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Efficient image reconstruction algorithm for the circle and arc cone beam computer tomography

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Methods, systems and processes for providing efficient, accurate and exact image reconstruction using portable and easy to use C-arm scanning devices and rotating gantries, and the like, that combines both a circle and a curve scan. The invention can provide exact convolution-based filtered back projection (FBP) image reconstruction by combining two curved scans of the object. The curved scan can be less than or greater than a full circle about an object being scanned. The invention can be done by a first curve within a first plane followed by a second curve within a second plane that is transversal to the first plane.
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Fig. 1

C-arm

x-ray tube

Patient Table

detector array

image reconstruction computer

display
**Step 10.** Load the current CB (cone beam) projection into computer memory. Suppose that the mid point of the CB projections currently stored in memory is $y(s_0)$.

**Steps 20 and 30.** Depending on where the x-ray source is located (on $C_1$ or on $C_2$), choose an appropriate family of filtering lines. Then, choose a discrete set of lines from the family.

**Step 40.** Preparing for filtering. Parameterize points on the said filtering lines selected in Steps 20 and 30 by polar angle $\gamma$. Compute the derivative of the CB data $(\partial / \partial q)D_y(y(q), \beta)|_{q=s}$ at points $\beta$ on the said lines that correspond to discrete values of the polar angle.

**Step 50.** Filtering. For each filtering line convolve the data for that line computed in Step 40 with filter $1 / \sin \gamma$.

**Step 60.** Back-projection. For each reconstruction point $x$, back-project the filtered data found in Step 50 according to equation (10). Then go to Step 10, unless there are no new CB projections to process or image reconstruction at all the required points $x$ have been completed.

**Reconstructed image**
Fig. 6

![Diagram showing a region of interest (ROI) with backprojection filtering line and a backprojection region]

Fig. 7

![Diagram showing a region of interest (ROI) with backprojection filtering line and a backprojection region]
Fig. 9

**Step 30.** Finding families of lines for filtering.

It is assumed the x-ray source is located on the arc $C_2$.

**Step 31.** Choose a discrete set of values of the parameter $s_i$ inside the interval $[s_i^{\text{min}}, s_i^{\text{max}}]$.

**Step 32.** For each $s_i$ chosen in Step 31 find a line tangent to the projected circle $\hat{C}_i$.

**Step 33.** The collection of lines constructed in Step 32 is the required set of lines.

**Step 40**
**Step 40. Preparation for filtering**

**Step 41.** If the x-ray source is located on the circle $C_1$, fix a filtering line $l_{flt} \in L_1$. If the x-ray source is on the arc $C_2$, fix $l_{flt} \in L_2$.

**Step 42.** Parameterize points on the said line by polar angle $\gamma$ in the plane through $y(s_0)$ and $l_{flt}$.

**Step 43.** Choose a discrete set of equidistant values $\gamma_j$ that will be used later for discrete filtering in Step 50.

**Step 44.** For each $\gamma_j$ find the unit vector $\beta_j$ which points from $y(s_0)$ towards the point on $l_{flt}$ that corresponds to $\gamma_j$.

**Step 45.** Using the CB projection data $D_f(y(q), \Theta)$ for a few values of $q$ close to $s_0$ find numerically the derivative 

$$(\partial / \partial q)D_f(y(q), \Theta) |_{q=s_0} \text{ for all } \Theta = \beta_j$$

**Step 46.** Store the computed values of the derivative in computer memory.

**Step 47.** Repeat Steps 41-46 for all lines $l_{flt}$ identified in Steps 20 and 30. This way we will create the processed CB data $\Psi(s_0, \beta_j)$
**Step 50. Filtering**

Step 51. Fix a filtering line $l_{fl}$.

- If the x-ray source is located on the circle $C_1$, take $l_{fl} \in L_1$.
- If the x-ray source is located on the arc $C_2$, take $l_{fl} \in L_2$.

Step 52. Compute FFT of the values of the said processed CB data computed in Step 40 along the said line.

Step 53. Compute FFT of the filter $1/\sin \gamma$.

Step 54. Multiply FFT of the filter $1/\sin \gamma$ (the result of Step 53) and FFT of the values of the said processed CB data (the result of Step 52).

Step 55. Take the inverse FFT of the result of Step 54.

Step 56. Store the result of Step 55 in computer memory.

Step 57. Repeat Steps 51-56 for all lines in the said family of lines. This will give the filtered CB data $\Phi(s_0, \beta_j)$.

**Step 60**
Step 60. Back-projection

Step 61. Fix a reconstruction point \( x \), which represents a point inside the patient where it is required to reconstruct the image.

Step 62. If \( s_0 \in I_1(x) \cup I_2(x) \), then the said filtered CB data affects the image at \( x \) and one performs Steps 63-68. If \( s_0 \not\in I_1(x) \cup I_2(x) \), then the said filtered CB data is not used for image reconstruction at \( x \). In this case go back to Step 61 and choose another reconstruction point.

Step 63. Find the projection \( \hat{x} \) of \( x \) onto the detector plane \( DP(s_0) \) and the unit vector \( \beta(s_0,x) \), which points from \( y(s_0) \) towards \( x \).

Step 64. Identify filtering lines \( l_{fl} \in L_1 \) or \( l_{fl} \in L_2 \) (depending on where the x-ray source is located) and points on the said lines that are close to \( \hat{x} \). This will give values of \( \Phi(s_0,\beta_j) \) for \( \beta_j \) close to \( \beta(s_0,x) \).

Step 65. With interpolation estimate the value of \( \Phi(s_0,\beta(s_0,x)) \) from the said values of \( \Phi(s_0,\beta_j) \) for \( \beta_j \) close to \( \beta(s_0,x) \).

Step 66. Compute the contribution from the said filtered CB data to the image being reconstructed at the point \( x \) by multiplying \( \Phi(s_0,\beta(s_0,x)) \) by \( -\delta_x(s_0,x)/(2\pi^2 |x - y(s_0)|) \) (see (8) for definition of \( \delta_x(s_0,x) \)).

Step 67. Add the said contribution to the image being reconstructed at the point \( x \) according to a pre-selected scheme for approximate evaluation of the integral in equation (10).

