The Nature Of Turbulence In A Narrow Apex Angle Isosceles Triangular Duct

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THE NATURE OF TURBULENCE IN A NARROW APEX ANGLE ISOSCELES TRIANGULAR DUCT

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy
in the Department of Mechanical, Materials, and Aerospace Engineering
in the College of Engineering and Computer Science
at the University of Central Florida
Orlando, Florida

Fall Term
2007

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ABSTRACT

An experimental investigation was performed to ascertain the nature of turbulence in a narrow apex angle isosceles triangular duct. The study involved the design and construction of a low noise, low turbulence wind tunnel that had an isosceles triangular test section with an apex angle of 11.5°. Experiments involved the measurement of velocity fluctuations using hot wire anemometry and wall pressure fluctuations using a condenser microphone.

Measurement of the velocity fluctuations reconfirms the coexistence of laminar and turbulent regions at a given cross section for a range of Reynolds numbers. The laminar region is concentrated closer to the apex while the turbulent region is found closer to the base. The point of transition is a function of the Reynolds number and moves closer to the apex as the flow rate is increased. Moreover, it was found in this investigation that traditional scaling of the turbulent statistical quantities do not hold good in this geometry.

Although velocity fluctuations showed distinctive flow regimes, no such distinction could be seen in the dynamic wall pressure data. The nature of the dynamic wall pressure was uniform throughout the entire cross section suggesting that wall pressure fluctuations, unlike the velocity fluctuations, are able to travel from the base to the apex, without being damped. This implies that the relationship between the velocity and the pressure fluctuations applicable in the other systems does not hold well in a narrow apex angle isosceles triangular duct. Further, the typical scaling relationships applied to wall pressure spectra of other geometries doesn’t apply in this scenario and the
ratio of the RMS pressure fluctuation to the mean shear is much higher compared to a flat plate or pipe flow situation.
ACKNOWLEDGMENTS

Several people have contributed to the success of this work and it gives me great pleasure to acknowledge them. First and foremost, I thank Professor Jayanta Kapat for serving as my academic advisor and for providing me this opportunity. In his capacity as my mentor, he has been a constant source of inspiration, and has offered me the very much needed insight and direction. His stimulating suggestions and encouragement have helped through my entire graduate work and I am highly indebted to him. I also acknowledge the financial support he provided during my course of study.

I would also like to thank Professors Ranganathan Kumar, Ruey Hung Chen, Eric Petersen and Bhimsen Shivamoggi for being kind enough to serve in my committee. They have been extremely cooperative and accommodating, particularly during the scheduling of my various presentations.

I am grateful to Ernesto, Mobin and Sameer for their help during the construction of the wind tunnel facility. Without their sincere efforts, this project could not have been completed in a timely fashion.

I am extremely grateful to my friends Sylvette, Chris, Quan, and Dr. Bharani, who have been with me since the day I started my graduate studies. I also like to acknowledge the support and companionship of my other lab mates. They have played a pivotal role in making my stay at UCF an enjoyable experience.

I would also like to express my gratitude to my friends outside school for helping me maintain my sanity.
The financial support given by Siemens Westinghouse Power Corporation (SWPC) in the form of Fellowship during my first three years in UCF is also acknowledged.

Last, but most important, I express my gratitude to my family for their continuing love and encouragement. I dedicate this dissertation to them.
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NOMENCLATURES

\( A \) area

\( D_h \) hydraulic diameter of duct

\( d \) diameter of hot wire sensor

\( dp \) pressure drop between two consecutive wall pressure taps

\( f \) frequency

\( Gr \) Grashof number

\( g \) acceleration due to gravity

\( H \) altitude of the triangular duct

\( h \) co-ordinate along the altitude of the triangle

\( m \) mass flow rate

\( P \) perimeter of duct

\( PSD \) power spectral density

\( p \) fluctuating pressure

\( R \) turbulent cross correlation

\( Re \) Reynolds number

\( Re_{D_h} \) Reynolds number based on hydraulic diameter

\( Re_{dw} \) Reynolds number based on hot wire diameter

\( Re_\tau \) shear Reynolds number

\( T \) static temperature

\( U \) time averaged x-component of velocity

\( u^* \) friction velocity

\( u \) fluctuating x-component of velocity
$x$ co-ordinate measured along the flow direction
$y$ co-ordinate measured along the side wall of the triangle
$z$ co-ordinate measured normal to the side wall of the triangle

**Greek Symbols**

$\alpha$ half apex angle
$\beta$ coefficient of volume expansion for air
$\mu$ dynamic viscosity
$\nu$ kinematic viscosity
$\rho$ density
$\tau$ shear stress

**Subscripts**

$a$ air
$av$ circumferentially averaged for shear and area averaged for velocity
$c$ cross-section
$crit$ critical
$hw$ hot wire
$max$ maximum value
$rms$ root mean square
$w$ wall
CHAPTER 1: INTRODUCTION

The phenomenon of transition from laminar to turbulent flow has been a subject of ever increasing interest since O. Reynolds[1] first performed his classical dye injection studies. Ever since, researchers in fluid dynamics have tried various methods to understand the complex flow physics associated with the process of transition. Innumerable experiments have been conducted to identify the reason for such a transition and to attribute a criterion, if any, for such a phenomenon to occur. The experimental investigations span a wide variety of geometries, both internal and external flows. External flow investigations range from flat plates to complex airfoil geometries. Traditional internal flow experiments have been on cylinders with circular cross sections. However, as the need arose for usage of non-circular cross sections of ducts in engineering applications, the flow characteristics in these ducts were investigated. These investigations also included square and triangular ducts, as ducts of these cross sections find wide use in heat exchangers and other applications.

Transition and turbulence play a vital role in many engineering applications. Although laminar flows are more convenient for mathematical investigations, they are not common in practice. Laminar flows tend to become turbulent as the flow rate is increased beyond a particular value, making them an exception rather than a rule. Conversely, when the flow rate is decreased, the flow transforms from turbulent to a laminar one. The point of demarcation between these kinds of flow is called laminar-to-turbulent transition or, in short, transition. Transition plays a very important role in many design processes, wherein it helps in deciding the usefulness of certain transport-enhancing schemes[2].

The first systematic study on transition was performed by O. Reynolds[1] for flows through a straight pipe. He injected dye into the flow stream and observed the path of the dye. He
found that as the flow rate was increased, the filament of dye changed from an orderly, well-defined thickness to a state of complete mixing, as shown in Figure 1.1. Reynolds correlated this transition from the orderly laminar state to the “sinuous” turbulent state with the non-dimensional parameter $\frac{U_{av} D}{\nu}$ (now known as the Reynolds number, Re). The value of the Reynolds number at which such a transition occurs is called the critical Reynolds number ($Re_{crit}$). Accordingly, flows for which the Reynolds number, $Re < Re_{crit}$, are supposed to be laminar and flows for which $Re > Re_{crit}$, are expected to be turbulent. He found that the numerical value of the Reynolds number at which transition occurs, for circular pipes, was approximately 2300. However, he also noticed that the numerical value of $Re_{crit}$ is very sensitive to the conditions which prevail in the initial length of the pipe as well as in the approach to it. He even thought that the critical Reynolds number would increase if the disturbances in the flow before the pipe are decreased. Barnes and Coker[3] confirmed this fact in their experiments on water flow through pipes.

Many experiments have ever since been reported wherein people have reported laminar flow at very high Reynolds numbers by carefully controlling the external conditions. Ekman[4] succeeded in maintaining laminar flow up to a critical Reynolds number of 40,000 by carefully providing an inlet which was made exceptionally free from disturbances. Albeit many such experiments have been performed, the upper limit to which the critical Reynolds number can be pushed if extreme care is taken to free the inlet from disturbances is not known at present. However, there exists, as demonstrated by numerous experiments, a lower bound of $Re_{crit}$ for plain circular ducts, which is approximately 2000 (Schlichting[5]). For Reynolds numbers below this value, the flow remains laminar even in the presence of severe external disturbances.
For a plane Poiseuille flow, it has been possible to maintain laminar flow at a Reynolds number as high as 12000 (Nishioka et al[6]), by reducing the level of external disturbances and the intensity of fluctuation at the entrance. However, the lower limit of Reynolds number below which turbulence was not observed is about 1500 (Potter and Foss[7], Shah and Bhatti[8]). Moreover, any modifications of the flow geometries mentioned above may change the critical Reynolds number. Kapat[2] did a systematic study of the impact of eddy promoters on the transition to turbulence in rectangular channels and their implications on the augmentation of heat transfer. He found out that depending on the pattern of the placement of the eddy promoters (six different arrangements), the Reynolds number values at transition could be significantly reduced. The critical Reynolds number (based on the average velocity and the channel height) ranges from 1500 for a plain channel to about 400 for the most unstable eddy promoter configuration. He further correlated the demarcation of transition using a Reynolds number based on the spatially averaged mean wall shear stress, Reₜ, and found that for all the cases tested the value was between 40~60. Further, the role of transition on the enhancement of heat transfer
is also discussed. Hence it can be seen that the presence of external disturbances and the modifications of flow geometry have a pronounced effect on the critical Reynolds number. Although there is no theory to predict transition to turbulence under all circumstances, stability analysis does provide some estimate for critical Reynolds number corresponding to transition to turbulence in certain types of flow. These predicted numbers, although not in perfect agreement with the experimental results, are of the right order of magnitude and are useful in engineering calculations.

