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The Silicon Solar Cell as an Optical Detector

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THE SILICON SOLAR CELL AS AN OPTICAL DETECTOR

BY

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RESEARCH REPORT

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ABSTRACT

The optical detector characteristics of a silicon solar cell are examined. A general equivalent circuit model is developed and typical parameter values are determined. A comparison is made between the photovoltaic and short circuit operating modes and the short circuit mode is shown to be preferable in terms of linearity, extended frequency response, and temperature stability. A method is developed to determine the noise characteristics of the amplifier-detector system used in the short circuit mode. The silicon solar cell is shown to be an economical alternative to standard photodiodes in low to medium data rate systems.
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The optical detector characteristics of a silicon solar cell are examined. A general equivalent circuit model is developed and typical parameter values are determined. A comparison is made between the photovoltaic and short circuit operating modes and the short circuit mode is shown to be preferable in terms of linearity, extended frequency response, and temperature stability. A method is developed to determine the noise characteristics of the amplifier-detector system used in the short circuit mode. The silicon solar cell is shown to be an economical alternative to standard photodiodes in low to medium data rate systems.
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PURPOSE AND SCOPE

Most studies of optical detectors for detection of modulated light in the visible and near infrared regions have been concerned with devices of high speed and sensitivity such as the avalanche and p-i-n photodiodes. There are many applications, however, that do not require extremely high response speeds and sensitivity. In these applications it may be more desirable to use the low cost, readily available, silicon solar cell.

In order to make a design decision on the suitability of the silicon solar cell for a specific application, it is necessary to determine the important optical detector characteristics. The performance of an optical detector may be defined by the following characteristics:

1) Sensitivity to incident optical signal.
2) Speed of response.
3) Linearity.
4) Temperature stability.
5) Noise characteristics.

The ability of silicon solar cells to function as an optical detector has been recognized for several years. Witherall and Faulhaber (1973) developed an equivalent circuit model from which many of the significant properties of the silicon solar cell as an optical detector could be deduced. However, their paper was concerned only with photometric applications and response times due to capacitive or diffusion effects was neglected. This paper is intended to extend their work by development of a more complete equivalent circuit model to include
frequency response and noise characteristics. Experimental measurements of spectral response and equivalent circuit parameters of a typical modern solar cell are analyzed in terms of the equivalent circuit model in order to determine expected solar cell detector characteristics. Both the photovoltaic and the short circuit mode of operation are discussed and the short circuit mode is shown to be preferable for most applications due to its linear response, high temperature stability, and extended frequency response compared to the photovoltaic mode.

DEFINITION OF PARAMETERS

There are many parameters which may be measured to determine the performance characteristics of an optical detector. Some of these parameters are not absolute quantities, but may depend on the conditions of measurement and on the environment in which the detectors are used. It is important in the characterization of optical detectors that the parameters used be carefully defined and measurement techniques clearly specified, so that the results may be readily interpreted and reproduced.

The sensitivity of a detector is measured in terms of either the voltage spectral responsivity \( R_{\lambda} \) or the current spectral responsivity \( R_{\lambda} \). The defining equations are:

\[
R_{\lambda} = \frac{V_s}{P_{s\lambda}},
\]

\[
R_{\lambda} = \frac{I_s}{P_{s\lambda}},
\]

where \( R_{\lambda} \) is the ratio of the rms signal voltage \( V_s \) (or current \( I_s \)) to the rms value of the monochromatic incident signal power \( P_{s\lambda} \) referred to an infinite load impedance in the case of \( V_s \) or to a zero
load impedance in the case of $I_s$. The spectral responsivity is expressed in terms of (volt/watt) or (amp/watt) and is a function of wavelength ($\lambda$), signal frequency ($f$), absolute temperature ($T$), and bias voltage.

A common method of determining the speed of response of a detector is a direct measurement of the decay or rise time characteristics of the detector signal when it is exposed to a square wave pulse of radiation. The parameter used is the pulse time constant ($\tau_p$), which is the time required for the signal to rise to 0.63 times its asymptotic value. Care must be taken to insure that the rise time of the radiation pulse is either negligible compared to the rise time of the detector or that it can be compensated for in the measurement. An alternate method is to measure equivalent circuit parameters that are indicative of the mechanisms responsible for the response time of the detector. This method requires that the physical diode mechanisms be well understood, but it has the advantage of being able to predict the detector response under varying conditions.

