Effects of Programming on Mathematics Achievement

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EFFECTS OF PROGRAMMING ON MATHEMATICS ACHIEVEMENT

BY

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CHAPTER I
INTRODUCTION

Background to the Study

In 1972, Paul G. Watson wrote "the question of whether the computer should be used in education and/or instruction is now no longer relevant ... the relevant question today is ... how they will affect the school's instructional processes in the next decade " (1). Now it is the next decade. Educators are still asking the same question. How can a computer be used most effectively in the classroom?

Is it sufficient for students to have a passing acquaintance with computers (computer literacy) (2-4)? Should everyone develop the skills of a programmer (5,6) (Or is this really asking the same question?) (7,8)? Will computers fulfill their claims of greater student achievement and motivation (9-11)?

While it is easier to ask questions than to provide answers, the need for answers has become acute. Some of the old problems inherent with large mainframe systems have been greatly reduced through the introduction of the microcomputer. Microcomputers are (relatively) inexpensive. A school can afford to own more than one machine. Through
the advances made in technology, the equipment is more reliable and less prone to breakdown (12). Thus it is no surprise that the number of computers in the classroom is on the rise (13-16). Computers are here to stay.

**Two Viewpoints of Computers in Education**

So far, the emphasis on computers in education has been upon the student as a passive recipient of knowledge. An important example of this is the work of Patrick Suppes. In the 1960s he wrote a series of computer programs for elementary school children designed to provide practice in arithmetic. While each child physically "pushed the buttons", the sequencing of lessons was predetermined. The machine was in charge. This is the concept inherent in most computer-aided instruction (CAI) materials.

Much research has been dedicated to determining how these materials affect a student's understanding of concepts (17). While this is important, it is not enough (18,19). A computer is more than an electronic workbook; it is a tool. Like a pencil, it can be manipulated by the student to explore ideas and make conclusions.

This idea of active student participation is not new. The University of Pittsburgh, supported by the National Science Foundation, initiated Project Solo (called Solo-works) in 1969. In each of the five laboratories developed (computer, dynamics, logical design, modeling/simulation,
and synthesis), a student could combine his knowledge of mathematics and computer science to design and test an experiment of his own (20). This time the student was in control of the machinery.

Other institutions have attempted to provide similar experiences. Thus courses in "computer science" and "computer mathematics" are a part of the curriculum in many high schools. Universities offer mathematics classes with a computer option.

Statement of the Problem

But how do students in these classes fare? Do students who "teach" the computer via algorithms develop a measurably better understanding of the concepts involved? Do their skills increase? Do they develop better problem-solving skills?

Research in this area is scant and the results conflicting. There have been very few studies whose aim was to detect the effects of programming on achievement.

This experiment was designed to consider three such issues:

1. Is there a difference between the mathematics achievement of students who write computer programs and those who do not?

2. Do programming students develop better computational skills than non-programming students?
3. Do programming students develop better problem solving skills than non-programming students?

**The High School Level**

Although previous research seems to have dealt with college classes and students, there are several reasons why a high school class was deemed more appropriate for the study.

High school mathematics is taught at a more leisurely pace than college mathematics. With a longer school year, the teachers are able to devote more time to each topic. So an experiment can contain only one topic and still last longer than one day.

The students tend to have comparable backgrounds in high school. When a new topic is introduced, it is often the first time a child has seen it. Then an experiment designed to test achievement need not be concerned with the effects of previous exposure to the topic.

Finally, there is the easy access to computer equipment. A high school campus tends to be smaller than a college campus. Computer equipment is often concentrated in one area. For an experiment which wishes to examine the relationship between computer use and mathematics achievement, this can be a real convenience.

However, it is also true that many high schools have no cross-over between the mathematics and computer science
departments. Mathematics teachers know little about computers and computer science teachers do not teach mathematics. This can be a severe limitation. The author's solution to this dilemma was to change the student's homework assignments rather than the teaching method already in use. It was thought that a student should not have to have a set time during which to write his programs (such as mathematics class). It is only important that the work be done and the child have access to the computers.

Choice of Topic

Before the details of the experiment could be worked out, it was first necessary to select a topic. The Law of Sines from elementary trigonometry was chosen. Again, there were several reasons for this selection.

A student taking trigonometry in high school is usually a junior or senior. Being older, he tends to be more serious than a freshman. His mathematical background is more complete. He may also, by this time, have some kind of computer mathematics/computer science background.

The Law of Sines is one of the last topics taught in an elementary trigonometry course. Thus the student has had ample time to become familiar with his teacher, trigonometry and the computer (assuming this is also his first exposure to a computer class) before the routine is disrupted by an experiment.
Perhaps, though, the most important consideration in choosing the Law of Sines was its substance. Taught with the ambiguous case, it requires approximately a week of high school instruction. This would imply that the experiment should not conclude too rapidly.

Plan of the Experiment

After choosing a topic, it was necessary to decide how to conduct the experiment. It was decided to choose trigonometry students who were presently taking or had taken a course in computer mathematics/science. These students would be divided into three distinct groups. Each group would have a different homework assignment. Some students would work only paper and pencil exercises while others would need the use of a microcomputer. At the end of the experimental period, the same test would be given to all students. The test would be considered in three ways: an overall score, the drill problem score, and the word problem score.

Research Hypotheses

Then the following hypotheses were formulated:

1. A student whose homework assignment includes programming a computer will gain a better understanding of the concept than a non-programming student

2. A student whose homework assignment includes programming a computer will develop better computational
skills than the non-programming student

3. A student whose homework assignment includes programming a computer will develop better problem-solving skills than the non-programming student
CHAPTER II

ESTIMATION OF HOMEWORK TIMES

Need for the Pilot Study

As indicated in the introduction, it was decided that this experiment would affect only the students' homework assignments, not the teaching method. The composition of each student's homework assignment would be determined by the group in which he was placed.

There were three basic types of homework problems that could be assigned in the main experiment--drill problems, word problems, and computer programs. Reinforcement of the basic concept could be provided through the use of drill problems, that is, problems which emphasize the mechanics of the topic. The value of the concept in practical situations could be shown in word problems and/or computer programs.

To illustrate these three types of problems, consider a simple topic: calculation of the perimeter of a rectangle. Then a drill problem might be:

\[ l = 5 \quad w = 7 \quad \text{find } P \]

A word problem could read:

A man's garden has been overrun by rabbits. He wants to put up a fence to keep them out. If his
garden is a rectangle 10 feet wide and 35 feet long, how much fencing should he buy?

And a programming assignment might say:

Write a program which will find the perimeter of any rectangle.

Of the three types of problems, it would seem reasonable to suppose that a drill problem is the easiest to solve and that a student would need to spend the least amount of time working on it. On the other hand, a computer programming assignment should require significantly more time.

It would also seem reasonable to think that the more time and effort a student spends on a topic, the better he will understand it. If this is so, then it would be difficult to tell if a programming student learned more because of the type of homework assignment he had or because of the length of time he spent on it.

The purpose of this study was to estimate the time a student devotes to each type of homework assignment—drill problem, word problem, and computer program. In that way, the homework assignments made for each group of students in the main experiment would be comparable. That is, the time a student could be expected to spend on his homework should be the same regardless of the group in which he was placed.

In keeping with the topic chosen for the main experiment, the problems used in this study involved only the Law of Sines.
Overview of the Pilot Study

This pilot study was performed at Easton High School, Easton, Maryland and was divided into two distinct parts. The first part measured the time a student spent writing computer programs while the second part measured the time he spent on drill problems and word problems.

A total of two teachers and fifty-seven students participated in the study. The students were seniors, juniors, and a sophomore who were taking a college preparatory mathematics class and/or a computer mathematics class.

Nine students participated in the first part of the study and were given the opportunity to work up to four computer programs. Consequently, some students worked all four of the programs while others did only two.

The students were allowed only one problem at a time and could not receive another until the previous problem had been completed to their satisfaction. The programs were assigned in a random manner. Each program slip had a space for the student's name, a space to record his starting time, the statement of the problem, and a space to record his ending time. A copy of the four programs, all on one sheet, can be found in Appendix A.

Of the twenty-five programs attempted, one could not be used because no final time had been recorded.

Fifty-six students participated in the second part of the study. They were given an in-class two page test on
the Law of Sines. Four forms of this test, each containing five problems, were used. The test forms were randomly distributed among the students.

The first page of each test had a space for the student's name, a space to record his starting time, four drill problems, and a space to record the time when these four problems were completed (intermediate time). The second page contained a space for the student's name, one word problem, and a space to record the student's final time. Half of the problems required the student to use the ambiguous case. A copy of all four tests with solutions may be found in Appendix B.

Of the fifty-six students who took the test, four did not record an intermediate time. Four noted the intermediate time but not the final time. Three recorded an intermediate time which was later than the final time. One student's work was rejected because she had taken the test twice, once with each teacher. Thus there were 48 observations of the drill times and 44 observations of the word problem times.

The data from the study was analyzed under release 79.5 of SAS (Statistical Analysis System) at the Northeast Regional Data Center. It was assumed that the times were normally distributed.
Analysis of Drill Problem Times

There were four different forms of the test, each containing four different drill problems. The test was administered by two different teachers. Was the length of time a student spent working the four drill problems influenced by either the test form he took or the teacher he had? In other words, were the test forms comparable? Were the groups of students (classes) comparable? Since two different questions needed to be answered, the drill problem times were analyzed as a two-by-four factorial experiment with two factors, teacher and test form. The means were determined for each factor-level combination (see Appendix C, Table 1). The design was unbalanced, so the GLM (general linear model) procedure was used to analyze the data. The following hypotheses were tested:

H: The mean times spent working drill problems do not vary between teachers.

K: The mean times are not all the same.

H: The mean times spent working drill problems do not vary among test forms.

K: The mean times are not all the same.

Both indications of global utility, the overall F value and R square, were very small (.78 and .067, respectively). The individual F values for teacher and test form were also small (1.00 and .70, respectively). Thus there was nothing to indicate either null hypothesis should be re-
ject at a 0.05 alpha level (see Appendix C, Table 2).

**Analysis of Word Problem Times**

Although each of the four test forms contained only one word problem, the concerns regarding test form and teacher remained the same. Was the time a student spent working a word problem influenced by either the test form he took or the teacher he had?

Thus the word problem times were analyzed in the same manner as the drill problem times—as an unbalanced two-by-four factorial experiment with the two factors, teacher and test form. Means were determined for each factor-level combination (see Appendix C, Table 3). The following hypotheses were tested:

- **H:** The mean times spent working a written problem do not vary between teachers.

- **K:** The mean times are not all the same.

- **H:** The mean times spent working a written problem do not vary among test forms.

- **K:** The mean times are not all the same.

With an F value of 2.39 and an R-square of .201, the overall usefulness of the model was not significant. Individual F values were also small enough to be insignificant (1.17 and 2.79, respectively). There was no reason to reject either null hypothesis at a 0.05 alpha level (see Appendix C, Table 4).
Analysis of Programming Times

Does one of the four programming problems require more time to solve than the others? To answer this (single) question, the programming times were analyzed as a completely randomized experiment with four treatments, the programs. Mean times were determined for each program (see Appendix D, Table 1). PROC ANOVA (analysis of variance procedure) was used to test the following hypothesis:

H: The mean time spent on a program does not vary according to the problem chosen.

K: The mean times are not all the same.

The overall F value was very small (.89) as was the R-square value (.118) which would indicate the model's lack of usefulness. Thus there was no reason to reject the null hypothesis at a .05 alpha level (see Appendix C, Table 2).

Comparison of Average Times

Because the time a student spent working a problem did not appear to be influenced by either the particular problem given or his class, an overall average was calculated for the time spent working each type of assignment. The data indicated that a student spends approximately 6.98 minutes working a drill problem, 8.51 minutes working a word problem and 30.38 minutes writing a computer program.

Are these times significantly different? GLM was
used to test the hypothesis:

H: The mean time spent working on a problem does not vary according to the type.

K: The mean times are not all the same.

This time the F-value was 38.59 and R-square was .408 showing the model to be useful. Thus the null hypothesis was rejected at a .05 alpha level (see Appendix E, Table 1).

To determine which means were different, Duncan's multiple comparisons test was run. It showed that differences existed between the mean programming time and the mean times for working drill problems and word problems. There was no statistical difference between the mean time for working a drill problem and the mean time for working a word problem (see Appendix E, Table 2).

Summary

This study determined the average time a student spends working three types of trigonometry problems involving the Law of Sines (drill problems, word problems, and computer programs). By conducting this study in a Maryland high school, the author felt sure that there would be no overlap between these students and those who were to participate in the main experiment. Thus actual test questions and computer programs could be pre-tested via this study. However, this study also reflects the author's assumption that all students spend (approximately) the same amount of
time working a particular type of problem. That is, a Maryland student and a Florida student will spend a comparable length of time doing homework.

Using these calculations, the author determined the composition of the homework assignments for each group of students in the main experiment. If computer programs were assigned, the number of word problems and drill problems given were decreased so the student would not be forced to spend an inordinate amount of time doing homework.

