Computationally Efficient Digital Backward Propagation For Fiber Nonlinearity Compensation

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COMPUTATIONALLY EFFICIENT DIGITAL BACKWARD PROPAGATION FOR FIBER NONLINEARITY COMPENSATION

by
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Fall Term
2011

Major Professor: Guifang Li
ABSTRACT

The next generation fiber transmission system is limited by fiber nonlinearity. A distributed nonlinearity compensation method, known as Digital Backward Propagation (DBP), is necessary for effective compensation of the joint effect of dispersion and nonlinearity. However, in order for DBP to be accurate, a large number of steps are usually required for long-haul transmission, resulting in a heavy computational load.

In real time DBP implementation, the FIR filters can be used for dispersion compensation and account for most of the computation per step. A method of designing a complementary filter pair is proposed. The individual errors in the frequency response of the two filters in a complementary filter pair cancel each other. As a result, larger individual filter error can be tolerated and the required filter length is significantly reduced.

Unequal step size can be used in DBP to minimize the number of steps. For unrepeatered transmission with distributed Raman amplification, the Raman gain as a function of the distance and the effective fiber length of each DBP step need to be calculated by solving the differential equations of Raman amplification. The split-step DBP is performed only for transmission where the signal power is high.

In comparison with solving the nonlinear Schrodinger equation (NLSE) for the total field of the WDM signal, solving the coupled NLSE requires a smaller step number and a lower sampling rate. In addition, the phase-locking between the local
oscillators is not necessary for solving the coupled NLSE. The XPM compensation of WDM long-haul transmission by solving the coupled NLSE is experimentally demonstrated.

At the optimum power level of fiber transmission, the total nonlinear phase shift is on the order of 1 radian. Therefore, for transoceanic fiber transmission systems which consist of many (>100) amplified fiber spans, the nonlinear effects in each span are weak. As a result, the optical waveform evolution is dominated by the dispersion. Taking advantage of the periodic waveform evolution in periodically dispersion managed fiber link, the DBP of $K$ fiber spans can be folded into one span with $K$ times the nonlinearity. This method can be called “distance-folded DBP”.

Under the weakly nonlinear assumption, the optical waveform repeats at locations where accumulated dispersions are identical. Consequently, the nonlinear behavior of the optical signal also repeats at locations of identical accumulative dispersion. Hence for a fiber link with arbitrary dispersion map, the DBP steps can be folded according to the accumulated dispersion. Experimental results show considerable savings in computation using this “dispersion-folded DBP” method. Simulation results show that the dramatically reduced computational load makes the nonlinearity-compensated dispersion-managed fiber link a competitive candidate for the next-generation transmission systems.
To my family
ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor, Dr. Guifang Li, for the continuous support of my Ph.D study and research, for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D study.

My sincere thanks also goes to my fellow group members, Ibrahim Ozdur, Neng Bai, Cen Xia, Erdem Erden, Bin Huang and former group members, Xiaoxu Li, Xiaobo Xie, Eduardo Mateo, Fatih Yaman, Inwoong Kim, Xin Chen, Kevin Croussore, Gilad Goldfarb, Gregory Eskridge, Andreas Schmidt, Ivan Carlos Vivanco, Guanmao Zhang, Jianfei Liu and Guijun Hu for their valuable suggestions and cooperation throughout the most unforgettable years of my life.

Lastly, and most importantly, I wish to thank my family for all their love, support and encouragement.
# TABLE OF CONTENTS

LIST OF FIGURES .......................................................................................................................... X

LIST OF ABBREVIATIONS ............................................................................................................... XIII

1. INTRODUCTION .................................................................................................................... 1

1.1. Impairments in Fiber-Optic Transmission ........................................................................... 1

1.2. Digital Backward Propagation ......................................................................................... 2

1.3. Dissertation Outline ......................................................................................................... 3

1.4. References ......................................................................................................................... 4

2. COMPLEMENTARY FILTER PAIR ............................................................................................ 8

2.1. Introduction ....................................................................................................................... 8

2.2. Theory of Complementary Filter Pair ............................................................................ 8

2.2.1. Error Accumulation in the Split-step Backward Propagation ..................................... 8

2.2.2. Analytical Illustration with Rectangular Windows .................................................... 10

2.2.3. Design a Complementary Filter Pair with Tukey Windows ....................................... 14

2.3. Impairment Compensation of a 16QAM/WDM System .................................................. 17

2.3.1. 16QAM/WDM Transmission System ......................................................................... 17

2.3.2. Digital Backward Propagation .................................................................................... 19

2.3.3. Simulation Results ....................................................................................................... 20

2.3.4. Computational Load ..................................................................................................... 22

2.4. References ......................................................................................................................... 23

3. NONLINEARITY COMPENSATION OF RAMAN LINK .......................................................... 26

3.1 Introduction ......................................................................................................................... 26
3.2. Theory of DBP for Raman Link ................................................................. 27
  3.2.1. Raman Amplification ........................................................................... 27
  3.2.2. DBP for Distributed Amplified Raman Link ....................................... 28
3.3. Simulation Results .................................................................................... 31
  3.3.1. System Configuration .......................................................................... 31
  3.3.2. Simulation Results ............................................................................... 34
3.4. References.................................................................................................. 36

4. XPM COMPENSATION USING COUPLED NLSE .................................... 39
  4.1. Introduction .............................................................................................. 39
  4.2. DBP with Coupled NLSE ......................................................................... 39
    4.2.1. Coupled NLSE .................................................................................. 39
    4.2.2. SSFM Step Size .............................................................................. 40
  4.3. Numerical Demonstration ....................................................................... 44
  4.4. Experimental Demonstration ................................................................... 46
    4.4.1. Experiment Setup ............................................................................. 46
    4.4.2. Results ............................................................................................... 48
  4.5. Reference .................................................................................................. 51

5. FOLDED-DBP FOR DISPERSION MANAGED FIBER LINK ..................... 53
  5.1. Introduction .............................................................................................. 53
  5.2. Distance-Folded DBP ............................................................................. 55
    5.2.1. Theory of Distance-Folded DBP ....................................................... 55
    5.2.2. Simulation Results .......................................................................... 59
LIST OF FIGURES

Figure 2-1: Convolution of $H_\delta(\omega)$ with $W(\omega)$ ................................................................. 11

Figure 2-2: Graph of $W(\omega)$ (solid line) and the Domains of Integration (between the dashed lines). ........................................................................................................... 12

Figure 2-3: Frequency Response of the Original and Shifted Filters ......................... 14

Figure 2-4: The Errors In Phase (a) and Magnitude (b) of an 83-tap FIR Filter Optimized with Tukey Windows. .............................................................. 15

Figure 2-5: Phase Error (a) and Magnitude Error (b) of the Filter Responses ......... 16

Figure 2-6: Block Diagram of the 16QAM/WDM Transmission System ................. 18

Figure 2-7: Block Diagram of the Split-Step Backward Propagation ....................... 19

Figure 2-8: The Constellations of the 7th Channel for 1200 km Transmission (a) with Impairment Compensation (Q=12.3 dB, $P_T$=10 dBm), (b) with Dispersion Compensation (Q=8.3 dB, $P_T$=6 dBm). ................................................................. 20

Figure 2-9: Q-value of the 7th Channel vs. Total Launching Power ...................... 21

Figure 2-10: Mean Q-value of the 12 channels vs. Filter Length (a) and Maximum Q-value of Each Channel (b). ................................................................................. 22

Figure 3-1: Block Diagram of the Unrepeatered Transmission System with Bi-directional Raman Amplification ................................................................. 32

Figure 3-2: Signal Power Profile of 310 km Unrepeatered Transmission (Forward Pump Power = 0.8 W, Backward Pump Power = 1 W, Signal Launching Power = -9 dBm) ................................................................. 33

Figure 3-3: Block Diagram of the Digital Backward Propagation ....................... 33
Figure 3-4: Constellations After 310 km Transmission and DSP, (a) Lumped Dispersion Compensation for 186 km and Split-Step DBP for 124 km, (b) Lumped Dispersion Compensation for the Entire Link, (c) Split-Step DBP for the Entire Link. ........................................................................................................................................35

Figure 3-5: Q-value versus Signal Launching Power after DBP (circles) or Lumped Dispersion Compensation (triangles and asterisks). ...................................................... 36

Figure 4-1: Block Diagram of the 16QAM/WDM Transmission System.......................... 44

Figure 4-2: Q-factor and Step Size for XPM and FWM Compensation for $\Delta f = 50$ GHz (black and green) and $\Delta f = 100$ GHz (blue and red). Dashed Lines Indicate the Characteristic Step Size for Each Case. .......................................................... 46

Figure 4-3: Experimental Setup of QPSK Transmission and Coherent Detection.... 48

Figure 4-4: Q-value versus Step Size of Digital Backward Propagation................... 49

Figure 4-5: Constellations after (a) Back-to-back Detection, (b) Dispersion Compensation, (c) SPM Compensation, (d) SPM+XPM Compensation, and (e) Nonlinearity Compensation via Total-field NLSE...................................................... 50

Figure 4-6: Q-value versus Transmission Distance.................................................. 51

Figure 5-1: Distance-Folded DBP for a Periodically Dispersion Managed Fiber Link with $M \times K$ Spans.................................................................................................. 59

Figure 5-2: Block Diagram of the Dispersion Managed WDM System..................... 60

Figure 5-3: (a) Q-value vs. Launching Power per Channel and (b) Q-value at Optimum Power vs. Folding Factor $K$ with RDPS = 0 ps/nm........................................ 61

Figure 5-4: (a) Q-value vs. Folding Factor $K$ and (b) Q-value vs. RDPS............ 62
Figure 5-5: Dispersion Managed Coherent Fiber Link Using Conventional DBP or D-folded DBP. .......................................................................................................................... 65
Figure 5-6: Q-value vs. RDPS After Dispersion-Folded DBP ......................... 68
Figure 5-7: (a) Q-value vs. Step Number per Span after Conventional DBP, (b) Q-value vs. Step Number After Dispersion-Folded DBP ....................................................... 69
Figure 5-8: (a) Q-value vs. Step Number per Span after Conventional DBP with Walk-off Integral, (b) Q-value vs. Step Number after Dispersion-Folded DBP with Walk-off Integral ........................................................................................................ 70
Figure 5-9: Experimental Demonstration of D-folded DBP. (a) Experimental Setup. Inset: Constellations after Back-to-back Detection, EDC and DBP at the Corresponding Optimum Power Levels. (b) Q-value as a Function of the Number of Steps Using Conventional DBP (green line) and D-folded DBP (blue line). (c) Q-value as a Function of Optical Launching Power after EDC (red line), 30-step D-folded DBP (blue line) and 1,300-step Conventional DBP (green line). ........................................... 71
## LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>Analog-to-digital conversion</td>
</tr>
<tr>
<td>AOM</td>
<td>Acousto-optic modulators</td>
</tr>
<tr>
<td>ASE</td>
<td>Amplified spontaneous emission</td>
</tr>
<tr>
<td>BPF</td>
<td>Band-pass filter</td>
</tr>
<tr>
<td>CD</td>
<td>Chromatic dispersion</td>
</tr>
<tr>
<td>DCF</td>
<td>Dispersion compensating fiber</td>
</tr>
<tr>
<td>D-folded</td>
<td>Dispersion-folded</td>
</tr>
<tr>
<td>DRS</td>
<td>Double-Rayleigh scattering</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital signal processor</td>
</tr>
<tr>
<td>EDC</td>
<td>Electronic dispersion compensation</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium-doped fiber amplifier</td>
</tr>
<tr>
<td>FEC</td>
<td>Forward error correction</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite impulse response</td>
</tr>
<tr>
<td>FWM</td>
<td>Four-wave mixing</td>
</tr>
<tr>
<td>IDF</td>
<td>Inverse dispersion fibers</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse discrete Fourier transform</td>
</tr>
<tr>
<td>ISE</td>
<td>Integral square error</td>
</tr>
<tr>
<td>LO</td>
<td>Local oscillator</td>
</tr>
<tr>
<td>MAC</td>
<td>Multiply-accumulate</td>
</tr>
<tr>
<td>MPI</td>
<td>Multipath interference</td>
</tr>
<tr>
<td>MPS</td>
<td>Multiplications per sample</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>NLC</td>
<td>Nonlinearity compensation</td>
</tr>
<tr>
<td>NLSE</td>
<td>Nonlinear Schrodinger equation</td>
</tr>
<tr>
<td>NZ-DSF</td>
<td>Non-zero dispersion-shifted fiber</td>
</tr>
<tr>
<td>NRZ</td>
<td>Non-return-to-zero</td>
</tr>
<tr>
<td>OSNR</td>
<td>Optical signal-to-noise ratio</td>
</tr>
<tr>
<td>PC</td>
<td>Polarization controllers</td>
</tr>
<tr>
<td>PD</td>
<td>Photo-detector</td>
</tr>
<tr>
<td>PDM</td>
<td>Polarization-division-multiplexing</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudo random bit sequence</td>
</tr>
<tr>
<td>PMD</td>
<td>Polarization mode dispersion</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature amplitude modulation</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature phase-shift keying</td>
</tr>
<tr>
<td>RDPS</td>
<td>Residual dispersion per span</td>
</tr>
<tr>
<td>SLA</td>
<td>Super-large-effective-area</td>
</tr>
<tr>
<td>SOA</td>
<td>Semiconductor optical amplifier</td>
</tr>
<tr>
<td>SPM</td>
<td>Self-phase modulation</td>
</tr>
<tr>
<td>SSMF</td>
<td>Standard single mode fiber</td>
</tr>
<tr>
<td>VOA</td>
<td>Variable optical attenuators</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength-division-multiplexing</td>
</tr>
<tr>
<td>XPM</td>
<td>Cross-phase modulation</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

1.1. Impairments in Fiber-Optic Transmission

In fiber communication systems, there are several kinds of transmission impairments: the ASE (Amplified Spontaneous Emission) noise from EDFA (erbium-doped fiber amplifier), SOA (semiconductor optical amplifier) or Raman amplifier; Chromatic Dispersion (CD) and Polarization Mode Dispersion (PMD); self-phase modulation (SPM), cross-phase modulation (XPM) and four-wave mixing (FWM) due to Kerr nonlinearity; Rayleigh scattering; Brillouin scattering; etc [1].

