Simulation of a Solar-Driven Thermoelectric Generator

Fall 1982

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SIMULATION OF A SOLAR-DRIVEN THERMEOLECTRIC GENERATOR

BY

IRAJ ANDAMPOUR
B.S., University of Florida, 1978

RESEARCH REPORT

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Graduate Studies Program of the College of Engineering at the University of Central Florida at Orlando, Florida

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SIMULATION OF A SOLAR-DRIVEN THERMOELECTRIC GENERATOR

BY

IRAJ ANDAMPOUR

ABSTRACT

With improvements of thermoelectric materials leading to higher figures of merit, interest has been developed in a broad spectrum of applications. In this study, the thermal performance of a solar-driven thermoelectric (TE) generator was examined by computer simulation and analytical formulations. The hot junction of the disk-shaped TE module is heated by a conical-shaped solar concentrator reflecting rays onto a cylindrical inner electrode. Controllable cooling water flow cools the outer P-N junctions to establish the necessary thermal potential for electric generation. Desired power output can be obtained from a number of TE modules in series and parallel.

The computer program was used to examine periodic and constant flow rate of the cooling water. It was found that the constant flow rate operation yielded the highest time-integrated TE thermal efficiency.

Other parametrical studies performed include the height of copper rod, the ratio of outer to inner diameters of the disks, the thickness of the disks, the solar influx and the heat transfer coefficient between cooling water and the modules.

The computer and analytic results on these studies show
similar behaviors.

It was found that the efficiency of the solar thermoelectric cogenerator ranges from 1.5 to 5.0 percent which is considerably lower than a photovoltaic system.

Dr. K. K. Chang
Director of Research Report
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LIST OF SYMBOLS

\( V \) = Electromotive Force (EMF)
\( \alpha \) = Seebeck Coefficient
\( \sigma \) = Stefan-Boltzman Constant
\( \rho \) = Electrical Resistivity
\( \rho_{cu} \) = Resistivity of the Copper Rod
\( \rho_{dcu} \) = Density of the Copper Rod
\( c_{pcu} \) = Specific Heat of the Copper Rod
\( T_h \) = Hot Junction Copper Rod Temperature
\( (T_h)_{high \ set} \) = High Temperature Setting of Hot Junction
\( (T_h)_{low \ set} \) = Low Temperature Setting of Hot Junction
\( T_1 \) = Cold Junction Temperature
\( T_{ss} \) = Steady State Temperature
\( T_{amb} \) = Ambient Air Temperature
\( T_{\infty} \) = Temperature of the Cooling Water
\( T_1 \) = Temperature of the First Inside Ring
\( T_{av} \) = Average Temperature of Hot and Cold Junctions
\( \Delta T \) = Temperature Difference \( (T_h - T_1) \) at any time
\( \Delta T_0 \) = Initial Temperature Difference \( (t = 0) \)
\( \Delta T_f \) = Final Temperature Difference \( (t = \infty) \)
\( \frac{dT}{dt} \) = Time Rate of Change of Copper Rod Temperature
(t)_{off} = Time Required for Cooling Water to Remain Off
(t)_{on} = Time Required for Cooling Water to Remain On
Q_h = Heat Absorbed at the Hot Junction from Hot Source
Q_1 = Heat Rejected from the Cold Junction into the Cold Sink
Q_{in} = Energy In
Q_{out} = Energy Out
Q_{gen} = Total Heat Generated
\Delta E = Change of Internal Energy
R = Thermal Resistivity
R_{tot} = Internal Resistance of the Whole Couple
K = Thermal Conductance of Copper
k_p = Thermal Conductivity of P Material
k_n = Thermal Conductivity of N Material
A = Effective Area Collecting the Solar Input
A_p = Cross Sectional Area of P Material
A_n = Cross Sectional Area of N Material
L = Height of P - N Junction
t = Thickness of P - N Junction (d)
\eta_{carnot} = Carnot Efficiency
\eta_{max} = Maximum Efficiency
\eta_{t(mp)} = Thermal Efficiency at Maximum Power Output
I = Current
W = Electrical Power Output
Z = Figure of Merit
\( \dot{Z} \) = Maximum Value of the Figure of Merit

\( H \) = Solar Input

\( M \) = Mass of the Copper Rod

\( M' \) = Ratio of the Load Resistance to the Internal Resistance

\( M'_{\text{opt}} \) = Optimum Ratio of the Load Resistance to the Internal Resistance

\( H_{\text{conv}} \) = Convection Heat Transfer Coefficient

\( h' \) = Convective Loss Coefficient of the Inner Electrode

\( \theta \) = Time Constant of the Copper Rod

\( r \) = Thermal Impedance of the Thermoelectric Module

\( T_{\text{hh}} \) = Height of Copper Rod

\( \frac{r_o}{r_i} \) = Outer to Inner Radius of P and N Disks
CHAPTER 1

INTRODUCTION

The first experiment in Solar Thermoelectric Generators (STEG) dates back to 1913 when Goblentz (Benson 1979) obtained a patent for his generator which used copper constantan. His generator had a very low efficiency because of the unavailability of good thermoelectric materials at that time. Little work was performed on STEG until the advance of semiconductors. In the early 1950s, Maria Telkes (1954) devoted considerable effort to a reassessment of the use of STEG. Because most of the currently available thermoelectric materials had not been developed at that time, Telkes concluded in 1953 that conversion efficiencies of only about 3% were possible, and that such low values of conversion efficiency did not justify the exploitation of this form of energy conversion. As a consequence, the interest in STEG remained dormant until the 1960s. By that time, many new thermoelectric materials and associated technologies had been developed and warranted a new look at the field. This evaluation showed improved efficiency, but low fossil fuel costs continued to favor conventional power generation systems. Toward the end of the 1960s and early 1970s, specialized fabrication techniques were developed for larger scale thermoelectric devices; however, most of these devices were intended for use with sources
of heat from fossil and nuclear fuels. There were studies of solar thermoelectric generation for near sun space missions. Past studies of the use of thermoelectric power generation have, in some cases, addressed the economics of this form of energy conversion. However, in all cases it has been assumed that fairly small quantities of electric power were to be generated. Such studies were aimed at determining the cost of specific STEG in a fairly low power output range. Today the thermoelectric field has a broader spectrum of thermoelectric materials and considerable experience has been accumulated in recent years in fabrication, testing, and operation of thermoelectric generators in terrestrial and space applications. Studies which consider the effect of large scale thermoelectric utilization on the cost of electricity explores perhaps for the first time the potential of solar thermoelectric generation deployed on a large scale. Renewed interest in STEG has developed since 1970 as a result of increasing costs of energy from conventional resources and the problem of importing and extracting fuels that are acceptable from environmental standpoints. STEG of several types have been proposed. Landecker (1976) introduced the concept of disk type STEG which shows thermoelements with radial flow of currents in coaxial disks and a copper rod in the center to receive solar energy reflected from a cone shaped reflector. His theoretical analysis led to an estimate of 18% thermal efficiency, based on a water control scheme resulting in burst type energy input. However, his results were based on mathematical modeling. The primary purpose of the present study is to investigate the transient behavior
of disk type thermoelectric generators using computer simulation. The simulations will enable us to obtain information about the dependence of the system efficiency on various parameters such as the water control schemes, height of copper rod, solar input, etc.

In Chapter 2, the fundamental theory of thermoelectrics (TE) is presented. In Chapter 3 system description and operation is discussed. Chapters 4 and 5 illustrate the theoretical analysis and computer simulation of a disk type TE generator. Chapter 6 presents the results and discussion.
CHAPTER 2

FUNDAMENTAL THEORY

Thermoelectric refers to effects that arise from an interaction between temperature difference and electrical current in any material. There are three basic thermoelectric phenomena; the Peltier, Seebeck, and Thomson effects. The Seebeck effect is created by joining dissimilar materials and maintaining a temperature difference between the junction and a resulting flow of electrical current in the system. Conversely, the Peltier effect is created when an electrical current passes through a junction of two dissimilar materials. A cooling or heating will take place on the junction. The Thomson effect consists of the generation or absorption of heat by the passage of a current through a homogeneous conductor in which there is a temperature gradient (Ioffe 1957).

Peltier Effect

When an electric current passes through a junction of two dissimilar materials, such as P type and N type semiconductors, a cooling or heating will take place on the junction, depending on the direction of the current. As shown in Fig. 2.1a, when a current passes from N material to a P material heat is removed from the cold junction and carried as kinetic energy of the electrons to the hot
Fig. 2.1a. Peltier effect

Fig. 2.1b. Peltier Effect

Fig. 2.2. Seebeck effect
junction where kinetic energy is given up to heat the hot junction. If the current is reversed, the hot and cold junctions are reversed and heat is pumped in the opposite direction as shown in Fig. 2.1b. Peltier heat removed or gained is proportional to the current and the absolute temperature of the junction \( Q_p = \alpha IT \) where \( \alpha \) is the Seebeck coefficient. Note that \( Q_p \) is a reversible term, that is, when one reverses the direction of the current, the direction of the heat flow will change without changing the magnitude. Mathematically, one may write (Chang 1980)

\[
Q_h = \alpha IT_h - K(T_h - T_c) + \frac{1}{2} I^2 R \quad \text{(2-1)}
\]

\[
Q_c = \alpha IT_c - K(T_h - T_c) - \frac{1}{2} I^2 R \quad \text{(2-2)}
\]

**Seebeck Effect**

Referring to Fig. 2.2 an electromotive force (EMF) is produced when two junctions formed by dissimilar materials P and N are maintained at different temperatures. The EMF may be expressed as:

\[
V = \alpha (T_h - T_c) \quad \text{(2-3)}
\]

where

\[
\alpha = \text{Seebeck coefficient}
\]

Notice in Fig. 2.2, the current generated by the EMF is in such a direction that it attempts to cool down the hot junction and warm up the cold junction. The heat absorbed at the hot junction \( Q_h \) from the hot source and the heat rejected from the cold junction \( Q_1 \) into the cold sink are respectively (Chang 1980)

\[
Q_h = \alpha IT_h + K(T_h - T_1) - \frac{1}{2} I^2 R \quad \text{(2-4)}
\]
\[ Q_1 = \alpha I T_1 + K(T_h - T_1) + \frac{1}{2} I^2 R \]  

(2-5)

where

- \( K \) = thermal conductance
- \( R \) = internal resistance
- \( I \) = current generated.

