The Theory and Application of Frequency Domain Deconvolution Using Optimum Compensation in Time Domain Measurement

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THE THEORY AND APPLICATION OF FREQUENCY
DOMAIN DECONVOLUTION USING OPTIMUM
COMPENSATION IN TIME DOMAIN MEASUREMENT

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ABSTRACT

This thesis describes a method of transforming a deconvolution problem into an equivalent convolution problem. A short discussion of the theory behind convolution and deconvolution is included, along with a survey of techniques now being used. A new method using a synthesized compensator is proposed and mathematically developed. Finally, the results of applying this method to several real and analytical signals are given.

Approved by: [Signature]
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CHAPTER 1

INTRODUCTION

This thesis describes the synthesis and software implementation of an optimized compensator in the frequency domain. This compensator is intended to perform the frequency domain equivalent of deconvolution in systems where classical methods are rendered untenable.

This thesis is divided into six chapters. A brief discussion of the theory behind frequency domain deconvolution and a survey of methods presently available to perform such a deconvolution form Chapter 2. The proposed technique is synthesized and mathematically justified in Chapter 3. Chapter 4 contains a description of an experiment which was performed using a PDP 11/34 computer to operationally verify the compensator algorithm. A hardware based experiment is described in Chapter 5 in which the optimum compensator technique was used to reconstruct the input waveform to a Tektronix S-1 oscilloscope sampling head. Chapter 6 discusses an experiment which was carried out at the Kennedy Space Center Laser Lab in which the optimum compensator method was used to extract the impulse response of an optical fiber. Finally, Chapter 7 discusses conclusions which may be drawn from the results of the three
experiments and presents suggestions for possible future research into the optimum compensator technique. Software support for the experiments described in Chapters 4 and 5 may be found in appendices 1, 2 and 3.
CHAPTER 2

THE DECONVOLUTION PROBLEM

This chapter presents the deconvolution problem as encountered in physical systems, as well as several techniques which have been previously used to solve this problem. The difficulties inherent in each of these techniques will be discussed and a new approach will be introduced.

THE PHYSICAL DECONVOLUTION PROBLEM

Deconvolution is often encountered in time and frequency domain measurement techniques as the process of separating the effect of a non-ideal test signal from the detected output of a linear system to yield the system impulse response and transfer function. In some cases, where the system transfer function is known, deconvolution may be used to gain information about the exciting input.

A pictorial representation of typical measurement technique for determining a system impulse response or transfer function is given in figure 2-1.
In this technique an input $x(t)$ is impressed upon the system, resulting in an output $y(t)$. The relation between these time functions and the system impulse response, $h(t)$ is given by

$$y(t) = x(t) * h(t)$$  \hspace{1cm} (2-1)$$

where $*$ denotes the convolution operation. This operation is defined by

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$  \hspace{1cm} (2-2)$$

In the frequency domain the above relation reduces to

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$  \hspace{1cm} (2-3)$$

where $Y(e^{j\omega})$, $X(e^{j\omega})$, and $H(e^{j\omega})$ are Fourier transforms of $y(t)$, $x(t)$, and $h(t)$, respectively. The Fourier transform itself is defined by

$$Y(e^{j\omega}) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t} dt$$  \hspace{1cm} (2-4)$$

The convolution process implies that $x(t)$, the input function, and $h(t)$, the system impulse response, are both known, and $y(t)$ is being sought. Unfortunately, this is seldom the case.
Occasionally, \( y(t) \) and \( h(t) \) are known and \( x(t) \) is being sought, as in the case of measuring instruments. More often \( x(t) \) and \( y(t) \) are given and the objective is to find \( h(t) \), as in the case of system or device characterization. The solution to either of these problems can be found in the operation which is the inverse of convolution, the so called "deconvolution". Furthermore, both of these problems are mathematically equivalent, with only a change of variables necessary to transform one problem statement into the other.

Assuming \( h(t) \) is being sought, the simplest way of deriving it is implied by equation 2-3. A harmonic input, \( X(e^{j\omega}) \) is applied to the system and the complex output, \( Y(e^{j\omega}) \), is measured for various frequencies. The ratio of these two quantities, \( Y(e^{j\omega}) / X(e^{j\omega}) \) gives \( H(e^{j\omega}) \). The inverse Fourier transform

\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(e^{j\omega}) \cdot e^{j\omega t} \, dw
\]

may then be applied to get \( h(t) \).

This method suffers from several serious drawbacks. To properly utilize the inverse Fourier transform, the bandwidth over which the measurements are taken must be as large as possible. This makes the process quite time consuming and usually necessitates the use of several different signal generators in the test setup to achieve the required bandwidth. This constant changing of the test setup makes experimental errors hard to
avoid. Further, at high frequencies erroneous data may be produced through mismatches in the test setup, connector and cable effects, and difficulties in measuring the relative phase of the input and output signals.

Another commonly used technique involves the use of an impulsive input for \( x(t) \). In the ideal case it can easily be shown that using a Dirac delta function characterized by

\[
\lim_{\Delta \to 0} U_0(t) = 0 \quad |t| > \Delta
\]

\[
\lim_{\Delta \to 0} \int_{t-\Delta}^{t+\Delta} U_0(t)dt = 1
\]

in the Fourier transform equation 2-4 yields a frequency domain function of unity. This may be interpreted as meaning that this function contains all frequencies at unit strength. In this case the output of the system is actually \( h(t) \).

The main problem in using the time domain technique lies in the impossibility of producing a true Dirac delta function which has infinite amplitude, zero transition duration and zero duration. Fortunately, finite amplitude pulses with non-zero transition durations are still quite rich in harmonics, and are relatively easy to produce with short enough duration to approximate the delta function for most applications. Due to thermal and high frequency circuit effects, successive pulse occurrences tend to fluctuate in time, an effect known as jitter. This can be overcome by using signal averaging techniques.
The most appealing feature of the time domain technique is the speed with which data may be obtained, since there is no need for repetitive measurements over various frequencies. For these reasons the impulse response technique is more attractive than the harmonic input method in many cases.

In cases where the available input signal is not a good approximation of the delta function for the system under study, problems can arise. The next section will define numerical deconvolution, the discrete form of deconvolution used on sampled data systems, and will investigate some of the problems which may arise from its use.

