification of Aggregate Planning Problems</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>A Two-Stage Serial Production System</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Goal Programming Approximation to a Quartic</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>Range Programming Approximation to a Quartic</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>Production Cost Functions</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>Inventory Cost Functions</td>
<td>36</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

The scheduling of aggregate production and workforce is a planning problem of major interest to many manufacturing concerns. Managers are keenly aware that production and workforce decisions in view of changing customer demands can have substantial economic impact on the firm. The problem of aggregate scheduling is concerned with management's response to these fluctuations in the demand pattern. Specifically, how can the production, manpower, and goods resources best be utilized in the face of changing demands in order to minimize the total cost of operations over a given planning horizon. To see this problem in its proper context, we identify three types of production systems:

1. continuous systems: where the demand for a product justifies its production on a continuous basis, but because of fluctuating demand it is desirable to adjust the production level from time to time.

2. job production: where a stream of orders has to be processed on common facilities or production centers, each job having its own unique specifications and requirements in terms of production resources. A job may consist of a single item or a batch of identical items. The scheduling problem here is concerned with setting the
sequence with which jobs should be processed at each production center.

3. batch production: where a continuous demand for certain products exists, but because the rate of production exceeds the rate of demand, there is a need to produce products in batches. The scheduling problem here is concerned with determining the batch sizes for products and the order in which they should be produced.

In this report the structure of the aggregate planning problem and a number of different approaches are reviewed and presented. Approaches are classified in three categories: a) workforce smoothing models, b) production smoothing models, and c) production and workforce smoothing models.

In the next chapter we define the problem. This is followed by the problem structure and an attempt to solve the problem. Chapter III presents a brief description of the structure of different basic approaches to the problem and compares different techniques with respect to the cost structure, parameter estimation, forecast requirements, decision variables, computability and optimization techniques.

A number of real world applications are reviewed in Chapter IV. Finally, the research report provides an extensive bibliography.
CHAPTER II

THE NATURE OF AGGREGATE PRODUCTION PLANNING

Aggregate planning and scheduling has to do with the overall planning and scheduling of the use of various sources of capacity in relation to demand. In responding to changing demands, management can utilize the following alternatives:

1. Adjust the workforce through hiring and layoff. In the literature this problem is called the workforce smoothing problem.

2. Adjust the production rate through overtime and undertime. This is called the production smoothing problem.

3. Adjust the production rate and the workforce which is called the production scheduling and workforce smoothing problem.

OBJECTIVES INVOLVED

Most of the models assume that the firm has a single objective of cost minimization or profit maximization. In this chapter we will discuss some evidence that, in practice, a firm has multi-objectives. But, because of the scope of this research, we will discuss the procedures which assumed a single objective.

Single-Objective

In general, there are seven common types of costs which may be involved in the single-objective type of problem.
1. Procurement costs for products purchased from outside sources.

2. Production costs which include any out-of-pocket costs that are associated with production under normal conditions and that vary with the production rate.

3. Inventory holding costs.

4. Shortage losses associated with backorders and lost sales.

5. Costs of increasing and decreasing workforce levels. These include hiring and training costs and separation pay and other losses associated with firing or laying off workers.

6. Costs of deviating from normal capacity through use of overtime or undertime. Wage rate premiums for overtime and work and opportunity losses because of underutilization of the workforce are examples.

7. Cost of changing production rates. Examples are machine setup and takedown costs, opportunity losses because of lost production during changeover, and losses because of quality problems and inefficiencies resulting from schedule changes.

As we shall see, only certain of these management alternatives and costs are considered in any particular model found in the literature. A brief summary of cost items involved in changing production levels (workforce and production rate) is given in Table 1.

After studying aggregate scheduling in different firms, Gordon found that different types of companies face different types of aggregate production problems. For example, in a shoe company and a
<table>
<thead>
<tr>
<th>Costs of Increasing Levels</th>
<th>Costs of Decreasing Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Employment and Training:</strong></td>
<td><strong>1. Unemployment compensation insurance</strong></td>
</tr>
<tr>
<td>a) Interviews &amp; selection</td>
<td>2. Contributions to union funds</td>
</tr>
<tr>
<td>b) New personnel records, physical examinations, payroll setup</td>
<td>3. Costs of employee transfer and retraining</td>
</tr>
<tr>
<td>c) Training new workers</td>
<td>4. Intangible effects on public relations</td>
</tr>
<tr>
<td><strong>2. Service and staff functions:</strong></td>
<td><strong>5. Production and inventory costs of revising schedules, order points etc.</strong></td>
</tr>
<tr>
<td>a) Production &amp; inventory control</td>
<td>6. Idle time costs due to lags in decisions and action</td>
</tr>
<tr>
<td>b) Purchasing, receiving, inspection &amp; materials handling</td>
<td></td>
</tr>
<tr>
<td><strong>3. Added shifts:</strong></td>
<td></td>
</tr>
<tr>
<td>a) Supervision</td>
<td></td>
</tr>
<tr>
<td>b) Shift premium</td>
<td></td>
</tr>
<tr>
<td><strong>4. Overtime costs related to the increased level</strong></td>
<td></td>
</tr>
</tbody>
</table>

dairy products company he found inventory is the problem. In a very large container plant which supplies the beverage industry, production smoothing scheduling is the problem because mechanization limits workforce fluctuations. In a medium-sized candy and chocolate producer, workforce variations were used extensively to absorb peaks and valleys.

Multiple Objectives

The assumption of single-objective has been seriously questioned by Chamberlain (2), Cyert and March (3), Morgenstern (4), Murweiz (5), among others. Bilkey (6) summarized the evidence that the firm in practice does not have a single objective of profit minimization. Starr (7), the past president of the Institute of Management Sciences, also stressed the opportunity for the application of management science to multiple criteria decision making problems.

There are at least three approaches for dealing with multiple objective decision problems: 1) a utility function approach, 2) a goal programming approach, and (3) a vector maximization approach.

Goal programming, an extension of linear programming, was developed by Charnes (8). A similar model was also represented by Jaaskelainen (9), with a goal programming model. The manager can handle decision problems which deal with a single goal and multiple subgoals. Unlike linear programming, an arbitrary conversion of other value measures to a single objective function is unnecessary.

The advantages of the goal programming technique are: 1) its flexibility in the formulation of the problem; that is, one can use goal programming for the problems of a single-objective firm with
a single subobjective, a single-objective firm with multiple subobjectives, a multiple-objective firm with either compatible or incompatible multiple objectives; and (2) it can be applied to most management decision problems with linear constraints.

While goal programming was developed a decade ago, the possible applications are demonstrated only in recent years. Lee (10, 11, 12, 13, 14) has several applications and publications in this field. In reference (11) he described briefly the application of goal programming in aggregate production planning and provided a computer program of the modified simplex method for goal programming models.

Lin (15) developed an APL conversational goal programming algorithm based on Lee's work. He indicated that a future research would be to develop a generalized goal programming algorithm which allows both linear and integer variables. Lin (16) formulated two planning models; one model is the goal programming with uncertain demand; the other one is a multiple-objective linear programming with uncertain demand. In each model he maximized the profit and the sales.

