The Development of a Computer Aided Design Program for Constant Group Delay Monolithic Crystal Filters

Spring 1983

Michael A. Robitaille
University of Central Florida

Find similar works at: https://stars.library.ucf.edu/rtd

University of Central Florida Libraries http://library.ucf.edu

Part of the Engineering Commons

STARS Citation

https://stars.library.ucf.edu/rtd/712

This Masters Thesis (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of STARS. For more information, please contact lee.dotson@ucf.edu.
THE DEVELOPMENT OF A COMPUTER AIDED DESIGN PROGRAM FOR CONSTANT GROUP DELAY MONOLITHIC CRYSTAL FILTERS

BY

MICHAEL ANDRE ROBITAILLE
B.S., University of Central Florida, 1982

THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering in the Graduate Studies Program of the College of Engineering, University of Central Florida, Orlando, Florida

Spring Term
1983
ABSTRACT

A computer program, which realizes monolithic crystal filters with constant group delay, is developed. This program analyzes and synthesizes a class of transfer functions developed by J. D. Rhodes called equidistant linear phase transfer functions. Using cascade synthesis, the program calculates the values of the inductor and capacitors of the ladder sections which realizes these functions. The ladder sections are implemented using monolithic crystal filters.
ACKNOWLEDGEMENTS

I would like to acknowledge and thank the many people who have contributed to the completion of this thesis. First, I would like to dedicate this thesis to my wife, Mary, because of the many long hours she spent helping me prepare it and also for her help during the past 4 years with all of my schoolwork. I can easily say that without her help I never would have finished as soon as I did.

Special thanks is extended to Piezo Technology Inc. for their financial support and also for allowing me the use of their equipment and knowledge.

I would like to thank my thesis advisor, Dr. Donald Malocha, for being so patient with me for the past six months and for keeping me motivated.

Thanks is also given to my thesis committee members, Mr. Bob Martin, Mr. Madjid Belkerdid, and Dr. Richard Miller for the time they spent evaluating this paper.

I would also like to thank my parents, Jean Paul and Marie Robitaille and my sister, Ginette LaForce, for their moral support and help during my time of need.
# TABLE OF CONTENTS

ACKNOWLEDGMENT ............................................................... iii

LIST OF ILLUSTRATIONS ............................................................ v

INTRODUCTION ........................................................................... 1

I. NON-MINIMUM PHASE TRANSFER FUNCTION ..................................... 3

II. EQUIDISTANT LINEAR PHASE TRANSFER FUNCTION ......................... 9

   How to Generate the Transfer Function ........................................... 9

   Properties of the Transfer Function .............................................. 12

III. MONOLITHIC CRYSTAL FILTER ............................................. 15

   Low-pass MCF Prototype .......................................................... 15

   Advantages of MCF ................................................................. 19

IV. SYNTHESIS PROCEDURE .......................................................... 20

V. CAD PROGRAM ........................................................................ 31

CONCLUSION .............................................................................. 39

APPENDICES

A. Program ..................................................................................... 40

B. Tables of Transfer Function Coefficients ....................................... 56

C. Graphs of Transfer Function Response .......................................... 71

REFERENCES .............................................................................. 83
LIST OF FIGURES

1. Group Delay Response of an Ideal Amplitude Filter (Redrawn from Rhodes, 1976)..................5

2. Amplitude Response of an Ideal Linear Phase (Redrawn from Rhodes, 1976)..................7

3. Phase Error Function for the Equidistant Linear Phase Polynomial of Order 6. (Redrawn from Rhodes, 1970)..........................12

4. (a)Amplitude and (b) Group Delay Response for a Fourth Order Transfer Function.............13

5. (a)Monolithic Crystal Filter and (b) Schematic Diagram (Redrawn and adapted from Smyth, 1972)....15

6. (a)Crystal Resonator Diagram (b) Equivalent Bandpass, and (c) Low-pass Networks (Redrawn and adapted from Holt, 1968)..............16

7. (a)MCF Equivalent Circuit with (b) Low-pass Lattice and (c) Ladder Prototype (Redrawn and adapted from Dillon, 1976)..................18

8. Impedance Invertor.............................................23

9. MCF Networks That Remove Zeros at Infinity..........................25

10. Narrow Band Low-pass to Bandpass Transformation.............................................29

11. Bandpass Monolithic Crystal Filter Equivalent Circuit for a Maximally Flat Delay Filter of Fourth Order..................30

12. Bandpass Monolithic Crystal Filter Equivalent Circuit for a Maximally Flat Delay Filter of Sixth Order..................38
INTRODUCTION

With the ever-increasing popularity of digital communication systems, such as frequency-shift keying (FSK), monolithic crystal filter (MCF) designers are faced with the problem of designing filters with constant group delay. Methods used for realizing constant group delay filters range from using delay equalizers to special transfer functions, such as Guassian to 6dB function (Zverev, 1967). Each of these methods have their drawbacks: for instance, equalizers increase the complexity of the filter, while the Guassian to 6dB function is very component sensitive. For these reasons, this thesis will look at a new transfer function developed by J. D. Rhodes (1976) known as equidistant linear phase transfer function. These transfer functions offer good delay response and selectivity.

In order to facilitate the design of these filters, a computer aided design (CAD) program, which realizes Rhodes' transfer function, is developed. The CAD program is constructed to assist an engineer in designing constant group delay filters by generating the Rhodes' transfer function, analyzing the function and realizing it using monolithic crystals. Since the Rhodes' functions are non-minimum phase functions, an unconventional synthesis procedure must be used. The method used is one that was presented by
Dillon and Lind (1976). This procedure involves the removal of transmission zero sections, which can be realized by monolithic crystals, from the input impedance of the transfer function. Herzig and Swanson (1978) have used this approach for implementation of a 6th order Rhodes' transfer function for use in a communications satellite.
I. NON-MINIMUM PHASE TRANSFER FUNCTION

It is well-known that in order to realize a filter with a constant group delay in the passband and a sharp cut-off, a non-minimum phase transfer function must be realized. In order to gain a better insight into the realization of a constant group delay filter, an argument given by J. D. Rhodes (1976) will be reviewed. This argument compares the amplitude response versus group delay for two minimum phase and two non-minimum phase filters.

A minimum phase system is defined as one having a cut-off frequency in which there are no poles or zeros in the right half plane (R.H.P.) of its root locus diagram. If the system has a transfer scattering parameter of the form

\[ S_{12}(s) = \frac{N(s)}{D(s)} \]

then \( N(s) \) and \( D(s) \) must be Hurwitz polynomials, i.e., no poles in the R.H.P.. The terminology, minimum phase, arises from the fact that the phase of the scattering parameter is a minimum over all frequencies (Rhodes, 1976).

For these minimum phase networks, A. Papoulis (1962) has shown that the unique relationship related by the Hilbert Transform, between the amplitude and phase response at real frequencies is

\[-\psi(\omega_0) = \frac{\omega_0}{\pi} \int_{-\infty}^{\infty} \frac{\alpha(\omega)}{\omega^2 - \omega_0^2} \, d\omega \]  

(1)
and

\[ a(\omega_0) = a(0) + \frac{\omega_0^2}{\pi} \int_{-\infty}^{\infty} \frac{\psi(\omega)}{\omega(\omega^2 - \omega_0^2)} \, d\omega \]  

(2)

where the transmission scattering parameter is given by

\[ S_{12}(j\omega) = \exp \left[ -a(\omega) + j\psi(\omega) \right] \]  

(3)

The group delay response for the minimum phase filters will now be discussed. First consider the ideal amplitude character of a low-pass filter. Let

\[ |S_{12}(s)| = 1 \quad |\omega| < 1 \]

\[ = A \quad |\omega| > 1 \]  

(4)

where A 1. From equation 1 and 3, since \( \psi(\omega) \) is an even function, we have

\[ -\psi(\omega) = -\frac{2\omega}{\pi} \int_{1}^{\infty} \frac{\ln A}{y^2 - \omega^2} \, dy \]

\[ = \frac{-\ln A}{\pi} \int_{1}^{\infty} \left( \frac{1}{y+\omega} - \frac{1}{y-\omega} \right) dy \]

which results in

\[ \psi(\omega) = \frac{-\ln A}{\pi} \ln \left| \frac{1-\omega}{1+\omega} \right| \]

The group delay, \( T_g \), is obtained by taking the derivative of \( \psi \) with respect to \( \omega \), resulting in

\[ T_g = \frac{-d\psi(\omega)}{d\omega} = \frac{-2\ln A}{\pi} \left[ \frac{1}{|1-\omega^2|} \right] \]  

(5)

As illustrated in Figure 1, this function increases to infinity at the cut-off frequency, \( |\omega| = 1 \), while in the passband, the function \( 1/T_g \) has a quadratic character.
Next consider the piecewise approximation to the Chebyshev filter given by

\[ |S_{12}(j\omega)| = \begin{cases} 1 & |\omega| < 1 \\ \omega^{-n} & |\omega| > 1 \end{cases} \]  

For this example, the group delay can be most easily obtained by first taking the derivative of equation 1, resulting in

\[ T_g(\omega_0) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\alpha(\omega)(\omega^2 + \omega_0^2)}{(\omega^2 - \omega_0^2)^2} \, d\omega \]

Substituting the requirements of equation 6 into equation 3, results in

\[ T_g(\omega) = \frac{1}{\pi} \int_{1}^{\infty} n \ln y \left[ \frac{1}{(y-\omega)^2} + \frac{1}{(y+\omega)^2} \right] \, dy \]

After integrating each term by parts, the following result is obtained

\[ T_g(\omega) = \frac{n}{n \pi} \ln \left| \frac{1 + \omega}{1 - \omega} \right| \]  

Comparison between the two group delays can be simplified by examining the power-series expansion for \(|\omega| < 1\) of equation 5,

\[ T_g(\omega) = T_{g0} \left( 1 + \frac{\omega^2}{3} + \frac{\omega^4}{5} + \frac{\omega^6}{7} + \ldots \right) \]
and of equation 7.

\[ T_g(\omega) = T_g(0) \left(1 + \omega^2 + \omega^4 + \omega^6 + \ldots\right) \]

From the above expansions, it is seen that the group delay response around \( \omega = 0 \) deviates from being constant as the rate of variation of the amplitude characteristic increases in the vicinity of the cut-off frequency; therefore, the better the amplitude response, the worse the group delay in the passband.

Now, let us examine the relationship between group delay response and the variation in amplitude characteristic for a non-minimum phase network. For the first non-minimum phase network, consider a filter with linear phase in the passband and constant phase in the stop band. For such a filter the phase response is given by

\[ -\psi(\omega) = \tau \omega \quad |\omega| < 1 \]

\[ = \tau \frac{\omega}{|\omega|} \quad |\omega| > 1 \]

which, when substituted in equation 2 results in

\[ \alpha(\omega) = \alpha(0) - \frac{2\tau \omega^2}{\pi} \left[ \int_0^1 \frac{dy}{y^2 - \omega^2} + \int_1^\infty \frac{dy}{y(y^2 - \omega^2)} \right] \]

\[ = \alpha(0) + \frac{\tau}{\pi} \left\{ (1+\omega) \ln|1+\omega| + (1-\omega) \ln|1-\omega| \right\} \]

Therefore, the rate of change in amplitude is

\[ \frac{d\alpha(\omega)}{d\omega} = \frac{\tau}{\pi} \ln \left| \frac{1+\omega}{1-\omega} \right| \]
As seen from Figure 2, the amplitude response is not constant in the passband as is desired. However, this figure also reveals that the change in attenuation around the cut-off frequency is large, which is needed for good filter performance.

![Figure 2. Amplitude response of ideal linear phase (Redrawn from Rhodes, 1976).](image)

Secondly, consider a filter with constant group delay in the passband

\[
T_g(\omega) = \tau \\
= \tau |\omega|^{-2}
\]

The corresponding phase response is

\[
-\psi(\omega) = \tau(\omega) \\
= 2\tau \frac{\omega}{|\omega|} - \frac{\tau}{\omega}
\]

which, when substituted in equation 2 results in

\[
\alpha(\omega) = \alpha(0) - \frac{\tau}{\pi} \left[ 2 + \frac{(1-\omega)^2}{\omega^2} \ln |1-\omega| - \frac{(1+\omega)^2}{\omega^2} \ln |1+\omega| \right]
\]

and

\[
\frac{d\alpha(\omega)}{d\omega} = \frac{\tau}{\pi} \left[ 2 \frac{\omega}{\omega^2} - \frac{(1-\omega)^2}{\omega^2} \ln \frac{|1+\omega|}{|1-\omega|} \right]
\]
Expanding the rate of change of the amplitude characteristic of the two non-minimum examples about \( \omega = 0 \), results in

\[
\frac{d\alpha(\omega)}{d\omega} = 2\omega \frac{T}{\pi} \left[ 1 + \frac{\omega^2}{3} + \frac{\omega^4}{5} + \ldots \right]
\]

for the first example, and

\[
\frac{d\alpha(\omega)}{d\omega} = 4\omega \frac{T}{\pi} \left[ \frac{1}{3} + \frac{1}{15} \omega^2 + \frac{1}{35} \omega^4 + \ldots \right]
\]

for the second.

From these two examples, one can deduce that for non-minimum phase systems, the greater the rate of change in group delay around the cut-off, the greater the change in attenuation around the origin.