In the internal flow systems discussed above, laminar-to-turbulent transition and subsequently fully turbulent flow is typically an entire cross-sectional phenomenon. Generally, the entire cross-section goes through the phase of laminar-to-turbulent transition and eventually becomes completely turbulent. However, in a narrow apex angle triangular duct, as pointed out by researchers[9], there is coexistence of both laminar and turbulent regions in the same cross-section, i.e. part of the flow is laminar and the remaining is either transitional or fully turbulent at a particular cross-section. These narrow apex angle triangular ducts find application in compact heat exchangers and in gas turbine internal cooling channels as shown in Figure 1.2. Therefore, it is necessary to understand the flow physics inside this special duct geometry as a stepping stone to understand their heat transfer characteristics. As the type of flow, laminar or turbulent, significantly affects the heat transfer characteristics, it becomes necessary to thoroughly understand the nature of flow before one proceeds to understand the transfer of heat.

(a)
The objective of this work was to experimentally study the nature of turbulence inside a narrow apex angle isosceles triangular duct. The study involved the design and construction of a low noise, low turbulence wind tunnel that had an isosceles triangular test section with an apex angle of 11.5°. Experiments involved the measurement of velocity fluctuations using hot wire anemometry and wall pressure fluctuations using a condenser microphone.

Chapter 2 is devoted to the review of past literature. We start with the literature on narrow apex angle triangular ducts and introduce the phenomenon of co-existence of laminar and turbulent flow in the same cross section. We then discuss the impact of this phenomenon on the pressure drop and heat transfer characteristics of these ducts reported by other researchers. Section 2.2 deals with the literature on wall pressure fluctuations. The difference in the nature of wall pressure fluctuations between laminar, transitional and turbulent flow and their physical implications are discussed.
The detailed design and construction of the wind tunnel facility is given in Chapter 3. A separate section in this Chapter deals with the construction of the anechoic chamber and its subsequent characterization.

In Chapter 4 we deal with the measurement procedure. Detailed description of the instrumentation used, data acquisition hardware/software and the data reduction methodology is provided in this Chapter.

The obtained results are discussed in detail in Chapter 5 and are compared with available data in the literature. The significant features of the results are pointed out. Attempts were made to apply existing scaling procedures to the current data.

Finally, in Chapter 6, the conclusions of the experimental results are presented.
CHAPTER 2: LITERATURE REVIEW

2.1 Triangular Ducts

In the past there have been quite a few investigations on narrow apex angle isosceles triangular ducts, particularly after Eckert and Irvine [9] found an interesting flow phenomenon inside these triangular ducts. They found that in these ducts there is coexistence of laminar and turbulent regions in the same cross section. In their work, they investigated two isosceles triangular ducts with apex angles, 11.5, 24.8 degrees and a height to base ratio (aspect ratio, AR) of 5 and 2.56 to 1, respectively. Flow-visualization techniques (cigar smoke), longitudinal pressure-drop measurements and velocity field characterization (total and static pressure probes) had been used to investigate the two ducts over a range of Reynolds numbers. Flow visualization studies showed that over a large range of Reynolds number, both laminar and turbulent flows exist side by side within the duct. They found that the laminar portion is more towards the apex and decreases with increasing Reynolds number as shown in Figure 2.1. The damping properties of the shear flow near the triangle apex are so pronounced that they do not permit the large turbulent fluctuations to spread into the corner region.

Following this investigation, Eckert and Irvine [11], analyzed the pressure drop and heat transfer characteristics inside a triangular duct. They found that the friction factors in the laminar flow region agreed well with analytical predictions. However, in the turbulent flow range they were 20% lower than the values calculated using circular tube correlations with the use of the “hydraulic diameter” as can be seen from Figure 2.2. The heat transfer coefficients averaged over the circumference of the duct, as shown in Figure 2.3, were only half as large as values calculated from round tube relations in the Reynolds number range of 4300 to 24000. Their
measurements also revealed that the thermal starting lengths were in excess of 100 diameters, while for the round counterpart it has been found that 10 to 20 diameters were sufficient to develop the temperature field.

![Figure 2.1: Transition in a triangular duct (Dh is the hydraulic diameter)[9]](image)

Subsequently, Carlson and Irvine [12] performed experiments to characterize the pressure drop in these ducts. They investigated isosceles triangles with 5 different apex angles. One of the objectives of this investigation was to check the validity of the concept of hydraulic diameter used in non circular cross sections. They found that the friction factors predicted theoretically (using the concept of hydraulic diameter) matched very well with the experimental data in the laminar region. However, they were off by 20% for the smaller apex angles in the turbulent regime in accordance to the finding of Eckert and Irvine [11]. Hence they proposed a correlation for the friction factor in order to incorporate the effect of the apex angle and found that the friction factor was proportional to $Re^{-0.25}$, where the constant of proportionality was a strong function of the apex angle. Figure 2.4 shows the variation of the constant of proportionality as a function of the apex angle.
Figure 2.2: Friction factors of a fully developed flow in a narrow apex angle isosceles triangular duct [11]

Figure 2.3: Average cross section Nusselt numbers [11]
In just over a decade after Eckert and Irvine[9] pointed out the coexistence of laminar and turbulent regions in a cross section of a narrow apex angle triangular duct, Hanks and Brooks[13]
performed a flow visualization study with an optically birefringent solution of milling yellow dye in water flowing through a transparent duct of isosceles triangular cross section. The work was performed to re-examine the conclusions drawn by earlier researchers[9], which did not agree with the then recent theoretical predictions[14] and to study the influence of bent injection needles on a flow field.

Based on theoretical considerations using a stability theory developed by Hanks[14], they predicted the following: Figure 2.5 is a schematic representation of the isosceles triangular cross section. The maximum velocity should occur around the centroid, here point M. They reported that the portion symmetric about the line joining the centroid to the apex, regions A in Figure 2.5, should become unstable to disturbances and undergo transition to turbulence at a mean velocity for which region B was still stable and the region closer to the base should be laminar for apex angles < 30 deg. For apex angles > 30 the situation reverses and the base, location B, becomes unstable and turbulent, while locations A are stable. Since they were trying to verify earlier finding on triangular ducts, the duct investigated had an apex angle of 12.4 deg, similar to the one used in [9]. For this configuration, the authors observed no turbulence at Reynolds numbers less than 1200. In the process of transition from laminar to turbulent flow, turbulence is initiated at the apex region, above the point of maximum velocity. Laminar and turbulent flow co-exist for a range of Reynolds numbers with turbulence occurring at the apex, while the zone near the base is laminar. Although the converging walls of the apex portion exert significant damping, turbulent fluctuations do penetrate all the way into the apex.

These findings contradict those reported in [9]. Moreover, they claim that the previously reported dual zone was a result of the influence of the smoke injection probe on the flow. Such
probes intrude into the flow and cause wakes and the smoke downstream could be capturing the wake and can be wrongly interpreted as turbulence.

Subsequently, Cope and Hanks[15] presented hot-wire anemometer traces in both isosceles (30° apex angle) and equilateral triangular ducts in order to explain the characteristics of the friction factor. They argued that the existence of a dual flow region implies that the frictional resistance of the duct should be increased over that due to laminar flow because of the turbulence present, but decreased from the turbulent value because of the laminar flow present. However, the hot wire data revealed that even at a Reynolds number of 1515, there was no sign of turbulent bursts, i.e. the entire cross-section was uniformly non-turbulent. From these observations it was concluded that the original model of a “two regime” flow ([9] & [13]) in the triangular cross section is essentially incorrect and that the observed increase in the frictional resistance data must be ascribed to a different phenomenon. Further they attribute this increase in frictional resistance to the presence of secondary circulation superimposed upon the primary flow in the transition region of the triangular ducts. The proof of presence of secondary flow is obtained indirectly from the displacement of the maximum velocity position from the altitude towards the corner region.

