Prediction And Measurement Of Radiated Emissions Based On Empirical Time Domain Conducted Measurements

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PREDICTION AND MEASUREMENT OF RADIATED EMISSIONS BASED ON EMPIRICAL TIME DOMAIN CONDUCTED MEASUREMENTS

by

LARRY FREEMAN
B.S.E.C.E, The Ohio State University

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the School of Electrical Engineering and Computer Science in the College of Engineering and Computer Science at the University of Central Florida, Orlando, Florida

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ABSTRACT

This thesis develops a novel method to predict radiated emissions measurements. The techniques used are based on standard Electromagnetic Compatibility (EMC) qualification test methods. The empirical data used to formulate the final results was restricted to pertinent data protocol waveforms however the entire method may be applied to any waveforms for which empirical radiated emissions have been measured. The method provides a concise means for predicting worst case radiated emissions profiles based on empirical measured data.
I would like to acknowledge Tom, Sean, Charlie, and Joe. Four wonderful supervisors; I am a better engineer and person for having known you all. Thank you for all your patience, humor, and inspiration. A special thanks to Prof. Thomas Wu, for all his guidance and support in this endeavor; he is a superb professional and scholar.
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1. INTRODUCTION

The profession of Electromagnetic Interference (EMI) and Electromagnetic Compatibility (EMC) engineering has long been governed by design practices established through empirical measurement. Often detailed analysis isn’t an option, due to the sheer complexity of the phenomena involved. The necessary parameters are either impossible to obtain or require a nearly complete design to be of any real pertinence. The end result is a design driven by what has worked in the past. This often leads to more stringent design guidelines than are necessary. Many times a design effort has been driven by these restrictive measures, that often have little or no basis, other than it is what has been done before.

1.1. Premise

All electrical devices sold in the United States for commercial or military use are required by law to undergo a battery of certification tests; to ensure their proper operation will not have undesirable electrical effects on the environment of their intended use. For example, the Federal Communications Commission (FCC) restricts the amount of radiated emissions allowed in order that other electrical devices, such as television transmitters, cell towers, etc., do not have their transmissions inadvertently hindered. Compliance to these requirements may have dire consequences in critical areas such as aerospace vehicle controls, medical devices, and communications.
Most all of these EMC standards contain a suite of various tests. The most common are Conducted Susceptibility (CS), Conducted Emissions (CE), Radiated Emissions (RE), and Radiated Susceptibility (RS). Most engineers over simplify these into two categories, “Stuff that gets out are emissions. Stuff that gets in is susceptibility”. However they are much more complex. For example in Figure 1.1, within a chassis or box one Printed Circuit Board (PCB) may have conducted emissions from its trace that radiate susceptibility to another PCB. From the first card’s point of view this is initially a conducted emissions problem that manifests itself into a radiated emission that causes a radiated susceptibility of the second PCB card.

Existing methods require the use of invasive tools. For example, current monitor probes are frequency dependent and must be wrapped around the conductor being tested. This may be impractical or even impossible. The only other alternative is to bring an Engineering Design Unit (EDU) into the test chamber to perform RE testing. This is particularly unappealing for several reasons; usually it will affect schedule and cost. Not to mention EDU units are never meant to be fully compliant (only functional), often they require significant modifications to meet their functional obligation.

Figure 1-1: Conducted Emission to Radiate Susceptibility Scenario
The objective of this thesis is to investigate an approach that seeks to bridge the gap between empirical measurement and derived analysis.

1.2. Past Research

A thorough review for similar research efforts was performed. This literature survey included several texts, the World Wide Web, and the IEEE EMC society archives dating back to 1955 [1]. Many topics covered some aspect of this research effort. For example, a myriad of papers discussing conducted emissions, radiated emissions, or the Fast Fourier Transform were found. Even several papers relating the two were found. Works by Professor Clayton Paul and Donald White detail theoretical aspects but do not correspond easily to measured parameters. Few papers sought to specifically relate measured data. Instead they chose to simply verify with measured results.