The noise characteristics of a detector may be specified by the spectral noise equivalent power ($\text{NEP}_\lambda$), which is an indication of the minimum detectable power. $\text{NEP}_\lambda$ is the value of monochromatic incident rms signal power of wavelength $\lambda$ required to produce an rms signal to rms noise ratio of unity.

$$\text{NEP}_\lambda = P_s \frac{V_n}{V_s} = V_n/R\lambda = I_n/R\lambda .$$

$\text{NEP}_\lambda$ is expressed in units of (watt) and the chopping frequency, the electrical bandwidth used in the measurement, and the detector area should be specified. For detectors that are background noise limited,
the field of view (Ω) and the background temperature (T_B) should also be specified. The spectral detectivity (D_λ) which is the reciprocal of NEP_λ is often used. A normalization of spectral detectivity (D^* (λ, f_o)) to take into account the area and electrical bandwidth dependence is given by:

\[ D^* (λ, f_o) = \sqrt{A Δf D_λ} \]

where \( f_o \) is the chopping frequency, \( A \) is the detector area, and \( Δf \) is the electrical bandwidth. It is expressed in units of \((\text{cm} \ (\text{Hz})^{1/2} \ \text{watt}^{-1})\).

The linearity of a detector can best be specified by determining how well the detector performance matches that of a linear circuit model, and the temperature stability is determined by observing how changes in detector temperature affect the other parameters.

GENERAL DESCRIPTION OF SILICON SOLAR CELL

The silicon solar cell is basically a p-n junction with a large surface area which is designed to convert solar radiation directly into electrical energy with high overall efficiency. The diode chosen as representative of a typical modern solar was the ST101-U made by Processor Technology. The physical construction of this diode is shown in Figure 1.

![Figure 1. ST101-U Solar Cell.](image)
The active area of the cell is 1 cm$^2$. The p region of the cell is on the light sensitive side with the p-n junction approximately 1 um beneath the surface. An epitaxial substrate is used to reduce series resistance which is a thin layer of lightly doped (donor impurities $\sim 10^{15}$ cm$^{-3}$) n type material on the heavily doped ($\sim 10^{19}$ cm$^{-3}$) n material. The light sensitive side of the cell is covered with a thin highly transparent epoxy coating. The experimental procedures described in this report were performed with several samples of this cell with consistent results.

The process by which the p-n silicon solar cell converts light energy into electrical energy may be explained by reference to the energy level diagram of Figure 2.

![Energy level diagram of silicon cell.](image)
A light source whose photons have energy $h\nu$ greater than 1.1 eV produce excess minority carriers in the p type material which, if they are within a diffusion length (average distance minority carriers diffuse before they recombine) of the potential barrier, will be "trapped" by the barrier and caused to flow across it in an attempt to reduce their energy. The excess electrons flow to the right and excess holes to the left. This flow constitutes an electric current, and when external connections are made, converts radiation into an electrical current. The current produced is proportional to the number of photons absorbed, and the voltage depends on the barrier height, which is always less than the energy gap depending on how heavily the p and n regions are doped.

**EQUIVALENT CIRCUIT MODEL**

The manufacturer states that the device has a "thin" epitaxial substrate of lightly doped n type silicon between the heavily doped p and n regions. No information on the actual thickness of this region was available. A diode of this type is referred to as a p-n-n device and postulates an additional series resistance ($R_\nu$) and a capacitance ($C_\nu$) in series with the junction capacitance where

$$R_\nu = \frac{W_\nu}{GA} = \frac{W_\nu}{I_F A} ,$$

and

$$C_\nu \approx \frac{\varepsilon_s}{W_\nu} .$$

where $W_\nu$ is the width of the lightly doped $\nu$ region and $\varepsilon_s$ is the permittivity of silicon. The junction capacitance may be given as

$$C_j = \frac{\varepsilon_s}{W_D} ,$$
where \( W_D \) is the depletion region width. \( W_D \) is proportional to the square root of the total junction potential and \( W_v \) is constant. If \( W_v < W_D \), the series capacitance of \( C_v \) and \( C_j \) reduces to essentially \( C_j \). Experimental data on the diode capacitance as a function of voltage and the I-V characteristics of the ST101-U indicate that the capacitance is proportional to the square root of the junction voltage and that the series resistance is essentially constant. Therefore, the assumption is made that the v region width is small enough that the diode may be modeled as a simple p-n abrupt junction.