Although the work presented by Cope and Hanks[15] provided very useful data on the frictional resistance and also demonstrated the usefulness of Hanks’ stability parameter, it created an ambiguity regarding the co-existence of laminar and turbulent flow. In order to resolve this ambiguity Bandopadhayay and Hinwood[16], performed hot-wire anemometry studies in a narrow apex angle isosceles triangular duct with an apex angle of 3.63 degrees. The hot wire traces reported showed the coexistence of laminar and turbulent regimes with the laminar region being closer to the apex. Although the region closer to the apex is termed laminar,
they found some weak fluctuations and hence suggested that the region be appropriately called viscous layer. They also reported the point of transition obtained using an intermittency meter. It is found that the point of transition varies as the inverse cube root of the pressure gradient, which is depicted in Figure 2.6. Based on this finding, using the laminar velocity profile obtained in [9] and a critical value of a stability parameter, they found that the friction factor varied as the inverse fourth root of the Reynolds number. This further led to the deduction of the location of transition to be proportional to $Re^{-7/12}$.

![Figure 2.6: Location of transition point $x_c$ as a function of pressure gradient $dp/dz$ along the duct; log scales[16]](image)

Although there have been many investigations on narrow apex angle isosceles triangular ducts, there have been only two investigations to the author’s knowledge so far which experimentally studied the turbulence structure in these ducts. Cremers[17] performed an experimental investigation of the turbulence characteristics using single wire and X-wire sensors.
He investigated two Reynolds numbers, namely 5480 and 10900 (both based on the hydraulic diameter). He found that the usual turbulence quantities, viz. root mean square velocity fluctuations, do not scale with the friction velocity u* at the lower Reynolds number. At the higher Reynolds number, he did find that the scaling was correct but claims that the result has to be considered fortuitous. In spite of the agreement with Laufer[18] at the higher Reynolds number the investigation concludes that the findings of Laufer[18] do not apply to three dimensional geometry. Hiromoto et al.[19] applied an electrochemical method to obtain the overall and local mass transfer coefficients. They also measured the time averaged and fluctuating wall shear stress and based on these measurements they ascertain the change in the turbulent characteristics along the side wall. The decrease in the averaged heat transfer coefficient, as reported in[11], was interpreted based on these measurements. Turbulence intensities and length scales near the wall were indirectly obtained from the shear stress measurement. They observed that a damping of the turbulence intensity and the increase of the turbulent scales near the wall over the whole side wall may correspond to the decrease in the overall or local mass transfer coefficient.

From the above discussion, it can be seen that there is little knowledge available in the open literature in regards to the structure of turbulence viz. intensities and spectral information in a narrow apex angle isosceles triangular duct. A systematic study of the phenomenon of laminar to turbulent transition and the nature of the turbulence structure would provide a better understanding of the flow physics inside these ducts. Moreover, there still exists a controversy between the theoretical prediction of Hanks’[14] and the experimental observations of Eckert & Irvine[9] and Bandopadhayay & Hinwood[16]. Although both the experiments showed similar behavior with regards to the co-existence of laminar and turbulent region, it is not in agreement
with the theoretical calculations performed by Hanks using his established stability parameter. Since the nature of the flow--laminar, transitional or turbulent-- has a major influence on the head loss and heat transfer characteristics of the flow, the understanding of the flow physics would be of practical as well as theoretical value.

2.2 Wall Pressure Fluctuations

In the turbulent or transitional regime, the flow exhibits fluctuations of the pressure field, which are particularly predominant at the wall. These pressure fluctuations induce local flow separation, which in turn produce eddies at the wall. Eddies so generated are carried into the mainstream and go through the cascading process and produce turbulence. Once turbulence is initiated, there is a continuous flow of eddies from the wall to the turbulent core. Thus it is seen that the pressure fluctuations play an important role in the mechanism of turbulence production and sustenance. In the past 30+ years there have been many investigations on the fluctuating pressure field in a turbulent boundary layer that were motivated by the desire to improve our understanding of the structure of the turbulence or to provide data needed in the solution of practical engineering problems.

The pressure field in a turbulent flow is produced by the summation of contributions from the turbulent velocity fluctuations. It is a well known fact that in incompressible flows the pressure fluctuations are related to the velocity fluctuations through Poisson’s equation, obtained from the divergence of the momentum equation.

\[
\frac{\partial^2 p}{\partial x_i^2} = -\rho \frac{\partial^2 (u_i u_j)}{\partial x_i \partial x_j}
\]  

(2.1)

The examination of the integral representation of the solution of the above equation, led to the fact that the pressure fluctuations at one point were produced by momentum fluctuations at
many other points. This led to the conclusion that the pressure at a given point will not be highly
correlated with the velocity at any one neighboring point. Batchelor[20] studied the correlation
between fluctuating pressures at two different points in a field of isotropic turbulence. He was
able to express the pressure correlation in terms of the fourth moment of the velocity
fluctuations, but wasn’t able to proceed further due to the inability to theoretically determine the
turbulent velocity field. The wall pressure and the pressure field in the boundary layer are more
complicated than the pressure field in isotropic turbulence. The complication comes due to the
fact that the turbulent velocity field in the boundary layer is anisotropic and the mean velocity
varies rapidly with distance normal to the wall. Kraichnan ([21], [22]) was the first to report
theoretical estimates of the mean-square wall pressure and spectra. He assumed that the turbulent
flow is homogeneous in planes parallel to the wall. Based on this assumption he arrived at a
result which predicted the root-mean-square (rms) wall pressure fluctuations to be of the order of
six times the mean wall shear stress, a value that had turned out to be much more accurate than
one might expect.

The first measurements of wall pressure fluctuations beneath turbulent boundary layers in
a wind tunnel were reported by Willmarth[23] and on an airplane wing by Mull & Algranti[24].
Willmarth[23] found that the ratio of rms wall pressure to dynamic pressure was approximately
0.0035, while Mull & Algranti[24] found that the ratio decreased as speed increased, becoming
constant and equal to 0.0013 above Mach number 0.5. A comparison of these measurements with
Kraichnan’s[22] theoretical estimate shows that the value reported in [23] was much closer than
used Skinner’s[26] time correlator to measure the longitudinal space-time correlation of the wall
pressure fluctuations produced beneath the boundary layer. Willmarth[25] used a specially
constructed low-noise wind tunnel with a sonic throat ahead of the diffuser. The quantity measured was the space-time correlation coefficient of the wall pressure:

\[
R(x, \tau) = \frac{p(x', t)p(x' + x, t + \tau)}{\sqrt{p^2(x', t)p^2(x' + x, t + \tau)}}
\]

(2.2)

Where \( p \) is a stationary, homogeneous random variable and \( x \) and \( x' \) represent vectors in the plane of the wall. Willmarth’s[25] measurements of \( R \) in the stream direction showed the convection and decay of the wall pressure fluctuations. These results played a very profound impact on practical application as it was realized that a convected pressure pattern on an aircraft skin (for example) might excite modes of vibration leading to the generation of noise. Moreover, it was clear that the convected pattern of wall pressure fluctuations must be related to the convected velocity fluctuations in the boundary layer.

Early investigations of Harrison[27] and Willmarth[25] established the fact that wall pressure fluctuations beneath turbulent boundary layers are convected past the measuring station at a speed of the order of 8/10 of the free-stream speed. At the time these measurements were made, interest in the results was very great for two reasons. It was realized that a convected pressure pattern might excite certain modes of vibration leading to the generation of noise. In addition, it was clear that the convected pattern of wall pressure fluctuations must be related to the convected velocity fluctuations measured by Favre et al. ([28], [29]) in the boundary layer.

Willmarth and Wooldridge[30] found that the convection velocity of the pressure fluctuation is dependent on the size of the fluctuation. Figure 2.7 shows the longitudinal space-time correlations in narrow frequency bands. The measurements show that for a band centered at low frequencies the convection velocity is higher than for a band centered at high frequencies. The primary reason for this is that the larger convected eddies are responsible for the majority of
the low-frequency contributions to the space-time correlation and they move more rapidly because they extend to larger distances from the wall where the mean velocity is higher. One can also approximately measure the decay of eddies of various sizes using space-time correlation measurements in narrow frequency bands. Figure 2.8 shows the results of early measurements of this type made by Willmarth and Wooldridge[30] in which the decay is scaled using the convection velocity, and longitudinal separation. An important interpretation of the results in Figure 2.8 is that an eddy of a given size decays, as it is convected, in a distance proportional to its size.