The other significant differences were the use of voltage measurements versus current measurements. This is attributed to the fact that 99% of these papers concerned themselves with power line measurements that had varying impedances. Current probes present other issues, these are discussed later. The other significant discriminator was the use of special equipment or measurement fixtures. The use of special equipment or fixtures was deemed much too restrictive to be of use for this effort. The result of this thesis is to provide the details by which an individual using simple techniques and equipment commonly found around an EMC laboratory can perform preliminary measurements and formulate a prediction of compliance to radiated emissions. This is best done using empirical measurement data.
1.3. **Outline**

First the overall process being followed is presented; step by step. Then an elementary EMC certification setup is discussed; this explains the rationale behind such an endeavor and highlights the conception of specific physical modeling discussed later. Then a wire coupling model is presented along with the justification and explanation for its expansion. The initial measurement collection and transformation processes are detailed. Next the entire process is demonstrated in its intended sequence. Finally, a comparison is made between the predicted emissions profile and an actual empirically measured emissions profile, along with an explanation or hypothesis for any deviation.

![Figure 1-2: Process Outline](image-url)
1.4. **Objective**

It is important to point out the overall objective of this research. The goal of this research is not to predict the precise emissions profile but rather to envelope its worst case profile, using relatively straightforward data measurements. This will give the EMC design engineer an early look at what is to be expected through the use of real measurement data. This will allow a design to have a much higher certainty of compliance to measured emissions standards. Ideally the measurement data gathered from consecutive measurements will be used to establish a database. Then an overall notion of accuracy can be assessed in conjunction with strong empirical data. The end result of this work is to formulate a process, which can be implemented continuously and enhanced each time it is employed.
2. WAVEFORM MEASUREMENT

During an EMC certification test, conducted emissions are most always directly related to radiated emission profiles. Radiated emissions from structures or a mechanical chassis are common, but radiated emissions from cabling are far more prevalent. This is the main reason behind the focused scrutiny on cabling of this paper. Conductor cabling handles two distinct signals, digital and analog.

Typical for digital lines the frequency content is simply derived from transition rates [9]. Figure 2.1 shows a standard transform table used to predict the potential frequency content using known transition rates. Analog transmissions are defined accordingly. However neither of these techniques account for the unexpected variations that are certain to occur. For example, ringing would not be accounted for using the transition rate technique discussed, see Figure 2.2. From the figure it is easy to see how inadvertent effects such as ringing can be overlooked by simply using the transition table. A better more definitive approach would be to simply measure each transmission line.

This may be accomplished using a current clamp or voltage probe. The current clamp is physically large and made of ferromagnetic material, it requires at least one turn for the transformer action to occur. Current clamps are also frequency dependent. All of this makes current clamps extremely cumbersome and intrusive. For that reason a voltage measurement was deemed more reasonable. Since the transmission line impedance is known it is a simple conversion to get the current value.
A simple waveform measurement of the conducted waveform taken in the time domain is easy to obtain using an oscilloscope. Certain oscilloscope measurement parameters such as sample rate and time reference must be established in order to guarantee a uniformed approach; these are discussed in a later section. The measured waveform can then be transformed into the frequency domain.

Figure 2-2: Time Domain Measurement with Ringing
Modern oscilloscopes have the capability to transform time domain measurements into the frequency domain, however not all oscilloscopes use the same Fourier transform techniques. While most all of the oscilloscope manufacturers use the Fast Fourier Transform, many use completely different weighting functions and versions of the mathematical technique. For the purposes of this effort it was deemed much too restrictive to rely on one particular manufacturer’s technique or method. Therefore each waveform measurement was exported into a standard ASCI text file format, interpolated and then converted into Matlab for manipulation. Figure 2.3 shows a sample of a data waveform that has been corrupted with random noise.

![Figure 2-3: Sample Corrupted Measured Waveform](image)
3. WAVEFORM TRANSFORM

The next step is to take the measured waveform data, shown in Figure 2.3 and interpolate it into Matlab. Figure 3.1 shows how the FFT can highlight a specific frequency of concern. The specific DFT methodology used is outlined in a later section. The end result is an accurate profile of all the frequencies that warrant consideration when deriving the emissions profile envelope.

![Frequency Content of Sample Waveform](image)

**Figure 3-1: Power Density of Frequency Content**

3.1. Technique to Convert into Matlab

A program, implemented in the Matlab programming language is listed in appendix A. As mentioned earlier, certain waveform parameters must be standardized, such as sample rate, time reference, and duration. The Matlab program imports the waveform data, translates from a standard ASCI text file and performs the DFT. The program outputs are
the vectors containing the DFT amplitude and frequency reference and plots of the various waveform data. A simple functional diagram is shown in Figure 3.2.