For the configuration shown in Fig. 2.2

\[ K = \frac{K_n A_n + K_p A_p}{L} \]  

(2-6)

\[ R = \left( \frac{\rho_n}{A_n} + \frac{\rho_n}{A_p} \right) \]  

(2-7)

where

- \( K_p \) = thermal conductivity of P material
- \( K_n \) = thermal conductivity of N material
- \( A_p \) = cross section area of P material
- \( A_n \) = cross section area of the N material
- \( \rho_p \) = electrical resistivity of P material
- \( \rho_n \) = electrical resistivity of N material
- \( L \) = height of P – N junction

For a disk type configuration as is shown in Fig. 2.3,

\[ K = \frac{2\pi tk}{r_o \ln\left(\frac{r_o}{r_1}\right)} \]  

(2-8)

\[ R = \frac{\rho}{\pi t} \ln\frac{r_o}{r_1} \]  

(2-9)

where

- \( t \) = thickness of P – N junction
Fig. 2.3. Disk type thermocouple
\[
\frac{r_o}{r_i} = \text{ratio of outer radius to inner radius}
\]

**NOTE:** The operation of the two types of generators is fundamentally the same. In Fig. 2.4, the current flows radially into the hot junction through the N disk and radially outward through the P disk to the cold junction. This is equivalent to the operation of Fig. 2.2.

In equations (2-4) and (2-5), the \( \frac{1}{2} r^2 R \) term is Joulean heating which is irreversible production of heat by the current in an electric circuit. The Joulean heat, \( \pm \frac{1}{2} I^2 R \), tends to warm up both junctions, \( K(T_h - T_1) \) is the thermal conduction heat which always flows from the high temperature region to the low temperature region without regard to the direction of the electrical current. The heat conduction tends to cool the hot junction and warm up the cool junction. The last two terms in equation (2-5) are positive which means that the waste heat increased due to irreversible processes.

**Efficiency**

Efficiency of a TE generator can be given as follows:

\[
\eta = \frac{Q_h - Q_1}{Q_h}
\]  
(2-10)

Substituting equations (2-4) and (2-5) into equation (2-10) we obtain

\[
\eta = \frac{I^2 R}{K \Delta T + \alpha I T_h - \frac{1}{2} I^2 R}
\]  
(2-11)

The maximum thermal efficiency can be shown to be (see detail in Appendix A) (Angrist 1976).
Fig. 2.4. Comparison between disk type and rectangular thermocouple
\[
\eta_{\text{max}} = \eta_{\text{carnot}} \frac{1 + \frac{Z}{2} (T_h + T_l)^{1/2} - 1}{1 + \frac{Z}{2} (T_h + T_l)^{1/2} + \frac{T_l}{T_h}}
\]  \hspace{1cm} (2-12)

where

\[
\eta_{\text{carnot}} = \frac{T_h - T_l}{T_h}
\]  \hspace{1cm} (2-13)

In equation (2-12) \( \eta = \eta_{\text{carnot}} \) when \( Z = \infty \) and \( \eta = 0 \). Thus, Z is a very important property of the TE material, and has been given the name "FIGURE OF MERIT", used to describe the efficiency of a TE material.

Presently, the materials most widely used in thermoelectric power generation are Bismuth Telluride, Lead Telluride, the Selenides and Silicon-Germanium alloys. Because of the very large number of elements needed, even for plants of moderate capacity, it is mandatory to use materials that are abundant and cheap and can be prepared by simple processes. These requirements make use of semiconductors such as Bismuth Telluride. Its useful range of operating temperature extends from room temperature to about 300°K.

Efficient thermoelectric materials exhibit a high Seebeck coefficient, high electrical conductivity, and low thermal conductivity. In earlier times Z values of \( 1.5 \times 10^{-4} \) were used, but with improvement of thermoelectrical materials Z values as high as \( 4.16 \times 10^{-3} \) have been achieved. Unfortunately, the Z value is still quite small.
CHAPTER 3

SYSTEM DESCRIPTION AND OPERATION

The system to be studied is the STEG suggested by Landecker (1976). A schematic diagram is shown in Fig. 3.1. The system uses two disks which are made of P and N type semiconductor material respectively. The disks are insulated from each other by rubber. A concentrating cone is used for collecting the solar radiation. This radiation is absorbed by the hot junction copper rod of a thermoelectric generator and converted into heat. The cold junction is surrounded by water control to regulate the temperature of the cold junction. With cooling water shut off, the TE disks would get quite hot as heat is conducted along the hot junction cylinder and radially out the disks. To get electric power the cooling water would be turned on causing a cooling of the cold junction. The temperature difference would cause electric current to flow. However, the output will soon diminish when the hot junction temperature drops due to the loss of heat to the cooling water. Thus, it might become necessary to turn off the cooling water periodically to allow reheating of the hot junction. The TE elements can be in various series and parallel combinations to provide desired voltage and current (see Fig. 3.2).
Fig. 3.1. Concentration of solar radiation on thermocouple with radial flow of current
Fig. 3.2. Modules in series and parallel
With a typical temperature difference of 300°C a module with a size of 0.0424 meter in diameter and 0.00156 meter in thickness can supply a voltage of about 0.03 volts and a current of about 17 amps. Specific power output can be met by various series and parallel combinations of modules, for example, a typical application of 65 volts and 40 amps will require 2,000 modules in series and 3 modules in parallel, with a total size of about 9 M².

The efficiency of the STEG system is affected by many parameters. These include solar input, height of copper rod, thickness of disk, etc. The efficiency dependence on these parameters can be expressed analytically. However, accurate results have to be obtained from computer simulations which include the transient behavior of the system.
CHAPTER 4

THEORETICAL ANALYSIS

Landecker Analysis

The procedure used in Landecker Analysis to determine the efficiency of STEG is done in the following two phases.

Phase 1

Phase 1 starts right after the cooling water has been turned off. The hot junction copper rod begins to heat up according to

\[ HA = MC_p \frac{dT}{dt} \] (4-1)

where

\[ H = \text{solar input per unit area} \]
\[ A = \text{aperture area of solar collector cone} \]
\[ M = \text{mass of the copper rod} \]
\[ C_p = \text{specific heat of the copper} \]
\[ \frac{dT}{dt} = \text{time rate of change of copper rod temperature}. \]

The time necessary for the copper rod to attain temperature \( T_h \) (limited by Bismuth Telluride material property) is thus

\[ t_1 = \frac{MC_p (T_h - T_\infty)}{HA} \] (4-2)

with

\[ M = \rho \pi (r_i)^2 T_{hh} \quad \text{and} \quad A = \pi (T_{hh})^2 \]

one obtains
Phase 2

When $T_h$ has reached the limiting temperature permitted by material properties (7000$^\circ$K), the cooling water is switched on. The temperature of the cold junction then drops near the temperature of the cooling water almost immediately. This is followed by the more gradual decrease of the hot junction temperature as heat is conducted from the hot junction cylinder to the disks. The temperature difference of the junctions falls exponentially as:

$$\Delta T = (\Delta T)_i e^{-\frac{t}{\theta}}$$

(4-5)

where

$$(\Delta T)_i = T_h - T_\infty = \text{initial temperature difference}$$

$t = \text{the time elapsed since the beginning of Phase 2}$

$\theta = \text{time constant of the process which is equal to}$

$$\theta = \dot{M}C_p\bar{r}$$

(4-6)

where

$\bar{r} = \text{thermal impedance chosen to be } \frac{3}{\delta n K_d}$ from Table 1 (see Appendix D).

When $t = 0$, $\Delta T = \Delta T_i$ and when $t = \infty$, $\Delta T = 0$, as is expected. In Landecker's model this phase continues until $\Delta T = 0.01\Delta T_i$ after
which the cooling water is turned off again. In other words, the length of Phase 2 is

\[ t_2 = \theta \ln 100 \]  

(4-7)

The total work accumulated during this phase is thus (see detail in Appendix A)

\[ W_{\text{out}} = \int_{t_1}^{t_1+t_2} \frac{v^2}{4R} \, dt \quad \text{or} \quad \int_{0}^{t_2} \frac{(\alpha \Delta T)^2}{4R} \, dt \]  

(4-8)

where

- \( t_2 \) is given by equation (4-7)
- \( t_1 \) is determined by equation (4-3)
- \( \Delta T \) is obtained by equation (4-5)
- \( \theta \) is from equation (4-6)

One may replace \( t_2 \) by \( \omega \) in equation (4-7) and obtain approximately (see detail in Appendix C)

\[ W_{\text{out}} = \frac{3(\Delta T_1)^2 \alpha^2 (C_p r_i^2) (T_{hh})}{64(KRd)} \]  

(4-9)

In Landecker’s analysis the efficiency is evaluated as the work output in Phase 2 divided by energy input in Phase 1, namely

\[ \eta = \frac{W_{\text{out}}}{Q_{\text{in}}} \]

where

- \( W_{\text{out}} \) is from equation (4-8)
- \( Q_{\text{in}} \) is from equation (4-4)

This results in:
In Landecker's analysis $R = \frac{3\sigma}{2\pi d}$, assuming the ratio of outer to inner radii $\left(\frac{r_0}{r_i}\right)$ of the two disks to be 4.5.

**Modified Analysis**

In Landecker's analysis two important assumptions have been made: (a) there is no solar input during the cooling period and (b) in the steady state, $\Delta T$ is smaller than $0.01(\Delta T_i)$.

Careful analysis will show, however, these might not be true. In this study, a more realistic analysis is made as follows. In the cooling period (Phase 2), instead of using equation (4-5) one puts:

$$\frac{\Delta T}{\Delta T_f} = (\Delta T_i - \Delta T_f) e^{\frac{-t}{\theta}}$$

(4-11)

where

$\Delta T_f = \text{temperature difference of the two junctions when the system has reached steady state.}$

When $t = 0$, $\Delta T = \Delta T_i$ and when $t = \infty$, $\Delta T = \Delta T_f$ as expected.

Taking the realistic approach that the solar input is available during the whole process, one replaces equation (4-4) by:

$Q_{in} = HA(t_1 + t_2)$

(4-12)

where $(t_1 + t_2)$ is the total time of operation in one cycle (Phase 1 plus Phase 2). Then the efficiency can be expressed as
The efficiency of the system for steady operation \((t_2 = \infty)\) is relatively simple to discuss and can be analytically expressed as (see Appendix B).

\[
\eta = \frac{\alpha^2 H(T_{hh})^2 (\ln \frac{r_0}{r_i})}{64(K)^2 \rho t} \tag{4-15}
\]
This program is intended to simulate the performance of a solar driven thermoelectric generator (Clay 1980). It has been suggested by Landecker (1976) that TE elements with radial flow of current are applicable for this usage.