Figure 2.7: Convection velocities: Experiments by Willmarth & Wooldridge[30]; Theory and figure from Landahl[31]
A theoretical estimate of the observed convection and decay of wall pressure fluctuations has been carried out by Landahl[31]. Landahl’s calculations use the non-homogeneous Orr-Sommerfeld equation for the behavior of linearized disturbances in a shear flow. The shear flow velocity distribution used in the analysis is the mean velocity profile in the turbulent boundary layer. This profile admits only stable damped disturbances. In order to obtain convection velocities, Landahl assumed that the cross-spectral density of the pressure attains its largest contribution from the least attenuated mode of a disturbance and that the most important modes were those propagating normal to the stream. The convection velocity and decay of the least attenuated low order modes were determined by use of a numerical computation method. Landahl’s results are also shown in Figure 2.7 and Figure 2.8 and indicate reasonable agreement with the experimental results.

The nature of the pressure fluctuations in the transitional regime, on a flat plate, has also been investigated. The statistics of the pressure fluctuations in a transitional regime are
essentially stationary in time, but inhomogeneous in the stream-wise direction. Fundamentally it has been argued that this region is capable of creating monopole sound radiation (Lauchle[32]). Also it has been suspected that a transitional boundary layer can induce wall vibrations. In order to address these issues, Lauchle and Josserand[33], performed a set of measurements on the space-time statistics of turbulent spots in a naturally occurring transition zone and from them developed an analytical model for the wavenumber-frequency spectrum of the pressure fluctuations. Based on this model, it appears that the transition zone wall pressure is less intense than that of a fully developed turbulent layer by a factor equal approximately to the intermittency factor. Thus the pressure fluctuations in a transitional boundary layer are significantly different from a completely turbulent boundary layer. The pressure fluctuations caused by this spatially bounded, and intermittent, transition phenomenon encompass a very wide range of spatial wave numbers and temporal frequencies. Lauche and Park[34], based on their measurements of wall pressure fluctuations on a transitional boundary layer, found that transition induces higher low-streamwise wave number wall pressure levels than does a fully developed turbulent boundary layer that might superficially exist at the same location and at the same Reynolds number. Their results suggest that transition may be more effective than the turbulent boundary layer in forcing structural excitation at low Mach numbers, and it may have a more intense radiated noise contribution on a per unit area basis.

The extensive literature on pressure fluctuations in the boundary layer has increased our knowledge on the characteristics of the transitional and turbulent boundary layer. These pressure fluctuations exist in transitional and turbulent flows, although they are significantly different from each other. However, no such fluctuations exist in the perfectly laminar regime. Everything is completely steady with respect to time in laminar flows. Although there are many
investigations on the behavior of the pressure fluctuations in a turbulent boundary layer, there is very little reported on internal flow geometries. Of particular interest is the behavior of the pressure fluctuations in a narrow apex angle isosceles triangular duct, where all three regimes, laminar-transition-turbulent exist at the same cross-section. Due to this special phenomenon found in these ducts some interesting questions arise. Do the pressure fluctuations, generated in the turbulent region dissipate while crossing over to the laminar one? If so what causes this dissipation. If not, what happens with those pressure fluctuations when they reach the wall of the laminar region? The current investigation is an attempt to find the answers to the above raised questions by measuring the wall pressure fluctuations. These measurements would help in understanding the nature of the flow inside these ducts. Moreover, it may pave the way for better understanding the frictional, heat transfer and acoustic characteristics inside a narrow apex angle isosceles triangular duct, which might be of immense help to a practicing engineer.
CHAPTER 3: EXPERIMENTAL FACILITY

The objective of this investigation was to map the turbulence quantities, including the turbulence intensity, velocity spectra and the wall pressure spectra in a cross section of a narrow apex angle triangular channel, for a range of Reynolds numbers. In order to achieve this goal, a low noise, low turbulence wind tunnel with a narrow apex angle triangular cross-section was designed and fabricated. The setup is explained in detail in the following sections.

3.1 The Wind Tunnel

The wind tunnel layout (schematic: top view) and a picture of the wind tunnel facility is shown in Figure 3.1. The facility was designed in such a way that the test section is under suction, to prevent blower noise from entering the test section. The open loop wind tunnel layout consists of three separate sections: an upstream section consisting of a honeycomb, screens and triangular ducts, an anechoic chamber test section, and a downstream section which includes a triangular-to-circular transition piece, a muffler, and the blower assembly. The design of these sections was influenced by three criteria: low-noise, low-turbulence, and a hydro-dynamically fully developed condition at the test section. The details of the design concept and the subsequent fabrication methodology are as follows.
3.1.1 Honeycomb

A honeycomb is typically used as a flow-straightening device. It helps in breaking down large-scale eddies and swirls which may be present at the inlet. However, it has also been reported ([35],[36]) that the honeycombs also help in attenuating the transmission of diffuse sound field. Thus, in addition to its use as a flow-straightening device, a honeycomb can be used to diminish noise transmitted through the inlet. However, the sound attenuating property of the honeycomb is highly dependent on the material used to make the honeycomb. Accordingly, a honeycomb made out of polypropylene was chosen as it not only helps in straightening the flow but is also effective in sound and vibration dampening. A commercially available (Plascore Inc. ®) polypropylene honeycomb is used in this investigation. The honeycomb is housed in a frame with a triangular cross section similar to the test section thereby making sure that the honeycomb is in the lowest dynamic head cross-section and hence minimizes the pressure drop.
3.1.2 Screens

Honeycombs are usually followed by a set of screens which further help in reducing the large eddies to small ones, which dissipate rapidly [37]. For the screens not to generate turbulence, the Reynolds number based on wire diameter must be less than about 70 [38]. Moreover, imperfections in screens have been reported to produce slowly decaying fluctuations [37]. Based on the above criteria, a set of three screens with wire diameter less than 0.2 mm was used in this investigation. This resulted in a Reynolds number (based on wire diameter and average velocity) of 52 for the largest flow rate tested. Similar to the honeycomb, the screens are also held in place by frames of triangular cross-section.

3.1.3 Ducts

Ducts made of acrylic are used to allow the flow to develop hydro-dynamically before it reaches the test section as one of the major design criteria is to have a fully developed velocity profile at the test section. Figure 3.2 shows the 3-D model of the ducts. The acrylic ducts were made of three different pieces (representing the three sides of the triangle). These three pieces were assembled in a triangular shape using a custom made aluminum frame (3-D model shown in Figure 3.2) and held in place by three wedges, for the three sides of the duct. The aluminum frame was machined using a CNC mill. This methodology ensured that the apex angle made by the assembled triangular duct was 11.5°. Eckert and Irvine [11] suggest that a length of more than 80 hydraulic diameters is necessary for the flow to be fully-developed for a duct having a narrow apex angle triangular cross-section. In the current investigation, the test section is around 158 hydraulic diameters away from the inlet of the ducts. Such length of duct work ensures that the flow is fully developed before it reaches the test section. Figure 3.4 shows a sample duct
assembly held by the aluminum frames, and Figure 3.5 shows a picture of three triangular ducts assembled together.

Figure 3.2: 3-D Model of the triangular ducts
Figure 3.3: External aluminum frames

Figure 3.4: A sample duct assembly
3.1.4 Test Section

The test section is a narrow apex angle triangular duct with the apex angle \(2\alpha\) of 11.5°, made out of acrylic. The assembly process of the test section is similar to the other triangular ducts. The assembled test section has a base of 0.0309m (1.18 in) and sides measuring 0.15875m (6.25in) which give an aspect ratio of 5. The test section has its own flanges and hence can be easily removed for modifications. As one of the objectives of this investigation was to ascertain the nature of the velocity fluctuations both along the triangle altitude and normal to the side walls, the test section was equipped with measurement ports on the side walls and the base of the triangle. Figure 3.6 shows the actual test section with the measurement ports plugged. The dummy plugs were made of dowels whose ends were smoothened using fine grit sand paper and were snugly fit into the ports. It was ensured that the plugs were flush with the inside wall of the duct. The outside of the plugs was properly sealed with silicone to prevent air leaks. The other measurement objective of this research was to ascertain the nature of the wall pressure fluctuations. To accomplish this objective the test section had to be housed in an anechoic chamber. The detailed description of the anechoic chamber is provided in section 3.2.
3.1.5 Transition Piece

From the test section, the flow goes through a triangular-to-circular transition piece. The transition piece was cut out of foam using a hot wire cutter. The foam transition piece is housed inside a MDF (Medium Density Fiber) box. One inch thick sound absorbing foam was sandwiched between the transition piece and the MDF box in order to acoustically insulate the transition piece from external noise.

