Individual Identification Of Polar Bears By Whisker Spot Patterns

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INDIVIDUAL IDENTIFICATION OF POLAR BEARS
BY WHISKER SPOT PATTERNS

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Biology in the College of Sciences at the University of Central Florida Orlando, Florida

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ABSTRACT

Many types of ecological studies require identification of individual animals. I developed and evaluated an automated identification system for polar bears (*Ursus maritimus*) based on their whisker spot patterns. First, I measured the reliability of using whisker spot patterns for identification from polar bear photographs taken in western Hudson Bay. This analysis involved estimating the complexity of each whisker spot pattern in terms of its information content. I found that 98% of patterns contained enough information to be reliable, and this result varied little among three different observers. Based on these results, I implemented a computer-aided identification system for polar bears based on whisker spot pattern recognition. I used standard computer vision techniques to pre-process images and the Chamfer distance transform to compute similarity scores between images. In addition, I evaluated the system by testing the effects of photographic quality and angle on system accuracy. I found that excellent and moderate quality/angle provided best results, with system accuracy of 90-95%. These findings suggest that individual identification of polar bears in the field based on whisker spot pattern variation is possible. Researchers studying polar bear behavior or estimating population parameters should benefit from this noninvasive technique.
To my parents
ACKNOWLEDGMENTS

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INTRODUCTION

Individual identification of animals is necessary in many types of ecological studies (Nietfeld, Barret and Silvy, 1994). Estimates of population size, survival and reproduction rates, and migration rates typically involve identifying previously marked or sighted individuals (Nichols, 1992). In addition, many studies in behavioral ecology depend on recognition of individual animals as variability in individual behavior is important in our understanding of the evolution these behaviors (Martin and Kraemer, 1987; Hayes and Jenkins, 1997).

Identification techniques based on natural characteristics (e.g., coat marking patterns, facial scar patterns) have advantages to techniques based on artificial markings (Pennycuick, 1978), which require capturing and handling individuals. Natural markings, however, are not guaranteed to be unique among individuals in a study population (Pennycuick, 1978), and are impossible to use if they are difficult to see or are lacking altogether. Furthermore, as the number of identified animals grows, the effort and probability of error in matching new individuals also grow (Hiby and Lovell, 1990; Hillman et al., 2003).

The purpose of my research was to develop an automated identification system for polar bears (*Ursus maritimus*) based on whisker spot patterns, which were observed to vary considerably in polar bears (J. M. Waterman and J. D. Roth, unpubl. data). Before developing the identification system, however, I first examined whether whisker spot patterns could be used to reliably identify individual polar bears. Once I demonstrated that polar bear whisker spot patterns were sufficiently unique (Chapter 2), I then developed a computer-aided identification system based on whisker spot pattern recognition (Chapter 3).
Pennycuick and Rudnai (1970) devised an information-theoretic method to assess the reliability of identifying individual lions (*Panthera leo*) based on their whisker spot patterns. Their approach was later generalized for use in other species (Pennycuick, 1978). I used this generalized approach to measure the reliability of using polar bear whisker spot patterns as an identification method (Chapter 2). This method assessed the reliability of an identification system by measuring the information (i.e., complexity) contained in each natural pattern (Pennycuick, 1978). Conceptually, the lower the probability a pattern occurs in a population, the more information it contains and, thus, the more reliable it is.

Finally, I used standard computer vision techniques to implement and evaluate a computer-aided identification system for polar bears based on whisker spot pattern recognition (Chapter 3). In addition, I tested the effects of photographic quality and angle on the accuracy of the system, which I estimated based on the maximum allowed probability of error. Researchers studying polar bear behavior or population parameters should benefit from this system. Additionally, the techniques described here may be useful to those interested in implementing their own identification system for their study animal.
References


Introduction

Identification of individual animals in the field is often necessary for studies involving population dynamics, movement patterns, and animal behavior (Nietfeld, Barret and Silvy, 1994). For example, estimates of population size, survival and reproduction rates, and immigration and emigration rates using capture-recapture models involve identifying previously marked or sighted individual animals (Nichols, 1992). Research in behavioral ecology also depends on recognition of individuals because animals differ greatly in their individual behavior, and identifying this variability aids our understanding of the evolution of these behaviors (Martin and Kraemer, 1987; Hayes and Jenkins, 1997).

Methods for identifying individual animals can be categorized as (1) invasive or (2) noninvasive. Invasive methods rely on artificial markings, such as ear tags, neck collars, transponders, tattoos, tissue removal, dyes, and chemical or radioactive markers (Nietfeld et al., 1994). These methods are very reliable as they afford unambiguous identification (Pennycuick, 1978), and are quite useful in studies where animals are routinely handled for physical measurements (e.g., mass or blood samples) or when noninvasive identification is unfeasible. However, applying such markers possibly could affect the behavior of handled animals (e.g., Rodda et al., 1988; but see Borges-Landaez and Shine, 2003), and if the study does not otherwise require capture and restraint, the difficulty and expense of such methods may be prohibitive.
Noninvasive methods of identification rely on natural markings, such as facial and body scars or coloration (e.g., Pennycuick and Rudnai, 1970; Pot and Noakes, 1985; Jarman et al., 1989; Miththapala et al., 1989; Bretagnolle, Thibault and Dominici, 1994; Gowans and Whitehead, 2001; Kelly, 2001; Eitam and Blaustein, 2002; Dixon, 2003; Bradfield, 2004), and thus minimize most drawbacks of invasive methods. However, they cannot guarantee that all individuals in a population will possess unique markings (Pennycuick, 1978), and are not feasible when natural markings are difficult to see or are lacking altogether. Nonetheless, noninvasive identification is a practical alternative to invasive methods, and has been used in estimating several population parameters (e.g., Hammond, Mizroch and Donovan, 1990; Karanth and Nichols, 1998; Langtimm et al., 2004; Stevick et al., 2006) and in studying animal behavior (e.g., Grinnell, Packer and Pusey, 1995; Mougeot, Thibaul and Bretagnolle, 2002; Dixon et al., 2006).

