Beam-Steerable and Reconfigurable Reflectarray Antennas for High Gain Space Applications

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BEAM-STEERABLE AND RECONFIGURABLE REFLECTARRAY ANTENNAS FOR HIGH GAIN SPACE APPLICATIONS

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Electrical Engineering and Computer Science in the College of Engineering & Computer Science at the University of Central Florida Orlando, Florida

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ABSTRACT

Reflectarray antennas uniquely combine the advantages of parabolic reflectors and phased array antennas. Comprised of planar structures similar to phased arrays and utilizing quasi-optical excitation similar to parabolic reflectors, reflectarray antennas provide beam steering without the need of complex and lossy feed networks. Chapter 1 discusses the basic theory of reflectarray and its design. A brief summary of previous work and current research status is also presented. The inherent advantages and drawbacks of the reflectarray are discussed.

In chapter 2, a novel theoretical approach to extract the reflection coefficient of reflectarray unit cells is developed. The approach is applied to single-resonance unit cell elements under normal and waveguide incidences. The developed theory is also utilized to understand the difference between the TEM and TE_{10} mode of excitation. Using this theory, effects of different physical parameters on reflection properties of unit cells are studied without the need of full-wave simulations. Detailed analysis is performed for Ka-band reflectarray unit cells and verified by full-wave simulations. In addition, an approach to extract the Q factors using full-wave simulations is also presented. Lastly, a detailed study on the effects of inter-element spacing is discussed.

Q factor theory discussed in chapter 2 is extended to account for the varying incidence angles and polarizations in chapter 3 utilizing Floquet modes. Emphasis is laid on elements located on planes where extremities in performance tend to occur. The antenna element properties are assessed in terms of maximum reflection loss and slope of the reflection phase. A thorough analysis is performed at K_a band and the results obtained are verified using full-wave simulations. Reflection coefficients over a 749-element reflectarray aperture for a broadside radiation pattern are presented for a couple of cases and the effects of coupling conditions in conjunction with incidence angles are demonstrated. The presented theory provides explicit physical intuition and guidelines for efficient and accurate reflectarray design.

In chapter 4, tunable reflectarray elements capacitively loaded with Barium Strontium Titanate (BST) thin film are shown. The effects of substrate thickness, operating frequency and deposition pressure are shown utilizing coupling conditions and the performance is optimized. To ensure minimum affects from biasing, optimized biasing schemes are discussed. The proposed unit cells are fabricated and measured, demonstrating the reconfigurability by varying
the applied E-field. To demonstrate the concept, a 45 element array is also designed and fabricated. Using anechoic chamber measurements, far-field patterns are obtained and a beam scan up to 25° is shown on the E-plane.

Overall, novel theoretical approaches to analyze the reflection properties of the reflectarray elements using Q factors are developed. The proposed theoretical models provide valuable physical insight utilizing coupling conditions and aid in efficient reflectarray design. In addition, for the first time a continuously tunable reflectarray operating at Ka-band is presented using BST technology. Due to monolithic integration, the technique can be extended to higher frequencies such as V-band and above.
Dedicated to my lovely family Raja Gopal Karnati, Rajya Lakshmi Karnati, and Anusha Karnati for all your love and support
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CHAPTER 1: INTRODUCTION

1.1. Motivation

One of the primary interests of 21st century is to understand the Earth and other planets of our solar system, and their interactions. This knowledge not only helps us understand the world better, but also predict the changes in the future [1]. These interactions occur from local, regional to global and range from interim weather to enduring climate, predictable cyclones to unpredictable tsunami and earthquakes, motion of the Earth and other planets, life outside the Earth etc. For precise characterization of these complex changes in the global system, next-generation tracking systems are required to be agile, multifunctional, and tunable with high data rates, particularly at frequencies of $K_a$ band and above. Antenna is one of the major components of a communication system. These antenna systems are able to use a common aperture for inter- and intra-planet communications, remote sensing, and radar applications. To enable the rapid and seamless switching between the aforementioned functions, electrically beam-steerable antennas with high tuning speeds are required. With sophisticated tracking systems and high-pixel cameras on board, antenna systems with high data rates are crucial to reduce the transmission time of data and high-quality images by more than one order of magnitude. At the same time, mechanical considerations and high launching costs demand for compact, flexible and deployable systems.

Parabolic reflectors, though highly efficient and wide band, are bulky and heavy. The complicated curvature makes reflector antennas expensive in both fabrication and deployment. Phased array antennas, on the other hand, can be planar and easily deployable. However, they suffer from the disadvantages of complex beam-forming manifold and expensive transmit/receive modules. Reflectarray antennas, an exquisite combination of reflectors and antenna arrays, utilize quasi-optical excitation on a planar aperture and are capable of electrically steering the beam without the need of lossy feed networks.

1.2. Introduction to Reflectarray antennas

Reflectarray antennas have been considered as a promising candidate for highly efficient, directive and beam scanning applications due to their unique advantage of combining reflectors and antenna arrays [2], [3], [4]. Berry et al first proposed the concept of reflectarray using waveguide elements [5] as early as 1963. Reflectarray with microstrip elements was first investigated as early as 1978 [6]. With the advancement in fabrication technology, microstrip
Reflectarray antennas started to become popular in early 1990’s due to their simple planar geometry and low profile [7], [8].

A plurality of microwave applications desire highly-directive antennas with the main beam focused at a specified direction. Reflectors and antenna arrays have been commonly employed to achieve the same goal. Reflector antennas can provide large bandwidths and high efficiencies, however, are bulky and expensive to fabricate and deploy. On the other hand, antenna arrays can be planar but suffer from additional losses inside complex feed networks. Reflectarray antenna is an exquisite solution that combines the advantages of quasi-optical excitation of parabolic reflectors and planar structure of antenna arrays. Similar to parabolic reflectors, reflectarray antennas can achieve very high efficiencies (>50 percent). Being flat and compact in form factor similar to phased arrays [9], reflectarray facilitates an inexpensive fabrication procedure and simpler mechanism to deploy with moderate installation costs. Reflectarray elements with active phase adjustment capability can provide tunable and beam scanning capabilities. Low profile, high gain, beam steering capabilities, conformal and compact forms make reflectarrays potential contenders for deep-space telecommunications and micro-spacecrafts, particularly at frequencies of K- and V-band [8, 10]. In addition, by adjusting the phase of the each element, contour beam shapes can also be achieved [11]. Recently, conformal and flexible reflectarray antennas started to become popular which could provide cost-effective packaging and launching for spacecrafts [12].

1.3. Operation

Figure 1-1 shows a reflectarray antenna illuminated by a feed. Each element in the array is excited by the incident wave coming from the feed at different phases. To achieve a constructive interference in the desired direction, every element of the array needs to provide the required phase compensating for the differential phase delay from the illuminating feed and also the linear phase variation required for beam scanning. From the array theory, the phase shifts desired on the element with coordinates \((m,n)\) for a beam scan angle of \((\theta_b,\varphi_b)\) can be calculated using:

\[
\phi_{des}(m,n) = k_0d_{mn} - k_0(m \cdot \sin \theta_b \cos \varphi_b + n \cdot \sin \theta_b \sin \varphi_b)
\]  \hspace{1cm} (1)

Unit cell configurations utilizing a variety of techniques were proposed to achieve the same goal and can be approximately classified into non-tunable and tunable reflectarray elements. Before discussing these techniques, it is important to introduce the performance parameters for a reflectarray design.
1.3.1. Bandwidth

Bandwidth is one of the major parameters in characterizing the performance of an antenna. There are two major factors that limit the bandwidth performance of a printed reflectarray [2]. The first limiting factor is the narrow bandwidth of the antenna element, and the other factor is the differential spatial phase delay from the feed. Bandwidth limitation due to differential spatial distances is significant only for antennas with very large electrical dimensions [2], and can be compensated by using antenna elements that provide true time delays (TTD) [13]. Reflectarrays designed using traditional techniques such as patches with variable-length stubs [8] and variable-size patches [7] have limited element bandwidth. To achieve wide bandwidth, techniques such as multiple stacked patches [14], multiple-resonance elements [15], and aperture-coupled antennas [16] were proposed.

1.3.1. Phase range

In a reflectarray design, it is crucial to achieve desired phase distributions in the array to account for the different spatial delays from the feed and linear phase variations required for beam forming. A larger phase range is beneficial to avoid phase errors and achieve higher efficiencies and lower side lobe levels. Traditional approaches of using a

Figure 1-1. Geometry of a printed reflectarray antenna with an illuminating feed.
single resonant element result in phase ranges of less than 360°. Design techniques using TTD, multiple-resonant elements can provide wider phase ranges but at the expense of increased design complexities and fabrication costs.

1.3.2. Aperture Efficiency
The overall aperture efficiency of a reflectarray is dependent on the illumination efficiency and spillover efficiency of the antenna [2]. Particularly, the directivity of the feed antenna and the \( f/D \) ratio determine the aperture efficiency of the reflectarray antenna. For an efficient reflectarray design, a detailed study on the effects of the feed and \( f/D \) is required [17].

1.3.3. Overall Efficiency
The overall efficiency of a reflectarray antenna is determined by the aperture efficiency, polarization efficiency, phase efficiency, blockage efficiency and the loss performance of the antenna elements at the operating frequency [18]. Unlike reflectors, reflectarray antenna consists of resonant radiating elements which suffer from higher losses. To achieve higher efficiency, low-loss antenna elements which can provide the desired phase shifts are desired.

1.3.4. Phase-sensitivity
Phase-sensitivity is derived from the slope of the reflection phase versus the tuning parameter employed to design the array. Large phase-sensitivity implies that the variation of phase with the tuning parameter is very high, which is undesirable because small errors in fabrication or approximations in the analysis might lead to significant phase errors and degrade the array performance.

1.4. Passive Reflectarray elements
As the name suggests, these unit cells usually employ some variations of passive tuning mechanisms and are commonly used for fixed-beam applications. Several techniques have been proposed out of which identical patches with variable-length stubs were used in [8]. The incident feed signal received by the patch enters the open-ended stub and gets reflected. This reflected signal re-enters the patch and gets re-radiated with the accumulated phase-shift determined by the stub length. Major drawbacks of this technique are higher cross-polarization levels and additional dissipative losses due to the stubs. A similar technique of achieving the phase delay is to use the aperture-coupled patches loaded with variable-length transmission lines [16, 19]. The energy incident on the antenna element is coupled to a transmission line through an aperture in the ground plane. By varying the length of the transmission line, desired
phase ranges from 0° to 360° can be easily achieved. In fact, phase ranges greater than 360° can also be achieved which will improve the bandwidth of the array by means of the true time delays [13]. Compared to the patches loaded with stubs on the reflecting surface, aperture-coupled antennas possess the advantage of having the transmission line not on the reflecting surface but beneath the ground plane. However, aperture-coupled antennas suffer with spurious radiation from the aperture, and increased design and fabrication complexities.

Techniques to control the reflection phase by varying the resonant dimension were introduced in [20], [7]. By changing the resonant length, the resonant frequency of the unit cell is varied and different phases can be achieved at the operating frequency of the array. In [20], printed dipoles with variable dipole lengths were used to achieve the desired phase shifts. Similarly in [7], variable patch lengths were used to control the re-radiated phase. These methods result in lower dissipative losses and reduced cross-polarization levels compared to patches with variable stub lengths. However, smaller phase ranges and highly non-linear phase variations are observed, which limit the operational bandwidth of the reflectarray.

Multi-layer reflectarray structures in order to improve the bandwidth performance were proposed in [14, 21]. The dimensions of the resonant patch elements on each layer are optimized to improve the bandwidth performance of the entire reflectarray structure. In [14], a two-layer reflectarray composed of two stacked arrays with variable-size patch elements was proposed. Further improvement in terms of bandwidth was shown in [21], where three-layer stacks of variable-size patch elements were used. Even though significant bandwidth improvement is achieved, the multi-layer structures suffer from major drawbacks of increased real estate of the antenna, complex design and expensive fabrications.

Another technique to improve the bandwidth is to use the multiple resonant antenna elements. The resonant structures are coupled together to enhance the bandwidth and phase-sensitivity of the unit cells. The inadequate phase range provided by a single resonant structure was improved in [22] by using square- and circular-shape multiple rings. Lower phase-sensitivity with increased overall phase-range has been achieved using this technique. In [23], a single-layer printed reflectarray with multi cross loop elements of variable loop lengths were used for enhanced bandwidth operation. A significant improvement in the gain bandwidth performance was shown.
Bandwidth enhancement was also shown using sub-wavelength elements in [24-27]. In [27], the gain bandwidth of a microstrip reflectarray was improved by using the artificial impedance surface consisting of closely-spaced electrically-small patches on a dielectric substrate. Using this technique, a 1-dB gain bandwidth in excess of 20% on a single-layer aperture was shown. In [25], a comparative study between two reflectarray antennas, one with \( \lambda/2 \) spacing and the other with \( \lambda/3 \) spacing, respectively, was done at 32 GHz. A peak gain of 28.92 dB with a 0.5 dB gain bandwidth of 6.25% for \( \lambda/2 \) grid spacing array and a maximum gain of 29.22 dB with a 0.5 dB gain bandwidth of about 9% for \( \lambda/3 \) grid spacing array were measured. Similarly, it was shown in [26] that sub-wavelength coupled resonant elements could significantly reduce losses while maintaining the reflection phase variation. The reduced losses were shown to be associated with the reduced frequency dispersion of the reflection phase from sub-wavelength resonant elements.

Other interesting developments in reflectarray antennas include inflatable antennas printed on thin membranes for spacecraft antenna applications. These inflatable antennas when deflated can be rolled and significantly reduce the transportation and deployment costs. A circularly polarized X-band deployable and low-mass reflectarray antenna was successfully demonstrated in [12]. This antenna exhibited surface tolerance of \( \pm 1.3 \) mm, mass of 1.2 kg, bandwidth of at least 3%, and a peak side-lobe and cross-pol level of -18 dB with a radiation efficiency of 37%. An extension to a \( K_a \)-band inflatable reflectarray consisting of 200,000 elements was shown in [28]. The antenna elements were printed on thin membranes that were mechanically supported at its perimeter by inflatable tubes. A dual \( \chi/K_a \)-band inflatable reflectarray using offset feed configuration and ring elements on thin membranes was shown in [29]. More than 50% efficiencies were demonstrated in both frequency bands.

A promising innovation in the reflectarray development is to integrate the reflectarray elements with the solar cells [30, 31]. This integration facilitates the combination of the two largest structures of a spacecraft and significantly reduces the deployment cost and the real estate. In [31], an inflatable reflectarray consisting of 408 crossed-dipole elements operating at X band was demonstrated. The antenna elements were printed on a Kapton substrate, which was then integrated with the solar panel. The integrated solar/antenna array consists of a solar panel with 198 silicon solar cells. A good solar performance was shown using the proposed structure, however, the RF performance was not as good with an aperture efficiency of merely 10%. In [32], a unit cell design employing square-shape ring element as the unit cell was shown on the glass substrate with copper metallization This solar/antenna array exhibited an average
optical transparency in the visible spectrum of 85% and an average loss of 0.25 dB at X band. The optical performance was improved up to 90% by using transparent conductors at the expense of increased RF losses up to 2.45 dB. In [33], a reflectarray integrated with solar cells operating at $K_a$ band was shown using crossed-dipole unit cell elements. An epitaxial layer consisting of InGaAs and InGaP for solar energy harvesting was grown on a 4-inch silver-backed germanium wafer. Grid electrodes were deployed to collect converted electricity. A cover glass, which was bonded to the grid electrodes and the epitaxial layer, served to protect them in space. On top of the cover glass, an array of gilded crossed-dipoles was printed. This solar/antenna array achieved an optical blockage of 17.6% and an antenna gain of 26.3 dBi which corresponded to an aperture efficiency of 29.8%.

1.5. Tunable Reflectarray elements

In order to realize a beam-steerable reflectarray antenna, the reflection phase of each antenna element has to be tunable. Antenna elements loaded with a variety of tunable components have been proposed to achieve the phase tuning. Each concept has its own pros and cons. In general larger phase ranges with low losses are desired. Also it is preferred to have a linear relationship between the tuning parameter and the reflection phase to decrease the phase-sensitivity. Reflectarrays with tunable elements using RF MEMS, varactors, PIN diodes, liquid crystal materials, complex oxide thin films, graphene, fluids, micro-motors and others have been proposed to achieve the phase tuning.

Phase shift elements based on MEMS technology for satellite telecommunications was shown in [34]. Tunable MEMS capacitors were integrated with the reflectarray element in [35]. A phase range of around 225° with a maximum reflection loss of approximately 3 dB was measured at 5.8 GHz. In [36], a patch element with a slot on the ground plane was used as the unit cell. RF MEMS switch was loaded across the slot to achieve tunability. By using four MEMS switches in the element, 10 independent states were realized with a limited phase range of 150° and a loss varying from 0.4 to 1.5 dB at 2 GHz. The major challenges of this technique are the back side radiation and the interconnects between the MEMS devices and the antenna unit cell, which will limit the range of operating frequency. Recently, a reflectarray monolithically integrated with RF MEMS switches operating at 26.5 GHz was demonstrated in [37]. Aperture-coupled microstrip antennas were used as the unit cell and the phase tuning was achieved by adjusting the length of the open-ended transmission lines via MEMS switches.