Prior to the advent of dispersion compensating fiber (DCF), chromatic dispersion was considered to be one of the key limitations for optical communication systems [2]. Currently, in most of the installed long-haul systems, dispersion is compensated by periodically cascading two or more kinds of fiber with inverse dispersion parameters [3]. With the inverse dispersion fibers (IDF), wide-band dispersion flatness has been obtained by compensating for both dispersion and dispersion slope while minimizing the total polarization mode dispersion (PMD). Some other approaches of dispersion compensation include pre-chirping of the signal wave [4], in-line chirped optical filters [5] and two-mode fiber near the cut-off [6].

Coherent fiber communication has regained significant attention thanks to the advances in integrated circuits and Digital Signal Processing technologies [7]. With coherent detection, the dispersion can be compensated in the digital domain with
Fast Fourier Transform (FFT) or digital filter [8-10]. Recently, coherent receivers based on digital signal processor (DSP) for chromatic dispersion compensation and PMD compensation have been reported [11], in which Finite Impulse Response (FIR) filters are used for real-time dispersion compensation.

With the dispersion compensated, the performance of a fiber link is mainly limited by optical signal-to-noise ratio (OSNR) and fiber nonlinearity [12-16]. The OSNR at the receiver can be enhanced by increasing the optical power; however, high optical power causes strong nonlinear effect in the fiber. Hence the nonlinear impairment determines the optimum power and the maximum capacity x distance factor of a fiber link. One approach of nonlinearity compensation is the use of mid-span phase conjugation [17]. The effectiveness of this method is limited by the asymmetric light intensity in the fiber. Another approach is the lumped electronic post-compensation of the nonlinear phase fluctuation due to SPM [18], in which the nonlinear phase shift is calculated with the optical power and the effective fiber length. In the scheme of lumped electronic post-compensation, the change of the waveform and the inter-channel nonlinear interaction are not taken into account. Hence this method does not work when there is significant interaction between nonlinearity and dispersion.

1.2. Digital Backward Propagation

Recently, our group proposed a universal post-processing scheme [19] where, in the context of Wavelength-Division-Multiplexing (WDM) transmission, dispersive and
nonlinear intra- and inter-channel impairments are fully compensated using electronic
digital backward propagation (DBP).

The optical propagation in fiber is governed by the nonlinear Schrödinger
equation (NLSE) which has no analytical solution. Thus a split-step method is
necessary to describe the joint effect of dispersion and nonlinearity. We can divide
the propagation distance into small steps. When the step size is small enough, the
dispersion and nonlinearity in each step can be de-coupled. That is, we can calculate
the dispersion and nonlinearity with a dispersion operator and nonlinearity operator
separately.

Using coherent detection, the complete information of the optical field can be
obtained at the receiver. In the digital domain, we can perform a similar split-step
calculation using inverse dispersion and nonlinearity parameters. This is equivalent to
optical propagation in the backward direction. The nonlinear effects can be effectively
compensated using this distributed nonlinearity compensation method.

1.3. Dissertation Outline

The computational requirement is a crucial consideration for DSP implementation of
the backward propagation. The total computational load is determined by the step
number and the computation per step. In real time DSP implementation, the FIR
filters can be used for dispersion compensation and account for most of the
computation per step. In Chapter 2, a method of designing a complementary filter
pair is presented. The individual errors in the frequency response of the two filters in a
complementary filter pair cancel each other. As a result, larger individual filter error can be tolerated and the required filter length is significantly reduced.

In Chapter 3, the nonlinearity compensation for a fiber link using distributed Raman pre-amplification is investigated. In Chapter 4, the XPM compensation of WDM long-haul transmission by solving the coupled NLSE is experimentally demonstrated.

In order for the split-step DBP method to be accurate, a large number of steps are needed especially for inter-channel nonlinearity compensation of WDM systems, resulting in a very heavy computational load. In Chapter 5, we propose a computational efficient folded-DBP method for dispersion-managed fiber-optic transmission systems. The waveform evolution in long distance fiber link is dominated by dispersion. Taking advantage of the periodic waveform evolution in periodically dispersion managed fiber link, we can fold the DBP of $K$ fiber spans into one span with $K$ times the nonlinearity. For a fiber link with arbitrary dispersion map, the DBP can be folded according to the accumulated dispersion. The required step number can be reduced by up to two orders of magnitude using the folded-DBP method.

1.4. References


2. COMPLEMENTARY FILTER PAIR

2.1. Introduction

Digital backward propagation by means of the split-step method is based on the division of the total transmission distance into short segments. A large number of segments are required for high bandwidth systems, where the effect of the walk-off between channels has to be properly described. In a real-time implementation, dispersion is compensated on every segment by using FIR filters [1]. For a large number of segments, the frequency response of each FIR filter has to be very accurate in order to minimize the error accumulation. Such accuracy is translated into a large filter length and hence, into a large computational load.

In this chapter, a method of designing a complementary filter pair is proposed. The individual errors in the frequency response of the two filters cancel each other. Consequently, larger individual filter error can be tolerated and the required filter length is significantly reduced. A 12x100 Gbit/s 16-QAM (16-ary Quadrature amplitude modulation) transmission system with non-zero dispersion-shifted fiber (NZ-DSF) is simulated using such a filter pair. The required filter length is halved by the complementary filter pair design for a transmission distance of 1200 km.

2.2. Theory of Complementary Filter Pair

2.2.1. Error Accumulation in the Split-step Backward Propagation
The nonlinear Schrödinger equation for the backward propagation can be written as
\[ \partial A / \partial z = (\hat{D}^{-1} + \hat{N}^{-1})A, \]
where \( \hat{D}^{-1} \) and \( \hat{N}^{-1} \) are the DC (dispersion compensation) and NLC (nonlinearity compensation) operators given by
\[ \hat{D}^{-1} = \frac{\alpha}{2} - i \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} - i \frac{\beta_3}{6} \frac{\partial^3}{\partial t^3}, \quad \hat{N}^{-1} = -i \gamma |A|^2. \] (2-1)

\( \alpha, \beta_2, \beta_3, \gamma, t \) and \( A \) are the absorption coefficient, 2nd and 3rd order dispersion, nonlinear parameter, retarded time and field amplitude, respectively [2]. In the real time DSP implementation, FIR filter and exponential operator can be used as the DC and NLC operators, respectively, in each step.

At a sampling frequency of \( \omega_s \), the desired frequency response of the FIR filter is a periodic function which. In one period, it is given by
\[ H_{DC}(\omega) = \exp\left[\frac{\alpha}{2} + i \frac{\beta_2 \omega^2}{2} + i \frac{\beta_3 \omega^3}{6}\right] h \left( -\frac{\omega}{\omega_s} < \omega < \frac{\omega}{\omega_s} \right), \] (2-2)

where \( h \) is the step size which should be small enough to describe the changing waveform in the fiber [3]. As a result, \( H_{DC}(\omega) \) has a limited maximum phase shift and is relatively slow varying with respect to \( \omega \).

The frequency response of an FIR filter has a finite bandwidth which is limited by the sampling rate. Ideally, the coefficients of an FIR filter are the IDFT (Inverse Discrete Fourier Transform) of the desired frequency response and represent the impulse response of the linear system in time domain. However, the impulse response has an infinite length when the bandwidth of the frequency response is finite.
In reality, an FIR filter with finite length is obtained by truncating or windowing the impulse response. Hence there is always an error between the desired frequency response and the actual frequency response of the filter [4]. This error accumulates in the split-step backward propagation where many identical FIR filters are used in series.

In the split-step backward propagation, the filter error does not accumulate in a linear manner because there are nonlinear operators between the FIR filters. The waveform distortion due to the accumulated error introduces additional errors to the NLC operations. However, the error in NLC operation is negligible when the waveform distortion is small.

2.2.2. Analytical Illustration with Rectangular Windows

To illustrate the method of complementary filter pair analytically, let us analyze a zero-phase-shift all-pass filter when rectangular windows are applied. An FIR filter with finite length can be obtained by truncating the impulse response, which is equivalent to applying a time-domain rectangular window. Such rectangular window has a frequency response given by

$$W(\omega) = \sin\left(\frac{2m+1}{2} \omega T\right) / \sin\left(\frac{\omega T}{2}\right)$$

(2-3)

where $2m+1$ is the filter length (tap number) and $T$ the sampling interval. The side lobes at $|\omega| > 2\pi / T$ give rise to the rippling in the filter response. These side lobes are
characterized by oscillations with zero crossings every \( \Omega = 2\pi / [(2m + 1)T] \) and decreasing amplitudes [4].

With a rectangular window applied in the frequency domain, the desired frequency response of the all-pass filter becomes

\[
H_D(\omega) = \begin{cases} 
1 & (-\omega_c < \omega < \omega_c) \\
0 & \text{elsewhere} 
\end{cases}
\]  

(2-4)

where \( \omega_c \) is the cut-off frequency. Note that there is a margin between the signal bandwidth \( B \) and the sampling frequency, so \( B / 2 < \omega_c < \omega_s / 2 \).

The frequency response of the resulting filter is the convolution of \( H_D(\omega) \) with \( W(\omega) \), which can be expressed as

\[
H_f(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} W(\omega) d\omega
\]  

(2-5)

![Figure 2-1: Convolution of \( H_D(\omega) \) with \( W(\omega) \)](image-url)
The convolution of \( H_D(\omega) \) with \( W(\omega) \) is illustrated in Figure 2-1, where it is shown that the ripples in the filter response is due the side lobes of the window response. For \(-\omega_c < \omega < 0\), the filter response can be expressed as

\[
H_f(\omega) = \frac{1}{2\pi} \int_{-\omega_c/2}^{\omega_c/2} W(\omega) d\omega + \frac{1}{2\pi} \int_{\omega_c}^{\omega} W(\omega) d\omega
\]

(2-6)

Figure 2-2: Graph of \( W(\omega) \) (solid line) and the Domains of Integration (between the dashed lines).

Figure 2-2 is the graph of \( W(\omega) \) and the domains of integration. It is shown that the integral from \( -\omega_c \) to \( -\omega_c/2 \) is much smaller than the integral from \(-\omega_c/2\) to \( \omega + \omega_c \). Hence the second term on the right hand side of Equation 2-5 can be neglected and the filter response can be expressed as

\[
H_f(\omega) = \frac{1}{2\pi} \int_{-\omega_c/2}^{\omega_c/2} W(\omega) d\omega
\]

(2-7)

According to Equation 2-7, the zero crossings of \( W(\omega) \) correspond to the maximum or minimum points of \( H_f(\omega) \). The \( n \)th maximum point of \( H_f(\omega) \) counting
from $\omega = -\omega_c$ is located at $\omega = -\omega_c + (2n-1)\Omega$, whereas the $n$th minimum point is at $\omega = -\omega_c + 2n\Omega$.

Likewise, for $0 < \omega < \omega_c$, the filter response can be expressed as

$$H_f(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega} W(\omega) d\omega + \frac{1}{2\pi} \int_{\omega_c}^{\omega+\omega_c} W(\omega) d\omega$$

(2-8)

where, again, the second term can be neglected. The $n$th maximum point of $H_f(\omega)$ counting from $\omega = \omega_c$ is at $\omega = \omega_c - (2n-1)\Omega$, whereas the $n$th minimum point is at $\omega = \omega_c - 2n\Omega$.

It has been shown that the maximum and minimum points of the filter response are approximately equally spaced. Note that within the signal bandwidth, the ripples near $\omega = \pm\omega_c$ have largest amplitude because the side lobes of $W(\omega)$ have the largest amplitude near $\omega = 0$, and to calculate the value of $H_f(\omega)$ at $\omega = \pm\omega_c$, the boundary of the integral in Figure 2-2 is at $\omega = 0$. The error is positive at the maximum points and negative at the minimum points. By increasing or decreasing the cut-off frequency $\omega_c$ by $\Omega$, the maxima of $H_f(\omega)$ can be shifted to the location of the original minimum points. As a result, within the signal bandwidth, the shifted and original filters have approximately opposite errors which cancel each other. The frequency responses of the two filters are illustrated in Figure 2-3, where the signal bandwidth is the region between the green dashed lines.
2.2.3. Design a Complementary Filter Pair with Tukey Windows

An FIR filter can be optimized by applying appropriate windows, such as Tukey windows in both frequency and time domains [5,6]. The equation for computing the coefficients of a Tukey window is

\[
    w(k) = \begin{cases} 
        \frac{1}{2} \left[ 1 + \cos \left( \frac{2\pi}{r} \frac{(k-1)}{(N-1)} - \pi \right) \right], & k < \frac{r}{2} (N-1) + 1 \\
        1, & \frac{r}{2} (N-1) + 1 \leq k \leq N - \frac{r}{2} (N-1) \\
        \frac{1}{2} \left[ 1 + \cos \left( \frac{2\pi}{r} - \frac{2\pi}{r} \frac{(k-1)}{(N-1)} - \pi \right) \right], & k > N - \frac{r}{2} (N-1)
    \end{cases} 
\]  

(2-9)

where \( r \) is the roll-off factor of the Tukey window.