**NUMERICAL DECONVOLUTION AND INSTABILITIES**

Data for real systems is seldom found in the continuous form which is implied by equation 2-2. Instead, the functions involved are usually manipulated as waveforms which have been sampled over discrete intervals of time and then stored as numerical arrays of values. This gives rise to the operations of numerical deconvolution. In the time domain, the discretized counterpart to equation 2-2 is

\[
y(k) = \sum_{n=-\infty}^{\infty} h(k-n)x(n)
\]

This may also be stated as a matrix equation

\[
y = h \, x
\]
where $y$ and $x$ are column vectors and $h$ is the matrix
\[
\begin{bmatrix}
    h(k) \\
h(2k) & h(k) \\
h(3k) & h(2k) & h(k) \\
    \vdots \\
h(nk) & \cdots & \cdots & h(2k) & h(k)
\end{bmatrix}
\]
[Ekstrom, 1969]. The frequency domain representation is of similar form to equation 2-3
\[
Y = X \cdot H
\]
except that $Y$, $X$, and $H$, are all vectors and the multiplication is performed element by element. Thus $Y(1) = X(1) \cdot H(1)$, $Y(2) = X(2) \cdot H(2)$, etc. Deconvolution in the time domain involves multiplying both sides of equation 2-9 by the appropriate inverse matrix or vector. In the frequency domain an element by element division is used.

If the measurements involved were pure mathematical entities no problem would exist. However, the presence of noise in the system causes grave difficulties. As Hunt [1972] clearly indicates, the matrix $h$ of equation 2-9 "is very nearly singular and ill conditioned." Indeed, he goes on to say that sampling the $h$ function more closely causes the problem to increase, as $h$ becomes "increasingly more ill-conditioned ...." Ekstrom [1973] further indicates that "small perturbations in $[y(k)]$ give rise to large perturbations in $[x(n)]"$. See equation 2-8. The
transform equation 2-9 runs into the same sort of problems, since transforming to the frequency domain does not change the stability of the system. In addition, if the time domain window over which the samples are taken does not contain an integral number of cycles, the fast Fourier transform (FFT), which is often used to perform the transformation into the frequency domain, may generate false frequency components due to the process known as leakage [Bertram, 1970].

These difficulties are serious but not insoluble. The next section will survey several methods which have been used with some degree of success to perform the deconvolution in physical problems. A new method is also proposed which appears to be potentially useful in a wide variety of cases.

SURVEY OF DECONVOLUTION METHODS

From the above discussion it is apparent that noise is the major cause of problems in physical deconvolution. Techniques which have been used to minimize the effect of noise in deconvolution include filtration, iterative predictor-corrector methods, and the use of optimal compensation.

The term filtration, as it is used here, will include any process which performs some arbitrary weighting of the frequency domain data. Techniques which fall under this classification include bandlimiting, sliding averages, and the use of window functions.
Perhaps because bandlimiting is the easiest process to perform, it has been used extensively on the deconvolution problem. This is often accomplished by the use of digital filtering techniques, but pure bandlimiting by simply truncating frequency components in the FFT of the signal is also possible. Since deconvolution represents an anti-integration operation, rapid discontinuities are enhanced by the operation. Noise fits this criterion and usually transforms as the upper frequency components of the signal. Bandlimiting eliminates these components and significantly reduces the noise problem. Examples of the use of this technique may be found in Papoulis [1975] and the work of Cohen-Sfetcu [1975], among others.

Another signal modification method which is a form of filtering is the method of sliding averages. This method seeks to smooth the time domain signal so that the FFT does not 'see' the noise. Gans [1972, p.19] describes a scheme in which the averaged value over a number of adjacent points on either side of the discontinuity is used only if the magnitude of the discontinuity exceeds a pre-defined limit. It is important to note that here we are talking about time averaging rather than ensemble averaging, a procedure which averages the waveform of interest over many samples of the entire waveform and which reduces uncertainty caused by jitter.

Both of the above filtering schemes seek to minimize noise found in the time domain, but windowing is an attempt to reduce
noise generated by the use of the FFT on signals with certain properties. Since the FFT assumes a periodic signal, errors will occur if the beginning and the end of the time domain waveform do not have exactly the same slope and amplitude. Failure to meet this condition causes the spectral components to 'leak' from their expected positions.

The windowing technique applies some weighting function to the time domain signal to assure that it meets the above stated condition. These functions may take various forms but in general they emphasize the middle of the time sample with respect to the two ends [Bertram, 1970]. A particularly elementary example which involves truncation of the time waveform outside of certain limits is described by Mersereau [1978].

Although the above filtering techniques are often used in combination with other deconvolution schemes, each one suffers serious drawbacks if used alone. For example, although some form of bandlimiting in sampled data systems is mandated by Shannon's Sampling Theorem which states that the highest frequency encountered in the signal may be no higher than one-half of the sampling frequency to eliminate the problem of aliasing, reduction in signal frequency below this point not only results in reduced resolution in the frequency domain [Mersereau, 1978] but may eliminate useful information as well. At this point it should be noted that, due to the properties of the FFT, the bandwidth of the transformed signal is
\[ \Delta F = \frac{1}{(2\Delta t)} \]

where

\[ \Delta t = \frac{T}{N} \]

for an N point sample over time T. [Andrews, 1977, p.78]

Similarly, signal averaging over time causes the loss of information about natural transients in the time domain signal. Finally, windowing suffers from the fact that the FFT returns the convolution of the window with the windowed function. Bertram [1970] notes that this may "accentuate the effect of the errors in the middle of the time sample." Andrews [1975, p.42] emphasizes that "the resulting spectrum must be in error to the extent that the weighting function distorted the original waveform data."

The inadequacy of filtering methods for deconvolution has led to extensive investigation into iterative techniques. In general these algorithms utilize some a priori information about the system of interest to make an initial guess for the waveform being sought. Subsequent iterations are used to modify this guess in the direction of the true waveform. An implementation of this technique when one of the known waveforms is of infinite duration and some finite portion is known has been proposed by Pearson [1973]. Papoulis [1975] addresses the situation where the signal of interest is bandlimited but only a portion of it is known. Data with a high signal to noise ratio has been deconvolved suc-
cessfully using an algorithm by Goutte [1977] and a process by Mersereau [1978] may be utilized if the unknown signal may be modelled by an approximation which is bandlimited, time limited, and positive.

In general, iterative methods suffer from at least one of two major faults. In some cases the a priori information or the constraints which must be put on the solution restrict the utility of the method to a small class of problems. In others the convergence of the solution is strongly dependent upon the waveform used for system excitation or convergence is so slow that finite word length errors in the computation produce a solution of limited viability. Thus, Papoulis [1975] mentions limiting the number of iterations to control noise effects, and Pearson [1973] speaks of "ad hoc methods for handling large amounts of noise."

The final technique which will be discussed here is the method of optimal compensation. In this technique a compensating system is postulated which possesses a transfer function of approximately $H^{-1}(e^{j\omega})$. Utilizing some a priori constraint or constraints upon this system, an error function is synthesized. This function is then minimized mathematically. If the constraint(s) are well posed the resulting function specifies the optimal compensating system. Thus, in essence this technique converts the deconvolution problem into one of convolution which is an integral, and therefore smoothing, process. Ekstrom [1969] presents an example of this process where the minimization is performed entirely
in the time domain using matrix derivatives.