To find the electrical power produced, the temperature difference between hot and cold junctions must be found. The process to determine the temperature distribution is to use an extended one-dimensional formulation. The disks are broken down into smaller circular rings, width $\Delta r$, and then an energy balance is performed on each ring.

The energy balance equation is established in the following manner (refer to Fig. 5.1).

\[ Q_{in} - Q_{out} + Q_{gen} = \Delta E \] (5-1)

where $Q_{in}$ is energy in, $Q_{out}$ is energy out, $Q_{gen}$ is energy generated internally from Joulean heating, and $\Delta E$ is the element's change in internal energy with time.

The total number of rings is defined as $N$. The position of each ring is defined by a node specified by $i$, the inside ring being $i = 1$ and the outside ring being $i = N$. 

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Fig. 5.1. Ring for \( i = 2 \) to \( i = N - 1 \)
Heat transferred through a solid due to conduction is

\[ Q = \frac{2\pi kt}{\ln\left(\frac{r_o}{r_i}\right)} \Delta T \]  

(5-2)

where \( k \) is the conductivity, \( t \) is thickness of P or N junctions, \( \frac{r_o}{r_i} \) is outer to inner radius and \( \Delta T \) is the temperature difference between the two rings. Therefore, heat conduction in the radial elements is

\[ Q_{in} = \frac{2\pi t(r_i + r_{i-1})}{2} \frac{(T_{i-1} - T_i)k}{\Delta r} \]  

(5-3)

\[ Q_{out} = \frac{2\pi t(r_i + r_{i+1})}{2} \frac{(T_i - T_{i+1})k}{\Delta r} \]  

(5-4)

where \( t \) is the thickness of each ring, \( r_i \) is the center radius of the ring in question, \( r_{i-1} \) is the center radius of the ring just inside \( r_i \), and \( r_{i+1} \) is the center radius of the ring just outside \( r_i \).

The electrical resistance of a ring \( (R_i) \) depends upon the material and geometric shape. Electrical resistance for a ring is found by

\[ R_i = \frac{\rho}{2\pi t} \ln \left[ \frac{r_{i+1} + r_i}{r_i + r_{i-1}} \right] \]  

(5-5)

where \( \rho \) is the electrical resistivity of the ring.

The electrical current produced depends on the temperature difference across the radial device, therefore, the current produced also has to be found in the same iterative process as the temperature distribution. Electrical current produced \( I \), is found by
\[ I = \frac{\alpha \Delta T}{R_{\text{tot}} + R_1} \]  

(5-6)

where \( R_1 \) is the resistance of the load and \( R_{\text{tot}} \) is the internal resistance of the whole couple and can be found to be:

\[ R_{\text{tot}} = \frac{\rho \ln(r_n/r_1)}{\pi t} \]  

(5-7)

It can be shown that under the optimized conditions, \( R_1 = R_{\text{tot}} \).

Therefore, we have

\[ I = \frac{\alpha \Delta T}{2R_{\text{tot}}} \]  

(5-8)

The total heat generated in each ring, \( Q_{\text{gen}} \), can be expressed as

\[ Q_{\text{gen}} = I^2 R_1 \]  

(5-9)

The internal energy of the ring is given by

\[ \Delta E = C_p M_i dT/dt \]  

(5-10)

where \( C_p \) is the specific heat. The mass of each ring, given as \( M_i \), is found by

\[ M_i = \rho_d t \left[ \pi \left( \frac{r_{i+1} + r_i}{2} \right)^2 - \pi \left( \frac{r_i + r_{i-1}}{2} \right)^2 \right] \]  

(5-11)

where \( \rho_d \) is the mass density of the rings.

Substituting all the above into equation (5-1), one obtains an equation governing the behavior of the ring.

The inside element includes the inside ring \( (i = 1) \) and the inner electrode, the energy balance equation (5-1) is used again where \( Q_{\text{in}} \) is the heat from the solar input minus convection and re-radiation losses. \( Q_{\text{out}} \) is heat lost through conduction to the
outer ring and through the Peltier effect. $Q_{\text{gen}}$ is from Joulean heating, and $\Delta E$ is the change in internal energy of the ring and inner electrode.

These are defined as:

\[
Q_{\text{in}} = HA - h' A' (T_1 - T_{\text{amb}}) - \sigma A' (T_1^4 - T_{\text{amb}}^4) \quad (5-12)
\]
\[
Q_{\text{out}} = 4\pi k t ((r_1 + r_2)/2)((T_1 - T_2)/\Delta r) + Ia T_1 \quad (5-13)
\]
\[
Q_{\text{gen}} = I^2 (\rho/\pi t) \ln \left( \frac{r_1 + r_2}{2r_1} \right) + I^2 \rho_{\text{cu}} r_i^2 \pi (T_{\text{hh}}) \quad (5-14)
\]
\[
\Delta E = \rho_{\text{d cu}} \rho_{\text{cu}} C_{\text{p cu}} r_i^2 \pi (T_{\text{hh}}) (dT/dt) \quad (5-15)
\]

where $H$ is the solar flux density, $A$ is the aperture area collecting the solar flux, $h'$ is the convective loss of the inner electrode, $T_{\text{amb}}$ is the ambient air temperature, $T_1$ is the temperature of the first inside ring, $T_2$ is the temperature of the next ring, $\sigma$ is the Stefan-Boltzmann constant for radiation, $\rho_{\text{d cu}}$ and $\rho_{\text{cu}}$ are the density and resistivity respectively of the inner electrode, $C_{\text{p cu}}$ is the specific heat of the electrode and $T_{\text{hh}}$ is the height of the inner electrode.

On the outside ring, $i = N$, the energy balance (equation (5-1)) is used again. $Q_{\text{in}}$ is the conduction heat from the center, $Q_{\text{out}}$ is the heat loss due to convection of the cooling water and removed due to the Peltier effect, $Q_{\text{gen}}$ is from Joulean's heating and $\Delta E$ is the change in internal energy. These are defined as:

\[
Q_{\text{in}} = 2\pi k t ((r_{n-1} + r_n)/2)((T_{n-1} - T_n)/\Delta r) \quad (5-16)
\]
\[
Q_{\text{out}} = 2\pi h r_n t (T_n - T_\infty) + Ia T_n \quad (5-17)
\]
\[
Q_{\text{gen}} = I^2 (\rho/2\pi k) \ln (2r_n/(r_n + r_{n-1})) \quad (5-18)
\]
\[ \Delta E = \rho_d t C_p r_n \pi r_n (\Delta r/2)(dT/dt) \]  

(5-19)

where \( T_\infty \) is the temperature of the cooling water.

Starting with the cooling water off, the solar input heats up the total thermoelectric assembly. When the inside temperature reaches a pre-determined temperature (\( T_{\text{max}} \)), the cooling water comes on. The outside temperature drops rapidly and a large temperature differential is developed which produces the electrical current. The inside eventually cools down, even with continuous solar flux. A new temperature (\( T_{\text{low}} \)) is reached with a small temperature differential. To re-attain the large temperature difference the cooling water is turned off, and the cycle is repeated.

The equations are solved by the implicit iteration formulation which proceeds as follows: The temperature of an element at a given time is expressed as a function of the temperature of the same element at a previous time as well as the temperature of other elements at the same time. In other words

\[ T_i(t) = f(T_i(t - \Delta t) + T_j(t))_{j \neq i} \]  

(5-19)

where \( i \) goes from 1 to \( N \). Since no stability criterion has to be satisfied, any time interval could be used. A \( \Delta t \) of one to two minutes is suggested. This was found to be short enough to give a representative time history of the temperature distribution along with not using an excessive amount of computer time.

The initial condition \( T_i(o) \) is first put into the program, one then "guesses" a set of \( T_i(1) \) to the right of the equation from which the left hand side \( T_i(1) \)'s are calculated. The calculated
The temperature of the hot junction and cold junction and the current are plotted as a function of time. The overall efficiency is printed out, by varying parameters, such as the limiting temperature of the hot junction rod, the diameter of the disks,
the size of the cone, outer to inner radius, heat transfer coefficient, height of copper rod, and solar input. A copy of this program is included in Appendix E.
CHAPTER 6

RESULTS AND DISCUSSION

The computer program was carried out to examine the effect of various parameters on the efficiency of the system. This includes the low temperature setting for the hot junction which controls the water cooling period, the height of copper rod \((T_{hh})\), the thickness of the disk \((t)\), the ratio of outer to inner radius \(\frac{r_o}{r_i}\), the solar input \((H)\), and the convection heat transfer coefficient. The results of varying lower temperature setting for the hot junction will be presented first followed by those for varying the other parameters. All results use the following constants:

- Electrical Resistivity \(\rho = 2 \times 10^{-6} \, \Omega \text{m}\)
- Thermal Conductivity \(k = 1.2 \, \frac{W}{\text{m}^\circ \text{C}}\)
- Seebeck Constant \(\alpha = 1.0 \times 10^{-4} \, \frac{\text{volt}}{\text{C}^\circ}\)
- Density of Copper Rod \(\rho_{cp} = 9000 \, \frac{\text{kg}}{\text{M}^3}\)
- Specific Heat of Copper Rod \(C_p = 386 \, \frac{\text{Joule}}{\text{kg} \circ \text{C}}\)
- Hot Temperature Setting \((T_{h})_{\text{high set}} = 427^\circ \text{C}\)

**Effect of Lower Temperature Setting**

The lower temperature setting for the hot junction \((T_{h})_{\text{low set}}\) was varied to determine the most efficient performance for water control. It was predicted by Landecker that the copper rod
temperature reaches the cooling water temperature during Phase 2 at which point the cooling water should be turned off again. Tables 1, 2 and 3 which resulted from the computer simulation would verify the necessity for water control. Table 1 gives the results of TE generator efficiency for various low hot junction temperature settings, based on parameter values chosen by Landecker (1976). Table 1 shows an inverse relationship between efficiency and hot junction lower temperature setting. This highest efficiency of 1.33% is observed at the lowest temperature setting of 350°K. At this temperature the system will remain in steady state without the cooling water shut off. Thus, it appears that steady state operation with steady flow of coolant yields the highest efficiency.