In [38], a slotted patch loaded with a varactor was used as the tunable element. Using an SMD capacitor, a phase range of about 200° was shown at 7 GHz. In [39], a varactor diode between the microstrip patch and ground plane was used
to achieve the phase agility. By varying the capacitance, a phase range close to 360° with a maximum reflection loss of 2.6 dB at 5 GHz was shown using waveguide measurements. In [40], two varactor diodes that connected two halves of a microstrip patch were used as the tuning mechanism. A phase range of over 320° at 5.5 GHz was discussed. Using the proposed unit cell, a 70-element electronically-tunable reflectarray was also demonstrated. Dipoles loaded with a tunable capacitor were analyzed for tunability in [41]. By applying a bias voltage from 0 to 12 V, a phase range of about 270° and a maximum reflection loss of about 4 dB were shown at 12 GHz. In addition, a theoretical model to represent the reflection coefficient of the unit cell was also proposed.

The usage of monolithic PIN diodes for beam-steerable reflectarray antennas was presented in [42]. The process was based on silicon-on-silicide-on-insulator (SSOI) technology. The buried silicide provided a low contact resistance and the buried oxide enabled a low parasitic capacitance. Using PIN diodes, an electronically switching-beam reflectarray antenna at X band was shown in [43]. To reduce the fabrications cost and complexity, aperture-coupled patches combined in pairs to a common delay line are used for the phase control. The antenna was capable of switching the beam between ±5°, in a scanning plane tilted 18.3° with respect to the YZ plane.

In [44], the application of liquid crystals to achieve phase tunability was shown. By applying an electrostatic field across the material, the relative permittivity was changed between two extreme values, thereby modifiying the electrical dimensions of the antenna element. Unit cells operating at X band were demonstrated with very high losses and moderate phase ranges. Tunable microstrip reflectarray elements based on the voltage-controlled dielectric anisotropy property of nematic state liquid crystals were discussed in [45]. For fabrications, the metallization of quartz/silicon wafers based on micromachining processes were used. By applying a low frequency AC bias voltage of 10 V, a phase shift of 165° with a loss of 4.5–6.4 dB at 102 GHz and a 130° phase shift with a loss variation between 4.3–7 dB at 130 GHz were obtained.

The use of graphene for fixed-beam reflectarray antennas at Terahertz (THz) was proposed in [46]. The designed reflectarray operated at 1.3 THz and comprised of more than 25000 elements of size about λ/16. Graphene’s unique bandgap structure leads to a complex surface conductivity at THz frequencies. This allows for the propagation of very slow plasmonic modes and significantly reduces the electrical size of the unit cell. When compared to gold, the graphene-based reflectarrays demonstrated better cross-polarization levels and bandwidths at these frequencies. This study is an important step in realizing reconfigurable THz reflectarrays based on graphene.
A novel photonically-controlled reflectarray antenna based on the variation of the slot length beneath a regular lattice of identical patches by generating a photo-induced plasma inside the silicon was presented in [47]. By changing the amount of plasma profile illuminated by the optical light, the slot length was varied and the desired phase distribution was achieved. A reflectarray prototype was demonstrated at 30 GHz. However, more work is necessary to improve the efficiency, linearity and bandwidth performance of the proposed structure.

Another reconfigurable reflectarray unit cell enabled by fluidic colloidal dispersions was proposed in [48]. The unit cell consisted of a microstrip patch on a dielectric substrate. The reactive loading mechanism was a coaxial stub microfluidic impedance transformer (COSMIX) centered about the patch width and placed at a distance from the radiating edges. By altering the dielectric properties of the fluidic materials that fill the COSMIX, the reactive loading to the microstrip patch was changed and the reflection phase was tuned. An overall phase range of 220° with a 2-dB maximum loss was measured at 3 GHz. The major disadvantages of using micro-fluidic techniques are less reliability, increased design and fabrication complexities, and slow switching speeds.

Semiconductor varactors and PIN diodes have degraded quality ($Q$) factors above 10 GHz making them less attractive for space applications. The reconfigurable reflectarray elements based on LCP usually suffer from very high losses and slow switching speeds. Even though the usage of graphene for THz applications is promising, the losses are still high at mm-wave frequencies. Compared to other tuning mechanisms, RF MEMS offers high $Q$ at mm-wave frequencies, however, increased complexity and slow switching speeds make them undesirable for many applications.

Reflectarray antennas based on complex oxide materials such as Barium Strontium Titanate (BST) are drawing attention due to their high power-handling capacity, negligible DC power consumption, fast switching speeds and monolithic integration capability [49-51]. The fast response time, on the order of nanoseconds, makes BST materials very suitable for fast beam scanning and multiple beam forming applications. The large dielectric constants of BST, in the range of hundreds, enable the miniaturization of microwave components and the fabrication of high-capacitance-density capacitors. In addition, BST also possess key advantages in view of practical space applications. In [52], it was shown that the variation of BST permittivity with temperature was small up to 650 K. All the aforementioned advantages make BST a promising candidate for space application operating at mm-wave frequencies.
A comparison of the different technologies available for the implementation of reconfigurable reflectarray antennas is presented in [53].

Table 1-1. Comparison of various reconfigurable technology for the implementation of tunable reflectarrays. (+”, ‘0’, and ‘-‘ refer to good, neutral and poor, respectively. [53]

<table>
<thead>
<tr>
<th>Type</th>
<th>Technology</th>
<th>1.6. Barium Strontium Titanate (BST)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumped elements</td>
<td>PIN diodes</td>
<td>Barium Strontium Titanate (BST) is the continuous solid solution of two common ferroelectric ceramics, barium titanate (BaTiO$_3$) and strontium titanate (SrTiO$_3$). In general a ferroelectric ceramic has a perovskite crystalline structure, consisting of tetravalent metal ions (titanium) in a lattice of large divalent metal ions (barium, strontium) and O$_2$- ions. A common perovskite structure (ABO$_3$) is shown in the Figure 1-2, where A represents the large cations positioned at the corner (yellow) and B represents smaller cation located at the center (black) and oxygen located at the face centers (white) of the unit cell.</td>
</tr>
</tbody>
</table>
The majority of the cubic perovskite structures are not symmetric due to distortions, where positively charged metal ions are displaced with respect to the negatively charged oxygen ions. This asymmetry results in the formation of polar axis and spontaneous polarization (ferroelectric property) [54]. A phase transition from ferroelectric to paraelectric is usually observed at the Curie temperature, where paraelectric property is observed at the temperatures above the Curie temperature [55]. The phase transitions and the corresponding crystal structures are shown in Figure 1-3.
In the paraelectric phase, the dielectric properties of BST can be varied by modifying the composition. In particular, processing gas pressure ($P_{sys}$) plays a significant role in the (Ba+Sr)/Ti ratio, which determines the dielectric constant, $Q$ factor, and tunability of the BST thin film [58]. By applying a variable DC voltage across the electrodes of the BST capacitor, the dielectric constant is varied and a tunable capacitor is realized.

Recently, reconfigurable reflectarray antennas based on ferro-electric materials such as Barium Strontium Titanate (BST) [59], [60] are becoming popular, due to the potential for monolithic integration, fabrication costs and moderate losses. A phase shifter based on BST for X-band reflectarray applications was presented in [59]. These tuning elements are typically fabricated on separate substrates and then diced and assembled on the reflectarray antenna panel. Due to the parasitic effects within the interconnects, e.g. wire bonding, between the tuning elements and antennas, the
maximum operating frequency is limited [61]. A tunable reflectarray unit cell designed on a thick BST film was presented in [60]. Maximum reflection losses in the range of 14-18 dB for varying bias voltages at X band were observed. In [61] efforts to reduce the losses by limiting the BST within the gap were attempted. However, un-optimized BST deposition and biasing scheme resulted in an overall phase range of only 80° at X band. Excessive losses and limited phase range makes any reflectarray element less useful. In [62], a similar unit cell operating at K_a band with an overall phase range of 114° was presented using simulations without any measurement data.

1.7. Reflectarray analysis techniques

As shown in Figure 1-1, reflectarray antenna consists of an array of radiating elements separated by a certain distance. A commonly employed technique to analyze the reflection phase of each unit cell is infinite array approach. Analysis of microstrip reflectarray element using infinite array approach was first attempted by [63]. This procedure is based on periodic boundaries, which simulates an infinite array of identical elements separated by a fixed distance. There have been different full-wave analysis methods to analyze a reflectarray element with periodic boundaries, out of which the popular ones being waveguide approach, parallel plate approach, and Floquet analysis.

There have been a few theoretical models to analyze the characteristics of reflectarray elements. The analysis based on theoretical models not only provides significant physical insight but also aid in optimizing the design with fewer computational resources. By using transmission line model, the reflection losses of the periodic structure were analyzed in [64]. In [65], an analytical model for a dipole loaded with a tunable varactor was presented. Similarly in [40], the modeling of a tunable reflectarray element based on varactor diode was presented. In this report, novel theoretical approaches to analyze the reflectarray elements and coupling conditions is presented using Q factors.
CHAPTER 2: REFLECTARRAY ANALYSIS UTILIZING Q FACTORS

2.1. Introduction

In this chapter, the reflection properties of reflectarray elements are analyzed using the Q factor theory. The analysis is first applied for fixed incidence angle cases. In a reflectarray design, the phase of each antenna element is compensated to account for the differential spatial lengths from the feed and achieve a desired phase front in the preferred direction. Thus, the knowledge of the phase response of the unit cell is a paramount for any reflectarray design. There exist many combinations of physical parameters such as dielectric properties, patch size, substrate thickness, and spacing between the elements, which could result in the same resonant frequency but with different reflection properties. The optimum design should make the best use of the aforementioned degrees of freedom in order to achieve the desired reflectarray performance. Among these parameters, effects of the substrate thickness have been investigated. It was reported in [66] that as the substrate thickness increases, the reflection loss decreases which is desirable. However, as this study was further expanded in [67], it was shown that thicker substrates exhibit smaller phase-swing which limits the reflectarray beam-steering angle. Recently, effects of the substrate loss on the reflection properties of reflectarray unit cells were reported in [68]. It was observed that for certain combinations of dielectric losses and substrate thicknesses, the unit cells exhibit anomalous phase-swing phenomenon. Therefore, it is necessary to study the effects of various physical parameters on the reflectarray unit cell performance such as phase-swing, bandwidth, and loss. In addition, a theoretical model is desirable to predict and avoid the misbehaved phase responses.
Figure 2-1. Waveguide simulator approach for studying a reflectarray unit cell (a) ideal TEM excitation (b) practical TE\textsubscript{10} excitation using metallic walls and (c) the electric field and magnetic current distribution on the reflectarray unit cell in (a) or (b).
In this section, an intuitive theoretical expression of the reflection coefficient of reflectarray unit cell is derived using its analogy to coupled resonators [69, 70].

This theory uses the quality ($Q$) factors to quantify the reflection characteristics of reflectarray unit cells. In this approach, there is no need for the intermediate lumped-element circuit model which still relies on full-wave simulations shown in [40, 71]. Using the theory developed here, the anomalous phase-swing phenomenon [68, 72] can be identified and avoided in reflectarray designs. Effects of substrate thickness ($h$), dielectric constant ($\varepsilon_r$), and patch width ($W$), dielectric and metallic losses on the reflection coefficient, phase-swing, and bandwidth are theoretically derived and verified using full-wave simulations. First, the expressions for reflection coefficients of unit cell excited by fixed incident angles are discussed. Next, a technique to extract the quality factor using full-wave simulations is also presented. Later, the difference between TEM and TE$_{10}$ modes of excitation is analyzed in terms of Q factors and coupling conditions. Finally, the effects of inter-element spacing on the unit cell’s performance are shown.

2.2. Theoretical approach

Waveguide excitation is commonly used to characterize and design reflectarray unit cells. In this approach, the reflectarray unit cell is placed inside a parallel-plate TEM waveguide as shown in Figure 2-1(a) or a rectangular metallic waveguide as shown in Figure 2-1(b) and the reflection properties of the unit cell are extracted. This method uses periodic boundaries to simulate an infinite reflectarray accounting for the mutual coupling. A parallel-plate TEM waveguide corresponds to a plane wave with fixed normal incidence. On the other hand, a rectangular waveguide corresponds to an oblique incidence angle that is dependent on the frequency of operation and waveguide dimensions. The TEM waveguide approach is generally limited to simulations as the structure is practically complicated. However, a metallic rectangular waveguide supporting a TE$_{10}$ mode is commonly employed for measurements.

In this section, a complete theoretical approach to extract the reflection properties of a unit cell is discussed. The reflection coefficient at the patch surface for a reflectarray element inside a waveguide can be expressed by coupled mode theory as [69]:

$$\text{Reflection Coefficient} = \frac{Z_{in} - Z_{out}}{Z_{in} + Z_{out}}$$
\[ \Gamma(f) = \frac{1}{Q_{rad}} - \frac{1}{Q_o} - \frac{2j(f - f_r)}{f_r} \]

where \( f_r \) is the resonant frequency and \( Q_{rad} \) is the radiation \( Q \) factor of the patch antenna. \( Q_o \) is the parallel combination of metallic loss \( (Q_c) \) and dielectric loss \( (Q_d) \) of the patch antenna given by [73, 74]:

\[ Q_c = h\sqrt{\pi f \mu \sigma} \]  
\[ Q_d = \frac{1}{\tan \delta} \]  
\[ Q_o = \frac{Q_c Q_d}{Q_c + Q_d} \]

First, a theoretical approach to obtain the radiation \( Q \) factor of a reflectarray unit cell is presented. For comparison purposes, a second method which is dependent on simulations [75] is also discussed. First, the expression for \( Q_{rad} \) of a unit cell inside a rectangular waveguide is discussed. The results obtained from theory are compared with simulations. Using the derived theory, different types of coupling modes for a reflectarray unit cell are discussed. A few cases are chosen for fabrications and the results obtained from theory, and simulations are compared with measurements. Along the discussion, the limitations of the model will be discussed with supporting examples.

Similarly, the expression of \( Q_{rad} \) for a unit cell inside a parallel-plate TEM waveguide is also shown. A few cases are chosen and compared with simulations.

2.2.1. \( Q_{rad} \) of a rectangular unit cell inside a metallic waveguide:

The reflectarray unit cell inside the waveguide involves two different modes which are coupled together; \( TM_{010} \) mode resonating inside the patch and \( TE_{10} \) mode propagating inside the waveguide. To formulate the coupling between these two modes, the structure shown in Figure 2-1(b) can be modeled by a transmission line (the waveguide) terminated with a resonator (the patch).
\(Q_{\text{rad}}\) is obtained by assuming the reflectarray unit cell as a patch antenna, where the source of excitation is a rectangular waveguide. An analysis based on the cavity model of a patch is used. Assuming that the dominant mode within the cavity is \(TM_{010}\) mode, the field distribution under the patch is given by [73]:

\[
E_z(y) = E_0 \sin \left( \frac{\pi y}{L} \right), \quad \left| x \right| \leq \frac{W}{2}, \left| y \right| \leq \frac{L}{2}
\] (6)

where \(E_0\) is the electric field amplitude at the radiating edges. \(L\) and \(W\) are the length and width of the patch respectively.

Utilizing field equivalence principle, the patch antenna is replaced with equivalent magnetic currents at its radiating edges as shown in Figure 2-1(c). Using image theory, and assuming a thin cavity, the equivalent magnetic currents are doubled and are now free to radiate in an infinite waveguide. The linear equivalent magnetic currents at the radiating edges are [73]:

\[
\overline{I}_m = -2hE_0 \hat{x}, \quad y = -\frac{L}{2}, \frac{L}{2}
\] (7)

At the non-radiating edges, we have:

\[
\overline{I}_m(y) = \pm 2hE_0 \sin \left( \frac{\pi y}{L} \right) \hat{y}, \quad x = \pm W / 2
\] (8)

The amplitude \(A^+\) of \(TE_{10}\) mode excited in the infinite waveguide by a volume magnetic current density \(\vec{M}\) can be found using [74]:

\[
A^+ = \frac{1}{P_1} \int \vec{H}_1^* \cdot \vec{M} \, dv
\] (9)

where \(\vec{H}_1\) is the magnetic field of the \(TE_{10}\) mode propagating in the negative \(z\) direction and \(P_1\) is the normalization constant given by:

\[
P_1 = 2 \int e_1 \times h_1 \cdot ds = \frac{ab}{Z_W}
\] (10)

where \(a\) and \(b\) are the waveguide dimensions and \(Z_W\) is the wave impedance of the \(TE_{10}\) mode. \(e_1\) and \(h_1\) are the modal transverse field components of the \(TE_{10}\) mode given by [74]:
\[ e_i = \hat{y} \sin \frac{n \pi y}{a} \]  
\[ h_i = -\frac{\hat{x}}{Z_w} \sin \frac{n \pi x}{a} \]  
and \( Z_w \) is given by:

\[ Z_w = \frac{\omega \mu}{\beta_w} = \frac{\omega \mu}{\sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n \pi}{a}\right)^2}} \]  

Noting that only the radiating edges contribute to the integral in (9), it simplifies to a line integral. Using \( \bar{I}_m \) in place of \( \bar{I} \) in (9), \( A^* \) is found as:

\[ A^* = \frac{8hE_0}{\pi b} \sin \left( \frac{\pi W}{2a} \right) \]  

The power carried away by the \( TE_{10} \) mode towards the waveguide port is:

\[ P_{rad} = \frac{1}{2} \int E \times H^* \, ds \]  
\[ = \frac{ab}{4Z_w} |A|^2 = \frac{16h^2 a E_0^2}{\pi^2 b Z_w} \sin^2 \left( \frac{\pi W}{2a} \right) \]  

The electric energy stored in the patch is given by:

\[ W_e = \frac{\varepsilon}{4} \iint |E|^2 \, dv = \frac{\varepsilon}{8} E_0^2 h W L \]  

where \( \varepsilon \) is the permittivity of the substrate material. Since the electric and magnetic energies are equal at resonance, the external quality factor \( Q_{rad} \) is given by:

\[ Q_{rad} = 2\pi \frac{\text{Energy Stored}}{E_{rad}} \]  
\[ = \frac{f_0}{32 h} \frac{W L b Z_w}{\sin^2 \left( \frac{\pi W}{2a} \right)} \]  

where \( W \) and \( L \) are width and length of the patch, \( h \) is the substrate thickness, \( a \) and \( b \) are the waveguide dimensions and \( Z_w \) is the wave impedance of the \( TE_{10} \) mode.
2.2.2. Coupling conditions

Using the expressions obtained for $Q_o$ and $Q_{rad}$, the reflection coefficient at the resonant frequency can be derived using (2):

$$
\Gamma(f_r) = \frac{1}{1 + \frac{Q_{rad}}{Q_o}}
$$

(18)

It is observed that based on the relative values of $Q_{rad}$ and $Q_o$, three different conditions exist:

1. $Q_{rad} = Q_o$; Critically-coupled
2. $Q_{rad} > Q_o$; Under-coupled
3. $Q_{rad} < Q_o$; Over-coupled

1) If $Q_{rad} = Q_o$, (2) reduces to

$$
\Gamma(f_r) = 0
$$

(19)

This corresponds to critically-coupled condition. In this case there is no reflection from the unit cell and all the energy is dissipated inside the resonator. This condition is not useful for reflectarray applications.