To optimize an FIR filter with Tukey windows, one can apply a Tukey window with a roll-off factor of \( r_f \) to the desired frequency response of the filter. Note that the sampling frequency should be larger than the optical signal bandwidth. Then the impulse response of the windowed frequency response can be calculated with IDFT. For a given filter length, another Tukey window with a roll-off factor of \( r_t \) can be
applied in the time domain. Then the value of $r_f$ and $r_t$ can be optimized to minimize the ISE (integral square error) of the filter response.

Optimization can only reduce the integral squared error (ISE) of an individual filter to a value correlated to the filter length. The errors in phase and magnitude of an 83-tap optimized filter are shown in the Figure 2-4.

There is no analytical solution for the frequency responses when Tukey window is applied. The theory of complementary filter pair design has been illustrated analytically when rectangular windows are applied, where the width of the rectangular window in the frequency domain is adjusted to shift the ripples. Similarly, with Tukey window...
windows applied, the ripples can be shifted by adjusting the width of the Tukey window in frequency domain, i.e. by adjusting the roll-off parameter $r_f$.

Additionally, modifying the time-domain Tukey window (adjusting $r_t$) can change the frequency response of this window, and consequently change the shape and position of the ripples in the filter response. A global search should be performed for $r_f$ and $r_t$ of both filters to minimize the ISE of the filter pair.

A complementary filter pair is designed for the backward propagation of a 16QAM/WDM system. The filter errors in phase and amplitude are shown in Figure 2-5. The black dotted lines are the accumulated errors of two identical filters optimized with Tukey windows. The green and blue dashed lines are the errors of the two filters of a complementary filter pair. Note that the complementary filter pair is designed using the identical filter by adjusting its roll-off parameters. The black solid lines are the total error of the complementary filter pair.

Figure 2-5: Phase Error (a) and Magnitude Error (b) of the Filter Responses.
The length (tap number) of each filter is 83. The roll-off parameters of the time-domain Tukey windows for filter 1, filter 2 and the identical filter are 0.360, 0.422 and 0.42, respectively. The roll-off parameters of the frequency-domain Tukey windows are 0.122, 0.146 and 0.14, respectively. It is shown that the two complementary filters have approximately the opposite error in both phase and amplitude within the signal bandwidth, which is -300 GHz to 300 GHz. Note that the errors are negligible from -150 GHz to 150 GHz. The ISE of the filter pair is $1.06 \times 10^{-10}$. The total ISE of two identical filters is $3.94 \times 10^{-9}$. This filter pair is used in the simulation of the 16QAM/WDM system.

2.3. Impairment Compensation of a 16QAM/WDM System

2.3.1. 16QAM/WDM Transmission System

The block diagram of the 16QAM/WDM transmission system is shown in Figure 2-6. Forward transmission of 12 WDM channels, with a channel spacing of 50 GHz, is simulated with the VPI Transmission Maker. In the 16-QAM transmitter of each channel, the laser output is split into in-phase (I) and quadrature (Q) components and modulated with Mach-Zehnder modulators at 100 Gbit/s in a 16-QAM format, corresponding to a symbol rate of 25 GBaud/s. Then the optical signals are combined with an optical multiplexer and launched into the fiber. The loss, dispersion, dispersion slope and nonlinearity of the non-zero dispersion-shifted fiber (NZ-DSF) are 0.2 dB/km, 4.4 ps/km/nm, 0.045 ps/km/nm$^2$ and 1.46 /W/km, respectively. The
span length between every two EDFAs is 100 km. The EDFAs operate in the power mode with a noise figure of 5 dB.

![Block Diagram of the 16QAM/WDM Transmission System.](image)

At the receiver, the optical signals are mixed with phase-locked local oscillators in a 90° hybrid. The I and Q components of each channel are detected with balanced detectors and sampled at a sampling rate of 25 GSa/s. Then the signal is up-sampled by DSP to a sampling rate of 800 GSa/s which is higher than the total optical bandwidth. Then the total optical field is reconstructed in the digital domain for backward propagation. The laser linewidth of the transmitter and the local oscillator is 500 kHz. After backward propagation and demultiplexing, phase estimation is performed by phase tracking and averaging the estimated phase of every 8 symbols [7,8].
2.3.2. Digital Backward Propagation

The block diagram of the backward propagation is shown in Figure 2-7, where an FIR filter and an exponential operator are used for DC and NLC in each step, respectively. The two filters of each complementary filter pair are used in adjacent steps. The attenuator is used to compensate for the EDFA gain in each span.

![Block Diagram of the Split-Step Backward Propagation](image)

Figure 2-7: Block Diagram of the Split-Step Backward Propagation

In [8] a symmetric split-step structure was used, where 3 DC operators and 2 NLC operators are needed to iteratively approximate the field amplitude within each step. When the step size is limited by the nonlinearity, this symmetric structure allows a larger step size so that the total number of operations could be reduced. However, in this WDM system using NZ-DSF, the step size is strongly limited by the walk-off due to dispersion. Hence we use a structure with one DC operator and one NLC.
operator per step which requires fewer operations. Additionally, fewer filters are used in this structure, leading to less error accumulation.

In order to determine the optimum step size, backward propagation was performed using split-step Fourier method, from which a maximum and stationary Q-value was obtained for $h \leq 200m$. As a tradeoff between the computational load and the Q-value penalty, the step size was chosen to be 500 m corresponding to a Q-value penalty of 0.5 dB. For $h = 500m$, an optimum filter length of 83 taps was obtained according to the design method described above. This filter length ensures a Q-value penalty due to filter error of less than 0.1 dB.

2.3.3. Simulation Results

Figure 2-8: The Constellations of the $7^{th}$ Channel for 1200 km Transmission (a) with Impairment Compensation ($Q=12.3$ dB, $P_T=10$ dBm), (b) with Dispersion Compensation ($Q=8.3$ dB, $P_T=6$ dBm).

The constellations for 1200 km transmission with and without NLC are shown in Figure 2-8 (a) and (b), respectively, where $P_T$ is the optimum total launching power.
The benefit of nonlinearity compensation is evident from comparison between the two constellations.

Figure 2-9: Q-value of the 7th Channel vs. Total Launching Power.

The system performance as a function of launching power is shown in Figure 2-9. With NLC, the Q-value is increased by 4 dB in comparison with the Q-value without NLC for a 1200 km link. The transmission distance can be increased from 500 km to 1200 km by NLC preserving the same Q-value.
Figure 2-10: Mean Q-value of the 12 channels vs. Filter Length (a) and Maximum Q-value of Each Channel (b).

The mean Q-value of the 12 channels as a function of the filter length is shown in Figure 2-10 (a), where the Q-value increases to a maximum and stationary value when the effect of the filter error becomes negligible. It is shown that in order to achieve the same accuracy, the required filter length is much shorter with the design of complementary filter pair. Figure 10 (b) shows the Q-values of all the channels after 1200 km transmission and impairment compensation. It is shown that the performance of the complementary filter pairs with a filter length of 83 is better than the performance of the identical filters with a filter length of 177. The complementary filter pair design reduces the required filter length and, consequently, the total computational load by a factor of 2.

2.3.4. Computational Load

The computational requirement is a crucial consideration for DSP implementation. The computation can be parallelized so that the DSP can operate at a speed lower
than the sampling rate [9]. Ignoring the computation and latency caused by up-sampling, demultiplexing and phase estimation, the number of multiply-accumulate (MAC) units is

\[ N_{MAC} = N_b \times N_s \times (4N_t + 16) \]  \hfill (2-10)

where \( N_b \) is the number of the parallelization branches, \( N_s \) is the number of steps and \( N_t \) is the filter length (tap number). Each FIR filter requires \( 4N_t \) multiplications and \( 4N_t - 2 \) summations. Each NLC operation includes 16 multiplications and 7 summations. The latency of the backward propagation is

\[ T_L = N_s (\lceil \log_2 N_t \rceil + 9) \times T / 2 \]  \hfill (2-11)

where \( \lceil x \rceil \) stands for the ceiling function of \( x \). It is assumed that each multiplication or summation requires half a clock cycle \( (T / 2) \). The computation time for DC and NLC in one step are \( (\lceil \log_2 N_t \rceil + 2) \times T / 2 \) and \( 7 \times T / 2 \), respectively [9].

By assuming a DSP speed of 25 GHz, the impairment compensation of a 1200 km link requires 22.3 M of MAC units. The computational efficiency is 464 kMAC/bit, which is defined as the total multiply-accumulate operation rate divided by the total bit rate. The latency is 10.2 \( \mu \)s which is small in comparison with the transmission latency of 6 ms in the fiber [10].

2.4. References


Photonics Technologies for Defense, Security, and Aerospace Applications

Conference, Orlando, April, 2010.
3. NONLINEARITY COMPENSATION OF RAMAN LINK

3.1 Introduction

The advent of high-power optical pump sources [1] makes the distributed Raman gain in the transmission fiber itself a realistic possibility for amplifying optical signals. Raman gain is attractive because it is obtainable in any conventional transmission fiber; there is no excess loss if the pump power is absent; and the gain spectrum is very broad band, allowing WDM transmission in the system. Distributed Raman amplification can also improve the noise performance in both unrepeatered and long-haul fiber-optic transmission systems over discrete amplification. In 1985 the experiments of remotely pumped amplification employing Raman gain was reported [2]. In 1995 the first transmission system employing remote amplification was commissioned [3].

In this section, we investigate DBP for distributed amplified transmission. In contrast to the discrete amplifiers, distributed Raman amplifiers utilize the transmission fiber as the gain medium. In order to compensate for the Kerr nonlinearity, the Raman gain as a function of the fiber length and the effective length of each fiber section in DBP should be calculated by solving the differential equations of Raman amplification prior to DBP.

As an example, we simulate the transmission performance of 100 Gb/s 16-QAM signal in an unrepeatered system using distributed Raman amplification. Unrepeatered transmission system with Raman amplification is an effective method
to simplify the system configuration and reduce the cost [4]. The transmission
distance of unrepeatered systems can be increased by compensating for the extra
span loss with higher launching power. However, when the launching power exceeds
a threshold, fiber nonlinearity can dramatically degrade transmission performance.
Hence the fiber nonlinear impairments have to be compensated in order to further
increase the transmission distance.

3.2. Theory of DBP for Raman Link

3.2.1. Raman Amplification

Raman amplification occurs due to the transition between vibrational modes in the
transmission fiber medium. The molecules are excited to a higher energy vibrational
state and at the same time a fraction of the incident pump light generates light that is
downshifted in frequency known as Stokes light. The coupled differential equations
modeling the Raman gain process are [4]

\[
\frac{dP_s}{dz} = C_R P_p P_s - \alpha_s P_s, \quad (2-1)
\]

\[
\frac{dP_p}{dz} = -\frac{\omega_p}{\omega_s} C_s P_s P_p - \alpha_p P_p, \quad (2-2)
\]

where subscripts S and P denote quantities of the signal and pump light. P is the
optical power, \( C_R \) is the Raman gain efficiency, \( \alpha \) is the fiber loss, and \( \omega \) is the optical
frequency.
At high Raman gain levels the Rayleigh backscattering becomes important. The backscattered optical signal may then undergo further Rayleigh scattering and be re-coupled into the forward direction. In conventional transmission fiber, the fraction of light undergoing double-Rayleigh scattering (DRS) is very small. However, with distributed amplification, the resultant multipath interference (MPI) can produce significant additional noise.

Spontaneous emission noise can also be backscattered and reach the receiver after being further amplified. The optical power propagation in the signal bandwidth can be expressed with the following differential equations

\[
\frac{dP^+}{dz} = C_R P_p(z) \cdot (P_S^+(z) + E_{\text{ph}} B_o) + r \cdot P_S^-(z) - \alpha_s \cdot P_S^+(z), \quad (2-3)
\]

\[
\frac{dP^-}{dz} = -C_R P_p(z) \cdot (P_S^-(z) + E_{\text{ph}} B_o) - r \cdot P_S^+(z) + \alpha_s \cdot P_S^-(z), \quad (2-4)
\]

where \( P_S^+ \) and \( P_S^- \) are the forward and backward traveling power levels, respectively. \( E_{\text{ph}} \) is the signal photon energy, \( B_o \) is the optical bandwidth, \( r \) accounts for the Rayleigh backscattering. The terms \( \pm C_R P_p(z) \cdot E_{\text{ph}} B_o \) describe spontaneous Raman emission into one axial mode. The terms \( r \cdot P_S^+(z) \) and \( r \cdot P_S^-(z) \) account for Rayleigh backscattering. In a Raman amplifier, the Raman gain shows no temperature dependence, whereas the rate at which the noise photons are emitted is dependent on the temperature [5].

3.2.2. DBP for Distributed Amplified Raman Link
Digital backward propagation is usually carried out using the split-step method, in which Kerr nonlinearity is compensated in each step according to the instantaneous signal power. In a discretely amplified system, the signal is attenuated uniformly in the fiber. However, with distributed Raman amplification, the signal experiences both constant fiber loss and spatially varying Raman gain along the fiber. The Raman gain in each fiber section therefore must be calculated so that the power evolution of the received signal can be traced in DBP.