**Single-Item and Multi-Item Aggregate Production Planning**

In an aggregate planning problem a single production variable is used to represent the total production of all products. This means that there must be some natural unit for measurement of aggregate output such as tons for a steel mill, cases for a bottling plant, barrels for a refinery, machine-hours for a job shop, or manhours for a maintenance department. In this report, it is called single-item aggregate planning. The solution to an aggregate model establishes the
production capacity and the aggregate production level for each period. Apparently a second-stage decision procedure is required to determine production quantities for individual products. Using the output of the aggregate model as constraints, this two-stage approach to planning production and workforce could result in an inferior solution to that obtained by combining decisions about individual item production quantities and the decision on workforce level into one model which will be illustrated in Chapter III. The principal disadvantage of multiple-item analysis over single-item aggregate analysis is the computational difficulty resulting from the size of the model. With advances in the size and speed of digital computers and improvements in mathematical programming software, this disadvantage is disappearing gradually.

Figure 1 shows the domain of the aggregate production planning problems. A general idea is given for the single-stage models, multi-stage models, effect of demand and price problems on the aggregate planning.

**Single-Stage and Multi-Stage Decision Models**

It is useful to establish the nature of the problem structure with which we are dealing in order to see the kinds of characteristics which must be modeled using informal solution methodologies and to appraise the available alternate solutions. The difference between single-stage and multi-stage is defined as the planning horizon. A single-stage is looking ahead only one period, while multi-stage is looking ahead several periods. So, the single-stage model is the one
Fig. 1. Classification of Aggregate Planning Problems
period planning horizon model which is in the finite horizon category. A stage may be thought of as a planning period or point in time when decisions are made concerning the use of resources. The details of these models are discussed by Buffa. But this definition in the literature is also mentioned as single-period and multi-period planning.

Another meaning of the stage is that the product should be processed in several plants (departments). For example, we consider the situation in Figure 2. A single finished product is manufactured in two stages. The first consists of a plant (department) which produces a semi-finished product, and an inventory of that commodity. The second stage contains a plant (department) which converts the semi-finished product into finished product, and the finished goods inventory which is subjected to a known time-varying demand schedule. Each plant can use overtime or subcontracting in any period to produce in excess of its regular time capacity.

![Stage 1](Plant 1 (Department 1)) ![Stage 2](Plant 2 (Department 2))

Raw Material \[\rightarrow\] Semi-finished Demand \[\rightarrow\] Demand

Fig. 2. A Two-Stage Serial Production System
Fluctuation of Demand

Demand for products may be random, seasonal or deterministic from one period to the next over a planning horizon. Fluctuations may be absorbed or smoothed by a variety of management strategies. In Chapter III we will discuss some models related to this problem.

Price is another variable which is considered as a constant variable in most of the models. In Section 3.3.2 a model with fluctuating price is mentioned.

The Aggregate Production Planning Process

Since numerous approaches have been proposed to aggregate production planning, this section is aimed at assisting the reader in finding the most appropriate model and solution technique for the type of situation he faces.

Step 1: Is your problem related to aggregate planning?
To answer the question read Chapter I and II

Step 2: What kind of aggregate planning problem do you have?

2a) production smoothing problems
2b) workforce smoothing problems
2c) production and workforce smoothing problems

Step 3: The problem which you are dealing with

3a) single-item aggregate planning
3b) multi-item aggregate planning

For example, we can consider an oil company as single-item aggregate planning because we can relate the barrels of oil to one unit, which is gallon. But, in an air conditioning factory we cannot relate fan
coil, unit heater or chiller to one unit. We also cannot relate the chiller to one unit because for each size of chiller we will need a certain expansion valve, solenoid valve, compressor, evaporator, etc. In this case you should find the optimal values of production rate and workforce level for each product or component.

Step 4: Choose the appropriate table according to your answers in Steps 2 and 3.

<table>
<thead>
<tr>
<th>Answers in Steps 2 and 3</th>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a, 3a</td>
<td>2</td>
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<tr>
<td>2a, 3b</td>
<td>3</td>
</tr>
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<td>2b, 3a</td>
<td>4</td>
</tr>
<tr>
<td>2b, 3b</td>
<td>5</td>
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<td>2c</td>
<td>6</td>
</tr>
</tbody>
</table>

Step 5: Then we should choose the model according to objective. The following questions will help you to find a desired model. The final decision depends on how important the answers are for you.

Question 1: What kind of cost structure do you have? The cost can be linear or nonlinear function of the variables. Also, the combination of the cost factors is important. For example, for one model we may consider the layoff and hiring costs and for another model we may consider the cost of fluctuations in production. The critical cost combinations are studied by Gordon (1).
Question 2: What kind of method do you have for forecast requirements?

Forecast requirements are in column 4 of the tables and parameter estimation in column 3 is the method which you use in order to forecast your requirements. They can be the historical curve, estimation techniques like moving average, exponential smoothing, regression methods, simulation, etc. In column 4, when we mention "as desired" it means that the problem can have several combinations and the appropriate combination depends on the kind of situation that the firm has. In dynamic programming techniques we use the term "depends on the problem." In Chapter 3.3.2 we will describe the dynamic programming model. For each specific situation, the parameters and their forecast will be different.

Question 3: What kind of computability does the method have?

For example linear programming is easy to apply but it must be run in each period. But, some methods are hard to apply. Also, we mention about the decision variables and optimization technique. Sometimes the employee is not aware of the technique or the manager is not interested in that technique.

Considering these factors, we will be able to select the desired method according to the degree of significance which the answers to the questions have for us.
CHAPTER III

APPROACHES TO AGGREGATE PRODUCTION PLANNING

3.1 Introduction

There are several approaches to the selection of production rates and workforce sizes. However, they can really be divided into two fundamental groups. The first group is non-quantitative methods. The philosophy here is that the decision maker is either unaware of analytical techniques available to his problem or he does not believe that the mathematical models are representative enough of the actual situation. The second group is quantitative methods. However, such models usually have built-in assumptions that may not be realistic.

3.2 Non-Quantitative Models

In this section three approaches used in industry are presented; non-quantitative haggling, adjust last year's plan, graphic methods.

Non-quantitative Haggling

There are conflicting objectives held by different departments of an organization when it comes to production smoothing and workforce balancing. One way to achieve a compromise of the conflicting desires is to bargain in a noneconomic manner. In general, this is not desirable. The policy usually is dictated by the most persuasive individual rather than being set in an objective manner.
Adjust Last Year's Plan

This approach often used in industry is to take the previous plan and adjust it slightly so as to meet this year's conditions. The danger here is the implicit assumptions that the previous plan was close to optimal; in this way management may get locked into a series of poor plans. This is not to say that one should never arrive at a new plan by adapting the old one. On the contrary, when the use of an appropriate mathematical model guarantees a close to optimal plan and when the computational procedure is iterative in nature, it makes good sense to use the old plan as an initial solution for the new plan.

