Examination of the Feasibility of an Earth Coolant Tube to Provide Residential Space Cooling

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EXAMINATION OF THE FEASIBILITY OF AN EARTH COOLANT TUBE TO PROVIDE RESIDENTIAL SPACE COOLING

BY

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ABSTRACT

A study was performed to gain an understanding of the feasibility of an Earth Coolant Tube for use in cooling and heating air for residences and industry. It was concluded that previous studies did not include the effect of coolant tube depth or coolant tube operation over long periods of time. A numerical methods approach using a finite difference form of the general energy conduction equation was used to evaluate these effects. It was concluded that a coolant tube 1 foot in diameter and 100 feet long could provide as much as 1/6 ton of refrigeration for a 4 month time period. It was also concluded that for coolant tubes below a depth of five feet, depth had little effect on coolant tube performance. This study also presents estimates on expected rates of energy transfer for coolant tubes, and recommends a simplified approach for designing coolant tubes.
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<td>area</td>
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<td>$[A]$</td>
<td>matrix A</td>
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<td>$[B]$</td>
<td>matrix B</td>
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<tr>
<td>$\text{Biot}$</td>
<td>dimensionless Biot number</td>
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<tr>
<td>$\text{CFM}$</td>
<td>volumetric flow rate</td>
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<td>specific heat capacity</td>
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<td>dimensionless number that relates coolant tube properties</td>
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<td>diameter</td>
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<tr>
<td>$m$</td>
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<td>$\text{NDAYS}$</td>
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<td>$\dot{Q}$</td>
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<td>pipe radius</td>
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<td>( \Delta T )</td>
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<td>( \lambda )</td>
<td>stability factor</td>
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<td>( \rho )</td>
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Subscripts and Superscripts

a       air
in      at pipe inlet
l       liquid
l       x coordinate counter
m       y coordinate counter
out     at pipe outlet
s       soil
sv      saturated liquid, vapor
v       vapor
v       time step counter
The purpose of this study was to study the feasibility of an earth coolant tube for use in cooling or warming air.

An earth coolant tube is a simple device consisting of a long pipe buried in the ground. Soil deep under the ground surface maintains a relative stable and moderate temperature year round. This underground energy reservoir could be used as an energy sink in the summer to cool air that passes through the buried pipe and as an energy source in the winter to heat air drawn through the pipe.

In this study, a mathematical model and computer simulation model were developed for a coolant tube system and the results of the simulation were compared to available data. For mass flow rates less than 2500 lbm/hr, the model varied from the experimental data by less than 20%.

As a result of the study, it was concluded that a coolant tube could provide cooling for an entire cooling season. It was also determined that there is little advantage to burying coolant tubes below 5 feet deep.
A literature search was conducted to locate previous work on the use of earth cooling tubes. Previous studies can be divided into two main categories. First, those papers directly related to modeling or evaluating the performance of an earth cooling tube system will be discussed. Second, those papers which are indirectly related to earth cooling tubes in that they discuss background information or talk about systems similar to earth cooling tube systems, will be discussed.

**Directly Related Models**

Five papers present information about the evaluation or modeling of earth cooling tube systems.

Two articles by Jan F. Krieder (1,2) were found that discussed and presented methods for evaluating cooling tube systems.

The first of the two articles presented a simple technique for determining the length of pipe needed to provide a desired amount of cooling (1). Krieder stated that the rate of energy transfer for a cooling tube could be described using an approach common to heat exchanger design. No development was shown for the relationship. The product of the overall heat transfer coefficient
and area (UA) that was used in the equation was stated as $0.07 \times L$. The log mean temperature difference was used to represent the average temperature difference, $\Delta T$.

A few months after the first article was published, due to popular demand, a second article by Krieder appeared, presenting a more complex method of performing earth cooling tube calculations (2). In this article, Krieder distinguished between laminar and turbulent flow in the cooling tubes and simplified his expression for the average temperature difference, replacing the log mean temperature difference by a simpler arithmetic temperature difference. The desired amount of cooling was calculated as:

$$Q = 1.08 \times \text{CFM} \left( T_{\text{in}} - T_{\text{out}} \right) \quad (2-1)$$

The length of pipe needed to provide the desired rate of heat transfer was obtained from equation 2-2,

$$Q = 0.13 \left( V \times D \right)^{0.8} \times L \times \Delta T \quad (2-2)$$

if the flow in the tube was turbulent, or equation 2-3,

$$Q = 0.16 \times L \times \Delta T \quad (2-3)$$

if the flow in the tube was laminar.

The paper also stated that if the pipe length is already known, the rates of heat transfer calculated in equations 2-1 and 2-2 or 2-3 would not be the same if the air exit temperature (used in equation 2-1) was inaccurately estimated. The rate of energy transfer leaving the air (found in equation 2-1) must equal the rate of energy transfer through the coolant tube (found in equations
2-2 or 2-3). Thus, the correct value of the air exit temperature would be obtained by an iterative technique. By performing this series of calculations, the rate of energy transfer provided by a given coolant tube system of known dimensions, could be calculated. No background on the derivation of the equations that were presented or the constants in them was given.

The Princeton Energy Group evaluated the cooling potential of an underground duct (coolant tube) (3). Their efforts can be broken down into four main categories: (1) experimental, (2) simulation, (3) results of the simulation and (4) future work.

The description of the experimental analysis described the efforts of the Princeton Energy Group to construct an underground duct for test purposes. Because of numerous problems, the experimenters were never successful in obtaining useful data from their test setup. The paper states that attempts to evaluate the test system were going to be made the following year; however, no more work was ever done on the project.

The simulation of a cooling tube system represented the majority of the work presented in the paper. The system that was analyzed consisted of a typical residence with a cooling tube to cool incoming air during the warmer summer months. The inside air temperature of the house was estimated hourly from the cooling or heating load on the house. If the temperature of the house \( T_h \) was less than 70°F, auxiliary heating was initiated. If the house temperature was between 70°F and 75°F, no heating or cooling took place. If the
house temperature was between 75°F and 80°F and the temperature of the cool-pipe air was at least 3°F less than the house temperature, the cooling pipe was assumed to be running. Finally, if the house temperature was greater than 80°F, auxiliary cooling was used to cool the house.

By evaluating the house for an entire season, using an incremental time step of one hour, the seasonal cooling provided by the cool-pipe was estimated. The simulation was performed 30 different times. Each time the simulation was performed, different key parameters were varied. The method of calculating the energy transfer from the cool-pipe was never discussed.

By analyzing the results of the simulation, an equation for calculating the total seasonal cooling provided by a single underground duct was developed. It was:

\[
\text{Cooling (million Btu)} = 0.00463 \times \text{air flow (cfm)} + 0.0220 \times \text{diameter (inches)} + 0.0168 \times \text{length (feet)} + 0.168 \times \text{soil conductivity (Btu/hr-F-ft)} + 0.0580 \times \text{soil heat capacity (Btu/lbm-F)} - 0.970 \text{ (million Btu)} \quad (2-4)
\]

Several assumptions and simplifications were made during the simulation. The assumptions included:

1. latent heat transfer in the cool-pipe was neglected
2. only one climatic region (New Jersey) was studied
3. only one type of cooling system was studied
4. the assumptions made in calculating the amount of energy transfer from the cool-pipe to the ground were not stated.

Another simulation developed by Abrams, Benton and Akridge (4) presented a performance model for earth cooling tubes. The purpose of the model was to gain understanding of the effect of significant variables on the rate of energy transfer from the air in a coolant tube to the earth.

Some of the simplifying assumptions made in the model were:

1. the pipe was assumed to be a point source of energy, with a surrounding heat sink of infinite size. This assumption ignored the effects of ground surface conditions on the heat sink,
2. energy transfer axially down the pipe was assumed to be negligible,
3. the earth mass was assumed to initially be isothermal with constant thermal conductivity and diffusivity, both radially and axially along the tube,
4. provisions for cycling run conditions were not incorporated in the model. Cycling run conditions refers to using the cooling tube system to cool air and then turning off the system in order to recharge the energy sink around the tube,
5. latent heat transfer was not specifically addressed.

In comparing this model with other models, the sensitivity study performed did the best job of representing the significant variables which effect the cooling tube performance. The variables analyzed in the model were tube length, soil conductivity, soil diffusivity, temperature difference between the air in the pipe and the earth surrounding the pipe, the convection heat transfer.
coefficient, the length of time the tube is used, thermal resistance of the tube wall, and tube diameter.

The sensitivity studies can be summarized by observing that the following variable changes will improve the performance of a cooling tube (increase the rate of energy transfer from the air in the tube to the earth around the tube).

1. increase the length of the tube
2. increase the soil thermal conductivity
3. decrease the soil diffusivity
4. increase the temperature difference between the earth and the air
5. increase the convection heat transfer coefficient
6. minimize the tube run time (as the length of time the tube is used increases, the rate of energy transfer from the air to the earth decreases)
7. increase the tube diameter

At the time this model was developed, efforts to verify the model with experimental data had been unsuccessful. One year after the paper was written, experimental data was obtained by Benton and Akridge. This experimental data was included in a report entitled, "Performance Study of a Thermal Envelope House" (5). This report will be discussed later.

