Model Studies Of Time-dependent Ducting For High-frequency Gravity Waves And Associated Airglow Responses In The Upper Atmosphere

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MODEL STUDIES OF TIME-DEPENDENT DUCTING FOR HIGH-FREQUENCY GRAVITY WAVES AND ASSOCIATED AIRGLOW RESPONSES IN THE UPPER ATMOSPHERE

by

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Summer Term
2007

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ABSTRACT

This doctoral dissertation has mainly concentrated on modeling studies of shorter period acoustic-gravity waves propagating in the upper atmosphere. Several cases have been investigated in the literature, which are focusing on the propagation characteristics of high-frequency gravity wave packets. The dissertation consists of five main divisions of which each has its own significance to be addressed, and these five chapters are also bridged in order with each other to present a theme about gravity wave ducting dynamics, energetics, and airglows.

The first chapter is served as an introduction of the general topic about atmospheric acoustic-gravity waves. Some of the historical backgrounds are provided as an interesting refreshment and also as a motivation reasoning this scientific research for decades. A new 2-D, time-dependent, and nonlinear model is introduced in the second chapter (the AGE-TIP model, acronymically named atmospheric gravity waves for the Earth plus tides and planetary waves). The model is developed during this entire doctoral study and has carried out almost all research results in this dissertation.

The third chapter is a model application for shorter period gravity waves ducted in a thermally stratified atmosphere. In spite of mean winds the thermal ducting occurs because ducted waves are fairly common occurrences in airglow observations. One-dimensional Fourier analysis is applied to identify the ducted wave modes that reside within multiple thermal ducts. Besides, the vertical energy flux and the wave kinetic energy density are derived as wave diagnostic variables to better understand the time-resolved vertical transport of wave energy in the presence of multiple thermal ductings.
The fourth chapter is also a model application for shorter period gravity waves, but it instead addresses the propagation of high-frequency gravity waves in the presence of mean background wind shears. The wind structure acts as a significant directional filter to the wave spectra and hence causes noticeable azimuthal variations at higher altitudes. In addition to the spectral analysis applied previously the wave action has been used to interpret the energy coupling between the waves and the mean flow among some atmospheric regions, where the waves are suspected to extract energy from the mean flow at some altitudes and release it to other altitudes.

The fifth chapter is a concrete and substantial step connecting theoretical studies and realistic observations through nonlinearly coupling wave dynamic model with airglow chemical reactions. Simulated O (^1S) (557.7 nm) airglow images are provided so that they can be compared with observational airglow images. These simulated airglow brightness variations response accordingly with minor species density fluctuations, which are due to propagating and ducting nonlinear gravity waves within related airglow layers. The thermal and wind structures plus the seasonal and geographical variabilities could significantly influence the observed airglow images. By control modeling studies the simulations can be used to collate with concurrent observed data, so that the incoherencies among them could be very useful to discover unknown physical processes behind the observed wave scenes.
ACKNOWLEDGMENTS

I express my gratitude to the people who supported me most to make it happen: my thesis advisor, Professor Michael P. Hickey and his beloved wife Diane C. Hickey, my wife Shaona Tan and my daughter Kejia Yu, my father Zirong Yu and my mother Fuying Weng, and my brother Yongfeng Weng.
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CHAPTER ONE:  
INTRODUCTION

Prelude

Every day we experience and are very familiar with acoustic waves in all aspects of our working, traveling, and living. Because these acoustic waves help us talk to each other with our ears detecting them and listen to what is going on around us. For example, during the summer season, especially when thunderstorm occurs, acoustic waves are produced and usually they have large wave amplitudes because we can hear the thunder. Normally what most people do not realize is that another kind of wave, the gravity wave, is also produced in the thunderstorm. The gravity waves have frequencies far below the audible frequencies for hearing, and so they probably go unnoticed by the community except being detected by professionals with sophisticated instruments. However, because the gravity waves play profound influences on the upper atmosphere, the atmospheric physics community takes even much more interest in these waves.

The whole spectrum of waves during the thunderstorm activity is also known as acoustic-gravity waves. The difference between acoustic waves and gravity waves is that, acoustic waves are compressional and longitudinal waves, wherein air parcels expand and contract along the direction of wave propagation; while gravity waves are basically vertically transverse waves, wherein air parcels oscillate within the vertical plane essentially perpendicular to the direction of wave propagation. There is a certain limitation for the gravity wave frequency. The shortest oscillation period for a gravity wave that the atmosphere will allow to propagate is typically
about 4 or 5 minutes. These frequencies are well below the minimum frequency with which most people are capable of hearing. Locally gravity waves have even lower frequencies at mid-latitudes and being even lower at equatorial latitudes.

One of the remarkable facts about gravity wave propagation concerns how far and how high they can propagate forward and upward and still remain coherent wave structure. Gravity waves generated by thunderstorms, which normally are high-frequency waves, are believed to be capable of propagating as high as 250 km altitude into the atmosphere before they dissipate away. To do so, they must have travelled many hundreds of kilometers horizontally. Gravity waves exist within a wide range of spatial and temporal scales. Observations reveal a continuous spectrum of gravity waves with horizontal wavelengths of a few to several thousand kilometers, periods ranging from several minutes to tens of hours depending on altitudes and latitudes, and a general phase downward motion. These gravity waves carry energy and momentum and also have secular influences to distribution of minor species. While still low in the atmosphere they seem not too important, but once it is recognized that, the atmospheric density decreases with increasing height by a factor of about one over a million between sea level and 100 km altitude, then even a relatively small amount of wave energy in the troposphere can have a tremendous influence on the upper atmosphere provided it can reach there. It is for this reason that atmospheric gravity waves are particularly important to the overall structure of the upper atmosphere.

This brings us to an important but largely unknown fact concerning the region near 90 km altitude also known as the mesopause region. In most regions of the atmosphere the summer polar regions are considerably warmer than their winter counterparts. However, it is an amazing
fact that the summer polar mesopause is considerably cooler than its winter counterpart. What is
even more remarkable is that the summer polar mesopause is the coldest region anywhere in the
terrestrial atmospheric environment. There must be a reason in the polar mesopause region that is
away from the local thermodynamic equilibrium. The answer is simply due to gravity waves that
could cause the polar mesopause region to be so unlike other regions of the Earth’s atmosphere.

Significance

The middle atmosphere, which ranges from 80 km altitude to 100 km altitude, is a region
full of active photochemical and dynamical interactions. It is also a region where waves and their
dissipative process play an important role in atmospheric energetics and dynamics. The
importance of momentum and energy balances due to gravity waves in the mesosphere has been
firmly established and recognized by numerous studies. Basically, gravity waves act as a
transporter for wave energy and momentum to be transported into the middle atmosphere from
the lower atmosphere. Therefore, understanding gravity waves is essential in atmospheric
dynamics, long-term minor species variations, and long distance energy and momentum
transport.

Gravity waves arise from a number of lower atmospheric sources like jet streams, tidal
waves, earthquakes, volcanic eruptions, nuclear explosions, latent heating, thunderstorms, and
from upper atmospheric sources like that associated with aurora. They can help the mixing of
minor species chemical reactions in the atmosphere. They propagate vertically as well as
horizontally, dissipate, interact nonlinearly, and profoundly influence the momentum, energy,
and the constituents in the atmosphere. Some gravity waves are observed to propagate
horizontally even for thousands of kilometers. Gravity waves may alter their environment and profoundly affect the circulation of the middle atmosphere on a global basis.

The atmosphere is basically a fluid that is acted upon by a force due to the Earth’s gravity. Under the influence of gravity, the background gas density decreases exponentially with increasing height, so does the background gas pressure. The amplitude of the gravity wave increases exponentially with increasing height. The physical interpretation is that, by maintaining the vertical flux of wave energy constant, the wave amplitude offsets the decrease of the background gas density. The mechanism of the gravity wave is that when the force of the Earth’s gravity and the stabilizing restoring force, which is produced by the atmospheric pressure gradient, become comparable with compressional forces, the resultant waves are gravity waves.

The produced waves may be termed internal or surface waves according to whether the vertical wave number is pure real or pure imaginary. For internal waves, in the high frequency limit, they behave like simple sound waves so they are called acoustic gravity waves. In the low frequency or long period range from several minutes to several hours, they are termed internal gravity waves. In 1960, Hines proposed a theory describing the observed atmospheric fluctuations as a manifestation of the propagation of internal gravity waves. Since then, this theory has been universally accepted and has provided a solid theoretical foundation for the middle and upper atmosphere studies due to its simplicity in mathematics.

**Mathematics**

The linear theory of acoustic gravity waves assumes that the single fluid background atmosphere is isothermal, stationary, and horizontally stratified. Superimposed wave motions are assumed to have only small perturbation magnitude and occur adiabatically. Forces due to
pressure gradients, gravity, and inertia are treated explicitly. The oscillations are governed by the linearized momentum, adiabatic state, and continuity equations [Hines, 1960]:

\[ \rho_0 \frac{\partial U}{\partial t} = \rho g - \nabla p, \quad (1.1) \]

\[ \frac{\partial p}{\partial t} + U \cdot \nabla p_0 = C^2 \left[ \frac{\partial p}{\partial t} + U \cdot \nabla p_0 \right], \quad (1.2) \]

\[ \frac{\partial p}{\partial t} + U \cdot \nabla p_0 + \rho_0 \nabla \cdot U = 0. \quad (1.3) \]

Those equations relate the perturbed velocity \( U (u, w) \), the perturbed atmospheric pressure \( p \) and density \( \rho \), the unperturbed atmospheric pressure \( p_0 \) and density \( \rho_0 \), the gravitational acceleration \( g \), and the sound speed \( C \). By assuming that plane wave solutions exist for (1.1), (1.2), and (1.3) we can write [Hines, 1960]

\[ \frac{p - p_0}{p_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{U_x}{X} = \frac{U_z}{Z} = A \exp \left( i \omega t - K_x x - K_z z \right), \quad (1.4) \]

where \( P, R, X, \) and \( Z \) are all complex constant amplitudes, and \( A \) is a real constant amplitude. Substituting (1.4) back to the linearized (1.1), (1.2), and (1.3), one builds up the matrix equation as

\[
\begin{bmatrix}
  i\omega & 0 & -iK_x & -(1/H + iK_z) \\
  0 & -iK_x C^2 / \gamma & i\omega & 0 \\
  g & -(1/H + iK_z) C^2 / \gamma & i\omega & 0 \\
  -i\omega C^2 & i\omega C^2 / \gamma & 0 & (\gamma - 1) g
\end{bmatrix} \cdot \Phi = 0, \quad (1.5)
\]

where \( \gamma \) is the ratio of specific heats and \( H \) is the scale height and
\[ \Phi = \begin{bmatrix} \frac{\delta p}{p_0} \\ \frac{\delta p}{p_0} \\ u \\ w \end{bmatrix} \exp i(\omega t - K_x x - K_z z). \] (1.6)

The wave angular frequency \( \omega \) and the horizontal wave number \( K_x \) are both real and constant, \( K_x = k \), but the vertical wave number \( K_z \) is complex, \( K_z = m + i/2H \), allowing for a change with height \( z \) in the effective wave amplitude. The plane wave solutions require the determinant of the matrix in (1.5) to be zero, then the wave numbers appearing in (1.4) are related to the wave angular frequency by the dispersion equation [Hines, 1960]:

\[ \omega^4 - \omega^2 C^2 \left( K_x^2 + K_z^2 \right) + (\gamma - 1) g^2 K_x^2 + i \gamma g \omega^2 K_z = 0. \] (1.7)

Observations

The mesosphere can be heated by several different processes: solar absorption in the Ozone bands; quenching of O metastable species; the release of significant amount of stored chemical potential energy; dynamical interaction of wave-wave and wave-mean flow in which the waves dissipate and a portion of this energy is transferred from macroscopic to microscopic motion; and adiabatic compressional heating due to vertical motion. The cooling rate in the mesosphere is largely dominated by radiative processes involving CO\(_2\), NO, O, and O\(_3\). A number of techniques are employed to obtain data on the motion and the structure of the middle and upper atmosphere. Optical and radio remote sensing instruments, rocket-based in situ measurements, aircraft, balloon, lidar, and all-sky airglow imagers from the ground are the most common sources of observational data. For example, rocket vapor trails and chemical releases...
are used to infer winds, wind shears, and molecular and turbulent diffusion; lidar is used to measure wind and temperature profiles and all-sky airglow imagers are used to photograph the structure of the gravity waves. Those observational data provides an excellent tool to test the validity of the existing models and help develop more accurate models to acquire a more complete understanding of the atmosphere.

The occurrence of horizontal background winds can affect the propagation of gravity waves and they are capable of exchanging energy with the waves. Under proper conditions, the atmosphere may be heated thus raising the temperature of the atmosphere. As gravity waves rise into the rarer atmosphere, the effects of viscosity and thermal conduction become increasingly severe. They also act to dissipate the energy of the waves, and convert them into heat. The gravity wave is unstable in the vicinity of critical layers where the wave’s horizontal phase velocity becomes equal to the mean flow velocity, in other words, the critical layer occurs when the Doppler-shifted frequency is zero. The gravity wave-critical layer interaction is known to have several properties that are important in the dynamics of the atmosphere. These are strong couplings between a gravity wave and the mean flow occurring at a critical layer. The interaction of a gravity wave with the mean flow near the critical layer results in a severe gravity wave attenuation with much of its energy and momentum being absorbed by the mean flow.

All-sky imaging of airglow emissions has become one of the major techniques used to quantify parameters of gravity waves propagating in and through the mesosphere and lower thermosphere (MLT) region. Such observations provide extrinsic horizontal phase speed and direction as well as horizontal wavelength of gravity wave disturbances in the airglow. Aided with numerical model simulations of the interaction between gravity wave dynamics and airglow chemistry, gravity wave amplitudes in the MLT region can also be inferred from the airglow
imaging observations. Thus, the imager measurements combined with suitable modeling have the potential to contribute greatly to our understanding of gravity wave fluxes of energy and momentum in the MLT region and fluxes of minor species. Ultimately, these efforts will contribute to a quantification of gravity wave influences on the global structure of the MLT region, including the diabatic circulation that drives the mesopause region away from local radiative equilibrium leading to the cold summer (warm winter) mesopause.
CHAPTER TWO:
NUMERICAL MODEL

Description

An acoustic gravity wave model is developed in this dissertation study and acronymically named atmospheric gravity waves for the Earth plus tides and planetary waves (AGE-TIP). The equations solved in the AGE-TIP model are the Navier-Stokes equations, which involve mass continuity, momentum, and thermodynamic energy equations, plus the definition of the potential temperature and the equation of state for an ideal gas. These highly coupled equations include dissipation due to eddy processes and molecular processes (viscosity and thermal conduction). The initial atmosphere is a non-isothermal one and horizontal mean winds can be included. The Coriolis force (owing to the rotation of the Earth) and ion drag are both negligible for high-frequency gravity waves. Ion drag is important for longer period gravity waves, but it will be unimportant for high-frequency waves considered here [Hines, 1968]. Francis [1973] also found that the effects of ion drag were nowhere particularly large for ducted waves that usually are shorter period gravity waves. Furthermore, ion drag usually maximizes at F-region altitudes (from about 250 km to 300 km altitude), which are far higher than the altitudes we are interested in here. In addition, for high-frequency gravity waves ($\omega >> \Omega_E$, where $\Omega_E$ is the angular frequency of the Earth) the effects of the Earth’s rotation can be safely neglected [e.g., Hickey and Cole, 1987]. Modifications to the dispersion equation including the Coriolis force and ion drag are given by Volland [1969], Francis [1973], and Hickey and Cole [1987]. Composition effects in the thermosphere associated with an altitude variation of mean molecular weight
[Walterscheid and Hickey, 2001] are also neglected because the primary region of interest is the atmospheric region below the thermosphere.

\[
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0, \quad (2.1)
\]

\[
\rho \frac{D\mathbf{v}}{Dt} + \nabla p - \rho g - \nabla \cdot (\rho \mathbf{v} \nabla \mathbf{v}) - \nabla \cdot (\rho \eta \nabla \mathbf{v}) + \rho K_r \mathbf{v} = 0, \quad (2.2)
\]

\[
\rho c_v \frac{DT}{Dt} + p \nabla \cdot \mathbf{v} - \nabla \cdot (\lambda_m \nabla T) - \frac{c_v T}{\theta} \nabla \cdot (\rho \kappa_e \nabla \theta) + c_v \rho K_s T = \rho c_v (Q_i + Q_w), \quad (2.3)
\]

\[
\theta = T \left( \frac{p_{60}}{p} \right)^{\kappa}, \quad (2.4)
\]

\[
p = \frac{\rho R_s T}{M}. \quad (2.5)
\]

The model domain is in two spatial dimensions and one temporal dimension, where \(x\) is the horizontal position, \(z\) is the vertical position, and \(t\) is time. The model extends vertically from the ground up to 250 km altitude and spans horizontally only one horizontal wavelength allowing periodic boundary conditions to be imposed. These nonlinear equations are used to describe fully compressible and non-hydrostatic plane wave motions. \(\mathbf{v}\) is the normal velocity vector with \(x\) (positive eastward) and \(z\) (positive upward) component \(u\) and \(w\), respectively; \(\rho\) is the atmospheric neutral density; \(p\) is the atmospheric pressure; \(g\) is the acceleration due to gravity; \(\nu\) is the molecular kinetic viscosity; \(\eta_e\) is the eddy momentum diffusivity; \(c_v\) and \(c_p\) are the specific heats at constant volume and constant pressure, respectively; \(Q_i\) is the atmospheric heat source initially introduced to balance the thermodynamic energy equation; \(Q_w\) is the wave thermal excitation; \(T\) is the atmospheric temperature; \(\lambda_m\) is the molecular thermal conductivity; \(\kappa_e\) is the
eddy thermal diffusivity; $M$ is the mean molecular weight; and $K_R$ and $K_N$ are Rayleigh friction and Newtonian cooling coefficients, respectively. The operator $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$ is the substantial derivative, where $\mathbf{v}(x, z, t)$ is the total velocity vector (mean plus perturbation). $\theta$ is the potential temperature, $\overline{p}_{\text{ref}} = 1000$ mbar (the over-bar represents a horizontally averaged value) is the reference pressure on the ground, $\kappa = R/c_p$, $R = R^*/M$, and $R^*$ is the universal gas constant. The governing equations (2.1) – (2.5) applied in the AGE-TIP model have been used in previous applications in Hickey et al. [2000, 2003], Hickey [2001], and Hickey and Yu [2005].