Graphic Methods

Graphic methods are discussed by Buffa (17, 18, 19, 20, 21, 22). The advantages of the graphical approaches are in its simplicity and in being able to visualize the effect of various proposals. The disadvantages are that the procedure requires the analyst to generate proposals which may or may not be good and there is no test for optimality. The procedure generates a static plan and is hard to apply when multiple products are competing for the use of the same productive facilities.

3.3 Quantitative Models

Mathematical approaches to the optimal solution to this problem are presently nonexistent for the general case. Several mathematical methods for specialized conditions are available. In this chapter as indicated earlier, our interest is not in solution techniques, but in the main assumptions, data requirements, special feature of the
problem, forecast requirement, results, optimization technique, computational efforts and performances of the various approaches.

3.3.1 Work Force Smoothing Models and Comparison

There have been a number of approaches to production planning when workforce level decisions are involved. In this section, we shall describe briefly some of the better known mathematical approaches.

**Linear Decision Rule (LDR)**

The linear decision rule was developed by Molt et al. (23, 24, 25). This work is a standard reference in management science. The LDR would be classified as a quadratic programming approach to the aggregate planning problem. The cost function is the sum of four types of costs: 1) regular payroll; 2) hiring and layoff; 3) overtime costs; and 4) inventory holding, backordering, and machine set-up costs.

The main assumptions of this approach are:

1. The cost of regular production is linearly related to the size of the workforce.

2. The cost of increasing or decreasing the workforce is assumed to take the form of the quadratic function. This cost is assumed to be symmetrical, namely, an increase or a decrease in the workforce by a given amount incurs the same cost.

3. It is assumed that for a given production level there is a corresponding desirable level of labour requirements and that the cost of overtime is the form of the quadratic function.
4. The minimum cost inventory is assumed to be linearly related to the demand. In fact, it is known from inventory theory that the optimal inventory level is proportional not to demand but to its square root, while in the LDR model it is assumed that the linear relationship is an adequate approximation.

5. LDR model is for the single-item aggregate planning.

6. We assume that the price does not change quickly from one period to the next period.

7. Demand is not assumed to be deterministic and we should forecast the demand over yearly horizon.

This method's objective is the derivation of linear equations or "decision rules" which can be used to specify the optimum production rate and workforce level over some prescribed production planning horizon. These two rules require as inputs the forecast for each period of the planning horizon in aggregate terms, the ending size of the workforce, and inventory level in the last period. Once the two rules have been developed for a specific situation, the computations required to produce the decisions recommended by the model require only a few minutes by manual methods. The optimization technique for this model is matrix inversion of differentiated cost.

**Multi-Item Linear Decision Rule**

Bergstrom and Smith (26) extended the LDR to a multi-item formulation which solves directly for the optimum sales, production and inventory levels for individual items in future periods.

To remove the restriction of specified demand, revenue curves
are estimated for each item in each time period. This model then seeks a solution to maximize profit for the firm over the time horizon by an application in a firm producing a line of electric motors.

The LDR is designed to make decisions on aggregate production rate and employment level for the upcoming period. Because of the aggregate nature of this formulation, it is not possible to solve directly for the optimum production rates for individual products. Therefore, in situations where no natural dimension for aggregation exists, the breakdown of an aggregate production plan into individual item plans may result in a schedule which is far from optimum. As an extreme example of this, consider the situation where a plant produces lawn mowers and snow blowers. In this case specification of an aggregate production plan neglects the most interesting question, namely, the correct production plan for each individual item. One of the goals of their method as we mentioned is to extend LDR model to enable determination of the optimum plan for each individual item to be produced in a facility. Their second goal is to remove the LDR restriction that demands or forecasts of demands for future period have been specified. This will be done by estimating a revenue curve for each product in each time period. Once revenue curves have been specified we can maximize profit across a time horizon of interest by calculating optimal sales quantities from the model at the same time we also determine the optimal production rates and inventory levels for this sales program on a product-by-product basis.

The model was applied to a firm producing electric motors. We
will present this application in Chapter IV. They argued that with current computer technology it is hard to justify the effect necessary to develop a closed form solution to their model, and suggest the use of standard computer codes for solving the equation system resulting from setting the first partial derivatives to zero. However, a careful examination of the structure of this equation system reveals that very substantial computational savings can be realized without attempting to derive a solution completely in closed form.

Welam (27) used the same symbols and assumptions as Bergstrom and Smith (26) but employed matrix notation in order to study the structural properties of the equation systems. His main objective is to reduce the computational efficiency in solving large-scale multi-item production smoothing models with quadratic costs. So, he considered two multi-item versions of the HMMS model and shows that their optimal solutions can be obtained in "almost" closed form. Numerical inversion of large matrices is therefore not necessary and considerable savings in terms of storage requirements are also possible. The existence of unique optimal solutions depends in a very simple way on the model parameters.

Goodman (28) used the sectioning search procedure to solve the workforce scheduling problem. His model is a single-item, multi-stage aggregate scheduling. He raised a new (albeit related) problem based on the LDR method. In this new problem, one must minimize a quartic (fourth order) cost function subject to linear constraints. The method is simple and can be understood by managers as well as
technicians; it is fast, and supplies an integer solution for the production quantity, workforce level and number of units to be sold in each period.

The goal programming approach to the problem of aggregate workforce scheduling was developed by Goodman (28). However, his goal programming approach is different from those of Lee (10, 11, 12, 13, 14). This approach is a linear approximation to a quartic cost element which was developed by Goodman (28). The idea is presented in Figure 3. He applied this model to the LDR model and derived the computational result. These results are compared with similar results derived using the Hanssmann et al. (29) approach. The two case applications demonstrate that the effectiveness of such an approach is highly dependent upon the degree of non-linearity which the goal programming model must approximate. The results suggest that, for relatively low degree models, goal programming may provide an efficient and effective solution approach, while for higher degree models the approach may be inappropriate.

Range programming is a technique which was developed by Laurent (30). This approach is a linear approximation to the same quartic as shown in Figure 4. A deviation from \( X' \) lower than a given threshold \( m \) is assumed here to entail no cost. A deviation larger than the threshold is assumed to entail a cost linear in the surplus. The rationale for this formulation is that the real cost increases only slightly when \( X \) varies in a sizeable range around its optimal value; for practical purposes, it is then possible to neglect these slight
Fig. 3. Goal Programming Approximation to a Quartic


Fig. 4. Range Programming Approximation to a Quartic

variations and thus to substitute the objective of keeping the variable in a satisfying range to the one of having it approach, as much as possible, its "optimal" value. More precisely, a quartic cost element: 
\[ Y = K(X-X')^4 \]
(where \( X \) is the variable whose variations entail costs, \( Y \) is the cost element, \( K \) is a positive constant, and \( X' \) is the value of \( X \) entailing the lowest cost) will be approximated by defining three deviation variables, \( \overline{X}, X^0, \) and \( X^+ \), through the new couple of constraints:

\[
X + \overline{X} - X^0 - X^+ = X' - M, \quad X^0 \leq 2M
\]

and replacing the quartic cost element \( Y \) by the following linear approximations: 
\[ r = CX^- + CX^+ \]
(where \( m \) is the largest deviation assumed to entail no cost, \( C \) is a positive constant). In practice, the range programming formulation makes it possible to introduce in a linear program, in a simple manner, the range of acceptable values for a variable derived by a manager from his experience. This possibly could offer bases for new, practical, applications of mathematical programming, taking advantage simultaneously of the optimizing and satisfying concepts.