The evolution of the pump and signal powers along the longitudinal axis of the fiber $z$ in a Raman amplified system is governed by the following differential equations.

$$\frac{\partial P_s}{\partial z} = g_R P_{pf} P_s + g_R P_{pb} P_s - \alpha_s P_s,$$

$$\frac{\partial P_{pf}}{\partial z} = -\frac{\omega_p}{\omega_s} g_R P_{pf} P_s - \alpha_p P_p,$$

$$\frac{\partial P_{pb}}{\partial z} = \frac{\omega_p}{\omega_s} g_R P_{pb} P_s + \alpha_p P_p \quad (2-5)$$

where $P_s$, $P_{pf}$, and $P_{pb}$ are the powers of the signal, the forward pump and the backward pump, $\alpha_{p/s}$ and $\omega_{p/s}$ are the attenuation coefficients and frequencies of the pump and the signal, respectively, and $g_R$ is the Raman gain coefficient. The time-averaged signal power as a function of $z$ can be obtained by solving the differential equations given the input power levels of the signal and pumps as the boundary conditions.
The backward propagation is described by the nonlinear Schrödinger equation,
\[ \frac{\partial A}{\partial z} = \left( \frac{\alpha_s - g(z)}{2} - i \beta_2 \frac{\partial^2}{\partial t^2} - i \beta_3 \frac{\partial^3}{\partial t^3} - \frac{\gamma}{6} \frac{\partial^4}{\partial t^4} \right) A, \] (2-6)
where \( g(z) \) is the Raman gain as a function of \( z \). \( \beta_2, \beta_3, \gamma, t \) and \( A \) are 2\(^{nd}\) and 3\(^{rd}\) order dispersion, nonlinear parameter, retarded time and field amplitude, respectively. The effect of Raman amplification is described through \( g(z) \) which is obtained by solving the system of equations given by Equation 2-5.

The nonlinearity compensation is performed using the following operator.
\[ \overline{N} = \exp(-i \gamma |A|^2 L_{\text{eff}}), \] (2-7)
where \( L_{\text{eff}} \) is the effective length of the fiber section. In a discrete amplified system where the power decays exponentially in the fiber, \( L_{\text{eff}} \) is given by \( L_{\text{eff}} = \frac{1 - \exp(-\alpha_s L)}{\alpha_s} \), where \( L \) is the fiber length. However, with distributed Raman amplification, the effective length becomes
\[ L_{\text{eff}} = \int_{z_1}^{z_2} \exp[\int_{z_1}^{z} (g(z) - \alpha_s)dz]dz, \] (2-8)
where \( z_1 \) and \( z_2 \) are the beginning and end of the fiber section. The fiber loss and the Raman gain of each fiber section are compensated with constants \( G \) given by
\[ G = \int_{z_1}^{z_2} (g(z) - \alpha_s)dz. \] (2-9)
Note that real-time computation is not required to obtain $g(z)$, $L_{eff}$, or $G$ because they are time invariant parameters for a fiber link with fixed pump and signal power.

3.3. Simulation Results

3.3.1. System Configuration

To verify the effectiveness of DBP for distributed Raman systems, we performed simulations on an unrepeatered link [6]. Forward transmission of the 100 Gb/s, 16-QAM signal through standard single mode fiber (SSMF) is simulated with VPItransmissionMaker. The block diagram of the system is shown in Figure 3-1. The fiber loss of the Raman pumps and the signal is 0.2 dB/km. The dispersion, dispersion slope, nonlinear parameter and Rayleigh backscatter coefficient of the SSMF are 16 ps/km/nm, 0.045 ps/km/nm$^2$, 1.46 /W/km and -82 dB, respectively. At the receiver, the optical signals are mixed with the local oscillator (LO) in a 90° hybrid. The linewidth and relative intensity noise of the lasers are 500 kHz and -155 dB/Hz, respectively. The I- and Q- components of each channel are detected at a sampling rate of 50 GSa/s. Then the optical field is up-sampled to 100 GSa/s to reduce the aliasing in DBP due to any spectral broadening created by the Kerr nonlinearity. Clock recovery and phase estimation are performed after DBP.
Figure 3-1: Block Diagram of the Unrepeatered Transmission System with Bi-directional Raman Amplification.

Rayleigh backscattering degrades the transmission performance of distributed Raman systems at very high pump power [7]. Studies have shown that the best performance can be obtained when the span is pumped above transparency, and the optimal percentage of the on-off gain from forward pumping is between 25% and 50% depending on system details [8]. We varied the pump power levels of this system and the maximum transmission distance was achieved with a backward pump of 1 W and a forward pump of 0.8 W. At these optimum pump power levels, the maximum transmission distance with a Q value above the forward error correction (FEC) threshold [9] after DBP was 310 km.
Figure 3-2: Signal Power Profile of 310 km Unrepeatered Transmission (Forward Pump Power = 0.8 W, Backward Pump Power = 1 W, Signal Launching Power = -9 dBm).

The signal power profile of a 310 km link with the bi-directional Raman pumps is shown in Figure 3-2. As can be seen from the power profile, nonlinear impairments are incurred mostly at the beginning of the transmission line where the signal power is much higher. Therefore, split-step DBP for impairment compensation can be performed only for transmission at the transmitter end of the link without significant penalty in performance while saving computational load.

Figure 3-3: Block Diagram of the Digital Backward Propagation.
The scheme of backward propagation is shown in Figure 3-3 where $L_{DC}$ is the length of fiber using lumped dispersion compensation only and $L_{DBP}$ is the length of fiber for which split-step DBP is employed. In the split-step DBP, there are two nonlinearity compensation (NLC) operators and one dispersion compensation (DC) operator in each step. This symmetric structure reduces the approximation error in each step of the split-step scheme [10].

3.3.2. Simulation Results

Impairment compensation for 310 km unrepeated transmission is performed as illustrated in Figure 3. In a first stage, the first 186 km from the receiver are assumed behave linearly and only dispersion compensation using a 161-tap FIR filter is performed. In a second stage, combined action of dispersion and nonlinearity is compensated for the remaining 124 km of the link. In order to determine the optimum step size, backward propagation was performed using split-step Fourier method with different step sizes. As a tradeoff between the computational load and the Q-value penalty, the step size was chosen to be 12.4 km corresponding to a Q-value penalty of 0.3 dB with respect to the maximum Q value obtained with very small step sizes. 

Within each step, the dispersion operator is implemented, in this case, by an FIR filter of 29 taps. All the filters have been optimized by Tukey windowing in both time and frequency domains [11]. The filter lengths are reduced until the Q value incurs a 0.1 dB penalty due to filter error.
Figure 3-4: Constellations After 310 km Transmission and DSP, (a) Lumped Dispersion Compensation for 186 km and Split-Step DBP for 124 km, (b) Lumped Dispersion Compensation for the Entire Link, (c) Split-Step DBP for the Entire Link.

The constellations after 310 km transmission and digital signal processing are shown in Figure 3-4. Figure 3-4 (a) is obtained after impairment compensation via DBP as illustrated in Figure 13, where $L_{DC}=186$ km, $L_{DBP}=124$ km. Figure 3-4 (b) is obtained after lumped dispersion compensation only. The benefit of nonlinearity compensation is evident from comparison between the constellations (a) and (b). Figure 3-4 (c) is obtained after split-step DBP for the entire 310 km link where the step size remains 12.4 km. The Q values of constellations (a), (b) and (c) are 11.9 dB, 9.2 dB and 11.9 dB, respectively. Almost no penalty is incurred by dividing the fiber into linear and nonlinear regions.

The system performance as a function of signal launching power is shown in Figure 3-5. As a result of DBP, the Q-value is increased from 7.2 dB to 11.9 dB for transmission of 310 km. Preserving the same Q-value, employing DBP increases the optimum signal power by 8 dB and consequently increases the transmission distance from 270 km to 310 km.
Figure 3-5: Q-value versus Signal Launching Power after DBP (circles) or Lumped Dispersion Compensation (triangles and asterisks).

Nonlinearity compensation via DBP increases the maximum distance of 100 Gb/s 16-QAM unrepeatered transmission by 40 km preserving the same Q-value. The optimum signal power is increased by 8 dB, which means that a much higher Kerr nonlinearity is tolerated. The optimum signal power is not further increased because the received signal has been distorted by noise and, consequently, there is error in the nonlinearity compensation via DBP.

Distributed Raman amplification is an attractive alternative for both unrepeatered and long-haul fiber transmission. This method of impairment compensation also applies in the WDM long-haul transmission using Distributed Raman amplification.

3.4. References


4. XPM COMPENSATION USING COUPLED NLSE

4.1. Introduction

Digital backward propagation can be implemented by solving the total-field NLSE or the coupled NLSEs. By solving the coupled NLSEs, XPM can be compensated while neglecting the effects of FWM. In comparison with XPM+FWM compensation by solving the total-field NLSE, XPM compensation via coupled NLSEs requires a smaller computational load [1]. Moreover, XPM compensation does not need phase-locked local oscillators to reconstruct the total optical field. Distributed nonlinearity and dispersion compensation by solving the total-field NLSE has been experimentally demonstrated [2]. In this section, we present the method of XPM compensation via coupled NLSEs for WDM transmission. A broadband WDM system is simulated to demonstrate the computational efficiency of the XPM compensation. The simulation results show that the computational load can be significantly reduced by the XPM compensation method in comparison with the DBP using total-NLSE. The effectiveness of XPM compensation is demonstrated experimentally. In the experiment, random phase shift is added to the field of each channel verifying that the phase-locking of the local oscillators is not necessary for XPM compensation. The performance of WDM transmission is significantly improved by XPM compensation.

4.2. DBP with Coupled NLSE

4.2.1. Coupled NLSE
In a coherent wavelength-division multiplexed (WDM) system, the total received field can be expressed as $A = \sum \hat{A}_m \exp(i m \cdot 2\pi \Delta f t)$, where $\hat{A}_m$ is the field envelope detected by the coherent receiver for the $m^{th}$ channel, $\Delta f$ is the channel spacing. The total-field NLSE for backward propagation is given by

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \frac{i \beta_z}{2} \frac{\partial^2 A}{\partial t^2} - i \gamma |A|^2 A = 0,$$  \hspace{1cm} (4-1)

where $\alpha$ is the fiber loss, $\beta_z$ is the group velocity dispersion, and $\gamma$ is the nonlinearity parameter [3]. Equation 4-1 governs the total-field backward propagation where dispersion, SPM, XPM and FWM are compensated.

By expanding $A$ in Equation 4-1 and neglecting the terms for FWM, a set of coupled equations for backward propagation can be derived as [1]

$$\frac{\partial \hat{A}_m}{\partial z} + \frac{\alpha}{2} \hat{A}_m + m \cdot 2\pi \Delta f \beta_z \frac{\partial \hat{A}_m}{\partial t} + \frac{i \beta_z}{2} \frac{\partial^2 \hat{A}_m}{\partial t^2} - i \gamma (2 \sum_{q \neq m} |\hat{A}_q|^2 - |\hat{A}_m|^2) \hat{A}_m = 0.$$  \hspace{1cm} (4-2)

It is clear from Equation 4-2 that changes in the relative phase among the channels do not affect the results of backward propagation. As a result, neglecting FWM alleviates the requirement of phase-locking among the local oscillators.

4.2.2. SSFM Step Size

One way to set the upper bound for the step size is to identify the characteristic physical lengths of the transmission system, which correlate the optical field fluctuations with the propagation distance. We define three physical lengths here,
namely, the nonlinear length $L_{NL}$, the walk-off length $L_{wo}$ and the four-wave mixing length $L_{fwm}$. The nonlinear length is defined as

$$L_{nl} = \frac{1}{\gamma P_T \frac{2N-1}{N}}, \quad (4-3)$$

where $P_T = \sum_m |A_m|^2$ is the total launching power, $N$ is the channel number. The nonlinear length is the length after which an individual channel experiences a 1 radian phase shift due to SPM and XPM. The walk-off length is defined as

$$L_{wo} = \frac{1}{2\pi |\beta_2| (N-1)\Delta f B}, \quad (4-4)$$

where $B$ is the symbol rate. The walk-off length is the distance after which the relative delay of pulses from the edge channels is equal to the pulse period [5].

When FWM is considered, the nonlinear and walk-off lengths are not enough to identify the propagation distance in which the field fluctuation due to FWM takes place. The nonlinear term of Equation 4-2 can be expressed as follows for the $m^{th}$ channel

$$-i\gamma \left( 2\sum_{q=1} |A_q|^2 - |A_m|^2 \right) A_m - i\gamma \left[ \sum_{\{r,s,l\} \in I} A_r A_s A_l^* \exp(i\delta k_{relm} z) \right], \quad (4-5)$$

with the following conditions: $l = r + s - m$, $[m,r,s] \in I$ and $r \neq s \neq m$. The first condition neglects fast time-oscillating terms. The second condition forces the newly generated waves to lay within the WDM band. Finally, the third condition excludes SPM and XPM terms. $\delta k_{relm}$ is the phase mismatch parameter given by
\[ \delta k_{\text{lin}} = k_r + k_s - k_t - k_m = \frac{1}{2} \beta_2 \Delta \omega^2 \left[ r^2 + s^2 - \left( r + s - m \right)^2 - m^2 \right] \quad (4-6) \]

with \( r = 1 \) and \( s = N \), the expression for the maximum phase-mismatch is given by,

\[ \delta k_{\text{max}} = \frac{1}{4} \beta_2 \left| (N - 1)^2 \Delta \omega^2 \right| \quad (4-7) \]

The above expression leads to the following definition for the FWM length

\[ L_{fwm} = \frac{1}{\pi^2 \left| \beta_2 \right| (N - 1)^2 \Delta f^2} \quad (4-8) \]

The FWM length defined above represents the length after which the fastest FWM term is shifted by 1 radian. The definition of the FWM length assumes that the FWM-induced variations on a given channel are governed by the linear (dispersive) phase mismatch. Nonlinearity also contributes to the overall phase mismatch through SPM and XPM. However, this contribution is only relevant in high power regimes and it is not expected to play a role in the analysis of fiber transmission.