This model provided a reasonable method of examining the performance of an earth cooling tube system. The model provides the designer with a sound feeling for how parameter variations affect the performance of a cooling tube system. A need for more accurate
models which will be able to better predict the performance of cooling tube systems still exists. One drawback of this model is that it requires the use of a small computer. No simple method was proposed in the paper that could be used by a designer to predict the performance of a cooling tube system. To make up for this deficiency, Abrams, Benton and Akridge included several graphs in their paper that presented estimations of coolant tube performance for typical variations in design parameters.

This paper by Benton and Akridge (5) was useful in understanding the performance that could be expected from an earth cooling tube system. This report presented actual experimental data obtained from a cooling tube system. This report contained the only set of experimental data from an earth cooling tube system that could be found.

A thermal envelope home in Canton, Georgia, which contained two coolant tubes, was monitored during the experimental system test. Different aspects of the home cooling system were tested; however, only the cooling tubes' performance will be discussed here.

The house used two 21 inch diameter, 100 feet long earth cooling tubes to supply earth tempered air to the house envelope. The cooling tubes were tested by installing temperature probes axially along the tubes and radially away from the tubes. This was done to measure how the air temperature varied as air was drawn through the tubes and to determine how the earth temperature around the tube was affected by the energy transfer that took place. The volumetric
flow rate through the tube was varied by adjusting the tube inlet size, or the speed of the fan used to draw air through the tube.

The data gathered were compiled and presented in several different forms. First, air temperature as a function of time for different positions along the tube was plotted. An example of this type of data is shown in Figure 2-1. Secondly, the ground temperature as a function of time for different radial distances from the tube was sketched (Figure 2-2). Data was also presented on air temperature as a function of distance along the cooling tube for different tube volumetric flow rates, Figure 2-3.

The data presented by the paper was useful as a standard of comparison when comparing various mathematical models (Chapter V). This was because it was actual experimental data measured from an existing coolant tube system, and not estimates on what a system might provide, derived from a mathematical model with simplifying assumptions.

A report by Nordham, entitled, "A Design Procedure for Underground Air Cooling Pipes Based on Computer Models" (6) described an attempt to evaluate the heat transfer related effects of water condensing on the inside of a cooling tube. The work is unique in that it is the only cooling tube study found that examined the effects of condensation.

In the model, conductance values for filmwise condensation of vapor on the inside of tubes were obtained using a modified integral analysis as shown by Chen (7). For the study, the equation:
Fig. 2-1. Temperature distribution along cooling tube - Day 2S (5).
Fig. 2-2. Ground temperature gradient perpendicular to the pipe - Day 28 (5).
Fig. 2-3. Effect on fan speed on temperature gradient along the cooling tube (5).
\[ h_c = \frac{0.725 \times \left[ \rho_1 (\rho_1 - \rho_v) \times g \times h_{fg} \times k^2 \right]}{[D \times U_1 \times (T_{sv} - T_s)]} \]  

(2-5)

was used to calculate the conductance values.

The description of the model used to evaluate the performance of the cooling tube was sketchy. Basically, a simple grid with 10 nodal points in the soil around the tube for every incremental segment of tube length was established. A theoretical temperature gradient between the nodes was estimated using conservation of energy considerations. The energy balance was repeated for each increment along the tube. From the temperature gradients found, the temperature and enthalpy of the air in the tube were estimated.

The results of a typical analysis were presented. The analysis concluded that 13,000 BTU/hr could be absorbed by the tube surrounding with an additional 8,000 BTU/hr being transferred to runoff water in the tube and lost through condensation. The additional 8,000 BTU/hr were attributed mostly to what the author called a misting chamber. The method of operation or a description of the misting chamber was not discussed in the paper.

The paper concluded that by increasing the unit surface heat conductance and by pre-humidification of air through the use of a misting chamber, the overall length of an underground tube could be reduced considerably. This was due to an increase in the ability to transfer energy from the air to the soil around the tube.

The validity of the results of this paper are doubtful and
need to be analyzed carefully. The paper indicates that because condensation of water vapor in the air is occurring, the air will be cooled more than if no condensation were occurring. This is not the case. The condensation does increase the ability to transfer energy from the air to the soil around a coolant tube by increasing the overall heat transfer coefficient. This is because of the higher convection heat transfer coefficient for condensation compared to that for energy transfer from air to a dry pipe wall. However, another factor must also be analyzed. The amount of energy that must be absorbed by the soil to cool air and condense water vapor is much greater than the amount of energy that must be absorbed to just cool the air. In other words, the condensation provides better energy transfer characteristics; however, more energy must be transferred. It is not known whether the author of this study analyzed both effects.

Indirectly Related Research

Papers that discuss topics related to earth cooling tubes will be presented in this section. An article by Bahadori discusses the passive cooling techniques used in Iranian architecture (8). Many methods for cooling air are discussed. Among them is the discussion of a cooling system similar to an earth cooling tube system.

The Iranian cooling system consists of a cooling tower (similar to what is known as a thermal chimney in the United States today) located about 50 meters from a building that is being cooled.
An underground tunnel is dug from the bottom of the tower to the basement of the building being cooled. The ground over the tunnel is planted with trees, shrubs and grass. When the ground is watered, water diffuses through the soil so that the tunnel walls are kept damp, and air coming through the tower and the tunnel is sensibly and evaporatively cooled.

Other systems take advantage of underground streams. The theory of operation is similar to the tunnel systems. As the air passes over the underground stream, it is cooled before it enters the house.

The article states that systems of this type have been used since around 300 A.D. From this paper, it is apparent that coolant tubes are not a new innovation in passive cooling.

It should be pointed out that the difference between the Iranian tunnels and the cooling tubes being designed today is the importance of evaporative cooling. Because of the low relative humidity in Iran, the increase in relative humidity caused by evaporative cooling is not an undesirable effect. The problem of analyzing cooling tube performance taking into account energy transfer effects related to condensation of liquid in the tube was discussed in the article entitled, "A Design Procedure for Underground Air Cooling Pipes Based on Computer Models" (6). This can be a major consideration that could be analyzed in areas with high humidity.

A study by Jacobs et al. (9) discussed a problem similar to an earth cooling tube problem; however, there was one important
difference. Both the pipeline considered and the earth cooling tube analysis were concerned with the amount of energy transfer into a buried pipeline. The difference between the two problems is that in the buried pipeline problem, all energy transfer took place between the buried pipe and the ground surface above the pipe. The earth is treated like a layer of insulation. The possibility of the earth being a heat sink for energy and the related effects on energy transfer out of the pipe were not considered. An important consideration of earth cooling tubes is to determine how the earth energy reservoir will affect the buried pipe.

Even though the pipeline problem is different than the cooling tube problem, several methods of calculating the energy flux into the pipe were applicable to cooling tubes. The temperature profile around the pipe was first estimated using a finite element model. Once the temperature was determined, the energy flux into the pipe was calculated using Fourier's Law. Different assumed temperature profiles between the nodes in the grid around the pipe resulted in different values of energy flux. Three temperature profiles were analyzed: (1) quadratic, (2) cubic, and (3) logarithmic. Each method was evaluated in terms of accuracy by applying each to a problem with a known solution. The logarithmic temperature profile gave the best results, while the cubic and quadratic fits varied by as much as 15% from the known solution.

Carlson and Jaeger's book (10) is a collection of purely mathematical solutions to conduction heat transfer problems in solids.
The book contains the solutions to problems similar to the earth cooling tube problem. The closest such related problem was of a long cylinder in infinite surroundings. Because ground surface effects cause a coolant tube to be in non-infinite surroundings, the purely mathematical solution was never used for the modeling of a coolant tube; however, the solution could possibly have more merit than given in this discussion.

The book, "Earth Sheltered Housing Design" (11), presented guidelines for earth sheltered housing design. It was prepared by the underground space center, University of Minnesota. The book discussed how soil temperatures were affected by daily and seasonal surface air temperature variations. According to the book, the wide daily air temperature swing has no effect below 0.2 m (8 inches). At greater depths, soil temperatures respond only to seasonal changes and change occurs after a considerable delay. An example for Minneapolis-St. Paul was given. At 5 to 8 meters (16.5 to 26.5 ft), ground temperature was almost constant year round.

An article by Spiegel (12) also contained information on how deep ground temperatures were affected by surface temperatures. The book stated that at a depth of ten feet below the surface, the ground temperature will vary annually on the order of magnitude of plus or minus 6°F from the average temperature for a given area. At a depth of approximately 30 feet, the variation will be less than 1°F. Going below 30 feet, the ground temperature will become warmer because of the warm earth core.
An article by Hartley and Black (13) was subdivided into two phases. The first phase consisted of formulating the conservation equations for simultaneous energy and mass transfer in a homogeneous, isotropic porous medium. The governing equations were non-dimen-

sionalized, and an order-of-magnitude analysis was performed to determine those dimensionless groups which had the most significant effect on the energy transfer and moisture movement in the soil surrounding a heat source. An implicit, finite difference scheme was used to solve the governing equations for transient temperature and moisture content distributions. The second phase of the study consisted of correlating the results of the analysis so that the conditions which lead to significant moisture migration away from the heat source in the soil could be predicted. The results of the analysis were compared with experimental measurements.