**TABLE 1**

<table>
<thead>
<tr>
<th>LOWER TEMPERATURE SETTING (K°)</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN ON (MIN)</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K°) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>1.33</td>
<td>∞</td>
<td>30</td>
<td>352.68</td>
</tr>
<tr>
<td>400</td>
<td>1.24</td>
<td>6</td>
<td>30</td>
<td>----</td>
</tr>
<tr>
<td>450</td>
<td>1.24</td>
<td>6</td>
<td>30</td>
<td>----</td>
</tr>
<tr>
<td>600</td>
<td>1.01</td>
<td>4</td>
<td>30</td>
<td>----</td>
</tr>
<tr>
<td>610</td>
<td>1.01</td>
<td>4</td>
<td>30</td>
<td>----</td>
</tr>
</tbody>
</table>

**COMPUTER RESULTS FOR VARIOUS** \((T_h)_{\text{low set}}\) **VALUES BASED UPON GIVEN INPUT DATA:** \(H = 800 \frac{W}{m^2} , t = 0.00776 \text{ M} , \frac{r_0}{r_1} = 4.68 , T_{hh} = 0.050 \text{ M} , (T_h)_{\text{high set}} = 700 \text{°K} , h = 2280 \frac{W}{m^2 \text{°K}} \)**
Different heights of copper rod and solar input were examined in order to see if this parameter would have any influence on the conclusion that a steady state operation is more efficient. For example, Tables 2 and 3 give the results of efficiency for various lower temperature settings of hot junction based upon selected heights of 0.1 and 0.12 meters with solar input of 300, 800 \( \frac{W}{m^2} \) respectively, with other variables remaining the same as before. It is to be noticed that Tables 2 and 3 show similar conclusions, namely steady state operation yields the best efficiency.

**TABLE 2**

**COMPUTER RESULTS FOR VARIOUS \((T_{lh})_{low}\) SET VALUES BASED UPON GIVEN INPUT DATA:** \( H = 300 \frac{W}{M^2}, t = 0.00776 \text{ M}, \frac{r_o}{r_i} = 4.68, \)

\( T_{th} = 0.1 \text{ M}, (T_h)_{high\ set} = 7000 \text{ K}, h = 2280 \frac{W}{m^2 \cdot \text{o}_K} \)

<table>
<thead>
<tr>
<th>LOWER TEMPERATURE SETTING (K)</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN ON (MIN)</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>1.92</td>
<td>( \infty )</td>
<td>20</td>
<td>401.25</td>
</tr>
<tr>
<td>380</td>
<td>1.92</td>
<td>( \infty )</td>
<td>20</td>
<td>401.25</td>
</tr>
<tr>
<td>400</td>
<td>1.92</td>
<td>( \infty )</td>
<td>20</td>
<td>401.25</td>
</tr>
<tr>
<td>450</td>
<td>1.82</td>
<td>6</td>
<td>20</td>
<td>------</td>
</tr>
<tr>
<td>550</td>
<td>1.49</td>
<td>4</td>
<td>20</td>
<td>------</td>
</tr>
</tbody>
</table>

The transient history of the junction temperature corresponding to \((T_{l})_{low\ set} = 450^0\text{K}\) and \((T_{l})_{low\ set} = 600^0\text{K}\) in Table 3 are shown in Figs. 6.1 and 6.2 respectively. Fig. 6.1 shows that when the lower temperature setting is set at 450^0\text{K}, steady state
Steady State Temperature = 595.700 K

Lower Temperature Setting = 450°K

Cold Junction Temperature = 300.71°K

Fig. 6.1. Steady state case when Tss > (Th)low set
temperature would take place at 595.7°K before reaching the setting temperature. In Fig. 6.2 it is shown that when the lower temperature is set at 600°K, which is above the steady state temperature of 595.7°K, the rod temperature drops to setting and raises again due to the turning off of the cooling water. The information on Figs. 6.1 and 6.2 are translated into Fig. 6.3 which shows \((\Delta T)^2\) as a function of time for the two temperature settings. As one may notice the total area under steady state operation is indeed greater. Therefore, the steady state operation offers larger energy output, since the total energy gain is proportional to \((\Delta T)^2\).

These results suggested that the water control is not needed. In other words, water should be on all the time. Thus, all the following analyses for parametric studies are based on constant water flow rate operation.

### TABLE 3

**COMPUTER RESULTS FOR VARIOUS** \((T_{h})_{low}\) **VALUES BASED UPON GIVEN INPUT DATA:** 

\[ H = 800 \text{ W m}^{-2}, \ t = 0.00776 \text{ M}, \ \frac{r_o}{r_1} = 4.68, \]

\[ T_{hh} = 0.12 \text{ M}, \ (T_{h})_{high} = 700°K, \ h = 2280 \frac{\text{W}}{\text{m}^2\cdot\text{°K}} \]

<table>
<thead>
<tr>
<th>LOWER TEMPERATURE SETTING (K°)</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN ON (MIN)</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K°) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>380</td>
<td>4.71</td>
<td></td>
<td>6</td>
<td>595.70</td>
</tr>
<tr>
<td>450</td>
<td>4.71</td>
<td></td>
<td>6</td>
<td>595.70</td>
</tr>
<tr>
<td>596</td>
<td>4.70</td>
<td>16</td>
<td>6</td>
<td>----</td>
</tr>
<tr>
<td>600</td>
<td>4.66</td>
<td>12</td>
<td>6</td>
<td>----</td>
</tr>
<tr>
<td>610</td>
<td>4.54</td>
<td>5</td>
<td>6</td>
<td>----</td>
</tr>
</tbody>
</table>
Fig. 6.2. Nonsteady state when $T_{ss} < (T_h)_{low\ set}$
Fig. 6.3. Comparison between figure 1 and 2

Nonsteady state temperature difference

Steady state temperature difference
Effect of the Height of the Copper Rod

Table 4 gives the computer results of steady-state temperature and efficiency for various heights of the copper rod.

TABLE 4

COMPUTER RESULTS FOR VARIOUS $T_{th}$ VALUES BASED UPON GIVEN INPUT DATA: $H = 800 \ \frac{W}{m^2}$, $t = 0.00776 \ \text{M}$, $\frac{r_o}{r_i} = 4.68$, $h = 2280 \ \frac{W}{m^2 \cdot \text{K}}$, $(T_h)_{\text{high set}} = 700 \ \text{K}$

<table>
<thead>
<tr>
<th>HEIGHT OF COPPER ROD (M)</th>
<th>EFFICIENCY (%)</th>
<th>TIME REMAINING STEADY STATE (MIN)</th>
<th>STEADY STATE TEMPERATURE $(K^\circ)$ OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>4.71</td>
<td>4</td>
<td>593.70</td>
</tr>
<tr>
<td>0.11</td>
<td>4.18</td>
<td>5</td>
<td>553.29</td>
</tr>
<tr>
<td>0.08</td>
<td>2.58</td>
<td>8</td>
<td>508.30</td>
</tr>
</tbody>
</table>

The parameters used by Landecker are used here for comparison purposes. The efficiency versus height of copper rod relation is shown in Fig. 6.4. The results show that when copper rod height increases, the efficiency increases as was predicted by equation (4-16) (efficiency is proportional to rod height). The equation of efficiency (dashed line) in Fig. 6.4 was obtained by substituting the following data into equation (4-16):

\[
\alpha = 1.0 \times 10^{-4} \ \frac{\text{volt}}{C^\circ}
\]
\[
\frac{r_c}{r_i} = 4.68
\]
\[
K = 1.2 \ \frac{W}{W^\circ \text{C}}
\]
\[
H = 800 \ \frac{W}{m^2}
\]
\[
t = 0.00776 \ \text{M}
\]
Fig. 6.4. Comparison between computer and analytical results of efficiency as a function of height of copper rod.

\[ \eta = 8.38 (h_m)^2 \]
\[ p = 2 \times 10^{-6} \ \Omega \ M \]

The two curves on Fig. 6.4 do not quite agree with each other. The reason is due to the simplification made in obtaining equation (4-16). In equation (4-16) we have neglected heat lost through radiation and convection. Although the analytical value is different, the simulation result shows similar behavior according to Fig. 6.4. The increase of efficiency due to the increase of rod height can be explained by the fact that a larger copper rod makes the area of the collector cone larger, thus more heat is absorbed by the solar collector. This increases the temperature of the hot junction and therefore permits a higher efficiency. Also shown in equation (2-12) is the fact that the higher the hot junction temperature the smaller the reduction of efficiency due to irreversible losses. Therefore, an increase of the hot junction temperature increases efficiency not only by increasing the value of the efficiency of a reversible engine \( \frac{T_h - T_1}{T_h} \) but also because of the simultaneous increase of resistance ratio at a given Figure ofMerit (see Appendix A). The analytical results on steady state temperature difference \( (\Delta T)_f \) and the time required for cooling water to remain off \( (t)_{off} \) can be predicted by equations (4-15) and (4-3) respectively. By substituting \( \Delta T = 412^\circ K \) (temperature difference at end of Phase 1), \( C_p = 386 \frac{\text{Joule}}{\text{kg}^\circ C} \), \( \rho_{cp} = 9,000 \frac{\text{kg}}{\text{m}^3} \), \( r_i = 0.0025 \ M \), and \( H = \frac{W}{m^2} \) into equations (4-15) and (4-3) one obtains

\[
(t)_{off} = \frac{0.1864}{T_{hh}} \quad \text{and} \quad (\Delta T)_{final} = 1.659 \times 10^4 T_{hh}^2
\]
The results obtained from these analytical equations by varying $T_{th}$ are in close agreement with the computer readings in Table 4.

**Effect of the Solar Input**

Table 5 gives the computer results of steady state temperature and efficiency for various solar inputs ranging from 400 to 1000 W/m². Other parameters remain fixed at previous values.

The efficiency versus solar input relationship is shown in Fig. 6.5. It is demonstrated that with higher solar input, the system has a higher steady state temperature difference which results in better efficiency. Note that both the dashed line and the solid line in Fig. 6.5 are linear, corresponding to equation (4-16). The observation is made that with solar input 800 W/m² the system is capable of operating with a 3.67% efficiency whereas the efficiency predicted by the analytical method is 8.88%. The difference between these two values of efficiency is mainly due to not accounting for losses.