The p-n silicon solar cell may be represented by the general equivalent circuit model shown in Figure 3.

\[
I_A \quad I_D \quad I_{sh} \quad I_L
\]

\[
+ \quad V_D \quad R_{sh} \quad C_{sh} \quad R_L
\]

**Figure 3.** General equivalent circuit model of silicon solar cell.
$I_\lambda$ is the photocurrent which is related to the incident radiant power $P_\lambda$ by the equation

$$I_\lambda = \eta q \frac{P_\lambda}{E_\lambda},$$

where $\eta$ is the effective quantum efficiency, $q$ is the electron charge, and $E_\lambda$ is the photon energy at a particular wavelength. The photon energy is given by:

$$E_\lambda = \frac{hc}{\lambda},$$

where $h$ is Planck's constant and $c$ is the velocity of light. If we substitute the values of the physical constants, we get:

$$I_\lambda = 8.06 \times 10^5 \eta \lambda P_\lambda.$$

$I_D$ is the current through the ideal diode, which is given by:

$$I_D = I_0 (e^{\alpha V_D} - 1),$$

where $I_0$ is the diode reverse saturation current, and

$$\alpha = \frac{q}{AKT},$$

where $K$ is the Boltzmann constant, $T$ is the absolute temperature in degrees Kelvin, and $A$ is an experimentally determined constant ranging in value between 1 and 3. When the recombination current dominates, $A = 2$, and $A = 1$ when the diffusion current dominates. If the solar cell contains many defects, $A$ may approach 3. At room temperature, $\alpha^{-1} = 26A$ (in millivolts).

$I_{sh}$ is the current through the parallel combination of the diode shunt resistance ($R_{sh}$) and the shunt capacitance ($C_{sh}$), and $I_L$ is the external load current. The total diode shunt capacitance is equal to the junction capacitance ($C_j$) in parallel with the diffusion capacitance ($C_d$). $C_d$ is proportional to the DC current through the diode:
where \( T_p \) is the hole lifetime in the n region of the cell. For silicon, \( T_p \approx 10^{-7} \) seconds and \( C_d \) is negligible compared to \( C_j \) for values of \( I_{\lambda} \) less than about 1 milliamp.

The junction capacitance may be modeled as:

\[
C_j = \frac{k_a}{(V_{BI} - V_D)^\gamma},
\]

where \( k_a \) is a constant and \( V_{BI} \) is the built-in junction potential. The constant \( \gamma \) is approximately \( 1/2 \) for an abrupt junction and \( 1/3 \) for a linearly graded junction. The equivalent circuit parameters of a number of silicon solar cells were determined experimentally. Detailed experimental procedures and results are given in the appendix. Typical values for the ST101-U solar cell are:

\[
\begin{align*}
R_s &= 5\Omega \\
R_{sh} &= 25\ \text{K}\Omega \\
I_0 &= 1.6\ \mu\text{A} \\
k_a &= 5 \times 10^{-9} \text{FV}^{1/2} \\
V_{BI} &= 0.5\ \text{V} \\
\gamma &= \frac{1}{2}
\end{align*}
\]

These parameters may vary significantly with different types of silicon solar cells, but are easily obtainable using the experimental techniques described in the appendix. The experimental values obtained for \( \alpha \) and \( \gamma \) indicated that recombination current is the dominant current and that the diode may be modeled as an abrupt junction.
LINEARITY

It is instructive to consider two different modes of operation commonly referred to as the photovoltaic mode and the short circuit mode. In the photovoltaic mode, $R_L$ is very large compared to the effective cell shunt resistance, and

$$I_\lambda = I_D + I_{sh}.$$ 

Substituting the equation for $I_D$ and solving for $V_D$, we have:

$$V_D = \frac{1}{\alpha} \log_e \left( \frac{I_\lambda - I_{sh}}{I_o} + 1 \right).$$

If $(I_\lambda - I_{sh})/I_o \gg 1$ and $I_\lambda \gg I_{sh}$, we can write:

$$V_D = \frac{1}{\alpha} \log_e \left( \frac{I_\lambda}{I_o} \right).$$

If we use a small signal approach, we can write:

$$\frac{V_s}{I_{\lambda s}} \approx \frac{\partial}{\partial I_\lambda} V_D = \frac{1}{\alpha} \frac{I}{I_\lambda},$$

where $V_s$ is the output signal voltage of the detector and $I_{\lambda s}$ is the portion of $I_\lambda$ related to the incident optical signal. Here we see that signal voltage gain is inversely related to the total radiant energy incident to the detector which is composed of both the carrier and background illumination. In many applications we would like to operate the detector under conditions of widely varying signal strength and background illumination, which would make this mode of operation undesirable.