Recalling that for minimum phase systems, as the rate of change in the amplitude characteristics around the cut-off increases, so does the rate of change in group delay. However, in non-minimum phase, the rate of variation in the amplitude and group delay around cut-off is large, while the change in amplitude around \( \omega = 0 \) is small; therefore, non-minimum phase networks must be used when constant group delay characteristics are required.
II. EQUIDISTANT LINEAR PHASE TRANSFER FUNCTION

Generating the Transfer Function

As stated earlier, the transfer function developed by J. D. Rhodes will be implemented. This type of transfer function represents a near optimum solution to the problem of optimizing the amplitude and phase response of a transfer function. Since these transfer functions are based on a polynomial whose phase response is equal to a linear phase at equal distance, they are known as equidistant linear phase transfer functions. The transfer function is constructed from a polynomial $D_n(p,\epsilon)$, which has the desired phase response, and $E_{2r}(p,\epsilon)$, which does not contribute to the phase but improves the amplitude response.

\[
S_{12}(p,\epsilon) = \frac{E_{2r}(p,\epsilon)}{D_n(p,\epsilon)D_n^*(p,\epsilon)} \tag{8}
\]

where the relationship between $D_n(p,\epsilon)$ and $E_{2r}(p,\epsilon)$ for an even order transfer function was shown to be

\[
E_{2r}(p) = \text{Im} \left[ D_n(p,\epsilon)D_n^*(p,\epsilon) \right] \tag{9}
\]

where * indicates complex conjugates (Rhodes, 1970). Only the even order transfer functions are considered, since the odd order functions have poor performance characteristics.

$D_n(p,\epsilon)$ is constructed from the equidistant linear phase polynomial $P_n(p,\epsilon)$, developed by Rhodes. The rela-
The relationship between \( D_n(p, \varepsilon) \) and \( P_n(p, \varepsilon) \) is given by
\[
D_n(p, \varepsilon) = 2P_n(p, \varepsilon) - (1-j)P'_n(p, \varepsilon)
\] (10)
where the prime indicates derivative with respect to \( p \).

\( P_n(p, \varepsilon) \) has an integral representation of the form
\[
P_n(p, \varepsilon) = \frac{e^{pA_{2n+1}(p)2^n}}{(2n)! \varepsilon^{2n+1}} \int_1^\infty e^{-px(\cos \varepsilon - \cos \varepsilon x)} x^n dx
\] (11)
evaluated for \( \text{Re}(p) > 0 \), and where
\[
A_{2n+1}(p, \varepsilon) = \prod_{m=1}^{m=n} (p^2 + (\varepsilon m)^2)
\]
Using integration by parts twice, on equation 11, the following recursion formula is obtained
\[
P_{n+1}(p, \varepsilon) = P_n(p, \varepsilon) + \left( \frac{\tan \varepsilon}{\varepsilon} \right)^2 \frac{(p^2 + (\varepsilon n)^2)}{(2n+1)(2n-1)} P_{n-1}(p, \varepsilon)
\] (12)
with the initial conditions
\[
P_0(p, \varepsilon) = 1
\] (13)
and
\[
P_1(p, \varepsilon) = 1 + \left( \frac{\tan \varepsilon}{\varepsilon} \right) p
\] (14)
When \( \varepsilon \) is set to zero in equation 8, the polynomial degenerates to the familiar maximally flat Bessel polynomial, that is,
\[
P_n(p, 0) = \frac{e^{p^2n+1}}{(2n)!} \int_1^\infty e^{-px(x^2-1)^n} dx
\]
which is the optimum maximally flat solution.

From the previous discussion, the steps involved in forming the scattering transfer coefficient are as follows:

1. Calculate the equidistant linear phase polynomial using equation 12, 13, and 14.
2. Calculate \( D_n(p) \) and \( D^*_n(p) \) via equation 10,
3. Substitute \( D_n(p) \) and \( D^*_n(p) \) in equation 8 and 9.
For example, if a 4th order transfer function was to be computed, the first step involves starting with the initial condition given by equations 13 and 14. Assuming that \(\tan(\epsilon) = 1.0\), which implies that \(\epsilon = 0.78540\), then the initial conditions are

\[ P_0(p) = 1 \]

and

\[ P_1(p) = 1 + 1.27324p \]

Substituting this result into equation 12 leads to

\[ P_2(p) = 1 + 1.27324p + 1.62114(p^2 + 0.61685)/3 = 1.33333 + 1.27324p + 0.54038p^2 \]

From equation 10, the polynomial \(D_2(p)\) is calculated to be

\[ D_2(p) = 2(1.33333 + 1.27324p + 0.54038p^2) - (1 - j)(1.27324 + 2(0.54038)p) = 1.39343 + 1.46572p + 1.08076p^2 + j(1.27324 + 1.08076p) \]

The denominator of the reflection coefficient (equation 8) is

\[ D_2(p)D_2(p) = 3.5628 + 6.8369p + 6.3283p^2 + 3.1682p^3 + 1.1680p^4 \]

and from equation 9 the numerator is

\[ E_2(p) = \text{Im}(D_n(p)D_n^*(p)) = 3.5483 - 4.1605p^2 \]

As can be seen from this example, if the order of the transfer function is \(N\), 4 in this example, the order of the polynomial, \(P_n(p)\), needed is equal to \(N/2\). In Appendix B are tables of coefficients for transfer functions of order four to twelve with selected values of \(A\).
Properties of the Transfer Function

One of the properties of these functions is that their phase response is equal to a linear phase response at equal intervals. This can be seen from examining the error function (see Fig. 3), which is a measurement of how much the phase deviates from a constant. The coefficient $e$, in equation 11, determines the spacing between the points where the error function is equal to zero. Varying $e$, alters the amplitude performance and the group delay response of the transfer function. Figure 4 demonstrates how the amplitude and group delay of a 6th order transfer function is affected when $A=\tan e$ is varied. As $A$ is increased, a transmission zero moves in from $\omega=\infty$, resulting in a better shape factor. However, this results in an increase in return loss which degrades the stop-band performance of the transfer function.

Figure 3. Phase error function for the equidistant linear phase polynomial of order 6 (Redrawn from Rhodes, 1970).
Figure 4. (a) Amplitude and (b) group delay response for a fourth order transfer function.
filter. Increasing the value A also results in an increase in the group delay variation. When A is restricted to values which result in acceptable return loss and shape factor, the group delay stays fairly flat over 80 percent of the passband.

When an equidistant linear phase transfer function is compared to other types of constant delay transfer functions, it performs very well. Even though it does not have a flat group delay response over as wide a portion of its bandwidth as the Bessel function does, it does possess an amplitude characteristic that is better. For certain values of A, it has a 60dB shape factor and group delay response which compares to the Gaussian to 6dB and Butterworth transfer functions. One of the advantages that these functions have over the Gaussian and Butterworth transfer functions is its improved amplitude performance in the bandpass region. For transfer functions of sixth order and higher, the insertion is less than 0.1dB for 80 percent of the passband. Also of great value is that the coefficient A gives an extra degree of freedom to the designer.
III. MONOLITHIC CRYSTAL FILTER

Low-pass MCF Prototype

A monolithic crystal filter is usually comprised of a piezoelectric plate and a pair of thin film electrodes deposited on each side. As seen from Figure 5, a pair of electrodes form the input and output electrical ports (Smyth, 1979). When a voltage is applied between the input electrodes, the quartz underneath the electrodes is excited into a mechanical mode of resonance. The standing waves set up in this region decay exponentially in the unplated quartz, which has a lower cut-off frequency. The evanescent waves acoustically couple to the output resonator and confine the waves to the output electrode. By reciprocity, there is acoustic to electric conversion via the piezoelectric effect.

Electrode Pair
Resonators

Resonator
Regions

Figure 5. (a) Monolithic crystal filter and (b) schematic diagram (Redrawn and adapted from Smyth, 1972).
The electrical equivalent circuit for the monolithic crystal can best be obtained by first considering the equivalent circuit of a single quartz resonator. The crystal resonator equivalent schematic diagram and electrical equivalence are shown in Figure 6. The capacitor, $C_0$, represents the capacitance formed by the electrodes and the piezoelectric material between them. The motional inductor, $L_m$, and the motional capacitor, $C_m$, are the electrical equivalence of the mechanical resonator formed between the electrodes (Humphreys, 1970). The resistor, which depicts the losses of the crystal, has been omitted since it is a small value.

![Figure 6.](image)

In conventional bandpass filter design, it is typical to synthesize a low-pass filter prototype and then transform the prototype network into the required bandpass form; therefore, we will develop a low-pass network for the crystal. This network can then be used to directly synthesize
the low-pass prototype and then be transformed into the bandpass filter.

For the equivalent crystal resonator circuit in Figure 5a, a low-pass equivalent circuit can be obtained by first considering the low-pass to bandpass transformation.

\[ \Omega = \frac{(\omega^2 - \omega_0^2)}{\omega} \]

where \( \omega \) and \( \Omega \) are the low-pass and bandpass frequencies, respectively. The relationship for \( \omega \) is be obtained by using the quadratic equation which results in

\[ \Omega = \Omega/2 \pm \sqrt{\Omega^2/4 + \omega_0^2} \]

and since \( \Omega >> \omega \) this reduces to

\[ \omega = \omega_0 + \Omega/2 \]

From this bandpass to low-pass transformation, the admittance of Figure 5b may be approximated by

\[ Y(j\omega) = j\omega_0C_0 - j\left(\frac{\omega}{2\Omega(\omega - \omega_S)}\right) \]

where

\[ \omega_s = 1/L_mC_m \]

This results in the low-pass crystal network shown in Figure 5c, with constant susceptance \( B \) and constant reactance \( X \) having the following value (Holt, 1968):

\[ B = \omega_0C_0 \]
\[ X = 2L_m(\omega_0 - \omega_s) \]

The monolithic low-pass network can now be obtained by observing that it can be modeled by crystal resonators ar-
ranged in a lattice network, as shown in Figure 7a. Using the same bandpass to low-pass transformation, the low-pass MCF prototype lattice network is obtained (Figure 7b), also has an equivalent ladder network as shown in Figure 7c. The transmission matrix for this network is

$$\begin{bmatrix} \overline{A}(p) & \overline{B}(p) \\ \overline{C}(p) & \overline{D}(p) \end{bmatrix} = \frac{1}{j(1-p^2/p_0^2)} \begin{bmatrix} \overline{a}p & \overline{h}(1+\overline{b}p^2) \\ (1-\overline{c}p^2/h) & \overline{a}p \end{bmatrix}$$

where

$$\overline{a} = L/X(BX-1)$$
$$\overline{b} = L^2/X^2$$
$$\overline{c} = B'L^2/(BX-1)^2$$
$$h = X/(BX-1)$$

and

$$p_0^2 = X(BX-1)/BL^2$$

This network possesses a transmission zero at \( p = \pm p_0 \). The model developed will be very useful when synthesizing a transfer function. It will be used to remove zeros from the input admittance of the transfer function.

![Figure 7](image-url)

Figure 7 (a) MCF equivalent circuit with (b) low-pass lattice and (c) ladder prototype (Redrawn and adapted from Dillon, 1976).
Advantages of MCF

Monolithic filters have a practical range of center frequencies of about 5 MHz to 180 MHz. With new developments in lapping and polishing techniques it is possible to obtain center frequencies of up to 250 MHz. Monolithic filters have the advantage of low cost, small size, and higher quality factor (Q) than conventional LC circuits. Due to the higher Qs, ranging from 25,000 to 100,000, MCF can attain bandwidths of 0.01 to 0.35 percent of center frequency. They are very stable and the frequencies vary less than 0.55 ppm/°C over the temperature range 0° to 60°C. Due to their high performance, size and weight, MCFs are used in hand-held two-way radios, satellites, paging receivers and land and marine mobile radios. They are also used in telephone-frequency multiplex equipment (Smyth, 1979).
IV. SYNTHESIS PROCEDURE

The synthesis procedure chosen to realize the equidistant linear phase transfer function is one that was used by Dillon and Lind (1976) to realize a non-minimum phase transfer function with a MCF. The procedure involves removal of zero producing sections from the driving-point admittance. This is accomplished by considering the transmission matrix of the network to be realized, which is

\[
\begin{bmatrix}
A'_{11} & A'_{12} \\
A'_{21} & A'_{22}
\end{bmatrix} = \frac{1}{F(p)} \begin{bmatrix}
A' & B' \\
C' & D'
\end{bmatrix}
\]

and also that of a zero producing network which is

\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} = \frac{1}{f(s)} \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\]

The function \( f(p) \) is the product of the transmission zeros and is a factor of \( F(p) \). By matrix algebra, the network remaining possesses the following matrix:

\[
\frac{1}{f(p)F(p)} \begin{bmatrix}
D & -B \\
-C & A
\end{bmatrix} \begin{bmatrix}
A' & B' \\
C' & D'
\end{bmatrix}
\]

\[
= \frac{1}{f(p)F(p)} \begin{bmatrix}
DA' & -BC' & DB' & -BD' \\
AC' & -CA' & AD' & -CB'
\end{bmatrix}
\]

In order for the new network to be void of the transmission zero \( f(p) \), each term of the matrix must possess a factor of \( f^2(p) \). For example, if a zero at \( p_0 \) is to be removed, the
term, \( AC' - CA' \), must possess a factor \( p_0 \). Scanlan and Rhodes (1970) have shown that this can be assured by setting

\[
\{A-CZ(p)\}_{p=p_0} = 0
\]

and its derivative

\[
d/d_p\{A-CZ(p)\}_{p=p_0} = 0
\]

where the relationship

\[
Z(p) = \frac{A_{11}}{A_{21}} = \frac{A'}{C'}
\]

was used. The driving-point impedance remaining, under the previous requirement, is

\[
Z_1(p) = \frac{D'Z(p) - B'}{A' - C'Z(p)}
\]

