An Economic Framework For Resource Management And Pricing In Wireless Networks With Competitive Service Providers

2007

Shamik Sengupta

University of Central Florida

Find similar works at: http://stars.library.ucf.edu/etd

University of Central Florida Libraries http://library.ucf.edu

Part of the Computer Engineering Commons

STARS Citation


This Doctoral Dissertation (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Electronic Theses and Dissertations by an authorized administrator of STARS. For more information, please contact lee.dotson@ucf.edu.
AN ECONOMIC FRAMEWORK FOR RESOURCE MANAGEMENT AND PRICING IN WIRELESS NETWORKS WITH COMPETITIVE SERVICE PROVIDERS

by

SHAMIK SENGUPTA
B.E. Jadavpur University, 2002

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the School of Electrical Engineering and Computer Science in the College of Engineering and Computer Science at the University of Central Florida Orlando, Florida

Fall Term
2007

Major Professor: Mainak Chatterjee
ABSTRACT

A paradigm shift from static spectrum allocation to dynamic spectrum access (DSA) is becoming a reality due to the recent advances in cognitive radio, wide band spectrum sensing, and network aware real-time spectrum access. It is believed that DSA will allow wireless service providers (WSPs) the opportunity to dynamically access spectrum bands as and when they need it. Moreover, due to the presence of multiple WSPs in a region, it is anticipated that dynamic service pricing would be offered that will allow the end-users to move from long-term service contracts to more flexible short-term service models.

In this research, we develop a unified economic framework to analyze the trading system comprising two components: i) spectrum owner–WSPs interactions with regard to dynamic spectrum allocation, and ii) WSP–end-users interactions with regard to dynamic service pricing. For spectrum owner–WSPs interaction, we investigate various auction mechanisms for finding bidding strategies of WSPs and revenue generated by the spectrum owner. We show that sequential bidding provides better result than the concurrent bidding when WSPs are constrained to at most single unit allocation. On the other hand, when the bidders request for multiple units, (i.e., they are not restricted by allocation constraints) synchronous auction mechanism proves to be beneficial than asynchronous auctions. In this regard, we propose
a winner determination sealed-bid knapsack auction mechanism that dynamically allocates spectrum to the WSPs based on their bids.

As far as dynamic service pricing is concerned, we use game theory to capture the conflict of interest between WSPs and end-users, both of whom try to maximize their respective net utilities. We deviate from the traditional per-service static pricing towards a more dynamic model where the WSPs might change the price of a service almost on a session by session basis. Users, on the other hand, have the freedom to choose their WSP based on the price offered. It is found that in such a greedy and non-cooperative behavioral game model, it is in the best interest of the WSPs to adhere to a price threshold which is a consequence of a price (Nash) equilibrium. We conducted extensive simulation experiments, the results of which show that the proposed auction model entices WSPs to participate in the auction, makes optimal use of the common spectrum pool, and avoids collusion among WSPs. We also demonstrate how pricing can be used as an effective tool for providing incentives to the WSPs to upgrade their network resources and offer better services.
Dedicated to,

My beloved parents

Pradip and Swapna Sengupta

and

My advisor

Dr. Mainak Chatterjee
ACKNOWLEDGMENTS

I am grateful to my advisor Dr. Mainak Chatterjee whose experience, guidance and inspiration were integral towards my doctoral degree. I could not have imagined having a better advisor and mentor for my Ph.D., and without his infinite support I would not have reached this destination. I would also like to thank my committee members Dr. Mostafa A. Bassiouni, Dr. Ujjayant Chakravorty, Dr. Ratan K. Guha, and Dr. Taskin Kocak for agreeing to serve in my committee. Their comments and suggestions helped me greatly in improving the quality of my thesis. I would like to thank all my colleagues at the NETMOC laboratory and all my friends who always inspired me while working as a Ph.D. student. It is them who created a nice ambience so that the long hours of working were always thoroughly enjoyable.

In addition I would like to thank the School of Electrical Engineering and Computer Science, University of Central Florida for their financial support. I would like to acknowledge the role of Jadavpur University in my life for building a strong initial base standing on which I could achieve my doctoral degree. Last, but definitely not the least, I would like to thank my parents for the constant emotional and moral support they have provided and without whom I would not have reached so far.

Finally, I again thank Dr. Mainak Chatterjee for being my friend, philosopher and guide and helping me immensely to complete my Ph.D. thesis.
# TABLE OF CONTENTS

LIST OF FIGURES ................................................................. xii

LIST OF TABLES ................................................................. xviii

CHAPTER 1  INTRODUCTION ................................................. 1

  1.1 Radio Spectrum Allocation to WSPs .................................... 2
  1.2 Service Provisioning by WSPs for End-Users ......................... 6
  1.3 Economic Paradigm Shift and Cyclic Dependency .................... 8
  1.4 Contributions of this Work ............................................ 12
  1.5 Organization of the Thesis ............................................ 16

CHAPTER 2  BASICS OF GAME AND AUCTION THEORY ................. 18

  2.1 Basics of Game Theory .................................................. 18
      2.1.1 Classification of Games ........................................ 20
      2.1.2 Analyzing Games and Nash Equilibrium ..................... 22
  2.2 Basics of Auction Theory .............................................. 23
2.2.1 Types of Auctions ........................................ 23
2.2.2 Auction Design ........................................ 25

CHAPTER 3 RELATED WORK ................................. 26
3.1 Related Work on Auction Theory ..................... 26
3.2 Related Work on Pricing using Game Theory .......... 28

CHAPTER 4 SPECTRUM MANAGEMENT THROUGH AUCTIONS . 32
4.1 Auction Models for DSA ................................ 33
4.2 Auction Design for Single Unit Grant ............... 35
   4.2.1 Sequential Auction for Substitutable Bands .... 37
   4.2.2 Concurrent Auction for Substitutable Bands .... 40
   4.2.3 Dominant Strategy – Sequential and Concurrent Auction ...... 42
   4.2.4 Concurrent and Sequential Auctions for Non-Substitutable Bands . 45
4.3 Auction Design for Multiple Unit Grant ........... 47
   4.3.1 Synchronous and Asynchronous Auctions .......... 50
   4.3.2 Performance Comparison .......................... 53
   4.3.3 Bidders’ Strategies ............................... 56

CHAPTER 5 SERVICE PROVISIONING USING GAME THEORY .... 61
5.1 Modeling Conflict between WSPs and End users ........ 61
5.2 Decision Model ............................................................. 64
5.3 Utility functions for WSPs and End users .......................... 66
  5.3.1 Existence of Channel Threshold ............................... 69
  5.3.2 Existence of Resource and Price (Nash) equilibrium ....... 74
5.4 Maximizing Providers’ and Users’ Utilities ......................... 78
  5.4.1 Finding Optimal Pricing for WSPs ............................ 78
  5.4.2 Estimation of Bandwidth Demand from End users .......... 81

CHAPTER 6 HETEROGENEOUS NETWORKS AND SERVICES .... 82

  6.1 Overlapped Heterogeneous Network Model ....................... 85
  6.1.1 Heterogeneity of Network ........................................ 85
  6.1.2 Heterogeneity of Service ......................................... 87
  6.1.3 Decision Problems for WSPs and End–users .................. 88
  6.1.4 Games for WSPs and End–users ............................... 89
6.2 Analyzing the Game for Differentiated Services ................. 90
  6.2.1 Utility Functions and Nash Equilibrium for Voice/Video Services 91
  6.2.2 Utility Functions and Nash equilibrium for Data Services .... 97
6.3 Network Selection .......................................................... 101
6.4 A Case Study: Heterogeneous Networks and Utility Maximization 103
6.4.1 User’s Utility ........................................ 105
6.4.2 Provider’s Utility .................................... 112

CHAPTER 7 COGNITIVE RADIO BASED OPPORTUNISTIC SPECTRUM ACCESS .................................................. 114

7.1 IEEE 802.22 System ....................................... 115
  7.1.1 The MAC Layer of IEEE 802.22 ........................ 117
  7.1.2 Drawbacks of the Existing 802.22 MAC ............... 119
7.2 Spectrum Allocation for Self-coexistence among 802.22 Networks .......... 121
  7.2.1 Objective Functions .................................. 124
  7.2.2 Spectrum Allocation through Utility Graph Coloring ............ 124
7.3 Enhanced MAC for Hidden Incumbent Problem .................. 129
  7.3.1 Aggregation / Fragmentation of Channel Carriers ............... 129
  7.3.2 Dynamic Multiple Broadcasting .......................... 130
  7.3.3 Contention Resolution among CPEs ...................... 132

CHAPTER 8 SIMULATION MODELS AND RESULTS ................... 134

8.1 Auction Models and Results .................................. 134
  8.1.1 Results for Single Unit Grant ........................... 135
  8.1.2 Results for Multiple Unit Grant ......................... 142
8.2 Service Pricing: Numerical results .............................................. 154

8.3 Simulation Model for Heterogeneous Networks ................................. 161
  8.3.1 Network Layout ............................................................... 162
  8.3.2 Perceived Bandwidth ......................................................... 163
  8.3.3 Cost and Utility of Provider ............................................... 163
  8.3.4 Per-user Utility ............................................................... 164
  8.3.5 Perceived Bandwidth with Radius of Cell ............................... 166

8.4 Cognitive Spectrum Access ...................................................... 168
  8.4.1 Simulation Results ........................................................... 169

CHAPTER 9 CONCLUSIONS .......................................................... 176

LIST OF REFERENCES ............................................................... 178
LIST OF FIGURES

Figure 1.1  White space in spectrum usage (Courtesy: Shared Spectrum Company) . 5

Figure 1.2  Strong coupling between user and WSP . . . . . . . . . . . . . . . . . . 6

Figure 1.3  Economic paradigm shift . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10

Figure 1.4  Cyclic dependency . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 11

Figure 2.1  Different types of auction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 24

Figure 4.1  Auction classifications (S : substitutable, NS : non–substitutable) . . . . 34

Figure 4.2  Asynchronous allocation in different intervals of time . . . . . . . . . . . 50

Figure 4.3  Synchronous allocation of spectrum in fixed intervals . . . . . . . . . . . 51

Figure 5.1  WSPs–End users association through service broker . . . . . . . . . . . 62

Figure 6.1  Heterogeneous Network and Service Model . . . . . . . . . . . . . . . . . 85
Figure 8.4  Auctioneer’s revenue with substitutable bands: 100 bidders and 50 spectrum bands ................................................................. 137

Figure 8.5  Auctioneer’s revenue with substitutable bands: 100 bidders and 90 spectrum bands ................................................................. 138

Figure 8.6  Substitutable bands: optimal bid for a specific bidder for sequential auction 139

Figure 8.7  Substitutable bands: optimal bid for a specific bidder for concurrent auction 140

Figure 8.8  Revenue in sequential auction with non-substitutable spectrum bands . 141

Figure 8.9  Revenue in concurrent auction with non-substitutable spectrum bands . 141

Figure 8.10  Auctioneer’s revenue with non-substitutable spectrum bands: 100 bidders and 10 spectrum bands ................................................................. 142

Figure 8.11  Auctioneer’s revenue with non-substitutable spectrum bands: 100 bidders and 50 spectrum bands ................................................................. 143

Figure 8.12  Auctioneer’s revenue with non-substitutable spectrum bands: 100 bidders and 90 spectrum bands ................................................................. 143

Figure 8.13  Revenue Maximization with auction rounds (with 15 bidders) ............ 146

Figure 8.14  Usage Maximization with auction rounds (with 15 bidders) ............ 146
Figure 8.15 Revenue maximization ........................................... 147

Figure 8.16 Usage Maximization ........................................... 147

Figure 8.17 Probability of winning with lowest spectrum request .......... 148

Figure 8.18 Probability of winning with lowest value bid .................... 149

Figure 8.19 Avg Revenue with and without collusion ....................... 150

Figure 8.20 Avg Usage with and without collusion ........................... 150

Figure 8.21 Avg probability of winning with and without collusion ........ 151

Figure 8.22 Revenue generated with auction rounds ......................... 152

Figure 8.23 Spectrum usage with auction rounds ............................. 152

Figure 8.24 Average revenue generated with number of service providers . 153

Figure 8.25 Average spectrum usage with number of service providers ... 153

Figure 8.26 Average probability of winning spectrum with number of bidders 154

Figure 8.27 Price per unit of resource with number of users fixed ........... 156

Figure 8.28 Total profit of a provider with number of users fixed ........... 157
Figure 8.29 Resource allocated to a user with number of users fixed .......................... 157

Figure 8.30 Net utility of a user with number of users fixed ................................. 158

Figure 8.31 Price per unit of resource with increasing number of users .................. 159

Figure 8.32 Total profit of a provider with increasing number of users .................... 160

Figure 8.33 Resource allocated to a user with increasing number of users ............... 160

Figure 8.34 Net utility of a user with increasing number of users ......................... 161

Figure 8.35 Net utility vs. numbers of users with resources fixed .......................... 162

Figure 8.36 Perceived system bandwidth: simulation result .................................. 164

Figure 8.37 Net utility of the provider ................................................................. 165

Figure 8.38 Relative increase in net utility (in %) with respect to the system without internetworking ................................................................. 165

Figure 8.39 per-user perceived bandwidth ........................................................... 166

Figure 8.40 per-user perceived utility ................................................................. 167

Figure 8.41 Perceived system bandwidth due to effect of radius ratio ..................... 167

xvi
Figure 8.42 Total utility achieved by all the BSs under the proposed collaborative approach and greedy non-collaborative approach .......................... 169

Figure 8.43 Total utility achieved by all the BSs under the proposed collaborative approach and greedy, traditional graph coloring approach .......................... 170

Figure 8.44 Jain’s fairness index for maximized utility objective function and proportional fair utility objective function ................................................. 171

Figure 8.45 Probability to tune to a BS (no contention) ............................... 172

Figure 8.46 Spectrum utilization efficiency .................................................... 173

Figure 8.47 Average initial delay to tune to a BS broadcasting frequency (no contention) 174

Figure 8.48 Average startup delay for existing MAC ................................. 174

Figure 8.49 Average startup delay for proposed MAC ................................. 175
LIST OF TABLES

Table 6.1 Data Rates and Ring Radius ............................................. 111

Table 8.1 Simulation Parameters for multi-unit grant auction .................. 145

Table 8.2 Simulation parameters for IEEE 802.22 system ..................... 168
In the recent times, the telecommunication industry is going through major advancements and innovations. Privatizations of the telecommunication sectors coupled with technological advancements and economic liberalization have stimulated competition among network service providers and driven down the price of network services. In addition, the transformation from second generation (2G) wireless cellular technology to third and fourth generation (3G and 4G) technologies has boosted this competition to a great extent resulting in numerous wireless service providers (WSP) in one geographic region.

The presence of multiple wireless service providers in every geographic region is creating an environment where every user has more freedom/flexibility to choose his service providers based on his needs. This multi-provider competition has also led to what is called churning, i.e., the migration of users from one provider to another owing mostly to dissatisfaction in the perceived quality of service (QoS), coverage, and the competitive offerings of new services by other providers. In the past, most providers experienced an average churn rate (i.e., the rate at which existing customers leave the current provider) between 2%–3% per month. In addition, the Federal Communications Commissions (FCC) mandated Wireless Local Number Portability (WLNP) [98] in the US, which is having a catalytic effect on
churning. Thus this is leading to new challenges for wireless service providers, who must not only focus on ensuring the absolute quality of their own service offerings (data rates and QoS guarantees) but also ensure competitive pricing. As far as the WSPs are concerned, the goal of every service provider is to maximize their profit and continue to enhance technology innovations. Every wireless service provider buys spectrum from the spectrum owner (for example, FCC) and then sells the spectrum to the subscribers (end users) in the form of various services. In such a scenario, the aim of each service provider is to get a large share of subscribers and a big spectrum chunk from the spectrum band to fulfill the demand of these subscribers. As end users and capacity of spectrum band are finite, this gives birth to an interrelated two–fold competitive behavior: (i) wireless service providers compete among themselves for acquiring spectrum and (ii) they compete for a larger customer base. To the best of our knowledge, there is no unified framework that attempts to analyze the economic aspects that arise due to the interactions between spectrum owner, wireless service providers, and users.

1.1 Radio Spectrum Allocation to WSPs

In most countries, the competitive behavior among wireless service providers for spectrum was initiated by spectrum auctions held in 2000 and 2001. Though the auctions were very successful in some countries (e.g., United Kingdom, Germany), they were open to criticism in others (e.g., Austria, Switzerland, Netherlands) [29]. Through FCC, the spectrum was
auctioned in the United States – the results of which are hotly debated. Moreover, FCC sets the rules and regulation which govern the access to spectrum. These rules have led to allocation of spectrum chunks for specific purposes; e.g., 824–849 MHz, 1.85–1.91 GHz, 1.930–1.99 GHz frequency bands are reserved for licensed cellular and PCS services and require a valid FCC license, whereas 902–928 MHz, 2.40–2.50 GHz, 5.725–5.825 GHz frequency ranges are reserved as free–for–all unlicensed bands [96]. These spectrum allocations are usually long–term and any changes are made under the strict guidance of FCC.

This kind of static allocation of spectrum has several disadvantages because of being time and space invariant. In static spectrum allocation, a large part of the radio bands are allocated to the military, government and public safety systems. However, the utilization of these bands are significantly low. One may argue that spectrum allocated to cellular and PCS network operators are highly utilized. But in reality, spectrum utilization even in these companies vary over time and space and undergo under–utilization. Often times, the usage of spectrum in certain networks is lower than anticipated, while there might be a crisis in others if the demands of the users using that network exceed the network capacity. Static allocation of spectrum fails to address this issue of spectrum sharing even if the service providers (with statically allocated spectrum) are willing to pay for extra amount of spectrum for a short period of time if there is a demand from the users it supports.

Another problem static spectrum allocation often faces is due to the modification of old technologies. For example, in case of VHF and UHF bands reserved for television broadcast in the United States, allocation of 6 MHz per TV channel was based on old analog NTSC system
even though better quality video can be now broadcasted with almost 50% less spectrum per channel [10]. Given the pervasive penetration of cable–TV, this precious spectrum, though allocated and owned, remains unused in most locations.

It has been well–established that there is a great amount of unused bands available sparsely which could be exploited by both licensed and unlicensed services. Thus uncoordinated, opportunistic deployment of spectrum chunks has led to an “artificial scarcity of spectrum”. An experimental study conducted by Shared Spectrum Company during the 2004 Republic National Convention [97] found that spectrum utilization is typically time and space dependent and there is a great amount of “white space” (unused bands) as shown in figure 1.1. As a result, it is intuitive that static spectrum allocation may not be the optimal solution toward efficient spectrum access for both licensed and unlicensed services. With the dis–proportionate and time–varying demand and hence usage of the spectrum, the notion of fixed spectrum assignment to providers is questionable. Though it might be argued that the implementation and administration is very easy, the fact remains that the current system is ineffective and deprives service providers, end–users, and FCC from maximizing their benefits.

In order to break away from the inflexibility and inefficiencies of static allocation, a new concept of Dynamic Spectrum Allocation (DSA) is being investigated by network and radio engineers, policy makers, and economists. In DSA, spectrum will be allocated dynamically depending on need of the service providers which in turn depends on end users’ demands in a time and space variant manner. Emerging wireless technologies such as cognitive radios
[49] is poised to make DSA a reality. This method of spectrum sharing is more efficient and will help service providers, and FCC to avoid any artificial scarcity. In this new approach, spectrum access barrier may be physically or virtually broken. In the physical approach, FCC may take back all the allocated spectrum that had been allocated and may merge these spectrum bands physically by moving in between spectrum bands (unlicensed bands) to create a common pool of open spectrum. In the virtual approach, spectrum already allocated to service providers are not taken back. Instead, parts of the spectrum band, which are no longer used or under-used, are made open to all the service providers. These parts of the band are known as the Coordinated Access Band (CAB) [11] and are not physically merged. Examples of such bands include the public safety bands (764–776 MHz, 794–806 MHz) and unused broadcast UHF TV channels (450–470 MHz, 470–512 MHz, 512–698 MHz, 698–806
MHz). As physical merging is not possible overnight and many business and political factors are attached to it, the virtual merging is gaining popularity and is the focus of this research.

1.2 Service Provisioning by WSPs for End–Users

Currently, a user is usually associated with one service provider, i.e., gets services from one provider for a period of time as per the contractual agreement which typically varies from one to two years. Thus, there is a strong coupling between the user and service provider as shown in figure (1.2) that does not allow much flexibility to the end users.

However, it is anticipated that in the near future, the concept of service brokers, technically known as Mobile Virtual Network Operators (MVNO), will evolve that will act as...
an interface between the providers and the users [93]. These service brokers will allow more flexibility to the end users with cognitive radio enabled devices to choose and connect to any WSP. Connections can be made depending on what the users’ preferences are – quality of service (QoS), quality of experience (QoE), coverage, price etc. In such a scenario, the end users will be the customers of one of the service brokers that provide the users: authentication credentials, billing account and access to any of the service providers. It can be noted that the physical existence of the service brokers is not important in this research as it is merely a way to allow the dynamic selection of the providers. In the new paradigm there is no strong association between an end user and provider as end user can choose any provider almost on a session by session basis.

In addition to new economic deregulations, wireless service providers have started to use a multitude of access technologies, operating on both licensed and unlicensed bands, to serve an increasing number of subscribers. It is likely that numerous types of wireless networks will soon prevail to support wireless services that have varied QoS requirements, for example, video and telephony services are more sensitive to delay than services such as file downloads that are affected by loss. Even for a particular kind of network i.e., cellular, WLAN or WMAN, it is not clear which technology will ultimately emerge as the global standard. For example, we do not know if CDMA (code division multiple access) or GPRS (GSM based general packet radio service) will become the de facto standard as far as cellular networks are concerned. To harness the wide variability of coverage, bandwidth, and reliability offered by the different technologies operating at different spectrum bands, the
network service providers are deploying heterogeneous wireless technologies in an over-laid fashion. Each of these heterogeneous networks provide different set of services governed by corresponding QoS capabilities. The most common example seen today is the accessibility of Wi-Fi hotspots on top of 3G services [89]. All services are traded as commodities where the users have the flexibility to get/buy any service from any service provider. In such a scenario, the user device is expected to have the capability to connect to different networks belonging to different service providers. In other words, we assume that the user device is equipped with cognitive radio enabled multi-mode network interface – an adaptive and extremely programmable radio that can learn user’s preference and automatically adjust to changes in the operating environment [49, 71, 92].

With such loose coupling between providers and end-users, the first question that arises is ‘how or which wireless service provider should be selected by a user’? Second, ‘what price per unit of resource should be offered by these WSPs such that profit is maximized and at the same time users are enticed’? By introducing all the providers and users in a market environment, it becomes convenient to leverage the concept of prices of services to regulate the demands of users who consume resources (bandwidth).

1.3 Economic Paradigm Shift and Cyclic Dependency

With dynamic spectrum access in effect for WSPs, and loose coupling between provider and end-users, it is clear that a new economic model needs to be investigated. A vast number
of options exist for dynamic spectrum allocation, where, static spectrum allocation and DARPA XG [90] are two extremes. Static spectrum allocation has been tried and tested, the results of which conclusively prove that static allocation leads to artificial scarcity. On the other extreme, DARPA XG has focused on free–for–all, opportunistic spectrum access for peer–to–peer ad–hoc communication, typically targeting military applications. In between exists one of the most interesting problem of controlled dynamic spectrum access for multi provider networks. Dynamic spectrum access takes a novel approach for spectrum allocation in which the providers are no longer restricted to use the statically partitioned spectrum. A *spectrum broker*, on behalf of FCC (spectrum owner), co–ordinates the allocation, usage, and pricing of the portions of the spectrum which is unallocated, unused or under-utilized. In such a dynamic setting, the question arises, how the spectrum will be allocated from the coordinated access band (CAB) to the service providers and how these service providers will determine the price of their services to the end users? Each provider gets a part of the spectrum as and when they need through the spectrum broker. The acquired spectrum is then used to offer services to end–users and generate revenue according to some business strategies.

End–users, on the other hand, are no longer restricted to a single provider but select their provider as per their requirements through a *service broker* almost on a session–by–session basis. The introduction of spectrum broker and service broker along with the lack of co–ordination and cooperation among providers has made an economic paradigm shift as
Figure 1.3: Economic paradigm shift

depicted in figure 1.3. Thus it is important to investigate the economic issues that has a profound impact on (i) spectrum broker–WSP and (ii) WSP–end users interactions.

Though the two problems have been addressed separately, i.e., how spectrum is acquired by the providers in a dynamic manner and how resources (bandwidth) are allocated to the end users, there is a strong correlation between the two. As mentioned earlier, the service providers compete against each other for their own profit. The goal of a service provider in such a situation is to grab as much spectrum as possible from spectrum broker with lowest possible price and then sell those spectrum to the end users to make maximum profit. In doing so, there are several factors that the service providers have to consider. To maximize profit in a real world environment, service providers must judge spectrum broker’s and other service providers’ strategies. In addition, the service providers need to consider the *amount* of spectrum that they must acquire and how much *price* they are willing to pay for that.
Figure 1.4: Cyclic dependency

spectrum – both of which are determined by the service demands and revenue generated from the users. In effect, estimation of the demand of bandwidth and expected revenue from the users become the prime factors in driving the service providers’ request for spectrum. Again on the other hand, depending on the spectrum acquired and the prices paid, the provider will decide about allocating bandwidth and pricing the services to the end users. Service pricing by the providers, in turn, will affect the demand for the services from the end-users, thus resulting in a cyclic dependency in a typical supply-demand scenario as depicted in Fig. 1.4. As a result, the relationship between spectrum owner and WSP has a strong correlation with the relationship between WSPs and end-users and must be analyzed together.
1.4 Contributions of this Work

In this work, we develop an economic framework for resource management and pricing for heterogeneous wireless networks in the presence of competing service providers. In particular, we use auction theory to address the issue of spectrum allocation and game theory to address the WSP–end-user interaction. For every geographic region, auction is conducted taking the service providers in that region who are the bidders in the auction. FCC or any equivalent spectrum owner in respective countries are the auctioneer or seller. From here onwards, we use the terms service providers and bidders interchangeably. Also, auctioneer and seller mean the same. The service providers buy spectrum from the auctioneer and sell the spectrum in some form of services to the end–users to make additional profits. The demands for spectrum and the revenue generated from the end users become the driving factors for the service providers while taking part in auction.