3.1.6 Muffler and Blower Assembly

After the transition piece, the flow goes through a muffler. It is a regular muffler found in a car. It helps in attenuating low frequency noise from entering the test section. With the use of a muffler there is an extra pressure drop in the tunnel. However, the blower has been oversized to overcome this pressure drop.

The flow through the test rig is supplied by a Spencer® Vortex® regenerative blower which can supply a flow rate of 108 scfm and can handle a pressure drop of 47 inches of water under suction. The blower sits on a 3/8 inch thick vibration damping pad which prevents any vibration from reaching the floor. The blower is housed inside a wooden box. The box is made of...
½ inch thick MDF wood. The inside of the box is acoustically sealed with one inch thick acoustic foam with an overall NRC (Noise Reduction Coefficient) of 0.9. This insulation absorbs most of the noise produced by the blower and hence the noise level outside the box is minimal. The entire MDF wooden box is placed on cinder blocks and is located outside the building, preventing any propagation of sound to the test section. Figure 3.7 shows the blower assembly. The blower is completely isolated from the test section and the other ducts using a ½ inch thick, 2 inch inner diameter and 8 inch long neoprene tube thereby reducing the vibration transferred by the blower to the test section.

![Blower Assembly](image)

The blower assembly, the muffler and the transition piece are connected to one another using 2 inch inner diameter PVC tubing. This ensures the smooth flow of air from the test section to the blower (since the test section is under suction). The PVC tubing is around 13 feet long to make sure that the blower assembly is located far from the test section, which in turn reduces the
intensity of the sound propagating toward the test section. Provision was made in the piping to
the blower to insert venturi meters to measure flow rates. The test rig is also equipped with a gate
valve to control the flow rate.

3.1.7 Support Structure

All the parts of the test rig, except for the blower assembly, are well supported by a
support structure which includes straight as well as cross members made of 2x4 lumbers. The
support members are nicely anchored to the concrete floor using Tapcon® screws. The legs of
the support structure are decoupled from the floor by the use of 2 inch thick foam pieces which
prevent vibration from traveling to the test section.

3.2 Anechoic Chamber

One of the major parts of this investigation was the design and construction of an
anechoic chamber to enable the accurate measurement of wall pressure fluctuations. For any
meaningful acoustic measurement, it is desirable to have the signal level to be at least 6dB[39]
above the background noise level. An anechoic chamber can be achieved by the classical wedge
treatment applied to the wall, ceiling and floor surrounding the test section or by using thick
fiberglass blankets slightly away from the walls, floor and ceiling. The advantage of using the
wedge treatment is that the cut-off frequency of the signal is sharp when compared to the
inexpensive alternative of hanging fiberglass blankets.

Figure 3.8 shows the picture of the anechoic chamber. The chamber is made of sound
insulating wedges called Max sound blocks purchased from Netwell Noise Control®. These
blocks are 1 foot X 1 foot square open cell polyurethane foam panels. The blocks are 6 inches
deep which makes the sound wave travel a greater number of times through the material before
escaping, thereby providing maximum dissipation. These foam panels, each arranged perpendicular to its neighboring panels, are glued to half inch thick plywood. The entire plywood assembly is held together by screws, except for one side which acts as the door to access the test section. All the sides, except for the top, of the chamber are at least a foot and a half away from the measurement plane. The exterior of this chamber is covered with 3 inch thick fiberglass. Figure 3.9 shows the external view of the anechoic chamber.

Great care was taken in making sure that no exterior noise enters the chamber from wind tunnel structure and electrical conduits. The two test section penetrations into the chamber are sealed using acoustical foams, including the ¼ inch penetration used for the microphone power lead. Moreover, the wind tunnel parts downstream of the test section are all acoustically sealed using fiberglass.
3.2.1 Anechoic Chamber Characterization

As mentioned earlier, in order to obtain meaningful wall pressure data, the background noise levels in the chamber should be at least 6dB lower than the actual signal measured. The main source of data pollution in experiments like these will be the noise produced by the blower itself. In the current facility, extra care has been taken to minimize this noise by placing the blower outside the room and also completely decoupling the blower from the rest of the test rig. Moreover, the entire test facility has been rigidly supported to prevent any vibration from polluting the data. In spite of all these precautionary measures, there would still be noise entering the chamber. Therefore, it was necessary to actually characterize the chamber performance under different blower operating conditions, to quantify the levels of background noise present in the
chamber. This background noise is to be compared to expected values of wall pressure fluctuation levels and make sure that they are within acceptable levels.

In order to accomplish this, a condenser microphone (measurement technique described in detail in Chapter 4) was used to ascertain the sound pressure levels at different operating conditions. All the microphone measurements were made inside the chamber but away from the duct. Firstly, the sound level measurement was made with the blower turned off. This is referred to as the “Background” noise. Then a measurement was made with the fan turned on, but the gate valve completely closed. In this situation, there is no flow going through the facility (confirmed by the zero differential pressure reading in the venturi). This noise level is given the name “Fan-no flow-background”. Later on the gate valve was completely opened to allow the maximum flow possible through the test facility and the sound pressure level recorded was named “Fan-full flow-background”. Figure 3.10 shows the sound pressure levels in decibels plotted versus the frequency for all the above mentioned cases. It can be clearly seen that the sound levels inside the chamber do not change for a range of frequencies between the different operating scenarios. Also shown in the same plot are measurements made at the location S2 (located at 1.45 inches from the base of the triangle on the side wall) on the duct. Two measurements were taken, one when the fan was on but the gate valve was closed (S2-with fan no flow) and the other when the gate valve was open such that the flow was turbulent. Figure 3.11 schematically shows the two measurement locations i.e. background and S2. As can be seen from Figure 3.10, the desired sound pressure measurement is more than 30dB at the lower frequencies. Overall sound pressure levels are at least more than 10dB for the entire range of frequencies shown here. It should be noted here that all the decibel calculations are based on a reference pressure of 20μ-pascal.
Figure 3.10: Anechoic chamber characterization; Sound pressure levels
Figure 3.11. Condenser microphone location for tunnel characterization
CHAPTER 4: MEASUREMENT PROCEDURE

Experiments were carried out in the wind tunnel described in the previous chapter to ascertain the velocity and pressure fluctuations in a narrow apex angle isosceles triangular duct. In this chapter, the instruments used to measure these flow parameters are described in detail along with the data acquisition and reduction procedures.

4.1 Instrumentation and Data Acquisition

The air temperature in the duct is measured and monitored using a type K thermocouple which is inserted through a hole in the base of the triangular test section. The thermocouple is connected to an Omega® HH203A digital meter which directly displays the temperature. Static pressures were measured using the TSI® Model 8710® micro-manometer capable of measuring pressures of 0.001 inches of water in the differential mode. This manometer was used to measure both the local static pressure as well as the differential pressure between the pressure ports.

Flow rates through the test facility are measured using two different flow meters. A McMillan® thermal mass flow meter is used to measure the very low flow rates (the low Reynolds number cases), while a Preso® venturi meter, in conjunction with an Omega® differential pressure transducer is used to measure the larger flow rates (the high Reynolds number cases).

Digital data acquisition was done using two different computers, one each for velocity and pressure fluctuation measurements. The velocity measurements were done using a National Instruments® 16 channel PCI 6034E® card which was connected to a SCXI 1001 chassis with a SCXI 1100 A/D converter module in conjunction with a SCXI-1303 accessory. This card is capable of sampling at frequencies up to 200 kHz. The data is collected using an in house
software program written in Python and stored for later processing. The dynamic pressure measurements were made using a Measurement Computing® PCIMDAS-16JR 16-channel data acquisition board capable of sampling data at a maximum frequency of 100 kHz. The data is collected using a Labview® program and stored for post processing.

Velocity fluctuations are measured using a TSI® model 1261A-T1.5 hot wire sensor. It is used in conjunction with a TSI® IFA-300 Constant Temperature Anemometer. The anemometer is controlled using the TSI® Thermal Pro® software. This software enables one to adjust the operating temperature of the sensor. It also helps in setting the low pass filter frequency while acquiring data. Before being put to use for acquiring velocity data in the test rig, the hot wire sensor is calibrated using a TSI® Model 1125 calibrator which can be used to calibrate for velocities ranging from 0.1 m/s to 300 m/s.