If the transmission zero to be removed is at real frequencies, \( p = j\omega_0 \), it is extracted via a Brune section whose transmission matrix is

\[
\frac{1}{f(p)} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{1-p^2/\omega_0^2} \begin{bmatrix} 1+ap^2 & bp \\ cp & 1+dp^2 \end{bmatrix}
\]

From the previous discussion, it is required that

\[
1 + ap^2 - cpZ(p) \big|_{p^2=-\omega_0^2} = 0
\]

and

\[
2ap - cZ(p) - cpZ'(p) \big|_{p^2=-\omega_0^2} = 0
\]

in order to remove this zero. This then reduces to

\[
1 - a\omega_0^2 + c\omega_0 X(\omega_0) = 0
\]

and

\[
2\omega_0 a - cX(\omega_0) - c\omega_0 X'(\omega_0) = 0
\]

since \( Z(j\omega_0) = jX(\omega_0) \), and \( Z'(j\omega_0) = X'(\omega_0) \) for lossless networks. Solving for the coefficient \( a \) and \( c \) results in

\[
a = \frac{\omega_0 X'(\omega_0) + X(\omega_0)}{\omega_0^2 \{\omega_0 X'(\omega_0) - X(\omega_0)\}}
\]
and

\[ c = \frac{2}{\omega_0 \{ \omega_0 X'(\omega_0) - X(\omega_0) \}} \]  

(20)

Using the same technique on DA'-BC', the following result is obtained.

\[ b = \frac{2X^2(\omega_0)}{\omega_0 \{ \omega_0 X'(\omega_0) + X(\omega_0) \}} \]
\[ d = \frac{\omega_0 X'(\omega_0) - X(\omega_0)}{\omega_0 \{ \omega_0 X'(\omega_0) + X(\omega_0) \}} \]  

(21)

Recalling the transmission matrix of the monolithic crystal filter, in equations 15 and 16, and adding an inverter at its' output, results in

\[
\begin{bmatrix}
\bar{A}(p) & \bar{B}(p) \\
\bar{C}(p) & \bar{D}(p)
\end{bmatrix}
= \frac{1}{J(1-p^2/p_0^2)} \begin{bmatrix}
ap & h(1+b p^2) \\
1+cp^2/h & \bar{ap}
\end{bmatrix} \begin{bmatrix}
o & Jk \\
j/k & 0
\end{bmatrix}
= \frac{1}{(1-p^2/p_0^2)} \begin{bmatrix}
1+ap & bp \\
cp & 1+dp^2
\end{bmatrix}
\]

where

\[ a = b = L^2/X^2 \]
\[ b = \bar{a}K = L/(BX-1)^2 \]
\[ c = \bar{a}/K = L/X^2 \]
\[ d = \bar{d} = B^2L^2/(BX-1)^2 \]

(22)

and

\[ K = h = X/(BX-1) \]

Setting K=h ensures that the network is real and transparent at dc. Comparing this matrix to that of the Brune
section, in equation 18, one can see that the monolithic filter can be used to realize such a section. By inspection, the element values for the prototype MCF section are found to be

\[ L = \frac{a}{c} \]
\[ X = \frac{a^\frac{1}{2}}{c} \]

and since from equation 16 \( \omega_0^2 = -p_0^2 = X(BX-1)/BL \), then

\[ B = \frac{c}{(1-\omega_0^2)a^\frac{1}{2}} \]

and

\[ K = \frac{(1-\omega_0^2)/\omega_0^2ca^\frac{1}{2}}{1} \]

The impedance inverter is realized by the network shown in Figure 8.

![Figure 8. Impedance inverter.](image)

For removal of zeros that are located along the real axis, \( p = \pm \sigma_0 \), the Darlington C-section, which possesses the following matrix, is used.

\[
\frac{1}{(1-p^2/\sigma_0^2)} \begin{bmatrix} 1+ap & bp \\ cp & 1+dp \end{bmatrix}
\]
By utilizing the same procedure as before, the following result is obtained.

\[
\begin{align*}
a &= \frac{\sigma_0 Z'(\sigma_0) + Z(\sigma_0)}{\sigma_0^2 \{Z(\sigma_0) - \sigma_0 Z'(\sigma_0)\}} \\
b &= \frac{2Z^2(\sigma_0)}{b\{Z(\sigma_0) - \sigma_0 Z'(\sigma_0)\}} \\
c &= \frac{2}{\sigma_0^2 \{Z(\sigma_0) - \sigma_0 Z'(\sigma_0)\}} \\
d &= \frac{Z(\sigma_0) - Z'(\sigma_0)}{\sigma_0^2 Z(\sigma_0) + Z'(\sigma_0)}
\end{align*}
\]  

The values for the MCF network, which will be used to realize this section, are as follows:

\[
\begin{align*}
L &= a/c \\
x &= a^{1/2}/c \\
B &= c/(1+a\sigma_0^2)a^{1/2} \\
K &= -(1+a\sigma_0^2)/c\sigma_0^2a^{1/2}
\end{align*}
\]  

The removal of a zero at infinity is usually realized by a series inductor, L, followed by a shunt capacitor, C. The MCF can be used to synthesize such a network by setting the shunt susceptance, B, to zero in equation 22. This results in the following transmission matrix:

\[
\begin{bmatrix}
L + \frac{L^2}{X^2} & Lp \\
\frac{L}{X^2} & p \\
\frac{L}{X^2} & 1
\end{bmatrix}
\]

where

\[
\begin{align*}
L &= L_1 \\
x &= (L_1/C_1)^{1/2} \\
R &= 0 \\
K &= X
\end{align*}
\]
Figure 9. MCF networks which remove zeros at infinity.

With the three zero removing sections, it is now possible to realize any low-pass transfer function with MCF, which possess real zeros, imaginary zeros, or double zeros at infinity. From the tables of coefficients in Appendix B, it is observed that it is possible to realize up to a 6th order transfer function. Synthesis of higher order functions would require the removal of zeros at complex frequencies and that is beyond the scope of this thesis.

As a simple example of the realization procedure, a fourth order equidistant linear phase transfer function is synthesized. For this example, the constant $A$ is set to zero resulting in the following maximally flat transfer function, which has been normalized to 3dB at $\omega=1$.

$$S_{12}(p) = \frac{2-1.18913p^2}{2 + 6.54284p + 9.51306p^2 + 7.78030p^3 + 3.18158p^4}$$

This transfer function possesses a pair of transmission zeros at $p=\pm\alpha_0=\pm1.29688$ and at infinity. The first step in synthesizing this transfer function is to obtain its input
impedance. The input impedance is related to the transmission parameter, $S_{11}(p)$, which can be obtained from the transmission parameter. For lossless networks, the relationship between the transmission and reflection parameters is

$$S_{11}(p)S^*_{11}(p) = 1 - S_{12}(p)S^*_{12}(p)$$

(28)

This expression can be simplified to

$$N_{11}(p)N^*_{11}(p) = D(p)D^*(p) - N_{12}(p)N^*_{12}(p)$$

where the following relationships were taken into consideration

$$S_{12}(p) = \frac{N_{12}(p)}{D(p)} \quad S_{11}(p) = \frac{N_{11}(p)}{D(p)}$$

The reflection parameter can now be obtained by finding the roots of the right side of equation 28 and assigning the left half zeros to $N_{11}(p)$ and the right half plane zeros to $N^*_{11}(p)$. Using this technique the reflection parameter is found to be

$$S_{11}(p) = \frac{-0.00014p^3 + p^4}{2 + 6.54284p + 9.51306p^2 + 7.78030p^3 + 3.18158p^4}$$

Now the input impedance can be calculated since

$$Z(p) = \frac{1 + S_{11}(p)}{1 - S_{11}(p)}$$

which results in

$$Z(p) = \frac{0.62862 + 2.05648p + 2.99004p^2 + 2.44528p^3 + p^4}{0.62862 + 2.05648p + 2.99004p^2 + 2.44556p^3}$$
To remove the zero at \( \sigma_0 = \pm 1.29688 \) we must use a Darlington C-section. From equation 28 the transmission matrix coefficients are found to be

\[
\begin{align*}
    a &= 2.08083 \\
    b &= 1.40201 \\
    c &= 2.45352 \\
    d &= 1.69888
\end{align*}
\]

since \( \mathbf{Z}(\sigma_0) = 1.36589 \) and \( \mathbf{Z}'(\sigma_0) = 1.93158 \)

The values of the low pass MCF prototype network used to realize this section are (see eq.26)

\[
\begin{align*}
    L &= 0.84810072 \\
    X &= 0.58793455 \\
    B &= 0.37799218 \\
    K &= 0.75592794
\end{align*}
\]

By use of equation 17 the remaining input impedance is found to be

\[
Z_1(p) = 0.53493 \left( \frac{0.65402 + 1.22264p + p^2}{1.22264 + p} \right)
\]

To remove the pair of zero at infinity, a series inductor and a shunt capacitor are removed. An inductor equal to

\[
L = \left[ \frac{Z(p)}{p} \right]_{p=\infty}
\]

is first removed, leaving an input impedance of

\[
Z_1(p) = Z(p) - pL
\]

Next a shunt capacitor equal to

\[
C = \left[ \frac{Y_1(p)}{p} \right]_{p=\infty}
\]
is removed, which leaves the following input admittance (Saal, 1958)

\[ Y_2(p) = Y_1(p) - pC \]

Removing the inductor and capacitor from the remaining input impedance results in

\[
L_2 = 1.86941278 \\
C_2 = 0.81790076
\]

The LC-circuit can now be realized by the MCF low-pass equivalent circuit shown in Figure 9. The circuit elements of the MCF network are (equation 27)

\[
L_2 = 1.86941278 \\
X_2 = 1.51182770 \\
B_2 = 0 \\
K_2 = 1.51182770
\]

Since the source load, \( R_S \), is one ohm, the impedance inverter is transparent at dc, and \( S_{12}(0)=1 \) the load resistance, \( R_L \), must also be equal to one ohm. The low-pass MCF prototype network must now be transformed to a bandpass network before it can be realized. The narrow band low-pass to bandpass transformation is a well known procedure; therefore, it is only summerized in Figure 10. Transforming the network to a center frequency of 10.7MHz with a 3dB bandwidth of 10KHz and load termination of 3kΩ results in the network shown in Figure 11.
LOW-PASS

\[ L \quad \rightarrow \quad L_{BP} \quad C_{BP} \]

\[ L_{BP} = \frac{L R_a}{2\pi f_{BW}} \quad C = \frac{1}{(2\pi f_0)^2 L_{BP}} \]

BANDPASS

\[ L_{BP} = \frac{1}{(2\pi f_0)^2 C_{BP}} \quad C_{BP} = \frac{C}{2\pi f_{BW} R_0} \]

\[ L_K = \frac{X R_0}{2\pi f_0} \]

\[ C_x = \frac{2\pi f_0 R_0 X}{2\pi f_0} \]

\[ C_B = \frac{2\pi f_0 R_0}{2\pi f_0} \]

\[ L_B = \frac{R_0}{2\pi f_0} \]

**BW = BANDWIDTH \quad R = TERMINATION**

Figure 10. Narrow band low-pass to bandpass transformation
Element values:

\[ (C_i = C_i + C_0) \]

\[ C_1' = 1.87399\text{pF} \]
\[ C_2' = 8.4329\text{pF} \]
\[ C_3' = 6.5589\text{pF} \]
\[ C_4' = 3.2794\text{pF} \]
\[ C_5' = 3.2794\text{pF} \]

\[ L_1 = 118.056\mu\text{H} \]
\[ L_2 = 33.732\mu\text{H} \]
\[ L_3 = 67.463\mu\text{H} \]
\[ R_S = R_L = 3\text{k}\Omega \]

\[ L = 40.4966\text{mH} \]
\[ L = 89.2566\text{mH} \]
\[ C = 5.4598 \times 10^{-3}\text{pF} \]
\[ C = 2.4769 \times 10^{-3}\text{pF} \]
\[ C_m = 8.43296\text{pF} \]
\[ C_m = 3.2795\text{pF} \]

Figure 11. Bandpass monolithic crystal filter equivalent circuit for a maximally flat delay filter of fourth order.
V. CAD PROGRAM

The computer aided design (CAD) program which was developed performs three main functions. First, it generates and outputs, in factored form, an equidistant linear phase transfer function given the order of the transfer function, and the coefficient A. After this is generated, the program can analyze the transfer function and output the amplitude, phase, or group delay response in tabular or graphical form. The third function synthesizes the transfer function. This function outputs the inductor and capacitor values for the Brune Section and Darlington C-section that are removed from the input admittance. It also removes series inductors and shunt capacitors, and outputs the equivalent MCF section. The program was developed on a Hewlett Packart 1000 computer using Fortran 77 language. A listing of the program can be found in Appendix A.

The different functions are selected via the menu shown below. The use of and operation of the program will be explained by going through an example of synthesizing a sixth order transfer function. The first step in the syn-

1) CALCULATE RHODE’S TRANSFER FUNCTION
2) CALCULATE PHASE, ATTEN., AND GROUP DELAY (LOSSLESS)
3) CALCULATE PHASE, ATTEN., AND GROUP DELAY (LOSSY)
4) PLOT ATTENUATION (LOSSY:-4)
5) PLOT PHASE (LOSSY:-5)
6) PLOT GROUP DELAY (LOSSY:-6)
7) SYNTHESIZE
8) END
thesis procedure is to generate the transfer function. This is accomplished by entering a "1" in response to the menu.