The nominal eddy diffusion coefficients are based on a profile due to Strobel [1989] and have large values in the mesopause region. The eddy momentum diffusivity maximizes with a value of 100 m$^2$ s$^{-1}$ at 90 km altitude, and the Prandtl number is 3. This maximum value for the eddy diffusivity is comparable to values derived from radar observations of Hocking [1987]. A small value of eddy diffusivity (0.1 m$^2$ s$^{-1}$) is used for the lower atmosphere. Molecular diffusion coefficients are taken from Rees [1989]. Rayleigh friction and Newtonian cooling provide artificial sponge layers near the upper boundary to simulate the radiation conditions. They have large effects near the upper boundary and exponentially decrease with lower altitudes away from the upper boundary. Relevant parameters can also be found in Walterscheid and Schubert [1990], Hickey et al. [2000], and Hickey and Yu [2005].

**Viscosity**

In a dissipative atmosphere, we apply molecular process including molecular kinematic viscosity $\nu$, which is expressed numerically as [Hickey and Yu, 2005]
\[ \nu = \frac{[N_2] \times 3.43 + [O_2] \times 4.03 + [O] \times 3.90}{\rho_0 \times ([N_2] + [O_2] + [O])} \times (T_0)^{0.69} \times 10^{-7}, \tag{2.6} \]

and it is plotted in Figure 1 as a function of altitude. Brackets \( [ \) in the equation denote the density of the major or minor species and they are also functions of altitude. \( T_0 \) is the atmospheric mean temperature, a function of altitude too. Another molecular process is involving the molecular thermal conductivity that is also expressed numerically as [Hickey and Yu, 2005]

\[ \lambda_m = \frac{[N_2] \times 56.0 + [O_2] \times 56.0 + [O] \times 75.90}{([N_2] + [O_2] + [O])} \times (T_0)^{0.69} \times 10^{-5}, \tag{2.7} \]

and it is plotted in Figure 2 as a function of altitude. Eddy diffusion process involves the eddy momentum diffusivity that is expressed numerically as [Hickey and Yu, 2005]

\[ \eta_e = \frac{2 \times 100}{\left( \exp(2.6 \times (z(km) - 90.0)/20) + \exp(-2.6 \times (z(km) - 90.0)/20) \right)} + 0.1, \tag{2.8} \]

and it is plotted in Figure 2 as a function of altitude. Another eddy diffusion process is involving the eddy thermal diffusivity \( \kappa_e = \eta_e/3 \), where we consider the Prandtl number as 3 and the plot for \( \kappa_e \) is neglected because of its similarity to \( \eta_e \).
Figure 1: The atmospheric mean density $\rho$ (kg m$^{-3}$) and the kinematic viscosity $\nu$ (m$^2$ s$^{-1}$).
Figure 2: The eddy momentum diffusivity $\eta_e$ (m$^2$ s$^{-1}$) and the molecular thermal conductivity $\lambda_m/\rho c_v$ (m$^2$ s$^{-1}$).

**Algorithm**

A time-splitting technique is used to integrate the finite difference equations that are derived from the model equations (2.1 – 2.5). Using an explicit second-order Lax-Wendroff scheme, the first-half integration is implemented in the convective part of the equations. The second-half integration is performed iteratively in the remainder part of the equations using an implicit Newton-Raphson scheme. The vertical momentum equation and the thermodynamic energy equation use both schemes, but the mass continuity equation and the horizontal momentum equation use the second-order Lax-Wendroff scheme only. The primary wave
variables are evaluated using a staggered grid technique similar to that demonstrated by Taylor [1984], by which the density and the pressure (or the temperature) are carried out at the center of a computational unit box, while the horizontal and vertical mass flux terms, $\rho u$ and $\rho w$, are computed at the midpoints of the lateral and top-bottom boundaries of a computational unit box, respectively. The application described above is similar to that described in Walterscheid and Schubert [1990].

The time-splitting technique employed here to solve the Navier-Stokes equations is described as follows. First we substitute equation (2.4) in (2.3), and solve equation (2.5) for $T$ and substitute it in (2.3). Then we regroup the governing equations (2.1) – (2.3) as (1) the mass continuity equation, (2) the thermodynamic energy equation, (3) the vertical momentum equation, (4) the horizontal momentum equation. Each equation numbered is splitted into two parts as the form as

\[
\frac{\partial \psi}{\partial t} = f_i(\psi) + f_e(\psi),
\]

(2.9)

where $f_i$ represents for those terms evaluated implicitly by the Newton-Raphson scheme, and $f_e$ represents for those terms evaluated explicitly by the Lax-Wendroff scheme. The $f_e$ terms of each equation are rewritten in the flux vector form as

\[
\frac{\partial \bar{\psi}}{\partial t} + \frac{\partial \bar{F}}{\partial x} + \frac{\partial \bar{G}}{\partial z} + \bar{S} = 0,
\]

(2.10)

where $\bar{\psi} = \{\rho, p, \rho w, \rho u\}$, $\bar{F} = \{\rho u, pu, \rho wu, \rho uu\}$, $\bar{G} = \{\rho w, pw, \rho ww, \rho uw\}$, and $\bar{S}$ terms are the remainder terms that depend on each numbered individual equation. The time integration is implemented in two steps, first the explicit one (Lax-Wendroff), then the implicit one (Newton-
Raphson). The lower boundary condition is a solid surface, the upper boundary condition is a sponge layer to simulate the radiation conditions, and the periodic boundary conditions are applied on the lateral boundaries. The horizontal and vertical grid spacings are 0.5 km and 1.0 km, respectively. The time step used is 0.7 sec.
CHAPTER THREE: THERMAL DUCT

Summary 1

In order to solve the Navier-Stokes equations in two spatial dimensions, the AGE-TIP model, a time-dependent and fully nonlinear numerical model, is employed here to describe the propagation of a Gaussian gravity wave packet generated in the troposphere. One dimensional Fourier spectral analysis is used to analyze the frequency power spectra of the wave packet, which propagates through and dwells within several thermal ducting regions. The frequency power spectra of the wave packet are derived at several discrete altitudes, which allow us to determine the evolution of the wave packet. The spectral analysis used also clearly reveals the existence of a stratospheric duct, a mesospheric and lower thermospheric duct, and a duct lying between the tropopause and the lower thermosphere. In addition, we determine the spatially localized wave kinetic energy density and the horizontally averaged, time-resolved, normalized vertical velocity. Examination of these diagnostic wave variables allows us to better understand the process of wave ducting and the vertical transport of wave energy among multiple thermal ducts. The spectral analysis allows us to unambiguously identify the ducted wave modes. These results compare favorably with those derived from a full-wave model.

Introduction

Theoretical and numerical studies have shown that atmospheric gravity waves (AGWs) can be ducted or trapped by the vertical variation of atmospheric temperature and winds
A large number of observations have also provided a better understanding of the ducting processes [Hines and Tarasick, 1994; Taylor et al., 1995a, 1995b; Isler et al., 1997; Hecht et al., 1997; Walterscheid et al., 1999]. The airglow imager measurements of Walterscheid et al. [1999] were interpreted as being due to quasi-monochromatic (QM) waves that were ducted or trapped in the lower thermospheric thermal duct, or between the ground and the evanescent layer above the duct. Subsequently, Hecht et al. [2001] observed periodic coherent wave structures propagating horizontally across the field of view of their airglow images (about 200 km) and reasoned that they were QM waves. The observed QM waves typically had horizontal phase speeds less than 100 m/s, horizontal wavelengths on an order of tens of kilometers, and periods of several minutes.

In the absence of ducting only freely propagating gravity waves would be observable as they propagate obliquely upward through airglow regions. In this case the waves could be observed in a fairly close proximity (about 100 – 200 km horizontally) to their source region in the troposphere. Walterscheid et al. [1999] argued that the QM waves seen in the airglow images over Adelaide, Australia (35° S, 138° E) might be ducted or trapped because there were no local sources that would have generated freely propagating waves. It was suggested that the wave source was most probably remote (several thousand kilometers away) and located over the northern Australian coast where intense convective activity often occurs. The possibility for such an explanation was explored using a full-wave model [Hickey, 1988a, 1988b; Hickey et al., 1997, 1998] that confirmed the existence of a lower thermospheric thermal duct lying between the
mesopause and an altitude of about 140 km. Moreover, the ducting was believed to be quite different from the Doppler ducts discussed by Isler et al. [1997].

In the past several modeling studies of gravity wave ducting have been presented by different authors. For example, Fritts and Yuan [1989] provided solutions to the 1-D Taylor-Goldstein equation to study waves ducted in thermal and Doppler ducts. Walterscheid et al. [2001] simulated the propagation of waves from a tropical convective storm and their subsequent ducting in a thermal duct using a 2-D cylindrical coordinate system. Most recently, Snively [2003] used Gaussian wave packets in a 2-D model to simulate nonlinear wave breaking in the far-field lower thermospheric thermal duct. Inspired by these previous studies, we perform simulations using the time-dependent, nonlinear, 2-D AGE-TIP model to better understand the ducting processes. To this end we specifically analyze the spatially localized wave kinetic energy density and the horizontally averaged, time-resolved, normalized vertical velocity. In addition, we perform a spectral analysis at various discrete altitudes to help identify those waves in the packet that are selectively ducted. In so doing we will learn more about the QM waves discussed by Walterscheid et al. [1999]. The AGE-TIP model clearly demonstrates the duct characteristics that include the evolution of the ducting, the coupling between ducts, and their persistency. The model is configured in a horizontally infinite domain to facilitate comparisons with a full-wave model [Hickey, 2001] and with the full-wave model results presented by Hecht et al. [2001].

Theory

Mean winds affect wave ducting by altering the intrinsic wave periods (and therefore the vertical wave numbers) and causing them to vary with height in the real atmosphere. In this chapter we have chosen to deliberately exclude the effects of mean winds so that we can focus
on the thermal ducting process alone. The neglect of mean winds also allows us to determine unambiguous frequency spectra at various heights. In a windless atmosphere the altitude variation of the Brunt-Väisälä frequency is responsible for the wave thermal ducting. Hines [1960] formulated a dispersion relation that can be solved for the square of the vertical wave number ($m^2$) as

$$
m^2 = \frac{(N^2 - \omega^2)}{\omega^2} k^2 + \frac{(\omega^2 - \omega_a^2)}{C^2},
$$

where $\omega$ is the extrinsic frequency observed on the ground, $C$ is the sound speed, $N$ is the Brunt-Väisälä frequency, $\omega_a$ is the acoustic-cutoff frequency, and $k$ is the horizontal wave number. An atmospheric gravity wave thermal duct could exist when two evanescent regions ($m^2 < 0$) sandwich an internal region ($m^2 > 0$), which from Equation (3.1.1) implies that the thermal duct is located in the vicinity of a region of a local maximum of $N$. Ducting also requires that a standing wave fits within the internal region ($m^2 > 0$) between the duct upper and lower boundaries with a half-integer number of local vertical wavelengths. Another condition required for strong ducting is that the evanescent regions below and above the duct are thick enough to efficiently reflect the ducted waves. Therefore, only certain combinations of wave periods and horizontal wavelengths favor strong ducting in atmospheric thermal ducts.

Although Equation (3.1.1) is based essentially on the linear gravity wave theory, according to Zhang et al. [2000] it can still be feasible in the nonlinear circumstances they considered. Fritts [1984] provided a measure of the importance of nonlinearity by considering the ratio of the horizontal perturbation velocity ($u'$) to the horizontal phase speed ($c$) of the wave. Waves for which $|u'| << c$ are linear, while waves for which $|u'| \sim c$ are nonlinear. Since most ducted waves are shorter period waves and much of the wave energy resides in high-frequency,
fast wave modes, so if the wave amplitude is kept relatively small the nonlinearity should not be an issue. Slower wave modes would be viscously damped at higher altitudes in the thermosphere. Our AGE-TIP model is a nonlinear model, and so it accounts for such effects without ever using a dispersion relation. In the present study, we use the dispersion relation only to estimate altitudes of evanescence most of which lie below the thermosphere.

Results

In Figure 3, we plot the Brunt-Väisälä frequency as a function of altitude. Also shown is the square of the vertical wave number, $m^2$, calculated for the primary period of 6.276 minutes and the horizontal wavelength of 35 kilometers. The atmospheric mean temperature and neutral density are defined with the MSIS-E-90 model [Hedin, 1991] for a date of 1993 Jan 15, a local time of 2200 hours, and a latitude and longitude of 18.5 deg. N and 0.0 deg., respectively. The Brunt-Väisälä frequency $N$ can be derived and it is defined by Fritts [1984] as

$$N^2(z) = \frac{g}{T} \left( \frac{\partial T}{\partial z} + \frac{g}{c_p} \right). \quad (3.1.2)$$

A previous simulation using the full-wave model [Hecht et al., 2001] to describe a wave generated in the lower troposphere has been used as a basis for the numerical experiments performed here. A wave thermal excitation is chosen explicitly to have a primary period of 6.276 min and a horizontal wavelength of 35 km. The results from the full-wave model indicate that there is a strong lower thermospheric thermal duct existing with this chosen wave mode. There are four wave ducting modes shown in Figure 3, depicting the stratospheric duct (period 5.07 min), the mesospheric duct (period 6.20 min), the lower thermospheric duct (also a period of 6.20 min), and the vertically extended duct (period 7.06 min). According to the $m^2$ plot derived
from Equation (3.1.1), the stratospheric duct is estimated to lie between altitudes of about 15 and 40 km, the mesospheric duct lies between altitudes of about 50 and 90 km, the lower thermospheric duct lies between altitudes of about 90 and 140 km, and the vertically extended duct lies between altitudes of about 12 and 150 km. These ducts will be discussed in more detail later.

Based on our previous experiences of wave ducting in the lower thermospheric thermal duct [Walterscheid et al., 1999; Hecht et al., 2001] we use the full-wave model to identify those wave parameters that will most likely lead to strong wave trapping in the thermal ducts of the mesosphere and lower thermosphere. Therefore, the AGE-TIP model shares the same wave source parameters and the same geophysical parameters as those used in the full-wave model (location, local time, atmospheric thermal structure, etc.). This also helps facilitate a comparison between the simulations of the two models. The wave thermal excitation varies sinusoidally and periodically over a horizontal wavelength, and periodic boundary conditions at the lateral boundaries imply an infinite wave train in the horizontal direction. The prescribed source is a Gaussian envelop over altitude of half-width $\Delta z = 0.8$ km, centered at altitude $\xi = 8$ km, and a Gaussian envelop over time of half-width $\Delta t = 6.276$ min, centered at time $\tau = 37.656$ min, and with an amplitude of $10^{-5}$ Ks$^{-1}$. It is described analytically as

$$Q_w(x,z,t) = 10^{-5} \exp\left(-\frac{(t-\tau)^2}{2\Delta t^2}\right) \exp\left(-\frac{(z-\xi)^2}{2\Delta z^2}\right) \sin(k_0 x - \omega_0 t), \quad (3.1.3)$$

where $\omega_0 = 2\pi/6.276$ min and $k_0 = 2\pi/35$ km.

The thermal excitation at a fixed position of $x = 17.25$ km and $z = 8$ km is plotted as a function of time in Figure 4. In the same figure we also plot the vertical velocity at the same position as a function of time. From the figure we see that the vertical velocity and the thermal
excitation both have periods close to the primary forcing period of 6.276 min and are centered at about 39.08 min. They share an amplitude envelop over time and have an inphase variation. The phase of the vertical velocity accords with the phase of the thermal excitation (keeping paces with each other). After the first hour of the simulation there is still a residual vertical velocity oscillation at altitude 8 km.

In Figure 5 we compare the vertical velocity \( w' \) derived from the AGE-TIP model with that derived from the full-wave model. Because the full-wave model is a steady-state model we show the amplitude and the phase of \( w' \). There is a general agreement between the two model simulations. A slight difference between the two sets of model results occurs in the thermosphere. This is largely due to the time-dependency of the wave packet simulated in the AGE-TIP model. The wave packet never reaches a steady-state and myriad frequency components exist at altitudes where the waves are freely propagating. In contrast to these the full-wave model solutions are steady-state solutions with only one wave frequency present. The overall agreement between the two models suggests that the AGE-TIP model is realistically simulating the propagation of a wave packet from the lower to the upper atmosphere.

