Analysis of a Solar Collector in an Absorption Cycle Including a Pump Operated by Thermoelectric Cells

Spring 1981

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ANALYSIS OF A SOLAR COLLECTOR IN AN ABSORPTION CYCLE INCLUDING A PUMP OPERATED BY THERMOELECTRIC CELLS

BY

STUART JEE-FONG CHEN
B. S., Feng-Chia Engineering and Commercial College, 1974

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Sciences in the Graduate Studies Program of the College of Engineering at the University of Central Florida at Orlando, Florida

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ABSTRACT

A mathematical computer model has been developed for the prediction of the thermal performance of a flat-plate solar collector used as the generator for an absorption cycle cooling system. The system employs thermoelectric cells to power its pump. The exponential relationship between the auxiliary energy and the collector area is used to optimize the system. It is predicted that the system will become feasible when the fuel price is raised to four times the current value.
ACKNOWLEDGEMENT

I appreciate the patient advice and able assistance of Dr. K.K. Chang who made this work possible. I am also indebted to Dr. P.J. Bishop and Professor Minardi for their generous comments and suggestions.
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<tr>
<td>( A_c )</td>
<td>Solar collector area</td>
</tr>
<tr>
<td>( A_{\text{Op}} )</td>
<td>Optimum collector area</td>
</tr>
<tr>
<td>( A_{\text{SC}} )</td>
<td>Solar cell area</td>
</tr>
<tr>
<td>( C_c )</td>
<td>Unit cost of solar collector panel</td>
</tr>
<tr>
<td>( C_f )</td>
<td>Unit fuel cost</td>
</tr>
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<td>( C_{\text{sc}} )</td>
<td>Solar cell cost</td>
</tr>
<tr>
<td>( C_y )</td>
<td>Other directly related collector costs</td>
</tr>
<tr>
<td>( e )</td>
<td>Electronic charge</td>
</tr>
<tr>
<td>( F_R )</td>
<td>Collector heat removal factor</td>
</tr>
<tr>
<td>( G )</td>
<td>Flow rate per unit collector area</td>
</tr>
<tr>
<td>( H )</td>
<td>Rate of incidence of beam or diffuse radiation on a unit area of surface of any orientation</td>
</tr>
<tr>
<td>( i )</td>
<td>Annual interest rate</td>
</tr>
<tr>
<td>( K )</td>
<td>Boltzmann's constant</td>
</tr>
<tr>
<td>( P_o )</td>
<td>Electrical power output</td>
</tr>
<tr>
<td>( Q_{\text{aux}} )</td>
<td>Annual auxiliary energy consumption</td>
</tr>
<tr>
<td>( Q_L )</td>
<td>Rate of energy losses from the collector to the surroundings</td>
</tr>
<tr>
<td>( Q_m )</td>
<td>Constant depending on solar system characteristics, location and operating conditions</td>
</tr>
<tr>
<td>( Q_S )</td>
<td>Rate of energy storage in the collector</td>
</tr>
<tr>
<td>( Q_u )</td>
<td>Rate of useful heat transfer to a working fluid in the solar exchanger</td>
</tr>
<tr>
<td>( R )</td>
<td>Factor to convert beam or diffuse radiation to that on the plane of the collector</td>
</tr>
<tr>
<td>( S )</td>
<td>Solar energy absorbed by the plate</td>
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LIST OF SYMBOLS (cont'd)

\( S' \) Scattering constant
\( T \) Temperature
\( T_a \) Ambient temperature
\( T_{f,i} \) Collector inlet fluid temperature
\( T_{f,o} \) Collector outlet fluid temperature
\( U_b \) Bottom loss coefficient
\( U_L \) Total loss coefficient
\( U_t \) Top loss coefficient
\( Z^* \) Figure of merit
\( \alpha \) Seebeck coefficient
\( \lambda \) The fuel consumption decay constant
\( \lambda' \) Thermal conductivity
\( \tau \alpha \) Transmittance - absorbance product of cover system for beam or diffuse radiation
\( \rho \) Electrical resistivity
\( \pi \) Peltier coefficient
\( \varepsilon \) Energy, electron volt
\( \eta_t \) Thermal efficiency
\( \gamma \) Saving to cost ratio
CHAPTER I
INTRODUCTION
CHAPTER I

INTRODUCTION

Renewed interest in solar energy has developed since 1970 as a result of increasing costs of energy from conventional resources and the problems of importing and extracting fuels that are acceptable from environmental standpoints.

Solar air conditioners of several types have been proposed. Löf [1] has described a dehumidification system in which the absorbent is regenerated by solar heated air. Intermittent absorption coolers regenerated by solar energy have been studied by Williams, et.al.[2] and cycles of this type could be applied to space cooling.

Intermittent cycles which can cool during regeneration, a necessary feature for solar air conditioning, must have storage capacities for spent and regenerated absorbent, and perhaps also for refrigerant. In principle, compression refrigeration systems can be operated by solar-generated electricity, but these systems appear impractical.

Florida has large solar radiation insolation and a long lasting summer. Cooling by solar energy appears desirable. A solar absorption cooling system was built in the Solar Energy Laboratory at Wisconsin in 1962 [3]. But this system did not take advantage of using solar cells to provide needed electricity to operate the pump. Today it is found that the thermoelectric cells will fit the application
of this kind of system for cooling purposes.

In this study the design of a absorption cycle using the solar collector as a generator with the pump supplied by thermoelectric cells will be investigated. A simulation computer program is developed for the system which is used to find the useful energy supplied by the solar collector and the auxiliary energy needed to supply a 2 ton cooling load under varying environmental conditions.

The solar collector area of a self-sufficient system is too big to be installed on an ordinary house. Different collector areas are used in the computer program to find the relationship between them and the auxiliary energy. The curve obtained can be used to predict the auxiliary energy needed at any collector area by interpolation.

Optimum area for the current interest rate, fuel cost and collector cost will be considered in the Economic Study section by applying the formulation derived by Chang and Minardi [4]. The computation result will enable us to foresee the potential of this solar cooling system in the future with respect to economic factors.

System Description

The principal function of a generator used in the absorption cycle is to drive off the refrigerant (steam) from the absorbent (Lithium bromide solution) by the heat supplied to it. A solar collector is used as a generator in this study and thermoelectric cells are set behind the absorbing plate to convert the temperature difference between the absorbing plate and the inlet fluid into electricity to drive the pump. A heat exchanger is placed in the
liquid solution circuit between the generator and absorber. The generator and condenser are on the high-pressure side of the system, while the evaporator and absorber are on the low-pressure side. The whole system is schematically shown in Fig. 1.
A computer program is presented in Appendix (I) to simulate the performance of the solar collector in the three months of June, July and August in 1979. An iteration method must be used to find the absorbing plate temperature before the useful energy can be calculated.

To make the system more practical auxiliary energy is needed to decrease the collector size. This auxiliary energy is calculated for the ambient temperature range from 298 K to 311 K.

The analysis on the performance of this system will be considered in three major parts -- the absorption cycle, solar collector and thermoelectric cells. Detailed calculations for these three major components are attached in Appendix (II) - (IV) for the purpose of illustrating those procedures used to find the collector size in the month of June 1979.

The two major portions of the study that will be discussed are:

(I) Absorption cycle: The fundamental physical properties of binary mixtures will be reviewed in terms of a simple test. The use of an h-x (enthalpy-concentration) diagram is highlighted as well. A theoretical simple absorption model is established to derive those formulas for this cycle. Calculations based upon the absorption cycle are shown in Appendix (II).

(II) Solar collector: Flat plate collectors are analyzed in detail as they are basic to the solar processes of most interest. Equations are developed based upon the work of H.C. Hottel in 1942 [5]. More environmental factors are considered than those in Hottel's study. Calculations for the solar collector are shown in Appendix
(III) for the purpose of showing the iteration method used to find the collector temperature, and in conjunction with the absorption cycle to find the area of the collector. In a later chapter the basic heat transfer concepts will be reviewed briefly. Direct energy conversion considerations are reviewed by applying a simple model. Applications that are important to solar heating are discussed by defining some pertinent energy conversion terms. Calculations of this device needed to find the maximum efficiency and maximum output are shown in Appendix (IV).
CHAPTER II
LITERATURE SURVEY
CHAPTER II

LITERATURE SURVEY

There has been less experience with solar cooling than solar heating. Several solar heated buildings have been designed, built and operated for extended periods, carefully studied, and the results published in the last several years. Modeling of cooling systems has been accomplished in a manner that does allow the evaluation and identification of critical design areas.