The SSFM step size will be correlated to the minimum characteristic length involved in each case; thus, for the compensation of SPM and XPM effects via coupled equations, the step size is limited by the walk-off length whereas the FWM length will be the parameter limiting the step size for the total-field NLSE. For WDM systems, the nonlinear length is longer than the FWM and walk-off lengths for the typical launching powers of interest in communications.
The following relationships define the characteristic step sizes for XPM and FWM compensation via the iterative symmetric SSFM, i.e,

\[ h_{wo} = \tau_r L_{wo}, \quad h_{fwm} = \varphi_{fwm} L_{fwm} \quad (4-9) \]

where the non-dimensional parameters \( \tau_r \) and \( \varphi_{fwm} \) represent, respectively, the maximum normalized inter-pulse relative delay (by the pulse width) and the maximum phase-mismatch allowed within one step. The values of \( \tau_r \) and \( \varphi_{fwm} \) may depend on several factors including the impact of non-deterministic fluctuations on the system (ASE and laser phase noise), the desired degree of accuracy and the numerical procedure. Thus, those parameters are usually determined a posteriori within the simulation results. In addition, provided that the SSFM is used for both the total-NLSE and the coupled-NLSE, the ratio \( k = \tau_r/\varphi_{fwm} \) can be regarded as a constant for any variant of the split-step Fourier method.

The computational load of DBP is determined by the computation per step and the step number. Note that the coupled equations for XPM compensation parallelize post-processing; thus, the total number of operations for XPM compensation is multiplied by the total number of channels. However, the required sampling rate is decided by the signal bandwidth. The sampling rate that can describe the signal in one channel is enough for XPM compensation. Thus the sampling rate for XPM compensation is reduced approximately by a factor of channel number in comparison with DBP using total-NLSE. As a result, the computational load is approximately
inversely proportional to the step size when comparing the methods of DBP with coupled-NLSE and total-NLSE.

4.3. Numerical Demonstration

![Block Diagram of the 16QAM/WDM Transmission System](image)

Figure 4-1: Block Diagram of the 16QAM/WDM Transmission System.

In this section, simulations are performed to evaluate the impact of XPM and FWM on backward propagation impairment compensation. For that, a 12 channel WDM is considered in which each channel is modulated at 100 Gb/s in a 16-QAM format [6,7], corresponding to a symbol rate of 25 Gbaud. The total transmission distance is 1000 km, divided into 10 spans of 100 km. Two channel spacing values will be considered according to the ITU-T standards for WDM systems, i.e. $\Delta f = 50, 100$ GHz.

The schematic of the WDM transmission system is shown in Figure 4-1 where post-compensation is performed in the digital domain after coherent detection. The
16-QAM/WDM signals are transmitted over multiple amplified fiber spans. At the receiver, the received signals are mixed in an optical hybrid with a set of co-polarized local oscillators. The in-phase and quadrature components of each WDM channel are obtained by balanced photo-detectors. Analog-to-digital (A/D) conversion is followed by DSP field reconstruction, backward propagation, demultiplexing and data recovery. We used non-zero dispersion shifted fiber with: \( \beta_2 = -5.63 \text{ ps}^2/\text{km}, \) \( \beta_3 = 0.083 \text{ ps}^3/\text{km}, \) \( \alpha = 0.046 \text{ km}^{-1}, \) and \( \gamma = 1.46 \text{ W}^{-1}\text{km}^{-1}. \) The signal is amplified after each span using an EDFA with a noise figure of 5 dB. For simplicity, the laser phase noise is neglected.

Forward transmission simulations have been performed using VPItransmissionMaker where two different channel spacings and nine different input power values had been considered. Backward propagation algorithms are developed in Matlab where C-NLSE and T-NLSE are solved with different step sizes for each case.

Figure 4-2 shows the Q-factor with respect to the step size for both channel spacings. It is shown that FWM has a negligible influence for \( \Delta f = 100 \text{ GHz}, \) having effectively the same Q-factor for XPM and FWM compensation. The larger channel spacing gives rise to a higher phase-mismatch, which rapidly averages to zero the contribution of the FWM products. Because of this small contribution, the optimum power is also effectively equal for both compensation schemes.
With $\Delta f = 100 \text{ GHz}$, the characteristic step sizes are $h_{wo} = 50 \text{ km}$ and $h_{wo} = 44 \text{ m}$. It is shown that with channel spacing of either $\Delta f = 100 \text{ GHz}$ or $\Delta f = 50 \text{ GHz}$, the step size can be increased significantly. As a result, the computational load of DBP was reduced.

4.4. Experimental Demonstration

4.4.1. Experiment Setup

The simulation results show that the computational load can be significantly reduced by DBP using coupled-NLSE. We also performed the experiment to further demonstrate the effectiveness of XPM compensation using coupled-NLSE. The experimental setup of the coherent transmission system is shown in Figure 4-3.
Three distributed-feedback lasers were used as the WDM carriers. Two QPSK modulators were driven by a pattern generator to modulate the central channel and two side channels. Two RF delay lines were used to align the in-phase (I) and quadrature (Q) inputs of the QPSK modulators. The same data pattern with a pseudo random bit sequence length of $2^{23}-1$ was used to modulate all the channels. An optical delay line was used to decorrelate the data sequence of the central channel and the side channels. The symbol rate was chosen to be 6 Gsymbols/s so that all three WDM channels could fit within the 24 GHz double-sided bandwidth of the real-time oscilloscope. The channel spacing of the WDM channels was set to be 6.8 GHz which made the three WDM channels nearly orthogonal, resulting in the lowest linear cross-talk.

The WDM signal was launched into a recirculating loop controlled by two acousto-optic switches. The length of the nonzero dispersion shifted fiber (NZ-DSF) spool in the loop was 80 km. The total launching power into the fiber was 6 dBm. Variable optical attenuators (VOA) and polarization controllers (PC) were used to balance the power levels and align the polarizations of the three channels. The local oscillator was tuned to the central channel and mixed with the signal in a 90° hybrid. The I and Q components of the received signal were detected with two photodetectors. The real-time oscilloscope was used for analog-to-digital conversion at 40 Gsamples/s and data acquisition.
4.4.2. Results

The three WDM channels were de-multiplexed using digital filters. Digital backward propagation was performed offline using the split-step Fourier method to solve the coupled NLSEs. Clock recovery, Phase estimation and Q-value calculation were performed after compensation of fiber impairments using DBP. The fiber parameters were determined by optimizing the performance in the training experiments. The dispersion parameter was found to be 3.9 ps/nm/km by performing dispersion compensation for 80 km transmission. The fiber loss and nonlinear parameter were found to be 0.2 dB/km and 1.4 /W/km by optimizing the nonlinearity compensation after 800 km transmission.
In DBP, the step size should be much smaller than the dispersion length
\[ L_D = (\beta_2 \Delta \nu^2)^{-1} \]
and nonlinear length
\[ L_{NL} = L_{sp} [\gamma P_T \int_0^{L_{sp}} \exp(-\alpha z) dz]^{-1} \]
where \( \Delta \nu \) is the total bandwidth, \( L_{sp} \) is the span length, and \( P_T \) is the total launching power. In this system, the dispersion and nonlinear length were 347 km and 675 km, respectively. The Q-value after 640 km transmission and DBP as a function of step size was calculated and shown in Figure 4-4. There is no appreciable penalty when the step size was smaller than 80 km. Thus the step size was chosen to be 80 km which was the length of one fiber span.

Figure 4-4: Q-value versus Step Size of Digital Backward Propagation

We used one local oscillator for coherent detection of all the three WDM channels. The relative phase information of the channels has been preserved in our experiment. In order to verify that the relative phase is not necessary for XPM compensation, we added a random phase shift (0.1\( \pi \) rad., 1.3\( \pi \) rad. and 0.6\( \pi \) rad.) to
$A_1$, $A_2$ and $A_3$ before solving the coupled NLSEs. The random phase shifts made no difference to the resulting Q-values.

Figure 4-5: Constellations after (a) Back-to-back Detection, (b) Dispersion Compensation, (c) SPM Compensation, (d) SPM+XPM Compensation, and (e) Nonlinearity Compensation via Total-field NLSE

To evaluate the effectiveness of XPM compensation, other compensation methods are included here for comparison. Figure 4-5 (a) shows the back-to-back constellation corresponding to a Q-value of 17.6 dB. The constellations after 800 km transmission and digital signal processing are shown in Figure 4-5 (b) to (e). Figure 4-5 (b) is obtained after dispersion compensation only and the corresponding Q-value is 8.1 dB. Figure 4-5 (c) is obtained after SPM compensation by neglecting both FWM and XPM terms in the coupled NLSEs and the corresponding Q-value is 8.5 dB. The constellations after SPM+XPM compensation via coupled NLSEs and nonlinearity compensation via total-field NLSE are shown in Figure 4-5 (d) and (e), respectively. The Q-values of the two constellations are both 13.0 dB.
As the transmission distance increases the combined effect of the ASE noise, dispersion and nonlinearity degrades the signal quality further. The dependence of Q-value with transmission distance is shown in Figure 4-6. For all transmission distances, the performance improvement due to SPM compensation is negligible but the improvement due to XPM compensation is significant. In this experiment, the performance improvement due to the inclusion of FWM is smaller than what can be expected theoretically [1]. This is due to the fact that FWM components outside the bandwidth of the real-time oscilloscope were not captured for DBP. In addition, FWM compensation via DBP is generally more sensitive to errors in the received waveform.

4.5. Reference


5. FOLDED-DBP FOR DISPERSION MANAGED FIBER LINK

5.1. Introduction

Optical signal is distorted by the joint effects of dispersion and nonlinearity during its propagation in optical fiber. In most of the installed long-haul systems, dispersion is compensated by periodically cascading two or more kinds of fiber with inverse dispersion parameters [1].

Fiber nonlinearity compensation via DBP is typically implemented using the split-step method. In order for the split-step method to be accurate, a large number of steps are needed especially for inter-channel nonlinearity compensation of WDM systems, resulting in a very heavy computational load. Some efforts have been devoted to increase the computational efficiency of DBP. In comparison with solving the nonlinear Schrodinger equation (NLSE) for the total field of the WDM signal, solving the coupled NLSE was suggested because it requires a smaller step number and lower sampling rate [2]. The step number can be further reduced by factorizing the dispersive walk-off effects in the DBP algorithm [3] and using variable step size [4]. In this chapter, we propose a computationally efficient method of nonlinearity compensation for dispersion managed fiber-optic transmission systems.

In dispersion managed long-haul transmission systems, waveform distortion is dominated by chromatic dispersion. In a periodic dispersion managed link, as a result of the periodic waveform evolution, the nonlinear behavior also repeats itself in every dispersion period. It is shown that, under the weakly nonlinear assumption, nonlinear
effects accumulated in a large number ($K$) of dispersion periods can be approximated by nonlinear effects accumulated in a single dispersion period with the same dispersion map and $K$ times the nonlinearity. Thus, significant savings in computational load can be achieved in digital compensation of fiber nonlinearity by folding digital backward propagation (DBP) of $K$ dispersion periods into one dispersion period. Because the DBP is folded in the distance domain, we call this method “distance-folded DBP”. Simulation results show, the required computation for DBP of dispersion managed transoceanic transmission systems can be reduced by up to 2 orders of magnitude with negligible penalty using distance-folded DBP.

When the residual dispersion per span (RDPS) is increased, the performance of distance-folded DBP degrades because the waveform evolutions in difference spans become less identical. Under the weakly nonlinear assumption in which waveform evolution is dominated by the dispersion, the optical waveform repeats at locations where accumulated dispersions are identical. Consequently, the nonlinear behavior of the optical signal also repeats at locations of identical accumulative dispersion. Hence the DBP steps can be folded according to the accumulated dispersion. Simulation results show that the penalty due to accumulated residual dispersion is mitigated using dispersion-folded (D-folded) DBP. Experimental results show considerable savings in computation using D-folded DBP. Comparative simulation also shows that the dramatically reduced computational load makes the nonlinearity-compensated dispersion-managed fiber link a competitive candidate for the next-generation transmission systems.
5.2. Distance-Folded DBP

5.2.1. Theory of Distance-Folded DBP

We focus on dispersion managed fiber-optic transmission systems. Without loss of generality, we assume that each fiber span with a length of \( L \) is a period of the dispersion map. For long-haul fiber-optic transmission, an optimum power exists as a result of the tradeoff between optical signal to noise ratio (OSNR) and nonlinear impairments. The total nonlinear phase shift at the optimum power level is on the order of 1 radian \([5]\). Therefore, for transoceanic fiber transmission systems which consist of many (>100) amplified spans, the nonlinear effects in each span is weak. As a result, chromatic dispersion is the dominant factor that determines the evolution of the waveform within each span.