If we consider the short circuit mode, in which $R_s + R_L$ is very small compared to the diode effective shunt resistance, then
and the small signal gain

\[ \frac{\partial I_s}{\partial I_\lambda} = 1 \]

This indicates a perfectly linear response with no dependence on signal strength and background illumination which is highly desirable. It should be noted that this is an ideal which is only approached in practice as will be shown later.

**TEMPERATURE STABILITY**

Temperature affects the silicon cell characteristics in two ways. First the diode saturation current depends strongly on temperature. Second, the light generated current, \( I_\lambda \), varies with temperature. The variation in \( I_\lambda \) occurs mainly because the spectral response of the cell is shifted toward the IR region as the temperature is increased. A typical value of a temperature coefficient for the spectral shift has been given as \(-0.1\%/{}^\circ\text{C}\) at a wavelength of 550 nm.

In the short circuit mode of operation, the silicon cell output is equal to \( I_\lambda \) and the device is relatively insensitive to temperature changes. A measure of the temperature sensitivity of the photovoltaic mode may be obtained by assuming the silicon cell output to be \( V_D \) and differentiating with respect to temperature:

\[ \frac{dV_D}{dT} = \frac{1}{\alpha} \left[ \frac{1}{I_\lambda} \frac{dI_\lambda}{dT} - \frac{1}{I_o} \frac{dI_o}{dT} \right] + \frac{V_D}{T} \]

since

\[ \frac{1}{I_\lambda} \frac{dI_\lambda}{dT} \ll \frac{1}{I_o} \frac{dI_o}{dT} \]
\[
\frac{dV_D}{dT} = \frac{V_D}{T} - \frac{1}{\alpha I_o} \frac{dI_o}{dT}
\]

For silicon, assuming room temperature and \( V_D = 0.4V \), typical values are:

\[
\frac{1}{I_o} \frac{dI_o}{dT} \approx 0.08/\degree C
\]

we obtain

\[
\frac{dV_D}{dT} \approx -0.003 \ V/\degree C
\]

which is approximately \(-.8%/\degree C\) or about ten times more sensitive than the short circuit current.

**SPEED OF RESPONSE**

The response speed of a photodiode can be limited either by the effective lifetime of the photoexcited carriers represented by the diffusion capacitance or by depletion layer effects represented by the junction capacitance. In the silicon solar cell, the diffusion capacitance is negligible compared to the junction capacitance for low levels of background illumination. The pulse time constant \( \tau_p \) is then simply

\[
\tau_p = R_T C_j
\]

where \( R_T \) is the total effective diode shunt resistance. The junction capacitance for an abrupt junction is given by:

\[
C_j = \frac{A}{V_T^{\frac{3}{2}}} \left[ \frac{\epsilon q N_n P_p}{N_n + P_p} \right]^{\frac{1}{2}}
\]

where \( V_T \) is the magnitude of the total voltage across the junction, \( \epsilon \) is the dielectric constant, \( N_n \) and \( P_p \) are majority electron and hole concentrations on the n and p sides of the junction, \( q \) is the electron charge, and \( A \) is the junction cross sectional area. This equation is
equivalent to the equation given earlier and shows the functional relationship of the doping concentrations and cross sectional area to the junction capacitance. The junction capacitance may be reduced by the application of a reverse bias to increase the junction voltage thereby increasing the frequency response. However, it will be shown that adequate frequency response can usually be obtained by operating in the short circuit mode with no external bias.

If the diode is operated in the short circuit mode, $R_T$ is effectively $R_s + R_L$. An operational amplifier configured as a current to voltage amplifier as shown in Figure 4 may be used to operate the cell in the short circuit mode.