In this study, I examined whether whisker spot patterns could be used to identify individual polar bears (Ursus maritimus) as part of a long-term study of polar bear behavior in western Hudson Bay (Eckhardt et al., 2002; Eckhardt, 2005). Previous studies of polar bear behavior have used invasive identification methods (e.g., Latour, 1981b) or facial scars and body shape or size to identify individuals (e.g., Eckhardt et al., 2002; Dyck and Baydack, 2004). However, logistical constraints prohibited immobilizing and capturing free-ranging bears, and scars are not always present on bears and body shape or size may not be reliable. Field observations and photographs (J. M. Waterman and J. D. Roth, unpubl. data) suggest that patterns of whisker spots (small, dark, circular areas around whisker follicles distinctively arranged on each side of the anterior end of the muzzle) of polar bears may be sufficiently
distinctive to use for noninvasive identification of individuals, as has been found for other large-bodied mammals (Pennycuick, 1978).

A method of identifying individuals based on whisker spot patterns was first developed for lions (Panthera leo; Pennycuick and Rudnai, 1970). This study assessed the reliability of the method by measuring the information (i.e., complexity) contained in each pattern. Conceptually, the lower the probability a pattern occurs in a population, the more information it contains and, thus, the more reliable it is. This information-theoretic approach of assessing identification reliability was later generalized for use in other species (Pennycuick, 1978), and has been applied to whisker spot patterns on leopards (Panthera pardus kotiya; Miththapala et al., 1989) and to several traits on two macropod species (Jarman et al., 1989). However, the utility of an identification method depends on the proportion of the population with reliable patterns. Reliable whisker spot patterns were found in 92% of lions examined (Pennycuick and Rudnai, 1970), and although using additional characters (e.g., sex, scar patterns) would improve the reliability of identification (Pennycuick, 1978), there has been little discussion in the literature about the frequency of reliable patterns needed for this method to be used with confidence. In this study, I formulated a criterion for determining the utility of an identification method. Using information theoretic techniques (Pennycuick and Rudnai, 1970; Pennycuick, 1978), I show that polar bear whisker spot patterns can be used to reliably identify individuals, and thus could be used to develop a noninvasive identification system based on whisker spot patterns for use in studies of behavior and population parameter estimates.
Methods

Study site and photograph collection

Polar bears were photographed about 30 km east of the town of Churchill, Manitoba, Canada (58º 45’ N, 93º 45’ W). The Hudson Bay sea-ice melts in August, forcing polar bears to aggregate along the coast until freeze-up in mid-November (Latour, 1981b). Access to this site was facilitated by a tundra vehicle (a large bus adapted to travel on tundra), normally used for polar bear viewing by ecotourists (Dyck and Baydack, 2004). No more than 18 tundra vehicles were permitted in this 8 km² area, and polar bears rarely responded to the approach of these vehicles (Eckhardt, 2005) and were free to leave the area at any time.

Photographs were taken daily (09:00–15:00 h) by trained volunteers and myself during October 18–November 11, 2003, October 18–November 10, 2004, and October 18–November 10, 2005. Nikon (Melville, NY, USA) D100 6.0-megapixel digital cameras equipped with 70–300 mm and 80–400 mm lenses were used to photograph bears. Polar bears were individually identified by distinct facial scars, sex, and body shape and size. Several photographs of the same bear were taken at different angles as the bear moved, especially as facial profiles came to view. Bushnell (Overland Parks, KS, USA) Yardage Pro 1000 laser range-finders were used to measure the distance to bears for some photographs.

Whisker spot selection

For my analyses, I selected the best 50 polar bears based on their photographic quality (determined by focus, clarity, and resolution) and angle (determined by the extent a bear’s facial profile was perpendicular to the camera’s axis). Photographs were enhanced with Adobe (San Jose, CA, USA) Photoshop 7.0 to improve brightness and contrast, and were rotated and/or
flipped so that the front corner of the eye and the notch of the nose were aligned horizontally with the nose pointing to the right, creating the abscissa for a relative coordinate system where the eye was at the origin and the nose at 1.0 (Fig. 2.1).

Whisker spot locations were marked on the highest quality and best angle image for each bear; additional photographs of the same bear were sometimes available to confirm spots locations. Polar bear whisker spots are found each side of the bear’s anterior end of the muzzle,

Figure 2.1. Grid superimposed on polar bear photograph. The grid divides a spot pattern into characters, with each character having a value of either “present” or “absent,” depending on whether a spot is present in its corresponding cell.
between the nose and the upper lip, roughly aligned into three to four rows. Dark bands and spots that blend with the black upper lip were not considered whisker spots. To reduce statistical bias caused by possible correlation between whisker spot patterns on each side of a bear, only one side was used in my analyses. The relative location of each whisker spot was determined by dividing the x and y coordinates (in pixels) by the distance between the eye and the nose (in pixels).

**Information content and reliability of spot patterns**

A pattern must be divided up into mutually independent characters, each taking at least two values, before its information content can be calculated (Pennycuick, 1978). By fitting a regular grid on the relative coordinate system described earlier, every spot pattern was divided into characters, with each character having a value of either “present” or “absent”. The size of each grid cell was 0.05 × 0.05 (relative units), a conservative size determined by the maximum distance found between the same whisker spots on two different photographs of the same bear. Character A1, for example, denotes whether at least one spot is present within the cell defined by $0 \leq x < 0.05$ and $0 \leq y < 0.05$ (Fig. 2.1). Similarly, character B2 denotes whether at least one spot is present within $0.05 \leq x < 0.1$ and $0.05 \leq y < 0.1$, and so on (spots with $x > 0.95$ were not used in my analyses because whisker spots present on that region were difficult to distinguish).