Among all the current studies on solar air conditioning two conducted by Chung, et al in 1963 [3] and InterTechnology/Solar Corporation in 1975 are similar to the system analyzed in this research. Their work will be outlined in this chapter for reference purposes.

The research reported by Chung et al [3] is one of the few experiments which have been done on the solar operation of continuous LiBr-H₂O absorption coolers with flat-plate collectors. In this study no auxiliary energy supply was used and no energy storage was provided. The cooler was an Arkla machine of nominal 3 ton capacity that was designed for steam heating of the generator. External modifications were made to the generator to allow solar heated water to be supplied.

InterTechnology/Solar Corporation was selected by the U.S. Energy Research and Development Administration early in 1975 to
design a solar heating and cooling system for the Page-Jackson Elementary School in Jefferson County, West Virginia [6]. This study was mostly concerned with the best load distribution and equipment combination for their specific application. According to their research a 28 ton solar absorption system was used.

(A) Absorption Cycle

Two approaches have been taken with regard to the solar operation of absorption coolers. The first is to use continuous coolers, similar in construction and operation to conventional gas or steam fired units, and with energy supplied to the generator from the solar collector-storage-auxiliary system. The second approach is to use intermittent coolers similar in concept to that of commercially manufactured food coolers.

Experience to date suggests that continuous absorption cycles may be adapted for operation from flat-plate collectors. An analytical study of the solar operation of a LiBr-H₂O cooler and flat-plate collector combination has been carried out by Duffie and Sheridan in 1965 [7] to identify critical design parameters and to assess the effects of operating conditions on integrated solar operation. They concluded that the generator design is more critical here than in fuel-fired coolers because of the coupled performance of the collector and cooler.

Farber et al in 1970 [8] have studied a series of solar-operated ammonia-water coolers, using flat-plate collectors, without storage. Operation was continuous and at varying rates, depending on energy
supply.

Intermittent absorption cooling may be an alternative to continuous systems. The limited work to date on these cycles has been largely directed at food preservation rather than comfort cooling.

Swartman and Swaminathan in 1970 [9] have been experimentally studying the operation of intermittent NH₃-H₂O machines in which flat-plate collectors have served as the energy supply. With cooling water available at about 30°C the effective cooling per unit area of collector surface per day for the experimental machine was in the range of 50 to 85 KJ/m² for clear days.

The study done by Bosnjakovic in 1965 [10] on absorption cycles is used in this paper to establish a simple theoretical absorption refrigeration system. By applying the derived formulas the performance of the refrigeration cycle with a solar collector as an energy supply will be analyzed.

(B) Solar Collectors

The great magnitude of solar insolation on the earth has stimulated a number of attempts at its more effective utilization during the past few decades.

Some flat plate collectors have been built and described. Brooks in 1936 [11] has studied several types of natural convection solar water heaters in use in California, and gives some quantitative data on performance but no correlation with measurements of solar intensity. Thus it is seen that previous work on this problem has not yielded quantitative relationships for predicting the effects
of the many factors which contribute to the overall performance of the flat plate collector.

The analysis presented in this paper follows the basic derivation by Hottel and Whillier in 1958 [12] and Bliss in 1959 [13]. These are the first studies to analyze the performance of a solar collector quantitatively. Current research on how to improve the solar collector performance will be outlined in the discussion chapter of this paper.

(C) Thermoelectric Cells

In the analysis of both the thermoelectric generator and cooler the importance of the materials parameter called the figure of merit will be demonstrated in Appendix (IV). The equations of performance and the plotted results show that even small increases in the figure of merit can produce significant changes in the thermal efficiency and coefficient of performance. But on the other hand we must recognize that the development of new thermoelectric materials to a certain extent is still more of an art than a science.

In 1965 Klem and Dingwall [14] developed a silicon-germanium alloy thermocouple for the RCA Company. The silicon-germanium has been used to form hot junction electrodes. The cold ends are connected to the heat rejection plate by means of an insulating disc. This device is well suited to operate at relatively high temperatures.

Nowadays many research groups in the world have been conducting experiments trying to discover the methods and any new material that could improve the performance of the thermoelectric cells.
The current research shows the promise of compounds $\text{Ag}_2\text{Se}$, $\text{Ag}_2\text{Te}$ and $\text{Cu}_2\text{Te}$ as a basis for obtaining composite alloys with high thermoelectric properties. After the thermoelectric material has been improved satisfactorily a thermoelectric cell possibly can compete with conventional cooling systems and also be used as a compact generator.

In the following chapter those formulas derived by Angrist in 1977 [15] will be used to calculate the electricity output of the thermoelectric cells attached behind the absorbing plate by establishing a thermoelectric generator model.

(D) Economics

Optimum area of a solar system is determined at the peak of a saving versus collector area curve. The procedures used to find this area are often tedious and time consuming. The optimization formulation developed by Chang and Minardi in 1979 [4] is used in this paper because of its simplicity.
CHAPTER III

MODELING
CHAPTER III

MODELING

The solar air conditioning system consists of three major components: the absorption cycle, solar collector and thermoelectric cells. The performance of each component is interlocked with that of the other two. In other words, one cannot find the physical properties at each state point without knowing the performance information of all the other state points. In this study the physical properties at the condenser and evaporator in the absorption cycle are assumed to be known (see Appendix II). Based on this information and the application of the h-x chart, the total energy needed by the absorption cycle to generate 2 tons of cooling under specific environmental conditions can be found. Obviously this amount of energy should be equal to the energy supplied by the solar collector.

For the purpose of better understanding the procedures used in this study a flowchart is shown in Fig. 2. In addition to the designated evaporating and condensing temperatures, environmental variables such as ambient temperature are substituted into the mathematical model to find pump work and energy needed by the absorption cycle. By applying the thermoelectric cell model, the amount of thermoelectric cells needed can be found from the known pump work.

Equivalence between heat supplied by the collector and energy needed
Fig. 2. Flow chart of calculation procedures
by the absorption cycle can be used to find the collector area from substituting solar data into the solar collector model.

(A) Introduction

Models for the three major components used in this system will be discussed in the following section.

Absorption Cycle

Since the most prevalent types of cooling or dehumidifying systems today employ a refrigerant compressor, the other methods used for refrigeration are not so well known. Of these other methods used, the most common is the absorption refrigeration system, developed by Ferdinand Carre in France. In the early years of the twentieth century absorption refrigeration gained considerable prominence, but after 1915 the electric-motor-driven, fully enclosed ammonia compressor was more actively promoted and received widespread acceptance. Development work was then concentrated on compression systems, and absorption systems were practically forgotten, except for domestic units, until the late 1930's. For the compression system, the energy input is shaft work, which is high-grade and very expensive. Large amounts of work are required because in compression the vapor undergoes a large change in specific volume.

The principal advantage of the absorption refrigeration cycle is that only a small amount of work is needed by the pump which in turn is supplied by the thermoelectric cells, but a heat input many times greater than the work input of the mechanical vapor-compression
cycle is required. Wherever heat is sufficiently abundant, the absorption cycle will be attractive economically.

Solar Collector

The performance of a solar collector is described by an energy balance that indicates the distribution of incident solar energy into useful energy gain and various losses. The energy balance on the whole collector can be written as [15]

\[ A_c \left( [HR(\tau \alpha)] + [HR(\tau \alpha)]_d \right) = Q_u + Q_L \quad (1) \]

The detailed analysis of a solar collector is a very complicated problem. Fortunately, a relatively simple analysis will yield very useful results. These results will show the important variables, how they are related, and how they affect the performance of a solar collector.

Thermoelectric Cells

A temperature difference existing between the absorbing plate and inlet working fluid makes the application of the thermoelectric semiconductors as an electrical power source possible. One other advantage of using the thermoelectric semiconductors is because of its low thermal conductance which can be used as insulation to prevent heat dissipation from the absorption plate. In the absorption cycle section the power needed to drive the compression pump is very small, which makes the application of solar cells practical and
reasonable. The calculation used to find the amount of solar cells needed is in Appendix (IV).

The first person to notice the deflection of a magnetic needle when held near a circuit made of two different conductors was Thomas Johann Seebeck. Today his observation is called the Seebeck Effect and is defined as \( \frac{dV_{A,B}}{dT} = \alpha_A - \alpha_B \).

Thirteen years after Seebeck reported his discovery a French watchmaker, Jean Charles Athanase Peltier, discovered that the passage of a current through a junction formed of two different conductors caused absorption or liberation of heat. Today his observation is known as Peltier Effect which is defined as

\[
\pi_T(p) = \frac{1}{e}(\epsilon_f + 2KT)
\]

for p-type material and

\[
\pi_T(n) = -\frac{1}{e}[(\epsilon_g - \epsilon_f) + (\frac{5}{2} - S)KT]
\]

for n-type material. Like Seebeck, Peltier failed to explain his observation correctly.