2) If $Q_{rad} > Q_o$,

$$
\Gamma(f_r) = \Gamma(f_r) e^{j\pi}
$$

(20)

This corresponds to under-coupled condition. At this condition, it can be observed that the reflection phase at resonance is always $180^\circ$, which represents anomalous phase phenomenon.

3) If $Q_{rad} < Q_o$,

$$
\Gamma(f_r) = \Gamma(f_r) e^{j0}
$$

(21)

This corresponds to over-coupled condition, which should be used for reflectarray designs.

2.2.3. Reflection loss, Phase-swing and Bandwidth

The effects from $h$, $\varepsilon_r$, and $W$ on the performance of the unit cell are evaluated in terms of maximum reflection loss, phase-swing ($\Delta$), and bandwidth ($\beta$), which can be quantified using the $Q$ factors described earlier.

The reflection phase of a unit cell is defined as:
\[ \Phi(f, f_r) = -\tan^{-1}\left( \frac{2(f - f_r)}{f_r \frac{1}{Q_{rad}} - \frac{1}{Q_o}} \right) - \tan^{-1}\left( \frac{2(f - f_r)}{f_r \frac{1}{Q_{rad}} + \frac{1}{Q_o}} \right) \]  

(22)

Phase-swing is defined as:

\[ \Delta = \Phi(f_0, f_2) - \Phi(f_0, f_1) \]

(23)

\[ f_1 = \frac{f_0}{1 + \delta L/L_0}; \quad f_2 = \frac{f_0}{1 - \delta L/L_0} \]

(24)

where \( f_0 \) is the center frequency of the reflectarray unit cell. \( L_0 \) is the patch length to achieve a resonant frequency equal to \( f_0 \). \( f_1 \) and \( f_2 \) correspond to the resonant frequencies by tailoring the patch length \( L_0 \) with an amount of \( \pm \delta L \).

Bandwidth is defined as the range of frequencies over which the reflection phase differs by \( \pm 45^\circ \) with respect to the center frequency \( f_0 \) [71] and can be derived as:

\[ \beta \approx \frac{1}{f_0} \frac{\pi / 2}{\sigma_f} \]

(25)

\[ \sigma_f = -\frac{\partial \Phi(f, f_0)}{\partial f} \bigg|_{f=f_0} = \frac{4Q_o^2Q_{rad}}{f_0(Q_o^2 - Q_{rad}^2)} \]

(26)

2.3. Analysis of coupling conditions and its effects of unit cell’s performance

The response of a unit cell is determined by the combined effect of the physical parameters on \( Q \) factors and their relative values. In this section, the effects of \( h, W, \varepsilon, \tan\delta, \) and \( \sigma \) on \( Q \) factors, coupling conditions and reflection properties are studied using the theory defined in Section 2.2. and compared with Ansoft High Frequency Structure Simulator (HFSS) full-wave simulations.

2.3.1. Effects of Substrate thickness (h)

In this section, unit cells with different substrate thicknesses are studied, while the substrate material (Rogers RO3010 with specified \( \varepsilon_r = 10.2, \tan\delta = 0.0035, \) ½ oz copper) and patch width \( (W = 2 \text{ mm}) \) are fixed. To facilitate adequate waveguide measurements of unit cells, a high dielectric constant is chosen. The patch length \( L_0 \) is adjusted to resonate at 32 GHz for every thickness [73]. Substrate thicknesses \( h \) ranging from 1 mil (25.4 \( \mu \text{m} \)) to 20 mil (0.508 mm) are considered for the study. Throughout the text, \( h \) is referred in mil in view of commercially-available units. \( Q \) factors are plotted in Figure 2-2 using the theory described in section 2.2. It is observed that as \( h \) increases, \( Q_o \) increases.
while $Q_{rad}$ decreases. For a substrate thickness of ~3 mil (76.2 µm), $Q_{rad}$ and $Q_o$ are equal, corresponding to the critically-coupled condition. The regions where the patch is under-coupled or over-coupled are also indicated.

![Figure 2-2. Q factors versus substrate thickness using the theory.](image)

To demonstrate these different coupling conditions, the reflection magnitude and phase of three reflectarray unit cells with $h = 1$ mil (25.4 µm), 3 mil (76.2 µm), and 5 mil (127 µm), respectively, are plotted in Figure. 2-3. For the critically-coupled condition, the maximum reflection loss is very high as shown in Figure. 2-3(a). For the under-coupled case, the unit cell exhibits anomalous phase characteristics as shown in Figure. 2-3(b). Despite having low reflection loss, the inadequate phase characteristics render this coupling condition inappropriate for use. It is therefore concluded that a reflectarray unit cell ought to be over-coupled to have useful reflection characteristics as observed for $h = 5$ mil.
Figure 2-3. Reflection (a) magnitude, and (b) phase versus frequency for different $h$ using the theory and full-wave simulations showing different coupling conditions.

Figure 2-4 shows the variation of reflection magnitude versus the substrate thickness with three coupling conditions labeled. Using (23)-(26), the phase-swing and bandwidth for different substrate thicknesses are plotted in Figure 2-5. It should be noted that these two parameters can only be defined in the over-coupled region. A patch length variation of $\delta L/L_0 = \pm 20\%$ is used to tailor the reflection phase and obtain the desired phase-swing with relatively small specular reflection effects. In the over-coupled region, thicker substrates lead to lower reflection loss (Figure 2-4) and wider
bandwidth (Figure 2-5). However, reduced phase-swing is observed in Figure 2-5. Hence, a compromise needs to be made to select a thickness which provides reasonable reflection loss, phase-swing and bandwidth at the same time. In both the figures, a good agreement between the theory and full-wave simulations has been observed. The slightly larger discrepancy for thicker substrates is due to the fact that $Q_{rad}$ formula does not consider the fringing field and surface wave effects. However, it should be noted that unit cells with very thick substrates are usually not preferred (in this study $h > 15$ mil) due to limited phase-swing.

![Figure 2-4. Reflection magnitude versus substrate thickness at 32 GHz using theory and full-wave simulations.](image1)

![Figure 2-5. Bandwidth and Phase-swing for $\delta L/L_0 = \pm 0.2$ versus substrate thickness at 32 GHz using theory and full-wave simulations.](image2)
2.3.2. Effects of Patch width (W)

Figure 2-6 shows $Q$ factors, reflection loss, phase-swing, and bandwidth versus patch width for the same substrate material with a thickness of 5 mil, and at 32 GHz. It is observed in Figure 2-6(a) that $Q_{\text{rad}}$ is the only $Q$ factor depending on $W$. All three coupling regions are labeled. In the over-coupled region, as $W$ increases, the reflection loss decreases and bandwidth increases, but at the expense of reduced phase-swing. Good agreement between theory and full-wave simulation is observed, particularly in the over-coupled region, which is the region of interest in reflectarray design. A difference of $\sim 0.3$ mm ($0.03\lambda_0$) in the value of $W$ corresponding to critical coupling is primarily due to the pronounced effect from the fringing field; this is especially true for narrow patches. However, it should be noted that the under-coupled region needs to be avoided in the reflectarray design.

2.3.3. Effects of Dielectric Constant ($\varepsilon_r$)

In this subsection, the effects from $\varepsilon_r$ on reflection properties of reflectarray unit cells are studied. Figure 2-7 shows $Q$ factors, reflection loss, phase-swing, and bandwidth versus dielectric constant for unit cells having patch width of 2 mm, substrate thickness of 5 mil, and loss tangent of 0.0035, at 32 GHz. It is observed that the only $Q$ factor depending on dielectric constant is $Q_{\text{rad}}$. Critical coupling is observed at $\varepsilon_r \sim 49$, which is beyond the conventional range of dielectric constants ($2.2 \leq \varepsilon_r \leq 12$). Thus for the chosen unit cells, any practical dielectric thickness can be chosen to ensure the over-coupled case. In the over-coupled region, as $\varepsilon_r$ decreases, the reflection loss decreases and the bandwidth increases, again with a reduced phase-swing. The theory matches the full-wave simulations very well, particularly in the useful over-coupled region. The discrepancy in $\varepsilon_r$ corresponding to the critical coupling is mainly from the surface wave effects.
Figure 2-6. (a) \( Q \) factors, (b) reflection magnitude, and (c) bandwidth and phase-swing for \( \delta L/L_0 = \pm 0.2 \) versus patch width (W) at 32 GHz.
Figure 2.7. (a) Q factors, (b) reflection magnitude, and (c) bandwidth and phase-swing for $\delta L/L_0 = \pm 0.2$ versus dielectric constant ($\varepsilon_r$) at 32 GHz.
2.3.4. Effects of Dielectric and Metallic Losses

Figure 2-8 and Figure 2-9 show $Q$ factors, reflection loss, phase-swing, and bandwidth versus loss tangent and metal conductivity, respectively. Substrates with a thickness of 5 mil, dielectric constant of 10.2, and patch width of 2 mm are chosen. Figure 2-8 is plotted for a fixed metal conductivity ($\sigma = 5.8 \times 10^7 \text{ S/m}$), and Figure 2-9 is plotted for a fixed loss tangent ($\tan\delta = 0.0035$). From Figure 2-8, critical coupling is observed at $\tan\delta \sim 0.009$. In the over-coupled region, as the loss tangent decreases, reflection loss decreases, bandwidth increases and phase-swing almost remains constant.

![Plot of Q factors, reflection magnitude, and bandwidth versus loss tangent](image)

Figure 2-8. (a) Q factors, (b) reflection magnitude, and (c) bandwidth and phase-swing for $\delta L/L_0 = \pm 0.2$ versus loss tangent ($\tan\delta$) at 32 GHz.
From Figure 2-9, critical coupling is observed at $\sigma = 5.7 \times 10^6 \text{ S/m}$. It is also observed that, in the over-coupled region as the metal conductivity increases, reflection loss decreases and bandwidth increases over a small range and almost remains constant for $\sigma > 3 \times 10^7 \text{ S/m}$. Also, the phase-swing remains constant for the chosen range of conductivities.

Figure 2-9. (a) Q factors, (b) reflection magnitude, and (c) bandwidth and phase-swing for $\delta L/L_0 = \pm 0.2$ versus metal conductivity ($\sigma$) at 32 GHz.
2.3.5. Combined Effects of Different Parameters

In the previous subsections, the effects from each physical parameter are studied, while the other parameters are fixed. By doing so, contributions from $h$, $W$, $\varepsilon_r$, $\tan\delta$, and $\sigma$ on the unit cell performance were studied separately. It is shown that there exists a certain value for each parameter which results in critical coupling. However, this position shifts if any other parameters are changed. To demonstrate this concept, combined effects from dielectric thickness and other four parameters on the location of critical coupling are studied herein.

Figure 2-10(a) shows the location of critical coupling for different substrate thicknesses and patch widths for a fixed substrate material ($\varepsilon_r = 10.2$, $\tan\delta = 0.0035$, ½ oz copper) and resonant frequency of 32 GHz. It is observed that for different values of $W$, the values of $h$ which result in critical coupling are different. As the substrate thickness increases, the ranges of useful patch widths increases and vice-versa. Similar demonstration is performed for the other three combinations in Figure 2-10(b)-(d).

Figure 2-10(b) is plotted for fixed resonant frequency of $f_0 = 32$ GHz, $W = 2$ mm, $\tan\delta = 0.0035$ and ½ oz copper. It is observed that as the dielectric constant increases, the choice of useful substrate thicknesses decreases. In addition, for this particular combination, all commercially-available dielectric constants fall in the over-coupled range for $h > 4$ mil. Figure 2-10(c) and Figure 2-10(d) are plotted for substrates with a dielectric constant of 10.2, and patch width of 2 mm, resonating at 32 GHz. Figure 2-10(c) is plotted for a fixed metal conductivity ($\sigma = 5.8 \times 10^7$ S/m), and Figure 2-10(d) is plotted for a fixed loss tangent ($\tan\delta = 0.0035$). It is observed that as the dielectric loss increases or metal conductivity decreases, the choice of substrate thicknesses which correspond to over-coupled region decreases. It is also observed that the unit cell is always under-coupled for $\sigma < 10^4$ S/m. The results of this subsection suggest simultaneous investigation of different physical parameters and serves as guideline for unit cell performance optimization.
2.3.6. Measurements

To prove the concept experimentally, a few case studies for different physical parameters including substrate thickness ($h$), patch width ($W$) and dielectric constant ($\varepsilon_r$) are chosen. For each physical parameter, sets of reflectarray unit cells are fabricated and measured at $K_a$ band. Figure 2-11 shows the fabricated unit cells and the $K_a$-band waveguide/coaxial adaptor used to characterize the reflectarray unit cells. A metallic holder was designed to keep the sample firm and aligned to the polarization of the incoming wave. For each configuration, a resonant patch is designed for 32 GHz and the length is then changed to achieve the phase-swing. Due to the fabrication tolerances, the measured
resonant frequencies are slightly different from the designed 32 GHz. To present a fair comparison with theory and simulations, the results from measurements are presented at their respective resonant frequencies.

Effects of substrate thickness on the reflection properties of a reflectarray unit cell are demonstrated first. Rogers RO3010 is used as the substrate and patch width is fixed at 2 mm. Since not all substrate thicknesses are commercially available, two available thicknesses of 5 mil (0.127 mm) and 10 mil (0.254 mm), corresponding to the over-coupled condition are used for illustration. To quantify the shift in measured resonant frequencies, reflection magnitudes versus frequency from theory, simulations and measurements are compared in Figure. 2-12(a). It is observed that the measured resonant frequencies of patches with thicknesses 5-mil and 10-mil are 32.4 GHz and 31.5 GHz respectively. The maximum reflection losses of the measured cases are compared to theoretical and simulation results in Figure. 2-12(b).

Figure. 2-12(c) shows the variation of bandwidth and phase-swing for different substrate thicknesses. A good match is observed among measurements, theory and simulation results for substrate thickness variation. Note that bandwidth and phase-swing are defined only in the useful over-coupled region where they exist.

Figure. 2-11. Fabricated Ka band reflectarray unit cells and measurement setup including waveguide adaptor and sample holder.
Figure 2-12. (a) Reflection magnitude versus frequency (b) reflection magnitude versus substrate thickness (fixed W and εr), (c) bandwidth and phase-swing versus substrate thickness using theory, HFSS and measurements.
Effects of patch width on reflection properties are demonstrated in Figure 2-13. Unit cells with patch widths of 0.2, 0.4, 1.5, 2 and 3.5 mm are fabricated on a 5-mil-thick RO 3010 substrate. Patch widths of 0.2 and 0.4 mm fall in the under-coupled condition while others correspond to over-coupled region. Reflection magnitude versus frequency for the over-coupled patches is shown in Figure 2-13(a). Again for comparison, maximum reflection loss is plotted in Figure 2-13(b), and bandwidth, phase-swing are plotted in Figure 2-13(c). A good agreement, in particular for bandwidth and phase-swing, is observed between the measurements, theory and simulation results.

Finally, effects of dielectric constant on reflection properties of reflectarray unit cells are presented through measurements along with theoretical and simulated results in Figure 2-14. 5-mil-thick Rogers RT6006 and RO3010 with dielectric constants of 6.15 and 10.2, respectively, are chosen for measurement purposes. Patch width of the unit cells is fixed at 2 mm. Figure 2-14(a) shows the reflection magnitude versus frequency for 5-mil-thick Rogers RT6006 with tanδ = 0.0019. Figure 2-14(b)-(c) shows the measured variation of maximum reflection loss, bandwidth, and phase-swing versus dielectric constant along with theory and simulations. Note that in Figure 2-14(b)-(c), the results from theory and simulations are plotted for a fixed tanδ = 0.0035. It is observed that the measured data points closely follow the two traces corresponding to simulations and theory.
Figure 2-13. (a) Reflection magnitude versus frequency, (b) reflection magnitude versus patch width (fixed $h$ and $\varepsilon$), (c) bandwidth and phase-swing versus patch width using theory, HFSS and measurements.
Figure 2-14. (a) Reflection magnitude versus frequency, (b) reflection magnitude versus dielectric constant (fixed $h$ and $W$), (c) bandwidth and phase-swing versus dielectric constant using theory, HFSS and measurements.
2.4. Simulation procedure to extract the $Q$-factors

In this section, a technique to extract radiation $Q$ factors using full-wave simulations is discussed with the aid of Ansoft HFSS [75]. A unit cell excited inside a parallel-plate TEM waveguide (Figure 2-15(a)) is chosen for the study. The setup consists of a rectangular patch weakly excited by a coaxial line and radiating into a TEM waveguide as shown in Figure 2-15(b). To extract $Q_{\text{rad}}$, the metallic and dielectric losses are set to zero in simulations. A typical reflection coefficient response at the coaxial port is shown in Figure 2-16. $S_{11}^{\text{min}}$ is the minimum reflection coefficient at the resonant frequency $f_r$. $f_1$ and $f_2$ correspond to the frequencies where $S_{11} = S_{11}^\Phi$, where $S_{11}^\Phi$ is defined as:

$$S_{11}^\Phi = \sqrt{\frac{1 + |S_{11}^{\text{min}}|^2}{2}}$$  \hspace{1cm} (27)
Figure 2-16. Reflection coefficient at the coaxial port on a linear scale.