![Figure 3-2: Matlab Program Functional Flowchart](image)

3.2. **Verification**

Before any further consideration a verification step was performed. Aside from the obvious sanity check, this step allowed for the identification of any unintentional frequency content. For example, if an unidentified frequency component is discovered it can be investigated. The measured waveform should be taken from preliminary engineering designs, even bench top models, to allow for adequate time to correct the design.
Unintentional frequency content may be a result of the preliminary design and not a part of the finished product. For example, the final design may be implemented using DC power from a vehicle battery, this source is by definition not likely to cause conducted transients. However the design model could be powered from a DC power supply with a switching rectifier that produces frequency content into the measured waveform. The verification step should consist of, as a minimum, a preliminary survey of the intended frequency content for analog transmissions and a comparison with Figure 2.1, for known digital transition rates.

3.3. *Matlab Transform Verification*

In order to verify the accuracy of the Matlab program a square wave was measured on the oscilloscope, imported and transformed using the Matlab program in appendix A. This same waveform was fed directly into an Agilent spectrum analyzer and measured directly across frequency. Each measurement was then captured as an image file; both files are shown below as Figures 3.5 and 3.6. This strong correlation demonstrates the accuracy of the Matlab implemented transform.
Figure 3-3: Measured Waveform

Figure 3-4: Matlab DFT Waveform
4. ANTENNA COUPLING

The principal used to formulate the emissions antenna model is to determine the induced voltage due to an incident electromagnetic wave upon a wire suspended overtop of a ground plane. This is pertinent because almost all EMC radiated emissions certification testing uses this setup or one similar to it. This is true for both forms of radiated emissions testing, commercial and military. The mathematical derivation of this technique was originally documented by Edward Vance [2] and Alberta Smith [3]. The formulas and derivation are delineated in the following sections with greater detail and clearer nomenclature added where deemed necessary.

4.1. EMC Certification Setup

In the interest of uniformed scrutiny almost all radiated emissions tests use the same setup approach, mainly with the Equipment Under Test (EUT) and its supporting conductors being suspended above a ground plane. In the interest of simplicity the typical setup used in Mil-Std-461E is used for this paper. Figure 4.1 below shows a diagram of this setup.

![Figure 4-1: Generic Radiated Emissions Test Setup](image)
4.2. Coupling Model Derivation

4.2.1. First Transmission Line Equation

The initial coupling model is derived from that of a two wire transmission line. Figure 4.2 shows the geometry of the two wire coupling model. This has a strong correlation to the eventual setup approach, a single wire above a ground plane, especially when considering the image plane induced by the ground plane. This correlation is expanded upon further in the next section.

![Two Wire Coupling Model Geometry](image)

**Figure 4-2: Two Wire Coupling Model Geometry**

Starting with Maxwell’s equation for the curl of the electric field over an incremental surface as shown in Figures 4.3 and 4.4, and using Stokes theorem to integrate, the induced voltage is derived as follows:

\[
\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = \oint_C \mathbf{E} \cdot d\mathbf{l} = -j \omega \int_S \mathbf{B} \cdot d\mathbf{S} 
\]

(1)

Evaluating the line integral over the contour that bounds the surface, using \( dS = dxdz \), we have

\[
\int_0^b \left[ E_X (x, z + \Delta z) - E_X (x, z) \right] dx - \int_z^{z+\Delta z} \left[ E_Z (b, z) - E_Z (0, z) \right] dz = -j \omega \int_z^{z+\Delta z} B_Y (x, z) dx \ dz
\]

(2)
Dividing by the incremental step and taking the limit as it approaches zero gives

\[
\frac{\partial}{\partial z} \int_0^b E_x(x, z) dx - [E_z(b, z) - E_z(0, z)] = -j \omega \int_0^b B_y(x, z) dx
\]  

(3)

The field terms delineated are the total scattered and incident fields. The voltage between the two wires is defined as
\[ V(z) = -\int_{0}^{b} E_x(x,z) \, dx \]  \hspace{1cm} (4)