For service pricing by the providers, we consider a given region that is served by the base stations of several WSPs. We consider an ideal market with perfect competition, where each WSP has its own set of services and establish their corresponding prices. A user agrees to pay a price to a provider for every unit of resource that is consumed. We deviate from the notion of per–service static prices [45, 56] and allow the providers to set the price it prefers to charge the users. The price is dynamically adjusted over time in order to maximize their profit and minimize resource wastage. Such a market mechanism is more flexible and realistic, as there does not exist centralized authorities to determine the price of a service
This multi-provider environment can be viewed from two different perspectives. First, from the perspective of users as they have the choice to select the most suitable provider. Suitability can be in terms of performance and cost. Second, from the perspective of the service providers, as they compete among themselves to get the business of the users by offering competitive prices. Their strategies to determine the prices affect their profits and losses. Thus, we are led to a situation where the goals of the parties involved are conflicting in nature.

In order to deal with such conflicting objectives we propose to use game theory in WSP-end-user interaction since it is an effective tool to deal with rational entities who make decisions to maximize their benefit with whatever little information they have. We develop a game theoretic framework to analyze the interplay among the service providers and their strategies to attract users by offering competitive prices. In this context, it becomes necessary to formulate games for pricing that would primarily involve the competitions and interactions among the service providers and end-users. One fundamental question that arises in such an environment is the existence of (Nash) equilibrium, where no provider finds it beneficial to change its price unilaterally [51]. The service providers will compete among themselves and the extent to which they will be willing to undergo negotiation will depend on their current load or their desperateness to attract new users. Obviously, the decision cannot be made unilaterally and the user must be involved. Through QoS and price negotiations, a user will select a provider based on a function that best characterizes his preferences, usually given by a utility function.
More specifically, the contributions of this work are as follows.

- We develop an auction model to address the dynamic spectrum access among the wireless service providers. Spectrum broker on behalf of FCC becomes the auctioneer and allocates spectrum to the WSPs who are the bidders. We investigate the bidding strategies and the reservation prices of WSPs under different allocation constraints. First, we investigate the special case where WSPs (bidders) are granted at most one spectrum chunk from the pool of spectrum chunks in each allocation period. We study both sequential and concurrent auctions, i.e., when bands are auctioned one after another and when all the bids for all the bands are submitted simultaneously. Substitutable and non–substitutable – both types of bands are considered and analyzed in this regard. As the more general case, we also consider the spectrum allocation where bidders are not constrained to single unit of spectrum. We devise a “dynamic spectrum allocator knapsack auction” mechanism with the help of sealed bid auction strategies that is used to determine the winning set of WSPs.

- For WSP–end-user interaction, we apply game theory to investigate the competitive behavior shown by end users and multiple WSPs. In this regard, we provide a conflict and decision model under an incomplete information game situation, and propose utility functions for both WSPs and end users. We assume the players (WSPs and end users) in this game to be rational and non-cooperative in nature.
• We estimate the demand for bandwidth from the end users and the pricing offered by service providers in this non-cooperative game model. We find the expected revenue generated from the end users under the equilibrium condition; we argue that the revenue generated from users is the prime factor that governs the reservation price for the WSPs. We investigate threshold conditions that determine the Nash equilibrium and propose a dynamic pricing policy that helps both providers and users to maximize their utilities.

• We also propose a dynamic game model to investigate the conflict between wireless service providers who provide services through heterogeneous access technologies. Specifically our focus is on a setting where end users may access any of the multiple heterogeneous wireless networks even under a single wireless service provider. We investigate how the presence of overlaid access networks may be exploited to allow spatial multiplexing (smoothing) for load balancing across resources with different coverage scales. In this regard, we conduct a case study where a service provider provides service through two wireless access networks: CDMA/HDR based 3G cellular network and Wi-Fi network. We capture the interplay among these heterogeneous networks under the same service provider.

• To address the issue of opportunistic spectrum access we focus on the enhancements of the existing IEEE 802.22 air-interface based on cognitive radio. We address the issue of self-coexistence among multiple overlapping 802.22 WSPs. In this regard, we propose a network controlled spectrum access mechanism where 802.22 BSs behave
collaboratively to minimize the interference and maximize the utility obtained from the system. We show how the collaborative approach among 802.22 BSs outperforms any other spectrum allocation mechanism.

- Through simulation experiments, we show that the proposed auction mechanisms entice the WSPs to participate in the auction, make optimal use of the common spectrum pool, and avoid collusion among WSPs. The dynamic spectrum access with the help of proposed auction models eliminates the “artificial scarcity of spectrum” and strikes a balance between demand and supply of spectrum. The results demonstrates the effectiveness of the proposed methods. Through numerical results, we demonstrate that there are incentives for the providers to upgrade their network and radio resources.

1.5 Organization of the Thesis

The rest of the thesis is organized as follows. In chapter 2, we discuss the basics of game theory and auction theory. In chapter 3, we discuss the significant relevant works that deal with spectrum auctions, service provisioning and pricing of network resources. Spectrum management with different allocation constraints is studied with the help of auction theory in chapter 4. Service provisioning among the end-users from the service providers are discussed in chapter 5. We analyze the game models and find the existence of Nash equilibriums. The demand for bandwidth is also estimated. In chapter 6, we extend our game model to formulate the interaction between end-users and providers with heterogeneous access.
networks and services as a dynamic game. We also propose the pricing strategies and study
the existence of Nash equilibrium for differentiated services. Opportunistic spectrum access
with the help of cognitive radio in the sub-900 MHz bands is discussed in chapter 7. We
use principles of graph coloring to efficiently allocate spectrum. In chapter 8, we present the
simulation models and discuss the results. Conclusions are drawn in the last chapter.
CHAPTER 2
BASICS OF GAME AND AUCTION THEORY

Game theory and auction theory are used to analyze problems with conflicting objectives among interacting decision-makers. These theories have become important to study the engineering problems from an economic point of view. They have been extensively used in various industries including the competitive energy market, airlines industry, and Internet services. Game and auction theories both have proved to be a very powerful tool to deal with distributed problems in networking and communications. In this chapter, we present a brief discussion on the basics of these theories.

2.1 Basics of Game Theory

Game theory [24, 25, 54] is a branch of applied mathematics, and it is used to analyze problems with conflicting objectives among interacting decision-makers. It has been used primarily in economics and has also been applied to other areas, including politics, biology and networking. A broad overview of game theory and its application to different problems in networking and communications can be found in [3, 40, 79] and the references therein. More recently, researchers are using game theory to deal with wireless networks and services: the decision makers in this game are the wireless service providers and end-users. These decision
makers have to deal with a limited network and radio resources (e.g., radio spectrum) that imposes a conflict of interest between them.

A game consists of players, the possible actions of the players, and consequences of the actions. For notational purpose, a game is always expressed by the \((\mathcal{N}, \mathcal{S}, \mathcal{U})\) tuple, where \(\mathcal{N}\) denotes the set of players, \(\mathcal{S}\) denotes the strategy space of the players and \(\mathcal{U}\) denotes the set of utility functions. The players are decision-makers, who choose how they act. Formally, a game can be defined by a conflict among several (two or more) players, where the players strive to ensure the best possible consequence according to their preferences. The preferences of a player are expressed through a \emph{utility function}, which maps every consequence to a real number, or with preference relations, which define the ranking of the consequences. An utility function can be defined as a mathematical characterization that represents the benefits and cost incurred by the players in the game.

The most fundamental assumption in game theory is \emph{rationality}. Rational players are assumed to always maximize their profit or payoff. If the game is not deterministic, the players maximize their expected payoff. The idea of maximizing the expected payoff was justified by the seminal work of von Neumann and Morgenstern in 1944 [52]. Maximizing one’s payoff is often referred to as selfishness. This is true in the sense that all players try to gain the highest possible utility. However, a high utility does not necessarily mean that the players act selfishly. Any kind of behavior can be modeled with a suitable utility function. A game describes the actions the players can take as well as the consequences of the actions. The solution of a game is a description of outcomes that may emerge in the game if the
players act rationally and intelligently. Generally, a solution is an outcome from which no player wants to deviate unilaterally.

2.1.1 Classification of Games

Before we proceed any further, let us discuss the classifications of games that are relevant to this research.

- **Cooperative Vs. Non-cooperative Games**: Game theory can be divided into non-cooperative and cooperative game theory. In cooperative games, the joint actions of groups are analyzed, i.e., what is the outcome if a group of players cooperate. Cooperative game theory looks at reasonable or fair outcomes when players form coalition and share resources. It answers questions such as which players will form a coalition and how will resources be divided within these coalitions. In non-cooperative games, the actions of individual players are considered where cooperation from each of the players must be self-enforcing. Most game theoretic research has been conducted using non-cooperative games, but there are also approaches using cooperative games.

- **Complete Vs. Incomplete–Information Games**: Depending on whether or not each player knows the other players’ payoff functions, a game can be formulated either as a complete or incomplete information game. If every player is aware of the strategies and utilities of all the other players, the game is said to have complete information. If not, the
game has incomplete information. Given a situation, i.e., some information, a game can be a complete or incomplete–information game depending on the goal we are seeking.

- **Pure strategy Vs. Mixed strategy:** If a player selects one of the strategies from his strategy set with probability 1, then the player is playing a pure strategy. In contrast, in mixed strategy profile, a player has several pure strategies in the strategy space and the player decides to play each of the pure strategies with some probability, i.e., the selection is randomized. Thus in mixed strategy, the strategy space has some probability distribution which corresponds to how frequently each of the strategies is chosen.

- **Static Games Vs. Dynamic Games:** In a static game, the players make decisions only once, i.e., the players have only one move. The strategies are chosen simultaneously by the players without knowledge of other players' strategies. Even though the decisions can be taken at different time instants, the game is simultaneous because each player has no information about the decisions of others; thus, it is as if the decisions are made simultaneously. In contrast to the static games, if the players interact multiple times by playing the game iteratively, the game is called a dynamic, or repeated game. Unlike static games, players may have some information about the strategy profiles of other players and thus may contingent their play on past moves.
2.1.2 Analyzing Games and Nash Equilibrium

Once the game is formulated, it needs to be solved. Solving a game means predicting the strategy of the players, considering the information the game offers and assuming that the players are rational. There are several possible ways to solve a game: *iterated dominance*, *best response*, *backward induction* and many more. A detailed study on these techniques can be found in [24, 25]. In this research, we focus on the best response strategy. The best response of a player $i \in \mathcal{N}$ is to choose a strategy $s_i \in \mathcal{S}$ when the strategy vector $s_{-i}$ is chosen by all the opponents. The objective of player $i$ is to maximize the utility $u_i$. More formally, the best response strategy can be defined as follows.

**Definition 2.1.2.1** The best response $br_i$ of player $i$ to the opponents’ strategy profile $s_{-i}$ is a strategy $s_i$ such that:

$$br_i = \arg \max_{s_i \in \mathcal{S}} u_i(s_i, s_{-i})$$ (2.1)

From the above definition, one can find that if the strategies taken by the players are mutual best responses to each other, then no player would like to deviate from the given strategy profile. To identify such strategy profiles, John Nash introduced the famous equilibrium concept known as *Nash equilibrium* [51]. The concept of Nash equilibrium can be formally defined as follows.

**Definition 2.1.2.2** The strategy profile $s^*$ constitutes a Nash equilibrium if and only if, for each player $i$,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in \mathcal{S}$$ (2.2)

The above definition means that in a Nash equilibrium state, none of the players would unilaterally change the strategy to increase the utility. Thus Nash equilibrium brings the
game to a steady state, from which the players would not like to deviate as that would not increase their benefits any more.

2.2 Basics of Auction Theory

An auction is the process of buying and selling goods by offering them up for bids (i.e., an offered price), taking bids, and then selling the item to the highest bidder. In economic theory, an auction is a method for determining the value of a commodity that has an undetermined or variable price. In some cases, there is a minimum or reserve price; if the bidding does not reach the minimum, there is no sale. Traditional auctions involve single seller and many buyers. The buyers compete among themselves to procure the goods of their choice by placing a bid, which they feel most appropriate.

2.2.1 Types of Auctions

There are several kinds of auction models as shown in figure 2.1. Depending on whether the bidding strategies of each of the bidders are disclosed to the other bidders, open and closed bid auctions are designed. In open auctions [5, 29], bids are open to everybody so that a player’s strategy is known to other players and players usually take their turns one by one until the winner(s) evolve. Bids generated by players in open bid auction can be either in increasing or decreasing order. Couple of famous increasing bid open auction are English auction [22, 29] and Yankee auction. Dutch auction on the other hand is a famous decreasing
open bid auction. Dutch-style auction satisfies the property that privacy of losing bids is preserved after auction closes [64, 65].

An important perspective of increasing auction is that it is more in the favor of bidders than the auctioneers. Moreover, increasing open bid auction helps bidders in early round to recognize each other and thus act collusively. Increasing auction also detract low potential bidders because they know a bidder with higher bid will always exceed their bids.

Closed bid (or sealed bid as they are more popularly known as) auctions are opposite to open bid auctions and bids/strategies are not known to everybody [27]. Only the organizer of the auction will know about the bids submitted by the bidders and will act accordingly. Bids are kept secret until the opening phase, and then all bids are opened and compared to determine the highest one. Thus, closed bid auctions do not promote collusion. Couple of the famous closed bid auctions are first price sealed bid auction and second price sealed bid auction. In a first price auction, the winners payment is equal to the winner’s bid while in a second price auction, the winners payment is equal to the second highest bid. Open
bid auctions are best generalized as complete information games while closed auctions are incomplete information games.

2.2.2 Auction Design

Good auction design is important for any type of successful auction and often varies depending on the item on which the auction is held. The auctions held in Ebay [95] are typically used to sell an art object or a valuable item. Bidding starts at a certain price defined by auctioneer and then the competing bidders increase their bids. If a bid provided by a bidder is not exceeded by any other bidder then the auction on that object stops and final bidder becomes the winner.

There are three important issues behind any auction design. They are (i) attracting bidders (enticing bidders by increasing their probability of winning), (ii) preventing collusion thus preventing bidders to control the auction and (iii) maximizing auctioneer’s revenue. It is not at all intended that only bidders with higher purchasing power should get most of the items. The goal is to increase competition among the WSPs and bring fresh new ideas and services. As a result, it is necessary to make even the low potential bidders, who have a low demand of items, interested to take part in the auction.
CHAPTER 3
RELATED WORK

In this chapter, we discuss the relevant works that deal with spectrum auctions and service pricing in a competitive multi-WSP scenario using games.

3.1 Related Work on Auction Theory

Auction theory is a mature field with a substantial literature. Our attempt is not to provide an exhaustive review of auction theories; interested readers are referred to [36] for an overview. Rather we try to discuss and present the ongoing interesting works, where researchers are investigating about the applications of auction theory in spectrum markets and resource pricing.

Currently, most auction sites (e.g., eBay [95]) support a basic bidding strategy through a proxy service [67] for a single-unit auction where ascending bidding continues till a winner emerges. In this type of auction, there is only one item for auction and all the bidders bid for that item only. In such single unit auction, Vickrey proved that “English” and “Dutch” type auctions yield the same expected revenue under the assumptions of risk neutral participants and privately known value drawn from a common distribution [77].
With the emergence of spectrum markets [1, 81], single unit auction models are no longer valid. Multi-unit auctions have been used to investigate pricing policies of network resources (e.g., transmission rate, bandwidth or link capacity) in [33, 41, 42, 43, 68, 75]. The key issue addressed in [33] concerns how the available bandwidth within the network should be shared between competing streams of elastic traffic; the stability and fairness of a class of rate control algorithms are also investigated. The implications of flat pricing and congestion pricing for capacity expansion are studied in [41]. A bandwidth pricing mechanism based on second-price auctions that solves congestion problems in communication networks has been proposed in [42, 43]. A decentralized auction-based approach to price edge-allocated bandwidth in a differentiated services Internet is presented in [68]. Most works done so far on auctions are extensions of classical auctions with somewhat strong assumptions. These auctions are designed in such a way such that the bidders with higher bids are always favored. But favoring higher bidders does not always necessarily maximize the revenue and/or usage. Moreover, spectrum owner’s intention is not only to maximize revenue but also to be fair to the market. Bidders in these auctions may either look for single unit or bundle of units from the available pool of multiple units of resources. Thus in contrast to the classical auctions, these auctions can be considered as a generalized knapsack problem [20, 34, 58, 70]. The resource auctioning mechanisms and the problem of determining winners are investigated with the help of forward auctions and reverse auctions in [20] which comes out to be knapsack problems. In [34], the author summarizes the connection between winner determination problems in multi-unit combinatorial auctions and generalized knapsack problems. In [58],
the authors model the multi–unit auction as a knapsack problem for the *selfish players* and investigate their truthful strategies. Though the above auction models take the right approach towards the revenue and usage maximization but they only consider identical items with fixed values of the goods. This unfortunately may not be true for spectrum auctions as spectrum bands may or may not be of same value to different WSPs. Also the valuation of the spectrum bands may constantly vary over time. This is because revenue generated from the same spectrum band through the services for end–users may be different for different WSPs due to many factors such as their locations, interference from others, end–users’ strategies etc. Considering such constraints, a spectrum architecture called DIMSUMnet was proposed in [10]. In [60, 61], the authors introduced a dynamic spectrum access scheme in which a spectrum manager periodically auctions short-term spectrum licenses. The spectrum is sold at a unit price, and the assumptions underneath is that a large number of spectrum buyers are present and none has enough power to influence the market clearing price. Spectrum auctioning mechanisms under heterogeneous wireless access networks have been investigated in [66].

### 3.2 Related Work on Pricing using Game Theory

Let us discuss the emerging body of work in game theory that deal with decision making in a multi–provider setting. In [13], a market in the form of a ‘bazaar’ is introduced where infrastructure–based wide area wireless services are traded in a flexible manner and at any
time scale. The mobile bazaar architecture allows fine-grained service through cooperative interactions based on user needs. The problem of dynamically selecting ISPs for forwarding and receiving packets has been studied in [78]. A multi-homed user, i.e., user with access to multiple ISPs, has the freedom to choose a subset of ISPs from the available ones. In [73], the authors examine how transit and customer prices and quality of service are set in a network consisting of multiple ISPs and the equilibrium can be achieved using threat strategies with multiple qualities of service.

Several interesting investigations on multi-access networks have also been conducted recently with game theory. Convergence of cellular networks with broadcast networks has been discussed in [31]. In [72], the authors present a resource allocation scheme for data traffic under heterogeneous cellular CDMA framework. Multi-radio channel allocation problem using game theory has been discussed in [23]. In [83], the authors assumed the existence of two orthogonal technologies that were overlaid. The network design in a wireless multi-provider setting using cooperative decision making was presented in [84]. In particular, cellular and WLANs are considered where users are vertically transferred from one network to another based on the load of each network. These works assume cooperation among the agents and make their decisions based on cooperative approaches. But in reality, entities who are involved in such pricing game approaches and decision making have always been greedy and selfish [14, 79] and they always act in a non-cooperative manner. In [39], an integrated admission and rate control framework for CDMA based wireless data networks is proposed. The providers define the admission criteria as the outcome of the game and the Nash equi-
librium is reached using pure strategy. Users are categorized into multiple classes and are offered different QoSs based on the price they pay and the service degradation they can tolerate. However, dynamic pricing was not explored in this research. In [15], a simple static pricing policy is adopted for service provisioning based on classes of service.

There is a large set of literature about Internet service pricing in [46, 47, 53, 74] which concentrate on customer pricing strategy and differentiated service pricing. In [50], the authors study the economic interests of a wireless access point owner and his paying client, and model their interaction as a dynamic game. Resource allocation and base-station assignment problems for the downlink in CDMA networks is studied based on dynamic pricing in [37]. Revenue maximization and pricing problems are discussed in [44, 80]. In [44], pricing for end-users with heterogeneous service requirements are investigated. In [26], the authors presented a non-cooperative game for pricing Internet services but concluded with an unfair Nash equilibrium where future upgradation of the networks were not discussed. These works primarily investigate the conflict between end-users and provider while the competition among multiple service providers are not discussed.

As far as the prices for the differentiated services are concerned, the wireless industry simply followed the charging model for voice services, i.e., free or flat-rate. One example of flat-rate pricing could be paying a certain amount of money every month and having unlimited usage facility. Flat-rate pricing worked so far for end-users and service providers because of its simplicity and predictability. Some of the variations of flat-rate pricing can be found in [17, 55]. However, the extensive increase in wireless service access and the
corresponding quality of service degradation have shown the need for a proper pricing model. Also, the current practice of offering different contract plans (based on minute usage) for voice services will no longer be valid for data services. There is still no distinction with respect to QoS, i.e., the service level agreement (SLA) is essentially the same. The same notion of resource sharing in voice networks is not valid because packet data systems are usually aimed at maximizing the throughput [86]. The goal is to allocate each user the maximum data rate based on the application needs and wireless channel conditions while maintaining fairness among users. This new way of looking at the resource allocation to wireless service users brings forth the requirement for proper pricing schemes for wireless services. By introducing the providers and users in a market like environment, it becomes convenient to leverage the concept of prices to regulate the demands of users who consume resources.
In this chapter, we analyze the interaction between spectrum broker working on behalf of the spectrum owner (FCC) and wireless service providers. This refers to the upper part of the cyclic dependency model presented in figure 1.4. Spectrum broker manages spectrum assignment to the service providers through auctions under different spectrum allocation constraints and procedures. Upon receiving the bid requests from service providers, the spectrum broker allocates the spectrum bands dynamically to the winners using some strategy. Service providers use the newly allocated spectrum to serve end–users and generate revenue. We assume that the assignments of the spectrum bands are done for a short duration called lease period. During this period, the spectrum chunk assigned to a service provider is dedicated to that service provider. When the lease period expires, service providers return their assigned spectrum chunks to the spectrum broker. Later, in chapter 7, we will analyze the extended economic model when even this dynamic spectrum assignment for a short duration is not dedicated for a service provider, i.e., service providers in such secondary market models may be asked to return the assigned spectrum chunk to the spectrum broker (or alternatively to the primary licensed owner of the bands) anytime.
4.1 Auction Models for DSA

Spectrum broker owns the coordinated spectrum band (CAB) and is the seller. Service providers on the other hand are the buyers. We assume that there are multiple service providers who are already overloaded i.e., they have little or no spectrum left from their static allocation. To serve the overloaded customer base and to make more profit, these service providers request additional spectrum from the CAB and advertise a price that they are willing to pay for that amount of spectrum for a certain period. Spectrum is then allocated dynamically by the spectrum broker depending on these advertised prices and the requested amounts of spectrum based on some winner determination strategy.

To determine which WSPs must get the requested spectrum, the auctioneer must answer couple of questions. First, what is the objective of the spectrum allocator (FCC for reference)? Apart from maximizing the revenue, the spectrum owner must also be fair in leasing out the unused spectrum bands for the purpose of self-coexistence. The second question that follows is what would be the pricing and market mechanisms? In such auction models, multiple spectrum chunks are available simultaneously and service providers are interested in one or multiple units of these chunks. Thus unlike classic single unit auction, multiple winners evolve constituting a winner set. The determination of winner set often depends on the auction design strategy taken by the spectrum owner. In figure 4.1, we present the broad classification of auctions based on the spectrum allocation constraints. The part shown within the dashed circle is the focus of this research where we consider the
multiple units are available for auction. We consider spectrum allocation mechanisms, where (i) bidders are granted at most one spectrum unit from the available pool, and (ii) bidders are not constrained and thus are granted multiple units. (The use of constrained auction mechanism, where bidders are granted at most one single unit is justified by the newly proposed IEEE 802.22 wireless network spectrum sharing model where IEEE 802.22 devices share the spectrum in the sub-900 MHz.) The spectrum band could be substitutable (S) or non-substitutable (NS). By substitutable band we mean a bidder will not care about which band(s) he gets as long as all the bandwidths are equal. (We ignore the physical characteristics of signals when they operate at different frequencies.) Non-substitutable bands are the ones with different bandwidths and thus different valuations. For the more generalized scenario, where bidders are granted multiple spectrum bands without any allocation constraint, we only analyze the non-substitutable bands. This is because substitutable bands
under multiple units grant is isomorphous to non-substitutable bands under the single unit
grant from multiple units for the bidders.

4.2 Auction Design for Single Unit Grant

Let $S = \{s_1, s_2, \cdots, s_m\}$ be a vector of $m$ substitutable spectrum bands and let there be
$n$ bidders engaged in the auction. For proper auction setting, we assume $n > m$. Without
loss of generality, we assume the WSPs to be greedy, i.e., they always try to maximize
profit. Let $B = \{b_1, b_2, \cdots, b_n\}$ denote the $n$-bid vector from the bidders submitted to the
auctioneer where $b_i$ is the bid from the $i$th bidder. After the auction is completed, winners
obtain the lease of the bands for a certain period. We follow the sealed-bid auction policy
to prevent collusion. We assume all the bidders are rational; losing bidders in any auction
round will increase their bids by certain amount in the next round if their bids were less
than the true valuation of the bands. Similarly, winning bidder(s) will decrease their bids by
certain amount in the next round to increase their payoff(s) till a steady state is reached. At
the end of each auction round, the auctioneer only broadcasts the information of minimum
bid submitted in that round. Note that, the justification behind not broadcasting any other
information (e.g., maximum bid) and only broadcasting minimum submitted bid information
in the proposed model is that bidders are only allowed to know the lower bound of the bids.
Knowing the lower bound will encourage only the potential bidders (bidders with reservation
price higher than or equal to the broadcasted bid information) to participate in the next auction rounds.