Figure 4. Operational amplifier for short circuit operation of silicon cell.
The objective of this circuit is to establish a convenient voltage level by passing the cell current through a large resistance while providing a low resistance load to the silicon cell. This is accomplished by using an amplifier with a high voltage gain (typically \(10^4 - 10^8\)) and high input impedance (typically \(10^5 - 10^7 \Omega\)). The high gain assures that the voltage between the input terminals will be very small (typically \(10^{-4} - 10^{-8} \text{ V}\)) and the high input impedance assures that nearly all of the silicon cell current passes through the feedback resistor. If the feedback resistor is \(R_f\) and the open loop gain of the amplifier is \(A\), the amplifier output is given by

\[
V = - I_c R_f
\]

while the load resistance on the silicon cell is

\[
R_L = \frac{R_f}{A}
\]

This means that an operational amplifier with \(A = 10^6\) can convert a silicon cell current of 1 \(\mu\)A into a 1 V signal by using a 1 M\(\Omega\) feedback resistor, and load the silicon cell by 1\(\Omega\). Using the typical experimental values for the ST101-U solar cell previously given with no external bias and assuming \(R_L = 1\Omega\), we get \(R_S + R_L = 6\Omega\), \(C_{j0} = 7 \times 10^{-9} \text{ F}\). In the short circuit mode, \(R_T = R_S + R_L\) and

\[
\tau_{psc} = 4.2 \times 10^{-8} \text{ sec}
\]

or a cutoff frequency of

\[
f_c = 3.75 \text{ MHZ}
\]

However, if we operate the circuit in the photovoltaic mode, \(R_T\) is effectively \(R_{sh} = 25 \text{ K\(\Omega\)}\) which gives

\[
\tau_p = 1.75 \times 10^{-4} \text{ sec}
\]
or

\[ f_c = 909 \text{ HZ} \]

We see a tremendous difference in the frequency response of the short circuit mode as compared to the photovoltaic mode since the short circuit mode frequency response is essentially limited only by the cell series resistance and the load resistance for small values of \( I_\lambda \).

**PRACTICAL CONSIDERATION**

It has been shown that the short circuit mode of operation offers many advantages. However, the assumptions made in deriving the short circuit mode results are only approximations and the complete equivalent circuit given in Figure 2 should be used to test the validity of the short circuit results for possible ranges of \( I_\lambda \) and \( R_L \). In particular, \( V_D \) must be kept well below the built in junction potential and \( I_\lambda \) must be kept below the point where the diffusion capacitance becomes significant. The following equation for the diode voltage is derived from the general equivalent circuit with \( R_X \) being the total shunt resistance:

\[
V_D = R_X \left[ I_\lambda - I_D \right] = R_X \left[ I_\lambda - I_o (e^{\alpha V_D} - 1) \right]
\]

where

\[
R_X = \frac{(R_S + R_L) R_{sh}}{(R_S + R_L + R_{sh})}
\]

This is a transcendental equation which can be solved iteratively by Newton's method. Once \( V_D \) is determined:

\[
I_L = \frac{V_D}{R_S + R_L}
\]

and the effective diode resistance \( (r_d) \) is given by:
The cutoff frequency is then determined by

\[ f_c^{-1} = 2\pi (C_j + C_D) \frac{(r_d R_X)}{(r_d + R_X)} \]

The cutoff frequency vs. \( I_\lambda \) as plotted for various values of load resistance using the experimentally determined parameters in the general model is shown in Figure 5.

Figure 5. Cutoff frequency vs. \( I_\lambda \) for ST101-U solar cell.

For values of load resistance less than about 10\( \Omega \), the frequency response is essentially flat and corresponds to the predicted results of the ideal short circuit model until the photocurrent forward biases the diode to a value approaching the built-in junction potential. At this point, the junction capacitance becomes very high and the diffusion capacitance becomes significant which causes the frequency response to drop off rapidly. For higher values of load resistance, the
frequency response drops slightly at first due to the increasing junction capacitance as the diode voltage rises, then starts to increase as the diode resistance starts to drop with increasing self bias until the built-in potential is approached where the junction capacitance and diffusion capacitance again rise very rapidly.

It should be noted that the frequency response shown in Figure 5 is only the frequency response determined by the RC components of the model. There is an additional variation in the overall signal gain due the current shunting effect of the diode as it becomes forward biased. Figure 6 shows a plot of load current vs. photocurrent for a range of load resistances.