In Figure 6 we plot the vertical velocity as a function of time at three discrete altitudes of 110, 130, and 150 km, respectively. At all altitudes considered residual oscillations are evident after the main wave packet has propagated through those regions. For times between 150 – 350 min, harmonic oscillations are seen at discrete altitudes of 110, 130, and 150 km. These harmonic oscillations are now spectrally analyzed to determine if they are ducted waves.

A Fourier analysis is performed that is inspired by that used by Alexander [1996], but here it is in one dimension only and applies to the wave frequency \( \omega \). We apply a step function to the other dimension for the horizontal wave number \( k \) to selectively choose \( \lambda_h = 35 \) km. The
normalized power spectral density (units in fractions of the sum of total power spectra present at each altitude considered), which results from a Discrete Fourier Transform (DFT) of a time series of the vertical velocity, is shown in Figures 7 and 8 for altitudes of 8, 30, 80, 100, 110, 130, and 150 km. The normalized power spectral density of the wave thermal forcing is also shown. The spectra are calculated using a sample of 5 seconds and over 6 hours of the simulation. In Figure 7 the three large peaks seen in the spectra at periods of 7.06 min, 6.20 min, and 5.07 min are identified as wave ducting modes. All three frequencies are seen to exist at 30 km altitude (Figure 7). The 5.07 min spectral peak has a spectral amplitude of about 25.7%, the 7.06 min spectral peak has a spectral amplitude of about 17.7%, and the 6.20 min spectral peak has a spectral amplitude of about 4.4%. The same spectral analysis is also applied to the vertical velocities at altitudes of 20 km and 40 km and it results in similar spectra (not shown) to that shown for the altitude of 30 km. These three wave modes exist clearly at altitudes of 20, 30, and 40 km. These results imply that the wave with a period of 5.07 min is ducted in the stratospheric duct, and another wave with a period of 7.06 min is ducted between the lower thermosphere and the tropopause. Because of its much smaller relative spectral power (only 4.4%) compared to the other two waves (25.7% and 17.7%), we believe that the wave with a period of 6.20 min is not trapped in the stratosphere and is instead freely propagating through this region of the atmosphere. A more detailed analysis of the 6.20 min wave with respect to stratospheric ducting is provided in the discussion section. A schematic diagram elucidating the wave ducting modes is shown in Figure 3.

The ducting mode with a period of 6.20 min is seen to be trapped in the mesosphere and lower thermosphere (MLT) region. It clearly resides at altitudes of 80, 110, and 130 km (shown in Figures 7 and 8), and has normalized spectral amplitudes at each of these altitudes of
approximate 37.4%, 29.1%, and 21.1%, respectively. A similar spectral analysis that is applied to
the vertical velocities at altitudes of 60, 70, 90, 120, and 140 km results in similar spectra (not
shown) to those derived at altitudes of 80, 110, and 130 km. The ducting mode with a period of
5.07 min is not evident at altitudes of 60 km and above. The fact that of all the altitudes
considered the 5.07 min wave is evident (through the spectral analysis) only at altitudes of 20,
30, and 40 km supports our belief that this wave is efficiently trapped and ducted in the
stratosphere. The ducting mode with a period of 7.06 min appears at all altitudes considered,
although some of them are clearly distinguishable (such as the one at 30 km altitude) while
others are weak and dominated by other wave modes. The plausibility that this ducting wave
mode resides in the duct between the lower thermosphere and the tropopause is supported by the
interpretation of Figure 3.

Although the 6.20 min wave mode appears at all altitudes considered its normalized
spectral amplitude has prominent peaks only in two discrete altitude regions. One of these is a
thin region centered near 75 km altitude, while the other is a broader region lying between about
90 and 140 km altitude. Inspection of Figure 3 shows that these regions correspond to the regions
of a local maximum in the Brunt-Väisälä frequency $N$. Therefore, this wave mode appears to be
ducted in these two discrete ducts, one is in the mesosphere and the other one is in the lower
thermosphere. They are separated by a thin region near 90 km altitude that corresponds to a local
minimum in $N$. The refractive index, $m^2$, becomes negative for this wave in the thin region
centered near 90 km altitude. The ducting wave mode is now further examined by considering
other wave diagnostic variables such as the wave kinetic energy density and the normalized
vertical velocity. The vertical energy flux is another wave diagnostic variable that will be
discussed in the second half of this chapter.
The square root of the wave kinetic energy density, \( (u'^2 + w'^2) \rho / 2 \) of the wave packet is shown across the spatial grid at times of about 94 min and about 191 min in Figures 9 and 10, respectively. It exhibits two maxima within a horizontal wavelength as expected for a second order quantity. By 94 min (Figure 9) the wave packet has reached the lower thermosphere. The wave kinetic energy appears to be concentrated in two separate regions near altitudes of 70 km and 130 km. A smaller amount of wave kinetic energy is seen near 20 km altitude. After about 3 hours (Figure 10) the wave kinetic energy is seen to be concentrated in two separate regions near 20 km and 75 km altitude. A smaller amount of wave kinetic energy resides near 130 km altitude in the lower thermosphere. Note that the scaling in Figures 9 and 10 is not identical.

The horizontally averaged wave vertical velocity normalized by the square root of the ratio of the densities \( \left( \frac{\rho}{\rho_0} \right)^{1/2} w' \), similar to the one used by Snively and Pasko [2003], is shown as a function of altitude and time for the 3rd hour of the simulation in Figure 11. Here, \( \rho \) is the atmospheric neutral density at altitude \( z \), and \( \rho_0 \) is the atmospheric neutral density on the ground (\( z = 0 \)). The wave packet is restricted to altitudes below about 150 km and standing waves can be clearly identified between altitudes of about 100 km and 120 km and for times between about 120 min and 180 min. Standing waves are also evident during this time at altitudes between about 20 km and 40 km. In Figure 12 we show the horizontally averaged wave vertical velocity normalized by the square root of the ratio of the densities \( \left( \frac{\rho}{\rho_0} \right)^{1/2} w' \) as a function of altitude and time for the 5th hour of the simulation. The fluctuations are concentrated in two different regions centered near altitudes of 30 km and 110 km. Note that the scaling in Figures 11 and 12 is not identical.
Figure 3: The Brunt frequency $N$ (units in rad s$^{-1}$, green line, lower x-axis) and the $m^2$ profile (units in m$^{-2}$, magenta line, upper x-axis) of the primary wave (period 6.276 min, $\lambda_h = 35$ km). The three vertical lines (red, dot-dot) identify waves of period 7.06, 6.20, and 5.07 min. Three pairs of blue arrows identify three individual ducts (the middle one is the mesosphere and lower thermosphere (MLT) duct). The vertical line (red, dash-dot) signifies $m = 0$ at a period of 6.276 min.
Figure 4: Wave thermal excitation and its resultant vertical velocity at a fixed position of $x = 17.25$ km and $z = 8$ km.
Figure 5: An overall comparison of the vertical velocities between the full-wave (1-D steady state) model and the AGE-TIP (2-D time-resolved) model.
Figure 6: The vertical velocities at discrete altitudes of 110, 130, and 150 km.
Figure 7: Spectral analysis at discrete altitudes of 8, 30, and 80 km.
Figure 8: Spectral analysis at discrete altitudes of 100, 110, 130, and 150 km.
Figure 9: The square root of the wave kinetic energy density, \((u'^2 + w'^2) \rho/2\) of the wave packet is shown across the spatial grid at a time of about 94 minutes.
Figure 10: The square root of the wave kinetic energy density, \((u'^2 + w'^2)\rho/2\) of the wave packet is shown across the spatial grid at a time of about 191 minutes.
Figure 11: The horizontally averaged wave vertical velocity normalized by the square root of the ratio of the densities \( \left( \frac{\rho}{\rho_0} \right)^{1/2} w' \) is shown as a function of altitude and time for the 3rd hour of the simulation.
Figure 12: The horizontally averaged wave vertical velocity normalized by the square root of the ratio of the densities \( \left( \frac{\rho}{\rho_0} \right)^{1/2} w' \) is shown as a function of altitude and time for the 5th hour of the simulation.

**Discussion**

As a consequence of periodic boundary conditions imposed at the lateral boundaries our model domain is essentially of infinite extent in the horizontal direction. Also the range of any possible horizontal wave numbers is restricted because we only prescribe one horizontal wavelength of 35 kilometers in our horizontal periodic domain. To some extent this restricts our analysis of the horizontal range of the ducted waves because the model precludes direct
observation of the horizontal group velocity. However, an estimate of the horizontal range can be made by considering the horizontal group velocity of the ducted wave based on the isothermal dispersion equation. A calculation of the horizontal group velocity from $d\omega/dk$ gives a most realistic value of about 23.5 m/s at 75 km altitude, suggesting that there would be a long range propagation (about 500 km over 6 hrs). The observations of Hecht et al. [2001] support our expectation of a long range propagation for the ducted waves.

Here we provide some discussion on the reason why the 6.2 min wave is imperfectly ducted in the stratospheric duct. According to the results of the spectral analysis described in the Results section, at stratospheric altitudes the 6.2 min wave was far weaker than the 5.07 min wave. We attribute this difference to the different vertical structures of the two waves. In particular, for the 6.2 min wave we find that an integer number of half vertical wavelengths do not fit into the stratospheric duct, whereas the converse is true for the 5.07 min wave. We can provide a rough estimate of the vertical wavelength $\lambda_z$ for the 6.2 min wave by calculating the $m^2$ at about 27.5 km altitude (close to the midpoint of the stratospheric duct, as shown in Figure 3), where a local minimum of the $m^2$ occurs. We obtain a value for the $m^2$ of about $2.17 \times 10^{-8}$ (units in $m^{-2}$), which results in a value of $\lambda_z = 42.65$ km and a half vertical wavelength of 21.32 km. In Figure 3, the vertical distance between the stratospheric duct boundaries for the 6.2 min wave is about 37 km (from about 15 km to 52 km altitude). Clearly, an integer number of half vertical wavelengths (21.32 km) will not fit in a duct of depth about 37 km, and so the 6.2 min wave is not efficiently trapped in the stratospheric duct. A similar analysis (not shown) indicates that the 5.07 min wave does fit well in the stratospheric duct, and so it is efficiently trapped there. We note that in the simulations of the second half of this chapter, although the 6.2 min wave isn’t efficiently trapped in the stratospheric duct, it later descends from the mesospheric duct into the
stratospheric duct, and eventually it propagates back to the mesospheric duct. The tunneling of this 6.2 min wave through the thin evanescent region near 50 km altitude is also seen in the simulations of the second half of this chapter. Similar wave coupling was described by Fritts and Yuan [1989], who used the Taylor-Goldstein equation for the vertical velocity and a WKB approximation for the vertical wave number to provide a detailed analysis of the ducted wave modes in thermal and Doppler ducts.

Some physical processes have been excluded from our analysis though. One is ion drag that is important for longer period gravity waves, but for high frequency waves considered here it will be unimportant [Hines, 1968]. Francis [1973] also found that the effects of ion drag were nowhere particularly large for the ducted waves that usually are shorter period gravity waves. Furthermore, ion drag usually maximizes at F-region altitudes (from about 250 km to 300 km altitude), which are far higher than the altitudes we are interested in here. In addition, for high frequency gravity waves (\(\omega >> \Omega_E\), where \(\Omega_E\) is the angular frequency of the Earth) the effects of the Earth’s rotation can be safely neglected [e.g., Hickey and Cole, 1987]. Modifications to the dispersion equation including the Coriolis force and ion drag are given by Volland [1969], Francis [1973], and Hickey and Cole [1987].

Because the amplitudes of gravity waves increase with altitude as they propagate upward in a dissipationless atmosphere, they may achieve nonlinear amplitudes and break [e.g., Fritts, 1984]. At altitudes below the breaking height the amplitudes may be large enough to invalidate the dispersion relation (Equation (3.1.1)) that is based on the linear gravity wave theory. A measure of the importance of nonlinearity is the ratio of the horizontal perturbation velocity to the horizontal phase speed of the wave [e.g., Fritts, 1984]. In Figure 6, if we assume that the horizontal perturbation velocity is on an order of the vertical velocity (about 0.8 m/s above 100
km altitude), the horizontal phase speed of the primary wave is about 92.95 m/s, and so the relevant ratio is about 0.0086 (considerably small). Because in our study much of the wave energy resides in high frequency, fast modes, nonlinear effects in the dispersion relation should never be an issue. Also Zhang et al. [2000] have demonstrated that the dispersion relation based on the linear gravity wave theory is applicable even for the nonlinear cases they considered.

Another process not considered here but which could possibly influence the applicability of Equation (3.1.1) is the effect of atmospheric baroclinicity [Jones, 2005, 2006]. Often the dispersion relation so derived by Jones [2005, 2006] ignores acoustic effects by setting the $1/C^2$ terms to zero, but these are important (a first order) for the high frequency waves ($\omega \leq N$) we are studying [Hickey, 2001]. In addition, in relation to his equations (29) and (31) Jones [2006] has concluded that “For practical purposes, we could probably neglect all of the terms except the first term”. This implies that for high frequency waves the baroclinicity is, in fact, of lesser importance. We also note that Equation (3.1.1) applies only to an isothermal atmosphere, but nonisothermal effects could only possibly become largest in the lower thermosphere. In any event, we use the dispersion relation to approximately delineate regions of propagation from regions of evanescence, a calculation that is made independent of our numerical AGE-TIP model. We should also mention that our full-wave and AGE-TIP models do not rely on the use of the WKB approximation. For fast acoustic-gravity waves of large vertical wavelengths dispersion relations like Equation (3.1.1) that are based on using the WKB approximation are likely to be suspect [Einaudi and Hines, 1971].

The thermosphere is diffusively separated and behaves as a multi-constituent gas where individual species in static equilibrium are each stratified according to their individual scale heights. In contrast the atmospheric region below the thermosphere is considered well mixed and
behaves like a single constituent gas with a constant molecular weight up to the turbopause near 105 km altitude. Gravity waves propagating in the thermosphere drive gases out of static equilibrium causing individual gases to oscillate with different amplitudes and phases. Mutual diffusion attempts to mitigate these differences and restore diffusive equilibrium. In the lower thermosphere where mutual diffusion occurs on timescales long compared to typical gravity wave periods, amplitude and phase differences between fluctuating species can be large [Del Genio et al., 1979]. The composition and specific heats of the total gas can be significantly perturbed, and the parcel buoyancy can be significantly affected [Walterscheid and Hickey, 2001]. These effects are beyond the scope of our present study, but we note that they may be important for the lower thermospheric ducted wave modes discussed here.

Because the wave source in our model is a Gaussian function of time, the wave packet generated never reaches a steady-state and hence fully ducted wave modes cannot be achieved. A simulation of fully ducted wave modes could be possibly performed with the use of a source that reaches constant amplitude for a sufficient length of time to approach a steady-state in the whole atmosphere. We have neglected variations that occur in the atmosphere in addition to gravity waves, such as those due to tides and planetary waves [e.g., Forbes et al., 2002]. They are responsible for height-dependent temperature and winds that vary with time. We believe that the basic atmospheric structure can plausibly support ducted wave modes, and the frequent observations of ducted waves that propagate coherently across the field of view of all-sky airglow images [e.g., Hecht et al., 2001] support that belief. They indicate that wave ducting occurs in spite of other variations that are also occurring. Mean winds should also be included in our analysis because our previous full-wave model simulations [Hecht et al., 2001] have shown
that winds will modify the ducting processes. We plan to examine wind effects on wave ducting using our AGE-TIP model in the next chapter.

Conclusion

Simulations of acoustic-gravity wave propagation in a non-isothermal, dissipative, and initially motionless atmosphere have been performed using a time-dependent, 2-D, non-hydrostatic gravity wave model (the AGE-TIP model). A Gaussian thermal excitation was applied in the troposphere and produced a wave packet that propagated upward. We found that different frequency components of the wave packet were thermally ducted at different altitudes. A spectral analysis applied at different altitudes has successfully identified three individual wave modes trapped in the stratospheric duct, the MLT duct, and the vertically extended duct lying between the tropopause and the lower thermosphere, respectively. A thin region of evanescence near 90 km altitude partitions the MLT duct into a lower duct (the mesospheric duct) and an upper duct (the lower thermospheric duct).

The periodic coherent QM wave structures that propagated horizontally across airglow images were interpreted by Hecht et al. [2001] as gravity waves ducted in the lower thermospheric thermal duct. Our current simulations suggest a possible alternative interpretation wherein the mesospheric duct dominates at airglow altitudes. In that case the lower thermospheric thermal duct, which is centered near 130 km altitude, would play a secondary role in airglow variations associated with ducted gravity waves. However, we note that waves with different combinations of horizontal wavelengths and periods may be predominantly ducted in the lower thermospheric thermal duct, and so a possible ambiguity may exist in the interpretation of some inferences of ducted waves from airglow observations. Note that the lifetime of ducted
waves in the mesospheric and lower thermospheric thermal duct certainly suggests that such waves could travel a long horizontal distance. By considering the horizontal group velocity of the wave packet at these altitudes it is quite realistic to estimate that the waves could be ducted over a horizontal distance of about 500 kilometers.