Wall pressure fluctuations were measured using a calibrated condenser microphone. The microphone used in this investigation is Model 7016 ACO Pacific, Inc® condenser microphone cartridge. This is used along with a Model PS9200 power supply system capable of providing two levels of preset output gains (20 and 40 dB). The microphone cartridge is used in conjunction with a Model 4016 preamplifier.

**4.2 Measurement and Data Reduction**

4.2.1 Measurement of Static Pressure at the Wall

The static pressure was measured at 7 different locations, closer to the test section using wall pressure taps. Figure 4.1 shows the location of these pressure taps in the test facility. The wall pressure taps were made of 1/16\textsuperscript{th} inch stainless steel tubes, which were mounted, flush on the walls. Before the ducts were assembled, holes were drilled from the inside out. This ensures that no burrs are formed on the inside of the duct that could potentially cause errors in the
measurement. Once the pressure taps were installed, they were sealed on the outside with silicone to prevent air leaks. Nylon tubing from the pressure taps is directly connected to a Model 8710 TSI® micro-manometer. This measurement serves two purposes: 1. it indicates whether or not the flow is fully developed entering the test section, and 2. it is used to calculate the average wall shear stress from the pressure gradient. The first pressure tap was located about 90 hydraulic diameters from the entrance of the test facility, and the subsequent taps were located 6 hydraulic diameters apart from each other. As the flow was expected to be fully developed by about 80 hydraulic diameters[11], the static pressure profile is expected to be a straight line.

Figure 4.1: Static pressure taps
4.2.2 Measurement of Velocity Fluctuations

Turbulent velocity fluctuations were measured using a TSI® 1261A-T1.5 hot wire sensor in conjunction with a TSI® IFA-300 constant temperature bridge. The TSI® hot wire system is shown in Figure 4.2. The sensor works on the principle of cross-flow convective heat transfer from a heated cylindrical element, which forms a part of a Wheatstone bridge circuit. The details of the working of such systems can be found in Bruun[40]. Before making use of the hot wire sensor in the actual test rig, the sensor is calibrated over a range of velocities inside a TSI® model 1125 calibrator. This calibrator is capable of providing a wide velocity range starting from as low as 0.01 m/s to 300 m/s. Since our flow ranges from about 0.3 m/s to 17 m/s, the sensor was calibrated from 0.1 m/s to about 25 m/s using the model 1125 calibrator. The calibration curve is shown in Figure 4.3. Once the sensor has been calibrated, it can be used in the actual test rig to measure the velocity fluctuations.

Figure 4.2: TSI® IFA 300 thermal anemometry system
Using the calibrated hot wire sensor, turbulent velocity measurements were made along the altitude of the triangular duct from the base of the triangle to the apex. Only the longitudinal component of the velocity was measured in this experiment. The probe was carefully positioned and traversed along the center line using a two axis traversing system. The traversing system is a linear stage purchased from Servo Systems® which is powered by a high torque, NEMA23 stepper motor obtained from Applied Motion Products®. The linear stage is a model LPS-18-20, shown in Figure 4.4, which has a total travel length of 18.77 inches. The bi-directional repeatability of this stage is 0.0005 inches. The motor and in turn the linear stage is controlled using a Model 3540i® stepper controller which is powered by an external power supply. The stepper controller, shown in Figure 4.5, connects to the PC through the serial port and is controlled by a SCL® utility program supplied by the manufacturer. Two such stages were used.
perpendicular to each other. This system helps in accurately positioning the hot wire sensor for obtaining high quality data.

Since we deal with very low flow rates, the effect of natural convection could be a source of error for the hot wire sensor data. However, according to the studies of Collis and Williams[41], buoyancy effects are important in air only if

\[
Gr > Re_{d_w}^3, \text{ where}
\]

\[
Gr = \frac{\beta g(T_{hw} - T_d) d^3}{\nu^2}
\]
A typical value of $\text{Gr/Re}^3$ in our experiments is $1.5 \times 10^{-4}$, which suggests that the effects of free convection can be neglected.

### 4.2.3 Measurement of Wall Pressure Fluctuations

Wall pressure fluctuations were measured using a PS9200 kit obtained from ACO Pacific®. Figure 4.6 shows the complete PS9200 kit. The kit comes with a model 4016 pre-amplifier, a PS9200 2 channel precision power supply, a Model 7016 ¼ inch microphone cartridge and a CA4012-5 cable which is terminated in a 5 pin male XLR. The output from the microphone system is directly fed into the data acquisition system through a BNC cable. The microphone system was calibrated by the manufacturer and hence no in house calibration was performed.

![Figure 4.6: Condenser microphone kit](image)

The cartridge is a condenser microphone which is shown in Figure 4.7. The microphone operates on the principle that when a sound pressure deflects the diaphragm, it changes the capacitance between the diaphragm and a flat electrode (back plate) parallel to the diaphragm. This change of capacitance is converted to and electrical signal by maintaining a constant charge on the capacitance. The output voltage thus varies and is proportional to the sound pressure.
The condenser microphone is flush mounted on one of the sides of the triangular duct to obtain the wall pressure fluctuations. Pressure measurements are made at 8 different locations along the side wall. Table 4.1 lists the different locations at which these measurements were made. S1 is the location closest to the base and S8 the farthest. Figure 4.8 shows the measurement locations S1-S8 in a schematic along with the co-ordinate system used. The side wall coincides with the y coordinate of the orthogonal co-ordinate system used in this investigation (x co-ordinate lines up with the flow direction, and z co-ordinate is perpendicular to the side wall) with the origin at the common vertex of the base and the side wall.

Table 4.1: Y co-ordinates for pressure measurement

<table>
<thead>
<tr>
<th>Locations</th>
<th>Distance from base, y (in)</th>
<th>y (meters)</th>
<th>y/y_max</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.730</td>
<td>0.019</td>
<td>0.117</td>
</tr>
<tr>
<td>S2</td>
<td>1.449</td>
<td>0.037</td>
<td>0.232</td>
</tr>
<tr>
<td>S3</td>
<td>2.043</td>
<td>0.052</td>
<td>0.327</td>
</tr>
<tr>
<td>S4</td>
<td>2.730</td>
<td>0.069</td>
<td>0.437</td>
</tr>
<tr>
<td>S5</td>
<td>3.449</td>
<td>0.088</td>
<td>0.552</td>
</tr>
<tr>
<td>S6</td>
<td>4.058</td>
<td>0.103</td>
<td>0.649</td>
</tr>
<tr>
<td>S7</td>
<td>4.761</td>
<td>0.121</td>
<td>0.762</td>
</tr>
<tr>
<td>S8</td>
<td>5.480</td>
<td>0.139</td>
<td>0.877</td>
</tr>
</tbody>
</table>
4.2.4 Calculation of Fluid Properties

The fluid properties that are needed are the density (\( \rho \)) and dynamic viscosity (\( \mu \)). The density is calculated from the measured value of the temperature and the local static pressure and the use of the ideal gas law. The uncertainty in the temperature measurement is \( \pm 1^\circ C \). The dynamic viscosity is obtained from the measured fluid temperature and the use of standard property tables[42].

4.2.5 Calculation of Reynolds number

The average velocity in the cross section is obtained from the measured mass flow rate and the cross sectional area. The flow rate through the test facility was measured by a venturi meter downstream of the test section. The venturi meter directly gives the mass flow rate through the facility. Based on this mass flow rate, the cross sectional area and the hydraulic diameter, the
Reynolds number was calculated. The formula used for the Reynolds number calculation is given in Equation 4.1. The error in the value of the Reynolds number is estimated to be a maximum of 8%. Unless otherwise noted, all Reynolds numbers are based on the hydraulic diameter

\[ \text{Re}_{Dh} = \frac{\dot{m} \cdot D_h}{A_c \cdot \mu} \]  

(4.1)
CHAPTER 5: RESULTS AND DISCUSSION

The first and foremost test was the measurement of the static pressure distribution. As mentioned in Chapters 3 and 4, it was necessary to have a fully developed flow condition at the test section. Once this condition was established, velocity and pressure fluctuation measurements were performed using hot wire anemometer and condenser microphone systems respectively.