Although the differences between the mean times for solving a word problem and working a drill problem did not test to be statistically significant, the author was conservative enough to use the mean times as determined here.

Then this study would indicate that, from the viewpoint of time, one computer programming problem could replace two drill and two word problems (This sums to slightly more than 30.38 minutes); or three drill and one word problem (This sums to slightly less than 30.38 minutes).
CHAPTER III
DEVELOPMENT OF THE CAI MATERIALS

Need for CAI Materials

In previous research, experimenters have only compared the achievement of programming students to non-programming students (21-26). There was no indication of any attempt to compensate for a possible Hawthorne effect resulting from the use of a computer.

The Hawthorne effect was first described in a study (Roethlisberger and Dickson, 1934) made at a Western Electric Company plant in Hawthorne, Illinois. The study showed that there is a "tendency of subjects in some experiments to respond to almost ANY change, apparently due to a feeling of appreciation that someone is paying attention to them" (27). In the study, it was found that production went up regardless of whether the working conditions were improved or made worse. Only the fact that a change had occurred seemed to matter.

It is possible that this phenomenon could apply to educational experiments as well. If this is so and a researcher deals only with an experimental group and a control group, then how is he to interpret his findings? Are his results due to a superior teaching method or merely a
different procedure?

To avoid this problem, the author wrote a series of three computer aided instruction (CAI) lessons on the Law of Sines to be used in the main experiment. The group of students using these lessons would provide an additional type of control. These students would be exposed to the computer (a different teaching technique); yet they would not be programmers. The Hawthorne effect is thus taken into account.

**General Guidelines for Developing the Lessons**

There are several characteristics of a microcomputer that make it a valuable teaching tool. It is interactive and patient. It can display pictures (graphics) to illustrate the point being presented. It can perform complex calculations quickly. It requires the student to pay attention.

Still, not all topics can (or probably should) be converted into CAI lessons. It is the responsibility of the author to choose a topic and method of presentation which utilizes as many of the special characteristics of the microcomputer as possible. Very little is gained by making a computer a mechanical page turner.

If enough time is spent analyzing a topic's suitability for the computer, then fewer problems will develop when the author transcribes these ideas into code. It is
important to remember the age and grade level of the intended user when planning each screen's presentation. Each screen needs to be clean, clear, concise and complete. Finally, if the lesson is designed to be teacher independent (no teacher intervention needed), it is vital that it be friendly. The lesson should ask for and frequently use the student's name. Safeguards should be built in so the lesson cannot be terminated by an unexpected response.

Once these conditions are fulfilled and the lesson is complete, it only remains to try it out. The Association for Educational Data Systems (AEDS) recommends a three-stage evaluation of materials (29):

1. One-on-one
2. Small group
3. Field trial

In a one-on-one evaluation, the author and user sit down together to discuss the lesson's vocabulary, flow, completeness, and embedded questions. The author may coach or help the user through the lesson.

For a small group evaluation, the author observes the experiences of several users without interfering in the process. He notes each user's attitude, performance, and the length of time it takes him to complete the lesson.

A field trial evaluation is conducted after the lesson has been revised according to the findings of the first two stages. This can be thought of as a large group "validation"
of the lesson. Again, attitude, performance and time are noted.

While following these procedures can result in a product which is both useful and effective, it is a time-consuming and often difficult process to complete. Finding sufficient numbers of students to participate in an evaluation is not a trivial problem. This is not to suggest, however, that the evaluation process can be eliminated. It has been noted that the maximum benefits occur during the one-on-one procedure. That much, at least, should be completed.

Then it can be seen that writing educational software is a three-stage process:

1. Selecting an appropriate topic
2. Writing the code
3. Field-testing the results

Selecting an Appropriate Topic

As stated in the introduction, the Law of Sines had already been selected as the topic for the main experiment. Now it was only necessary to determine how to approach the topic. The author decided that it would be most effective to develop a series of three lessons. Each lesson considered a different facet of the Law of Sines: its derivation, the non-ambiguous case and the ambiguous case. The following assumptions were made:
1. The user would know and understand the parts of a right triangle
2. The user would know and understand the sine function
3. The user would have had some exposure to the arcsine relation
4. The user would have had some exposure to the Law of Sines

Thus these lessons were designed as a review of, not a replacement for, the teacher's regular instruction. The lessons would check for this prerequisite knowledge. If found lacking, the user would not be permitted to continue with the lesson.

Within the latter two CAI lessons would be imbedded the four computer programs assigned to the programming group of students (group A). The author referred to these sections of the lessons as student interactions because the user can input any (meaningful) data he desires. It was recommended that the student use these sections as opportunities to check homework or to explore the topic without having to do tedious calculations. Thus it would be possible to determine if the use of canned programs is as effective as the writing of the programs.

There were two student interactions in Lesson II. In the first, after the user types in a number between 0 and 1, the computer returns the two arcsine values between 0 and
180 degrees. It also draws a graph to illustrate the relationship between the number inputted and the solutions. In the second interaction, the student types in two angles and one side of a triangle and the computer solves for the missing parts.

There were also two student interactions in Lesson III. In the first, the user types in two sides and one angle. The computer then determines the number of possible triangles. In the second interaction, the student again types in two sides and one angle. This time the computer not only determines the number of possible triangles but also solves for the missing parts in all cases.

A sample run of all three lessons can be found in Appendix F.

Writing the Code

During Spring 1981, the author began by sketching (on paper) each screen (or frame) for each lesson. These frames were modified until a continuity of thought and a pleasing appearance were achieved. Then rough flowcharts were drawn.

From mid to late summer 1981, the author wrote the code for the lessons on an S-100 bus microcomputer (Polymorphic 8080 CPU with North Star disk drive).

The reasons for choosing to develop the lessons on this system were:
1. The author was familiar with the system
2. It was easily accessible; located in the author's home
3. The disk drive allowed for rapid storage and retrieval and reliability
4. The large amount of available memory (40K) allowed for experimentation
5. A printer was available

However, the school system uses APPLE II and Radio Shack microcomputers. North Star BASIC is not compatible with either system. It would be necessary to choose one of these systems and translate the code already developed. The Radio Shack was chosen.

During the fall of 1981, the three lessons were translated to Radio Shack BASIC so they could run on either the Model I or Model III in 16K of memory.

The reasons for choosing the Radio Shack over the APPLE II were:
1. The screen format was similar to the Polymorphic
2. The resolution matched
3. It was easy to mix graphics and text anywhere on the screen
4. Through the kind permission of Dr. Christian S. Bauer, Jr., the author had ready access to the Radio Shack machines in the engineering department's microlab
5. It was less difficult to translate to Radio
Field-Testing the Results

By late October 1981, the lessons had been translated and all major bugs removed. Then these lessons underwent three stages of testing.

They were first examined by the author's husband, John E. Harrison. He reviewed them for continuity (Do the lessons make sense? Do they flow naturally from one screen to the next, from one lesson to the next?). He checked them for accuracy (Are there misspelled words? Do the lessons work when you follow the directions?). Finally, he examined them for bullet-proofing (Is the BREAK key disabled? Can the program be stopped prematurely by giving an unexpected answer? Are there a sufficient number of error messages to cover all possibilities and are they complete? Can the program be crashed?).

The second stage of testing was the review by the author's advisor, Dr. Lee H. Armstrong. He also checked the lessons for continuity and accuracy. However, he also examined them for pedagogical content (Are the lessons mathematically accurate? Are the presentations sound from a teaching point of view?).

The final stage of testing was done with eight students from Dr. Armstrong's college trigonometry class. These college freshmen had just finished studying the Law
of Sines. They reviewed the lessons for clarity of presentation (Was the purpose of each lesson clear? Were there enough examples? Were explanations complete enough? Were error messages comprehensive?). Two of these students looked at two of the lessons while the other six students only did one of the three lessons.

As they worked, the students were timed to provide an indication of each lesson's length.

**Results and Conclusions**

Mean times (and the corresponding standard deviations) were calculated for each lesson (see Appendix G, Table 1). Because the sample size was so small and the author only needed an estimate of the lesson length, no hypotheses were formulated nor tests run.

However, it was assumed that the time these college freshmen spent on the lessons would be comparable to that spent by the students involved in the main experiment. Both the age level and mathematical background of the two groups were similar.

Favorable comments made by the students as they reviewed the lessons led the author to believe that minimal change was necessary before progressing to the main experiment. Thus no more timing studies were done.

So this study would indicate that, from the viewpoint of time, these three lessons (total time 50.7 min-
utes) could replace six word problems (total of 51.066 minutes) or seven drill problems (total of 48.89 minutes).
CHAPTER IV

THE MAIN EXPERIMENT

Overview

Three Orange County high schools agreed to participate in this experiment: Edgewater, Apopka, and West Orange. Before the experiment was begun, the author visited each of the schools several times to discuss the purpose and conduct of the experiment. In addition, a packet of information was left with the mathematics teacher. The packet contained an overview of the experiment, reviewed the procedures to be followed, and included a copy of all materials needed for each group of students; including the final test. A copy of these materials may be found in Appendix H.

Most of the materials used in the experiment were pre-tested in the pilots. Thus the four programs available for the computer programming group of students were the same programs used in the first pilot study to determine times. Four of the five test questions can be found (verbatim) on the tests used in the first pilot while the fifth is similar to one of the other problems. Most of the extra homework problems were collected from other elementary trigonometry textbooks.
Construction of the Test

Because the test was the only means of evaluation for the experiment, definite objectives guided its construction. A listing of these objectives follows:

The student should be able to:

1. Recognize problems in which the Law of Sines is applicable
2. State the Law of Sines
3. Recognize problems involving the non-ambiguous case of the Law of Sines
4. Solve for the missing parts of a triangle given two angles and one side
5. Recognize problems involving the ambiguous case of the Law of Sines
6. Determine how many solutions are possible given two sides and an angle opposite one of them
7. Solve for the remaining parts of all triangles given two sides and an angle opposite one of them

Each question on the test addressed some combination of these objectives. Each objective was used at least twice on the test. The scoring was based both on the type of problem (drill or word problem) and the objectives it covered. The first problem was a drill problem involving the non-ambiguous case (objectives 1,2,3,4). It was worth five points. The second problem was a five-point
drill problem involving the ambiguous case (objectives 1,2, 5,6,7). The third problem was a drill problem requiring a very basic understanding of the ambiguous case (objectives 1,5,6). It had a value of eight points. The fourth problem was a two-step word problem, of which only one part required the use of the Law of Sines (objectives 1,2,3,4). It was worth twelve points. The fifth problem was a ten-point problem requiring the use of the ambiguous case (objectives 1,2,5,6,7).

The Schools

Thirty-four trigonometry students from the three different Orange County high schools were chosen to participate in the six day experiment. All of these students were currently taking or had taken a computer programming course at the school.

Edgewater

There were six students who participated in the experiment at Edgewater High School. Instead of using the provided test as is, the instructor did the following:

1. Changed some of the test problems
2. Increased the length of the test
3. Divided the test into two parts and gave it over a two-day period

It was not possible to adopt the changed test for use at the other test sites for the following reasons:
1. The added length made the test too long for a one-period test.
2. The test no longer covered all the intended objectives.

Therefore, the results from Edgewater High School could not be used.

Apopka

There were nine students who participated in the experiment at Apopka High School. Apparently, there was a misunderstanding as to the conduct of the experiment. The teachers involved assumed that the experiment was a variation of the PLATO system developed in the early 1960s.

Only the control group of students were allowed to attend the trigonometry classes. The other two groups of students went directly to the computers without receiving any instruction from the mathematics teacher.

Although the correct test was given, in its entirety, this improper procedure invalidated the results.

West Orange

Originally planned to run before December 1981, the teacher was not ready to conduct the experiment until February 1982. During the change of semesters, the composition of the trigonometry class changed. The number of students available to participate in the experiment dropped to fourteen.
Procedure

The students were divided into three groups based upon their previous semester's grade in trigonometry. Each group contained students who had received high grades and students who had received low grades. All three groups were taught the Law of Sines (including the ambiguous case) by their regular teacher. Only their homework assignments differed. Using the results of the pilot studies, the composition of each group's assignments were adjusted so they were approximately equal in length (timewise).

The control group (group C) had standard, paper and pencil, hand-written assignments consisting of drill and word problems. They were assigned a total of thirteen drill and five word problems over the length of the experiment. There were six students in this group.

The computer programmers (group A) were assigned only six drill problems and no word problems. Rather, they were given a page with four programming problems concerning the Law of Sines. Each student chose three programs to write and run. There were four students in this group.

The computer users (group B) worked some, but not all, of the problems given to the control group. They were assigned a total of eight drill and three word problems. In place of the other problems, the students in this group reviewed the day's lesson with the CAI materials written by the author. There were four students in this group.
In addition, all students kept an activity log where all time spent on each type of assignment was recorded. This was provided as a check on the pilot study results. By knowing the actual time each child spent on his assignments, it would be possible to compensate for this variable in the analysis later.

At the end of the experiment, all students took the same forty-point, five-question test described earlier in this chapter. The test contained no programming questions. The first three problems tested for mechanical skills and were worth a total of eighteen points. The last two questions were word problems worth a total of twenty-two points.