Figure 6. Load current vs. photocurrent for ST101-U solar cell.
The slope of the $I_L$ vs. $I_\lambda$ curve corresponds to the small signal current gain $A_I$. $A_I \approx 1$ until the photocurrent forward biases the diode to the point where the diode current becomes significant. When we consider the results of Figure 5 and Figure 6, we see that the ideal short circuit model is valid for a load resistance less than $10\Omega$ as long as $I_\lambda$ is less than about 1 milliamps. It will be shown later that this value of $I_\lambda$ corresponds to about 4 milliwatts of incident radiant energy at spectral peak. It may be necessary to use a filter to keep $I_\lambda$ below this value when the detector will be subjected to strong background illumination.

SPECTRAL RESPONSE

The silicon cell is sensitive to radiant energy with wavelengths between approximately 350 nm and 1200 nm. The response at the long wavelengths is limited by the band gap of silicon since radiation at long wavelengths does not have sufficient energy to break a valence bond in silicon. Short wavelength photons are absorbed near the surface of the cell and the photon generated electron-hole pairs recombine before reaching the p-n junction. A shallower junction depth would increase the short wavelength sensitivity, however, this would also increase the series resistance of the cell. The spectral response of the ST101-U solar cell is shown in Figure 7.

This spectral response is flatter than typical response curves for silicon shown in other sources and the absolute response at spectral peak of 0.25 amp/watt is about half of what is typically given. I suspect this is due to the clear epoxy protective coating on the front surface of the cell. The spectral response curve of Figure 7 can
be used to predict the short circuit output current of the ST101-U solar cell for any illuminant whose spectral distribution is known.

![Spectral response of ST101-U solar cell.](image)

**Figure 7.** Spectral response of ST101-U solar cell.

**NOISE CHARACTERISTICS**

The signal-to-noise ratio and minimum detectable signal are the major criteria by which the sensitivity of detector systems to weak optical signals is judged. In any practical detector system, the noise characteristics of both the detector and amplifier must be considered. The mean square signal power at the output terminal of a photodetector is given as

\[ S = \frac{1}{2} \left( \langle m I_{ph} \rangle \right)^2 \left| H(\omega) \right|^2, \]
where $m$ is the modulation index, $I_{ph}$ is the signal generated photocurrent, and $M(\omega)$ is the internal detector current gain (Melchior 1970). The mean square noise power at the output terminal is given as

$$N = B \left[ 2q (I_{ph} + I_B + I_D) M(\omega) \left| \frac{2}{F(M)} \right| \frac{4}{R} \right],$$

where $B$ is the electrical bandwidth of the detector system, and $I_B + I_D$ are the background radiation induced photocurrent and dark current. In this equation, the mean square shot noise is represented by

$$\overline{i_s^2} = 2q (I_{ph} + I_B + I_D),$$

and the effective mean square thermal noise current is represented by

$$\overline{i_t^2} = \frac{4}{R}.$$

$R$ is the equivalent resistance of the diode and output circuit and the factor $F(M)$ accounts for the increase in noise that is produced by the current gain. $F = 2$ for unbiased photodiodes at low frequencies because the magnitude of the radiation induced generation-recombination noise is twice as large as the shot noise.

An operational amplifier is used to operate the diode in the short circuit mode, as was described earlier. The mean square voltage noise at the output of the amplifier, due to internal amplifier noise, may be described by

$$\overline{E_o^2} = \int_{f_1}^{f_2} \left( G_e + G_i \left| Z_I \right|^2 \right) df,$$
where $G_e$ is the input noise voltage power spectral density and $G_i$ is the input noise current power spectral density (Wait 1975). $Z_i$ is the Thevenin equivalent impedance in the inverting terminal branch. If the solar cell presents an input impedance to the amplifier of $R_c$, then
\[ Z_i = \frac{R_c R_F}{R_c + R_F} . \]

The noise gain ($A_{n}$) is given by
\[ A_{n} = \frac{A}{1 + \beta} , \]

where $A$ is the open loop voltage gain, and
\[ \beta = Z_i / R_F . \]

Manufacturer data sheets for operational amplifiers give plots of $G_e$ and $G_i$.