Using this relation the first term of (3) may be re-written as

\[ \frac{\partial}{\partial z} \int_{0}^{b} E_x(x,z) \, dx = -\frac{\partial}{\partial z} V(z) \]  \hspace{1cm} (5)

Using the definition for incremental voltage

\[ E_z \Delta z = I \left( \frac{R_1}{2} \right) \Delta z \]  \hspace{1cm} (6)

and substituting this relation into (3), we have

\[ E_z(b, z) - E_z(0, z) = R_1 \left( \frac{I_2 - I_1}{2} \right) \]  \hspace{1cm} (7)

where \( R_1 \) represents the distributed resistance in resistance per length, \( I_1 \) and \( I_2 \) represent the total current within each wire. From the convention shown in Figure 4.5, the common mode (CM) and differential mode (DM) currents are separated as

\[ I_{DM} = \left( \frac{I_2 - I_1}{2} \right), \quad I_{CM} = \left( \frac{I_2 + I_1}{2} \right) \]  \hspace{1cm} (8)

Since the measured time domain data is always differential mode; this is because common mode carries no information; it is convenient to use (7) and the first part of (1) to restrict the second term of (2) to DM current with [6][8]

\[ I(z) = I_{DM}(z) \]  \hspace{1cm} (9)

\[ E_z(b, z) - E_z(0, z) = R I(z) \]  \hspace{1cm} (10)

Finally the third term of (3) can be divided into its incident and scattered components as follows

\[ j \omega \int_{0}^{b} B_x(x,z) \, dx = j \omega \int_{0}^{b} B_x^i(x,z) \, dx + j \omega \int_{0}^{b} B_x^s(x,z) \, dx \]  \hspace{1cm} (11)
The reason for this is that the magnetic field originating from the DM current is what causes the scattered magnetic field component. The reasoning behind this phenomenon is that DM fields cancel, while CM fields combine. The difference between the two differential fields results in a scattered element.

![Figure 4-5: DM and CM Current Action](image)

Next the inductance per unit length of transmission line $\Delta z$ is given by

$$L_i \Delta z = -\frac{\Phi'_y}{I(z)}$$  \hspace{1cm} (12)

where $L_i$ is the distributed inductance per unit length and $\Phi'_y$ is the incremental surface scattered flux from between the conductors. By rearranging (12) to

$$\frac{\Phi'_y}{\Delta z} = -L_i I(z)$$  \hspace{1cm} (13)

in terms of the flux

$$\Phi'_y = \int \int_s B'_y ds = \int_z^{z+\Delta z} \int_0^b B'_y dx dz = \Delta z \int_0^b B'_y(x,z) dx$$  \hspace{1cm} (14)

we have
\[
\frac{\Phi_y^s}{\Delta z} = \int_0^b B_y^s(x,z)dx
\]  
(15)

Therefore, (13) and (15) give

\[
\int_0^b B_y^s(x,z)dx = -L_1 I(z)
\]  
(16)

By substituting (16) into (11), we have

\[
j \omega \int_0^b B_y(x,z)dx = j \omega \int_0^b B_y^i(x,z)dx - j \omega L_1 I(z)
\]  
(17)

Inserting (5) and (17) into (3), we get the first transmission line equation with voltage source as

\[
\frac{dV(z)}{dz} + Z_1 I(z) = V_s(z)
\]  
(18)

where

\[
V_s(z) = j \omega \int_0^b B_y^i(x,z)dx
\]  
(19)

and \(Z_1 = j \omega L_1\) is series impedance per unit length.

4.2.2. Second Transmission Line Equation

From Maxwell’s equations, for the scattered field, we have

\[
\nabla \times \mathbf{H}^s = j \omega \varepsilon \mathbf{E}^s
\]  
(20)

from which we obtain

\[
E_x^s = \frac{1}{j \omega \varepsilon} \left[ \frac{\partial H_y^s}{\partial y} - \frac{\partial H_z^s}{\partial z} \right]
\]  
(21)