Lord Kelvin (William Thomson) realized that a relation between the Seebeck and Peltier effects should exist, and proceeded to derive this relation from thermodynamic arguments. The reasoning that he used led him to conclude that there must be a third thermoelectric effect, the Thomson Effect, which is defined as

\[
\pi_T' = -\frac{V_D}{\Delta T} + 2K/e \quad \text{for the p-type material.}
\]

The basic theory of thermoelectric generators and refrigerators was derived in 1901 and 1911 by Alfendirch. His work indicated that for both applications materials were needed with high Seebeck coefficients, high electrical conductivities to minimize Joule heating, and low thermal conductivities to reduce heat transfer
through the devices. Though Alfenkirch enumerated the desirable properties for materials to be used in thermoelectric devices, 50 years passed before those materials became known and widely available. Shortly after semiconductors initiated the technological revolution of the 1950's thermoelectricity was, in a sense, re-discovered in the old physics texts and put to use in a number of ways.

The structure of the collector and the location of the thermoelectric cells are shown in Fig. 3.

(B) Simulation Model

The analysis can be broken down into three parts which are discussed as follows:

Absorption Cycle

The fundamental characteristics of binary mixtures will be introduced in this section. In a homogeneous binary mixture, the quantitative composition is described in terms of the concentration \( x \) which is the mass of one arbitrary constituent divided by the mass of the mixture. Knowledge of \( p, t \) and \( x \) enables us to establish the thermodynamic state of the mixture.

The \( h-x \) (enthalpy-concentration) diagram for a homogeneous binary mixture is shown in Appendix (VIII). Thus, binary mixtures, as contrasted with pure substances, do not have a single boiling temperature or condensing temperature for a given pressure. Hence some assumptions must be made to simplify the derivation process.

It is assumed that the absorbent does not vaporize in the
Fig. 3. Structure of solar collector
generator. Thus, only pure refrigerant flows through the condenser and evaporator, and these two components may be identical to those used in a mechanical vapor-compression system. The vapor leaving the evaporator is mixed with a weak liquid solution in the absorber. Then by the removal of heat the refrigerant vapor could be absorbed by the weak liquid, producing a liquid solution stronger in the refrigerant. The pressure of the liquid solution is then raised to the generator pressure by the pump. By the addition of heat in the generator, which is the flat-plate solar collector in the system used in this research to absorb the solar energy, refrigerant vapor is driven out of the liquid solution. A heat exchanger is placed in the liquid solution circuit between the generator and absorber. The generator and condenser are on the high-pressure side of the system, while the evaporator and absorber are on the low-pressure side.

The Bosnjakovic's method [10] was used to establish the maximum attainable C.O.P. for an absorption system. If the pump work \( W_p \) is assumed negligible, then

\[
\text{C.O.P.} = \frac{Q_E}{Q_G} \leq \frac{T_E(T_G-T_0)}{T_G(T_0-T_E)} \tag{2}
\]

and for a completely reversible system

\[
(\text{C.O.P.})_{\text{max}} = \frac{T_E(T_G-T_0)}{(T_0-T_E)T_G} \tag{3}
\]

Equation (3) shows the maximum attainable coefficient of
performance for an absorption system is equal to the coefficient of performance for a Carnot refrigerating cycle working between the temperatures, $T_E$ and $T_0$, multiplied by the efficiency of a Carnot engine working between the temperatures, $T_G$ and $T_0$. Equation (3) also shows that for a given environmental temperature, $T_0$, the C.O.P. will increase with an increase of the generator heating medium temperature, $T_G$, and with an increase of temperature of the refrigerated region, $T_E$. In practice, however, the C.O.P. is much less than that given by Eq. (3).

Solar Collector

To establish a model for the solar collector, a number of simplifying assumptions were made without obscuring the basic physical situation. These important assumptions are:

1. Performance is steady-state.
2. There is no absorption of solar energy by covers insofar as it affects losses from the collector.
3. There is one-dimensional heat flow through the covers.
4. There is a negligible temperature drop through the covers.
5. There is a one-dimensional heat flow through back-insulation.
6. The sky can be considered as a blackbody for long-wavelength radiation of an equivalent sky temperature.
7. Properties are independent of temperature.
8. Loss through front and back are to the same ambient temperature.
9. Dust and dirt on the collector are negligible.
10. Shading of the collector absorbing plate is negligible.

As mentioned before, the heat needed by the absorption cycle should be equal to the heat absorbed by the solar collector, i.e. the useful energy gain of the collector.

By following the study done by Duffie and Beckman [16] the following equations are used for determining the useful energy gain by a flat-plate solar collector.

\[ Q_u = A_c F_R [S - U_L (T_{f,i} - T_a)] \]  \hspace{1cm} (4)

where

\[ F_R = \frac{G C_p (T_{f,o} - T_{f,i})}{[S - U_L (T_{f,i} - T_a)]} \]

The useful energy gain is calculated as a function of the inlet fluid temperature. However, it must be realized that losses based on the inlet temperature are very small since losses occur all along the collector and the fluid has an ever increasing temperature in the flow direction. The effect of the multiplier, \( F_R \), is to reduce the calculated useful energy gain from what it would have had if the whole collector had been at \( T_{f,i} \) to what it actually is using a fluid that increases in temperature as it flows through the collector. As the mass flow rate through the collector increases, the temperature rise through the collector decreases. If the flow rate becomes very large, the temperature rise from inlet to outlet decreases toward zero but the temperature of the absorbing surface
will still be higher than the fluid temperature. This temperature difference is accounted for by the collector efficiency factor, $F'$.  

Thermoelectric Cells

To obtain an analysis that highlights the phenomena taking place in a thermoelectric generator it is necessary to make certain simplifying assumptions. Our analysis will be based on the model illustrated in Fig. 4. It consists of two semiconductor elements, one of which is a p-type material and the other an n-type material. The following assumptions have been made:

1. The generator works between the two temperatures $T_H$ and $T_C$, which are the actual temperatures at the junctions between the active semiconducting materials and the straps at reservoirs to which they are connected.

2. There is no heat transfer between the reservoirs at $T_H$ and $T_C$ except through the thermoelectric elements with no lateral heat transfer from the arms of the device.

3. The junction electrical contact resistance is negligible compared with the bulk resistance of the arms.

4. The arms are of constant cross-sectional area.

5. The electrical resistivity $\rho$, the thermal conductivity $\lambda'$, and the Seebeck coefficient, $\alpha$, of the material are independent of temperature.

6. Thermal contact resistance between the source and connecting bus bar between the p- and n-type elements can be made small.
Fig. 4 The model of the thermoelectric generator

while providing good electrical insulation. A similar assumption holds for the sink.

Assumption number (5) is probably the weakest one in the list. While convenient for making calculations, it can sometimes lead to significant errors in predicting results.

After introducing the grouping of properties called the figure of merit, $Z^*$, can be derived. It is given as the maximum thermal efficiency, which is:

$$
\eta_t^{(\text{max})} = \frac{(M'_\text{opt} - 1)\left(\frac{\Delta T}{T_H}\right)}{M'_\text{opt} + \frac{T_C}{T_H}} \quad (5)
$$

where

$$
M'_\text{opt} = (1 + Z^*T_{av})^\frac{1}{2} \quad (6)
$$

$$
Z^* = \frac{(|\alpha_n| + |\alpha_p|)^2}{[(\rho_n\lambda_n)^{\frac{1}{2}} + (\rho_p\lambda_p)^{\frac{1}{2}}]^2} \quad (7)
$$

The efficiency of a generator operating at a maximum power is derived in the same fashion by using
In designing a generator for maximum power output one tries to attain the given power with minimum volume, minimum weight, or minimum thermoelectric material. To obtain these objectives it is necessary to maximize the power output per unit of total cross-sectional area, \( \frac{P_o}{A_{tot}} \), where it is defined as

\[
\frac{P_o}{A_{tot}} = \frac{(\alpha \Delta T)^2}{4\varepsilon \left(\frac{\rho_n}{A_n} + \frac{\rho_p}{A_p}\right)[A_n + A_p]}
\]

and \( A_{tot} = A_n + A_p \). It is assumed that the lengths of the n- and p-type elements are the same. Taking the derivative of the denominator with respect to \( \frac{A_n}{A_p} \), the area ratio that maximizes Eq. (9) is:

\[
\frac{A_n}{A_p} = \left(\frac{\rho_p}{\rho_n}\right)^{1/2}
\]
Economics

The derivation done by Chang and Minardi in 1979 [4] is applied in this paper. According to their study the exponential relation between $Q_{aux}$ and $A_c$ can be used in finding the optimum area. For simplicity the tax benefits, other governmental advantages, and inflation rate have been neglected, because of their unpredictability.