Step 68. Go to Step 61 and choose a different reconstruction point \( x \).
EFFICIENT IMAGE RECONSTRUCTION ALGORITHM FOR THE CIRCLE AND ARC CONE BEAM COMPUTER TOMOGRAPHY


FIELD OF INVENTION

This invention relates to computer tomography, and in particular to processes, methods and systems for reconstructing three dimensional images from the data obtained by a circle and arc scan of an object, such as when the C-arm rotates around an object within a first plane, and then the C-arm rotates around the object within another plane that is transversal to the first plane.

BACKGROUND AND PRIOR ART

Over the last thirty years, computer tomography (CT) has gone from image reconstruction based on scanning in a slice-by-slice process to spiral scanning to also include non-spiral scanning techniques such as those performed with C-arm devices, with all techniques and devices experiencing problems with image reconstruction.

From the 1970s to 1980s the slice-by-slice scanning was used. In this mode the incremental motions of the patient on the table through the gantry and the gantry rotations were performed one after another. Since the patient was stationary during the gantry rotations, the trajectory of the x-ray source around the patient was circular. Pre-selected slices through the patient were reconstructed using the data obtained by such circular scans.

From the mid 1980s to present day, spiral type scanning has become the preferred process for data collection in CT. Under spiral scanning a table with the patient continuously moves at a constant speed through the gantry that is continuously rotating about the table. At first, spiral scanning has used one-dimensional detectors, which receive data in one dimension (a single row of detectors). Later, two-dimensional detectors, where multiple rows (two or more rows) of detectors sit next to one another, have been introduced. In CT there have been significant problems for image reconstruction especially for two-dimensional detectors. Data provided by the two-dimensional detectors will be referred to as cone-beam (CB) data or CB projections.

In addition to spiral scans there are non-spiral scans, in which the trajectory of the x-ray source is different from spiral. In medical imaging, non-spiral scans are frequently performed using a C-arm device, which is usually smaller and more portable than spiral type scanning systems. For example, C-arm scanning devices have been useful for being moved in and out of operating rooms, and the like.

FIG. 1 shows a typical prior art arrangement of a patient on a table that moves through a C-arm device, that is capable of rotating around the patient, having an x-ray tube source and a detector array, where cone beam projection data sets are received by the x-ray detector, and an image reconstruction process takes place in a computer with a display for the reconstructed image.

There are known problems with using C-arm devices to reconstruct data. See in particular for example, pages 755-759 of Kudo, Hiroyuki et al., Fast and Stable Cone-Beam Filtered Backprojection Method for Non-planar Orbits, IOP Publishing LTD, 1998, pages 747-760. The Kudo paper describes image reconstruction using C-arm devices for various shift-variant filtered back projection (FBP) structures, which are less efficient than convolution-based FBP algorithms.

For three-dimensional (also known as volumetric) image reconstruction from the data provided by spiral and non-spiral scans with two-dimensional detectors, there are two known groups of algorithms: Exact algorithms and Approximate algorithms, that each have known problems. Under ideal circumstances, exact algorithms can provide a replication of an exact image. Thus, one should expect that exact algorithms would produce images of good quality, even under non-ideal (that is, realistic) circumstances.

However, exact algorithms can be known to take many hours to provide an image reconstruction, and can take up great amounts of computer power when being used. These algorithms can require keeping considerable amounts of cone beam projections in memory.

Approximate algorithms possess a filtered back projection (FBP) structure, so they can produce an image very efficiently and using less computing power than Exact algorithms. However, even under the ideal circumstances these algorithms produce an approximate image that may be similar to but still different from the exact image. In particular, Approximate algorithms can create artifacts, which are false features in an image. Under certain circumstances and conditions these artifacts could be quite severe.

To date, there are no known algorithms that can combine the beneficial attributes of Exact and Approximate algorithms into a single algorithm that is capable of replicating an exact image under the ideal circumstances, uses small amounts of computer power, and reconstructs the exact images in an efficient manner (i.e., using the FBP structure) in the cases of complete circle and arc and incomplete circle and arc scanning.

If the C-arm rotates 360 degrees around the patient, this produces a complete circle. If the C-arm rotates less than 360 degrees around the patient, this produces an incomplete circle. In what follows, the word circle covers both complete and incomplete cases. Here and everywhere below by the phrase that the algorithm of the invention reconstructs an exact image we will mean that the algorithm is capable of reconstructing an exact image. Since in real life any data contains noise and other imperfections, no algorithm is capable of reconstructing an exact image.

Image reconstruction has been proposed in many U.S. patents. See for example, U.S. Pat. Nos. 5,663,995 and 5,706,325 and 5,784,481 and 6,014,419 to Hu; U.S. Pat. Nos. 5,881,123 and 5,926,521 and 6,130,930 and 6,233,303 and 6,292,525 to Tam; U.S. Pat. No. 5,906,055 to Samaresekera et al.; U.S. Pat. No. 5,995,580 to Schaller; U.S. Pat. No. 6,009,142 to Sauber; U.S. Pat. No. 6,072,851 to Sivers; U.S. Pat. No. 6,173,032 to Besson; U.S. Pat. No. 6,198,789 to Dafini; U.S. Pat. Nos. 6,315,541 and 6,266,388 to Hsieh; Other U.S. patents have also been proposed for image reconstruction as well. See U.S. Pat. No. 6,504,892 to Ning;
SUMMARY OF THE INVENTION

A primary objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects that have been scanned with two-dimensional detectors.

A secondary objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that is able to reconstruct an exact image and not an approximate image.

A third objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that creates an exact image in an efficient manner using a filtered back projection (FBP) structure.

A fourth objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that creates an exact image with minimal computer power.

A fifth objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that creates an exact image utilizing a convolution-based FBP structure.

A sixth objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that is CB projection driven allowing for the algorithm to work simultaneously with the CB data acquisition.

A seventh objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that does not require storing numerous CB projections in computer memory.

An eighth objective of the invention is to provide improved processes, methods and systems for reconstructing images of objects scanned with a circle and arc x-ray source trajectory that allows for almost real time imaging to occur when images are displayed as soon as a slice measurement is completed.