**Linear Programming Approach**

Hanssman and Hess (29) give a linear programming formulation of the workforce scheduling problem. They assume that:

1. There is no cost for a change in the production rate.
2. Demand is deterministic.
3. Each period is split into subperiods, e.g., regular, overtime, and third shift.
4. The cost of producing each unit in the jth subperiod of period i is given, i.e., the marginal cost is constant for a subperiod.

5. The inventory carrying cost for period i is given.

6. There are restrictions on the total amount that can be produced in each subperiod.

7. The model may give a solution that puts the inventory at a higher than physically allowable level, i.e., there is no simple way to include a bound on the inventory level.

Parameter estimation for this model is historical curve fitting or estimates (with piecewise linear approximation). The optimal technique to solve the problem is the transportation technique. Computer programs for this model are available; but, it must be run each time period. Once the problem is formulated, any standard linear programming techniques, such as the simplex method, can be used to obtain the solution. Linear programming models have been reviewed in many books, e.g., Magee and Boadman (31).

Search Decision Rule (SDR)

SDR is a simulation search procedure. As in any simulation procedure the approach is to systematically vary the variables (e.g., the workforce sizes and production rate) until a reasonable (and hopefully near optimal) solution is obtained. Normally a computer is required to make the approach feasible, even under the assumption of no uncertainty in the forecasts of demand. Taubert (32, 33) has investigated several different heuristics for searching the total cost. The idea is to get reasonably close to the global minimum of total cost.
Both Vergin (34) and Taubert (32, 33) have found that simulation search procedures perform extremely well. When the true cost structure closely resembles that assumed by one of the optimizing procedures (e.g., linear programming), the search procedures do essentially as well as the true optimizing procedure. When the true cost structure differs from that assumed by the optimizing procedures, the simulation search approach generally outperforms the optimization strategy. As an example, Taubert (32) applied the search procedures to the paint factory mentioned earlier. The results are extremely close to the LDR model with perfect forecasts. SDR does not provide forecast weights as do the LDR model. Computation time for SDR was very short for the paint factory data. However, the procedure varies from difficult to easy, depending on the cost structure. The optimization technique used is the gradient search.

Comparison

Single-item workforce scheduling models are compared in Table 2 and multi-item cases are compared in Table 3.

By "not required" for parameter estimation we mean any method can be used. For the search decision rule cost structure is arbitrary, it means that any combination of the costs can be used. The forecast requirement depends on the cost structure and the problem, so we indicate "as desired." The sectioning search is relatively easy with respect to the linear decision rule. The other notations are clear.
<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Decision Rule (LDR)</td>
<td>Quadratic approximate for layoff &amp; hiring, overtime &amp; regular payroll, inventory &amp; shortages</td>
<td>Historical curve fitting or estimates</td>
<td>Monthly shipments or over yearly estimates</td>
<td>Linear decision rule for P_t, W_t as function of W_{t-1}</td>
<td>Difficult</td>
<td>Matrix inversion of differentiated cost</td>
</tr>
<tr>
<td>David A. Goodman (Sectioning Search)</td>
<td>Minimize production &amp; inventory costs over N stages quartic (fourth order) cost function subject to linear constraints</td>
<td>Not required</td>
<td>Demand</td>
<td>P_s = production</td>
<td>Relatively Sectioning search</td>
<td></td>
</tr>
</tbody>
</table>

\[ P_t = \text{production} \]
\[ W_t = \text{work force level at} \]
\[ t; C_t = \text{number of units to be sold in period} t \]
<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goodman (Goal Programming Model)</td>
<td>Linear approximate to quartic (fourth order) cost as above</td>
<td>Historical curve fitting or estimates</td>
<td>Same as LDR</td>
<td>$P_t = \text{production in period } t$; $W_t = \text{workforce level in period } t$</td>
<td>Must be run each time</td>
<td>Linear programming</td>
</tr>
<tr>
<td>Laurent, G. Range Programming (sectioning search)</td>
<td>Same as Goodman (sectioning search)</td>
<td>Not required</td>
<td>Demand</td>
<td>$P_t, W_t$</td>
<td>Easier than Goodman</td>
<td>Range programming (transportation technology)</td>
</tr>
<tr>
<td>Henssmann &amp; Hess, linear program</td>
<td>Linear approximate to layoffs &amp; hiring, overtime &amp; regular, inventory &amp; shortages</td>
<td>Same as LDR except piecewise linear approximation</td>
<td>Monthly shipments over 6-month horizon multi-periods increase computational difficulty</td>
<td>Optimal values of $P_t, W_t$</td>
<td>Must be run each time, period, easy</td>
<td>Linear programming (transportation technology)</td>
</tr>
</tbody>
</table>

TABLE 2 - Continued
<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taubert, Search Decision Rule</td>
<td>Arbitrary</td>
<td>Historical curve fitting or estimates</td>
<td>As desired ( P_t W_t )</td>
<td></td>
<td>Difficult to easy depending on cost structure, must be run each time period</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 3

COMPARISON OF MULTI-ITEM WORKFORCE SCHEDULING MODELS

<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bergstrom and Smith</td>
<td>Minimize the profit = revenue-cost) cost is the same as LDR model</td>
<td>Historical curve fitting or estimates</td>
<td>The same as LDR model; also it needs estimation of revenue curve for each product in each time period</td>
<td>$p_t^<em>$, $s_t^{<strong>}$, $w_t^{</strong></em>}$</td>
<td>Difficult to apply, easy to compute</td>
<td>Calculus Easier Matrix notation</td>
</tr>
<tr>
<td>Peter Welam</td>
<td>Same as Bergstrom</td>
<td>Same as Bergstrom</td>
<td>Same as Bergstrom</td>
<td>Same as Bergstrom</td>
<td>Easier than Bergstrom</td>
<td>Matrix notation</td>
</tr>
</tbody>
</table>

$P_{it}$ = production of product i in period t

$s_{it}$ = number of units of product i to be sold in period t

$w_t$ = workforce level in period t
3.3.2 Production Smoothing Model

A different approach to the aggregate production planning problem is the minimization of the fluctuations in production and inventory levels. For the single-item production smoothing problem there is the DE decision rule. Pekelman's model, linear programming models and a general dynamic programming approach are suitable for the special cases.

The DE Decision Rule

This approach is exemplified by the method investigated by Deziel and Eilon (35) which will be referred to as the DE rule. Their model is based on the following assumptions:

1. The decision variable is the production quantity $Q_t$ and this decision is made at the beginning of period $t$. There is no separate decision for the workforce level.