Since the wireless service providers use the acquired spectrum to provide services to the end users, the revenue generated from the end–users gives an indication of the true valuation price of the band. Providers use this valuation price profile to govern their future bidding strategy for forthcoming auction periods. To complicate matters in real-world scenario, the revenue generated even from one particular spectrum band can be different for different service providers depending on company policy and pricing for the end-users. Note that, this assumption does not contradict the definition of substitutable band. (With the substitutable band assumption, one single provider sees no difference between any two bands but two providers can have different revenues from same band.) As a result, the common valuation price for a spectrum band will also be different for different service providers. We present this true valuation price of a substitutable band as a vector, \( V = \{ V_1, V_2, \ldots, V_n \} \) for \( n \) bidders. Later, in section 4.2.4, we will reform the valuation price vector for the non-substitutable bands. With the valuation price and bids from a bidder formally defined, payoff of a bidder is given by,

\[
\begin{aligned}
V_i - b_i & \quad \text{if } i\text{th bidder wins} \\
0 & \quad \text{if } i\text{th bidder loses}
\end{aligned}
\]  
(4.1)

We analyze and compare the sequential and concurrent bid mechanisms under the above mentioned auction setting. Auctions among the WSPs occur periodically with the periods being the *dynamic spectrum access* (DSA) period. In the sequential mechanism, spectrum
bands are auctioned one after another in one DSA period and each winning bidder gets at most one spectrum band, i.e., winning bidder is not allowed to participate for the remaining auction rounds in that DSA period. Thus each DSA period consists of \( m \) auction rounds with decreasing number of bidders. In contrast to the sequential bid, in concurrent bidding, each DSA period consists of only one auction round. All the bidders submit their bids concurrently at the beginning of each DSA period. When one DSA period expires, all the bands are returned to the auctioneer and the process repeats for both sequential and concurrent bids.

### 4.2.1 Sequential Auction for Substitutable Bands

In sequential auction, \( m \) spectrum bands are auctioned one after another. First, \( n \) bidders submit their sealed bids for band \( s_1 \) and the winner is determined. Winner of \( s_1 \) does not participate for the rest of the auction in that DSA period. Remaining \( (n - 1) \) bidders then bid for spectrum band \( s_2 \) and so on till all the spectrum bands are auctioned. Let us analyze the properties of sequential auction.

#### 4.2.1.1 Probability of winning

We assume a time instance when the auction for \( k \) spectrum bands are over and \( k \) winners have emerged. As a result, there are \( (n - k) \) bidders participating for \( (m - k) \) spectrum bands. We assume that bids from all the bidders are uniformly distributed. The probability
density function of bid submissions in sequential auction mechanism can be given by,

\[ f(b) = \frac{1}{V_{\text{max}} - b_{\text{min}}} \]  \hspace{1cm} (4.2)

where, \( V_{\text{max}} \) is the maximum valuation possible of a spectrum band and \( b_{\text{min}} \) is the minimum bid of all the bids submitted by the existing bidders.

Now, let us assume that bidder \( i \) submits a bid \( b_i \) at the beginning of \((k+1)\)th band auction. All the other \((n-k-1)\) bidders also submit their corresponding bids for the \((k+1)\)th band. Bidder \( i \) will win the \((k+1)\)th band if and only if all the \((n-k-1)\) bidders’ bids are less than \( b_i \). Let us first find the probability that any other bid \( b_j \), \((j \in (n-k-1) \text{ bidders})\) is less than \( b_i \). The probability that any bid \( b_j < b_i \), such that, \( j \neq i; j, i \in (n-k) \text{ bidders} \), can be given by

\[ P(b_j < b_i \mid j \neq i; j \in (n-k-1) \text{ bidders}) = \int_{b_{\text{min}}}^{b_i} f(b)db \]  \hspace{1cm} (4.3)

Substituting \( f(b) \) and integrating, we obtain,

\[ P(b_j < b_i \mid j \neq i; j \in (n-k-1) \text{ bidders}) = \frac{b_i - b_{\text{min}}}{V_{\text{max}} - b_{\text{min}}} \]  \hspace{1cm} (4.4)

If bidder \( i \) is to win the \((k+1)\)th band, we need to calculate the probability that all the \((n-k-1)\) bidders’ bids are lower than the bid \( b_i \). Thus probability of bidder \( i \) winning the \((k+1)\)th band can be given by,

\[ P(\forall b_j < b_i \mid j \neq i; \forall j \in (n-k-1) \text{ bidders}) = \prod_{n-k-1}^{n-k-1} P(b_j < b_i \mid j \neq i; j \in (n-k-1) \text{ bidders}) \]  \hspace{1cm} (4.5)
Using equation 4.4 in equation 4.5, we obtain the probability of a bidder winning the 
\((k + 1)\)th auction round as

\[
P_{\text{seq}}(i^{\text{th}} \ \text{bidder winning}) = \left( \frac{b_i - b_{\text{min}}}{V_{\text{max}} - b_{\text{min}}} \right)^{(n-k-1)}
\]

(4.6)

### 4.2.1.2 Optimal bid analysis

We define *optimal bid* of *i*th bidder as the bid that wins a band and maximizes the payoff for 
*i*th bidder. In other words, optimal bid denotes the reservation bid of a bidder, exceeding 
which, the bidder is in the risk of obtaining low payoff. If on the other hand, the bid 
submitted is less than the optimal bid, probability of winning also decreases.

The *i*th bidder’s expected payoff is given by,

\[
E_i = (V_i - b_i) \times P(i^{\text{th}} \ \text{bidder winning})
\]

(4.7)

Substituting \(P_{\text{seq}}(i^{\text{th}} \ \text{bidder winning})\) from equation 4.6 into equation 4.7, we obtain,

\[
E_i = (V_i - b_i) \left( \frac{b_i - b_{\text{min}}}{V_{\text{max}} - b_{\text{min}}} \right)^{(n-k-1)}
\]

(4.8)

Let us evaluate bid \(b_i^*\) that will maximize \(E_i\). To maximize \(E_i\), we equate the first derivative 
of \(E_i\) to 0, i.e.,

\[
\frac{\partial E_i}{\partial b_i} = \frac{(V_i - b_i)(n - k - 1)(b_i - b_{\text{min}})^{(n-k-2)}}{(V_{\text{max}} - b_{\text{min}})^{(n-k-1)}} \frac{(b_i - b_{\text{min}})^{(n-k-1)}}{(V_{\text{max}} - b_{\text{min}})^{(n-k-1)}} = 0
\]

(4.9)

We obtain the optimal bid for *i*th bidder in \((k + 1)\)th auction round as

\[
b_{i,\text{seq}}^* = \frac{(n - k - 1)V_i + b_{\text{min}}}{(n - k)}
\]

(4.10)
In our auction formulation, as all the bidders are rational, the natural inclination of the losing bidders would be to increase their bids (if the bids are less than the bidders’ true valuation prices). As the auction progresses, $b_{\text{min}}$ will be non-decreasing. Thus in the steady state, with increase in auction rounds, $b_{\text{min}} \to V_{\text{min}}$, where $V_{\text{min}}$ is the minimum true valuation price of the bands.

4.2.2 Concurrent Auction for Substitutable Bands

In concurrent auction, $m$ spectrum bands are auctioned concurrently where all the $n$ bidders submit their bids together at the beginning of a DSA period. As all the bands are substitutable, each bidder submits just one bid. Each of the highest $m$ bidders win a spectrum band. Let us analyze the properties of concurrent auction here.

4.2.2.1 Probability of winning

In concurrent auction setting, a bidder’s choice would be to be among the highest $m$ bidders and to maximize the payoff profit. The probability of winning would then boil down to the probability of generating a bid such that all the bids from $(n - m)$ losing bidders are below this bid.

The probability of bidder $i$ winning a band in concurrent auction can be given by,

$$P_{\text{con}}(i^{\text{th}} \text{ bidder winning}) = \prod_{j \neq i}^{n-m} P(b_j < b_i \mid j \neq i; j \in (n - m) \text{ bidders}) \quad (4.11)$$
As a greedy bidder, the aim of the bidder is not only to win but also to maximize the profit. In other words, the aim is to win with the lowest possible bid.

Simplifying and expanding equation 4.11, we obtain the probability of a bidder winning in concurrent auction with maximized profit as

\[ P_{\text{con}}(i^{th} \text{ bidder winning}) = \left( \frac{b_i - b_{\text{min}}}{V_{\text{max}} - b_{\text{min}}} \right)^{(n-m)} \]  \hspace{1cm} (4.12)

### 4.2.2.2 Optimal bid analysis

The expected payoff is given by

\[ E_i = (V_i - b_i) \times P_{\text{con}}(i^{th} \text{ bidder winning}) \]  \hspace{1cm} (4.13)

Substituting \( P_{\text{con}}(i^{th} \text{ bidder winning}) \) from equation 4.12 into equation 4.13, we obtain,

\[ E_i = (V_i - b_i) \left( \frac{b_i - b_{\text{min}}}{V_{\text{max}} - b_{\text{min}}} \right)^{(n-m)} \]  \hspace{1cm} (4.14)

To maximize \( E_i \), we take the first derivative of \( E_i \) and equate to 0,

\[ \frac{\partial E_i}{\partial b_i} = \frac{(V_i - b_i)(n - m)(b_i - b_{\text{min}})^{(n-m-1)}}{(V_{\text{max}} - b_{\text{min}})^{(n-m)}} - \frac{(b_i - b_{\text{min}})^{(n-m)}}{(V_{\text{max}} - b_{\text{min}})^{(n-m)}} = 0 \]  \hspace{1cm} (4.15)

Solving equation (4.15), we obtain the optimal bid for \( i^{th} \) bidder in concurrent auction as

\[ b_{i,\text{con}}^* = \frac{(n - m)V_i + b_{\text{min}}}{(n - m + 1)} \]  \hspace{1cm} (4.16)

This bid is optimal in the sense that this is the minimum bid to maximize the probability of winning a spectrum band and thus maximizes the expected payoff. Next, we present a
comparison between optimal bids for both sequential and concurrent auction to study the dominant strategies for bidders.

4.2.3 Dominant Strategy – Sequential and Concurrent Auction

The optimal bids for sequential and concurrent auctions are given in equation (4.10) and (4.16) respectively. Let us consider their difference as

\[ b_{diff} = b_{i_{seq}}^* - b_{i_{con}}^* \]  

(4.17)

We consider two cases. First, under the transient state and second, when steady state has been reached. We define steady state as the state when all the bidders eventually settle down to their corresponding fixed bids and after that bidders will have no extra payoff in unilaterally changing their bids. Transient state is the learning phase where bidders have not reached the steady state and are willing to experiment with their bids. Under the transient state, we again consider two possibilities. One at the beginning of the allocation period (even before the first band auction in sequential setting: all \( m \) bands remaining) and the other after \( k \) spectrum bands auctions are over.

**Transient state – No bands auctioned so far:** The difference in optimal bids \( b_{i_{seq}}^* \) and \( b_{i_{con}}^* \) is

\[ b_{diff} = \frac{(n - 1)V_i + b_{\min}}{n} - \frac{(n - m)V_i + b_{\min}}{n - m + 1} \]  

(4.18)
Simplifying we obtain,

\[ b_{\text{diff}} = \frac{(m - 1)(V_i - b_{\text{min}})}{n(n - m + 1)} \]  \hspace{1cm} (4.19)

We know that for a bidder to win a spectrum band, the following conditions must be true.

\[ V_i \geq b_{i_{\text{seq}}}^* > b_{\text{min}} \quad \text{and} \quad V_i \geq b_{i_{\text{con}}}^* > b_{\text{min}} \]  \hspace{1cm} (4.20)

From conditions presented in equation 4.20 and for \( m > 1 \), we can conclude that \( b_{\text{diff}} \) in equation 4.19 is a positive quantity (\( b_{\text{diff}} > 0 \)). This establishes the fact that optimal bid (reservation price of the bidder) to win in sequential auction setting is more than that in concurrent auction. It is also clear from equation 4.19 that with increase in the number of available bands, \( m \), while keeping \( n \) fixed, \( b_{\text{diff}} \) increases, i.e., the difference between reservation prices in sequential auction and concurrent auction increases. Thus increasing available spectrum bands for auction, which should have been an incentive for the auctioneer, does not benefit auctioneer in real world scenario in concurrent auction setting.

**Transient state – \( k \) bands auctioned so far:** All bidders participating in \((k+1)\)th auction round have the chance to iterate their bids thus increasing the minimum bid. Note that, compared to concurrent auction, in sequential auction, bidders get the opportunity to revisit their bids \((m - 1)\) times more in each DSA period. Then in concurrent auction, as the bidders have less number of chances to resubmit their bidding strategies, it is clear that minimum bid submitted in concurrent auction would be less than the minimum bid submitted in sequential auction.
After $k$ spectrum bands auctions are over let the minimum bids in sequential and concurrent auctions be $b_{\text{min}1}$ and $b_{\text{min}2}$ respectively; such that $b_{\text{min}2} \leq b_{\text{min}1}$. Substituting values of $b^*_{\text{seq}}$ and $b^*_{\text{con}}$ in equation (4.17), we get the difference in optimal bids between sequential and concurrent auction as,

$$b_{\text{diff}} = \frac{(n - k - 1)V_i + b_{\text{min}1}}{(n - k)} - \frac{(n - m)V_i + b_{\text{min}2}}{(n - m + 1)} \quad (4.21)$$

Simplifying equation 4.21, we obtain,

$$b_{\text{diff}} = \frac{(m - k - 1)(V_i - b_{\text{min}1})}{(n - k)(n - m + 1)} + \frac{(n - k)(b_{\text{min}1} - b_{\text{min}2})}{(n - k)(n - m + 1)} \quad (4.22)$$

As all the terms in equation 4.22 are positive, it can be concluded that optimal bids in sequential auction setting is more than that in concurrent auction setting. Thus, from the auctioneer’s perspective, it is more beneficial to follow the sequential bidding mechanism for substitutable bands.

**Steady state reached:** In this case, we assume that the auction has been run for sufficient large number of times to reach the steady state both for sequential and concurrent mechanisms. As we mentioned previously, auctioneer broadcasts the minimum bid submitted so the history of minimum bids are known to all the bidders. Thus as we assume the auction model to achieve the steady state, minimum bid submitted both for sequential and concurrent mechanism would be the same.
Then the difference in optimal bids between sequential and concurrent auction is given as,

\[ b_{\text{diff}} = \frac{(m - k - 1)(V_i - b_{\text{min}})}{(n - k)(n - m + 1)} \]  

(4.23)

As all the terms in equation 4.23 are positive, it can be concluded again that optimal bids in sequential auction setting is more than that in concurrent auction setting.

### 4.2.4 Concurrent and Sequential Auctions for Non-Substitutable Bands

In this section, we present the concurrent and sequential auction models for \( m \) non-substitutable bands. For every bidder, the value of each of these \( m \) bands is different. We assume that bidders have complete information about the valuation and rankings of the bands. Under the complete information scenario, \( n \) bidders submit bids concurrently at the beginning of the allocation period.

Let the true valuation price be in the form of a vector of vectors,

\[ V = \{\{V_1\}, \{V_2\}, \cdots, \{V_n\}\} \]  

(4.24)

where \( \{V_i\} \) is the valuation price vector of \( i \)th bidder for all \( m \) spectrum bands, i.e.,

\[ V_i = \{V_{i1}, V_{i2}, \cdots, V_{im}\} \]  

(4.25)

Let the reservation price of \( i \)th bidder for all \( m \) spectrum bands be

\[ R_i = \{r_{i1}, r_{i2}, \cdots, r_{im}\} \]  

(4.26)
With all the values for bands known, it is obvious that a bidder $i$ will choose to submit bid for *that* spectrum band which will maximize his payoff profit,

$$U_i = V_{ij} - r_{ij}; \quad j \in m$$

(4.27)

The dominant strategy of bidder $i$ in concurrent auction would be to choose the band which will provide him the maximum payoff profit $U_i$. Thus it may happen that $j$th band provides the maximum payoff profit for $l$ bidders which will result in $l$ bidders competing for $j$th band excluding all other bands from the spectrum band list. Moreover, in concurrent auction the losing bidders do not have chance to revisit their bid strategy even if there might be less valuation bands unoccupied by any bidder. This problem does not happen if the auction is sequential as bidders get chances to revisit their bid strategies. We compare concurrent and sequential auction revenue generation from the auctioneer’s perspective.

### 4.2.4.1 Concurrent auction

Before we calculate the aggregate revenue for the auctioneer, let us first analyze only one band $j$. If $l > 1$ bidders aim for this band $j$, then the revenue $Rev_{ij}$ generated from this band would be the maximum bid submitted from all these $l$ bidders. If only 1 bidder aims for the band $j$, the revenue generated will be the bid submitted by the sole bidder. If no bidder aims at the band $j$, the revenue generated will be zero from band $j$. 

46
Then the total revenue generated from all the \( n \) bidders and \( m \) bands in the concurrent auction setting can be expressed in the following recursive way

\[
Rev_{con}[n, m] = Rev_l + Rev_{con}[n - l, m - 1]
\]  

where \( Rev_{con}[n, m] \) is the total revenue generated from \( n \) bidders and \( m \) bands and \( l \) can take values from 0 to \( n \). The disadvantage in such a concurrent setting is that \((n - l)\) may be 0 even if some of the bands are still left unoccupied. Thus all the bands are not sold out in auction even if \( n > m \) and thus auctioneer does not get full benefit of all the bands.

4.2.4.2 Sequential auction

Similarly, we formulate the revenue generated from the sequential auction. The total revenue generated can be presented as a recursive expression

\[
Rev_{seq}[n, m] = Rev_l + Rev_{seq}[n - 1, m - 1]
\]  

where \( l \) can take values from 0 to \( n \). We find that as the bands are sequentially auctioned, all the bands are sold out thus providing better revenue possibility than concurrent auction.

4.3 Auction Design for Multiple Unit Grant

So far, we analyzed the scenario where only a single band would be assigned to a service provider from the multiple bands available. In this section, we relax this constraint and investigate the more generalized case where service providers can win multiple spectrum bands.
available from the common spectrum pool. We propose our auction model and formulate the conflict among the service providers and spectrum broker under such multiple units grant.

To ensure successful auction design, we consider three important issues on which the success of the auction depends. They are (i) maximizing auctioneer’s revenue, (ii) attracting bidders by increasing their probability of winning, and (iii) preventing collusion so that bidders can not control the auction. It is not at all intended that only big companies with high spectrum demand should acquire these additional spectrum bands. The goal is to increase competition and bring fresh new ideas and services. As a result it is necessary to make the small companies, who also have a demand of spectrum, interested to take part in the auction. This way, revenue can be maximized and maximum use of the available spectrum from the CAB can be made.

The situation described above maps directly to the 0-1 knapsack problem, where the aim is to fill the sack as much as possible maximizing the valuations of the sack. Here, we compare the spectrum bands present in CAB as the total capacity of the sack and the bids presented by service providers as the valuations for the spectrum amount they request. We propose this auction procedure as “Dynamic Spectrum Allocator Knapsack Auction”.

We formulate the above mentioned knapsack auction as follows. Let us consider that there are \( n \) bidders looking for the additional amount of spectrum from the CAB. All the bidders submit their demand through sealed bids. We follow sealed bid auction strategy because sealed bid auction has shown to perform well in all–at–a–time auction bidding and has a tendency to prevent collusion. Note that, each service provider has knowledge about
its own bidding quantity and bidding price but do not have any idea about any other service providers’ bidding quantity and price. We assume that the spectrum band available in CAB is $W$. Now, if the spectrum requests submitted by some or all of the service providers exceed the spectrum available in CAB then the auction is held to solve the conflict among these providers.

Let, $i = 1, 2, \cdots, n$ denote the bidders (service providers). We denote the strategy taken by service provider $i$ as $q_i$, where $q_i$ captures the demand tuple of this $i$th service provider and is given by

$$q_i = \{w_i, x_i\} \quad (4.30)$$

where, $w_i$ and $x_i$ denote the amount of spectrum and bidding price for that spectrum respectively.

Auction is best suited when the total demand is more than the supply, i.e.,

$$\sum_{i=1}^{n} w_i > W \quad (4.31)$$

Our goal is to solve the dynamic spectrum allocation problem in such a way so that earned revenue is maximized from the spectrum owner’s point of view, by choosing a bundle of bidders, subject to condition such that total amount of spectrum allocated does not exceed $W$. Thus the allocation policy of the spectrum owner would be,

$$\text{maximize}_i \sum_{i} x_i \quad (4.32)$$
subject to the condition,

\[ \sum_i w_i \leq W \]  

(4.33)

### 4.3.1 Synchronous and Asynchronous Auctions

Spectrum allocation with the help of proposed sealed bid knapsack auction can be done either synchronously or asynchronously \([69]\). In synchronous auction, bids from all the bidders are taken simultaneously and allocation/de-allocation of spectrum from and to the CAB are done only at fixed intervals. On the other hand, in asynchronous auction, bids are submitted by bidders asynchronously and allocation/de-allocation of spectrum from and to the CAB are not done at fixed intervals.

**Asynchronous auction:** As the name suggests, this auction procedure of spectrum is asynchronous among the service providers as shown in figure 4.2. Whenever a service provider...
comes up with a request for spectrum from the CAB, the spectrum owner checks to see if that request can be serviced from the available pool of CAB. If the requested amount of spectrum is available, spectrum owner assigns this chunk to the service provider for the requested time and declines if the spectrum requested is not available. Similarly, if more than one service provider come up with requests for spectrum from the CAB, the spectrum owner checks to see if all the requests can be serviced from the available pool of CAB. If they can be serviced, the spectrum is assigned but if all the requests can not be granted, then auction is initiated. We denote the strategy taken by service provider \( i \) as \( q^a_i \). \( q^a_i \) captures the demand tuple of this \( i \)th service provider in asynchronous allocation mode and is given by

\[
q^a_i = \{w_i, x_i, T_i\}
\]  

(4.34)

where, \( w_i \) and \( x_i \) denote the amount of spectrum and bidding price for that spectrum respectively. \( T_i \) is the duration for which the spectrum amount is requested. The numbers inside the parenthesis in the figure 4.2 denote the duration \( T_i \) of the spectrum lease allocated.
to the corresponding bidders. As the decision about whether to allocate or not to allocate spectrum to a service provider is taken instantly in this allocation procedure by looking at the available pool only this allocation procedure is not very effective and may not maximize the earned revenue. It may happen that a service provider $B$ is willing to pay a higher price than a service provider $A$ who paid a lower price for the same demand, but unfortunately $B$’s request came up after $A$’s request. In this allocation procedure, as the spectrum owner does not have any idea about the future, $A$’s request will be processed and $B$’s will be declined (assuming that the available pool does not change at the time of $B$’s arrival). Thus revenue could not be maximized through this allocation procedure.

**Synchronous auction:** In synchronous auction, spectrum bands are allocated and deallocated at fixed intervals as shown in figure 4.3. All the service providers with a demand present their requests to the spectrum owner and the price they are willing to pay. Spectrum owner takes all the requests, processes them using some strategy and then allocates the spectrum bands to the providers at the same time for the same lease period. When the lease period expires, all the allocated spectrum chunks are returned to the common pool for future use. For example, lease periods for all the bidders are indicated as 1 in the figure 4.3.
4.3.2 Performance Comparison

We analyze and compare the synchronous and asynchronous strategies with the help of knapsack auction. Below, we present two lemmas to show the performance comparison between synchronous allocation coupled with knapsack auction and asynchronous allocation of spectrum.

**Lemma 1** Average Revenue generated in asynchronous allocation through knapsack auction procedure can not be better than average revenue generated in synchronous allocation for a given set of biddings.

**Proof:** We assume that there are $n$ bidders competing for $W$ amount of spectrum. In asynchronous allocation mode, the bid strategies taken by $i$th service provider is given by tuple $q_i^a$, while in synchronous mode, the tuples are represented by, $q_i$.

We prove the above proposition with the help of counter-example. We arbitrarily decide two time intervals, $t_j$ and $t_{j+1}$ for the asynchronous mode allocation. We assume that first deallocation(s) of spectrum and new allocation(s) are happening at time $t_{j+1}$ after time $t_j$. Moreover, we assume that the asynchronous allocation at time $t_j$ is maximal and provide the maximum revenue. Let, $m$ be the number of bidders who were granted spectrum at time $t_j$. Then, the maximum revenue generated at time $t_j$ can be given by,

$$\sum_{i=1}^{m} x_i$$

(4.35)
Now, we assume \( l \) of \( m \) bidders de-allocate at time \( t_{j+1} \) and rest \((m - l)\) bidders continue to use their spectrum. Then the revenue generated by these \((m - l)\) bidders is given by,

\[
\sum_{i}^{m-l} x_i
\]

(4.36)

Moreover, the \((n - m)\) bidders, who were not granted spectrum at time \( t_j \), will also compete for the rest of the spectrum,

\[
W - \sum_{i}^{m-l} w_i
\]

(4.37)

Now, we need to find, whether the revenue generated in this asynchronous mode at time \( t_{j+1} \) can exceed the synchronous mode revenue at the same time by the same set of bidders. For simplicity, we assume that the bidders do not change their bidding requests in time intervals \( t_j \) and \( t_{j+1} \).

By the property of 0-1 knapsack auction, we know that the revenue generated by a subset (we denote this subset by \( Q \)) of \( n - l \) bidders will be a local maxima, if only the revenue obtained from all the \((n - l)\) bidders are considered simultaneously, i.e., synchronous allocation of spectrum to \((n - l)\) interested bidders (note that \( l \) is the set of bidders de-allocating their spectrum at time \( t_{j+1} \) and are not taking part in auction at time \( t_{j+1} \)). But on the other hand, in the asynchronous mode, \((m - l)\) bidders are already present and thus knapsack auction is conducted among \((n - m)\) bidders for the spectrum \( W - \sum_{i}^{m-l} w_i \). Then, it can be easily said from the property of 0-1 knapsack auction that, this asynchronous mode will generate the same local maxima as the synchronous mode, if and only if all \((m - l)\) bidders (who are already present from the previous time interval) fall under the optimal
subset $Q$. If any of the bidders out of $(m - l)$ bidders do not fall under the optimal subset $Q$, then it is certain that asynchronous mode allocation will not be able to maximize the revenue for that given set of biddings.