We can analyze the nonlinear behavior of the optical signal using a perturbation approach. The NLSE governing the propagation of the optical field, \( A_j(z,t) \), in the \( j^{th} \) fiber span can be expressed as

\[
\frac{\partial A_j(z,t)}{\partial z} = [D + \varepsilon \cdot N(\left|A_j(z,t)\right|^2)] \cdot A_j(z,t), \quad (5-1)
\]

where \( 0 < z < L \) is the propagation distance within each span, \( D \) is the linear operator for dispersion, fiber loss and amplifier gain, \( N(\left|A_j(z,t)\right|^2) \) is the nonlinear operator. \( \varepsilon \) (to be set to unity) is a parameter indicating that the nonlinear perturbation is small for the reasons given above. The boundary conditions are

\[
A_j(0,t) = a(0,t), \quad (5-2)
\]
\[ A_j(0, t) = A_{j-1}(L, t) \quad \text{for} \quad j \geq 2, \quad (5-3) \]

where \( a(0, t) \) is the input signal at the beginning of the first span. We assume that the solution of Equation 5-1 can be written as,

\[ A_j(z, t) = A_{j,j}(z, t) + \varepsilon \cdot A_{j,nl}(z, t). \quad (5-4) \]

Substituting Equation 5-4 into Equation 5-1 and expanding the equation in power series of \( \varepsilon \), we have

\[ \frac{\partial A_{j,j}(z, t)}{\partial z} - D \cdot A_{j,j}(z, t) + \varepsilon \cdot \left[ \frac{\partial A_{j,nl}(z, t)}{\partial z} - D \cdot A_{j,nl}(z, t) - N\left( A_{j,j}(z, t) \right)^2 \right] \cdot A_{j,j}(z, t) + O(\varepsilon^2) = 0. \quad (5-5) \]

Equating to zero the successive terms of the series, we have

\[ \frac{\partial A_{j,j}(z, t)}{\partial z} = D \cdot A_{j,j}(z, t), \quad (5-6) \]

\[ \frac{\partial A_{j,nl}(z, t)}{\partial z} = D \cdot A_{j,nl}(z, t) + N\left( A_{j,j}(z, t) \right)^2 \cdot A_{j,j}(z, t). \quad (5-7) \]

The boundary conditions are

\[ A_{j,j}(0, t) = a(0, t), \quad (5-8) \]

\[ A_{j,j}(0, t) = A_{j-1}(L, t) \quad \text{for} \quad j \geq 2, \quad (5-9) \]

and

\[ A_{j,nl}(0, t) = 0. \quad (5-10) \]
First, we assume that dispersion is completely compensated in each span. As a result, at the end of the first span,

\[ A_{1,j}(L,t) = a(0,t), \]  

(5-11)

and

\[ A_2(0,t) = A_1(L,t) = a(0,t) + \varepsilon \cdot A_{1,nl}(L,t), \]  

(5-12)

where \( A_{i,nl}(z,t) \) is the solution of Eq. (30) with \( j = 1 \). For the second span,

\[ A_{2,j}(z,t) = A_{i,j}(z,t) + \varepsilon \cdot \bar{A}(z,t), \]  

(5-13)

where the first and second terms are solutions to Equation 5-6 with boundary conditions \( A_{2,j}(0,t) = a(0,t) \) and \( A_{2,j}(0,t) = \varepsilon \cdot A_{i,nl}(L,t) \), as a result of the principle of superposition. At the end of the second span, because of the complete dispersion compensation,

\[ A_{2,j}(L,t) = a(0,t) + \varepsilon \cdot A_{i,nl}(L,t). \]  

(5-14)

The nonlinear distortion in the second span is governed by Equation 5-7 with \( j = 2 \). Since

\[ \left| A_{2,j} \right|^2 = \left| A_{i,j} + \varepsilon \cdot \bar{A} \right|^2 = \left| A_{i,j} \right|^2 + O(\varepsilon), \]  

(5-15)

the differential equation, i.e. Equation 5-7, and the boundary conditions for \( A_{2,nl}(z,t) \) and \( A_{i,nl}(z,t) \) are identical, so

\[ A_{2,nl}(L,t) = A_{i,nl}(L,t). \]  

(5-16)
As a result, the optical field at the end of the second span is given by

\[ A_2(L,t) = A_{2,1}(L,t) + \varepsilon \cdot A_{2,\text{nl}}(L,t) = a(0,t) + \varepsilon \cdot 2A_{1,\text{nl}}(L,t). \]  

(5-17)

That is, the nonlinear distortion accumulated in 2 spans is approximately the same as nonlinear distortion accumulated in 1 span with the same dispersion map and twice the nonlinearity. It follows that, assuming weak nonlinearity and periodic dispersion management, the optical field after \( K \) spans of propagation can be written as

\[ A_K(L,t) = a(0,t) + \varepsilon \cdot KA_{1,\text{nl}}(L,t), \]  

(5-18)

which is the solution of the NLSE

\[ \frac{\partial A_j(z,t)}{\partial z} = [D + \varepsilon \cdot KN([A_j(z,t)]^2)] \cdot A_j(z,t). \]  

(5-19)

Equation 5-19 describes the fiber propagation in a fiber span where the nonlinearity is \( K \) times of that in the original fiber.

The equivalence described above suggests that DBP for \( K \) spans can be folded into a single span with the same dispersion map and \( K \) times the nonlinearity. Assuming that the step size for the split-step implementation of DBP is unchanged, computational load for the folded DBP can be saved by the folding factor of \( K \). We will call this method distance-folded DBP.

The above derivation is based on the assumption that waveform distortion due to nonlinearity and the residual dispersion per span (RDPS) is negligible, and consequently the nonlinear behavior of the signal repeats itself in every span. This
assumption is not precisely valid because first, fiber nonlinearity also changes the waveform, and secondly, dispersion is not perfectly periodic if the RDPS is non-zero or higher-order dispersion (dispersion slope) is not compensated. These effects accumulate and as a result, the waveform evolutions are not identical between two spans that are far away from each other.

In order for the nonlinearity compensation to be accurate, it might be necessary to divide the entire long-haul transmission system into segments of multiple dispersion-managed spans so that the accumulated nonlinear effects and residual dispersion is small in each segment. Moreover, in order to minimize the error due to residual dispersion, distance-folded DBP should be performed with a boundary condition calculated from lumped dispersion compensation for the first half of the segment. For a fiber link with $M \times K$ spans, the distance-folded DBP is illustrated in Figure 5-1.

![Distance-Folded DBP for a Periodically Dispersion Managed Fiber Link with $M \times K$ Spans](image)

**5.2.2. Simulation Results**
We simulate a WDM system with quadrature phase-shift keying (QPSK) modulation at a bit rate of 56 Gbits/s using VPItransmissionMaker. The simulation setup is shown in Figure 5-2. 12 channels of NRZ (non-return-to-zero) QPSK signal were transmitted with 50 GHz channel spacing. The linewidth of the lasers was 100 KHz. The dispersion managed fiber link consisted of 140 spans of 50 km of the OFS UltraWave SLA/IDF Ocean Fiber combination. In each span, the SLA fiber with a large effective area was used near the EDFA, followed by the IDF fiber with inverse dispersion and dispersion slope. The EDFA noise figure was 4.5 dB. The loss, dispersion, relative dispersion slope and effective area of the SLA fiber are 0.188 dB/km, 19.5 ps/nm/km, 0.003/nm and 106 μm². The corresponding parameters for the IDF fiber are 0.23 dB/km, -44 ps/nm/km, 0.003/nm and 31 μm², respectively. The RDPS was determined by the proportion of SLA fiber to IDF fiber in each span. A piece of fiber at the receiver was used to compensate for the residual dispersion. After de-multiplexing and coherent detection, DSP was performed in Matlab.

Figure 5-2: Block Diagram of the Dispersion Managed WDM System

The DBP was performed as illustrated in Figure 5-1. Without loss of generality, we solved the coupled NLSE with the non-iterative asymmetric split-step Fourier
method (SSFM). After matched filtering, phase estimation and clock recovery, the Q-value averages of the WDM channels were estimated.

![Figure 5-3: (a) Q-value vs. Launching Power per Channel and (b) Q-value at Optimum Power vs. Folding Factor $K$ with RDPS = 0 ps/nm.](image)

We first simulated the transmission with full inline dispersion compensation, i.e., RDPS = 0. The Q-value as a function of the launching power is shown in Figure 5-3 (a). Without nonlinearity compensation, the maximum Q-value was 10.8 dB. With conventional DBP in all spans, the Q-value was increased to 13.3 dB. With distance-folded DBP with a folding factor of 140 (i.e., $M=1$, $K=140$), the maximum Q-value was 13.1 dB. The 0.2 dB Q-value penalty was due to the accumulated nonlinear waveform distortion which reduced the accuracy of nonlinearity compensation. There was almost no penalty when the folding factor was 70 (i.e. $M=2$, $K=70$).

In the split-step implementation DBP, the step size has to be small enough so that the dispersion and nonlinear effects can be properly de-coupled. In long-haul
WDM fiber links, the step size is usually limited by dispersion [6]. In the simulation, we use the same number of steps in SLA fiber and IDF fiber in each fiber span, so that the dispersion in each step is approximately the same. Figure 5-3 (b) shows the Q-value as a function of step number per span. The computational efficient method did not reduce the required step size.

The nonlinear impairments of a dispersion managed fiber link can be suppressed with inline residual dispersion [7,8]. But non-zero RDPS can induce penalty in the distance-folded DBP. Figure 5-4 (a) shows the Q-values obtained at optimum power levels as functions of the folding factor. With a RDPS of 5 ps/nm (20 ps/nm), the maximum Q-value can be approached using a folding factor of 20 (5). Figure 5-4 (b) shows the Q-values as functions of the RDPS. With conventional DBP in all spans, the Q-value increases with RDPS and approaches the maximum value when RDPS is larger than 10 ps/nm. When distance-folded DBP is used, the Q-value
penalty increases with RDPS. For a fiber link with non-zero RDPS, there is a tradeoff between computational load and system performance.

In the simulation, each amplified span contains one period of the dispersion map. The folding factor can be further increased by shortening the dispersion map so that each amplified spans may consist of several dispersion periods. Folded DBP can also be applied when each dispersion period consists of several amplified spans.

5.3. Dispersion-Folded DBP

5.3.1. Theory of Dispersion-Folded DBP

In cases where there is residual dispersion, distance-folded DBP is not optimal as optical waveforms do not repeat at identical spatial locations in all amplified spans. In long-haul fiber transmission, the optical waveform evolution along the fiber is dominated by the chromatic dispersion. Under the weakly nonlinear assumption, the optical waveform repeats at locations where accumulated dispersions are identical. Since the Kerr nonlinear effects are determined by the instantaneous optical field, the nonlinear behavior of the optical signal also repeats at locations of identical accumulative dispersion. Hence the DBP steps can be folded according to the accumulated dispersion.

Under the weakly nonlinear approximation, the propagation of the optical field, $E(z,t)$, is governed by the nonlinear Schrödinger equation (NLSE)

$$\frac{\partial E(z,t)}{\partial z} = \left[D + \varepsilon \cdot N(|E(z,t)|^2)\right] E(z,t),$$

(5-20)
where $D$ is the linear operator for dispersion, fiber loss and amplifier gain, $N(|E(z,t)|^2)$ is the nonlinear operator, $\varepsilon$ (to be set to unity) is a parameter indicating that the nonlinear perturbation is small for the reasons given above.

The solution of Equation 5-20 can be written as,

$$E(z,t) = E_i(z,t) + \varepsilon \cdot E_{nl}(z,t). \quad (5-21)$$

Substituting Equation 5-21 into Equation 5-20, expanding the equation in power series of $\varepsilon$, and equating to zero the successive terms of the series, we have

$$\frac{\partial E_i(z,t)}{\partial z} = D \cdot E_i(z,t), \quad (5-22)$$

$$\frac{\partial E_{nl}(z,t)}{\partial z} = D \cdot E_{nl}(z,t) + N(|E_i(z,t)|^2) \cdot E_i(z,t), \quad (5-23)$$

which describe the linear evolution and the nonlinear correction, respectively. It is noted that the nonlinear correction $E_{nl}(z,t)$ is governed by a linear partial differential equation with nonzero forcing which depends on the linear solution only.

Based on the principle of superposition, the total nonlinear correction is the sum of nonlinear corrections due to nonzero forcing at each fiber segment. In conventional DBP, the contribution from each fiber segment is computed separately. However, it is advantageous to calculate the total nonlinear correction as the sum of nonlinear corrections due to nonzero forcing at different accumulated dispersion divisions, each having multiple fiber segments. This is because, with the exception of the input power and possibly effective length, the dispersion operator $D$ (to reach the
end of transmission) and the linear component $E_i(z,t)$ that determine the nonzero forcing are identical for the fiber segments with the same accumulated dispersion. Therefore, the nonlinear corrections due to these multiple fiber segments with the same accumulated dispersion are identical except a constant and can be calculated all at once.

Figure 5-5: Dispersion Managed Coherent Fiber Link Using Conventional DBP or D-folded DBP.

A typical long-haul dispersion-managed fiber-optic transmission system is illustrated in Figure 5-5. In each amplified fiber span, standard single mode fiber (SSMF) with positive dispersion is followed by DCF with negative dispersion. After coherent detection, DBP is performed in the digital signal processor.
In conventional DBP, multiple steps are required for each fiber span, resulting in a large number of steps. Dispersion compensation ($D$) and nonlinearity compensation ($NL$) are performed in each step.

In D-folded DBP, the DBP steps are defined and folded according to the accumulated dispersion. For example, the blue solid lines in the dispersion map correspond to some fiber sections which can be folded into one step. The optical field ($E_t$) for the first step of DBP can be obtained with a lumped dispersion compensator ($D_{lumped}$). Then the nonlinearity compensation is performed in each step using a weighting factor ($W_m$). The total number of steps for D-folded DBP is equal to the number of dispersion divisions.