2. There is lead time of $L$ periods for implementing the production decision, so that the production level $P_t$ in period $t$ is

$$ P_t = Q_{t-L} $$

3. Production orders already in the pipeline cannot be altered.

4. Orders that cannot be filled at the end of a period are backlogged.

The decision rule takes the form

$$ Q_t = K \left[ R_{t-1} - \frac{1}{t-1} \sum_{i=t-L}^{t-1} (Q_i - D) \right] + D $$
where

\[ Q_t = \text{decision made in period } t \text{ for reorder quantity} \]

\[ R = \text{safety stock} \]

\[ I_{t-1} = \text{stock level at the end of period } t-1 \]

\[ D = \text{expected demand level} \]

\[ K = \text{a smoothing constant } (0 \leq K \leq 1) \]

\[ R - I_{t-1} \] describes the amount by which the stock level falls below the safety stock requirement and the third term in the square brackets represents the cumulative excess of production over demand during the lead time. The smoothing factor \( K \) gives a weight to the total inventory balance in the square brackets. If \( K = 0 \) is taken then the decision rule is reduced to ordering an amount equivalent to the expected demand.

If in place of the expected demand \( D \) the forecast \( F_t \) is substituted into the decision rule becomes

\[ Q_t = K \left[ R - I_{t-1} - \sum_{i=t-1}^{t-1} Q_i \right] + (1+KL) F_t \]

Here \( F_t \) represents the forecast for demand per period and it may be derived from some forecasting procedure; for example, if simple exponential smoothing is used then

\[ F_t = D_{t-1} + (1-\alpha) F_{t-1} \]

where

\[ F_t = \text{updated demand forecast} \]

\[ F_{t-1} = \text{previous demand forecast} \]
D_{t-1} = \text{actual demand in period } t-1 \\
\alpha = \text{the smoothing constant in forecasting} \\
\left(0 \leq \alpha \leq 1\right)

Thus, the decision involves two smoothing parameters \( K \) and \( \alpha \). The performance of the system can be described by several measures, such as the following:

1. Fluctuations in the inventory level, measured by the standard deviation \( \sigma_I \).

2. Fluctuations in the reorders level, measured by the standard deviation \( \sigma_Q \).

3. If a sudden increase in demand occurs, the level of stock runout increases. The additional amount of stock that cannot be supplied as a result of this sudden impulse in the demand is defined as

\[ \varsigma = \phi' - \phi \]

where \( \phi \) is the expected level of future runout when demand is stationary and \( \phi' \) is the level of future runouts when the mean demand is subject to a sudden increase (both \( \phi \) and \( \phi' \) are measured over a given horizon). Thus \( \varsigma \) expresses (in terms of runouts) the consequence of a "disturbance" in the demand pattern.

4. If a disturbance occurs (for example, a discrete increase in the demand level), the system reacts by supplying the demand from stock and by issuing orders to increase the production level. After awhile the stock is replenished sufficiently for the production level to settle down to the expected demand level. The time that it takes the system to reestablish the inventory level \( I_t \) to a value within a
given margin of the level prior to the disturbance is called the "rise
time" $T_r$. The purpose of this procedure is to provide a mechanism
that will respond quickly enough to abrupt changes in the mean demand
level and yet protect the production rate from being affected by spuri-
ous demand fluctuations and this is the essence of the smoothing prob-
lem.

Three alternative objectives are considered in the DE model:

1. Minimize $c = a \sigma_I + b \sigma_Q$ where $a$ and $b$ are constants.

2. Minimize $c = a \sigma_I + b \sigma_Q + c$ where $a$, $b$ and $c$ are constants.

3. Same as (1) and subject to a given maximum value of $T_r$ to
ensure that the system recovers from an abrupt disturbance
within a reasonable period.

Production Smoothing with Fluctuating Price

The model presented by Pekelman (36) deals with a firm facing a
known price which varies over time during some finite period $(Q, T)$.
The firm wishes to determine the production rate at each instant of
that interval which will maximize profit, when adjustment of output
incurs additional cost. He characterized the optimal solution and
constructed a forward algorithm which is shown to converge to the
unique optimal solution. He also specified the conditions for plan-
ing and forecast horizons, both of which can be identified by the
described algorithm. Control theory was used to achieve these results.

Pekelman (36) deals with an individual firm in a purely competi-
tive industry so that prices in each period are already determined by
industry supply and demand conditions. Hence, each firm can sell as much as it wishes at the given price, but will restrict its production due to an increasing marginal cost function as well as an asymmetric smoothing cost function. The principal difference is that he mentioned price rather than demand is the exogenous element so that the objective function is maximization of profit rather than minimization of cost.

A General Dynamic Programming Model for Production Smoothing Models

This model appears in Johnson and Montgomery (37).

Let

\[ X_t = \text{production scheduled for period } t \]
\[ (t=1,2,\ldots,T) \]
\[ D_t = \text{expected demand in period } t \]
\[ I_t = \text{net inventory at the end of period } t \]
\[ K_t(X_t, I_t, X_{t-1}) = \text{cost of production, inventory, shortages,} \]
\[ \text{and production change in period } t, \text{ as a function of the production level and ending inventory in period } t \text{ and the production level in period } t-1 \]
\[ f_t(I, X) = \text{the minimum cost attainable over periods } t, t+1,\ldots, T, \text{ when the net inventory at the start of period } t \text{ is } I \text{ and the production level in period } t-1 \text{ was } X. \]
\[ C_t(X_t) = \text{production cost for period } t \]
\[ H_t(I_t) = \text{inventory cost for period } t \]
Typically we can write

\[ K_t(X_t, I_t, X_{t-1}) = C_t(X_t) + H_t(I_t) + V_t(X_t, X_{t-1}) \]

Here we assume that the production, inventory related, and smoothing costs are separable.

Consider the decision to be made in period \( t \). If the inventory at the start of period \( t \) is \( I_{t-1} = I \) and the production decision is \( X_t \), and the previous production is \( X_{t-1} \), the resulting cost in period \( t \) is \( K_t(X_t, I_{t-1}, X_{t-1}) \). Furthermore, the ending inventory, \( I_t \) which is strictly determined by \( I, X_t \), and \( D_t \), affects the minimum costs that can be obtained in the periods following \( t \). Assuming that an optimal policy is followed after period \( t \), the decision \( X_t \) results in a cost over the lost \( T-t+1 \) periods of \( K_t(X_t, I_t, X_{t-1}) + f_{t+1}(I_t, X_t) \). The minimum value of this cost is \( f_t(I_{t-1}, X_{t-1}) \), defined by the following recursive equation:

\[ f_t(I_{t-1}, X_{t-1}) = \min_{X_t \geq 0} \left[ K_t(X_t, I_t, X_{t-1}) + f_{t+1}(I_t, X_t) \right] \]

where

\[ I_t = I_{t-1} + X_t - D_t \]
The model has two stated variables, $I_{t-1}$ and $X_{t-1}$. The following special forms can be considered:

(1) $C_t(X_t)$ Linear $\Rightarrow$ $C_t(X_t) = c_t X_t$, for $X_t \geq 0$ ($C_t > 0$)

Convex $\Rightarrow$ $C_t(X_t) \geq 0$ convex for $X_t > 0$

Piecewise concave $\Rightarrow$ $C_t(X_t) \geq 0$ concave for $X_t > 0$

Production cost functions are shown in Figure 5.

