Design and Analysis of an Explosive Driven Hydrodynamic Conical Shock Tube

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DESIGN AND ANALYSIS
OF AN EXPLOSIVE DRIVEN
HYDRODYNAMIC CONICAL SHOCK TUBE

BY

LEONARD W. CONNELL
B.S.M.E., Michigan Technological University, 1976

RESEARCH REPORT
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in the Graduate Studies Program of the College of Engineering
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ABSTRACT

An explosive driven, water filled, conical shock tube was designed and evaluated regarding its ability to amplify a charge weight and to produce hydrodynamic spherical shock waves. The results show that the shock waves in the tube are essentially spherical in nature—with an initial exponential shape, peak pressure attenuation as \( (1/R)^{1.13} \) and the time constant spreading roughly as \( (R)^{22} \).

The charge weight was amplified by a factor of 3600 compared to a theoretical amplification of 7770. An estimate of the energy absorbed by the breach plug (which houses the charge) during an explosion was performed.

The peak pressure data taken from the detonation of number 8 strength blasting caps were seen to satisfy the semiempirical scaling law. However, with the addition of plastic explosive to the blasting cap, peak pressure lower than that predicted by the scaling law was observed. At this time it is felt that a decreasing amplification factor with charge weight is the cause for the lower than predicted peak pressure. More data are needed to verify this hypothesis.
ACKNOWLEDGMENTS

I wish to thank Dr. C. E. Nuckolls and Dr. R. L. Phillips for their guidance and helpful discussions; Madjid Belkerdid for the many days spent in assisting with the shock tube tests; and Joseph Haibach for the construction of much of the mechanical hardware.

The original design calculations for the shock tube were done by Dr. Nuckolls. They have been included here in an expanded and somewhat modified form. Some of the electronics related to the explosive detonation and measurement were designed and assembled by Dr. Phillips and his assistants.

Special thanks are given to my wife Sandra for her moral support and for the typing of this paper.
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I. INTRODUCTION

A shock tube was developed under the auspices of the Naval Research Laboratory, Underwater Sound Reference Division, Orlando, Florida. Its function was to test the structural integrity of a particular underwater device against the shock wave generated by 125 pounds of TNT detonated at a range of 11.0 feet.

Filler (1) has demonstrated that a conical shock tube driven by a relatively small amount of explosive will produce shock waves whose characteristics are representative of a much larger charge weight. This amplification of charge weight makes the shock tube a useful laboratory tool in simulating the effects of a large weight of high explosives which could otherwise be tested only in free field at considerable cost and difficulty.

This paper will deal only with the design and performance of the shock tube. The performance characteristics to be considered are as follows:

1. Spherical nature of the generated shock wave.
2. Magnitude of the amplification factor.
3. Conformity of the shock waves to the scaling laws.
4. Amount of energy dissipated to the shock tube.

The design of the tube involved an iterative procedure in which preliminary dimensions were chosen and then checked to ensure
satisfactory performance. Only the final design and supporting calculations will be discussed. The major design concern was to prevent fully plastic deformation of the tube walls which would cause relatively large wall deflections, absorb large amounts of energy and perhaps destroy the shock wave by creating rarefaction waves that could overtake and interfere with the shock front.
II. DESIGN CALCULATIONS

Amplification Factor

The fundamental principle behind the conical shock tube is that a small conical charge placed at the vertex of the conical tube produces the same shock effect as the spherical charge of equal radius. The shock tube merely isolates this portion of the wave from the rest and ideally has no affect on the wave characteristics. Ideally, the shock waves in the tube should be spherical in nature and the explosive energy liberated by the conical charge will be concentrated into the solid angle of the cone rather than radiating in all directions. The small amount of explosive thus behaves like a much larger amount and an amplification is realized.

The amplification factor (AF) may be defined as the weight of apparent spherical charge to that of the actual conical charge. Figure 1 illustrates this concept. The AF may be calculated as follows:

\[ W_{\text{APPARENT}} = \frac{4}{3} \rho \pi r^3 \]  
\[ W_{\text{ACTUAL}} = \frac{\phi}{4\pi} W_{\text{APPARENT}} \]  
\[ \text{AF} = \frac{W_{\text{APPARENT}}}{W_{\text{ACTUAL}}} = \frac{4\pi}{\phi} \]

where

\( \phi \) = solid angle of cone
DEFINITIONS:

\( \alpha \) - plane angle of the cone
\( \phi \) - solid angle of the cone
\( \rho \) - charge density
\( W_{\text{true}} \) - actual weight of the conical charge
\( W_{\text{apparent}} \) - weight of the apparent spherical charge
\( r \) - radius of the apparent spherical charge

Fig. 1. Amplification of a Conical Charge
\[ \rho = \text{mass density of charge} \]

\[ W_{\text{TRUE}} = \text{actual weight of conical charge} \]

\[ W_{\text{APPARENT}} = \text{weight of apparent spherical charge} \]

\[ r = \text{radius of apparent spherical charge} \]

The AF then can be viewed as the ratio of the solid angle of a sphere (4\pi steradians) to that of the cone. The solid angle of the cone \( \phi \) can be expressed in terms of the plane angle \( \alpha \) by the relation

\[ \phi = 2 \left( 1 - \cos \left( \frac{\alpha}{2} \right) \right) . \]

The AF can therefore be simplified to

\[ \text{AF} = \sin^{-2}(\alpha/4). \]

This expression will be used later in the design calculations.

**Scaling Laws**

The scaling laws will be briefly introduced here since they are used in determining shock tube dimensions. A complete discussion of these laws will be covered in chapter IV.

The form of the shock wave may be approximated by a discontinuous rise in pressure followed by an exponential decay \(2\)

\[ P(t) = P_m \exp(t/\theta) . \]

The scaling laws are empirical correlations relating the peak pressure \( P_m \), time constant \( \theta \), and other shock wave parameters to the charge weight \( W \) and the range \( R \) from the charge center. For TNT the following relations apply \(3\):

\[ P_m = 2.16 \times 10^4 (W^3/R)^{1.13} \]

\[ \theta = 58 \left( \frac{1}{W^3} \right)^{0.22} \]

where

\[ \rho = \text{mass density of charge} \]

\[ W_{\text{TRUE}} = \text{actual weight of conical charge} \]

\[ W_{\text{APPARENT}} = \text{weight of apparent spherical charge} \]

\[ r = \text{radius of apparent spherical charge} \]
$W = \text{charge weight in pounds}$

$R = \text{distance from charge in feet}$

$P_m = \text{pressure in psi}$

$\theta = \text{time constant in microseconds}$

**Shock Tube Design**

The tube is to be designed to generate and withstand a shock wave whose characteristics are equivalent to that of a 125 pound spherical charge of TNT up to a range of 11.0 feet. The scaling laws show that to match peak pressure the same value of reduced distance $\frac{1}{(W^3/R)}$ is required. Obviously, this places no constraint on the length of the tube. However, to match the time constant ($\theta$), the same apparent weight must be used since

$$\theta = W^3f(W^3/r).$$

Therefore, to match both the peak pressure and the time constant an apparent weight of 125 pounds must be used ($W_{\text{APPARENT}} = A F(W)_{\text{TRUE}}$) and the tube length must be at least 11.0 feet from the center of the apparent charge to the muzzle end.

The tube was constructed from 8 inch diameter steel round using Electric Discharge Machining (EDM) to form the interior conical surface. The minimum wall thickness which occurs at the muzzle end was chosen to be 1.0 inch and to satisfy the scaling laws the "effective" length of the tube was made 11.0 feet. Figure 2 shows a schematic of the tube.

These dimensions fix the cone angle

$$\tan \alpha/2 = (3)\text{in}/(11)(12)\text{in},$$
Fig. 2. Preliminary Sketch of Shock Tube
\[ \alpha = 2 \arctan (0.0227) = 2.6^\circ. \]

Theoretical Amplification Factor and Required Charge Weight

The theoretical AF can now be determined from equation II. 5

\[ AF = \sin^{-2}(2.6/4) = 7770. \]

Assuming for now that this level of amplification is achievable the true weight of explosive required is

\[ W_{\text{actual}} = \frac{W_{\text{apparent}}}{AF}, \]

\[ = \frac{(125 \text{ lb})}{(7770)} \frac{(454 \text{ gm})}{(1\text{ lb})}, \]

\[ = 7.3 \text{ gm TNT}. \]

The Dupont Company manufactures a flexible tubular explosive with trade name Detaprime which is available in 3.0 inch lengths and a TNT equivalent weight of approximately 6.0 grams per inch. Number 8 strength blasting caps with a TNT equivalent weight of 0.73 grams were used to initiate the Detaprime. The length of Detaprime required is then

\[ (7.3 - 0.73) \text{ gm} \frac{(1.0 \text{ in})}{(6.0 \text{ gm})} = 1.1 \text{ in. Detaprime}. \]

The blasting cap is cylindrical in shape and is fitted inside the 1.1 inch tube of Detaprime. Figure 3 shows the arrangement.