Results
Main Hypotheses

Analysis of covariance

The data from this experiment was analyzed under release 79.5 of SAS (Statistical Analysis System) at the Northeast Regional Data Center.

Because test scores tend to be normally distributed, the data was first examined by an analysis of covariance procedure where the covariate was the total time spent on the homework assignments.

The first set of data analyzed involved the total test score. The following hypothesis was tested:

$H_0$: The mean scores on the entire test do not vary
among groups.

K: The mean scores are not all the same.

The overall F-value was small (1.55) which would indicate the model's lack of usefulness. Although the means appeared to be different, the differences were not significant at a .05 alpha level (see Appendix I, Tables 1 and 2).

Even though the overall test scores did not test to be (statistically) different, it was thought that differences may exist in the groups' ability to work drill problems or solve word problems. Such differences (if they exist) might be hidden within the overall test scores.

Thus the following set of hypotheses were tested:

H: The mean scores for the drill problems do not vary among Groups.

K: The mean scores are not all the same.

H: The mean scores for the word problems do not vary among groups.

K: The mean scores are not all the same.

Again, no statistical differences could be found in the means. For the drill scores, the F-value was 1.37; too small for the model to be useful (see Appendix I, Tables 3 and 4). For the word problem scores, the F-value was .92; again showing a model of limited usefulness (see Appendix I, Tables 5 and 6).
Non-parametric analysis

Because the numbers of students involved in each group was small and the variability in the scores large, the author was concerned that the assumption of normality was invalid. Thus it was decided to apply a non-parametric test, the Kruskall-Wallis H test. In this procedure, all the scores are ranked in order of magnitude. Since it is the ranking which is examined, not the actual scores, the effects of "wild points" are minimized.

This change of test requires a change in the way the hypotheses are stated:

H: The population probability distributions of total test score for the three groups are identical.
K: At least two of the groups have probability distributions with different locations.

H: The population probability distributions of drill problem score for the three groups are identical.
K: At least two of the groups have probability distributions with different locations.

H: The population probability distributions of word problem score for the three groups are identical.
K: At least two of the groups have probability distributions with different locations.

In all three instances, the calculated H statistic was very small. There was no reason to reject any of the null hypotheses at a .05 alpha level (see Appendix J, Tables 1 and 2).
Comparison of Homework Times

To provide a check upon the results of the two pilot studies, the actual time spent of homework assignments was examined. It was expected that each group would spend about the same amount of time on homework. However, an examination of the activity logs showed a large discrepancy: Group A, 209.5 minutes, Group B, 105.25 minutes and Group C, 111.5 minutes (see Appendix K, Table 1).

It appeared that these times were badly skewed; that is, a student could have spent a longer period of time doing homework merely because he happened to be in a particular group. Thus it was thought proper to use a non-parametric test to analyze these times.

The Kruskall-Wallis H test was used to test the following hypothesis:

H: The population probability distributions of total homework time for the three groups are identical.

K: At least two of the groups have probability distributions with different locations.

The calculated H statistic was 8.024, larger than the chi-square value of 5.99147. Thus the null hypothesis could be rejected at a .05 alpha level (see Appendix K, Table 2).

Now it was necessary to determine which of the three groups was different from the others.

Since the programming students worked longer than anyone else, the author began by comparing the CAI and
control groups. This was done with the Wilcoxon rank sum test, a non-parametric test similar in concept and procedure to the Kruskall-Wallis H test. The following hypothesis was tested:

H: The two populations of homework times corresponding to the CAI group and the control group have the same probability distribution.

K: The probability distribution for the CAI group is shifted to the right or left of the probability distribution corresponding to the control group.

The calculated T value of 21 lay between the critical values of 12 and 32. Thus there was no reason to reject the null hypothesis at a .05 alpha level (see Appendix K, Table 3). This implies that the differences indicated by the Kruskall-Wallis H test must be between the group of programming students and the other two groups. That is, the programming students spent (statistically) significantly more time on their homework than either of the other two groups.
CHAPTER V

SUMMARY AND RECOMMENDATIONS

Summary

None of the differences in the test score means tested to be statistically significant; that is, any of the differences observed among the three groups of students (computer programmers, computer users, and control) could be due to random chance rather than a superior method of instruction. So there is nothing to indicate that those students who used the computer as a part of their mathematics homework developed better computational or problem-solving skills. However, the lack of significance also indicates that these techniques did not adversely affect their achievement.

It can be noted that, despite working a smaller quantity of paper and pencil exercises:

1. Programming students had at least as good an understanding of the Law of Sines as the control group.

2. The replacement of paper and pencil exercises with computer exercises did not detract from the students' ability to perform the calculations required for the Law of Sines.
Recommendations for Further Research

This experiment involved only fourteen students, one topic, and six days. Were these numbers too small to have had an effect upon the results? Previous research involved more students for a longer period of time. Perhaps this experiment should be repeated with the following modifications:

1. Use more students. By increasing the sample size, the effect of individual student differences would be minimized. Extreme values in the data set would be less noticeable. It may be possible to avoid a non-parametric analysis—a less powerful technique.

2. Expand the time and content. Then, student absences and unexpected classroom interruptions would be less important. More homework problems and computer programs could be assigned. More tests could be given. With more information available, it is possible that a more complete evaluation would result.

However, this would first require a reworking of the pilot studies. It was seen that the three groups of students did not spend the same amount of time on their homework. Neither did the times match what was expected. While these differences can be compensated for in the analysis, it might be better if the assignments were adjusted so such differences were minimized. A student may resent being a member of the group which always has the longest homework
assignments.

It would also be necessary to spend a great deal of time developing CAI lessons. There is a dearth of educational materials available for microcomputers (the equipment available to most schools) particularly at the secondary level. To the author's knowledge, there is nothing on the market today which could satisfy the needs of a semester-long course in trigonometry.

In the effort to balance homework times, the students who had access to the microcomputers were assigned very few (or no) written problems. It may be that the thinking that occurs when a student works a word problem is different from that associated with writing a computer program. Perhaps the design of this experiment should be adjusted so that the number of drill and word problems assigned is controlled; not the times. Thus all students would begin by working the same paper and pencil exercises. However, those students who were in the two computer groups would have additional assignments involving the computer.

The author has assumed a need for the computer in the classroom. This study raises a question concerning the role of CAI in education. Although none of the means were statistically different, they were highest for the CAI group. Should computer-aided instruction materials be used as an adjunct to rather than a replacement for regular classroom instruction? Additional study could be
conducted to determine which of the following methods results in optimal effectiveness of CAI materials:

1. The student would work a prescribed number of paper and pencil exercises at the end of a lesson

2. The student would review a topic via a CAI lesson and then work a reduced number of paper and pencil exercises

3. The student would rely totally upon the CAI materials for instruction and homework

Perhaps such a study could be combined with a more extensive examination of the effects programming has upon achievement. The results from such work may help to define the proper role of computers in the classroom.
APPENDIX A

COMPUTER PROGRAMS
PROBLEM:
For any positive value of \( w < 1 \), the equation:
\[ \sin(x) = w \]
has two solutions between 0 and \( \pi \). Write a program to find both of these values for a given \( w \).

TIME FINISHED: ______________________

PROBLEM:
Given two angles and one side of a triangle, write a program to find the other two sides.

TIME FINISHED: ______________________

PROBLEM:
Given two sides of a triangle and an angle opposite one of them, write a program to find the other two angles.

TIME FINISHED: ______________________

PROBLEM:
Given two sides and an angle opposite one of them, write a program to determine if none, one, or two triangles can exist.

TIME FINISHED: ______________________
APPENDIX B

PILOT STUDY TESTS
Directions: Show all work on this paper. (Use the back if necessary.) Solve all lengths to the nearest tenth and all angles to the nearest 10 minutes. Be sure to record your starting time, the time after you have completed the first four problems, and your ending time.

TIME: 

1) Given triangle ABC where $a=10$, $\sin(A)=\frac{3}{4}$ and $\sin(B)=\frac{4}{5}$ find $b$.

\[
\frac{10}{\frac{3}{4}} = \frac{b}{\frac{4}{5}} \quad \Rightarrow \quad \frac{3}{4} b = 8 \quad \Rightarrow \quad b = 10.7
\]

2) Find the remaining parts of triangle ABC if $A=34^\circ10'$, $b=25$ and $C=54^\circ$.

\[
\frac{25}{\sin(54)} = \frac{a}{\sin(34^\circ10')} \quad \Rightarrow \quad a = 14.047243
\]

\[
\frac{25}{\sin(54)} = \frac{c}{\sin(91^\circ50')} \quad \Rightarrow \quad c = 20.235783
\]

3) Find the remaining parts of triangle ABC if $b=6$, $a=2$ and $A=37^\circ$.

\[
\frac{2}{\sin(37)} = \frac{6}{\sin(B)} \quad \Rightarrow \quad \sin(B) = 1.805
\]

No triangle \( \varnothing \)

4) Find the remaining parts of triangle ABC if $a=41$, $c=55$ and $C=64^\circ40'$.

\[
\frac{41}{\sin(A)} = \frac{55}{\sin(64^\circ40')} \quad \Rightarrow \quad \frac{b}{\sin(72^\circ58.5')} = \frac{55}{\sin(64^\circ40')}
\]

\[
A = 42^\circ21.5' \quad b = 58.18519 \quad B = 73^\circ
\]
5) Given a triangle ABC where \( a = 70 \) and \( B = 120^\circ \) determine the values of \( b \) for which \( A \) has (a) no value (b) one value (c) two values.

\[
\frac{70}{\sin(A)} = \frac{b}{\sin(120)}
\]

\[
\sin(A) = \frac{70 \sin(120)}{b}
\]

\[
b = \frac{60 \cdot 60}{b}
\]

\( b \) is at least \( > 60.6 \)

\( b \) is actually \( > 70 \) since

\( 120 \) is the largest angle.

\[ a) \quad b \leq 70 \]

\[ b) \quad b > 70 \]

\[ c) \quad \text{not possible} \]
Directions: Show all work on this paper. (Use the back if necessary.) Solve all lengths to the nearest tenth and all angles to the nearest 10 minutes. Be sure to record your starting time, the time after you have completed the first four problems, and your ending time.

TIME: ____________

1) Given triangle ABC where B=75°40', C=52°30' and c=30 find b.

\[
\frac{b}{\sin(75°40')} = \frac{30}{\sin(52°30')} \Rightarrow b = 36.6
\]

2) Find the remaining parts of triangle ABC if A=50°, B=55° and c=25

\[
\frac{a}{\sin(50°)} = \frac{25}{\sin(75°)} \Rightarrow a = 19.8 \quad \frac{b}{\sin(55°)} = \frac{25}{\sin(75°)} \Rightarrow b = 21.2 \quad \frac{c}{\sin(50°)} = \frac{25}{\sin(75°)} \Rightarrow c = 19.8
\]

3) Find the remaining parts of triangle ABC if c=6, a=10, and C=30°

\[
\frac{6}{\sin(30°)} = \frac{10}{\sin(A)} \Rightarrow A = 57° \quad \frac{6}{\sin(30°)} = \frac{b}{\sin(93°33')} \Rightarrow b = 11.9 \quad \frac{6}{\sin(30°)} = \frac{b}{\sin(26°30')} \Rightarrow b = 9.3
\]

4) Find the remaining parts of triangle ABC if B=125°40' c=6.3 and b=12

\[
\frac{12}{\sin(125°40')} = \frac{6.3}{\sin(C)} \Rightarrow C = 25°14.8' \quad \frac{12}{\sin(125°40')} = \frac{a}{\sin(29°5')} \Rightarrow a = 7.2 \quad A = 29° \quad C = 25°10'
\]
5) A surveyor who was running a line due north avoided an impassable area by moving on bearing N 42°20' E for 315 feet and then on bearing N 38°30' W. How far must he move on the last mentioned bearing in order to reach a point on the original north line extended?

\[
\frac{x}{\sin(42°20')} = \frac{315}{\sin(38°30')}
\]

\[
x = 340.77022
\]

\[
x = 340.8
\]
Name: KEY #3

Directions: Show all work on this paper. (Use the back if necessary.) Solve all lengths to the nearest tenth and all angles to the nearest 10 minutes. Be sure to record your starting time, the time after you have completed the first four problems, and your ending time.