As a result of determining the influence of the dimensionless coefficients in the coupled equations used to describe the temperature and moisture distributions, it was concluded that the conduction equation with moisture dependent thermal properties is adequate to determine the temperature distribution. It was also concluded that the dimensionless temperature profile would increase linearly with respect to the Fourier Number until a critical time was reached when significant moisture movement occurred. The value of Fourier Number where the moisture movement occurred was found to depend only on the initial soil moisture content and the heat flux from the source. Soon after the moisture movement occurred, the
soil would become dry and assume new thermal properties. After some soil drying, the dimensionless temperature was observed to again increase linearly with Fourier Number, except at a faster rate. For low values of heat flux, it was observed that drying of the soil never occurred and constant thermal properties could be used throughout the problem.

The paper proposed simple tests that could be conducted to evaluate the thermal properties of moist soil, the onset of drying at the surface of a source, and the time when drying at the surface of the source was complete. These techniques would be useful for coolant tube experiments.

**Summary: The Need for Future Work**

Of the papers that were studied, all had merit in that they discussed some unique aspect of an earth coolant tube. The following major aspects were presented or examined: (1) a simplified method to determine the size of an earth cooling tube, (2) the seasonal cooling that could be expected from a cooling tube, (3) the effect of key parameters on cooling tube performance, (4) the effect of latent heat on cooling tube performance, and (5) experimental data obtained from a cooling tube.

Many areas of cooling tube performance have been studied; however, there are several areas that still need to be understood. The major areas that will be discussed in the chapters that follow are: (1) the effect of using a cooling tube for a long period of
time to determine the ability of the tube to continue to cool air for a cooling season, (2) the effect of surface temperature variations on the cooling tube, (3) the effect of pipe depth on cooling, and (4) a comparison of experimental data with available simulation models to determine their abilities to predict cooling tube performance.
CHAPTER III

MATHEMATICAL MODEL AND COMPUTER SIMULATION MODEL

After studying the different papers and models discussed in Chapter II of this thesis, it was concluded that a more general mathematical model needed to be developed. This model would be designed to include some of the characteristics of an earth cooling tube that other models had ignored. Two characteristics that were of particular interest were (1) the effect of cyclical use (drawing air through the tube during the warm hours of a day, and non-use of the tube during the cool night) of an earth coolant tube over an entire season, and (2) the effect of surface temperature fluctuations on the coolant tube's ability to perform.

This chapter will describe the steps that were taken to develop such a mathematical model. The following areas will be discussed: (1) formulation of governing equations, (2) development of the computer simulation model, and (3) selection of properties and dimensions.

Formulation of Governing Equations

The differential equation which describes the transfer of energy in the soil around a coolant tube is the non-steady state form of the general energy conduction equation without energy generation:
When first studying the model, it would appear that the selection of a cylindrical coordinate system would be most convenient for a pipe buried in an infinite environment; however, in order to study the effect of changing ground surface conditions on the buried pipe, it was concluded that a rectangular coordinate system would be easier to work with. Writing the three dimensional energy conduction equation for a rectangular coordinate system yielded the equation:

\[
K \nabla^2 T = \rho C_p \frac{\partial T}{\partial t}
\]  

(3-1)

Figure 3-1 illustrates the coordinate system orientation chosen.

The next step in the model formulation was to non-dimensionalize equation 3-2. This was done by making the following substitutions:

\[
u = \frac{T - T_\infty}{T_0 - T_\infty}
\]

(3-3)

\[
\bar{t} = \frac{t}{t_\infty}
\]

(3-4)

\[
\bar{X} = \frac{X}{X_\infty}, \quad \bar{Y} = \frac{Y}{Y_\infty}, \quad \bar{Z} = \frac{Z}{Z_\infty}
\]

(3-5)

The reference temperatures selected for the non-dimensional temperature were chosen so that the non-dimensional temperature would vary between 0.0 (when the soil had not been affected by the air) and 1.0 (when the soil was completely saturated with energy and could
Fig. 3-1. Coordinate system orientation.
no longer cool air). The reference time was selected so that one-
half cycle of operation would be equal to one time unit.

The non-dimensional form of equation 3-2 became:

\[ \nabla^2 u = F_0 \frac{\partial u}{\partial t} \quad (3-6) \]

Because the pipe is long compared to its radius, the temperature
gradient in the \( z \) direction was assumed to be negligible. Assuming
that temperature was independent of length (\( z \)), equation 3-6 became:

\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = F_0 \frac{\partial u}{\partial t} \quad (3-7) \]

Now that the governing differential equation was obtained, it
was written in finite difference form to make it more readily sol-
vable using a computer. Equation 3-7 was rewritten using a central
difference form. An equally spaced nodal system (or grid) was
used where "1" was used as the \( x \) coordinate counter and "m" was
the \( y \) coordinate counter. Using the Crank-Nicolson method of solu-
tion (14), equation 3-7, when rewritten for a node not on the boun-
dary, as shown in Figure 3-2, of the nodal grid became:

\[
\begin{align*}
  u_{l,m}^{(v+1)} &= u_{l,m}^{(v)} + \frac{\Delta t}{F_0 \Delta x} \left[ (u_{l-1,m} + u_{l+1,m} + u_{l,m-1} + u_{l,m+1} - \\
  4u_{l,m})^v + (u_{l-1,m} + u_{l+1,m} + u_{l,m-1} + u_{l,m+1} - 4u_{l,m})^{v+1} \right]
\end{align*}
\]

For a node on the vertical axis of symmetry of the nodal grid, equa-
tion 3-7 became:
For a node on a vertical boundary where the temperature was assumed to no longer change with respect to position, equation 3-7 became:

\[ u_{1,m}^{(v+1)} = u_{1,m}^{(v)} + \frac{\Delta T}{F_o \Delta X} [(2u_{1+1,m}^{(v)} + u_{1,m-1}^{(v)} + u_{1,m+1}^{(v)} - 4u_{1,m}^{(v)})^{v+1} + (2u_{1+1,m}^{(v)} + u_{1,m-1}^{(v)} + u_{1,m+1}^{(v)} - 4u_{1,m}^{(v)})^{v+1}] \] (3-9)

Writing equations similar to equations 3-8, 3-9 and 3-10 for all temperature nodes in the grid, and then writing the system of equations in matrix form yielded the following results:

\[ [A][u]^{(v+1)} = [B][u]^{(v)} \] (3-11)

Equation 3-11 was the finite difference form of the general energy equation that needed to be solved for each time step during the coolant tube simulation. It should be noted that equation 3-11 only accounts for one slab of earth perpendicular to the coolant tube, and assumes that temperature did not vary through the thickness of the slab. Later temperatures were solved for in terms of previous known temperatures, starting with the initial condition for the first iteration. The solution to the matrix equation describes the temperature profile in the soil around the coolant tube.
After determining the temperature profile in the soil around the coolant tube using equation 3-11, the temperature profile was curve fit assuming a logarithmic temperature profile (8). Once an equation for the temperature profile was known, Fourier's Law of heat conduction was used to estimate the energy flux at the coolant tube surface. With the energy flux known, the amount of cooling that took place was calculated and the change in the temperature of the air in the pipe due to the energy flux was found.

Computer Simulation Model

This section of Chapter III will discuss more details as to the solution procedure of the mathematical model. The following will be discussed: (1) the nodal system analyzed, (2) the computer program flow chart, and (3) program body and subroutines used. A program listing and sample output are contained in Appendix A.

Nodal System

The nodal system was established as shown in Figure 3-2. The temperature values to the right and left (x direction) of the pipe were assumed to be identical. Symmetry above and below (y direction) the pipe could have been assumed if ground surface effects were to be ignored; however, since surface effects are of interest to the operation of a cooling tube, only one axis of symmetry was defined. The region along the length (z direction) of the pipe was divided into ten slabs perpendicular to the pipe. Each slab was
Fig. 3-2. Node system (grid).
5 columns wide and 20 rows deep. The nodes were numbered as shown in Figure 3-2. The method of numbering was selected to make the diagonals in matrices $A$ and $B$ as close together as possible.

Computer Program Flow Chart

The following flow chart represents the basic logic sequence used in the computer program.

```
Initialize program:
1. dimension arrays
2. define initial values
3. define parameters

Read matrix elements

System is on, check to see if system is to be turned off
  no
  Calculate soil temperature around pipe
  Calculate flux into pipe
  Calculate air temperature
  Define new boundary conditions
  Increment time step
  Let old soil temperature equal new soil temperature

yes

Print temperature data

System is off, check to see if system is to be turned on
  no
  Calculate soil temperature around the pipe
  Increment time step
  Let old soil temperature equal new soil temperature
  Print temperature data
  Increment number of days system has been in use
  Check to see if simulation is over

yes

End Program
```
Program Body and Subroutines

A subroutine was written to load the values of the elements into the two matrices A and B. Both A and B were five line diagonal matrices.