A preferred embodiment of the invention uses a seven overall step process for reconstructing the image of an object under a circle and arc scan. In a first step a current CB projection is measured. Next, a family of lines is identified on a detector according to a novel algorithm. Next, a computation of derivatives between neighboring projections occurs and is followed by a convolution of the derivatives with a filter along lines from the selected family of lines. Next, using the filtered data, the image is updated by performing back projection. Finally, the preceding steps are repeated for each CB projection until an entire object has been scanned. This embodiment works with keeping several (approximately 2 to approximately 4) CB projections in memory at a time and uses one family of lines.

The invention is not limited to an object that undergoes a scan consisting of a single circle and a single arc. The invention can be applied to trajectories consisting of several circles and arcs by applying it to various circle and arc pairs, and then combining the results.

The circle and arc scanning can include a partial planar curve scan before or after an arc scan. The planar curved scan can be less than a full circle and even greater than a full circle. Additional and subsequent circle and arc scans can be done consecutively after a first circle and arc scan.

Unlike the prior art, the subject invention does not require the patient having to be moved during the measurements. Patients already hooked up to intravenous feeding lines and/or other machines are not easily moveable. Thus, the subject invention can be used as a portable device where the patient does not need to be moved while measurements are taking place.

Further objects and advantages of this invention will be apparent from the following detailed description of the presently preferred embodiments, which are illustrated schematically in the accompanying drawings.

BRIEF DESCRIPTION OF THE FIGURES

FIG. 1 shows a typical prior art view arrangement of a patient on a table that moves through a C-arm device, that is capable of rotating around the patient, having an x-ray tube source and a detector array, where cone beam projection data sets are received by the x-ray detector, and an image reconstruction process takes place in a computer with a display for the reconstructed image.

FIG. 2 shows an overview of the basic process steps of the invention.

FIG. 3 shows mathematical notations of the circle and arc scan.

FIG. 4 illustrates a stereographic projection from the current source position on to the detector plane used in the algorithm for the invention.

FIG. 5 illustrates finding of a filtering line for a reconstruction point x when the x-ray source is on the circle.

FIG. 6 illustrates finding of a filtering line for a reconstruction point x when the x-ray source is on the arc.

FIG. 7 illustrates a family of lines used in the algorithm of the invention corresponding to the case when the x-ray source is on the circle.

FIG. 8 illustrates a family of lines used in the algorithm of the invention corresponding to the case when the x-ray source is on the arc.

FIG. 9 is a three substep flow chart for identifying the set of lines, which corresponds to step 30 of FIG. 2.

FIG. 10 is a seven substep flow chart for preparation for filtering, which corresponds to step 40 of FIG. 2.

FIG. 11 is a seven substep flow chart for filtering, which corresponds to step 50 of FIG. 2. FIG. 12 is an eight substep flow chart for backprojection, which corresponds to step 60 of FIG. 2.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

Before explaining the disclosed embodiments of the present invention in detail it is to be understood that the invention is not limited in its application to the details of the
particular arrangements shown since the invention is capable of other embodiments. Also, the terminology used herein is for the purpose of description and not of limitation.

This invention is a Continuation-In-Part of PCT (Patent Cooperation Treaty) Serial No. PCT/US04/12536 filed Apr. 23, 2004, which is a Continuation-In-Part of U.S. patent application Ser. No. 10/728,136, filed Dec. 4, 2003 which claims the benefit of priority to U.S. Provisional Application Ser. No. 60/430,802 filed, Dec. 4, 2002, and is a Continuation-In-Part of U.S. patent application Ser. No. 10/389,534 filed Mar. 14, 2003, now U.S. Pat. No. 6,804,321, which is a Continuation-In-Part of Ser. No. 10/389,090 filed Mar. 14, 2003, now U.S. Pat. No. 6,771,733, which is a Continuation-In-Part of Ser. 10/143,160 filed May 10, 2002 now U.S. Pat. No. 6,574,299, which claims the benefit of priority to U.S. Provisional Application 60/312,827 filed Aug. 16, 2001, all of which are incorporated by reference.

As previously described, FIG. 1 shows a typical prior art view arrangement of a patient on a table that moves through a C-arm device such as the AXIOM Artis MP, manufactured by Siemens, that is capable of rotating around the patient, having an x-ray tube source and a detector array, where CB projections are received by the x-ray detector, and an image reconstruction process takes place in a computer 4 with a display 6 for displaying the reconstructed image. For the subject invention, the detector array can be a two-dimensional detector array. For example, the array can include two, three or more rows of plural detectors in each row. If three rows are used with each row having ten detectors, then one CB projection set would be thirty individual x-ray detections.

Alternatively, a conventional gantry, such as ones manufactured by Siemens, Toshiba, General Electric, and the like, can be used, as shown by the dotted concentric lines, for the subject invention, the detector array can be a two-dimensional detector array. For example, the array can include two, three or more rows of plural detectors in each row. If three rows are used with each row having ten detectors, then one CB projection set would be thirty individual x-ray detections.

We say that x admits a line segment one end-point of which is on C 1 , y(s)EC 1 (see FIG. 3). Next, pick a reconstruction point x=(x1,x2,x3)EU,x3~0. The shape of C 1 , y(s)EC 1 is the convolution of the computed CB data with a filter along lines of interest. This set is the union of surfaces U=Us,E\[(o.s,=)S(s 2 )], and the other--on C 2 =y(s 2 )EC 2 (see FIG. 3) it-line x=(x1,x2,x3)ECU,x3~0. admit unique line segments. The line segment L 1 (x) determines two parametric intervals. The first one I 1 (x)ECU,x3~0. The second one I 2 (x)=I 1 (x) corresponds to the section of C 2 between y 0 and y(s 2 )EC 2.

We use the following notations in equations 2 and 3 as follows:

\[D_f(y,\theta) := \int_0^1 f(y+\theta s, s)ds; \quad (2)\]

\[\beta(s, x) := \frac{x-y(s)}{|x-y(s)|}; \quad (3)\]

where S 2 is the unit sphere, f is the function representing the distribution of the x-ray attenuation coefficient inside the object being scanned, \(\Theta\) is a unit vector, \(D_f(y,\Theta)\) is the cone beam transform of f, \(\beta(s, x)\) is the unit vector from the focal point y(s) pointing towards the reconstruction point x.