Since the current flows in \(z\) direction, we can assume \(\partial / \partial x = \partial / \partial y = 0\) inside the transmission line, we obtain

\[
E_x^s = \frac{1}{j \omega \varepsilon} \left[ -\frac{\partial H_y^s}{\partial z} \right]
\]  
(22)
Since the voltage on the transmission line can be expressed as
\[ V(z) = -\int_{0}^{b} E_x(x,z)dx = -\int_{0}^{b} E'_x(x,z)dx - \int_{0}^{b} E'_z(x,z)dx \]  \hspace{1cm} (23)

inserting (22) into (23) yields
\[ V(z) = -\int_{0}^{b} E'_x(x,z)dx + \frac{1}{j\omega\varepsilon} \frac{d}{dz} \int_{0}^{b} H'_z(x,z)dx \]  \hspace{1cm} (24)

Since \( B = \mu H \), the integration in the second term of (24) becomes
\[ \int_{0}^{b} H'_z(x,z)dx = \frac{1}{\mu} \int_{0}^{b} B'_y(x,z)dx = -\frac{L_1 I(z)}{\mu} \]  \hspace{1cm} (25)

(16) is also used to derive (25). Inserting (25) into (24), we can obtain the second transmission line equation as
\[ \frac{dI(z)}{dz} + Y_i V(z) = I_s(z) \]  \hspace{1cm} (26)

where
\[ Y_i = \frac{j\omega\mu\varepsilon}{L_1} = j\omega C_1 \]  \hspace{1cm} (27)

and
\[ I_s(z) = -Y_i \int_{0}^{b} E'_x(x,z)dx \]  \hspace{1cm} (28)

4.3. Solution of Transmission Line Equations

In the following, we will discuss solution procedure for transmission line equations (18) and (26). The circuit representation for these two equations are shown in Figure 4.6.
We are going to discuss the solution with current source $I_s(z)$ only at first and then discuss the solution with voltage source $V_s(z)$. And the total solution is the superposition of these two cases.

When there is only current source, we have

$$
\begin{cases}
\frac{dV(z)}{dz} + Z_1 I(z) = 0 \\
\frac{dI(z)}{dz} + Y V(z) = I_s(z)
\end{cases}
$$

The solution of current in (29) can be obtained using Green’s function as

$$I(z) = \int_0^z I_s(z') I_G^I(z, z')dz' + \int_z^L I_s(z') I_G^{II}(z, z')dz'$$

where the Green’s functions are given by

$$I_G^I = -\frac{A_1}{Z_0} e^{jkz} + \frac{\Gamma Z_0}{A_1} e^{-jkz}$$

$$I_G^{II} = \frac{A_2}{Z_0} e^{-jkz} - \frac{\Gamma Z_0}{2A_2} e^{-jkl} e^{jk(z-L)}$$
where

\[ A_1 = \frac{Z_0 e^{-jz'}\left[1 + \Gamma_2 e^{2j(z-L)}\right]}{2\left(1 - \Gamma_1 \Gamma_2 e^{-2j\Gamma L}\right)} \]  
(33)

\[ A_2 = \frac{Z_0 e^{jz'}\left[\Gamma_1 e^{-2jz'} + 1\right]}{2\left(1 - \Gamma_1 \Gamma_2 e^{-2j\Gamma L}\right)} \]  
(34)

Since we are interested in the solution at \( z = L \), from (30) we have

\[ I(L) = \int_0^L I_s(z') I^H_G(z,z')dz' \]  
(35)

Assuming the transmission line is matched at both ends, which means

\[ \Gamma_1 = \Gamma_2 = 0 \]  
(35)

Equation (34) for derived current reduces to

\[ A_2 = \frac{Z_0 e^{jz'}}{2} \]  
(36)

Therefore (35) becomes

\[ I(L) = \frac{1}{2} \int_0^L I_s(z')e^{jk(z-L)}dz' \]  
(37)

In order to derive an empirical solution for experimental results, we assume the transmission line is short so that \( I_s(z') = I_s(L) \) is a constant, then (37) becomes

\[ I(L) = \frac{V(L)}{Z_0} = I_s(L)\frac{1 + e^{-j\Gamma L}}{2jk} \]  
(38)

Likewise, when there is only current source, we have
\[
\begin{align*}
\begin{cases}
\frac{dV(z)}{dz} + Z_I I(z) = V_s(z) \\
\frac{dI(z)}{dz} + Y I(z) = 0
\end{cases}
\end{align*}
\] (39)

The solution for \( V(L) \) of this case should be dual to solution for \( I(L) \) of (29). Therefore, we have

\[
V(L) = V_s(L) \frac{1 + e^{-jkL}}{2jk}
\] (40)