A standard subroutine, LINV1F, was used in solving the matrix equation, equation 3-11. It was obtained from the International Math and Statistics Library, supported by the IBM 360 computer.

Before calculating the new temperatures of the nodes in the grid around the pipe, the program was designed to check to see if the coolant system was being used to cool air or if the coolant system was sitting stagnant. The program was designed to use the coolant tube for 12 hours during the day and to let the energy reservoir recharge for 12 hours during the night.

New nodal temperatures were solved for in terms of previously determined nodal temperatures at earlier time steps. This calculation was done for each of the 10 slabs along the pipe.

The temperatures found from solving the matrix equation were used in a curve fitting sub-routine to estimate the best logarithmic temperature profile for the temperatures around the coolant tube. The energy flux at the pipe surface was calculated using Fourier's Law of heat conduction once the temperature profile was known. A logarithmic temperature profile was assumed based on the results of information found in reference 8 and from a knowledge that many solutions to the steady state general energy conduction equation in cylindrical coordinates yield logarithmic solutions.
The temperature profile was assumed to be of the general form:

\[ u = a + b \ln(x) \]  

(3-12)

The quantity \( \frac{du}{dx} \) was found by differentiating equation 3-12. The quantity \( \frac{du}{dx} \) was evaluated at the surface of the pipe. These calculations yielded the equation:

\[ \frac{du}{dx} = \frac{b}{r} \]  

(3-13)

where \( r \) was the radius of the pipe. The constant \( b \) was found from the equation:

\[ b = \frac{\sum u_i \ln x_i - \frac{1}{N} \sum \ln x_i \sum u_i}{\sum (\ln x_i)^2 - \frac{1}{N} (\sum \ln x_i)^2} \]  

(3-14)

Equation 3-14 was obtained from reference 15 and is based on least square techniques for curve fitting.

Only the nodes in the immediate vicinity of the coolant tube were used to estimate the temperature profile near the tube. The nodes chosen are shown in Figure 3-3.

![Fig. 3-3. Nodes used to find dimensionless temperature profile and heat flux.](image)
Once the flux from the ground into the pipe was known, the change in air temperature due to the flux was calculated. As the air passed through one slab of earth, of thickness $\Delta z$, the change in air temperature due to the flux was calculated using the equation:

$$ Q = m c_p (T_{out} - T_{in}) = -KA \frac{dT}{dx} \bigg|_{x = r} $$

(3-15)

After equation 3-15 was non-dimensionalized and rearranged, and solved for the air temperature leaving one slab, the equation became:

$$ u_{z+\Delta z} = u_z + \frac{2\pi r z K}{m c_p} \frac{du}{dx} \bigg|_{x = r} $$

The quantity $\frac{2\pi rz K}{m c_p}$ was a dimensionless number which consisted mostly of coolant tube related properties. It was given the name coolant tube number ($C_t$). The value of $C_t$ was varied in order to evaluate the effect of pipe length and radius on the performance of a coolant tube.

The air temperature leaving each pipe segment was calculated knowing the air temperature entering the pipe segment. The values of the air temperatures were stored and later printed.

The first time the model was used, the air was cooled to the extent that the 2nd law of thermodynamics was violated. In other words, the conservation of energy principles used in determining the air temperature indicated that the ground could absorb so much
energy from the air, that the air could be cooled to below the ground temperature. To ensure that the air was not excessively cooled (2nd law violated), a conditional statement was used to set the air temperature equal to the ground temperature if the predicted air temperature was less than the ground temperature. This meant that the results of the air temperature calculations could obtain a Carnot value, rejecting energy with no temperature difference. This was a non-conservative assumption.

Different boundary conditions were used for different situations studied. The boundary condition that applied when the surface temperature effects were being ignored was that the ground temperature adjacent to the coolant tube was dependent on the air temperature in the tube. After the air temperature in the pipe was calculated, it was assumed that the soil temperature adjacent to the tube was the same as the air temperature in the tube.

When the ground surface temperature was significant, a value of ground surface temperature was assigned. The ground surface temperature was assumed to vary sinusoidally over a 24 hour period. The surface temperature was calculated using the equation:

\[ u = 0.5 \sin(\pi t) + 0.5 \sin\left(\frac{2\pi N \text{ Days}}{365}\right) \]  

Equation 3-18 was formulated so that the ground surface temperature would vary between 0.0 and 1.0 on the hottest day of the summer and between 0.0 and -1.0 on the coldest day of winter.
At the end of each series of calculations, the time variables were then incremented and the iterative process restarted if the entire simulation was not over. If the coolant tube had reached the end of a 12 hour period of operation, the program would change to a new mode of operation (if the coolant tube had been in use, it was turned off, or if it had been sitting stagnant, it was turned on), and output data was printed.

The following information was obtained as output from the simulation model:

1. time of day
2. day of simulation
3. air temperatures along the length of the pipe
4. soil temperature around the pipe
   a. at the pipe entrance
   b. at the pipe exit

Selection of Properties and Dimensions

Table 3-1 includes the thermophysical properties and dimensions used in the coolant tube model.
<table>
<thead>
<tr>
<th>Thermophysical Property or Dimension</th>
<th>Low Value</th>
<th>Nominal Value</th>
<th>High Value</th>
<th>References and Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_s$ (lbm/ft$^3$)</td>
<td>70</td>
<td>100</td>
<td>130</td>
<td>Ref. 16</td>
</tr>
<tr>
<td>$c_{ps}$ (Btu/lbm-F)</td>
<td></td>
<td>0.175</td>
<td></td>
<td>Ref. 16</td>
</tr>
<tr>
<td>$K_s$ (Btu/hr-ft-F)</td>
<td>0.50</td>
<td>1.0</td>
<td>1.5</td>
<td>Ref. 16</td>
</tr>
<tr>
<td>$x_\infty$ (ft)</td>
<td>1</td>
<td></td>
<td></td>
<td>Note 1</td>
</tr>
<tr>
<td>$t_\infty$ (hr)</td>
<td>12</td>
<td></td>
<td></td>
<td>Note 2</td>
</tr>
<tr>
<td>$\overline{\Delta x}$ (unitless)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{\Delta E}$ (unitless)</td>
<td>0.0417</td>
<td>0.1667</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ (ft$^2$/hr)</td>
<td>0.02198</td>
<td>0.05714</td>
<td>0.12245</td>
<td>Note 3</td>
</tr>
<tr>
<td>$F_0$ (unitless)</td>
<td>0.68055</td>
<td>1.4583</td>
<td>3.7913</td>
<td>Note 4</td>
</tr>
<tr>
<td>$SP$ (unitless)</td>
<td>0.04396</td>
<td>0.14288</td>
<td>0.2449</td>
<td>Note 5</td>
</tr>
<tr>
<td>$\lambda$ (unitless)</td>
<td></td>
<td>0.75</td>
<td>0.944</td>
<td>Note 6</td>
</tr>
<tr>
<td>$r$ (ft)</td>
<td>0.25</td>
<td>0.5</td>
<td>1.0</td>
<td>Ref. 3,4,5</td>
</tr>
<tr>
<td>$V_a$ (ft/sec)</td>
<td>1.0</td>
<td>4.0</td>
<td>8.0</td>
<td>Ref. 3,4,5</td>
</tr>
<tr>
<td>$K_a$ (Btu/hr-ft-F)</td>
<td></td>
<td>0.01516</td>
<td></td>
<td>Ref. 17</td>
</tr>
<tr>
<td>$\rho_a$ (lbm/ft$^3$)</td>
<td></td>
<td>0.0735</td>
<td></td>
<td>Ref. 17</td>
</tr>
<tr>
<td>$c_{pa}$ (Btu/lbm-F)</td>
<td></td>
<td>0.2402</td>
<td></td>
<td>Ref. 17</td>
</tr>
<tr>
<td>$h_{film}$ (Btu/lbm-ft$^2$-F)</td>
<td>0.5</td>
<td>1.0</td>
<td>2.5</td>
<td>Ref. 3,4,18</td>
</tr>
</tbody>
</table>
TABLE 3-1 (Continued)

<table>
<thead>
<tr>
<th>( \dot{m} ) (l bm/hr)</th>
<th>52</th>
<th>831</th>
<th>6650</th>
<th>Note 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (( \text{ft}^2 ))</td>
<td>0.1963</td>
<td>0.7853</td>
<td>3.1416</td>
<td>Note 8</td>
</tr>
<tr>
<td>( \Delta z ) (unitless)</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C_t ) (unitless)</td>
<td>0.0787</td>
<td>0.1574</td>
<td>0.2361</td>
<td>Note 9</td>
</tr>
<tr>
<td>Biot (unitless)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.667</td>
<td>Note 10</td>
</tr>
</tbody>
</table>

NOTES:

1. The standard length, \( x_0 \), was picked to be 1 foot. This was so the node system would be 5 feet wide. From experimental data (5), it was shown that the soil temperature 5 feet from the pipe should not be affected by the pipe.