Depending on the location of the coaxial feed, the amount of energy coupled from the coaxial feed to the patch antenna will be identified. The loaded $Q$, taking into account the effects of external coaxial feed and measured at the coaxial port can be defined as [76]:

$$ Q_L = \frac{f_L}{f_2 - f_1} $$

(28)

In order to calculate the unloaded $Q$ of the patch, which is equal to the radiation $Q$, the following coupling coefficient is used [76]:

$$ k = \frac{1 \mp S_{11}^{\text{min}}}{1 \pm S_{11}^{\text{min}}} \quad \text{(undercoupled/overcoupled)} $$

(29)

Finally, $Q_{rad}$ can be calculated using:

$$ Q_{rad} = (1 + k) \frac{f_L}{f_2 - f_1} $$

(30)

To demonstrate the validity of the proposed approach, reflection coefficient of a unit cell derived using the presented theory is compared with HFSS full-wave simulations. A 32 GHz rectangular patch of $L \times W (1.3 \times 2 \text{ mm}^2)$ is designed on a 10-mil-thick Rogers RO3010 substrate ($\varepsilon_r = 10.2$, $\tan\delta = 0.0035$). The air-filled coaxial feed is positioned at the center of the patch along the $x$-axis and is offset by 0.125 mm from the center along the $y$-axis. The radiuses of the inner and outer conductors used are 0.1 and 0.2 mm, respectively. It should be noted that the position of the coaxial feed does not play a significant role here and the same results can be achieved for other feeding location by properly
extracting $k$ values as described in [76]. For this particular configuration, $Q_{\text{rad}}$ is found to be 24. Using (3)-(5), the $Q$ factors due to losses are computed as $Q_c = 687; Q_d = 285; \text{ and } Q_o = 201$.

Finally, by substituting these $Q$ values in (2), the reflection coefficient of the unit cell at the waveguide port in Figure 2-15(a) is extracted and plotted in Figure 2-17 and compared with HFSS results. It can be observed that the matching is excellent and the two results are in perfect agreement. This proves the validity of the presented analogy between a reflectarray unit cell and the coupled resonators theory and provides a new perspective into reflectarray antenna design.

Figure 2-17. Reflection magnitude and phase for a 10-mil thick substrate using theory and HFSS full wave simulations at the waveguide port ($W = 2 \text{ mm}, \varepsilon_r = 10.2, \tan\delta = 0.0035$).

2.5. Comparison between TEM and TE$_{10}$ modes of excitation

In this subsection, the difference between TEM and TE$_{10}$ modes of excitation are studied utilizing Q factors. First, the analytical expression for the radiation Q factor of the unit cell excited inside a TEM waveguide is presented. Following the procedure mentioned earlier, the power carried by the TEM mode of the waveguide is:

$$P_{\text{rad}} = \frac{1}{2} \int E \times H^* \, ds = \frac{a}{2 b \eta_0} |A^+|^2$$  \hspace{1cm} (31)

where $a$ and $b$ are the dimensions of the waveguide and $\eta_0$ is the wave impedance of the TEM mode. The amplitude $A^+$ of TEM mode excited in the infinite waveguide by the volume magnetic current density can be found as:
\[ A^+ = \frac{2hE_0W}{a} \]  

At resonance, as the electric and magnetic energies are equal, \( Q_{\text{rad}} \) is derived by as:

\[ Q_{\text{rad}} = \frac{f_0\pi L}{4h} \frac{1}{W ab\eta_0} \]  

To comprehend the difference between the two modes of excitation (TEM and \( TE_{10} \)), a unit cell resonant at 32 GHz is considered. RO3010 (\( \epsilon_r = 10.2; \tan\delta = 0.0035 \)) is used as the substrate and thicknesses ranging from 1 to 20 mil are considered. For each dielectric thickness, length of the patch is tailored to achieve a fixed resonant frequency of 32 GHz.

![Figure 2-18. Q factors versus substrate thickness at 32 GHz for two modes (TEM and \( TE_{10} \)) of excitation.](image)

Figure 2-18 shows the comparison of \( Q \) factors obtained for normal incidence and waveguide incidence (41° at 32 GHz). Note that \( Q_o \) for both the modes of excitation is assumed to be same. The position of critical coupling, where \( Q_{\text{rad}} \) becomes equal to \( Q_o \), happens at 4 mil for normal incidence and 3.25 mil for RWG incidence. The difference between \( Q_{\text{rad}} \) of TEM and \( TE_{10} \) is larger for unit cells near critical coupling. Note that these differences decrease as the unit cell becomes more over-coupled. For 3.25 mil < \( h < 4 \) mil, the unit cell could fall into either under- or over-coupled region depending on the incident angle.

To verify the loss performance of the unit cells, reflection loss versus substrate thickness at 32 GHz resonance is plotted in Figure 2-19. It can be observed that at critical coupling, maximum reflection loss is observed for both cases.
Even though very thin substrates have lower loss, anomalous phase response prohibits the use of this region. Also in the useful over-coupled region, loss of the unit cells excited by normal incidence is slightly higher than waveguide incidence. Thus, non-negligible variations in reflection properties of unit cells, particularly for elements near critical coupling, are observed for the two modes of excitation. Excellent match between simulations and theoretical results is observed.

![Figure 2-19. Reflection loss versus substrate thickness at 32 GHz for two modes of excitation.](image)

To study the effects from the two modes of excitation on the reflection phase, phase-sensitivity is defined. Phase-sensitivity is defined as the negative of the slope of reflection phase at resonance. The phase of the reflection coefficient can be expressed as:

$$
\Phi(f, f_r) = -\tan^{-1}\left( \frac{2(f - f_r)}{f_r \left( \frac{1}{Q_{rad}} - \frac{1}{Q_o} \right) } \right) - \tan^{-1}\left( \frac{2(f - f_r)}{f_r \left( \frac{1}{Q_{rad}} + \frac{1}{Q_o} \right) } \right)
$$

Phase-sensitivity is defined as:

$$
\sigma_f = -\frac{\partial \Phi(f, f_r)}{\partial f} \bigg|_{f=f_r}
$$

which is derived in terms of Q factor and the final expression is given as:
\[ \sigma_f = \frac{4Q \alpha^2 Q_{rad}}{f_0 \left( Q^2 - Q_{rad}^2 \right)} \]  

Figure 2-20 shows the variation of \( \sigma_f \) versus substrate thickness for the modes of excitation. It is observed that the singularity in the phase-sensitivity is observed at the critical coupling conditions for both the cases. Again, the difference between the two modes of excitation is dominant near the critically-coupled condition. In the useful over-coupled region, it is observed that the TEM excited unit cells exhibit higher phase-sensitivity than TE\(_{10}\) excited elements. This increased sensitivity corresponds to decreased bandwidth. In addition, it is also observed that for smaller substrate thicknesses, phase-sensitivity is negative confirming the under-coupled condition. Again excellent match between theory and simulations is observed.

Figure 2-20. Phase-sensitivity versus substrate thickness at 32 GHz for two modes of excitation.

2.6. Effects of Inter-element spacing

A parallel-plate waveguide setup shown in Figure. 2-21 is used to characterize the reflectarray elements with different inter-element spacing. Using Ansoft High Frequency Structure Simulator (HFSS), a 2.83 \times 3.7 \text{mm}^2 (L\times W) patch is designed on a 10-mil Rogers RT Duroid 5880 substrate (\( \varepsilon_r = 2.2; \ tan\delta = 0.0009; \ ½ \ oz \ copper \)) having a center frequency \( f_0 \) of 32GHz. The inter-element spacing of A=7.112 mm (0.76\( \lambda_0 \)) and B=3.556 mm (0.38\( \lambda_0 \)) used in this
design are equal to the cross-sectional dimensions of the standard $K_a$-band waveguide. The fractional bandwidth (FBW) of the reflectarray unit cell is found to be 3.3%.

Figure. 2-21. Reflectarray unit cell simulation setup and the corresponding infinite array configuration

To study the effects of the inter-element spacing for a fixed patch dimensions ($L$ and $W$), $A$ ($B$) is swept from $0.45\lambda_0$ ($0.35\lambda_0$) to $0.95\lambda_0$ ($0.85\lambda_0$) in HFSS simulations. The change of $A$ ($B$) results in the mutual coupling level change in $H$ ($E$) plane. Figure. 2-22 presents the resonant frequency versus inter-element spacing. It is noted that when the spacing between elements is decreased, the change in the resonant frequency is more pronounced. For inter-element spacing larger than $0.6\lambda_0$, resonant frequency of the antenna element is relatively insensitive to the spacing. Across all values of inter-element spacing ($A$ and $B$) studied here, more than 6% variations in the resonant frequency is observed, which is noticeable considering the FBW of the unit cell is only $\sim$3%. 
As the resonant frequency of the reflectarray unit cell changes, the reflection phase at $f_0$ changes as well. This concept is illustrated in Figure. 2-23. It is observed that the reflection phase changes around 140° within the aforementioned range of inter-element spacing. As the inter-element spacing increases, variations in both resonant frequency and phase become less significant. In this regard, reflectarray unit cells are less vulnerable to fabrication errors if the spacing between the elements is greater than $0.5\lambda_0$. In addition, it is also essential to understand the effects of inter-element spacing on other important parameters like bandwidth, maximum loss and phase-swing.

Figure. 2-22. Resonant frequency versus inter-element spacing at Ka-band.

Figure. 2-23. Reflection phase versus inter-element spacing at $f_0 = 32$ GHz.
Maximum reflection loss and bandwidth for different inter-element spacing are plotted in Figure 2-24(a) and Figure 2-24(b), respectively. It is noted that the variation in reflection loss and bandwidth with inter-element spacing is close to linear. As the inter-element spacing is increased, reflection loss is increased from 0.2 to 1.5 dB, and the FBW is decreased from ~5 to ~1%. The increase in total reflection loss was found to be a result of increase in both metallic and dielectric losses. Figure 2-25 shows the contribution from different losses in unit cell for two chosen cases of A=B=0.45λ₀ and A=B=0.75λ₀. Note that as the separation between the elements is increased, both conductor and dielectric losses are increased. This is due to the increase in radiation Q-factor of the unit cell with spacing between the elements.

Figure 2-24. (a) Reflection loss, and (b) FBW for different inter-element spacing at their respective resonant frequency.

Figure 2-25. Reflection loss showing dielectric, conductor and total loss for different inter-element spacing.
Figure 2-26 shows reflection phase versus patch length, which demonstrates the achievable phase-swing. To enable comparisons, three different values of inter-element spacing, $A=B=0.45\lambda_0$, $A=B=0.60\lambda_0$ and $A=B=0.75\lambda_0$, are chosen and plotted at their respective resonant frequencies. It is observed that the phase-swing increases as the inter-element spacing is increased from 0.45 to 0.75$\lambda_0$.

The preceding studies prove that small inter-element spacing is beneficial if bandwidth and reflection loss of the unit cell are crucial. However, this comes at costs in excessive mutual coupling, increased fabrication tolerance sensitivity, and reduced phase-swing.

A waveguide measurement setup is commonly used to characterize reflectarray unit cells. Due to the fixed dimensions of standard waveguides, this setup facilitates measurements only for a fixed separation between the elements, which is equal to the waveguide dimensions. In order to experimentally observe the unit cell performance for different inter-element spacing, customized waveguides can be designed, fabricated and measured. However, due to multiple modes in the waveguide, special considerations should be taken into account to ensure that the energy is coupled to the dominant mode. Another simpler way is to approximate the infinite array by a finite sub-array of identical elements.

In this study, two 5x5 sub-arrays with $A=B=0.45\lambda_0$ and $A=B=0.75\lambda_0$ corresponding to minimum and maximum inter-element spacing are designed and fabricated as shown in Figure 2-27(a). It is noted that the total substrate areas are identical for both cases. Each sub-array is excited by a radiating source, in this case a $K_a$-band waveguide as shown in
Figure 2-27(b). The sub-array is placed at the far field of the feed, which creates a plane wave above the patch elements. Approximately same amount of mutual coupling will be experienced by the center element in both finite sub-array and infinite array. Maximum reflection occurs at the resonant frequency of the patch antenna. Time-domain gating is utilized to isolate the undesired reflections from the transmit/receive antenna.

Figure 2-27. (a) Fabricated 5×5 array elements approximating an infinite array, and (b) measurement setup using a waveguide feed.

The measured reflection magnitudes for the two cases are shown in Figure 2-28. When the inter-element spacing is changed from chosen minimum to maximum values, the resonant frequency is shifted from 31.73 to 32.4 GHz. This is equal to 2.1% variation with respect to the center frequency of 32 GHz, which agrees very well with the simulated 2.25% frequency shift. For the same variation in inter-element spacing, the reflection loss is increased by 2.4 dB
compared with 0.63 dB observed in simulations. This implies that not all the reflected energy is captured at the waveguide port due to the non-broadside diffractions from the finite array. The measured reflection phases for the two sub-arrays are plotted in Figure 2-29. A phase difference (ΔPhase) of 130° is observed at 32 GHz compared with simulated 75° phase shift.

![Figure 2-28](image-url)  
**Figure. 2-28.** Measured reflection magnitude in dB for minimum and maximum inter-element spacing.

![Figure 2-29](image-url)  
**Figure. 2-29.** Measured reflection phase in degrees for minimum and maximum inter-element spacing.
The measured results reveal the successful performance of the proposed setup in predicting the resonant frequency behavior of reflectarray unit cells. Due to the approximate nature of the measurement setup and difficulties in phase measurements in free-space, the measured and simulated ∆Phase are slightly off. Nevertheless, it is apparent that the simplified finite sub-array approach can demonstrate the variations in reflection properties and provide valuable insight to the underlying physics without the need of an infinite array.
CHAPTER 3: **Q FACTOR ANALYSIS INVESTIGATING THE EFFECTS FROM ANGLE OF INCIDENCE USING FLOQUET MODES**

3.1. **Introduction**

A complete analytical approach to extract the reflection properties of unit cells using $Q$ factors is discussed in the previous chapter. The reasons for anomalous phase responses and the effects from the aforementioned physical parameters on reflection properties were shown. However, the theory was developed for a single incident mode or fixed incidence angle and therefore finds limited applications.

In this chapter, the $Q$ factor analysis is extended and a theoretical model accounting for incidence angle and plane of incidence are presented [77]. In general, the incident and reflected fields of a reflectarray are expressed in terms of orthogonal Floquet modes. By expanding the coupled-mode theory, generic expressions for reflection coefficient of multiple orthogonal modes are derived. The results, applied to reflectarrays, provide the reflection coefficients in terms of the $Q$ factors of the unit cell radiating into the Floquet modes. A rectangular patch antenna is chosen to illustrate the concept, and closed-form expressions of the $Q$ factors are derived in terms of the unit cell’s physical parameters and angle of incidence. This allows the investigation of the reflection properties and coupling conditions of the reflectarray without the need of full-wave simulations.

For demonstration, the effects of incidence angle for a single linearly-polarized (LP) incident wave are investigated by choosing cases corresponding to different coupling conditions. The reasons for varied sensitivity of elements located on different planes are explained. The presented theory expedites the design process of reflectarrays and provides definitive physical perception.

3.2. **Theoretical derivation**

In this section, the reflection coefficients of reflectarray elements are presented in terms of $Q$ factors. An arbitrary incident plane wave with an oblique incidence angle can be expressed in terms of Floquet space harmonics [78]. The analysis of an infinite array of identical elements shown in Figure. 3-1(a) is simplified into a single unit cell with periodic boundaries as shown in Figure. 3-1(b). This model can be interpreted as an antenna element inside a waveguide supporting Floquet modes. From a practical perspective where the grating lobes are undesired, dominant Floquet modes $TE_{00}$ (transverse electric to $z$) and $TM_{00}$ (transverse magnetic to $z$) suffice in the analysis.
Figure 3-1. (a) Reflectarray excited by a feed showing reference plane and angles of incidence. (b) Unit cell inside a waveguide excited by Floquet modes. (c) Electric field and magnetic current distribution on the patch excited in (b) obtained from cavity model.

The waveguide shown in Figure 3-1(b) involves two orthogonally-polarized fundamental Floquet modes that are coupled to the resonator; TE\(_{00}\) and TM\(_{00}\) of the waveguide coupled to TM\(_{010}\) mode of the patch. For an incidence angle of \(\theta\) in the Floquet waveguide analysis, where both the incident and reflected waves are expressed in terms of fundamental Floquet modes, the angle of reflection is limited to \(-\theta\). Due to the existence of two simultaneous modes inside the waveguide, both co-coupling and cross-coupling of the modes occur. Co-coupling corresponds to the coupling between the same modes, while cross-coupling corresponds to coupling between the orthogonal modes.