This technique is a poor one when performed in the time do-
main, since good resolution demands the utilization and subsequent
inversion of a very large matrix. This causes problems with com-
puter system storage and with the time needed to perform this pro-
cess on even a moderately fast machine. Furthermore, if the large
matrix is decomposed into smaller sub-matrices to aid in the in-
version process, problems with singularities may still be encount-
ered.

Optimum compensation retains many attractive features in
spite of these drawbacks, however. A high degree of selective
control over the final output function is available through the
intelligent selection of constraint weighting, and yet no overt
changes in the input data are performed. Thus, this technique
avoids some of the pitfalls inherent in iterative and filtering
techniques. Finally, if all functions of interest are transformed
to the frequency domain before the error function is synthesized
the need for an N x N point matrix is obviated and an N point
vector may be used to contain the necessary data.

This is the technique which is mathematically developed in
the next chapter. Besides the advantages already noted, it offers
the benefits of being quite straight-forward conceptually, and be-
ing very easy to test for analytic functions. Further, the par-
ticular implementation studied presents only one constraint, and
this constraint is easily fulfilled by most signals of interest.
CHAPTER 3

THE OPTIMAL COMPENSATOR IN THE FREQUENCY DOMAIN

In the search for the optimal compensator in the frequency domain, it would seem that any parametric statement of the problem must include some criterion of optimality, as well as some constraints upon the solution with a view towards realizability. Finally, a compensator producing algorithm must be generated and optimized. In this chapter these requirements are fulfilled as the proposed compensator technique is mathematically developed. Also, a short discussion concerning the choice of a good initial guess for the constraint parameter is presented.

THE PROBLEM STATEMENT

The task at hand is to reduce the process of deconvolution to a process of convolution. Since convolution is an integral process this should significantly reduce the effects of noise produced random discontinuities in the system function. A diagram of the process described is given in figure 3-1.
Fig. 3-1. The optimum compensation process

Given a known system transfer function \( H(e^{j\omega}) \) which may have no inverse by virtue of its high noise content, we seek to synthesize some optimum compensating system, \( D(e^{j\omega}) \). Upon excitation by the output of the known system, \( Y(e^{j\omega}) \), this compensator must produce an output \( X'(e^{j\omega}) \) which is in some sense a close approximation to the original system input, \( X(e^{j\omega}) \). In other words, the system \( D(e^{j\omega}) \) may be thought of as a deconvolver for \( H(e^{j\omega}) \). Of course, if \( H(e^{j\omega}) \) possesses an inverse \( H^{-1}(e^{j\omega}) \), then the optimum compensator \( D(e^{j\omega}) \) is just equal to \( H^{-1}(e^{j\omega}) \). This fact will be used later to help test the algorithm.

In order to optimize the compensator, a "goodness of fit" function must be described. Here we use the integral square error criterion. Thus, we define

\[
E = \int_{0}^{\text{BW}} |X'(e^{j\omega}) - X(e^{j\omega})|^{2} d\omega
\]

where \( \text{BW} \) stands for the bandwidth of the system under study and \( E \) is the error function. From Parseval's Theorem [Tranter, 1976] it is clear that \( E \) is proportional to the energy difference between \( X(e^{j\omega}) \) and \( X'(e^{j\omega}) \). To help insure realizability we constrain
the compensator to be a finite energy function where the energy $Q$, is indicated by

$$ Q = \int_0^{BW} |D(e^{j\omega})|^2 d\omega $$

and $Q < \infty$. This is the so-called stability condition.

From the stability constraint it is further apparent for a real, finite input signal $X(e^{j\omega})$ that the operation $X(e^{j\omega}) \cdot D(e^{j\omega})$ yields finite results. We therefore define

$$ Q^* = \int_0^{BW} |X(e^{j\omega}) \cdot D(e^{j\omega})|^2 d\omega $$

Since this leads to no loss of generality, the finiteness of $Q^*$ will be used as our constraint condition.

The problem may then be stated as follows. Defining the Lagrange multiplier equation

$$ J = E + \lambda Q^* $$

the task is to minimize $E$, the error energy, under the $\lambda$ weighted stability condition $Q^*$. 
THE ALGORITHM

Expanding equation 3-4

\[ J = \int_{0}^{BW} \left\{ \left| X(e^{j\omega}) - X(e^{j\omega}) \right|^2 + \lambda \left| X(e^{j\omega}) \cdot D(e^{j\omega}) \right|^2 \right\} d\omega \] 3-5

From figure 3-1 it is seen that

\[ X(e^{j\omega}) = Y(e^{j\omega}) \cdot D(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega}) \cdot D(e^{j\omega}) \] 3-6

Thus

\[ J = \int_{0}^{BW} \left\{ \left| X(e^{j\omega}) \cdot H(e^{j\omega}) \cdot D(e^{j\omega}) - X(e^{j\omega}) \right|^2 + \lambda \left| X(e^{j\omega}) \cdot D(e^{j\omega}) \right|^2 \right\} d\omega \] 3-7

Now, from complex variable theory we know that

\[ |AB|^2 = (AB) \cdot (AB)^* = (AB) \cdot (A^*B^*) = AA^* \cdot BB^* = |A|^2 \cdot |B|^2 \] 3-8

where * denotes the complex conjugate.

Thus, we may factor

\[ J = \int_{0}^{BW} \left\{ \left| X(e^{j\omega}) \right|^2 \cdot \left| H(e^{j\omega}) \cdot D(e^{j\omega}) - 1 \right|^2 + \lambda \left| D(e^{j\omega}) \right|^2 \right\} d\omega \] 3-9

But since

\[ H(e^{j\omega}) = H_R(\omega) + jH_I(\omega) \] 3-10

and

\[ D(e^{j\omega}) = D_R(\omega) + jD_I(\omega) \] 3-11
then we may expand to obtain
\[
J = \int_0^{BW} \left| X(e^{j\omega}) \right|^2 \left[ H_R(\omega) D_R(\omega) - H_I(\omega) D_I(\omega) - 1 \right]^2 d\omega \quad 3-12
\]
\[
+ \int_0^{BW} \left| X(e^{j\omega}) \right|^2 \left[ H_R(\omega) D_I(\omega) + H_I(\omega) D_R(\omega) \right]^2 + D_R^2(\omega) + D_I^2(\omega) ] d\omega
\]