The program now prompts the user for the order and the phase flatness coefficient, A.

INPUT ORDER OF THE RHODES TRANSFER FUNCTION(MAX.12)?
INPUT THE PHASE FLATNESS CONSTANT A?

For this example, the order 6 and A=1.35, is entered in the computer. From these two entries, the computer calculates the transfer function. This is accomplished in the program from lines 54 to 85 via the recurrence formula (equation 12). The program then normalizes the transfer function to 3dB at $\omega=1$, lines 86 to 154 of the program. This part of the program uses the subroutine PLYFRO which calculates the complex response of the transfer function. The output of this subroutine, array H, can then be used to calculate the amplitude, phase, and group delay response at any given frequency.

After the transfer function is normalized, the program finds the roots of the function and outputs the transfer function in factored form, which, for this example, is

$$KK=-0.00727472$$

<table>
<thead>
<tr>
<th>i</th>
<th>a(i)</th>
<th>b(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(.8293480143, j0.0000000000)</td>
<td>(-.412881273, j.5983415884)</td>
</tr>
<tr>
<td>2</td>
<td>(-.829348014, j0.0000000000)</td>
<td>(-.412881273, j-.598341588)</td>
</tr>
<tr>
<td>3</td>
<td>(0.0000000000, j4.861132000)</td>
<td>(-.456633694, j1.791018443)</td>
</tr>
<tr>
<td>4</td>
<td>(0.0000000000, j-4.861132000)</td>
<td>(-.456633694, j-.179101844)</td>
</tr>
<tr>
<td>5</td>
<td>(0.0000000000, j0.0000000000)</td>
<td>(-.170235601, j.9494062166)</td>
</tr>
<tr>
<td>6</td>
<td>(0.0000000000, j0.0000000000)</td>
<td>(-.170235601, j-.949406217)</td>
</tr>
</tbody>
</table>
The roots of the function are found by the subroutine ROOT. This subroutine uses a modified Newton's method. The subroutine, by use of the random number generator, first guesses at the location of the root of a polynomial. If the polynomial is \( M(p) \) and the guess is at \( p = \tau \), then a better guess is calculated at \( p = \tau + \delta \tau \), where \( \delta \tau = -M(p)/m'(p)|_{p=\tau} \). This process is called Newton's method and is performed until the root is found to a given degree of accuracy. To make sure that the same root will not be found, the method is now modified. The increment, \( \delta \tau \), is now defined as 
\[
\delta \tau = -\frac{N(p)/N'(p)|_{p=\tau}}{M'(p) - \frac{G'(p)M(p)}{G(p)}}|_{p=\tau}
\]
This modification makes the new \( \tau \) move away from any previously found roots (Skwirzynski, 1971).

Once the transfer function is output in factored form, the user can return to the main menu by entering a "1" or terminate by entering anything other than a "1". The menu is reprinted and the user can again choose any of the seven options. Functions two to six are designed to help the user to determine when he has generated the transfer function which will accomplish his design requirements.

Function two will generate, in tabular form, the amplitude, phase and group delay of a transfer function for a
given set of frequencies. Once this function is selected, the program will give the option of linear or logarithmic frequency increments. It will also ask for the starting and finishing frequencies and the increment stops. An example of such a run is shown below:

LOG OR LINEAR FREQ. SCALE \((1=\text{LINEAR}, 2=\text{LOG})\).
ENTER THE LOWER FREQ.
ENTER THE UPPER FREQ.
ENTER THE FREQ. INCREMENTS LIMIT.
PRINT DATA (Y/N)?

<table>
<thead>
<tr>
<th>FREQUENCY (HZ)</th>
<th>ATTENUATION (DB)</th>
<th>PHASE (DEG)</th>
<th>GROUP DELAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000000</td>
<td>0.003896</td>
<td>0.000000</td>
<td>0.5724E+01</td>
</tr>
<tr>
<td>1.000000</td>
<td>3.000102</td>
<td>-8.967745</td>
<td>7.641E+01</td>
</tr>
<tr>
<td>2.000000</td>
<td>37.555191</td>
<td>116.13733</td>
<td>6.445E+00</td>
</tr>
<tr>
<td>3.000000</td>
<td>56.110466</td>
<td>139.11703</td>
<td>2.523E+00</td>
</tr>
<tr>
<td>4.000000</td>
<td>72.424210</td>
<td>149.73395</td>
<td>1.364E+00</td>
</tr>
<tr>
<td>5.000000</td>
<td>95.404800</td>
<td>-24.07182</td>
<td>8.576E-01</td>
</tr>
<tr>
<td>6.000000</td>
<td>82.789734</td>
<td>-19.99720</td>
<td>5.899E-01</td>
</tr>
<tr>
<td>7.000000</td>
<td>82.006805</td>
<td>-17.10841</td>
<td>4.310E-01</td>
</tr>
<tr>
<td>8.000000</td>
<td>82.676590</td>
<td>-14.95178</td>
<td>3.288E-01</td>
</tr>
<tr>
<td>9.000000</td>
<td>83.760971</td>
<td>-13.27949</td>
<td>2.591E-01</td>
</tr>
<tr>
<td>10.000000</td>
<td>84.968811</td>
<td>-11.94450</td>
<td>2.095E-01</td>
</tr>
</tbody>
</table>

HIT ONE RETURN TO CONTINUE.

Functions 4, 5 and 6 output the data in graphical form. Examples of such output are shown in Appendix B. The plot routines used for this program were developed at Piezo Technology Inc. Functions 3, 4, 5 and 6 insert uniform losses by inserting a real part in the complex frequency equal to the inverse of the normalized Q. This option allows the designer to foresee the response of the real lossy network that will be built.
After the user has chosen the transfer function which will meet his design requirements, the next step is to synthesize it. This is accomplished by responding with a "7" to the prompt from the menu. This results in the subroutine SYN being called by the main program. In the subroutine SYN, the reflection coefficient, \( S_{11}(p) \), is calculated from which the input admittance is generated. This is accomplished in lines 363 to 420 as is prescribed in Section IV. The input admittance for this example is

**INPUT ADMITTANCE**

<table>
<thead>
<tr>
<th>i</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.14750817E+00</td>
<td>1.21835587E+00</td>
</tr>
<tr>
<td>1</td>
<td>6.77156797E+00</td>
<td>6.77156797E+00</td>
</tr>
<tr>
<td>2</td>
<td>1.61199239E+01</td>
<td>1.94050165E+01</td>
</tr>
<tr>
<td>3</td>
<td>2.81784074E+01</td>
<td>2.81784074E+01</td>
</tr>
<tr>
<td>4</td>
<td>2.16216246E+01</td>
<td>3.92915623E+01</td>
</tr>
<tr>
<td>5</td>
<td>2.07950114E+01</td>
<td>2.07950114E+01</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000E+00</td>
<td>2.000000000E+01</td>
</tr>
</tbody>
</table>

After the input admittance is printed the following menu appears

1) REMOVE C-SECTION (ZERO AT IMAGINARY FREQ.)
2) REMOVE BRUNE SECTION (ZERO AT REAL FREQ.)
3) REMOVE ZEROS AT INFINITY
4) RESTART SYNTHESIS PROCEDURE
5) END SYNTHESIS.

This menu offers the user the choice of removing any of the three zero producing sections presented in Section IV. Since the network component values will vary depending on the order that the sections are removed, the filter designer may wish to try different removal sequences and select the best result. Option "4" makes it possible for the user to regen-
erate the original input admittance and try a new removal sequence. Selecting option "5" will end the synthesis portion of the program and return to the main menu.

If any of the first three functions are selected, the program outputs the low-pass component values for each section and the remaining input admittance. If the remaining admittance did not reduce in complexity, the synthesis procedure will have to be restarted. The component values are not transformed to bandpass components since some simplification may be done in the low-pass domain before it is transformed.

From the transfer function coefficients on page 32, it is observed that there is a pair of zeros on the real axis. Therefore, a "1" is entered in response to the menu on page 35 in order to remove these zeros.

**ZERO TO BE REMOVED**
The program now asks the user which zero should be removed. In this case the proper response is "1", since the first zero is to be removed.

**LOW-PASS COMPONENT VALUES FOR THE C-SECTION**

<table>
<thead>
<tr>
<th>i</th>
<th>a(i)</th>
<th>b(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.31721463E+00</td>
<td>.18882191E+00</td>
</tr>
<tr>
<td>1</td>
<td>.91427669E+00</td>
<td>.63117812E+00</td>
</tr>
<tr>
<td>2</td>
<td>.10394897E+01</td>
<td>.12164161E+01</td>
</tr>
</tbody>
</table>

**REMAINING INPUT ADMITTANCE**

K = .56063579
The program has output the component values for the C-section, the remaining input admittance, and returned to the menu. Next, the zeros at real frequencies will be removed using the Brune section, which results in the following:

**LOW-PASS COMPONENT VALUES FOR THE BRUNE SECTION**

<table>
<thead>
<tr>
<th>i</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.0000000E+01</td>
<td>1.0394897E+01</td>
</tr>
<tr>
<td>4</td>
<td>0.0000000E+00</td>
<td>1.0000000E+00</td>
</tr>
<tr>
<td>5</td>
<td>0.0000000E+00</td>
<td>0.0000000E+00</td>
</tr>
<tr>
<td>6</td>
<td>0.0000000E+00</td>
<td>0.0000000E+00</td>
</tr>
</tbody>
</table>

**REMAINING INPUT ADMITTANCE**

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0394897E+01</td>
<td>6.0921618E+00</td>
</tr>
<tr>
<td>1</td>
<td>1.0000000E+01</td>
<td>1.0394897E+01</td>
</tr>
<tr>
<td>2</td>
<td>0.0000000E+00</td>
<td>1.0000000E+00</td>
</tr>
</tbody>
</table>

Now all that remains is to remove the zeros at infinity. This is accomplished by selecting option "3". The results are shown below with the admittance remaining.

**THE COMPONENT VALUES FOR THE EQUIVALENT SERIES L SHUNT C ARE**

<table>
<thead>
<tr>
<th>( L )</th>
<th>( X )</th>
<th>( B )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.81162008</td>
<td>1.41401117</td>
<td>0</td>
<td>1.41401117</td>
</tr>
</tbody>
</table>

**REMAINING INPUT ADMITTANCE**

\( Y(0) = 0.94184978 \)

Transforming these results to a bandpass filter with a center frequency of 10.7 MHz, a 3dB bandwidth of 10 KHz and a load termination of 3K\( \Omega \) results in the circuit shown in Figure 12.
Element values:

\[ (C_i' = C_i + C_o) \]

\[
\begin{align*}
C_1' &= 2.554 \text{pF} & L_1 &= 86.618 \mu\text{H} & L &= 52.103 \text{mH} & L &= 85.179 \text{mH} & L &= 86.498 \text{mH} \\
C_2' &= 8.005 \text{pF} & L_2 &= 40.59 \mu\text{H} & C_m &= 8.005 \text{pF} & C_m &= 4.992 \text{pF} & C_m &= 4.349 \text{pF} \\
C_3' &= 5.385 \text{pF} & C_8 &= 6.638 \times 10^{-2} \text{pF} & C &= 4.244 \times 10^{-3} \text{pF} & C &= 2.596 \times 10^{-3} \text{pF} & C &= 2.556 \times 10^{-3} \text{pF} \\
C_4' &= 4.992 \text{pF} & L_4 &= 43.738 \mu\text{H} & & & & \\
C_5' &= 5.058 \text{pF} & L_5 &= 50.915 \mu\text{H} & & & & \\
C_6' &= 3.5064 \text{pF} & R_L &= R_S &= 3 \text{K}\Omega & & & \\
C_7' &= 3.5064 \text{pF} & & & & & \\
\end{align*}
\]

Figure 12. Bandpass monolithic crystal filter equivalent circuit for a maximally flat delay filter of sixth order.
CONCLUSION

The equidistant linear phase transfer function offers a suitable alternative for filter designers when implementing a constant group delay filter. They have excellent passband performance and group delay response, while having good shape factors.

These transfer functions can be easily synthesized, using MCF, by utilizing Brune sections and Darlington C-sections to remove the finite zeros of transmission. With the aid of the CAD program developed in this thesis, the synthesis procedure is considerably reduced in complexity. The CAD program greatly reduces the time spent designing the filter and offers the designer good flexibility. With the aid of the CAD program, the designer can generate several alternative designs in a fraction of the time it would have taken him to tabulate one such design without the aid of the program. Thus, the cost is greatly reduced.
APPENDIX A

PROGRAM
PROGRAM RHODES(30425.0955)

THIS PROGRAM ANALYSES, AND SYNTHESIZES RHODES' EQUIDISTANT LINEAR PHASE TRANSFER FUNCTION

DOUBLE PRECISION A(-2:7,-2:7),E,EE,C,DA<0:7>,DNSQ<8:13>,FW<20i>,
XEPS,FWEXL,DELEXP,DFWPC,FWUNC,H(2,201),NUM(0:13),AA,WN,FWT(201)

DOUBLE COMPLEX Z,DNC<0:7>,H<201>,DHC<201>,FNUM<14>,F1<EN<14>

INTEGER FLAG1,FUNC,FLAG2

EQUIVALENCE (H,HH)

COMMON IHP,LUPLT

IMP=1 $ FLAGi=0

DO 3 I=0,22
3 PRINP
PRINT'(19X,•************')
PRINT'(19X,•$ RHODE'S TRANSFER FUNCTION SYNTHESIS PROGRAM $')
PRINT'(19X,•***********************************************')
PRINT'(19X,•$ CALCULATE RHODE'S TRANSFER FUNCTION $')
PRINT'(19X,•$ CALUCLATE PHASE, ATTEN., AND GROUP DELAY (LOSSLESS) $')
PRINT'(19X,•$ CALCULATE PHASE, ATTEN., AND GROUP DELAY (LOSSY) $')
PRINT'(19X,•$ PLOT ATTENUATION (LOSSY:-4) $')
PRINT'(19X,•$ PLOT PHASE (LOSSY:-5) $')
PRINT'(19X,•$ PLOT GROUP DELAY (LOSSY:-6) $')
PRINT'(19X,•$ SYNTHESIZE *')
PRINT'(19X,•$END$') $ READi,FUNC

DO TO(1,50,50,50,50,50,170),ABS(FUNC)

CALL INPUT ORDER OF THE RHODES TRANSFER FUNCTION(MAX.12)?...