It is interesting to note that with higher solar input the length of time for termination of water flow is reduced. For example, with a solar input of 400 W/m² it takes 16 minutes before the cooling water is turned on. If the solar input is doubled it takes half the time (8 minutes) for the hot junction to reach the high temperature setting (700°K) and the water to flow back to the system. The analytical results on the time required for cooling water to remain off $(t)_{off}$ and the steady state temperature difference $(\Delta T)_f$
Fig. 6.5. Comparison between computer and analytical results of efficiency as a function of solar input, based on selected parameter values
can be calculated by equations (4-12) and (4-15) respectively. Substituting the input data into equations (4-12) and (4-15) one obtains:

\[(t)_{off} = \frac{1490.925}{H}\]

\[(\Delta T)_{final} = 0.207 \ H\]

The results of these analytical equations for various values of H do not match the computer readings in Table 5, but indicate similar behavior.

**TABLE 5**

**COMPUTER RESULTS FOR VARIOUS SOLAR INPUTS BASED UPON GIVEN INPUT DATA:** \(t = 0.00776 \text{ M, } \frac{r_0}{r_1} = 4.68, \ T_{hh} = 0.11 \text{ M, } h = 2280 \frac{W}{m^2 \cdot \text{K}}, (T_h)_{high \ set} = 700^\circ \text{K, } (T_l)_{low \ set} = 350^\circ \text{K}\)

<table>
<thead>
<tr>
<th>SOLAR INPUT (\frac{W}{m^2})</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K°) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>2.24</td>
<td>16</td>
<td>409.57</td>
</tr>
<tr>
<td>500</td>
<td>2.61</td>
<td>12</td>
<td>434.08</td>
</tr>
<tr>
<td>700</td>
<td>3.32</td>
<td>8</td>
<td>488.47</td>
</tr>
<tr>
<td>800</td>
<td>3.67</td>
<td>8</td>
<td>513.14</td>
</tr>
<tr>
<td>900</td>
<td>3.98</td>
<td>6</td>
<td>537.34</td>
</tr>
<tr>
<td>1000</td>
<td>4.29</td>
<td>6</td>
<td>560.89</td>
</tr>
</tbody>
</table>

**Effect of the Disk Thickness**

Tables 6 and 7 give the computer results for steady-state temperature and efficiency as a function of various thicknesses of the disks where other parameters remain the same as before and the
**TABLE 6**

COMPUTER RESULTS FOR VARYING DISK THICKNESS BASED UPON GIVEN INPUT DATA: $H = 300 \text{MW}, \frac{r_o}{r_i} = 4.68$, $T_{th} = 0.08 \text{M}$

$\frac{W}{m^2 K}$, $(T_h)_{\text{low set}} = 350\text{K}$, $(T_h)_{\text{high set}} = 700\text{K}$

<table>
<thead>
<tr>
<th>THICKNESS OF DISK (M)</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K$^\circ$) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00756</td>
<td>1.27</td>
<td>34</td>
<td>356.80</td>
</tr>
<tr>
<td>0.00656</td>
<td>1.52</td>
<td>30</td>
<td>361.07</td>
</tr>
<tr>
<td>0.00556</td>
<td>1.73</td>
<td>26</td>
<td>372.28</td>
</tr>
<tr>
<td>0.00356</td>
<td>2.38</td>
<td>18</td>
<td>414.37</td>
</tr>
<tr>
<td>0.00256</td>
<td>2.94</td>
<td>16</td>
<td>457.83</td>
</tr>
<tr>
<td>0.00156</td>
<td>3.98</td>
<td>12</td>
<td>550.07</td>
</tr>
</tbody>
</table>

**TABLE 7**

COMPUTER RESULTS FOR VARYING DISK THICKNESS BASED UPON GIVEN INPUT DATA: $H = 800 \text{MW}, \frac{r_o}{r_i} = 4.68$, $T_{th} = 0.08 \text{M}$

$\frac{W}{m^2 K}$, $(T_h)_{\text{low set}} = 350\text{K}$, $(T_h)_{\text{high set}} = 700\text{K}$

<table>
<thead>
<tr>
<th>THICKNESS OF DISK (M)</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K$^\circ$) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00756</td>
<td>2.37</td>
<td>8</td>
<td>396.30</td>
</tr>
<tr>
<td>0.00656</td>
<td>2.77</td>
<td>8</td>
<td>444.02</td>
</tr>
<tr>
<td>0.00556</td>
<td>3.37</td>
<td>8</td>
<td>492.03</td>
</tr>
<tr>
<td>0.00356</td>
<td>4.57</td>
<td>8</td>
<td>587.42</td>
</tr>
<tr>
<td>0.0030</td>
<td>5.10</td>
<td>8</td>
<td>633.62</td>
</tr>
<tr>
<td>0.0256</td>
<td></td>
<td>8</td>
<td>681.46</td>
</tr>
</tbody>
</table>
thickness varies from 0.00756 to .00156 meters. These results are also illustrated in Figs. 6.6 and 6.7. It is observed that the efficiency increases as the thickness decreases. Although the efficiency improves as the disk is made thinner there is a practical limit to how thin they may be since high temperatures will result in thermal stresses and cracking.

Comparing Tables 6 and 7, it is noted that solar input has a great effect on the efficiency of the system. The efficiency is much higher with a 800 $\frac{W}{m^2}$ solar input than for a 300 $\frac{W}{m^2}$ solar input. It is also illustrated in Tables 6 and 7 that it takes less time to heat up the hot junction as the thickness of the disk increases.

The analytical results for steady-state temperature difference can be achieved using equation (4-15). These results are expected to correlate well with those obtained from the computer simulation.

By substituting the input data into equation (4-15) one obtains:

$$(\Delta T)_{\text{final}} = \frac{0.3086}{t} \quad \text{when } H = 300 \frac{W}{m^2}$$

$$(\Delta T)_{\text{final}} = \frac{0.823}{t} \quad \text{when } H = 800 \frac{W}{m^2}$$

**Effect of the Convection Coefficient**

Tables 8 and 9 show the computer results of steady-state temperature and efficiency for various convection coefficients, with $h$ varying from 500 through 8000 $\frac{W}{m^2K}$. The results are also depicted in Fig. 6.8 and show that increasing the heat transfer coefficient causes only a slight decrease
Fig. 6.6. Comparison between computer and analytical results of efficiency as a function of thickness of disk

\[ \eta = \frac{1.56 \times 10^{-4}}{t} \]
Fig. 6.7. Comparison between computer and analytical results of efficiency as a function of thickness of disk.
Fig. 6.8. Computer results of efficiency as a function of convection coefficient
of steady state temperature and a small increase in thermal efficiency. Thus, the cooling water speed is not a major designing factor from the standpoint of generator efficiency. Note that as temperature difference decreases efficiency increases. It is because the higher flow of cooling water causes both $T_h$ and $T_l$ to decrease where the difference $(T_h - T_l)$ remains fairly constant. Therefore, for the same $\Delta T$, the lower $T_h$ will cause better efficiency.
TABLE 9

COMPUTER RESULTS FOR VARYING CONVECTION COEFFICIENT BASED UPON

GIVEN INPUT DATA: \( H = 800 \), \( \frac{W}{m^2} = 4.68 \), \( t = 0.030 \) M,

\( T_{th} = 0.08 \) M, \((T_h)_{high set} = 700^\circ K\), \((T_h)_{low set} = 350^\circ K\)

<table>
<thead>
<tr>
<th>CONVECTION COEFFICIENT</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K°) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>4.76</td>
<td>8</td>
<td>676.66</td>
</tr>
<tr>
<td>1000</td>
<td>4.97</td>
<td>8</td>
<td>658.85</td>
</tr>
<tr>
<td>2280</td>
<td>5.10</td>
<td>8</td>
<td>651.00</td>
</tr>
<tr>
<td>3000</td>
<td>5.12</td>
<td>8</td>
<td>630.72</td>
</tr>
<tr>
<td>4000</td>
<td>5.14</td>
<td>8</td>
<td>628.43</td>
</tr>
<tr>
<td>6000</td>
<td>5.16</td>
<td>8</td>
<td>626.25</td>
</tr>
<tr>
<td>8000</td>
<td>5.17</td>
<td>8</td>
<td>625.10</td>
</tr>
</tbody>
</table>

Effect of the Outer to Inner Radius

Tables 10 and 11 give the results for steady state temperature and efficiency for \( \frac{r_o}{r_i} \) varying from 1.5 through 12.5. These results are also shown in Figs. 6.9 and 6.10. As the radius ratio increases, steady state temperature increases and consequently the efficiency increases. As was indicated in equation (4-14), efficiency is proportional to \( \log \frac{r_o}{r_i} \) and this is shown as the dashed line in Figs. 6.9 and 6.10. The increase of efficiency due to the increase of \( \frac{r_o}{r_i} \) is understandable. Large \( \frac{r_o}{r_i} \) enables the system to
TABLE 10

COMPUTER RESULTS FOR VARYING $\frac{r_0}{r_i}$ BASED UPON GIVEN INPUT DATA:

\[ H = 300 \frac{W}{m^2}, \; T_{hh} = 0.08 \; M, \; t = 0.0030 \; M, \; h = 2280 \frac{W}{m^2 \cdot ^\circ K}, \]

\[ (T_h)_{\text{high set}} = 700^\circ K, \; (T_h)_{\text{low set}} = 350^\circ K, \; r_i = 0.00256 \; M \]

<table>
<thead>
<tr>
<th>OUTER TO INNER RADIUS $r_0/r_i$</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K$^\circ$) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.56</td>
<td>8</td>
<td>367.00</td>
</tr>
<tr>
<td>2.5</td>
<td>2.94</td>
<td>10</td>
<td>469.76</td>
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<td>3.5</td>
<td>3.65</td>
<td>12</td>
<td>516.02</td>
</tr>
<tr>
<td>5.5</td>
<td>4.21</td>
<td>14</td>
<td>571.71</td>
</tr>
<tr>
<td>6.5</td>
<td>4.42</td>
<td>16</td>
<td>596.00</td>
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<td>16</td>
<td>612.00</td>
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<tr>
<td>8.5</td>
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<td>16</td>
<td>633.00</td>
</tr>
<tr>
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<td>4.68</td>
<td>20</td>
<td>648.37</td>
</tr>
<tr>
<td>10.5</td>
<td>4.72</td>
<td>20</td>
<td>659.60</td>
</tr>
</tbody>
</table>
TABLE 11