As discussed by Pennycuick and Rudnai (1970), if a whisker spot is present in character $i$ in $n_i$ patterns, the frequency of occurrence $f_i$ of a spot for that character is defined as

$$f_i = \frac{n_i}{N}$$ (1)
where $N$ is the number of patterns (i.e., polar bears) in the sample. Assuming that the characters in a whisker spot pattern are mutually independent (an assumption I examine later), the probability of occurrence of the spot pattern in the study population is

$$p = f_a \times f_b \times f_c \times \ldots \times (1 - f_q) \times (1 - f_r) \times (1 - f_s) \times \ldots \quad (2)$$

where characters $a$, $b$, $c$, etc. of the pattern have spots, and characters $q$, $r$, $s$, etc. do not (Pennycuick and Rudnai, 1970). For each whisker spot pattern in the sample, the value of $p$ was calculated and expressed in terms of its information content $I = -\log_2 p$ (Pennycuick and Rudnai, 1970).

Identification is considered “reliable” if the probability that two or more indistinguishable individuals exist in the study population is less than some arbitrary value $\varepsilon$ (Pennycuick and Rudnai, 1970). Thus, the probability that at most one individual in the population has a particular pattern must be $> 1 - \varepsilon$ (Pennycuick and Rudnai, 1970). This relationship can be expressed as

$$(1 - p)^M + Mp(1 - p)^{M-1} > 1 - \varepsilon, \quad (3)$$

which represents the probability that at most one individual in a population of $M$ individuals has a pattern with probability $p$ (Pennycuick and Rudnai, 1970).

For a polar bear whisker spot pattern to be reliable, I required that its probability of duplication in the western Hudson Bay population of 1,000 polar bears (Regehr et al., 2005) be less than 1% ($\varepsilon = 0.01$). Using the above equation, the maximum value of $p$ was estimated to be $1.4862 \times 10^{-4}$ or, in terms of information, 12.72 bits. Hence, for an individual bear to be reliably identified in the study population, its whisker spot pattern must contain $> 12.72$ bits of information.
**Mutual independence of spots**

Calculation of the probability of occurrence of a spot pattern in a study population requires that all characters of the pattern be mutually independent (Pennycuick, 1978). A set of events \( E = \{E_1, E_2, \ldots, E_n\} \) is mutually independent if for every subset of the events, their joint probability is equal to the product of their individual probabilities (Larson, 1982). In other words, \( P(E_i \cap E_j) = P(E_i)P(E_j) \) must hold for all distinct \( i \) and \( j \); \( P(E_i \cap E_j \cap E_k) = P(E_i)P(E_j)P(E_k) \) must hold for all distinct \( i, j, \) and \( k \); and so on until \( P(E_1 \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n) \). To determine whether all characters of a spot pattern were mutually independent, I defined \( E_i \) as the event in which character \( i \) had the value “present.” Because there were not enough spot patterns to satisfy all combinations of spot occurrences required for a test of mutual independence, I only tested whether characters were pairwise independent, which is always satisfied when characters are mutually independent.

Therefore, I tested whether the joint probability of characters \( i \) and \( j \) having a value of “present” was equal to the individual probability of character \( i \) having a value of “present” multiplied by the individual probability of character \( j \) having a value of “present”. I called the joint probability “observed” because it was determined from the observed proportion of spot patterns that contained spots at locations \( i \) and \( j \), and I called the product of the two individual probabilities “expected” because it was determined from the spot probability distribution (Fig. 2.2). In addition, because each character could also have the value of “absent”, I tested for the events in which one or both characters in each pairwise comparison had the value of “absent.”
To test for significant differences between the observed and expected probabilities, I performed randomization tests (Quinn and Keough, 2002) for each possible pair of characters. I randomly generated 5,000 samples of 50 spot patterns such that each sample retained the probability distribution determined from the original sample of bears (Fig. 2.2). For each pair of characters in the randomized samples, I calculated their observed probabilities and determined the proportion that deviated from their expected probability at least as much as did the nonrandomized sample. If this proportion was < 0.01 (see Quinn and Keough, 2002), the true deviation between the observed and expected probabilities for that particular pair of characters was too great to be explained by chance, and so those characters were nonindependent.

Therefore, I eliminated the character that, on average, contributed the least amount of information to a pattern, and thus preserving the more useful character.
Utility of identification method

My *a priori* criterion for confidence in using this identification method was that > 95% of whisker spot patterns in the sample must be reliable (i.e., their information content was > 12.72 bits). To account for sample error, I also calculated the proportion of reliable patterns from 10,000 whisker spot patterns randomly generated from the sample spot probability distribution (Fig. 2.2).

Consistency of analyses

The best image selected for each polar bear and the whisker spots marked on each image were chosen by a judge (me). To test whether my analyses were contingent upon the observer who selected the images and marked the whisker spots, two additional judges were provided the same images that the first judge used. Like the first judge, they selected the image they thought had the highest quality and angle for each polar bear, and marked the locations of the eye, nose, and whisker spots on those images. Judges were allowed to use additional images for the same bear (if available) that the first judge used for confirmation of spot locations. For each set of whisker spot patterns, the same analyses were performed: a character set was derived and nonindependent characters were removed, the information content of each pattern was calculated for the sample and randomized patterns, and the proportions of those that were reliable were determined.

Results

Over 10,000 polar bear photographs were taken for all years combined, of which about 10% were appropriate for identification (i.e., polar bear’s face was clearly visible). Over 200 individual polar bears were identified based on facial scars, sex, and body shape and size. From
the 50 polar bears selected for this study, I chose 167 photographs of relatively high quality to use to identify whisker spots.