Shock Tube; Detailed Drawing

Since the explosive material is cylindrical instead of conical it is not feasible to run the cone back to the vertex. Instead, the 11.0 foot cone was truncated 1.0 foot from the vertex and the breach end of the tube was recessed 3.0 inches to allow for a breach plug which housed the explosive material. Figure 4 shows a detailed
Fig. 3. Physical Arrangement of the Explosives
drawing of the shock tube and breach plug.

As mentioned previously, the tube was fabricated by the EDM process. This was accomplished by machining the tube in 2.0 foot sections and then welding the sections together. Flanges were welded at the breach and muzzle end to facilitate attachment of the breach plug and closure head respectively. An intermediate flange was located 4.0 feet from the breach end to simplify placement and cleaning of the tube. A 2.0 inch thick annular steel ring was bolted to the muzzle flange and fitted with fill and drain apparatus.

The lead wires from the blasting cap were fed down the length of the tube where they penetrate the closure head. This served a twofold purpose:

1. the shock wave peak pressure will be much lower at the muzzle end and leakage through the wire penetration will be less of a problem
2. energy losses to the packing gland will be minimized

Figures 5 through 9 are photographs of the shock tube and related apparatus.

Beam Directivity

By using a cylindrical rather than conical charge a non spherical characteristic is introduced. Linear acoustic theory will be used to obtain a rough approximation of the effect on the shock wave. From Kinsler and Frey (4) the pressure waves radiated by a circular piston oscillating harmonically are

\[
P(r, \theta, t) = \frac{\rho_{0}ckQ}{2\pi r} \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \sin (\omega t - kr),
\]
Fig. 5. The shock tube; an overall view
Fig. 6. Breach plug assembly
Fig. 9. Oscilloscope and firing circuit
where

\[ \rho_0 = \text{undisturbed fluid density} \]
\[ c = \text{acoustic speed} \]
\[ k = \text{wave constant} = \frac{2\pi}{\lambda} \]
\[ Q = \text{source strength} = \pi a^2 U_0 \]
\[ U_0 = \text{maximum piston speed} \]
\[ a = \text{piston radius} \]
\[ J_1 = \text{Bessel function of the first kind} \]
\[ \omega = \text{angular frequency}. \]

Figure 10 describes the piston and defines the variables in equation II. 14. This expression is identical to that describing pressure about a hemispherically radiating source of equal strength (Q) except for the term

\[ \frac{J_1(ka \sin \theta)}{ka \sin \theta} \]

called the directivity. This function introduces a non-spherical characteristic in that at a constant radius the pressure varies with the polar angle (\( \theta \)) and first drops to zero when

\[ ka \sin \theta = 3.83, \]

or

\[ \frac{2\pi a}{\lambda} \sin \theta = 3.83. \]

A shock wave is a transient phenomenon and it must be represented by a continuous spectrum of frequency rather than a single frequency as in the simple harmonically oscillating piston. The wavelength used in equation II. 17 was chosen to be that value corresponding to the
$\rho_0$ - undisturbed fluid density

$c$ - acoustic speed

$k$ - wave constant $= \frac{2\pi}{\lambda}$

$Q$ - source strength $= \pi a^2 U_0$

$U_0$ - maximum piston speed

$a$ - piston radius

$J_n$ - Bessel function

$\omega$ - angular frequency

Fig. 10. Oscillating Piston in an Infinite Baffle
natural frequency of the pressure transducer, a logical choice since frequencies above this value will not be accurately sensed due to mechanical resonance of the pressure transducer and the effect of finite transducer size. The following calculations are based on a natural frequency of 170 KHZ corresponding to the pressure transducer used in the tests

\[
\lambda = \frac{c}{f} = \frac{(5000 \text{ ft/sec})}{1.70 \times 105 \text{ cycles/sec}} = 2.94 \times 10^{-2} \text{ft/cycle.}
\]

For a piston radius corresponding to the size of the breach plug cavity

\[
2\pi (0.332) \sin \theta = (3.83)(2.94 \times 10^{-2})(12),
\]

\[
\theta = 40.3, \quad 2\theta = 80.6.
\]

The angle \(2\theta\) is called the beam width and represents the angular region the boundary of which is at zero pressure. Figure 11 is a plot of the directivity function versus polar angle (\(\theta\)). The cone angle of 2.6° is superimposed on this plot showing that over this range of polar angle the beam directivity has little effect on the wave's sphericality. Beam directivity was therefore not considered to be a problem.

**Elastic-Plastic Behavior of the Shock Tube**

It is desirable that the shock tube maintain its conical dimensions after repeated explosions, i.e. to behave elastically. To check this it is necessary to compute the expected peak shock wave pressure versus position as it propagates down the tube for the apparent weight of 125 pounds of TNT. This pressure must then be
Fig. 11. Directivity Function compared with shock tube cone angle (angle scale factor = 2).
compared with the static pressure that would cause fully plastic flow. Comparing the shock wave peak pressure with the static failure pressure would appear to be conservative. It is not likely that the mild steel used in the manufacture of the tube would fail in a brittle manner as a result of the impulse imparted by the shock waves passage. Furthermore, since the peak pressure is applied for only an instant its ability to cause fully plastic flow is more limited than a static pressure of equal value. The prime concern for limiting plastic flow of the shock tube walls is the draining away of energy and the abnormal attenuation of peak pressure this could produce. Another related effect mentioned by Filler (1) is the possibility that radial wall deflections could generate rarefaction waves that could overtake and destroy the shock front.

The mild steel was modeled as being elastic-perfectly plastic. This model assumes that once the yield stress ($Y$) is reached in a uniaxial tension test the material will continuously deform (i.e. flow) at constant stress. Figure 12 compares the stress-strain diagram for mild steel with that of the model. The model is seen to be a reasonable approximation out to a strain of about 1 percent, with the effect of strain hardening coming into play at larger strain values. Mild steel follows the failure criterion of Tresca (5) which states that failure (which will be defined here as yielding) occurs in shear when the maximum shearing stress reaches a limiting value. This condition can be expressed mathematically in terms of the maximum and minimum principle stresses and the yield stress as
Fig. 12. Stress-Strain Curve for Mild Steel and Model
\[ \sigma_1 - \sigma_3 = Y, \]

where

- \( \sigma_1 \) = maximum principle stress
- \( \sigma_3 \) = minimum principle stress
- \( Y \) = yield stress
- \( \sigma_2 \) = intermediate principle stress.

Den Hartog (6) determines the fully plastic or collapse pressure by first obtaining the elastic stress distribution. Since the cone angle is small \((2.6^\circ)\) the shock tube may be modeled in this analysis as a straight cylindrical tube, as shown in Figure 13. Ignoring end effects and observing symmetry, the governing equations are as follows:

1. Equilibrium

\[
\frac{\partial \sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r}
\]

2. Compatibility

\[
\epsilon_r = \frac{du}{dr}
\]
\[
\epsilon_\theta = \frac{u}{r}
\]
\[
\epsilon_z = C \text{ (constant)}
\]

3. Constitutive Relations

\[
\epsilon_r = \frac{1}{E} \left[ \sigma_r - \nu \sigma_\theta - \nu \sigma_z \right]
\]
\[
\epsilon_\theta = \frac{1}{E} \left[ \sigma_\theta - \nu \sigma_r - \nu \sigma_z \right]
\]
\[
\epsilon_z = C = \frac{1}{E} \left[ \sigma_z - \nu \sigma_r - \nu \sigma_\theta \right]
\]

4. Boundary Conditions

\[ \sigma_r(a) = -p \]
Fig. 13. Shock Tube Model for Stress Analysis

\( \sigma_r \) - radial stress
\( \sigma_\theta \) - tangential stress
\( \varepsilon \) - strain
\( u \) - radial displacement
\( E \) - Young's modulus
\( \gamma \) - Poisson's ratio
\( a \) - internal radius
\( b \) - external radius
\( \sigma_r(b) = 0 \) (external atmospheric pressure is negligible)

\[
p\pi a^2 = 2\pi \int_a^b \sigma_z \, dr
\]

Upon combining the equilibrium, compatibility and constitutive relations the following differential equation is obtained

\[
\frac{d^2 \sigma_r}{dr^2} + 3 \frac{d\sigma_r}{r \, dr} = 0 \quad \text{II. 25}
\]

Solving this subject to the boundary conditions yields the Lame' Equations

\[
\sigma_r = -p\left[ \frac{(b/r)^2 - 1}{(b/a)^2 - 1} \right] \quad \text{II. 26}
\]

\[
\sigma_\theta = \frac{p\left[ (b/r)^2 + 1 \right]}{\left[ (b/a)^2 - 1 \right]}.
\]

By combining equations II. 23, 24, and 26 the axial stress (\( \sigma_z \)) may be determined. Its value is constant and equal to the average of the radial and tangential normal stresses

\[
\sigma_3 = \frac{\sigma_r + \sigma_\theta}{2} = \frac{pa^2}{b^2 - a^2} \quad \text{II. 27}
\]