Under time-harmonic steady state condition, the co-coupled and cross-coupled reflection coefficients at the patch surface are derived as follows using the coupled-mode theory:

Coupled-mode theory utilizing time reversibility and energy conservation are applied to derive the expressions for reflection coefficients of multiple modes inside a waveguide. As shown in Figure 3-1(b), if a resonating patch is connected to a waveguide with two modes or two waveguides, the decay rate of the positive frequency mode amplitude is given as [30]:

\[
\frac{da}{dt} = j\omega_0 a - \left( \frac{1}{\tau_0} + \frac{1}{\tau_{et}} + \frac{1}{\tau_{es}} \right) a + k_1 s_{+1} + k_2 s_{+2}
\]  

(37)
where \( a \) is the mode amplitude of the positive frequency, \( \frac{1}{\tau_0} \) is the decay rate due to losses (conductor and dielectric losses), \( \frac{1}{\tau_{e1}} \) and \( \frac{1}{\tau_{e2}} \) are decay rates due to escaping power into waveguide mode 1 and 2, respectively, \( k_1 \) and \( k_2 \) are coupling coefficients between the resonator and waves \( s_{+1} \) and \( s_{+2} \), respectively given by:

\[
k_1 = \frac{2}{\sqrt{\tau_{e1}}} \quad k_2 = \frac{2}{\sqrt{\tau_{e2}}}
\]  

(38)

The reflected waves \( s_{-1} \) and \( s_{-2} \) are given as:

\[
s_{-1} = -s_{+1} + \frac{2}{\sqrt{\tau_{e1}}} a, \quad s_{-2} = -s_{+2} + \frac{2}{\sqrt{\tau_{e2}}} a
\]  

(39)

For a wave source at frequency \( \omega \) (\( s_+ \propto \exp(j\omega t) \)), (37) reduces to:

\[
a = \frac{k_1 s_{+1} + k_2 s_{+2}}{j(\omega - \omega_0) + \left( \frac{1}{\tau_0} + \frac{1}{\tau_{e1}} + \frac{1}{\tau_{e2}} \right)}
\]  

(40)

Now the co-coupled and cross-coupled reflection coefficients can be obtained from (38), (39) and (40), and the relation \( Q = \frac{\omega \tau}{2} \) as:

\[
\Gamma_{11}(f) = \left| \frac{S_{-1}}{S_{+1}} \right|_{s_{+2}=0} = \frac{1 - \left( \frac{1}{Q_1} + \frac{1}{Q_2} \right)}{\frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_0} + \frac{2j(f - f_0)}{f_0}}
\]  

(41)

and similarly,

\[
\Gamma_{22}(f) = \frac{1 - \left( \frac{1}{Q_1} + \frac{1}{Q_2} \right)}{\frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_0} + \frac{2j(f - f_0)}{f_0}}
\]  

(42)

Cross-coupled reflection coefficient:

\[
\Gamma_{12}(f) = \Gamma_{21}(f) = \frac{2}{\sqrt{Q_1 \cdot Q_2}} \left[ \frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_0} + \frac{2j(f - f_0)}{f_0} \right]
\]  

(43)

In general, for a waveguide supporting \( N \) orthogonal modes, the co-coupled reflection coefficients are given as:
Cross-coupled reflection coefficients are given as:

\[ \Gamma_{ij}(f) = \left. \frac{S_{-i}}{S_{+j}} \right|_{S_{+i}=0 \text{ for } j \neq i} \]

\[ = \frac{1}{Q_i} - \left( \frac{1}{Q_o} + \sum_{j=1, j \neq i}^{N} \frac{1}{Q_j} \right) - \frac{2j(f - f_0)}{f_0} \]

Cross-coupled reflection coefficients are given as:

\[ \Gamma_{ij}(f) = \left. \frac{S_{-i}}{S_{+j}} \right|_{S_{+i}=0 \text{ for } k \neq j} \]

\[ = \frac{2}{\sqrt{Q_i \cdot Q_j}} \left( \frac{1}{Q_o} + \sum_{j=1}^{N} \frac{1}{Q_j} \right) + \frac{2j(f - f_0)}{f_0} \]  

For the Floquet analysis with the dominant TE\(_{00}\) and TM\(_{00}\) modes propagating, the final expressions for co- and cross-coupled components are given as:

TE co-coupled reflection coefficient (\(\Gamma_{TEco}\)) is defined as the fraction of reflected power into TE mode, when the incident mode is TE polarized:

\[ \Gamma_{TEco}(f) = \frac{1}{Q_{radTE}} - \left( \frac{1}{Q_{radTM}} + \frac{1}{Q_o} \right) - \frac{2j(f - f_0)}{f_0} \]

TM co-coupled reflection coefficient (\(\Gamma_{TMco}\)) is defined as the fraction of reflected power into TM mode, when the incident mode is TM polarized:

\[ \Gamma_{TMco}(f) = \frac{1}{Q_{radTM}} - \left( \frac{1}{Q_{radTE}} + \frac{1}{Q_o} \right) - \frac{2j(f - f_0)}{f_0} \]

Cross-coupled reflection coefficient (\(\Gamma_{cross}\)) is defined as the fraction of reflected power into TE (TM) mode, when the incident mode is TM (TE) polarized:

\[ \Gamma_{cross}(f) = \frac{2}{\sqrt{Q_{radTE} \cdot Q_{radTM}}} \]

where \(f_0\) is the resonant frequency. \(Q_{radTE}\) and \(Q_{radTM}\) are the radiation \(Q\) factors of the TE and TM modes, respectively, and \(Q_o\) is the combined \(Q\) factor due to dielectric and conductor losses given as:
\[ Q_c = d \sqrt{\pi f \mu \sigma}; \quad Q_d = \frac{1}{\tan \delta}; \quad Q_o = \frac{Q_c Q_d}{Q_c + Q_d} \tag{49} \]

3.2.1. Derivation of \( Q_{\text{radTE}} \) and \( Q_{\text{radTM}} \):

Floquet-mode and the cavity model of the patch antenna are utilized to derive the expressions for radiation \( Q \) factors dependent on incidence angles \((\theta, \varphi)\). In general, the radiation \( Q \) factor is given by:

\[ Q_{\text{rad}} = 2\pi f_0 \frac{\text{Energy stored}}{P_{\text{rad}}} \tag{50} \]

For all incidence angles, the dominant mode within the cavity is assumed to be \( TM_{010} \) mode as shown in the Figure. 3-1(c). Applying cavity model, image theory, and assuming a thin cavity, the patch antenna can be represented in terms of equivalent magnetic currents at the radiating edges given as:

\[ \tilde{I}_m = -2dE_0 \hat{x}, \quad y = -\frac{L}{2}, \quad x = \frac{W}{2} \tag{51} \]

And at the non-radiating edges:

\[ \tilde{I}_m(y) = \pm 2dE_0 \sin \left( \frac{\pi y}{L} \right) \hat{y}, \quad x = \frac{W}{2} \tag{52} \]

If the field distribution is assumed to be confined underneath the patch, the stored electric energy is given by:

\[ W_e = \frac{\varepsilon}{4} \iint |E|^2 dv = \frac{\varepsilon}{8} E_0^2 dWL \tag{53} \]

The modal transverse fields of \( TE \) and \( TM \) Floquet harmonics (with \( z \)-components suppressed) are given as [78]:

\[ \bar{e}_{TE} = \frac{\hat{x} k_{xmn} - \hat{y} k_{ymn}}{\sqrt{ab(k_{xmn}^2 + k_{ymn}^2)}}; \]

\[ \bar{h}_{TE} = \frac{\hat{x} k_{xmn} + \hat{y} k_{ymn}}{Z_{TE} \sqrt{ab(k_{xmn}^2 + k_{ymn}^2)}} \tag{54} \]

similarly,

\[ \bar{e}_{TM} = \frac{\hat{x} k_{xmn} + \hat{y} k_{ymn}}{\sqrt{ab(k_{xmn}^2 + k_{ymn}^2)}}; \]

\[ \bar{h}_{TM} = \frac{-\hat{x} k_{ymn} + \hat{y} k_{xmn}}{Z_{TM} \sqrt{ab(k_{xmn}^2 + k_{ymn}^2)}} \tag{55} \]
where \(a, b\) are the waveguide dimensions, \(Z_{TE}\) and \(Z_{TM}\) are the mode impedances of \(TE\) and \(TM\) modes, and \(k_{xmn}, k_{ymn}\) are the wave numbers corresponding to \((m, n)\) Floquet mode given by:

\[
k_{xmn} = k_0 \sin \theta \cos \varphi + \frac{2m\pi}{a}
\]
\[
k_{ymn} = k_0 \sin \theta \sin \varphi + \frac{2n\pi}{b}
\]

The amplitude \(A^+\) of \(TE\) and \(TM\) modes excited in the infinite waveguide by the volume magnetic current density can be found using:

\[
A_{TE}^+ = \frac{1}{P_{TE}} \int \vec{H}_{TE} \cdot \vec{M} \, dv;
\]
\[
A_{TM}^+ = \frac{1}{P_{TM}} \int \vec{H}_{TM} \cdot \vec{M} \, dv
\]

where \(\vec{H}_{TE}, \vec{H}_{TM}\) are the magnetic fields of the \(TE\) and \(TM\) modes and \(P_{TE}, P_{TM}\) are the normalization constants given by:

\[
P_{TE} = 2 \int \vec{e}_{TE} \times \vec{h}_{TE} \cdot \hat{z} \, ds = \frac{2}{Z_{TE}}
\]
\[
P_{TM} = 2 \int \vec{e}_{TM} \times \vec{h}_{TM} \cdot \hat{z} \, ds = \frac{2}{Z_{TM}}
\]

Substituting equations (51), (52), (54), (56) in (57):

\[
A_{TE}^+ = \frac{4dE_0 W k_{xmn}}{P_{TE} Z_{TE} \sqrt{ab(k_{xmn}^2 + k_{ymn}^2)}}
\]

similarly,

\[
A_{TM}^+ = \frac{4dE_0 W k_{ymn}}{P_{TM} Z_{TM} \sqrt{ab(k_{xmn}^2 + k_{ymn}^2)}}
\]

The power carried by the \(TE\) and \(TM\) modes of the waveguide is:

\[
P_{radTE} = \frac{1}{2} \int \vec{E}_{TE} \times \vec{H}_{TE}^* \cdot \hat{z} \, ds = \frac{|A_{TE}^+|^2}{2Z_{TE}}
\]
\[
P_{radTM} = \frac{1}{2} \int \vec{E}_{TM} \times \vec{H}_{TM}^* \cdot \hat{z} \, ds = \frac{|A_{TM}^+|^2}{2Z_{TM}}
\]

The mode impedances of fundamental \(TE\) and \(TM\) modes, \(Z_{TE}\) and \(Z_{TM}\) are expressed in terms of free space wave impedance \(\eta_0\) given by:
\[ Z_{TE} = \frac{\eta_0}{\cos \theta}; \quad Z_{TM} = \eta_0 \cdot \cos \theta \]  \hspace{1cm} (63)

At resonance, as the electric and magnetic energies are equal, \( Q_{radTE} \) and \( Q_{radTM} \) for the fundamental Floquet modes \((m = n = 0)\) are derived as:

\[ Q_{radTE} = \frac{f_0 \pi \eta_0 L}{4d} \frac{ab}{\cos \theta \cos^2 \varphi} \]  \hspace{1cm} (64)

and

\[ Q_{radTM} = \frac{f_0 \pi \eta_0 L}{4d} \frac{ab}{\sin^2 \varphi} \]  \hspace{1cm} (65)

Finally, the reflection coefficients of unit cell are determined by substituting the \( Q \) factors (49), (64) and (65) in (46), (47) and (48). Using the developed theory, the reflection from a unit cell excited by a wave of any angle of incidence and polarization can be analyzed.

For an arbitrary incident field on the array expressed in terms of Floquet harmonics \((E_{inc}^{TE}, E_{inc}^{TM})\), the reflected TE and TM components are given as:

\[
\begin{bmatrix}
E_{ref}^{TE} \\
E_{ref}^{TM}
\end{bmatrix} =
\begin{bmatrix}
\Gamma_{TEco} & \Gamma_{cross} \\
\Gamma_{cross} & \Gamma_{TMco}
\end{bmatrix}
\begin{bmatrix}
E_{inc}^{TE} \\
E_{inc}^{TM}
\end{bmatrix}
\]  \hspace{1cm} (66)

Even though, the total reflected fields are a combination as shown in (65), it is still beneficial to analyze each reflection coefficient separately and illustrate the characteristics.

To comprehend the effects from angles of incidence and polarization, the antenna elements are assessed using two important parameters for each wave component, reflection loss at resonance which determines the maximum loss, and phase sensitivity (negative slope of the reflection phase at resonance) which determines the bandwidth, phase range and sensitivity of the unit cells. Maximum reflection loss can be obtained from equations (46), (47) and (48). The phase-sensitivity at \( f_0 \) for co- and cross-coupled wave components is derived as:

For co-coupled wave components:

\[
\sigma_{TEco} = \left. -\frac{\partial \varphi_{TEco}}{\partial f} \right|_{f=f_0} \approx \frac{4/Q_{radTE}}{f_0 \left( \frac{1}{Q_{radTE}} \right)^2 - \left( \frac{1}{Q_{radTM}} \right)^2} \]  \hspace{1cm} (67)

similarly,
\[ \sigma_{TMco} = -\frac{\partial \Phi_{TMco}}{\partial f} \bigg|_{f=f_0} \approx \frac{4}{Q_{radTM}} f_0 \left( \frac{1}{Q_{radTM}} - \left( \frac{1}{Q_{radTE}} + \frac{1}{Q_o} \right)^2 \right) \]  

(68)

And for cross-coupled wave components:

\[ \sigma_{cross} = -\frac{\partial \Phi_{cross}}{\partial f} \bigg|_{f=f_0} \approx \begin{cases} 0 & \text{if } Q_{radTE} = \infty \text{ or } Q_{radTM} = \infty \\ \frac{2}{f_0 \left( \frac{1}{Q_{radTE}} + \frac{1}{Q_{radTM}} + \frac{1}{Q_o} \right)} & \text{(else)} \end{cases} \]  

(69)

where \( \Phi_{TEco}, \Phi_{TMco}, \) and \( \Phi_{cross} \) are the reflection phases of \( \Gamma_{TEco}, \Gamma_{TMco}, \) and \( \Gamma_{cross} \) respectively.

### 3.3. Coupling conditions:

Depending on the relative values of \( Q \) factors, which are dependent on antenna physical parameters and incidence angle, a unit cell can be over-coupled, critically-coupled, or under-coupled. Under TE excitation, an antenna element is:

1. **TE over-coupled**, if

\[
\frac{1}{Q_{radTE}} > \frac{1}{Q_{radTM}} + \frac{1}{Q_o} ; \quad \sigma_{TEco} > 0
\]  

(70)

2. **TE critically-coupled**, if

\[
\frac{1}{Q_{radTE}} = \frac{1}{Q_{radTM}} + \frac{1}{Q_o} ; \quad \sigma_{TEco} = \infty
\]  

(71)

3. **TE under-coupled**, if

\[
\frac{1}{Q_{radTE}} < \frac{1}{Q_{radTM}} + \frac{1}{Q_o} ; \quad \sigma_{TEco} < 0
\]  

(72)

A special case of under-coupled occurs when \( Q_{radTE} = \infty \), which corresponds to specular reflection or no coupling.

\[
Q_{radTE} = \infty ; \quad \sigma_{TEco} = 0
\]  

(73)

similarly for TM excitation.
3.4. Analysis and validation

In this section, the effects of incidence angle on reflection properties are studied using the theory developed in section 2.2. for a single LP case. HFSS applying master-slave boundaries and Floquet port is used for simulation purposes.

A rectangular patch of dimensions 1.3x2 mm² (LxW) on a 10-mil-thick RO3010 substrate (\(\varepsilon_r = 10.2, \tan\delta = 0.0035\), \(\frac{1}{2}\) oz copper) resonant at 32 GHz for normal incidence is used for demonstration. An inter-element spacing of \(a = b = 0.5\lambda_0\), where the higher-order Floquet modes are cutoff for all scan angles, is chosen.

For an array of reflectarray elements, there exists infinite sets of scan planes. However, the extremity in performance is most likely to occur in cardinal (\(\phi = 90^\circ, \varphi = 0^\circ\)) and inter-cardinal planes (\(\varphi = 45^\circ\)) [79]. Thus, elements located on the principal planes or cardinal planes are chosen first. Later, the reflection properties of elements located on inter-cardinal plane are also discussed.

3.4.1. H-plane (\(\varphi = 0^\circ\)) & E-Plane (\(\varphi = 90^\circ\)):

From Figure. 3-1(c), it can be observed that the polarization of the incident wave has to be y-polarized for the magnetic currents on the radiating edges of the dominant mode to be along x. Thus for elements located on x-axis (\(\varphi = 0^\circ\)), which corresponds to H-plane, (64) and (65) reduce to:

\[
Q_{radTE} = \frac{f_0 \pi \varepsilon_r}{4d} \frac{L}{W} ab \frac{\eta_0}{\cos \theta}
\]

and

\[
Q_{radTM} = \infty
\]

Thus, equations (46), (47) and (48) reduce to:

\[
\Gamma_{TE0}(f) = \frac{1}{Q_{radTE}} - \frac{1}{Q_0} - \frac{2j(f - f_0)}{f_0}
\]

\[
\Gamma_{TM0}(f) = -1
\]

\[
\Gamma_{cross}(f) = 0
\]

The coupling conditions (70)-(72) reduce to:
1. Over-coupled, if $Q_o > Q_{radTE}$.
2. Critically-coupled, if $Q_o = Q_{radTE}$.
3. Under-coupled, if $Q_o < Q_{radTE}$.