2. The standard time, \( t_0 \), was picked as 12 hours or 1/2 day for simplicity in calculations. The coolant tube was simulated 12 hours on and 12 hours off.

3. The thermal diffusivity was calculated from the equation:

\[
\alpha = \frac{K}{\rho_c \beta} \quad (3-19)
\]

4. The Fourier number was calculated from the equation:

\[
F_o = \frac{\rho_c x_\infty^2}{t_0 K} \quad (3-20)
\]

5. The dimensionless quantity \( SP \) (Soil Property Number) was calculated from the equation:

\[
SP = \frac{\Delta t}{F_o \Delta x^2}
\]

The first nominal value for \( SP \) was obtained for a 2 hour time step. The second nominal value was for a 1/2 hour time step.
6. The stability factor ($\lambda$) was needed to determine if the selection of thermophysical properties and dimensions would yield a stable solution when used in the mathematical model. Reference 13 stated that if $\lambda$ is calculated using equation 3-22 and if it has a value greater than zero and less than one, the solution would be stable.

$$\lambda = \frac{1 - SP}{1 + SP} \quad (3-22)$$

7. The air mass flow rate was calculated using the equation:

$$m_a = \rho_a V_a A$$  \quad (3-23)

8. The area of the pipe was calculated using the equation:

$$A = \pi r^2$$  \quad (3-24)

9. The dimensionless number $C_t$ (sometimes referred to as DIM2) was calculated using the equation:

$$C_t = \frac{\frac{2\pi r \Delta z K_s}{m c_p a}}{x_\infty}$$  \quad (3-25)

10. The dimensionless Biot number was calculated using the equation:

$$\text{Biot} = \frac{h_{\text{film}} x_\infty}{K_s}$$  \quad (3-26)
CHAPTER IV
RESULTS OF SIMULATION

This chapter will present and explain the results of the computer study described in Chapter III. The results will be discussed by examining the following topics: (1) the effect of time on the capacity of a coolant tube to cool air, (2) the effect of coolant tube and soil parameters on a coolant tube's performance, and (3) discussion of the model's potential to analyze an entire season of coolant tube use.

The Effect of Time on a Coolant Tube

One of the important considerations that should be accounted for by coolant tube designers is determination of the time it will take until the soil around a coolant tube can no longer absorb energy from the air in the pipe. Of equal concern is whether the energy sink around the tube can recharge itself when a coolant tube is used for a few hours during the day when cooling is needed, and then allowed to sit stagnant. These concerns will be discussed in this section of Chapter IV.

Two computer simulations were conducted to answer the above questions. The results of the first simulation are shown in Figures 4-1 and 4-2. Figure 4-1 demonstrates that the ability of the coolant tube to cool air deteriorates with time. Figure 4-2 shows that
Fig. 4-1. Air temperature as a function of position and time (constant inlet air temperature).
Fig. 4-2. Soil temperature as a function of depth and time (constant inlet air temperature).
the soil close to the pipe is able to recharge itself by a limited amount if the tube is left unused for a short period of time (12 hours).

To help interpret the figures of this chapter, example problems will be worked. Example 1 is found in Table 4-1.

Similar calculations to those found in Example 1 were performed for various numbers of days of coolant tube operation. The results are graphed in Figure 4-3. From Figure 4-3 and Example 1, it can be seen that the coolant tube's ability to cool air decreased from 0.132 tons to 0.097 tons of refrigeration over the one month time period, a decrease of 27%.

From Figure 4-2, it can be seen that the soil close to the tube recharges itself during the 12 hour time period when the coolant tube was not being used; however, seasonally the soil around the coolant tube was slowly warmed up and would eventually not serve as an appropriate energy sink. It is interesting to note that the energy reservoir around the tube recharges itself after the 30th day as much as it does after 5 days.

In an attempt to evaluate if a coolant tube would be able to provide cooling for an entire season under more realistic air temperatures (in the first case studied, the inlet air temperature was assumed to be a constant maximum value), a four month simulation was performed where the inlet air temperature varied sinusoidally throughout the day. Figure 4-4 illustrates how the inlet air temperature varied with respect to time.
TABLE 4-1
EXAMPLE 1

LOCATION: Orlando, Florida
SOIL TEMPERATURE: 70°F
AIR ENTERING TEMPERATURE: 90°F
DATE: 5 and 30 days
MASS FLOW RATE: 831 lbm/hr
Cp : 0.2402 Btu/lbm-F

From Figure 4-1:
\[ u_{in} = 1.0 \]
\[ u_{out} = 0.60299 \]

From Equation 3-3:
\[ T_{out} = u(T_o - T_\infty) + T_0 = 0.60299 (90-70) \]
\[ + 70 = 82.06°F \]
\[ Q = mc_p (T_{out} - T_{in}) \]
\[ = 831 \text{ lbm/hr} \times (0.2402 \text{ Btu/lbm-F}) \times (90 - 62.2) \]
\[ Q = 1585 \text{ Btu/hr} \]

Equivalent to 0.132 tons of refrigeration after 5 days of operation
0.097 tons of refrigeration were provided after 30 days of operation
Fig. 4-3. Results of Example 1.
Fig. 4-4. Inlet air temperature as a function of time.
Figures 4-5 and 4-6 illustrate how the air and soil temperatures varied with respect to time. It can be seen from Figure 4-5 that the soil around the coolant tube is almost completely saturated at the end of 4 months, and that after 90 days, the soil near the entrance to the tube becomes so warm during the hot part of the day that the air entering the tube during the later, cooler time of the day (shown in Figures 4-5 and 4-6) is first warmed, and then cooled by the soil.

A second example, Example 2, is presented in Table 4-2 to illustrate the significance of the data in Figure 4-5.

On the basis of the simulations conducted, it can be concluded that a coolant tube could provide cooling, even after four months of use. After 4 months of use, with the soil in the saturated condition (so warm that it could no longer cool air), the energy reservoir would be in an ideal condition to begin a winter season where the soil would be used to warm cool air. After using the system during the winter months to warm air, the energy reservoir would be left in a condition where the dimensionless soil temperature would be less than that for undisturbed soil. The energy reservoir would be in an ideal condition to begin a season of cooling the pipe air.

In interpreting the results of Example 2, it should be noted that the information used was not at the time when coolant tube performance was a maximum, but at a time soon before the coolant tube was to be turned off when the soil conditions were an extreme due to 12 hours of system use.
Fig. 4-5. Air temperature as a function of position and time (varying inlet air temperature).
Fig. 4-6. Soil temperature as a function of depth and time (varying inlet air temperature).
TABLE 2
EXAMPLE 2

LOCATION: Orlando, Florida
SOIL TEMPERATURE: 70°F
AIR ENTERING TEMPERATURE: 90°F
MASS FLOW RATE: 831 lbm/hr
\(c_p:\) 0.2402 Btu/lbm-F

From Figure 4-5:
\[u_{in} = 7.5\]
\[u_{out} = \text{variable}\]

From equation 3-3:
\[T = u(T_0 - T_\infty) + T_0\]
\[Q = mc_p(T_{out} - T_{in})\]

The rate of energy transfer \(Q\) was calculated for various days of coolant tube operation. The results are graphed in Figure 4-7.

Effect of Coolant Tube Dimensions and Soil Parameters on Coolant Tube Performance

It was desired to understand how coolant tube dimensions and soil parameters affected the performance of a coolant tube. In order to understand the effects of soil parameters and tube dimensions, several different types of simulations were conducted. The studies will be discussed by addressing the three areas: (1) soil parameters, (2) coolant tube dimensions, and (3) coolant tube depth.
Fig. 4-7. Results of Example 2.
Soil Parameters

The Fourier number ($F_o$), which is a combination of the soil properties, reference time, and reference distance, was varied to observe the effects of varying soil parameters on the ability of a coolant tube to cool air. In the program the soil property number (SP), which was a combination of $F_o$, the dimensionless time step and nodal spacing, was varied during different simulations. Figure 4-8 illustrates the effect of the variable SP on the coolant tube performance (tube exit temperature).

Calculations for Orlando, Florida similar to those found in Examples 1 and 2 were conducted using the data in Figure 4-8. The rate of energy transfer as a function of SP is graphed in Figure 4-8.

From both Figures 4-8 and 4-9, it can be seen that decreasing the Soil Property Number (SP) increases the performance of a coolant tube. The Soil Property Number will be examined to determine the effect of coolant tube parameters on tube effectiveness. Recall that:

$$SP = \frac{\Delta t}{F_o \Delta x^2}$$  \hspace{1cm} (4-1)

$$F_o = \frac{\rho S c_p S x_s^2}{t_s K_s}$$  \hspace{1cm} (4-2)

To obtain a small SP, a large $F_o$ is desired. Time and position were fixed variables in the simulation; therefore, only the
Fig. 4-8. Air temperature as a function of position and soil property number (constant inlet air temperature, after 30 days).
Fig. 4-9. Rate of energy transfer as a function of soil property number (SP).
effect of varying soil properties will be studied. The coolant tube performance could be increased by making the following parameter changes (they would make $SP$ smaller):

1. increase soil density ($p_s$)
2. increase soil specific heat capacity ($c_{ps}$)
3. decrease soil thermal conductivity ($K_s$)

The results of the simulation indicate that the above listed changes will increase coolant tube performance. Physically, increasing soil density and specific heat capacity increases the total amount of energy that the soil can absorb. This is desirable and should increase the coolant tube's performance. The physical significance of decreasing the soil thermal conductivity is more difficult to understand. One possible explanation is that soil with a low thermal conductivity slows the process of the soil absorbing the energy from the air, thus increasing the usable time of the coolant tube. The importance of soil thermal conductivity will be discussed more later in this paper.