TIME: _________

1) Given triangle ABC where A=60°, B=40° and b=5; find a.

\[
\frac{a}{\sin(60°)} = \frac{5}{\sin(40°)}
\]

\[
a = 6.7364918
\]

2) Find the remaining parts of triangle ABC if a=28, B=71°30' and C=46°40'.

\[
\frac{b}{\sin(61°50')} = \frac{28}{\sin(61°50')} = \frac{c}{\sin(46°40')}
\]

\[
A = 61°50',
\]

\[
b = 30.1179
\]

\[
c = 23.1022
\]

3) Find the remaining parts of triangle ABC if B=21°, a=5 and b=8.

\[
\frac{c}{\sin(21°)} = \frac{5}{\sin(21°)} = \frac{c}{\sin(116°31')} = \frac{a}{\sin(116°31')}
\]

\[
A = 116°31',
\]

\[
c = 12.4669
\]

\[
A = 13°,
\]

\[
c = 14°10'\text{ nonsense}
\]

4) Find the missing parts of triangle ABC if C=131°, a=19 and c=17.

\[
\frac{17}{\sin(131°)} = \frac{19}{\sin(a)}
\]

\[
\text{not possible}
\]

\[
A = 57°31'
\]

TIME: _________
5) From a point some distance away from the base of a sequoia tree, a man measures the angle of elevation of the top of the tree as $37^\circ$. Walking 153 feet further on, he measures the angle of elevation as $28^\circ$. Find the height of the tree.

\[
\frac{153}{\sin(37)} = \frac{y}{\sin(28)}
\]

\[y = 459.10447\]

\[
\sin(37) = \frac{x}{153.19447}
\]

\[x = 276.33208\]
Name: ____________________________

Directions: Show all work on this paper. (Use the back if necessary.) Solve all lengths to the nearest tenth and all angles to the nearest 10 minutes. Be sure to record your starting time, the time after you have completed the first four problems, and your ending time.

TIME: ____________________________

1) Given triangle ABC where \( \sin(C) = .3 \), \( \sin(B) = .6 \) and \( c = 12 \) find \( b \).

\[
\frac{b}{\sin(B)} = \frac{c}{\sin(C)}
\]

\[
.3b = 7.2
\]

\[
b = 24
\]

2) Find the remaining parts of triangle ABC if \( c = .8 \), \( B = 70^\circ 50' \) and \( A = 230^\circ 30' \)

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}
\]

\[
\begin{align*}
a &= .3465 \\
b &= .7602
\end{align*}
\]

3) Find the remaining parts of triangle ABC if \( C = 210^\circ 20' \) \( a = 10 \) and \( c = 3.7 \)

\[
\frac{a}{\sin(A)} = \frac{3.7}{\sin(21^\circ 20')}
\]

\[
\begin{align*}
a &= 3.7 \\
b &= 9.989
\end{align*}
\]

4) Find the remaining parts of triangle ABC if \( A = 12\degree 50' \) \( a = 30 \) and \( b = 40 \)

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)}
\]

\[
\begin{align*}
a &= 30 \\
b &= 8.731
\end{align*}
\]

TIME: ____________________________
5) Steaming down the center of a channel, a ship sights two buoys ahead of it, one on either side. The buoys are 800 yards apart and the distance from the ship to the farther buoy is 1200 yards. How far is the ship from the nearer buoy if the angle between the lines of sight to the buoys is 35°?

\[
\frac{800}{\sin(35°)} = \frac{1200}{\sin(85°)}
\]

\[
\theta_1 = 59.357° \quad \text{or} \quad 120.642°
\]

\[
\theta_2 = 85.642° \quad \text{or} \quad 24.358°
\]

\[
\frac{x}{\sin(35.64°)} = \frac{800}{\sin(35°)} \quad \text{or} \quad \frac{x}{\sin(24.357°)} = \frac{800}{\sin(35°)}
\]

\[
x = 1390.7 \quad \text{or} \quad x = 575.239
\]

\text{not correct answer}

\[
x = 575.2
\]
APPENDIX C

STATISTICS FOR DRILL AND WORD PROBLEM TIMES
### TABLE 1

STATISTICS FOR DRILL PROBLEM TIMES

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Test</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min. Value</th>
<th>Max. Value</th>
<th>Std. Error</th>
<th>Sum</th>
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<tbody>
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<td>24.167</td>
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<td>30.000</td>
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### TABLE 2

ANOVA FOR DRILL PROBLEM TIMES

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<th>Source</th>
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<td>49.86199911</td>
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<td>Corrected Total</td>
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<td>2298.81250000</td>
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<th>R-Square</th>
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<th>Drill Mean</th>
<th>C.V.</th>
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<th>Pr &gt; F</th>
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### TABLE 3

**STATISTICS FOR WORD PROBLEM TIMES**

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<th>Teacher</th>
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<th>Mean</th>
<th>Std. Dev.</th>
<th>Min. Value</th>
<th>Max. Value</th>
<th>Std. Error</th>
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<th>Var.</th>
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<td>7</td>
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<td>10.000</td>
<td>0.778</td>
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<td>1</td>
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</tr>
<tr>
<td>1</td>
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<td>6</td>
<td>9.500</td>
<td>2.588</td>
<td>5.000</td>
<td>13.000</td>
<td>1.057</td>
<td>57.000</td>
<td>6.700</td>
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TABLE 4
ANOVA FOR WORD PROBLEM TIMES

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<th>Source</th>
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<th>Sum of Square</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
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<tbody>
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<td>Model</td>
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<td>97.71873805</td>
<td>24.42968451</td>
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<td>0.0681</td>
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<td>38</td>
<td>389.02544800</td>
<td>10.23751179</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>42</td>
<td>486.74418605</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
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<td>2.79</td>
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<td>0.1371</td>
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<td>Form</td>
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<td>85.71232978</td>
<td>2.79</td>
<td>0.0535</td>
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</tbody>
</table>

R-Square: 0.200760
Std. Dev.: 3.19961119
Word Mean: 8.51162791
C.V: 37.5911

56
APPENDIX D

STATISTICS FOR PROGRAMMING TIMES
<table>
<thead>
<tr>
<th>Program</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max. Value</th>
<th>Min. Value</th>
<th>Sum Var.</th>
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<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>39.333</td>
<td>27.274</td>
<td>90.000</td>
<td>13.000</td>
<td>743.86</td>
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<td>2</td>
<td>6</td>
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<td>F Value</td>
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<td>-----</td>
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</tr>
<tr>
<td>Model</td>
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<td>536.48611111</td>
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<td>0.4613</td>
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<td>20</td>
<td>11998.16666661</td>
<td>599.90833333</td>
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<td></td>
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</table>

**TABLE 2**

ANOVA FOR PROGRAMMING TIMES

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<tr>
<th>R-Square</th>
<th>Std. Dev.</th>
<th>Time Mean</th>
<th>C.V.</th>
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<tr>
<td>0.118276</td>
<td>24.49302622</td>
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<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Program</td>
<td>3</td>
<td>1609.45833333</td>
<td>0.89</td>
<td>0.4613</td>
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</tbody>
</table>
APPENDIX E

COMPARISON OF HOMEWORK TIMES
### TABLE 1

ANOVA FOR ASSIGNMENT TIMES

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<tr>
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<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
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<td>4906.09273374</td>
<td>38.59</td>
<td>0.0001</td>
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<tr>
<td>Error</td>
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<td>127.12540149</td>
<td></td>
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<tr>
<td>Corrected Total</td>
<td>114</td>
<td>24050.23043478</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>R-Square</th>
<th>Std. Dev.</th>
<th>Time Mean</th>
<th>C.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.407987</td>
<td>11.27499009</td>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>2</td>
<td>9812.18546749</td>
<td>38.59</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>Type IV SS</th>
<th>F Value</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>2</td>
<td>9812.18546749</td>
<td>38.59</td>
<td>0.0001</td>
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TABLE 2
DUNCAN'S MULTIPLE RANGE TEST
(Time in minutes)

<table>
<thead>
<tr>
<th>Grouping</th>
<th>Mean</th>
<th>N</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30.375</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>8.511</td>
<td>43</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>6.984</td>
<td>48</td>
<td>1</td>
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</table>

Alpha Level = .05  Df = 112  MS = 127.125
APPENDIX F
SAMPLE RUN OF CAI LESSONS
THE LAW OF SINES

\[
\frac{SA}{\sin(A)} = \frac{SB}{\sin(B)} = \frac{SC}{\sin(C)}
\]

(PRESS (ENTER) TO GO ON.)?
So that I can better help you, please type in your first name and then press (Enter).

Name: Mary

Mary, you remember that a right triangle is one which has a 90 degree angle. Let's begin by reviewing the parts of a right triangle.

(Press (Enter) to go on.)?
WHAT SIDE IS OPPOSITE ANGLE B? F
GOOD, MARY!

(PRESS ENTER TO GO ON.)

MARY, WHAT ANGLE IS OPPOSITE SIDE E? C
VERY NICE, MARY!

(PRESS ENTER TO GO ON.)

MARY, WHAT IS THE SINE OF A?
(A) F/E
(B) D/E
(C) E/F
(ANSWER: A, B, C)? A

GOOD, MARY

LET'S DO ANOTHER ONE.

(PRESS ENTER TO GO ON.)
MARY, WHAT IS THE SINE OF C?
(A) E/F
(B) F/E
(C) D/E
(Answer: A, B, C)? B

GOOD, MARY

LET'S DO ANOTHER ONE.

(PRESS (ENTER) TO GO ON.)

MARY, WHAT IS THE SINE OF A?
(A) E/F
(B) F/E
(C) D/E
(Answer: A, B, C)? B

GOOD, MARY

(PRESS (ENTER) TO GO ON.)

In class, you have used the sine function to find the missing parts of a right triangle. For example, let's find the length of the hypotenuse in the above triangle where
B = 25 degrees, A = 65 degrees, and BC = 6

\[ \sin(25) = \] (A) X/6
(B) 5/X
(C) 9/X

(Answer: A, B, C)? B

GOOD, MARY

(PRESS (ENTER) TO GO ON.)
\[
\sin(25^\circ) = \frac{6}{x} \\
\sin(25^\circ) = 0.4226 \\
0.4226 = \frac{6}{x} \\
x = 6 / 0.4226 \\
x = 14.1978
\]

(Table Value)

(Substituting into the Equation)

(Solve for x)

(Simplify the expression)

(Press (Enter) to go on.)

Although this is fun, it is of limited use.

What can we do if the triangle is not a right triangle?

Well, Mary, you are going to see that there is a very nice relationship between the sine of an angle and the length of its opposite side. This relationship is called the law of sines:

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

(Press (Enter) to go on.)

Mary, let's begin our work with an acute triangle. By drawing in an altitude, \( h \), we have formed two right triangles. One of them is \( \text{AXB} \). What is the other? \( \text{AXC} \)

Good, Mary.

Now we will find the sine of \( B \) and the sine of \( C \).

(Press (Enter) to go on.)
\[
\begin{align*}
\sin(B) &= \frac{H}{SC} & \sin(C) &= \frac{H}{SB} \\
(\text{SC})\sin(B) &= H & (\text{SB})\sin(C) &= H \quad \text{(Simplifying)} \\
(\text{SC})\sin(B) &= (\text{SB})\sin(C) & (\text{Combining Equations}) \\
\frac{\text{SB}}{\sin(B)} &= \frac{\text{SC}}{\sin(C)} & \text{(Standard Form)}
\end{align*}
\]

(PRESS (ENTER) TO GO ON.)

---

A similar equation can be obtained regardless of the altitude used. Suppose we draw in a different altitude and work through the problem again. One right triangle is ABC. What is the other? BIC

Right, vary
Let's find \(\sin(A)\) and \(\sin(C)\).

(PRESS (ENTER) TO GO ON.)

\[
\begin{align*}
\sin(A) &= \frac{H}{SC} & \sin(C) &= \frac{H}{SA} \\
(\text{SC})\sin(A) &= H & (\text{SA})\sin(C) &= H \quad \text{(Simplifying)} \\
(\text{SC})\sin(A) &= (\text{SA})\sin(C) & (\text{Combining Equations}) \\
\frac{\text{SA}}{\sin(A)} &= \frac{\text{SC}}{\sin(C)} & \text{(Standard Form)}
\end{align*}
\]

(PRESS (ENTER) TO GO ON.)
MARY, REMEMBER THAT WE FIRST SAW THAT:
\[ \frac{SB}{\sin(B)} = \frac{SC}{\sin(C)} \]

AND WE JUST SHOWED THAT:
\[ \frac{SA}{\sin(A)} = \frac{SC}{\sin(C)} \]

THESE TWO EQUATIONS ARE USUALLY COMBINED.

THE LAW OF SINES:
\[ \frac{SA}{\sin(A)} = \frac{SB}{\sin(B)} = \frac{SC}{\sin(C)} \]

(PRESS <ENTER> TO GO ON.)

MARY, YOU HAVE JUST SEEN THE LAW OF SINES DERIVED USING AN ACUTE TRIANGLE.

BUT THE LAW OF SINES CAN BE USED WITH ANY TRIANGLE PROVIDED YOU KNOW TWO ANGLES AND ONE SIDE OF A TRIANGLE, OR TWO SIDES AND AN ANGLE OPPOSITE ONE OF THE SIDES.

WOULD YOU LIKE TO SEE A DERIVATION USING AN OBTUSE TRIANGLE (YES/NO)  ? YES

FIRST, MARY, WE'LL DRAW AN ALTITUDE TO MAKE TWO RIGHT TRIANGLES. ONE RIGHT TRIANGLE IS AXC. WHAT IS THE OTHER?

? BXC

RIGHT, MARY

NOW WE'LL FIND \(\sin(A)\) AND \(\sin(C)\).