The assumption that the detector system is either detector noise limited or amplifier noise limited cannot be arbitrarily made as is done by many authors. The following example of a typical detector system will illustrate this point. The values for the detector parameters are assumed to be the values given previously for the typical ST101-U solar cell, and a Fairchild UA741 operational amplifier is used in the short circuit mode amplifier stage. The output of the amplifier is coupled through a narrow band filter with a 6 HZ bandwidth. In addition, the following conditions are assumed:

\[ f_s = 10 \text{ KHZ} , \]
\[ m = 1 , \]
\[ M(\omega) = 1 , \]
\[ I_{ph} = 10^{-6} \text{ A} , \]
\[ I_B + I_D = 10^{-3} \text{A} , \]
\[ R_F = 1 \text{K}\Omega . \]

The amplifier induced output noise voltage will be calculated first. Using the short circuit assumption, the diode resistance is found to be:

\[ R_d = \left. \frac{d V_D}{d I_D} \right|_{V_D = 0} = \frac{1}{\alpha I_o} \approx 37 \text{K}\Omega . \]

Neglecting the series resistance, the cell resistance is \( R_d \) in parallel with \( R_{sh} \), so:

\[ R_c = 15 \text{K}\Omega . \]

Calculations of the other parameters give:

\[ A_n \approx 1.0 \]
\[ Z_I \approx 940 \text{\Omega}. \]

from the UA741 data sheet (given in the appendix) at 10 KHZ with a bandwidth of 6 HZ:

\[ G_e = 6 \times 10^{-16} \text{V}^2/\text{HZ} , \]
\[ G_i = 4 \times 10^{-25} \text{A}^2/\text{HZ} . \]

Calculation of \( E_0^2 \) with these values gives:

\[ E_0^2 \approx 3.6 \times 10^{-15} \text{V}^2 . \]

A calculation of the mean square current noise from the detector at the given background illumination and at room temperature yields:

\[ N_D \approx 2 \times 10^{-21} \text{A}^2 . \]

This is amplified to give a mean square voltage noise at the amplifier output of:

\[ E_{OD}^2 \approx 1.9 \times 10^{-15} \text{V}^2 . \]
The total mean square voltage noise at the amplifier output due to both detector and amplifier noise is therefore:

\[ E_{ON}^2 = 5.5 \times 10^{-15} \text{ V}^2 \]

The mean square signal voltage referred to the amplifier output due to an incident signal power of \( P_{S\lambda} \) is

\[ E_{OS}^2 = \frac{1}{2} (m P_{S\lambda} R_{I\lambda})^2 \left| M(\omega) \right|^2 R_F^2 \]

NEP_\lambda is then given by:

\[ \text{NEP}_\lambda = \sqrt{2} \frac{E_{ON}}{m \left| M(\omega) \right| R_F R_{I\lambda}} \]

If we assume the current spectral responsivity to be \( R_{I\lambda} = 0.25 \), and use the values in the example:

\[ \text{NEP}_\lambda \approx 4.2 \times 10^{-10} \text{ watts} \]

We see that the amplifier noise, as well as the detector noise, must be considered in determining the total noise of the system and that detailed information including expected background illumination and amplifier characteristics must be used to accurately evaluate the performance of the detector system.

**CONCLUSION**

The silicon solar cell in the unbiased short circuit mode has been shown to have characteristics which make it suitable for many optical detector applications in the 350 nm to 1200 nm range. The silicon solar cell operated in the short circuit mode has excellent temperature stability and has a linear response for a wide range of illumination intensities. The frequency response of a typical solar cell has been shown to be in the MHz range, which makes it suitable
for application in low to medium data-rate systems. Unfortunately, little information is available from manufacturers regarding detector performance of their solar cells since they were originally developed solely for energy conversion, but the procedures outlined in this report may be used to determine the detector characteristics of most solar cells. In addition to being economical, the solar cell is available in a wide range of shapes and sizes and has many potential applications as a single or multi-cell array in systems requiring large detector areas.
APPENDIX A: DIODE PARAMETER MEASUREMENTS

The I-V characteristics of the solar cells are determined by using a HP576 transistor curve tracer. The cells are masked from any illumination during all parameter measurements to insure that only the electrical characteristics of the diode are determined. The series resistance and shunt resistance are measured by taking the slope of the I-V curve at high forward and reverse biases. Data are taken from several points on the curve and corrected for series and shunt resistance to obtain the true I-V characteristic of the diode. The corrected diode voltage and current are given by:

\[ V_D = V - IR_s, \]

and

\[ I_D = I - \frac{V}{R_{sh}}. \]