Effect of Coolant Tube Dimensions on Coolant Tube Performance

In the simulation, the dimensionless number, $C_t$ (coolant tube number), was used in calculating air temperatures in the tube. $C_t$ was a variable that represented the tube dimensions and air mass flow rate.

$$C_t = \frac{2\pi r \Delta z K_s}{\dot{m} c_p a}$$
Figures 4-10 and 4-11 illustrate the effect of $C_t$ on coolant tube performance. In Figure 4-10, pipe length is assumed to be held constant. From Figure 4-10, it can be seen that increasing $C_t$ increases coolant tube performance. For constant tube length, $C_t$ could be increased by increasing pipe radius or decreasing air mass flow rate.

Figure 4-12 assumes pipe radius is constant, and only pipe length is varied. Figure 4-12 indicates that a shorter pipe is more effective than a long pipe which was also noted by the Princeton Group in their paper (3).

Calculations similar to Examples 1 and 2 were performed to determine the rate of energy transfer as a function of $C_t$. Figure 4-12 presents the results.

From Figure 4-12, it can be seen that increasing $C_t$ increases the performance of a coolant tube. From equation 4-3, it can be seen that $C_t$ can be made larger by making the following changes:

1. increasing pipe radius ($r$)
2. increasing pipe length ($\Delta z$)
3. increasing soil thermal conductivity ($K_s$)
4. decreasing mass flow rate

The physical significance of the above changes are that increasing the pipe length or radius increases the heat transfer area between the soil and the air. It appears that increasing the soil thermal conductivity would lead to better tube performance. This is the exact opposite of the effect seen when the Soil Property
Fig. 4-10. Air temperature as a function of position and $C_t$ (constant pipe length and inlet air temperature) after 30 days of operation.
Fig. 4-11. Air temperature as a function of position and $C_t$ (constant pipe diameter and air temperature) after 30 days of operation.
Fig. 4-12. Rate of energy transfer as a function of $C_t$. 
Number in Figure 4-9 was varied. It appears that high soil thermal conductivities have both positive and negative effects on tube performance.

Simulations were performed to help understand the effect of varying soil thermal conductivity. The results are presented in Figure 4-13. Recall that from Figure 4-9 it was concluded that decreasing thermal conductivity (SP decrease) led to an increase in tube performance and from Figure 4-12 that increasing thermal conductivity (Ct increasing) also led to increased tube performance. Is the role of thermal conductivity more dominant in the Soil Property Number, or the Coolant Tube Number? Figure 4-13 indicates that increasing the thermal conductivity in both SP and Ct caused a decrease in coolant tube performance. This indicates that having an optimum thermal conductivity in the Soil Property Number is most important, and that thermal conductivity plays a minor role in the Coolant Tube Number.

Coolant Tube Depth

To understand the importance of coolant tube depth, the effect of the ground surface conditions on the coolant tube needed to be included in the model. The model was modified to include soil surface temperature and the tube inlet temperature variations as shown in Figure 4-14. The depth of the pipe was varied from 5 feet deep to 15 feet deep. The results of the simulation are illustrated in Figure 4-14 and 4-15.
Fig. 4-13. Air temperature as a function of position and soil properties (constant inlet air temperature)

NOTE:  
1. \( SP = 0.04396 \) \( C_t = 0.0787 \)  
2. \( SP = 0.14288 \) \( C_t = 0.1574 \)  
3. \( SP = 0.2449 \) \( C_t = 0.2361 \)
Fig. 4-14. Air temperature as a function of position and tube depth (varying inlet temperature after 30 days).
Fig. 4-15. Soil temperature as a function of depth and coolant tube depth (varying inlet temperature, after 30 days).
Figure 4-14 indicates that the 5 foot deep coolant tube would perform better than the 15 foot deep coolant tube; however, the difference in cooling caused by varying tube depth is very small and less than 2%. Figure 4-15 gives insight into what might cause the shallower tube to appear to perform better.

From Figure 4-15, it can be seen that during the stagnant time period (coolant tube not in use) the soil around the coolant tube, as well as the soil close to the ground surface is cooled. Because the shallower tube is closer to the surface, it may be that the cooling of the ground surface affects the shallow coolant tube more than it affects deeper tubes. This would lead to better performance for the shallower tube; however, this would only be the case if the average soil temperature between the pipe and ground surface was greater than the average environmental air temperature. From Figure 4-15, it can be seen that the average air temperature is greater than the soil temperature so there must be some other explanation of the results. The boundary condition on the bottom edge of the nodal system forced the temperature gradient on that edge to be zero. This is seen in Figure 4-15. Because of this boundary condition, non-uniform temperature profiles below the tubes were generated for the three cases studied. Because all three tubes would see an infinite surrounding below them, the temperature profiles below the coolant tubes should have been uniform for the different depths of tubes.
Another explanation to the phenomena could be the selection of soil properties for the simulation. In an actual coolant tube system, as the tube depth increases, the moisture content and, thus, the specific heat capacity \( c_p \) and the density \( \rho_s \) of the soil increases. As seen in the discussion of the importance of SP, increasing \( c_p \) and \( \rho \) would lead to better tube performance. In the simulation, the soil properties were assumed to be the same for all depths. Thus, the advantage held by the deeper tube due to its more ideal soil properties was not considered. This could explain why the model did not predict better performance of the deeper tube compared to that of the shallower tube.

A point of interest that has not been discussed yet, but that should be noted, is the change in temperature profiles in the soil around the coolant tube from the inlet of the tube to the tube exit. Figure 4-16 illustrates how the soil temperature changes. The soil near the entrance to the coolant tube is affected more by the air than the soil near the exit of the tube.

**One Year Simulation**

An attempt was made to simulate an entire year of coolant tube performance. For this simulation, the air and ground surface temperatures would need to vary both seasonally and daily. It was determined that the mathematical model would have to be modified so that it would simulate an entire year of coolant tube operation.
Fig. 4-16. Soil temperature as a function of depth and length (after 30 days).
One of the key elements in the model was the curve fitting of the temperature profile in order to obtain the temperature gradient and thus the energy flux. A logarithmic temperature profile was assumed for the curve fitting process. The assumption of a logarithmic temperature profile yielded accurate results if the air in the coolant tube was being warmed or cooled; however, when both phenomena occurred in rapid succession, the logarithmic curve fit could not accurately fit the data. A quadratic curve fit might be necessary here to yield usable information over a one year period.
CHAPTER V

COMPARISON WITH AVAILABLE MODELS AND DATA

The purpose of this chapter is to evaluate the different coolant tube models to see how well they can predict the performance of a coolant tube system and to estimate the economic feasibility of a coolant tube.

Comparison of Different Models

The different models proposed for estimating the performance of a coolant tube system, references 2, 3, and 4, and the model proposed by this paper will be compared with the experimental data presented in reference 5. Reference 5 was picked as the standard of comparison because it had the only experimental data available.

The difficulty in comparing the different models came in finding some common ground on which they could be compared. Each model presented some unique aspect of coolant tube operation. It was determined that all models except (3) could be used to predict the rate of energy transfer to the soil for different air mass flow rates in the pipe. The results of these predictions are presented in Figure 5-1. The important variable used in the calculations, obtained from reference 5, are listed below:

1. pipe diameter 21 inches

65
Fig. 5-1. Comparison of different models with experimental data.
2. pipe length 100 feet
3. initial soil temperature 71°F
4. initial air temperature 81°F
5. soil thermal conductivity 0.84 Btu/hr-ft-F
6. air specific heat capacity 0.24 Btu/lbm-F

The coolant tube data (Figure 5-1) seemed to be best approximated by the simple equations presented by Jan F. Kreider (2), which were presented in Chapter II. The two, more complex, computer models do a fair job of predicting the coolant tube performance at low mass flow rates; however, at higher mass flow rates, as much as 50% error is encountered. Both computer models illustrate the effect of time on the capacity of a coolant tube to cool air. The Kreider equation has no time dependence. It is unknown how long the coolant tube tested in reference 5 had been in operation before the data was measured. It is possible that the coolant tube had been sitting stagnant for a long period of time and that data was taken when the system was first turned on. If this is the case, the computer models may be more accurate than it may initially appear.