(PRESS <ENTER> TO GO ON.)
\[ \sin(a) = \frac{h}{sc} \quad \sin(c) = \frac{h}{sa} \]

\[ (sc)\sin(a) = h \quad (sb)\sin(c) = h \quad (simplifying) \]

\[ (sc)\sin(a) = (sa)\sin(c) \quad (combining equations) \]

\[ sa / \sin(a) = sc / \sin(c) \quad (standard form) \]

"(press enter) to go on."

Mary, I'm sure you remember that to obtain the rest of the law, we must use a different altitude. Again we have formed two right triangles. One of them is AxB. What is the other? AxC.

Good, Mary. Let's find \( \sin(c) \) and \( \sin(b) \)."  

\[ \sin(c) = \frac{h}{sb} \quad \sin(b) = \frac{h}{sc} \]

\[ (sb)\sin(c) = h \quad (sc)\sin(b) = h \quad (simplifying) \]

\[ (sb)\sin(c) = (sc)\sin(b) \quad (combining equations) \]

\[ sc / \sin(c) = sb / \sin(b) \quad (standard form) \]

"(press enter) to go on."

71
FIRST WE FOUND THAT:

\[ \frac{a}{\sin(A)} = \frac{c}{\sin(C)} \]

THEN WE SHOWED THAT:

\[ \frac{c}{\sin(C)} = \frac{b}{\sin(B)} \]

COMBINING THE EQUATIONS WE HAVE:

THE LAW OF SINES

\[ \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \]

(PRESS (ENTER) TO GO ON.)

NOW THAT YOU HAVE SEEN THE LAW OF SINES DEVELOPED, LET'S LOOK AT AN EXAMPLE WHICH USES IT.

(PRESS (ENTER) TO GO ON.)

LET'S FIND THE LENGTH OF THE SIDE OPPOSITE ANGLE C; THAT IS, THE LENGTH OF \( x \); USING THE LAW OF SINES.

\[ \sin(B) / \overline{AB} = \sin(C) / \overline{AC} \]

\[ \sin(73^\circ) / 1 = \sin(21^\circ) / x \]

(\( \sin(73^\circ) = 0.9563 \) \( \sin(21^\circ) = 0.3583 \) \( \overline{AB} = 1 \) \( \overline{AC} = x \) \( \) (\( \) (SUBSTITUTING GIVEN VALUES) \( \) (VALUES FROM TABLE)

\[ x = (1)(0.3583) / 0.9563 \]

\[ x = 0.3746 \]

(PRESS (ENTER) TO GO ON.)
MARY, YOU HAVE SEEN THAT FOR ANY TRIANGLE
THERE IS A RELATIONSHIP BETWEEN THE SINE OF AN ANGLE AND ITS
OPPOSITE SIDE:

THE LAW OF SINES

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}
\]

(PRESS (ENTER) TO GO ON.)

WILL YOU LIKE TO REVIEW THE DERIVATION USING
THE ACUTE TRIANGLE (YES/NO) ? NO

WILL YOU LIKE TO REVIEW THE DERIVATION USING
THE OBTUSE TRIANGLE (YES/NO) ? NO

WILL YOU LIKE TO SEE ANOTHER EXAMPLE USING THE LAW
OF SINES? (YES/NO)? NO

MARY, YOU HAVE COMPLETED TODAY'S LESSON ON THE
DERIVATION OF THE LAW OF SINES. I'LL SEE YOU AGAIN TOMORROW.

READY

)
THE LAW OF SINES
(HOW TO USE IT)

\[
\frac{SA}{\sin(A)} = \frac{SB}{\sin(B)} = \frac{SC}{\sin(C)}
\]

(PRESS (ENTER) TO GO ON.)

IN PART I, YOU EXAMINED THE DERIVATION OF THE LAW OF
SINES. TODAY, WE WILL START WITH A REVIEW OF THE INVERSE
SINE-(ARCSINE) RELATION. THEN YOU WILL SEE A SITUATION
WHERE THE LAW OF SINES CAN BE USED.

WOW, IF YOU WILL PLEASE TYPE IN YOUR FIRST NAME,
WE CAN BEGIN.

NAME: ? MARY

MARY, YOU REMEMBER THAT THE SIX TRIGONOMETRIC FUNCTIONS
ARE FORMED BY COMPARING THE LENGTHS OF THE SIDES OF A RIGHT
TRIANGLE. IN PARTICULAR, THE SINE OF AN ANGLE IS THE RATIO OF
THE LENGTH OF THE OPPOSITE SIDE TO THE LENGTH OF THE?
(A) ADJACENT SIDE (B) HYPOTENUSE
(ANSWER: A OR B) ? B
CORRECT!

THE SINE RATIO IS WRITTEN AS EITHER A DECIMAL OR A FRACTION.
IT CAN NEVER HAVE A VALUE SMALLER THAN -1 OR LARGER THAN;
(ANSWER WITH A NUMBER.) ? 1
GOOD, MARY

(PRESS (ENTER) TO GO ON.)
BUT SOMETIMES, MARY, WE NEED TO FIND THE ANGLE THAT CORRESPONDS TO A PARTICULAR FRACTION (OR DECIMAL).

FOR EXAMPLE:

SUPPOSE A LOGGING COMPANY WANTS TO BUILD A ROAD UP A HILL. THE BUILDER DOES NOT WANT THE ROAD TO HAVE AN ANGLE STEEPER THAN 30 DEGREES. A SURVEYOR HAS MEASURED THE TRAIL AND DETERMINED THAT FOR EVERY 3 FEET ALONG THE HILL, THE LAND RISES 1 FOOT. IS THE LAND TOO STEEP TO USE?

WE WILL BEGIN WITH A PICTURE.

(PRESS (ENTER) TO GO ON.)

\[ \frac{3}{x} = \frac{1}{1} \]

WE NEED TO KNOW IF \( x < 30 \) DEGREES. WE DO KNOW THAT:

\[
\sin \text{ INCREASES IN THE FIRST QUADRANT} \\
\sin(x) = \frac{1}{3} = 0.333 \quad \text{AND} \quad \sin(30) = 0.5
\]

SINCE 0.333 < 0.5 WE KNOW THAT X < 30. (X = 19.5 DEGREES.)

THE LAND CAN BE USED.

TO SOLVE PROBLEMS LIKE THIS ONE, WE REALLY NEED TO USE THE INVERSE SINE RELATION.

(PRESS (ENTER) TO GO ON.)

\[ y = \arcsin(x) \]

NOTE THAT THE DOMAIN OF \( \arcsin(x) \) IS (-1,1) AND THE RANGE IS (-90,90).

(PRESS (ENTER) TO GO ON.)
BUT THE ANGLES OF A TRIANGLE MUST HAVE POSITIVE VALUES BETWEEN 0 AND 180 DEGREES. SO WE ONLY NEED A PORTION OF THE ARCSINE RELATION:
WHERE X IS BETWEEN 0 AND 1 AND Y IS BETWEEN 0 AND 180.

(PRESS (ENTER) TO GO ON.)?

NOTICE THAT FOR ANY VALUE OF X (EXCEPT 1), THERE ARE TWO VALUES OF Y THAT CORRESPOND TO X. FOR EXAMPLE: ARCSIN(.5) CROSSES THE GRAPH AT TWO PLACES. LOOKING AT THE Y-AXIS, WE CAN SEE THAT THE TWO ANGLES WHOSE SINE IS .5 ARE 30 AND 150 DEGREES.

(PRESS (ENTER) TO GO ON.)?

NOW, MARY, LET'S FIND ARCSIN(.7071)
THEN, ONE VALUE OF ARCSIN(.7071) IS ? 45 AND THE OTHER IS ? 135
GOOD, MARY

(PRESS (ENTER) TO GO ON.)?
Now it's your turn, Mary. After you press (Enter), I will clear the screen. Then you can choose any decimal number between 0 and 1 (such as .24), and I will plot the arcsine on the graph. I will then tell you the angles (between 0 and 180) whose sine is the number you chose. By typing (Go) instead of a number, you can go on to the next part of this lesson.

(Press (Enter) to go on.)?

![Graph showing arcsine values]

Type (Go) when you've seen enough.

Number: ? .83

\[ \text{ARCSIN(.83)} = 56 \text{ or } 124 \]

(Press (Enter) to go on.)?

Type (Go) when you've seen enough.

Number: ? Go.
Earlier, we said that the Law of Sines allows you to solve for the parts of any triangle (a side or an angle) even if the triangle does not contain a right angle. One way this law can be used is when two angles and one side of a triangle are known.

(PRESS (ENTER) TO GO ON.)

For example:

A

\[ \text{SA} \]

\[ \text{SC} \]

\[ 120 \]

\[ 10 \]

\[ B \]

\[ C \]

The Law of Sines may be used here.

The two known angles are \( A = 40 \) degrees and \( B = 120 \) degrees.

The known side is \( SC = 10 \)

(PRESS (ENTER) TO GO ON.)

A

\[ \text{SB} \]

\[ \text{SC} \]

\[ B \]

\[ \text{SA} \]

\[ C \]

The Law of Sines may be used when two angles and one side are known.

Given: \( A = 125 \) degrees \( C = 5 \) degrees \( SC = 15 \)

Can the missing parts be found using the Law of Sines?

(Answer Yes or NO.)? Yes

Good, Mary, let's do another one.

(PRESS (ENTER) TO GO ON.)
THE LAW OF SINES MAY BE USED WHEN TWO ANGLES AND ONE SIDE ARE KNOWN.

GIVEN: SA = 10  SC = 9  SB = 5
CAN THE MISSING PARTS BE FOUND USING THE LAW OF SINES?
(ANSWER YES OR NO.)  ? NO
GOOD, MARY, LET'S DO ANOTHER ONE.
(PRESS (ENTER) TO GO ON.)?

THE LAW OF SINES MAY BE USED WHEN TWO ANGLES AND ONE SIDE ARE KNOWN.

GIVEN: A = 154 DEGREES  C = 5 DEGREES  SC = 1
CAN THE MISSING PARTS BE FOUND USING THE LAW OF SINES?
(ANSWER YES OR NO.)  ? YES
GOOD, MARY, LET'S DO ANOTHER ONE.
(PRESS (ENTER) TO GO ON.)?

GIVEN: SA = 14  SC = 2  SB = 5
CAN THE MISSING PARTS BE FOUND USING THE LAW OF SINES?
(ANSWER YES OR NO.)  ? NO
GOOD, MARY, LET'S DO ANOTHER ONE.
(PRESS (ENTER) TO GO ON.)?
GIVEN: $A = 141$ DEGREES  $B = 3$ DEGREES  $C = 36$ DEGREES
CAN THE MISSING PARTS BE FOUND USING THE LAW OF SINES?
(ANSWER YES OR NO.)  NO
GOOD, MARY, LET'S DO ANOTHER ONE.

(PRESS (ENTER) TO GO ON.)?

GIVEN: $SA = 15$  $SC = 1$  $SB = 12$
CAN THE MISSING PARTS BE FOUND USING THE LAW OF SINES?
(ANSWER YES OR NO.)  NO

(PRESS (ENTER) TO GO ON.)?

MARY, LET'S FIND THE LENGTH OF SIDES SB AND SC
AND THE MEASURE OF ANGLE C.
HOW MANY DEGREES ARE IN A TRIANGLE?  $180$
RIGHT, MARY
SO, WHAT IS THE MEASURE OF ANGLE C?  $20$
GOOD, MARY

(PRESS (ENTER) TO GO ON.)?
NOW LET'S FIND THE LENGTH OF SIDE SB.

\[
\frac{\text{SA}}{\sin(A)} = \frac{\text{SB}}{\sin(B)}
\]
\[
10 / \sin(40) = \text{SB} / \sin(120)
\]
\[
10 / .6428 = \text{SB} / .8660
\]
\[
.6428 (\text{SB}) = (.8660)(10)
\]
\[
\text{SB} = 13.47
\]

(SUBSTITUTE GIVEN VALUES)
(SINE TABLE VALUES)
SIMPLIFY
(SOLVE FOR SB)
(PRESS (ENTER) TO GO ON.)?

LET'S FIND THE LENGTH OF SIDE SC.

\[
\frac{\text{SA}}{\sin(A)} = \frac{\text{SC}}{\sin(C)}
\]
\[
10 / \sin(40) = \text{SC} / \sin(20)
\]
\[
10 / .6428 = \text{SC} / .3420
\]
\[
.6428 (\text{SC}) = (10)(.3420)
\]
\[
\text{SC} = 5.32
\]

(SUBSTITUTE GIVEN VALUES)
(SINE TABLE VALUE)
SIMPLIFY
(SOLVE FOR SC)
(PRESS (ENTER) TO GO ON.)?

MARY, NOW IT'S TIME TO LET ME DO SOME WORK.
AFTER YOU PRESS (ENTER) I WILL SHOW YOU AN EXAMPLE OF HOW
YOU CAN TYPE IN THE NAMES AND MEASURES OF TWO ANGLES AND
ONE SIDE TO MAKE ME FIND THE OTHER THREE PARTS OF THE
TRIANGLE. WHEN YOU HAVE SEEN ENOUGH, TYPE (GO) TO CONTINUE
WITH THE NEXT PART OF THE LESSON.