Let us provide an illustrative example here to clarify the proof. Let us consider that 5 bidders who compete for a total capacity of 14 and the bid tuples generated by them at time interval $t_j$ are $(6, 10, 2), (5, 9, 3), (7, 14, 1), (2, 8, 2)$ and $(3, 9, 3)$ respectively. The first number of the tuple denotes spectrum amount requested, while the second and third numbers denote the price willing to pay for that spectrum request and time duration for which the spectrum request is done respectively. As we can see from the above tuples that bidder 3’s request has duration 1, that means, bidder 3 will de-allocate first at time $t_{j+1}$.

We execute both asynchronous and synchronous knapsack auction. In asynchronous mode, the revenue generated at time $t_j$ is 31 with the optimal subset of bidders given by bidders 2, 3, and 4. Now at time $t_{j+1}$, bidder 3 exits, while bidders 2 and 4 continue. The remaining spectrum left in the CAB is 7 for which the bidders 1 and 5 compete. Then the revenue generated at time $t_{j+1}$ is 27 and the bidders granted are 1, 2, and 4.

On the other hand, in synchronous allocation, each of the providers are allocated and de-allocated at fixed time intervals. Then with the same set of bid requests of spectrum amount and price, it is seen that maximum possible revenue generated at time $t_{j+1}$ out of the bidders 1, 2, 4 and 5 (as bidder 3 is not interested to take part in auction at time $t_{j+1}$) is 28, while the optimal subset of bidders is given by $Q = \{1, 2, 5\}$. This clearly shows that asynchronous auction may not provide the maxima all the time depending on the bidders
de-allocating and requesting.

**Lemma 2**  *Asynchronous allocation through knapsack auction procedure is sub-optimal while synchronous allocation is optimal.*

**Proof:** We define a process as optimal that always provides a local maxima for a given set of values, while a sub-optimal process may or may not achieve that local maxima with the same set of values. With the help of this definition and the proof provided in *lemma 1*, we can similarly prove *Lemma 2*.

### 4.3.3 Bidders’ Strategies

In knapsack auction, we investigate bidders’ strategies for both first and second price bidding. In first price auction, bidder(s) with the winning bid(s) pays their winning bid(s). In contrast, in second price auction, bidder(s) with the winning bid(s) do not pay their winning bid but pay some other lower winning bid according to the strategy fixed by the auctioneer.

For investigating the bidders’ strategy, we consider a particular bidder \( j \). Let each bidder \( i \) submit the demand tuple \( q_i \). Then the optimal allocation of spectrum to the bidders is done by the auctioneer taking all the demand tuples into consideration. We denote this optimal spectrum allocation as \( M \), where \( M \) incorporates all the demand tuples \( q_i \) and is subject to conditions presented in equation (4.32) and (4.33). Moreover, we assume that the \( j \)th bidder’s request falls among the optimal allocation \( M \), i.e., \( j \)th bidder has been granted
the spectrum. Then the revenue generated by auctioneer is given by,

$$\sum_{i \in M} x_i$$

where, all the bids of bidders present in the optimal allocation $M$, are summed.

In contrast, let us assume a case where $j$th bidder does not exist at all and the auction is held among the rest of the bidders. Let the optimal allocation be denoted by $M^*$ and is again subject to conditions presented in equation (4.32) and (4.33). Then the revenue generated by auctioneer in this case is given by,

$$\sum_{i \neq j, i \in M^*} x_i$$

Then the minimum winning price charged to $j$th bidder can be given by,

$$a_j = \sum_{i \neq j, i \in M^*} x_i - \sum_{i \neq j, i \in M} x_i$$

It is clear from the above equation that bidder $j$’s request is granted if

$$x_j > a_j,$$

bidder $j$’s request is not granted if

$$x_j < a_j$$

and bidder $j$ is indifferent between winning and losing if

$$x_j = a_j$$
With these insights, we try to find the bidders’ strategies in first price and second price bidding under the knapsack auction model.

Lemma 3  In second price bidding, the dominant strategy of the bidder is to bid their reservation price.

Proof: Before proving this lemma, let us explain the reservation price or true evaluation price of the bidder. When a service provider (bidder) buys spectrum from the spectrum broker, the service provider needs to sell that spectrum in form of some service to the end users who are willing to pay for that service. The revenue generated from the end users for that amount of spectrum can be the true evaluation price or reservation price for that service provider (bidder).

Let us assume $j$th service provider (bidder) has the demand tuple $q_j = \{w_j, x_j\}$ and its reservation price for that amount of spectrum requested be $r_j$. As per equation (4.40), $j$th bidder’s request will be granted and hence be in the optimal allocation $M$, only if the bid generated by $j$th bidder is more than $a_j$. Then according to the second price bidding policy, $j$th bidder will pay the second price which is $a_j$. The expected payoff obtained by $j$th bidder is given by,

$$E_j = r_j - a_j$$

We proceed to show that $j$th bidder’s true bid is its reservation price $r_j$ as claimed in the lemma using counter proof approach.
We assume that \( j \)th bidder does not bid its true evaluation of the spectrum requested, i.e., \( x_j \neq r_j \). Two cases might arise depending on the relative values of \( x_j \) and \( r_j \).

**Case 1:** Bid is less than the reservation price, i.e., \( x_j < r_j \).

- \( r_j > x_j > a_j \): bidder \( j \) falls inside the optimal allocation \( M \) and its request is granted. The expected payoff obtained by \( j \)th bidder is still given by \( (r_j - a_j) \).

- \( r_j > a_j > x_j \): bidder \( j \) loses and its request is not granted. Accordingly, the expected payoff is 0.

- \( a_j > r_j > x_j \): bidder \( j \) still loses and the expected payoff is again given by zero.

**Case 2:** Bid is more than the reservation price, i.e., \( x_j > r_j \).

- \( x_j > r_j > a_j \): bidder \( j \) falls inside the optimal allocation \( M \) and its request is granted. The expected payoff obtained by \( j \)th bidder is still given by \( (r_j - a_j) \).

- \( x_j > a_j > r_j \): though bidder \( j \) wins but the expected payoff becomes negative in this case. The expected payoff obtained by \( j \)th bidder is now given by \( (r_j - a_j) < 0 \). Bidder \( j \) definitely will not be interested in this scenario.

- \( a_j > x_j > r_j \): bidder \( j \) loses and the expected payoff is again 0.
Thus it is clear that if bidder $j$ wins, then the maximum expected payoff this bidder can obtain is given by $E_j = r_j - a_j$ and bidding any other price above or below its reservation price $r_j$ will not increase the payoff. Thus the dominant strategy of the bidders in second price bidding is to bid their reservation prices.

**Lemma 4** *In first price bidding, the bid is upper bounded by the reservation price.*

**Proof:** In contrast to the Lemma 3, in first price bidding, the expected payoff obtained by $j$th bidder can be given by,

$$E_j = r_j - x_j$$

(4.45)

as the price paid by the bidder is the same as the bid. Then, to increase the expected payoff, i.e., to keep $E_j > 0$, $x_j$ must be be less than $r_j$.

At the same time, for winning, bid $x_j$ must be greater than $a_j$, as specified in equation (4.40). Thus dominant strategy for the bidders in first price auction is $r_j > x_j > a_j$. 

60
CHAPTER 5
SERVICE PROVISIONING USING GAME THEORY

In this chapter, we analyze the interactions between wireless service providers and end-users. This is depicted as the lower part of the cyclic dependency model presented in figure 1.4. We consider the most generic abstraction of “always greedy and profit seeking” real-world network service model. In this model, multiple wireless service providers exist and provide service to a common pool of users. The providers are in a competitive environment and must make profit. Users on the other hand choose service providers depending on benefit they obtain for the prices they pay. If users must pay for their services, it is quite natural that the aim of the users will be to maximize their benefit and always act greedy. Thus, we have conflicting agents (multiple wireless service providers competing among themselves for users) who may or may not have information about others; but the aim of these agents is to maximize their own benefits. Next, we discuss this conflict model.

5.1 Modeling Conflict between WSPs and End users

We consider a simple network model as shown in figure 5.1, where any user can access any wireless service provider (WSP). These users can be interpreted as potential buyer of the service, while the WSPs can be interpreted as sellers. Each WSP buys its own spectrum
resources from FCC or some other regulatory body in respective countries and sells this resource in some form of services (e.g. bandwidth) to the subscribers (users). There are a finite number of users who compete among themselves for the available resources. In the existing static models, subscribers (users) have to subscribe to a wireless service provider for a certain period (of the order of a year) and can not change the provider for this period. In our proposed dynamic model users will subscribe to a wireless service provider in a service basis and can change its WSP upon completion of that service. Accordingly, users will pay for the consumed resources from the chosen service providers. But in this new model as pricing is not static, the users do not have any information about other users’ strategies i.e., the price the other users are willing to pay for a service or estimate of demand of resources from other users. In such an incomplete information scenario, the benefit of a user depends not only on its own strategy but also on those of other users. Since we assume that every user is selfish (all trying to pay the minimum for a service), the problem is modeled as a non-cooperative game.
Service providers, very much like the users, also act in their self-interest. As a seller of the service, the service providers determine the price for its services depending on the amount of spectrum acquired from the spectrum owner and the price paid for it and also estimating other service providers' strategies. Similar to the non-cooperative incomplete information game among the users, the service providers also do not have any information about other providers' strategies, such as, price assigned to resources, allotted resource, remaining resource, existing load, etc. Based on this conflict model, we need to define the decisions that we need to make. Before we present our decision model, let us explicitly state the underlying assumptions.

Assumptions:

1. We consider a scenario where a spatially distributed population of customers are equipped with devices that can be served by all the service providers in that region. For the sake of simplicity, we consider only one type of technology being used by the providers, i.e., all the providers provide cellular coverage. Services using other technologies such as WiFi are not considered.

2. The devices carried by the users have the capability of measuring channel condition experienced from each of the service providers. Channel condition can easily be measured in terms of interference or received signal strength from other users and base stations.
3. Wireless service providers are selected on a session by session basis. For every session, a user chooses one of multiple service providers that have the capability of providing the resource (bandwidth) demanded by that user meeting its expectation.

4. Each service provider is capable of serving multiple customers simultaneously – the exact number depends on the total resource available to that provider.

5. Service providers and users are honest in this model, i.e., users request their true demands and wireless service providers advertise the true existing load in its network in addition to the price offering per unit of resource.

### 5.2 Decision Model

As a user (or potential buyer), the decision problem is to select the best service provider for the session requested. Now the question arises, how to select the best service provider or rather what criteria determines the best. One answer to this question might be to select the service provider that offers the lowest price. In that case it might be assumed that for any service that a user needs, he sends out a request stating resource amount needed, and the service providers respond by advertising a price (this price could be price of the whole service or price per unit of resource) – the lowest of which is selected by the user. Though this approach yields the lowest price to the user, it does not necessarily mean that this is the best approach for the users to follow. The above approach does not consider the quality of service that is expected. It might easily happen that price advertised by a provider is
very low but quality of service is not met thus making the service useless for the users. The quality of service perceived by a user in a network will depend on the instantaneous traffic load and channel state in that network. Therefore, we must consider the load and channel condition in the decision making process and perform a cost benefit analysis to find the best service provider. A natural question that arises in such settings is the existence of Nash equilibrium. In other words, we are interested in finding an equilibrium such that no user finds it beneficial to change its strategy to choose a wireless service provider.

As a service provider (or potential seller), the decision problem is to advertise a price for a service without knowing what prices are being advertised by its competitors. The optimization is to find a price such that the provider is able to sustain profit in spite of offering a low price i.e., is there any price threshold to reach Nash equilibrium and at the same time maximize profit? With fixed resources and operational costs, prices exhibited cannot be too low, as that would attract too many users leading to degradation in performance because of resource sharing. It is to be noted that such a price (equilibrium) might not even exist. This is because a provider might have to advertise such a low price to attract users that it might not make any profit—contradicting the goal of profit maximization. Moreover, when a provider is abiding by some pricing constraint to reach Nash equilibrium, the sole objective is to find a strategy that would help the provider to maximize its own profit.

Let us now formulate the games for both the providers and the users. Here, we assume each wireless service providers has its own resources and provides this resource to the users upon receiving demands from the users in a service basis. Users subscribe to the best (best
chosen by users according to some criteria) provider and use the resource for both uplink and
downlink purpose. First, we define the utility functions for the users and providers that we
use for decision making process including price determination and choosing wireless service
providers. Then we investigate the existence of some system thresholds that must be agreed
upon in order to reach Nash equilibrium.

5.3 Utility functions for WSPs and End users

In our network model, the consistent objective of any greedy, selfish entity is to maximize
its profit for every session that it is active. For mathematical characterization, a utility
function is needed to represent the potential benefits (i.e., revenue earned) and losses (i.e.,
cost incurred).

We assume that there are $L$ service providers that are trying to cater to a common pool of
$\mathcal{N}$ users. Let the price per unit of resource advertised by the service provider $j$, $1 \leq j \leq L$, at
time $t$ be $p_j(t)$. Moreover, $b_{ij}(t)$ is the resource consumed by user $i$, $1 \leq i \leq \mathcal{N}$ and allocated
by the $j$th provider. We further assume that the total resource (capacity) of provider $j$ is
$C_j$.

Then, the empirical utility obtained by a user $i$ under the provider $j$ can be given by,

$$a_{ij} \log(1 + b_{ij}(t))$$

(5.1)

Note that, we could have chosen any other form for the empirical utility that increases with
$b_{ij}(t)$. But we chose the $\log$ function because the empirical benefit increases quickly from
zero as the total throughput increases from zero and then increases slowly. This reflects the intuition that the initial increase in the perceived throughput is more important to a user. Moreover, \( \log \) function is analytically convenient, increasing, strictly concave and continuously differentiable. The coefficient \( a_{ij} \) is a positive parameter that indicates the relative importance of empirical benefit and acts as a weightage factor.

Next, we consider the cost components that the user needs to encounter for obtaining the empirical benefit. The first cost component is the direct cost paid to the provider for obtaining \( b_{ij}(t) \) amount of resource from that provider. If \( p_j(t) \) is the price per unit of resource for \( b_{ij}(t) \) amount of resource, then the direct cost paid to the \( j \)th provider is given by,

\[
p_j(t) b_{ij}(t) \tag{5.2}
\]

This direct cost component will try to decrease the user \( i \)'s empirical utility. Note that in the above equation (6.4), both price per unit resource and the resource amount requested are variable quantity, where price per unit of resource is offered by providers and resource amount is requested by users.

The second cost component encountered by the user is purely due to the queuing delay under the \( j \)th service provider, which depends on the resources obtained by all the users from the total available. This queuing delay can be twofolds. One, for the downlink process, queuing delay at the base station and second for the uplink process when the users are sharing the uplink. We assume the queuing process to be \( M/M/1 \) at both the links. Using these above assumptions, the delay cost component can be given by,

\[
\psi \left( \frac{1}{C_j - \sum_{i}^{N_j} b_{ij}(t)} \right) \begin{cases} 
\frac{1}{C_j - \sum_{i}^{N_j} b_{ij}(t)} & \text{if } \sum_{i}^{N_j} b_{ij}(t) < C_j \\
\infty & \text{if } \sum_{i}^{N_j} b_{ij}(t) \geq C_j 
\end{cases} \tag{5.3}
\]

Where, \( N_j \) is the number of users under \( j \)th provider.
The last cost component is the cost due to the inherent characteristics of the wireless medium, viz channel condition. Due to the relative distances of the user from the base stations of different networks, and due to the various radio propagation effects, the signal received from different base stations will be different. If the channel quality is good, then the loss due to the channel will be less leading to higher empirical benefit to the users. On the other hand, if the quality of the channel is poor, then loss probability will be more leading to lower empirical benefit. Without loss of generality, we assume that \( Q_j \) denotes the wireless channel quality from the receiving base station from \( j \)th provider. We model this cost component as an inverse function of \( Q_j \) and write as,

\[
\phi\left(\frac{1}{Q_j}\right)
\] (5.4)

With all the components of the user utility known, we write the net utility by combining the components as obtained in equations (6.3), (6.4), (5.3), and (5.4) as

\[
U_{ij}(t) = a_{ij} \log(1 + b_{ij}(t)) - p_j(t)b_{ij}(t) - \psi\left(\frac{1}{C_j - \sum_{i}^{N_j} b_{ij}(t)}\right) - \phi\left(\frac{1}{Q_j}\right)
\] (5.5)

where, \( N_j \) is the number of users currently with service provider \( j \) including user \( i \).

We also obtain the utility as obtained by the service providers. The utility of service provider \( j \) at time \( t \) is given by

\[
V_j(t) = p_j(t) \sum_{i}^{N_j} b_{ij}(t) - K_j
\] (5.6)

where, \( K_j \) is the cost incurred to provider \( j \) just for maintaining the resources. For the sake of simplicity, we assume this cost to be a constant regardless of the amount of resources handled by provider \( j \).
Once we have formulated the utility equations of users and providers, it is now time to look at the game where both the users and the providers try to maximize their utility at every time. In this model, as providers do not have any information about other providers’ strategies, i.e., the traffic load and price level set at time $t$, and users also are ignorant of other users demands, we have an incomplete information game in our hand.

In this incomplete information game we try to find out if there exist any strategy(s) that will help the users and providers to reach any equilibrium(s). Since, there is component in users’ utility functions that accounts for the channel conditions, let us investigate if there exists any bound on the channel quality for end users. In addition to the channel threshold strategy, we also examine if any pricing threshold strategy is there. In section 5.3.2, we investigate if there exists any pricing constraint from the users’ and provider’s point of view.

5.3.1 Existence of Channel Threshold

We first analyze whether there exist any strategy for the users in choosing wireless service providers with respect to channel condition. We investigate if there exists any channel quality threshold i.e., any minimum acceptable channel quality below which it will not be beneficial to select a network. Also, we show that any unilateral decision to deviate from the minimum threshold will not help a user i.e., the threshold is the Nash equilibrium.
Theorem: Under varying channel conditions, a rational user should be active (transmit/receive) only when the channel condition is better than the minimum channel quality threshold set by the system to achieve Nash equilibrium. The user can not increase its benefit any more by unilaterally increasing or decreasing its minimum channel quality threshold.

Proof: To prove the above theorem, we modify the net utility equation given by equation (5.5) to emphasize the channel quality cost component. We present the modified equation as,

\[ U_{ij}(t) = \begin{cases} u_i(b_i, b_{-i}) - \phi\left(\frac{1}{Q_j}\right) & \text{if active} \\ 0 & \text{if not active} \end{cases} \] (5.7)

where,

\[ u_i(b_i, b_{-i}) = a_{ij} \log(1 + b_{ij}(t)) - p_j(t)b_{ij}(t) - \psi\left(\frac{1}{C_j - \sum_{i,j}^{N_j} b_{ij}(t)}\right) \] (5.8)

The notation \( u_i(b_i, b_{-i}) \) emphasizes that the \( i \)th user’s utility not only depends upon its own strategy but also on the strategies taken by the rest of the users denoted by \( b_{-i} \). For the notational simplification purpose, we use \( Q \) instead of \( Q_j \) throughout the proof for this theorem, where, \( Q \) here defines the channel condition perceived by the user when the intended wireless service provider is the \( j \)th service provider.

5.3.1.1 Channel (Nash) Equilibrium

From the equation established in (5.7), it is clear that due to the inverse nature of cost due to channel condition, utility decreases monotonically with decrease in channel condition. So we hypothesize that a user should be active with \( j \)th service provider, only if its channel
quality is better than a given threshold. Let this threshold be $Q_T$. Note that this threshold is effective with regard to $j$th wireless service provider only. The threshold might be different for different WSPs. Therefore, the probability that a user is active with $j$th provider is,

$$\int_{Q_T}^{\infty} f_Q(x) dx = p'(Q_T) \quad (5.9)$$

Let $f_Q(x)$ is the probability density function of $Q_T \leq Q < \infty$. Now, if we assume that all the other users in the network act rationally and maintain the minimum channel quality threshold $Q_T$, then the probability that $l$ users out of $N$ other users in the $j$th network will be active is given by,

$$p_l = \binom{N}{l} (p'(Q_T))^l (1 - p'(Q_T))^{N-l} \quad (5.10)$$

Then, the expected net utility of $i$th user (if the user is active) is given by,

$$E[U_{ij}(t)] = \sum_{l=0}^{N} (u_i(b_i, b_{-i}) - \phi(\frac{1}{Q}))p_l \quad (5.11)$$

As $\sum_{l=0}^{N} \binom{N}{l} (p'(Q_T))^l (1 - p'(Q_T))^{N-l} = 1$, the above equation can be rewritten as

$$E[U_{ij}(t)] = \sum_{l=0}^{N} u_i(b_i, b_{-i})p_l - \phi(\frac{1}{Q}) \quad (5.12)$$

If we define $u_i'(Q_T)$ as

$$u_i'(Q_T) = \sum_{l=0}^{N} u_i(b_i, b_{-i})p_l \quad (5.13)$$

then the expected net utility of user $i$ is given by

$$E[U_{ij}(t)] = u_i'(Q_T) - \phi(\frac{1}{Q}) \quad (5.14)$$
If the user is active, then the expected net utility is given by equation (5.14). If the user is not active then by definition the expected net utility is 0. Thus, the achievable gain (net utility considering both modes— the user is active and the user is not active) obtained by user $i$ is

$$G'_i(Q_T) = \int_{Q_T}^{\infty} [u'_i(Q_T) - \phi(\frac{1}{x})]f_Q(x)dx = u'_i(Q_T)p'(Q_T) - B'(Q_T)$$  

(5.15)

where $B'(Q_T) = \int_{Q_T}^{\infty} \phi(\frac{1}{x})f_Q(x)dx$.

Now we will show that if the users act rationally and be active only when the channel condition is better than $Q_T$, then Nash equilibrium can be reached, i.e., they will reach a stable state where the gain of a user cannot be increased further by unilaterally changing the strategy of that user. For a user, the expected net utility for being active and not being active should be equal at the threshold. Therefore, the solution to the equation,

$$u'_i(Q_T) - \phi(\frac{1}{Q_T}) = 0$$  

(5.16)

gives the value of the threshold. We will now show why maintaining this threshold will help reach the Nash equilibrium.

Let $Q_1$ be the solution to equation 5.16. Then, the average achievable gain of $i$th user is given by

$$G'_i(Q_1) = \int_{Q_1}^{\infty} [u'_i(Q_1) - \phi(\frac{1}{x})]f_Q(x)dx = u'_i(Q_1)p'(Q_1) - B'(Q_1)$$  

(5.17)
Suppose, this user now unilaterally changes his strategy and decides the threshold to be $Q_2$.

Then the average achievable gain for $i$th user will be

$$G_i'(Q_2) = \int_{Q_2}^{\infty} [u_i'(Q_1) - \phi(\frac{1}{x})] f_Q(x) dx$$

$$= u_i'(Q_1)p'(Q_2) - B'(Q_2) \tag{5.18}$$

The difference, $(G_i'(Q_1) - G_i'(Q_2))$, in the gain is given by,

$$[u_i'(Q_1)p'(Q_1) - B'(Q_1)] - [u_i'(Q_1)p'(Q_2) - B'(Q_2)]$$

$$= u_i'(Q_1)[p'(Q_1) - p'(Q_2)] - [B'(Q_1) - B'(Q_2)]$$

$$= \phi(\frac{1}{Q_1})[p'(Q_1) - p'(Q_2)] - [B'(Q_1) - B'(Q_2)]$$

Two cases might arise depending on the relative values of $Q_1$ and $Q_2$.

Case 1: $Q_1 > Q_2$

$$G_i'(Q_1) - G_i'(Q_2) = -\left[\int_{Q_2}^{Q_1} [\phi(\frac{1}{Q_1}) - \phi(\frac{1}{x})] f_Q(x) dx\right]$$

$$> 0 \tag{5.19}$$

Case 2: $Q_1 < Q_2$

$$G_i'(Q_1) - G_i'(Q_2) = \left[\int_{Q_1}^{Q_2} [\phi(\frac{1}{Q_1}) - \phi(\frac{1}{x})] f_Q(x) dx\right]$$

$$> 0 \tag{5.20}$$

Thus, we find that a user cannot increase his gain by unilaterally changing his strategy.

As a result, it becomes evident that a channel quality constraint exists for the users and maintaining this threshold will help the users to reach Nash equilibrium.
5.3.2 Existence of Resource and Price (Nash) equilibrium

Now, we investigate the resource and pricing constraint from the users’ and providers’ point of view to reach Nash equilibrium. We assume that user $i$ wants to connect to a provider at time $t$ with a certain requested resource amount. All the providers advertise their price per unit of resource amount and the existing load. As user $i$ wants to maximize its net utility (potential benefit subtracted by cost incurred), he computes the resource vector that would maximize utilities from all the providers and the corresponding maximized utility vector.

$$\{b_{i1}(t), b_{i2}(t), \cdots, b_{iL}(t)\} \quad (5.21)$$

$$\{U_{i1}(t), U_{i2}(t), \cdots, U_{iL}(t)\} \quad (5.22)$$

User $i$ would then connect to provider $j$ if $U_{ij}(t)$ gives the maximum value in the maximized utility vector and $b_{ij}(t)$ is the requested resource amount. This $b_{ij}(t)$ is the optimal amount of resource to be consumed by user $i$ from provider $j$ with advertised price $p_j(t)$. If all the users follow this strategy of optimal resource consumption then it can be shown to be a Nash equilibrium. To simplify our analysis, we now assume all the users maintain the channel quality threshold. Then without loss of generality, we combine the cost components $\psi\left(\frac{1}{C_j - \sum_i^N b_{ij}(t)}\right)$ and $\phi\left(\frac{1}{Q_j}\right)$ and modify equation (5.5) as

$$U_{ij}(t) = a_{ij}\log(1 + b_{ij}(t)) - p_j(t)b_{ij}(t) - \xi\left(\frac{1}{C_j - \sum_i^N b_{ij}(t)}\right) \quad (5.23)$$

where,

$$\xi\left(\frac{1}{C_j - \sum_i^N b_{ij}(t)}\right) = \psi\left(\frac{1}{C_j - \sum_i^N b_{ij}(t)}\right) + \phi\left(\frac{1}{Q_j}\right). \quad (5.24)$$
Basically, the function $\xi(\cdot)$ absorbs $\psi(\cdot)$ and $\phi(\cdot)$, and we assume that $b_{ij}(t)$ in $\xi(\cdot)$ captures the behavior of channel quality. If the channel quality is good, less amount of $b_{ij}(t)$ (resource) would be required; and a possibility of serving more number of users by that provider. On the other hand, if the channel quality is poor, then more resources (eg., more time slots) will be required to maintain the QoS of user $i$.