Under further expansion, and after cancelling like terms, this yields
\[
J = \int_0^{BW} \left| X(e^{j\omega}) \right|^2 D_R^2(\omega) [H_R^2(\omega) + H_I^2(\omega) + \lambda] d\omega \quad 3-13
\]
\[
+ \int_0^{BW} \left| X(e^{j\omega}) \right|^2 D_I^2(\omega) [H_R^2(\omega) + H_I^2(\omega) + \lambda] + 2H_I(\omega) D_I(\omega) - 2H_R(\omega) D_R(\omega) - 1 \right] d\omega
\]

From the calculus of variations, to minimize $J$ over all $D_R$ and all $D_I$ set
\[
\frac{\partial J}{\partial D_R} = \int_0^{BW} \left| X(e^{j\omega}) \right|^2 \left( 2D_R(\omega) [H_R^2(\omega) + H_I^2(\omega) + \lambda] - 2H_R(\omega) \right) d\omega = 0 \quad (3-14)
\]

and
\[
\frac{\partial J}{\partial D_I} = \int_0^{BW} \left| X(e^{j\omega}) \right|^2 \left( 2D_I(\omega) [H_R^2(\omega) + H_I^2(\omega) + \lambda] + 2H_I(\omega) \right) d\omega = 0 \quad (3-15)
\]
Since the right hand sides of both equations are equal to zero we may multiply equation 3-14 by one-half and equation 3-15 by one-half \( j \) and sum both equations to yield

\[
\int_{0}^{BW} |X(e^{j\omega})|^2 \left[ (D_{R}(\omega)+jD_{I}(\omega))(H_{R}(\omega)^2+H_{I}(\omega)^2+\lambda) \right] d\omega = 0
\]

But, from complex variable theory

\[
H_{R}^2(\omega)+H_{I}^2(\omega) = |H(e^{j\omega})|^2
\]

and

\[
H_{R}(\omega)-jH_{I}(\omega) = H^*(e^{j\omega})
\]

so utilizing the equality stated in equation 3-11, equation 3-16 reduces to

\[
\int_{0}^{BW} |X(e^{j\omega})|^2 \left[ D(e^{j\omega}) \left[ |H(e^{j\omega})|^2+\lambda \right] - H^*(e^{j\omega}) \right] d\omega = 0
\]

Since this must hold for any \( X(e^{j\omega}) \)

\[
D(e^{j\omega}) = \frac{H^*(e^{j\omega})}{|H(e^{j\omega})|^2+\lambda}
\]
This is the defining function for $D(e^{j\omega})$.

Some motivation for a good choice of $\lambda$ may be gained from equation 3-4. Under the minimization procedures used above, $J$ becomes small, and in the limit

$$\lim_{J \to 0} \lambda = \frac{E}{Q^{-}} \quad 3-22$$

Thus $\lambda$ should approximately represent the ratio of the error energy to the compensator response energy. It should be noted that, due to the physical nature of the quantities involved, $\tau$ must always remain positive. Further

if $\lambda = 0$

then $D(e^{j\omega}) = H^{-1}(e^{j\omega}) \quad 3-23$

As previously stated, this is the result expected in the case where no noise is present e.g. analytic functions.
CHAPTER 4

THE IDEALIZED TRANSFER FUNCTION EXPERIMENT

In the previous chapter the optimum compensator was derived and mathematically justified. However, the mathematical model includes many inherent assumptions concerning the accuracy of the operations performed. Certain real-world phenomena such as pulse jitter, and finite word length errors in signal quantification or frequency domain transformation, may so severly restrict accuracy that the method becomes unusable. This chapter describes an experiment which was designed to test the algorithm under some of these less than ideal conditions.

The program which was designed to implement the experiment allowed the user to create two separate waveforms, one of which would be the transfer function waveform, and the other would be the input, or forcing function for the system. Various amounts of noise could be added to both waveforms so that the algorithm did not see completely smooth input functions. Given the transfer function waveform and the output function (e.g. the convolution of the two waveforms) and with the proper choice of a weighting constraint \( \lambda \), the compensator routine should be able to reconstruct the forcing function.
THE SOFTWARE

The routine to accomplish this is listed in appendix 1 as NSTDCN. Although the programming is rather straightforward, a few points deserve clarification. A synthesis routine which begins on line 390 is used to generate both waveforms. Basically both are constrained to be of the form \( \sin^2 \) and magnitude 1, but the user has control over the beginning and ending points of each waveform, and thus controls its period. The \( \sin^2 \) form was chosen for its symmetry and because of the fact that it is zero based. These factors help to avoid possible errors in the frequency transformation routine which could arise if the waveform of interest exhibited a jump discontinuity between its beginning and end points.

The parameter NM is equivalent to the algebraic ratio between the maximum noise amplitude and the maximum signal amplitude. Parameters \( R_1 \) and \( R_2 \) are seed parameters which insure reproducibility in the random number noise generation scheme. Finally, some control over the portion of the waveform which is evaluated to produce an error parameter may be exerted by the operator using parameter NC.

It should be noted that from line 890 to line 1000 the program basically follows the procedure devised in Chapter 3. A transfer function is produced and stored in array T in line 890, and this function is used to produce the compensator in lines 930 and 940. A complex multiplication in the frequency
domain, which corresponds to convolution in the time domain, is
used to produce a predicted input function in lines 970 and 980.
Finally, the so called "error energy" parameter is generated by
taking the root mean square value of the difference between the
predicted input function and the true input function, and mul-
tiplying the result by the number of points the operator wishes
to observe in lines 1010 - 1030. Once an optimum value for \( \lambda \)
is found, the operator may jump to line 1140 to investigate the
differences between his predicted input function and the true
input function.

THE DATA

To test the conjecture that a noiseless waveform will have
a progressively smaller "error energy" as \( \lambda \) approaches 0, the
noise magnitude parameter was set to 0 and the error energy check-
ed for various values of \( \lambda \). The results are graphed over three
decades of variation in \( \lambda \) in figure 4-1. Note that a definite
decreasing trend is observable as \( \lambda \) approaches 0.

The experiment was also run for the cases where \( NM = 0.1 \)
and where \( NM = 1.0 \). The true input function, impulse response
and the input function predicted by the optimal compensator for
the case \( NM = 0.1 \) (e.g. max noise amplitude / max signal ampli-
tude = 0.1) are given in figures 4-2, 4-3, and 4-4 respectively.
It can be seen that there is a very high degree of resemblance
between the predicted and the real input functions,
The waveforms for the case where $NM = 1.0$ are given in figures 4-5, 4-6, 4-7, and 4-8. Figures 4-5 and 4-6 are the true input function and the impulse response, respectively. Figure 4-7 shows the output of the system for this case. This figure was plotted in an effort to determine why figure 4-8, the predicted input function, shows significantly less noise than the true input function.