By inspection it is clear that the stresses \( \sigma_\theta, \sigma_r, \) and \( \sigma_z \) are the principle stresses with the tangential stress and the radial stress being the maximum and minimum values respectively. The Tresca yield condition may then be written as, \( \sigma_\theta - \sigma_r = Y. \)

The Lame' Equations are valid so long as the material remains within the elastic limit. As the internal pressure builds, the material will first yield at the inner wall surface where the stresses are highest. The pressure to cause the onset of yielding is determined by replacing \( \sigma_1 \) and \( \sigma_3 \) in equation II. 20 by the Lame' Equations evaluated at \( r = a \).
\[ P_{oy} = \frac{Y}{2} (1 - (a/b)^2). \]

As the pressure increases above this value a plastic zone forms and spreads radially outward as pressure increases. Outside the plastic zone the material continues to deform elastically. Figure 14 shows a sketch of this elastic-plastic behavior. The radius, \( c \), locates the interface between the plastic and elastic zones. Large plastic deformations are restricted by the elastic zone surrounding the plastic core. The plastic collapse pressure is that value of pressure which pushes the interface, \( c \), to the outer tube wall and allows plastic flow to occur uninhibited. The derivation of the collapse pressure is included in the appendix, the result being

\[ P_c = Y \ln \frac{b}{a}. \]

The collapse pressure for a given material depends only on the ratio of the inner and outer radii and holds irrespective of any residual stress profile, which is nullified as the plastic zone sweeps through the tube wall.

Though the scaling law for peak pressure (Equation II. 7) is a useful formula, it is accurate only at distances greater than 10 charge radii (7). Since a 125 pound spherical charge of TNT of density 1.6 gm/cm\(^3\) has a radius of 0.67 feet the scaling law for peak pressure will not be applicable within about 6 feet from the breach plug. Measurements of the peak pressure at very close range have been made at the Explosives Research Laboratory, Bruceton, Pennsylvania (8) using photographic techniques to determine the shock front velocity from which the peak pressure can be determined via the Rankine-Hugoniot relations. Table 1 lists these experimental
Fig. 14. Elastic-Plastic Behavior
peak pressures covering a range from 1.0 to 5.33 charge radii. The rest of the pressures are determined from the peak pressure scaling law. Tabulated along with the peak pressures are the collapse pressure \( (P_c) \) and the onset of yielding pressure \( (P_{oy}) \) both of which decrease with position down to the tube due to the decreasing wall thickness. Figure 15 is a plot of the data in Table 1. According to the graph the peak pressure exceeds the collapse pressure over the first 1.5 feet of the tube. This clearly could not be avoided since to prevent collapse at 70 kilobars would require a ratio of \( b/a \) exceeding \( 10^9 \). However, as previously mentioned, since the duration of the peak is very small and at any instant is applied only locally, fully plastic flow is not expected. Elastic-plastic behavior is expected, though, and is welcomed, since pressurization above \( P_{oy} \) (but less that \( P_c \)) leaves a residual stress profile which allows the tube to respond elastically to repeated pressurization up to the same pressure level. This phenomenon is called shakedown and is used in the manufacture of gun barrels. Elastic behavior is most desirable in preventing large wall deflections and the accompanying energy drain from the shock front.

**Joint Integrity**

A bolt pattern of 12 equally spaced 3/4 inch SAE grade 5 bolts were chosen to secure the breach plate to its flange on the shock tube. In Figure 4 an illustration of the breach plate and bolt pattern was shown. From Shigley (9) the maximum allowable bolt torque is
<table>
<thead>
<tr>
<th>Distance From Charge Center (Charge Radii)</th>
<th>Distance From Breach Plug (ft)</th>
<th>Peak Pressure (ksi)</th>
<th>Onset Of Yielding Pressure $P_c$ (ksi)</th>
<th>Fully Plastic Pressure $P_c$ (ksi)</th>
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</table>
Fig. 15. Shock Tube Pressure Distributions
\( T_m = c F_{im} d, \)

where

\( T_m = \) maximum bolt torque
\( F_{im} = \) allowable preload, 27,000 lb
\( d = \) nominal bolt diameter, 0.75 in
\( c = \) torque coefficient, 0.2.

Therefore the maximum bolt torque is 337 ft-lb.

The free length of the breach plug is slightly longer than the breach cavity. The preload induced by the applied bolt torque therefore places the plug in compression. The external load created by the charge detonation will be transmitted partly to the plug and partly to the bolt provided the joint does not separate. The total force in the plug will then be

\[
F_P = \frac{K_p}{K_p + K_b} F_e - F_i,
\]

where

\( K_p = \) plug stiffness, \((AE/L)_p\)
\( K_b = \) combined bolt stiffness, \((AE/L)_b\).

At the onset of separation between the breach plug and flange the total force in the plug will be zero. The external load is carried entirely by the bolts

\[
F_P = 0 = \frac{K_p}{K_p + K_b} F_e - F_i_c,
\]

where \( F_{i_c} \) is the critical preload at the onset of separation

\[
F_{i_c} = \frac{K_p}{K_p + K_b} F_e
\]
\[ K_p = \frac{AE}{L_p} = \frac{(19.6 \text{ in}^2)(30 \times 10^4 \text{psi})}{(2.5 \text{ in})} = 2.36 \times 10^8 \text{ lb/in} \]
\[ K_b = \frac{AE}{L_b} = \frac{(.334 \text{ in}^2)(30 \times 10^6 \text{psi})(12 \text{ bolts})}{(3 \text{ in})} \]
\[ K_b = 4.01 \times 10^7 \text{ lb/in} \]
\[ \frac{K_p}{K_p + K_b} = 0.855 \]

The actual peak pressure developed in the breach plug is unknown. However, as sited previously (8) the peak pressure at the surface of a spherical charge of TNT is approximately 70 kilobars or $10^6$ psi. Using this value along with the projected cavity area of 0.315 in$^2$ the external applied load is calculated to be
\[ F_e = (.315 \text{ in}^2)(1 \times 10^6 \text{psi}) = 3.15 \times 10^5 \text{ lb} \]
and the preload at the onset of separation is
\[ F_{ic} = \frac{(.855)(3.15 \times 10^5)}{(12 \text{ bolts})} = 2.24 \times 10^4 \frac{\text{lb}}{\text{bolt}} \]

The bolt torque at the onset of joint separation is
\[ T_c = cF_{ic}d = (.2)(2.24 \times 10^4 \text{ lb/bolt})(0.0625 \text{ ft}), \]
\[ = 280 \text{ ft-lb}. \]

Since the critical bolt torque ($T_c$) is lower than the maximum allowable torque ($T_m$) joint separation should not occur at the breach end.

A similar analysis may be performed at the muzzle end. By the scaling laws the peak pressure at the muzzle may be calculated as follows:
\[ P_m = 2.16 \times 10^4 \left( \frac{(125)}{11} \right)^{1.13} = 8,900 \text{ psi} \]
then
\[ F_e = (8,900 \text{ psi})(\pi/4)(6 \text{ in})^2 = 2.51 \times 10^5 \text{lb} \]

\[ \frac{K_p}{K_p + K_b} = 1 \]

\[ \bar{F_i} = \frac{(1)(2.51 \times 10^5 \text{lb})}{10 \text{ bolts}} = 2.51 \times 10^4 \text{lb/bolt} \]

\[ T_c = (0.2)(2.51 \times 10^4 \text{lb/bolt})(0.0625 \text{ ft}) = 314 \text{ ft-lb} \]

Since the maximum allowable bolt torque is the same as for the breach plate \( T_m = 337 \text{ ft-lb} \), joint separation will again be prevented.
III. SHOCK WAVE PROPAGATION THEORY

Kirkwood-Bethe Theory

The most well known and accepted theory describing shock wave propagation is that developed by J. G. Kirkwood and H. A. Bethe (10). In their analysis of shock wave propagation the hydrodynamic equations of momentum and continuity are written in terms of the kinetic enthalpy

\[ \Omega = w + \frac{u^2}{2} \approx \frac{p}{\rho} + \frac{u^2}{2}, \]

where

- \( w \) - enthalpy excess
- \( dw = \int ds + \frac{dp}{\rho} \)
- \( w = \int_{a}^{p} \frac{dp}{\rho} \approx \frac{p}{\rho} \)

- \( p \) - shock wave pressure excess
- \( \rho \) - local fluid density
- \( u \) - local particle velocity.