Let us investigate if there exists any optimal resource amount for the users to reach the Nash equilibrium and any pricing bound from the providers that will maximize the users utility. To do so we need to find out whether the net utility given in equation (5.23) can be maximized with respect to the resource amount. If so, then a unique maximization point exists for $U_{ij}(t)$ with respect to $b_{ij}(t)$. We need to find out whether there exist any Nash equilibrium at this point. Differentiating equation (5.23) with respect to $b_{ij}(t)$, we get

$$U'_{ij}(t) = \frac{a_{ij}}{1 + b_{ij}(t)} - p_{j}(t) - \xi'(\frac{1}{C_{j}} - \sum_{i}^{N_{j}} b_{ij}(t))$$

(5.25)

Similarly, the second derivative is

$$U''_{ij}(t) = -\frac{a_{ij}}{(1 + b_{ij}(t))^2} - \xi''(\frac{1}{C_{j} - \sum_{i}^{N_{j}} b_{ij}(t)})$$

(5.26)

If we assume delay and congestion component, such that, $\xi''(\frac{1}{C_{j} - \sum_{i}^{N_{j}} b_{ij}(t)}) > 0$, then,

$$U''_{ij}(t) < 0$$

(5.27)

It is clear from the above condition that $U_{ij}(t)$ is strictly concave in the region bounded by $\sum_{i}^{N_{j}} b_{ij}(t) = C_{j}$; and $U_{ij}(t) \to -\infty$ as $\sum_{i}^{N_{j}} b_{ij}(t) \to C_{j}$. Moreover, it can be inferred from equation (5.26) that as $U''_{ij}(t) < 0$, $U_{ij}(t)$ contains a unique maximization point.
Thus, equating equation (5.25) to 0, we get the desired maximization point,

\[
\frac{a_{ij}}{1 + b_{ij}(t)} - p_j(t) - \xi'(\frac{1}{C_j - \sum_{i}^{N_j} b_{ij}(t)}) = 0
\]  

(5.28)

It is clear that solving the above equation 5.28 for \(b_{ij}(t)\) will give us the optimal amount of resources needed by the users for a certain price \(p_j(t)\) from provider \(j\) and this resource amount will maximize the utility of the user. If the users follow this strategy of demanding this optimal amount of resources for a certain advertised price, then Nash equilibrium will be achieved, where changing this strategy unilaterally by a single user will always give him the utility lesser than the maximum possible value.

From the reverse point of view, it is also clear from the above equation 5.28 that there exists a maximum threshold for the price \(p_j(t)\) which is in the region bounded by \(\frac{a_{ij}}{1 + b_{ij}(t)} - \xi'(\frac{1}{C_j - \sum_{i}^{N_j} b_{ij}(t)})\). For a certain demand \(b_{ij}(t)\) from a user, provider \(j\) must obey this pricing constraint to maximize net utility for user \(i\). Otherwise the user will not connect to this provider thus leading to a loss of user and thus loss of profit for the provider.

To find a more compact form of pricing constraint, we proceed this way. We know as the users are homogeneous here, to maximize users’ utility, first derivative of all the users can be equated to zero,

\[
U'_{1j}(t) = U'_{2j}(t) = \cdots = U'_{N_{ij}}(t) = 0
\]

(5.29)

Recall, \(N_j\) is the number of users currently served by provider \(j\). Then it can be easily said that,

\[
\frac{a_{1j}}{1 + b_{1j}(t)} = \frac{a_{2j}}{1 + b_{2j}(t)} = \cdots = \frac{a_{N_{ij}}}{1 + b_{N_{ij}}(t)}
\]

(5.30)
Letting $1 + b_{ij}(t) = m_{ij}(t)$ and with the help of identity, we can say,

$$\frac{a_{ij}}{m_{ij}(t)} = \frac{\sum_{i}^{N_j} a_{ij}}{\sum_{i}^{N_j} m_{ij}(t)} \quad (5.31)$$

For notational simplicity, we represent $a_{j} = \sum_{i}^{N_j} a_{ij}$ and $m_{j}(t) = \sum_{i}^{N_j} m_{ij}(t)$. Thus, equation (5.31) can be simply written as

$$\frac{a_{ij}}{m_{ij}(t)} = \frac{a_{j}}{m_{j}(t)} \quad (5.32)$$

Putting the above form into equation (5.25), we get

$$U_{ij}'(t) = \frac{a_{j}}{m_{j}(t)} - p_{j}(t) - \xi'(\frac{1}{C_{j} + N_{j} - m_{j}(t)}) \quad (5.33)$$

It is clear that $U_{ij}'(t)$ is strictly decreasing with the values of $m_{j}(t)$ lying in the interval $(C_{j}, C_{j} + N_{j})$. Then for achieving the Nash equilibrium by the providers, the pricing constraint $p_{j}(t)$ is upper bounded by,

$$\frac{a_{j}}{m_{j}(t)} - \xi'(\frac{1}{C_{j} + N_{j} - m_{j}(t)}) \quad (5.34)$$

This pricing upper bound helps the provider to reach the Nash equilibrium. If all of the rest of the providers and users keep their strategies of maintaining pricing threshold unchanged, then it is quite clear that, if a provider changes its strategy unilaterally and decides not to maintain its pricing upper bound, then that provider won’t be able to maximize its users’ utility and thus users will not connect to this provider. Thus unilaterally changing the strategy will not increase the profit of the provider and Nash equilibrium will not be achieved.
5.4 Maximizing Providers’ and Users’ Utilities

Now that we have found the existence of Nash equilibrium, we analyze the maximization point from the providers’ and end users’ perspective. That is, we need to solve for the total optimal amount of resources consumed by the users and the price for which the provider’s utility will be maximized. This total optimal amount of resource is nothing but the estimated demand of resource of the providers at the beginning of every session.

5.4.1 Finding Optimal Pricing for WSPs

Our objective is thus to maximize $V_j(t)$ abiding by the equation 5.34. Replacing $\sum_i N_j b_{ij}(t)$ by $m_j(t) - N_j$, we get,

$$V_j(t) = \left( \frac{a_j}{m_j(t)} - \xi'(\frac{1}{C_j + N_j - m_j(t)}) \right) (m_j(t) - N_j) - K_j$$

(5.35)

Differentiating equation (5.35) with respect to $m_j(t)$, we get,

$$V'_j(t) = \left( \frac{a_j}{m_j(t)} - \xi'(\frac{1}{C_j + N_j - m_j(t)}) \right) + \left( -\frac{a_j}{(m_j(t))^2} - \xi''(\frac{1}{C_j + N_j - m_j(t)}) \right) (m_j(t) - N_j)$$

(5.36)

Differentiating equation (5.36) again, we get

$$V''_j(t) = \left( -\frac{a_j}{(m_j(t))^2} - \xi''(\frac{1}{C_j + N_j - m_j(t)}) \right) + \left( -\frac{2a_j}{(m_j(t))^3} - \xi'''(\frac{1}{C_j + N_j - m_j(t)}) \right) (m_j(t) - N_j)$$

(5.37)
As we have assumed earlier, if $\xi(\cdot)$ is such that both $\xi''(\cdot)$ and $\xi'''(\cdot)$ are positive, then $V_j''(t) < 0$; which again indicates that $V_j(t)$ has a unique maximum. We can infer that with a constraint on the price, the providers can maximize their utility too. We now try to find the exact price that will maximize the provider’s utility.

For finding the maxima, we equate equation (5.36) to 0, which gives the optimal value of $m_j(t)$. Equation (5.36) is not in closed form because the exact nature of $\xi(\cdot)$ is not known. We assume the solution of the above equation to be $m_{j(opt)}(t)$. Of course, for a given $\xi(\cdot)$, the value of $m_{j(opt)}(t)$ can always be obtained.

The optimal price that will maximize provider $j$’s utility can be obtained by substituting $m_{j(opt)}(t)$ in equation (5.34). Thus, we get the optimal price as

$$p_{j(opt)}(t) = \frac{a_j}{m_{j(opt)}(t)} - \xi'(\frac{1}{C_j + N_j - m_{j(opt)}(t)})$$

(5.38)

To have a better insight into the analysis, we assume a simple closed form of $\xi(\frac{1}{C_j + N_j - m_{j(t)}})$ which we write as $\frac{1}{(C_j + N_j - m_{j(t)})^\alpha}$, where $\alpha$ is a power coefficient in the delay and congestion component.\(^1\) Rewriting equation (5.36), we get

$$V_j'(t) = \left(\frac{a_j}{m_j(t)} - \frac{\alpha}{(C_j + N_j - m_j(t))^{\alpha+1}}\right) + \left(-\frac{a_j}{(m_j(t))^2} - \frac{\alpha(\alpha + 1)}{(C_j + N_j - m_j(t))^{\alpha+2}}\right) (m_j(t) - N_j)$$

(5.39)

\(^1\)While taking an exact form of $\xi(\cdot)$, we made sure that it satisfies the constraint of its 1st, 2nd and 3rd derivatives to be positive. Any other form of $\xi(\cdot)$ would also suffice if the derivatives are positive.
Simplifying and equating equation (5.39) to 0, we get
\[ a_j N_j (C_j + N_j - m_j(t))^{\alpha+2} - \alpha^2 m_j(t)^3 - \alpha m_j(t)^2 (C_j + N_j) + \alpha (\alpha + 1) N_j m_j(t)^2 = 0 \] 

(5.40)

To obtain a closed form solution of \( m_j(t) \), we further assume, \( \alpha = 1 \) and \( N_j = C_j \). The above equation simplifies to
\[ (2C_j - m_j(t)) \sqrt[3]{a_j C_j} = m_j(t) \] 

(5.41)

which we solve for optimal \( m_j(t) \) to get
\[ m_{j(opt)}(t) = \frac{2C_j \theta}{1 + \theta} \] 

(5.42)

where, \( \theta = \sqrt[3]{a_j C_j} \).

Using the optimal value of \( m_j(t) \), we get the optimal value of \( p_j(t) \) as
\[ p_{j(opt)}(t) = \frac{a_j}{2C_j} \left( 1 + \frac{1}{\theta} \right) - \left( \frac{1 + \theta}{2C_j} \right)^2 \] 

(5.43)

which can be simplified and rewritten as
\[ p_{j(opt)}(t) = \frac{a_j}{2C_j} \left( 1 + \frac{1}{\theta} \right) - \left( \frac{1 + \theta}{2C_j} \right)^2 \] 

(5.44)

Thus, we see that the providers can achieve Nash equilibrium under the given pricing constraint and at the same time they can maximize their utility if the price is set as given by equation (5.44). Next, we use this pricing strategy as an incentive for the providers to upgrade their resources and users to improve their utility.
5.4.2 Estimation of Bandwidth Demand from End users

With the strategies for prices set and the expected profit known, we also need to calculate the optimal amount of resources that the users should demand to maximize their utility.

Substituting the value of \( m_{j(\text{opt})}(t) \) in equation (5.32), we get

\[
m_{ij}(t) = \frac{a_{ij}}{a_{j}} \left( \frac{2C_j \theta}{1 + \theta} \right)
\]

(5.45)

Now, we know, \( m_{ij}(t) = 1 + b_{ij}(t) \); then the optimal resource consumed by user \( i \) under provider \( j \) is given by

\[
b_{ij}(t) = \begin{cases} 
\frac{a_{ij}}{a_{j}} \left( \frac{2C_j}{1 + \theta} \right) - 1 & \text{if } b_{ij}(t) > 0 \\
0 & \text{if } b_{ij}(t) \leq 0 
\end{cases}
\]

(5.46)

Moreover, the profit (or total utility experienced) by provider \( j \) can be written as,

\[
p_{j(\text{opt})}(t) = C_j
\]

(5.47)

where, for simplicity we exclude the cost \( K_j \), which is nothing but a constant.
CHAPTER 6
HETEROGENEOUS NETWORKS AND SERVICES

So far, we have considered the scenario where a spatially distributed population of end users are equipped with devices that can be served by all the service providers in that region. But the underlying assumption was that all the service providers would use homogeneous services (one example of such scenario might be where all the WSPs provide cellular coverage only). Services using other access networks such as WiFi are not considered.

With technological advancements, it is anticipated that in the near future, the wireless service providers will use a multitude of access technologies, operating on both licensed and unlicensed bands, to serve an increasing number of end users. These heterogeneous networks would be capable of providing different sets of services governed by their corresponding quality–of–service (QoS) capabilities. The most common example seen today is the accessibility of Wi-Fi hotspots on top of third generation (3G) cellular services [89]. In such a scenario, the user device is also expected to have multi-mode network interface capable of connecting to different networks [49, 92]. In this chapter, we extend our economic model to heterogeneous networks and services to analyze the WSP–end users relationship. Note that in such a model, the decision problem is not only from end users’ perspective to choose a service provider but also from the service providers.
As far as the prices for the various services are concerned, the wireless industry simply followed the charging model for voice services, i.e., free or flat-rate. One example of flat-rate pricing could be paying a certain amount of money every month and having unlimited usage facility. (Some of the variations of flat-rate pricing can be found in [17, 55].) As a result, wireless users community have exhibited selfishness towards resource usage. However, the extensive increase in wireless service access and the corresponding quality of service degradation have demonstrated the need for a proper pricing model. Also, the current practice of offering different contract plans (based on minute usage) for voice services will no longer be valid for data services. The same notion of resource sharing in voice networks cannot be used because packet data systems are usually aimed at maximizing the throughput. The goal is to allocate each user the maximum data rate based on the application needs and wireless channel conditions while maintaining fairness among users. This new way of looking at the resource allocation brings forth the requirement for proper pricing schemes for wireless services. By introducing the providers and users in a market like environment, it becomes convenient to leverage the concept of prices to regulate the demands of users who consume resources.

We deviate from the concept of flat pricing towards more dynamic pricing. We study the use of dynamic pricing in the presence of multiple competing service providers who cater to end-users through overlaid heterogeneous networks. In future, end-users are anticipated to have more freedom in dynamically connecting to any service provider for any service through their multi-mode terminals [93]. In such a scenario, this calls for some incentive
to service providers also. For being fair to both service providers and users, we investigate
the extremity of dynamic pricing in this research. Thus we do not consider the traditional
concept of per-service static pricing from WSPs’ perspective. In contrast, we assume that
there will be an economic paradigm shift where service providers will also have more freedom
in terms of choosing the price to be charged to their end-users. Thus in this new model,
a WSP has freedom to change (increase or decrease) the price of a service for a user even
after admitting the user with another advertised price depending on changing load, revenue
etc. Note that with this freedom gained, a WSP can become malicious also to start with a
low price to admit users and then increase the price in the midway. Thus it is necessary for
end-users also to develop their strategies.

We investigate the decision problem of optimal network selection and pricing from the
both service provider and end-users’ point of views. The questions we answer are ‘what
would be the dominant best response strategies from both users and service providers for
the selection of provider’, ‘once a user request is admitted to a WSP, how or which network
is selected for user requests’, and ‘what price to charge such that the profit for the provider
is maximized’? In this regard, we use game theory to analyze the competitions and inter-
actions between service providers and users. We find the best response of the users and
the competitive pricing strategies adopted by the service providers to attract more users.
We also investigate the existence of Nash equilibrium, where no provider or user finds it
beneficial to change the strategy unilaterally [51].
6.1 Overlapped Heterogeneous Network Model

In this work, we consider the generic abstraction of “always greedy and profit seeking” providers and users. We assume a heterogeneous network model as shown in figure 6.1.

In this model, every provider is in a market-like scenario trying to maximize their revenue by providing heterogeneous services through heterogeneous networks while competing for a common pool of users.

6.1.1 Heterogeneity of Network

To exploit the relative advantages of different kinds of technologies, different technologies are deployed in an overlaid manner. For example, cellular networks provide better coverage while Wi-Fi technology provides higher data rate. These technologies offer their advantages in terms of bit-rate and coverage. Thus the frequency spectrum at which a technology
operates affects the QoS capabilities; for example, high frequencies impose restrictions on
the coverage area but can support very high speed data transfer. Going down the hierarchy of
the frequency spectrum, we get higher coverage with low speed data transfer. With respect to
figure 6.1, the different technologies are shown as Access Network 1, 2, · · · , X. Each of these
access networks is capable of supporting a variety of services. As an illustrative example, we
consider the situation where there are heterogeneous access networks (e.g., WLAN, WiMax,
3G cellular) belonging to a provider as shown in figure 6.2. With different technologies being

![Figure 6.2: Heterogeneous Networks](image)

laid over each other to provide improved services, there is no clear resource map, i.e., the
available bandwidth at a given location in space is not known with certainty. The knowledge
of the available resources must be considered and taken advantage of for offering services
to users. The challenge is to coordinate the various available radio access technologies such
that a user is served through the network that fits him best in terms of terminal capabilities,
service requirements, and most importantly the price.
6.1.2 Heterogeneity of Service

By heterogeneous services, we mean different QoS enabled wireless applications such as, telephony, video, file transfer etc. Though in figure 6.1 we show service classes 1, 2, · · · , Y for generalization under every access network, we classify all services into two broad classes each having a totally different quality of service [21].

• Class A – Voice/Video service: In this class, we consider non–elastic services like voice/video services. These services are time–critical and therefore delayed packets are of no significance here. Users pay for the duration they are connected to the network and do not pay if negotiated QoS is not met at any point during the session. This generic model can be extended to include any popular real time services such as, voice, streaming video, VoIP, and teleconferences.

• Class B – Data service: In this class, we consider the elastic services (popularly known as data services) that have less strict delay requirement. Packets are not discarded even if they are delayed. Services such as web downloads, FTP, emails are typical examples for this class. In contrast to the non–elastic model, users obtain full utility only when 100% of the data is downloaded successfully; else utility is 0.
6.1.3 Decision Problems for WSPs and End-users

We consider two major decision making aspects – one by the WSP in the presence of other competitive WSPs, and the other by the end users in deciding which WSP to pick for their service.

As a service provider, the first decision problem is to advertise a price for a service without knowing what prices are being simultaneously advertised by its competitors at that instant. Note that providers have the knowledge of distribution of advertised price but not the exact price at any instant. The optimization is to find different strategies for different services and find the price such that the provider is able to entice the end users and still sustain profit. With fixed resources and operational costs, prices exhibited cannot be too low, as that would attract too many users leading to degradation in performance because of resource sharing. At the same time, prices can not be too high because that will lead to user churn [39].

Second decision-making problem from the provider’s point of view is related to admission control and network selection in heterogeneous networks. When a user requests for a particular service, a provider has to decide which network to use to serve this request. Identification of the access network and allocation of resources for the services should be such that number of users being served can be increased without violating the QoS expected by the users.

As a user, the decision problem is to select the best service provider for the particular service request. All WSPs advertise their prices for the user request. The end-user will
either reject all the prices and will not receive any service or the end–user might connect to
the best service provider according to some criteria of best.

6.1.4 Games for WSPs and End–users

The interaction (decision problems presented in the earlier section) of the service providers
and the end users can be modeled formally using non-cooperative games. The game starts
with a user making a request for an application service (voice/video or data). Each service
requires a specific expected QoS in terms of bandwidth to satisfy user’s request. User in turn
pays for the service to the provider from whom the service request is granted. The consistent
objective of any service provider in this game is to maximize profit for the service provided.
Similarly, the users try to maximize their benefits from the service for the price they pay.

The duration of the game is the entire time the user is connected to a service provider
and the game ends once user or provider disconnects. We assume that the exact duration of
the game (how much time user would like to get the service) is not known to provider but
only a distribution might be known. As earlier mentioned, in this work, we investigate the
extremity of dynamic pricing. Thus in our model, user can connect to any of the WSPs for
their desired services at any time. Similarly, WSPs can change their price of service for the
users at any point of time even inside the game.

With respect to end–users’ and WSPs’ best response, to better analyze the above game,
we introduce the concept of sub–games in a game. We define sub–games as the discretized
smaller unit time epochs of the whole game played between end–user and WSP. We assume that inside a sub–game there is no change of pricing or strategy from any of the end–users or WSPs. But across the sub–games there might be changes as new users might be admitted or users may finish their session or may also churn out. Thus across sub–games there might be a change in the load of the WSPs as a result of which the WSPs might decide to reassign their prices of the services.

Let us explain why discretization of the game is necessary. Let us assume without any loss of generality that user $i$ was admitted to provider $j$ with a low price for the service. If we assume the game is no longer discretized into sub-games, the price negotiation will be done only at the beginning of the game and would be done for the entire service. A malicious user would then definitely try to hold onto the bandwidth acquired. If the provider has other service requests with willingness to pay higher price than the malicious user, even then the provider can not end the connection as the malicious user would claim as the entire service requested not being completed and thus deny to pay even for the service consumed as the price negotiation was done for the entire service.

### 6.2 Analyzing the Game for Differentiated Services

Once the problems are identified and the game is formalized, we need to solve the game for both service providers and end–users. Solving a game means predicting the strategy of each player (service provider and user in this case) considering the information the game
offers and assuming that the players are rational. One can see that if the strategies from the players are mutual best responses to each other, no player would have a reason to deviate from the given strategies and the game would reach a steady state. In this regard, we need to study the existence of equilibrium from both user and service provider’s perspectives where deviating unilaterally from the equilibrium will not maximize benefit. By definition this equilibrium is known as the Nash equilibrium [51]. Next, we provide the utility functions for voice/video and data services and analyze the dominant strategies for both users and service providers.

### 6.2.1 Utility Functions and Nash Equilibrium for Voice/Video Services

An utility function is a mathematical characterization that represents the benefits obtained and cost incurred by the players playing the game. We assume that there are $M$ service providers that are trying to cater to a common pool of $N$ users. Let us assume that one of the $N$ users is requesting for a non–elastic voice/video service.

The methodology of mapping the user satisfaction derived from the bandwidth received is well established. We follow a variant of the sigmoid functions [94] to model the non–elastic service user’s satisfaction. In figure 6.3, we present an example of the nature of the user satisfaction. The normalized user satisfaction is modeled as,

$$US(b) = \frac{1}{1 + e^{\left(\frac{b_{max} + b_{min}}{2} - b\right)}}$$  \hspace{1cm} (6.1)
where, \( U_S(b) \) is the user satisfaction perceived for bandwidth \( b \), \( b_{\text{min}} \) is the minimum bandwidth required to maintain the service, while \( b_{\text{max}} \) is the maximum bandwidth above which the user perceives no significant improvement in the QoS.

![Figure 6.3: User satisfaction as a function of bandwidth for non–elastic service](image)

The utility function of the user \( i \) served by provider \( j \) can be expressed as,

\[
U_i(b_{ij}) = \frac{a_{ij}}{1 + e^{\left(\frac{(b_{\text{max}} + b_{\text{min}})}{2} - b_{ij}\right)}}
\]

where, \( U_i(b_{ij}) \) is the utility perceived, \( b_{ij} \) is the bandwidth received by \( i \)th user from \( j \)th provider. The coefficient \( a_{ij} \) is a positive parameter that indicates the relative importance of empirical benefit and acts as a weightage factor. It is a simple scaling parameter that maps user satisfaction to a dimension equitable to the price paid to the WSP as the cost of the service. Note that, \( a_{ij} \) could be even a function of bandwidth perceived; for simplicity, we consider \( a_{ij} \) to be a constant.

User satisfaction for non–elastic service is a non-decreasing function of the bandwidth received. However the satisfaction and thus the utility remains almost close to zero unless a minimum bandwidth \( (b_{\text{min}}) \) is received. This is because all real time services require a minimum amount of resource to sustain the application at the minimum QoS level. On
the other hand, with a bandwidth more than maximum needed for the service \( (b_{\text{max}}) \), the improvement in the service quality is almost not recognizable. Thus at very low and very high bandwidth, the marginal utility is almost 0. On the contrary, when the allocated bandwidth is in between the minimum and maximum, the marginal utility changes significantly.

With the user satisfaction model is formalized, it is necessary to evaluate the price that must be advertised by the providers for every service request. As the duration of the service requested by the user is not known beforehand to the providers, it is not possible for the providers to set the price for the entire service duration. We propose to divide the service pricing in smaller unit time periods, defined earlier as sub–games. The advertised price of the service from the providers can be then expressed as per unit of bandwidth, per unit time period.

If user \( i \) is connected to provider \( j \) for \( L \) time epochs, the cumulative utility obtained by user \( i \) can be given by,

\[
\sum_{l=1}^{L} \frac{a_{ij}}{1 + e^{(b_{\text{max}} + b_{\text{min}}/2 - b_{ij}(l))}}
\]

(6.3)

where, \( b_{ij}(l) \) is the bandwidth consumed in \( l \)th time period.

**Net utility:** We must also consider the cost incurred by user \( i \) for this game. The cost is the price paid to the provider for obtaining the required amount of bandwidth. Let the price per unit of bandwidth in \( l \)th time epoch set by service provider \( j \), \( 1 \leq j \leq M \), be \( p_j(l) \). The cumulative cost component for \( L \) time periods is

\[
m_j = \sum_{l=1}^{L} p_j(l)b_{ij}(l)
\]

(6.4)
Thus, the net utility from the user’s perspective can be given by,

$$U_{ij} = \sum_{l=1}^{L} \frac{a_{ij}}{1 + e^{\left(\frac{b_{\text{max}} + b_{\text{min}}}{2} - b_{ij}(l)\right)}} - \sum_{l=1}^{L} p_{j}(l)b_{ij}(l)$$  \hspace{1cm} (6.5)$$

Recall, the dimension of the constant $a_{ij}$ is adjusted so as to make both the right hand side terms of equation (6.5) dimensionally equitable.

The net utility for service provider $j$ due to user $i$, if user $i$ is connected for $L$ time periods, is given by

$$V_j = \sum_{l=1}^{L} m_j(l) = \sum_{l=1}^{L} p_{j}(l)b_{ij}(l)$$  \hspace{1cm} (6.6)$$

**Strategies for Nash (Price) Equilibrium for non–elastic Service:** To investigate the existence of Nash equilibrium, we first need to consider if the strategies taken by user and service provider are dominant best response to each other or not, and if they are, then whether deviating from those strategies unilaterally will have any impact or not. In this regard, we also need to see why discretization of the game into sub-games is necessary for voice service.