X $ READi,NN
N=NN/2
PRINT('" *')
PRINT('"INPUT THE PHASE FLATNESS CONSTANT A?_"') $ READ*,AA
PRINT('" ")
E=ATAN(AA**.5)

C
CALCULATING Pn(π) USING THE RECURRANCE FORMULA

A(0,0)=ID0 $ A(1,0)=ID0 $ A(1,1)=(AA**.5)/E
DO 40 J = 1,N-1
EE=(J*EE)**2 $ C=AA/((E**2)*(2D0*J+1)*(2D0*J-1))
DO 35 I = 0,J+1
A(I+1,I)=A(J,I)+C*(A(I,J)*EE+A(J-1,I-2))
35 CONTINUE
40 CONTINUE

***CALCULATING THE EVEN ORDER TRANSFER FUNCTION***

Z=(1.0D0,-1.0D0)

CALCULATING THE DERIVATIVE OF Pn(π)
AND Dn(p)=2nPn(π)-(1-j)Pn'(π)

DO 60 I=0,N
DA(I)=(I+1)*A(N,I+1)
DN(I)=2*A(N,I)-2*DA(I)
60 CONTINUE

CALCULATING THE DENOMINATOR, Dn(π)=Dn(π)*CONJG(Dn(π)), AND
THE NUMERATOR, Nn(π)=IMIDn(π)*Dn(-π), OF THE TRANSFER FUNCTION

DO 115 I=0,N
115 CONTINUE

FINDING THE NORMALIZATION CONSTANT

EPS=0D0 $ FWEXL=0D0 $ DELEXP=0D0 $ DFWPC=0.1D0 $ FWUNC=1D0
WN=1D0
DO 160 I=1,200
FW(I)=0D0 $ GDPY(I)=0 $ ATLEN(I)=0 $ H(I)=(0D0,0D0)
DH(I)=(0D0,0D0)
160 CONTINUE

PHASE(I)=0 $ H(I)=(0D0,0D0)

DO 160 CONTINUE

NW=(FWUNC-FWEXL)/DFWPC+1.0

FW(1)=FWEXL
DO 8 I=2,NW
8 FW(I)=FW(I-1)+DFWPC

CALL PLYFRQCNW(FW,H,DH,EPS,NUH,DNSQ,NN,WN)

DO 12 I=1,NW
12 AMPL=HH(I,1)**2+HH(I,2)**2
ATTLEN(I)=-1D0*ALOGT(AMPL)
IF (ATTEN(NW), LT, 3.0) THEN
FWEXL = FWUNC $ FWUNC = FWUNC + 1.0D0
GO TO 210
ELSE
DO 220 K = 1, 5
DU = 3.0 + 1.0 * 10.0 ** (-K)
DL = 3.0 - 1.0 * 10.0 ** (-K)
IBE = 1 $ IEND = NW
250 IF (IEND, EQ, IBEG + 1.0) GO TO 260
MID = (IBE + IEND) / 2
IF (DU, LT, ATTEN(MID)) THEN
IEND = MID $ GO TO 250
ELSE
IF (DL, GT, ATTEN(MID)) THEN
IBE = MID $ GO TO 250
ENDIF
ENDIF
260 IF (ATTEN(IEND), GT, 3.00001) GO TO 230
IF (ATTEN(IBEG), LT, 2.99999) GO TO 230
WN = FW(IEND)
GO TO 290
230 FWEXL = FW(IBEG) $ FWUNC = FW(IEND) $ DFWPC = 10.0 ** (-K - 1)
WN = 1.0
IF (K, EQ, 5) THEN
WN = FW(IEND) $ GO TO 290
ELSE
NW = (FWUNC - FWEXL) / DFWPC + 1.0
IF (NW, GT, 200) NW = 200
FW(1) = FWEXL
DO 280 II = 2, NW
FW(II) = FW(II - 1) + DFWPC
CALL PLYFRQ(NW, FW, H, EPS, NUM, DNSQ, NN, WN)
DO 270 J = 1, NW
AMPL = HH(J, J)**2 + HH(2, J)**2
ATTEN(J) = -I0D0 * ALOGT(AMPL)
ENDIF
290 IF (CHN.LT, I) GOTO 15
DO 4 I = 0, NN
NUM(I) = NUM(I) * WN ** (I)
IF (ABS(NUM(I)), LT, 1D-10) NUM(I) = 0D0
4 CONTINUE
DO 5 I = 0, NN
DNSQ(I) = DNSQ(I) * WN ** (I)
IF (ABS(DNSQ(I)), LT, 1D-10) DNSQ(I) = 0D0
5 CONTINUE
KK = NUM(NN - 2) / DNSQ(NN) $ WN = 1.0
C 1157  CALCULATE THE ROOT OF THE TRANSFER FUNCTION
C 1158  CALL ROOT(NH-2,NX,FNUM,IZ,NI3)
C 1159  CALL ROOT(NH,DSQ,FDEN,ON,NI3)
C 1160  C 1161  OUTPUT
C 1162  C 1163  PRINT('" "')
C 1164  PRINT('"35X,"KK=",F10.8)"',KK
C 1165  PRINT('" "')
C 1166  PRINT('"8X," i",16X,"a(i)",29X,"b(i)")' 
C 1167  PRINT('" "')
C 1168  DO 13 I=1,NH
C 1169  PRINT('"9X,i2," ("F11.10","j",F11.10,") ("F11.10,
C 1170  X","j",F11.10,")"),I,FNUM(I),FDEN(I)
C 1171  13 CONTINUE
C 1172  C 1173  PRINT('" "')
C 1174  PRINT('"HI1 ONE RETURN TO CONTINUE._")? $ READ*,CONT
C 1175  IF(CONT.EQ.1) GO TO 110
C 1176  C 1177  500 CALL SYM(NH,NX,DSQ,FNUM)
C 1178  C 1179  GO TO 110
C 1180  C 1181  C 1182  INITIALIZING VARIABLES
C 1183  50 IF(FLAG1.EQ.0) THEN
C 1184  NFIN=1H $ GOTO 202 $ENDIF
C 1185  C 1186  PRINT('" "')
C 1187  PRINT('"USE PREVIOUS FREQ. INCREMENTS (Y/N)?_")? $READ',A2)',NFIN
C 1188  IF(NFIN/400B.EQ.131B) THEN
C 1189  NW=NWT
C 1190  DO 201 I=1,NW
C 1191  201 FW(I)=FWT(I)
C 1192  GOTO 203
C 1193  ELSE
C 1194  DEEXP=0DO
C 1195  DO 200 I=1,200
C 1196  200 FW(I)=0DO
C 1197  ENDF
C 1198  203 DO 204 I=1,200
C 1199  GDLY(I)=0 $ ATTEN(I)=0 $ H(I)=(0D0,0D0) $ DH(I)=(0D0,0D0)
C 2000  PHASE(I)=0
C 2001  204 CONTINUE
C 2002  IF(FUNC.GT.3.0R.FUNC.EQ.2) THEN $ EPS=0DO
C 2003  ELSE
C 2004  C 2005  C 2006  EVALUATING THE TRANSMISSION LOSS, PHASE, GROUP DELAY,
C 2007  C 2008  PRINT('" "')
C 2009  PRINT('"INPUT THE NORMALIZED Q FACTOR._")? $ READ*,Q
PRINT"(" ")'
EPS=1D0/0
ENDIF
IF(NFIN/400B.EQ.131B) GOTO 11
PRINT"(" ")'
PRINT(""LOG OR LINEAR FREQ. SCALE (1=LINEAR,2=LOG)"")'
X $ READ*,LNLP
GO TO (2,6),LNLP
2 PRINT"(" ")'
PRINT(""ENTER THE LOWER FREQ._")' $ READ*,FWEXL
PRINT"(" ")'
PRINT(""ENTER UPPER FREQ. LIMIT._")' $ READ*,FWUNC
PRINT(""ENTER FREQ. INCREMENTS SIZE._")' $ READ*,DFWPC
C C
CALCULATING LINEAR FREQUENCY INCREMENTS
NW=(FWUNC-FWEXL)/DFWPC+1.0
IF(NW.GT.200) NW=200
FM(1)=FWEXL
DO 7 I=2,NW
FW(I)=FW(I-1)+DFWPC
GO TO 11
6 PRINT"(" ")'
PRINT(""INPUT LOWER FREQ. (INPUT X WHERE F=10**X)._")'
X $ READ*,FWEXL
PRINT"(" ")'
PRINT(""HOW MANY LOGARITHMIC CYCLES DO YOU DESIRE._")' $ READ*,FWUNC
PRINT("" HOW MANY POINTS DO YOU WHICH EVALUATED PER CYCLE? _")'
READ*,DFWPC
C C
CALCULATING THE LOGARITHMIC FREQUENCY INCREMENTS
NW=FWUNC*DFWPC+1.0
IF (NW GT 200) NW=200
DELEXP=1.0/DFWPC
DO 9 I=1,NW
FM(I)=10D0*FWEXL
FMEXL=FWEXL+DELEXP
9 CALL PLYFRG(NW,FM,H,DH,EPS,NUM,DNSQ,NN,WN)
C C
COMPUTING THE TRANSFER PARAMETERS
DO 14 I=1,NW
AMPL=HH(1,I)**2+HH(2,I)**2
ATTEN(I)=-10D0*ALOGT(AMPL)
PHASE(I)=-57.2957795*ATAN2(HH(2,I),HH(1,I))
GPDLY(I)=(HH(2,I)*REAL(DH(I))-HH(1,I)*IMAG(DH(I)))/AMPL
C C
OUTPUT RESULTS
100 IF(ABS(FUNC).GT.3) GOTO 190
PRINT('" "')
PRINT('"PRINT DATA (Y/N)?")' $ READ'(A2)',NDAT
IF(NDAT.400B.EQ.131B) THEN
PRINT('" "')
PRINT('"H",12X,"(DB)",13X,"(DEG)",13X,"DELAY")
PRINT('" "')
DO 30 I=1,NW
PRINT'I(9X,F9.6,12X,F9.6,9X,F9.6,10X,E9.6)',FW(I),ATTEN(I),PHASE(I),GPDLY(I)
CONTINUE
END
C CHANGING FW TO SINGLE PRECISION FOR THE PLOT ROUTINE
NWT=NW
DO 150 II=1,NW
FWW(II)=FW(II) $ FWT(II)=FW(II)
CONTINUE
150 IF(ABS(FUNC).GT.3) THEN
GOTO(120,130,140),ABS(FUNC)-3
ENDIF
C PRINT('" "')
PRINT('"HIT ONE RETURN TO CONTINUE._")' $ READ*,CONT
IF(CONT.EQ.1) GO TO 110
STOP
C CALLING G PLOT
CALL GPLOT(NW,FWW,ATTEN,TRANSFER FUNCTION ANALYSIS_.15FREQUENCY)
XY (HZ)_,11HATTEN (DB)_8H ATTEN ,0.,0.)
GO TO 110
CALL GPLOT(NW,FWW,PHASE,TRANSFER FUNCTION ANALYSIS_.15FREQUENCY)
XY (HZ)_,12HPHASE (DEG)_6H PHASE ,0.,0.)
GO TO 110
CALL GPLOT(NW,FWW,GPDLY,TRANSFER FUNCTION ANALYSES_.15FREQUENCY)
XY (HZ)_,18HGROUP DELAY (SEC)_BHG. DELAY,0.,0.)
GO TO 110
170 END
C SUBROUTINE PLYFRO-COMPUTES REAL AND IMAGINARY PARTS OF A
TRANSFER FUNCTION
C
DOUBLE PRECISION FW(201),EPS,NUN(0:12),DNSQ(0:12),WN,AD(2,42)
X,BD(2,42)
SUBROUTINE PLYFRO(FW,H,DH,SC,NUM,NUN,DNSQ,WN,AD)
DOUBLE PRECISION FX(201)
NORMALIZING THE TRANSFER FUNCTION

DO 80 IF(NA.LT.2) GO TO 3

DO 2 I=2,NA
  A(I)=A(I)**NN**(I-1)
DO 3  I=2,NB
  B(I)=B(I)**NN**(I-1)

DO 4 I=1,NA
  AA=AA#WW**(I-1)
  DA=DA#WW+DA(L)
DO 5 I=1,NB
  BB=BB#WW+BB(L)