A possible explanation may be that the very high frequency components which the noise represents are truncated through the bandwidth limitations of the data representation. That is, since convolution is an integral process and therefore inherently smoothing, and since the output waveform, which again is much smoother than the input waveform or the impulse function since it represents the convolution of the two, is convolved with the compensator function to produce the predicted input waveform, the compensator function must be very noisy to yield noise in the predicted function. However, the frequency of the noise in any of these waveforms is limited by the sampling rate, which translates to array size, so the noise in the compensator function is probably limited by this. It is possible that the same effect helps to account for the relatively smooth output function.

This experiment indicates that the compensator producing algorithm is behaving in general as predicted. The error energy does indeed decrease as $\lambda$ approaches 0, and the second portion
of the experiment indicates that a workable prediction of the input function can be produced by the compensator, even in the presence of considerable noise. In the next chapter a physical system will be treated using the optimal compensator technique, to determine what physical limitations might apply to the technique.
Fig. 4-2. Input reference waveform for NM = 0.1
Fig. 4-3. System impulse response waveform for NM = 0.1
Fig. 4-4. Predicted reference waveform for NM = 0.1
Fig. 4-5. Input reference waveform for $NM = 1.0$
Fig. 4-7. System reference response waveform for NM = 1.0
Fig. 4-8. Predicted reference waveform for NM = 1.0
CHAPTER 5

THE OSCILLOSCOPE HEAD EXPERIMENT

Although the optimum compensator performed as expected for the purely imaginary case of a $\sin^2$ response in the previous chapter, it was felt that some testing of the algorithm for real world cases would be desirable. In order to more easily test the response of the algorithm in a real environment a fairly ideal test case was chosen. This chapter describes the experiment which resulted. Included also is a description of the software which was used to perform the signal processing and some observations concerning experimental results.

THE EXPERIMENT

In an effort to restrict the complexity of the waveforms which would have to be digitized and processed by the software a search was conducted for a system with a fairly flat frequency response upon which to test the routine. An excellent choice immediately presented itself in the form of a Tektronix S-1 sampling head which was being used in the 7S12 sampling unit of a Tektronix 7000 series digital processing oscilloscope which digitized the input waveform and stored it in mainframe memory with a resolution of 512 points over the total time window.
Although the 7S12-S1 combination seemed quite favorable since its output information was already in a form optimum for computer acquisition, the choice of this system did pose some problems. A major consideration was the fact that the S-1 head has a specified rise time of 350 picoseconds or less. In order to observe the impulse response of the head a signal generator had to be found which could generate a pulse with a rise time of 30 picoseconds or less. Fortunately, the same system included a S-52 pulse generator head, which has a specified rise time of 25 picoseconds, so this was used to approximate the impulse response of the system. The same system provided a considerably slower pulse generator head, the S-56, with a specified rise time of 1000 picoseconds, so this was used to generate the input waveform to the S-1 "system". Finally, a very fast sampling head, the S-6, which is accurate down to a 30 picosecond rise time, was used to generate accurate characterisations of the reference, and impulse waveforms.

THE SOFTWARE

The information from these devices was fed directly into a PDP 11/34 minicomputer and was operated upon by a program written using Tektronix SPS Basic. Each point in the final input waveforms was averaged over 20 successive samples to reduce the effects of jitter.
Memory limitations in the system in conjunction with the relatively large amount of memory required to store and manipulate the signal arrays necessitated a modular programming approach. Two routines were used to manipulate the data. These routines, which may be found in appendices 2 and 3, are entitled WNDW1 and WNDW2.

WNDW1 is basically a bookkeeping routine. The small section which starts at line 80 opens each of the data files in turn and stores the information found there in an array called WT. The format of the digital processor is such that the first 3 values stored in the mainframe memory are the number of points, the horizontal data interval, and the vertical scale factor respectively. After each waveform is acquired the program branches to a graphic subroutine which starts on line 360 and gives a graph of the waveform. This provides a check of whether the waveform data has been correctly read.

The operation performed at line 160 deserves special attention. This technique, which was devised by William L. Gans and N. S. Nahman of the National Bureau of Standards especially to facilitate the generation of frequency domain transfer functions from time domain waveforms via the fast Fourier transform, may best be described as turning the waveform off in the same way that it was turned on (Gans, 1972). The technique basically consists of concatenating an inverted version of the waveform to the original waveform. This makes the signal appear periodic to the fast Fourier transform, thus satisfying the requirement that the ampli-
tude and slope of the initial and final portions of the transform input waveform be the same. Although the technique doubles the number of points in the time domain waveform, thereby effectively doubling the frequency resolution, it also makes the waveform appear to be an odd function to the Fourier transform. Consequently, all even harmonics of the frequency domain representation go to zero. To facilitate storage, these zero components are discarded and the remaining information is compressed in line 230 and line 240.

The frequency domain magnitude function is generated in line 260 and graphed in line 290. Finally, the real and imaginary components of the frequency domain waveform are saved in two arrays in the block between line 300 and line 330 and the next program, WNDW2, is called in line 350. The output waveforms from the WNDW1 routine are shown in figures 5-1 through 5-12.

The optimum compensator algorithm is actually implemented in the routine called WNDW2. The module starting at line 30 graphs the input waveforms to insure that no data was lost from WNDW1. An initial guess for the constraint parameter, as well as the amount to increment it for the next guess, is input in lines 110 through 140. The magnitude portion of the original transfer function is graphed in line 200. At this point the user is expected to determine which portion of the transfer function represents useful information and which portion is inaccurate due to noise or computational error. The inaccurate portion is then set to
zero so that the optimization routine does not "see" it.

A graph of the transfer function of the S-1 head is shown in figure 5-13. The data is fairly smooth up to the 10 gigahertz point but becomes somewhat uncertain after that point. For this reason it was decided to set the array to zero after the 10 gigahertz point. This is equivalent to bandlimiting the transfer function, but this will not reduce the validity of the experiment since the transfer function and the output are the known quantities and it is the input function which is sought.

The magnitude transfer function waveform of the optimizer is generated and stored in WT at line 240. This function is convolved with the real and imaginary parts of the system output function and the magnitude of the resulting optimizer output function is stored in WT in line 280. Line 310 gives some measure of the error energy by taking the root mean square value of the difference between the predicted and the input functions over the bandwidth of interest and the routine which starts in line 340 saves both the optimum constraint value and the so called "error energy" for that constraint value.

Finally, the portion of the program which begins at line 690 is used after an optimum compensator is found. It expands the frequency compressed transfer functions of both the true and the predicted input functions, and utilizes a reverse Fourier transform to bring them both back to the time domain.
The actual frequency domain representation of the optimized compensator is presented in figure 5-14. The bandlimiting above approximately 10 gigahertz is quite evident in this waveform, which was obtained for \( \lambda = 7.8 \times 10^{-3} \). At this value the so called error energy was 412.688. A plot of the real time domain input waveform, as determined by the inverse Fourier transform, appears in figure 5-15 and the predicted input waveform is shown in figure 5-16. Finally, the two are plotted together on the same scale for purposes of comparison in figure 5-17.