Because of the special shape of the thermal structure in the upper and lower atmosphere, a mesospheric plus a lower thermospheric thermal duct, a stratospheric thermal duct, and a vertically extended thermal duct lying between the lower thermosphere and the tropopause (or the ground) must have existed. Some QM waves and only these certain QM waves are determined to be ducted in those regions, even though the propagating wave packet comprises a full spectrum of myriad continuous frequencies. These individual waves will be sorted out and filtered (becoming QM waves) by the thermal structure of the atmosphere. Under certain circumstances, the QM waves containing the resonant frequency (with which their vertical wavelengths satisfy the ducting conditions) could be ducted and trapped in the lower thermospheric thermal duct and/or other thermal ducts. They could be responsible for the transport of wave-associated energy and momentum over large horizontal distances of several hundreds of kilometers.

**Summary 2**

A new 2-D time-dependent model is used to simulate the propagation of an acoustic-gravity wave packet in the atmosphere. A Gaussian tropospheric heat source is assumed with a forcing period of 6.276 minutes. The atmospheric thermal structure creates three discrete wave ducts in the stratosphere, mesosphere, and lower thermosphere, respectively. The horizontally averaged vertical energy flux is derived over altitude and time in order to examine the time-
resolved ducting. This ducting is characterized by alternating upward and downward energy fluxes within a particular duct, which clearly show the reflections occurring from the duct boundaries. These ducting simulations are the first that resolve the time-dependent vertical energy flux. They suggest that when ducted gravity waves are observed in the mesosphere they may also be observable at greater distances in the stratosphere.

Introduction

Gravity waves propagating upward through the atmosphere are strongly influenced by the mean thermal structure. Thermal ducting may occur in a region of a local maximum in the Brunt-Väisälä frequency, particularly for shorter period gravity waves. In this instance a region of internal wave propagation (the duct) is sandwiched between regions of evanescence. Observations of wave events in the nightglow and theoretical analysis confirm that atmospheric gravity waves may be ducted in the lower thermospheric thermal duct [Tuan and Tadic, 1982; Hines and Tarasick, 1994; Taylor et al., 1995; Isler et al., 1997; Hecht et al., 2001; Hickey, 2001; Walterscheid et al., 1999, 2001; Snively and Pasko, 2003]. In addition to the mean thermal structure, winds may also play a significant role in wave ducting [Isler et al., 1997; Walterscheid et al., 1999; Hecht et al., 2001; Hickey, 2001].

Walterscheid et al. [1999] interpreted quasi-monochromatic (QM) waves seen in airglow images as ducted or trapped waves in the lower thermospheric thermal duct. Coherent periodic structures of observed QM waves were shown by Hecht et al. [2001] to propagate horizontally across airglow images. Those QM waves typically had horizontal wavelengths of tens of kilometers and periods of several minutes. A numerical simulation of ducted waves requires information regarding the directions of wave propagation, the wave intrinsic periods, and their
horizontal wavelengths. The intrinsic wave period is a wave parameter that varies with height in the real atmosphere due to the effects of mean winds. Because these mean winds affect wave ducting, we have chosen to deliberately exclude their effects so that we can focus on the thermal ducting alone.

In the past several authors have modeled gravity wave ducting. Fritts and Yuan [1989] studied wave ducting with 1-D Taylor-Goldstein solutions. Walterscheid et al. [2001] simulated thermal ducting above a convective storm with a system of 2-D cylindrical coordinates. Hickey [2001] simulated steady-state gravity wave ducting and the airglow response to the wave using a full-wave model. Snively [2003] used a 2-D model to simulate ducted waves in the far-field lower thermospheric thermal duct excited by linear tropospheric forcing, and also by nonlinear breaking of tropospherically generated waves [Snively and Pasko, 2003]. Although inspired by previous studies, the present study uses a distinctive, nonlinear 2-D model (the AGE-TIP model) to elucidate the wave ducting by examining its vertical energy flux \( p'w' \).

Theory

In a windless atmosphere the dispersion relation given by Hines [1960] can be solved for the vertical wave number squared \( (m^2) \) which is then given by

\[
m^2 = \frac{(N^2 - \omega^2)}{\omega^2} k^2 + \frac{(\omega^2 - \omega_a^2)}{C^2}, \tag{3.2.1}
\]

where \( \omega \) is the observed frequency, \( C \) is the sound speed, \( N \) is the Brunt-Väisälä frequency, \( \omega_a \) is the acoustic-cutoff frequency, and \( k \) is the horizontal wave number. The existence of thermal gradients implies that \( N \) varies with altitude, which introduces the possibility of ducting. A
thermally ducted wave will be internal \((m^2 > 0)\) in the region near a local maximum of \(N\), and become evanescent \((m^2 < 0)\) at some vertical distances either side of this where \(N\) has decreased \([\text{Hines, 1960; Walterscheid et al., 1999; Hecht et al., 2001; Hickey, 2001}]\).

Alternatively, a gravity wave may be ducted between the ground and some higher altitude where the wave becomes locally evanescent \([\text{Tuan and Tadic, 1982}]\). However, while these are necessary conditions for wave ducting, they are not sufficient conditions. Whether or not wave ducting is strong or leaky depends on the distance between the duct boundaries and the vertical structure of the wave. Perfect thermal ducting requires that an integer number of half vertical wavelengths fit exactly in the internal wave region \((m^2 > 0)\), so the vertical energy flux could approach zero at the duct boundaries and strong standing wave behavior will result \([\text{Hickey, 2001; Walterscheid et al., 2001}]\). Another condition required for strong ducting is that the evanescent regions below and above the duct are thick enough to efficiently reflect the ducted waves. Therefore, only certain combinations of wave periods and horizontal wavelengths favor strong ducting in atmospheric thermal ducts.

Results

The current study of thermal ducting is implemented with the 2-D AGE-TIP model. The atmospheric mean state was specified by using the MSIS-E-90 model \([\text{Hedin, 1991}]\) for 1993 Jan 15, 18.5° N latitude and 0.0° longitude, and the local time of 2200 hours. The solar and geomagnetic indices were \(F_{10.7} = F_{10.7A} = 87\) and \(a_p = 12\) for moderately disturbed conditions. Mean winds were excluded from this analysis. A full-wave model was first used to determine the parameters of a strongly ducted gravity wave in the lower thermospheric thermal duct \([\text{Hickey, 2001}]\). This analysis produced a wave period of 6.276 min and a horizontal wavelength of 35.0
km (equally a horizontal phase speed of 92.95 m s\(^{-1}\)). They are used to excite a gravity wave packet thermally forced in the troposphere. The thermal excitation \(Q_w\) (K s\(^{-1}\)) is prescribed by the following equation

\[
Q_w(x,z,t) = 10^{-5} \exp(-(t-\tau)^2/2\Delta t^2) \exp(-(z-\zeta)^2/2\Delta z^2) \sin(k_0 x - \omega_0 t),
\]

(3.2.2)

where \(\Delta z = 0.8\) km, \(\zeta = 8\) km, \(\Delta t = 6.276\) min, and \(\tau = 37.656\) min. The thermal forcing frequency and horizontal wave number are given by \(\omega_0 = 2\pi/6.276\) min and \(k_0 = 2\pi/35\) km, respectively. The magnitude of the excitation is chosen to be small so that the resulting gravity wave amplitudes remain small and linear at all altitudes. Note that this excitation is similar in a form to that of Snively and Pasko [2003], but theirs appears in the vertical momentum equation as a mechanical standing wave oscillator.

The height variation of the Brunt-Väisälä frequency \(N\) is shown in Figure 13. Also shown are three ducting regions and the associated ducted waves for the horizontal wavelength of 35 km. The stratospheric duct is estimated to lie between about 16 and 40 km altitude, and the mesospheric duct is between about 52.5 and 91.5 km altitude. The periods of the ducted wave modes identified in the figure were determined through a spectral analysis of time series of resulting wave fields. The stratospheric duct supports a wave mode of period 5.07 min, the duct between the lower thermosphere and the tropopause supports a wave mode of period 7.06 min, and the mesosphere and lower thermosphere (MLT) duct supports a wave mode of period 6.20 min. Figure 13 also shows the square of the vertical wave number, \(m^2\), calculated for the primary period of 6.276 min. For this wave the \(m^2\) is negative in several regions, which implies that the wave is evanescent there. The wave is evanescent below about 15 km altitude, over a 10 km region centered near 55 km altitude, over a 3 km region centered near 90 km altitude, and above
about 140 km altitude. Therefore, this wave can be ducted in three different internal wave regions lying between these evanescent regions.

The horizontally averaged vertical energy flux $\bar{p}'w'$ is shown as a function of height and time in Figures 14 through 17. Note that the scaling in these figures is not identical. Figures 14 and 15 share the same scaling, while Figures 16 and 17 share a different scaling.

The strongest wave forcing occurs in the first hour of the simulation in the troposphere near 38 min (Figure 14). The maximum upward energy flux is about $5.0 \times 10^{-7}$ W m$^{-2}$. Prior to 38 min the energy flux is positive (upward) above the source, and negative (downward) below the source. Below the source region and for times $38 \text{ min} \leq t \leq 48 \text{ min}$, the direction of the energy flux changes to upward after reflecting from the ground. Above the source the energy flux remains positive for times less than 50 min, while for subsequent times a weaker downward energy flux is evident, which becomes especially noticeable near 1 hr and $z = 30$ km. The downward directed energy flux is a consequence of partial reflections due to evanescence of the high frequency ($\tau$ about 5 min) component of the wave packet, which is associated with the height variation of the $N$ or $m^2$ shown in Figure 13. It is also evident that the upward energy flux remains approximately constant up to about 40 km altitude, after which it decreases due to partial reflections and propagation through the evanescent region. The energy flux that remains directed upward from the source in Figure 14 has a vertical group velocity of about 42.3 m s$^{-1}$. By comparison, using the thermal forcing frequency, the wave numbers and the mean state parameters at 30 km altitude in Equation (3.2.1) yields a vertical group velocity ($\partial \omega / \partial m$) of about 42.4 m s$^{-1}$. 
Figure 15 shows the wave vertical energy flux for the second hour of the simulation. The main wave packet continues propagating upward into the thermosphere with an approximate vertical group velocity of about 32.2 m s\(^{-1}\), which is smaller than the value given above for the lower part of the atmosphere (Figure 14). There are two reasons for this. First, a spectral analysis confirms that some of the higher frequency (and faster) components of the wave packet are reflected in the upper stratosphere, impeding their propagation to greater altitudes. Second, the mean atmospheric stability (\(N\)) is lower in the mesosphere, which reduces the vertical wave numbers of the waves thereby reducing the vertical group velocity. The results shown in Figure 15 also clearly indicate that the downward reflected part of the wave packet tends to remain in the stratospheric duct. At later times (about 70 min) this trapped wave is reflected upward and continues propagating upward until a time of about 90 min. Weak partial reflections of this part of the wave packet subsequently occur, with some upward (and weaker) penetration into the mesosphere at times greater than 90 min. The primary upward propagating wave packet weakens as it approaches the thermosphere, primarily as a consequence of partial reflections occurring in the mesopause region. Wave trapping is evident in the mesosphere centered at about 75 km altitude, with a maximum energy flux (upward) occurring at a time of about 100 min and minima (downward) occurring at times of about 80 and about 120 min. Assuming a vertical group velocity of about 32.2 m s\(^{-1}\) for the wave packet in this region of the atmosphere leads to an estimated duct depth of about 38.6 km. This value compares favorably with the result shown in Figure 13 (about 39.0 km) for this region of the atmosphere.

Figures 16 and 17 show the vertical energy flux plotted on a magnified scale to more clearly reveal the long-term behavior of the wave ducting. The results shown in Figure 16 reveal ducting occurring in the stratosphere (about 30 km altitude) and in the mesosphere (about 75 km
altitude), identified by the alternating direction of the vertical energy flux within each of the ducts. In the mesospheric duct the alternating motion occurs with a period of about 40 min while in the stratospheric duct it occurs with a period of about 20 min for times between about 2 and about 3 hrs (Figure 16) and of about 40 min for times beyond 3 hrs (Figure 17). The results in Figure 16 also show that the thin evanescent region near 90 km altitude (see Figure 13 too) is strong enough to support ducting below (in the mesosphere) and above (in the lower thermosphere). At longer times in the simulation (beyond 3 hrs, Figure 17) most signs of MLT wave ducting have largely disappeared, but stratospheric ducting persists at least out to 4 hrs. The waves ducted in the stratosphere at times between 2 and 3 hrs dissipate and weaken (comparing the energy flux at about 128 min with that at about 165 min). Wave energy in the MLT duct begins descending into the stratospheric duct at about 170 min. At subsequent times the ducting strengthens in the stratospheric duct and the energy flux exhibits a similar characteristic period (about 40 min), as the “precursor” MLT region ducted waves. We believe that during this time period the waves ducted in the stratosphere originated from the overlying MLT region. We base this on two observations. First, the energy flux associated with the waves ducted in the stratosphere displays the same 40 min period variations as those of the mesosphere. Second, the energy flux associated with the waves ducted in the stratosphere strengthens during the fourth hour, which can only occur by energy entering the stratospheric duct. Note, however that after the first three hours in the simulation the slower waves descending from the overlying MLT region superimpose on the faster waves initially trapped in the stratospheric duct. At very long times (longer than 4 hrs) our simulations (not shown) reveal that wave energy leaks upward from the stratospheric duct to the MLT region. However, the energy of these waves is considerably reduced and is very small (about \(10^{-8}\) W m\(^{-2}\)).
Figure 13: The Brunt frequency $N$ (units in rad s$^{-1}$, green line, lower x-axis) and the $m^2$ profile (units in m$^{-2}$, magenta line, upper x-axis) of the primary wave (period 6.276 min, $\lambda_h = 35$ km). The three vertical lines (red, dot-dot) identify waves of period 7.06, 6.20, and 5.07 min. Three pairs of blue arrows identify three individual ducts. The vertical line (red, dash-dot) signifies $m = 0$ at a period of 6.276 min.
Figure 14: Vertical energy flux (W m$^{-2}$) during the 1$^{\text{st}}$ hour of simulation.
Figure 15: Vertical energy flux (W m$^{-2}$) during the 2$^{nd}$ hour of simulation.
Figure 16: Vertical energy flux (W m\(^{-2}\)) during the 3\(^{rd}\) hour of simulation.
Figure 17: Vertical energy flux (W m\(^{-2}\)) during the 4\(^{th}\) hour of simulation.

Discussion

The shorter period waves considered here are all confined to the atmospheric region below about 150 km altitude due to evanescence at greater altitudes and associated reflections. The fast waves reach the thermosphere quickly (taking less than about 90 min), but they suffer partial reflections at all heights along the way. The downward propagating wave energy is subsequently trapped at lower levels where it resides in two main ducts, one in the mesosphere and the other in the stratosphere. The ducting in the stratosphere is the most persistent, with significant energy remaining trapped there for times of about 4 hrs.
In Figure 16 a clear ducting signature was seen between altitudes of about 52.5 km and about 91.5 km identified by the alternating direction of the vertical energy flux. The thin evanescent region near 90 km altitude, identified by the thin horizontal line in the figure, causes partial reflection, while also allowing partial transmission of wave energy. The energy flux is zero in the 90 km region. Although the typical periods of the individual waves within the wave packet are short and of several minutes, the energy flux associated with trapped wave packet motions exhibits much longer periods of about 40 min in the mesosphere (Figure 16), and also in the stratosphere (Figure 17). This behavior can be explained by non-ideal ducting in which cancellation between upward and downward waves is incomplete. It raises the possibility that measurements of low temporal resolution (below the Nyquist frequency) could be misinterpreted in terms of long period motions.

We have focused our attention on the energy flux, but we note that this is not a wave diagnostic variable that can be easily measured. In contrast, the wave-associated momentum flux is commonly estimated from radar and lidar observations of horizontal and vertical velocities, and also from airglow imaging measurements [Gardner et al., 1999]. The momentum flux derived from our simulations (not shown) behaves similar to the vertical energy flux. Our findings will apply equally well to the momentum flux and can, in principle, be observationally confirmed.

Although we have focused on the vertical energy flux it should be recognized that the waves also have an associated horizontal energy flux. Our simulations reveal that the horizontal energy flux (not shown) has a one-to-one correlation with the vertical energy flux, and both are approximately zero at the duct boundaries. For perfectly ducted waves we expect perfect cancellation of the upward and downward energy fluxes so that the vertical energy flux within
such a duct would be zero (but the horizontal energy flux would be non-zero). However, our wave train is too short to fill the entire vertical extent of the atmosphere because our wave source generates a Gaussian wave packet and therefore we never generate fully ducted wave modes. Airglow imagers frequently observe coherent monochromatic structures in the mesopause region that can be followed as they propagate across the field of view (typically about 200 km), which implies that many “bounces” must be occurring (see the discussion in Hecht et al. [2001]). These are usually “far-field” observations, so interference effects associated with freely propagating waves or waves ducted in other atmospheric regions are usually unimportant.

Walterscheid et al. [2001] used a model with a cylindrical symmetry to simulate acoustic-gravity waves generated by a convective source due to the latent heat release from rainfall associated with a Hector event in northern Australia. They also examined the effects of using different source characteristics and found that a quasi-monochromatic (QM) source generated ducted waves that were similar to those of the more realistic Hector forcing. Notable differences included a stronger standing wave behavior and less weakening with radial propagation distances associated with the QM source. Because in our simulations the tropospheric wave forcing was characterized by a monochromatic variation of fixed horizontal wavelength and period modulated by a Gaussian variation in time, the results of Walterscheid et al. [2001] suggest that we may be overestimating the strength of the ducting in our simulations.