Computer results for varying $\frac{r_o}{r_i}$ based upon given input data:

$$H = 800 \frac{W}{m^2}, \ T_h = 0.08 \ M, \ t = 0.0030 \ M, \ h = 2280 \frac{W}{m^2K}, $$

$(T_h)$ high set = 700°F, $(T_h)$ low set = 350°F, $r_i = 0.00256 \ M$

<table>
<thead>
<tr>
<th>OUTER TO INNER RADIUS $\frac{r_o}{r_i}$</th>
<th>% EFFICIENCY</th>
<th>TIME REMAIN OFF (MIN)</th>
<th>STEADY STATE TEMPERATURE (K) OF COPPER ROD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.91</td>
<td>4</td>
<td>449.20</td>
</tr>
<tr>
<td>2.5</td>
<td>3.68</td>
<td>4</td>
<td>460.80</td>
</tr>
<tr>
<td>3.5</td>
<td>4.53</td>
<td>6</td>
<td>570.30</td>
</tr>
<tr>
<td>4.5</td>
<td>5.10</td>
<td>6</td>
<td>583.70</td>
</tr>
<tr>
<td>5.5</td>
<td>5.39</td>
<td>6</td>
<td>603.10</td>
</tr>
<tr>
<td>6.5</td>
<td>5.73</td>
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<td>620.80</td>
</tr>
<tr>
<td>7.5</td>
<td>5.89</td>
<td>8</td>
<td>631.70</td>
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<tr>
<td>8.5</td>
<td>6.09</td>
<td>8</td>
<td>645.80</td>
</tr>
<tr>
<td>9.5</td>
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<td>648.40</td>
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<td>8</td>
<td>651.60</td>
</tr>
<tr>
<td>11.5</td>
<td>6.30</td>
<td>8</td>
<td>671.70</td>
</tr>
<tr>
<td>12.5</td>
<td>6.35</td>
<td>8</td>
<td>692.00</td>
</tr>
</tbody>
</table>
Fig. 6.9. Comparison between computer and analytical results of efficiency as a function of outer radius/inner radius.
Fig. 6.10. Comparison between computer and analytical results for efficiency as a function of outer/inner radius.
maintain a larger temperature difference and thus a better Carnot efficiency. The analytical results on steady state temperature difference and the time it takes for the cooling water to remain off can be predicted by equations (4-15) and (4-3) respectively.

By substituting the input data into equations (4-15) and (4-3) one obtains:

when \( H = 300 \ \text{W/m}^2 \)

\[
(t)_{\text{off}} = 9.93 \times 10^5 \ (r_i^2)
\]

\[
(\Delta T)_{\text{final}} = 133.33 \ (\ln \frac{r_o}{r_i})
\]

when \( H = 800 \ \text{W/m}^2 \)

\[
(t)_{\text{off}} = 3.727 \times 10^5 \ (r_i^2)
\]

\[
(\Delta T)_{\text{final}} = 355.5 \ (\ln \frac{r_o}{r_i})
\]

The results of these analytical equations for various of \( \frac{r_o}{r_i} \) appear similar to those obtained by computer readings displayed in Tables 10 and 11.
CONCLUSIONS AND RECOMMENDATIONS

1. STEG is most efficient at steady state operation mode, therefore, water control is not needed.
2. Speed of cooling water is not an important design factor.
3. Efficiency of steady state is proportional to the square of copper rod height, therefore, the height of the rod should be as large as possible subject to the temperature limitation by material. This is the most important parameter which controls the efficiency of STEG.
4. Efficiency is linearly proportional to solar input.
5. Efficiency is inversely proportional to thickness, therefore, better efficiency will be accomplished at smallest possible thickness.
6. As radius ratio increases efficiency increases.
7. The error between computer and analytical results was introduced because of the assumptions which were made in the analytical calculation. These assumptions were a) neglect heat losses through convection and radiation b) the use of \( (\Delta T - \Delta T_f) = (\Delta T_i - \Delta T_f) e^{t/T} \) which gives only an approximate result of temperature difference.
8. The efficiency of STEG is too low to make it practical at the present time. It may become both technically and economically feasible with a reduction in material costs and an increase in the operating figure of merit.
APPENDIX A

MAXIMUM EFFICIENCY AND POWER OUTPUT

Maximum Thermal Efficiency of a Generator

The suitability of any material for direct thermoelectric energy conversion is examined by means of a parameter called the thermoelectric figure of merit $Z$ (Angrist 1976).

$$Z = \frac{\alpha^2}{KR} \quad (A-1)$$

where

$$K = \frac{K_n A_n + K_p A_p}{L} \quad (A-2)$$

$$R = \frac{\rho_n A_n + \rho_p A_p}{L} \quad (A-3)$$

which is true for the straight couples as it is shown in Fig. 2.2.

The essential difference between the "radial" and "straight" couples is the resistance of the disks to radial currents (Landecker 1976)

$$R = \left(\frac{\rho}{\pi t}\right) \ln \left(\frac{r_0}{r_1}\right) \quad (A-4)$$

and the heat conducted per unit time under the influence of a radial temperature gradient,

$$Q = \frac{2 K t}{\ln \left(\frac{r_0}{r_1}\right)} \Delta T \quad (A-5)$$
The maximal values of the efficiency obtained by variation of the shape of the bars are independent of the shape (Boerdijk 1958) which means the same efficiency equation is applied for both configurations. We now may calculate the efficiency of a thermoelectric generator by application of the laws of thermodynamics. The thermal efficiency is defined as the ratio of the electrical power output \( W \) to the thermal power input \( Q_h \) to the hot junction.

\[
\eta_t = \frac{W}{Q_h}
\]  

(A-6)

The thermal input to the hot junction is given by equation (2-4).

\[
Q_h = \alpha I T_h + K \Delta T - \frac{1}{2} I^2 R
\]  

(A-7)

where the amount of heat energy received by the hot junction due to the Peltier effect is \( \alpha I T_h \) and the heat flux transferred from the hot junction to the cold junction is \( K \Delta T \). Of the total Joulean heat \( I^2 R \) generated in the thermoelement, half passes to the hot junction and half passes to the cold junction. The output power is simply the current squared times the load resistance \( R_o \).

\[
W = I^2 R_o
\]  

(A-8)

The thermal efficiency may be computed from equation (A-6).

\[
\eta_t = \frac{I^2 R_o}{K \Delta T + \alpha I T_h - \frac{1}{2} I^2 R}
\]  

(A-9)

We would like to maximize the thermal efficiency. The feasibility of doing this becomes clear if we introduce a new variable, \( M' \), which represents the ratio of the load resistance to the internal resistance of the device.
\[ \eta_t = \frac{M' (\Delta T/T_h)}{\frac{(1 + M')^2}{T_h} \cdot \frac{RK}{\alpha^2} + 1 + M') - 1/2 \frac{\Delta T}{T_h}} \]  \hspace{1cm} (A-10)

The appearance of the product RK in the denominator of equation (A-10), all other things being equal, the smaller RK is the higher the efficiency. Let us form this product from the definitions given by equations (A-2) and (A-3).

\[ RK = K_n \rho_n + K_n \rho_p \left( \frac{\alpha_n}{\alpha_p} \right) + K_p \rho_n \left( \frac{\alpha_p}{\alpha_n} \right) + K_p \rho_p \]  \hspace{1cm} (A-11)

where

\[ \alpha = \frac{A}{L} \]

We minimize this product by taking the derivative of equation (A-11) with respect to \( \frac{\alpha_n}{\alpha_p} \) and set the result equal to zero to obtain

\[ \frac{\alpha_n}{\alpha_p} = \left( \frac{\rho_n K_n}{\rho_p K_p} \right)^{1/2} \]  \hspace{1cm} (A-12)

which will optimize the cross sectional areas. The value of RK when \( \frac{\alpha_n}{\alpha_p} \) has this magnitude is

\[ (RK)_{\text{min}} = \left( (\rho_n K_n)^{1/2} + (\rho_p K_p)^{1/2} \right)^2 \]  \hspace{1cm} (A-13)

Utilizing the minimum value of the RK product given in equation (A-13) in the definition of Figure of Merit, we find the maximum value of the Figure of Merit for any combination of N and P type materials to be:

\[ Z^* = \frac{\left( |\alpha_n| + |\alpha_p| \right)^2}{(\rho_n K_n)^{1/2} + (\rho_p K_p)^{1/2})^2} \]  \hspace{1cm} (A-14)
We may now express the thermal efficiency of a thermoelectric generator which has had its geometry optimized by means of equation (A-12) as

\[
\eta_t = \frac{M' \left( \frac{\Delta T}{T_h} \right)}{(1 + M')^2 \left( z^* T_h \right)^{1/2} + (1 + M') - 1/2 \frac{\Delta T}{T_h}} \tag{A-15}
\]

The value of the resistance ratio \( M' \) that maximizes the thermal efficiency is found by taking the derivative of equation (A-15) with respect to \( M' \) and setting the result equal to zero to obtain

\[
M'_{\text{opt}} = (1 + z^* T_{\text{av}})^{1/2} \tag{A-16}
\]

where

\[
T_{\text{av}} = 1/2(T_c + T_h).
\]

The thermal efficiency with both geometry and load resistance optimized by substituting equation (A-16) into equation (A-15) is

\[
\eta_t^{\text{(max)}} = \frac{(M'_{\text{opt}} - 1)(\Delta T/T_h)}{M'_{\text{opt}} + T_c/T_h} \tag{A-17}
\]

By substituting for \( M' \) and \( T_{\text{ave}} \) in the above equation the new form for maximum efficiency is obtained as

\[
\eta_t^{\text{(max)}} = \eta_{\text{carnot}} \left[ \left( \frac{1 + \frac{Z}{2} (T_h + T_1)}{1 + \frac{Z}{2} (T_h + T_1)} \right)^{1/2} - 1 \right] \tag{A-18}
\]

where

\[
\eta_{\text{carnot}} = \frac{T_h - T_1}{T_h} \tag{A-19}
\]

The first factor represents the thermal efficiency of a reversible engine (Carnot), and the second describes the reduction
of this efficiency as a result of irreversible losses due to heat conduction and Joulean heat entering into the expression for $Z$. If values of thermal conductance or resistivity approach zero, the $Z$ factor will become infinity and the efficiency will reach its Carnot value. Therefore, the thermal efficiency of a thermoelectric generator depends on $K$ and $R$ as well as the temperature difference.