DO 6 K=1,N-1
  L=NA-K
  AA=AA#WW+(L)
  DA=DA#WW+DA(L)

DO 7 K=1,L-1
  L=N1-K
  BB=BB#WW+BB(L)

DO 8 K=I,H
  L=NB-K
  HH=HH#WW+(L)
  DB=DB#WW+DB(L)

CONTINUE

RETURN

END

SUBROUTINE SYN-REALIZES THE TRANSFER FUNCTION USING CASCADE

SUBROUTINE SYNTt£SIS

REAL B NUM(0:13),DEN(0:13),H(0:13),NY(0:9),DY(0:9),Y1,DEY1,
XA,B,C,D,L,X,BB,K,RFNUM,NYR(0:9),DWR(0:9),RREP(2,0:14),KR,DEL
INITIALIZING VARIABLES

FUNC2=0

FLAG=1 $ \text{N13=13}$ $ \text{N14=14}$ $ \text{N9=9}$ $ \text{ZE=0}$ $ \text{ON=1}$ $ \text{FLAG2=0}$ $ \text{N18=10}$

DO 505 I=0,13

CONTINUE

DO 508 I=0,8

CONTINUE

IF(FUNC2.EQ.4) GO TO 535

CALCULATING THE REFLECTION COEFFICIENT

DO 515 I=0,NH

DO 510 J=0,NH

IF(OR.LT.0) GO TO 510

H(I+J)=DEN(I)*DEN(J)*(-DO)**J-H(I+J)

X=NUM(I)*NUM(J)*(-DO)**J+H(I+J)

CONTINUE

CALL ROOT(2*NN,H,REFN,ZE,N13)

SORTING THE LEFT HALFTH PLANE ROOT'S

DO 530 I=3,2*NN,2

DEL=ABS(REFN(FLAG)-REFN(I))

IF(DVLT.1E-5) GOTO 525

DO 520 J=1,FLAG-2

DEL=ABS(REFN(J)-REFN(I))

IF(DEL.LT.1E-5) GOTO 525

CONTINUE

FLAG=FLAG+2

REFN(FLAG)=REFN(I)

REFN(FLAG+1)=CONJG(REFN(I))

IF(FLAG+1.EQ.NN) GOTO 531

CALL POLYCN13+1,N14,REFN,REP,ZE,NH

CALCULATING THE INPUT ADMITTANCE

DO 540 I=0,NH-1
NY(I)=(DEN(I)/DEN(NN))-RREP(I,I)
DY(I)=(DEN(I)/DEN(NN))+RREP(I,I)

PRINT(" ")
PRINT("INPUT ADMITTANCE")
PRINT(" ")
PRINT("Gx," ,10X,"a(i)",12X,"b(i)")
PRINT(" ")
DO 545 I=0,NN
PRINT("Gx,12,Sx,E14.11,4X,E14.11",I,NY(I),DY(I)
545 CONTINUE

PRINT(" ")$ PRINT(" ")$ PRINT(" ")
PRINT("4X," ,1) REMOVE C-SECTION (ZERO AT IMAGINARY FREQ.)")
PRINT("4X," ,2) REMOVE BRUNE SECTION (ZERO AT REAL FREQ.)")
PRINT("4X," ,3) REMOVE ZEROS AT INFINITY")
PRINT("4X," ,4) RESTART SYNTHESIS PROCEDE")
PRINT("4X," ,5) END SYNTHESIS.")$ READ*,FUNC2
GOTO(547,547,580,591,599),FUNC2
PRINT(" ")$ PRINT(" ")$ PRINT("4X," ,"ZERO TO BE REMOVED")
READ*,II
EVALUATING THE INPUT ADMITTANCE AND IT'S DERIVATIVE AT THE ROOT FNUM(I)
DNY=FLAG2*NY(FLAG2) $ DDY=FLAG2*DY(FLAG2) $ NY1=NY(FLAG2)
DY1=DY(FLAG2)
DO 549 J=FLAG2-1,1,-1
DDY=DDY$FNUM(I)+J*DY(J)
DNY=DNY+NY(I)
NY1=NY1+FNUM(I)+NY(J)
DY1=DY1+FNUM(I)+DY(J)
549 CONTINUE

IF(FUNC2.EQ.2) THEN
Y1=IMAG(KR*NY1/DY1)
ELSE
Y1=KR*NY1/DY1
ENDIF
DEY1=KR*(DNY$DY1-DDY$NY1)/DY1**2
GOTO(550,560),FUNC2
C REMOVAL OF DARLINGTON C-SECTION

RFNUM=FNUM(I)
IF(RFNUM.LT.1E-5) THEN
PRINT(" ")
PRINT("CAN NOT REMOVE THIS ZERO USING A C-SECTION")$ GOTO 546
ENDIF
A=(Y1-RFNUM$DEY1)/((RFNUM**2)*(Y1+RFNUM$DEY1))
B=2.0D0/(RFNUM*(Y1-RFNUM$DEY1))
C=2.0D0*Y1**2/(RFNUM*(Y1+RFNUM$DEY1))
D=(Y1+RFNUM$DEY1)/((Y1-RFNUM$DEY1)*RFNUM**2)
L=A/C
X=SQR(A)/C
BB=C/((1+A*RFNUM**2)*SQR(A))
K=(1+A*RFNUM**2)/(SQR(A)*C*RFNUM**2)
PRINT("")
PRINT("LOW-PASS COMPONENT VALUES FOR THE C-SECTION")
PRINT("")
PRINT(4X,"L=",F14.10),L
PRINT(4X,"X=",F14.10),X
PRINT(4X,"BB=",F14.10),BB
PRINT(4X,"K=",F14.10),K
GOTO 570
C
REMOVAL OF BRUNE SECTION
C
RFNUM=IMAG(FNUM(II))
IF(RFNUM.LT.1E-5) THEN
PRINT(" ")
PRINT("THIS ZERO CAN NOT BE REMOVED USING A BRUNE SECTION.")
GOTO 546
ENDIF
A=(RFNUM*DEY1-Y1)/((RFNUM**2)*(Y1+RFNUM*DEY1))
B=2.000/(RFNUM*(RFNUM*DEY1-Y1))
C=2.000*Y1**2/(RFNUM*(Y1+RFNUM*DEY1))
D=(Y1+RFNUM*DEY1)/((RFNUM*DEY1-Y1)*RFNUM**2)
L=A/C
X=SQR(A)/C
BB=C/((1-A*RFNUM**2)*SQR(A))
K=(1-A*RFNUM**2)/(SQR(A)*C*RFNUM**2)
PRINT(" ")
PRINT("LOW-PASS COMPONENT VALUES FOR THE BRUNE SECTION")
PRINT(" ")
PRINT(4X,"L=",F14.10),L
PRINT(4X,"X=",F14.10),X
PRINT(4X,"BB=",F14.10),BB
PRINT(4X,"K=",F14.10),K
GOTO 570
C
CALCULATING THE INPUT ADMITTANCE OF THE REMAINING NETWORK
C
NYR(0)=KR*NY(0)+NYR(1)=KR*NY(1)-C*DY(0)
DYR(0)=DY(0) $ DYR(1)=DY(1)-B*KR*NY(0)
DO 555 J=0,FLAG2
NYR(J+2)=NYR(J)+C*DY(J+1)
DYR(J+2)=DYR(J)+DY(J+2)-B*KR*NY(J+1)
DO 565 J=0,FLAG2+2
NY(J)=NYR(J) $ DY(J)=DYR(J) $ NYR(J)=0D0 $ DYR(J)=0D0
CONTINUE
KR=NY(FLAG2+1)/DY(FLAG2+2)
CALL ROOT(FLAG2+1,NY,YRF,ON,N9)
FLAG1=1
DO 568 I=1,FLAG2+2
DEL=ABS(YRF(I))-ABS(FNUM(II))
IF(ABS(DEL).LT.1.E-5) THEN
  YRF(I)=(0.D0,0.D0)
ELSE
  YRF(FLAG1)=YRF(I)
ENDIF
IF(FLAG1.NE.I) YRF(I)=(0.D0,0.D0)
FLAG1=FLAG1+1
END IF
CONTINUE
RT=FLAG1-1
CALL POLY(N10,N14,YRF,REP,ZE,RT)
DO 561 I=0,RT
  MYR(I)=REO(I)
  CALL RDOT(FLAG2+2,DY,YRF,ON,N9)
  FLAG1=I
  DO 565 I=I,FLAG2+2
    DEL=ABS(YRF(I))-ABS(REO(I))
    IF(ABS(DEL).LT.1.E-5) THEN
      YRF(I)=(0.D0,0.D0)
    ELSE
      YRF(FLAG1)=YRF(I)
    IF(FLAG1.NE.I) YRF(I)=(0.D0,0.D0)
    FLAG1=FLAG1+1
    END IF
  565 CONTINUE
  FLAG2=FLAG2-2
  RT=FLAG1-1
  CALL POLY(N10,N14,YRF,REP,ZE,RT)
  DO 567 J=I,FLAG2+2
    MYR(J)=MYR(J)+DY(J)=DYR(J) $ NY(J)=0.D0 $ DYR(J)=0.D0
  PRINT'(8X,I2,5X,E13.10,4X,E13.10),J,MY(J),DY(J)
  567 CONTINUE
  DO 581 J=FLAG2+3,8
    DY(J)=0.D0 $ NY(J)=0.D0
GOTO 546
C
C
REMOVAL OF SERIES INDUCTOR
C
L=DY(FLAG2)/(KR*NY(FLAG2-1))
DO 585 I=I,FLAG2
  DY(I)=DY(I)-L*KR*NY(I-1)
  585 CONTINUE
C
C
REMOVAL OF SHUNT CAPACITOR
C
C
C=KR*NY(FLAG2-1)/DY(FLAG2-2)
0573NY(O)=KRNY(O) * KR=1D0
0574DO 586 I=1,FLAG2-1
0575586NY(I)=KRNY(I)-DY(I-1)*C
0576X=SQRT(L/C) $ B=0D0 $ K=X
0577PRINT((" "))
0578PRINT("THE COMPONENT VALUES FOR THE EQUIVALENT SERIES L SHUNT C A XRE.")
0579DO 588 I=1-FLAG2-1
0580PRINT(" ") PRINT(4X,"L=",F10.0),L
0581PRINT(4X,"X=",F10.0),X $ PRINT(4X,"B=0")
0582PRINT(4X,"K=",F10.0),K
0583FLAG2=FLAG2-2
0584PRINT(4X,"REMAINING INPUT ADMITTANCE")
0585PRINT(" ") PRINT(9X,"Y(0)=",F10.0),NY(0)/DY(0)
0586GOTO 546
0587590RETURN
0588599END
0590C****************************************************************
0591C*SUBROUTINE TO CALCULATE THE ROOTS OF THE TRANSFER FUNCTION*
0592C****************************************************************
0593C****************************************************************
0594SUBROUTINE ROOT(NN,H,T,FLAG2,N>
0595INTEGER ITIME(5),RT,N,T,TAG2,ADEL,FLAG1,FLAG
0596DOUBLE PRECISION H(0:13),DH(0:13),TEST,RE
0597DOUBLE COMPLEX TH(N+1),NDEL,DDEL,DEL,G(0:13),G1,NN,
0598DG(0:13),DG1
0599IF(NN.LE.0)GOTO 495
0600FLAG=0 $ TEST=1D-18 $ RT=0 $ HR=0 $ FLAG3=0 $ FLAG1=1
0601I=-1)*WNN
0602IF(I.LT.0) FLAG3=1
0603DO 410 I=0,13
0604NH=0D0 $ G(I)=(0D0,0D0) $ DG(I)=(0D0,0D0)
0605410CONTINUE
0606DO 412 I=1,N+1
0607412TH(I)=(0D0,0D0)
0608CALL EXEC(11,ITIME)
0609CALL SSEEED(ITIME(2)*100+ITIME(1))
0610T(I)=DCMPLX(2*URAN(1)-1,2*URAN(1)-1)
0611DO 415 I=1,NN
0612415DH(I)=0D0 $ G(I)=(0D0,0D0) $ DG(I)=(0D0,0D0)
0613NDEL=H(NN) $ DDEL=DH(NN-1) $ FLAG=FLAG+1
0614DO 425 J=NN-1,1,-1
0615NDEL=NDEL*T(I)+H(J)
0616425DDEL=DDEL*T(I)+DH(J-1)
0617NDEL=NDEL*T(I)+H(0)
0618DEL=NDEL/DDEL
0619T(I)=T(I)-DEL
0620IF(ABS(DEL).GT.TEST) THEN
0621IF(FLAG.GT.50.AND.FLAG.LT.60) TEST=10*TEST
0622IF(FLAG.GT.150) THEN
0623IF(FLAG.LT.3) THEN
0624DEL=DEL/2D0 $ FLAG=60 $ FLAG1=FLAG+1
0625 T(1)=T(1)+DEL
0626 GOTO 420
0627 ELSE
0628 ADEL=LOG10(ABS(DEL))
0629 PRINT("MINIMUM ROOT ACCURACY TO ONLY ",I4),ADEL
0630 GOTO 430
0631 ENDIF
0632 ELSE
0633 GOTO 420
0634 ENDIF
0635 ENDIF
0636 RT=1
0637 GOTO 430
0638 435 RT=RT+1 $ TEST=10-20 $ FLAG=0 $ FLAG1=1
0639 T(RT)=DCMPLX(2*URAN(1)-1,2*URAN(1)-1)
0640 440 GI=G(RT-1) $ DG1=DG(RT-2) $ FLAG=FLAG1
0641 DO 445 J=RT-2,1,-1
0642 G1=G1*T(RT)+G(J)
0643 445 DG1=DG1*T(RT)+DG(J-1)
0644 G1=G1*T(RT)+G(0)
0645 NDEL=HNN X DDEL=DH(NN-1)
0646 DO 450 J=NN-1,1,-1
0647 NDEL=NDEL*T(RT)+H(J)
0648 450 DDEL=DDEL*T(RT)+DH(J-1)
0649 NDEL=NDEL*T(RT)+H(0)
0650 DEL=NDEL/(DDEL-((DG1*NDEL)/G1))
0651 T(RT)=T(RT)-DEL
0652 IF(ABS(DEL).GT.TEST) THEN
0653 IF(FLAG.GT.50.AND.FLAG.LT.60) TEST=10*TEST
0654 IF(FLAG.GT.150) THEN
0655 IF(FLAG1.LT.3) THEN
0656 DEL=DEL/200 $ FLAG=60 $ FLAG1=FLAG1+1
0657 T(RT)=T(RT)+DEL
0658 GOTO 440
0659 ELSE
0660 ADEL=LOG10(ABS(DEL))
0661 PRINT("MINIMUM ROOT ACCURACY TO ONLY ",I4),ADEL
0662 GOTO 430
0663 ENDIF
0664 ELSE
0665 GOTO 440
0666 ENDIF
0667 ENDIF
0668 430 IF(FLAG3.EQ.1)GOTO 465
0669 IF(FLAG2.EQ.1) THEN
0670 T(RT)=DCMPLX(RE(T(RT)),ABS(IMAG(T(RT))))
0671 T(RT+1)=CONJG(T(RT))
0672 ENDIF
0673 RT=RT+1
0674 ELSE
0675 IF(ABS(RE(T(RT))).LT.1D-6) THEN
0676 T(RT)=DCMPLX(0D1,ABS(IMAG(T(RT))))
T(\text{RT}+1)=-1\times\text{RT} \quad \text{RT}:=\text{RT}+1