The theory shows that the principle propagation function, \( G(r,t) \) is propagated at a variable speed \( \bar{c} \) where

\[ G(r,t) = r\bar{\Omega}(r,t) \]

\( r \) = radial distance from the spherical charge center
\( \bar{c} = c + \sigma \)
\[ c = \text{local acoustic velocity} = \frac{\partial P}{\partial \rho} \mid_s \]

\[ \sigma = \text{Riemann function} = \int_{\rho_o}^\rho \frac{c d\rho}{\rho} \]

\[ \rho_o = \text{undisturbed density}. \]

The Riemann function is integrated over an isentropic path using the relation

\[ p = \frac{\rho \rho_o c_o^2}{n} \left[ \left( \frac{\rho}{\rho_o} \right)^n - 1 \right], \quad \text{III. 2} \]

where

\[ n = \text{a constant} \]

\[ \rho_o, c_o = \text{undisturbed values of density and acoustic velocity}. \]

Equation III. 2 is obtained from the adiabatic Tait equation of state for water. Integration of the Riemann function yields

\[ \sigma = \frac{2c_o}{n - 1} \left[ \left( \frac{\rho}{\rho_o} \right)^{\frac{n-1}{2}} - 1 \right]. \quad \text{III. 3} \]

The Riemann function turns out to be a convenient independent variable to use in expressing other shock wave parameters

\[ c = c_o \left[ 1 + \left( \frac{n - 1}{2 c_o} \right) \sigma \right] \]

\[ w = c_o \sigma \left[ 1 + \left( \frac{n - 1}{4 c_o} \right) \sigma \right] \]

\[ \Omega = c_o \sigma \left[ 1 + \left( \frac{n + 1}{4 c_o} \right) \sigma \right] \]

\[ U - \text{shock front velocity} = c_o \left[ 1 + \left( \frac{n + 1}{4 c_o} \right) \sigma \right]. \quad \text{III. 4} \]

The main facility in using the function, \( G(r,t) \), is that its value
at any location and time can be related to the value of $G$ on the gaseous sphere of explosion products at a retard time ($\tau$). That is

$$G(r, t) = G(a, \tau) \text{ or } G_a(\tau),$$

where "$a$" is the radius of the gas sphere. The retard time for the shock front is defined by the relation

$$\tau_0 = \int_{a_0}^{R} \frac{dr}{U(r, \tau)} - \int_{a(\tau_0)}^{R} \frac{dr}{c(r, \tau_0)},$$

where

$U(r, t) = \text{shock front velocity}$

$R = \text{location of the shock front relative to the center of the charge}$

$a_0 = \text{initial charge radius}$

$a(\tau_0) = \text{radius of the gaseous explosion products evaluated at time } t = \tau_0.$

The first integral represents the time required for the spherical shock front to arrive at the location $R$. The second integral is the time required for the value of $G$ which is presently existing at the shock front to travel from the gas sphere of radius $a(\tau_0)$ to the position $R$. The second integral is evaluated at constant $\tau_0$. Since $G(r, t) = G(\tau)$ this means a fixed value $G(a, \tau_0)$ (or $G_a(\tau_0)$) is followed from the gas sphere to location $R$. It can be shown (7) that shock wave quantities behind the front propagate faster than the front itself ($\bar{c} > U$). This so called "overtaking effect" results from the dissipation at the shock front which limits its rate of advance. Thus a value of $G$ behind the front will in time overtake and become the shock front only to be overtaken and destroyed by another value. As a result, the shock front delay time ($\tau_0$) is
positive and increases monotonically as the wave moves outward. The
shock front parameters are therefore related to values of \( G \) on the
gas sphere at successively later and later times.

The Kirkwood-Bethe theory "simplifies" the problem down to
determining the conditions of the gas sphere. Several approximations
are made in view of the overwhelming complexities, only the major
approximations will be discussed.

A detonation wave forms in the explosion products. This wave
is very similar to a shock wave in that a sharp discontinuity in
pressure exists. However, the pressure and speed of the detonation
wave is maintained by the release of energy at the front caused by
the explosive chemical reaction. A more exact theory would follow
the detonation wave to the boundary of the explosive upon which a
shock wave would be transmitted to the water and a rarefaction wave
reflected back to the center of the gas sphere. Kirkwood and Bethe
neglected the head and form of the detonation wave and approximated
the initial conditions with an adiabatic explosion at constant
volume. This is a reasonable approximation in that the head of the
shock wave formed by the head of the detonation wave is rapidly
destroyed by dissipation within a distance of two charge radii (7).

To determine the initial conditions of pressure \( (P(0)) \), par-
ticle velocity \( (u(0)) \) and enthalpy \( (w(0)) \) in the water at the
boundary of the gas sphere continuity of pressure and particle ve-
locity was employed along with the Rankine-Hugoniot conditions
which relate \( P(0) \) to \( u(0) \). With these values known it remains to
determine their variation with time at the boundary. Here another
important approximation was employed. The function $G(a,t)$ is assumed to initially decay exponentially

$$G(a,t) = G(a_0)\exp(-t/\theta) = a_0[w(o) + \frac{u(o)^2}{2}]\exp(-t/\theta).$$

The time constant ($\theta_1$) is chosen to coincide with the initial rate of pressure drop at the gas sphere boundary, which in turn is obtained from the continuity and momentum equations applied to the boundary. This technique of representing the initial variations at the boundary by an exponential is called the "peak approximation". This approach becomes increasingly in error after one time constant after which the pressure-history curve deviates significantly from an exponential and internal reflections in the gas sphere which were ignored in the theory come into play (7).

Determination of the pressure-history curve depends on further approximations regarding calculation of the retard time ($\tau$) and the dissipation factor ($\chi$) to be introduced shortly. As previously defined, the retard time is given by the expression

$$\tau(r,t) = t - \int_{a(\tau)}^{r} \frac{dr}{c(r,\tau)},$$

with

$$\tau_o(R,t_o) = t_o - \int_{a(\tau)}^{R} \frac{dr}{c(r,\tau)},$$

where

$$c(r,\tau) = c + \sigma - \text{speed of propagation of } G,$$

$$t_o = \int_{a_0}^{R} \frac{dr}{U} - \text{shock front arrival time at position } R,$$

$$\tau_o - \text{as before, the shock front retard time.}$$
The retard time for points not far behind the head of the shock wave can be approximated by a Taylor's series expansion about $\tau_0$

$$\tau(R, t) = \tau(R, t_0) + \frac{\partial \tau}{\partial t} \bigg|_R (t - t_0), \quad \text{III. 9}$$

$$= \tau_0 + 1/\gamma (t - t_0),$$

$$\gamma = \frac{\partial \tau}{\partial t} \bigg|_R$$

- the time spread parameter.

The time spread parameter ($\gamma$) relates the time scale at the shock front to that on the gas sphere. To obtain $\gamma$ Equation III. 7 is differentiated. The actual mechanics of obtaining $\gamma$ are complicated and tedious, the result being that $\gamma$ is a function of the ratio $a_0/R$.

The principle propagation function and the kinetic enthalpy can now be written

$$G(r, t) = G(a, \tau),$$

$$R\Omega(R, t - t_0) = G(a_0) \exp \left[ - \tau_0 + \frac{t - t_0}{\gamma \theta_1} \right].$$

Therefore

$$R\Omega(R, t - t_0) = G(a_0) \exp (-\tau_0) \exp \left[ - \frac{(t - t_0)}{\gamma \theta_1} \right].$$

However

$$G(a_0) \exp (-\tau_0),$$

$$x(\tau_0) = \frac{G(a, \tau_0)}{G(a_0)} = e^{-\tau}. \quad \text{III. 10}$$

The dissipation factor ($x$) is determined by evaluating Equation III. 8 for $\tau_0$. The result being that $x$ like $\gamma$ is a function of the ratio $a_0/R$. The expression for the kinetic enthalpy now becomes
\[ \Omega(R, t - t_0) = \frac{a_0}{R} \eta(0) x e^{-\frac{(t - t_0)}{\gamma \theta_1}} , \]

where
\[
\Omega = \frac{p}{\rho} + \frac{u^2}{2} .
\]

At distances far from the charge center \((R/a_0 > 25)\) the particle velocity is small and can be ignored relative to the pressure term. With this approximation the expression becomes

\[ P(R, t - t_0) = P_m e^{-\frac{(t - t_0)}{\theta}} , \]

\[ P_m = \rho(0) \frac{a_0}{R} \eta(0) x \]

\[ \theta = \gamma \theta_1 . \]

The Kirkwood-Bethe theory thus predicts an exponential decay for the shock wave with the peak pressure \((P_m)\) attenuating as a function of the ratio \(a_0/R\). Close to the explosion the particle velocity cannot be ignored and the pressure must be determined from the kinetic enthalpy and the equation of state for water.

Figure 16 shows the variation of \(x\) and \(\gamma\) with reduced distance \((R/a_0)\) for TNT as determined from the theory. Table 2 shows data for the peak pressure and time constant for TNT at various values of reduced distance.