I found a total of 39 characters from the sample of whisker spot patterns (Fig. 2.2). Characters S6 and S4 had the highest probability of spot occurrence, 0.66, which indicated that 66% of polar bears had at least one spot within those cells. Thus, the presence of a spot within cells S6 or S4 adds only 0.60 bits of information to a pattern. Conversely, characters with spot occurrence probabilities of 0.02 indicated that only one polar bear had a spot within those cells, whose presence adds 5.64 bits of information to a pattern. The amount of information that other characters add to a pattern if a spot is present there can be calculated using the equation $I = -\log_2 p$, where $p$ is the corresponding probability value from Fig. 2.2.

Four randomization tests of pairwise independence were performed on all 741 possible pairs of characters in which: (1) characters $i$ and $j$ had the value of “present,” (2) character $i$ had the value of “present” and character $j$ had the value of “absent,” (3) character $i$ had the value of “absent” and character $j$ had the value of “present,” and (4) characters $i$ and $j$ had the value of “absent.” Based on each test, I found that 5 pairs of characters were nonindependent: N7-O7, R3-P6, P6-M7, Q6-S7, and S8-Q9. Thus, characters N7, R3, Q6, M7, and Q9 were removed from my analyses because they contributed less average information than their pair.

I found that 49 (98%) of 50 whisker spot patterns contained > 12.72 bits of information, which means they were reliable (for all patterns: median = 18.24 bits, range = 12.00–43.43 bits). Of the 10,000 generated patterns, 9,812 (98.12%) were reliable (for all patterns: median = 18.94 bits, range = 11.61–42.43 bits) (Fig. 2.3). Because both proportions were > 95%, I feel confident in this identification method.
For the whisker spot patterns from judge 2, I found 35 characters, but removed three due to nonindependence. For the 50 patterns, the median information content was 17.67 bits (98% were reliable) and, for the 10,000 generated patterns, the median information content was 17.60 bits (92.44% were reliable). For judge 3, I found 49 characters, but removed one due to nonindependence. For the 50 patterns, the median information content was 21.39 bits (100% were reliable) and, for the 10,000 generated patterns, the median information content was 22.31 bits (99.92% were reliable) (Table 2.1).

Figure 2.3. Probability distribution of information content for 50 polar bear whisker spot patterns and for 10,000 randomly generated spot patterns. The arrow indicates the minimum information content required for a pattern to be reliable.
Discussion

These results indicate that polar bear whisker spot patterns vary sufficiently to be used reliably to identify individuals. Thus, an identification system that takes advantage of the complexity of whisker spots will be successful. An information theoretic approach to measuring the reliability of whisker spot patterns has been used in previously. For example, 23 of 25 lions could be reliably identified assuming a probability of duplication of 1% and a study population of 50 lions, but if any unusual features on the two “substandard” lions were considered or the probability of duplication was relaxed to 1.5%, then all 25 lions could be reliably identified (Pennycuick and Rudnai, 1970). Similarly, 19 of 21 leopards could be reliably identified at a 5% probability of duplication, but only 15 at a 1% probability of duplication (Miththapala et al., 1989). Based on my results, 49 of 50 polar bears could be reliably identified at a 1% probability of duplication in a study population of 1,000 individuals. In addition, because > 95% of polar bear whisker spot patterns were reliable, I was confident in this identification method.

Differences in photograph and whisker spot selections among three judges did not affect my general results (one exception was that the proportion of reliable randomized patterns from
judge 2 did not quite meet the criterion for identification utility). Different images of the same bear were sometimes taken at different angles, so apparent positions of whisker spots varied slightly among images. Also, not all whisker spots within a pattern were equally discernible, so where one judge selected a faint spot another judge did not. These findings suggest that when measuring the reliability of a whisker spot pattern, one must (1) work with only high-quality images of perpendicular angle and (2) clearly define what should be considered a whisker spot so that future identification by multiple observers is consistent. These results corroborate with the findings of Friday et al. (2000), who recommend evaluating the reliability of photographic quality and ability of judges before using natural marks for identification.

An important difference in the derivation of characters in this study and others is that a reference row of whisker spots was used for lions (Pennycuick and Rudnai, 1970) and leopards (Miththapala et al., 1989) to position other spots relative to the reference row. While most polar bears clearly had three to four rows of whisker spots, I did not find a row that was consistent enough to be used as reference. In fact, the variation in the number and spacing of spots within a row added to the complexity of each spot pattern. As a result, I used a relative coordinate system and a regular grid to determine the location of each spot. Although the use of a grid could have introduced some discrepancies in the location of whisker spots if the bear’s profile was not exactly perpendicular to the camera viewpoint, the chosen grid cell size should have minimized any effect on the information content of each spot pattern.

In any identification system based on natural patterns, it is important that characters do not change over time (Pennycuick, 1978). There are at least three high-quality photographic records of known polar bears (identified through scar patterns and other body features) that have
returned to the field site in different years. Qualitative observations suggest that whisker spot patterns of the same bear do not change much from year to year. However, I do not know whether whisker spot patterns in polar bears change with the bear’s maturation or whether pattern similarities exist among related bears.

The use of high-quality photographs for the identification of individual whales has been shown to reduce the number of errors in photographic matching (Agler, 1992; Gowans and Whitehead, 2001). In addition, digital photography has improved the image quality and increased the efficiency of analyses in the identification of several species of dolphins (Markowitz, Harlin and Wursig, 2003). In this study, high-quality and perpendicular photographs allowed me to better discriminate between actual spots and shadows, and enabled me to discern spots that were close together. The use of digital cameras increased the number of images that could be obtained in the field, thereby increasing the probability of obtaining good photographs. Digital photographs also increased the speed at which they could be loaded into a computer for analysis while preserving their quality.