go to 465

\text{ENDIF}

IF(ABS(\text{IMAG}(\text{RT}))) \lt 10^{-6} \text{ THEN}

T(\text{RT})=\text{DCHPLX(ABS(REAL(\text{RT}))),0.0)}

T(\text{RT}+1)=-T(\text{RT}) \quad \text{RT}:=\text{RT}+1

\text{ELSE}

T(\text{RT})=\text{DCHPLX(ABS(REAL(\text{RT}))),ABS(\text{IMAG}(\text{RT})))}

T(\text{RT}+1)=\text{CONJG(}\text{RT}\text{))} \quad T(\text{RT}+2)=-100 \times T(\text{RT}+1)

T(\text{RT}+3)=\text{CONJG(}(\text{RT}+2)) \quad \text{RT}:=\text{RT}+3

\text{ENDIF}

\text{ENDIF}

IF(\text{RT} \lt \text{NH} \text{ THEN}

\text{NZ}=\text{NR} \quad \text{NZ}:=13

\text{CALL POLY(N+1,MI3,T,G,NZ,\text{RT})}

\text{DO} 490 \text{ K}=1,\text{RT}

490 \quad D(\text{G(K-1)})=K \times D(G(K))

\text{DO} 495 \text{ NR=}\text{RT} \quad \text{GO TO} 435

\text{ENDIF}

\text{IF} \text{.LT.} \text{NH} \text{ THEN}

\text{NZ}=\text{NR} \quad \text{NZ}:=13

\text{CALL POLY(N+1,MI3,T,G,NZ,\text{RT})}

\text{DO} 490 \text{ K}=1,\text{RT}

490 \quad D(\text{G(K-1)})=K \times D(G(K))

\text{DO} 495 \text{ NR=}\text{RT} \quad \text{GO TO} 435

\text{ENDIF}

\text{RETURN}

\text{END}

\text{FUNCTION TO} \text{ CALCULATE THE REAL PART OF A COMPLEX NUMBER}

\text{REAL*8 FUNCTION RE(Z)}

\text{DOUBLE COMPLEX Z}
DOUBLE COMPLEX Z
RE=Z
RETURN
END
APPENDIX B

TABLES OF TRANSFER FUNCTION COEFFICIENTS

\[ S_{12}(p, \epsilon) = \prod_{i=1}^{2r} \frac{\prod_{i=1}^{2n} (p - d_i)}{(p - a_i)} \]
<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.761014516</td>
<td>0.223277334</td>
<td>-0.560098887</td>
<td>0.254735502</td>
</tr>
<tr>
<td>2</td>
<td>0.761014516</td>
<td>-0.223277334</td>
<td>-0.560098887</td>
<td>-0.254735502</td>
</tr>
<tr>
<td>3</td>
<td>-0.761014516</td>
<td>0.223277334</td>
<td>-0.580184370</td>
<td>0.088913820</td>
</tr>
<tr>
<td>4</td>
<td>-0.761014516</td>
<td>-0.223277334</td>
<td>-0.580184370</td>
<td>-0.088913820</td>
</tr>
<tr>
<td>5</td>
<td>0.637384149</td>
<td>0.684301603</td>
<td>-0.526037343</td>
<td>0.432719679</td>
</tr>
<tr>
<td>6</td>
<td>0.637384149</td>
<td>-0.684301603</td>
<td>-0.526037343</td>
<td>-0.432719679</td>
</tr>
<tr>
<td>7</td>
<td>-0.637384149</td>
<td>0.684301603</td>
<td>-0.456244811</td>
<td>0.600786936</td>
</tr>
<tr>
<td>8</td>
<td>-0.637384149</td>
<td>-0.684301603</td>
<td>-0.456244811</td>
<td>-0.600786936</td>
</tr>
<tr>
<td>9</td>
<td>3.120696888</td>
<td>0.000000000</td>
<td>-0.369473411</td>
<td>0.781040158</td>
</tr>
<tr>
<td>10</td>
<td>-3.120696888</td>
<td>0.000000000</td>
<td>-0.369473411</td>
<td>-0.781040158</td>
</tr>
<tr>
<td>11</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.160344392</td>
<td>0.967947481</td>
</tr>
<tr>
<td>12</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.160344392</td>
<td>-0.967947481</td>
</tr>
</tbody>
</table>

\[ A = .5700 \]

\[ KK = .00036966 \]
12TH ORDER

\[ A = 0.6000 \]

\[ KK = -0.00392841 \]

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.726901338</td>
<td>0.218704617</td>
<td>-0.440576424</td>
<td>0.604986709</td>
</tr>
<tr>
<td>2</td>
<td>0.726901338</td>
<td>-0.218704617</td>
<td>-0.440576424</td>
<td>-0.604986709</td>
</tr>
<tr>
<td>3</td>
<td>-0.726901338</td>
<td>0.218704617</td>
<td>-0.358564578</td>
<td>0.787324938</td>
</tr>
<tr>
<td>4</td>
<td>-0.726901338</td>
<td>-0.218704617</td>
<td>-0.358564578</td>
<td>-0.787324938</td>
</tr>
<tr>
<td>5</td>
<td>0.615111371</td>
<td>0.672282709</td>
<td>-0.540691631</td>
<td>0.256313035</td>
</tr>
<tr>
<td>6</td>
<td>0.615111371</td>
<td>-0.672282709</td>
<td>-0.540691631</td>
<td>-0.256313035</td>
</tr>
<tr>
<td>7</td>
<td>-0.615111371</td>
<td>0.672282709</td>
<td>-0.560275401</td>
<td>0.090388717</td>
</tr>
<tr>
<td>8</td>
<td>-0.615111371</td>
<td>-0.672282709</td>
<td>-0.560275401</td>
<td>-0.090388717</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>4.830313624</td>
<td>-0.508608667</td>
<td>0.437128109</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>-4.830313624</td>
<td>-0.508608667</td>
<td>-0.437128109</td>
</tr>
<tr>
<td>11</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.152705239</td>
<td>0.970386855</td>
</tr>
<tr>
<td>12</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.152705239</td>
<td>-0.970386855</td>
</tr>
</tbody>
</table>

\[ A = 0.6300 \]

\[ KK = 0.00737521 \]

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.717856331</td>
<td>0.217778993</td>
<td>-0.535216142</td>
<td>0.256736104</td>
</tr>
<tr>
<td>2</td>
<td>0.717856331</td>
<td>-0.217778993</td>
<td>-0.535216142</td>
<td>-0.256736104</td>
</tr>
<tr>
<td>3</td>
<td>-0.717856331</td>
<td>0.217778993</td>
<td>-0.554664686</td>
<td>0.090799656</td>
</tr>
<tr>
<td>4</td>
<td>-0.717856331</td>
<td>-0.217778993</td>
<td>-0.554664686</td>
<td>-0.090799656</td>
</tr>
<tr>
<td>5</td>
<td>0.608656590</td>
<td>0.669498834</td>
<td>-0.355459309</td>
<td>0.789076113</td>
</tr>
<tr>
<td>6</td>
<td>0.608656590</td>
<td>-0.669498834</td>
<td>-0.355459309</td>
<td>-0.789076113</td>
</tr>
<tr>
<td>7</td>
<td>-0.608656590</td>
<td>0.669498834</td>
<td>-0.503684362</td>
<td>0.438343682</td>
</tr>
<tr>
<td>8</td>
<td>-0.608656590</td>
<td>-0.669498834</td>
<td>-0.503684362</td>
<td>-0.438343682</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>3.575643999</td>
<td>-0.436130951</td>
<td>0.606139534</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>-3.575643999</td>
<td>-0.436130951</td>
<td>-0.606139534</td>
</tr>
<tr>
<td>11</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.150537666</td>
<td>0.971062931</td>
</tr>
<tr>
<td>12</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.150537666</td>
<td>-0.971062931</td>
</tr>
</tbody>
</table>
### 12TH ORDER

\[ A = 0.7000 \]

\[ KK = -0.01501203 \]

<table>
<thead>
<tr>
<th></th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>REAL</td>
<td>IM</td>
</tr>
<tr>
<td>1</td>
<td>0.594170892</td>
<td>0.664082029</td>
</tr>
<tr>
<td>2</td>
<td>0.594170892</td>
<td>-0.664082029</td>
</tr>
<tr>
<td>3</td>
<td>-0.594170892</td>
<td>0.664082029</td>
</tr>
<tr>
<td>4</td>
<td>-0.594170892</td>
<td>-0.664082029</td>
</tr>
<tr>
<td>5</td>
<td>0.698375501</td>
<td>0.216090303</td>
</tr>
<tr>
<td>6</td>
<td>0.698375501</td>
<td>-0.216090303</td>
</tr>
<tr>
<td>7</td>
<td>-0.698375501</td>
<td>0.216090303</td>
</tr>
<tr>
<td>8</td>
<td>-0.698375501</td>
<td>-0.216090303</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>2.580277346</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>-2.580277346</td>
</tr>
<tr>
<td>11</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>12</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>
### 10TH ORDER

\[A = 0.7000\]

\[\text{KK} = -0.00040218\]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(a(i))</th>
<th>(b(i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(REAL)</td>
<td>(IM)</td>
</tr>
<tr>
<td>1</td>
<td>0.765242060</td>
<td>0.000000000</td>
</tr>
<tr>
<td>2</td>
<td>-0.765242060</td>
<td>0.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.682768949</td>
<td>0.572152340</td>
</tr>
<tr>
<td>4</td>
<td>0.682768949</td>
<td>-0.572152340</td>
</tr>
<tr>
<td>5</td>
<td>-0.682768949</td>
<td>0.572152340</td>
</tr>
<tr>
<td>6</td>
<td>-0.682768949</td>
<td>-0.572152340</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>16.217922291</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>-16.217922291</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>

\[A = 0.7250\]

\[\text{KK} = 0.00320005\]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(a(i))</th>
<th>(b(i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(REAL)</td>
<td>(IM)</td>
</tr>
<tr>
<td>1</td>
<td>0.757290928</td>
<td>0.000000000</td>
</tr>
<tr>
<td>2</td>
<td>-0.757290928</td>
<td>0.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.676605005</td>
<td>0.569707074</td>
</tr>
<tr>
<td>4</td>
<td>0.676605005</td>
<td>-0.569707074</td>
</tr>
<tr>
<td>5</td>
<td>-0.676605005</td>
<td>0.569707074</td>
</tr>
<tr>
<td>6</td>
<td>-0.676605005</td>
<td>-0.569707074</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>5.802263031</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>-5.802263031</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>
**10TH ORDER**

\[ A = .7500 \]

\[ KK = -.00593000 \]

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.749685110</td>
<td>0.000000000</td>
<td>-0.515359681</td>
<td>0.325498864</td>
</tr>
<tr>
<td>2</td>
<td>-0.749685110</td>
<td>0.000000000</td>
<td>-0.515359681</td>
<td>0.325498864</td>
</tr>
<tr>
<td>3</td>
<td>0.670597500</td>
<td>0.567482211</td>
<td>-0.378562681</td>
<td>0.753149778</td>
</tr>
<tr>
<td>4</td>
<td>0.670597500</td>
<td>-0.567482211</td>
<td>-0.378562681</td>
<td>0.753149778</td>
</tr>
<tr>
<td>5</td>
<td>-0.670597500</td>
<td>0.567482211</td>
<td>-0.535813300</td>
<td>0.101152435</td>
</tr>
<tr>
<td>6</td>
<td>-0.670597500</td>
<td>-0.567482211</td>
<td>-0.535813300</td>
<td>0.101152435</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>4.299164643</td>
<td>-0.454672799</td>
<td>0.529135980</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>-4.299164643</td>
<td>-0.454672799</td>
<td>0.529135980</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.160454818</td>
<td>0.966725923</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.160454818</td>
<td>0.966725923</td>
</tr>
</tbody>
</table>

\[ A = .8000 \]

\[ KK = .01119595 \]