Winds have significant effects on gravity wave propagation in the atmosphere but we have neglected them here in order to focus on the thermal ducting. Strong wind shears will alter the details of the ducting by impacting the wave parameters (the wave period and the vertical wave number) that satisfy the ducting criteria. Numerical simulations including the effects of mean winds that include shears have been discussed by Hecht et al. [2001]. We know that
ducting occurs in spite of mean winds because ducted waves are fairly common occurrences in airglow observations. Later we will perform a detailed study including mean winds and shears.

Conclusion

We have used a new time-dependent model to simulate the propagation of a wave packet from the troposphere into the overlying atmosphere. By examining the horizontally averaged vertical energy flux we have elucidated the coupling of wave energy between different atmospheric regions and also revealed the trapping of wave energy in various atmospheric thermal ducts. Although the waves we have studied are all of short period (about 6 min), the energy flux exhibits longer period (about 20 min and about 40 min) behavior associated with the times taken for the waves to traverse between the lower and upper duct boundaries. Ducting persists in the mesospheric duct till about 3 hrs and in the stratospheric duct till about 4 hrs. Eventually (for times longer than 4 hrs) the waves in the stratospheric duct leak upward to the mesosphere, but the relatively small amount of wave energy would likely make the waves unobservable. Our findings should apply equally well to the momentum flux and so can be observationally confirmed. Our simulations suggest that when ducted gravity waves are observed in the mesosphere they may also be observable at greater distances in the stratosphere.
CHAPTER FOUR: 
DOPPLER DUCT

Summary 3

We employ a new nonlinear and time-dependent model to derive the horizontally averaged vertical wave action of two linear gravity wave packets propagating from the troposphere to the lower thermosphere. These two wave packets are excited in the troposphere by a Gaussian heat source varying in space and time and with a primary forcing period of 6.276 minutes and a horizontal wavelength of 35 km. They respectively propagate in an eastward and westward direction and in the presence of a zonal wind. Analysis of the refractive index, the power spectra and the wave action allows us to correctly interpret the propagation characteristics of these two wave packets. We find that the westward propagating waves are largely trapped in the stratosphere. Analysis of the time-resolved and horizontally averaged vertical wave action reveals that, the alternating upward and downward total perturbation energy fluxes are associated with reflections from the stratospheric duct boundaries. The lower frequency components of the eastward propagating waves remain internal and freely propagating at all heights and so propagate unimpeded and quickly through the atmosphere. These lower frequency components do not contribute substantially to the wave spectrum in the upper mesosphere, and their short residence times in this region would probably allow them to go largely undetected. The higher frequency components of the eastward propagating waves remain partially trapped within the stratosphere and also near the upper mesosphere. Because the higher frequency components spend more time in the airglow region, they substantially contribute to the wave power spectrum.
and would also be more likely to be observed by optical techniques. The contribution of the wave momentum flux to the wave action is also described and analyzed.

Introduction

Upward propagating gravity waves in the atmosphere are strongly influenced by the thermal structure [Tuan and Tadic, 1982; Hines and Tarasick, 1994; Walterscheid et al., 1999], and also by the height variation of the background winds [Chimonas and Hines, 1986; Wang and Tuan, 1988; Hecht et al., 2001]. For shorter period gravity waves the possibility of trapping and ducting arises wherein a local region of propagation is sandwiched between two regions of evanescence. Alternatively, ducting can also occur when a region of propagation is sandwiched between the ground and a higher region of evanescence. In the case of thermal ducting the region of propagation occurs in the vicinity of a local maximum in the Brunt-Väisälä frequency. The inclusion of winds with shears will cause a height-dependent Doppler shift of the wave frequency for wave propagation with a component parallel to the winds which may either reinforce or destroy the ducting depending on the properties of the wave and winds [Jones, 1972; Walterscheid et al., 1999; Hecht et al., 2001]. The ducting that is facilitated by mean winds is commonly referred to as Doppler ducting. Observations of waves in the airglow and simulations of waves from analytic and numerical models verify that atmospheric gravity waves may be thermally ducted [Taylor et al., 1995a; Hecht et al., 2001; Hickey, 2001; Walterscheid et al., 1999, 2001; Snively and Pasko, 2003, 2005] and Doppler ducted [Isler et al., 1997; Walterscheid et al., 1999; Hecht et al., 2001; Hickey, 2001].

Isler et al. [1997] examined airglow observations of gravity wave events and with measurements of mean winds they were able to determine that about 75% of the observed waves
were ducted or evanescent and the remainder is freely propagating. Quasi-monochromatic (QM) waves observed in airglow images were interpreted by Walterscheid et al. [1999] as waves ducted or trapped in the lower thermospheric thermal duct. Hecht et al. [2001] observed periodic structures of QM waves propagating horizontally and coherently across airglow images. They typically had horizontal wavelengths on an order of tens of kilometers and periods of about several minutes. More recently, simulations of ducted gravity waves produced antiphase fluctuations of the OH and OI airglow emissions [Snively and Pasko, 2005]. Thermal ducting in the presence of multiple ducts (in the stratosphere, mesosphere, and lower thermosphere) has been described in the modeling studies in the previous chapter.

The objective of this chapter is to study the effects of mean zonal wind on the propagation of an eastward and westward respectively propagating atmospheric gravity wave packet by analysis of the time-resolved, horizontally averaged wave action. A spectral analysis of the wave vertical velocity is performed at discrete altitudes, and the refractive index is used to help quantify and differentiate the freely propagating waves from those that are trapped and ducted. A 2-D nonlinear, time-dependent model (the AGE-TIP model) described in the second chapter is used in these simulations.

Theory

With the inclusion of mean winds the dispersion relation given by Hines [1960] can be solved for the vertical wave number squared ($m^2$) which is then given by

$$m^2 = \frac{(N^2 - \Omega^2)}{\Omega^2} k^2 + \frac{(\Omega^2 - \omega_c^2)}{C^2}. \quad (4.1.1)$$
Here $\Omega = \omega - k \cdot \bar{U}$ is the intrinsic (Doppler shifted) wave frequency, $\omega$ is the extrinsic (observed) wave frequency, $k$ is the horizontal wave number ($k$ represents the horizontal wave number vector), $\bar{U}$ is the horizontal mean wind vector, $C$ is the sound speed, $N$ is the Brunt-Väisälä frequency, and $\omega_a$ is the acoustic-cutoff frequency. We use the definition of the non-isothermal $N$ as extensively discussed by Einaudi and Hines [1971]. It is given by

$$N^2 = g \frac{d \ln \theta}{dz} = (\gamma - 1 + \gamma \frac{dH}{dz}) \frac{g^2}{C^2},$$

where $g$ is the gravitational acceleration, $\gamma$ is the ratio of specific heats, $H$ is the atmospheric scale-height, and $\theta$ is the potential temperature [Einaudi and Hines, 1971; Holton, 1992]. For internal acoustic and gravity waves $m^2 > 0$, while for evanescent waves $m^2 < 0$. Because $\bar{U}$, $C$, $N$, and $\omega_a$ all vary with height in the atmosphere so does $m^2$, and so upward propagating waves can be internal at some heights and evanescent at others. This introduces the possibility of wave trapping and thermal ducting in which an internal wave region is sandwiched between regions of evanescence. For some waves the ground acts as a lower boundary to facilitate ducting [Tuan and Tadic, 1982; Wang and Tuan, 1988]. For high-frequency gravity waves ducting can occur in a region where a local maximum of $N$ occurs (for which $m^2 > 0$), and where above and below this region the decrease of $N$ causes $m^2$ to become negative. Height-dependent winds Doppler shift the waves so that their intrinsic frequencies $\Omega$ vary with height. Waves propagating with a component in a direction opposite to the wind will be Doppler shifted to higher frequencies so that their $m^2$ decreases and the internal waves may become evanescent. Waves propagating with a component in the same direction as the wind will be Doppler shifted to lower frequencies so that their $m^2$ increases and the evanescent waves may become internal. Therefore in the atmosphere the combinations of height-dependent winds and
mean temperature determine wave Doppler ducting [Walterscheid et al., 1999; Hecht et al., 2001; Hickey, 2001].

The existence of a region of internal wave propagation sandwiched between two regions of evanescence is not by itself sufficient for wave ducting, although some trapping of wave energy may still occur. Strong ducting also requires that nodes exist at the duct boundaries, constraining the allowed vertical wavelengths of ducted waves [Hickey, 2001; Walterscheid et al., 2001]. If a half integer number of local vertical wavelengths fit perfectly in the internal wave region, the vertical components of the vertical velocity and wave fluxes at the duct boundaries are zero and reflection and standing wave behavior occur. Hence, strong ducting in the atmosphere requires certain combinations of wave parameters (wave period, horizontal wavelength, and propagating direction) for a given vertical structure of the atmospheric mean temperature and winds.

Here we study propagating and ducting of gravity waves in a non-isothermal and windy atmosphere using the 2-D AGE-TIP model previously described in the second chapter. We specifically examine the different propagation characteristics of eastward and westward propagating gravity wave packet with the inclusion of mean winds. In an atmosphere free of dissipation the wave vertical energy flux is not a conserved quantity for gravity waves propagating through regions of wind shear [Hines and Reddy, 1967]. Instead it is the wave action that is conserved in this circumstance and so this is the wave diagnostic variable that we consider here. For a monochromatic gravity wave the wave action is defined as

\[ F_T = (\omega/\Omega) \langle w' p' \rangle, \]  

(4.1.2)
where $\omega$ and $\Omega$ are the extrinsic and intrinsic wave frequency, respectively, as previously defined, $w'$ is the vertical velocity perturbation, $p'$ is the wave pressure perturbation, and the angle brackets denote a horizontal average. Equation (4.1.2) was applied by Hickey and Brown [2002] using a full-wave, monochromatic wave model to evaluate the mean state forcing associated with the propagation of waves observed during the ALOHA-93 campaign. An alternative form of the wave action for gravity waves is [Hines and Reddy, 1967; Lindzen, 1990]

$$F_r = \langle w' p' \rangle + \bar{\rho} \bar{U} \langle u' w' \rangle + \bar{\rho} \bar{V} \langle v' w' \rangle,$$

(4.1.3)

where $\bar{U}$ and $\bar{V}$ are the mean horizontal wind components, respectively, $u'$ and $v'$ are the corresponding velocity perturbations, and $\bar{\rho}$ is the mean atmospheric density. Equation (4.1.3) explicitly demonstrates how the wave momentum flux (e.g., $\bar{\rho} \langle u' w' \rangle$) contributes to the wave action. The contribution of the wave momentum flux to the wave action can be thought of as a consequence of a coupling between the wave perturbation and the mean flow [Hines and Reddy, 1967]. Because we are considering a time-dependent wave packet with myriad frequency components the above expression (4.1.3) for the wave action will be used. We will designate the wave momentum flux contribution to the wave action as $F_C$, so that the expression (4.1.3) can be written as $F_r = \langle w' p' \rangle + F_C$.

Results

The MSIS-E-90 model [Hedin, 1991] and the Horizontal Wind Model (HWM93) [Hedin et al., 1996] are used to define the mean atmospheric thermal/density structure and winds, respectively. Simulations are performed for the date of 1993 Jan 15 at 18.5° N latitude and 0.0°
longitude, and for the local time of 2200 hours. The solar and geomagnetic indices used are $F_{10.7} = F_{10.7A} = 87$ and $a_r = 12$ for moderately disturbed conditions.

An eastward propagating wave packet with a primary period of 6.276 min and a horizontal wavelength of 35 km (equally a horizontal phase speed of 92.95 m s$^{-1}$) is used here and has been used in several previous studies. These parameters were first determined using a full-wave model to study a strongly ducted gravity wave in the lower thermospheric thermal duct for a windless atmosphere [Hickey, 2001]. An eastward propagating gravity wave packet is excited in the troposphere by a heat source $Q_w$ prescribed by the following equation

$$ Q_w(x, z, t) = 10^{-5} \exp(-t - \frac{\tau}{t}) \exp(-z - \xi) \sin(k_0 x - \omega_0 t), \quad (4.1.4) $$

where $\Delta z = 0.8$ km, $\xi = 8$ km, $\Delta t = 6.276$ min, and $\tau = 37.656$ min. The thermal forcing horizontal wave number and frequency are given by $k_0 = \frac{2\pi}{35}$ km and $\omega_0 = \frac{2\pi}{6.276}$ min, respectively. The amplitude of the thermal forcing is chosen to be small ($10^{-5}$ K s$^{-1}$) so that the resulting wave amplitudes remain linear at all heights. It is much smaller than the value of about 0.05 K s$^{-1}$ used by Alexander et al. [2004] in their simulations of convectively driven waves during the DAWEX campaign. An expression similar to Equation (4.1.4) but with a negative sign replacing the positive sign in the horizontal wave number $k_0$ is used to thermally force a westward propagating gravity wave packet.

The Brunt-Väisälä frequency, $N$, and the mean zonal wind, $U$ (positive eastward), are shown in the left panel of Figure 18. The refractive indices $m^2$ (calculated using Equation (4.1.1)) for eastward and westward propagation and for several waves are shown in the right panel of Figure 18. The Brunt-Väisälä period ($2\pi/N$) has a local maximum of about 13 min at near 10 km altitude, and decreases to about 4.5 min at near 20 km altitude. Thereafter it reaches a local
maximum of about 6.5 min at near 60 km altitude, and throughout much of the mesosphere it has a value near 5.8 min. It has a local minimum of about 3.7 min at near 110 km altitude, above which it monotonously increases with increasing altitude and becomes asymptote to a value of about 9.5 min in the upper thermosphere (not shown). The zonal wind (left panel) is predominantly eastward, except in narrow regions centered near 40 km altitude and 100 km altitude where it is westward with a magnitude of about 15 m s\(^{-1}\). At altitudes of about 72.5 km and about 117.5 km the zonal wind is a local maximum (eastward). It tends to increase monotonously with increasing height above about 130 km for the altitudes shown. Because the zonal wind is predominantly eastward, the eastward propagating waves (such as the 7.06 min wave shown in the right panel of Figure 18) tend to be internal \((m^2 > 0)\) at most heights. The higher frequency components of the eastward propagating waves are exception to this and exhibit regions of weak evanescence near 55 km altitude and 90 km altitude (e.g., the 5.51 min wave shown in the right panel of Figure 18). Based on the \(m^2\) profile for the 5.51 min wave we would expect it to be weakly trapped in the mesosphere and also in the stratosphere. The westward propagating 6.31 min wave is evanescent \((m^2 < 0)\) in most of the mesosphere, and is internal in the stratosphere (from about 15 to about 50 km altitude) and also in the uppermost mesosphere and lower thermosphere (from about 90 to about 120 km altitude). Based on the \(m^2\) profile for this wave we would expect it to be strongly trapped in the stratosphere. The longer period westward propagating waves (e.g., the 9.58 min wave shown in the right panel of Figure 18) remain internal at most heights. The eastward propagating waves encounter a critical level (where \(\Omega = 0\)) near 180 km altitude (not shown).

The normalized power spectra derived from the vertical velocity fluctuations calculated for the entire simulation time (about 6 hrs) at altitudes of 8, 30, and 80 km and also the
normalized source spectrum are shown in Figure 19 for eastward (left panel) and westward (right panel) wave propagation. These spectra have been adjusted for the Doppler shifting due to the mean winds to help facilitate a meaningful comparison between spectra at different altitudes and between eastward and westward propagating waves. The adjusted frequency $\omega^*$ we so calculate is in the reference frame of the source region at 8 km altitude, and is given by

$$\omega^* = \Omega + k(\overline{U}(z) - \overline{U}_s),$$

where $\overline{U}_s$ is the zonal wind at the center of the source region (the center altitude $z = 8$ km), $\overline{U}(z)$ is the zonal wind at the altitude $z$, and all other variables are as previously defined. The frequency $\omega^*$ so obtained is not the extrinsic frequency, and so it cannot be used to compare with observations. For clarity, the spectra are normalized at each altitude (taking as fractions of the sum of total power spectral amplitudes), and so the magnitudes of the normalized power spectra calculated at different altitudes cannot be directly compared. Our intent is to emphasize the frequency content at each altitude.

For the westward propagating wave packet (Figure 19, right panel) the spectrum is dominated by high frequency components in the stratosphere (30 km altitude, green curve). Low frequency components are seen in the upper mesosphere spectrum (80 km altitude, blue curve) but high frequency waves are absent due to the efficient trapping in the stratosphere, as discussed previously with respect to the refractive indices. Because the source spectrum (represented by the black smooth curve in the Figure 19) does not efficiently generate the low frequency waves, very little wave energy reaches the upper mesosphere for westward propagation.

For the eastward propagating wave packet (Figure 19, left panel) the spectrum is dominated by higher frequency components in the upper mesosphere but high and low frequency components are well distributed in the stratosphere. Examination of the refractive index curves
(Figure 18, right panel) shows that the lower frequency waves (such as the 7.06 min wave) remain internal at most heights. These lower frequency waves propagate through the mesosphere quite quickly, and so do not contribute substantially to the spectra that are based on time series of 6 hours in length. A higher frequency eastward propagating wave (e.g., the 5.51 min wave) has regions of weak evanescence near 50 km altitude and near 90 km altitude. This wave is weakly trapped and ducted near 75 km altitude. Therefore, it spends more time in this region than it would if it was a freely propagating internal wave, and hence it dominates the spectrum in this region.