Next, we suppose s\in I 1 (x). Project x,C 1 , C 2 onto the detector plane DP(s) as shown in FIG. 4. We assume that for each y(s)EC 1 \cup C 2 , DP(s) contains the origin O and is perpendicular to the line through y(s) and O. The shape of C 2 on DP(s) depends on s. If 0<s<s 0 \epsilon f, then C 2 is bent to the right as shown in FIG. 5. If\(s=s_0\epsilon f,\) then C 2 is bent to the left. Here and everywhere below we use the convention that whenever a geometrical object is projected onto the detector plane (e.g., point x, circle C 1 , arc...
The corresponding projection is denoted with hat \( \hat{\langle y(s, x) \rangle} \).

We now pick a source position \( y(s) \) on the arc \( y(s) \in C_2 \), \( s \in S_2 \). Find a line through \( y(s) \), which is tangent to \( C_2 \), at some \( \gamma(s(x)), s(x) \in I_2(x) \). This determines another unit vector by equation 5.

\[
u_2(s(x)) = \left( \frac{(y(s(x)) - y(s)) \times \beta(s(x))}{||y(s(x)) - y(s)||} \right) \quad \text{x} \in U, \ s \in I_2(x).
\]

The resulting family, which is one-parametric, is denoted \( L_1 \). Any source position \( y(s) \) and a filtering line from the corresponding family (either \( L_1 \) or \( L_2 \), depending on whether \( y(s) \in C_1 \) or \( y(s) \in C_2 \)), determine a plane. We call it a filtering plane. Since \( e(s, \beta) \hat{\beta} = 0, \hat{e}(s, \beta) = 1 \), we can write equations 11 and 12.

\[(s, \beta) = \sum_{y(q) \in I_2} D_f(y(q), (\cos(\theta + y), \sin(\theta + y))) \frac{1}{\sin(\theta)} dy,
\]

for all \( \beta \) confined to a filtering plane. Here \( \theta \) denotes polar angle within the filtering plane. Therefore, \( e(s, \beta) \hat{e}(s, \beta) = \hat{\beta} \).

Equation (12) is of convolution type and one application of Fast Fourier Transform (FFT) gives values of \( \Psi_f(s, \beta) \) for all \( \beta \) confined to the filtering plane at once. Equations (10) and (12) would represent that the resulting algorithm is of the convolution-based FBP type. This means that processing of every CB projection consists of two steps. First, shift-invariant and \( x \)-independent filtering along a family of lines on the detector is performed. Second, the result is back-projected to update the image matrix. A property of the back-projection step is that for any point \( \hat{x} \) on the detector the value obtained by filtering at \( \hat{x} \) is used for all points on the line segment connecting the current source position \( y(s) \) with \( \hat{x} \). Since \( \hat{\Psi}_f(s, \beta) \beta = 0, \hat{\Psi}_f(s, \beta) \beta = 1 \), we can write equations 11 and 12.

\[(s, \beta) = \sum_{y(q) \in I_2} D_f(y(q), (\cos(\theta + y), \sin(\theta + y))) \frac{1}{\sin(\theta)} dy,
\]

for all \( \beta \) confined to a filtering plane. The detector plane \( D(s) \) corresponding to the source \( y(s) \) and a filtering line from the corresponding family (either \( L_1 \) or \( L_2 \), depending on whether \( y(s) \in C_1 \) or \( y(s) \in C_2 \)), determine a plane. We call it a filtering plane. Since \( e(s, \beta) \hat{\beta} = 0, \hat{e}(s, \beta) = 1 \), we can write equations 11 and 12.

\[(s, \beta) = \sum_{y(q) \in I_2} D_f(y(q), (\cos(\theta + y), \sin(\theta + y))) \frac{1}{\sin(\theta)} dy,
\]

for all \( \beta \) confined to a filtering plane. The detector plane \( D(s) \) corresponding to the source \( y(s) \) and a filtering line from the corresponding family (either \( L_1 \) or \( L_2 \), depending on whether \( y(s) \in C_1 \) or \( y(s) \in C_2 \)), determine a plane. We call it a filtering plane. Since \( e(s, \beta) \hat{\beta} = 0, \hat{e}(s, \beta) = 1 \), we can write equations 11 and 12.

\[(s, \beta) = \sum_{y(q) \in I_2} D_f(y(q), (\cos(\theta + y), \sin(\theta + y))) \frac{1}{\sin(\theta)} dy,
\]

for all \( \beta \) confined to a filtering plane. The detector plane \( D(s) \) corresponding to the source \( y(s) \) and a filtering line from the corresponding family (either \( L_1 \) or \( L_2 \), depending on whether \( y(s) \in C_1 \) or \( y(s) \in C_2 \)), determine a plane. We call it a filtering plane. Since \( e(s, \beta) \hat{\beta} = 0, \hat{e}(s, \beta) = 1 \), we can write equations 11 and 12.

\[(s, \beta) = \sum_{y(q) \in I_2} D_f(y(q), (\cos(\theta + y), \sin(\theta + y))) \frac{1}{\sin(\theta)} dy,
\]

for all \( \beta \) confined to a filtering plane.
Referring to FIG. 8, the set of lines can be selected by the following substeps 31, 32, and 33.

Step 31. Choose a discrete set of values of the parameter \( s \) inside the interval \([s_{\text{min}}, s_{\text{max}}]\).

Step 32. For each \( s \) chosen in Step 31 find a line tangent to the projected circle \( C_1 \).

Step 33. The collection of lines constructed in Step 32 is the required set of lines (see FIG. 8 which illustrates the family of lines used in the algorithm of the invention).

Step 40. Preparation for Filtering

FIG. 10 is a seven step subflow chart for preparation for filtering, which corresponds to step 40 of FIG. 2, which will now be described.