It can be shown that the D-folded DBP is equivalent to the conventional DBP under the weak nonlinearity assumption. First we perform the conventional DBP in the backward direction of fiber propagation. As shown in Figure 5-5, the dispersion map is divided into $M$ divisions with the horizontal dashed lines. And we use the fiber sections between the neighboring horizontal lines as the steps of conventional DBP. In each step, a small term for nonlinearity compensation is generated as a function of the optical field $E(t)$ and the effective fiber length $L_{eff}$. For simplicity, we assume equal effective length and optical power level for all the steps. For a step within the $m$th dispersion division in the $k$th fiber span, this small term can be expressed as $N[E_{m,k}(t)] = E_{m,k}(t) \cdot \left[\exp(j\gamma|E_{m,k}(t)|^2L_{eff}) - 1\right]$, where $\gamma$ is the fiber nonlinear parameter. Under the weak nonlinearity assumption, the optical field
where \( E_{m,k}(t) \) is the input field of the first step of DBP which is in the \( f \)th dispersion division, and \( h_{jm}(t) \) is the impulse response of the accumulated dispersion difference between the \( f \)th and the \( m \)th division. \( E_{m,k}(t) \) is the same for any \( k \) and thus can be expressed as \( E_{m}(t) \). In the following conventional DBP steps, this small nonlinearity compensation term experiences dispersion and negligible nonlinearity. At the last step of DBP in the \( l \)th dispersion division, the nonlinearity compensation term becomes \( N[E_{m}(t)] \otimes h_{ml}(t) \), which is identical for all the steps in the \( m \)th division. At the output of the conventional DBP, the summation of the nonlinearity compensation terms generated in all the steps can be expressed as \( \sum_{m=1}^{M} S_m \cdot N[E_{m}(t)] \otimes h_{ml}(t) \), where \( S_m \) is the number of the conventional DBP steps in the \( m \)th division.

In D-folded DBP, only one step is needed for each dispersion division. For a fiber link with positive RDPS, we can perform a lumped dispersion compensation to obtain the waveform \( E_i(t) \) in the top dispersion division. Then dispersion compensation and nonlinearity compensation are performed for each of the subsequent dispersion divisions. For the \( i \)th dispersion division, the nonlinear phase de-rotation is given by \( \varphi_i = j \gamma \cdot S_i \left| E_i(t) \right|^2 \cdot L_{eff} \). Then the nonlinearity compensation term generated in the \( i \)th folded DBP step is given by \( E_i(t) \cdot \left[ \exp(j \gamma \cdot S_i \left| E_i(t) \right|^2 \cdot L_{eff} ) - 1 \right] \approx S_i \cdot E_i(t) \cdot \left[ \exp(j \gamma \left| E_i(t) \right|^2 L_{eff} ) - 1 \right] \). Again, this small term experiences dispersion and negligible nonlinearity in the following folded DBP steps. At the output of the D-folded DBP, the summation of the nonlinearity compensation terms is \( \sum_{i=1}^{M} S_i \cdot N[E_i(t)] \otimes h_{li}(t) \), which is equal to that for the conventional DBP.
Our assumption that all the fiber sections have the same effective length and power level is usually not true. Thus we replace $S_m$ with a weighting factor $W_m = \sum_{k=1}^{K} \gamma \int P_{m,k}(z)dz$, where $P_{m,k}(z)$ is the power level as a function of distance within the $k$th fiber section in the $m$th dispersion division. The nonlinear phase shift in the $m$th step of D-folded DBP is given by $\varphi_m = W_m \left| E_m(t) \right|^2$, where $E_m(t)$ is the optical field with the power normalized to unity. Note that calculating the weighting factors does not require real-time computation.

5.3.2. Numerical Demonstration of Dispersion-Folded DBP

We performed VPI simulation of the forward fiber transmission as described in section 5.3.2. Again, without loss of generality, we solved the coupled NLSE with the non-iterative asymmetric split-step Fourier method in DBP.

![Figure 5-6: Q-value vs. RDPS After Dispersion-Folded DBP](image)

Figure 5-6 shows the Q-values at the optimum power levels as functions of the RDPS after dispersion-folded DBP. In comparison with the results after
distance-folded DBP shown in Figure 5-4 (b), the penalty due to RDPS is removed. When the dispersion-folded DBP was performed in one segment ($M=1$, $K=140$) with RDPS = 30 ps/nm, there is a 0.3 dB Q-value penalty in comparison with the conventional DBP.

Figure 5-7: (a) Q-value vs. Step Number per Span after Conventional DBP, (b) Q-value vs. Step Number After Dispersion-Folded DBP

Figure 5-7 shows the Q-values as functions of the step number after conventional DBP and dispersion-folded DBP. It is shown in Figure 5-7 (a) that with conventional DBP, a larger step number is required when RDPS=0. That is because the nonlinear effects, including the un-compensated FWM effect [9], are enhanced when the dispersion is completely compensated in each span. In Figure 5-7 (b), it is shown that with RDPS=0, dispersion-folded DBP does not change the required step size in comparison with the conventional DBP. With RDPS>0, a larger step number is required when the RDPS is larger. The reason is that a larger RDPS means a larger difference between $D_{\text{max}}$ and $D_{\text{min}}$, and the dispersion limitation on the step size
results in a larger step number. When RDPS = 10 ps/nm, the required step number after dispersion-folded DBP was 400. In comparison with the conventional DBP, the computation for DBP was saved by a factor of 14. When RDPS=0, the computation for DBP was saved by a factor of 140.

Figure 5-8 shows the Q-values as functions of the step number when walk-off integral is applied [3]. When RDPS = 30 ps/nm, the required step number after dispersion-folded DBP was 150. In comparison with the conventional DBP, the computation for DBP was saved by a factor of 19. When RDPS=10 ps/nm, the computation for DBP was saved by a factor of 56. It is shown in Figure 5-7 and Figure 5-8 that there is almost no performance penalty due to the dispersion-folded DBP.

5.3.3. Experimental Demonstration of Dispersion-Folded DBP
Figure 5-9: Experimental Demonstration of D-folded DBP. (a) Experimental Setup. 

Inset: Constellations after Back-to-back Detection, EDC and DBP at the Corresponding Optimum Power Levels. (b) Q-value as a Function of the Number of Steps Using Conventional DBP (green line) and D-folded DBP (blue line). (c) Q-value as a Function of Optical Launching Power after EDC (red line), 30-step D-folded DBP (blue line) and 1,300-step Conventional DBP (green line).

To demonstrate the effectiveness of the D-folded DBP, we performed the experiment of single-channel 6084 km transmission of NRZ-QPSK signal at 10 Gbaud. The experimental setup is shown in Figure 5-9a. At the transmitter, carrier
from an external-cavity laser is modulated by a QPSK modulator using a $2^{23}-1$ pseudo random bit sequence (PRBS). The optical signal is launched into a recirculating loop controlled by two acousto-optic modulators (AOMs). Two erbium doped fiber amplifiers (EDFAs) are used to completely compensate for the loss in the loop. An optical bandpass filter (BPF) is used to suppress the EDFA noise. At the receiver, the signal is mixed with the local oscillator from another external cavity laser in a 90° hybrid. The I and Q components of the received signal are detected using two photo-detectors (PDs). A real-time oscilloscope is used for analog-to-digital conversion and data acquisition at 40 Gsamples/s. The DSP is performed off-line with Matlab.

The recirculating loop consists of 82.6 km SSMF with 0.2 dB/km loss and 70 $\mu$m$^2$ effective area, and 11 km DCF with 0.46 dB/km loss and 20 $\mu$m$^2$ effective area. By optimizing the performance in the training experiments of electronic dispersion compensation, the dispersion of the SSMF and the DCF are determined as 17.06 ps/nm/km and -123.35 ps/nm/km, respectively. The residual dispersion per span is 53 ps/nm.

Without loss of generality, we solve the NLSE using the asymmetric split-step algorithm with one dispersion compensator per step [10]. For long-haul transmission, the DBP step size is usually limited by dispersion [6]. In this paper, we use DBP steps with equal dispersion for simplicity. The Q-value as a function of the number of steps is shown in Figure 5-9b. The required number of steps can be reduced from 1,300 to 30 by using the D-folded DBP. The number of multiplications per sample is reduced
by a factor of 43. The calculation of the number of multiplications is given in the method section. Figure 5-9c shows the Q-value as a function of the launching power. With only electronic dispersion compensation (EDC) for the accumulated residual dispersion, the maximum Q-value is 9.1 dB. With nonlinearity compensation using D-folded DBP, the maximum Q-value is increased to 10.7 dB. The performance using the 30-step D-folded DBP is almost the same as that using the 1,300-step conventional DBP.

5.3.4. Dispersion Managed Systems vs. Dispersion Unmanaged Systems

Enabled by the high-speed DSP technology, EDC has been implemented in coherent fiber communication systems [10,11]. With EDC, dispersion unmanaged systems outperform dispersion managed systems because the DCF with high loss and nonlinearity can be removed [12,13]. However, the D-folded DBP method can reduce the DBP computational load of dispersion managed systems to a level that may be practical for the current electronic technique. On the contrary, DBP for long-haul dispersion unmanaged systems cannot be folded and remains impractical. In order to find the optimum solution for the next generation systems, it is necessary to compare the performance and computational load of the dispersion managed system with D-folded DBP and the dispersion unmanaged system with EDC.
Figure 5-10: Comparison of Dispersion Managed System with D-folded DBP and Dispersion Unmanaged System with EDC. (a) Block Diagram of the Dispersion Managed/Unmanaged WDM System. (b) Q-value as a Function of the Number of Steps Using D-folded DBP (blue line) and Conventional DBP (green line) for the Dispersion Managed System. (c) Q-value as a Function of Optical Launching Power after Dispersion Managed Transmission with EDC (red line) and 35-step D-folded DBP (blue line), and after Dispersion Unmanaged Transmission with EDC (black line).

We performed the simulation of WDM transmission with VPItransmissionMaker as illustrated in Figure 5-10a. At the transmitter, 12 channels of 56 Gb/s NRZ-QPSK signal are transmitted with 50 GHz channel spacing. The linewidth of the lasers is 100 kHz. For the dispersion unmanaged transmission, we
use the OFS UltraWave Super-Large-Effective-Area (SLA) fiber with loss, dispersion, relative dispersion slope and effective area of 0.188 dB/km, 19.5 ps/nm/km, 0.003/nm and 106 μm², respectively. For the dispersion managed transmission, the SLA fiber is cascaded with OFS HOM fiber. The corresponding parameters of the HOM fiber are 0.56 dB/km, -835 ps/nm/km, 0.003 /nm and 61 μm², respectively [14]. Each amplified fiber span consists of two identical dispersion periods. And each dispersion period has a residual dispersion of 25 ps/nm. The fiber span length is 100 km for both dispersion managed and dispersion unmanaged transmission. The noise figure of the EDFAs is 4.5 dB. After de-multiplexing and coherent detection, the signal of each channel is sampled at 56 Gsamples/s.

In DBP, coupled NLSE is solved with the asymmetric split-step method [15]. The inter-channel walk-off effects are considered in the nonlinearity compensation operators in order to increase the required step size [3]. For each channel, the XPM from two neighboring channels and the SPM are compensated. The XPM from the other channels is weak because of the walk-off effect. Simulation results show that using more channels in the XPM compensation does not improve the performance significantly.

The Q-value averages of the WDM channels are calculated after 5,000 km transmission. Figure 5-10b shows the Q-value of the dispersion managed system as a function of the number of steps using conventional DBP and D-folded DBP. The Q-values after 35-step D-folded DBP and 2000-step conventional DBP are 14.7 dB and 15.5 dB, respectively. The number of multiplications per sample is reduced by a
factor of 57 with a penalty of 0.8 dB in Q-value. Note that the distance-folded DBP is not effective because of the large residual dispersion per span.

The Q-value as a function of the launching power per channel is shown in Figure 3c. The maximum Q-value of the dispersion managed system after EDC and 35-step D-folded DBP are 10.7 dB and 14.7 dB, respectively. The nonlinearity compensation using D-folded DBP increased the Q-value by 4 dB. The maximum Q-value of the dispersion unmanaged system with EDC is 13.4 dB. The dispersion managed system with D-folded DBP outperforms the dispersion unmanaged system with EDC by 1.3 dB in Q-value. The number of multiplications per sample for the 35-step D-folded DBP and the EDC of the dispersion unmanaged link are 544 and 95, respectively. The computational load for the D-folded DBP is less than 6 times of that for the EDC.

5.3.4. Computational Load

The computational load can be associated to the number of complex multiplications per sample (MPS) involved in the operation. We perform the filtering with the computationally efficient overlap-add method [3]. For an overhead length P, a signal block length $M$ is chosen to minimize the computational load using the radix-2 Fast Fourier Transform (FFT) with an FFT block size of $(M+P)$. For dispersion compensation, the overhead is approximately given by $P = 2\pi |\beta_2| B \cdot h \cdot S$ where $\beta_2$, $B$, $h$ and $S$ are the dispersion, signal bandwidth per channel, fiber length and sampling rate, respectively. The MPS for the overlap-add filtering is given by
For the EDC of the 5,000 km dispersion-unmanaged link, we consider an FFT block size of $2^{10}$ which is practical for the current electronic technology. By using 4 overlap-add filtering operators each with $M = 475$ and $P = 549$, the minimum possible MPS is 95.

In the nonlinearity operator of DBP, the calculations of the optical intensity and the nonlinear phase shift each costs one complex multiplication. The value of the nonlinear phase shift can be obtained using a lookup table. For the 6,084 km single channel transmission, the MPS for DBP is given by

$$n_s \cdot [(M + P) \log_2 (M + P) + (M + P) + 2M] / M,$$

where $n_s$ is the step number. The minimum MPSs of the 1,300-step conventional DBP and the 30-step D-folded DBP are 11,300 and 260, respectively.