(2) $H_t(I_t)$ Linear $\Rightarrow$ $H_t(I_t) = h_t I_t$, for $I_t > 0$ ($h_t > 0$)

Convex $\Rightarrow$ $H_t(I_t) \geq 0$ convex for $I_t < 0$ ($h_t < 0$)

Inventory cost functions are shown in Figure 6.
Dynamic programming approaches are not attractive because there are two state variables. However, there are several more efficient algorithms for solving a few special cases. We next describe some of these results.

An example for a simple case when the production smoothing cost is a quadratic form is discussed by Wagner (38). Zangwill (39) considers a situation where the cost functions $C_t$ and $H_t$ are concave and $V_t$ is piecewise concave. No backlogging is allowed and the requirements are assumed nondecreasing; that is, $D_t \leq D_{t+1}$, for all $t$. He characterizes the optimal solutions as having the following properties:

1. If $X_t > 0$ and $X_{t-1} > 0$, then $X_t = X_{t-1}$

2. If $X_t > 0$ and $X_{t-1} = 0$, then $I_{t-1} = 0$

Thus, production occurs in runs, where a run is a sequence of periods having the same positive production level in each period. Also no inventory is on hand at the start of the first period of the run. He
then develops a solution algorithm for the special case.

\[
V_t(X_t, X_{t-1}) = \begin{cases} 
V_t, & \text{if } X_t > X_{t-1} \\
0, & \text{if } X_t = X_{t-1} \\
V_t', & \text{if } X_t < X_{t-1}
\end{cases}
\]

The algorithm can be made more efficient for the simple production cost function

\[
C_t(X_t) = \begin{cases} 
A_t + CX_t, & \text{if } X_t > 0 \\
0, & \text{if } X_t = 0
\end{cases}
\]

Sobel (40, 41) analyzes the problem of smoothing start-up and shutdown costs, using a smoothing cost structure \( V_t \) which is

\[
V_t(X_t, X_{t-1}) = \begin{cases} 
V_t, & \text{if } X_t > 0 \text{ and } X_{t-1} = 0 \\
V_t', & \text{if } X_t = 0 \text{ and } X_{t-1} > 0 \\
0, & \text{otherwise}
\end{cases}
\]

and assuming production and inventory costs are concave. He develops shortest-path network algorithms for solution of two forms for \( V_t \) and points out simplifications for special production cost functions.

**Linear Programming Techniques**

In the case of

\[
V_t(X_t, X_{t-1}) = \lambda_t \left| X_t - X_{t-1} \right| \quad \text{and}
\]

\[
V_t'(X_t, X_{t-1}) = \lambda_t (X_t - X_{t-1}) + \lambda_t' (X_t - X_{t-1})^{-}
\]
Problem can be solved by linear programming techniques. This model is efficient when the costs are linear.

Comparison

Single-item production smoothing models are compared in Table 4 and multi-item production smoothing model is shown in Table 5. The reason is that the different characteristics will be more clear for the reader. Again, we used the same notations as in Tables 2 and 3.

3.3.3 Production and Workforce Models and Comparison

In these models we adjust the production rate and the workforce which is called the production scheduling workforce smoothing problem.

The combined optimization problem for a single product has been treated by Lippman et al. (42, 43). In this model, the cost is the sum of production, employment smoothing, and inventory costs subject to a schedule of known demand requirements over a finite time horizon. The three instrumental variables are workforce producing at regular-time, workforce producing on overtime, and the total workforce. Overtime is limited to be not more than a fine multiple of regular time. Production costs are convex, smoothing costs are V-shaped, and holding costs are increasing; both the production and holding cost functions need not be stationary. They consider upper and lower bounds on the cumulative regular time plus overtime workforce for any sequence of demand requirements. They also give the form of an optimal policy when demands are monotone (either increasing or decreasing). All of these results, which convey information about the numerical values of optimal policies, given specific demands and an initial level of
### TABLE 4

**COMPARISON OF SINGLE-ITEM PRODUCTION SMOOTHING MODELS**

<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>The DE Decision Model</td>
<td>Minimize cost &amp; fluctuation in production and inventory levels</td>
<td>Any forecasting procedure, e.g., exponential smoothing</td>
<td>Demand</td>
<td>Production quantity ( Q_t )</td>
<td>Will respond quickly enough to abrupt change in the mean demand level</td>
<td>Algebraic method</td>
</tr>
<tr>
<td>DOV Pekelman</td>
<td>Minimize the profit with consideration of (Revenue at ( t ) - production cost at ( t ) - smoothing costs) production cost is assumed to be convex</td>
<td>Industry supply, demand conditions</td>
<td>Price</td>
<td>( P_t = \text{production rate at } t )</td>
<td>Difficult, is not practical</td>
<td>Control theory</td>
</tr>
<tr>
<td>Dynamic Programming Models Johnson &amp; Montgomery</td>
<td>Minimize cost of production, inventory shortages, &amp; production change</td>
<td>Historical curve, estimation</td>
<td>Depends on the problem</td>
<td>Production rate at ( t )</td>
<td>Is not efficient, difficult</td>
<td>Dynamic programming</td>
</tr>
</tbody>
</table>
TABLE 4 - Continued

<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Models</td>
<td>Minimize sum of production &amp; inventory related cost, and production costs over the planning horizon</td>
<td>Estimation ( h_{t'-t'} )</td>
<td>( X_1, X_2, \ldots )</td>
<td>Efficient, Linear programming</td>
<td>Run every period</td>
<td></td>
</tr>
<tr>
<td>Johnson &amp; Montgomery</td>
<td>Estimation ( t'-t' )</td>
<td>( t*t )</td>
<td>( X_t, ) production quantity for rate change each period</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( h_t \) = inventory carrying cost per unit held from period \( t \) to \( t+1 \)

\( t \) = backorder cost per unit carried from period \( t \) to \( t+1 \)

\( t \) = cost to decrease the production rate by one unit from period \( t-1 \) to period \( t \)

\( t \) = cost to increase the production rate by one unit from period \( t-1 \) to period \( t \)

\( D_t \) = demand in period \( t \)
### TABLE 5

**MULTI-ITEM PRODUCTION SMOOTHING MODEL**

<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Parameter Estimation Requirement</th>
<th>Forecast Requirement</th>
<th>Result</th>
<th>Computability</th>
<th>Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Models</td>
<td>Maximize profit or minimize cost</td>
<td>Estimation techniques</td>
<td>( r_{it} ) ( C_{ijt} ) ( X_{ijt} )</td>
<td>Efficient</td>
<td>Linear programming</td>
</tr>
<tr>
<td></td>
<td>( Z=\text{total profit} )</td>
<td>( \text{Manager's opinion} ) ( F_{it}, F'<em>{it} ) ( S</em>{it} )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
Z = \sum_{t=1}^{T} \sum_{i=1}^{I} \left( r_{it} S_{it} - \sum_{j=1}^{J} C_{ijt} X_{ijt} - h_{it} I_{it} - I'_{it} \right)
\]