Though the theory of Kirkwood and Bethe accurately predicts shock wave behavior it fails to provide a simple relation between shock wave parameters \((P_m, \theta, \text{etc.})\) and the range \((R)\) except for asymptotic conditions. For this reason the theory is often discarded for the empirical power law relationships called the scaling law.
Fig. 16. Dissipation and Time Spread Parameter (TNT)
**TABLE 2**

Shock Wave Parameters From The Kirkwood-Bethe Theory

<table>
<thead>
<tr>
<th>$R/a_o$</th>
<th>1</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1.00</td>
<td>0.399</td>
<td>0.300</td>
<td>0.258</td>
<td>0.231</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.00</td>
<td>4.04</td>
<td>6.43</td>
<td>8.11</td>
<td>9.55</td>
</tr>
<tr>
<td>$e/a_o$ (10^5 sec/cm)</td>
<td>0.344</td>
<td>1.39</td>
<td>2.21</td>
<td>2.79</td>
<td>3.29</td>
</tr>
<tr>
<td>$P_m$ (ksi)</td>
<td>538</td>
<td>20.2</td>
<td>6.10</td>
<td>2.62</td>
<td>1.17</td>
</tr>
</tbody>
</table>
Similarity and the Scaling Laws

The principle of similarity states that the pressure produced by two charges of the same explosive material and geometry will be the same if measured at equal values of reduced distance \((a_o/R)\) and reduced time \((t/a_o)\); where \(a_o\) is a characteristic dimension of the charge; \(R\) the position at which pressure is measured; and \(t\) is the time elapsed after the shock front arrival. Experiments conducted at Woods Hole (11) have verified the similarity principle for spherical charges.

The theoretical justification of similarity can be obtained by observing the characteristics of the governing hydrodynamic equations. For symmetrical spherical waves ignoring viscosity these equations are

\[
\frac{\rho du}{dt} = -\frac{\partial P}{\partial r} \quad \text{momentum,}
\]

\[
\frac{d \rho}{dt} = -\frac{\partial}{\partial r} \left( \frac{\rho}{r^2} \frac{\partial}{\partial r} (r^2 u) \right) \quad \text{continuity,}
\]

where, as before

\(\rho\) - fluid density
\(u\) - particle velocity
\(P\) - fluid pressure
\(r\) - radial position

and

\[
\frac{d(\_\_)}{dt} = u \frac{\partial (\_\_)}{\partial r} + \frac{\partial (\_\_)}{\partial t}
\]

is the substantial derivative.

If the scale is changed by the constant

\(r' = \lambda r\),
\(t' = \lambda t\).
Then by the chain rule

\[ \frac{du}{dt} = \frac{\partial u'}{\partial t'} \frac{dt'}{dt} + u' \frac{\partial u'}{\partial r'} \frac{dr'}{dr}, \]

where

\[ u' = u(r',t') \]

\[ \frac{du}{dt} = \lambda \frac{\partial u'}{\partial t'} + \lambda u' \frac{\partial u'}{\partial r'} = \lambda \frac{du'}{dt'}. \]

Similarly

\[ \frac{\partial P}{\partial r} = \frac{\partial P'}{\partial r'} \frac{dr'}{dr}, \]

\[ = \lambda \frac{\partial P'}{\partial r'}. \]

Therefore

\[ \rho \frac{du'}{dt'} = -\frac{\partial P'}{\partial r'}, \]

and the momentum equation is again satisfied by the primed coordinates. By the same argument the continuity equation can also be shown to satisfy the similarity principle. As a result the solution \( P(r,t) \) for a spherical charge of radius \( a_1 \) and the function \( P(\lambda r,\lambda t) \), both satisfy the governing equations. The function \( P(\lambda r,\lambda t) \), however, is the solution for a charge of radius \( \lambda a_1 \). Therefore

\[ P(r_1,t_1) = P(\lambda r_1,\lambda t_1). \]

Let

\[ r_2 = \lambda r_1 \]

\[ t_2 = \lambda t_1 \]

\[ a_2 = \lambda a_1 \]

then
Thus the pressures are equal at equal values of $a/r$ and $t/a$. The similarity principle therefore suggests a functional relationship for peak pressure of the form

$$P_m = f(a/R),$$

or

$$P_m = f(W^{3/2}/R).$$

since the radius of a spherical charge is proportional to the cube root of its weight. For the time constant we have

$$\theta = g(a/R),$$

or

$$\theta = W^{3/2}g(W^{3}/R).$$

The Kirkwood-Bethe theory satisfies these requirements, which should not be surprising since the theory is based on the same hydrodynamical equations.

To determine the actual relations that fit the described general forms one must resort to experiment. This has been done by Arons (3) and others. The well known equations for peak pressure and time constant are

$$P_m = k_1(W^{3}/R)^{\alpha},$$

$$\theta = k_2(W^{3})(W^{3}/R)^{\beta}.$$
\[ k_1 = 2.16 \times 10^4 \]
\[ \alpha = 1.13 \]
\[ k_2 = 58 \]
\[ \beta = -0.22 \]

R - range in feet
W - charge weight in pounds

These relations are called the scaling laws. The peak pressure power law follows the experimental data much better than does the scaling law for the time constant. In Figure 17 the dashed line represents the scaling law relation and the solid line represents the Kirkwood-Bethe theory. Arons (3) has shown that the peak pressure scaling law is accurate over a wide range from 10 to 10,000 charge radii.

The scaling laws and the principle of similarity are applicable to spherical charges detonated in free field i.e. in an infinite medium with no boundaries. These principles have not previously been verified inside a shock tube. The similarity principle is based on the assumption that fluid pressure is the only significant force acting. The interaction between the fluid and the tube walls will introduce other forces which may invalidate the similarity principle. Another factor to consider is the energy loss to the breach plug during detonation. Ideally this loss would be small. It is expected, however, that a considerable amount of work will be done on the breach plug during detonation. Whether this energy drain scales geometrically remains to be seen. Quite understandably, the validity of similarity and the scaling laws in this case can be best determined by experiment.
Fig. 17. Comparison of the Scaling Law and Kirkwood-Bethe Theory
IV. EXPERIMENTAL PROCEDURE AND RESULTS

Instrumentation

In order to check the scaling laws inside the shock tube the shock wave pressure-history curve must be measured and recorded at various ranges. Figure 18 is a schematic representation of the measurement system used in the experiment to fulfill this requirement.

The pressure transducer used in the tests was composed of a 0.25 inch diameter tourmaline disk. The assembly of this tourmaline disk into functional pressure gauge was performed by personnel at the Naval Research Laboratory (NRL), Underwater Sound Reference Division (USRD), Orlando, Florida. USRD also calibrated the gauge in their calibration tank facility yielding a calibration factor of $3.9 \times 10^{-4}$ volts/psi. There is some doubt as to the accuracy of this factor since the calibration tank generates a continuous sine wave with peak pressure not exceeding a few atmospheres while in actual testing the gauge was exposed to a transient signal consisting of a sharp rise in pressure followed by an exponential decay. To obtain more confidence in this calibration factor other methods were attempted to calibrate the gauge under conditions more similar to actual operating conditions. One method employed the crushing of a light bulb in the water filled shock tube by pressurization up to
Fig. 18. Schematic representation of the measurement system.
the bulbs crush strength at which point the collapsing vacuum produces a known pressure drop easily calculated from the initial and final volumes of the bulb and the isothermal compressibility of water. This attempt did not produce useful results because the ordinary light bulbs used crushed at too low a pressure (100 psi). Furthermore this collapse pressure was not consistent and did not occur stepwise as had been hoped. Another calibration technique which shows promise involves a free field detonation of a blasting cap in close proximity of the pressure gauge. Poche' (12) has demonstrated that these small detonators obey the scaling laws. He obtained the following peak pressure scaling law for M-6 detonators
\[ P_m = 2.29 \times 10^4 (W^3/R)^{1/2} \text{psi.} \]  
By placing the pressure gauge a known distance from the M-6 blasting cap \((W = 0.00193 \text{ lb})\) the output from the gauge in millivolts can be compared to the pressure predicted by Equation IV. 1. This procedure was carried out at 1.0 and 2.0 foot locations with the faces of the tourmaline disk oriented parallel to the direction of the on-coming shock wave. The axis of the gauge was placed perpendicular to the axis of the blasting cap. Figure 19 is a plot of the results. The slope of this line yields a calibration factor of 0.37. The piezoelectric tourmaline gauge has been the standard shock wave pressure transducer for some time due to its ruggedness and reliability. More recently, the semiconductor piezoresistive transducer has been developed which boasts a high natural frequency (395 KHz) and low output impedance. This prompted the purchase of two high pressure transducers from Kulite Semiconductor—model
Fig. 19. Free field calibration curve for the tourmaline pressure transducer.
HKM-375, capable of handling peak pressures of 30,000 psi. These Kulite gauges did not perform adequately; with ringing large enough to mask the desired signal. Subsequently, after a few shots, the Kulite gauges failed altogether. The tourmaline gauge was, therefore, used for all shock wave pressure measurement.