However, obtaining high-quality photographs in the field usually requires proximity to the focal animal. With the 400 mm camera lens, for example, I found that whisker spots were most distinguishable in photographs taken < 50 m from the polar bear. At distances of about 75–100 m, only the largest spots were visible, and at distances > 150 m, spots were too blurry to recognize. Close-up photographs of polar bears were possible because the tundra vehicle permitted us to safely approach bears. In practice, however, such flexibility is not always feasible. For example, other researchers typically observe polar bears from distances of about 200–1500 m, and usually from a fixed location (e.g., Knudsen, 1978; Latour, 1981a; Latour,
1981b; Lunn and Stenhouse, 1985; Lunn, 1986; Derocher and Stirling, 1990; Brook and Richardson, 2002; Dyck and Baydack, 2004). Because the reliability of identification depends on the recognition of whisker spots, I recommend using this system only with relatively high-quality photographs.

This study has shown that an identification system for polar bears based on the complexity of whisker spot patterns is reliable. The grid-based system described here could be used as an identification method, but would not be practical if whisker spots were not defined clearly or if photographs were not perpendicular to the camera viewpoint. In addition, such a system would be tedious and time-consuming if used manually. In light of my findings and recent successful automated identification systems for various taxa (e.g., Kelly, 2001; Arzoumanian, Holmberg and Norman, 2005), I developed a computer-aided identification system for polar bears based on whisker spot pattern recognition (Chapter 3). I anticipate this system will be useful to researchers interested in studying polar bear behavior or population parameter estimates based on capture-recapture models.
References


COMPUTER-AIDED IDENTIFICATION OF INDIVIDUAL POLAR BEARS BASED ON WHISKER SPOT PATTERNS

Introduction

In many types of ecological studies, individual identification of animals is essential (Nietfeld, Barret and Silvy, 1994). Identification techniques based on natural characteristics (e.g., coat marking patterns, facial scar patterns) eliminate most drawbacks of techniques based on artificial markings (Pennycuick, 1978). However, natural markings are not guaranteed to be unique among individuals in a study population (Pennycuick, 1978), and their reliability should therefore be evaluated. In addition, as the number of identified animals grows, the effort and probability of error in matching new individuals also grow (Hiby and Lovell, 1990; Hillman et al., 2003). Therefore, an increasing number of automated identification systems for various taxa have been developed (e.g., Hiby and Lovell, 1990; Mizroch, Beard and Lynde, 1990; Whitehead, 1990; Huele and de Haes, 1998; Kelly, 2001; Hillman et al., 2003; Burghardt et al., 2004; Arzoumanian, Holmberg and Norman, 2005; Foster, Krijger and Bangay, in press).

The purpose of any automated identification system is to inform the user the identity (e.g., identification number) of the animal in question (typically its photograph), if it has been identified before. To accomplish this task, the system compares the input image with every image in a reference library (i.e., a database of known individuals) and compute a “similarity score” for each comparison. A score above or below a “similarity threshold” should indicate whether the related comparison is a match or not. Although no system is perfect and visual
inspection is always necessary (Kelly, 2001), reducing this effort makes the identification system worthwhile (Hiby and Lovell, 1990).

Anderson, Roth and Waterman (in press) suggested that an identification system for polar bears (*Ursus maritimus*) based on the variation of their whisker spot patterns should be reliable. Because of the advantages of automated identification systems, I implemented and evaluated a computer-aided identification system for polar bears based on whisker spot pattern recognition. I used several standard computer vision techniques to extract whisker spots from an image and calculate similarity scores. In addition, I tested the effects of photographic quality and angle on comparisons of the same polar bear. Finally, I estimated the accuracy of the system based on the maximum allowed probability of incorrectly matching two different individuals. Researchers studying polar bear behavior or population parameters based on capture-recapture models may benefit from this system. Additionally, the techniques described here may be useful to those interested in implementing their own identification system for other species.

**Methods**

*Collection of polar bear photographs*

Polar bears were photographed about 30 km east of the town of Churchill, Manitoba, Canada (58° 45’ N, 93° 45’ W), on the western coast of Hudson Bay (see Anderson *et al.*, in press). Photographs were taken daily (09:00–15:00 h) during October 18–November 11, 2003, October 18–November 10, 2004, and October 18–November 10, 2005. Nikon (Melville, NY, USA) D100 6.0-megapixel digital cameras equipped with 70–300 mm and 80–400 mm lenses were used to photograph bears. Polar bears were individually identified by distinct facial scars, sex, and body shape and size. Several photographs of the same bear were taken at different
angles as the bear moved. Bushnell (Overland Parks, KS, USA) Yardage Pro 1000 laser rangefinders were used to measure the distance to polar bears for some photographs.

**Computer-aided identification system**

The polar bear identification system consists of three main components: the database, the image pre-processing method, and the matching algorithm. The database stores images of known polar bears, and is used to match new bears. The image pre-processing method automatically extracts whisker spots from an image by standardizing and enhancing the image. Finally, the matching algorithm computes the similarity score between two images. The user is only required to select three locations on an input image: the eye, nose, and mouth. These points are used by the system to automatically orient the image and find the whisker spot area. The identification system was written in Microsoft Visual C# Express Edition (.NET Framework 2.0).

**Image pre-processing**

The input image was converted to grayscale and standardized by an affine transformation such that the eye, nose, and mouth points selected by the user were relocated to (0, 0), (225, 0), and (133, 120), respectively (Fig. 3.1). These standard locations were obtained from a manually rotated polar bear photograph of typical size. The input image was then cropped around the whisker spot pattern area (Fig. 3.2a) and enhanced by (1) histogram specification and (2) logarithmic transformation (Fig. 3.2b) (Gonzalez and Woods, 2002). The histogram specification adjusted the histogram of pixel values of the image to match that of a manually enhanced image (cropped around the whisker spot pattern area). The logarithmic transformation applied a special logarithm function to every pixel value of the image, which enhanced dark pixels (i.e., whisker
spots). The cropped image was then smoothed by neighborhood averaging with radius 2
(Gonzalez and Woods, 2002) to remove fur details (Fig. 3.2c).