**Maximum Power Output of a Generator**

Instead of maximizing the efficiency, it is sometimes desirable to maximize the power output of a generator. If we express the power output in terms of the resistance ratio $M'$ and the current as given

$$I = \frac{\alpha \Delta T}{R + R_o} \quad (A-20)$$

we obtain

$$P_o = \frac{(\alpha \Delta T)^2 M'}{(1 + M')^2 R} \quad (A-21)$$

which may be maximized by the usual procedure. This operation yields the well known result that for maximum power, the resistance ratio $M'$ is identically equal to unity, that is $R_o = R$. Thus, the efficiency of a generator operating at maximum power is given by equation (A-15) with $M'$ set equal to one:

$$\eta_{t(mp)} = \frac{\Delta T/T_h}{4/(Z^*T_h) + 2 - 1/2 (\Delta T/T_h)} \quad (A-22)$$

which is the same equation that was given by Landecker (1976). In
designing a generator for maximum power output one tries to attain the given power with minimum thermoelectric material. To obtain these objectives it is necessary to maximize the power output per unit of total cross sectional area, \( \frac{P_o}{A_{\text{tot}}} \). In addition, the thickness of element should be kept as small as is practicable. A lower limit on this factor is imposed, because with extremely small elements the contact resistance is no longer negligible in comparison with the resistance of the elements.

The power output per unit of cross sectional area may be found by setting \( M' \) equal to one in equation (A-15) and dividing by the area of the elements \( A_n \) and \( A_p \), thus

\[
\frac{P_o}{A_{\text{tot}}} = \frac{(a\Delta T)^2}{4L((\rho_n/A_n) + (\rho_p/A_p))(A_n + A_p)}
\]  

(A-23)

where \( R \) is given by equation (A-3) and \( A_{\text{tot}} = A_n + A_p \), such that

\[
P_o = \frac{(a\Delta T)^2}{4R}
\]  

(A-24)

which is the same equation as given by Landecker (1976), where we have assumed that the thickness of the N and P type elements are the same. The power output will be a maximum when the denominator of equation (A-24) is a minimum. Taking the derivative of the denominator with respect to \( \frac{A_n}{A_p} \) we obtain an area ratio which maximizes equation (A-24).

\[
\frac{A_n}{A_p} = \left( \frac{\rho_n}{\rho_p} \right)^{1/2}
\]  

(A-25)

The significant difference between the criterion of equation
(A-25) and the one that maximizes the thermal efficiency is that equation (A-25) does not involve the thermal conductivity.
APPENDIX B

THERMAL EFFICIENCY OF STEG USING MODIFIED ANALYSIS

The thermal efficiency of a thermoelectric generator is

\[ \eta = \frac{W}{Q_{in}} \]

where

\[ W = \frac{V^2}{4R} \]  \hspace{1cm} \text{equation (A-24)}

\[ Q_{in} = H\alpha T_{\infty} \]  \hspace{1cm} \text{equation (4-12)}

and

\[ V = \alpha \Delta T \]  \hspace{1cm} \text{equation (2-3)}

where

\[ \Delta T = (\Delta T_i - \Delta T_f) e^{-\frac{t}{\theta}} + \Delta T_f \]  \hspace{1cm} \text{equation (4-11)}

substituting

\[ V = \alpha((\Delta T_i - \Delta T_f) e^{-\frac{t}{\theta}} + \Delta T_f) \]

and

\[ W = \int_0^\infty \frac{V^2}{4R} dt = \int_0^\infty \frac{\alpha^2}{4R} ((\Delta T_i - \Delta T_f) e^{-\frac{t}{\theta}} + \Delta T_f)^2 dt \]
or

\[ W = \int_{0}^{\infty} (\Delta T_i - \Delta T_f)^2 e^{-\frac{t}{\theta}} + (\Delta T_f)^2 + 2\Delta T_f (\Delta T_i - \Delta T_f) e^{\frac{-t}{\theta}} \frac{\alpha^2}{4R} dt \]

Thus the solution becomes

\[ W = \frac{\alpha^2}{4R} \left\{ (\Delta T_i - \Delta T_f) ((\Delta T_i - \Delta T_f)^2 + 2\Delta T_f \theta) + (\Delta T_f)^2 t_{\infty} \right\} \]

The first two terms are neglected since they are very small, so we can write

\[ W = \frac{\alpha^2}{4R} (\Delta T_f)^2 t_{\infty} \]

Therefore

\[ \eta = \frac{\frac{\alpha^2}{4R} (\Delta T_f)^2 t_{\infty}}{HA t_{\infty}} \]

where

R is given by equation (2-9) and \( A = \pi T_h^2 \)

\( \Delta T_f \) can be found writing

\[ Q_{\text{in}} = Q_{\text{out}} \]
\[ Q_{\text{in}} = HA t_{\infty} \]
\[ Q_{\text{out}} = (\sigma(T_h^4 - T_{\infty}^4) + \frac{2\pi k t}{r_o} (T_h - T_{\infty}) + HA(T_h - T_{\infty})) \]

The first and third term of the above equation can be omitted, assuming heat losses through convection and radiation are very small.

Thus, the \( Q_{\text{out}} \) becomes

\[ Q_{\text{out}} = \frac{4\pi k t}{r_o} \Delta T_f \]

\[ \frac{r_o}{\ln r_i} \]
Thus the energy balance becomes

$$H_A = \frac{4\pi kt \Delta T_f}{\frac{r_o}{\ln r_o/r_i}}$$

where

$$\Delta T_f = \frac{H(T_{thh})^2(\ln r_o/r_i)}{4kt}$$

Upon substituting the values of $\Delta T_f$ and $R$ into the equation of efficiency the solution then becomes

$$\eta = \frac{\alpha^2 H(T_{thh})^2(\ln r_o/r_i)}{64k^2 \rho t}$$
The thermal efficiency of a thermoelectric generator is

\[ \eta = \frac{W}{Q_{\text{in}}} \]

where

\[ W = \frac{V^2}{4R} \]
equation (A-24)

\[ Q_{\text{in}} = M \rho \frac{\Delta T}{} \]
equation (4-4)

and

\[ V = \alpha \Delta T \]
equation (2-3)

where

\[ \Delta T = (\Delta T)_1 e^{-\frac{t}{\theta}} \]
equation (4-5)

substituting

\[ V = \alpha (\Delta T_1 e^{-\frac{t}{\theta}}) \]

and

\[ \theta = \frac{M \rho \overline{r}}{p} \]
equation (4-6)

\( \overline{r} \) is chosen to be \( \frac{3}{8\pi K_d} \) from Table 1 in Appendix D therefore

\[ \theta = \frac{3p C_p (r_i)^2 T_{hh}}{8K_d} \]
and

\[ W = \frac{v^2}{4R} \int_0^\infty (\Delta T_i)^2 e^{-\frac{2t}{\theta}} \]

or

\[ W = \frac{(\Delta T_i)^2 \alpha^2 \theta}{8R} \]

Upon substituting the values of \( Q_{\text{in}} \) and \( W \) into the equation of efficiency the solution then becomes

\[ = \frac{3\alpha^2 \Delta T}{64\pi Kd} \]
APPENDIX D

GIVEN INTERNAL RESISTANCE, THERMAL IMPEDANCE
AND MAXIMUM POWER BASED UPON DIFFERENT
OUTER TO INNER RADIUS

TABLE D.1

<table>
<thead>
<tr>
<th>(r_0/r_1)</th>
<th>Internal resistance</th>
<th>Thermal impedance</th>
<th>Maximum power</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_0/r_1 \neq 4.5)</td>
<td>(R_1 = \frac{\rho}{\pi d} \ln(r_0/r_1))</td>
<td>(y^* = \frac{\ln (r_0/r_1)}{4\pi Kd})</td>
<td>(p = \frac{V^2\pi d}{4\rho \ln (r_0/r_1)})</td>
</tr>
<tr>
<td>(r_0/r_1 = 4.5)</td>
<td>(R^* = \frac{3\rho}{2\pi d})</td>
<td>(y^* = \frac{3}{8\pi Kd})</td>
<td>(p = \frac{V^2\pi d}{6\rho})</td>
</tr>
<tr>
<td>(r_0/r_1 = 4.5) and hyperbolic boundary</td>
<td>(R^*_\text{hy} = \frac{0.77\rho}{\pi d})</td>
<td>(y^*_\text{hy} = \frac{0.194}{2\pi Kd})</td>
<td>(p^*_\text{hy} = \frac{V^2\pi d}{3.1\rho})</td>
</tr>
</tbody>
</table>
APPENDIX E

COMPUTER PRINT OUT

FORTRAN IV Q1 RELEASE 2.0 MAIN DATE = 81341 15/3/11 PAGE 0001

0001 REAL K, N, L, I, J, \n0002 DIMENSION X(I), Y(I), Z(I), XJ(I), XI(I), YI(I), ZI(I), R(I), \n0003 X(20), Y(20), Z(20), XJ(20), XI(20), YI(20), ZI(20), R(20) \n0004 CHARACTER LINE(100), STAN, BLANK

0004 DATA LINE/100" /

0005 DATA STAR/"*/

0006 DATA BLANK/"/ 

0007 READ STATEMENTS TO STORE INPUT DATA \n0008 READ, N, TTH, TTL, MOUNT \n0009 READ, TH, R(1), R(N), TZERO, TINFIN, K, TOL, DELTIM, RHOD, RHOD, H, CP, \n0010 ALPHA, HSOLAR, RHODCU, CPCU, THM \n0011 ASOLAR=THM*THM*PI

0012 WRITE STATEMENTS TO PRINT INPUT DATA \n0013 WRITE((6,301),R(I),R(N),TZERO,TINFIN,K,TOL,DELTIM,RHOD,RHOD,H,CP, \n0014 ALPHA, HSOLAR, ASOLAR)

0015 WRITE((6,401),TTH)

0016 WRITE((6,401),TTL)

0017 WRITE((6,401),MOUNT)

0018 32 FORMAT((10X,"DENSITY OF COPPER RHODCU="",F9.2, \n0019 1/10X,"SPECIFIC HEAT OF COPPER CPCU="",F9.3) \n0020 2/10X,"HEIGHT OF COPPER ROD THM","FB",5)

0021 WRITE(6,400) THM

0022 WRITE(6,400) F13 3)

0023 WRITE((6,401),TTL)

0024 401 FORMAT((10X,"TTL="",F13 3))