For the 5,000 km WDM transmission, the length of the inter-channel walk-off filter is given by

$$P = 2\pi |\beta_2| \Delta f \cdot h \cdot S \text{ where } \Delta f \text{ is the channel spacing. The MPS for DBP is given by } n_s \cdot [2(M + P) \log_2 (M + P) + 3(M + P) + 2M] / M.$$

The minimum MPS for the 2,000-step conventional DBP is 31,100. For the 35-step D-folded DBP, the minimum MPS is 544.

5.3.5. Conclusion and Discussion

In conclusion, the dispersion-folded DBP method can dramatically reduce the computational load of DBP for fiber nonlinearity compensation at the receiver. This folded DBP method can also be applied to pre-compensation of fiber nonlinearity at the transmitter. Experimental results show that the computation for DBP of 6,084 km
transmission can be reduced by a factor of 43 with negligible penalty in performance. We have also compared two 5,000km WDM transmission systems: the dispersion unmanaged system with EDC and the dispersion managed system with D-folded DBP. Simulation results show that the dispersion managed system with D-folded DBP outperforms the dispersion unmanaged system by 1.3 dB in Q-value. The computational load for the D-folded DBP is 6 times of that for EDC. Since DSP in the coherent receiver includes phase estimation, clock recovery, matched filtering, polarization demultiplexing, etc., EDC only occupies a small portion of the DSP chip area [11,16]. Thus replacing the EDC with the D-folded DBP would increase the total chip area by a factor of much less than 6. Using the D-folded DBP method, fiber nonlinearity compensation for the deployed dispersion managed long-haul WDM links is becoming practical [17]. At the same time, dispersion managed system becomes a competitive candidate for the next generation systems.

5.4. References


6. SUMMARY AND FUTURE WORK

6.1. Summary

The fiber nonlinear effects including SPM, XPM and FWM need to be compensated in order to further increase the capacity and transmission distance of next generation fiber transmission system. In long-haul broadband transmission where the dispersion causes significant pulse reshaping and inter-channel walk-off, a distributed nonlinear compensation method, known as digital backward propagation, is necessary for the effective compensation of the joint effect of dispersion and nonlinearity. In order for DBP to be accurate, a large number of steps are usually required for long-haul transmission, resulting in a prohibiting computational load for the current electronic technique.

In real time DBP implementation, the FIR filters can be used for dispersion compensation. Since the nonlinearity compensation can be performed with a simple exponential operator, the dispersion compensation accounts for most of the computation per step. For a large number of DBP steps, the frequency response of each FIR filter has to be very accurate in order to minimize the error accumulation. Such accuracy is translated into a large filter length and hence, into a large computational load.

In order to reduce the required FIR filter length, a method of designing a complementary filter pair is proposed. An FIR filter can be optimized by applying appropriate windows. By adjusting the windowing in both time domain and frequency
domain, the individual errors in the frequency response of the two filters in a complementary filter pair can cancel each other. As a result, larger individual filter error can be tolerated and the required filter length is significantly reduced.

A 12-channel 16QAM/WDM transmission system is simulated to demonstrate the effectiveness of the complimentary filter pair method. Simulation results show that the performance of the complementary filter pairs with a filter length of 83 is better than the performance of the identical filters with a filter length of 177. The complementary filter pair design reduces the required filter length and, consequently, the total computational load by a factor of 2 [1].

For a fiber link with distributed Raman amplification, the signal experiences both constant fiber loss and spatially varying Raman gain along the fiber. Hence the Raman gain as a function of the distance and the effective fiber length of each DBP step need to be calculated by solving the differential equations of Raman amplification.

To verify the effectiveness of DBP for distributed Raman systems, we performed simulations on a dual-pump unrepeatered Raman link. In DBP, the dispersion and the nonlinear effects has to be small enough in each step so that they can be de-coupled. The nonlinear effects are dependent on the optical power which changes along the fiber. Thus in order to minimize the number of steps, unequal step size can be used in DBP. As can be seen from the power profile, nonlinear impairments are incurred mostly at the beginning of the transmission line where the
signal power is much higher. Therefore, split-step DBP for impairment compensation can be performed only for transmission at the transmitter end of the link without significant penalty in performance while saving computational load.

As a result of DBP, the Q-value is increased from 7.2 dB to 11.9 dB for transmission of 310 km. Preserving the same Q-value, employing DBP increases the optimum signal power by 8 dB and, consequently, increases the transmission distance from 270 km to 310 km [2,3].

Digital backward propagation can be implemented by solving the total-field NLSE or the coupled NLSEs. By solving the coupled NLSEs, XPM and SPM can be compensated while neglecting the effects of FWM. In comparison with compensating for the FWM, compensating for the XPM requires a larger step size and a smaller step number. Solving the coupled NLSE also requires a lower sampling rate as the total field of all the WDM channels is not constructed. Moreover, XPM compensation does not need phase-locked local oscillators to reconstruct the total optical field [4].

The simulation results show that the computational load can be significantly reduced by DBP using coupled-NLSE. To further demonstrate the effectiveness of XPM compensation using coupled-NLSE, we also performed the WDM long-haul transmission experiment. With XPM compensation, the transmission distance can be increased from 320 km to 1440 km preserving the same Q-value [5].

At the optimum power level of fiber transmission, the total nonlinear phase shift is on the order of 1 radian. Therefore, for transoceanic fiber transmission systems
which consist of many (>100) amplified fiber spans, the nonlinear effects in each span is weak. In periodically dispersion managed long-haul transmission systems, waveform distortion is dominated by chromatic dispersion. As a result of the periodic waveform evolution, the nonlinear behavior also repeats itself in every dispersion period. Taking advantage of the periodic waveform evolution in periodically dispersion managed fiber link, the DBP of $K$ fiber spans can be folded into one span with $K$ times the nonlinearity. This method can be called “distance-folded DBP”.

Significant savings in computational load can be achieved in digital compensation of fiber nonlinearity using distance-folded DBP. We simulate a 12-channel QPSK/WDM system with 56 Gb/s per channel. Simulation results of 7000 km transmission and DBP show that the required computation for DBP of dispersion managed transoceanic transmission systems can be reduced by up to 2 orders of magnitude with negligible penalty using folded DBP.

This assumption of repeating linear and nonlinear behavior is not exactly valid because first, fiber nonlinearity also changes the waveform, and secondly, dispersion is not perfectly periodic if the RDPS is non-zero or the dispersion slope is not compensated. Most of the deployed fiber links has non-zero residual dispersion per span in order to avoid the resonant nonlinear effects. However, the non-zero RDPS can induce penalty in the folded DBP. With a RDPS of 5 ps/nm (20 ps/nm), the maximum Q-value can be approached using a folding factor of 20 (5) [6,7].
In order to solve this problem, we propose and demonstrate a computationally efficient dispersion-folded (D-folded) DBP method that is effective for fiber links with arbitrary dispersion maps. Under the weakly nonlinear assumption, the optical waveform repeats at locations where accumulated dispersions are identical. Since the Kerr nonlinear effects are determined by the instantaneous optical field, the nonlinear behavior of the optical signal also repeats at locations of identical accumulative dispersion. Hence we can fold the DBP according to the accumulated dispersion.

The dispersion map of a dispersion managed fiber link can be divided into $m$ divisions by the horizontal lines. The fiber segments between the neighboring horizontal lines have the same accumulated dispersion, and can be folded into one step of the D-folded DBP. For a fiber link with positive residual dispersion per span, a lumped dispersion compensator can be used to obtain the optical field in the first dispersion division. Then dispersion compensation and nonlinearity compensation are performed for each of the subsequent dispersion divisions. To take into account the different power levels and effective lengths of the fiber segments, a weighting factor is used in the nonlinearity compensator of each step. The total number of steps for D-folded DBP is equal to the number of dispersion divisions.

To demonstrate the effectiveness of the D-folded DBP, we performed the experiment of single-channel 6,084 km transmission of non-return-to-zero (NRZ) QPSK signal at 10 Gbaud. The required number of steps can be reduced from 1,300 to 30 by using the D-folded DBP with negligible performance penalty. With only EDC
for the accumulated residual dispersion, the maximum Q-value is 9.1 dB. With nonlinearity compensation using D-folded DBP, the maximum Q-value is increased to 10.7 dB.

With EDC, dispersion-unmanaged systems outperform dispersion-managed systems because the DCF with high loss and nonlinearity can be removed. However, the D-folded DBP method can reduce the DBP computational load of dispersion-managed systems to a level that may be practical for the current electronic technology.

In order to find the optimum solution for the next-generation transmission systems, we performed comparative simulations of a dispersion-managed system with D-folded DBP and a dispersion-unmanaged system with EDC. After 5000 km 12-channels 56 Gb/s NRZ-QPSK, coupled NLSE is solved with D-folded DBP. The dispersion-managed system with D-folded DBP outperforms the dispersion-unmanaged system with EDC by 1.3 dB in Q-value. The computational load for the D-folded DBP is less than 6 times of that for the EDC.

Since EDC only occupies a small portion of the DSP chip area, replacing the EDC with the D-folded DBP would increase the total chip area by a factor of much less than 6. Using the D-folded DBP method, fiber nonlinearity compensation for the deployed dispersion-managed long-haul WDM links is becoming practical. At the same time, dispersion-managed system becomes a competitive candidate for the next generation systems.
6.2. Future Work

6.2.1 Demonstration of Computationally Efficient DBP Using PDM System

We have presented several methods for the computationally efficient DBP for nonlinearity compensation including the complementary FIR filter pair, the variable step size for repeaterless Raman system, the XPM compensation using coupled NLSE, the distance-folded DBP and the dispersion-folded DBP. All of these methods have been demonstrated numerically or experimentally using single-polarized transmission systems.

The next generation 100G coherent transmission systems will probably use polarization-division-multiplexing (PDM) technique. The nonlinear effects in PDM systems are governed by the vectorial form of the NLSE as follows [9].

\[
\frac{\partial A_x}{\partial z} = -\frac{\alpha}{2} A_x + \frac{i\beta_x}{2} \frac{\partial^2 A_x}{\partial t^2} + i\gamma (|A_x|^2 + \frac{2}{3} |A_y|^2) A_x + \frac{i\gamma}{3} A_x^* A_y^2, \tag{6-1}
\]

\[
\frac{\partial A_y}{\partial z} = -\frac{\alpha}{2} A_y + \frac{i\beta_y}{2} \frac{\partial^2 A_y}{\partial t^2} + i\gamma (|A_y|^2 + \frac{2}{3} |A_x|^2) A_y + \frac{i\gamma}{3} A_y^* A_x^2. \tag{6-2}
\]

\( A_x \) and \( A_y \) are the two orthogonal polarization components of the electric field. The residual birefringence of the optical fibers randomly scatters the polarization. The fast change of the polarization state results in nonlinearity as an average over the entire Poincare sphere. The average nonlinear effects can be described with the Manakov equation given by [10]

\[
\frac{\partial A_x}{\partial z} = -\frac{\alpha}{2} A_x + \frac{i\beta_x}{2} \frac{\partial^2 A_x}{\partial t^2} + \frac{8i\gamma}{9} (|A_x|^2 + |A_y|^2) A_x, \tag{6-3}
\]
\[
\frac{\partial A_y}{\partial z} = -\frac{\alpha}{2} A_y + \frac{i\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} + \frac{8i\gamma}{9} \left( |A_y|^2 + |A_z|^2 \right) A_y.
\] (6-4)

After the coherent detection, the DBP can be performed by solving the Manakov equation. The DBP for PDM transmission has been experimentally demonstrated [11,12], where coupled NLSE was solved for XPM compensation.

The residual birefringence randomly scatters the electric field in lengths scales less than 100 m [13], which is much smaller than the typical DBP step size. In DBP of the PDM transmission, the average nonlinear effects in each step are described with the Manakov equation.

The linear operators for dispersion compensation for PDM transmission can be implemented using FIR filter. The complementary filter pair design method can be used to reduce the number of operations. Similar to single polarization system, the DBP step size for PDM transmission is also limited by the dispersion length and the nonlinear length. Variable step size can also be applied to PDM system since the nonlinear length is dependent on the signal power which changes along the fiber.

The folded DBP method can also be applied to PDM system because the optical field, except the polarization, also repeats at locations of identical accumulated dispersion. The effect of PDM on folded DBP should be similar to regular DBP described in [12].

We expect the computationally efficient methods including complementary filter pair design and folded DBP are effective for DBP of PDM transmission. In the
future, numerical or experimental demonstration of these methods can be performed in PDM systems.

6.2.2 Real-Time Implementation of DBP

We have experimentally demonstrated the effectiveness of DBP and the computationally efficient methods using coherent fiber transmission system and off-line digital signal processing. Using the dispersion-folded DBP method, the computational load for the DBP can be reduced to less than 6 times of that for the EDC in a 5000 km WDM system. Real-time implementation of EDC has been realized in high speed coherent systems. Since EDC only occupies a small portion of the DSP chip area, replacing the EDC with the D-folded DBP would increase the total chip area by a factor of much less than 6. Hence it should be possible that the DBP for long haul transmission be implemented using high speed ASIC chip.

EDC for each of the WDM channel does not require information from the neighboring channels. However, XPM compensation for WDM transmission requires the exchange of information between channels, which may increase the complexity of the ASIC design.

6.2.3 New Computationally Efficient DBP Methods

Using the Computationally Efficient DBP methods such as dispersion-folded DBP, the computational load for the DBP has been reduced by orders of magnitude. However, the computational load may be further reduced.
For example, in long-haul fiber transmission, the waveform evolution is dominated by chromatic dispersion. It is known that Gaussian pulses remain Gaussian during the propagation in dispersive fiber. By taking advantage of this unchanged pulse shape, a method may be developed to further simplify the nonlinearity compensation of long-haul transmission.

6.3. References