Let us assume that user $i$ requests for voice/video service. We consider the 1st sub-game, where providers advertise a price for the first time for this user $i$. Being rational, the aim of user $i$ is to shortlist the service providers providing the user with positive net utility such that,

$$\frac{a_{ij}}{1 + e^{\left(\frac{(b_{\text{max}} + b_{\text{min}})}{2} - b_{ij}(l=1)\right)}} \geq m_j(l = 1); \hspace{1cm} \text{where } j \in \mathcal{M}$$  \hspace{1cm} (6.7)$$

We claim that this is the necessary condition for the user towards choosing the best response but not the sufficient. User $i$’s aim would be to maximize the net utility. Thus the dominant
best response from the user $i$’s perspective is to choose the provider that not only provides positive net utility but also maximizes the net utility of all the providers. Let us assume that provider $j$ is the desired service provider in this case. For generalization purpose, we assume that user $i$ has been connected to the provider $j$ for $n^*$ consecutive sub-games, where, $n^* < n$ and $n$ is the expected number of sub-games for the service requested. Then net utility obtained by user $i$ over these $n^*$ consecutive sub-games is given by,

$$\sum_{l=1}^{n^*} U_{ij}(l)$$  \hspace{1cm} (6.8)

Suppose, at this point, provider $j$ changes its price strategy (i.e., increase the price), such that, user $i$’s net utility is not positive for $(n^* + 1)$th sub-game. If user $i$ still decides to connect to provider $j$ at the $(n^* + 1)$th sub-game, the net utility of user $i$ will then be given by,

$$\sum_{l=1}^{n^*} U_{ij}(l) + U_{ij}(l = n^* + 1)$$  \hspace{1cm} (6.9)

As, $U_{ij}(l = n^* + 1)$ is a negative quantity, it is evident that user $i$’s dominant best response would be to withdraw from the game with provider $j$ at $n^*$th sub-game itself i.e., to disconnect from the service and leave with net utility $\sum_{l=1}^{n^*} U_{ij}(l)$.

As far as the best response of provider $j$ is concerned, we consider the 1st sub-game of this session (game). Suppose, provider $j$ advertises a price $\hat{p} = p_j(l = 1)$ per unit bandwidth for the 1st sub-game and user $i$ connects to this provider. This implies that the price advertised by provider $j$ maximizes the net utility for user $i$. Then, for the existence of Nash equilibrium, we need to check if provider $j$ or user $i$ wants to change their strategies unilaterally. In this
regard, without loss of generality, we assume that all other service providers and users keep their strategies unchanged.

As user $i$ has accepted the connection with provider $j$, the provider knows for sure that if price $\hat{p}$ is charged for subsequent sub-games, user $i$ would continue to play the game (i.e., remain connected). Thus, at equilibrium, the lower bound on the pricing can be given by $\hat{p}$. As a greedy player, provider $j$’s strategy would be to charge higher and thus maximize its expected net utility over the entire service. The expected net utility of the provider for the voice service can be then given by,

$$\sum_{l=1}^{n} m_j(l) P(U_{ij}(l) \geq 0) P(U_{ij}(l) > U_{ik}(l)) \text{ such that, } \forall k \in \mathcal{M}, k \neq j$$  \hspace{1cm} (6.10)

where, $m_j(l)$ is the revenue generated from user $i$ at the $l$th sub-game. $P(U_{ij}(l) \geq 0)$ denotes the probability of generating positive net utility for user $i$ and $P(U_{ij}(l) > U_{ik}(l))$ denotes the probability of generating maximum net utility for user $i$ with provider $j$ than any other provider. Now, if equation (6.10) is strictly concave and continuously differentiable upto atleast 2nd order, then it is clear that a maximum point exists.

As provider now knows the lower bound of the price charged at equilibrium, the natural inclination of the provider would be to charge the user with a non-decreasing pricing sequence over the consecutive sub-games. Let us assume that the non-decreasing pricing sequence at equilibrium is given by,

$$p_j(l = 1) \leq p_j(l = 2) \leq \cdots \leq p_j(l = n)$$ \hspace{1cm} (6.11)
would maximize the expected net utility of the provider $j$ as was given in equation (6.10). Provider’s best response would be to choose the maximum price from this non-decreasing pricing sequence given in equation (6.11). Let us denote this maximum price by $\hat{p}_{\text{max}}$. It is evident from equation (6.10), that $\hat{p}_{\text{max}}$ is upper bounded by the condition $P(U_{ij}(l) \geq 0)$ and $P(U_{ij}(l) > U_{ik}(l))$. Provider’s best response would then be to charge $\hat{p}_{\text{max}}$ from the very 1st sub-game. But earlier, we have seen that provider has charged $\hat{p}$ in the 1st sub-game, which is the lower bound of the prices charged. Thus it is clear that $\hat{p}_{\text{max}} = \hat{p}$, i.e., at equilibrium provider’s best response would be to adhere to the price agreement made at the time of admission for any user.

Once, we have found the best responses for both the user and the provider, it is rather easy to show that both players are affected by deviating from the equilibrium unilaterally. If user $i$ deviates from its dominant strategy unilaterally, the net utility obtained by the user will not be maximized (as explained in equation (6.9)). Similarly, if provider $j$ deviates from its best response by charging some other price than the non-decreasing pricing scheme, then it will not increase the expected net utility which proves the existence of Nash equilibrium.

### 6.2.2 Utility Functions and Nash equilibrium for Data Services

In data services, a user gets utility only when the whole file is downloaded completely (e.g., web download, FTP etc.); else the utility is 0. In such a scenario, we modify the user
satisfaction and express it as

\[ US(b) = \begin{cases} 
  k_s \log(1 + b) & 0 < b < b_{\text{max}} \\
  1 & b \geq b_{\text{max}} 
\end{cases} \]  

(6.12)

The parameter \( k_s \) is used for normalization and is defined such that,

\[ US(b_{\text{max}}) \approx 1 \]  

(6.13)

The normalized user satisfaction presented for elastic service can be modeled as in figure 6.4.

The utility function for user \( i \) from provider \( j \) can then be given by

\[ \begin{cases} 
  a_{ij}k_s\log(1 + b_{ij}) & \text{if transaction is complete,} \\
  0 & \text{if transaction is incomplete} 
\end{cases} \]  

(6.14)

where \( a_{ij} \) is a simple scaling parameter that maps user satisfaction to a dimension equitable to the price paid to the WSP as the cost of the service.

**Net utility:** The net utility for user \( i \) for data service upon completion can be expressed as,

\[ U_{ij} = a_{ij}k_s\log(1 + b_{ij}) - \hat{m}_j \]  

(6.15)
where, \(\hat{m}_j\) is the total price charged.

**Strategies for Nash (Price) Equilibrium for data service:** To investigate the existence of Nash equilibrium for data service, we proceed with the analysis similar to the voice/video service, i.e., we assume that the session (game) is discretized into sub-games and user \(i\) uses the same best response strategy proposed for the voice/video service, i.e., negotiate price at the beginning of each sub-game.

Suppose, user \(i\) requests for a data service at the beginning of the 1st sub-game and after QoS negotiation and price agreement, service provider \(j\) is selected as the desired provider. Let the expected number of game sessions to complete the entire data service be \(n\) \((n > 1)\). Then, at the 1st game session, user \(i\) would pay \(m_j(l = 1)\) and will receive bandwidth \(b_{ij}(l = 1)\). At the end of the 1st sub-game, the cost incurred by user \(i\) would be \(m_j(l = 1)\), whereas, the utility obtained is still 0; thus conflicting the nature of a rational player. At this point, service provider \(j\) will become malicious and will charge a high price for bandwidth provided from 2nd sub-game onwards. If user \(i\) quits the game at this session and tries to connect to some other provider, the net utility perceived so far will be negative as the service has not been completed thus making the user a loser in the game. If user persists to be in the game with the losing strategy, even then the net utility over the \(n\) sub-games will be negative as provider continues to charge at a high rate. Thus it is clear that discretization of prices over sub-games will not be a choice for end-users in this case. Though discretization was necessary for voice services, for data services discretization in terms of pricing is not at all desired to prevent provider to be malicious. So user’s best response would be to quit the
data service game if it is discretized pricing. Thus from user \( i \)'s perspective, the equilibrium strategy would be to negotiate price at the beginning for the entire data service if and only if net utility for the entire data service is positive and maximized. Following this strategy will enable the user to quit the game at any point of time if price advertised by the provider does not maximize his net utility. This prevents provider to be malicious as user quitting the game means provider would also loose the game obtaining zero revenue though bandwidth is wasted.

Once we have the equilibrium strategy from user’s point of view, it is easy to define the dominant strategy from service provider’s point of view. As user would pay only once, service provider’s aim is to maximize the expected net utility

\[
m_i \sum_{l=1}^n a_{ij} k_s \log (1 + b_{ij}) \geq \hat{m}_j \sum_{l=1}^n a_{ik} k_s \log (1 + b_{ik}) - \hat{m}_k
\]

such that, \( \forall k \in \mathcal{M}, k \neq j \) \hspace{1cm} (6.16)

where, \( \hat{m}_j \) is the price charged for the entire service (game). \( P\left( \sum_{l=1}^n a_{ij} k_s \log (1 + b_{ij}) \geq \hat{m}_j \right) \) denotes the probability that the service provider has to provide positive net utility to the user \( i \) with the one time charge \( \hat{m}_j \) and the second probability in the above equation denotes the probability that the provider has to provide the maximized net utility for this service than any other provider. Then, if equation (6.16) is concave and continuously differentiable upto atleast 2nd order, then maximizing point and Nash equilibrium exist.
6.3 Network Selection

In this section, we consider the problem of selecting the best network once a service request is admitted. For generalization, we assume that each service provider is equipped with $X$ heterogeneous networks, whereas each of these networks provide $Y$ different services as was shown in figure 6.1. Then total bandwidth capacity of a service provider can be given by $\sum_{x=1}^{X} \sum_{y=1}^{Y} B_{xy}$ where, $B_{xy}$ is the bandwidth capacity of service $y$ under network $x$.

Moreover, let us assume that the existing load (bandwidth already assigned to the existing users) under this service provider is $\sum_{x=1}^{X} \sum_{y=1}^{Y} B^*_{xy}$ where, $B^*_{xy}$ is the bandwidth allocated from service $y$ under network $x$. Thus, we define QoS affect ratio and denote it as,

$$Q = \frac{\sum_{x=1}^{X} \sum_{y=1}^{Y} B^*_{xy}}{\sum_{x=1}^{X} \sum_{y=1}^{Y} B_{xy}}$$

(6.17)

Note that, $0 \leq Q \leq 1$ is directly related to the relative usage of the bandwidth. As evident from equation (6.17), there is a chance that QoS of the service will be degraded as more and more users are served by this service provider and more bandwidth is allocated. On the other hand, QoS of the service will be improved if the bandwidth capacity of the service provider is improved.

It is clear that provider’s sole interest will be to increase the usage efficiency, as revenue earned by service provider is directly proportional to the existing usage efficiency, provided QoS of the existing load is maintained. Otherwise, users will be churning out decreasing provider’s revenue. In other words, the game from the service provider’s point of view can
be formalized as follows,

\[
\text{maximize } R = \text{maximize } f\left(\sum_{x=1}^{X} \sum_{y=1}^{Y} B_{xy}^*\right)
\]

(6.18)

where, \( R \) is the revenue earned by the service provider and \( f(\cdot) \) is some increasing function.

Thus a service provider’s best bet would be to admit the incoming service request to a network which would introduce minimum relative increase in QoS affect ratio.

**Best response strategy from service provider:** When a new service request arrives, provider’s aim is to serve the request to maximize the revenue, but at the same time maintain the QoS of the on–going services. The important question that arises is, which network (among the multiple overlaid networks) to select for servicing a particular service request such that the relative increase in QoS affect ratio is minimum. Note that, a service request can potentially be served by more than one network. For example, a user trying to make a voice call can use the Wi-Fi network or the cellular network.

We assume that user \( i \) requests for service \( y \) from provider \( j \). The aim of the provider would be to admit this request so that expected revenue can be increased by,

\[
m_{ij} P(\text{user } i \text{ connects to provider } j)
\]

(6.19)

where, \( m_{ij} \) is the price paid by the user \( i \) to provider \( j \). As a result provider’s intention would be to follow a pricing mechanism such that probability of user \( i \) connecting to provider \( j \) is maximized. We consider that provider \( j \) admits this service request and allocates bandwidth \( b_{ij} \). Then the relative increase in QoS affect ratio in service \( y \) under network \( x \) is given by

\[
\frac{d}{db_{ij}} \left(Q_{xy}\right) = \frac{1}{B_{xy} - B_{xy}^*}
\]

(6.20)
where $Q_{xy}$ denotes the QoS affect ratio of service $y$ under network $x$. Similarly, relative increase in QoS affect ratio in network $x$ considering all services, $y \in \mathcal{Y}$, is given by

$$\frac{d}{db_{ij}}(Q_x) = \frac{1}{\sum_{y=1}^{\mathcal{Y}} B_{xy} - \sum_{y=1}^{\mathcal{Y}} B^*_{xy}}$$

(6.21)

Then the ratio of relative increase in QoS affect ratio for a particular service $y$ under network $x$, and all services, $y \in \mathcal{Y}$ under network $x$ can be given by,

$$Q^r_x = \frac{\sum_{y=1}^{\mathcal{Y}} B_{xy} - \sum_{y=1}^{\mathcal{Y}} B^*_{xy}}{B_{xy} - B^*_{xy}}$$

(6.22)

A provider’s aim would be to select the network $x$ from $\mathcal{X}$, that would have the least value of $Q^r_x$ so that users experience least QoS affect and more and more users can be served without users churning out.

### 6.4 A Case Study: Heterogeneous Networks and Utility Maximization

In this section, we investigate the decision making approaches and net utility obtained by both service providers and users. We consider a system architecture where a wireless service provider has two networks – 3G network based on CDMA/HDR [88, 6] and 802.11 based Wi-Fi network. The coverage area in a 3G network is large but the perceived QoS degrades as user goes far from the 3G base station. On the other hand, Wi-Fi network provides uniform coverage for a small area. A user’s hand-held device is a multi-mode terminal and is thus capable of connecting to any of the networks provided it is inside the coverage area of
that particular network. Note that 3G channels are more expensive because of the licensing fee for 3G spectrum as opposed to the unlicensed Wi-Fi band.

In a time-slotted CDMA/HDR system, the base station transmits to one user at full power using one of the 11 pre-defined modulation and coding schemes. Thus, we can think of the cell being divided into 11 concentric rings, each receiving a unique data rate [6]. To analyze the performance of this system, we consider a 3G cell of area $A_{3G}$ with users uniform randomly distributed over the entire cell. Let $\rho$ be the density of active users and $A_1, A_2, \cdots, A_i, \cdots, A_k$ be the areas of the $1, 2, \cdots, i, \cdots, k$ rings respectively. The number of expected users in the $i$th ring is $\rho A_i$. The expected number of total active users ($N$) in the cell can then be given by $\sum_{i=1}^{k} \rho A_i$. We assume data rate perceived by a user in the $i$th ring as $C_i$ Kbit/sec ($i = 1, \cdots, k$). Similarly, we define $A_{AP}$ to be the the area covered by Wi-Fi access point, where $A_{AP} \ll A_{3G}$ and $C$ Kbit/sec is the uniform data rate perceived inside the Wi-Fi cell. For our analysis, we assume, $C = \max_{i=1}^{k} \{C_i\}$. Note that, we could have chosen $C$ to be much higher than $C_i$’s but as the intention of this research is not to show which network is better, but to show the benefit due to the overlapping heterogeneous networks; we consider $C$ to be comparable with 3G data rates.

It can be argued that the providers’ utility (which in turn depends on the users’ utility) depends on proper utilization of the radio resources, the placement of the APs becomes an important issue. In this regard, we consider two different kinds of placement: ideal placement and random placement. It is assumed that in the ideal placement of the APs, the distribution function of the user’s locations are known beforehand and more number of
APs are placed at the high density areas. Since the ideal placement is not possible in every premise due to incomplete information of user’s profiles, random placement is the alternative solution where APs are placed uniform randomly inside the 3G cell.

### 6.4.1 User’s Utility

For the users, the utility is expressed in terms of perceived bandwidth, blocking probability, and expected delay.

#### 6.4.1.1 Perceived Bandwidth

We define perceived bandwidth as the total data (in Kbits) received by users per unit time considering the effects due to fading and shadowing. We consider a cell with $N$ active users and $L$ Wi-Fi APs. Therefore, the expected number of users that fall within the coverage area of $L$ APs are $\rho_{AP} L$. These $\rho_{AP} L$ users then have the choice of connecting either to the 3G base station or Wi-Fi AP. We assume these $\rho_{AP} L$ users receive data at highest rate either from Wi-Fi AP or 3G base station. Moreover, we assume that $\rho_{AP} L < N$, i.e., not all users have both the connection. The remaining $(N - \rho_{AP} L)$ active users then must receive their signals from the 3G base station during their allocated time slot. If we assume a fair round robin scheduling policy, then the average perceived system bandwidth in the
presence of Wi-Fi APs can be given by,

\[ B_W = \frac{\rho A_{AP} L C \tau}{N \tau} + \sum_{j=1}^{N} \rho A_{AP}^L C_j \tau \]  

(6.23)

where \( \tau \) is the duration of each slot, \( C \) is the highest data rate (either from AP or 3G base station) and \( C_j \) is the 3G data rate received by each of the \( j \) users who are not connected to APs. In comparison, the average perceived system bandwidth in the absence of Wi-Fi APs can be given by,

\[ B_{3G} = \frac{\sum_{i=1}^{k} \rho A_i C_i \tau}{\tau \times \sum_{i=1}^{k} \rho A_i} = \frac{\sum_{i=1}^{k} \rho A_i C_i}{\sum_{i=1}^{k} \rho A_i} \]  

(6.24)

We consider a single-cell system with \( N \) as 1000. The radius of the 3G cell and the Wi-Fi cells are considered 2000 and 100 meters respectively. We considered received data rates and corresponding distances from Table 6.1 as given in [9]. Figure 6.5 shows that with increase in number of APs the system performance is much better than the existing system with only 3G network. In an ideal situation, the value of \( L \) is the saturation point after which the performance does not increase any more.

### 6.4.1.2 Blocking Probability

We define blocking probability as the probability with which a user’s request is rejected i.e., the request is denied. We consider both cases – without and with APs.
Without APs: We first consider a 3G system without Wi-Fi APs. In such systems, each user is guaranteed a minimum data rate regardless of his location. If the server has enough bandwidth available to serve all the existing users along with the new user providing at least their guaranteed data rates, then a new user is admitted into the server. Otherwise, the new user is queued, after which, upon availability of the bandwidth the new user is admitted. We assume a queue of size $s$.

We denote the data rate perceived by a user in the $i$th ring in his own slot as $C_i$ Kbit/sec ($i = 1, \cdots, k$) and the guaranteed minimum data rate provided to all the users as $C_{\text{min}}$ Kbit/sec for the entire time, where $C_{\text{min}} << C_i$, for $i = 1, \cdots, k$, i.e., if there are total $N$ slots for all the users present in the server, and if a user receives data only in 1 slot, then...
this user’s actual perceived data rate can be given by $C_i/N$, assuming the user is in the $i$th ring. Then the condition of guaranteed minimum data rate states that, $C_{min} \leq C_i$.

We argue that the capacity of system varies, i.e., the number of users served depends on the locations of the users. If all the users are situated in the very first ring and receive data at highest possible rate, then the maximum capacity of the server can be given by $(C_1/C_{min})$, whereas, if at least one user is situated in the last ring of the cell then the maximum capacity of the server reduces drastically to provide all the users guaranteed minimum data rate. Thus it can be concluded that the maximum capacity of the server varies and depends on the farthest user from the base station. We assume that the farthest user suffers the most due to fading and shadowing effects and receives the minimum number of slots.

We find the probability of the farthest user from the base station being in the $i$th ring as $P'_i = \frac{A_i}{A_{3G}}$, where, $A_i$ is the area of the $i$th ring and $A_{3G}$ is the total area of the cell. The expected maximum capacity of the server is given by, $S = \sum_{i=1}^{k} P'_i \frac{C_i}{C_{min}}$.

We assume that the arrival process of originating users in a cell is Poisson with rate $\lambda$. We define the state $j$ ($j = 0, 1, 2, \cdots, S + s$) of a cell as the number of total users present in the server and the queue. We represent the system as an one-dimensional Markov chain, where state transitions are given in figure 6.6.
The service rate in this model is different for different states and depends entirely on the perceived data rates of the users and the number of users present in the server. Thus the service rate for this system is

$$
\mu_j = \begin{cases} 
\frac{\sum_{i=1}^l c_i}{j^F} & \text{if } j \leq S \\
\mu_S & \text{if } j > S
\end{cases}
$$

(6.25)

For simplicity, we assume every user intends to download file of size $F$. Note that, $\mu_S$ is not a constant but rather depends on the perceived data rates of $S$ users being served in the server in time slotted mechanism. Then, the equilibrium probabilities $P(j)$ can be given by,

$$
\left\{ \begin{array}{ll}
\mu_j P(j) = \lambda P(j - 1) & \text{if } 1 \leq j \leq S \\
\mu_S P(j) = \lambda P(j - 1) & \text{if } S < j \leq S + s
\end{array} \right.
$$

(6.26)

Using the above equations recursively along with the normalization condition we get, $\sum_{j=0}^{S+s} P(j) = 1$. Then the steady state probabilities can be given by,

$$
P(j) = \begin{cases} 
\frac{\lambda^j}{\prod_{i=1}^j \mu_i} P(0) & \text{if } 1 \leq j \leq S \\
\frac{\lambda^j}{(\mu_S)^{j-S}(\prod_{i=1}^S \mu_i)} P(0) & \text{if } S < j \leq S + s
\end{cases}
$$

(6.27)

where,

$$
P(0) = \left[ 1 + \sum_{j=1}^S \frac{\lambda^j}{\prod_{i=1}^j \mu_i} + \sum_{j=S+1}^{S+s} \frac{\lambda^j}{(\mu_S)^{j-S}(\prod_{i=1}^j \mu_i)} \right]^{-1}
$$

(6.28)

Based on the above analysis and probability definitions, we can define $P(S+s)$ as the blocking probability, $P[\text{block}_{3G}]$, which is given by,

$$
P[\text{block}_{3G}] = P(S + s) = \frac{\lambda^{S+s}}{(\mu_S)^s(\prod_{i=1}^S \mu_i)} P(0)
$$

(6.29)
Similarly, expected delay experienced by each user is

$$\text{Delay}[3G] = \frac{1}{\mu_j - \lambda}$$  \hspace{1cm} (6.30)

**With APs:** With Wi-Fi APs, the service rate can be given by,

$$\mu_j = \begin{cases} \frac{\sum_{i=1}^{j} C_{\text{max}}}{jF} & \text{if } j \leq S \\ \mu_S & \text{if } j > S \end{cases}$$  \hspace{1cm} (6.31)

where, $C_{\text{max}}$ is the maximum possible data rate perceived by the users under the direct influence of the APs and moreover, we assume here that sufficient number of APs are laid so that all the users can connect to both APs and 3G base station. Then blocking probability, $P[\text{block}_{AP}]$ is given by

$$P[\text{block}_{AP}] = P(S + s) = \frac{\lambda^{S+s}}{(\mu_S)^s(\prod_{i=1}^{S} \mu_i)} P(0)$$  \hspace{1cm} (6.32)

and the corresponding expected delay is given by,

$$\text{Delay}[AP] = \frac{1}{\mu_j - \lambda}$$  \hspace{1cm} (6.33)

where, $\mu_j$’s are given by equation (6.31).

To see the effect of APs on blocking probabilities and expected delay, we consider the system with 3G cell radii as shown in Table 6.1 [9]. We assume the queue size as 15.

In figures 6.7 and 6.8, we plot the blocking probability and expected delay experienced per user with and without APs. As evident from the figures, blocking probability and delay decrease with introduction of APs. Of course, the cost of deploying the APs is not considered here.
Table 6.1: Data Rates and Ring Radius

<table>
<thead>
<tr>
<th>Ring k</th>
<th>Data rate (Kbit/sec)</th>
<th>Radius (m.) (α = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>38.4</td>
<td>2820</td>
</tr>
<tr>
<td>10</td>
<td>76.8</td>
<td>2370</td>
</tr>
<tr>
<td>9</td>
<td>102.6</td>
<td>2210</td>
</tr>
<tr>
<td>8</td>
<td>153.6</td>
<td>2000</td>
</tr>
<tr>
<td>7</td>
<td>204.8</td>
<td>1860</td>
</tr>
<tr>
<td>6</td>
<td>307.2</td>
<td>1680</td>
</tr>
<tr>
<td>5</td>
<td>614.4</td>
<td>1410</td>
</tr>
<tr>
<td>4</td>
<td>921.6</td>
<td>1280</td>
</tr>
<tr>
<td>3</td>
<td>1228.8</td>
<td>1190</td>
</tr>
<tr>
<td>2</td>
<td>1843.2</td>
<td>1070</td>
</tr>
<tr>
<td>1</td>
<td>2457.6</td>
<td>1000</td>
</tr>
</tbody>
</table>

Figure 6.7: Blocking Probability with and without relay nodes
6.4.2 Provider’s Utility

As discussed in equation (6.18), we assume that the revenue generated by each provider is directly proportional to the bandwidth usage efficiency. We assume the capacity of the 3G network to be fixed and thus the spectrum maintenance cost due to this 3G network is also assumed constant. We consider $E_{3G}$ to be the cost for the 3G network. As far as the cost of the Wi-Fi APs are concerned, the cost is mainly due to deployment, infrastructure maintenance and cost due to the backhaul link allocation. For simplicity, we assume cost incurred due to each of the APs is fixed and is given by $e_{AP}$. Then the provider’s net utility without internetworking is

$$V_j[3G] = f(B_{3G}^*) - E_{3G}$$  \hfill (6.34)
where, $B^*_{3G}$ is the allocated bandwidth from the service provider only from the 3G network. In contrast to equation (6.34), the net utility for the service provider with 3G and Wi-Fi internetworked is

$$V_j[AP] = f(B^*_{AP}) - E_{3G} - L e_{AP}$$  \hspace{1cm} (6.35)$$

where, $B^*_{AP}$ is the allocated bandwidth from the service provider with 3G and Wi-Fi internetworked and in the presence of $L$ APs. The relative increase in net utility in the internetworked system from the service provider’s point of view is given by,

$$\frac{(f(B^*_{AP}) - E_{3G} - L e_{AP}) - (f(B^*_{3G}) - E_{3G})}{f(B^*_{3G}) - E_{3G}}$$  \hspace{1cm} (6.36)$$

Note that, relative increase in net utility depends heavily on $f(B^*_{AP})$ and the ratio $\frac{E_{3G}}{e_{AP}}$. With increase in the allocated bandwidth in the internetworked system and increase in the ratio, relative increase in net utility is also increased.
In the previous chapters, we have considered concept of dynamic spectrum access, where the WSPs dynamically seek additional spectrum in addition to the spectrum that has already been allocated statically. This model works well when there is a pre-defined common pool of spectrum (e.g., CAB) that is being shared by the WSPs. However, situations might arise when there is availability of spectrum that is not being used by the primary (licensed) user for a period of time. This sudden availability of portions of spectrum is not known a priori and is almost random with respect to space and time. Such occurrence (i.e., sudden availability of spectrum) can be exploited in an opportunistic manner. This leads to a new model called opportunistic spectrum access where secondary (unlicensed) users can access and use spectrum that is not being used by the primary. The usage of the spectrum by the secondary users must be such that the primary user is not affected at all.