Similarly, for elements located on the principal E-plane ($\varphi = 90^\circ$), (64), (65) reduces to:

$$Q_{radTE} = \infty$$  \hspace{1cm} (79)$$

and

$$Q_{radTM} = \frac{f_0\pi E}{4d} \frac{W}{ab} \eta_0 \cos \theta$$  \hspace{1cm} (80)$$

Reflection coefficients (46), (47) and (48) reduce to:

$$\Gamma_{TEco}(f) = -1$$  \hspace{1cm} (81)$$

$$\Gamma_{TMco}(f) = \frac{1}{Q_{radTM}} - \frac{1}{Q_o} - \frac{2j(f - f_0)}{f_0}$$

$$\Gamma_{TMco}(f) = \frac{1}{Q_{radTM}} + \frac{1}{Q_o} + \frac{2j(f - f_0)}{f_0}$$  \hspace{1cm} (82)$$

$$\Gamma_{cross}(f) = 0$$  \hspace{1cm} (83)$$

The coupling conditions reduce to:

1. Over-coupled, if $Q_o > Q_{radTM}$.
2. Critically-coupled, if $Q_o = Q_{radTM}$.
3. Under-coupled, if $Q_o < Q_{radTM}$.

Note that cross-coupled reflection coefficients are zero for the elements on principal planes. Equations (77) and (81) correspond to reflection coefficients from specular reflection. Thus, only $\Gamma_{TEco}(f)$ and $\Gamma_{TMco}(f)$ given by (76) and (82) are the coefficients of interest for elements on H- and E-planes respectively.
Figure 3.2. $Q$ factors versus $\theta$ showing different coupling regions at 32 GHz for (a) $\varphi = 0^\circ$ and (b) $\varphi = 90^\circ$ using theory.
Figure. 3-3. (a) Reflection magnitude and (b) phase sensitivity vs. $\theta$ at resonance for $\varphi = 0^\circ$ and $\varphi = 90^\circ$ using theory and HFSS simulations.
Q factors versus incidence angle $\theta$ for $\varphi = 0^\circ$ & $90^\circ$ are shown in Figure. 3-2(a) and Figure. 3-2(b) respectively. It is observed that for $\varphi = 0^\circ$, critical coupling ($Q_o = Q_{radTE}$) is observed at $\theta = 78^\circ$. Unit cells are over-coupled ($Q_o > Q_{radTE}$) for $\theta < 78^\circ$ and under-coupled ($Q_o < Q_{radTE}$) for $\theta > 78^\circ$. Reflectarray elements exhibit anomalous phase response when under-coupled and cannot be used in any design [70]. Thus, the maximum incidence angle or scan angle is limited by the critically-coupled condition of the antenna element. For $\varphi = 90^\circ$ as shown in Figure. 3-2(b), $Q_o$ is greater than $Q_{radTM}$ for all incidence angles, corresponding to over-coupled condition. On both planes for smaller incidence angles ($\theta < 40^\circ$), the changes in radiation Q factors are small. However, for $\theta > 45^\circ$, a noticeable change in radiation Q factors w. r. t $Q_o$ is observed, particularly for elements on H-plane. This explains a similar behavior observed in [80], where elements on the H-plane are found to be more sensitive to incidence angles than elements on E-plane. These variations in Q factors render non-negligible effects on the reflection properties of unit cells which are demonstrated in Figure. 3-3.

Maximum reflection loss and slope of the reflection phase at resonance versus $\theta$ for the two planes of incidence are shown in Figure. 3-3(a) and Figure. 3-3(b), respectively. For $\varphi = 0^\circ$, it can be observed that as $\theta$ increases, reflection loss and phase sensitivity ($\sigma_{TEco}$) remains almost constant for smaller incidence angles, and starts to increase drastically for angles greater than $45^\circ$. When the unit cell is critically coupled ($\theta = 78^\circ$), we note that maximum reflection loss and singularity in $\sigma_f$ are observed. For $\varphi = 90^\circ$, it is observed that as $\theta$ increases reflection loss and $\sigma_{TMco}$ decrease. This decrease in $\sigma$ signifies the reduced sensitivity which implies increased bandwidth. It is noted that on $\varphi = 90^\circ$ plane, $\sigma_{TMco}$ is positive for all the chosen $\theta$ implying over-coupled condition. For angles less than $40^\circ$, it is observed that the variation in $|S_{11}|$ and $\sigma$ versus $\theta$ for both the planes is small. A very good match between the full-wave simulations and theoretical results is observed.

### 3.4.2. Inter-cardinal plane ($\varphi = 45^\circ$)

To study the effects of incidence angles on elements off the principal planes, $\varphi = 45^\circ$ is chosen. Since the radiation Q factors for both TE and TM modes are not infinite (except for grazing angles), both co-coupling and cross-coupling conditions need to be addressed.
Figure 3-4. $Q$ factors vs. $\theta$ showing coupling regions for (a) TE co-coupled wave component and (b) TM co-coupled wave component at 32 GHz resonance for $\phi = 45^\circ$ using theory.
Figure 3-5. (a) Reflection magnitude and (b) phase sensitivity vs. θ for co-coupled and cross-coupled wave components at 32 GHz resonance for φ = 45° using theory and HFSS simulations.

First, the coupling conditions for each case given by (70)-(72) are studied using Q factors. Figure 3-4(a) shows $Q_{radTE}$ and $Q_{radTM} // Q_o$ versus θ for φ = 45°. It can be observed for the chosen physical parameters, that the unit cell is always under-coupled ($Q_{radTE} > Q_{radTM} // Q_o$) for TE co-coupled wave component as indicated in (72). Similarly, $Q_{radTM}$ and $Q_{radTE} // Q_o$ versus θ are shown in Figure 3-4(b). Note that for TM co-coupled wave component, all the
three coupling-conditions exist. Critical-coupling is observed at \( \theta = 36^\circ \). Under-coupled and over-coupled conditions are also indicated.

To verify the loss performance, reflection magnitude for both co- and cross-coupled wave components are shown in Figure. 3-5(a). For TE co-coupled wave component, even though loss decreases as \( \theta \) increase, the unit cell exhibits under-coupled condition. In the over-coupled region of the TM co-coupled wave component, loss is particularly high for angles less than \( 50^\circ \), and makes this region less useful as well. Thus, for \( \varphi = 45^\circ \), the co-coupled reflection coefficients suffer with anomalous phase responses or high loss for incidence angles less than \( 50^\circ \). In fact, the significant contribution on this plane comes from the cross-coupling between the modes, where the unit cell is over-coupled for all \( \theta \), as shown in (69).

For cross-coupled wave component, reflection loss varies very slightly for angles less than \( 40^\circ \), and increases drastically particularly for \( \theta > 50^\circ \). It is also observed that at grazing angle \( \theta = 90^\circ \), cross-coupled reflection coefficient becomes zero.

Figure. 3-5(b) shows the variation of \( \sigma \) versus \( \theta \) for both co- and cross-coupled wave components. It is observed that \( \sigma_{TEco} \) is negative for all incidence angles, confirming the under-coupled condition. In the over-coupled region of TM co-coupled wave component, \( \sigma_{TMco} \) decreases as \( \theta \) increase. For the cross-coupled wave component, it is observed that \( \sigma_{cross} \) is positive, varies slightly and tends toward zero. Again theoretical results closely follow simulation results.

3.4.3. Normal incidence (\( \theta = \varphi = 0^\circ \))

The \( Q \) factors and reflection coefficient for unit cells excited by a normal incident plane wave are obtained by substituting \( \theta = \varphi = 0^\circ \) in (64) and (65). Under normal incidence, cross-coupled reflection coefficient becomes zero and the expression for radiation \( Q \) factor simplifies to:

\[ Q_{radTEM} = \frac{f_0 \eta_0 L}{4d W ab \eta_0} \]  

(84)

Reflexion coefficient simplifies to:

\[ \Gamma(f) = \frac{1}{Q_{radTEM}} - \frac{1}{Q_0} \frac{2j(f-f_0)}{f_0} \]  

(85)
The expression for radiation $Q$ factor obtained from Floquet modes is in agreement with the $Q$ factor derived from actual modal fields of normal incident plane wave reported in [81]. Equations (84) and (85) can be used for analyzing the performance of reflectarray unit cells under normal incidence.

3.4.4. Combined effects of incidence angle and coupling conditions

In the previous sub-sections, the effects of incidence angle on a particular antenna element with fixed physical parameters are shown. However, the sensitiveness to incidence angles and the reflection properties will vary with coupling conditions, if any physical parameter is changed. In this sub-section, the effects of varying substrate thicknesses on coupling regions are shown intuitively using graphical approach. In addition, the design curves showing the coupling conditions for varying patch widths, dielectric constants and losses are also discussed.

First, the position of critical coupling and coupling regions for varying substrate thicknesses ranging from 1 mil (25.4 μm) to 20 mil (0.508 mm) are studied in Figure 3-6. A resonance of 32 GHz is maintained by tailoring the patch length, while the other dimensions are fixed ($W = 2$ mm, $\varepsilon_r = 10.2$, $\tan\delta = 0.0035$, ½ oz copper). On H-plane, since $Q_{radTE}$ is inversely proportional to $\cos\theta$, the under-coupled regions are observed for larger incidence angles as shown in Figure 3-6(a). Conversely on E-plane as shown in Figure 3-6(b), the under-coupled regions are observed for smaller incidence angles because $Q_{radTM}$ is directly proportional to $\cos\theta$. On both the planes, it is also noted that as the substrate thickness decreases, the available range of incidence angles for over-coupled condition decreases. It is worth mentioning that even in the useful over-coupled region, the characteristics of the resonant patch element vary with the $Q$ factors.
Figure 3-6. Demonstration of coupling regions of unit cells for varying substrate thicknesses and incidence angle $\theta$ on (a) $\varphi = 0^\circ$ (H-plane) and (b) $\varphi = 90^\circ$ (E-plane).

To demonstrate this concept, two RO3010 substrates marked in Figure 3-6 with thicknesses of 5 mil (0.127 mm), 20 mil (0.508 mm) corresponding to different levels of over-coupling under normal incidence ($\theta = \varphi = 0^\circ$) are chosen. For reference, a center-fed circular reflectarray with a diameter of $15.5\lambda_0$ consisting of 749 elements and $a = b = 0.5\lambda_0$ is chosen. A $y$-polarized ideal feed modeled as $\cos^q \theta$, with $q = 1.5$ operating at 32 GHz is chosen as the feed.
with $f/D = 0.25$. This setup corresponds to a maximum incidence angle of $63^\circ$. The ideal phase shifts required on the array surface to compensate for the differential spatial distances from the feed and achieve a broadside radiation pattern are shown in Figure 3-7(a).

To highlight the effects of incidence angle, the array elements over the aperture are designed to achieve the phases shown in Figure 3-7(a) using variable patch lengths and normal incidence (84), (85). Later, the Floquet theory developed in the earlier section is utilized to extract the actual reflection coefficients on the antenna elements for the incident polarization accounting for real incidence angles. The effects of incidence angle are first shown on reflection phase, next on reflection magnitude and finally on the radiation patterns.

To comprehend the phase differences, the desired ideal phases are subtracted from the actual achieved phases and the results are plotted. For $d = 5$ mil as shown in Figure 3-7(b), considerable phase differences on the aperture are observed. In particular, on $\varphi = 0^\circ$ plane, under-coupled condition is observed beyond 10$^{th}$ element from the array center corresponding to an incidence angle of $53^\circ$ which is in agreement with Figure 3-6(a). Figure 3-7(c) corresponds to $d = 20$ mil, which is over-coupled for all the chosen range of incidence angles. However, noticeable differences between the achieved and desired phase shifts are observed particularly for edge elements nearby principal planes. A maximum phase variation of $43^\circ$ is recorded over the array.
Figure 3-7. (a) Ideal phase shifts required on each element, (b), (c) difference between the phase shifts obtained using Floquet theory and (a), for $d = 5, 20$ mil and $f/D = 0.25$. 
Similarly, the losses (in dB) for the chosen cases under normal incidence and accounting actual incidence angles are plotted in Figure 3-8. Figure 3-8(a) and Figure 3-8(b) correspond to losses under normal incidence for 5-mil- and 20-mil-thick substrates respectively. Similarly, Figure 3-8(c) and Figure 3-8(d) correspond to losses computed using Floquet theory. It is observed that for both the cases, on the E-plane ($\varphi = 90^\circ$), the losses computed using Floquet theory are less than that of TEM theory. Conversely, on the H-plane ($\varphi = 0^\circ$), the losses calculated considering actual incidence angles are higher than normal incidence. In addition, very high losses are observed for thin 5-mil-substrate. These variations in reflection magnitude and phase will affect the far-field performance of the antenna array.
Figure 3-8. Reflection magnitude obtained for \( f/D = 0.25 \) by considering (a), (b) a fixed normal TEM incidence for \( d = 5 \) and 20 mil substrates. (c), (d) Floquet theory for \( d = 5 \) and 20 mil substrates.
Figure 3-9 shows the radiation patterns on principal planes for both cases, computed using aperture fields from Floquet theory and TEM normal incidence. From Figure 3-9(a), for 5-mil case, even though considerable phase-differences between the Floquet theory and TEM (Figure 3-7(b)) are observed, due to very high losses (Figure 3-8(a) & Figure 3-8(c)) the radiation patterns are closely matched. Conversely, for 20-mil case in Figure 3-9(b), where the
phase differences are smaller than the 5-mil case, considerable differences in the side lobe levels of the radiation pattern between Floquet and TEM theory are observed because of the low loss performance of the antenna elements. The chosen scenario is a simple case study to demonstrate the effects of incidence angles using $Q$ factors. The study can be further extended to analyze other reflectarray configurations and antenna elements that are more sensitive to incidence angles with high $Q$.

Lastly, the effects of other physical parameters, patch width ($W$), dielectric constant ($\varepsilon_r$), loss tangent ($\tan\delta$) and metal conductivity ($\sigma$) in conjunction with incidence angle on coupling conditions are studied in Figure.3-10(a)-(h). Figure.3-10(a) and Figure.3-10(b) show the coupling regions for varying patch widths and $\theta$ for a fixed substrate RO3010 ($\varepsilon_r = 10.2$, $\tan\delta = 0.0035$, ½ oz copper) of thickness 5 mil and 32 GHz resonance for H- and E- plane, respectively. Figure.3-10(c) and Figure.3-10(d) correspond to varying dielectric constants at 32 GHz resonance, while the other parameters are fixed ($d = 5$ mil, $W = 2$ mm, $\tan\delta = 0.0035$, ½ oz copper). Similarly, Figure.3-10(e)-(h) correspond to varying loss tangents of the substrate ($\varepsilon_r = 10.2$, $d = 5$ mil, $W = 2$ mm, ½ oz copper) and metal conductivities ($\varepsilon_r = 10.2$, $d = 5$ mil, $W = 2$ mm, $\tan\delta = 0.0035$) on both principal planes.

For all cases, it is confirmed that on H-plane the under-coupled conditions happen at larger incidence angles, and for unit cells with small widths, and/or high dielectric and/or high loss substrates, and/or metals with low conductivities. Few substrates and metals that could potentially lead into under-coupled condition for the chosen combination are listed in Figure.3-10(e) and Figure.3-10(g). In addition, it is observed that on H-plane, the useful range of physical parameters under consideration decrease as $\theta$ increase. Conversely on E-plane, the under-coupled condition occurs at small incidence angles and the useful range increases as $\theta$ increase. Also observe that at $\theta = 0^\circ$ for both planes, the transition from under-coupled to over-coupled happens at the same point for all cases, such as $W = 1.2$ mm for varying patch widths.
Figure 3.10. Demonstration of coupling regions of unit cells for varying patch widths, dielectric constants, substrate loss tangents, metal conductivities vs. incidence angle $\theta$ on (a), (c), (e), (g) $\varphi = 0^\circ$ (H-plane) and (b), (d), (f), (h) $\varphi = 90^\circ$ (E-plane).
3.4.5. Circuit model

In this section, a circuit model for the patch antenna excited by dominant Floquet TE and TM modes inside a waveguide is presented as shown in Figure 3-11. First, the unloaded patch antenna is represented by a parallel RLC circuit model whose parameters are given by:

\[
C_p = \frac{\varepsilon_0 \varepsilon_r L W}{2d}, \quad L_p = \frac{1}{\omega_0^2 C_p}; \quad \text{and} \quad R_{\text{int}} = \frac{Q_o}{\omega_0 C_p} \tag{86}
\]

Next, the coupling between the patch antenna and the Floquet modes are represented through transformers with ratios \(n_1\) and \(n_2\), which can be expressed in terms of radiation quality factors as:

\[
Q_{\text{radTE}} = \omega_0 \cdot n_1^2 \cdot \frac{\eta_0}{\cos \theta} \cdot C_p \tag{87}
\]

\[
Q_{\text{radTM}} = \omega_0 \cdot n_2^2 \cdot \eta_0 \cos \theta \cdot C_p \tag{88}
\]

By equating (64), (65), (87) and (88), the transformer ratios are derived as:

\[
n_1^2 = \frac{ab}{4W^2 \cos^2 \varphi}; \quad n_2^2 = \frac{ab}{4W^2 \sin^2 \varphi} \tag{89}
\]

It can be observed that by using the quality factors, the circuit parameters are completely expressed in terms of physical parameters of the patch, substrate properties and waveguide parameters.