The formula to find the optimum area is:

$$A_{op} = \frac{1}{\lambda} \ln \gamma = \frac{1}{\lambda} \ln \left[ \frac{Q_m C_f}{(C_c + bC_t + C_y) \gamma} \right]$$

(11)

where

$$\lambda = \frac{\ln \left( \frac{Q_{aux1}}{Q_{aux2}} \right)}{A_2 - A_1}$$

(12)

$$Q_m = Q_{aux1} e^{\lambda A_1} = Q_{aux2} e^{\lambda A_2}$$

(13)

and

$$\gamma = \frac{Q_m C_f}{(C_c + bC_t + C_y) \gamma}$$

(14)

The saving to cost ratio, $\gamma$, indicates economic feasibility of a solar system if its value is equal to or bigger than one. A solar system is not feasible when its $\gamma$ value is less than one, since
this will cause a negative optimum area which is physically im-
possible.

Annual operating cost, $, is found by applying the following
equation:

\[
S = (C_c A + C_{sc} A_{sc} + C_y A) I + Q_{aux} C_f + M
\]  

(15)

where

\[
I = \frac{i(1 + i)^n}{[(1 + i)^n - 1]}
\]  

(16)

As illustrated in Chapter II a simulation computer program is
compiled to supply the data needed by the economic study to predict
the financial potential of this system. Any two sets of computer
data at the same ambient temperature and different collector areas
can be substituted into the above two equations to find \( A_{op} \) and \( \lambda \). Because an exponential relation exists between auxiliary energy
and collector area, obviously this relation could be used to test
the accuracy of the simulation program. By interpolation the opti-
mum combination of auxiliary energy and collector area would be
found for any average ambient temperature.
CHAPTER IV

RESULTS AND DISCUSSION
CHAPTER IV

RESULTS AND DISCUSSION

In the previous sections, a computer simulation for a solar flat plate absorption air-conditioning unit has been described. This program allows the calculation of useful energy from the system based on collector area, wind speed and ambient temperature. Detailed calculations based on this mathematical model are given in Appendices (II) through (IV). Pertinent computer results are summarized in Table 1 and Fig. 5a to 5c. Fig. 5a-5c illustrates the relationship between the average ambient temperature and the auxiliary energy for selected collector areas. One may see that the auxiliary energy is inversely proportional to the ambient temperature. As the temperature increases, the auxiliary energy decreases for all collector areas.

This result is reasonable since for a given load at higher temperatures, the amount of useful energy will increase as the auxiliary energy decreases. Fig. 5 also shows that the auxiliary energy is not proportional to collector area. In fact, it can be verified easily that the auxiliary energy decays exponentially as the area increases. This behavior is in agreement with the results found by Chang and Minardi [4].

Table 1 shows the annual 90 day operational results based on an
TABLE 1

COMPUTER PRINT-OUT DATA

<table>
<thead>
<tr>
<th>Collector Area (in²)</th>
<th>Annual Useful Energy (Joule)</th>
<th>Annual Auxiliary Energy (Joule)</th>
<th>Annual Pump Work (Joule)</th>
<th>Thermal Cell Area (cm²)</th>
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Fig. 5(a) Auxiliary energy versus ambient temperature for June 1979
Fig. 5(b) Auxiliary energy versus ambient temperature for July 1979
Fig. 5(c) Auxiliary energy versus ambient temperature for August 1979
assumed average solar insolation of 370.5 W/m² and an average ambient temperature of 27.2 C for three different collector areas.

Substituting \( C_c = 65$/$m², \( C_f = 1.1 \times 10^{-5}$/$KJ, \( i = 0.09, \( n = 20, \( M = 0, \) and \( C_y = b = 0 \) into Eqs. 12 through 14, the economic feasibility factor, \( \gamma \), was found to be 0.278. Since \( \gamma < 1 \) the system is not feasible. In other words, over the life cycle, the unit will cost more money than the energy saved. However, under different economic conditions the system may become economically worthwhile. There are two reasonable ways in which the economics of the system can be improved. One way is to improve the performance of the system and thus increase the useful energy for the same collector area. An increase in fuel cost will also increase \( \gamma \). The second possibility is most likely considering today's rapidly increasing fuel costs. Based on the above data, the system will become feasible when the fuel price becomes larger than $0.12/Kwh.

For example when the price becomes $0.2/Kwh, the optimum collector area would be 70m² and the correspondent annual operational cost would then be $2300.

The collector area is large. But a large size is anticipated for cooling loads in Florida. A similar study by Chung in Wisconsin [3] found an optimum collector area which is 30% smaller. This result is in reasonable agreement when the different weather conditions are counted for. Wisconsin has a much milder summer climate and therefore a smaller cooling load requirement than Florida.

From Table 1, it can be observed that a solar cell of 87 cm² is able to provide enough electricity to completely operate the pump
(17 Kwh). The small energy requirement for the pump makes the use of the solar thermoelectric cells unjustified, considering the fact that the cells are not commercially available. The situation may be different in the future when the cells are economically and widely used. Under those conditions, the output of the cells can also be used to supply other utility uses in addition to the pump.
CHAPTER V

CONCLUSION AND RECOMMENDATIONS
CHAPTER V

CONCLUSION AND RECOMMENDATIONS

1. A computer model for describing the performance of a solar flat-plate absorption cycle cooling system has been developed.
2. The system is found to be unfeasible for the current fuel cost.
3. The system will become feasible when the fuel price becomes larger than 0.12$/Kwh.
4. When the fuel price becomes 0.2$/Kwh, the optimum collector area will be 70 m² with a corresponding annual operation cost of $2300.
5. The use of thermoelectric cells is unjustified due to the small pump work and economic considerations.
6. The results are in reasonable agreement with similar work done by Chung.
7. The model can be used to do parametric studies on solar input, ambient temperature, wind speed, and house insulation. Future studies are recommended.
8. Since the computer program can only handle the steady state, future work is needed to upgrade the program so that the transient state can be studied.
APPENDICES

Appendix I: The Computer Print-out
Appendix II: Calculation of Absorption Cycle
Appendix III: Calculation of Solar Collector
Appendix IV: Calculation of the Power Generated by Thermoelectric Cells
Appendix V: Collector Efficiency Factors
Appendix VI: The Meteorological Data
Appendix VII: Mean Solar Radiation for Orlando, Florida
Appendix VIII: The h-x Chart of Lithium-Bromide Water Solution
Appendix IX: The Thermoelectric Properties of Semiconductors
/* WATFIV */

1 3JO8

C PLENIT----PLATE RADIANCE
2 GLENIT----GLASS RADIANCE
3 WINSP-----WIND SPEED, M/SEC
4 TAMB-----AMBIENT AIR TEMPERATURE, DEG K
5 COMINS-----INSULATION CONDUCTIVITY (AT THE BACK OF THE SOLAR COLLECTOR)
6 CULTIL-----COLLECTOR U.LT ANGLE, DEG
7 H-----HEAT LOSS COEFFICIENT
8 N-----NUMBER OF LAYER PLATE
9 I-----"GUESS" PLATE TEMPERATURE, DEG K
10 WICK-----THICKNESS OF THE WICK INSULATION
11 NPC-----CONVECTION COEFF. BETWEEN WICK AND GLASS
12 J-----"GUESS" COVER TEMPERATURE, DEG K
13 UT15-----EXPERIMENTAL VALUE OF TOP LOSS COEFFICIENT
14 UT-----COND. VALUE THROUGH THE CONSIDERATION OF HEAT TRANSFER
15 G-----FLOW RATE, KG/SEC
16 AC-----COLLECTOR AREA, SQ M
17 S-----SOLAR INSULATION, W/(SQ M*DEG)
18 TFL-----INLET WORKING FLUID TEMPERATURE, DEG K
19 YCO-----OUTLET TEMP. OF THE WORKING FLUID FLOWING THROUGH THE SOLAR COLLECTOR

C THE ITERATION ADEQUATE TO FIND THE TOP LOSS COEFFICIENT & PLATE TEMP.