In conclusion, the easiest method for predicting the performance of a coolant tube would be to use the equations proposed by Jan F. Kreider (2); however, the model proposed by this paper is more useful when the deterioration due to time or surface conditions needs to be understood.
Economic Considerations

A simple economic study was done to estimate the feasibility of using a coolant tube system. It was assumed that a one foot diameter, one hundred foot long coolant tube could provide one-sixth ton of refrigeration. A sewer contractor in Orlando, Florida (19) was consulted to estimate the cost of buying and installing 600 feet of pipe (enough to provide 1 ton of refrigeration). The contractor estimated the cost at about $12 to $14 per foot of pipe, or about $7800 for a one ton system. Estimates were made of the cost to blow air through the pipe. The cost was minimal and assumed to be negligible.

Using a water cooled heat pump for house cooling, the capital investment for a 3 ton system would be $3,000. Operating and maintenance costs are estimated at about $30 with a cooling load of about 2 tons.

Ignoring the time value of money, the above findings indicate that it would take at least 13 years before the cost of installing the coolant tube would be equal to the cost of installing the heat pump plus its cost of operation. It is the author's opinion that at this time, it is not economically feasible to use earth coolant tubes in most situations. Conventional methods of heating and cooling air are more economical.
CHAPTER VI
CONCLUSION

Based on the literature search that was performed, it was determined that two aspects of coolant tube operation had never been studied: (1) the effect of operating coolant tubes for long periods of time and (2) the importance of coolant tube depth. In order to address these two aspects, a computer simulation model was formulated. As a result of the computer simulation, it was concluded that a coolant tube system could provide a significant amount of cooling or heating for long periods of time (as long as 4 months). It was also concluded that for coolant tubes below 5 feet deep, depth had little effect on coolant tube performance.

Another useful result of this thesis came from the comparison of the different models that have been proposed by other researchers, for predicting coolant tube performance, with a set of performance data from an operating coolant tube. It was concluded that an accurate model for predicting coolant tube performance, when long term effects are not considered, was the model proposed by Jan F. Kreider (2). Kreider's model is also the easiest to use, requiring only a basic understanding of algebra.

As a result of the study done, it was also concluded that a good rule of thumb for predicting coolant tube performance would
be that one, twelve inch diameter, one hundred foot long coolant
tube could provide about one-sixth of one ton of refrigeration, or
the equivalent amount of heating, for an entire heating or cooling
season.
APPENDIX
List of symbols used:

- **IDGT**: variable used in the subroutine LINVI1F
- **SP**: variable used to assign the elements of \([A]\) and \([B]\)
- **M**: number of nodes
- **N**: number of perpendicular slabs
- **KDEEP**: depth of coolant tube
- **RADIUS**: pipe radius (ft)
- **TIME**: time counter
- **DELT**: time step
- **NCOUNT**: counter used in printing data
- **NDAY**: day counter
- **NDAYS**: number of days to be simulated
- **BIOT**: Biot number
- **DIM2**: \(C_t\) (dimensionless number)
- **UAIR**: air temperature array
- **UNEW**: soil temperature array at new time step
- **UOLD**: soil temperature array at old time step
- **A**: \([A]\)
- **B**: \([B]\)
- **FLUX**: energy flux from the soil to the coolant tube
MAIN
DIMENSION A(100,100), B(100,100), C(100,100)
DIMENSION UOLD[100,100], UNEW[100,100]
DIMENSION WKAREA[100]
IDGT=0
SP=-0.14288
M=100
N=10
KDEEP=10
RADIUS=0.5
TIME=0.0
DELTIT=0.16667
NCOUNT=1
NDAY=0
NDAYS=30
BIOT=1.0
RADIUS=0.5
TIME=KDEEP+1
DO 2 J=1,M
   UAIR(J)=0.0
   DO 1 I=1,M
      UOLD(I,J)=0.0
      UOLD(I,J)=1.0
      UAIR(I,J)=1.0
   CALL PLOT(N,J,UOLD,UAIR,TIM,NDAY)
   NDAY=1
   AA=1.0+4.0*SP
   AB=-SP
   BCA=-SP
   BB=SP
   RC=SP
   CALL LOAD(A,AA,AB,BCA)
   CALL LOAD(B,AA,AB,BCB)
   CALL LINV1F(M,M,AA,AB,BCA,DKT,WKAREA,IER)
   CALL MULT(M,M,AA,AB,BCA)
   CALL TEMAIR(IN,J,TIM,TIME,UAIR,UNEW,FLUX)
   CALL BOUND(UKDEEP,N,KDEEP,KDEEP,UAIR,UNEW,FLUX)
   TIME=TIME+DELII
   DO 10 L=1,N
      UOLD(K,L)=UNEW(K,L)
   GO TO 4
2   IF(NCOUN'l ..LT. 5) GO TO 9
   CALL PLOT(N,J,UOLD,UAIR,TIM,NDAY)
   IF(TIME ..GE. 2.0) GO TO 11
   CALL SOLVE(M,N,UOLD,UNEW)
   TIME=TIME+DELI
   DO 10 L=1,N
      UOLD(K,L)=UNEW(K,L)
   GO TO 9
10  IF(NCOUN'r ..LT. 5) GO TO 13
   CALL PLOT(N,J,UOLD,UAIR,TIM,NDAY)
   NCOUNT=NCOUNT+1
   IF(ND DAY ..GE. NDAYS) GO TO 12
   NDAY=NDAY+1
   TIME=0.0
   GO TO 4
11  CONTINUE
STOP
END
LOAD
SUBROUTINE LOAD(Z, A, Z, B, A, C)
DIMENSION Z(100,100), A(100,100)
DO 728 I=1,100
DO 729 J=1,100
725 Z(I,J)=A
DO 726 J=1,100
726 Z(I,J)=B
END

MULT
SUBROUTINE MULT (Y, A, Z, C)
DIMENSION Z(100,100), A(100,100)
DO 746 I=1,100
DO 747 J=1,100
748 C(I,J)=A
DO 751 J=1,100
751 C(I,J)=B
CONTINUE
ENC

SOLVE
SUBROUTINE SOLVE (Y, N, C, U, D, L, E, W)
DIMENSION C(100,100), U(100,100), L(100,100), E(100,100)
DO 761 I=1,N
DO 766 K=1,N
760 U(I,J)=E
DO 761 K=1,N
761 V(K,J)=E
CONTINUE
ENC

SUBROUTINE FLUX (N+1, RADIUS, TNEW, FLUXX)
DIMENSION TNEW(100,10), FLUXX(10), X(5)
DELTA = 1.0
K3 = (KDEEP+5)*1.0
X3 = KK-5
K4 = KK-4
X4 = KK-1
K5 = KK-6
K6 = KK-7
X6 = KK-10
K7 = KK-15
X7 = KK-11
K8 = KK-17
X8 = KK-10
X9 = X8+DELTX
X10 = X9+DELTX
X11 = SORT(X(1),3,FLUXX(2,3))
C1 = ALG0G(X(1))**2+ALG0G(X(2))*2+ALG0G(X(3))**4+ALG0G(0(4))**2
+ALG0G(RADIUS)***2
C2 = ALG0G(X(1))**2+ALG0G(X(2))**2+ALG0G(X(3))**2+ALG0G(RADIUS)***2
C3 = TNEW(K1,N)+TNEW(K2,N)+TNEW(K3,N)+TNEW(K4,N)+TNEW(K5,N)
+TNEW(K6,N)+TNEW(K7,N)+TNEW(K8,N)+TNEW(K9,N)+TNEW(K10,N)
+TNEW(K11,N)+TNEW(K12,N)+TNEW(K13,N)
C4 = TNEW(K1,N)+ALG0G(X(1))+TNEW(K2,N)+ALG0G(X(1))
+TNEW(K3,N)+TNEW(K4,N)+ALG0G(X(1))
+TNEW(K5,N)+TNEW(K6,N)+ALG0G(X(1))
+TNEW(K7,N)+TNEW(K8,N)+ALG0G(X(1))
+TNEW(K9,N)+TNEW(K10,N)+ALG0G(X(1))
+TNEW(K11,N)+ALG0G(X(1))+TNEW(K12,N)+ALG0G(X(1))
XY = TNEW(K11,N)+ALG0G(RADIUS)
C5 = TNEW(K1,N)+TNEW(K2,N)+TNEW(K3,N)+TNEW(K4,N)+TNEW(K5,N)
+TNEW(K6,N)+TNEW(K7,N)+TNEW(K8,N)+TNEW(K9,N)+TNEW(K10,N)
+TNEW(K11,N)+TNEW(K12,N)+TNEW(K13,N)
C6 = TNEW(K11,N)+ALG0G(RADIUS)
DUMMY = (C5+C6)*13.0)/(C7-C1**2.0/13.0)
RETURN
END

SUBROUTINE TEMAIR (II,N+1,TIME,DIM2,UAIR,FLUXX, UNEW)
DIMENSION UAIR(10), FLUXX(10), UNEW(100,10)
UAIR(I,1) = 0.0
DO 2600 I = 1, N
UAIR(1) = UAIR(1)+DIM2*FLUXX(I)
IF(UAIR(1).LT.UNEW(II,1)) UAIR(1) = UNEW(II,1)
CONTINUE
RETURN
END