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.735356872</td>
<td>0.000000000</td>
<td>-0.507560060</td>
<td>0.326920879</td>
</tr>
<tr>
<td>2</td>
<td>-0.735356872</td>
<td>0.000000000</td>
<td>-0.507560060</td>
<td>0.326920879</td>
</tr>
<tr>
<td>3</td>
<td>0.659020418</td>
<td>0.563574157</td>
<td>-0.447443385</td>
<td>0.530475257</td>
</tr>
<tr>
<td>4</td>
<td>0.659020418</td>
<td>-0.563574157</td>
<td>-0.447443385</td>
<td>0.530475257</td>
</tr>
<tr>
<td>5</td>
<td>-0.659020418</td>
<td>0.563574157</td>
<td>-0.527465221</td>
<td>0.101135003</td>
</tr>
<tr>
<td>6</td>
<td>-0.659020418</td>
<td>-0.563574157</td>
<td>-0.527465221</td>
<td>0.101135003</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>3.178326614</td>
<td>-0.373339329</td>
<td>0.755729227</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>-3.178326614</td>
<td>-0.373339329</td>
<td>0.755729227</td>
</tr>
<tr>
<td>9</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.156833624</td>
<td>0.967742785</td>
</tr>
<tr>
<td>10</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.156833624</td>
<td>0.967742785</td>
</tr>
</tbody>
</table>
**10TH ORDER**

\[ A = 0.9000 \]

\[ KK = -0.02101585 \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 )</td>
<td>0.709628380</td>
<td>0.000000000</td>
</tr>
<tr>
<td>( 2 )</td>
<td>-0.709628380</td>
<td>0.000000000</td>
</tr>
<tr>
<td>( 3 )</td>
<td>0.637552147</td>
<td>0.557435040</td>
</tr>
<tr>
<td>( 4 )</td>
<td>0.637552147</td>
<td>-0.557435040</td>
</tr>
<tr>
<td>( 5 )</td>
<td>-0.637552147</td>
<td>0.557435040</td>
</tr>
<tr>
<td>( 6 )</td>
<td>-0.637552147</td>
<td>-0.557435040</td>
</tr>
<tr>
<td>( 7 )</td>
<td>0.000000000</td>
<td>2.382184929</td>
</tr>
<tr>
<td>( 8 )</td>
<td>0.000000000</td>
<td>-2.382184929</td>
</tr>
<tr>
<td>( 9 )</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>( 10 )</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>
### 8TH ORDER

\[ A = 0.9000 \]

\[ \text{KK} = -0.00008934 \]

<table>
<thead>
<tr>
<th>(a(i))</th>
<th>(b(i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(\text{REAL})</td>
</tr>
<tr>
<td>1</td>
<td>0.760607741</td>
</tr>
<tr>
<td>2</td>
<td>0.760607741</td>
</tr>
<tr>
<td>3</td>
<td>-0.760607741</td>
</tr>
<tr>
<td>4</td>
<td>-0.760607741</td>
</tr>
<tr>
<td>5</td>
<td>-38.156085872</td>
</tr>
<tr>
<td>6</td>
<td>-38.156085872</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>

### A = 0.9250

\[ \text{KK} = 0.00249092 \]

<table>
<thead>
<tr>
<th>(a(i))</th>
<th>(b(i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>(\text{REAL})</td>
</tr>
<tr>
<td>1</td>
<td>0.753712773</td>
</tr>
<tr>
<td>2</td>
<td>0.753712773</td>
</tr>
<tr>
<td>3</td>
<td>-0.753712773</td>
</tr>
<tr>
<td>4</td>
<td>-0.753712773</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>
### 8TH ORDER

**A** = 0.9500

**KK** = -0.00501550

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.747036685</td>
<td>0.386597632</td>
<td>-0.511159509</td>
<td>0.146513550</td>
</tr>
<tr>
<td>2</td>
<td>0.747036685</td>
<td>-0.386597632</td>
<td>-0.511159509</td>
<td>-0.146513550</td>
</tr>
<tr>
<td>3</td>
<td>-0.747036685</td>
<td>0.386597632</td>
<td>-0.400831557</td>
<td>0.697358318</td>
</tr>
<tr>
<td>4</td>
<td>-0.747036685</td>
<td>-0.386597632</td>
<td>-0.400831557</td>
<td>-0.697358318</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>5.147246864</td>
<td>-0.466563805</td>
<td>0.406015905</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000</td>
<td>-5.147246864</td>
<td>-0.466563805</td>
<td>-0.406015905</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.169212985</td>
<td>0.960180097</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.169212985</td>
<td>-0.960180097</td>
</tr>
</tbody>
</table>

### A = 1.000

**KK** = 0.00990488

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.734282416</td>
<td>0.383550054</td>
<td>-0.459637234</td>
<td>0.406709715</td>
</tr>
<tr>
<td>2</td>
<td>0.734282416</td>
<td>-0.383550054</td>
<td>-0.459637234</td>
<td>-0.406709715</td>
</tr>
<tr>
<td>3</td>
<td>-0.734282416</td>
<td>0.383550054</td>
<td>-0.503918217</td>
<td>0.147321745</td>
</tr>
<tr>
<td>4</td>
<td>-0.734282416</td>
<td>-0.383550054</td>
<td>-0.503918217</td>
<td>-0.147321745</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>3.697692382</td>
<td>-0.395574462</td>
<td>0.699594586</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000</td>
<td>-3.697692382</td>
<td>-0.395574462</td>
<td>-0.699594586</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.165527112</td>
<td>0.961102348</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.165527112</td>
<td>-0.961102348</td>
</tr>
</tbody>
</table>
**8TH ORDER**

\[ A = 1.100 \]

\[ \mathbf{K} \mathbf{K} = -0.01909237 \]

<table>
<thead>
<tr>
<th></th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.710900741</td>
<td>0.378665772</td>
</tr>
<tr>
<td>2</td>
<td>0.710900741</td>
<td>-0.378665772</td>
</tr>
<tr>
<td>3</td>
<td>-0.710900741</td>
<td>0.378665772</td>
</tr>
<tr>
<td>4</td>
<td>-0.710900741</td>
<td>-0.378665772</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>2.706762441</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000</td>
<td>-2.706762441</td>
</tr>
<tr>
<td>7</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>8</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>
### 6TH ORDER

<table>
<thead>
<tr>
<th></th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.952343834</td>
<td>0.0000000000</td>
<td>-0.451306490</td>
<td>0.584074209</td>
</tr>
<tr>
<td>2</td>
<td>-0.952343834</td>
<td>0.0000000000</td>
<td>-0.451306490</td>
<td>-0.584074209</td>
</tr>
<tr>
<td>3</td>
<td>2.412849904</td>
<td>0.0000000000</td>
<td>-0.503419098</td>
<td>0.179615274</td>
</tr>
<tr>
<td>4</td>
<td>-2.412849904</td>
<td>0.0000000000</td>
<td>-0.503419098</td>
<td>-0.179615274</td>
</tr>
<tr>
<td>5</td>
<td>0.0000000000</td>
<td>0.0000000000</td>
<td>-0.198189003</td>
<td>0.942803419</td>
</tr>
<tr>
<td>6</td>
<td>0.0000000000</td>
<td>0.0000000000</td>
<td>-0.198189003</td>
<td>-0.942803419</td>
</tr>
</tbody>
</table>

KK = -.02735710

<table>
<thead>
<tr>
<th></th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1.250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.857599415</td>
<td>0.0000000000</td>
<td>-0.422973039</td>
<td>0.594631334</td>
</tr>
<tr>
<td>2</td>
<td>-0.857599415</td>
<td>0.0000000000</td>
<td>-0.422973039</td>
<td>-0.594631334</td>
</tr>
<tr>
<td>3</td>
<td>9.931980407</td>
<td>0.0000000000</td>
<td>-0.468883062</td>
<td>0.179269130</td>
</tr>
<tr>
<td>4</td>
<td>-9.931980407</td>
<td>0.0000000000</td>
<td>-0.468883062</td>
<td>-0.179269130</td>
</tr>
<tr>
<td>5</td>
<td>0.0000000000</td>
<td>0.0000000000</td>
<td>-0.177609860</td>
<td>0.947673500</td>
</tr>
<tr>
<td>6</td>
<td>0.0000000000</td>
<td>0.0000000000</td>
<td>-0.177609860</td>
<td>-0.947673500</td>
</tr>
</tbody>
</table>

KK = .00171879
### 6TH ORDER

**A = 1.300**

**KK = -.00285760**

<table>
<thead>
<tr>
<th></th>
<th>REAL</th>
<th>IM</th>
<th></th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.842998109</td>
<td>0.000000000</td>
<td>2</td>
<td>-0.842998109</td>
<td>0.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>7.731267580</td>
<td>4</td>
<td>0.000000000</td>
<td>-7.731267580</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>6</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>REAL</th>
<th>IM</th>
<th></th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>-0.462658831</td>
<td>4</td>
<td>0.000000000</td>
<td>0.462658831</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>-0.462658831</td>
<td>6</td>
<td>0.000000000</td>
<td>-0.462658831</td>
</tr>
</tbody>
</table>

**A = 1.350**

**KK = .00727478**

<table>
<thead>
<tr>
<th></th>
<th>REAL</th>
<th>IM</th>
<th></th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.829250946</td>
<td>0.000000000</td>
<td>2</td>
<td>-0.829250946</td>
<td>0.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>4.861149181</td>
<td>4</td>
<td>0.000000000</td>
<td>-4.861149181</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>6</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>REAL</th>
<th>IM</th>
<th></th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.829250946</td>
<td>0.000000000</td>
<td>3</td>
<td>-0.417847831</td>
<td>0.417847831</td>
</tr>
<tr>
<td>4</td>
<td>-0.000000000</td>
<td>-0.417847831</td>
<td>5</td>
<td>0.000000000</td>
<td>-0.173867713</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000</td>
<td>-0.173867713</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**6TH ORDER**

\[ A = 1.400 \]

\[ KK = .01154151 \]

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.816529486</td>
<td>0.000000000</td>
<td>-0.408066339</td>
<td>0.600105345</td>
</tr>
<tr>
<td>2</td>
<td>-0.816529486</td>
<td>0.000000000</td>
<td>-0.408066339</td>
<td>-0.600105345</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>3.870223581</td>
<td>-0.450798326</td>
<td>0.179014816</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000</td>
<td>-3.870223581</td>
<td>-0.450798326</td>
<td>-0.179014816</td>
</tr>
<tr>
<td>5</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.166708564</td>
<td>0.950235854</td>
</tr>
<tr>
<td>6</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.166708564</td>
<td>0.950235854</td>
</tr>
</tbody>
</table>
### 4TH ORDER

**A = 1.500**

\[ KK = -0.05516380 \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{REAL} )</td>
<td>( \text{IM} )</td>
<td>( \text{REAL} )</td>
</tr>
<tr>
<td>1</td>
<td>2.288072525</td>
<td>0.000000000</td>
</tr>
<tr>
<td>2</td>
<td>-2.288072525</td>
<td>0.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>

**A = 2.000**

\[ KK = 0.00844848 \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{REAL} )</td>
<td>( \text{IM} )</td>
<td>( \text{REAL} )</td>
</tr>
<tr>
<td>1</td>
<td>5.553912363</td>
<td>0.000000000</td>
</tr>
<tr>
<td>2</td>
<td>-5.553912363</td>
<td>0.000000000</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>

**A = 2.150**

\[ KK = 0.00341966 \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a(i) )</th>
<th>( b(i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{REAL} )</td>
<td>( \text{IM} )</td>
<td>( \text{REAL} )</td>
</tr>
<tr>
<td>1</td>
<td>0.000000000</td>
<td>8.622408193</td>
</tr>
<tr>
<td>2</td>
<td>0.000000000</td>
<td>-8.622408193</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
</tbody>
</table>
### 4th Order

**A** = 2.200

**KK** = -0.00719853

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000000</td>
<td>5.919960628</td>
<td>-0.397191626</td>
<td>0.367402765</td>
</tr>
<tr>
<td>2</td>
<td>0.000000000</td>
<td>-5.919960628</td>
<td>-0.397191626</td>
<td>-0.367402765</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.152983107</td>
<td>0.930364395</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.152983107</td>
<td>-0.930364395</td>
</tr>
</tbody>
</table>

**A** = 2.500

**KK** = 0.02824759

<table>
<thead>
<tr>
<th>i</th>
<th>REAL</th>
<th>IM</th>
<th>REAL</th>
<th>IM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000000</td>
<td>2.926621474</td>
<td>-0.375720175</td>
<td>0.374885936</td>
</tr>
<tr>
<td>2</td>
<td>0.000000000</td>
<td>-2.926621474</td>
<td>-0.375720175</td>
<td>-0.374885936</td>
</tr>
<tr>
<td>3</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.135353570</td>
<td>0.936276839</td>
</tr>
<tr>
<td>4</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>-0.135353570</td>
<td>-0.936276839</td>
</tr>
</tbody>
</table>
APPENDIX C

GRAPHS OF TRANSFER FUNCTION RESPONSE
REFERENCES