(PRESS (ENTER) TO GO ON.)?
For example, you type in:

ANGLE NAME: A
MEASURE: 30
ANGLE NAME: B
MEASURE: 120
SIDE NAME: SB
LENGTH: 10

And I'll reply:

ANGLE: C = 30
SIDE: SB = 5.7734
SIDE SC = 5.7734

(PRESS (ENTER) TO GO ON.)

Type (GO) when you've seen enough.

ANGLE NAME: A
MEASURE: 45
ANGLE NAME: B
MEASURE: 32
SIDE NAME: SA
LENGTH: 7.39

ANGLE: C = 103
SIDE: SB = 5.9129
SIDE: SC = 10.8721

(PRESS (ENTER) TO GO ON.)

Type (GO) when you've seen enough.

ANGLE NAME: GO
TODAY, MARY, YOU SAW HOW TO USE THE LAW OF SINES TO FIND THE MISSING PARTS OF A TRIANGLE WHEN YOU WERE GIVEN TWO ANGLES AND ONE SIDE OF THE TRIANGLE. YOU ALSO SPENT SOME TIME REVIEWING THE ARCSINE RELATION.

TOMORROW, WE WILL CONTINUE TO SEE HOW THE LAW OF SINES CAN BE USED TO SOLVE FOR MISSING PARTS OF TRIANGLES WHEN YOU ARE GIVEN TWO SIDES AND AN ANGLE OPPOSITE ONE OF THEM. THEN YOU WILL SEE HOW THE ARCSINE RELATION IS USED TO FIND A MISSING ANGLE.

THIS CONCLUDES TODAY'S LESSON, MARY. I LOOK FORWARD TO SEEING YOU AGAIN TOMORROW.

READY

>
THE LAW OF SINES
(THE AMBIGUOUS CASE)

\[ \frac{SA}{\sin(A)} = \frac{SB}{\sin(B)} = \frac{SC}{\sin(C)} \]

(PRESS (ENTER) TO GO ON.)

In the first lesson, we examined the derivation of the law of sines. In the second lesson, we saw that one case involving the law occurred when two angles and one side of a triangle are given.

In this final lesson, we shall look at the case where two sides and an angle opposite one of them is given.

Please type in your first name.

NAME: MARY

You may recall from geometry that you can show two triangles are congruent if you are given two sides and the included angle:

SAS

(PRESS (ENTER) TO GO ON.)
BUT YOU CAN'T SHOW CONGRUENCY IF YOU ARE GIVEN TWO SIDES AND AN ANGLE OPPOSITE ONE OF THEM:

ASS:

\[ \begin{align*}
\text{SC} & \quad \text{C} \\
\text{SA} & \\
\end{align*} \]

(PRESS (ENTER) TO GO ON.)

YET THE LAW OF SINES CAN BE USED IN EITHER CASE. SINCE SOME OF THESE TRIANGLES DO NOT MEET THE REQUIREMENTS OF CONGRUENCY, THE TRIANGLE 'FOUND' BY THE LAW OF SINES MAY:

1. NOT EXIST
2. BE FORMED IN 2 WAYS
3. BE FORMED 1 WAY

THIS IS WHY THE LAW OF SINES IS SAID TO HAVE AN AMBIGUOUS CASE.

(PRESS (ENTER) TO GO ON.)

OBTUSE TRIANGLES

(PRESS (ENTER) TO GO ON.)
GIVEN:
ANGLE A ) 90 DEGREES
SIDE SC
SIDE SA

TO USE THE LAW OF SINES, WE MUST COMPARE THE
LENGTHS OF THE TWO GIVEN SIDES.
SUPPOSE THE SIDE OPPOSITE ANGLE A IS SHORT. (SA \neq SC)
CAN A TRIANGLE BE FORMED USING THESE THREE PARTS?
(ANSWER (YES) OR (NO))? NO
RIGHT, MARY

(PRESS (ENTER) TO GO ON.?)

GIVEN:
ANGLE A ) 90 DEGREES
SIDE SC
SIDE SA

SUPPOSE THE SIDE OPPOSITE THE ANGLE IS VERY LONG. (SA \equiv SC)
CAN A TRIANGLE BE FORMED USING THESE THREE PARTS?
(ANSWER (YES) OR (NO))? YES
GOOD, MARY

HOW MANY TRIANGLES CAN BE FOUND (BE CAREFUL) ? 1
GOOD, MARY

(PRESS (ENTER) TO GO ON.?)

SUMMARY

WHEN YOU ARE GIVEN AN OBTUSE ANGLE, ITS OPPOSITE SIDE
AND ONE OTHER SIDE, TWO POSSIBILITIES EXIST:

(1) THE SIDE OPPOSITE THE ANGLE IS SHORTER THAN THE OTHER SIDE.
HOW MANY TRIANGLES CAN BE FOUND? 0
GOOD, MARY

(2) THE SIDE OPPOSITE THE ANGLE IS LONGER THAN THE OTHER SIDE.
HOW MANY TRIANGLES CAN BE FOUND? 1
RIGHT, MARY

(PRESS (ENTER) TO GO ON.?)
WOULD YOU LIKE TO REVIEW THESE PICTURES BEFORE WE PRACTICE?

(ANSWER (YES) OR (NO))?

NO

I WILL DRAW AN OBTUSE ANGLE AND ITS OPPOSITE SIDE. I WILL TELL YOU THE LENGTH OF THE SIDE OPPOSITE THE OBTUSE ANGLE.

YOU TELL ME HOW MANY WAYS A TRIANGLE CAN BE FORMED USING THESE PARTS. THERE ARE 15 PROBLEMS AVAILABLE FOR YOU TO WORK; BUT YOU MAY GO ON TO THE NEXT PART OF THE LESSON AFTER YOU HAVE DONE 3 CORRECTLY — JUST TYPE (GO).

(PRESS (ENTER) TO GO ON.)?

ANGLE: A = 107 DEGREES
SIDE: SA = 10
SIDE: SC = 6

HOW MANY TRIANGLES CAN BE FOUND?
(ANSWER 0,1) ? 1

GOOD, MARY

(PRESS (ENTER) TO GO ON.)?
ACUTE TRIANGLES

(PRESS (ENTER) TO GO ON.)?

GIVEN:
ANGLE A
SIDE SC

IT IS A LITTLE MORE DIFFICULT TO DETERMINE HOW MANY
TRIANGLES CAN BE FORMED WHEN YOU ARE GIVEN AN ACUTE ANGLE RATHER
THAN AN OBTUSE ANGLE. THE LENGTH OF THE SIDE OPPOSITE THE GIVEN
ANGLE MUST BE COMPARED TO THE LENGTH OF THE ALTITUDE
OPPOSITE THAT ANGLE. LET'S FIND THE LENGTH OF THAT ALTITUDE IN
TERMS OF SIN(A).
\[ \sin(A) = \frac{H}{SC} \quad \text{OR} \quad H = (SC)\sin(A) \]

(PRESS (ENTER) TO GO ON.)?

GIVEN:
ANGLE A
SIDE SC
SIDE SA

ALTITUDE = (SC)\sin(A)

SUPPOSE THE SIDE OPPOSITE ANGLE A IS VERY SHORT
(SA < 4)
CAN A TRIANGLE BE FORMED USING THESE THREE PARTS
(ANSWER (YES) OR (NO) )? NO
RIGHT, MARY

(PRESS (ENTER) TO GO ON.)?
Given:
angle A
side SC
side SA

Altitude = (SC)sin(A)

Suppose the side opposite angle A is 'long'
(SB > SA > H)
Can a triangle be formed using these three parts?
(Answer (Yes) or (No))? Yes
Correct, Mary
How many triangles can be found? 2
Good, Mary

(Press (Enter) to go on.)

Given:
angle A
side SC
side SA

Altitude = (SC)sin(A)

Suppose the side opposite angle A has a length
equal to the distance from B to line AX, (SA = H)
Can a triangle be formed using these three parts?
(Answer (Yes) or (No))? Yes
Right, Mary
How many triangles can be found? 1
Good, Mary

(Press (Enter) to go on.)

Given:
angle A
side SC
side SA

Altitude = (SC)sin(A)

Suppose the side opposite angle A is 'very long' (SA > SC)
Can a triangle be formed using these three parts?
(Answer (Yes) or (No))? Yes
Right, Mary
How many triangles can be found? 1
Good, Mary

(Press (Enter) to go on.)
SUMMARY

WHEN YOU ARE GIVEN AN ACUTE ANGLE, ITS OPPOSITE SIDE, AND ANOTHER SIDE; AND ASKED TO FORM A TRIANGLE, THREE CASES EXIST:

1. THE SIDE OPPOSITE THE ACUTE ANGLE IS SHORTER THAN THE ALTITUDE. HOW MANY TRIANGLES CAN BE FOUND? 0
   GOOD, MARY

2. THE SIDE OPPOSITE THE ANGLE IS LONGER THAN THE ALTITUDE BUT SHORTER THAN THE OTHER GIVEN SIDE. HOW MANY TRIANGLES CAN BE FOUND? 2
   CORRECT, MARY

3. THE SIDE OPPOSITE THE ANGLE IS EQUAL TO THE ALTITUDE OR LONGER THAN THE OTHER GIVEN SIDE. HOW MANY TRIANGLES CAN BE FOUND? 1
   RIGHT, MARY

(PRESS (ENTER) TO GO ON.)?

WOULD YOU LIKE TO REVIEW THESE PICTURES BEFORE WE PRACTICE? (ANSWER (YES) OR (NO)). NO.

I WILL DRAW AN ACUTE ANGLE, ONE OF ITS ADJACENT SIDES, AND AN ALTITUDE. I WILL TELL YOU THE LENGTH OF THE SIDE OPPOSITE THE ACUTE ANGLE.

YOU TELL ME HOW MANY WAYS A TRIANGLE CAN BE FORMED USING THESE PARTS. THERE ARE 15 PROBLEMS AVAILABLE FOR YOU TO WORK; BUT YOU MAY GO ON TO THE NEXT PART OF THE LESSON AFTER YOU HAVE DONE 3 CORRECTLY -- JUST TYPE (GO).

(PRESS (ENTER) TO GO ON.)?
ANGLE: C = 15 DEGREES
SIDE: SC = 7
SIDE: SB = 2
H = (2) SIN(15) = 0.517638

MARY, HOW MANY TRIANGLES CAN BE FOUND? 1
GOOD, MARY

(PRESS (ENTER) TO GO ON.)?

IF YOU WISH, YOU CAN HAVE ME CALCULATE THE
NUMBER OF POSSIBLE TRIANGLES. TYPE IN THE MEASURE OF ANY ANGLE
(IN DEGREES) AND THE LENGTHS OF AN ADJACENT AND OPPOSITE SIDE.

TYPE (GO) WHEN YOU HAVE SEEN ENOUGH.

(PRESS (ENTER) TO GO ON.)?

TYPE (GO) WHEN YOU'VE SEEN ENOUGH.

ANGLE: ? 35
ADJACENT SIDE: ? 3.786
OPPOSITE SIDE: ? 49.97

NUMBER OF POSSIBLE TRIANGLES: 1

(PRESS (ENTER) TO GO ON.)?
NOW LET'S WORK THROUGH AN EXAMPLE

ANGLE $B = 30^\circ$  SIDE $SB = 4$  SIDE $SC = 6$

SOLVE FOR THE REMAINING PARTS OF TRIANGLE $ABC$.

(1) DETERMINE THE NUMBER OF POSSIBLE TRIANGLES.

\[ \sin(30^\circ) = \sin(45^\circ) = \frac{1}{2} \]

THE OPPOSITE SIDE IS LARGER THAN THE ALTITUDE BUT SMALLER THAN THE ADJACENT SIDE—2 TRIANGLES ARE POSSIBLE.

(PRESS (ENTER) TO GO ON.)

\( \angle 1 \) \hspace{2cm} \( \angle 2 \)

(2) DRAW THE TWO POSSIBLE FIGURES.

(PRESS (ENTER) TO GO ON.)

\( \angle 1 \) \hspace{2cm} \( \angle 2 \)

(3) SOLVE FOR ANGLE $C$ USING THE LAW OF SINES.

\[ \frac{SB}{\sin(B)} = \frac{SC}{\sin(C)} \]

\[ \frac{4}{1/2} = \frac{6}{\sin(C)} \]

\[ \sin(C) = \frac{1}{2} \]

\[ C = \arcsin(0.5) \]

\[ C = 30^\circ \hspace{2cm} 60^\circ \]

(PRESS (ENTER) TO GO ON.)
(4) FIND ANGLE A FOR EACH POSSIBLE TRIANGLE.

180 - 30 - 48.5 = 101.5
180 - 30 - 131.5 = 18.5

(PRESS (ENTER) TO GO ON.)