The oscilloscope used was a Tektronix model 7704-A. The scope was triggered externally by an accelerometer mounted on the shock tube. The output from the accelerometer was sent to an operational amplifier circuit which greatly amplified and then clipped the incoming signal. This output signal (basically a step signal) was then sent to the trigger circuit of the scope, and provided a reliable and repeatable trigger. A delay time was also placed in the circuit in an amount equal to the shock front arrival time to the pressure transducer.

The evolution of events during a typical shot are as follows:

1. Run blasting cap lead wires down the tube and feed them through the muzzle end plate and packing gland. Secure muzzle end plate and check continuity of the lead wires. Shunt the wires.

2. Load blasting cap and explosive into breach plug. Secure the breach plug.

3. Fill the tube.

4. Adjust vertical sensitivity, sweep rate and delay time for the scope. Test the triggering mechanism by jolting the shock tube and observing a trace.

5. Attach firing circuit to the lead wires.

6. Fire

7. Photograph

The scope used in the tests had storage capability which facilitated
the photographing of the shock wave.

**Data Reduction**

To test the similarity principle and the scaling laws the shock wave pressure-history curve was measured at various locations ranging from 2.0 to 9.0 feet from the breach plug at 1.0 foot increments. Figure 20 is a photograph of the apparatus employed in locating and securing the pressure transducer at a prescribed position. Eight different lengths of 5/8 inch aluminum conduit were employed (called stingers) with each successive stinger being one foot longer than its predecessor. With the longer stingers a small plexiglass wheel was designed to keep the pressure gauge centered in the tube. As can be seen in Figure 20 the wheel took quite a beating but still managed to function.

At each location the same shot was run two to three times. The photograph of each shot was placed on a drafting table and with the help of a T-square and triangle, pressure time coordinates were obtained. These data points were then plotted on semi-log paper to check for exponentiality and to determine the peak pressure and time constant. Figure 21 shows a sample of the results. The photograph is of two shock wave pressure-history curves both taken 4.0 feet from the breach plug with number 8 strength Dupont blasting caps (no plastic explosive). The semi-log plot of these curves is also shown in Figure 21. Circular and triangular data points are used to distinguish the two shots. The straight line running through the data points is a visual best fit.
Fig. 20. Stinger and Support Wheel
Fig. 21. Shock wave pressure-history curve
Data points were plotted at 10.0 microsecond intervals starting at the "peak". Data points past one time constant were not plotted since free field spherical shock waves no longer follow an exponential decay after one time constant (7). No attempt was made to "smooth out" the pressure-history curve. The data points obtained from the photographs were actual values regardless of whether the points fell on a peak or trough. The fact that the data points follow the straight line relationship on semi-log paper demonstrates that the waves are decaying exponentially, a characteristic of spherical shock waves.

The procedure discussed above was performed at each of the eight locations. From each of the semi-log plots of pressure versus time the peak pressure and time constant were determined. Table 3 summarizes the results. Determination of the peak pressure involved some approximations. As seen in Figure 21 an apparent peak and estimated peak are defined. The apparent peak is defined as the highest pressure which would have been sensed in the absence of any interfering inputs. In Figure 22 taken at the 6.0 foot location the peak of the wave appears to have been interfered with. At this time no systematic study has been conducted to determine the source of the interference, though radial wall oscillation and mechanical vibration of the pressure gauge due to the impulse of the shock wave are suspected as the main culprits. The apparent peak is determined by continuing the rise time line out and intersecting it with a line drawn tangent to the initial decay portion of the wave.

The estimated peak pressure is defined as the best estimate
### Table 3

**Summary of Shock Tube Data**

<table>
<thead>
<tr>
<th>Range (ft)</th>
<th>Weight (lb) x 10^3</th>
<th>Reduced Range (lb ft)</th>
<th>Peak Pressure (ksi)</th>
<th>Time Constant (µ sec)</th>
<th>Reduced Time Constant (µ sec / lb ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.6</td>
<td>0.058</td>
<td>19</td>
<td>87</td>
<td>740</td>
</tr>
<tr>
<td>3.0</td>
<td>1.6</td>
<td>0.038</td>
<td>12</td>
<td>94</td>
<td>800</td>
</tr>
<tr>
<td>4.0</td>
<td>1.6</td>
<td>0.029</td>
<td>8.4</td>
<td>100</td>
<td>850</td>
</tr>
<tr>
<td>5.0</td>
<td>1.6</td>
<td>0.023</td>
<td>6.5</td>
<td>140</td>
<td>1200</td>
</tr>
<tr>
<td>6.0</td>
<td>1.6</td>
<td>0.019</td>
<td>5.4</td>
<td>140</td>
<td>1200</td>
</tr>
<tr>
<td>7.0</td>
<td>1.6</td>
<td>0.016</td>
<td>4.9</td>
<td>130</td>
<td>1100</td>
</tr>
<tr>
<td>7.5</td>
<td>8.9</td>
<td>0.028</td>
<td>6.8</td>
<td>200</td>
<td>1700</td>
</tr>
<tr>
<td>8.0</td>
<td>1.6</td>
<td>0.014</td>
<td>4.0</td>
<td>170</td>
<td>1400</td>
</tr>
<tr>
<td>9.0</td>
<td>1.6</td>
<td>0.013</td>
<td>3.2</td>
<td>150</td>
<td>1300</td>
</tr>
</tbody>
</table>
Fig. 22. Shock Wave Pressure-History Curve Showing Interference
to the true peak pressure. As mentioned previously, the face of the circular tourmaline element is oriented parallel to the motion of the shock front. As a result, the true peak pressure is averaged with pressures immediately behind the front as the shock wave traverses the diameter of the disk. The estimated peak pressure was determined by an approximation suggested by Cole (7), in which the semi-log plot of the pressure-history curve is extrapolated back in time from the apparent peak an amount equal to the transit time from the edge to the center of the tourmaline disk. For a 0.25 inch disk and a shock front velocity of roughly 5000 ft/sec an extrapolation time of 2.1 microseconds is obtained. This correction amounted to less than a 3.0 percent increase over the apparent peak, and in light of the uncertainty involved in constructing the apparent peak may not be worth the effort.

Determination of the time constant proceeded as follows:

\[ P(t) = P_m e^{-\frac{t}{\theta}} \]

\[ \log P = \log P_m - (\log e) \frac{t}{\theta} \]

\[ \theta = \frac{t \log e}{\log \left( \frac{P_m}{P} \right)} \]

\[ \text{IV. 2} \]

The time constant was obtained by evaluating Equation IV. 2 using the maximum and minimum pressures on the semi-log plot for the ratio \( P_m/P \), with time, \( t \), equal to the base of the plot.

**Scaling Laws**

To check the scaling laws the estimated peak pressure and the reduced time constant are plotted versus reduced distance as in
Figures 23 and 24 respectively. It was originally planned to run several shots using different weights of explosive with the goal of covering a larger range of reduced distance ($\frac{W^{\frac{3}{2}}}{R}$) and to more convincingly verify similarity and the scaling laws. However, due to technical difficulties and time constraints only one shot was successfully completed using a value of weight other than a blasting cap. The circular data point in Figure 23 is this particular shot which utilized 3.36 grams of Detaprime along with the standard number 8 blasting cap. For this shot the pressure transducer was located 7.5 feet from the breach plug. The data point falls below the straight line. Though it is perhaps inappropriate to draw conclusions from a single shot, it seems logical that the increased weight of explosive caused losses inside the breach plug which did not scale properly. Obviously more data is required before any definite conclusions can be drawn.

The line in Figure 23 was drawn with a slope of 1.13 to coincide with the free field scaling law relationship. The attenuation of peak pressure as $\left(\frac{W^{\frac{3}{2}}}{R}\right)^{1.13}$ is characteristic of spherical shock waves in free field and in this respect the shock waves in the tube are behaving in the same manner. The scaling law relationship for the blasting cap data can easily be determined from the plot in Figure 23, the result being

$$P_m = 4.72 \times 10^5 \left(\frac{W^{\frac{3}{2}}}{R}\right)^{1.13}.$$  

where

$W =$ true weight of the explosive, 16

= 0.001616 for number 8 caps
Fig. 23. Shock tube peak pressure data.
Reduced time constant, $\theta / \lambda^{\frac{1}{3}}$ ($\mu$ sec/1 lb$^{\frac{1}{3}}$)

Reduced Distance, $\frac{w^{\frac{1}{3}}}{R}$, $\left(\frac{1}{1\text{b}^{\frac{1}{3}}}\right)$

Fig. 24. Shock tube time constant data
R = range, ft

\[ P_m = \text{peak pressure, psi} \]

The constant, \(4.72 \times 10^5\), is larger than that observed in free field, \(216 \times 10^4\), due to the amplification effect.

The time constant data plotted in Figure 24 does not follow the scaling law relationship nearly as well as the peak pressure data. However, a similar condition exists for the free field scaling laws in which the scatter of time constant data is considerably greater than in the case of peak pressure. In any case the trend of values does indicate the spreading of the shock wave and a corresponding increase in the time constant, typical of spherical shock wave in free field.