5.1 Static Pressure Distribution

As mentioned in Chapter 4, the wall static pressure was measured at 7 longitudinal locations using pressure taps. Since the first pressure tap was placed about 90 hydraulic diameters away from the inlet of the test rig, the subsequent taps being 6 hydraulic diameters apart, the pressure distribution was expected to be a linear one. Figure 5.1 shows the static pressure distribution as a function of the longitudinal distance x, made non-dimensional using the hydraulic diameter for the range of Reynolds number tested here. It can be seen that for all the Reynolds number shown, the pressure distribution is very linear. It shows that the flow is fully developed when it enters the test section. It should be noted here that the lowest Reynolds number shown, 1530, the flow is predominantly laminar. The slope of these lines should give the pressure drop for the particular Reynolds number. The slope was calculated by doing a linear curve fit to these pressure profiles. This slope was later checked with differential pressure measurements between two successive pressure taps and the values agreed with each other. This pressure gradient is then used to evaluate the average shear stress in the duct, which can be obtained using Equation 5.1.

$$\tau_{w,av} = \frac{A_c \cdot \frac{dp}{dx}}{P}$$

(5.1)
The average wall shear stress is later used to evaluate the friction velocity.
5.2 Coexistence of Laminar and Turbulent Flow Regimes

Once it was established that the flow is fully developed for all the Reynolds numbers tested, it was necessary to verify the coexistence of both laminar and turbulent regimes of flow in the cross section for a range of Reynolds numbers as reported first by Eckert and Irvine[9]. Identification of the flow regime is by itself a challenge. There are numerous ways for experimentally distinguishing a flow as a laminar or a turbulent one. Kapat[2] describes in detail the most common methods used to experimentally determine transition. In this investigation, we use the traversing of a hot wire sensor along the altitude of the triangle for Reynolds numbers ranging from 1500 to 15000. The hot wire signals were sampled at 200 kHz with a low pass filter setting following the Nyquist criterion. The data thus obtained is the time series representation of the hot wire sensor output. However, to ascertain the point of transition, the time series data is converted to its corresponding frequency spectra using a Fourier transform. The final form of the frequency domain plot is the power spectral density of the fluctuations.

The advantage of using the power spectral densities (PSD) of the fluctuations to identify transition is that the presence of energy cascading in a turbulent flow leads to a continuous broad-band spectra, a feature not seen in laminar flows. Moreover, the energy levels of the PSD’s of a turbulent flow are not only remarkably similar\(^1\) for different flow situations, but are also much higher than their laminar counterparts.

Yet another aspect of the identification process is the definition of the point of transition. It is a well known fact that the laminar-to-turbulent transition is strongly affected by the external conditions. Therefore, if transition were viewed as the point of demarcation as approached from

---

\(^1\) True except for large scale or slow oscillation, since larger eddies retain much of the information about the boundary conditions [43. Batchelor, G.K., *The Theory of Homogeneous Turbulence*. 1953: Cambridge University Press.]
the laminar nature of the flow, i.e. onset of turbulence, the point of transition would be a very strong function of the ambient disturbances. However, if the point of transition were viewed as the cessation of turbulence rather than as its onset, it would only be a function of the flow system parameters[44]. This implies that as the flow rate is reduced, there would be a point beyond which the flow wouldn’t be able to sustain turbulence. This particular definition of transition is used in this study to identify the point of transition.

Figure 5.2 shows samples of PSD vs frequency plots for a given Reynolds number (3560) as the hot wire sensor was traversed vertically along the altitude from the base to the apex. Four vertical locations are shown in this plot. The first plot is the PSD at $h=0.2H$, where $h$ is the distance from the base to the apex along the altitude of the triangle ($h=0$ is the base, $h=H$ is the apex). This is a nice broad-band spectrum, denoting that the flow is a turbulent one. As the sensor was traversed further down to a location of 0.52H from the base, there was little change in the spectrum implying that even at this location the flow is turbulent. Further down, at 0.68H, the first perceivable change in the PSD was noticed. As the probe is moved toward the apex the spectrum changes significantly. As we reach 0.76H, the spectrum resembles that of a pure laminar flow. Since transition has been defined as the cessation of turbulence, the spectrum shown for 0.68H is taken to be the point of transition. Therefore, for a Reynolds number of 3560, the point of transition happens to be in the vicinity of 0.68H from the base.

Figure 5.3 shows the consolidated plot of the obtained PSD for a Reynolds number of 3560. The change in the nature of the PSD can be clearly seen in this plot as we go from the base to the apex. Closer to the base we can see that the PSD trend is very similar and falls on top of each other (obeying the -5/3 law for turbulent flow). However, as we proceed toward the apex, we can observe the nature of the PSD plot deviating. As mentioned above, the first such
deviation occurred at 0.68H and hence was named the point of transition. This traverse verifies
the co-existence of laminar and turbulent regions in the same cross section at a particular
Reynolds number. Similar hot wire probe traverse was performed for other Reynolds numbers
and the PSD data thus obtained were analyzed. It was found that the co-existence occurs for a
wide range of Reynolds numbers.

As reported by earlier researchers, the point of transition moves towards the apex as the
Reynolds number is increased. Figure 5.4 shows the line of transition as a function of Reynolds
number obtained in this investigation. Also shown is the plot obtained by Eckert and Irvine. It
can be seen from this plot that depending on the method used to identify transition, the line of
transition varies. Eckert and Irvine[9] used two different methods to identify this phenomenon;
one using a smoke probe where transition was identified as the point where the smoke started to
diffuse randomly and the other using the mean velocity profile where the point of transition was
characterized by the change in the nature of the velocity profile, i.e. the point where it ceased to
be parabolic. The former was called the line of instability and the latter the line of transition.
Here we use the PSD of the velocity signal obtained using a hot wire anemometer and defined
transition as the cessation of turbulence. Although, the different methods predict the line of
demarcation between laminar and turbulent flow to be different, it should be noted here that the
general trend is maintained.
Figure 5.2: Power spectral density of x-velocity fluctuation: Re=3560 (inset show location along altitude)
Figure 5.3: Power spectral density: Consolidated plot for Re=3560
5.3 Turbulent Velocity Spectra

The turbulent velocity measurements were made using a hot wire anemometer system operating in a constant temperature mode. Spectra were measured along the altitude for Reynolds numbers ranging from 3000 to 11000. A sample of the PSD data so obtained was already shown in Figure 5.2. Similar PSD curves were obtained for the other Reynolds numbers in the range mentioned above. Along with the PSD data, the root-mean-square fluctuations of the longitudinal velocity were obtained. In typical pipe or duct flow systems, the intensity of the fluctuation ($u_{rms}$) scales with average friction velocity $u_*$ (obtained from the average wall shear stress and density) as seen by Laufer[18]. We therefore apply the same scaling in our case. Figure 5.5 shows the
normalized intensity variation along the altitude of the triangle for the Reynolds numbers investigated here.

![Figure 5.5: Root mean square longitudinal velocity fluctuation along the altitude of the duct](image)

It can be readily seen from the above plot that the scaling used for duct and pipe flow systems is not directly extendable to a three dimensional flow situation like the one inside a narrow apex angle isosceles triangle. Although they don’t scale with the average friction velocity, all the plots have similar trends. We notice here that the intensities have a small peak very close to the base (h/H=0+), similar to any pipe or duct flow system, and then start to fall as we go towards the apex. At a distance of h/H=0.1, the value starts to plateau. The curves then eventually drop to much lower values as we approach the apex. The interesting aspect is that the h/H value at which the intensity value drops varies with the Reynolds number. This value is farthest from the apex for the lowest Reynolds number and vice versa. This sudden fall in the intensity values is due to the laminar portion present in the flow. The point where the dip in the
intensity occurs can then be characterized as the point of transition. This finding is in accord with the conclusion of Cremers[17], who measured the longitudinal velocity fluctuations along the center line for two representative Reynolds number although at the higher Reynolds number also we find here that the scaling doesn’t apply.