2
3
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19

C BEGIN NEW PAGE AND SKIP LINES
C WRITE('G, 10
C BEGIN GIVING VALUES TO TAB
C READ, PLENIT, GLENIT, WINSP, COMINS, CULTIL, S, THICK,
$G, AC, S, TFL
TAMB=298.
$S
3=10
100  N=1-L*Y
11
12
13
14
15
16
17
18
19

C

SIMULATION COMPUTER PROGRAM

APPENDIX (I)

44
THE ITERATION TO FIND THE COVER PLATE X EMP

```c
20 200 J=J+J,J
21 IF PCT+1.61*[(I+J)**0.261]/2.54**0.157
22 1.8UW+[1.1-UW]/*[(I+J)/2.0-10.0]
23 1.8HPCT=5.672-8*(I**2+2*J**2)*[11+J]/[[1/PLFM2]]
24 1.8HRC51=5.672-8*(I**2+2*K**2)*[11+K]/[[1/PLFM2]]
25 1.8UT=1./[(1.1/HPCT+1.1/HPRC51)]
26 TC=UT*[(1.1-TABH)/(1.1/HPCT)]
27 DIF=ABS(TC-J)
28 IF(TC.LT.ELJ) GO TO 60
29 J=TC
30 1. JU TO 200
31 1. TC=TC*72.0
32 1. HPCT=1.61*[(I+J)**0.261]/2.54**0.157
33 1. HPCT=1.61*[(I+J)**0.261]/2.54**0.157
34 1. HPCT=1.61*[(I+J)**0.261]/2.54**0.157
35 1. HPCT=1.61*[(I+J)**0.261]/2.54**0.157
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54 1. HPCT=1.61*[(I+J)**0.261]/2.54**0.157
55 1. HPCT=1.61*[(I+J)**0.261]/2.54**0.157
56 TEST IF TU AILS
57 IF [ABS(TANG).LE.290] GO TO 300
58 LINE(1)=OUT
59 LINE(1)=OUT
60 LINE(1)=OUT
61 LINE(1)=OUT
62 LINE(1)=OUT
63 LINE(1)=OUT
64 LINE(1)=OUT
65 LINE(1)=OUT
66 LINE(1)=OUT
67 STOP
68 END
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**Graph of Outlet Temp. v. TAMB (At Area of 20 sq ft)**

June '79
### Table: Graph of Outlet Temp. v. Tard (At Area of 10 SQ ft)

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### Graph of Aux Energy vs. Tabs (at Area of 15 sq in)

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APPENDIX (II)

Calculation of Absorption Cycle

A computer program compiled to simulate the solar collector performance is shown in Appendix (I). For the purpose of showing the applications of those formulas derived in Chapter (III), an example calculation is demonstrated in this section. For the purpose of convenience Fig. 1 is reproduced here.

The following data are assumed to be known for a lithium bromide-water system:

condensing temperature = 120°F

 evaporating temperature = 40°F

temperature of strong solution leaving absorber = 77°F
temperature of strong solution entering generator = \(150^\circ F = 65.56^\circ C\)
generator temperature = \(162.6^\circ F = 72.56^\circ C\)

Assume saturated conditions for states 3, 4, 8 and 10. Neglect the pressure drop in components and lines. Assume a system capacity of 2 tons of refrigeration. Determine the

(a) Thermodynamic properties \(p, t, x,\) and \(h\) for all necessary state-points of system.

(b) Mass rate of flow in lb per minute for each part of the system.

(c) System coefficient of performance.

(d) System refrigerating efficiency.

By referring to the h-x chart in Appendix (VIII) the following state properties are determined:

**TABLE II**

**THERMODYNAMICS PROPERTIES AND FLOW RATES OF THOSE STATE POINTS ABSORPTION CYCLE**

<table>
<thead>
<tr>
<th>State Point</th>
<th>Pressure (P) (\text{mmHg})</th>
<th>Temperature (t) (\text{\degree F})</th>
<th>Concentration (\frac{\text{lbLiBr}}{\text{lbmix}})</th>
<th>Enthalpy (\frac{\text{Btu}}{\text{lbmix}})</th>
<th>Flow Rate (\frac{\text{lbmix}}{\text{(min)}(\text{ton})})</th>
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</thead>
<tbody>
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<td>0.515</td>
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</table>
For the evaporator, the mass flow rate is

\[ M_9 = \frac{2 \times 200}{h_{10} - h_9} = \frac{2 \times 200}{1079 - 68} = 0.396 \text{ lb/min} \]

For the absorber, the mass flow rate is

\[ M_6 = M_{10} \frac{x_1 - x_{10}}{x_1 - x_6} = 0.396 \frac{0.515}{0.565 - 0.515} = 4.08 \text{ lb/min} \]

The mass flow rate leaving the pump is

\[ M_1 = M_6 + M_{10} = 4.08 + 0.396 = 4.476 \text{ lb/min} = \frac{2.032 \text{ kg}}{60 \text{ sec}} = 0.034 \text{ kg/sec} \]

For the generator, the heat available is

\[ q_G = M_4 h_4 + M_7 h_7 - M_3 h_3 = 4.08(-41) + 0.396(1132) - 4.476(-42) = 468.98 \text{ Btu/min} = 8244.7\text{W} \]

Pump Work

The superheated water vapor at state (10) is absorbed by a weak solution at state (6) in the absorber; and in turn this fluid mixture is handled by the pump.

The incompressible work of the fluid is:

\[ W = \frac{\Delta p v}{XJ} = \frac{(49.22 - 6.3) \times 0.0193 \times 144 \times 0.016}{0.515 \times 778} = 4.76 \times 10^{-3} \text{ Btu/LbLiBr} \]
at \( p_1 = 6.3 \text{ mmHg} = 0.121 \text{ psia}, \) the specific volume, \( v_f \) is 0.01602 \( \text{ft}^3/\text{Lbm} \)

\[ p_2 = 49.22 \text{ mmHg} = 0.95 \text{ psia}, \] the specific volume, \( v_f \) is 0.01613 \( \text{ft}^3/\text{Lbm} \)

The specific volume of the Li-Bromide water mixture is 0.016 \( \text{ft}^3/\text{lbm} \). Since the pump work is very small it could be neglected. By neglecting the pump work, the C.O.P. (coefficient of performance) is:

\[
C.O.P. = \frac{q_E}{q_G} = \frac{2 \times 200}{468.98} = 0.853
\]

The energy needed by the generator, \( Q_G \), should be equal to the energy supplied by the solar collector, \( Q_u \). In this system the solar collector is used as the generator to evaporate the working fluid (steam) from the lithium bromide liquid.
Fig. 6  H-x diagram for absorption cycle
APPENDIX (III)

Calculation of Solar Collector Performance

According to reference [16], the solar collector in Orlando must be oriented toward the equator and tilted 15° for summer air conditioning. It is assumed that the monthly average of total integrated daily insolation on horizontal surfaces is the same value as the insolation at the mid day of the month. Also, from the same reference in the month of June the declination angle is $\delta = 23^\circ$ and the latitude is $\phi = 28.4^\circ$N. It is found that the average day length in this particular month is 13.8hr. Before the performance of the solar collector is calculated, the following data from reference [5] and Appendix (VII) must be known:

- plate to cover spacing: 1 inch
- plate emittance: 0.95
- glass emittance: 0.96
- ambient air and sky temperature: $81^\circ$F = 27.2°C (June)
- wind speed: 10.71 ft/sec = 3.26 m/sec
- back insulation thickness: 5.5 in (from reference [5])
- insulation conductivity: 0.051 Btu/°F - ft - hr
  $= 0.088 \text{ W/m}^2 \cdot \text{C}$ (from reference [5])
- collector tilt: 15°
The solar insolation is

\[
S = 1623 \text{ Btu/ft}^2\text{-day} = \frac{1623 \text{ Btu}}{13.8 \text{ ft}^2\text{-hr}} = 117.6 \frac{\text{Btu}}{\text{ft}^2\text{-hr}} = 370.5 \frac{\text{W/m}^2}{\text{m}^2}
\]

The wind speed is

\[