Whisker spots were extracted from the cropped image by thresholding, so each pixel was
changed to either black, if its value was less than a threshold, or white, otherwise. I did not use
global thresholding (i.e., one threshold for the entire image) because it failed when the
illumination varied across the image (Fig. 3.3a). Instead, I used adaptive thresholding (see
Appendix; Davies, 2004), which accounted for illumination gradients by dynamically changing
the threshold for each pixel based on the average value of the pixel’s neighborhood of radius 4
(Fig. 3.3b). In addition, I applied adaptive thresholding (see Appendix) on the image multiple
times while varying the constant parameter until the final number of black pixels on the image
was just under 300 (actual number varied slightly per image). I found that fixing the number of
black pixels accounted for different image qualities, and thus produced more consistent whisker

Figure 3.1. Image pre-processing (standardization). Input image was converted to grayscale and
standardized by an affine transformation using the eye, nose, and mouth points (indicated by the
crosses).
spot extraction. Finally, the image was cropped again to eliminate undesirable edge effects left by adaptive thresholding.

**Similarity score**

I used the Chamfer distance transform (Borgefors, 1986) to compute the similarity score between two pre-processed images. My implementation of the Chamfer distance transform (see Appendix) looked at every black pixel of the first image and calculated the distance to the nearest black pixel of the second image, and used the median of these distances as the score. Because this score depended on the image considered first, I also computed the similarity score of the images in reverse order, and calculated their average (i.e., undirected Chamfer distance).

![Figure 3.2](image1.png)  ![Figure 3.3](image2.png)

Figure 3.2. Image pre-processing (enhancement). (a) Input image was cropped around whisker spot area, (b) enhanced by histogram specification and logarithmic transformation, and (c) smoothed by neighborhood averaging.

Figure 3.3. Image pre-processing (extraction). (a) Global thresholding vs. (b) adaptive thresholding.
Finally, because photographs of the same polar bear were probably taken at different angles, the pre-processed images may not correctly align with one another, resulting in higher scores than expected. Thus, scores were computed many times while moving one of the images 16 pixels up, down, left, and right in 2 pixel increments, and the minimum of these scores was used as the final similarity score.

Reliability of identification system

Photographs were categorized according to angle (excellent, moderate, and poor) and quality (excellent, moderate, and poor) by trained volunteers and myself. Angle and quality of photographs categorized by volunteers were verified by me and re-categorized if necessary. Angle was based on the perpendicularity of a polar bear’s facial profile to the camera axis: excellent deviated < 15° away from the camera axis, moderate deviated 15°–30°, and poor deviated 30°–45° (photographs with angles > 45° were excluded because whisker spots were difficult to see). Degree deviations were estimated using a life-like ceramic polar bear model to match the photograph viewpoint of the bear’s face. Quality was based on focus, clarity, and resolution (see Kelly, 2001). Significance tests among categories were performed using a one-way ANOVA, and means (reported as mean ± S.E.) were compared using Tukey-Kramer HSD. Statistical significance was set to $\alpha = 0.05$ and statistical tests were conducted with JMP IN 5.1.
I used the identification system to compute similarity scores for different photographs of the same polar bear at various angle and quality categories. First, I examined the effect of quality on similarity score by comparing photographs of only excellent angle while varying the quality of one photograph (and leaving the other photograph with excellent quality). Also, I examined the effect of angle on similarity score by comparing photographs of only excellent quality while varying the angle of one photograph (and leaving the other photograph with excellent angle). Furthermore, I computed scores for photographs of different polar bears, which served to estimate system accuracy. I defined system accuracy as the proportion of matches that were

![Bar chart](image)

**Figure 3.4.** Mean similarity scores of comparisons of different images of the same polar bear when image angles are excellent and the quality of one image varies (left bars), and when qualities are excellent and the angle of one image varies (right bars). Numbers above bars indicate sample size, error bars represent standard error, and bars that share a line (bottom) are not statistically different (Tukey-Kramer HSD).
below the similarity threshold, determined based on the desired probability of incorrectly matching two different bears (i.e., a false positive). I estimated system accuracy based on two similarity thresholds that resulted in a 1% and 5% probability of obtaining a false positive. Finally, I averaged the scores of comparisons of the same pair of bears with identical image quality and angle, and thus avoided pseudoreplication of image comparisons.

Results

Over 10,000 polar bear photographs were taken for all years combined, of which about 10% were appropriate for identification (i.e., polar bear’s face was clearly visible). Over 200 individual polar bears were identified based on facial scars, sex, and body shape and size.

I found a significant difference among similarity scores for the three quality categories (excellent: $0.70 \pm 0.14, n = 11$; moderate: $0.98 \pm 0.11, n = 18$; and poor: $1.57 \pm 0.19, n = 6$; $F_{2,32} = 6.67, P = 0.004$), where excellent and moderate quality did not differ, but poor quality differed from the other groups (Tukey-Kramer HSD) (Fig. 3.4). I also found a significant difference among similarity scores for the three angle categories (excellent: $0.70 \pm 0.14, n = 11$; moderate: $1.22 \pm 0.16, n = 13$; and poor: $1.45 \pm 0.20, n = 9$; $F_{2,30} = 4.37, P = 0.022$), where excellent and moderate angle and moderate and poor angle did not differ in scores, but excellent and poor angle differed from each other (Tukey-Kramer HSD) (Fig. 3.4). In general, I found that similarity scores increased as quality or angle worsened from excellent to poor. For the rest of the analyses, I used photographs of only excellent and moderate angle and quality as the scores between these photographs were not significantly different.
The mean similarity score for comparisons of different images of the same bear was 1.06 ± 0.06 ($n = 58$); for comparisons of different bears, the mean score was 2.45 ± 0.02 ($n = 556$) (Fig. 3.5). The similarity threshold based on a 1% probability of obtaining a false positive was 1.53, resulting in 90% system accuracy (i.e., proportion of same-bear comparisons that were below the similarity threshold). Based on a 5% probability of obtaining a false positive, the similarity threshold was 1.95, resulting in 95% accuracy (Fig. 3.6).
Discussion