5.4 Wall Pressure Spectra

Dynamic wall pressure measurements were made at 8 different locations (S1 to S8) by mounting a condenser microphone flush on the side wall. The details of the measurement locations were discussed in Chapter 4. The output of the condenser microphone is a time series representation of the dynamic wall pressure. This data is then converted to the frequency domain by using a Fourier transform algorithm. Measurements were made for Reynolds numbers ranging from 1500 to 15000. Figure 5.6 shows the power spectral density of the wall pressure as a function of the frequency at all the locations at a Reynolds number of 10960. At this Reynolds number, the flow in the entire cross section is expected to be turbulent. It can be seen from this plot that the spectral information contained in location S1 (location closest to the base of the triangle) is the same as that at location S8 (location closest to the apex of the triangle). This implies that the fluctuations present in the base are immediately convected to the apex of the triangle. Although the velocity fluctuations at this Reynolds number predicted the existence of a very small region of laminar flow close to the apex (h/H~0.9), the pressure fluctuations do not seem to confirm this. This implies that although the shear stresses at the apex are able to damp the velocity perturbations and prevent them from destabilizing and becoming turbulent, no such destabilizing effect of the shear stress is seen on the pressure fluctuations. One might argue that the portion of the laminar region is so insignificant at this flow rate that the acoustic fluctuations easily travel from the base to the apex without hindrance. However, if this argument were true, at
lower Reynolds numbers there should be a difference in the spectra between S1 and S8. Figure 5.7 shows the PSD Vs frequency for a Reynolds number of 3560. Earlier it was established that at this Reynolds number there is a significant portion in the cross section of the duct where the flow is not turbulent. Here we can see that there is still no difference in the spectral information between S1 and S8. This confirms the fact that the pressure signal inside the cross section is uniform in spite of the coexistence of two different flow regimes. This further implies that the relationship between velocity fluctuations and pressure fluctuations does not necessarily hold in all scenarios. Also shown in both the plots is the typical level of the background noise spectrum. It can be seen that the signal levels are well above the noise level, except for very high frequencies.

![Figure 5.6: Power spectral density of wall pressure; Re=10960](image-url)
Similar results were obtained for the other Reynolds numbers tested (Re=5600 and Re=8400) i.e. no difference in spectra was noticed between the location close to the base and apex. A closer look at Figures 5.6 and 5.7 suggests that the spectra are very similar to one another. This implies that the spectra at S1 (S1 is chosen as this represents the fluctuation at a point where the flow is turbulent for the Reynolds numbers reported here) could be scaled using flow variables to make it independent of the Reynolds number.

Typically, for wall pressure measurements made in a pipe or in a turbulent boundary layer (TBL), the power spectral density is made non-dimensional using a typical velocity scale, the average wall shear stress ($\tau_w$) and a typical length scale. The usual velocity scales are the mainstream velocity ($U_\infty$) for the TBL and the area averaged cross sectional mean velocity ($U_{avg}$). The length scales that are used are the boundary layer thickness ($\delta$) and the diameter (D)
for the TBL and pipe flow respectively. The frequency is made non-dimensional using a typical time scale. The choice of the time scale depends on the desired interpretation of the data and the region of interest. If we want to resolve the larger scales of motion, the typical choice of the time scale, in a pipe flow system will be the ratio of the diameter (D) to the average velocity (U_{avg}). This would be used particularly to interpret the data in the region where the inertial forces are expected to dominate. If on the other hand if we are interested in the smaller scales of motion, the choice of the time scale would be based on the wall variables i.e. the friction velocity (u*) and the kinematic viscosity (ν). Both these time scales have been used in the past to scale the wall pressure spectra in an attempt to eliminate the dependence of the spectra on the Reynolds number of the flow ([45], [46]) with considerable success. Figure 5.8 shows typical representation of wall pressure spectra, adapted from Corcos[45] and Lauchle & Daniels[46].
In the current investigation we attempt to scale the obtained wall pressure spectra using both the above mentioned time scales. Figure 5.9 shows the non-dimensional PSD of the wall pressure plotted against the non-dimensional frequency (made non-dimensional using the wall variables $u^*$ and $\nu$). It can be seen from this Figure 5.9 that the scaling used for TBL and pipe cannot be directly extended to a three dimensional flow situation similar to the one present in a narrow apex angle isosceles triangular duct. The spectral content, particularly at the higher frequencies, should collapse on top of each other for this choice of scaling variables. From Figure 5.9 one can observe that although the PSD can be scaled at higher Reynolds numbers, they fail to do so at the lower ones. Figure 5.10 shows a similar pattern even while using the other time scale ($D_h$ and $U_{avg}$). It should be noted here that there is better agreement at the lower...
frequencies when we use $D_h$ and $U_{avg}$ for most of the Reynolds numbers. However, in general the scaling doesn’t hold well.

Figure 5.9: Frequency spectra of wall pressure fluctuation at S1: Viscous time scale
The main reason for the usual scaling to fail in this case is the fact that in a narrow apex angle triangular duct, the wall shear stress is non-uniform as opposed to the uniform shear in a regular pipe flow. In order to obtain proper scaling we might have to resort to local wall variables rather than the average wall variables. However, obtaining the local shear stress distribution is beyond the scope of this investigation.

The invalidity of the usual scaling variables for the particular case of a narrow apex angle isosceles triangular duct raises yet another interesting question; the scaling of the root mean square pressure fluctuation i.e. $p_{\text{rms}}$. In a TBL or in a pipe flow, the variation in the intensity of the pressure fluctuation ($p_{\text{rms}}$) with respect to the Reynolds number is minimized when the intensity is normalized with the average wall shear stress. A typical non-dimensional $p_{\text{rms}}$ distribution as a function of the Reynolds number is shown in Figure 5.11 (adapted from [45]).
similar distribution is plotted in Figure 5.12 using the data obtained in the current investigation. Since the pressure fluctuations were similar at all the 8 locations, the representative root mean square value for a given Reynolds number was taken to be the average $p_{rms}$ values of all the 8 locations. This average intensity, made non-dimensional using the average wall shear stress, is plotted as a function of Reynolds number in Figure 5.12. As can be seen from the plot, the intensity is almost a constant for a range of Reynolds numbers and drops as the Reynolds number increases. Moreover, the ratio of the intensity to the average wall shear is much higher than the corresponding values for a TBL or pipe flow. This fact can possibly explain why the traveling pressure fluctuations are not damped by the shear stresses at the apex unlike the velocity fluctuations.

![Figure 5.11: Typical root-mean-square pressure distribution at the wall in a turbulent pipe flow[45]](image)

Figure 5.11: Typical root-mean-square pressure distribution at the wall in a turbulent pipe flow [45]
Figure 5.12: Root-mean-square pressure at the wall as a function of Reynolds number
CHAPTER 6: CONCLUSIONS

Flow in a narrow apex angle isosceles triangular duct was investigated. Earlier research by other investigators suggested a coexistence of both laminar and turbulent regions side by side in the same cross-section. However, a detailed study of the turbulence quantities inside the duct was missing and the validity of the existing scaling laws were unknown. This study was an attempt to fill this gap by providing the necessary turbulence information.

Experiments were carried out in a narrow apex angle isosceles triangular duct with an aspect ratio of 5. Detailed longitudinal velocity fluctuation measurements and dynamic wall pressure measurement were made using a hot wire anemometer and a condenser microphone respectively. The following can be concluded from this study

1. The co-existence of a laminar and a turbulent region in the same cross section has been reconfirmed based on the hot wire measurements made along the altitude of the triangle. Hot wire data, reduced in terms of the power spectral density (PSD) suggests two separate regimes of flow; one in which the PSD exhibits a broad band structure similar to a turbulent flow while the other having no definite structure. The line of transition thus obtained and was found to be in good agreement with data from the open literature.

2. The intensity of the longitudinal velocity fluctuations obtained along the altitude of the triangle was normalized in a fashion similar to other pipe/duct flow systems i.e. the intensity was normalized with respect to the average friction velocity. It was found that the scaling applicable to other internal flow
geometries does not extend to the narrow apex angle isosceles triangle. This is mainly due to the existence of a laminar region.

3. For the range of Reynolds numbers tested, the velocity fluctuation measurements suggest the existence of a laminar region close to the apex (characterized by the difference in the structure of the velocity fluctuation). However, no such distinction was discernable from the wall pressure measurements. The wall pressure spectra were identical to each other immaterial of the location they were measured at, i.e. the spectra was the same close to the base and the apex. This suggests that, although the velocity fluctuations traveling from the base to the apex are damped by the wall shear stress, such an influence on the wall pressure is not possible. This implies that the relationship between the velocity and the pressure fluctuations applicable in the other systems does not hold well in a narrow apex angle isosceles triangular duct.

4. Further, the typical scaling applied to the wall pressure spectra in a turbulent boundary layer or inside a pipe does not apply to a narrow apex angle triangular duct.

5. Moreover, the relationship between the intensity of the wall pressure fluctuation and the average shear stress doesn’t hold well in this situation. The ratio is much higher in our case than the value reported in literature. The reason for such an anomaly is attributed to the variation in the wall shear stress.
LIST OF REFERENCES