hw = 5.7 + 3.8 \times V = 5.7 + 3.8 \times 3.26 = 18.1 \text{ W/m}^2\text{C}
\]

The formula for \( f \) is given as:

\[
f = (1.0-0.04hw + 5.0 \times 10^{-4}hw^2)(1 + 0.058N)
\]

\[
= (1.0-0.04 \times 18.1 + 5.0 \times 10^{-4} \times 18.1^2)(1 + 0.058 \times 2)
\]

\[
= 0.491
\]

The top loss coefficient is

\[
U_t = \frac{N}{344 \left[ \frac{T_p - T_a}{N + f} + \frac{1}{hw} \right]}^{-1} + \frac{o(T_p + T_a)(T_p^2 + T_a^2)}{\left[ \varepsilon_p + 0.0425N(1 - \varepsilon_p) \right]^{-1} + \left[ \frac{2N + f - 1}{\varepsilon_g} \right] - N}
\]

\[
= \frac{2}{344 \left[ \frac{82.2 - 27.22}{2 + 0.491} \right] 0.31 + \frac{1}{18.1}}^{-1} + \frac{5.67 \times 10^{-8} \times (355.34 + 300.36)(355.34^2 + 300.36^2)}{\left[ 0.95 + 0.0425 \times 2 \times (1-0.95) \right] + \left[ \frac{2 \times 2 + 0.491 - 1}{0.96} \right] - 2}
\]

\[
= 4.17 \text{ W/m}^2\text{C.}
\]
From reference [5], the insulation consisted of an air gap and 4" glass wool and 1½" of mineral wool. The total thermal conductivity is $K = 0.051 \text{ Btu/F ft}^2\text{hr} = 0.088 \text{ W/m}^2\text{C}$. The thickness of the whole insulation is $5.5" \times 2.5 = 0.14\text{m}$.

The bottom loss is found as:

$$U_b = \frac{1}{R_1} = \frac{K}{L} \frac{0.088}{0.14} = 0.63 \text{ W/m}^2\text{-°C}$$

For the collector with 15° tilt

$$U_t(15°) = U_t(45°) = 1 - (15 - 45)(0.00259 - 0.00144\epsilon_p)$$

$$= 1 - (-30)(1.22 \times 10^{-3}) = 1.037$$

$$U_t(15°) = 1.037 \times U_t(45°) = 1.037 \times 4.17 = 4.32 \text{ W/m}^2\text{-°C}$$

Finally, the overall loss coefficient, $U_L$, is found by adding together the top and bottom coefficients:

$$U_L = U_t + U_b = 4.32 + 0.63 = 4.95 \text{ W/m}^2\text{-°C}$$

The edge losses for large collectors are always negligible. Now the iteration method is used in Section 3-5 to find the mean plate temperature.

The first guess we make is $T_p = 180\text{F} (82.2 \text{C})$. 
For the upper surface of heated plates or the lower surface of cooled horizontal plates the $N_u_f$ is given by [17]

$$G_{f}Pr_{f} = 2 \times 10^7 ; \; C = 0.54; \; M = \frac{1}{4}$$

$$N_u_f = C(G_{f}Pr_{f})^M = 0.54 \times (2 \times 10^7)^{\frac{1}{4}} = 36.1$$

The thermal conductivity of water at the average temperature of state (3) and state (7) is 0.617 W/m$^2$-C. The heat transfer coefficient is than calculated as

$$h = \frac{N_u K}{x} = 0.617 \times 36.1 = 22.27 \text{ W/m}^2\text{-C}$$

The top loss coefficient is

$$U_t = \frac{1}{R_3 + R_4 + R_6} = \left(\frac{1}{h_{p-c_1} + h_{r,p-c_1}} + \frac{1}{h_{c_1-c_2} + h_{r,c_1-c_2}} + \frac{1}{h_w + h_{r,c_2-S}}\right)$$
In the space between two cover plates, the radiation is always much larger than the conduction term, so we neglect the $h_{c_1-c_2}$ term.

From reference [16]:

$$h_{r,p-c_1} = \frac{\sigma(T_p^2 + T_{c_1}^2)(T_p + T_{c_1})}{\left(\frac{1}{\epsilon_p} + \frac{1}{\epsilon_{c_1}}\right) - 1}$$

$$h_{r,c_2-S} = \epsilon_{c_2} \sigma(T_{c_2}^2 + T_S^2)(T_{c_2} + T_S)$$

$$h_{r,c_1-c_2} = \frac{\sigma(T_{c_1}^2 + T_{c_2}^2)(T_{c_1} + T_{c_2})}{\left(\frac{1}{\epsilon_{c_1}} + \frac{1}{\epsilon_{c_2}}\right) - 1}$$

Now from the energy balance between the two cover glasses and the absorber plate, it is noticed that the energy from plate to cover 1 is the same as the energy from the plate to the surroundings and is also the same as plate to cover 2. So there are two equations and two unknowns.

From plate to cover 1:

$$U_t(T_p - T_a) = (h_{p-c_1} + h_{r,p-c_1})(T_p - T_{c_1}) \quad (16)$$

$$U_t(T_p - T_a) = (h_{p-c_2} + h_{r,p-c_2})(T_p - T_{c_2}) \quad (17)$$
After those known values, and the $U_t$ value from the last iteration step have been substituted into the above equations, the Equations 16 and 17 become

\[
3.795(82.2 - 27.22) = (h_{p-c_1} + h_{r,p-c_1})(355.34 - T_{c_1}) \quad (18)
\]

\[
3.795(82.2 - 27.22) = (h_{p-c_2} + h_{r,p-c_2})(355.34 - T_{c_2}) \quad (19)
\]

Now it is assumed that the temperature of cover glass 1 is 84°C. The heat transfer coefficients are

\[
h_{r,p-c_1} = \frac{5.67 \times 10^{-8}(355.34^2 + 357.14^2)(355.34 + 357.14)}{\frac{1}{0.95} + \frac{1}{0.96} - 1} = 8.33
\]

\[
h_{r,p-c_2} = \frac{5.67 \times 10^{-8}(355.34^2 + 300.36^2)(355.34 + 300.36)}{2.083 - 1}
\]

from Eq. (4-11,3a):

For horizontal planes and $10^4 < Gr < 10^7$ the Nu number can be conveniently expressed in dimensional form as:

\[
h_{10,p-c_1} = 1.613 \Delta T^{0.281} = 1.613 \frac{0.281}{0.517} = 1.64 \text{ W/m}^2 \text{C}
\]
from Eq. (4, 11, 7) of reference [15]:

\[ h_{p-c_1} = [1 - 0.0018(T - 10)] \times 1.64 = 1.424 \]

\[ h_{10,p-c_2} = 1.613 \frac{\Delta T^{0.281}}{\xi^{0.157}} = 1.613 \frac{(355.34 - T_2)^{0.281}}{(2.54)^{0.157}} \]

\[ h_{p-c_2} = [1 - 0.0018\left(\frac{3.55.34 + T_2}{2} - 10\right)] \times 1.613 \frac{(355.34 - T_2)^{0.281}}{(2.54)^{0.157}} \]

then from Eqs. (18) and (19),

\[
(\frac{h_{p-c_2} + h_{r,p-c_1}}{(355.34 - T_1)}) = (h_{p-c_1} + h_{r,p-c_2})(355.34 - T_{c_2})
\]

\[
(0.7 + 8.33)(355.34 - 357.14) = 16.25 = \left\{1.613 \times \frac{(355.34 - T_2)^{0.281}}{(2.54)^{0.157}}\right\} \times \left[1 - 0.0018\left(\frac{355.34 + T_{c_2}}{2} - 10\right)\right]
\]

\[
5.67 \times 10^{-8} \left(\frac{355.34^2 + T_{c_2}^2}{2.083 - 1}\right) \left(\frac{355.34 + T_{c_2}}{2}\right)
\]

Corresponding to every mean plate temperature there are two cover plate temperatures, \(T_{c_1}\) and \(T_{c_2}\). After the first cover plate temperature is assumed, one can use the above equation to find the second cover plate temperature, \(T_{c_2}\). Then the above equation is solved by a trial and error method. It is found that \(T_{c_2} = 76.86\) C.
At $T_{c2} = 76.86 \, ^\circ C$,

$$h_{r,c2-S} = 0.96 \times 5.67 \times 10^{-8} \times 212716.13 \times 650.36 = 7.53 \, W/m^2C$$

By applying Eq. (7.4.7) of reference [16]:

$$U_t = \left( \frac{1}{h_{p-c1} + h_{r,p-c1}} + \frac{1}{h_w + h_{r,c-S}} \right)^{-1} = \left( \frac{1}{0.7 + 8.33} + \frac{1}{18.1 + 7.53} \right)$$

$$= 6.71 \, \frac{m^2C}{W}$$

After this new top loss coefficient has been found, it is substituted back into Eqs. (16) and (17) to find the new cover plate temperatures. From Eq. (16),

$$6.71(355.34 - 300.36) = (0.7 + 8.33)(355.34 - T_{c1})$$

and

$$T_{c1} = \frac{-368.92}{9.03} + 355.34 = 41.36 \, ^\circ C$$

From Eq. (17),

$$6.71(355.34 - 300.36) = (0.85 + 9.186) \times (355.34 - T_{c2})$$

and
\[ T_{c2} = \frac{-368.92}{10.03} + 355.34 = 45.42 \text{ C} \]

From Appendix (V), Fig. (e) is the system adopted in this research:

\[ h_r = \frac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)}{\varepsilon_1 + \varepsilon_2 - 1} \]