An exponential curve fit is used on the corrected data points to obtain the diode current equation:

\[ I_D = I_o \left( e^{V_D} - 1 \right). \]

The curve fit must be used iteratively to determine \( I_o \). A typical result for an ST101-U solar cell is:

\[ R_s = 6 \, \Omega, \]
\[ R_{sh} = 25 \, K\Omega, \]
\[ I_D = 1.63 \times 10^{-6} \left( e^{16.7 \, V_D} - 1 \right), \]
\[ r^2 = 0.997. \]

The correlation coefficient \( r^2 \) of 0.997 indicates a very good curve
The circuit used to measure the capacitance of the silicon cell is shown in Figure A-1.

**Figure A-1. Capacitance measurement circuit**

$R_p$ is chosen so that $R_s \ll R_p \ll R_{sh}$. $R_c$ and $C_c$ are chosen to provide coupling and to keep the peak value of the square wave signal small.

When $R_p$ is chosen to meet the above condition, the pulse time constant observed by monitoring $V_D$ on an oscilloscope can be approximated by:

$$\tau_p \approx R_p C_c$$

The total diode capacitance can be represented by a junction capacitance proportional to the square root of the voltage (abrupt junction
assumption) in series with a capacitance due to the intrinsic layer in the p-v-n diode. The diffusion capacitance, which is proportional to the diode DC current, may be assumed to be negligible. Capacitance measurements of the cells were taken for several values of reverse bias and power curve fits were made on the recorded data. The curve fits indicated that the capacitance can be fitted to the equation:

$$C = \frac{k_a}{(V_{BI} - V_D)^{1/2}}$$

This indicates that the diode can be modeled as an abrupt junction and that the series intrinsic capacitance may be neglected. Therefore, the total junction capacitance may be assumed to be effectively the junction capacitance $C_j$. The constant $V_{BI}$ represents the built in junction potential of the cell. A linear regression analysis of $1/C^2$ was performed on the capacitance data of a number of cells with consistent results. A typical result is:

$$\frac{1}{C_j^2} = 1.81 \times 10^{16} - 3.90 \times 10^{16} V_D,$$

$$r^2 = 0.998.$$ 

This corresponds to:

$$k_a = 5.06 \times 10^{-9} FV^{1/2},$$

$$V_{BI} = 0.46 V.$$
APPENDIX B: SPECTRAL RESPONSIVITY MEASUREMENTS

The set up shown in Figure B-1 was used to provide a calibrated source and modulated optical signal for use in the spectral responsivity measurements.

![Figure B-1. Calibrated source and chopper.](image)

The source was calibrated with a UDT-40X photometer for 500 nm < \( \lambda \) < 1000 nm, with the photometer detector flush against the output part of the spectrometer.

In order to measure the current spectral responsivity of the silicon solar cell, the set up shown in Figure B-2 was used.

![Figure B-2. Spectral responsivity measurement.](image)
The solar cell is placed in the same physical location that the photometer detector was and is shielded from ambient light by a hood. A current to voltage amplifier (which is described in the main body of the paper) provides a signal to the HP302 wave analyzer. The wave analyzer has a built in rms voltmeter, so the diode signal current may be read directly. The oscilloscope is used to monitor the output of the wave analyzer to insure that spurious signals are not included in the reading. The current to voltage amplifier is a Fairchild UA741 operational amplifier with a 100 KΩ feedback resistor. At the 200 Hz chopping frequency, the UA741 has an input impedance of 6 MΩ with an open loop voltage gain of $10^4$ at a supply voltage of 15 V. The operational amplifier in this configuration will load the cell by approximately 10 Ω and provide a current to voltage gain of 100,000. Detector current output measurements were made on several ST101-U solar cells at several wavelengths with consistent results. Since the incident signal power was known at each wavelength measured, the absolute current spectral responsivity curve shown in the main body of this paper could be determined.
APPENDIX C: FAIRCHILD DATA SHEET

TYPICAL PERFORMANCE CURVES FOR µA741A, µA741, µA741E AND µA741C
LIST OF REFERENCES