SUBROUTINE ROUNDI (KDEEP,N,BIOT,UAIR, UNEW, FLUXX)
DIMENSION UAIR(10), UNEW(100,10), FLUXX(10)
I = 1
DO 4000 I = 1, N
UNEW(I,1) = UAIR(1)
CONTINUE
RETURN
END
SUBROUTINE PLOT(N, UOLD, UIAIR, TIME, NDAY)

CALL PLOT(100, 15), UIAIR(10)

PRINT 1101, TIME
PRINT 1102, NDAY

DO 1000 I = 1, N

PRINT 1103, I, UIAIR(I)

PRINT 1104

ON 1000 I = 1, "", 5

1000 PRINT 1105, (UOLD(K,E), K = I, J), (UOLD(L,5), L = I, J)

1101 FORMAT(10X, "THE TIME IS", 10X, F4.1)

1102 FORMAT(10X, "THE AIR TEMPERATURE IN THE PIPE IS", /)

1103 FORMAT(10X, 13, 10X, F10.2)

1104 FORMAT(/, 10X, "THE TOIL TEMPERATURE AROUND THE PIPE IS", /)

1105 FORMAT(10X, "PIPE ENTERANCE", 5X, "END OF PIPE")

1106 FORMAT(10X, 3(5(F5.3, 2X), "X")

RETURN

END
MAIN

DIMENSION A(100,100), B(100,100), C(100,100)
DIMENSION UOLO(100), UNEW(100), UAIR(100), FLUX(100)
DIMENSION WKAREA(100)

IDG = 0
SP = 0.1
DELTT = 0.16667
NCOUNT = 1
NDAYS = 0
NDAYS = 30
BIOT = 1
DI2 = 0.157
JL = KOLDP + 1
DO 2 J = 1, N
UAIR(J) = 0
DO 2 I = 1, M
UOLO(I,J) = 0
UNEW(I,J) = 0
UOLO(I,J) = 1
UAIR(I) = 1
CALL PLOT(NU, UOLO, UAIR, TIME, NDAY)
NDAY = 1
AAS = 0.0 + 4.0 * SP
BCA = 0.0
NAC = 0.0 + 4.0 * SP
BCB = 0.0
CALL LOAD(A, AA, AB, CBA)
CALL LINVIF(A, MM, MA, 100, IDG, WKAREA, IER)
CALL MULT(M, A2, BCA, BC)
IF (TIME .GE. 1.0) GO TO 8
CALL SOLVE(M, UOLO, UNEW)
CALL FLUX(A, RADIUS, UNEW, FLUX)
CALL TEMAI(I, N, TIME, UOLO, UAIR, FLUX, UNEW)
CALL BOUND2(TIME, BIOT, UAIR, UNEW, FLUX)
TIME = TIME + DELTT
DO 5 K = 1, M
DO 5 L = 1, N
UOLO(K, L) = UNEW(K, L)
GO TO 6
P
IF (NCOUNT .LT. 5) GO TO 9
CALL PLOT(NU, UOLO, UAIR, TIME, NDAY)
IF (TIME .GE. 2.0) GO TO 11
CALL SOLVE(M, UOLO, UNEW)
CALL BOUND2(TIME, BIOT, UAIR, UNEW, FLUX)
TIME = TIME + DELTT
DO 10 K = 1, M
DO 10 L = 1, N
UOLO(K, L) = UNEW(K, L)
GO TO 9
11
IF (NCOUNT .LT. 5) GO TO 13
NCOUNT = 0
CALL PLOT(NU, UOLO, UAIR, TIME, NDAY)
13
IF (NDAY .GE. NDAYS) GO TO 12
NDAY = NDAY + 1
TIME = 0.0
GO TO 4
12
CONTINUE
STOP
PA

TEMAI

SUBROUTINE TEMAI(I, N, TIME, DIM2, UAIR, FLUX, UNEW)
DIMENSION UAIR(100), FLUX(100), UNEW(100)
UAIR(I) = 0.5 + 0.5 * SIN(3.14 * TIME)
DO 2000 I = 2, N
J = I
UAIR(J) = UAIR(J) + DIM2 * FLUX(I)
IF(UAIR(I) .LT. UNEW(I, J)) UAIR(I) = UNEW(I, J)
2000 CONTINUE
RETURN
END

BOUND2

SUBROUTINE BOUND2(TIME, BIOT, UAIR, UNEW, FLUX)
DIMENSION UAIR(100), UNEW(100), FLUX(100)
UAIR(I) = 0.5 + 0.5 * SIN(3.14 * TIME)
IF(TIME .GT. 1.0) GO TO 4100
DO 4000 I = 1, N
UNEW(I, J) = UAIR(I)
GO TO 4300
4100 70 4200 I = 1
70 4200 J = 1
4200 CONTINUE
RETURN
END
MAIN

DIMENSION $1(100,100), S(100,100), TA(100,100), U(100,100)
DIMENSION UOLD(100), UNEW(100), UAIR, FLUX(100)
DIMENSION WKAREA (100)

IDG=5
SP = 2E57
M=10
X=10
Y=0.5
T1=0.1
TIME=0
DELT=0.553
MCOUNT=1
NDAYS=1

BIOT=0.1
C2=0.15
J=0.5
DO = 0

UAIR(J) = 1
DO 2 I = 1, M
UOLD(I) = 0.0

3 IDEA(1) = 1.0

DO 7 PLOT(N+M, UOLD, UAIR, TIME, NDAYS)
NDAY = 1
AB1 = 0.4 + SP
BEL = A4 + SP
K2 = A4 + SP
K3 = A4 + SP
CALL LOAD (A, A, A, B, A, A, B)
CALL MUL (M, A, A, B, C)
4 IF (TIME = 2.0) GO TO 11
CALL SOLVE (TIME + TOL, UOLD, UNEW)
CALL FLUX(N+M, DEEP, RADIUS, NEW, FLUX)
CALL TEMP (TIME + NDAYS, DIM2, UAIR, UNEW, FLUX)
CALL ROUND (TIME, NEW, UAIR, TEMP)
TIME = TIME + DELT
DO 5 K=1, M
DO 5 L=1, N
5 UOLD(K, L) = NEW(K, L)
DO 10 K = 1
6 IF (MCOUNT < M) GO TO 9
7 IF (TIME > 2.0) GO TO 11
CALL SOLVE (TIME + TOL, UOLD, UNEW)
TIME = TIME + DELT
DO 10 K=1, M
DO 10 L=1, N
10 UOLD(K, L) = NEW(K, L)
DO 11 K = 1
11 IF (MCOUNT < M) GO TO 13

CALL PLOT(N+M, UOLD, UAIR, TIME, NDAYS)
MCOUNT = MCOUNT + 1
IF (NDAYS = NDAYS) GO TO 12
TIME = 0.0
GO TO 6
12 CONTINUE
STOP
END

SUBROUTINE TIME12 (TIME, UAIR, DIM2, UAIR, UNEW, FLUX)
TIME = TIME + UNNEW100)
WORK = 9.0
11=01
N2=10
2000 DO ZD = 0.0, 1.0
2000 UNNEW = UNNEW11, 1)
2050 D = UNNEW1X) = 0.5*UNNEW1X) = 0.5*TIME1
2050 IF (OLT = 0.5, UAIR1) GO TO 2000
2050 D = 0.5*TIME1
2050 IF (UIT = 0.5, UAIR1) UAIR1 = UNNEW(11, 1)
2050 IF (UIT = 0.5, UAIR1) UAIR1 = UNNEW11, 1)
2050 CONTINUE
RETURN
END

SUBROUTINE TIME204 (TIME, NEW, UAIR, UNEW)
TIME = TIME + UNNEW100)
NEW = 9.0
1 = 0.0
NEW = NEW11, 1)
2000 NEW = NEW1X) = 0.5*NEW1X) = 0.5*TIME1
2000 IF (OLT = 0.5, UAIR1) GO TO 2000
2000 D = 0.5*TIME1
2000 IF (UIT = 0.5, UAIR1) UAIR1 = UNNEW(11, 1)
2000 IF (UIT = 0.5, UAIR1) UAIR1 = UNNEW11, 1)
2000 CONTINUE
RETURN
END

SUBROUTINE TIME305 (TIME, NDAYS, UAIR, UNEW)
TIME = TIME + UNNEW100)
NDAYS = 0.0
NEW = 9.0
1 = 0.0
NEW = NEW11, 1)
2000 NEW = NEW1X) = 0.5*NEW1X) = 0.5*TIME1
2000 IF (OLT = 0.5, UAIR1) GO TO 2000
2000 D = 0.5*TIME1
2000 IF (UIT = 0.5, UAIR1) UAIR1 = UNNEW(11, 1)
2000 IF (UIT = 0.5, UAIR1) UAIR1 = UNNEW11, 1)
2000 CONTINUE
RETURN
END
### THE TIME IS 1:3
THE DAY IS 20

### THE AIR TEMPERATURE IN THE PIPE IS

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### THE TIME IS 2:0
THE DAY IS 20

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