- \( X_{ijt} \): the number of units of product \( i \) produced by process \( j \) in period \( t \)
- \( S_{it} \): the number of units of product \( i \) planned to be sold in period \( t \)
- \( r_{it} \): revenue, net of variable selling expense, from selling one unit of product \( i \) in period \( t \)
- \( C_{ijt} \): unit variable cost of producing a unit of product \( i \) by process \( j \) in period \( t \)
- \( F_{it} \): maximum projected sales potential for product \( i \) in period \( t \)
- \( F'_{it} \): minimum (required) level of planned sales for product \( i \) in period \( t \)
- \( h_{it} \): inventory carrying cost per unit of product \( i \) held from period \( t \) to \( t+1 \)
- \( I_{it} \): backorder cost per unit of product \( i \) carried from period \( t \) to \( t+1 \)
- \( r^+_{it} \): on-hand inventory of product \( i \) at the end of period \( t \)
- \( I^-_{it} \): backorder position of product \( i \) at the end of period \( t \)
inventory, depend only on the shape characteristic of the cost functions. This model was developed for the multiple product by Yuan et al. (44). They related monotonicity in each product's demand with monotonicity of production for each item in an optimal schedule. Because of the variety of the assumptions we didn't include these two models in Table 6.

**Parametric Production Planning**

Jones (45), in his parametric production planning, avoided the aforementioned dimensionality difficulty by postulating the existence of two linear feedback rules, one for the workforce, the second for the production rate. Each rule contains two parameters. For a likely sequence of forecasts and sales the rules are applied with a particular set of the four parameters, thus generating a series of workforce levels and production rates. The relevant costs are evaluated using the annual cost structure of the firm under consideration. By a suitable search technique the best set of parameters is determined. Cost function can be arbitrary. Forecast requirements depend on the cost structure and the problem. Decision variables are:

\[
\begin{align*}
        P_t &= \text{production rate in period } t \\
        W_t &= \text{workforce level in period } t \\
        P_t \text{ and } W_t \text{ are computed as a function of } W_{t-1}, I_t \\
        I_t &= \text{inventory level at the beginning of period } t \\
        W_{t-1} &= \text{workforce level in period } t-1
\end{align*}
\]
Management Coefficients Approach

Bowman (46, 47) developed a procedure for modeling management decision making with an illustration in the area of production smoothing and workforce balancing. Using statistical regression analysis the scheduling rules are fitted to the simple expressions, such as

\[ P_t = a_1 F_t + a_2 W_{t-1} - a_3 I_t + a_4 \]

\[ W_t = b_1 F_t + b_2 W_{t-1} - b_3 I_t + b_4 \]

where the a's and the b's are derived from the regressions. The assumption here is that management's decisions are in the main governed by the current workforce, by the forecast for demand in period t and by the inventory level.

There are, of course, many alternative multiple regression models that may be examined. For example, an attempt to account for the forecast in period t+1 as well may take the form

\[ P_t = a_1 F_t + a_2 F_{t+1} + a_3 W_{t-1} - a_4 I_{t-1} + a_5 \]

The purpose of this approach is that "experienced managers are quite aware of and sensitive to the criteria of a system." Bowman (42) proceeds to argue that managerial decisions are basically sound and that what is needed is to eliminate the "erratic" element by making them more consistent.

Linear Programming Techniques

Linear programming techniques can also be applied for the combined problem of production rate change and workforce rate change. These models for single-item and multi-item are formulated by Johnson
and Montgomery (37).

Comparison

Techniques are compared in Table 6. Because of the complexity of the model of Lipmann et al. (43) and Yuan et al. (44), we do not compare them in Table 6. Notations are the same as the previous table.
<table>
<thead>
<tr>
<th>Author or Model</th>
<th>Cost Structure</th>
<th>Parameter Estimation</th>
<th>Forecast Requirement</th>
<th>Decision Variables</th>
<th>Computability</th>
<th>Optimization Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jones, Parametric Production Planning (single-item)</td>
<td>Quadratic or linear layoffs &amp; hiring, overtime &amp; regular, inventory &amp; storages or arbitrary</td>
<td>Historical curve fitting or estimates with piecewise linear approximation</td>
<td>Depends on the problem</td>
<td>( P_t ) and ( W_t )</td>
<td>Difficult first time</td>
<td>Parameter search for good parameter values</td>
</tr>
<tr>
<td>Bowman, Management Coefficients (single-item)</td>
<td>Not required</td>
<td>Management past decision on ( P, W )</td>
<td>Depends on the problem</td>
<td>Smoothing rule for ( P_t ) and ( W_t )</td>
<td>Relatively easy</td>
<td>Regression for smoothing equations</td>
</tr>
<tr>
<td>Linear Programming Techniques (single- or multi-item costs)</td>
<td>Minimize production, change cost &amp; work- related</td>
<td>Estimation</td>
<td>Depends on the problem</td>
<td>( P_t ) and ( W_t )</td>
<td>Production Easy and efficient in period ( t ), and number of workers at facility ( K ) having work schedule ( r ) in period ( t )</td>
<td>Linear programming</td>
</tr>
</tbody>
</table>
CHAPTER IV

APPLICATIONS OF QUANTITATIVE MODELS

In this chapter we will review some applications of the quantitative models. This hopefully will give the reader a general idea of how they are applied. If a method is not used, it does not mean that the method is not practical. In summary the following cases are found in the literature:

Paint factory
Electric motor company
Chain brewing company
Transportation to and from the West Coast and Hawaii
The Northrop Corporation
The Camtor Company
Naval Ordnance System Command (NOSC)
Textile mill company
Blast furnace production factory
Packing industry
Post Office
Wool textile production

The Paint Factory

Linear decision rule was developed to a paint factory and applied to a six-year record of known decisions in the company. Two
kinds of forecasts were used as inputs; a perfect forecast and a moving average forecast. The actual order pattern was extremely variable, involving both the 1949 recession and the Korean war. The linear rules of the following form were found for the paint factory.

\[
P_t = \begin{cases} 
+0.463 F_t \\
+0.234 F_{t+1} \\
+0.111 F_{t+2} \\
+0.046 F_{t+3} \\
+0.013 F_{t+4} \\
-0.002 F_{t+5} \\
-0.008 F_{t+6} \\
-0.010 F_{t+7} \\
-0.009 F_{t+8} \\
-0.008 F_{t+9} \\
-0.007 F_{t+10} \\
-0.005 F_{t+11} 
\end{cases} + 0.993 W_{t-1} + 153 - 0.464 I_{t-1}
\]

\[
W_t = 0.743 W_{t-1} + 2.09 - 0.0101 I_{t-1} + \begin{cases} 
+0.0101 F_t \\
+0.0088 F_{t+1} \\
+0.0071 F_{t+2} \\
+0.0054 F_{t+3} \\
+0.0042 F_{t+4} \\
+0.0031 F_{t+5} \\
+0.0023 F_{t+6} \\
+0.0016 F_{t+7} \\
+0.0012 F_{t+8} \\
+0.0009 F_{t+9} \\
+0.0006 F_{t+10} \\
+0.0005 F_{t+11} 
\end{cases}
\]

where

\( P_t \) is the number of units of product that should be produced during the forthcoming month \( t \).
\( W_{t-1} \) is the number of employees in the workforce at the beginning of the month.