Step 41. If the x-ray source is located on the circle \( C_1 \), fix a filtering line \( \Gamma_1 \subseteq \Gamma_1 \), from the set of lines obtained in Step 20. If the x-ray source is located on the arc \( C_2 \), fix a filtering line \( \Gamma_2 \subseteq \Gamma_2 \) from the set of lines obtained in Step 30.

Step 42. Parameterize points on the said line by polar angle \( \gamma \) in the plane through \( y(s) \) and \( y_1 \).

Step 43. Choose a discrete set of equidistant values \( \gamma \) that will be used later for discrete filtering in Step 50.

Step 44. For each \( \gamma \) find the unit vector \( \beta_\gamma \) which points from \( y(s) \) towards the point on \( \Gamma_1 \) that corresponds to \( y_\gamma \).

Step 45. Using the CB projection data \( D_{\gamma}(y(q), \Theta) \) for a few values of \( q \) close to \( s \), find numerically the derivative \( \partial D_{\gamma}(y(q), \Theta) \partial s \) for \( \Theta = \beta_\gamma \).

Step 46. Store the computed values of the derivative in computer memory.

Step 47. Repeat Steps 41-46 for all lines \( \Gamma_1 \). This way we will create the processed CB data \( \Psi(s, \beta_\gamma) \) corresponding to the x-ray source located at \( y(s) \).

Step 50. Filtering

FIG. 11 is a seven step subflow chart for filtering, which corresponds to step 50 of FIG. 2, which will now be described.

Step 51. Fix a filtering line \( \Gamma_1 \). If the x-ray source is located on the circle \( C_1 \), we take \( \Gamma_1 = \Gamma_1 \). If the x-ray source is located on the arc \( C_2 \), we take \( \Gamma_1 = \Gamma_2 \).

Step 52. Compute FFT (Fast Fourier Transform) of the values of the said processed CB data computed in Step 40 along the said line.

Step 53. Compute FFT of the filter \( 1/\sin \gamma \).

Step 54. Multiply FFT of the filter \( 1/\sin \gamma \) (the result of Step 53) and FFT of the values of the said processed CB data (the result of Step 52).

Step 55. Take the inverse FFT of the result of Step 54.

Step 56. Store the result of Step 55 in computer memory.

Step 57. Repeat Steps 51-56 for all lines in the said family of lines. This will give the filtered CB data \( \Phi(s, \beta_\gamma) \).

By itself the filtering step can be well known in the field and can be implemented, for example, as shown and described in U.S. Pat. No. 5,881,123 to Tam, which is incorporated by reference.

Step 60. Back-Projection

FIG. 12 is an eight step subflow chart for backprojection, which corresponds to step 60 of FIG. 2, which will now be described.

Step 61. Fix a reconstruction point \( x \), which represents a point inside the patient where it is required to reconstruct the image.

Step 62. If \( s \) belongs to \( I_1(x) \cup I_2(x) \), then the said filtered CB data affects the image at \( x \) and one performs Steps 63-68.

If \( s \) is not inside \( I_1(x) \cup I_2(x) \), then the said filtered CB data is not used for image reconstruction at \( x \). In this case go back to Step 61 and choose another reconstruction point.

Step 63. Find the projection \( \gamma_1 \) of \( x \) onto the detector plane \( D(s, \beta) \) and the unit vector \( \beta(s, x) \), which points from \( y(s) \) towards \( x \).

Step 64. Identify filtering lines \( \Gamma_1 \subseteq \Gamma_1 \) or \( \Gamma_2 \subseteq \Gamma_2 \) (depending on where the x-ray source is located) and points on the said lines that are close to the said projection \( \gamma_1 \). This will give a few values of \( \Phi(s, \beta) \) for \( \beta \) close to \( \beta(s, x) \).

Step 65. With interpolation estimate the value of \( \Phi(s, \beta) \) from the said values of \( \Phi(s, \beta) \) for \( \beta \) close to \( \beta(s, x) \).

Step 66. Compute the contribution from the said filtered CB data to the image being reconstructed at the point \( x \) by multiplying \( \Phi(s, \beta(s, x)) \) by \( \delta(s, x) / (2\pi x - y(s)) \). The quantity \( \delta(s, x) \) is defined by equation (8).

Step 67. Add the said contribution to the image being reconstructed at the point \( x \) according to a pre-selected scheme (for example, the Trapezoidal scheme) for approximate evaluation of the integral in equation (10).

Step 68. Go to Step 61 and choose a different reconstruction point \( x \).

Step 70. Go to Step 10 (FIG. 2) and load the next CB projection into computer memory. The image can be displayed at all reconstruction points \( x \) for which the image reconstruction process has been completed (that is, all the subsequent CB projections are not needed for reconstructing the image at those points). Discard from the computer memory all the CB projections that are not needed for image reconstruction at points where the image reconstruction process has not completed. The algorithm concludes when the scan is finished or the image reconstruction process has completed at all the required points.

The invention is not limited to an object that undergoes a scan consisting of a single circle and a single arc. The algorithm can be applied to trajectories consisting of several circles and arcs by applying it to various circle and arc pairs, and then combining the results. The algorithm can be applied to trajectories in which a planar curve is not necessarily a circle, but, for example, an ellipse, and the like.

Other Embodiments of the invention are possible. For example, one can integrate by parts in equation (6) as described in the inventor’s previous U.S. patent application Ser. No. 10/143,160 filed May 10, 2002 now U.S. Pat. No. 6,574,299, now incorporated by reference, to get an exact convolution-based FBP-type inversion formula which requires keeping only one CB projection in computer memory. The algorithmic implementation of this alternative embodiment can be similar to and include the algorithmic implementation of Embodiment Two in the inventor’s previous U.S. patent application Ser. No. 10/143,160 filed May 10, 2002 now U.S. Pat. No. 6,574,299, now incorporated by reference. The corresponding equations will now be described. Introduce the following notations in equation 13:

\[
\Psi_k(s, \beta) = \int_0^{\rho} D_I(y(s), \Theta_k(s, \beta, \gamma)) \frac{dy}{\sin \gamma},
\]

\[
\Psi_k(s, \beta) = \int_0^{\rho} (\nabla_{y_0, y_0} D_I(y(s), \Theta_k(s, \beta, \gamma)) \cos \gamma) dy, \]

\[
\Psi_k(s, \beta) = \int_0^{\rho} (\nabla_{y_0, y_0} D_I(y(s), \Theta_k(s, \beta, \gamma)) dy, \]

\[
\Psi_k(s, \beta) = \int_0^{\rho} \left( \frac{\partial}{\partial \gamma} D_I(y(s), \Theta_k(s, \beta, \gamma)) \right) dy \] 

\[
k = 0, 1, 2.\]
Here $\nabla_s D_f$ denotes the derivative of $D_f$ with respect to the
angular variables along the direction $u$ by equations 14 and 15:

$$\nabla_s D_f (y(s), \theta) = \frac{\partial}{\partial \theta} D_f (y(s), \sqrt{1 - \theta^2 + m^2}) |_{\theta = u^t}, \theta \in U^t.$$  