**True Amplification Factor and Explosive Efficiencies**

The true amplification factor can be determined from the shock tube scaling law by comparing it with the free field TNT scaling law. The following notation will be used:

\[ P_T = K_T \left( \frac{W_T}{R} \right)^{\frac{1}{2}} = \text{shock tube scaling law} \]  

\[ P_F = K_F \left( \frac{W_F}{R} \right)^{\frac{1}{2}} = \text{free field scaling law} \]  

The true amplification factor \((AF_{\text{TRUE}})\) is defined as the ratio of the equivalent weight of TNT required in free field to that required in the shock tube to produce the same peak pressure at the same range:

\[ AF_{\text{TRUE}} = \left. \frac{W_F}{W_T} \right|_{P_m, R} \]
This true amplification factor can be obtained by setting the two peak pressure scaling laws equal

\[ K_T \frac{W_T^3}{R} = K_F \frac{W_F^3}{R} \quad \text{IV. 7} \]

In solving for the ratio \( W_F/W_T \) the following relationship is obtained

\[ AF_{\text{TRUE}} = \frac{K_T}{K_F} \frac{3}{\alpha} \quad \text{IV. 8} \]

Upon substituting the following values:

\[ K_T = 4.72 \times 10^5 \]
\[ K_F = 2.16 \times 10^4 \]
\[ W_T = 0.0016 \text{ lb (Dupont #8 cap)} \]
\[ \alpha = 1.13 \]

the expression becomes

\[ AF_{\text{TRUE}} = 3600 \quad \text{IV. 9} \]

With regard to peak pressure, therefore, the 0.0016 pound blasting cap behaves as though it weighed 5.8 pounds.

It is of interest also to determine if the time constants in the shock tube compare favorably with those from a 5.8 pound charge. For a 5.8 pound charge in free field the time constant scaling law is

\[ \theta = (58) (5.8)^{\frac{1}{3}} \left( \frac{(5.8)^{\frac{1}{3}}}{R} \right)^{-0.22} \]
\[ \theta = 91.6 \text{ (R)}^{0.22} \quad \text{IV. 10} \]

In order to graphically compare this relationship with the shock tube data \( \theta \) must be divided by the weight of the blasting cap\( (W_T) \) and then plotted versus reduced distance \( (W_T/R) \). The line drawn through the data in Figure 24 is a plot of Equation IV. 10 and is
seen to be in general agreement with the shock tube data.

The explosive efficiency is defined as the true amplification factor divided by the theoretical amplification factor. The theoretical amplification factor depends only on the cone angle and was calculated in Equation II. 11 to the 7770. With a true amplification factor of 3600 the explosive efficiency is

\[ \eta = \frac{A_{\text{TRUE}}}{A_{\text{THEROY}}} = \frac{3600}{7770} = 46\% \]

The shock tube developed by Filler (1) produced an efficiency of 21%. The improved efficiency of this tube may be attributed to the overall larger stiffness associated with a 75% thicker breach plug and nearly a 200% increase in the minimum shock tube wall thickness. The higher stiffness minimizes wall deflections and the accompanying energy absorption.

Energy Loss Calculation

It is believed that the major energy loss (and therefore the principle cause for not achieving 100% efficiency) occurs in the breach plug. This is apparent since, as mentioned by Filler (1), any significant energy loss from the shock wave in the tube would have produced an abnormal attenuation of peak pressure with reduced distance -- a situation that was clearly not observed. In the calculations that follow the breach plug will be modeled as a right circular cylinder, 2.6 inches in diameter and 2.5 inches in length, with a 0.665 diameter hole drilled through the center. In reality the breach cavity does not extend completely through the plug; a
0.5 inch thickness is left to back up against the breach plate.

The energy absorbed by the breach plug is composed of elastic strain energy which is stored reversibly and plastic strain energy which is irretrievable. Though the elastic energy is returned to the shock wave it is taken from the shock front and returned to the tail thus lowering the peak pressure. The elastic energy is, however, relatively small in comparison to the plastic strain energy. It is calculated as follows:

\[ W_E = \left[ \frac{1}{2} \bar{\sigma}_r \bar{\varepsilon}_r + \frac{1}{2} \bar{\sigma}_\theta \bar{\varepsilon}_\theta \right] dV, \]  

where

- \( W_E \) = elastic strain energy
- \( \bar{\varepsilon}_r, \bar{\varepsilon}_\theta \) = elastic strains at the onset of yielding
- \( \bar{\sigma}_r, \bar{\sigma}_\theta \) = radial and circumferential stresses at the onset of yielding
- \( V \) = volume of the breach plug

These elastic strains are determined from the constitutive relations, the Tresca yield condition and the fact that, at a given location, \( r \), the radial stress must be equal in magnitude to the onset of yielding pressure (Equation II. 28) with "a" replaced by \( r \). After some manipulation the following expressions are obtained:

\[ \bar{\varepsilon}_r = -\frac{2}{3} \frac{Y}{E} \left[ 1 - \frac{1}{3} (r/b)^2 \right] \]

\[ \bar{\varepsilon}_\theta = \frac{2}{3} \frac{Y}{E} \left[ 1 + \frac{1}{3} (r/b)^2 \right] \]  

The radial and circumferential stresses at the onset of yielding are

\[ \bar{\sigma}_r = -\frac{Y}{2} \left[ 1 - (r/b)^2 \right], \]

\[ \bar{\sigma}_\theta - \bar{\sigma}_r = Y. \]
Therefore
\[ \sigma_\theta = \frac{Y \Gamma_1}{2} + (r/b)^2 \] . \hspace{1cm} \text{IV. 14}

Applying these relations to Equation IV. 14 yields the following relation for the elastic work:

\[ W_E = \frac{Y^2 \pi L b^2}{bE} \int_0^1 (2 + x^2) \, dx \] \hspace{1cm} \text{IV. 15}

where

\[ x = (r/b)^2 \]

\[ Y = 50,000 \text{ psi} \]

\[ L = 2.5 \text{ in. breach plug length} \]

\[ b = 2.6 \text{ in.} \]

\[ E = 30 \times 10^6 \text{ psi} \]

Upon integrating Equation IV. 17 a value of 141 ft-lb is obtained for the elastic work \( W_E \).

The plastic work can be evaluated by the expression

\[ W_P = \int_V \left[ \int_0^{\Delta \varepsilon_r} \sigma_r \delta \varepsilon_r + \int_0^{\Delta \varepsilon_\theta} \sigma_\theta \delta \varepsilon_\theta \right] \, dV \] \hspace{1cm} \text{IV. 16}

where

\[ \Delta \varepsilon_r, \Delta \varepsilon_\theta - \text{total plastic strain increment}, \]

\[ \delta \varepsilon_r, \delta \varepsilon_\theta - \text{infinitesimal plastic strain increments}. \]

The integrals inside the brackets represent the plastic strain energy density, which when integrated over the volume gives the total plastic strain energy. These energy density terms are integrated from zero plastic strain (i.e. from the onset of yielding) out to the final amount of plastic strain \( (\Delta \varepsilon_r, \Delta \varepsilon_\theta) \). Therefore, the plastic work calculation starts where the elastic work calculation left off.
Integration of the plastic work expression is simplified by the following principles:

$$\varepsilon_r + \varepsilon_\theta = 0$$ constant volume deformation

$$\sigma_\theta - \sigma_r = Y$$ Tresca yield condition \[IV. 17\]

With the application of Equation IV. 17 the plastic work expression becomes

$$W_p = \int_V \left[ \int_0^\Delta \varepsilon_\theta \right] dV,$$

$$= \int_a^b Y \Delta \varepsilon_\theta 2\pi r dr. \quad \text{IV. 18}$$

The quantity $\Delta \varepsilon_\theta$ is a function of the radial position, $r$, and can be determined as follows:

$$\varepsilon_r = \frac{du}{dr} \quad \text{small strains}$$

$$\delta \varepsilon_r = \delta \frac{du}{dr} = \frac{d}{dr} (\delta u)$$

$$\varepsilon_\theta = \frac{u}{r}$$

$$\delta \varepsilon_\theta = \frac{\delta u}{r} \quad \text{IV. 19}$$

Therefore, by Equation IV. 19

$$\frac{d}{dr} (\delta u) + \frac{\delta u}{r} = 0,$$

upon integrating

$$\delta u = \frac{c}{r} \text{ or } \delta u = b \frac{\delta u_b}{r} ,$$

where

$$\delta u_b = \delta u \bigg|_r = b.$$

Therefore

$$\delta \varepsilon_\theta = b \frac{\delta u_b}{r^2} ,$$
Combining Equations IV. 19 and IV. 20 and integrating gives the following result for the plastic work

\[ W_p = 2\pi L b Y \Delta u_b \ln \frac{b}{a}. \]

Plastic radial deformations (\( \Delta u_b \)) of 0.015 inches were measured after the detonation of 1.5 inches of Detaprime with a number 8 strength blasting cap. With this value of radial displacement and with the other parameters as defined previously in the elastic work calculation a plastic work of 5,270 ft-lb is obtained. Combining this with the elastic work and converting to BTU's gives a total strain energy storage of 6.9 BTU.