Opportunistic spectrum access models are being explored by FCC. Recently, FCC defined provisions that allow unlicensed devices to operate in the licensed bands so long the unlicensed devices do not create interference for licensed services [91]. Initially, FCC has opened sub–900 MHz TV band to unlicensed services because of high under-utilization of this band. But for unlicensed devices to gain access to the sub–900 MHz band, FCC requires that these
devices must be able to detect licensed users and avoid interference. In this regard, a new standard called IEEE 802.22 based on cognitive radio (CR) has been proposed.

As per IEEE 802.22, a cognitive radio [90] can operate at any unused frequency in the licensed TV band, regardless of whether the frequency is assigned to licensed services or not. The most important regulatory aspect is that cognitive radios must not interfere with the operation in licensed bands and must identify and avoid such bands in timely manner. Cognitive radio systems continuously perform spectrum sensing, dynamically identify unused ("white") spectrum, and operate in this spectrum band when it is not used by the incumbent radio systems – who are the primary users of this band. Upon detecting incumbents in any band cognitive radio must automatically switch to another channel or mode.

7.1 IEEE 802.22 System

The core components of IEEE 802.22 system are the base stations (BSs) and the Consumer Premise Equipments (CPEs) as shown in figure 7.1. The BS transmits in the downstream direction to the various CPEs, which in turn respond back to the BS in the upstream direction. Based on the feedback received from the CPEs, if any, the BS decides its actions [18]. A BS typically manages its own cell by controlling on-air activity within the cell, including access to the medium by CPEs, allocations to achieve quality of service (QoS) and admission to the network based on network security mechanisms. The operations of BS/CPEs can be divided into two major categories in IEEE 802.22: sensing and transmitting/receiving data.
Sensing and avoiding incumbent transmission at any cost is the most prioritized task of all 802.22 enabled devices. If any of the channels used by 802.22 network is occupied by the licensed incumbents, the primary task of 802.22 devices would be to vacate the channels within channel move time (approximately 2 seconds) and switch to some other unused channel. To get the knowledge of the presence of licensed incumbents and their used channels, BS and CPEs do channel sensing periodically. Also self co-existence among 802.22 networks is an open problem. In areas with significant high incumbents (licensed services), open channels will be a sparse commodity. Therefore, channel allocation among 802.22 BSs will be of utmost importance so that the interference among the users under these base stations can be minimized. Let us now discuss the media access control (MAC) layer of IEEE 802.22 and its drawbacks.
7.1.1 The MAC Layer of IEEE 802.22

The existing MAC of 802.22 has most of the features similar to the MAC of 802.11 and 802.16. However, few distinguishing features make the 802.22 MAC worth mentioning. We propose our enhancements for these features. But before that, we explain the existing version of 802.22 MAC.

7.1.1.1 Initial connection establishment

Initial connection establishment in 802.22 differs from that of the previous IEEE 802 standards such as 802.11 or 802.16. Though connection establishment in a true centralized network, should be simple, it is not so for 802.22. In 802.22 there is no pre-defined channel for the CPEs to establish connection with BS as 802.22 networks share the spectrum band with licensed devices. Thus there is no way for a CPE to know which channel to use to establish the initial connection with a BS as that might introduce interference to an incumbent who is using the same channel.

In 802.22, when a CPE is switched on, it follows the mechanism of *listen before talk* by scanning all the channels in the licensed TV band to find out whether any incumbent in the interfering zone is using any particular channel and builds a spectrum usage report of vacant and used channels. BS on the other hand also follows the same mechanism of sensing spectrum bands and periodically broadcasts with an unused frequency channel. The broadcast from 802.22 BS is differentiated from other TV broadcasts by the preamble sent
at the start of each OFDMA frame. If a CPE can locate the broadcast sent from the BS, it then tunes to the frequency and then transmits back in the uplink direction with an identifier of the CPE; BS then becomes aware of the existence of the CPE. Authentication and connection registration is then done gradually. The spectrum usage report is then sent back to the BS from the CPE in the form of feedback. The BS upon acceptance of the feedback takes decision on spectrum usage. When more than one CPE tries to establish an initial connection, then contention-based connection setup similar to that of the 802.11 takes place after all the CPEs tune to the broadcasted channel.

7.1.1.2 Incumbent detection

Much of the standard of 802.22 is dependent on incumbent sensing and detection. At any point of time, a number of incumbents (TV broadcasting, wireless microphones etc.) may be operating in a region same as that of the 802.22 network. To co-exist with the incumbents, it is mandatory that incumbent sensing is done by both the BS and CPEs. CPEs in turn send their spectrum usage reports to the BS in the form of feedbacks. Depending on the incumbent detection algorithms proposed and their efficiencies, the general spectrum sensing process in divided into two categories: fast sensing and fine sensing. Fast sensing is done fairly quickly and it is carried out in-band at the same time of carrying out data transmissions. Though fast sensing is not very accurate, the advantage of fast sensing is that data transmission time is not wasted as channel sensing is done at the rate of 1ms/channel. Fine sensing, on
the other hand, is done out-of-band. BS periodically quiets the channel so that no network traffic is generated and incumbents are sensed. This sensing method provides more accuracy.

### 7.1.2 Drawbacks of the Existing 802.22 MAC

**Self Co-existence:** In a system like 802.22 where unlicensed devices are sharing the spectrum under the presence of licensed incumbents, the issue of self co-existence among multiple 802.22 operators in an overlapping region is very significant. In areas with high analog/digital TV transmissions and wireless microphone services, unused channels are already commodities of demand. Therefore, when multiple unlicensed operators are operating using a small available band of frequency, there is a chance that the operators will try to act greedy and hog the available bandwidth. As all the operators will act in the same way, this may result in interference among 802.22 networks themselves. Thus an efficient channel allocation method needs to be invoked in order to use the channels with least interference. Although the exact methodology for interference mitigation in 802.22 networks is yet unknown, we propose an algorithm that increases the spectrum utilization.

**The hidden incumbent problem:** Let us assume that a BS and a CPE are communicating using a specific frequency channel and an incumbent starts up with the same frequency channel near the CPE but outside the BS sensing region (refer Fig. 7.2 – hidden incumbent region). The CPE can detect the incumbent transmission in-band, but the BS can not. The
BS will then continue transmission and might interfere with the incumbent. The CPE will have no way to report this licensed incumbent. If it transmits with the same frequency with which it was connected to the BS, that will cause interference to the incumbent. On the other hand, due to the centralized nature of the 802.22 network (on-air activities of CPE is controlled by BS), the CPE can not choose any other channel to connect to the BS as it is not permitted to use any other channel unless BS provides the permission.

Figure 7.2: IEEE 802.22 hidden incumbent scenario

The problem even worsen as the CPEs do not have any reporting period. Instead what they do have is a channel move time which means that if they sense any incumbent present in the same frequency band they have to move within a stipulated channel move time. This channel move time currently is 2 seconds. If CPE assumes this 2 seconds as reporting period also, there will be a collision problem (in addition to the interference problem) with the primary incumbent (which is transmitting with the same frequency in the region) thus resulting in a corrupted signal. BS might not be able to decode this signal due to loss or corruption. Thus the hidden incumbent problem persists.
In another similar situation, let us assume that a CPE just turns on and wants to connect to a BS. According to the existing MAC proposal, the CPE will then scan all the channels to look for an 802.22 periodic broadcast. But if there is a nearby incumbent already transmitting with the same frequency as the BS periodic broadcast frequency, and is outside the BS sensing region but inside the CPE receiving region (hidden incumbent region as shown in Fig. 7.2), the CPE will not be able to decode the BS broadcasting frequency due to the interference from the incumbent. This results in a two-fold problem. The CPE might think that there is no 802.22 BS transmitting at that time and might switch off. Similarly, if the BS does not receive any feedback, it might think that there is no CPE alive and might stop broadcasting after a certain number of broadcasting periods, thus resulting in wasted control signaling and low spectrum utilization.

7.2 Spectrum Allocation for Self-coexistence among 802.22 Networks

When multiple IEEE 802.22 WSPs with their BSs operate in close proximity in an overlapping region, each of the BSs’ aim is to grab as much spectrum as possible to serve its corresponding CPEs without coordinating with other BSs. This greedy approach leads to interference to the operating BS and the surrounding neighboring BSs thus degrading the performance of the system.

In this regard, we propose an efficient spectrum allocation algorithm on behalf of FCC to increase the spectrum utilization of the BSs and reduce the interference. Note that increasing
the spectrum utilization of the BSs will increase the revenue for FCC proportionally. We frame a graph coloring model on spectrum allocation and study the spectrum access problem in a time and space variant manner under different objectives. We study a coordinated spectrum allocation approach instead of the greedy approach taken by the BSs.

We assume that there are $N$ 802.22 BSs competing for unused licensed spectrum. The amount of the unused spectrum is time variant. The key concept behind spectrum allocation efficiently is to find appropriate chunks of spectrum in such a manner so that BSs can coexist without interfering neighboring networks and the objectives are met. We will discuss about the objectives in a short while.

We consider that the utility achieved by the BSs depends directly on the throughput obtained, which in turn depends on the bandwidth of the interference-less frequency band the BS is operating on. Thus we define utility achieved by BS $i$ as,

$$U_i = B_{i_2} - B_{i_1}$$

(7.1)

where, $B_{i_2} - B_{i_1}$ is the spectrum range that BS $i$ is operating on and no other interfering neighbor is using that. $B_{i_1}$ and $B_{i_2}$ are the lower and upper bounds respectively of the spectrum range.

Then the revenue generated by FCC is given by

$$R_o = \sum_{i=1}^{N} \psi(U_i)$$

(7.2)
where, $R_o$ is the revenue generated in this opportunistic spectrum access and $\psi(\cdot)$ is an monotonically increasing function with $U_i$ and simply maps the utility dimension of $U_i$ to monetary dimension.

We follow a simple interference model among the overlapping BSs. We assume when two BSs are within a certain physical proximity and transmitting using the same frequency band or overlapping frequency bands, interference will occur and both the transmissions will fail. If the frequency bands are completely same, the utility achieved by both the BSs will be zero; otherwise, if the frequency bands are partially overlapping then the achieved utility will be due to the non-overlapping frequency bands only.

We consider the above scenarios in the multiple overlapping 802.22 networks using a graph theoretic model. We define an undirected graph $G = \{V, E, B\}$, where $V$ is the set of vertices denoting all BSs in the region. $E$ is the set of all undirected edges denoting the interference constraints among the BSs, i.e., if any two distinct vertices have an edge in between them, they are in the risk of interfering each other if using the same frequency band at the time of transmission. $B$ is the total available spectrum band not used by the incumbents and is usable by the 802.22 networks. Moreover, without loss of generality, we assume that the topology information of this overlapping region is known to all the 802.22 BSs (as BSs are static) and the BSs will be honest in providing all their acquired graph model information.
7.2.1 Objective Functions

We define three kinds of objective functions.

1. Maximize revenue and utility: The aim is to maximize the total utility achieved by all the BSs and thus the revenue earned by FCC. We impose a constraint that BS $i$ must get at least a certain amount of spectrum, which we denote as $B_{i_{min}}$. The objective function can be then expressed as

$$\text{maximize } \sum_{i=1}^{N} U_i$$

which is equivalent to maximize $R_o$.

2. Proportional fair utility: The aim here is to divide the spectrum bands under some proportional fairness criteria. The criteria we follow is to prioritize the BSs which interfere least number of other BSs. This mechanism of allocating spectrum will not let BSs to follow any greedy approach that may harm the system performance.

3. Complete fairness utility: In this mechanism all the BSs are treated equally. The problem in this approach is known as *tragedy of the commons* [24].

7.2.2 Spectrum Allocation through Utility Graph Coloring

We model the appropriate division problem of the whole spectrum band among BSs using the graph coloring technique. Though optimal graph coloring problem is known to be NP-
hard in searching and NP-complete in decision, it can be shown easily that at any point of
time, in any overlapping region, at most $6 - 10$ BSs will coexist due to the area coverage
capacity of the 802.22 networks and thus NP-complexity of the graph coloring problem is
not at all a hindrance for this proposed mechanism.

The traditional graph coloring problem [19] is to color each vertex using a color taken
from existing color list. The constraint in such coloring is that if an edge exists between
any two distinct vertices, then those two vertices can not be same colorable. With this
constraint, the aim of the traditional algorithm is to minimize the number of colors to color
all the vertices.

We propose an extension of the above graph coloring algorithm and call it Utility Graph
Coloring (UGC). The aim is to find divisions of spectrum band under the objective func-
tions defined, such that the system utility achieved can be maximized. In contrast to the
traditional graph coloring algorithm where the colors do not carry any value and thus each
color is equal in its weightage, in the UGC, we consider heterogeneity in the colors. A color
assigned to a vertex (BS) becomes associated with a spectrum band assigned to the BS and
the utility achieved by that BS is the bandwidth of that spectrum band if the band is not
interfered.

The proposed UGC algorithm is divided into two phases.

**Phase 1:** In this phase, we follow the principle of traditional graph coloring algorithm
to find the minimum number of colors to color all the vertices. We do not associate any
value to any color and thus keep colors homogeneous. Let us assume, $C_1, C_2, \cdots, C_m$ are
minimum colors to color all the vertices. With the completion of first phase, we get to
know that the graph is \( m \)-colorable and the available spectrum band needs to divided into
\( m \) chunks depending on the objective functions negotiated by the BSs at the beginning.

**Phase 2:** In this phase, we follow the mechanism of UGC. We first find the occurrences
of the colors in the graph. Let us assume the occurrences of the colors \( C_1, C_2, \cdots, C_m \) are
\( N_1, N_2, \cdots, N_m \) respectively, where, \( N_1 + N_2 + \cdots + N_m = N \), the total number of base
stations. Without loss of generality, let us assume that \( C_1 \) has appeared maximum number
of times, i.e., \( N_1 \) is the highest number among \( N_1, N_2, \cdots, N_m \). Then for each of the colors,
we run a progressive algorithm as presented in algorithm 1. For each iteration, we keep the
information which color has occurred the maximum number of times and how many times.
Let us assume that after all the color iterations, we find that, color \( C_m \) has the maximum
occurrence of \( N_m^* \) in iteration \( i \). We then choose this iteration \( i \) and redefine the occurrences
of colors \( C_1, C_2, \cdots, C_m \) as, \( N_1^*, N_2^*, \cdots, N_m^* \) respectively.

Note that, using traditional graph coloring, we are finding the total minimum number
of colors needed. The physical significance is that, in this way we can find out how many
minimum divisions we have to make out of the unused spectrum band to avoid interference.
But traditional graph coloring does not maximize the spectrum utilization. What UGC
provides us is another hashing of the traditional graph coloring taking traditional graph
coloring as the input trying to maximize the number of occurrences of a particular color
while keeping the total number of colors minimum.
**Algorithm 1** Maximum utility graph coloring algorithm

**INPUT:** Graph G

PARSE 1:
1. Color G with traditional graph coloring heuristic of coloring nodes with descending order of degree
2. G is $m$ colorable

PARSE 2:
1. FOR (each color $i$) {
   check each node in G if it can be made color $i$ without conflict to the other nodes’ colors made from PARSE 1
2. Store the information of occurrence of each color after this iteration $i$
}
3. Select the iteration with maximum occurrence of a color among all iterations and assign bandwidths to the nodes accordingly under the objective functions defined

In Fig. 7.3, we present an illustrative example to explain how the UGC works. With the traditional graph coloring algorithm, we find that the graph under consideration is a 3-colorable graph and we have colored the vertices accordingly. The left-hand graph (traditional graph coloring) in Fig. 7.3 shows that $C_1$, $C_2$ and $C_3$ appearing 2, 3 and 1 times respectively. After we parse this graph with our utility graph coloring (UGC) algorithm, we find that (right-hand side) $C_1$ appears once, $C_2$ appears once, and $C_3$ appears 4 times. This implies that UGC maximizes the spectrum reuse.

In general, depending on the objective functions, the actions taken for spectrum allocation are as follows:
For objective function 1: The whole spectrum band is divided into $m$ chunks such that the vertices with the color label $C_m$ (maximum number of vertices in the graph) are assigned the maximum possible spectrum as they would interfere least. The essence of UGC under objective 1 is to maximize the system utility only. The rest of the vertices (BSs) will be assigned the minimum threshold frequency ($B_{min}$) to operate on. This mechanism clearly reduces the interference to the least, as the BSs with interference risk (vertices with existing edge between them) now operate on different parts of the spectrum band. Moreover, as maximum number of BSs in the graph obtain the maximum possible spectrum band, the system utility is maximized. The only drawback in this scheme is that fairness is not maintained and the BSs with color labels $C_2, C_3, \ldots, C_m$ are all treated equally.

For objective function 2: We try to improve the fairness while maximizing the system utility. We divide spectrum bands in $m$ different parts in the ratio of $N_1^* : N_2^* : \cdots : N_m^*$ and assign them to vertices with color bands $C_1, C_2, \cdots, C_m$ respectively. Thus we maintain a proportional fairness criteria through a simple trade-off. As the BSs are now ranked according to number of neighbors they are interfering with, this mechanism will help the BS not to act greedy. This will result in least interference and increased system utility.
For objective function 3: Our aim is to provide complete fairness among all the BSs. Thus in this mechanism, we divide the available spectrum band in $m$ equal parts and assign each part to each of the non-interfering BSs.

### 7.3 Enhanced MAC for Hidden Incumbent Problem

In this section, we propose some additional functionalities for MAC of 802.22 to address the problem of hidden incumbent situation.

#### 7.3.1 Aggregation / Fragmentation of Channel Carriers

Since 802.22 PHY is capable of bonding channel carriers or fragmenting them to operate over the channels more flexibly and adaptively, we make use of this to utilize the channel bandwidths dynamically and in an efficient manner. For maximizing data transmission rates, we aggregate 2 to 3 channels, (either contiguous channels in the spectrum bands or separately placed in the band) to provide data rate upto a maximum of 71Mbps. Separate sets of OFDMA carriers are used on each channel to increase the data transmission rate at the time of channel bonding. In general, approximately 2K carriers are used for each channel of 6 MHz, thus making it 6K carriers to transmit data at a high rate while bonding 3 channels together.
We also fragment channel carriers so that 802.22 devices can operate over channel bandwidths of 1 to 6 MHz. This would allow 802.22 devices to share the spectrum with incumbent devices such as wireless microphones that only use 1 or 2 MHz of the entire channel assigned. Moreover, this ability of aggregating and/or fragmenting carriers would also allow to selectively tune out partial channels avoiding interference and cross talk and thus making more spectrum utilization. The ability to adapt to the number of carriers dynamically will make 802.22 more resilient to interference and spectrum utilization efficiency can also be increased.

### 7.3.2 Dynamic Multiple Broadcasting

To address the hidden incumbent problem, unlike the existing concept where BS periodically broadcasted using only one single frequency channel, we propose to use dynamic multiple outband broadcasting in different frequencies (*candidate frequencies*) periodically. The number of broadcast messages by BS is updated dynamically depending on the feedback received from the CPEs. BS decreases the number of candidate channels if all the candidate channels are decodable by the CPEs (implying less probability of hidden incumbent situation) and increases the number of broadcasting channels changing the candidate frequencies, if most of the previous candidate channels are not tuned up by CPEs. The reason behind broadcasting at multiple frequencies is that even if a CPE encounters an in-band licensed incumbent transmission (hidden to the BS), it still has ways to report this incumbent transmission to
the BS using other candidate channels. The BS then changes the service channel to some other unused band thus overcoming the problem of hidden incumbent.

Let us analyze the probability of success of a CPE to synchronize to a BS under the presence of single broadcasting and dynamic multiple broadcasting. For simplicity, we assume the occupancy of all the channels by the licensed incumbents are equally likely. Let us assume that the probability of occurrence of an incumbent transmission in a particular frequency channel is \( p \). Then the probability of successful synchronization of a CPE to a BS with single broadcasting is

\[
P_s = 1 - p
\]  

(7.4)

On the other hand, if a BS uses dynamic multiple broadcasting to synchronize to a CPE, then the probability of successful synchronization of a CPE to a BS is

\[
P_k = 1 - p^k
\]  

(7.5)

where \( k \) is the number of different frequencies that the BS is using. Since, \( p < 1 \), \( P_k > P_s \).

Thus the probability of successful synchronization of a CPE to a BS increases with the dynamic multiple broadcasting.

One may argue that multiple broadcasting would require more control signaling, thereby decreasing the spectrum utilization for data transmission. However, in practical cases it is not so. With single frequency broadcasting scheme, under the presence of hidden incumbents, on average, broadcasting is needed more number of times to synchronize with the CPE, which in turn decreases the actual spectrum utilization for data transmission. Moreover, we divide the
BS/CPE transmissions in two categories: (1) connection establishment or channel hopping with the help of control signaling and (2) data transmission with the help of data signaling. For control signaling, we use fragmentation of channel carriers to minimize the wastage of spectrum band and aggregation of channel carriers for data transmission to maximize the bandwidth and data rate. In our simulation of 802.22 network (discussed later), we use typical 1 or 2 MHz bands for control signaling.

7.3.3 Contention Resolvement among CPEs

Another functionality that we add to the 802.22 MAC is the addition of spectrum usage report inside the periodic broadcasting from BS to the CPEs. Currently, CPEs send the spectrum usage reports to the BS but not the other way. We mirror the spectrum usage report in all the multiple broadcasting from the BS. This spectrum usage report contains the information of all control frequencies that the CPEs can tune to in the uplink to the BS. Thus, in contrast to the existing connection establishment procedure in 802.22 where CPEs tune to the single broadcasting frequency and then follow the contention resolving mechanism similar to the IEEE 802.11 for initial connection establishment, we propose that CPEs obtain complete information about all control frequencies that they can utilize in the uplink connection establishment and then follow the contention resolving mechanism.

As CPEs have information about multiple control frequencies for uplink, CPEs will randomly select control channels and will randomly select random backoff numbers as in IEEE
802.11. The CPEs then will wait for the chosen backoff period and will try to establish connection with the chosen frequency. If the collision still occurs with another CPE, the affected CPEs will again randomly select the available frequency from the spectrum usage report list and another random backoff period. Next, we show that the probability of collision with other CPEs decreases with the proposed MAC thus resulting in lesser delay in connection establishment.

We assume that spectrum usage report contains \( m \) available frequencies to be used by CPEs for control signaling in the uplink direction. Let \( x \) be the size of the random backoff window, i.e., CPEs take a random backoff number between 0 and \((x - 1)\), both inclusive. Then the probability of two CPEs colliding, when they try to tune to the single broadcasting frequency can be given by,

\[
P_{c_1} = x \times \frac{1}{x} \times \frac{1}{x} = \frac{1}{x}
\]  \hspace{1cm} (7.6)

while the probability of two CPEs colliding under the proposed scheme is

\[
P_{c_2} = x \times \frac{1}{x} \times \frac{1}{x} \times m \times \frac{1}{m} \times \frac{1}{m} = \frac{1}{mx}
\]  \hspace{1cm} (7.7)

It is obvious that \( P_{c_2} < P_{c_1} \) thus proving that contention resolution is done better in the newly proposed scheme.
To validate and test the performance of the proposed auction and game models, we conducted extensive simulation experiments. All simulations were done on UNIX based platform. Our intention was to generate and test situations that represent the real-world scenario as closely as possible. Our simulation study is broadly divided into four parts. In section 8.1, we present the results of the proposed auction models between spectrum owner and WSPs. Similarly, in section 8.2, we demonstrate the results of the pricing game between WSPs and end-users. The game model is extended in section 8.3, where heterogeneous access networks are considered and results are presented. Finally in section 8.4, we demonstrate and discuss the results for cognitive radio based opportunistic spectrum access.

8.1 Auction Models and Results

To prove the effectiveness of the proposed auction models and the bidding strategies for dynamic spectrum access and allocation, we divided the experiments into two broad categories. Auctions with the allocation constraint of at most one spectrum band grant are discussed in subsection 8.1.1. In subsection 8.1.2, we present the auction model and the results where bidders are granted multiple spectrum bands.
8.1.1 Results for Single Unit Grant

For single unit grant, we present a comparison between sequential and concurrent bidding for both substitutable and non-substitutable bands. We assume the number of bands to be less than the number of bidders for the auction to take place.