The wave action calculated using Equation (4.1.3) is shown as a function of time and altitude in Figure 20 for eastward (left panel) and westward (right panel) propagation. As it propagates upward from the troposphere the eastward propagating wave packet weakens, significantly so near 50 km altitude and again near 90 km altitude. The lower frequency components of this wave packet are able to reach the upper mesosphere and lower thermosphere essentially unimpeded because they remain internal at most heights (see Figure 18, right panel). The regions of significant weakening of the wave action (about 50 and 90 km altitude) correspond to regions where $m^2 < 0$ for higher frequency components of the wave packet (see Figure 18, right panel, E 5.51 blue curve). The mean zonal wind impedes the propagation of the higher frequency components of the eastward propagating wave packet to higher altitudes by Doppler shifting them to evanescence within certain altitude ranges. This leads to the partial trapping of wave energy near the upper mesosphere. In so doing, it leads to a large spectral signature of these waves at 80 km altitude (Figure 19, left panel, the blue curve of the 5.51 min wave).
The wave action for the westward propagating wave packet is shown in the right panel of Figure 20. The mean zonal wind significantly impedes the propagation of this westward propagating wave packet into the mesosphere, and most of the wave energy remains strongly trapped in the stratosphere. The small amount of wave energy reaching the upper mesosphere and lower thermosphere (about 10% of the original wave energy) resides mainly in the long-period waves in the spectrum (about 7 – 13 min), as discussed in relation to Figure 19 (right panel, the 9.58 min wave). A long period (about 40 min) variation of wave action is seen to occur in the stratosphere, as identified by the alternating direction of the wave action in Figure 20 (right panel). Such variation is associated with the finite time taken for reflections between the lower and upper boundaries of the duct, which in turn depends on the vertical group velocity.

In Figure 21 we show the contribution of the wave momentum flux, $F_C$, to the wave action. As the eastward propagating wave packet (left panel) propagates upward, $F_C$ changes sign as a consequence of the changes in the direction of the zonal wind (see Equation (4.1.3) and Figure 18, left panel). When the zonal wind is eastward (that is, in the same direction as the eastward propagating waves), the waves extract energy from the mean flow, whereas when the zonal wind is westward (that is, in the opposite direction to the eastward propagating waves) the mean flow extracts energy from the waves [e.g., Hines and Reddy, 1967]. $F_C$ changes sign as time increases at a fixed altitude due to the partial trapping of the wave packet and the associated sign changes of the zonal momentum flux. In the upper troposphere and lower stratosphere $F_C$ contributes about 20% to the wave action, but in the lower thermosphere it contributes as much as 40% to the wave action because of the large zonal wind (about 60 m s$^{-1}$ at about 160 km altitude) in this region. For the westward propagating wave packet (right panel of Figure 21) $F_C$ is significant only in the stratosphere due to wave trapping, and it changes sign with increasing
time due to reflections and the associated sign changes of the zonal momentum flux. Comparison of the left and the right panel of Figure 21 shows that at a particular time and altitude (e.g., 50 min and 40 km altitude) $F_C$ has the opposite sign for the eastward and westward propagating wave packet due to their momentum fluxes being oppositely directed.

![Figure 18: Atmospheric stability (N, green) and zonal wind (U, blue) are shown in the left panel. The refractive indices, $m^2$, for wave periods of 5.51 min (E 5.51, blue) and 7.06 min (E 7.06, green) for eastward propagation, and for wave periods of 6.31 min (W 6.31, red) and 9.58 min (W 9.58, black) for westward propagation are shown in the right panel.](image-url)
Figure 19: Wave spectra derived from the vertical perturbation velocities are shown for eastward propagation (left panel) and westward propagation (right panel) at discrete altitudes of 8 km (red), 30 km (green), and 80 km (blue). The spectrum derived from the thermal source at 8 km altitude is also shown (black).
Figure 20: The wave total perturbation energy flux $F_T$ (W m$^{-2}$), also known as the wave action, is shown for eastward propagation (left panel) and westward propagation (right panel).
Figure 21: The contribution of the wave momentum flux $F_C$ (W m$^{-2}$) is shown for eastward propagation (left panel) and westward propagation (right panel).

Discussion

The vertical velocity amplitude is also a good indicator of the extent of the ducting region [Snively and Pasko, 2003; Snively et al., 2007]. We note that the Gaussian wave train in (4.1.4) is too short to fill the entire vertical extent of the atmosphere leading to incomplete cancellation of the upward and downward propagating waves. Hence fully ducted waves aren’t generated. If complete cancellation had been achieved then the energy flux would have approached zero once the waves filled the ducting region, while the vertical velocity would exhibit nodes and antinodes. The thermal forcing parameters for our wave packets were based on simulations of
strongly ducted waves in the MLT region in the absence of winds. We have found that with the inclusion of background mean winds only a small fraction of the original wave energy can reach the MLT duct and the duct is also considerably weakened.

*Hecht et al.* [2001] studied the propagation of monochromatic gravity waves in the atmosphere and showed that waves having a certain direction of propagation are more favorably ducted in the lower thermospheric thermal duct and therefore observable in the nightglow. For other directions of propagation, they found that large regions of evanescence in the middle atmosphere inhibited the propagation of the waves to the lower thermosphere. Here we have considered the propagation of a fairly high frequency wave packet through a moving background atmosphere and as expected, we have found that certain waves are inhibited from propagating to the lower thermosphere. However, we find that not all frequency components of a wave packet are inhibited from propagating to the lower thermosphere. For the eastward propagating wave packet the spectrum at 80 km altitude is dominated by shorter period waves (about 5 – 6 min). This is because the longer period waves are internal and propagate unimpeded through the upper mesosphere region quickly, with the result that their spectral signatures are small. The higher frequency components of this wave packet undergo partial reflections and trapping, with the result that they spend more time in the upper mesosphere and so they dominate the spectrum. For the westward propagating wave packet longer period waves (about 7 – 13 min) dominate the spectrum in the upper mesosphere. However, in the upper mesosphere the energy (and the wave action) of these westward propagating waves is considerably smaller than that of the eastward propagating waves.

The downward propagating wave energy for the westward propagating wave packet seen at about 65 min and about 40 km altitude in Figure 21 (right panel) is a sign of wave trapping in
the stratosphere. The wave action alternates direction periodically in time as a result of reflections from regions of evanescence at the duct boundaries, leading to a long period (about 40 min) fluctuation. This phenomenon, also has been discussed in last chapter, continues for longer times (not shown) during which the trapped waves finally dissipate.

Conclusion

We have simulated the propagation of two high-frequency gravity wave packets from the troposphere to the lower thermosphere in the presence of background mean winds. The eastward propagating wave packet is able to reach the upper mesosphere and lower thermosphere, but the longer period components of the wave packet will propagate quickly through this region of the atmosphere and hence might go undetected in the observations of the nightglow. The higher frequency components of the eastward propagating wave packet spend a longer time in the upper mesosphere as a consequence of partial trapping. Hence, these waves are more likely to be observed in the nightglow, and they also dominate our calculated spectrum at the relevant altitude. The westward propagating wave packet is significantly weakened as a result of a preferential reflection of the high-frequency components of the wave packet, which remain trapped in the stratospheric duct. Although longer period components of this wave packet can reach the upper mesosphere, they do so with significantly reduced amplitudes, essentially precluding their detection by optical techniques.
Summary 4

A control simulated study of upward propagating gravity waves in the presence of a zonal wind is performed to further understand the role played by wind shears. A Gaussian gravity wave packet excited by a heat source in the troposphere is seen to propagate upward through the middle atmosphere. Along its way of upward and eastward propagation, the zonal wind is seen to either Doppler shift the waves to higher intrinsic frequencies if it flows head on to the waves, or Doppler shift the waves to lower intrinsic frequencies if it flows along the waves. These wind shear effects can be observed in the simulations through the fluctuations in the atmospheric density, temperature, and velocities. In a dissipative atmosphere the linear waves are also observed in the simulations to deposit their energy and momentum up to the ionospheric heights. This simulated wave packet is supposed to be trapped in a lower thermospheric thermal duct according to the previous study in chapter 3, but now due to a zonal wind included, the ducting conditions are released and their energy and momentum are raised to greater altitudes rather than being carried horizontally in a lower thermospheric thermal duct. These simulations are among the first that resolve the time-dependent depositions of energy and momentum over altitudes, and they demonstrate that a surf front flows inside the wave momentum flux and a stratified energy “cloud” spreads around in the lower thermosphere.

Introduction

Atmospheric gravity waves (AGWs) have been widely studied since the study by Hines [1960], and also have been recognized as a crucial role player in transporting energy and momentum from the lower atmosphere, where the wave source locates, to the middle and upper
atmosphere [Hodges, 1967, 1969; Fritts, 1978; Lindzen, 1981; Holton, 1982, 1983; Fritts and Dunkerton, 1984; Fritts, 1984]. A considerable number of investigations have been carried out to propose enhanced understanding for gravity waves generated by convective processes according to Fritts and Alexander [2003]. Those gravity waves are estimated either through an obstacle effect [Clark et al., 1986], or by a mechanical oscillator effect [Fovell et al., 1992], as well as by a thermal excitation [Alexander et al., 1995; Pandya and Alexander, 1999; Piani et al., 2000]. Particularly, the force mechanism concerned with energy release has been explored by a significant amount of numerical modelings. Not only are these modelings in a planetary-scale [Manzini and Hamilton, 1993], but also in a small-scale as well [Walterscheid et al., 2001; Beres et al., 2004; Alexander et al., 2004]. Observational hints for heat releasing mechanism have also been provided by Mclandress et al. [2000].

Manzini and Hamilton [1993] investigated equatorial planetary waves and inertia gravity waves excited by latent and convective heating that propagate through the troposphere, stratosphere, and mesosphere using a comprehensive SKYHI general circulation model (GCM). Their studies appreciated that latent and convective heating had been an important wave source, and this kind of wave source was indicated to be a dominant mechanism in producing equatorial wave activities. Alexander et al. [1995] studied mesoscale atmospheric gravity waves that were convectively forced and with high frequency oscillations in the stratosphere above a simulated storm. They showed that this kind of gravity waves had a surprisingly approximate correlation between the heating depth and their vertical wavelengths. Spectral analysis was applied by Pandya and Alexander [1999] to further investigate the stratospheric gravity waves above a convective thermal excitation linearly as well as nonlinearly, and they clearly revealed that there
was a resemblance between the dominant frequency of the stratospheric gravity waves and the oscillating frequency of the time-varying tropospheric thermal excitation.

Walterscheid et al. [2001] used a cylindrically axisymmetric, f-plane model to study small-scale gravity waves up to the mesosphere and lower thermosphere (MLT) region, the waves were generated by energy deposition in the thunderstorm. The impulsive storm, the Hector event, and the Quasi-Monochromatic (QM) storm were analyzed individually in their studies, but with the exclusion of the background winds because of the azimuthally symmetric nature of the model. Beres et al. [2004] introduced a method of identifying the spectral characteristics of convectively excited multifrequency gravity waves if they had the knowledge of latent heating properties and the background winds provided. The wind effects were significant and the symmetry was altered between the eastward and westward propagating stratospheric gravity waves. Alexander et al. [2004] presented a thorough modeling study of stratospheric gravity waves generated by tropical convection near Darwin during the Darwin Area Wave Experiment. In their studies, radar reflectivity characterized by latent heating distributions has been converted to better describe the most realistic spatial and temporal representations of the wave excitation.

Upon those historical researches about the latent and convective heating introduced above, our current study is to undertake a control simulation on the propagating characteristics of atmospheric gravity waves, which are excited by a Gaussian heating function. This control study is implemented with a time-dependent, nonlinear 2-D model (which has been called as the AGE-TIP model). In the third chapter we used a combination of certain wave period (6.276 min) and horizontal wavelength (35 km) to probe a lower thermospheric thermal duct, in which the wave kinetic energy has been found to be trapped horizontally for a long distance. The control
experiment pursued here is under the otherwise same simulated conditions in case 1 in the third chapter except for including a zonal wind structure.

Theory

Latent heating, due to the release and absorption of energy through atmospheric dynamic activities, is a tropospheric excitation mechanism for internal gravity waves. This mechanism provides a distributed power spectrum centered at a primary frequency and a localized energy source initiates gravity waves propagating upward. The presentation in case 1 in the third chapter used a thermal excitation (a simulated heating source in the atmosphere excluding background winds) to initiate an atmospheric gravity wave packet, and clearly demonstrated that (also predicted by Walterscheid et al. [2001]) a small scale atmospheric gravity wave, initially ducted and finally trapped in a thermal duct of the lower thermosphere, like a long wave train traveling for a large horizontal distance of about 500 kilometers and persisting over several hours. In this current study, by turning on the zonal wind otherwise based on the same study in case 1 in the third chapter, we try to perceive the wave ducting conditions (either thermal or Doppler ducting), and to understand the effects caused by the wind shears towards the upward propagating atmospheric gravity waves.

The existence of an altitude variation of the Brunt-Väisälä frequency \( N \) introduces the possibility of ducting. A thermally ducted wave (without winds) will be internal \( (m^2 > 0) \) in the near region of a local maximum of \( N \), but at some vertical distances of both sides, either upward or downward, where \( N \) has decreased, and the wave becomes evanescent \( (m^2 < 0) \). The vertical structure of the waves in case 1 in the third chapter provided a strong wave ducting in the lower thermospheric thermal duct. Since efficient thermal ducting or strong standing wave behavior
resulted in the lower thermospheric thermal duct, there must be an integer number of half vertical wavelengths fitting exactly in the internal wave region (where \( m^2 > 0 \)), and the nodes occurred at the duct boundaries where \( m = 0 \) [Hickey, 2001; Walterscheid et al., 2001].

However, once the horizontal winds are turned on (in our current case it is the zonal wind only), the wave extrinsic frequency will be Doppler-shifted to the wave intrinsic frequency, where \( \Omega \) is not equal to \( \omega \) any more because of the subtracted term \( kU \) (where \( \Omega = \omega - kU \)). The vertical wave number \( m \) also changes with \( \omega \) changing to \( \Omega \), and the condition of an integer number of half vertical wavelengths might not valid any more. So that by a control simulation of including background winds in the study of case 1 in the third chapter, we expect that the lower thermospheric thermal duct won’t exist any more since the support of the vertical wave structure is released. Furthermore, the wave packet will be expected not to be ducted horizontally, and will instead propagate upward until the momentum and energy carried by the waves deposit in the upper atmosphere.

**Results**

In Figure 22, we plot the Brunt-Väisälä frequency, the zonal wind, and the wave intrinsic frequency as functions of altitude. The atmospheric mean temperature, which determines the altitude variation of the Brunt-Väisälä frequency, is specified from the MSIS-E-90 model [Hedin, 1991] for the date of 1993 Jan 15, the location of latitude N 18.5 deg. and longitude 0.0 deg., and the local time of 2200 hours. The zonal wind structure is specified from the Horizontal Wind Model (HWM93) [Hedin et al., 1996] at the same geophysical location and on the same date mentioned above. Imposed upon the background mean zonal wind, a localized thermal
excitation $Q_w$ (K s$^{-1}$) is prescribed. The prescribed source is a Gaussian envelop over altitude of half-width $\Delta z = 0.8$ km, centered at altitude $\xi = 8$ km, and a Gaussian envelop over time of half-width $\Delta t = 6.276$ min, centered at time $\tau = 37.656$ min, and with an amplitude of $10^{-5}$ Ks$^{-1}$. It is described analytically as

$$Q_w(x, z, t) = 10^{-5} \exp\left(-\left(t - \tau\right)^2 / 2\Delta t^2\right) \exp\left(-\left(z - \xi\right)^2 / 2\Delta z^2\right) \sin\left(k_0 x - \omega_0 t\right),$$

where $\omega_0 = 2\pi/6.276$ min and $k_0 = 2\pi/35$ km.

As shown in Figure 22, the applied zonal wind structure starts from the ground, and has a positive wind shear ($dU/dz > 0$) from $z = 0$ to about $z = 12.5$ km, so as from about $z = 43.5$ km to about $z = 72.5$ km and from about $z = 98.5$ km to about $z = 117.5$ km. From the altitude about $z = 12.5$ km to about $z = 43.5$ km, the zonal wind has a negative wind shear ($dU/dz < 0$), so as from about $z = 72.5$ km to about $z = 98.5$ km and from about $z = 117.5$ km to about $z = 133.5$ km. Eventually, from the altitude about $z = 133.5$ km up to altitudes near the upper boundary in the model, the magnitude of the zonal wind increases monochromatically (positive wind shear, $dU/dz > 0$). The wave packet initiates from the altitude of 8 km and propagates upward and eastward with a horizontal phase speed of 92.95 m/s ($V_p = \omega_0 / k_0$). At the altitude of about 12.5 km the zonal wind reaches a positive (eastward) local maximum, so that at the altitude of about 12.5 km the zonal wind is a tail wind with respect to the wave packet, so as at altitudes of about 72.5 km and about 117.5 km. At the altitude of about 43.5 km the zonal wind reaches a negative (westward) local minimum, so that at the altitude of about 43.5 km the zonal wind is a head wind with respect to the wave packet, so as the altitude of about 98.5 km. The reason we distinguish the altitudes of tail wind and head wind is that, the wave packet will be Doppler-shifted to lower frequency if it has a tail wind along, likewise the wave packet will be Doppler-shifted to higher
frequency if it has a head wind head on. This argument can be made clearly by the equation \( \Omega = \omega - kU \). In Figure 22, the wave intrinsic frequency \( \Omega \) is plotted with the lower x coordinate over altitude and it shows a symmetric feature comparing with the zonal wind structure.