\( I_{t-1} \) is the number of units of inventory minus the number of units on backorder at the beginning of the month.

Jones (45) also applied the parametric production planning approach to the paint factory. The model had the following results for \( P_t \) and \( W_t \):

\[
W_t = W_{t-1} + 0.2685 \left[ \frac{\text{EXPB}}{5.67} - 4.59 - W_{t-1} + 0.2364 \right] \\
(320 - I_{t-1}) / 5.67
\]

\[
P_t = 5.67(W_t + 4.59) + 0.9475 \left[ \text{EXPD} - 5.67(W_t + 4.59) + 0.5309(320 - I_{t-1}) \right]
\]

where

\[
\text{EXPB} = \begin{bmatrix}
0.2364 & F_t \\
0.1831 & F_{t+1} \\
0.1418 & F_{t+2} \\
0.1099 & F_{t+3} \\
0.0851 & F_{t+4} \\
0.0659 & F_{t+5} \\
0.0511 & F_{t+6} \\
0.0396 & F_{t+7} \\
0.0307 & F_{t+8} \\
0.0238 & F_{t+9} \\
0.0184 & F_{t+10} \\
0.0143 & F_{t+11}
\end{bmatrix}
\]

\[
\text{EXPD} = \begin{bmatrix}
0.5309 & F_t \\
0.2491 & F_{t+1} \\
0.1169 & F_{t+2} \\
0.0548 & F_{t+3} \\
0.0257 & F_{t+4} \\
0.0121 & F_{t+5} \\
0.0057 & F_{t+6} \\
0.0027 & F_{t+7} \\
0.0012 & F_{t+8} \\
0.0005 & F_{t+9} \\
0.0003 & F_{t+10} \\
0.0001 & F_{t+11}
\end{bmatrix}
\]
Jones (45) compared the parametric production planning and the linear decision rule for the paint factory with perfect forecast. The results were very close. Actually, the paint factory has become a standard reference for comparison of the other approaches. $W_t$ is the number of employees that will be required for the current month $t$. The number of employees that should be hired is therefore $W_t - W_{t-1}$. $F_t$ is a forecast of number of units of product that will be ordered for shipment during the current month $t$. $F_{t+1}$ is the same for the next month, $t+1$, etc. These equations would be used at the beginning of each month; the equations are simple to compute.

The Electric Motor Company

Bergstrom (26) developed and applied the linear decision rule to a multiple product case. To illustrate the feasibility of the multi-item decision rule the model was applied to a firm producing electric motors. The company is one of the largest producers of electric motors in India. This firm is faced with significant seasonal trends in demand for their six types of motor, and the firm has difficulty in finding a suitable measure of unit for aggregation as required by the linear decision rule approach. In applying the multi-item decision rule to this firm, a one-year time horizon was used with decisions being made on a quarterly basis. With six products and four quarterly time periods, this results in a problem with fifty-two unknown variables. A total of six solutions for the multi-item model were obtained under slightly different assumptions of starting conditions and coefficients.
A management coefficient model was developed by Gordon (1) for a chain brewing company. The model for this company had the following form for the $P_t$ and $W_t$:

$$P_t = 6.98 + 1.66 W_{t-1} - 0.12 I_{t-1} + 0.44 F_{t+1}$$

$$W_t = 4.20 + 0.63 W_{t-1} + 0.17 F_{t+1}$$

where

- $P_t$ = production in period $t$
- $W_t$ = workforce in period $t$
- $I_{t-1}$ = inventory in the distributor system at the beginning of period $t$
- $F_{t+1}$ = sales forecast in period $t+1$
- $t$ = week

The coefficients were developed by multiple regression based on past managerial experience.

Transportation to and from the West Coast and Hawaii

This application is discussed by Olson et al. (46). The aggregate planning problem for this firm was a multi-period, multi-item transportation problem with queuing at the origins and destinations and requiring integer solutions. Since this is difficult to optimize directly, a simulation approach was used.

The Northrop Corporation

Schussel and Price (47) applied the dynamic programming approach to several plants of the Northrop Corporation. In this particular use,
dates for factory requirements are known and various vendors submit bids with amounts offered, if delivery is taken in certain sized lots or at certain times. The total cost functions may not be strictly concave. Several interesting methods for overcoming this problem are discussed.

The Camtor Company

Forgusan and Sargent (48) gave a case example of the Camtor Company which places the emphasis on the balance between inventory and outside purchase costs for a closed job shop kind of operation. The resulting master plan indicates which parts to purchase and which to manufacture internally by quarters. Some parts are scheduled to be manufactured earlier than actually needed and are carried in inventory, in order to achieve minimum cost for the criteria selected.

The Naval Ordnance Systems Command

Weston (49) developed linear programming model for the Naval Ordnance Systems Command (NOSC). He illustrated a production (ordnance) scheduling problem where more than one organization is involved. It presents the production line of several activities. Several ordnance type products are being produced with constraints as to which activities and production lines can produce each item. He discussed a situation where assignment of workload, and thus funding, is controlled at a central source. This central source is responsible for meeting workload schedules and also minimizing total costs of production lines within the designated organization.
Industrial applications of linear programming to aggregate planning problems are reported by Eisemerm and Young (50) in a study of a textile mill, by Fabian (51) in a study of blast furnace production, and by Greene (52) in a study of packing industry. Galbraith (53) gave examples of production smoothing occurring in a post office. Hurst (54) also used a heuristic model for a wool textile production case.
CHAPTER V

SUMMARY AND CONCLUSIONS

Aggregate production planning models are of the greatest import to operations management, since these plans enable management to deploy the major resources at its command. Management's interest therefore is focused on the most important aspects of this deployment process, such as employment levels, activity or production rates, and inventory levels. We have discussed the structure of the aggregate planning problem and a number of different approaches that have been designed to meet the needs of aggregate planning. We classified the approaches in three categories: a) smoothing models, b) production smoothing models, and c) production and workforce smoothing models. The models are compared with respect to the cost structure, parameters estimation, forecast requirement, decision variables, computability and optimization technique. The main assumptions of the techniques are presented. We have also reviewed some case studies.

Some authors discuss the advantages and disadvantages of each technique. But actually each method is based on different assumptions. Thus, it is difficult to compare all the methods developed. For example, Eilon (55) commented on the linear programming model, indicating that cases where the cost is not linear or when the demand forecasts are not accurate, the method is not useful.

53
For further research it is recommended to formulate new objective functions. There are some factors which are related to the profit but they are not considered in the existing models. The incorporation of some of these factors seem worthwhile:

1. Speed of service is one of the most critical competitive factors in some industries. A study of this effect for the bank check printing industry is published by Clark (56).

2. Effect of rate on profit, Flora (57) claims that tones on profits as well as the loss of an opportunity to invest should be used to compute the inventory carrying change rate.

3. Value of information: A behavioral experiment by Moskowitz (58) which emphasizes that information is the main factor for the aggregate planning model.
REFERENCES