The computer-aided identification system was very accurate (> 90%) and robust to moderate angle and quality photographs. However, photographs of poor angle or quality were not reliable, reaching similarity scores close to the similarity threshold, making them almost
indistinguishable from photographs of different polar bears. Photographs with poor angle foreshortened the bear’s muzzle, which compressed its whisker spot pattern and therefore resulted in strong image misalignment. Similar conclusions were found by Arzoumanian et al. (2005), who found that as whale sharks (*Rhincodon typus*) moved away from the camera, foreshortening distorted the spot patterns on the sharks, causing mismatches for photos > 30°. Low-quality photographs of polar bears created errors in the extraction of whisker spots, effectively changing the pattern. Quality is difficult to compare among studies because its definition is largely subjective, and quantitative measures of quality have been difficult to establish (Hillman et al., 2003; but see Friday et al., 2000). Nevertheless, I concur with the recommendation by others to use photographs of even poor quality and angle (e.g., Kelly, 2001; Hillman et al., 2003; Arzoumanian et al., 2005), as it is unlikely that an individual will not be matched with any photograph of itself (Kelly, 2001). In fact, having polar bear photographs taken at various angles would allow a wider range of future photographs to be recognized.

Because users are mostly concerned with avoiding missing a match, system accuracy is typically measured as the proportion of correct matches. A correct match is a comparison of the same individuals whose similarity score satisfies a certain threshold. This threshold is usually chosen such that the probability of obtaining a false negative is low, but this requirement inevitably confers high system accuracy. Thus, I specifically defined the similarity threshold such that the probability of false positives satisfies some low value. I estimated the accuracy of the system for two different desired probabilities of false positives, 1% and 5%, resulting in 90% and 95% accuracy, respectively. Thus, one can expect that from a database of 200 bears, one would need to visually cross-check about 10 bears (for 95% accuracy) to make sure a true match
is not missed. Fig. 3.6 shows that as the proportion of correct matches of the same bear increases, the proportion of incorrect matches of different bears also increases. In other words, there is a trade-off in the two types of error: a low probability of false negatives corresponds to a high probability of false positives and vice versa. Another approach in choosing a similarity threshold is to minimize both error types, which is where the lines cross in Fig. 3.6. Regardless, I recommend that future evaluations of system accuracy choose a similarity threshold based on a desired probability of obtaining false positives, or at least report this probability at the chosen threshold.

False negatives are clearly caused by poor angle and quality in photographs, but false positives are likely caused by similarity in whisker spots of different polar bears. I found that some bears had slight differences in the positions or presence and absence of one or two spots. Because the identification system was robust (e.g., moving images many times to find minimum score), these slight differences were not seen by the matching algorithm, and so images of different, but similar, polar bears obtained a low score. As images of the same bears may also have slight differences in positions due to angle or differences in spot numbers due to noise (e.g., dirt on whisker spot area), it was desirable that these differences were not seen by the system. Here is another trade-off between the probability of obtaining false negatives and false positives.

I found that photographs taken < 50 m from a polar bear had excellent and moderate quality. The identification system was not very robust to poor quality photographs, so I am unable to guarantee high system accuracy unless excellent and moderate photographs are provided. Anderson et al. (in press) indicated that this constraint may preclude effective use of the system in remote locations, where attaining proximity to polar bears is difficult. However,
improvements in digital photography (e.g., 16-megapixel cameras are now available) and optical lenses should allow more distant polar bears to be reliably identified.

The identification system took up to 5 s to calculate a similarity score for a comparison of two images. Thus, matching one photograph to a database of 200 would take about 17 min. Because < 10 new polar bears were observed in the field per day, the system is fast enough to allow use in the field (i.e., on a laptop). Inputting a photograph into the computer took < 1 min, mostly spent in choosing the eye, nose, and mouth of the polar bear. In comparison, the system by Arzoumanian et al. (2005) took about 10 minutes to input a photograph because the user was required to perform most of the pre-processing (i.e., rotation and image enhancement). However, the system by Burghardt et al. (2004) did not require any user input, as it recognized the location of spot patterns on African penguins (*Spheniscus demersus*) automatically through sophisticated computer vision techniques, and thus permitted identification from a video stream.

The computer vision method of comparing whisker spot patterns in the system was based on image-to-image comparison using Chamfer distance transform. Other systems have used “blob extraction” to transform a set of spots into a set of \((x, y)\) points (e.g., Burghardt et al., 2004; Arzoumanian et al., 2005) and then apply a point pattern matching algorithm to compute a similarity score. I chose an image-to-image comparison because polar bear whisker spots are not so well-defined as to obtain consistent point patterns. For example, I often found that small or faded spots would sometimes not come out on the pre-processed image, and noise due to fur texture would sometimes show up as superfluous black pixels. In addition, image-to-image comparison was desired because part of the variability in whisker spots was in the size of spots, and such information is not contained in coordinate points.
Before an identification system based on natural markings is developed, it is important to know whether the use such natural markings will be reliable (Pennycuick, 1978). The success of the system described here is attributed to the fact that whisker spot patterns were shown to be variable enough to reliably identify individuals (Anderson et al., in press). At the same time, the results of the system support the claim that whisker spot patterns are virtually unique. In addition, it is also important that natural markings do not change over time (Pennycuick, 1978), as this could cause a match to be missed (Kelly, 2001). Qualitative observations suggest that whisker spot patterns of the same bear did not change much within my study period. However, I do not know whether whisker spot patterns in polar bears change with the bear’s maturation or whether pattern similarities exist among related bears. Burghardt et al. (2004) found that chest spots on juvenile penguins were not stable, and so could not use their system to identify them. Kelly (2001) found no difference between similarity scores of related cheetahs (*Acinonyx jubatus*) than unrelated cheetahs.