Figures 23, 24, 25, and 26 demonstrate the wave fluctuations in the atmospheric density, temperature, horizontal velocity, and vertical velocity at different times of about 47 minutes, 1 hour 9 minutes, 1 hour 31 minutes, and 5 hours 48 minutes. Figure 23 is a snapshot at 47 minutes and 4 seconds to the fractional density perturbation \( \rho' / \rho_0 \) (units in %). From the altitude of about 12.5 km to the altitude of about 43.5 km, the zonal wind switches from a tail wind to a head wind with respect to the eastward propagating wave packet. As a consequence the intrinsic frequency of the primary wave is Doppler-shifted to higher frequency; the appearance of this Doppler shift can be illuminated by the scene that the local vertical wavelength increases accompanying with the Doppler-shifted, faster primary wave. So as in the circumstance from the altitude of about 72.5 km to the altitude of about 98.5 km the wave packet becomes faster too. But in the circumstance from the altitude of about 43.5 km to the altitude of about 72.5 km the intrinsic frequency of the primary wave is Doppler-shifted to lower frequency and the primary wave becomes slower as shown by the decrease of the local vertical wavelength.

Similar wind shear effects can also be seen in Figure 24 simulated at 1 hour 9 minutes and 2 seconds with the fractional temperature perturbation \( T' / T_0 \) (units in %). For instance, from the altitude of about 98.5 km to the altitude of about 117.5 km the primary wave becomes slower wave, the sign appears as shorter vertical wavelength comparing to faster wave with longer vertical wavelength from the altitude of about 72.5 km to the altitude of about 98.5 km. In Figure 25 that shows the horizontal velocity perturbation simulated at 1 hour and 31 minutes, the primary wave has been slowered down from the altitude of about 98.5 km to the altitude of about
117.5 km where the wind shear is positive (dU/dz > 0). From the altitude of about 117.5 km to
the altitude of about 133.5 km the zonal wind structure appears as a tail wind with respect to the
eastward propagating wave packet, but the wind shear is negative (dU/dz < 0) within this vertical
zone and the primary wave just becomes slightly faster. In Figure 26, we show the vertical
velocity simulated at 5 hours, 48 minutes, and 19 seconds. From the altitude of about 133.5 km
to all the altitudes above, the wind shear becomes positive (dU/dz > 0). Once the wave packet
reaches the altitude of about 185 km, a critical level occurs because the speed of the zonal wind
equals to the wave horizontal phase speed (92.95 m/s), and also they both have the same
eastward direction (see Figure 22). At the critical level the upward propagation of the primary
wave stops because of its zero intrinsic frequency that is Doppler-shifted by the zonal wind and
the primary wave has been absorbed by the mean flow since then.

The velocity correlation u’w’ (momentum flux per unit density m²s⁻²) can be used to
estimate the vertical flux of the horizontal momentum carried by the upward propagating wave
packet. Figure 27 is a snapshot at the time of 47 minutes and 4 seconds for a spatially localized
u’w’. This second order, nonlinear term shows a highly fluid wave packet with double maximum
points within a horizontal wavelength. Those downward momentum fluxes are divided among
those upward momentum fluxes. After the velocity correlation u’w’ has been taken a horizontal
average within a horizontal wavelength as <u’w’>, a time-resolved <u’w’> during the times
from 240 minutes to 310 minutes is plotted in Figure 28. As shown in Figure 28 during the 5th
hour of the simulation, most of the horizontal momentum carried by the waves is deposited
within the altitudes from about 105 km to about 185 km. This horizontally averaged <u’w’>
shows a surf front with an interval of about 25 minutes (by lining up the altitude 150 km the
interval of the surf front is between about 268 minutes to about 293 minutes). There is also a period of several minutes among the upward and downward momentum flux.

The wave kinetic energy density \((u'^2+w'^2)/2\) (energy per unit mass \(J \, kg^{-1}\)) can be used to estimate most of the energy carried by the linear waves. Figure 29 is a snapshot at the time of 47 minutes and 4 seconds for a spatially localized \((u'^2+w'^2)/2\). Not only does this second order, nonlinear term show a highly fluid wave packet, but also it displays double maximum points within a horizontal wavelength. The wave kinetic energy (shown in Figure 29) and the horizontal momentum (shown in Figure 27) both carried by the waves share a similarity in a way of their upward propagation. After the wave kinetic energy density \((u'^2+w'^2)/2\) has been taken a horizontal average within a horizontal wavelength as \(<(u'^2+w'^2)/2>\), a time-resolved \(<(u'^2+w'^2)/2>\) during the times from 70 minutes to 140 minutes is plotted in Figure 30. In Figure 30 the wave kinetic energy centers at the altitude of about 130 km and at the time of about 110 minutes. A large amount of the wave kinetic energy carried by the waves is deposited around the altitudes from about 100 km to about 150 km and during the times from about 70 minutes to about 140 minutes, although there is still some residue energy remaining within the energy “cloud” after the 2nd hour of the simulation.
Thermal excitation locates at the altitude 8 km, centered at Period 6.276 min and Horizontal wavelength 35 km.

Figure 22: Atmospheric stability ($N$ green), zonal wind ($U$ blue) and wave intrinsic frequency ($\Omega$ magenta). The left vertical straight line (red, dash-dot) signifies a period of 6.276 min, and the right vertical straight line (red, dash-dot) signifies a horizontal phase speed of 92.95 m/s.
Figure 23: A spatially localized fractional density perturbation $\rho'/\rho_0$ (%) at a simulation time of 47 minutes and 4 seconds.
Figure 24: A spatially localized fractional temperature perturbation $T'/T_0$ (%) at a simulation time of 1 hour, 9 minutes, and 2 seconds.
Figure 25: A spatially localized horizontal velocity perturbation $u'$ (m s$^{-1}$) at a simulation time of 1 hour and 31 minutes.
Figure 26: A spatially localized vertical velocity $w'$ (m s$^{-1}$) at a simulation time of 5 hours, 48 minutes, and 19 seconds.
Figure 27: A spatially localized velocity correlation \( u'w' \) (m\(^2\)s\(^{-2}\)) at a simulation time of 47 minutes and 4 seconds.
Figure 28: A horizontally averaged velocity correlation $<u'w'>$ (m$^2$s$^{-2}$) during the times from 240 minutes to 310 minutes.
Figure 29: A spatially localized kinetic energy density \( (u'^2 + w'^2)/2 \) (Jkg\(^{-1}\)) at a simulation time of 47 minutes and 4 seconds.
Discussion

In this study a control simulation is carried out under the same situation with case 1 in chapter 3 except for the inclusion of a mean zonal wind. The propagating gravity wave packet is excited to proceed eastward (and upward) so that the zonal wind is the only wind shear efficiently acting on the waves. The meridional wind is perpendicular to the horizontal direction of the wave propagation so that it plays no effects to the propagation of the waves, and it will not be considered in the current study any more. The zonal wind is a function of altitude, and also has a positive or negative gradient over height, so that it plays a role of either Doppler-shifting
the waves to higher frequency, if the zonal wind direction is opposite to the horizontal wave propagating direction, or Doppler-shifting the waves to lower frequency, if the zonal wind direction is as the same as the horizontal wave propagating direction. These kind of Doppler-shifting features can be illuminated clearly by the wave intrinsic frequency plotted in Figure 22. These wind anisotropic effects to the wave packet have also been shown clearly in Figures 23, 24, 25, and 26, in which the major wave fluctuations among the atmospheric density, temperature, horizontal and vertical velocities have been snapshot at different times of the simulation within a horizontal wavelength.

In the study of case 1 in chapter 3, a lower thermospheric thermal duct had been found and the waves were seen to be ducted and trapped inside the duct. However, in the current study with a zonal wind included, the wave intrinsic frequencies have been Doppler-shifted away from their extrinsic frequencies, and the wave ducting conditions seem not be able to be maintained any more. Through observing Figure 22 in the current study, it is found that the wave intrinsic frequencies are less than the Brunt-Väisälä frequencies at altitudes greater than about 50 km, from which up to the altitude of the critical level (about 185 km), the waves are in an internal region of free propagation (according to Equation (4.1.1), for $\Omega < N$ and usually $V_p = \Omega / k < C$, so that normally $m^2 > 0$). Since the release of ducting conditions by the inclusion of the zonal wind, the waves can propagate upward to greater altitudes until they encounter the critical level, and deposit their energy and momentum further in the upper atmosphere.

As shown clearly as in Figures 27 and 29, the momentum and energy carried by the waves also propagate upward consistently as well as the wave propagating fluctuations in the atmospheric density, temperature, and velocities. This linear wave propagation can transport wave momentum and energy even further in depth in the upper atmosphere, since they do not
break down at the lower atmosphere. As illustrated by the time-resolved plots in Figures 28 and 30, the wave momentum and energy depositions show rather depending on the altitudes and the times. Especially in Figure 28, a highly consistent structure shown at the ionospheric heights in the velocity correlation $u'w'$ is estimated to attribute to the traveling ionospheric disturbances (TIDs), which are often observed by the radar community. The interval of about 25 minutes in this highly consistent structure (as we so called a surf front) is clearly revealed, and the intensity of this velocity correlation $u'w'$ is shown to slowly dissipate away with times (by comparing the intensity before 268 minutes with that after 293 minutes). The theoretical discovery of this momentum surf front can be practically observed and can also be confirmed with modeling studies such as the one presented here. Another finding provided here is an energy “cloud” shown in Figure 30, which is derived over altitude and time; it not only quantifies but also stratifies the process of the wave kinetic energy deposition. The deposition plot of the wave kinetic energy furnishes enormous opportunities to study the dissipation of atmospheric gravity waves in the upper atmosphere.

Conclusion

A control study presented here with a zonal wind included demonstrates that a Gaussian gravity wave packet forced by a latent heating propagates upward quite freely into the lower thermosphere, and deposits its momentum and energy around its environment. Otherwise, this wave packet is supposed to be ducted within the lower thermospheric thermal duct if without the zonal wind involved (like the case 1 in chapter 3). The wind shear effects illustrated by the fluctuations in the atmospheric density, temperature, and velocities play enormous influences to the waves propagating in the atmosphere. Moreover, with their propagation further in depth into
the lower thermosphere, the waves transport momentum and energy from the lower atmosphere where they are excited by a latent heating up to the upper atmosphere where their most of carried momentum and energy are deposited. Depending on altitudes and times, a surf front shown with the velocity correlation is a significant finding here. For an analyzed altitude of 150 km, the surf front has an interval of about 25 minutes after approximate 4 hours since the waves were excited. With the ionospheric heights where this kind of shorter period waves (6.276 min) can reach, the modeling discovery here can be applied to study the traveling ionspheric disturbances (TIDs), which are mostly caused by various gravity wave activities. In addition, because what we study here are linear waves and their wave kinetic energy is considered as most of the energy carried by the waves, we can analyze the wave-associated energy through a time-resolved way over altitudes. The energy “cloud” appearing in the lower thermosphere shows a well concentrated feature rather than a periodic one in the momentum flux, and also it is characterized with localized altitudes and certain times plus its quantities are stratified and spreading out from the center. Ultimately, the control numerical experiment reported here emphasizes a crucial role played by the wind shears and opens a door to study nonlinear and time-resolved gravity wave activities in a near future.
CHAPTER FIVE: SIMULATED AIRGLOW

Introduction

During the daylight hours energetic radiation from the Sun is absorbed in the atmosphere. In the upper atmosphere, extreme ultraviolet (EUV) radiation and certain bands in the ultraviolet (UV) are absorbed. Molecular oxygen (O₂) is readily dissociated by the radiation, so that during the daylight hours atomic oxygen (O) is produced. In effect, solar energy radiating from the Sun has been converted to chemical energy storing in the atomic oxygen. Once produced, the atomic oxygen recombines through various chemical channels, but during the daylight hours it is still being produced faster than being destroyed. During the nighttime hours in the absence of the solar radiation the atomic oxygen recombines with other minor and/or major species to form new molecules. Because the atomic oxygen is really a reservoir of chemical energy and there is always excess energy in these reactions (they are exothermic), so that the newly formed molecule is usually formed in some excited states.

The new molecule will not stay in these excited states for a very long time, but instead it will give up the excess energy in one of these several ways. First, it may react with another species, forming new products which themselves may be in some excited states. Second, it may collide with a major gas molecule (N₂ or O₂) and gives its energy to them in a so called “quenching” reaction. Third, it may simply radiate the excess energy away in a form of visible light. This last process produces the airglow.
Atmospheric gravity waves are ubiquitous phenomena in the upper and also in the middle atmosphere and so manifest themselves in many types of observations. These waves can set the atmosphere into a local oscillatory motion, so that produce fluctuations in the atmospheric pressure, density, temperature, and winds. Airglow is one of the most interesting interactions occurring between atmospheric gravity waves and chemically reactive species in the upper atmosphere. In this kind of interaction, fluctuations in wind speed, temperature, and density in the major gas constituents (molecular nitrogen and oxygen up to about 105 km altitude) all contribute to fluctuations in the minor species number densities. The so called airglows, which are optical emissions arising from the chemical reactions between certain minor constituents in regions of the upper atmosphere, can therefore be also affected by these atmospheric gravity waves. Sensitive optical instruments can measure these emissions, and the waves cause changes in airglow brightness that can also be easily detected.

Reactions

When either excited atoms or molecules consisting of like atoms (e.g., O₂) are produced in a reaction the excited states must be purely electronic. However, when excited molecules consisting of unlike atoms (e.g., OH) are produced in a reaction, the excited state may be a rotational and vibrational one. Generally speaking, electronic excitations are of higher energy than the rotational and vibrational one, so that during relaxation to some lower energy levels the light emitted by the former is usually of a higher frequency than that of the latter. So, the emission spectrum of the OH molecule lies in the range near the far infrared regions (long wavelengths). Other emissions related to electronic excitation often lie in the ultraviolet. Other emissions lie in the visible part of the spectrum.
O (1S) (557.7 nm) airglow, also known as the atomic oxygen green line, can be observed by several ways such as through Rocket, ground-based, and satellite. Its volume emission rate has a maximum around 96 km altitude, so that O (1S) (557.7 nm) airglow observation can provide a significant source of information about the wave activity around this altitude. All-sky CCD imaging system can observe O (1S) (557.7 nm) airglow brightness fluctuations and wave parameters including wave periods, horizontal wave numbers, and wave propagating directions. The observed wave information can be used in numerical models to simulate the propagation of gravity waves and to infer the wave characteristics and parameterizations in the atmosphere.

The O (1S) (557.7 nm) airglow chemistry is responsible for the green line emission at the wavelength of 557.7 nm. In accordance with the theory of Bates [1988] and the reaction rates provided by Torr et al. [1985], the production of O (1S) (557.7 nm) is in a two-step process in which the intermediate state is \( \text{O}_2 \left( c^l \sum_u^- \right) \), and the chemical reactions and their reaction rates are described in Table 1.

We assume the initial atmospheric mean state is a steady state and the chemical reactions in Table 1 can facilitate to calculate the mean state densities for \( \text{O}_2 \left( c^l \sum_u^- \right) \) and O (1S).

\[
\bar{n}(\text{O}_2 \left( c^l \sum_u^- \right)) = \zeta k_i \bar{n}^2 (O) \frac{\bar{n}(M)}{(k_2 \bar{n}(\text{O}_2) + k_3 \bar{n}(O) + A_1)} \\
\bar{n}(O(1S)) = \delta k_3 \bar{n}(O) \frac{\bar{n}(\text{O}_2 \left( c^l \sum_u^- \right))}{(k_6 \bar{n}(\text{O}_2) + A_2 + A_3)}
\]
Table 1: Chemical reactions and kinetic constants for O (\(^1\)S) 557.7 nm airglow

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>(O + O + M \rightarrow O_2 + M)</td>
<td>(k_1 = 4.7\times10^{-13} (300 / T)^2)</td>
</tr>
<tr>
<td>(O + O + M \rightarrow O_2 \left( c^1 \sum_u^- \right) + M)</td>
<td>(k_4 = \zeta k_1, \zeta = 0.8)</td>
</tr>
<tr>
<td>(O_2 \left( c^1 \sum_u^- \right) + O_2 \rightarrow O_2 \left( b^1 \sum_g^+ \right) + O_2)</td>
<td>(k_2 = 5.0\times10^{-13})</td>
</tr>
<tr>
<td>(O_2 \left( c^1 \sum_u^- \right) + O \rightarrow O_2 + O)</td>
<td>(k_3 = 3.0\times10^{-11})</td>
</tr>
<tr>
<td>(O_2 \left( c^1 \sum_u^- \right) + O \rightarrow O_2 + O ((^1)S))</td>
<td>(k_5 = \delta k_3, \delta = 0.01)</td>
</tr>
<tr>
<td>(O_2 \left( c^1 \sum_u^- \right) \rightarrow O_2 + h\nu)</td>
<td>(A_1 = 2.0\times10^{-2})</td>
</tr>
<tr>
<td>(O ((^1)S) + O_2 \rightarrow O ((^3)P) + O_2)</td>
<td>(k_6 = 4.0\times10^{-12} \exp(-865 / T))</td>
</tr>
<tr>
<td>(O ((^1)S) \rightarrow O + h\nu (557.7nm, 297.2nm))</td>
<td>(A_2 = 1.105)</td>
</tr>
<tr>
<td>(O ((^1)S) \rightarrow O + h\nu (557.7nm))</td>
<td>(A_3 = 1.06)</td>
</tr>
</tbody>
</table>

* Units are s\(^{-1}\), cm\(^3\)s\(^{-1}\) and cm\(^6\)s\(^{-1}\) for unimolecular, bimolecular, and termolecular reactions, respectively.