(5) SOLVE FOR SIDE SA IN EACH TRIANGLE USING THE LAW OF SINES.

\[ \frac{a}{\sin(B)} = \frac{b}{\sin(A)} \]

\[ \frac{4}{\sin(30)} = \frac{a}{\sin(101.5)} \]

\[ a = \frac{4 \times \sin(101.5)}{\sin(30)} \]

\[ a = 4.9739 \times 2 \]

\[ a = 7.9518 \]

\[ \frac{4}{\sin(30)} = \frac{a}{\sin(18.5)} \]

\[ a = \frac{4 \times \sin(18.5)}{\sin(30)} \]

\[ a = 4 \times \frac{0.3173}{0.5} \]

\[ a = 2.537 \]

(PRESS (ENTER) TO GO ON.)

THAT WAS A LOT OF WORK, WASN'T IT? WHY DON'T YOU LET ME WORK THE NEXT FEW PROBLEMS FOR YOU? TYPE IN ANY SIZE ANGLE (IN DEGREES), THE LENGTH OF THE SIDE OPPOSITE THE ANGLE, AND THE LENGTH OF A SIDE ADJACENT TO THE ANGLE. THEN I WILL TELL YOU HOW MANY TRIANGLES ARE POSSIBLE AND SOLVE FOR ANY MISSING PARTS. WHEN YOU HAVE SEEN ENOUGH TYPE (30).

(PRESS (ENTER) TO GO ON.)
TYPE (GO) WHEN YOU'VE SEEN ENOUGH.

ANGLE: ? 3.76

ADJACENT SIDE: ? 123

OPPOSITE SIDE: ? .876

NO TRIANGLE IS POSSIBLE.

(PRESS (ENTER) TO GO ON.)
APPENDIX G

STATISTICS FOR CAI TIMES
TABLE 1
MEAN AND STANDARD DEVIATION FOR THE CAI LESSONS

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson I</td>
<td>10.0</td>
<td>0</td>
</tr>
<tr>
<td>Lesson II</td>
<td>14.4</td>
<td>9.23</td>
</tr>
<tr>
<td>Lesson III</td>
<td>26.3</td>
<td>15.58</td>
</tr>
</tbody>
</table>
APPENDIX H

MAIN EXPERIMENT MATERIALS
Purpose:

The purpose of this experiment is to show that the composition of a student's homework assignment will affect his mastery of a topic. In particular, if two students take the same test on the Law of Sines, the child who has written computer programs involving this topic will score significantly better on the test.
Procedure:

(1) Divide the trigonometry students who are also taking computer science into three groups:

   Group A - computer programmers
   Group B - computer users
   Group C - control

   (For ease of teaching, the remainder of the class may be treated as members of Group A.)

(2) All students are presented material at the same time, in the same way, for the same length of time.

(3) Assign homework as specified.

(4) Give the test.

(5) Assemble all materials. I will pick them up the day the test is given.
Length of experiment:

I anticipate a five day experiment.

Day 1: Development (derivation) of the Law of Sines. Presentation of the non-ambiguous case.

Day 2: Continuation of the non-ambiguous case. Presentation of the ambiguous case.

Day 3: Continuation of the ambiguous case.

Day 4: Review

Day 5: Test

Homework will be assigned on days 1-3. Of course this is meant only as a guide. If your class requires a different schedule, homework assignments can be adjusted accordingly.
Group A - Computer Programmers

The core of their homework assignments is the enclosed list of computer programs. Each child in this group should write and run three of the four programs. You may assign three particular programs or give the student all four and let him choose the three he wishes to do.

I recommend that all three programs be assigned the first day and be due the day of the test.

The average child should require about 90 minutes to write and run three of these programs.

Group B - Computer Users

The core assignments for this group are the computer-aided instruction (CAI) lessons on the enclosed cassette tapes. These lessons are designed to be used sequentially, after each day's class presentation. The lessons are self-contained; that is, you will not need to supervise the student as he works.

The average child should spend a total of 50 minutes on these lessons.

Lesson I : 10 minutes
Lesson II : 14.4 minutes
Lesson III : 26.3 minutes
Group C - Control

The homework problems given this group are meant to balance, in a time sense, the work done by the other two groups.

A child should spend an average of 7 minutes on a drill problem and 8.5 minutes on a word problem.
Sample Homework Assignment (Total)

<table>
<thead>
<tr>
<th>Group</th>
<th>Task Description</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group A</td>
<td>3 programs</td>
<td>90</td>
</tr>
<tr>
<td>Group B</td>
<td>CAI lessons</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>3 word problems</td>
<td>25.5</td>
</tr>
<tr>
<td></td>
<td>@ 8.5 min.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 drill problems</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>@ 7 min.</td>
<td>89.5</td>
</tr>
<tr>
<td>Group C</td>
<td>6 word @ 8.5 min.</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>6 drill @ 7 min.</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>93</td>
</tr>
</tbody>
</table>

This sample will cause a student to spend approximately the same length of time on homework regardless of the group to which he is assigned.

Of course, you may wish to make the homework assignment longer or change the number of specific problems assigned. That is no problem so long as the three day time totals remain approximately the same.

Since the students will be asked to keep track of the time they spend on their assignments, the statistical analysis will be able to compensate for any inequalities of time.
COMPUTER PROGRAMS

For any positive value of \( w < 1 \) the equation:
\[
\sin(x) = w
\]
has two solutions between 0 and 180 degrees. Write a program to find both of these values for a given \( w \).

Given two angles and one side of a triangle, write a program to find the other two sides.

Given two sides of a triangle and an angle opposite one of them, write a program to find the other two angles.

Given two sides and an angle opposite one of them, write a program to determine if none, one or two triangles can exist.

Note: \( \arcsin(x) = \arctan\left(\frac{x}{\sqrt{1-x^2}}\right) \)
The purpose of this series of computer lessons is to reinforce your understanding of the Law of Sines. Please read this handout BEFORE you begin the lesson. It describes the notation used throughout the series and contains directions for loading the tapes.

The machine does not draw a smooth diagonal line. (Vertical and horizontal lines are no problem.) You may need to use your imagination to visualize this:

![Diagram of a triangle]

as a triangle.

The following notation is used:

Angles are denoted by the capital letters: A,B,C
Sides are labeled: D,E,F or SA,SB,SC (when it is important to designate a side opposite a given angle.)

example:

```
      A
     / |
    /  |
   /   |
   E   F
B-----D-----C
```

angles: A,B,C
sides: D,E,F

```
      A
     / |
    /  |
   /   |
     SC
B-----SA-----C
     SB
```

angles: A,B,C
sides: SA,SB,SC
side SA is opposite angle A
side SB is opposite angle B
side SC is opposite angle C

Note: Sometimes you will be asked a question. There is a difference between the number 0 and the letter O. Be careful to use the proper character in your answers.
LESSON I:

This first lesson shows a derivation of the Law of Sines. It begins with a review of the parts of a right triangle and the formation of the sine function. This should be familiar material, and you will be asked to demonstrate your knowledge. However, if you answer incorrectly too many times, the lesson will be stopped and you will be advised to get extra help before attempting the lesson again.

Then the Law of Sines is derived using an acute triangle. If you want, it will demonstrate a derivation using an obtuse triangle. The lesson concludes with an example showing how the Law of Sines can be used.

How to Begin:

(1) Turn the computer on. The on/off switch is under the right-hand edge of the machine.

(2) The word MEMORY ? will appear. Press <ENTER>

(3) Find the cassette tape marked:
   LAW OF SINES
   LESSON I / LESSON II

(4) Put the cassette into the tape recorder so that side A is up. Check to see that the volume control is on 6.

(5) Rewind the tape to the beginning, it it is not there.

(6) Type: CLOAD "l"
    Press: <ENTER>

(7) Within ten seconds, two stars will appear in the
upper right hand corner of the computer screen. The right one will flash. When it stops flashing, the computer will print: READY. (Be patient. It takes a while.)

(8) Press the STOP button on the tape recorder.

(9) Type: RUN
    Press: <ENTER>

(10) Enjoy the lesson.

How to Stop:

    At the end of the lesson, the computer will say goodbye and then print: READY

(1) Rewind the tape. Turn off the computer.

(2) Put the cassette back into its plastic box.

(3) Return the tape to your teacher.

(4) Go back to class and tell everyone (later) what a wonderful time you had.
LESSON II:

The main purpose of the second lesson is to examine the non-ambiguous case of the Law of Sines.

It begins with a review of the inverse sine relation. Then the non-ambiguous case is presented and drilled.

You may find it instructive to bring your homework with you. There are two sections in this lesson which allow you to enter information. In the first, the machine will tell you the two angles (between 0 and 180, rounded to the nearest degree) whose sine is the number you choose. In the second, after you type in two angles and one side, the machine will solve for the missing parts of the triangle.

Note: You cannot enter minutes directly but must convert them to hundredths of a degree. (Remember there are 60 minutes in a degree.)

For example:  
60°15' = 60.25°  
123°40' = 123.66°

How to Begin:

(1) Turn the computer on. The on/off switch is under the right-hand edge of the machine.

(2) The word MEMORY ? will appear. Press <ENTER>.

(3) Find the cassette tape marked:  
LAW OF SINES  
LESSON I / LESSON II

(4) Put the cassette into the tape recorder so that side B is up. Check to see that the volume control is on 6.

(5) Rewind the tape to the beginning, if it is not there.
(6) Type: CLOAD "2"
   Press: <ENTER>

(7) Within ten seconds, two stars will appear in the upper right hand corner of the computer screen. The right one will flash. When it stops flashing, the computer will print: READY. (Be patient. It takes a while.)

(8) Press the STOP button on the tape recorder.

(9) Type: RUN
   Press: <ENTER>

(10) Enjoy the lesson.

How to Stop:

At the end of the lesson, the computer will say goodbye and then print: READY

(1) Rewind the tape. Turn off the computer.
(2) Put the cassette back into its plastic box.
(3) Return the tape to your teacher.
(4) Go back to class and tell everyone (later) what a wonderful time you had.
LESSON III:

The last lesson of the series develops the ambiguous case of the Law of Sines. Obtuse and acute angles are considered separately. After an explanation of how to determine the number of possible triangles, drill is provided. An example involving the ambiguous case is worked out completely.

Again there are two sections of the lesson which allow you to enter information. (Bring your homework!) In the first, after you type in two sides and one angle, the machine computes the number of possible triangles. In the second, it not only computes the number of possible triangles, but also solves for the missing parts.

Note: You cannot enter minutes directly but must convert them to hundredths of a degree. (Remember there are 60 minutes in a degree.)
For example: \[60^\circ 15' = 60.25^\circ\]
\[123^\circ 40' = 123.6\overline{6}^\circ\]

How to Begin:

(1) Turn the computer on. The on/off switch is under the right-hand edge of the machine.

(2) The word MEMORY ? will appear. Press \textless \text{ENTER\textgreater}\)

(3) Find the cassette tape marked:
\text{LAW OF SINES}
\text{LESSON III}

(4) Put the cassette into the tape recorder so that side A is up. Check to see that the volume control is on 6.
(5) Rewind the tape to the beginning, if it is not there.

(6) Type: CLOAD "3"
    Press: <ENTER>

(7) Within ten seconds, two stars will appear in the upper right hand corner of the computer screen. The right one will flash. When it stops flashing, the computer will print: READY. (Be patient. It takes a while.)

(8) Press the STOP button on the tape recorder.

(9) Type: RUN
    Press: <ENTER>

(10) Enjoy the lesson.

How to Stop:

At the end of the lesson, the computer will say goodbye and then print: READY

(1) Rewind the tape. Turn off the computer.
(2) Put the cassette back into its plastic box.
(3) Return the tape to your teacher.
(4) Go back to class and tell everyone (later) what a wonderful time you had.
EXTRA HOMEWORK PROBLEMS

Drill:

1) \( \sin(C) = 0.3 \) \( \sin(B) = 0.6 \) \( c=12 \) find \( b \)

2) \( a=10 \) \( \sin(A) = 3/4 \) \( \sin(B) = 4/5 \) find \( b \)

3) \( \sin(A) = 2/3 \) \( \sin(C) = 5/6 \) \( a=12 \) find \( c \)

Solve for the remaining parts of triangle ABC.

4) \( a=36 \) \( A=138^\circ \) \( B=17^\circ \)

5) \( A=60^\circ \) \( B=40^\circ \) \( b=5 \)

6) \( B=75^\circ 40' \) \( C=52^\circ 30' \) \( c=29.80 \)

7) \( A=34^\circ 20' \) \( b=52.5 \) \( C=54^\circ \)

8) \( a=28.34 \) \( B=71^\circ 30' \) \( C=46^\circ 40' \)

9) \( A=50^\circ \) \( B=55^\circ \) \( c=25 \)

10) \( B=125^\circ 10' \) \( c=6 \) \( b=12 \)

11) \( C=131^\circ \) \( a=19 \) \( c=17 \)

12) \( A=125^\circ \) \( a=30 \) \( b=40 \)

13) \( b=6 \) \( a=2 \) \( A=37^\circ \)

14) \( c=6 \) \( a=10 \) \( C=30^\circ \)

15) \( B=21^\circ 40' \) \( a=5 \) \( b=8.4 \)

16) \( C=21^\circ 50' \) \( a=10 \) \( c=3.7 \)

17) \( C=75^\circ 10' \) \( c=14.7 \) \( a=8.2 \)

18) \( a=33.7 \) \( b=52 \) \( A=21^\circ 30' \)

Word Problems:

19) If \( c \), \( C \) and \( B \) are to be parts of triangle ABC, determine the value of \( c \) for which \( B \) has two values if \( b=6 \) and \( C=30^\circ \)

20) If \( a \), \( A \) and \( C \) are to be parts of triangle ABC, determine the value of \( a \) for which \( C \) has one value if \( c=12 \) and \( A=135^\circ \)
21) If b, B and A are to be parts of triangle ABC, determine the value of b for which A has no value (no such triangle exists) if a=20 and B=120°

22) While on opposite ends of a 7.4 mile beach, two men saw a ship. If the lines of sight from the men to the ship were 59° and 40° respectively, how far was the ship from the nearer man?