An automated identification system should not be expected to perfectly match individuals (Kelly, 2001), but to greatly reduce the time it takes to match them manually (Hiby and Lovell, 1990). Thus, the user may have to browse through several potential matches to find the right one (Hillman et al., 2003). Consequently, in addition to providing the similarity score between two images, I sorted the scores in ascending order such that the most likely matches were listed first. In addition, for each potential match, the system provided a way to automatically align the input image to any database image based on the chosen anchors, which helped to visually recognize whisker spots and other facial features.
In conclusion, the automated identification system described here was very accurate at matching polar bears that were the same, while discounting comparisons of bears that were different. The system required very little user input, and therefore reduced the probability of error due to observer inexperience in photo-identification. Thus, hours of user training in evaluating photographic quality or distinctiveness (Friday et al., 2000) were also eliminated. In addition, this system is currently being used in continuing studies of polar bear behavior (e.g., Eckhardt, Waterman and Roth, 2002), and several bears have been identified that were unknown due to lack of distinct markings. This system may also be useful in capture-recapture studies, provided that photographs are of good quality and angle. Finally, I believe this system could be extended for use in other species displaying facial or coat patterns, assuming the patterns are reliable.
References


CONCLUSION

Researchers who study or manage polar bear populations often must identify individual bears in the field. But polar bears can be difficult to tell apart. Consequently, researchers often apply artificial marks, such as ear tags and body paint, on polar bears for future identification. Applying artificial marks, however, requires immobilizing and handling the bears—a process that may be difficult, expensive, and even harmful to the animals. Thus, some researchers use instead a variety of natural marks—facial scars, sex, and body shape and size—to recognize individual polar bears. Unfortunately, not all bears have scars, the sex of an individual is often difficult to determine, and body shape and size are unreliable identifiers.

In the first part of this study, I examined whether individual polar bears could be identified by the pattern of their whisker spots. I calculated the information content, a mathematical way of measuring complexity, of 50 whisker spot patterns, which I obtained from photographs of 50 different polar bears. I then calculated the minimum amount of information required for a pattern to be reliable, and used that amount to determine the proportion of patterns—from the 50 I selected and 10,000 I randomly generated—that were reliable. Finally, I tested the consistency of my results by repeating my analyses with whisker spot patterns (of the same 50 polar bears) selected by two other judges. I found that about 98% of whisker spot patterns were reliable, which meant that whisker spots are sufficiently distinct to identify individual polar bears.

Polar bear whisker spots are, however, complex. Consequently, identifying individual bears based on their whisker spots would be tedious. In the second part of this study, therefore, I
developed an automated system of identification for polar bears. I used standard computer vision
techniques to extract the whisker spots from an input photograph and compute a similarity score
with each known polar bear in a reference library. In addition, I developed a way of assessing the
meaning of a similarity score based on the probability of a matching error. I also examined how
photographic quality and camera angle affected similarity scores. I found that system accuracy
was about 90-95% and that photographs of excellent and moderate, but not poor, quality and
angle were the most reliable.

The identification system described here should be useful to researchers who can obtain
polar bear photographs of good quality. In behavioral studies near tourist areas, where proximity
to polar bears is often achieved, good quality photographs should not be difficult to obtain.
Finally, the methods described here should be useful to those who wish to develop their own
identification system for their study species.
APPENDIX: PSEUDOCODE
Adaptive thresholding

Input: integer array $A$ (grayscale image); integer $r$ (neighborhood radius); integer $c$

Output: an integer array $B$ (black and white image)

Set $B =$ integer array with dimensions of $A$

For $i = 1, \ldots$, width of $A$

For $j = 1, \ldots$, height of $A$

Set mean = mean of $A_{i,j}$ neighborhood ($r$)

If $A_{i,j} < mean - c$ then

Set $B_{i,j} = \text{black}$

Else

Set $B_{i,j} = \text{white}$

End if

End for

End for

Notes: Input image $A$ was $225 \times 120$ pixels in size, neighborhood radius $r$ was 4 pixels, and constant $c$ varied (see iterative adaptive thresholding).
Iterative adaptive thresholding

Input: integer array $A$ (grayscale image); integer $r$ (neighborhood radius); integer $\text{maxcount}$

(maximum number of black pixels desired)

Output: an integer array $B$ (black and white image)

$\text{set } c = 0$

$\text{set } \text{count} = \text{infinity}$

$\textbf{while } \text{count} > \text{maxcount}$

$\text{set } B = \text{adaptive thresholding } (A, r, c)$

$\text{set } \text{count} = \text{number of black pixels in } B$

$\text{set } c = c + 1$

$\textbf{end while}$

Notes: Input image $A$ was $225 \times 120$ pixels in size, neighborhood radius $r$ was 4 pixels, and $\text{maxcount}$ was 300 pixels.
Chamfer distance

Input: integer array $A$ (black and white image); integer array $B$ (black and white image)

Output: a real $s$ (similarity score)

set $mindistances = \text{real array of size equal to the number of black pixels in } A$

set $n = 1$

for $i = 1, \ldots$, width of $A$

    for $j = 1, \ldots$, height of $A$

        if $A_{i,j} =$ black then

            set $mindistance =$ infinity

        for $k = 1, \ldots$, width of $B$

            for $l = 1, \ldots$, height of $B$

                if $B_{i,j} =$ black then

                    set $distance =$ Euclidean distance between $(i, j)$ and $(k, l)$

                    if $distance < mindistance$ then

                        $mindistance =$ distance

                    end if

                end if

        end for

    end for

$mindistances_n = mindistance$

$n = n + 1$
end if

end for

end for

set \( s \) = median of \( \text{mindistances} \)

Notes: Images \( A \) and \( B \) were 225 \( \times \) 120 pixels in size.