(14)

Here $u^t$ denotes the set of unit vectors perpendicular to $u$. Denote also

$$\mu_k (s, x) = \frac{1}{\|x - y(s)\|} \left. \mu_k (s, x) = \frac{\beta_k (s(x), u_k (s, x)) \cdot u_k (s, x)}{\|x - y(s)\|} \right|_{s(x)}$$  

$$\mu_k (s, x) = \frac{(\epsilon_k (s(x), u_k (s, x)) \cdot u_k (s, x)) \cdot u_k (s, x)}{\|x - y(s)\|}, \mu_k (s, x) = \frac{\beta_k (s(x), \epsilon_k (s, x)) \cdot e_k (s, x)}{\|x - y(s)\|}.$$  

(15)

Here $\beta_k = \partial \beta / \partial s$ and $e_k = \partial e / \partial s$.

Let $s_k (x)$ and $s_{\beta_k} (x)$ denote the end-points of $l_k (x)$, $k=1,2$. More precisely, $l_k (x) = [s_{\beta_k} (x), s_k (x)]$.

Integrating by parts with respect to $s$ in equation (6) and using that $\delta_k (s, x)$ is a constant with respect to $s$ within each $l_k (x)$ (so it can be replaced by $\delta_k (x)$) we get similarly to equation (17) in U.S. Pat. No. 6,574,299 to Katsevich and the same assignee of the subject invention which is incorporated by reference:

$$f(x) = -\frac{1}{2\pi^2} \sum_{k=1}^{4} \int_{s_{\beta_k} (x)}^{s_k (x)} \left\{ \frac{\Phi_k (s, x) \cdot e_k (s, x)}{\|x - y(s)\|} \right\} + \int_{s_{\beta_k} (x)}^{s_k (x)} \sum_{k=1}^{4} \mu_k (s, x) \Phi_k (s, x) \, ds.$$  

(16)

While the invention has been described with rotating C-arm type devices, the invention can be used with rotating gantry devices.

Furthermore, the amount of rotating can include a single rotational curve of at least approximately 5 degrees up to approximately 360 degrees or greater. Theoretically, there is no limit on the minimum range of rotation. Under realistic practical circumstances, a minimum range of rotation is between approximately 10 to approximately 20 degrees.

The circle and arc scanning of an object can have an arc scanning before or after a single rotational curve scan as defined above.

Subsequent circle and arc scanning can occur as needed for image reconstruction.

The invention can allow for two scanning curves to be used which are transversal to one another. One of the scanning curves can be sized anywhere from an arc up to a full circle, and the other scanning curve can also be sized anywhere from an arc up to a full circle.

In medical applications, the length of one curve is generally shorter than the length of the other curve. The planes of the curves can cross each other at a transversal angle of approximately 45 to approximately 135 degrees. Alternatively, the transversal angle can be in the range of approximately 80 to approximately 100 degrees. Still furthermore the second curve can be approximately perpendicular to the first curve.

Although the preferred embodiments describe applications of using x-ray sources for creating data for image reconstruction, the invention can be applicable with other sources that create line integral data for image reconstruction, such as but not limited to early arriving photons.

Other computational schemes for evaluating the integrals in equations 9, 10, 12, 13, and 16 are possible. For example, one can choose to perform filtering using native geometries, as explained in the paper by F. Noo, J. Pack, D. Heuscher “Exact helical reconstruction using native cone-beam geometries”, published in the journal “Physics in Medicine and Biology”, 2003, volume 48, pages 3787-3818.

Introduce the coordinates in the detector plane as follows. $u$ is the horizontal coordinate, and $w$ is the vertical coordinate. If the detector is flat, then computation of $f$ using equations 9-12 can be replaced by the following steps.

Step 1. Distance weighting.

$$g_1 (s, u, w) = \frac{1}{\sqrt{u^2 + w^2} + 1} \frac{\partial}{\partial u} D_f (q, u, w) |_{q=x}.$$  

(17)

Step 2. Hilbert transform filtering of $g_1$ along filtering lines on the detector:

$$g_2 (s, u, \rho) = \int_{-\infty}^{\infty} K_{H}(\rho - u') g_1 (s, u', w(\rho, u')) \, du',$$  

(18)

where $K_{H}$ is the Hilbert transform kernel, $ho$ is the variable that parametrizes filtering lines, and $w(\rho, u')$ is the $w$-coordinate of the point, whose $u$-coordinate equals $u'$, and which belongs to the filtering line determined by the parameter $ho$.


$$g_3 (s, u, w) = g_2 (s, u, \rho(u, w)),$$  

(19)

where $\rho(u, w)$ is the function that gives the value of the parameter $\rho$, corresponding to the filtering line that contains all $x$ projected onto the point on the detector with coordinates $(u, w)$.

Step 4. Backprojection. Evaluate the integrals

$$\int_{l_k (x)}^{V^k (s, x)} g_3 (s, u', s, x, w(s, x)) \, ds, k = 1, 2,$$  

(20)

where $u'^k(s,x), w'^k(s,x)$ are the coordinates of the point $x$ projected onto the detector plane $DP(s)$.

$V^k(s,x)=1-(x_1 y_1(s)+x_2 y_2(s)+x_3 y_3(s))$ if $y(s)$ is on the arc, $V^k(s,x)=1-(x_1 y_1(s)+x_2 y_2(s))$ if $y(s)$ is on the circle.