The energy lost in the explosion can be calculated by another means and compared with the preceding result. According to Arons and Yennie (13) roughly 53\% of the chemical energy liberated during an underwater explosion in free field is delivered to the initial shock wave. Therefore for perfect amplification the maximum radiated shock wave energy (MRE) would be

\[ \text{MRE} = (9.73 \text{ gm})(1060 \frac{\text{cal}}{\text{gm}})(0.53) = 5466 \text{ cal.} \]

The estimated radiated shock wave energy (ERE) can be determined by assuming an AF of 2000, which is approximately the AF achieved for the circled data point in Figure 23.

\[ \text{ERE} = (9.73 \text{ gm})(2000)(1060 \frac{\text{cal}}{\text{gm}})(0.53)/7770 = 1407 \text{ cal.} \]

The divisor of 7770 appears in the ERE because though the 9.73 gm charge is amplified by 2000 the energy received is only a fraction
The energy lost from the shock wave is then

\[ E_{\text{LOST}} = M_{\text{RE}} - E_{\text{RE}}, \]

\[ = 5466 - 1407 = 4059 \text{ cal.}, \]

\[ = 16.1 \text{ BTU}. \]

Comparing this with the breach plug work calculation of 6.9 BTU we see that approximately 9 BTU of energy has not been accounted for. Possible explanations are as follows:

1. The work calculation is in error, due to the fact that the calculation was based on the deformation of the outside fibers rather than the deformation on the inside of the breach cavity.

2. The true amplification factor is higher than 2000 causing the ERE calculation to be too low. This does not seem likely since for the shot run with 4.1 gm of TNT equivalent explosive produced an AF of 2200.

3. The 9 BTU of unaccounted energy was transferred from the breach plug into the shock tube as a compression wave.
V. CONCLUSIONS AND RECOMMENDATIONS

For the tests run with the small number 8 strength detonators the shock tube performed admirably. The 0.0016 pound detonator generated peak pressure and time constant data characteristic of a 5.8 pound spherical charge of TNT which translates into a weight amplification \( \text{AF}_{\text{TRUE}} \) of 3600. This true amplification factor amounted to 46% of the theoretical amplification factor of 7770, the difference being accounted for by the energy lost to the breach plug. That the shock waves were spherical in nature was attested by the fact that the pressure-time curves were exponential and that the peak pressure decayed as \( (1/R)^{1.13} \), both characteristics of shock waves from spherical charges of TNT.

The peak pressure and time constant data from the small detonators were shown to satisfy the scaling laws. However, the addition of 3.37 grams of plastic explosive (6 times the weight of the cap) to the blasting cap produced a peak pressure 12% lower than the value predicted from the shock tube scaling law based on blasting cap data alone. The lower than predicted peak pressure at higher weight was expected for the following reason. In order for the scaling law to be applicable over a large range of charge weights the energy loss to the breach plug would have to scale in proportion to the charge weight; only then would the true amplification factor
be a constant. Apparently this was not the case and a larger charge weight produces disproportionately larger energy losses to the breach plug, resulting in a true amplification factor which decreases with the charge weight rather than being constant. It is believed, therefore, that though the shock waves which leave the breach plug are spherical in nature, their apparent weight \( W_{\text{APPARENT}} = \frac{AF}{W_{\text{TRUE}}} \) will be dependent on the magnitude of the energy lost to the breach plug. The affect of this variable AF on the peak pressure scaling law is as follows:

\[
P_m = K \left[ \frac{(AF W_{\text{TRUE}})^{\frac{3}{2}}}{R} \right]
= K(\text{AF})^{\alpha/3} \left( \frac{W_{\text{TRUE}}}{R} \right)^{\alpha}
\]

Due to the non-linear relationship between charge weight and energy loss the AF will be a function of the charge weight

\[
P_m = g(W_{\text{TRUE}}) \left( \frac{W_{\text{TRUE}}}{R} \right)^{\alpha}
\]

The function \( g(W_{\text{TRUE}}) \) would account for the expected decrease in the AF with charge weight.

This expression can be used to explain the observed data. For the shots that employed a single blasting cap \( W_{\text{TRUE}} \) is fixed and therefore \( g(W) \) is a constant thus yielding the typical scaling law relationship that was determined in Equation IV. 4. However, for the one shot that utilized the extra weight of Detaprime, \( g(W) \) decreased resulting in a lower value of \( P_m \) than anticipated from a constant AF. Without a doubt, more empirical data on peak pressure with variable weight is needed to verify this hypothesis. It is however,
a logical deduction from the sparse data now available.

If indeed the AF is verified as being largely variable with weight, then the scaling laws will be less general. In any case a more efficient explosive geometry needs to be designed that will limit the energy losses to the breach plug, improve the explosive efficiency and hopefully insensitize the AF to the charge weight.

Improvements also need to be made in the measurement system and in the data reduction. The average scatter of data points in plotting the pressure time curves on semi-log paper amounted to \( \pm 14\% \) deviation from the straight line. As previously mentioned it is believed that the cause of the scatter is due to mechanical vibration of the pressure transducer which produce the random "glitches" in the pressure time record as seen in Figure 21. These random irregularities became more pronounced when the gauge was placed closer to the muzzle end where the thinner tube wall allowed greater wall deflection. To improve the accuracy of the time constant (and to some extent the peak pressure) data a means of isolating the transducer package from the shock tube needs to be designed.

Uncertainty in the peak pressure measurement arose mainly from the occasional clipping of the peak as in Figure 22. Though the cause of the problem has not been determined a possible explanation would implicate the oscilloscope. The Tektronix 7704-A digitizes the signal by sampling 512 points across the screen regardless of sweep rate. Though this sampling rate is more than adequate for capturing the decay of the shock wave the very rapid rise and initial fall of pressure may have been too rapid for the scope. For a
sweep rate of 50 microseconds per division the sampling interval is roughly 1 microsecond. In Figure 21 the initial rate of pressure rise is approximately 800 psi/microsecond. It is recommended that further tests be conducted with a standard continuous reading oscilloscope and any improvement in the peak pressure measurement be verified.

If more time had been available a least squares linear regression would have been done on the pressure-history curves to determine the time constant and on the peak pressure versus reduced distance curve to obtain K and \( \alpha \) for the scaling law. It would also be appropriate to provide confidence limits for these parameters using common statistical techniques.
APPENDIX A

Derivation of the Plastic Collapse Pressure

See Figure 13 for a schematic of the shock tube during elastic-plastic deformation. The plastic collapse pressure ($P_c$) can be determined from the following equations:

\[ \frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} \quad \text{Equilibrium} \]

\[ \sigma_\theta - \sigma_r = Y \quad \text{Tresca yield condition} \quad A. 1 \]

Combining these equations and integrating gives the following:

\[ \sigma_r = Y \ln r + K \quad a < r < c \]

\[ K = \text{integration constant} \quad A. 2 \]

The following boundary condition may be used to determine the value of $K$:

\[ \sigma_r \bigg|_{r = c} = \frac{-Y}{2} \left[ 1 - \frac{c^2}{b^2} \right] \quad A. 3 \]

In other words, at the interface between the elastic and plastic zone ($r = c$) the material is at the onset of yielding. The elastic zone is unable to distinguish between the plastic zone and a fluid applying a pressure equal to the onset of yield pressure ($P_{oy}$, Equation II. 28).

Applying the boundary condition to Equation A. 2 results in the following expression for the radial stress in the plastic zone:
\[ \sigma_r = \frac{-Y}{2} \left[ 1 - \frac{c^2}{b} \right] - \frac{Y}{r} \ln \frac{c}{r} \]

\[ a < r < c \quad \text{A. 4} \]

Upon evaluating Equation A. 4 at the inner wall \( r = a \) the following expression is obtained:

\[ \sigma_r \bigg|_{r = a} = -P = \frac{-Y}{2} \left[ 1 - \frac{c^2}{b} \right] - \frac{Y}{a} \ln \frac{c}{a} \]

\[ P = Y \ln \frac{c}{a} + \frac{Y}{2} \left[ 1 - \frac{c^2}{b} \right] \quad \text{A. 5} \]

Equation A. 5 gives the pressure required to produce a plastic zone out to the radial location \( r = c \). The plastic collapse pressure is obtained from Equation A. 5 by setting \( c = b \):

\[ P_c = Y \ln \frac{b}{a} \quad \text{A. 6} \]

This same expression may be obtained more directly by an energy method.
APPENDIX B

Shock Wave Oscilloscope Photographs

Note: In the photographs that follow the range at which each photograph was taken is listed in the caption with the first number referring to the top picture.
7 ft, 8 ft
REFERENCES