Although the two waveforms of figure 5-17 are quite similar, there appears to be a slight difference in phase. Since the phase information was not investigated per se during the course of the experiment, this is not a particularly surprising result. If a reduction in this error energy is necessary, the error energy function could easily be modified to include some indication of phase error. Figure 5-17 also shows both functions riding on 0 d.c. offset voltage, although the original input function shown in figure 5-3 rides on a negative d.c. offset. This phenomenon is a result of the Nahman - Gans algorithm. Since point zero is an even point, it is assumed to be zero and discarded. Then, during the unpacking part of the time domain transformation, point zero is set to 0 for both waveforms, yielding no d.c. offset.
Although these results were quite promising, it was felt that the routine needed a more stringent real world test in a less ideal case. The next chapter discusses the use of the optimum compensator method to find the transfer function of an optical fiber.
Fig. 5-2. Test impulse response
Fig. 5-3. System input reference waveform
Fig. 5-4. System response to input reference waveform
Fig. 5-6. Nahman-Gans windowed test impulse response
Fig. 5-7. Nahman-Gans windowed reference waveform
Fig. 5-8. Nahman-Gans windowed system reference response
Fig. 5-9. Frequency domain magnitude of windowed test impulse
Fig. 5-10. Frequency domain magnitude of windowed impulse response
Fig. 5-11. Frequency domain magnitude of windowed reference waveform
Fig. 5-12. Frequency domain magnitude of windowed system reference response
Fig. 5-13. Calculated system transfer function magnitude
Fig. 5-14. Optimum compensator transfer function magnitude
Fig. 5-15. Reference waveform regenerated through inverse Fourier transform
Fig. 5-16. Predicted reference waveform
Fig. 5-17. Regenerated reference waveform plotted on same axis as predicted reference waveform.
CHAPTER 6

THE OPTICAL FIBER EXPERIMENT

The true test of any compensation scheme must be its ability to adequately perform in a non-ideal (e.g. real) environment. One which presented itself quite readily for the purposes of this thesis was to be found in the Kennedy Space Center Laser Lab. Work was being performed there under a government grant to determine the pulse dispersion characteristics of an optical fiber. This involved the extensive use of time domain measurements and techniques and thus was ideally suited to test the proposed algorithm.

THE EXPERIMENT

The experimental set up consisted of a SG 2002 semiconductor injection laser diode which fed pulses of approximately 100 picosecond duration and 2 watt optical power into a 4 meter mode scrambler filter. This mode scrambler consisted of a piece of graded index fiber directly coupled between two pieces of step-index fiber. It was used to make the resulting optical pulse more independent of launch conditions.

The mode scrambler fed the test fiber and from the test fiber the optical pulse went to a TIED 56 semiconductor avalanche photodiode (APD). The APD output signal was acquired using a
Tektronix 7001 series digital processing oscilloscope of the same type described in Chapter 5 using an S6-7S12 sampling head. As mentioned in Chapter 5, the oscilloscope digitized the information into 512 points covering the time window. The digitized signal was then fed into an HP 9825 computer and, after averaging 50 times, was processed using the compensator algorithm.

Since the compensator routine requires at least two waveforms and since the transfer function was the desired output, both input and output signals were recorded. The APD was coupled to the mode scrambler directly to ascertain the input, or reference signal, and was coupled through the test fiber for the response waveform measurement.

**THE DATA**

The fiber chosen for the experiment was a 4 kilometer length of graded index fiber. Figure 6-1 shows the pulse which was launched into the fiber by the mode scrambler and figure 6-2 depicts the fiber response waveform, in which considerable pulse dispersion is evident.

The quantity of interest, decibel fiber attenuation, is defined by

$$\text{ATTEN} = -10 \log_{10} H(e^{j\omega})$$

and is shown over three decades of frequency for several choices of $\lambda$ in figure 6-3. Although the plot extends well into the 2
gigahertz region, inherent limitations in the equipment used se- 
v erly restricted the accuracy of the data in this region. The 
criterion for optimum choice of \( \lambda \), the error weighting parameter, 
thus became that choice of \( \lambda \) which produced a maximum attenuation 
in the response signal above 2 gigahertz while leaving the lower 
frequency signal components virtually unchanged from their \( \lambda = 0 \) 
values. It may be recalled that the transfer function generated 
for \( \lambda = 0 \) corresponds to that which would be found using classical 
deconvolution techniques.

A lack of time prevented the determination of \( \lambda \) any closer 
 than to within the correct order of magnitude, but even at this 
 coarse resolution the results were impressive. Figure 6-4 shows 
the fiber impulse response for \( \lambda = 0 \) as reconstructed by the com-
puter through the use of the inverse Fourier transform. This is 
the classical impulse response found by normal methods. It can 
be seen that the response is almost completely obscured by noise. 
The step response of figure 6-5, generated by having the computer 
integrate figure 6-4 is considerably more recognizable, and in-
dicates that an impulse response is indeed hidden within all that 
noise.

The corresponding waveforms for \( \lambda = 10 \) are given in figure 
6-6 and figure 6-7. For this choice of \( \lambda \) the impulse response 
shown in figure 6-6 is recognizable as such although resolution 
of fine detail is still lacking. At this point the step response 
has settled down to essentially its final value. This may be at-
tributed to the fact that it is produced by integration, which is essentially a smoothing process.

The impulse response for the final choice-of $\lambda, \lambda = 100$, is given in figure 6-8. The response is now sufficiently smooth to allow it to be used in dispersion measurements. The corresponding step response is shown in figure 6-9. Predictably, it is not significantly different from the response given in figure 6-7. Finally, figure 6-10 gives the attenuation and phase response of the test fiber versus frequency.

Although no single experiment can validate or invalidate a technique, these results show that the optimal compensator technique is viable in a real-world situation. As this experiment indicates, an essential factor in the use of this technique is the intelligent choice of a constraint parameter characterization. Although the constraint parameter, $\lambda$, was chosen only to within an order of magnitude, the results were still quite favorable. The next and final chapter will expand upon these conclusions and present observations concerning the potentiality of this technique for further research.
Fig. 6-1. Reference waveform for testing 4 km fiber
Fig. 6-2. Response waveform of 4 km fiber
Fig. 6-3. 4 km fiber attenuation for various values of $\lambda$
Fig. 6-6. 4 km fiber impulse response as computed using $\lambda = 10$
Fig. 6-7. 4 km fiber step response as computed using $\lambda = 10$
Fig. 6-8. 4 km fiber impulse response as computed using $\lambda = 100$
Fig. 6-9. 4 km fiber step response as computed using $\lambda = 100$
Fig 6-10. 4 km fiber attenuation and phase functions using optimum value of $\lambda$ (= 100)
CHAPTER 7

CONCLUSION

The optimum compensator technique appears to be quite viable in a large variety of situations. Probably the factor which most significantly enhances the flexibility of the method is the high degree of operator control concerning the definition of an optimum value for \( \lambda \). However, it must be highly stressed that, as in any other optimization technique, the success or failure of the method rests upon the intelligent characterization of the qualities necessary to define a good \( \lambda \). No general rule can be stated for this since it will vary with the problem, but it may be noted that when the capability of graphing the output of the compensator is available, this probably provides the most information regarding the optimum choice of \( \lambda \). A case in point may be found in figure 6-3 where the graphic representation of the attenuation function is used to determine the optimal choice for \( \lambda \).