Similarly, the circuit model for TEM or TE\(_{10}\) excitation is shown in Figure 3-12, where

\[
n_{TE10} = \frac{\pi}{5.657 \cdot \sqrt{\frac{b}{a}}} \cdot \frac{1}{\sin \frac{\pi W}{2a}} \tag{90}
\]
\[ n_{TEM} = \frac{\sqrt{ab}}{2W} \]  

(36)

Figure 3-12. Equivalent circuit model of a patch antenna excited by a single incident wave (TE_{10} or TEM)

So far, the analysis is carried out for a rectangular patch elements using Q factors and coupling conditions. In the next chapter, the analysis of a tunable reflectarray element required to realize a beam-steerable reflectarray antenna is presented and the performance is optimized utilizing the coupling conditions.
CHAPTER 4: REFLECTARRAY BASED ON BST INTEGRATED CAPACITIVELY LOADED PATCH

4.1. Introduction

To realize a beam-steerable reflectarray antenna, the phase of each antenna element has to be controllable. In this chapter, first the integration of BST with the reflectarray element is discussed. Next, the biasing schemes for the normal operation of the antenna elements are discussed. Finally, a reflectarray prototype capable of electrically scanning the beam is presented. The Unit cell design chosen in this design is similar to [60] and its performance is improved considerably by optimizing the BST deposition and biasing scheme. Utilizing the coupling regions introduced in [70], a detailed analysis on the effects of coupling conditions on the tunable unit cell’s performance is also shown.

A 400 nm thick BST thin-film grown by RF magnetron sputtering is used as the tuning mechanism compared to 5 μm thick film in [60]. This enables the possibility of the unit cell being transferred to a flexible substrate [82]. The designed BST-integrated X and $K_a$ band reflectarray elements are fabricated using micro-fabrication techniques and measured via waveguide approach. To author’s knowledge, this report showcases a continuously tunable BST-integrated reflectarray antenna at $K_a$ band for the first time, and an X-band unit cell with enhanced performance.

4.2. Analysis using full-wave simulations

The reconfigurable reflectarray element chosen in this study is based on a complex oxide BST thin-film varactor. $\text{Ba}_x\text{Sr}_{1-x}\text{TiO}_3$ is the solid solution of barium titanate and strontium titanate. By applying a variable DC voltage across the electrodes of the BST capacitor, the dielectric constant is varied and a tunable capacitor is realized.

A schematic of the reflectarray using the proposed unit cell is shown in Figure 4-1(a). The reconfigurable reflectarray element consists of two quarter-wavelength-long patches coupled through the $\text{Ba}_{0.5}\text{Sr}_{0.5}\text{TiO}_3$ capacitor as shown in Figure 4-1(b). The antenna elements are designed to operate at X and $K_a$ bands in view of space applications. Fixed gap dimensions of 20 μm for X band and 10 μm for $K_a$ band between the patch elements are chosen, respectively. The width of the BST layer is slightly greater than the gap dimension to allow overlap between the patch and BST in order to form reliable electrical connection. Rectangular metallic waveguide excitation shown in Figure 4-1(c), with the dominant $TE_{10}$ mode propagating is used for the extraction of reflection properties.
The dielectric constant and loss tangent of BST are functions of applied voltage and frequency. The varying DC voltages are simulated by tuning the relative dielectric constant and loss tangent of the BST. This facilitates the study of coupling conditions and the effects of BST film properties. In general, higher permittivity and loss are observed for lower DC voltage and vice-versa. For the analysis, dielectric properties of the BST are extracted from the unit cell.
measurements which are presented in the later section of the report. A relative dielectric constant swept from 500 to 250 for the \( K_a \)-band and 600 to 300 for the \( X \)-band unit cells are used in the simulations.

BST being a high permittivity material, usually in the ranges of hundred, when integrated with an antenna element will affect the \( Q \) factors and the coupling conditions. In addition, the substrate properties are also shown to significantly affect the performance of a reflectarray. As shown in [70], under-coupled unit cells exhibit anomalous phase responses. Unit cells close to critically-coupled condition suffer with very high losses but provide maximum phase sensitivity and phase range. On the other hand, over-coupled unit cells usually have reasonable losses but with limited phase range. In this paper, sapphire (\( \varepsilon_r = 10 \)) is chosen as the substrate for its favorable properties of BST growth and very low loss [83]. A detailed analysis on the effects of substrate thickness on coupling conditions and unit cell performance is shown in the following subsection.

In the first step, two extreme cases of \( \varepsilon_r = 500, \tan \delta = 0.16 \) and \( \varepsilon_r = 250, \tan \delta = 0.04 \) corresponding to lowest and highest applied electric field at \( K_a \) band are chosen for the BST. Substrate thicknesses ranging from 50 to 500 \( \mu \)m are used. For simplicity, dielectric constant and loss of the BST are assumed to be constant \( w. r. t \) frequency and substrate thickness. The width of the antenna element is fixed at 2.3 mm and 800-nm-thick copper is used for the metal layer. For each substrate thickness, the unit cell is designed to resonate at a fixed frequency of 29 GHz for a BST \( \varepsilon_r = 500 \) by tailoring the resonant dimension from 2.75 mm (\( h = 50 \mu \)m) to 1.42 mm (\( h = 500 \mu \)m).

Reflection loss at resonance (Min(|\( S_{11} \)|)), which corresponds to the maximum loss of the unit cell, and phase sensitivity (\( \sigma_f \)) defined as negative of the slope of the reflection phase at resonance, are used to study the coupling regions and evaluate the performance of the antenna element. Throughout the paper, Min(|\( S_{11} \)|) is used to study the coupling conditions. Whereas |\( S_{11} \)| is used to quantify the loss of the unit cell at a fixed frequency, which in general is the center frequency of the array.
Figure 4.2 (a) Reflection magnitude and (b) phase sensitivity vs. substrate thickness at resonant frequency of the unit cells showing the coupling regions.

Figure 4.2 (a) shows the maximum reflection loss versus substrate thickness for the two chosen permittivities at their respective resonant frequencies. It is observed that for the higher dielectric constant case ($\varepsilon_r = 500$, $\tan\delta = 0.16$), all the three coupling regions; under-coupled, critically-coupled and over-coupled are observed. However, for lower
dielectric constant case ($\varepsilon_r = 250$, $\tan\delta = 0.04$), the patch elements are over-coupled for the entire range of substrate thicknesses. Therefore, a unit cell corresponding to a substrate thickness in the under-coupled region can become over-coupled for higher bias voltages. Thus, under-coupled unit cells are not completely undesirable for the presented antenna element. It is important to note that for a passive reflectarray element, a unit cell in the under-coupled region is not useful because its $Q$ factors will not change considerably by adjusting its resonant dimension. In the under-coupled region of the high dielectric constant case, reflection loss increases as the substrate thickness increases. Maximum loss is observed at the critically-coupled condition. As expected, in the over-coupled region for both cases, the reflection loss decreases as the substrate thickness increases.

Phase sensitivity ($\sigma_f$) versus substrate thickness is shown in Figure 4-2(b). For the high dielectric constant case, it is observed that the phase sensitivity, which determines the bandwidth and phase range, is higher around the critically-coupled region. Antenna elements operating at this region will provide maximum phase range at the expense of additional losses. As the substrate thickness increases in the over-coupled region, phase sensitivity decreases, which implies increased bandwidth and reduced phase range. On the other hand, for the low dielectric constant case, all the antenna elements are over-coupled and phase sensitivity decreases as substrate thickness increases.
To achieve an optimal performance in terms of loss and phase range, unit cells on three substrate thicknesses $h = 175, 300, 450$ μm, corresponding to different levels of coupling conditions are studied. Again for each substrate thickness, the unit cell is designed to resonate at 29 GHz for a BST permittivity of $\varepsilon_r = 500$. The resonant dimension are found to be 2 mm, 1.75 mm and 1.5 mm for $h = 175, 300, 450$ μm respectively. To demonstrate the tunability of the antenna
element, BST permittivity is varied from 500 to 250 in steps of 25. The loss tangent of the BST film is swept from 0.16 to 0.04 in steps of 0.012, where high loss tangent (0.16) corresponds to high BST permittivity ($\varepsilon_r = 500$) and vice-versa. For the chosen range of relative dielectric constants, a frequency tunability of 7.4%, 5.7% and 4% are observed for 175, 300 and 450 $\mu$m respectively.

Figure 4-3(a) shows the maximum reflection loss observed for each dielectric constant. It is observed that the unit cells designed on 175 $\mu$m substrate undergo all the three coupling conditions. As pointed out earlier, even though 175 $\mu$m case is under-coupled for high BST permittivities (low DC voltage), as the applied voltage increases its permittivity decreases and the antenna becomes over-coupled. On the other hand, relatively thicker substrates 300 and 450 $\mu$m are always over-coupled. As expected, thicker substrates exhibit lower losses. However, thicker substrates possess smaller phase sensitivity ($\sigma_f$) implying lower phase range as shown in Figure 4-3(b).

To quantify these effects, design curves showing permittivity versus reflection magnitude and phase, at fixed operating frequencies are shown in Figure 4-4. For 175-$\mu$m-thick substrate, the curves are plotted at two different frequencies of 30.34 GHz and 30.8 GHz, corresponding to maximum phase range of 277° and a phase range of 250° respectively. For the maximum phase range case, the losses are extremely high as seen in Figure 4-4(b). Hence, a different frequency of 30.8 GHz is picked, where losses are considerably reduced by limiting the phase range to 250°. For 300 $\mu$m and 450 $\mu$m thick substrates, curves are shown at their respective frequencies corresponding to maximum phase range. Unit cells designed on 300 $\mu$m substrates have slightly better loss performance, but the overall phase range is observed to be less than 200°. Similarly, unit cells on 450 $\mu$m substrate exhibit considerably-lower losses, however suffer from a very low phase range of 102°. Therefore, 175-$\mu$m-thick substrate operating near the critically-coupled region is chosen for the measurement purposes at $K_a$ band. A similar analysis is performed at $X$ band, and a substrate thickness of 430 $\mu$m which is over-coupled and operates close to the critically-coupled condition is used.
In order to apply DC bias voltages to the BST thin film, and at the same time have the least effect on the total performance, special design is needed for DC bias lines. Three different configurations for biasing lines as shown in Figure 4-5 are discussed. Figure 4-5(a) corresponds to the antenna element without any bias lines; Figure 4-5(b), Figure 4-5(c) and Figure 4-5(d) correspond to the configurations where bias lines are connected to the edge, middle
and minimum E-field point on each patch, respectively. Moreover, high-impedance lines are defined by using highly-resistive chromium for X-band unit cell. For the sake of brevity, the simulations are performed for a fixed permittivity ($\varepsilon_r = 300$) and loss tangent ($\tan\delta = 0.105$) at X band. A square patch of 6.5×6.5 mm$^2$ on a 430-μm-thick sapphire substrate operating at X band is chosen. For $K_a$ band, another biasing scheme based on radial stubs is presented.

Figure 4-5. Different bias line configurations. (a) No bias lines, (b) horizontal bias lines at the corner of each patch, (c) horizontal bias lines at the middle of each patch, and (d) horizontal bias lines at the $E_{\text{min}}$ of the patch (1.26mm from the center of the unit cell).

Figure 4-6. Current density for different bias lines. (a) No bias lines, (b) horizontal bias lines at the corner of each patch, (c) horizontal bias lines at the middle of each patch, and (d) horizontal bias lines at the $E_{\text{min}}$ of the patch.

To evaluate the biasing schemes, the surface current density at resonance for each case is plotted in Figure 4-6. The corresponding reflection magnitude and phase are shown in Figure 4-7. It is obvious that Case (d) with horizontal bias
lines connected to the minimum E-field point on the patch share the most similarity of current density distribution as Case (a) without bias lines. However, biasing schemes (b) and (c) disturb the mode of the patch as observed from the current densities and result in undesired responses as shown in Figure 4-7(a) and Figure 4-7(b). It is also noted that when the biasing lines are connected to the edge of the patch, where the E-field is maximum, the affects from the biasing are dominant. Thus, the horizontal bias lines connected to the minimum E-field point is the best choice as confirmed from Figure 4-7(a) & Figure 4-7(b).

For $K_a$-band unit cell, the presence of biasing pads inside the waveguide is not practical due to the waveguide dimension limitations. Keeping this in mind, an optimized biasing scheme based on the radial stub is proposed. In the first step of bias line optimization, the biasing lines are connected to the minimum E-field points on each quarter-wavelength-long patches. Additionally, a half-wave length radial stub is used to provide a very high impedance (open-circuit) looking from the patch. Then, the vertical biasing lines start from the short-circuit point of the radial stub and run all the way towards the waveguide edge, which can be connected to soldering pads during measurements, as shown in Figure 4-8(a). For measurement purposes, a customized waveguide setup as shown in Figure 4-8(a) is designed. It is important to note that an intermediate waveguide holder with grooves aligned to the bias-lines is fabricated to avoid short circuit between the patch element and $K_a$-band waveguide adaptor. The comparison between the reflection properties of the $K_a$-band unit cell with and without the radial stub biasing scheme is shown in the measurement section.
Figure 4-7. A reflectarray unit cell loaded with a BST layer acting as a distributed capacitance and corresponding reflection (a) magnitude and (b) phase for different biasing schemes.
4.3. Unit cell fabrications and measurements

In this section, the proposed BST-loaded reflectarray unit cell fabrication is discussed following the steps shown in Figure 4-8(b). Next, the measurement results for different cases are shown. The Sapphire wafer is cut to the same size as the cross section of X- and Ka-band sample holders using MTI Precision CNC Dicing/Cutting Saw (SYJ-400). Then, a BST thin-film layer of 400 nm thick is deposited using RF sputtering system (AJA ATC 1800). Two targets from different vendors are used for X- and Ka-band designs. A system pressure ($P_{sys}$) of 5 mtorr is used for the X-band unit.
cell fabrication on a 430 μm thick sapphire substrate. For the Ka-band unit cell fabrications, two system pressures of 20 mtorr and 30 mtorr are used.

![Figure 4-9. Measured reflection magnitude and phase of the unit cells with and without the biasing lines at Ka band.](image)

After deposition, the BST layer is etched in diluted HF solution to a size that is slightly larger than the gap in both horizontal and vertical directions. A 2% solution of commercially-available HF is used for etching. Next the sample is annealed at 900° for 20 hours (X band) / 12 hours (Ka band) in a box oven with an oxygen flow under atmosphere pressure. For X-band unit cell, Cr bias lines of 20 μm wide and 300 nm thick are then deposited and patterned using liftoff. The resistance of each bias line is about 3 kΩ (measured using a digital multi-meter). After that, 1.3-μm-thick copper patch and soldering pads are deposited in the same run using an E-beam evaporator and patterned using liftoff. For the Ka-band unit cell, patterns for both patch and bias lines are defined using a single layer mask. A 750-nm-thick copper layer is deposited using E-beam evaporation and patterned using lift-off. A 50-nm-thick chromium layer is thermally evaporated to promote the adhesion between copper and substrate. Finally, 1.3-μm-thick copper is deposited on the back side of the substrate to form the ground plane.

The fabricated BST-loaded reflectarray unit cells and the measurement setup are illustrated in Figure 4-8. The measured results at Ka band obtained for the BST-loaded patch element with and without the bias lines on a 175-μm-
thick substrate at 20 mtorr system pressure are shown in Figure 4-9. It is clearly observed that the effects from biasing line are minimal.

![Graph showing reflection magnitude vs. frequency for varying bias E-field.](image1)

Figure 4-10. Measured reflection magnitude vs. frequency for varying bias E-field.

![Graph showing reflection magnitude vs. bias E-field showing the coupling conditions for two system pressures.](image2)

Figure 4-11. Reflection magnitude vs. bias E-field showing the coupling conditions for two system pressures.

Figure 4-10 shows the resonant frequency variation of the $K_a$-band unit cell with the applied bias E-field for 20 mtorr system pressure. A resonant frequency shift from 29.22 GHz to 30.93 GHz corresponding to 5.8% change in resonant frequency is observed for an applied E-field variation of 0-35 V/μm. As expected, it is also observed that the unit cells are under-coupled for the applied E-fields lower than 1.3 V/μm. Unit cells corresponding to critically-coupled and
over-coupled condition are also marked in the figure. To better demonstrate these coupling conditions, maximum reflection loss of the unit cell for each bias voltage is shown in Figure 4-11 for both 20 mtorr and 30 mtorr system pressures. It is clearly observed that unit cells deposited at 20 mtorr have lower loss for most of the bias E-field ranges. In addition, increased over-coupled region is observed for 20 mtorr system pressure compared to 30 mtorr.

![Graph](image1)

(a)

![Graph](image2)

(b)

Figure 4-12. Reflection (a) magnitude and (b) phase vs. bias E-field at fixed operating frequencies for Ka-band unit cell on 175µm thick sapphire.
Figure 4-13. Reflection (a) magnitude and (b) phase vs. bias E-field at fixed operating frequencies for $K_a$-band unit cell on 250µm thick sapphire.
To evaluate the performance of the unit cells, design curves showing reflection magnitude and phase range for two cases are shown in Figure 4-12. While presenting the reflection magnitude, additional losses from the sample holder are subtracted from the measured data. Case 1 corresponds to the frequency, where maximum phase range for the over-coupled unit cells is observed. Case 2 corresponds to the frequency where a phase range of 250° is recorded. Frequencies corresponding to Case 1 for 20 mtorr and 30 mtorr system pressures are found to be 30.17 GHz and 30.33 GHz, respectively. Similarly for Case 2, the graphs are plotted at 30.7 GHz and 30.88 GHz for 20 mtorr and 30 mtorr pressures, respectively. It is clearly observed from Figure 4-12(a) and Figure 4-12(b) that the reflection loss is very high for Case 1, even though a phase range close to 300° is observed for both system pressures. Specifically, a phase range of 283° and 298° are observed for 20 mtorr and 30 mtorr system pressures. On the other hand, if the unit cells are operated at frequencies corresponding to a phase range of 250° (Case 2), losses are significantly reduced for both system pressures. A maximum reflection loss of 5.6 dB and 7.1 dB are observed for 20 mtorr and 30 mtorr pressures, respectively. Comparing the losses between the two system pressures, unit cells fabricated at 20 mtorr have on average approximately 1-dB less loss than those at 30 mtorr for a phase range of 250°. Thus lowering the system pressure will improve the loss performance. However, it is important to note that the tunability of the BST thin-film decreases if the system pressure during deposition is reduced. A compromise between the phase range and reflection loss is required for the optimal performance. Similarly, another unit cell operating at $K_a$ band on a 250µm sapphire at 20 mtorr system pressure is fabricated and the results are plotted in Figure 4-13. A maximum phase range of 222° with a maximum reflection loss of 7.5 dB at 30.16 GHz is measured.