Step 5. Multiply the integrals in (20) by $\delta_k(s)(2\pi), s \in I_k(x)$, and add them to obtain $f(x)$.

Integrating by parts with respect to $s$ in equation 20 we can obtain another set of formulas, in which the derivative along the source trajectory is avoided. In this case the filtering step will involve computing the following quantities.
A method of reconstructing images from data provided by at least one detector, comprising the steps of:

- moving a scanner along a first curve within a plane relative to a stationary object while scanning the object;
- scanning the object along a second curve within a plane transversal to the plane of the first curve;
- storing at least one cone beam projection in memory at a time;
- using at least one family of lines for the step of reconstructing;
- applying a convolution based shift invariant Filtered Back Projection algorithm to the cone beam projections of the first single curve and the second single curve;
- back projecting the image of the scanned object and reconstructing an exact image of the scanned object with the convolution based Filtered Back Projection algorithm for circle and cone beam computed tomography.

2. The method of claim 1, wherein a single curve includes the step of:

- rotating a C-arm device about a portion of the object.

3. The method of claim 1, wherein a single curve includes the step of:

- rotating a gantry about a portion of the object.

4. The method of claim 1, wherein a single curve includes the step of:

- rotating between approximately 5 degrees up to approximately 360 degrees.

5. The method of claim 4, further comprising the step of:

- rotating over approximately 360 degrees around the object.

6. The method of claim 1, wherein the step of reconstructing includes the steps of:

- storing approximately 2 to approximately 4 cone beam (CB) projections in memory at a time; and
- using one family of lines for the step of reconstructing.

7. A method of reconstructing images from two planar curve scans of an object, comprising the steps of:

- (a) collecting cone beam (CB) data from a detector during two planar curve scans of the object;
- (b) identifying lines on a plane II intersecting the cone beam, wherein the step (b) of identifying lines includes the steps of:
  - (b1) when an x-ray source belongs to the curve scan A, project the curve scan B onto II and choose a discrete set of lines tangent to that projection; and
  - (bii) when the x-ray source belongs to the curve scan B, project the curve scan A onto II and choose a discrete set of lines parallel to that projection;
- (c) preprocessing and shift invariant filtering said data along said lines, wherein the step (c) of preprocessing includes computing the derivative \( \frac{\partial I(s, u)}{\partial s} \) of the CB data to obtain a pre-selected reconstruction scheme; and
- (d) repeating steps a, b, c, and d until an exact image of the object is reconstructed.

8. The method of claim 7, wherein shift-invariant filtering in step (c) includes convolving the derivative \( \frac{\partial I(s, u)}{\partial s} \) with kernel \( \frac{1}{\sin(\gamma)} \) within a filtering plane containing \( y(s) \) and a line, identified in step (b) above, where \( \gamma \) is polar angle in the plane.

9. The method of claim 7, wherein a planar curve scan includes:

- a complete circle about the object.

10. The method of claim 7, wherein a planar curve scan includes:

- less than complete circle about the object.

11. The method of claim 7, wherein the back-projection step (d) includes the steps of:

- (d) fixing a reconstruction point \( x \), which represents a point inside the object being scanned, to reconstruct the image:
  - (dii) when \( s \) belongs to \( I(x) \), then the said filtered CB data affects the image at \( x \) and one performs Steps (diii) to (dvi); and when \( s \) is not inside the interval \( I(x) \), then the said filtered CB data is not used for the image reconstruction at \( x \) and go back to step (d) and choose another reconstruction point, wherein \( I(x) \) is the parametric interval corresponding to the section of the scan path bounded by the PI-line of \( x \); PI-line of \( x \) is the line segment containing \( x \), one endpoint of which belongs to the first planar curve scan, and the other endpoint of which belongs to the second planar curve scan;
  - (d) estimating a value of \( \Phi(s, \beta(s,x)) \), where \( \Phi(s, \beta(s,x)) \) is the filtered CB data corresponding to the source position located at the point \( y(s) \) and direction \( \beta(s,x) \);
  - (dvi) add the said contribution to the image being reconstructed at the point \( x \) according to a pre-selected scheme; and
  - (dvi) go to step (d) and choose a different reconstruction point \( x \).
12. The method of claim 7, further comprising the steps of:

storing approximately 2 to approximately 4 cone beam (CB) projections in memory at a time; and

using one family of lines for each x-ray source position for the step of filtering.

13. The method of claim 7, wherein the step (c) of preprocessing includes distance weighting of the derivative \((\partial/\partial s)D_f(y(s), \Theta)\).

14. The method of claim 13, wherein shift-invariant filtering in step (c) includes convolving the distance weighted derivative \((\partial/\partial s)D_f(y(s), \Theta)\) with a Hubert transform kernel along said lines.

15. The method of claim 13, wherein the step (c) of preprocessing includes distance weighting of the data \(D_f(Y(s), \Theta)\).

16. The method of claim 15, wherein shift-invariant filtering in step (c) includes convolving the distance weighted data \(D_f(Y(s), \Theta)\) with the derivative of a Hilbert transform kernel along said lines.

17. A method of computing images derived from two planar curve scans, being called curve scan A and curve scan B, for circle and are cone beam computed tomography comprising the steps of:

(a) collecting cone beam (CB) data from a detector during two planar curve scans of an object;

(b) identifying lines on a plane intersecting the cone beam, wherein the step (b) of identifying lines includes the steps of:

(i) when an x-ray source belongs to the curve scan A, project the curve scan B onto a plane and choose a set of lines tangent to that projection; and

(ii) when the x-ray source belongs to the curve scan B, project the curve scan A onto a plane and choose a set of lines parallel to that projection;

(c) preprocessing and shift invariant filtering said data along said lines, wherein the step (c) of preprocessing includes computing the distance weighted data \(D_f(y(s), \Theta)\) with respect to \(\Theta\) along a direction non-parallel to the plane determined by \(y(s)\) and a filtering line, the said plane being a filtering plane, wherein

s is parameter along the scan path, which determines point \(y(s)\) on the said path,

\(D_f(Y(s), \Theta)\) is the cone beam transform of \(f\) corresponding to the x-ray source located at the point \(y(s)\) and the direction \(\Theta\).