It should be noted that the final compensator is not a function of the input to the system. This means that once the optimum compensator is determined for a system, it should remain fixed for all types of input signals. This means that this com-
pensator could conceivably be implemented on a high speed signal processing system to provide real-time operation. Further investigation into this possibility seems warranted.
APPENDIX 1

NSTDCN
5 REM CLEAR LAST RUN
10 DELETE WA, UR, UI, A, AR, AI, ER, BI, CR, CI
20 DELETE HR, HI, VZ, VI, UR, UI, DR, DI, PR, PI
30 DELETE ER, EI, UT, TR
40 WAVEFORM WA IS A(511), HA, HAS, VAS
50 HAS = "S"
60 VAS = "V"
70 HA = 10/12
75 REM SET WAVEFORM LIMITS
80 PRINT "START END FOR WA & UB"
90 INPUT H1, H2, N3, N4
95 REM NM = (MAX NOISE AMPLITUDE)/(MAX SIGNAL AMPLITUDE)
100 PRINT "NOISE MAGNITUDE RATIO:"
110 INPUT NM
115 REM INITIALIZE RANDOM NUMBER GENERATOR
120 PRINT "SEED VALUES (GIVE TWO):"
130 INPUT RI, R2
140 RANDOM RI, R2
150 N3 = N3 * NM = N2
155 REM SUBROUTINE 399 CREATES SIN SQUARED WAVEFORM^n
160 COSUB 390
170 DIM R(511)
175 REM R IS NOISE ARRAY
180 R = RND(R)
190 R = R - NER(R)
200 A = A + NM * R * MAX(A)
210 PAGE
220 PRINT "WA"
230 GRAPH WA
240 WAIT
250 PAGE
260 WAVEFORM UR IS HR(256), HF, HFS, VFS
270 WAVEFORM UI IS HI(256), HF, HFS, VFS
280 REFT UR, UP, WI
290 NB = N3 * NE = N4
300 COSUB 390
310 R = RND(R)
320 R = R - NER(R)
330 A = A + NM * R * MAX(A)
340 PRINT "UB"
350 GRAPH UB
360 WAIT
370 PAGE
380 GOTO 450
390 END SIN SQUARED WAVEFORM SYNTHESIS ROUTINE
394 A = 0
400 P = 4 * ATAN(1) / (NE - MB)
410 FOR I = NB TO NE
420 X = SIN(((I - N3) * P)
430 A(I) = X * X
440 NEXT I
450 RETURN
430 REM END OF SYNTHESIS ROUTINE
435 WAVEFORM VR IS BR(256), HF,HFS,VFS
470 WAVEFORM VI IS CI(256), HF,HFS,VFS
430 FFT WA,VR,VI
435 WAVEFORM UR IS BR(256), HF,HFS,UFS
530 WAVEFORM VI IS DI(256), HF,HFS,UFS
510 UR = UR*VR-UI*VI
520 UI = UR*VR+UI*VR
530 FFT WA,UR,VI,"INV"
540 PRINT "CONVOLVED WAVEFORM"
550 GRAPH WA
560 WAIT
570 PAGE
580 R=RND(R)
590 R=R-RND(R)
600 A=A+8+RandMax(A)
610 PRINT "EFFECT OF CONVOLUTION NOISE"
620 GRAPH UANPAGE
630 FFT WA,UR,UI
535 REM REGENERATE WA WITH NOISE
640 KG=MINS=K2
650 GOSUB 390
655 REM RESET NOISE GENERATOR
660 RANDOM R1,R2
670 R=RND(R)
680 R=R-RND(R)
690 A=A+8+RandMax(A)
700 GRAPH UANPAGE
710 FFT WA,UR,VI
715 REM REGENERATE WB WITH NOISE
720 NB=M3=ME=N4
730 GOSUB 390
740 P=RND(R)
750 R=R-RND(R)
760 A=A+8+RandMax(A)
770 GRAPH UANPAGE
780 FFT WA,VR,VI
790 DELETE WA,A,R
833 PRINT "COMPARISON POINTS"
810 INPUT MC
820 PRINT "LA = "
833 INPUT LA
840 PRINT "INCREMENT = ";
833 INPUT DL
853 SI=1E+20
870 K=0
880 DIM T(256)
895 REM TRANSFER FUNCTION MAGNITUDE
900 T=SCR(DM=8+HI=H1)
909 WAVEFORM ER IS BR(256), HB,HBS,VSS
910 WAVEFORM EI IS BI(256), HB,HBS,VSS
925 REM COMPENSATOR SYNTHESIS EQUATIONS
930 BR=HR/(T*T+LA)
940 BI=-HI/(T*T+LA)
950 WAVEFORM PR IS AR(255),HP,HPS,VPS
955 WAVEFORM PI IS AI(255),HP,HPS,VPS
960 REM INPUT WAVEFORM PREDICTION EQUATIONS
970 PR=ERROR-UI
980 PI=ERROR+UI+ERROR
990 WAVEFORM UT IS T,HP,HPS,VPS
1000 T=SQRT(AR*AR+AI*AI)
1020 REM ERROR INDICATION PARAMETER SYNTHESIS
1030 LR=PR-VR/MSQ(RS(Q*NC))
1040 DI=PI-VI/MSQ(SI(Q*NC))
1050 S=(SR-SR+SI*SI)*(NC+1)
1060 PRINT "ERROR ENERGY = ";S;" FOR LAMBDA = ";LA
1080 IF S>S1 THEN 1090
1090 S=S-L=LA
1100 IF K>10 THEN 1110
1110 K=K+1=LA+DL
1120 GOTO 290
1130 PRINT "ERROR ENERGY = ";S;" FOR OPTIMAL LAMBDA = ";L1
1140 END
1155 REM TRANSFORMATION TO TIME DOMAIN
1160 DELETE EI,ER,UI,UR,UI,UR,UT,BI,DI,DR,HI,HR,T
1170 DIM RR(S11),PP(S11)