Now design curves for the $X$-band unit cell on 430-µm-thick substrate are shown in Figure 4-14. An electric field variation of 0-20 V/µm is used in the measurements. Case 1 corresponds to maximum phase range while Case 2 corresponds to a phase range of 250°. It is observed that when the unit cells are operating at the frequency corresponding to maximum phase range (10.15 GHz), a maximum reflection loss of 8.9 dB and phase range of 263° are observed. In Case 2 (10.27 GHz), the maximum reflection loss is reduced to 5.8 dB.
4.4. BST properties and unit cell analysis

In this subsection, the design of a 45-element Ka-band reflectarray prototype based on the unit cell presented in the earlier section is described. Earlier, the effects of system pressure, operating frequency and substrate thickness on the unit cell performance were shown. It is important to note that the $S$-curves presented in Figure 4-12 were based on

Figure 4-14. Reflection (a) magnitude and (b) phase vs. bias E-field at fixed operating frequencies for X-band unit cell.
rectangular metallic waveguide simulations with an inter element spacing of 7.112 mm and 3.556 mm. However, for an array design, it is common to extract the S-curves from TEM waveguide with an inter-element spacing equal to the actual spacing between the elements in the final array. Hence the dielectric properties of the BST thin film for different bias E-field are estimated from the measured data for a system pressure of 20 mtorr and plotted in Figure 4-15. This enables the study of the reflection properties of the unit cell for any case of interest using full-wave simulations. From Figure 4-15, it is observed that the dielectric constant varies from 500 to 275, while loss tangent varies from 0.12 to 0.03 for an E-field variation of 0 – 35 V/μm.

![Figure 4-15. Estimated BST properties extracted from measurements at Ka band.](image)

4.4.1. TEM Analysis

First, the analysis of unit cell is performed using a parallel-plate TEM waveguide in High Frequency Structure Simulator (HFSS). An operating frequency of 31.6 GHz and a fixed inter-element spacing of $a = b = 4$ mm is chosen. The dimensions of the patch element are 2.3 mm (W) and 1.9 mm (L) with a fixed gap size of 10 μm. The dimensions of BST are slightly larger than the gap size to ensure a reliable electrical connection between the patch elements and the BST thin film.

The analysis is carried out by varying the BST dielectric constant from 275 to 500 in steps of 25, and loss tangent from 0.03 to 0.12 in steps of 0.01. The resulting S-curves showing the variation of reflection magnitude and phase
versus BST permittivity are shown in the Figure 4-16. It is important to note that the frequency of operation is picked to achieve an overall phase range close to 240°. From Figure 4-16(a) it is observed that the reflection loss varies from 1 dB to 7.5 dB for the chosen range of dielectric properties of BST. Maximum reflection loss is observed for a BST relative permittivity of 375. An overall phase variation of approximately 240° is observed as shown in Figure 4-16(b).

![Figure 4-16. Reflection (a) magnitude, and (b) phase vs. BST dielectric constant at 31.6 GHz for normal TEM incidence](image)
4.4.2. Floquet Analysis

To understand the effects from incidence angles on the reflection properties of the antenna elements, Floquet analysis is used. HFSS simulations employing master-slave boundaries are used to extract the S-curves. Incidence angles up to 40° for both TE and TM polarizations are simulated and compared with the normal incidence. From Figure 4-17, it is observed that the reflection losses of the unit cells excited by TE waves (elements on the H-plane) are higher than the normal incidence. It is also observed that on H-plane, the reflection loss increases as incidence angles increases as explained in the previous chapter. On the other, for the elements on the E-plane (TM), reflection loss is smaller than the normal incidence case, and decreases as the incidence angle increase. The variation of reflection phase versus BST dielectric constant for different incidence angles and polarization are shown in Figure 4-18. It is seen that the overall phase range decreases very slightly for the elements located on the E-plane (TM) compared to the normal incidence. On the other hand for the elements on the H-plane (TE), the overall phase range increases with the incidence angles. This behavior is due to increase in the phase-sensitivity. Overall, the effects from varying incidence angle are smaller for incidence angles up to 30° and start to increase for higher incidence angles particularly on H-plane.

![Figure 4-17](image)

Figure 4-17. S-curves showing reflection magnitude for varying incident angles using Floquet analysis.
4.5. Array analysis using full-wave simulations

In this section, first the biasing schemes for the unit cell described in the previous section are discussed. Later, a reflectarray prototype is analyzed using full-wave simulations. A simplified phase assignment to reduce the number of voltage sources for E-plane scanning is also discussed. Finally, the beam-scanning capability of the array is demonstrated on H-plane using individual phase control.

4.5.1. Biasing schemes

Ideally, every element in the array will have an individual phase control to achieve beam scanning. However, such a demonstration is particularly expensive since it requires a power supply for every single antenna element. For this purpose, two simplified biasing schemes shown in Figure 4-19 are studied. Type-1 and type-2 correspond to the cases where the antenna elements are connected along the E-plane, and H-plane, respectively. By using such a simplified biasing scheme, every antenna along each row or column can be controlled by a single power supply and beam scanning along a single plane can be demonstrated. This simplified biasing scheme helps to demonstrate the beam-scanning capability of the array with reasonable costs.

The bias lines are defined using high impedance chromium lines connected to the minimum E-field point on each quarter wavelength patch. The study is carried out for a fixed normal incidence and polarization with E-field along y-axis. The comparison between the S-curves with and without the biasing lines is shown in Figure 4-20. It is clearly
observed, for type-2, where the elements are connected along H-plane the affects from biasing are minimal. On the other hand, for type-2 biasing scheme, significantly higher losses with completely different phase response is observed. Hence, type-2 biasing scheme is chosen for the array design.

Figure 4-19. Two types of biasing schemes considered to minimize their affects.

Figure 4-20. S-curves for different biasing schemes using full-wave simulations
4.5.2. Array analysis using full-wave simulations

For the demonstration, a 5×9 (45) element array with an offset feed configuration of $\theta_f = 25^\circ$ along y-axis is chosen. A Ka-band horn antenna with the beam pointing towards the center of the array is applied. For biasing, type-2 biasing scheme where the elements along H-plane are controlled by a single voltage is used. By choosing a large $f/D$ and placing the feed along E-plane (y-axis), similar phase-delays for the elements along each column can be achieved and beam-scanning along E-plane can be demonstrated. For this reason, a large $f/D = 1.95$ is chosen. This corresponds to an aperture efficiency of ~27% and an incidence angle of 36°. For the analysis and design of the array, the unit cell simulations based on TEM waveguide are used. HFSS using Finite Element Boundary Integral (FEBI) boundary conditions is used to analyze the array performance. By using FEBI, the radiation boundary can be defined just around the feed and the array. The structures inside the FEBI are analyzed using FEM and the interactions between the structures are analyzed using boundary integral equations, which significantly reduces the simulation complexity compared to a standard radiation boundary.

In addition, due to the small size of the gap (10 μm) and high dielectric constants of BST it is computationally very expensive to perform the full-wave simulations. To simplify the simulation complexity, BST material is approximated by the HFSS lumped element model. The varying BST permittivity and loss tangent are simulated by changing the capacitance and resistance values of the parallel RC lumped elements. For the chosen BST dielectric property variations, capacitance ranging from 0.42 – 0.23 pF and resistance from 105 – 720 ohms are obtained.

For the array analysis, first the lumped element parameters of the BST are chosen to achieve the phase distributions required to scan the beam along E-plane. The elements along each column are assigned the same lumped element parameters to replicate the simplified biasing scheme as shown in Figure 4-21.
Figure 4-21. A schematic of a 5x9 reflectarray using the BST-integrated capacitively loaded patches.

Figure 4-22(a) shows the normalized radiation patterns for E-plane scan up to 25°. Only the scan cases along the image side of the feed are shown due to the feed blockage. In Figure 4-22(b), the E- and H-plane patterns for the broad-side case are shown. From Figure 4-22, it is observed that the narrow main lobes are observed for the E-plane patterns compared to H-plane patterns as expected due to the larger size along E-plane. Higher side lobe levels are observed for the E-plane radiation patterns, particularly for the broad-side case due to the simplified biasing scheme and also the scan angle being close to the feed. It is also important to note that better side-lobe levels can be achieved by improving the overall phase-range of the unit cell.

To demonstrate the beam scanning along H-plane, the simplified biasing scheme is not sufficient for the chosen configuration. Hence, individual phase assignment for each element in the array is used to demonstrate H-plane scanning. The simulated radiation patterns showing scan angles up to ± 25° along H-plane are plotted in Figure 4-23. Overall, employing full-wave simulations the beam-scanning capability of the array up to ± 25° on the principle planes is demonstrated.
Figure 4-22. Normalized (a) E-plane radiation patterns for various scan angles, and (b) principle plane patterns achieved by the simplified phase-assignment using full-wave simulations.
4.6. Array fabrications and measurements

In this section, the fabrication steps of the proposed array are discussed. The fabrication process is similar to the recipe discussed for the unit cell fabrication. First, a 400 nm BST film is deposited on 2in C-plane sapphire wafer using RF magnetron sputtering. A system pressure of 20mtorr and a deposition temperature of 400°C with a constant flow of Ar and O₂ gases are maintained during the deposition process. The deposited BST is patterned using photolithography and undesired BST is etched off using 2% HF solution. The patterned BST is annealed at 900°C with a constant O₂ flow. Next, 125 nm thick and 35 μm wide high resistive chromium is deposited using thermal evaporation and patterned using liftoff. A third mask layer is used to pattern the patches and biasing pads, where 750 nm copper is deposited via E-beam evaporation along with 40 nm chromium for adhesion. Finally another 800 nm copper is deposited as the ground plane. The fabricated 45 element array with the simplified biasing scheme is shown in Figure 4-24. Once the biasing pads are soldered with the DC cables, the array is mounted on a test fixture. To avoid undesired reflections from the biasing pads, thin foam sheet is mounted along the edges of the wafer. The measurement setup including the array, feed, biasing lines and the support structures is shown in Figure 4-25. All the supporting structures used in the measurement setup are fabricated using 3D printer. The measurements are carried out in a standard
anechoic chamber using far-field setup. The biasing lines run all the way across the wafer and connect to the biasing pads at the edges of the wafer.

![Biasing Pads](image1.png)

**Figure 4-24.** Fabricated 45-element reflectarray prototype showing the biasing pads and soldered DC cables.

To minimize the feed blockage during the array measurement, an offset feed configuration with $\theta_f = 25^\circ$ is used. A $K_a$-band horn antenna with $f/D = 1.95$ and the beam pointing towards the center of the array is applied. Due to the simplified biasing scheme used in the design, only E-plane scan is possible. The simplified biasing scheme significantly reduces the number of voltage sources and the associated costs. To achieve the far-field radiation pattern, phase distribution on the array elements is first calculated. Utilizing the data from the S-curves, the corresponding relative permittivity of BST is obtained. Once the required BST permittivity is known, Figure 4-15 is used to obtain the desired bias voltages. The calculated bias voltages are further tuned to optimize the performance during the measurements. Due to gap size of 10$\mu$m, DC voltages up to 350 volts are required to achieve the overall phase range of 240$^\circ$. Commercially available DC-DC converters are used to transform the low input voltages from the power supply (0 - 5 V) to the desired high voltages (0 – 350 V). The adjustable DC-DC converters along with the digital multi-meters to monitor the output voltages are shown in the Figure 4-26.
Figure 4-25. Measurement setup of the array showing the supporting structures, array feed and DC cables inside the anechoic chamber.
The reflectarray is mounted to the turntable and a receiving antenna is placed at the far-field of the array to measure the power. The turntable is rotated in short increment of angles from -90° to 90° and the corresponding S-parameter are used to extract the radiation patterns. The measured co- and cross-polarization levels of the reflectarray on both the principle planes when the biasing is configured for a scan angle of 0° is shown in Figure 4-27. It is clearly observed that narrow beam is achieved on the E-plane compared to H-plane. A cross-polarization level better than -20 dB is measured on the principle planes. Higher side lobe levels are observed for the E-plane pattern as predicted from the full-wave simulations.
Figure 4-27. Radiation patterns on the principle planes showing both co- and cross-polarization components.

Figure 4-28. Radiation patterns for different scan-angles along E-plane.

Figure 4-28 shows the radiation patterns for three scan angles of 0°, 13° and 25° along E-plane. It is observed that better side-lobe levels (10 dB less than the main beam) are measured for 13° and 25° scan angles compared to broadside case. The presented 45-element tunable reflectarray with simplified biasing scheme showcases the feasibility of achieving beam-scanning using BST loaded patch antennas. Figure 4-29 shows the variation of array gain with the
frequency for broad-side case. A 1-dB gain bandwidth of about 1.8 % is measured. Finally, the loss budget analysis of the array is listed in Table 4-1. The difference of 1.16 dB between the measured and calculated losses can be attributed to the measurement tolerances such as polarization errors, absorber sheet, and the supporting structures.

![Graph showing measured gain vs. frequency for broadside radiation pattern.](image)

**Figure 4-29.** Measured gain vs. frequency for broadside radiation pattern.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directivity (calculated)</td>
<td>20 dB</td>
</tr>
<tr>
<td>Aperture efficiency (calculated)</td>
<td>~27.14% (-5.66 dB)</td>
</tr>
<tr>
<td>Average element loss (calculated)</td>
<td>~4.88 dB</td>
</tr>
<tr>
<td>Total calculated losses</td>
<td>10.54</td>
</tr>
<tr>
<td>Gain (measured)</td>
<td>8.3 dB</td>
</tr>
<tr>
<td>Measured losses (Directivity – Gain)</td>
<td>11.7 dB</td>
</tr>
<tr>
<td>Efficiency</td>
<td>6.76 %</td>
</tr>
<tr>
<td>Measured-calculated losses</td>
<td>~1.16 dB</td>
</tr>
</tbody>
</table>

Table 4-1. Loss budget of the array for the broadside case
The presented 45-element tunable reflectarray with simplified biasing scheme showcases the feasibility of achieving beam-scanning using BST loaded patch antennas. Higher scan angles with better side-lobe levels and improved efficiencies can be realized by using larger apertures and optimizing the BST growth to minimize the losses from BST. In addition, the bandwidth performance can be improved by choosing broad-band antenna elements or multi-resonant elements.
The proposed reconfigurable reflectarray design using BST technology demonstrates the possibility of realizing next generation beam-steerable antenna systems. Future extensions include variations and improvements at both unit cell and array level, which are briefly discussed in this section.

To provide high date rates desired for space and commercial applications, antenna systems operating at mm-wave frequencies and beyond are desired. Due to monolithic integration, the proposed BST integrated antenna element can be extended to higher frequency ranges such as V-band. By changing the geometry of the gap between the patches, a variety of capacitance ranges can also be achieved. The major challenge for the operation at such higher frequencies will be to design a compact biasing network without effecting the antenna performance.

Reconfigurable antennas on flexible substrate are desired for cost effective space-deployment. The growth of BST on flexible substrate and the possibility of monolithically integrating with the antenna element opens up the possibilities of having foldable and electronically controllable antenna systems. To realize this, a possible route is to first fabricate the BST integrated antenna element on a rigid substrate such as Silicon and then use the wafer bonding to transfer the electronics from the rigid substrate onto the foldable substrate.

It is also important to reduce the dielectric loss of the BST to improve the overall efficiency of the array. The quality of BST film is dependent on the growth process and the process parameters. A detailed study is required to understand the tradeoff between different performance criteria and a collaboration between the material science and the antenna teams is desired for this purpose. In addition, BST can be doped with other materials to optimize the performance. For example techniques such as Mn-doped BST/MgO composites can be used to reduce the loss tangent of the BST.

The performance improvement of the BST loaded unit cells in terms of bandwidth and the phase range is also desired. This can be achieved by using multi-resonant elements loaded with BST or by using TTD’s integrated with BST. For the multi-resonant configuration, superior performance in terms of bandwidth and phase range can be achieved due to the multiple resonant modes and the coupling between the resonators. The major challenge is the biasing circuitry that is required to control each resonant element, separately, and have minimum effects on the electrical performance. It is also complicated to characterize the coupling mechanism between the resonators which will vary with the bias voltages.
Another extension will be to design individual phase control for each element to minimize the phase-errors and achieve higher efficiencies. However, for the proposed design this may result in extremely large number of biasing lines over the array aperture and result in spurious reflections and higher losses. Unit cell designs with the biasing circuity not on the active region of the array can provide individual phase control and have least effects from biasing. In this regard, aperture-coupled antenna with BST loaded microstrip transmission lines beneath the ground plane are advantageous.

In addition, for the applications where larger antenna aperture with higher gains are desired, several wafers can be fabricated separately and joined together. Such a design needs further study in terms of the fabrications and alignment between the wafers.

For an efficient array design, it is also crucial to understand the underlying physics of the antenna. The Q factor theory introduced in this study to analyze the reflectarray elements is proven to be a very effective design tool. This theory can be extended for other unit cell configurations such as multi-resonant antenna elements, aperture-coupled elements and other tunable elements. In addition, the theory can also be applied to other array configurations such as phased-array and lens antennas.
REFERENCES


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