0025 WRITE((6,401),MOUNT)

0026 30 FORMAT((10X,"AREA OF SOLAR FLUX COLLECTORS",F9.7) \n0027 7/10X,"NO OF NODES IN TE","14, \n0028 7/10X,"NO OF TIME PERIODS",14, \n0029 M SET TO 2 BECAUSE CONDITIONS AT M=1 IS KNOWN \n0030 M=2 \n0031 DISKS ARE BROKEN DOWN INTO PIECES EACH WITH A WIDTH OF DELR
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0022 DELR=I(R(N)-R(I-1))/(N-1)
0023 STARTING TEMPERATURE OUTSIDE OF RING FOUND
0024 TR=1/(T(1)+2*PI)*TH/(ALOG(R(N)/R(I-1))+R(2)+PI*R(N)*TH+TINF)/(2*PI
0025 +TH*ALOG(R(N)/R(I-1))+R(2)+PI*R(N)*TH)
0026 TOTAL POWER (PDT) AND TOTAL SOLAR INPUT (TSOLAR) SET TO ZERO
0027 PDT=0
0028 TSOLAR=0.
0029 DO 200 I=1,N
0030 DISK BROKEN INTO N SECTIONS WITH EQUAL THICKNESS WITH THE RADIUS OF
0031 EACH NODE FOUND. THE STARTING TEMPERATURE DISTRIBUTION (TR) IS
0032 FOUND FOR EACH NODE. THIS IS THE INITIAL GUESS FOR THE NEXT TIME
0033 PERIOD (M=2) TEMPERATURE DISTRIBUTION
0034 R(I)=R(I-1)+DELR
0035 TR=I/2*(T(1)+T(1)+1)*DELR
0036 DO 200 I=1,N
0037 200 T(I)=T(I-1)
0038 TOTAL ELECTRICAL RESISTANCE FOUND.
0039 R(0)=R(0)/DELTIN
0040 STARTING ELECTRIC CURRENT IS FOUND (ICT) AND USED FOR THE INITIAL
0041 GUESS FOR THE NEXT TIME PERIOD (M=2)
0042 C1=IC+T(1.1)-T(I.1)/RTOT
0043 C2=C(1)
0044 COMPUTER GUESS OF A NEW TEMPERATURE DISTRIBUTION FOUND
0045 600 C1=IC+T(I.1)+R(2)/(DELR
0046 C1=C(M)+C(M)+RHO*ALOG(2*(R(2)/(R(I-1)+R(2)))/(PI)*TH
0047 C3=Halog+Aalog
0048 C4=-RHO*TH*CPI*R(1)+DELR/DELTIN
0049 C5=C5
0050 C6=RHO*CU*CPUI*PI*R(1)+TH/DELTIN
0051 C7=C7
0052 C8=C(M)+ALPHA
0053 C9=C(M)+ALPHA
0054 CO=CO
0055 FORTRAN IV 91 RELEASE 2.0 MAIN DATE = 81341 12/30/11 PAGE 0003
0056 DO 201 I=2,NN
0057 TN(I)=C(I)*T(I.1)+C2+C3+C4+C5*TN(I.1-1)+C6)/(C5+C1+C6)
0058 TN(I)=C(I)*T(I.1)+C2+C3+C4+C5*TN(I.1-1)+C6)/(C5+C1+C6)
0059 C5=C(M)+C(M)+RHO*DELTIN*ALOG(1+R(I-1)+R(1))/TH
0060 C3=C3
0061 C4=2*CP*HDO*R(I-1)*DELTIN
0062 C5=C5
0063 C6=C6
0064 201 IF CABS(CN(IM)-CN(IM-1))/CN(IM-1) .GT. TOL GO TO 701
0065 IF CABS(CN(IM)-CN(IM-1))/CN(IM-1) .GT. TOL GO TO 701
0066 IF CABS(CN(M)-CN(M-1))/CN(M) .GT. TOL GO TO 701
0067 IF CABS(CN(M)-CN(M-1))/CN(M) .GT. TOL GO TO 701
0068 IF CABS(CN(M)-CN(M-1))/CN(M) .GT. TOL GO TO 701
0069 IF CABS(CN(M)-CN(M-1))/CN(M) .GT. TOL GO TO 701
0070 COMPUTER GUESS OF A NEW CURRENT FOUND
0071 CN(M)=ALPHA*(C(M)-T(N-M))+(RTOT+2)
0072 CN(M)=CN(M)+CN(M)*RTOT
0073 CN(M)=CN(M)+CN(M)*RTOT
0074 CN(M)=CN(M)+CN(M)*RTOT
0075 CN(M)=CN(M)+CN(M)*RTOT
0076 CN(M)=CN(M)+CN(M)*RTOT
0077 CN(M)=CN(M)+CN(M)*RTOT
0078 IF (ABS(TN(J,M)-T(J,M)) .GT. TOL) GO TO 202
0079 IF (ABS(CN(M)-CN(M)) .GT. TOL) GO TO 202
0080 IF (ABS(CN(M)-CN(M)) .GT. TOL) GO TO 202
0081 IF (ABS(CN(M)-CN(M)) .GT. TOL) GO TO 202
0082 IF (ABS(CN(M)-CN(M)) .GT. TOL) GO TO 202
0083 IF (ABS(CN(M)-CN(M)) .GT. TOL) GO TO 202
0084 IF (ABS(CN(M)-CN(M)) .GT. TOL) GO TO 202
0085 PROCESS CONTINUED TO ALL COMPARISONS WITHIN TOLERANCE
0086 DO 202 J=1,N
0087 202 TN=TN(J.1)-T(J.1)
0088
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0061  25 T(I,H)+TN(I,H)  
0062  C(I)+C(M)  
0070  DO 602 I=1,N  
0071  C  
0072  601 DO 602 I=1,N  
0074  FOR THE NEXT TIME PERIOD, THE LAST TEMPERATURE DISTRIBUTION AND CURRENT  
0075  C IS USED AS THE INITIAL GUESS FOR THE NEW COMPARISON PROCESS  
0076  FORTRAN IV 01 RELEASE 2.0  
0077  TOTAL POWER AND SOLAR INPUT BURNED  
0078  POT+PTOT*PTH(8+DELTM)  
0079  ISOLAR*ISOLAR+SOLAR+ASOL+DELTM  

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602  T(I,H)+T(I,H)  
603  C(I)+C(M)  

TOTAL POWER AND SOLAR INPUT BURNED  
PTOT+PTOT*PTH(8+DELTM)  
ISOLAR*ISOLAR+SOLAR+ASOL+DELTM

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IF (H.E.Q.MCOUN& ) GO TO 603
H=M  

CALL PLOT (C,MOUNT,DELTM)  
CALL PLOT (C,MOUNT,DELTM)  
EFF+PTOT/TISOLAR
WRITE (16,31) PLOT  
WRITE (16,32) TISOLAR
WRITE (16,32) EFF
50 FORMAT (10X,'TOTAL SOLAR=',F13.3)
31 FORMAT (10X,'TOTAL ENERGY GAINED=',F13.3)
32 FORMAT (10X,'EFF=',F9.4)
STOP
END

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*OPTIONS IN EFFECT*  
NOTEST, TRIM, BCD, SOURCE, NOLIST, NODATE, LOAD, NODATE, NOTEST
*OPTIONS IN EFFECT*  
NAME = MOUNT, LINEOUT = 50
*STATISTICS*  
SOURCE:  TAMELIS  92: PROGRAM SITE = 117B
*STATISTICS*  
COMPUTATIONAL CYCLES: HIGHEST SEVERITY CODE IS 8

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PLOT DATE = B1341  19/52/11

SUBROUTINE PLOT (C,MOUNT,DELTM)
REAL LDW
DIMENSION R(30), T(30,30), TN(30,30), C(30), CN(30), P(30)
CHARACTER LINE(100), STAR, BLANK
DATA LINE/100*,, /
DATA STAR/,, /
DATA BLANK/,, /
LDW=T(I,1)
HI=T(I,1)
DO 20 J=2,N
DO 20 J=2,MOUNT
IF (HI<T(I,1), HI=HI+1)
20 CONTINUE
PRINT 21
1 FORMAT (10X,'TOTAL=',1X,'TIME(SEC)','=4X,'TEMPERATURE')
17 NN=1
18 DO 20 J=1,MOUNT
19 DO 20 J=1,N
20 TIME=TIME+DELTM
21 B=99*(1(J,1)-LDW)/(HI-LDWW)+1
22 B=B/2
STOP
0023  LINE(B)=STAR
0024  PRINT 205, TIME, T(I), LINE
0025  205 FORMAT('O:FB,2.7x,F7.4x,100A1')
0026  LINE(B)=BLANK
0027  30 CONTINUE
0028  RETURN
0029  END
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OPTIONS IN EFFECT:  NOSTAT, NOID, BCD, SOURCE, NODECK, LOAD, NOMAP, NOTEST
*OPTIONS IN EFFECT = NAME = PLOT*  LINES =  50
*STATISTICS*  SOURCE STATEMENTS = 29  PROGRAM SIZE = 303B
*STATISTICS*  PRINT DIAGNOSTICS GENERATED. HIGHEST SEVERITY CODE IS B
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0001  SUBROUTINE PLOTC
0002  CALL COUNT, DELTIN
0003  DIMENSION R(30), T(30,30), TN(30,30), C(30), CN(30)
0004  CHARACTER LINE(100), STAR, BLANK
0005  DATA LINE+/1000', '6"
0006  DATA STAR/"/6"
0007  DATA BLANK/"/6"
0008  LOW(C(I))
0009  H(I)=C(I)
0010  DO 20 I=1,MCOUNT
0011   IF (C(I).LT.LOW) LOW=C(I)
0012   IF (C(I).GE.HI) HI=C(I)
0013  20 CONTINUE
0014  PRINT 211
0015  211 FORMAT('O:FB,2.7x,'TIME(SEC)',6x,'CURRENT')
0016  DO 30 I=1,MCOUNT
0017    TIME=DELTIN-DELTIM
0018    B=V0 *(C(I)-LOW)/(HI-LOW)+1.
0019    B=4/2+B
0020    LINE(B)=STAR
0021  PRINT 205, TIME, C(I), LINE
0022  205 FORMAT('O:FB,2.7x,F7.4x,100A1')
0023  LINE(B)=BLANK
0024  30 CONTINUE
0025  RETURN
0026  END
FORTRAN IV 01 RELEASE 2.0  PLOT  DATE = 81341  15/52/11  PAGE 0002
LIST OF REFERENCES