8.1.1.1 Substitutable spectrum bands

The parameters for this auction setting are as follows. We assume all the spectrum bands are of equal value to all the bidders. Note that throughout this simulation model, we use the notation $unit$ instead of any particular currency. The reservation price for each bidder is assumed to follow a uniform distribution with minimum and maximum as 250 and 300 units respectively. Moreover, the bids presented by the bidders are also assumed to follow a uniform distribution between 100 and 300 units.

In figures 8.1 and 8.2, we compare the auctioneer’s revenue for both sequential and concurrent bidding with varying number of bidders and spectrum bands. As discussed earlier, the revenue generated in the sequential auction setting is more than that in the concurrent one. In fact, with increase in number of bands and bidders, revenue generated in sequential setting is almost 200% more than the revenue in concurrent setting, thus proving that sequential auction to be more beneficial from the auctioneer’s perspective.
Figure 8.1: Revenue in sequential auction with substitutable spectrum

Figure 8.2: Revenue in concurrent auction with substitutable spectrum
Figure 8.3: Auctioneer’s revenue with substitutable bands: 100 bidders and 10 spectrum bands

Figure 8.4: Auctioneer’s revenue with substitutable bands: 100 bidders and 50 spectrum bands
In figures 8.3, 8.4 and 8.5, we present the revenue generated by auctioneer in both sequential and concurrent biddings with increase in DSA periods. We assume that the bidders use auction histories of previous rounds to submit their bids in future rounds. Thus a winning bidder in one DSA period will try to submit a lower bid in next DSA period to increase his surplus profit whereas a losing bidder will increase his bid provided the previous bid was less than his reservation price. For all three results, we fixed the number of bidders as $n = 100$ but varied the number of bands as $m = 10$, $m = 50$ and $m = 90$. We find that the difference in the revenue generated between sequential setting and concurrent setting increases with number of bands (note the y-axis scale value in figures 8.3, 8.4 and 8.5). Thus sequential auction provides more revenue than the concurrent auction. Moreover, we find that with increasing number of bands, sequential auction reaches steady state much faster than the concurrent auction. This happens due to the fact that as more and more number of bands
Figure 8.6: Substitutable bands: optimal bid for a specific bidder for sequential auction are available in the common pool for the auction \((m \rightarrow n)\), greedy bidders will get more incentive bidding less than their true valuation prices as was proved earlier. This of course will not happen in the sequential auction. Thus sequential auction is clearly a better choice for auctioneer to generate higher revenue.

Next, we present the optimal bid for a specific bidder to win a spectrum band for both sequential and concurrent bidding in figures 8.6 and 8.7. It can be observed that the optimal bid for the concurrent auction is less than the optimal bid for the sequential auction and even decreasing with \(m \rightarrow n\). Thus in concurrent auction setting, auctioneer will not receive any incentive increasing the number of bands in the common pool thus reducing the whole purpose of dynamic spectrum allocation.
8.1.1.2 Non-substitutable spectrum bands

For non-substitutable bands, the bands are not of equal value. We assume the band’s true value follow a uniform distribution with minimum and maximum being 450 and 500 units respectively. We follow the same distribution of bids as mentioned in the previous subsection.

In figures 8.8 and 8.9, we present the revenue with varying number of bidders and bands. It is clear that sequential auction provides better revenue for the auctioneer than the concurrent setting for non-substitutable bands.

In figures 8.10, 8.11 and 8.12, we present the revenue generated by auctioneer in both sequential and concurrent biddings with increase in auction rounds. Similar to the previous case, we assume that the bidders use auction histories of previous rounds to submit their bids in future rounds. We find that the difference in the revenue generated between sequential
Figure 8.8: Revenue in sequential auction with non-substitutable spectrum bands

Figure 8.9: Revenue in concurrent auction with non-substitutable spectrum bands
setting and concurrent setting under non-substitutable bands is even more than that of the substitutable bands of previous case (note the y-axis scale changes in figures 8.10, 8.11 and 8.12). Thus sequential auction setting is clearly a better choice for auctioneer to generate better revenue for both types of bands.

### 8.1.2 Results for Multiple Unit Grant

We simulate the dynamic spectrum allocator knapsack auction model. The objective of developing the simulation model in this work is to show how the proposed knapsack model provides better results than classical highest bid favoring auctions. Next, we compare the synchronous allocation of spectrum with the asynchronous allocation under the knapsack
Figure 8.11: Auctioneer’s revenue with non-substitutable spectrum bands: 100 bidders and 50 spectrum bands

Figure 8.12: Auctioneer’s revenue with non-substitutable spectrum bands: 100 bidders and 90 spectrum bands
auction model. We show how the synchronous allocation outperforms the asynchronous allocation when bidders are granted multiple non-substitutable spectrum bands.

8.1.2.1 Spectrum auctioning methodology and parameters

The main factors that we consider for comparing the performance issues between any two auction methodologies are the revenue generated by spectrum owner, total spectrum usage, and probability of winning for bidders. We consider the following for the simulation model.

- **Bid tuple**: The bid tuple $q_i$ generated by bidder $i$ in synchronous auction consists of amount of spectrum requested, $w_i$ and the price the bidder is willing to pay, $x_i$. In asynchronous auction, the duration is also advertised in addition to the above two. Each bidder has a reservation or evaluation price for the amount of spectrum requested and the bid is governed by this reservation price.

- **Bidders’ strategies**: We follow second price sealed-bid mechanism, i.e., bidder(s) with the winning bid(s) do not pay their winning bid but pay the second winning bid. We could have chosen the first price bidding policy; the only reason for choosing second price policy is that it has more properties than first price in terms of uncertainty [77]. After each round of auction, the only information bidders know is whether their request is granted or not. We assume that all the bidders are present for all the auction rounds; bidders take feedback from previous rounds and generate the bid tuple for next round.
Table 8.1: Simulation Parameters for multi-unit grant auction

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total amount of spectrum</td>
<td>125</td>
</tr>
<tr>
<td>Minimum amount of spectrum that can be requested</td>
<td>11</td>
</tr>
<tr>
<td>Maximum amount of spectrum that can be requested</td>
<td>50</td>
</tr>
<tr>
<td>Minimum bid for per unit of spectrum</td>
<td>25</td>
</tr>
<tr>
<td>Minimum time requested for spectrum leasing in asynchronous allocation</td>
<td>1</td>
</tr>
<tr>
<td>Maximum time requested for spectrum leasing in asynchronous allocation</td>
<td>5</td>
</tr>
<tr>
<td>Fixed time for spectrum leasing in synchronous allocation</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Auctioneer’s strategies:** Spectrum owner tries to maximize the revenue generated from the bidders.

Simulation parameters are shown in table 8.1. We first compare the proposed knapsack auction model with the classical second price auction.

**Revenue and spectrum usage:** Figures 8.13 and 8.14 compare revenue and spectrum usage for the proposed knapsack auction model with the classical second price auction for each auction round. The number of bidders considered is 15. Note that, both revenue and usage are low in the beginning and subsequently increases with rounds. In the initial rounds, bidders are dubious and make low bids. With increase in rounds, potential bidders emerged as expected and raised the generated revenue. We observe that the proposed auction generates 10-15% more revenue compared to the classical model and also reaches steady state faster.

Figures 8.15 and 8.16 show the average revenue and spectrum usage with varying number of bidders. Comparing revenue and spectrum usage in figures 8.15 and 8.16, it is evident that
knapsack auction generates more revenue than the classical auction and use the spectrum band more efficiently thus optimizing spectrum usage throughout the auction period.

Figure 8.13: Revenue Maximization with auction rounds (with 15 bidders)

Figure 8.14: Usage Maximization with auction rounds (with 15 bidders)
Figure 8.15: Revenue maximization

Figure 8.16: Usage Maximization
**Bidder participation:** In figures 8.17 and 8.18, we look at our auction model from the bidders’ perspective. Higher revenue requires high participation in number of bidders. However, classical auctions always favor bidders with high spectrum request and/or high bid, thus discouraging low potential bidders and giving the higher potential bidders a chance to control the auction. In order to evaluate the bidder participation, we consider two cases: a) bidder with the lowest spectrum request and b) bidder with the lowest bid. For these two cases, we compare the two strategies in terms of probabilities to win a bid. We observe that the proposed auction strategy has a significantly high probability of winning compared to classical strategy. Note that probability of winning in classical strategy almost reaches zero with increase in bidders.

![Graph showing probability of winning with lowest spectrum request](image)

**Figure 8.17:** Probability of winning with lowest spectrum request

**Collusion prevention:** The occurrence of collusion must be prevented in any good auction so that a subset of bidders can not control the auction that might decrease the spectrum
broker’s revenue. We consider two cases: i) when bidders collude and ii) when bidders do not collude. In our simulation model, we assume bidders randomly collude in pair in all possible combinations with others.

In figure 8.19, we show the average revenue generated by spectrum broker with increase in number of bidders both in presence and absence of collusion. Though at the beginning with less number of bidders, presence of collusion reduces the average revenue slightly, but with increase in number of bidders the effect due to collusion decreases. Thus with increase in number of bidders, i.e., with increase in (perfect) competition, revenue generated even in the presence of collusion reaches almost the same value as that of without collusion. Figure 8.20 presents the usage of spectrum in the presence and absence of collusion. The most interesting result from bidders’ perspective is shown in figure 8.21. When the number of bidders is low (less than or equal to 4 in our case) collusion provides better probability of winning but as
the number of bidders increases, probability of winning with the help of collusion decreases, discouraging bidders to collude.

Next, we analyze the proposed knapsack auction model with both synchronous and asynchronous allocations. Figures 8.22 and 8.23 compare revenue and spectrum usage for both
the strategies (synchronous and asynchronous) with increase in auction rounds. The number of bidders considered in this simulation is 15. Note that, both revenue and usage are low at the beginning and subsequently increases with rounds. When auction starts, bidders always act skeptical, thus initial bids are always much lower than their true potential bids. With the increase in auction rounds, bidders get an idea of the bids of other bidders and thus try to increase or decrease their bids accordingly.

Figures 8.24 and 8.25 show the average revenue and spectrum usage with varying number of bidders for both the auction strategies. We observe that the proposed synchronous knapsack auction generates approximately 10% more revenue compared to the asynchronous knapsack auction and also reaches steady state faster. The average spectrum usage is also more with the synchronous allocation policy.
Figure 8.22: Revenue generated with auction rounds

Figure 8.23: Spectrum usage with auction rounds
Figure 8.24: Average revenue generated with number of service providers

Figure 8.25: Average spectrum usage with number of service providers
In figure 8.26, we look at the auction model from the bidders’ perspective. Higher revenue requires more number of bidders. We compare the two strategies in terms of the probabilities to win a bid. We observe that the proposed synchronous auction strategy has a significantly higher probability of winning compared to asynchronous auction strategy. This implies that bidders will be encouraged to take part in the synchronous knapsack auction; thus increasing the competition among the providers and increasing the chance to generate more revenue.

8.2 Service Pricing: Numerical results

In this section, we provide some insights on how the pricing strategies proposed for WSP and end-users work as incentives for both. Of course, the values depend on the parameters chosen. Interesting observations from the equation (5.44) and equation (5.46) are that, with
increase in resources of a provider, the optimal price per unit of resource decreases which acts as an incentive for the users. At the same time, as the total resource is increasing, this gives the provider an incentive to increase its total profit. The increase in total profit of the provider will be more in the situation where the number of users increase with increasing resources. Note that it is not necessary for the providers to take in more number of users even if they have more resources i.e., they can decide to serve the same number of users with better resources for each.

First, we consider a scenario where the number of users remain the same even if the resource of the provider increases. We recapitulate from equations (5.44) and (5.46) that $a_{ij}$ is fixed when the number of users are fixed and $C_j$ is increasing. We assume the number of users already present is 5. Moreover, we consider all the users homogeneous i.e., $a_{ij} = 1.5$ for all users. We increase the value of $C_j$ from 1 to 100 units. It is to be noted that the values used for obtaining the numerical results are arbitrary and are merely for the sake of demonstration. Any other values of $a_{ij}$ and $C_j$ can be used as long as they satisfy the constraint that the price per unit resource as positive.

In figure 8.27, we show how the provider must decrease the the price per unit of resource if the total amount of resources increases with the same user base. This decrease in unit price is necessary if resource utilization is to be maximized. This certainly acts as an incentive for the users.

The total profit of the provider is presented in figure 8.28. With the number of users fixed, we observe that the total profit of the provider increases till a certain resource and
then decreases. We can infer that the provider has some incentives to acquire a certain amount of resources. For a fixed number of users, this plot allows us to estimate the amount of resource that the provider must have such that its profit is maximized. Though obvious, we show how the resource allocated to users increases with increase in total resource of the provider in figure 8.29.

We show how the net utility of users increases with more resources in figure (8.30). From the graph it is clear that for fixed number of users, increasing resources is certainly an incentive for the users. An important aspect to note here is that, for initial increase in resource the utility increased very quickly from zero but the utility slowly saturates indicating that more resources have limited value beyond a certain point i.e., the users will not find ways to utilize abundant resources.
Figure 8.28: Total profit of a provider with number of users fixed

Figure 8.29: Resource allocated to a user with number of users fixed
Next, we consider that the number of users is increasing which is typical of any market, along with the increase in resources. We start with only 1 user under a provider. For fair comparison with the previous case (i.e., with fixed number of users), we vary resources from 1 to 100 units. Note that, $a_j$ is no longer fixed and increases with increasing number of users. We assume $a_{ij}$’s to be 1.5 and the number of users are such that the ratio of $a_j$ and $C_j$ is fixed with increase in both users and resource. In figure (8.31), the price per unit of resource is presented. As the initial number of users are very few, increasing resource necessitates an initial increase in price per unit of resource. But as the number of users increase, it is imperative that price per unit resource decreases providing incentive for the users.

In figure 8.32, we present the total profit of the provider. Unlike the previous case (figure 8.28), we see that with users increasing proportionally with resources, the total profit is always increasing which presents a better incentive for the providers than the previous
case. It becomes quite evident that to increase their total profit, the providers would prefer more number of users, each getting less amount of resource, rather than having less number of users, each having more amount of resource.

Figure 8.33 justifies the hypothesis that with both increasing number of users and increasing resources, the increase in the amount of resource for users is much less than as presented for fixed number of users in figure 8.29.

In figure 8.34, we show the net utility of the users. We see that the net utility increases with increasing resources; thus providing incentive for the users but unlike figure 8.30, the increase in net utility saturates early. Thus comparing the two figures 8.30 and 8.34, it becomes quite evident that to increase their net utility, users would like to be in a lightly loaded system (i.e., with less number of users), thereby getting a larger share of the resources,
Figure 8.32: Total profit of a provider with increasing number of users

Figure 8.33: Resource allocated to a user with increasing number of users
rather than having to share the resources with more number of other users. This presents the classic case of conflict between the providers and users and thus is the basis of our work.

To justify the above claim, we present figure 8.35, where we assume that the resource is fixed at 100 units. We calculated the net utility of each user. We see that with increase in number of users, the resource allocated to each user is proportionally less, but the resulting decrease in net utility is more rapid.

### 8.3 Simulation Model for Heterogeneous Networks

To verify our proposed model for heterogeneous networks, we consider a system where a wireless service provider has two access networks – 3G network based on CDMA/HDR [88, 6] and 802.11 based Wi-Fi network. The coverage area of the 3G network is large.
but the perceived QoS degrades as user goes far from the 3G base station. On the other hand, Wi-Fi network coverage is provided by Wi-Fi access point (AP) that provides uniform coverage for a small area. We considered that the users are scattered randomly over the cell.

8.3.1 Network Layout

In the time-slotted CDMA/HDR system, the base station transmits to one user at full power using one of the 11 pre-defined modulation and coding schemes. Thus, we can think of the cell being divided into 11 concentric rings, each receiving a unique data rate [6]. It can be argued that proper utilization of the radio resources depends on the placement of the APs. In this regard, we consider two different kinds of placement: ideal placement and random placement. It is assumed that in the ideal placement, the distribution of users’ locations
are known and more number of APs are placed at the high density areas. Since the ideal placement is not possible in every premise due to incomplete information of user’s profiles, random placement is the alternative solution where APs are placed uniform randomly inside the 3G cell. For our simulation, we consider the radius of the 3G cell and the Wi-Fi cell as 2000 and 100 meters respectively. The data rates and corresponding distances are from Table 6.1 as given in [9]. We consider $N = 1000$.

8.3.2 Perceived Bandwidth

In figure 8.36, we compare the perceived system bandwidths of the systems with and without internetworking. For our proposed system we follow the round robin scheduling scheme for fairness criteria, whereas for the existing system we follow both the round robin and opportunistic scheduling scheme. (We follow a simple opportunistic scheme, where priorities are given to the users with better signal strength.) With increase of Wi-Fi APs, more and more number of users come under the direct influence of Wi-Fi coverage area and get the highest data rate. After a certain point, when all the users have the option of getting the highest rate for the service requested, the system performance reaches its maximum value.

8.3.3 Cost and Utility of Provider

In figure 8.37, we present the net utility obtained by the service provider with increase in number of Wi-Fi APs, keeping the capacity and thus the cost of the 3G network fixed. For
Figure 8.36: Perceived system bandwidth: simulation result

our simulation, in addition to the above parameters given, we assume the cost due to the 3G network is $E_{3G} = 1000$ unit. The cost incurred due to each AP is $e_{AP} = 20$ unit. In figure 8.38, we show the relative increase in net utility of the internetworked system, with respect to the system without internetworking with varying cost per AP.

8.3.4 Per-user Utility

Here, we study the per-user perceived bandwidth and utility obtained for both with and without internetworked system. We used the same parameters as mentioned earlier but considered but considered number of APs as 100 and 200. The results obtained are shown in figures 8.39 and 8.40 respectively. As expected, with the increase in number of active users, the per-user perceived bandwidth decreases because of resource sharing, but the rate
Figure 8.37: Net utility of the provider

Figure 8.38: Relative increase in net utility (in %) with respect to the system without internetworking
of decrease is lesser for the case with internetworked system. With more number of APs, this decreasing rate can be further reduced.

### 8.3.5 Perceived Bandwidth with Radius of Cell

So far, the radii of the cell and the APs were considered constant. In figure 8.41, we keep the radius of the Wi-Fi coverage area constant (100m) and vary the radius of the cell from 1000m to 3000m so as to get a varying radius ratio. We observe that when the radius of the 3G cell is small, throughput with and without APs does not vary much because users are situated very near to the base station and get very high rate. But when the 3G cell radius increases, more and more number of users get lesser signal strength and hence lesser data rate, resulting in decreased system performance for system without internetworking.
Figure 8.40: per-user perceived utility

Figure 8.41: Perceived system bandwidth due to effect of radius ratio
8.4 Cognitive Spectrum Access

We evaluate the improvements achieved by the IEEE 802.22 enhanced PHY/MAC air interface and UGC algorithm. Evaluations for enhanced and existing schemes were done for a fair comparison. We also present how the utility graph coloring algorithm (UGC) outperforms any other existing spectrum allocation. We compare our proposed UGC algorithm under all three objectives. For the topology, we have assumed a 100 km radius region where multiple overlapping 802.22 networks and licensed incumbents reside and share the spectrum from the licensed spectrum band. Moreover, we have assumed BS and CPEs use directional antenna for transmission/receiving purpose and omni-directional antenna for incumbent sensing. In table 8.2, we present the simulation parameters for our experiments.

Table 8.2: Simulation parameters for IEEE 802.22 system

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total licensed spectrum band</td>
<td>54 - 806 MHz</td>
</tr>
<tr>
<td>Number of overlapping BSs</td>
<td>8</td>
</tr>
<tr>
<td>BS/CPE receiving radius</td>
<td>30 - 50 km</td>
</tr>
<tr>
<td>BS/CPE sensing radius</td>
<td>30 - 50 km</td>
</tr>
<tr>
<td>TV transmission receiving radius</td>
<td>30 km</td>
</tr>
<tr>
<td>$B_{\text{min}}$</td>
<td>30 MHz</td>
</tr>
<tr>
<td>Control signal frequency</td>
<td>1 - 2 MHz</td>
</tr>
<tr>
<td>Data signal frequency</td>
<td>1 - 18 MHz</td>
</tr>
<tr>
<td>Broadcast control signaling interval</td>
<td>20 ms</td>
</tr>
<tr>
<td>Number of broadcast control signals transmitted at any instant</td>
<td>2 - 6</td>
</tr>
</tbody>
</table>
8.4.1 Simulation Results

In Fig. 8.42, we compare the total system utility achieved by the 802.22 BSs under utility graph coloring spectrum allocation mechanism and greedy non-collaborative spectrum hogging. In the greedy non-collaborative approach, most of the spectrum bands are wasted due to interference among the greedy and selfish base stations, whereas under the collaborative utility graph coloring mechanism, system utility is improved. Moreover, it is clear from Fig. 8.42 that with the increase in usage of the licensed spectrum band, proposed utility graph coloring method provides even better result than the non-collaborative approach. For a comprehensive performance evaluation of the proposed scheme, we present the results under all three objective functions and show that system utility is always better under the proposed scheme than the non-collaborative approach.

Figure 8.42: Total utility achieved by all the BSs under the proposed collaborative approach and greedy non-collaborative approach
In Fig. 8.43, we compare the performance of utility graph coloring with the traditional graph coloring method of spectrum allocation for all three objective functions. It is clear that proposed utility graph coloring mechanism outperforms the traditional graph coloring for objective functions 1 and 2. For objective function 3, i.e., the complete fairness, any of the either methods would provide same result.

![Graph comparing utility and spectrum usage](image)

**Figure 8.43:** Total utility achieved by all the BSs under the proposed collaborative approach and greedy, traditional graph coloring approach

Next is the fairness criteria that we investigated in figure 8.44 for UGC objective functions 1 and 2 for which we use Jain’s fairness index [30]. After taking average from 1000 simulation runs, we find that in addition to providing better system utility than any other spectrum allocation mechanisms, both the objective functions show fairness index more than 0.5, which is considered to be a good fairness index. (This fairness index lies between 0 and 1 with 0 being most unfair and 1 being absolutely fair.) Moreover, the objective function
for proportional fair utility maintains an excellent fairness index of 0.86 which exceeds the fairness index provided by maximum utility objective function.

![Fairness index vs Primary incumbent usage](image)

Figure 8.44: Jain’s fairness index for maximized utility objective function and proportional fair utility objective function

Next, we present the results to provide some insights on how our enhanced air-interface would improve the performance for hidden incumbents scenario. In Fig. 8.45, we present the probability of a CPE (which just switched on) to connect to a BS. For this scenario, we assumed there is no contention from other CPE but licensed incumbents are active and operating. The probability to connect to a BS with the dynamic multiple candidate channels is more than the existing single frequency broadcast signaling thus proving the improvement of the proposed scheme.

Fig. 8.46 presents an important result in terms of spectrum utilization for the 802.22 system. With increase in the usage by licensed incumbents (x-axis), the spectrum utilization
for data transmission (y-axis) from the residue spectrum band in the 802.22 networks is shown in this figure. As evident from the figure, the proposed mechanism of enhancing the MAC layer increases the spectrum utilization than the existing MAC layer.

The average initial delay (under no contention from other CPE) to tune to a BS broadcasting signal frequency against the licensed spectrum usage by incumbents is shown in Fig. 8.47. As obvious from the figure, with increase in licensed spectrum usage by incumbents (e.g., TV transmission, wireless microphones etc.), the average delay increases, but the average delay with the proposed scheme is always less than the average delay with the existing MAC.

In figures 8.48 and 8.49, we present a more comprehensive result of connection establishment under contention with other CPEs and licensed incumbents together. The average startup delay (delay between switching on and start of data transmission and receiving)
under the presence of contention is presented for number of CPEs and licensed spectrum usage by incumbents. We calculate the combined delays to tune to a BS broadcasting frequency signal and then successful uplink transmission (transmission of connection identifier and spectrum usage report) through contention resolution mechanism. It is evident from the figures that enhanced MAC (Fig. 8.49) provides better result in terms of delay to initiate data transmission.
Figure 8.47: Average initial delay to tune to a BS broadcasting frequency (no contention)

Figure 8.48: Average startup delay for existing MAC
Figure 8.49: Average startup delay for proposed MAC
CHAPTER 9
CONCLUSIONS

In this research, we propose a unified framework that is emerging due to the dynamic interactions between spectrum owner, WSPs, and end-users, where spectrum and services are traded in a market like scenario. We use auction theory to analyze the dynamic spectrum allocation process that the spectrum owner can potentially use to allocate spectrum bands to competing WSPs. Using game theoretic tools, we capture the WSPs–end-users interactions where all WSPs try to efficiently manage their resources, offer competitive prices, and maintain quality of service.

To address the allocation process of dynamic spectrum access (DSA), we propose and study different auction models where chunks of spectrum bands are auctioned to multiple WSPs. First, we consider the scenario where WSPs are granted no more than one band and analyze cases where auctions are held sequentially and concurrently. We show that sequential auction is a better choice for DSA when there is a constraint on the allocation. On the other hand, when there is no constraint, Knapsack based auction perform well. We study and compare both synchronous and asynchronous auction strategies and show that synchronous auction makes optimal use of the spectrum and maximizes the revenue for the spectrum owner. The proposed sealed bid knapsack auction mechanism also yields higher
probability of winning for the service providers and thus encourage them to participate in the auction.

As far as the WSP–end-users interactions are concerned, we devise utility functions for both WSPs and end–users. We propose a dynamic pricing strategy that helps the service providers and end–users to maximize their utilities. We show that the price (Nash) equilibrium exists in this non–cooperative pricing game. Also, we demonstrate how the proposed pricing can provide incentives to providers to upgrade their resources and users to opt for better services.

For opportunistic spectrum access, we expose the drawbacks of the newly proposed cognitive radio based IEEE 802.22 networks. We provide solutions that enable self–coexistence of multiple networks and allow hidden incumbents to be detected. We propose a utility based graph coloring algorithm that not only achieves better spectrum efficiency but also incurs lower connection establishment delay.
LIST OF REFERENCES


[88] 3GPP TR25.848 V0.5.0, “Physical Layer Aspects of UTRA High Speed Downlink Packet Access(HSDPA)”, ANNEX B, TSGR1#18(01)186, Jan 18, 2001.