\[ 5.67 \times 10^{-8}(355.34^2 + 300.36^2)(355.34 + 300.36) = 7.25 \text{ W/m}^2\text{C} \]

The collector efficiency factor is:

\[ F' = \frac{1}{1 + \frac{1}{h_1 + \frac{1}{h_2 + \frac{1}{h_r}}} + \frac{1}{7.34}} = 0.79; \]

The heat removal factor is:

\[ F_R = \frac{GCP}{U_L}(1 - e^{-\frac{U_L F'}{GCP}}) \]

\[ = \frac{0.03}{176} \times 39.52 \times 10^3 (1 - e^{-\frac{7.34 \times 0.03}{176 \times 39.52 \times 10^3}}) \]

\[ = 0.918(1 - e^{-0.861}) \]

\[ = 0.53 \]
The useful energy collected by the solar collector is

\[ Q_u = A_c F_R [S - U_L (T_{fi} - T_a)] \]

\( Q_u \) should be equal to the energy needed by the absorption cycle to generate 2 tons of refrigeration, where \( Q_G \) is 8300 W.

\[ 8300 = A_c \times F_R [S - U_L (T_{fi} - T_a)] \]

\[ = A_c \times \frac{G C_p}{U_L} (1 - e^{-\frac{U_L F'}{G C_p}}) \times [S - U_L (T_{fi} - T_a)] \]

\[ = A_c \times \frac{0.03 \times C_p \times 10^3}{7.34} \times (1 - e^{-\frac{7.34 \times 0.79}{0.03 \times C_p \times 10^3}})[370.5 - 7.34(65.56 - 27.22)] \]

(16)

The energy balance is

\[ T_{c,o} - T_{c,i} = \frac{Q_u}{\dot{m} C_p} \]

\[ \frac{8300}{0.03 \times C_p \times 1000} = T_{c,o} - T_{c,i} \]

(17)

By solving Eq. (17), the \( C_p \) is obtained as
\[ C_p = \frac{8300}{0.03 \times 1000 \times 7} = 39.52 \text{ KJ/Kg } \text{C} \]

Substituting \( C_p \) into Eq. (16) the collector area is calculated as:

\[ 8300 = A_c \times \frac{0.03 \times 39.52 \times 10^3}{7.34} \quad (1 - e^{-\frac{0.03}{A_c} \times 39.52 \times 10^3}) \times \]

\[ [370.5 - 7.34(65.56 - 27.22)] \]

\[ A_c = \frac{0.8604}{4.89 \times 10^{-3}} = 176 \text{ m}^2 \]

The mean fluid temperature is

\[ T_{f,m} = T_{f,i} + \frac{Q_u}{A \cdot U_L F_R} \left[ 1 - \frac{F_R}{F_I} \right] \]

\[ = 65.56 + \frac{8300}{175 \times 4.95 \times 0.53} [1 - 0.53] \]

\[ = 71.5 \text{ C} \]

The mean plate temperature is:

\[ T_{p,m} = T_{f,m} + Q_u R_{p-f} = 71.5 + 8300 \times \frac{1}{22.27} \times \frac{1}{175} = 73.63 \text{ C} \]
The useful energy, $Q_u$, is the energy that this solar collector can supply and it is needed by the whole system to generate 2 tons of cooling in an ordinary residential house in Florida.

The difference between this new mean plate temperature and the first guess mean plate temperature is 8.57 °C and the difference between the new $U_t$ and the first $U_t$ is 2.54 so that the calculated mean plate temperature is very close to the exact value. If the exact $T_p$ value is needed, the above procedures must be repeated until the calculated $T_p$ is equal to the last one. The iteration is performed by computer and the results are shown in Appendix (I) for the months of June, July, and August of 1979.
APPENDIX IV

Calculation of the Power Generated by Thermoelectric Cells

Those cells, located at the inlet part of the solar panel, operate between 82.2 °C and 65.56 °C.

From Appendix (IX) at the mean temperature of the generator, 73.88 °C (346.88 K), the n- and p-type materials with the highest figures of merit are:

<table>
<thead>
<tr>
<th>Material</th>
<th>n-type</th>
<th>p-type</th>
</tr>
</thead>
<tbody>
<tr>
<td>BioTe3</td>
<td>75%</td>
<td>25%</td>
</tr>
<tr>
<td>BioSe3</td>
<td>25%</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>175 x 10^{-6}</td>
<td>210 x 10^{-6}</td>
</tr>
<tr>
<td></td>
<td>1.1 x 10^{-3}</td>
<td>1.2 x 10^{-3}</td>
</tr>
<tr>
<td></td>
<td>2.2 x 10^{-3}</td>
<td>3.1 x 10^{-3}</td>
</tr>
</tbody>
</table>

Equation 4-46 of reference [15] expresses the figure of merit for a combination of materials that make up a couple. The figure of merit for a single material is generally defined as

\[ Z = \frac{\alpha^2}{\rho A} \]

Using this definition to obtain the thermal conductivity for the two materials
The optimal figure of merit with respect to geometry adjustments for the combination of materials is found from Eq. 4-46 of reference [15] as

\[
Z^* = \frac{\alpha^2}{[(\rho_n \lambda_n)^{\frac{1}{2}} + (\rho_p \lambda_p)^{\frac{1}{2}}]^2} = \frac{[(175 + 210) \times 10^{-6}]^2}{[(1.1 \times 10^{-3} \times 0.013)^{\frac{1}{2}} + (1.2 \times 10^{-3} \times 0.012)^{\frac{1}{2}}]^2}
\]

\[
= \frac{1.482 \times 10^{-7}}{5.74 \times 10^{-5}} = 2.582 \times 10^{-3}(^\circ\text{K})^{-1}
\]

The ratio of areas to lengths is found from Eq. 4-44 of reference [15] to be:

\[
\frac{\gamma_n}{\gamma_p} = \left[\frac{\rho_n \lambda_p}{\rho_p \lambda_n}\right]^{\frac{1}{2}} = \left[\frac{1.1 \times 10^{-3} \times 0.013}{1.2 \times 10^{-3} \times 0.012}\right]^{\frac{1}{2}} = 0.92
\]

A further assumption is made. The ratio of the area of the n-type
element to its length is 1 cm and the length of each element is

\[ \gamma_n = \frac{A_n}{l_n} = 1 \text{ cm} \]

and thus

\[ \gamma_p = \frac{1}{0.92} = 1.087 \text{ cm} \]

The thermal conductance from Eq. 4-36 [14] is

\[ K = \frac{\lambda \gamma_n + \lambda \gamma_p}{2} = (0.013 \times 1) + (0.012 \times 1.087) = 0.026 \text{ W/C} \]

and the internal electrical resistance may be calculated from Eq. 4-37 [14] as

\[ R = \frac{\rho_n}{\gamma_n} + \frac{\rho_p}{\gamma_p} = \frac{1.1 \times 10^{-3}}{1.087} = 2.204 \times 10^{-3} \Omega \]

The optimum resistance ratio is found from Eq. 4-48 [14] as

\[ m_{opt} = (1 + Z T_{av})^{1/2} = (1 + 2.582 \times 10^{-3} \times 346.88)^{1/2} = 1.377 \]

and thus the external resistance is

\[ m'_{opt} = 1.377 = \frac{R_o}{2.204 \times 10^{-3}} \quad ; \quad R_o = 3.035 \times 10^{-3} \Omega \]
The open circuit voltage is simply the Seebeck coefficient for both legs combined times the temperature difference across the legs,

\[ V_{oc} = \alpha \Delta T = [(175 + 210) \times 10^{-6}] \times 16.64 = 6.41 \times 10^{-3} \text{ Volts} \]

and the optimum current may be found from Eq. 4.39 [14] as

\[ I_{opt} = \frac{\alpha \Delta T}{R(m_{opt} + 1)} = \frac{6.41 \times 10^{-3}}{2.204 \times 10^{-3}(1.377 + 1)} = 1.22 \text{ Amp.} \]

\[ P_0 = I_{opt}^2 R_0 = (1.22)^2 \times 3.035 \times 10^{-3} = 4.52 \times 10^{-3} \text{W} \]

The power density is found by computing the total cross-sectional area of the semiconducting material. It was assumed earlier that \( \gamma_n = 1 \text{cm} \) and the length of both elements is 1cm. Thus

\[ A_n = 1 \text{ cm}^2 \]

and from the calculated value for the p-type element of \( \gamma_p = 1.027 \text{cm} \) then

\[ A_p = 1.087 \text{ cm}^2 \]

Thus the total cross-sectional area of both legs is
Atotal = 2.087 cm²

and the power density is

\[
\frac{P_o}{A_{tot}} = \frac{4.52 \times 10^{-3}}{2.087} = 2.17 \times 10^{-3} \text{ W/cm}^2