23) A tower 100 feet tall is located at the top of a hill. At a point 500 feet down the hill, the angle between the surface of the hill and the line of sight to the top of the tower is 10°. Find the inclination of the hill to a horizontal plane.

24) A boat is steaming northeast at 15 mph. At 2 P.M. a lighthouse bears N10°W and at 4 P.M. it bears S58°W. How far was the boat from the lighthouse at 4 P.M.?

25) A surveyor who was running a line due north avoided an impassable area by moving on bearing N42°20'E for 315 feet and then on bearing N38°30'W. How far must he move on the last mentioned bearing in order to reach a point on the original north line extended?

26) In triangle ABC, A=62°20' B=41°40' and BC=7.41. Find the length of the bisector of angle C.

27) ABC is an equilateral triangle whose side is 18 inches long. Lines AD and AE are drawn trisecting angle A and intersecting side BC is points D and E. Find the lengths of segments BD, DE, EC.

28) If the approximate distances from the Sun (S) to Earth (E) and Venus (V) were 9.3 x 10^7 miles and 6.7 x 10^7 miles, respectively, when angle EVS measured 41° how far was it from earth to Venus?

29) A playground is in the shape of an isosceles triangle. The base has a length of 56 feet. If the legs meet at an angle of 62° how long are they?

30) The radius of a circle is 25 inches. Find the length of a chord of the circle whose central angle is 72°

31) The distance between the two points B and C cannot be measured directly, but it is known to be less than a mile. The distance from point A to point B is 6340 feet. The distance from A to C is 4520 feet and angle ABC is 34°20'. Find BC.
32) A gun having an elevation of 30° has a range of 8000 yards. Assuming the trajectory of a shell to be very nearly an isosceles triangle, what is the altitude of the shell at its highest point and what is the total distance it travels?

33) Radio station A is 130 miles due north of station B. Station A receives a distress message from a ship at a bearing of 130° while station B receives the same message at a bearing of 47° (Bearings are measured in a clockwise direction from the north-south line.) How long would a helicopter flying at 110 mph take to reach the ship from station A?

34) A point of land is located 20 miles NE of a dock. A ship leaves the dock at 10 A.M. traveling east at 12 mph. At what time is the ship 15 miles from the point?

35) Prove:

\[ D = \frac{a}{\sin(A)} \]

where \( D \) = diameter of circle \( O \)

(Hint: \( A = \text{BOY} \)
\[ \frac{1}{2}a = (r)\sin(\text{BOY}) \]

where \( r \) = radius
<table>
<thead>
<tr>
<th>Activity</th>
<th>Starting Time</th>
<th>Ending Time</th>
</tr>
</thead>
</table>

Student Log
TEST

Name: ____________________________

Directions: Show all work on this paper. (Use back if necessary.) Solve all lengths to the nearest tenth and all angles to the nearest 10 minutes.

1) Find the remaining parts of triangle ABC if $a=28$, $B=71^\circ \ 30'$ and $C=46^\circ \ 40'$

\[
\begin{align*}
\frac{179^\circ \ 60'}{11^\circ \ 10'} &\quad \frac{28}{\sin (111^\circ \ 50')} = \frac{b}{\sin (71^\circ \ 30')} = \frac{c}{\sin (46^\circ \ 40')} \\
A &= 61^\circ \ 50' \quad (1 \ pt.) \\
b &= 30.1 \quad (2 \ pts.) \\
c &= 23.1 \quad (2 \ pts.) \\
\end{align*}
\]

2) Find the remaining parts of triangle ABC if $C=131^\circ$, $a=19$ and $c=17$

\[
\begin{align*}
\text{17} &\times \text{19} \\
\phi &\quad (5 \ pts.)
\end{align*}
\]

3) Given a triangle ABC where $a=70$ and angle $B=30^\circ$, determine the values of $b$ for which $A$ has (a) no value (b) one value (c) two values.

\[
\begin{align*}
h = \frac{35}{\sin (30^\circ)} \\
\sin (30^\circ) &= \frac{h}{a} \\
a) \ b < 35^\circ \quad (2 \ pts.) \\
b) \ b > 70 \quad (2 \ pts.) \\
c) \ 35^\circ < b < 70 \quad (2 \ pts.)
\end{align*}
\]

\[
\begin{align*}
a) \ b < h \\
b) \ h = b \\
c) \ h < b < a \\
\end{align*}
\]

\[
\begin{align*}
h > a \\
\end{align*}
\]
4) From a point some distance away from the base of an oak tree, a man measures the angle of elevation of the top of the tree as $37^\circ$. Walking 153 feet further on, he measures the angle of elevation as $28^\circ$. Find the height of the tree.

\[
\frac{153}{\sin(28^\circ)} = \frac{y}{\sin(37^\circ)}
\]

\[y = 459.16\]

\[\sin(37^\circ) = \frac{x}{459.16}\]

\[x = 276.3\]

---

5) Steaming down the center of a channel, a ship sights two buoys ahead of it, one on either side. The buoys are 800 yards apart and the distance from the ship to the farther buoy is 1200 yards. How far is the ship from the nearer buoy if the angle between the lines of sight to the two buoys is $35^\circ$?

This uses the ambiguous case. Please give no hints. I need to see who works this correctly.

\[\frac{800}{\sin(35^\circ)} = \frac{1200}{\sin(85^\circ)}\]

\[x = 575.2\]

Not correct.
APPENDIX I

STATISTICS FOR TEST SCORES

(ANALYSIS OF COVARIANCE)
<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max. Value</th>
<th>Sum Variance</th>
<th>Sum Value</th>
<th>C.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>6</td>
<td>19.833</td>
<td>8.134</td>
<td>28.000</td>
<td>119.000</td>
<td>66.167</td>
<td>41.013</td>
</tr>
<tr>
<td>Group 2</td>
<td>4</td>
<td>26.250</td>
<td>7.932</td>
<td>33.000</td>
<td>105.000</td>
<td>62.917</td>
<td>30.217</td>
</tr>
<tr>
<td>Group 3</td>
<td>4</td>
<td>22.500</td>
<td>12.234</td>
<td>39.000</td>
<td>90.000</td>
<td>149.667</td>
<td>54.373</td>
</tr>
</tbody>
</table>
### TABLE 2

**ANALYSIS OF COVARIANCE**  
**TOTAL TEST SCORE**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Sum of Square</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5</td>
<td>525.32711042</td>
<td>105.06542208</td>
<td>1.55</td>
<td>0.2766</td>
</tr>
<tr>
<td>Error</td>
<td>8</td>
<td>542.10146101</td>
<td>67.76268263</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>13</td>
<td>1057.42857143</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

R-Square 0.492143

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<th>Type I SS</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>1</td>
<td>52.68434207</td>
<td>0.78</td>
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<td>Group</td>
<td>2</td>
<td>172.40915132</td>
<td>1.27</td>
<td>0.3314</td>
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<tr>
<td>Time*Group</td>
<td>2</td>
<td>300.23361703</td>
<td>2.22</td>
<td>0.1715</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
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<tr>
<td>Time</td>
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<td>113.87080166</td>
<td>1.36</td>
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</tr>
<tr>
<td>Group</td>
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<td>421.58044930</td>
<td>3.11</td>
<td>0.1001</td>
</tr>
<tr>
<td>Time*Group</td>
<td>2</td>
<td>300.23361703</td>
<td>2.22</td>
<td>0.1715</td>
</tr>
<tr>
<td>-------</td>
<td>----</td>
<td>-------</td>
<td>-----------</td>
<td>------------</td>
</tr>
<tr>
<td>1</td>
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<td>4.367</td>
<td>6.000</td>
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<tr>
<td>2</td>
<td>4</td>
<td>13.250</td>
<td>3.202</td>
<td>10.000</td>
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<tr>
<td>3</td>
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<td>12.750</td>
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<td>5.000</td>
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<td>F Value</td>
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<tr>
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<td>---------------------</td>
<td>-----------------</td>
<td>---------</td>
</tr>
<tr>
<td>Model</td>
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<td>123.71823477</td>
<td>24.74364695</td>
<td>1.37</td>
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<tr>
<td>Error</td>
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<td>18.03522065</td>
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<td>258.00000000</td>
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<th>Drill Mean</th>
<th>C.V.</th>
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<tr>
<td>0.461635</td>
<td>4.24678945</td>
<td>12.00000000</td>
<td>35.3899</td>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
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<td>Group</td>
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<th>Pr &gt; F</th>
</tr>
</thead>
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<tr>
<td>Group</td>
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<td>Time*Group</td>
<td>2</td>
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<td>2.01</td>
<td>0.1966</td>
</tr>
<tr>
<td>Group</td>
<td>N</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Min. Value</td>
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<tr>
<td>1</td>
<td>6</td>
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<td>1.000</td>
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<td>4</td>
<td>13.000</td>
<td>7.394</td>
<td>6.000</td>
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<tr>
<td>3</td>
<td>4</td>
<td>9.750</td>
<td>7.500</td>
<td>6.000</td>
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### TABLE 6

**ANALYSIS OF COVARIANCE**

**WORD PROBLEM, SCORE**

<table>
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<tr>
<th>Source</th>
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<tr>
<td>Model</td>
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<td>335.95508120</td>
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<tr>
<td>Corrected Total</td>
<td>13</td>
<td>529.42857143</td>
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<table>
<thead>
<tr>
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<th>Word Mean</th>
<th>C.V.</th>
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<tbody>
<tr>
<td>0.365438</td>
<td>6.4803749</td>
<td>10.42857143</td>
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</tbody>
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<tr>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
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<td>24.67283127</td>
<td>0.59</td>
<td>0.4654</td>
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<tr>
<td>Group</td>
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<td>44.15991467</td>
<td>0.53</td>
<td>0.6102</td>
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<tr>
<td>Time*Group</td>
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<td>124.64074428</td>
<td>1.48</td>
<td>0.2830</td>
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</table>

<table>
<thead>
<tr>
<th>Source</th>
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<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
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<td>0.2830</td>
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</table>

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124
APPENDIX J
STATISTICS FOR TEST SCORES
(NON-PARAMETRIC ANALYSIS)
<table>
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<tr>
<th></th>
<th>Total</th>
<th>Drill</th>
<th>Word</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std. dev.</td>
<td>mean</td>
</tr>
<tr>
<td>Group A</td>
<td>22.50</td>
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<td>12.75</td>
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<tr>
<td>(programmers)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group B</td>
<td>26.25</td>
<td>7.93</td>
<td>13.25</td>
</tr>
<tr>
<td>(users)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group C</td>
<td>19.83</td>
<td>8.13</td>
<td>10.67</td>
</tr>
<tr>
<td>(control)</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2**

**KRUSKALL-WALLIS H TEST**
(rank sum for each group)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>H-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
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<td>1.39</td>
</tr>
<tr>
<td>Drill</td>
<td>32.5</td>
<td>34.5</td>
<td>38</td>
<td>0.8452</td>
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<tr>
<td>Word</td>
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<td>36</td>
<td>44</td>
<td>0.8810</td>
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\[ \chi^2 = 5.99147 \]
APPENDIX K

STATISTICS FOR HOMEWORK TIMES
TABLE 1
TIME COMPARISONS

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<tr>
<th></th>
<th>Expected</th>
<th>Actual</th>
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<tr>
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<td>209.5</td>
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<tr>
<td>Group B</td>
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<tr>
<td>Group C</td>
<td>133.347</td>
<td>111.5</td>
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</table>

TABLE 2
KRUSKALL-WALLIS H TEST
(rank sum for each group)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>H-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>21</td>
<td>34</td>
<td>8.024</td>
</tr>
</tbody>
</table>

TABLE 3
WILCOXON RANK SUM

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>34</td>
</tr>
</tbody>
</table>

\[ T=21 \quad T_L=12 \quad T_U=32 \]
FOOTNOTES


SELECTED BIBLIOGRAPHY


Bell, Frederick H. "Why is Computer Related Learning So Successful?" Educational Technology 14 (December 1974): 15-18


Schloss, Lisa, and Ball, Leslie D. "Computerized Education: Should We or Shouldn't We?" AEDS Monitor 20 (July/August/September 1981):18-21.