1180 RFFT RR,CR,C1,"INV"
1190 RFFT PP,AR,A1,"INV"
1200 END
5 REM CLEAR LAST RUN
10 DELETE UT,UA,UB,UC,WD,AR,AI,BR,BI,CR,CI,DR,D1,ER,EL,HR,HI,PR,PI
20 DIM WT(1023),TR(512),TI(512)
25 REM READ INPUT DATA FROM FILE
30 FOR J=1 TO 4
40 IF J<>1 THEN GOTO 50, FS="SS235.DAT"; TS="UT"
50 IF J<>2 THEN GOTO 60, FS="SS231.DAT"; TS="UB"
60 IF J<>3 THEN GOTO 70, FS="SS436.DAT"; TS="UC"
70 IF J<>4 THEN GOTO 80, FS="SS431.DAT"; TS="WD"
80 OPEN #1 AS FS FOR READ
85 REM FILE MANAGEMENT INFORMATION
90 INPUT #1,UT(0:511)
100 CLOSE #1
120 ZS=""
130 K=511
135 REM GRAPH TIME DOMAIN FUNCTION
140 GOSUB 400
145 REM PERFORM HANHAN-GANS WINDOWING ROUTINE
150 FOR J=512 TO 1023
160 UT(J)=UT(0)+UT(511)-UT(J-512)
170 NEXT J
180 ZS="HANHAN-GANS WINDOWED"
190 K=1023
200 GOSUB 400
210 RFFT UT,TR, TI
215 REM COMPRESS FREQUENCY DOMAIN DATA
220 FOR I=1 TO 256
230 TR(I-1)=TR(2*I-1)
240 TI(I-1)=TI(2*I-1)
250 NEXT I
255 REM MAGNITUDE FUNCTION
260 UT(0:255)=SOR(TR(0:255)*TR(0:255)+TI(0:255)*TI(0:255))
270 K=256
280 ZS="FREE DOMAIN"
290 GOSUB 400
295 REM OUTPUT MATRIX STORAGE ROUTINE
300 IF J<>1 THEN 320, D IN AR(255),AI(255)
310 AR=TR(0:255)\AI=TI(0:255)
320 IF J<>2 THEN 340, D IN AR(255),BI(255)
330 BR=TR(0:255)\BI=TI(0:255)
340 IF J<>3 THEN 360, D IN CR(255),CI(255)
350 CR=TR(0:255)\CI=TI(0:255)
360 IF J<>4 THEN 380, D IN DR(255),DI(255)
370 DR=TR(0:255)\DI=TI(0:255)
380 NEXT J
385 REM CALL NEXT ROUTINE
390 CHAIN "WNDW2.BAS"
395 REM GRAPH ROUTINE
400 PAGE=PRINT ZS; TS
410 SET GRANT 4,4
420 GRAPH UT(0:K) WAIT
430 RETURN
10 DELETE UT
20 DIM UT(255)
25 REM GRAPH INPUT DATA
30 FOR I=1 TO 4
40 GOSUB 1 OF 410, 470, 530, 550
50 NEXT I
60 DIM HR(255), HI(255)
70 UT=AR+AR+AI*AI
75 REM TRANSFER FUNCTION EQUATIONS
80 HR=(BR+AR+BI*AI)/UT
90 HI=(BI*AR-BR*AI)/UT
100 DELETE AR, AI, BR, BI
110 PRINT "LAMBDA = ";
120 INPUT LA
130 PRINT "INCREMENT = ";
140 INPUT DL
150 Z=“FREE DON RATIO”
160 TS=“U3/VA“
170 S1=1E+20
180 K=0
190 DIM ER(255), EI(255)
195 REM TRANSFER FUNCTION MAGNITUDE
200 UT=SQRT(HR*AR+HI*AI)
210 IF S1=1E+20 THEN GOSUB 520
215 REM OPTIMIZER SYNTHESIS
220 ER=HR/(UT+UT+LA)
230 EI=HI/(UT+UT+LA)
240 UT=SQRT(ER*ER+EI*EI)
250 DIM PI(255), PI(255)
255 REM PREDICTED INPUT FUNCTION
260 PR=ER+DR-EI*DI
270 PI=ER+DI-EI*DR
280 UT=SQRT(PR+PR+PI+PI)
290 UT=SQRT(PR+PR+PI+PI)
295 REM ERROR PARAMETER SYNTHESIS
300 ER=PR-CR/SR=RMS(ER(0: 49))
305 EI=PI-CI/SI=RMS(EI(0: 49))
310 S=(CR+SR+SI+SI)*50
320 PRINT "ERROR ENERGY = ";S;" FOR LAMBDA = ";LA
330 IF S>S1 THEN 350
340 S1=S*L1=LA
350 IF K>10 THEN 300
360 K=K+1; LA=LA+DL
370 GOTO 200
380 PRINT "PRINT"
390 PRINT "ERROR ENERGY = ";S1;" FOR OPTIMAL LAMBDA = ";L1
400 END
410 TS=“U3“
420 UT=CR+AR+AR+AI*AI
430 GOTO 620
440 TS=“U3“
450 UT=U3
460 GOTO 620
470 T$="UD"
480 UT=SU(OR*SR+8R*8I)
490 GOTO 620
500 T$="UC"
510 UT=MC
520 GOTO 620
530 T$="UC"
540 UT=SU(OR*CR+DI*CI)
550 GOTO 620
560 T$="UD"
570 UT=UD
580 GOTO 620
590 T$="UD"
600 UT=SU(OR*CR+DI*CI)
610 GOTO 620
615 REM GRAPHICS ROUTINE
620 PAGE
630 VIEWPORT 131,901,142,634
640 SETGR VIEW
650 PRINT ZS;TS
660 GRAPH UT\PRINT
670 PRINT "WAIT"
680 WAIT
690 RETURN
700 REM FREQUENCY DOMAIN DATA RECOMPRESS
710 DELETE AR,AI,ER,BI,DR,DI,ER,EL,HR,HI,UT,TL,TR
720 DIM R(512),RI(512),SR(512),SI(512)
730 RR=0
740 RI=0
750 SR=0
760 SI=0
770 FOR I=2 TO 512 STEP 2
780 J=(I/2)-1
790 RR(I-1)=CR(J)
800 RI(I-1)=CI(J)
810 SR(I-1)=PR(J)
820 SI(I-1)=PI(J)
830 NEXT I
840 REM TIME DOMAIN TRANSFORMATION
850 DELETE CR,CI,PR,PI
860 DIM R(1023)
870 RFFT R,RR,RI,"INV"
880 DELETE RR,RI
890 DIM S(1023)
900 RFFT S,SR,SI,"INV"
910 END
LIST OF REFERENCES