\]

The power input to the device is found from Eq. 4-35 [14] as

\[
q_H = K\Delta T + \alpha T_H I - \frac{1}{2} I^2 R
\]

\[
= 0.026 \times 16.64 + (385 \times 10^{-6}) \times 355.34 \times 1.22 - \frac{1}{2} \times (1.22)^2 \times 2.204 \times 10^{-3}
\]

\[
= 0.6 \text{ W}
\]

The thermal efficiency may be computed from Eq. 4-40 [14] as

\[
\eta_t = \frac{P_o}{q_H} = \frac{4.52 \times 10^{-3}}{0.6} = 7.53 \times 10^{-3} = 0.7\%
\]

The accuracy of these calculations is checked by comparing the above efficiency with the one obtained from Eq. 4-49 [14]
\[ n_{t\text{(max)}} = \frac{(m'_{\text{opt}}-1)(\Delta T)}{m'_{\text{opt}} + \frac{T_c}{T_H}} \]

\[ = \frac{16.64}{1.377 + \frac{338.56}{355.34}} = 0.018 \]

\[ = 7.73 \times 10^{-3} = 0.7\% \]

**ii) Maximum Power Conditions**

Consider next the problem of operating this same generator at maximum power. Under this condition the load resistance has the same value as the internal resistance, i.e. \( R = R_0 \) thus the current is

\[ I_{mp} = \frac{\alpha \Delta T}{2R} = \frac{6.41 \times 10^{-3}}{2 \times 2.204 \times 10^{-3}} = 1.45 \text{ amp} \]

The power delivered is

\[ P_0 = I^2R_0 = (1.45)^2 \times (3.035 \times 10^{-3}) = 6.38 \times 10^{-3} \text{ W}, \]

The power density is

\[ \frac{P}{A_{\text{tot}}} = \frac{6.38 \times 10^{-3}}{2.087} = 3.06 \times 10^{-3} \text{ W/cm}^2 \]

The generator's thermal efficiency is calculated from Eq. 4-51 [14] to be
\[ n \text{t(mp)} = \frac{\Delta T}{T_H} \left( \frac{4}{2 \pi T_H} + 2 - \frac{1}{2} \frac{(\Delta T)}{T_H} \right) = \frac{16.64}{355.34} \]

\[ = \frac{0.047}{4.36 + 2 - 0.023} = 7.42 \times 10^{-3} \]

(iii) The maximum power density:

Consider the possibility of designing the generator to yield the maximum power density. Eq. 31 gives the area ratio which maximizes the power per total cross-sectional area. It is calculated as

\[ \frac{A_n}{A_p} = \left[ \frac{\rho_n}{\rho_p} \right]^{1/2} \left[ \frac{1}{1.2} \right]^{1/2} = 0.96 \]

If \( \gamma_n = 1 \, \text{cm} \) (with \( A_n = 1 \, \text{cm}^2 \) and \( L_n = 1 \, \text{cm} \)), then \( \gamma_p = \frac{1}{0.92} = 1.09 \) from Eq. 4-37 [6], and the electrical resistance is

\[ R = \frac{\rho_n}{\gamma_n} + \frac{\rho_p}{\gamma_p} = \frac{1.1 \times 10^{-3}}{1} + \frac{1.2 \times 10^{-3}}{1.09} = 2.2 \times 10^{-3} \Omega \]

and where the maximum power is also the external load resistance, \( R_0 \). Thus the current is

\[ I_{\text{mpd}} = \frac{\alpha \Delta T}{2R} = \frac{6.41 \times 10^{-3}}{2 \times 2.204 \times 10^{-3}} = 1.45 \, \text{amp} \]
The power output is

\[ P_{o\text{(mpd)}} = I^2R_0 = (1.45)^2 \times 2.2 \times 10^{-3} = 4.63 \times 10^{-3} \text{ W} \]

The total cross-sectional area of the elements is

\[ A_{\text{tot}} = A_n + A_p = 1 + 1.087 = 2.087\text{cm}^2 \]

The power density becomes

\[ \frac{P_{o\text{(mpd)}}}{A_{\text{tot}}} = \frac{4.63 \times 10^{-3}}{2.087} = 2.22 \times 10^{-3} \text{ W/cm}^2 \]

According to the calculation on the absorption cycle it is known that the work needed by the pump is \( 4.76 \times 10^{-3} \text{ Btu/Lb of LiBr} \). At state point (1), the total mass flow rate is 4.476 lb/min and the concentration at this state (1) is 0.515 LbLiBr/Lbmix. That means the mass flow rate of LiBr at that point is:

\[ 4.476 \times 0.515 = 2.305 \text{ LbLiBr/min} \]

Then the power the pump needs is:

\[ (4.76 \times 10^{-3} \text{ Btu/lb of LiBr}) \times 2.305 \text{ LbLiBr/min} \]
\[ = 0.011 \text{ Btu/min} = 0.66 \text{ Btu/hr} = 0.193\text{W} \]
So the solar cells needed to generate enough power to drive the pump will be

\[ A_{\text{sol}} = \frac{0.193 \times 10^{-3}}{2.22 \times 10^{-3}} = 87 \, \text{cm}^2 \]
APPENDIX (V)
Collector Efficiency Factors

\[ U_L = U_t + U_b \]

\[ F' = \frac{1}{1 + \frac{U_L}{h_1 + \frac{1}{h_2} + \frac{1}{h_r}}} \]

\[ h_r = \frac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]
## APPENDIX (VI)

### MONTHLY SUMMARIZED STATION AND DIVISIONAL DATA

**TEMPERATURE**

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<th>APR</th>
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**PRECIPITATION**

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## APPENDIX (VII)

**FLORIDA SOLAR ENERGY CENTER, CAPE CANAVERAL, FLORIDA**

**SOUTH FACING TILTED SURFACES**

**ORLANDO (BELLE ISLE) C.A.U.S. LAT 28.4° 11 YEARS DATA REFLECTANCE 0.0**

<table>
<thead>
<tr>
<th>MONTH</th>
<th>HORIZNTL DIRECT</th>
<th>HORIZNTL DIFFUSE</th>
<th>TOTAL LATITUDE</th>
<th>TOTAL 15 DEGREE</th>
<th>TOTAL 30 DEGREE</th>
<th>TOTAL 45 DEGREE</th>
<th>DIR NORMAL TRACKING</th>
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</table>

### LATITUDE

| MONTH | BEAM BEAM BEAM BEAM BEAM BEAM |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1     | 1206.325        | 1022.340        | 1224.322        | 1342.295        | 1342.295        | 1342.295        |
| 2     | 1104.402        | 1144.501        | 1200.475        | 1174.435        | 1174.435        | 1174.435        |
| 3     | 1198.479        | 1143.505        | 1097.555        | 978.500         | 978.500         | 978.500         |
| 4     | 1105.557        | 1009.634        | 975.602         | 802.550         | 802.550         | 802.550         |
| 5     | 990.606         | 969.624         | 817.561         | 660.568         | 660.568         | 660.568         |
| 6     | 824.626         | 868.644         | 745.609         | 758.557         | 758.557         | 758.557         |
| 7     | 982.613         | 1078.641        | 945.609         | 981.467         | 981.467         | 981.467         |
| 8     | 1014.579        | 1008.605        | 1002.575        | 862.526         | 862.526         | 862.526         |
| 9     | 1032.515        | 1000.539        | 1020.511        | 964.467         | 964.467         | 964.467         |
| 10    | 1185.423        | 1084.443        | 1193.420        | 1221.384        | 1221.384        | 1221.384        |
| 11    | 1224.341        | 1052.356        | 1240.338        | 1344.310        | 1344.310        | 1344.310        |
| 12    | 1261.301        | 1054.315        | 1282.299        | 1422.273        | 1422.273        | 1422.273        |
| AVERAGE| 1094.481        | 1059.503        | 1094.477        | 1058.436        | 1058.436        | 1058.436        |

**UNITS ARE BTU PER SQUARE FOOT - DAY (MONTHLY AVG.)**

**FOR K-JOULES PER SQUARE METER - DAY MULTIPLY BY 11.35**

APPENDIX (VIII)

The h-x Chart of Lithium-Bromide Water Solution

APPENDIX (IX)

The Thermoelectric Properties of Semiconductors
REFERENCES