Then, the solution of

\[
\begin{align*}
\begin{cases}
\frac{dV(z)}{dz} + Z_I I(z) = V_s(z) \\
\frac{dI(z)}{dz} + Y I(z) = I_s(z)
\end{cases}
\end{align*}
\] (41)

is superposition of (38) and (40), which results in

\[
\frac{V(L)}{Z_0} = \left[ I_s(L) + \frac{V_s(L)}{Z_0} \right] \frac{1 + e^{-jkL}}{2jk}
\] (42)

From (19) and (28), we know that \( V_s \) and \( I_s \) both come from the incident fields. This means that they are correlated. We can generally assume

\[
E_s^i = \alpha(\omega) Z_0 H_y^i
\] (43)

For uniform plane wave normal incidence \( \alpha(\omega) = 1 \), otherwise, it’s just a general constant. From (19) and (28), we have

\[
I_s = \frac{\alpha(\omega) V_s}{Z_0}
\] (44)

which means (42) can be expressed in a format as

\[
\frac{V(L)}{Z_0} = \frac{1 + e^{-jkL}}{jk} \frac{\alpha(\omega) + 1}{2\alpha(\omega)} I_s(L) = Y(\omega) \int_0^L E_s^i(x, L) dx
\] (45)
where
\[
Y(\omega) = -\frac{1 + e^{-j \omega}}{jk} \alpha(\omega) + 1 \quad \text{and} \quad C_1 = -\frac{1 + e^{-j \omega}}{Z_0} \frac{\alpha(\omega) + 1}{2 \alpha(\omega)} \tag{46}
\]

4.4. Measurement Parameters

By the reciprocal property of antennas the incident electric field may be interpolated to a
distance of one meter; the standardized measurement distance. The driving current
source is the measured oscilloscope waveform, initially interpolated into the frequency
domain, using the technique outlined. From (45), because we are interested in E-field
radiation, we begin by taking the absolute value of each component.

\[
\left| Z(\omega) \left( \frac{V(L)}{Z_0} \right) \right| = \left| \int_0^L E_x(x,z) dx \right| \tag{47}
\]

Note that (47) has only a vertical component of the E-field. This is consistent with the
measurement setup and the definition for the E-field component. By definition the
measured radiated emission is a measurement of the E-field component, it has no phase
component and correlates to the absolute value of the E-field component.

However, the time domain measurement is a voltage measurement. This was deemed the
most unobtrusive since it is relatively simple and requires no correction other than to
divide by the characteristic impedance of the transmission line. Equation (47) divides the
current source by the characteristic impedance and solves for the impedance factor in
terms of the time domain voltage waveform.
\[
|Z(\omega)| = \left( \frac{\int_{0}^{b} E_i^i(x,z)dx}{\left( \frac{V(L)}{Z_0} \right)} \right)
\] (48)

However, the E-field component in (47) does not directly correlate to the radiated measurement. This is because the radiated emissions measurement uses antenna factors that account for the antenna loss. These factors are simply programmed into the receiver system and added to the detected E-field signal. These factors and their impact on the measurement data are discussed in more detail in a following section.

The final step is to establish a measurable sample test setup. The challenge is to assure direct correlation between the measurable controlled setup and that of the standard EMC qualification setup. For the purposes of this work, the Mil-Std-461 military test setup is used. However the controlled setup is just as applicable to virtually any standard radiated emissions qualification test setup, commercial or military.

4.5. Measurement Controlled Setup

This setup is a simple representation of a standard Electromagnetic Compatibility qualification test. Its main function is to provide an empirically measurable cable antenna, so that the radiated emissions profile may be measured and then directly correlated in terms of the parameters outlined above. This measurement data can then be interpolated in terms of the derived mathematical form. Finally, resultant profiles based in terms of the initial waveform capture, in the time domain, can be used to furnish a useful prediction of the radiated emissions profile, based mainly on measured waveforms easily captured in the time domain.
Measurements across the frequency range were made using the Figure 4-7 setup. The cabling stimulus used for this evaluation is a standard square wave pulse with the characteristics shown in Figure 4-8.
4.6. **Empirical Measurements**