(f) is a function describing the object being scanned;

(d) back projecting said filtered data to form a precursor of said image; and

(e) repeating steps a, b, c, and d until an exact image of the object is reconstructed.

18. The method of claim 17, wherein shift-invariant filtering in step (c) includes convolving the data \(D_f(Y(s), \Theta)\) with kernel \(1/\sin(y)\) within a filtering plane, where \(y\) is polar angle in the plane.

19. The method of claim 17, wherein shift-invariant filtering in step (c) includes convolving the data \(D_f(Y(s), \Theta)\) with kernel

\[
\frac{\partial}{\partial y} \sin(y)
\]

within a filtering plane, where \(y\) is polar angle in the plane.

20. The method of claim 17, wherein shift-invariant filtering in step (c) includes convolving the derivative of \(D_f(Y(s), \Theta)\) with a kernel within a filtering plane, the derivative of \(D_f(Y(s), \Theta)\) is the derivative with respect to \(\Theta\) along a direction non-parallel to the filtering plane.

21. The method of claim 20, wherein \(y(s)\) belongs to the curve scan A.

22. The method of claim 20, wherein \(y(s)\) belongs to the curve scan B.

23. The method of claim 17, wherein a planar curve scan includes:

(a) complete circle about the object.

(b) a less than complete circle about the object.

24. The method of claim 17, wherein a planar curve scan includes:

(i) a complete circle about the object.

(ii) a less than complete circle about the object.

25. The method of claim 17, wherein the back-projection step (d) includes the steps of:

(i) fix a reconstruction point \(x\) which represents a point inside the object being scanned, to reconstruct the image;

(ii) when \(s\) belongs to \(I(x)\), then the said filtered CB data affects the image at \(x\) and one performs Steps (dii) to (dvii) and when \(s\) is not inside the interval \(I(x)\), then the said filtered CB data is not used for the image reconstruction at \(x\) and go back to step (d) and choose another reconstruction point, wherein

\(I(x)\) is the parametric interval corresponding to the section of the scan path bounded by the PI-line of \(x\); and

PI-line of \(x\) is the line segment containing \(x\), one endpoint of which belongs to the curve scan A, and the other endpoint of which belongs to the curve scan B;

(iii) find the projection \(\tilde{x}\) of \(x\) onto a detector plane \(DP(s)\) and unit vector \(\beta(s,x)\), which points from \(y(s)\) towards \(x\); (div) estimate a value of \(\Phi(s,\beta(s,x))\), where \(\Phi(s,\beta(s,x))\) is the filtered CB data corresponding to the source position located at the point \(y(s)\) and direction \(\beta(s,x)\);

(dv) determine contribution from filtered CB data to the image being reconstructed at the point \(x\) by multiplying \(\Phi(s,\beta(s,x))\) by a weighting factor;

(dvi) add the said contribution to the image being reconstructed at the point \(x\) according to a pre-selected scheme; and

(dvii) go to step (di) and choose a different reconstruction point \(x\).

26. The method of claim 17, further comprising the steps of:

storing 1 cone beam (CB) projection in memory at a time; and

using one family of lines for each x-ray source position for the step of filtering.

27. A method of reconstructing images from data provided by at least one detector, comprising the steps of:

scanning an object with two planar curve scans by at least one detector;

storing at least one cone beam projection in memory at a time;

using at least one family of lines for the step of reconstructing;

applying a convolution based shift invariant Filtered Back Projection algorithm to the two planar curve scans;

back projection updating the image of the scanned object and reconstructing an exact image of the scanned object with a convolution based FBP (Filtered Back Projection) algorithm for circle and arc cone beam computed tomography.

28. The method of claim 27, wherein the scanning step includes the step of:

scanning by the two planar curves being transversal to one another.
29. The method of claim 27, wherein the scanning step includes the step of:
scanning by the object with a first curve followed by
scanning the object with another curve being transverse
to the first curve.
30. The method of claim 27, further comprising the step of:
providing a C-arm device for the scanning of the object.
31. The method of claim 27, further comprising the step of:
providing a gantry for the scanning of the object.
32. The method of claim 27, wherein at least one scan
curve includes:
at least a full circle scan about the object.
33. The method of claim 27, wherein at least one scan
curve includes:
less than a full circle scan about the object.
34. The method of claim 27, further comprising the step of:
consecutively scanning the object with another planar
curve scan.
35. A method of reconstructing images from data provided
by a detector, comprising the steps of:
first scanning an object with a first curve moving along a
first plane relative to the object;
second scanning the object with a second curve moving
within a second plane that is transversal to the first plane;
detecting data from the first scanning and the second
scanning steps with at least one detector; and
reconstructing an exact image of the scanned object with
the detected data with a convolution based FBP (Filtered
Black Projection) algorithm for circle and arc
cone beam computed tomography.
36. The method of claim 35, wherein the second plane
crosses at a transversal angle of approximately 45 to
approximately 135 degrees relative to the first plane.
37. The method of claim 35, wherein the second plane
crosses at a transversal angle of approximately 80 to
approximately 100 degrees relative to the first plane.
38. The method of claim 35, wherein the second plane
crosses at a transversal angle of approximately perpendicular
to the first plane.
39. The method of claim 35, wherein one of the curves
includes a full circle.
40. The method of claim 35, wherein one of the curves
includes less than a full circle.
41. The method of claim 35, wherein one of the curves is
shorter than the other curve.
42. The method of claim 35, wherein the first scanning
step and the second scanning step include the step of:
providing one C-arm scanning device to provide both the
first and the second scanning steps.

* * * * *
It is certified that error appears in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

Col. 1, line 5 should read as follows:

STATEMENT REGARDING FEDERALLY SPONSORED RESEARCH OR DEVELOPMENT

This subject invention was made with government support under the National Science Foundation, federal contract number DMS0104033. The government has certain rights in this invention.

Signed and Sealed this
First Day of March, 2011

[Signature]

David J. Kappos
Director of the United States Patent and Trademark Office