**Schemes**

The AGE-TIP model solves the coupled continuity equations with a time-splitting technique for several minor species including advection. The vector equation describing the time evolution of the unknown number density vector, \(n_i\), is written as
\[ \frac{\partial n_i}{\partial t} = f_i(n_i) + f_e(n_i), \]  

(5.3)

where \( f_i \) represents for those terms evaluated implicitly with the Newton-Raphson scheme, and \( f_e \) represents for those terms evaluated explicitly with the Lax-Wendroff scheme. The \( f_e \) terms of each minor species are written as (for a single species, \( f_e \) contributes as the continuity equation)

\[ f_e(n_i) = -\frac{\partial}{\partial x}(n_i u) - \frac{\partial}{\partial z}(n_i w), \]  

(5.4)

and the \( f_i \) terms of each minor species are written as

\[ f_i(n_i) = P(n_i) - L(n_i), \]  

(5.5)

where \( P(n_i) \) and \( L(n_i) \) are the production rate and the lost rate for the minor species \( n_i \), respectively.

Technically the \( f_i \) and \( f_e \) represent terms to be solved using implicit and explicit finite difference methods, respectively. In the time-splitting technique, Equation (5.3) is solved at each time step by first assuming that only the explicit terms exist on its right hand side. The integration is achieved by employing the Lax-Wendroff method. The Lax-Wendroff method allows us to solve for \( n_i \) over the entire 2-D grid. Next, the implicit terms on the right hand side of the equation are evaluated using the new values of \( n_i \), and then a fully implicit Newton-Raphson method is used to compute the final values of \( n_i \) at the end of the time step. Our implicit method solves over a 1-D vertical grid only, and it is applied in turn for each horizontal grid position. This method of time-splitting produces solution vector that converges to a limit as the time step is halved. In practice we find that a time step of 0.7 seconds works very well.
Deciding which terms should be solved either explicitly or implicitly depends on the stability of the method for the particular terms involved. Experimentation revealed that the chemistry terms required extremely small time steps in order to ensure numerical stability using the explicit method. Therefore, these chemistry terms ($P(n_i)$ and $L(n_i)$) are included for solution in the implicit method, which is inherently more stable than the explicit method. Of the remaining terms, the advection terms, those that could be included as flux terms or source terms, are solved using the explicit method. Sponge upper boundary condition is employed and periodic boundary conditions apply at the lateral boundaries, which are separated by a horizontal wavelength, to simulate an infinite wave train in the horizontal direction. With the airglow chemistry involved, we also could use a far-field horizontal domain in this chapter that is quite different from the horizontal infinite domain used in the previous chapters.

Results

The atmospheric mean temperature and neutral density are defined by the MSIS-E-90 model [Hedin, 1991] for the date of 1993 Jan 15, the local time of 2200 hours, and the latitude and longitude of 18.5 deg. N and 0.0 deg., respectively. The effects of mean winds are excluded from this analysis. The wave source is centered at a horizontal distance of 400.5 km and an altitude of 8 km; the horizontal Gaussian half-width for the source is 70 km. The prescribed source is a Gaussian envelop over altitude of half-width $\Delta z = 0.8$ km, centered at altitude $\xi = 8$ km, and a Gaussian envelop over time of half-width $\Delta t = 6.276$ min, centered at time $\tau = 37.656$ min, and with an amplitude of $10^{-3}$ Ks$^{-1}$. It is described analytically as

$$Q_w(x,z,t) = 10^{-3}G_w(x,z)\exp(-(t-\tau)^2/2\Delta t^2)\sin(k_v x - \omega_v t),$$

(5.6)
\[ G_{\text{ad}}(x,z) = \exp\left(-\left(x-x_0\right)^2 / 2\Delta x^2\right) \exp\left(-\left(z-\xi\right)^2 / 2\Delta z^2\right), \tag{5.7} \]

where \( \omega_0 = 2\pi/6.276 \text{ min} \) and \( k_0 = 2\pi/35 \text{ km} \), \( x_0 = 400.5 \text{ km} \) and \( \Delta x = 70 \text{ km} \).

Figure 31 is the figure we provide in a horizontal far-field domain at a simulation time of 1 hour and 31 minutes. The OI (557.7 nm) airglow volume emission rate (\( \# \text{ m}^{-3} \text{ s}^{-1} \)) plotted in Figure 31 has been subtracted from the initial mean state. In this figure there are four significant sharp fronts arraying from a horizontal distance of about 500 km to about 600 km and at an altitude of about 95 km. These bright “wall” fronts are tailed by a number of weak wave crests extending from a horizontal distance of about 500 km to about 300 km. Note that this simulated O (\(^1\text{S}\)) (557.7 nm) nightglow image in Figure 31 is pictured from the sky (e.g., satellite) instead of the ground and it may be referenced as a side-viewed O (\(^1\text{S}\)) (557.7 nm) airglow image simulated. This simulated O (\(^1\text{S}\)) (557.7 nm) airglow image is observed at about 1 hour and 31 minutes since a simulated heating source has been initiated in the troposphere. The nonlinear wave breaking and ducting signatures shown in Figure 31 may partially explain the cause of the mesospheric bore-like “wall” events [Taylor et al., 1995; Swenson et al., 1998].

Figure 32 is the figure we provide in a horizontally infinite domain during the second hour of the simulation. The periodic boundary conditions apply to the lateral boundaries that are separated by a horizontal wavelength. The horizontal variation in the wave source function (5.7) is dropped and the wave source amplitude is reduced 100 times (to be \( 10^{-5} \text{ K s}^{-1} \)). The horizontally averaged, atomic oxygen net upward density flux (\( <(n_t-n_0)w> \), units in \( \# \text{ cm}^{-2} \text{ s}^{-1} \)) plotted in Figure 32 displays an obviously transient upward flux from about 70 minutes to about 90 minutes and above the peak of the O profile (around 96 km altitude). A periodic variation of about 3 min is vaguely and obliquely shown in the horizontally averaged, upward O density flux.
(related to \(<n_0 \nu_0\)w>, the 3 min is almost a half of the primary wave period of about 6 min). This transient upward atomic oxygen flux is calculated with the propagating and ducting high-frequency gravity wave packet and the O (\(^1S\)) (557.7 nm) airglow chemistry. The secular variations of the O downward transport driven by a quasi-steady state gravity wave packet will be shown and discussed later [Hickey et al., 2000; Huang and Hickey, 2007].

The Gaussian gravity wave packet used here is a dissipative and transient wave packet that has violated the non-acceleration conditions, so the net cycle-averaged effects of the waves are expected to be non-zero. The reason for the O transient upward transport driven by a Gaussian high-frequency gravity wave packet is likely due to high-frequency waves consisting of longitudinal and compressional acoustic and fast wave components, while long period gravity waves (e.g., 20 min) are predominantly vertically transverse waves [Hickey et al., 2000; Huang and Hickey, 2007]. The O (\(^1S\)) (557.7 nm) chemistry effect to the O vertical distribution is expected to be relatively smaller if it is driven by a high-frequency gravity wave packet than that if it is driven by a long-period gravity wave packet, because the time constant in this volatile airglow chemistry is comparable to the shorter periods in high-frequency waves. In other words, this volatile airglow chemistry will react less complete if it is influenced by high-frequency waves than that if it is influenced by long period waves. However, the secular downward O transport seems independent of the airglow chemistry according to the studies between the O (\(^1S\)) (557.7 nm) chemistry in Hickey et al. [2000] and the OH chemistry in Huang and Hickey [2007].

In Figure 33 we provide the vertical distribution of atomic oxygen at different simulation times. Again the model is in a horizontally infinite domain during the entire simulation. The periodic boundary conditions apply to the lateral boundaries that are separated by a horizontal wavelength. The horizontal variation in the wave source function (5.7) is dropped because of the
horizontal symmetry and the wave source amplitude is maintained to be $5 \times 10^{-4}$ Ks$^{-1}$. Once the wave source amplitude approaches the maximum ($5 \times 10^{-4}$ Ks$^{-1}$) at the time of $\tau = 37.656$ min, it will keep this constant amplitude till the end of the simulation so that a quasi-steady state gravity wave packet ducting and propagating in the whole atmosphere is generated. The initial atomic oxygen profile has a peak value at about 96 km altitude. The linear and quasi-steady state gravity wave packet used in this simulation has maximum influence to the upper slope of the O profile and minimum influence to the lower slope. Because of the nature of the faster waves the dissipation due to the high-frequency wave packet is expected to be the smallest in the lower slope of the O profile. The secular variation in the vertical distribution of atomic oxygen caused by this quasi-steady state and high-frequency gravity wave packet is a downward transport of atomic oxygen, which is agreeable to the results shown by Hickey et al. [2000] and Huang and Hickey [2007].
Figure 31: Nonlinear breaking and lower thermospheric ducted gravity waves are signalized themselves in the O (1S) (557.7 nm) airglow emission as well-known as the airglow green line. They are observed at about 1 hour and 31 minutes since a simulated heating source has been initiated in the troposphere.
Figure 32: Upward O net density flux driven by a high-frequency, linear Gaussian gravity wave packet during the second hour of simulation.
Discussion

We have performed extensive modeling studies of ducted gravity waves using our AGE-TIP model. This model is able to explain the variations with season-preferred gravity wave propagation directions at different observing stations (in chapter four). We also found that only certain combinations of wave parameters produced ducted waves in the lower thermospheric thermal duct, and that for the atmospheric conditions considered at least one efficiently ducted wave having a period of 6.276 minutes and a horizontal phase speed of 92.95 m/s. In addition, the AGE-TIP model is used in this chapter to simulate the effects of nonlinear ducted gravity
waves and freely propagating gravity waves on the O (1S) (557.7 nm) airglow emission. Of the particular relevance to this simulation is the fact that those observations reported by Taylor et al. [1995], Swenson et al. [1998], Nielsen et al. [2006], and Li et al. [2007] of a bright gravity wave event revealed sharp fluctuations occurred in the airglow. A detailed modeling study of this gravity wave “wall” event here using the AGE-TIP model is well suited to studies involving wave reflection and ducting. Our practice in the AGE-TIP model included the airglow responses to the simulated waves by including a fairly complete chemistry for the airglow emission processes. The applied airglow chemistry must be properly accounted for when comparing dynamical signatures in different airglow emissions that are basically maximized at different atmospheric regions.

Because of the modeling studies performed in the previous chapters and here an alternative explanation is proposed for those dramatic wave “wall” events [Taylor et al., 1995; Swenson et al., 1998] involving ducted gravity waves. Because a duct is involved, there is some similarity to the bore model of Dewan and Picard [1998]; however, the physics is different for the two processes. The AGE-TIP model, which could describe nonlinear, time-dependent acoustic-gravity wave motions, has been used in conjunction with the OI 5577 airglow chemistry in the mesopause region to simulate the ducting effects of upward propagating gravity wave packet. This model has shown that transient effects associated with the leading edge of the wave packet can cause discontinuous variations of minor species and rapid brightening of airglow emissions in the MLT region. The AGE-TIP model has previously been used to study wave ducting also it is ideally suited to the problem described here. In combination with multiple airglow chemistries (e.g., the OH airglow chemistry other than the OI 5577 airglow chemistry)
we have the means to perform a detailed and exhaustive study of the interactions between ducted gravity waves and multiple layer airglow emissions.

Numerical experiments could supply an alternative mechanism for the mesospheric bores involving a ducting region to provide nonlinear gravity waves propagating and breaking. The proposed physical mechanism behind the wave scenes here is rather different from the theory of an undular internal bore provided by Dewan and Picard [1998, 2001]. An advantage of the employed AGE-TIP model here in the present research is that, the model can provide simulated airglow images flexibly and objectively according to different geographical locations, which makes the comparisons between all-sky CCD airglow images and modeling results more efficiently, and results in a side-by-side and image-to-image analysis. Multiple simulated airglow images, which include simulated airglow images of the OI (557.7 nm), OH (8, 3) Meinel band, and O₂ atmospheric (0-1) band, could be derived simultaneously under a provided real observational or simulated atmospheric mean state with determined temperature and winds and other parameters. This successful model approach can be applied to different atmospheric regions (ranging from low and mid latitudes to high latitudes) to further contribute great chances for airglow image science to cooperate with modeling study, and also can provide theoretical modifications and parameterizations through comparisons between observations and modelings.

**Conclusion**

By using the newly developed AGE-TIP model, we have already described a time-dependent OI (557.7 nm) chemical nonlinear response to a high-frequency gravity wave packet. Our major finding in this chapter is that, in addition to fluctuating the minor species densities, the Gaussian high-frequency gravity wave packet also causes a transient upward transport of atomic
oxygen driven by faster wave components. The quasi-steady state and high-frequency gravity wave packet contributes a secular downward transport of atomic oxygen driven by the OI (557.7 nm) chemical nonlinear responses. The dissipation due to the high-frequency wave packet is expected to be the smallest at the airglow altitudes where we are interested in. Further investigations could be considered by incorporating the OH chemistry and initiating a high-frequency gravity wave packet at different latitudes, so that the airglow chemical effects to the vertical distribution of atomic oxygen can be understood in a more complete and more global scale.

Although the simulated airglow image provided in this chapter lacks of observational data to be compared, it still imposes a perspective to a future work. All-sky imagers occasionally observe a remarkable phenomenon in the airglow, e.g., a “spectacular gravity wave event” reported by Taylor et al. [1995] and later by Swenson et al. [1998]. This wave event resembles a fast moving bright “wall” stretching from horizon to horizon with several waves propagating behind. The leading edge or front of the disturbance can propagate with a high speed and the entire disturbance can propagate over large horizontal distances. Additional wave disturbances are seen behind the front and observed to move with the speed of the front and locked to it. The mesospheric bore-like “wall” events once seemed to be rare and unusual, till now not only have they been observed at mid latitudes by Smith et al. [2003], She et al. [2004], and Brown et al. [2004], but also at low latitudes by Smith et al. [2005] and Medeiros et al. [2005], as well as at high latitudes over Antarctica by Stockwell et al. [2006] and Nielsen et al. [2006]. Further more according to Fechine et al. [2005], a large quantity of the mesospheric bore-like wave events (over 60) were observed at equatorial latitudes over a three year period, which suggested that the mesospheric bores are far from shortage.
One plausible explanation of these wave events has been proposed involving an internal mesospheric undular bore. This explanation has its roots in tidal bores observed in rivers and canals, and also in undular bores in the troposphere (the “morning glory” described by Clark [1972] and Smith [1988]). Mesospheric bores believed by theorists as Dewan and Picard [1998, 2001] and later as Seyler [2005], are due to mesospheric temperature inversions described by Huang et al. [1998, 2002] providing a channel for the nonlinear wave propagation. Nielsen et al. [2006] summarized that mesospheric temperature inversions are comparatively normal circumstances at mid and low latitudes, while an infrequent occurrence for these inversions at high latitudes could possibly explain just a few bore observations at Polar Regions noted by Cutler et al. [2001] and Nielsen et al. [2006]. However, the theory is not fulfilled till a bore observation simultaneously concurs with an available temperature and wind data. Li et al. [2007] recently reported a “wall” wave event observed at Maui, Hawaii, on the night of 11–12 August 2004. This bright airglow event was observed with multiple instruments including a Na wind/temperature lidar, an airglow imager, and a mesospheric temperature mapper. Analysis showed that this event was caused by large amplitude, upward propagating gravity waves with induced dramatic changes in temperature, airglow intensity, and Na abundance. It experienced strong dissipation and induced large downward heat flux and large momentum flux.

Not only is it important to understand the wave “wall” phenomenon by airglow observations, it is equally important to use these wave dramatic events as a test of our gravity wave-airglow interaction models. The models (e.g., the full-wave model and the AGE-TIP model developed and used in this dissertation) are critical to the interpretation of the airglow observations because without them one cannot confidently determine the gravity wave amplitude in the major gas, a key requirement in the wave flux determinations. This is because the airglow
observations constitute a height integral of minor species perturbations. Comprehensive and conclusive modelings of these wave events would provide more confidence to our interaction models thereby providing more confidence in wave fluxes derived from the combination of airglow observations and modelings. The importance of our present work also derives from the expected and directed fluxes of minor constituents in our gravity wave hypothesis, something not expected in the undular bore hypothesis. In addition, ducted gravity waves transport energy and momentum horizontally and thereby impact the interpretation of airglow imager measurements. A better understanding of these ducting processes will improve the interpretation of airglow imager measurements. Also our present work will tell us what parts of the gravity wave spectrum comprising a wave packet do get trapped in the ducts and do not continue to vertically propagate. The seasonal and geographical parameters implemented in the AGE-TIP model simulations can be used to con-proof the mesospheric bore theory with analyzed data from participating airglow imagers, lidars, and radars.


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