Laboratory measurements always have an unavoidable degree of uncertainty. The typical radiated emissions equation is shown in (49) below. Note (49) has units in decibels.

\[
E_{\text{Measured}} = E_{\text{Antenna}} + AF + \text{Loss} \tag{49}
\]

The two field components are divided into the measured electric field and the incident electric field on the measurement antenna. The additional term is the pre-calibrated antenna factor of the measurement antennas; these are given in appendix B. The final loss term is due to various measurement attenuations, i.e. Component Insertion Loss, Cable loss, etc.
4.6.1. Antenna Factor Interpolation

The measurement antenna factors require linear interpolation between any two measurement points. For example, the antenna factors measurement file may not have the exact number of measurement points that the time domain waveform will. Nor are the files likely to have the required corresponding frequency point. Therefore a program that first determines the closest measurement points and then linear interpolates between them, in order to calculate a corresponding frequency component.

This interpolation technique has been implemented using a Matlab program, given in appendix C. This technique inherently introduces a margin of error, but this margin is low, approximately 0.01%. Once the antenna factor has been determined, it can be subtracted from the measured field along with the loss and then the measured electric field can be interpolated to the electric field incident on the cabling.

![Antenna Factor Interpolation](image)

**Figure 4-9: Antenna Factor Interpolation**
4.7. Impedance Factor

From equation 37 the impedance factor has been empirically measured for a distinct setup scenario, a twisted pair wire over a ground plane. Both conductors are 22 AWG; this is the most commonly used conductor size for known protocols such as Mil-Std-1553, RS-422, RS-485, and Low Voltage Differential Signal (LVDS) signal interfaces. Distinct waveform measurements with varying rise time, fall time, and pulse width were measured for both setups. These are listed below.
<table>
<thead>
<tr>
<th>Setup</th>
<th>Rise/Fall Time</th>
<th>Pulse Width</th>
<th>Designator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient</td>
<td>NA</td>
<td>NA</td>
<td>AMB</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>10ns</td>
<td>400ns</td>
<td>TP-1</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>10ns</td>
<td>1us</td>
<td>TP-2</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>10ns</td>
<td>10us</td>
<td>TP-3</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>10ns</td>
<td>100us</td>
<td>TP-4</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>100ns</td>
<td>1us</td>
<td>TP-5</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>100ns</td>
<td>10us</td>
<td>TP-6</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>100ns</td>
<td>100us</td>
<td>TP-7</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>1us</td>
<td>10us</td>
<td>TP-8</td>
</tr>
<tr>
<td>Twisted Pair</td>
<td>1us</td>
<td>100us</td>
<td>TP-9</td>
</tr>
</tbody>
</table>

The impedance factor for each measurement is shown in appendix D.
5. RADIATED EMISSIONS PROFILE PREDICTION

Once the incident electric field has been measured and the attenuation loss factors have been accounted for, the radiated emissions profile can be predicted. The emissions profile prediction uses equations (48) and (49), defined in terms of the measurement parameters [5] [6]. The final equation is shown as equation (50) below

\[
|Z| \times \left( \frac{V_0}{Z_0} \right) + A F_{\text{interpolated}} = E_{\text{Radiated Emission}}
\]  

5.1. Prediction Example

For simplicity we will use discrete measurement setup waveform TP-2T. Beginning with a scope measurement shown in Figure 5.1 for waveform TP-2T the FFT was taken.

![Scope Capture of TP-2T Waveform](image)

Figure 5-1: Time Domain of TP-2T Waveform
Next the amplitude in voltage was divided by 50 ohms to get the current value; this was the size of the termination impedance used. The impedance factor was calculated already, this is displayed in Figure D-2. From this figure the impedance factor from low to high frequency ranges in amplitude from $10^{-1.5}$ to $10^2$. However this impedance factor cannot be assumed, since the objective is to predict the radiated emissions profile. Therefore a justifiable alternative would be to use the next highest impedance factor; this is the TP-1T impedance factor waveform. Therefore using the TP-1T impedance factor waveform and the calculated FFT from the TP-2T time base measurement we are able to predict the emissions profile. Figure 5-3 shows the predicted versus measured radiated emissions profile.
Notice the predicted emission envelope differs from the measured profile by 2dB. This emission profile tells the cognizant design engineer precisely how much shield attenuation this cable will require, relative to the specification limit.

Another alternative would be to simply choose a threshold impedance factor value. Figure 5-4 shows a predicted emissions profile based on a nominal impedance factor value of 10. Notice the peak emission is still well within the expectable margin.
Figure 5-4: Predicted Radiated Emission Profile Using Nominal Value for Impedance Factor
6. CONCLUSIONS

6.1. Overall Technique

The overall technique is sound and intuitive. However the actual implementation was fraught with logistics type issues. Empirical data measurements were required and this corresponds directly to budgetary constraint on man hours, equipment time, and lab use. This thesis is the non-recurring engineering portion of the process. From here on data will be taken in conjunction with routine testing efforts; however this effort laid the ground work.

6.2. Matlab Implementation

Matlab cannot support data manipulation of matrices larger than $2^{25}$. This is a fundamental design concept and cannot be overcome. The FFT requires more data samples for better accuracy so this was a natural inhibitor. However $2^{20}$ was deemed sufficient, any more would have diminishing returns.

6.2.1. Matrices Manipulation in Matlab

Matrices require distinct mathematical techniques. For example, indexes must correspond. This is impossible with empirical data. Receivers and Oscilloscopes have predefined interval measurements based on environment, span, etc. Therefore interpolation was required for all of the empirical data; this was unforeseen and led to a lengthy delay.
6.3.  *Empirical Measurements*

Empirical measurements by definition have many inherent aspects that are otherwise over
looked or just understood. However every aspect must be accounted for in a theoretical
derivation. For example, antenna correction factors are measured quantities. Antennas
are required to undergo a routine annual calibration. However the calibration is not
recorded as continuous, instead it is made at discrete points along the frequency
spectrum.

Another, severely limiting factor was the inability to view real-time data collection of the
oscilloscope measurements. Because the scope waveforms were so tightly sampled;
simple programs such as Microsoft Excel were unable to view them. Microsoft Excel is
limited to 65,536 values. This was a real problem later because some data had become
corrupted or was not recorded correctly and needed to rerecord. However the opportunity
to use laboratory time and equipment had passed.

6.4.  *Recommendations*

The author’s goal was to establish a process, the non-recurring portion at least, so that
progressively more and more recorded measurements could be used in conjunction with
these techniques to further bolster the accuracy of this approach. The only
recommendation for improvement would be delineate according to rise time and pulse
width as much as possible. This would allow more direct comparison, even though as
this paper demonstrated it is not necessary. Also, the author would like to see the overall
technique re-programmed into an alternate program language and made into an
executable file for distribution. As it stands now each program component is not well
meshed with its predecessor. However, much more fluent programming is well beyond the scope of this effort and the author’s skill.
Figure A-1: Rod Antenna Factor

Figure A-2: Bi-conical Antenna Factor
Figure A-3: Double Ridge Horn Antenna Factor

Figure A-4: Horn Antenna Factor
APPENDIX B: SAMPLE OF MEASURED WAVEFORMS
Figure B-1: Sample 1 Scope Capture, FFT of Scope Capture, and RE Measurement
Figure B-2: Sample 2 Scope Capture, FFT of Scope Capture, and RE Measurement
APPENDIX C: MEASUREMENT SETUP PICTURES
Figure C-1: 10 kHz-30MHz Measurement Setup

Figure C-2: 30 MHz-200MHz Measurement Setup
Figure C-3: 200MHz-1GHz Measurement Setup

Figure C-4: Waveform Generator
Figure C-5 Termination Shielding

Figure C-6: Feed and Bulkhead
APPENDIX D: IMPEDANCE FACTOR PLOTS FOR TWISTED PAIR MEASUREMENTS
Figure D-1: Impedance Factor for TP-1T Waveform

Figure D-2: Impedance Factor for TP-2T Waveform
Figure D-3: Impedance Factor for TP-3T Waveform

Figure D-4: Impedance Factor for TP-4T Waveform
Figure D-5: Impedance Factor for TP-5T Waveform

Figure D-6: Impedance Factor for TP-6T Waveform
Figure D-7: Impedance Factor for TP-7T Waveform

Figure D-8: Impedance Factor for TP-8T Waveform
Figure D-9: Impedance Factor for TP-9T Waveform


LIST OF REFERENCES


